Q.1-Identify which of the following statements are propositions-

1. France is a country.
2. 2020 will be a leap year.
3. Sun rises in the west.
4. P(x) : x + 6 = 7
5. P(5) : 5 + 6 = 2
6. Apples are oranges.
7. Grapes are black.
8. Two and two makes 4.
9. x > 10
10. Open the door.
11. Are you tired?
12. What a bright sunny day!
13. Mumbai is in India.
14. I always tell truth.
15. I always tell lie.
16. Do not go there.
17. This sentence is true.
18. This sentence is false.
19. It will rain tomorrow.
20. Fan is rotating.

Solutions-

1. Proposition (True)
2. Proposition (True)
3. Proposition (False)
4. Not a proposition (Predicate)
5. Proposition (False)
6. Proposition (False)
7. Proposition (False)
8. Proposition (True)
9. Not a proposition (Predicate)
10. Not a proposition (Command)
11. Not a proposition (Question)
12. Not a proposition (Exclamation)
13. Proposition (True)
14. Proposition (True)
15. Not a proposition (Inconsistent)
16. Not a proposition (Command)
17. Proposition (True)
18. Not a proposition (Inconsistent)

19. Proposition (Will be confirmed tomorrow whether true or false)
20. Proposition (True if fan is rotating otherwise false)

PRACTICE PROBLEMS BASED ON CONVERTING ENGLISH SENTENCES-

Problem-01:

Write the following English sentences in symbolic form-

1. If it rains, then I will stay at home.
2. If I will go to Australia, then I will earn more money.
3. He is poor but honest.
4. If a = b and b = c then a = c.
5. Neither it is hot nor cold today.
6. He goes to play a match if and only if it does not rain.
7. Birds fly if and only if sky is clear.
8. I will go only if he stays.
9. I will go if he stays.
10. It is false that he is poor but not honest.
11. It is false that he is poor or clever but not honest.
12. It is hot or else it is both cold and cloudy.
13. I will not go to class unless you come.
14. We will leave whenever he comes.
15. Either today is Sunday or Monday.
16. You will qualify GATE only if you work hard.
17. Presence of cycle in a single instance RAG is a necessary and sufficient condition for deadlock.
18. Presence of cycle in a multi instance RAG is a necessary but not sufficient condition for deadlock.
19. I will dance only if you sing.
20. Neither the red nor the green is available in size 5.

Solution-

Part-01:

We have-

- The given sentence is- “If it rains, then I will stay at home.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is **p** → **q** where-

p : It rains

q : I will stay at home

Part-02:

We have-

- The given sentence is- “If I will go to Australia, then I will earn more money.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is **p** → **q** where-

p : I will go to Australia

q : I will earn more money

Part-03:

We have-

- The given sentence is- “He is poor but honest.”
- We can replace “but” with “and”.
- Then, the sentence is- “He is poor and honest.”

So, the symbolic form is **p ∧ q** where-

p : He is poor

q : He is honest

Part-04:

We have-

- The given sentence is- “If a = b and b = c then a = c.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is **(p ∧ q)** → **r** where-

p : a = b

q : b = c

r : a = c

Part-05:

We have-

- The given sentence is- “Neither it is hot nor cold today.”
- This sentence is of the form- “Neither p nor q”.
- “Neither p nor q” can be re-written as “Not p and Not q”.

So, the symbolic form is $\sim p \wedge \sim q$ where-

p : It is hot today

q : It is cold today

Part-06:

We have-

- The given sentence is- “He goes to play a match if and only if it does not rain.”
- This sentence is of the form- “p if and only if q”.

So, the symbolic form is $p \leftrightarrow q$ where-

p : He goes to play a match

q : It does not rain

Part-07:

We have-

- The given sentence is- “Birds fly if and only if sky is clear.”
- This sentence is of the form- “p if and only if q”.

So, the symbolic form is $p \leftrightarrow q$ where-

p : Birds fly

q : Sky is clear

Part-08:

We have-

- The given sentence is- “I will go only if he stays.”
- This sentence is of the form- “p only if q”.

So, the symbolic form is **p → q** where-

p : I will go

q : He stays

Part-09:

We have-

- The given sentence is- “I will go if he stays.”
- This sentence is of the form- “q if p”.

So, the symbolic form is **p → q** where-

p : He stays

q : I will go

Part-10:

We have-

- The given sentence is- “It is false that he is poor but not honest.”
- We can replace “but” with “and”.
- Then, the sentence is- “It is false that he is poor and not honest.”

So, the symbolic form is **∼(p ∧ ∼q)** where-

p : He is poor

q : He is honest

Part-11:

We have-

- The given sentence is- “It is false that he is poor or clever but not honest.”
- We can replace “but” with “and”.
- Then, the sentence is- “It is false that he is poor or clever and not honest.”

