UNIT-3 (2 D Transformation)

INTRODUCTION:

Computer Graphics provide the facility of viewing object from different angles. The architect can study building from different angles i.e.

1. Front Evaluation
2. Side elevation
3. Top plan

A Cartographer can change the size of charts and topographical maps. So if graphics images are coded as numbers, the numbers can be stored in memory. These numbers are modified by mathematical operations called as Transformation.

The purpose of using computers for drawing is to provide facility to user to view the object from different angles, enlarging or reducing the scale or shape of object called as Transformation.

Two essential aspects of transformation are given below:

1. Each transformation is a single entity. It can be denoted by a unique name or symbol.
2. It is possible to combine two transformations, after connecting a single transformation is obtained, e.g., A is a transformation for translation. The B transformation performs scaling. The combination of two is C=AB. So C is obtained by concatenation property.

There are two complementary points of view for describing object transformation.

1. Geometric Transformation: The object itself is transformed relative to the coordinate system or background. The mathematical statement of this viewpoint is defined by geometric transformations applied to each point of the object.
2. Coordinate Transformation: The object is held stationary while the coordinate system is transformed relative to the object. This effect is attained through the application of coordinate transformations.

An example that helps to distinguish these two viewpoints:

The movement of an automobile against a scenic background we can simulate this by

- Moving the automobile while keeping the background fixed-(Geometric Transformation)
- We can keep the car fixed while moving the background scenery- (Coordinate Transformation)

TYPES OF TRANSFORMATION

There are various types of 2D transformation:

1. Translation
2. Scaling
3. Rotation
4. Reflection
5. Shearing

1. TRANSLATION

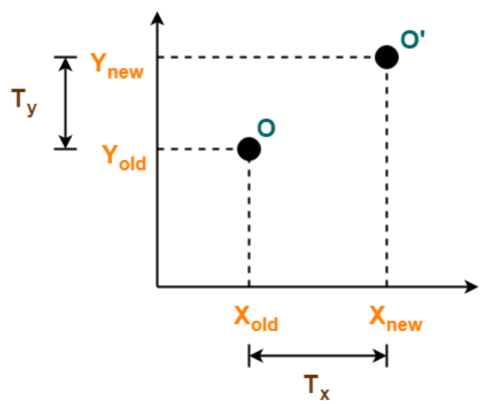
It is the straight line movement of an object from one position to another is called Translation. Here the object is positioned from one coordinate location to another.

Consider a point object O has to be moved from one position to another in a 2D plane.
Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the object O after translation = (X_{new}, Y_{new})
- Translation vector or Shift vector = (T_x, T_y)

Given a Translation vector (T_x, T_y)

- T_x defines the distance the X_{old} coordinate has to be moved.
- T_y defines the distance the Y_{old} coordinate has to be moved.



2D Translation in Computer Graphics

This translation is achieved by adding the translation coordinates to the old coordinates of the object as-

- $X_{new} = X_{old} + T_x$ (This denotes translation towards X axis)
- $Y_{new} = Y_{old} + T_y$ (This denotes translation towards Y axis)

In Matrix form, the above translation equations may be represented as-

- The homogeneous coordinates representation of (X, Y) is (X, Y, 1).
- Through this representation, all the transformations can be performed using matrix / vector multiplications.

The above translation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Translation Matrix
(Homogeneous Coordinates Representation)

EXAMPLE 1:

Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

SOLUTION:

Given-

- Old center coordinates of C = $(X_{old}, Y_{old}) = (1, 4)$
- Translation vector = $(T_x, T_y) = (5, 1)$

Let the new center coordinates of C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 1 + 5 = 6$
- $Y_{new} = Y_{old} + T_y = 4 + 1 = 5$

Thus, New center coordinates of C = (6, 5).

Alternatively,

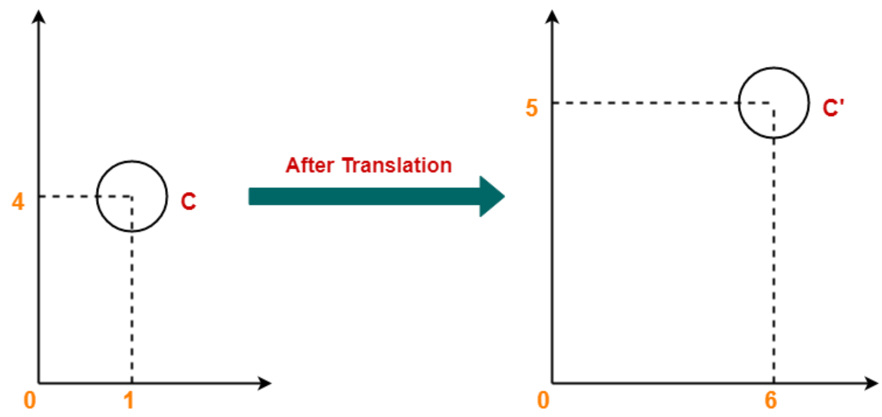
In matrix form, the new center coordinates of C after translation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Thus, New center coordinates of C = (6, 5).



