Scalars: A quantity that has magnitude only is known as a scalar.
e.g.: mass, length, time, temperature, density etc.

Vectors: A quantity that has magnitude as well as direction is called a vector.
e.g.: force, velocity, accelaration, momentum, etc.

Remark: A directed line segment is called a vector.
A directed line segment $A \longrightarrow B$ with initial point A and the terminal point B is the vector $\overrightarrow{AB}$ and its magnitude is denoted by $|\overrightarrow{AB}|$

Unit Vector: A vector $\vec{a}$ is called a unit vector if $|\vec{a}| = 1$ and it is denoted by $\hat{a}$.

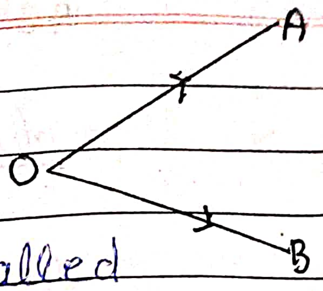
Equal vectors: Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal if they have the same magnitude and the same direction regardless of the position of positions of their initial points.

Negative of a vector: A vector having the same magnitude as that of a given vector $\vec{a}$ and the direction opposite to that of $\vec{a}$ is called the negative of $\vec{a}$ to be denoted by $-\vec{a}$.
Thus, if $\overrightarrow{AB} = \vec{a}$ then $\overrightarrow{BA} = -\vec{a}$

Zero or Null vector: A vector whose initial and terminal points coincide is called a zero vector, denoted by $\vec{0}$.
Magnitude of a zero vector is 0 but it cannot be

assigned a definite direction.

Thus, $\vec{AA} = \vec{O}$



Coinitial Vectors: Two or more vectors having the same initial point are called coinitial vectors.

$\vec{OA}$ and $\vec{OB}$ are the two coinitial vectors having the same initial point O.

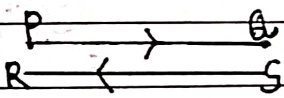
Collinear Vectors: Vectors having the same or parallel supports are known as collinear vectors.

Here, $\vec{AB}$, $\vec{BC}$ and $\vec{AC}$ are collinear vectors.

Like Vectors: Collinar vectors having the same direction are called like vectors.

Thus, $\vec{AB}$, $\vec{BC}$ and $\vec{AC}$ are like vectors.

Unlike Vectors: Collinear vectors having opposite directions are known as unlike vectors. $\vec{PQ}$ and $\vec{SR}$ are unlike vectors.

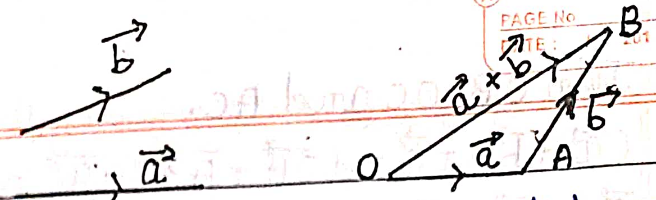


Free Vectors: If the initial point of a vector is not specified then it is said to be free vector.

Localised Vectors A vector drawn parallel to a given vector through a specified point as the initial point is called a localised vectors.

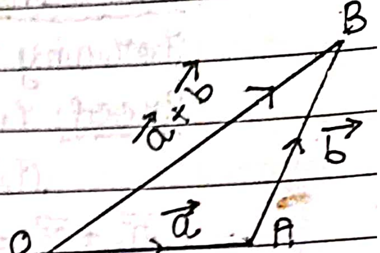
Coplanar Vectors: Three or more nonzero vectors lying in the same plane or parallel to the same plane are said to be coplanar, otherwise they are called non coplanar.

Vector Addition:



Let $\vec{a}$ and $\vec{b}$ be any two vectors. Taking any point O and drawing segments $\vec{OA}$ and $\vec{AB}$ such that $\vec{OA}=\vec{a}$ and $\vec{AB}=\vec{b}$. Join OB. Then, $\vec{OB}$ is called the sum or resultant of $\vec{a}$ and $\vec{b}$.