So, the symbolic form is **∼((p ∨ q) ∧ ∼r)** where-

p : He is poor

q : He is clever

r : He is honest

Part-12:

We have-

- The given sentence is- “It is hot or else it is both cold and cloudy.”
- It can be re-written as- “It is hot or it is both cold and cloudy.”

So, the symbolic form is **p ∨ (q ∧ r)** where-

p : It is hot

q : It is cold

r : It is cloudy

Part-13:

We have-

- The given sentence is- “I will not go to class unless you come.”
- This sentence is of the form- “p unless q”.

So, the symbolic form is **∼ q → p** where-

p : I will go to class

q : You come

Part-14:

We have-

- The given sentence is- “We will leave whenever he comes.”

- We can replace “whenever” with “if”.
- Then, the sentence is- “We will leave if he comes.”
- This sentence is of the form- “q if p”.

So, the symbolic form is **p → q** where-

p : He comes

q : We will leave

Part-15:

We have-

- The given sentence is- “Either today is Sunday or Monday.”
- It can be re-written as- “Today is Sunday or Monday.”

So, the symbolic form is **p ∨ q** where-

p : Today is Sunday

q : Today is Monday

Part-16:

We have-

- The given sentence is- “You will qualify GATE only if you work hard.”
- This sentence is of the form- “p only if q”.

So, the symbolic form is **p → q** where-

p : You will qualify GATE

q : You work hard

Part-17:

We have-

- The given sentence is- “Presence of cycle in a single instance RAG is a necessary and sufficient condition for deadlock.”
- This sentence is of the form- “p is necessary and sufficient for q”.

So, the symbolic form is **p ↔ q** where-

p : Presence of cycle in a single instance RAG

q : Presence of deadlock

Part-18:

We have-

- The given sentence is- “Presence of cycle in a multi instance RAG is a necessary but not sufficient condition for deadlock.”
- This sentence is of the form- “p is necessary but not sufficient for q”.

So, the symbolic form is **(q → p) ∧ ~(p → q)** where-

p : Presence of cycle in a multi instance RAG

q : Presence of deadlock

Part-19:

We have-

- The given sentence is- “I will dance only if you sing.”
- This sentence is of the form- “p only if q”.

So, the symbolic form is **p → q** where-

p : I will dance

q : You sing

Part-20:

We have-

- The given sentence is- “Neither the red nor the green is available in size 5.”
- This sentence is of the form- “Neither p nor q”.
- “Neither p nor q” can be written as “Not p and Not q”.

So, the symbolic form is **~p ∧ ~q** where-

p : Red is available in size 5

q : Green is available in size 5

Problem-02:

Consider the following two statements-

S1 : Ticket is sufficient to enter movie theater.

S2 : Ticket is necessary to enter movie theater.

Which of the statements is/ are logically correct?

1. S1 is correct and S2 is incorrect.
2. S1 is incorrect and S2 is correct.
3. Both are correct.
4. Both are incorrect.

Solution-

Statement S1 : Ticket is Sufficient To Enter Movie Theater-

This statement is of the form- “p is sufficient for q” where-

p : You have a ticket

q : You can enter a movie theater

So, the symbolic form is **p → q**

For p → q to hold, its truth table must hold-

p **(Ticket)**	**q** **(Entry)**	**p → q** **(Ticket is sufficient for entry)**
F	F	T
F	**T**	**T**
T	**F**	**F**

T	T	T

Here,

- Row-2 states it is possible that you do not have a ticket and you can enter the theater.
- However, it is not possible to enter a movie theater without ticket.
- Row-3 states it is not possible that you have a ticket and you do not enter the theater.
- However, there might be a case possible when you have a ticket but do not enter the theater.
- So, the truth table does not hold.

Thus, the statement- “Ticket is sufficient for entry” is logically incorrect.

Statement S2 : Ticket is Necessary To Enter Movie Theater-

This statement is of the form- “q is necessary for p” where-

p : You can enter a movie theater

q : You have a ticket

So, the symbolic form is **p → q**

For p → q to hold, its truth table must hold-

p **(Entry)**	**q** **(Ticket)**	**p → q** **(Ticket is necessary for entry)**
F	F	T
F	T	T
T	F	F
T	T	T

Converse, Inverse and Contrapositive-

For a conditional statement $p \rightarrow q$,

- The converse statement is $q \rightarrow p$
- The inverse statement $\sim p \rightarrow \sim q$
- The contrapositive statement is $\sim q \rightarrow \sim p$

- For conditional statements ($p \rightarrow q$) only, the converse, inverse and contrapositive statements can be written.

For example-

- Inverse of converse is contrapositive.
- Inverse of contrapositive is converse.
- Converse of inverse is contrapositive.
- Converse of contrapositive is inverse.
- Contrapositive of inverse is converse.
- Contrapositive of converse is inverse.