EXAMPLE 2:

Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

SOLUTION:

Given-

- Old coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Translation vector = $(T_x, T_y) = (1, 1)$

<u>For Coordinates A(0, 3)</u>

Let the new coordinates of corner A = (X_{new}, Y_{new}).
Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner A = (1, 4).

<u>For Coordinates B(3, 3)</u>

Let the new coordinates of corner B = (X_{new}, Y_{new}).
Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner B = (4, 4).

<u>For Coordinates C(3, 0)</u>

Let the new coordinates of corner C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner C = (4, 1).

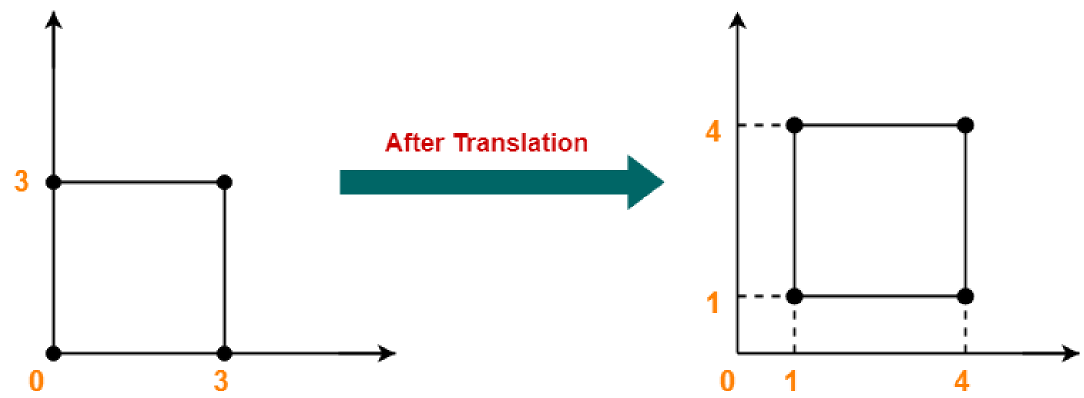
For Coordinates D(0, 0)

Let the new coordinates of corner D = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner D = (1, 1).
Thus, New coordinates of the square = A (1, 4), B(4, 4), C(4, 1), D(1, 1).



2. **ROTATION:**
It is a process of changing the angle of the object. Rotation can be clockwise or anticlockwise. For rotation, we have to specify the angle of rotation and rotation point. Rotation point is also called a pivot point. It is print about which object is rotated.

Types of Rotation: There are two types of rotation:

a. Anticlockwise
b. Counterclockwise

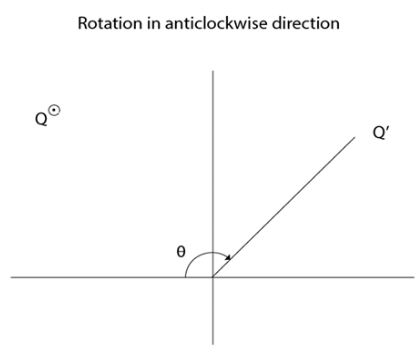
The positive value of the pivot point (rotation angle) rotates an object in a counter-clockwise (anti-clockwise) direction.
The negative value of the pivot point (rotation angle) rotates an object in a clockwise direction.
When the object is rotated, then every point of the object is rotated by the same angle.

Straight Line: Straight Line is rotated by the endpoints with the same angle and redrawing the line between new endpoints.

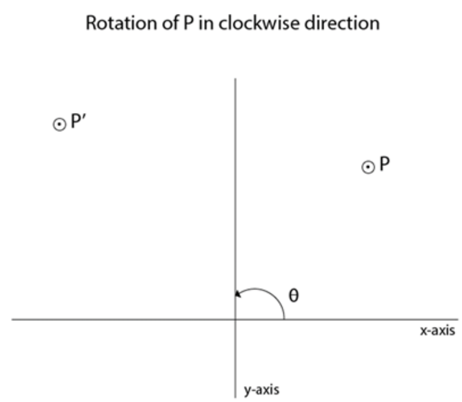
Curved Lines: Curved Lines are rotated by repositioning of all points and drawing of the curve at new positions.