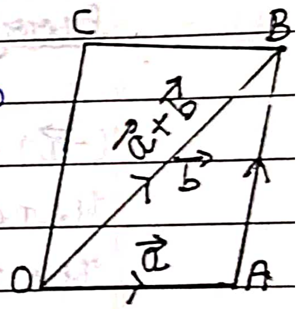
$\therefore \vec{a}+\vec{b} = \vec{OA}+\vec{AB} = \vec{OB}$



Triangle law of addition of vectors:

In a ΔOAB, if $\vec{OA}$ and $\vec{AB}$ represent $\vec{a}$ and $\vec{b}$ respectively, then $\vec{OB}$ represents $(\vec{a}+\vec{b})$. This is known as triangle law of addition of vectors.

Parallelogram law of addition of vectors:



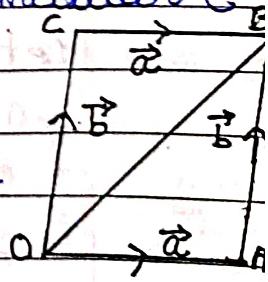
In a parallelogram OABC, if $\vec{OA}$ and $\vec{AB}$ represent $\vec{a}$ and $\vec{b}$ respectively, then $\vec{OB}$ represents $(\vec{a}+\vec{b})$.

This is known as parallelogram law of addition of vectors.

Laws of addition of vectors:

(1) Commutative law: Vector addition is commutative

i.e., $\vec{a}+\vec{b}=\vec{b}+\vec{a}$



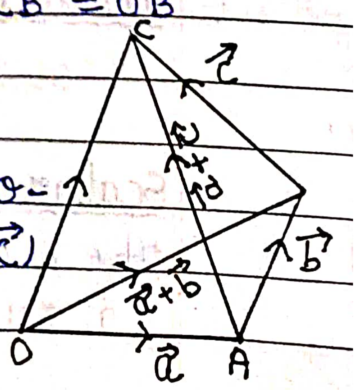
Proof: Let $\vec{a}$ and $\vec{b}$ be the given vectors represented by $\vec{OA}$ and $\vec{AB}$ respectively. Complete the parallelogram OABC.

Then $\vec{OC}=\vec{AB}=\vec{b}$ and $\vec{CB}=\vec{OA}=\vec{a}$

$\therefore \vec{a}+\vec{b}=\vec{OA}+\vec{AB}=\vec{OB}$ and $\vec{b}+\vec{a}=\vec{OC}+\vec{CB}=\vec{OB}$

Hence, $\vec{a}+b=\vec{b}+\vec{a}$

(2) Associative law: Vector addition is associative i.e, $(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$



Proof: Let $\vec{OA}=\vec{a}$, $\vec{AB}=\vec{b}$ and $\vec{BC}=\vec{c}$

Join OB, OC and AC.

$(\vec{a}+\vec{b})+\vec{c} = (\vec{OA}+\vec{AB})+\vec{BC} = \vec{OB}+\vec{BC} = \vec{OC}$

$\vec{a}+(\vec{b}+\vec{c}) = \vec{OA}+(\vec{AB}+\vec{BC}) = \vec{OA}+\vec{AC} = \vec{OC}$

$\therefore (\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

(3) **Existence of additive identity:**

For any vector $\vec{a}$, $\vec{a}+\vec{0} = \vec{0}+\vec{a} = \vec{a}$

Proof: Let $\vec{OA} = \vec{a}$. Then, $\vec{a}+\vec{0} = \vec{OA}+\vec{AA} = \vec{OA} = \vec{a}$ and

$\vec{0}+\vec{a} = \vec{OO}+\vec{OA} = \vec{OA} = \vec{a}$

$\therefore \vec{a}+\vec{0} = \vec{0}+\vec{a} = \vec{a}$

The vector $\vec{0}$ is called the additive identity for vectors

(4) **Existence of additive inverse:**

For any vector $\vec{a}$, $\vec{a}+(-\vec{a}) = (-\vec{a})+\vec{a} = \vec{0}$

Proof: Let $\vec{OA} = \vec{a}$ then $\vec{AO} = -\vec{a}$

$\therefore \vec{a}+(-\vec{a}) = \vec{OA}+\vec{AO} = \vec{OO} = \vec{0}$ and

$(-\vec{a})+(\vec{a}) = \vec{AO}+\vec{OA} = \vec{AA} = \vec{0}$

Hence, $\vec{a}+(-\vec{a}) = (-\vec{a})+\vec{a} = \vec{0}$

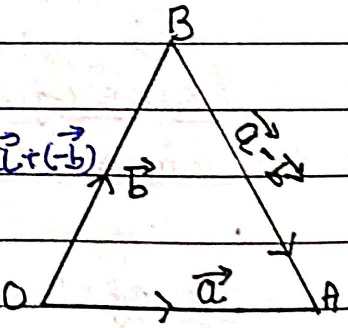
The vector $-\vec{a}$ is called the additive inverse of $\vec{a}$.