Q.1- Write the converse, inverse and contrapositive of the following statements-

1. If today is Sunday, then it is a holiday.
2. If $5x - 1 = 9$, then $x = 2$.
3. If it rains, then I will stay at home.
4. I will dance only if you sing.
5. I will go if he stays.
6. We leave whenever he comes.
7. You will qualify GATE only if you work hard.
8. If you are intelligent, then you will pass the exam.

Solution-

Part-01:

We have-

- The given sentence is- “If today is Sunday, then it is a holiday.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is **p** $\rightarrow$ **q** where-

p : Today is Sunday

q : It is a holiday

Converse Statement- If it is a holiday, then today is Sunday.

Inverse Statement- If today is not Sunday, then it is not a holiday.

Contrapositive Statement- If it is not a holiday, then today is not Sunday.

Part-02:

We have-

- The given sentence is- “If 5x – 1 = 9, then x = 2.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is **p** → **q** where-

p : 5x – 1 = 9

q : x = 2

Converse Statement- If x = 2, then 5x – 1 = 9.

Inverse Statement- If 5x – 1 ≠ 9, then x ≠ 2.

Contrapositive Statement- If x ≠ 2, then 5x – 1 ≠ 9.

Part-03:

We have-

- The given sentence is- “If it rains, then I will stay at home.”
- This sentence is of the form- “If p then q”.

So, the symbolic form is **p** → **q** where-

p : It rains

q : I will stay at home

Converse Statement- If I will stay at home, then it rains.

Inverse Statement- If it does not rain, then I will not stay at home.

Contrapositive Statement- If I will not stay at home, then it does not rain.

Part-04:

We have-

- The given sentence is- “I will dance only if you sing.”
- This sentence is of the form- “p only if q”.

So, the symbolic form is **p** → **q** where-

p : I will dance

q : You sing

Converse Statement- If you sing, then I will dance.

Inverse Statement- If I will not dance, then you do not sing.

Contrapositive Statement- If you do not sing, then I will not dance.

Determining Nature Of Proposition-

Tautology-

- A compound proposition is called **tautology** if and only if it is true for all possible truth values of its propositional variables.
- It contains only T (Truth) in last column of its truth table.

Contradiction-

- A compound proposition is called **contradiction** if and only if it is false for all possible truth values of its propositional variables.
- It contains only F (False) in last column of its truth table.

Contingency-

- A compound proposition is called **contingency** if and only if it is neither a tautology nor a contradiction.
- It contains both T (True) and F (False) in last column of its truth table.

Valid-

- A compound proposition is called **valid** if and only if it is a tautology.
- It contains only T (Truth) in last column of its truth table.

Invalid-

- A compound proposition is called **invalid** if and only if it is not a tautology.
- It contains either only F (False) or both T (Truth) and F (False) in last column of its truth table.

Falsifiable-

- A compound proposition is called **falsifiable** if and only if it can be made false for some value of its propositional variables.
- It contains either only F (False) or both T (Truth) and F (False) in last column of its truth table.

Unfalsifiable-

- A compound proposition is called **unfalsifiable** if and only if it can never be made false for any value of its propositional variables.
- It contains only T (Truth) in last column of its truth table.

Satisfiable-

- A compound proposition is called **satisfiable** if and only if it can be made true for some value of its propositional variables.
- It contains either only T (Truth) or both T (True) and F (False) in last column of its truth table.

Unsatisfiable-

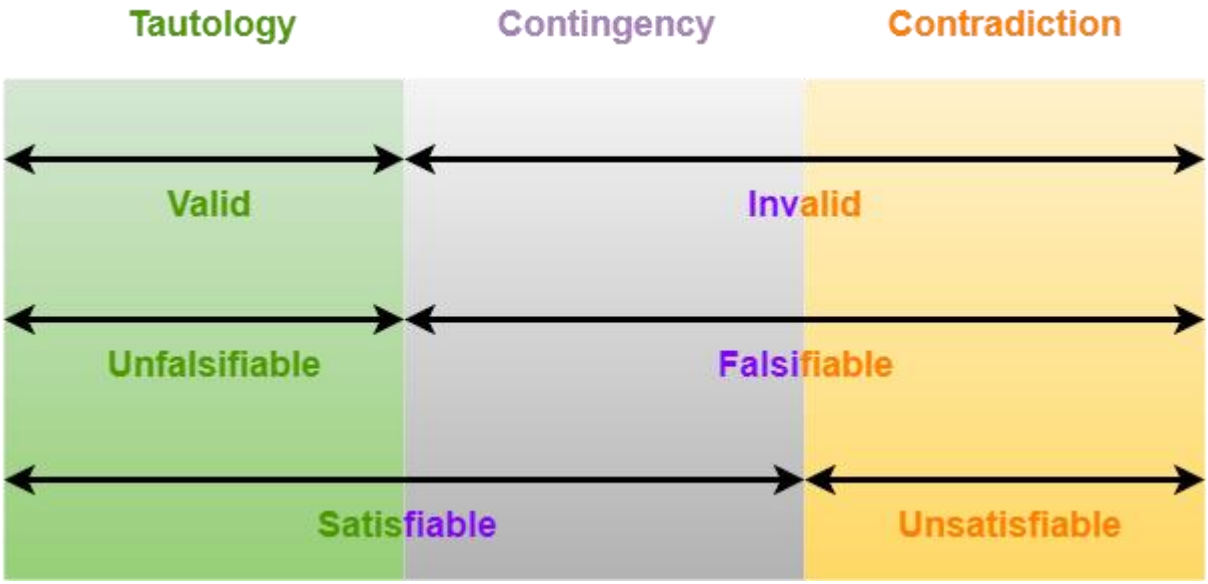
A compound proposition is called **unsatisfiable** if and only if it can not be made true for any value of its propositional variables.