Circle: It can be obtained by center position by the specified angle.

Ellipse: Its rotation can be obtained by rotating major and minor axis of an ellipse by the desired angle.



Q is original position
Q ' is final rotated position



P is original Position
P' is final position or position after rotation
where θ is angle of rotation

Matrix for rotation is a clockwise direction.

$$R=\begin{bmatrix}\cos\ \theta & \sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$$

Matrix for rotation is an anticlockwise direction.

$$R=\begin{bmatrix}\cos\ \theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$

Matrix for homogeneous co-ordinate rotation (clockwise)

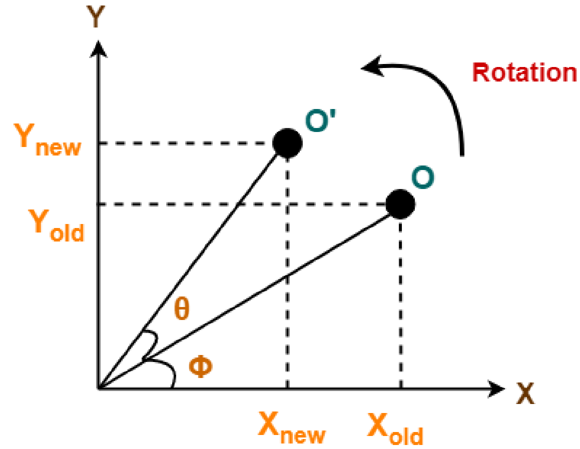
$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Matrix for homogeneous co-ordinate rotation (anticlockwise)

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Consider a point object O has to be rotated from one angle to another in a 2D plane.
Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Initial angle of the object O with respect to origin = Φ
- Rotation angle = θ
- New coordinates of the object O after rotation = (X_{new}, Y_{new})



2D Rotation in Computer Graphics

This rotation is achieved by using the following rotation equations-

- $X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Rotation Matrix

For homogeneous coordinates, the above rotation matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Rotation Matrix

(Homogeneous Coordinates Representation)

EXAMPLE:

Rotate a line CD whose endpoints are (3, 4) and (12, 15) about origin through a 45° anticlockwise direction.

SOLUTION:

The point C (3, 4)

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\theta = 45°$$

Let

$$R = \begin{bmatrix} \cos 45° & \sin 45° \\ -\sin 45° & \cos 45° \end{bmatrix}$$

$$R = \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & -0.707 \end{bmatrix}$$

The point A (3, 4) after rotation will be

$$[x, y] = [3, 4] \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & -0.707 \end{bmatrix}$$

$$= [3 * 0.707 - 4 * 0.707 \qquad 3 * 0.707 + 4 * 0.707]$$

$$[2.121\text{-}2.8282.21\text{+}2.828]$$

$$[-.707 \qquad 4.949]$$

The rotation of point B (12, 15)

$$R_1 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0.707 & 0.707 \\ -0.707 & -0.707 \end{bmatrix}$$

The point B (12, 15) will be

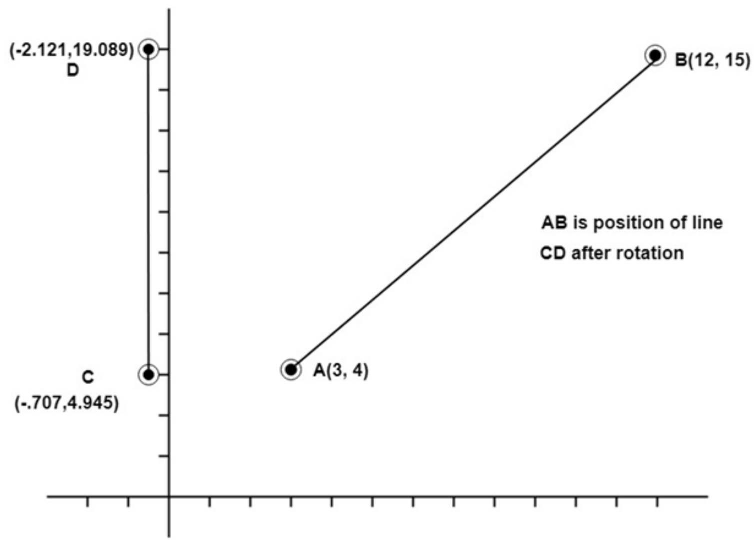
$$[x, y] = [12, 15]\begin{bmatrix} 0.707 & 0.707 \\ -0.707 & -0.707 \end{bmatrix}$$

$$= [12 * 0.707 + 15 * 0.707 \qquad 12 * 0.707 + 15 * 0.707]$$

$$= [(8.484-10.605)\ (8.484 + 10.605)]$$

$$= [-2.121 \quad 19.089]$$

So line AB after rotation at 45^o become [.707, 4.945]   and [-2.121, 19.089]



