Econ 4140 Assignment 2

The files SP500TBTESLA.xlsx and SP500TBTESLA.txt contain daily observations on three months T-bill rates, and daily closing prices of the S&P 500 index and TESLA stock between 2019/01/02 and 2021/12/31.

Use the code in file Assgn2.txt to calculate the excess returns on S&P 500 and TESLA and estimate the CAPM model, as shown on page 239 of the textbook.

1. Write the estimated regression model with and without the intercept. Refer to them as Model 1 and Model 2.

Model 1:

We can derive the estimated regression model without intercept from the security characteristic line:

$$R_{tsl} - \mu_{f,t} = +\beta_{j,t}(R_{M,t} - \mu_{f,t}) + \epsilon_{j,t}$$

We want to express it as a linear equation so we can set our excess returns to the following:

1. $R^*_{tsl} = R_{j,t} - \mu_{f,t}$
2. $R^*_{M,t} = R_{M,t} - \mu_{f,t}$

Then we get:

$$R^*_{tsl} = \beta_{j,t} R^*_{M,t} + \epsilon_{j,t}$$

Which is our general linear regression equation **WITHOUT** intercept. We will refer to this as **model 1**.

Running our code gives us the following estimation:

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
exretsp	1	1.35297	0.09671	13.99	<.0001

$\widehat{\beta}_{1,tsl} = 1.35297$
Substituting into our equation gives us:

$$R^*_{1,tsl} = 1.35297 R^*_{M,t} + \epsilon_{j,t}$$

Model 2:

We can simply add an intercept to our general equation earlier to get:

$$R^*_{j,t} = \alpha_j + \beta_{j,t} R^*_{M,t} + \epsilon_{j,t}$$

Which is our linear regression equation **WITH** intercept. We will refer to this as **model 2**

Running our code again gives us:

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.00264	0.00138	1.91	0.0561
exretsp	1	1.34232	0.09670	13.88	<.0001

$\widehat{\beta}_{2,tsl} = 1.34232$
$\widehat{\alpha}_{2,tsl} = 0.00264$
Substituting into our equation gives us:

$$R^*_{2,tsl} = 0.00264 + 1.34232 R^*_M + \epsilon_{j,t}$$

2. Explain what parameters are estimated and what hypotheses are tested in the computer output for Models 1 and 2.

Model 1: No intercept

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
exretsp	1	1.35297	0.09671	13.99	<.0001

Parameters:

1. $\widehat{\beta}_{1,tsl} = 1.35297$ - the slope of the regression - the beta of the stock is being estimated - measures how "aggressive" tesla is as an asset compared to the market.
   $\beta_j > 1 \implies$ "aggressive"
   $\beta_j = 1 \implies$ "average risk"
   $\beta_j < 1 \implies$ "not aggressive"
   Because $\beta_{tsl} > 1 \implies$ it has a greater risk and return than the market risk and return.
   To calculate/estimate our beta:

$$\beta_{1,tsl} = \frac{\sigma_{tsl,M}}{\sigma^2_M}$$

Hypothesis:
If the CAPM holds then the excess returns on tesla should satisfy a regression with no intercept. Therefore model 1 is estimating to see what our beta on tesla is if CAPM is satisfied.

Model 2:

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.00264	0.00138	1.91	0.0561
exretsp	1	1.34232	0.09670	13.88	<.0001

Parameters:

1. $\widehat{\beta}_{2,tsl} = 1.34232$ - Again the beta of the stock, same interpretation as before.
2. $\widehat{\alpha}_{2,tsl} = 0.00264$ - the intercept - if it is greater than zero the security is mispriced - more on this in question 6.

Hypothesis:
If the model has an intercept, CAPM does not hold and the security is mispriced. Therefore model 2 is estimating to see if tesla is properly priced and if CAPM holds.

3. Comment on the values of the estimated parameters and the outcomes of tests of their statistical significance for Models 1 and 2.

Model 1:

Parameters

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
exretsp	1	1.35297	0.09671	13.99	<.0001

1. $\widehat{\beta}_{1,tsl} = 1.35297$

As mentioned before tesla's beta is aggressive as it is greater than 1. We also reject the null hypothesis that $\widehat{\beta}_{1,tsl} = 0$ because $Pr > |t| = .0001 < 0.05$ and is inside our rejection region. This is no surprise though as we aren't trying to verify that $\widehat{\beta}_{1,tsl} = 0$. But this means that $\widehat{\beta}_{1,tsl}$ is statistically significant.