- It contains only F (False) in last column of its truth table.

Important Points-

It is important to take a note of the the following points-

- All contradictions are invalid and falsifiable but not vice-versa.
- All contingencies are invalid and falsifiable but not vice-versa.
- All tautologies are valid and unfalsifiable and vice-versa.
- All tautologies are satisfiable but not vice-versa.
- All contingencies are satisfiable but not vice-versa.
- All contradictions are unsatisfiable and vice-versa.



PRACTICE PROBLEMS BASED ON DETERMINING NATURE OF PROPOSITIONS-

Problem-01:

Determine the nature of following propositions-

1. $p \wedge \sim p$
2. $(p \wedge (p \rightarrow q)) \rightarrow \sim q$
3. $[(p \rightarrow q) \wedge (q \rightarrow r)] \wedge (p \wedge \sim r)$
4. $\sim(p \rightarrow q) \vee (\sim p \vee (p \wedge q))$
5. $(p \leftrightarrow r) \rightarrow (\sim q \rightarrow (p \wedge r))$

Solution-

Let us solve all the parts one by one-

Part-01:

Method-01: Using Truth Table-

p	~p	p ∧ ~p
F	T	F
T	F	F

Clearly, last column of the truth table contains only F.

Therefore, given proposition is-

- Contradiction
- Invalid
- Falsifiable
- Unsatisfiable

Part-02:

Method-01: Using Truth Table-

p	q	p → q	p ∧ (p → q)	~q	(p ∧ (p → q)) →~q
F	F	T	F	T	T
F	T	T	F	F	T

T	F	F	F	T	T
T	T	T	T	F	F

Clearly, last column of the truth table contains both T and F.

Therefore, given proposition is-

- Contingency
- Invalid
- Falsifiable
- Satisfiable

Part-03:

Method-01: Using Truth Table-

Let [(p → q) ∧ (q → r)] ∧ (p ∧ ~r) = R (say)

p	**q**	**r**	**p → q**	**q → r**	**(p → q) ∧ (q → r)**	**p ∧ ~r**	**R**
F	F	F	T	T	T	F	F
F	F	T	T	T	T	F	F
F	T	F	T	F	F	F	F
F	T	T	T	T	T	F	F
T	F	F	F	T	F	T	F
T	F	T	F	T	F	F	F

T	T	F	T	F	F	T	F
T	T	T	T	T	T	F	F

Clearly, last column of the truth table contains only F.

Therefore, given proposition is-

- Contradiction
- Invalid
- Falsifiable
- Unsatisfiable

Part-04:

Method-01: Using Truth Table-

Let $\sim(p \rightarrow q) \vee (\sim p \vee (p \wedge q)) = R$ (say)

p	**q**	**~p**	**p → q**	**~(p → q)**	**p ∧ q**	**~p ∨ (p ∧ q)**	**R**
F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T
T	F	F	F	T	F	F	T
T	T	F	T	F	T	T	T

Clearly, last column of the truth table contains only T.

Therefore, given proposition is-

- Tautology
- Valid

- Unfalsifiable
- Satisfiable

Part-05:

Method-01: Using Truth Table-

Let $(p \leftrightarrow r) \rightarrow (\sim q \rightarrow (p \wedge r)) = R$ (say)

p	q	r	~q	p → r	r → p	p ↔ r	p ∧ r	~q → (p ∧ r)	R
F	F	F	T	T	T	T	F	F	F
F	F	T	T	T	F	F	F	F	T
F	T	F	F	T	T	T	F	T	T
F	T	T	F	T	F	F	F	T	T
T	F	F	T	F	T	F	F	F	T
T	F	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T	T
T	T	T	F	T	T	T	T	T	T

Clearly, last column of the truth table contains both T and F.

Therefore, given proposition is-

- Contingency
- Invalid

- Falsifiable
- Satisfiable