EXAMPLE:

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

SOLUTION:

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

Given-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (4, 4)
- Rotation angle = θ = 30º

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Applying the rotation equations, we have-

X_{new}

= X_{old} x cosθ – Y_{old} x sinθ

= 4 x cos30º – 4 x sin30º

= 4 x (√3 / 2) – 4 x (1 / 2)

= 2√3 – 2

= 2(√3 – 1)

= 2(1.73 – 1)
= 1.46

Y_{new}
= X_{old} x sinθ + Y_{old} x cosθ
= 4 x sin30º + 4 x cos30º
= 4 x (1 / 2) + 4 x (√3 / 2)
= 2 + 2√3
= 2(1 + √3)
= 2(1 + 1.73)
= 5.46

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

Alternatively,

In matrix form, the new ending coordinates of the line after rotation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

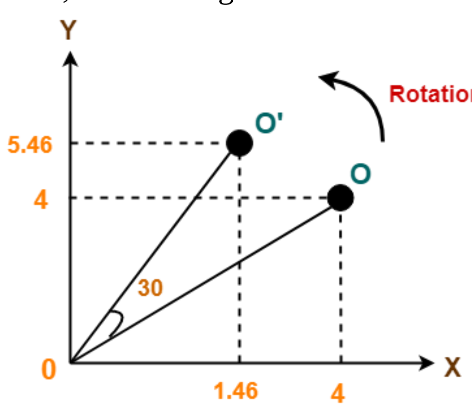
$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).



EXAMPLE: Rotate line AB whose endpoints are A (2, 5) and B (6, 12) about origin through a 30° clockwise direction.

SOLUTION: For rotation in the clockwise direction. The matrix is

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Step1: Rotation of point A (2, 5). Take angle 30°

$$R = \begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix}$$

$$= [2, 5] \begin{bmatrix} \cos 30° & -\sin 30° \\ -\sin 30° & \cos 30° \end{bmatrix}$$

$$= [2, 5] \begin{bmatrix} .866 & -0.5 \\ .5 & .866 \end{bmatrix}$$

$$= [2 * .866 + 5 * .5 \qquad 2 * (-.5) + 5\ (.866)]$$

$$= [(1.732 + 2.5 \qquad -1 + 4.33]$$

$$= [4.232 \qquad 3.33]$$

A point (2, 5) become (4.232, 3.33)

Step2: Rotation of point B (6, 12)

$$= [6 \ \ 12] \begin{bmatrix} \cos 30° & -\sin 30° \\ \sin 30° & \cos 30° \end{bmatrix}$$

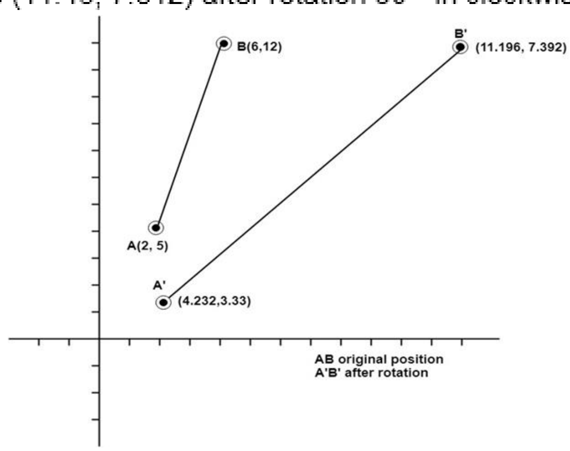
$$= [6\ 12] \begin{bmatrix} .866 & -0.5 \\ .5 & .866 \end{bmatrix}$$

$$= [6 * .866 + 12 * .5 \qquad 6 * (-.5) + 12 * (.866)]$$

$$= [5.196 + 6 \qquad -3 + 10.392]$$

$$= [11.196 \qquad 7.392]$$

B point (6, 12) becomes (11.46, 7.312) after rotation 30° in clockwise direction.



3. SCALING:

It is used to alter or change the size of objects. The change is done using scaling factors. There are two scaling factors, i.e. S_x in x direction S_y in y-direction. If the original position is x and y. Scaling factors are S_x and S_y then the value of coordinates after scaling will be x_1 and y_1.

If the picture to be enlarged to twice its original size then $S_x = S_y = 2$. If S_x and S_y are not equal then scaling will occur but it will elongate or distort the picture.