Model 2:

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	0.00264	0.00138	1.91	0.0561
exretsp	1	1.34232	0.09670	13.88	<.0001

1. $\widehat{\beta}_{2,tsl} = 1.34232$
2. $\widehat{\alpha}_{2,tsl} = 0.00264$
   Because $\alpha > 0$ the security is underpriced;

   BUT if we do not reject the null hypothesis that α is zero, then there's no evidence that Tesla was mispriced.
   Looking at our $Pr > |t|$ we find that $0.0561 > 0.05$ therefore we do not reject our null hypothesis of $\alpha = 0$.

So our data is consistent with CAPM because our intercept is not statistically significant and Tesla is properly priced.

4. Define and write the correlation coefficient. Use your output to demonstrate that there exists a statistically significant correlation between the excess returns on TESLA and excess market returns in Models 1 and 2.

Not 100% sure what you mean here but I have 3 guesses.

1. The correlation between the excess returns on TESLA and the excess market returns is just $\beta_{tsl,M}$ via the SCL (Q1) which we already saw were statistically significant in Models 1 and 2 earlier in Q3.

$$R_{tsl} - \mu_{f,t} = +\beta_{j,t}(R_{M,t} - \mu_{f,t}) + \epsilon_{j,t}$$

2. **F-test**

Model 1:

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.28035	0.28035	195.73	<.0001
Error	749	1.07280	0.00143		
Uncorrected Total	750	1.35315			

Model 2:

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	0.27504	0.27504	192.71	<.0001
Error	748	1.06758	0.00143		
Corrected Total	749	1.34262			

Our F-tests in Model 1 and 2 are both statistically significant due to us having a high F value, and a low $PR > F$ value.

3. Relationship between beta and the correlation coefficient $\rho_{tsl,M}$

$$\rho_{tsl,M} = \frac{\sigma_{tsl,M}}{\sigma_{tsl}\sigma_M} = \frac{Cov(tsl,M)}{\sigma_{tsl}\sigma_M}$$

$$\beta_{1,tsl} = \frac{\sigma_{tsl,M}}{\sigma_M^2} = \frac{Cov(tsl,M)}{Var(M)}$$

The difference between calculating the correlation coefficient is therefore in the denominator. Instead of multiplying by σ_{tsl} you multiply by σ_M to get σ_M^2

We proved earlier in our F-test that our ANOVA is statistically significant so you should just be able to input them and get to the correlation coefficient, which means that it would have to be statistically significant as well.

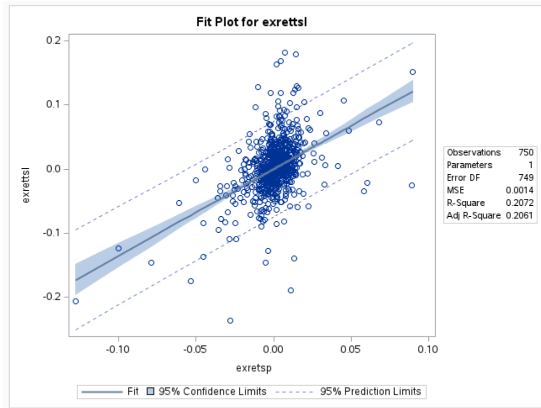
5. Are the results of the estimation without an intercept consistent with those when the intercept is included?

Yes, the results are quite consistent when considering both our values and our tests.

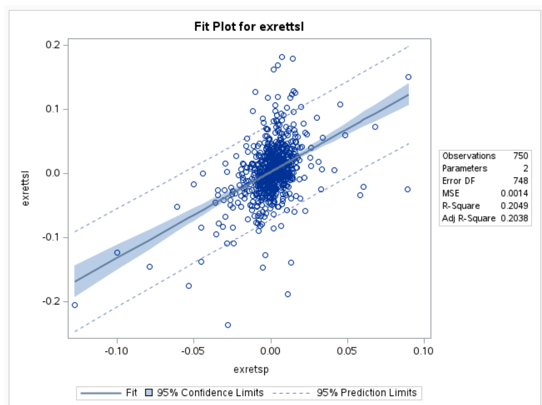
1. α isn't statistically significant because we didn't reject the null hypothesis that $\alpha = 0$, therefore Model 1 $\sim$ Model 2
2. $\widehat{\beta}_{1,tsl} \sim \widehat{\beta}_{2,tsl} = 1.35297 \sim 1.34232$. The 0.01065 difference doesn't make much of a difference at all.