Difference of two vectors:

For any two vectors, we define $\vec{a}-\vec{b} = \vec{a}+(-\vec{b})$

Let $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$. Then, $\vec{BO} = -\vec{b}$

$\therefore \vec{a}-\vec{b} = \vec{a}+(-\vec{b}) = \vec{OA}+\vec{BO} = \vec{BO}+\vec{OA} = \vec{BA}$



$\therefore \vec{OA}-\vec{OB} = \vec{BA}$

In a similar way, $\vec{OB}-\vec{OA} = \vec{AB}$

Remarks: (i) $\vec{AB}$ = (position vector of B) − (position vector of A)

(ii) $\vec{BA}$ = (position vector of A) − (position vector of B)

Scalar multiplication of a vector:

The scalar multiple of $\vec{a}$ by a scalar k is the vector $k\vec{a}$ such that (i) $|k\vec{a}| = |k||\vec{a}|$

(ii) direction of $k\vec{a}$ is the same as that of $\vec{a}$, when $k > 0$ and opposite to that of $\vec{a}$ when $k < 0$.

Components of a vector: Let O be the origin and let $P(x, y, z)$ be any point in space.
Let $\hat{i}, \hat{j}, \hat{k}$ be unit vectors along the x-axis, y-axis and z-axis respectively. Let the position vector of P be $\vec{r}$. Then, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$
This form of a vector is called its component form.
x, y, z are called the scalar components of $\vec{r}$ and $x\hat{i}, y\hat{j}, z\hat{k}$ are called its vector components.
Also, $|\vec{r}| = |x\hat{i} + y\hat{j} + z\hat{k}| = \sqrt{x^2 + y^2 + z^2}$

Direction Cosines and Direction ratios of a vector
Let $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$ be a vector. Then, the numbers a, b, c are called the direction ratios of $\vec{r}$.
Direction cosines of $\vec{r}$ are given as

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

Note: (1) If l, m, n are direction cosines of a vector then $l^2 + m^2 + n^2 = 1$

(2) If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be any two points in space then direction ratios of $\vec{AB}$ are $(x_2 - x_1)$, $(y_2 - y_1)$ and $(z_2 - z_1)$ and direction cosines of $\vec{AB}$ are
$\frac{x_2 - x_1}{r}, \frac{y_2 - y_1}{r}, \frac{z_2 - z_1}{r}$ where $r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

Remarks: If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ then

(i) $\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$

(ii) $\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$

(iii) $\vec{a} = \vec{b}$ iff $a_1 = b_1, a_2 = b_2, a_3 = b_3$

(iv) $\lambda\vec{a} = (\lambda a_1)\hat{i} + (\lambda a_2)\hat{j} + (\lambda a_3)\hat{k}$

Remarks: Let $\vec{a}$ and $\vec{b}$ be any two vectors and let k and m be any two scalars. Then,

(i) $k\vec{a}+m\vec{a}=(k+m)\vec{a}$ (ii) $k(m\vec{a})=(km)\vec{a}$

(iii) $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$

Remarks: (i) Whatever be the value of k, the vector $k\vec{a}$ is always collinear to the vector $\vec{a}$.

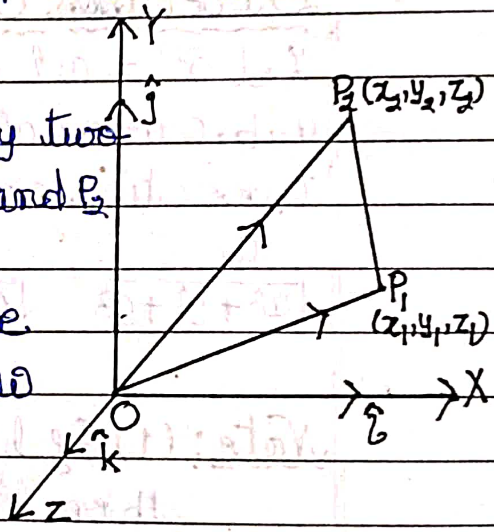
(ii) Two vectors $\vec{a}$ and $\vec{b}$ are collinear iff $\exists$ a non zero scalar k such that $\vec{b}=k\vec{a}$