- If scaling factors are less than one, then the size of the object will be reduced.
- If scaling factors are higher than one, then the size of the object will be enlarged.

If S_x and S_y are equal it is also called as Uniform Scaling. If not equal then called as Differential Scaling. If scaling factors with values less than one will move the object closer to coordinate origin, while a value higher than one will move coordinate position farther from origin.

Consider a point object O has to be scaled in a 2D plane.

Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Scaling factor for X-axis = S_x
- Scaling factor for Y-axis = S_y
- New coordinates of the object O after scaling = (X_{new}, Y_{new})

This scaling is achieved by using the following scaling equations-

- $X_{new} = X_{old} \times S_x$
- $Y_{new} = Y_{old} \times S_y$

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

For homogeneous coordinates, the above scaling matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Scaling Matrix

(Homogeneous Coordinates Representation)

EXAMPLE:

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

SOLUTION:

Given-

- Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Scaling factor along X axis = 2
- Scaling factor along Y axis = 3

For Coordinates A(0, 3)

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9).

For Coordinates B(3, 3)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

For Coordinates D(0, 0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

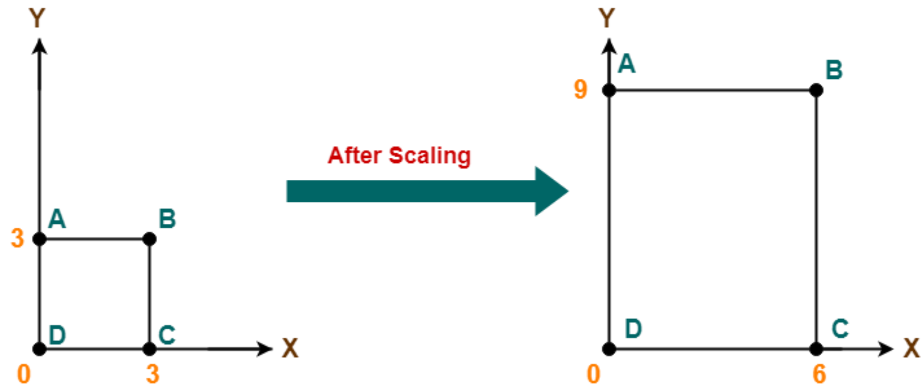
Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).



4. **Reflection:**
   - Reflection is a kind of rotation where the angle of rotation is 180 degree.
   - The reflected object is always formed on the other side of mirror.
   - The size of reflected object is same as the size of original object.

Consider a point object O has to be reflected in a 2D plane.
Let

- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the reflected object O after reflection = (X_{new}, Y_{new})

Reflection on X-Axis

This reflection is achieved by using the following reflection equations-

- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

(Homogeneous Coordinates Representation)

Reflection on Y-Axis

This reflection is achieved by using the following reflection equations-

- $X_{new} = -X_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

(Homogeneous Coordinates Representation)

EXAMPLE: Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

SOLUTION: Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the X axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).
Applying the reflection equations, we have-

- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$

Thus, New coordinates of corner A after reflection = (3, -4).

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).
Applying the reflection equations, we have-

- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$

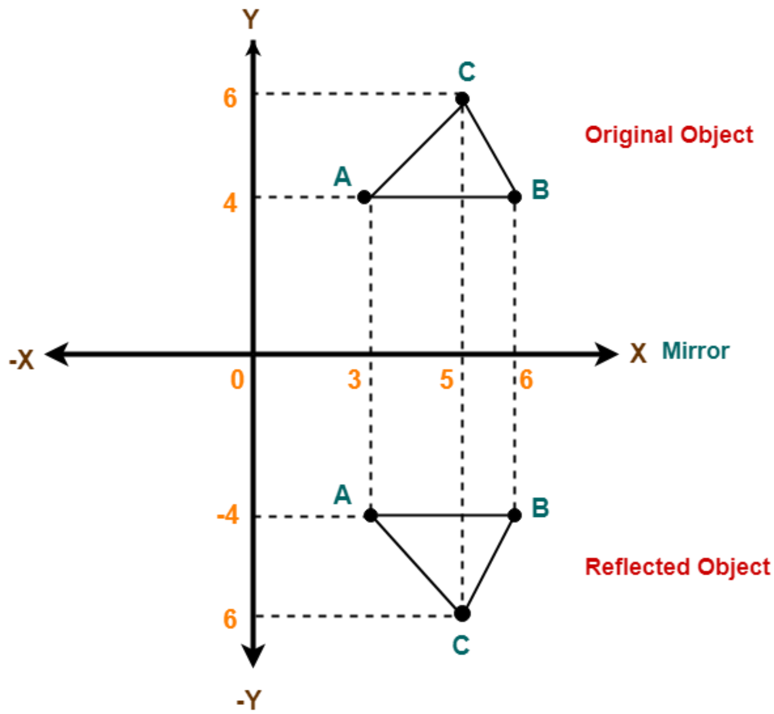
Thus, New coordinates of corner B after reflection = (6, -4).

