Spectrographs

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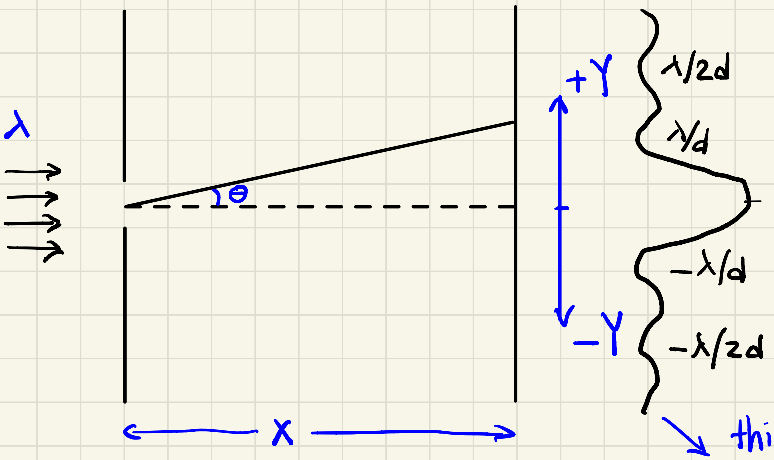
Diffraction:

The diffraction grating is at the heart of most optical spectrographs. Thing of a spectrograph as an extension of a two-slit interferometer.

Diffraction of a single slit:



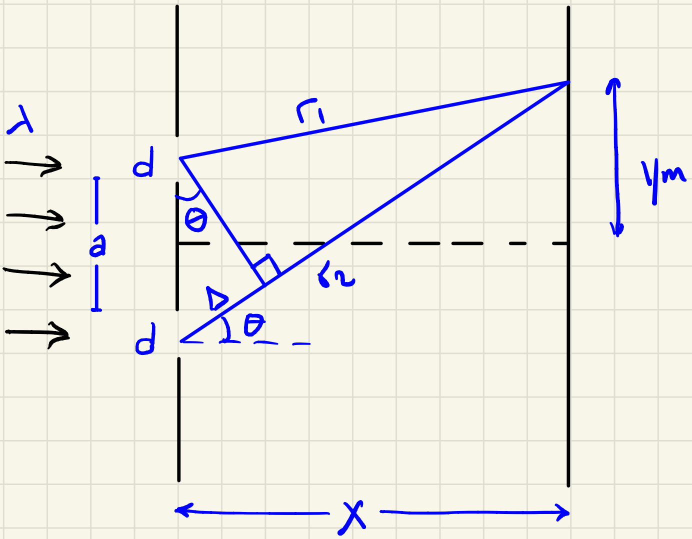
- single slit produces a series of fringes if slit width is similar to wavelength of light
- get dark fringes where light waves interfere:

$$\boxed{\sin\theta = m\frac{\lambda}{d}}$$

this is a sinc function, the Fourier Transform of a square aperture

- wide slits lead to narrow peaks and narrow slits lead to broad peaks
- fundamentally, this is why interferometers w/ long baselines obtain fine angular resolution!

Interference from multiple apertures:



- for light passing through a double slit and falling on a screen at distance x, constructive interference occurs when the path difference is an integer number of wavelengths:

$$\Delta = r_1 - r_2 = m\lambda$$

By geometry:

$$a\sin\theta \approx a\theta = \Delta$$

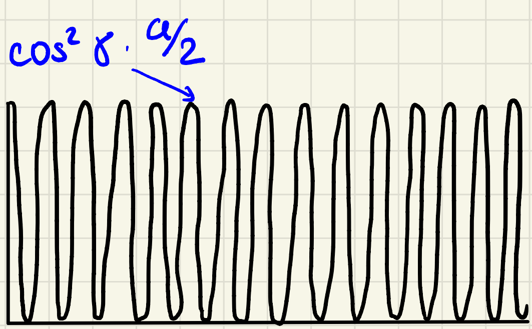
$$a\sin\theta \approx a\theta = m\lambda$$

Also: $\tan\theta \approx \theta = \frac{y_m}{x}$

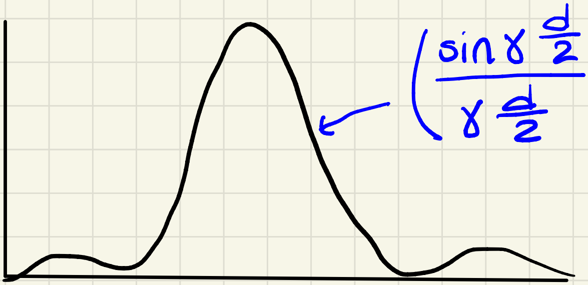
$$\Longrightarrow \quad \theta = \frac{y_m}{x} = \frac{\Delta}{a} = \frac{m\lambda}{a}$$