(iii) If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are two vectors, are collinear iff $b_1\hat{i}+b_2\hat{j}+b_3\hat{k}=\lambda(a_1\hat{i}+a_2\hat{j}+a_3\hat{k})$

$\Leftrightarrow b_1\hat{i}+b_2\hat{j}+b_3\hat{k}=(\lambda a_1)\hat{i}+(\lambda a_2)\hat{j}+(\lambda a_3)\hat{k}$

$\Leftrightarrow b_1=\lambda a_1,\ b_2=\lambda a_2,\ b_3=\lambda a_3 \Leftrightarrow \frac{b_1}{a_1}=\frac{b_2}{a_2}=\frac{b_3}{a_3}=\lambda$

Vector joining two points:



If $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ are any two points, then the vector joining P_1 and P_2 is the vector $\overrightarrow{P_1P_2}$.

Joining the points P_1 and P_2 with the origin O, and applying triangle law of vector addition in ΔOP_1P_2,

$\overrightarrow{OP_1}+\overrightarrow{P_1P_2}=\overrightarrow{OP_2} \Rightarrow \overrightarrow{P_1P_2}=\overrightarrow{OP_2}-\overrightarrow{OP_1}$

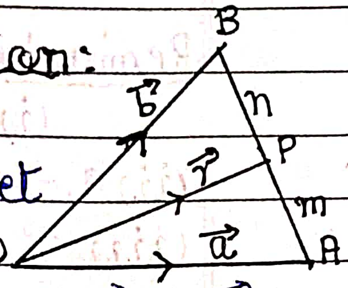
i.e., $\overrightarrow{P_1P_2}=(x_2\hat{i}+y_2\hat{j}+z_2\hat{k})-(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})$

$=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$ and

$|\overrightarrow{P_1P_2}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Section formulae

(1) **Section formula for internal division:**



Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and let P be a point dividing AB internally in the ratio m:n. Let $\overrightarrow{OP}=\vec{r}$. Then $\vec{r}=\frac{m\vec{b}+n\vec{a}}{m+n}$

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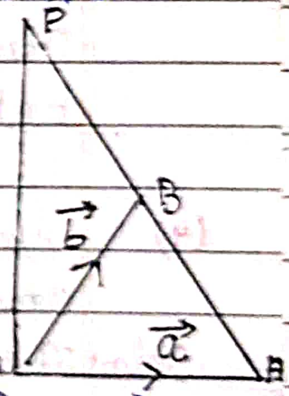
Proof: Let O be the origin. Then, $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$
Let P be a point on AB such that $\frac{AP}{PB} = \frac{m}{n}$
Then, $\frac{AP}{PB} = \frac{m}{n} \Rightarrow n \cdot AP = m \cdot PB \Rightarrow n(\vec{AP}) = m \cdot (\vec{PB})$
$\Rightarrow n(\vec{OP} - \vec{OA}) = m(\vec{OB} - \vec{OP}) \Rightarrow (m+n)\vec{OP} = m\vec{OB} + n \cdot \vec{OA}$
$\Rightarrow (m+n)\vec{OP} = m\vec{b} + n\vec{a} \Rightarrow \vec{OP} = \vec{r} = \frac{mb + n\vec{a}}{m+n}$

Corollary: The position vector of the mid-point of the join of two points with vectors $\vec{a}$ and $\vec{b}$ is $\frac{1}{2}(\vec{a} + \vec{b})$

Proof: Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively.

Section formula for external division:

Let A and B be two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and let P be a point dividing AB externally in the ratio m : n.
Let $\vec{OP} = \vec{r}$. Then, $\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$



Proof: Let O be the origin. Let $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$. Let AB be produced to P such that AP : BP = m : n.
Now, $\frac{AP}{BP} = \frac{m}{n} \Rightarrow n \cdot AP = m \cdot BP \Rightarrow n \cdot \vec{AP} = m \cdot \vec{BP}$
$\Rightarrow n(\vec{OP} - \vec{OA}) = m(\vec{OP} - \vec{OB}) \Rightarrow (m-n)\vec{OP} = (m \cdot \vec{OB} - n\vec{OA})$
$\Rightarrow (m-n)\vec{OP} = m\vec{b} - n\vec{a} \Rightarrow \vec{OP} = \frac{m\vec{b} - n\vec{a}}{m-n} \Rightarrow \vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$