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).
Applying the reflection equations, we have-

- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$

Thus, New coordinates of corner C after reflection = (5, -6).
Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).



EXAMPLE: Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the Y axis and obtain the new coordinates of the object.

SOLUTION:

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the Y axis

For Coordinates A(3, 4)

Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -3$
- $Y_{new} = Y_{old} = 4$

Thus, New coordinates of corner A after reflection = (-3, 4).

For Coordinates B(6, 4)

Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).

Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -6$
- $Y_{new} = Y_{old} = 4$

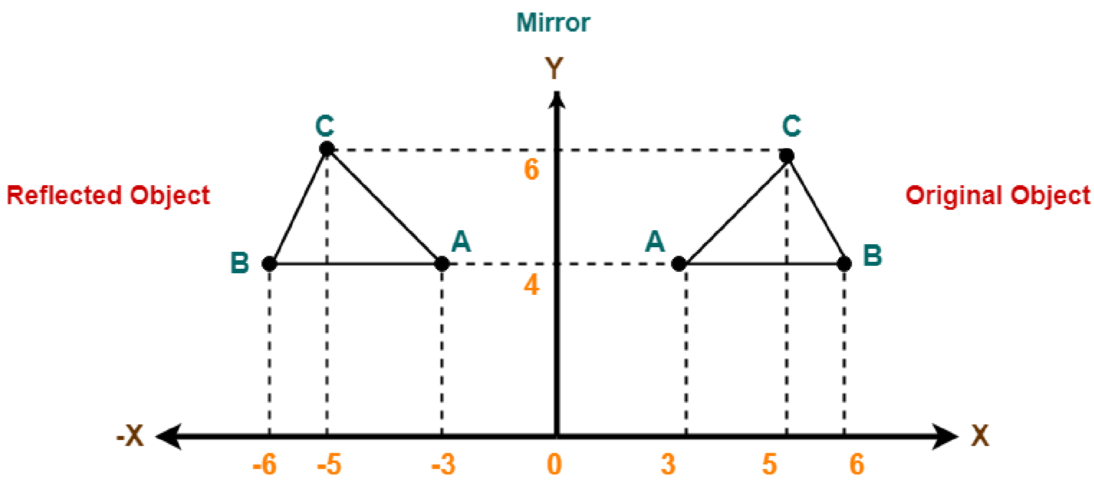
Thus, New coordinates of corner B after reflection = (-6, 4).

For Coordinates C(5, 6)

Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).
Applying the reflection equations, we have-

- $X_{new} = -X_{old} = -5$
- $Y_{new} = Y_{old} = 6$

Thus, New coordinates of corner C after reflection = (-5, 6).



5. **Shearing:**

In Computer graphics, 2D Shearing is an ideal technique to change the shape of an existing object in a two dimensional plane.

In a two dimensional plane, the object size can be changed along X direction as well as Y direction. So, there are two versions of shearing-

a. Shearing in X-Axis
b. Shearing in Y-Axis

Consider a point object O has to be sheared in a 2D plane.
Let-

- Initial coordinates of the object O = (X_{old}, Y_{old})
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y
- New coordinates of the object O after shearing = (X_{new}, Y_{new})

Shearing in X Axis-

Shearing in X axis is achieved by using the following shearing equations-

- $X_{new} = X_{old} + Sh_x \times Y_{old}$
- $Y_{new} = Y_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In X axis)

(Homogeneous Coordinates Representation)

Shearing in Y Axis-

Shearing in Y axis is achieved by using the following shearing equations-

- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$

In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In Y axis)

(Homogeneous Coordinates Representation)

EXAMPLE:

Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

SOLUTION:

Given-

- Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \text{ x } Y_{old} = 1 + 2 \text{ x } 1 = 3$
- $Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \text{ x } Y_{old} = 0 + 2 \text{ x } 0 = 0$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 0 = 1$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).

Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shearing in X Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$

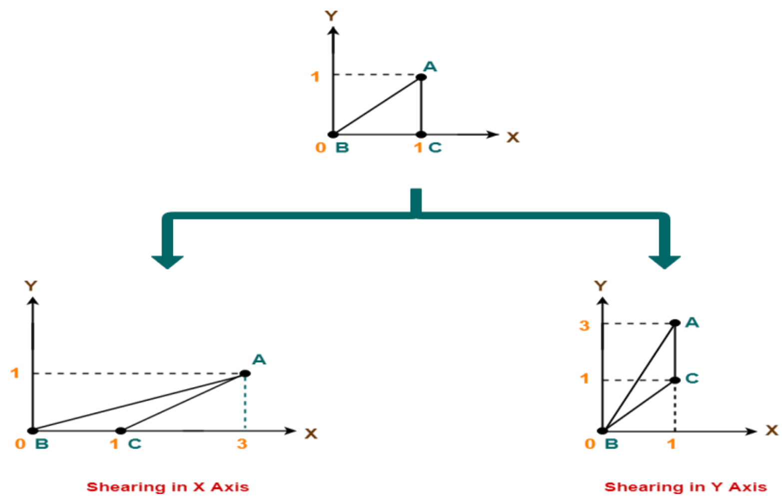
Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).
Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).
Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).



Matrix Representation of 2-D Transformation:

1. Scaling $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

2. Rotation (clockwise) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

3. Rotation (anti-clock) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

4. Translation $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_x & t_y \end{bmatrix}$

5. Reflection $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(about x axis)

6. Reflection $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

(about y axis)

7. Reflection $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(about origin)

8. Reflection about Y=X $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

9. Reflection about Y= –X $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

10. Shearing in X direction $\begin{bmatrix} 1 & 0 \\ Sh_x & 1 \end{bmatrix}$

11. Shearing in Y direction $\begin{bmatrix} 1 & Sh_y \\ 0 & 1 \end{bmatrix}$

12. Shearing in both x and y direction $\begin{bmatrix} 1 & Sh_y \\ Sh_x & 1 \end{bmatrix}$

Homogeneous Coordinates of 2 –D Transformation

The rotation of a point, straight line or an entire image on the screen, about a point other than origin, is achieved by first moving the image until the point of rotation occupies the origin, then performing rotation, then finally moving the image to its original position.

The moving of an image from one place to another in a straight line is called a translation. A translation may be done by adding or subtracting to each point, the amount, by which picture is required to be shifted.

Translation of point by the change of coordinate cannot be combined with other transformation by using simple matrix application. Such a combination is essential if we wish to rotate an image about a point other than origin by translation, rotation again translation.

To combine these three transformations into a single transformation, homogeneous coordinates are used. In homogeneous coordinate system, two-dimensional coordinate positions (x, y) are represented by triple-coordinates.

Homogeneous coordinates are generally used in design and construction applications. Here we perform translations, rotations, scaling to fit the picture into proper position.

Example of representing coordinates into a homogeneous coordinate system: For two-dimensional geometric transformation, we can choose homogeneous parameter h to any non-zero value. For our convenience take it as one. Each two-dimensional position is then represented with homogeneous coordinates (x, y, 1).

Following are matrix for two-dimensional transformation in homogeneous coordinate:

1. Translation $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$

2. Scaling $\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3. Rotation (clockwise) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

4. Rotation (anti-clock) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5. Reflection against X axis $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. Reflection against Y axis $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. Reflection against origin $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8. Reflection against line Y=X $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

9. Reflection against Y= –X $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

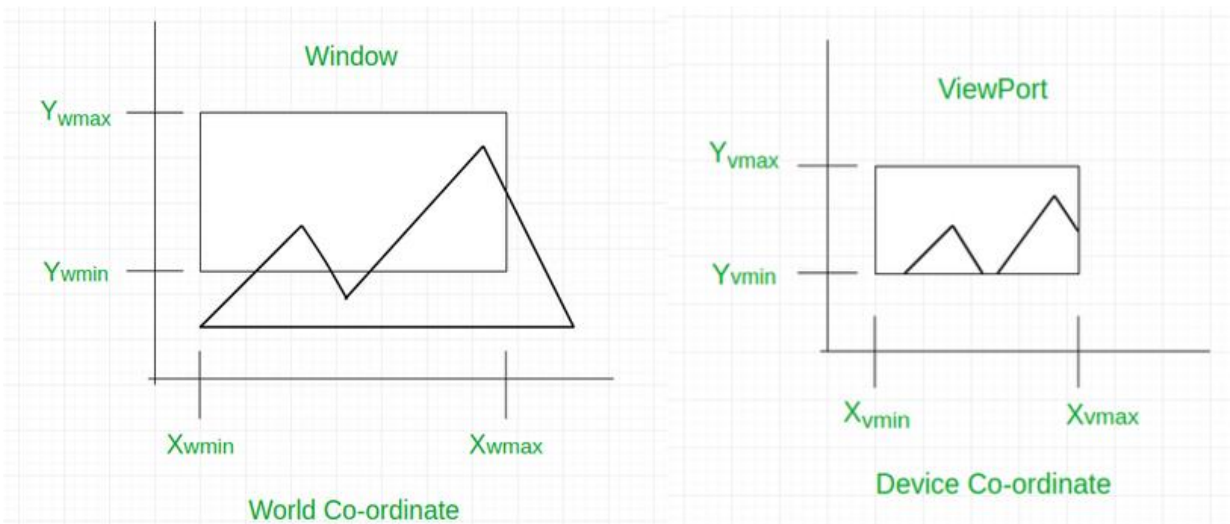
10. Shearing in X direction $\begin{bmatrix} 1 & 0 & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. Shearing in Y direction $\begin{bmatrix} 1 & Sh_y & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