$y_m = \frac{x}{a} m\lambda$ this is where the mth bright fringe occurs

What does the interference pattern look like?



double-slit interference pattern for two narrow slits



$$\left(\frac{\sin \gamma \frac{d}{2}}{\gamma \frac{d}{2}}\right)^2$$

single-slit diffraction pattern

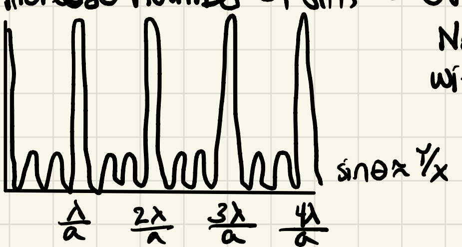
actual two-slit pattern product of single-slit diffraction and double-slit interference

$$\text{Intensity} = \underbrace{\frac{\sin^2 \gamma \frac{d}{2}}{[\gamma\, d/2]^2}}_{\text{diffraction term}} \cdot \Big(\underbrace{4C^2 \cos^2 \gamma \frac{a}{2}}_{\text{interference term}}\Big), \quad \gamma = \frac{2\pi}{\lambda} \cdot \frac{Y}{X}$$

- increase slit width → diffraction pattern narrows
- increase slit separation → fringe spacing decreases
- increase wavelength → diffraction pattern and fringe spacing increase
- increase number of slits → every other fringe is suppressed

$N_{slit} - 1$ minima between suppressed maxima

with four slits, two maxima are suppressed



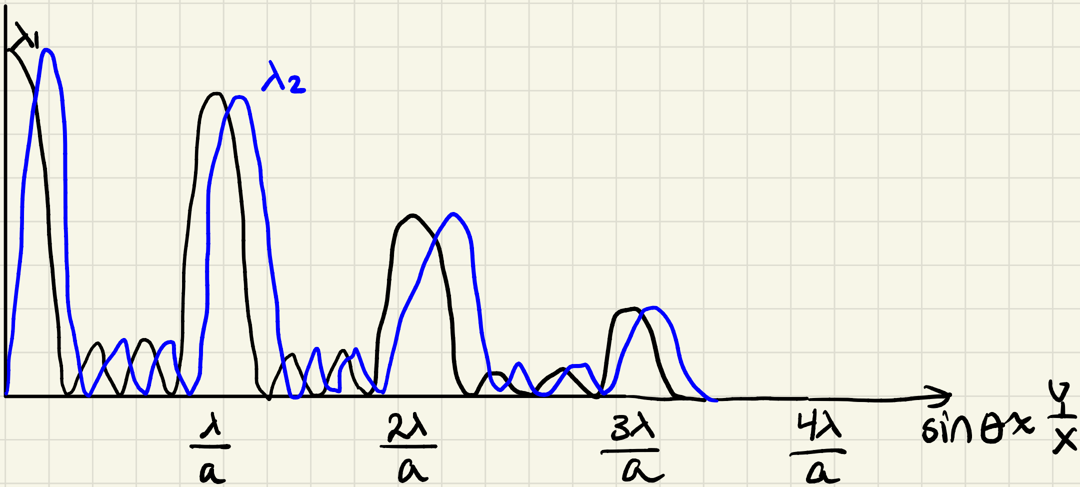
Interference pattern for 4 slits

With multiple slits, you approach the case of a diffraction grating. A diffraction grating is just a bunch of slits (grooves, really) that either transmit or reflect light (in the case of transmission or reflection gratings, respectively).

For a diffraction grating, extrapolate from 4 slits to hundreds or thousands of grooves (i.e. slits) per millimeter.

1. The intervening minima between bright maxima become many.
2. The separation of the principle maxima grows.
3. The width of the principle maxima narrows (i.e. the spectral resolving power increases).

Now imagine you have light of λ_1 and λ_2:



Extrapolate to white light, and each order (or peak) is a spectrum of light. Eventually a blue wavelength from the m+1 order will overlap a red wavelength from order m.

The "free spectral range", $\Delta\lambda_{FSR}$, is the range of wavelengths that are not blocked.

$m\lambda_m = (m+1)\lambda_{m+1}$ → this is the issue

Suppose your detector responds to light up to wavelength λ_{max}. You record light with λ_{max} in order m. The spectrum from order $m+1$ will overlap it and at the same position on the detector, deposits light with $m\lambda_{max}/(m+1)$. For example, 8000Å light in order 1 is contaminated by 4000Å light from order 2. You can use an order blocking filter that blocks all light shorter than $m\lambda_{max}/(m+1)$. The free spectral range, or the wavelengths that are not blocked, are:

$$\Delta\lambda_{FSR} = \lambda_{max} - \frac{m}{m+1}\lambda_{max} = \frac{\lambda_{max}}{m+1}$$

The free spectral range becomes small when hi L orders are used.