Furthermore if we look at our fit plot we find that model 1 looks very similar to model 2:

Model 1:



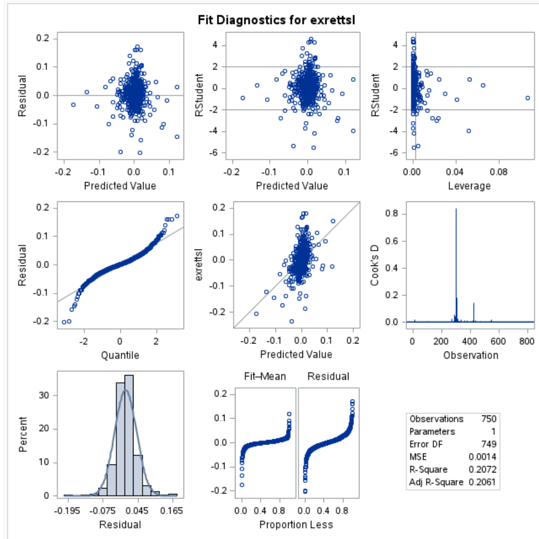
Model 2:



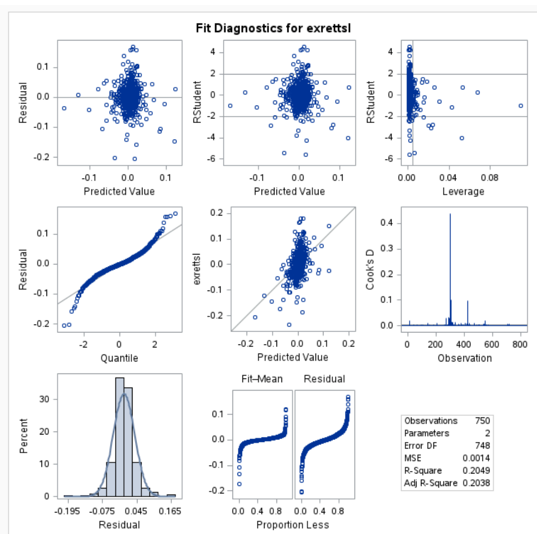
With our MSE being identical and R/ADJ R-Square being very similar as well.

Also, our fit diagnostics are nearly identical as well.

Model 1:



Model 2:



All in all, what this means is that the intercept really isn't of enough value to cause a significant difference between the two models.
It is very close to zero and we saw it isn't statistically significant.

$\widehat{\alpha}_{2,tsl} = 0.00264 \sim 0$

For larger and larger values of alpha that are statistically significant, Model 1 and Model 2 would deviate from each other, CAPM wouldn't hold and tesla would be mispriced.

6. What is the financial interpretation of test of the null hypothesis “intercept = 0” ?

$$H_0 : \alpha_{tsl} = 0$$

$$H_A : \alpha_{tsl} \neq 0$$

The financial interpretation of testing the null hypothesis that "intercept = 0" is to find mispriced securities under CAPM. If $\alpha \neq 0$ and it is statistically significant, then CAPM doesn't hold and the security is mispriced.

If $\alpha > 0$ the security is underpriced.
If $\alpha < 0$ then the security is overpriced.

But if $\alpha = 0$ and the we do not reject the null hypothesis that "intercept = 0" then CAPM does hold and the security is properly priced.

As said before, for us we found that even though α > 0, it wasn't statistically significant so CAPM holds and Tesla is properly priced.

7. Do the data provide sufficient evidence that the CAPM holds?

Yes the data does provide sufficient evidence that CAPM holds.

When we regressed Model 1 and Model 2, we found both regressions have statistically significant betas, and the alpha in our regression with intercept isn't because we didn't reject our null hypothesis that "intercept = 0".

This means we would use the model without intercept to regress tesla as it follows CAPM and would be more slightly more accurate but there isn't a very big difference at all between the 2 because alpha is so close to zero.