12. Shearing in both x and y direction $\begin{bmatrix} 1 & Sh_y & 0 \\ Sh_x & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Window to Viewport Transformation:

Window to Viewport Transformation is the process of transforming 2D world-coordinate objects to device coordinates. Objects inside the world or clipping window are mapped to the viewport which is the area on the screen where world coordinates are mapped to be displayed.



General Terms:

- **World coordinate –** It is the Cartesian coordinate w.r.t which we define the diagram, like X_{wmin}, X_{wmax}, Y_{wmin}, Y_{wmax}
- **Device Coordinate –**It is the screen coordinate where the objects are to be displayed, like X_{vmin}, X_{vmax}, Y_{vmin}, Y_{vmax}
- **Window –**It is the area on the world coordinate selected for display.
- **ViewPort –**It is the area on the device coordinate where graphics is to be displayed.

Mathematical Calculation of Window to Viewport:

It may be possible that the size of the Viewport is much smaller or greater than the Window. In these cases, we have to increase or decrease the size of the Window according to the Viewport and for this, we need some mathematical calculations.

(x_w, y_w): A point on Window
(x_v, y_v): Corresponding point on Viewport

We have to calculate the point **(x_v, y_v)**

Normalized Point on Window $$\left(\frac{X_w - X_{wmin}}{X_{wmax} - X_{wmin}}, \frac{Y_w - Y_{wmin}}{Y_{wmax} - Y_{wmin}} \right)$$

Normalized Point on Viewport $$\left(\frac{X_v - X_{vmin}}{X_{vmax} - X_{vmin}}, \frac{Y_v - Y_{vmin}}{Y_{vmax} - Y_{vmin}} \right)$$

Now the relative position of the object in Window and Viewport are same.

For x coordinate,

$$\frac{X_w - X_{wmin}}{X_{wmax} - X_{wmin}} = \frac{X_v - X_{vmin}}{X_{vmax} - X_{vmin}}$$

For y coordinate,

$$\frac{Y_w - Y_{wmin}}{Y_{wmax} - Y_{wmin}} = \frac{Y_v - Y_{vmin}}{Y_{vmax} - Y_{vmin}}$$

So, after calculating for x and y coordinate, we get

$$X_v = X_{vmin} + (X_w - X_{wmin})\, S_x$$

$$Y_v = Y_{vmin} + (Y_w - Y_{wmin})\, S_y$$

Where s_x is the scaling factor of x coordinate and s_y is the scaling factor of y coordinate

$$S_x = \frac{X_{vmax} - X_{vmin}}{X_{wmax} - X_{wmin}} \qquad S_y = \frac{Y_{vmax} - Y_{vmin}}{Y_{wmax} - Y_{wmin}}$$

Example: Let us assume,

- for window, $\mathbf{X_{wmin} = 20, X_{wmax} = 80, Y_{wmin} = 40, Y_{wmax} = 80}$.
- for viewport, $\mathbf{X_{vmin} = 30, X_{vmax} = 60, Y_{vmin} = 40, Y_{vmax} = 60}$.
- Now a point (X_w, Y_w) be (30, 80) on the window. We have to calculate that point on the viewport
  i.e **(X_v, Y_v)**.
- First of all, calculate the scaling factor of x coordinate S_x and the scaling factor of y coordinate S_y using the above-mentioned formula.

  S_x = (60 - 30) / (80 - 20) = 30 / 60
  S_y = (60 - 40) / (80 - 40) = 20 / 40
- So, now calculate the point on the viewport (X_v, Y_v).

  X_v = 30 + (30 - 20) * (30 / 60) = 35
  Y_v = 40 + (80 - 40) * (20 / 40) = 60
- So, the point on window (X_w, Y_w) = (30, 80) will be **(X_v, Y_v) = (35, 60)** on viewport.