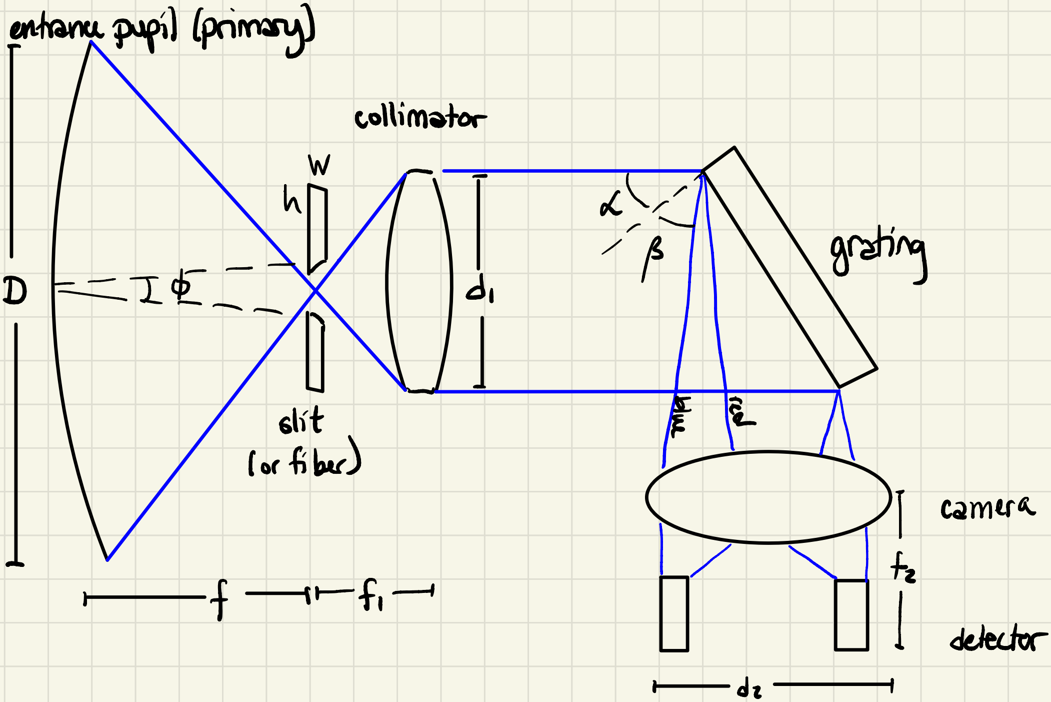
Slit and fiber spectrometers:

- address several issues common in slitless spectrometers:
  e.g. overlapping spectra, lack of convient wavelength calibration
- basic idea of improvement is to limit light that reaches the disperser, cost is that you lose all light that doesn't come through slit/fiber
- as long as slit is smaller than typical seeing, it limits the pority of light entering system



w - physical slit width
h - physical slit height
$\phi = w/f$ - angular width of slit from small angle approx.
$\phi' = h/f$ - angular height of slit from small angle approx.
w' = reimaged slit width = $w \times$ magnification $= w\, f_2/f_1$
h' = reimaged slit height = $h \times$ magnification $= h\, f_2/f_1$

That's without a disperser. Adding a disperser magnifies the image of the slit.
α - angle of collimated rays to grating normal
β - angle of reflected rays to grating normal

Design so: $w' = r w\, f_2/f_1$, where r = anamorphic mag. = $\cos\alpha / \cos\beta \approx 1$

To avoid losing light at the top/bottom of the collimator or making it bigger than needed, it needs the same focal ratio as the primary:

$$\frac{D}{f} = \frac{d_1}{f_1} \Rightarrow f_1 = \frac{d_1 f}{D}$$

Then the slit dimensions can be written:

$$h' = h\,(f_2/f_1) = h\,\frac{D f_2}{d_1 f} = \left(\frac{h}{f}\right)\frac{D f_2}{d_1} = \phi' \,\frac{D f_2}{d_1}$$

$$w' = w\,(f_2/f_1) = w\,\frac{D f_2}{d_1 f} = \left(\frac{w}{f}\right)\frac{D f_2}{d_1} = \phi\,\frac{D f_2}{d_1}$$

where $\phi = w/f$ is the angular size of the slit width on the sky and $\phi' = h/f$ is the angular height.

2.B. You are designing a spectrograph for use with a 5m f/12 telescope with 1" seeing. How wide should the slit width be in μm?

Set the slit width equal to typical seeing so the system is not seeing limited.

$$f = D \cdot \text{fratio} = 5m \cdot 12 = 60m$$

$$\text{platescale} = \frac{206265''}{f} = \frac{206265''}{60 \times 10^6 \mu m} = 0.003 \frac{''}{\mu m}$$

$$w = 1'' \cdot \left(0.003 \, ''/\mu m\right)^{-1} = 333 \mu m$$

Let's say the above system is used with a detector with 24 μm pixels. What camera-collimator focal length ratio is needed to Nyquist sample a monochromatic image of the slit?

$$w' = 2 \text{ pix} = 2 \cdot 24 \mu m = 48 \mu m$$

$$w' = f_2/f_1 \, w \, , \quad w = 333 \mu m$$

$$f_2/f_1 = w'/w = 48 \mu m / 333 \mu m = 0.14$$

What happens if you want high dispersion?

Recall, linear dispersion depends only on camera focal length, f_2.

$$\frac{dl}{d\lambda} = \frac{f_2 \, m}{\sigma \cos\beta}$$

High dispersion means big f_2 means big optics.

Spectral resolution:

Spectral resolution requires matching w' to the pixel size at the detector. For Nyquist sampling, w' should cover two pixels. For two spectral features to be resolved, they must be seperated by at least w'. Thus, the slit size determines spectral resolution.

Since $w' = w(f_2/f_1) = \phi\, Df_2/d_1$, fixed w' and ϕ implies that

$$Df_2/d_1 = \text{constant} = \left(\frac{d_1}{d_2}\right) Df_2/d_1 = D\frac{f_2}{d_2} = D\,(\text{camera } f\#)$$

So, for a larger telescope, D, the ratio f_2/d_2 must get small, implying a short focal length camera (i.e. fast, $f/2$ = fast, $f/8$ = slow)

The limit of spectral resolution can be obtained by considering a slit of width w illuminated by light of wavelength λ to $\lambda+\Delta\lambda$. The slit image at each wavelength w' and the centers of the images are seperated by

$$\Delta l = f_2 \underbrace{A}_{\text{angular dispersion, } d\beta/d\lambda} \Delta\lambda$$

The limit of spectral resolution $\Delta\lambda$ is then the difference for which $\Delta l = w'$.

$$\Delta\lambda = \underbrace{\frac{1}{f_2 A}}_{\text{linear dispersion } d\lambda/dl} \Delta l = \frac{1}{f_2 A} w' = \frac{1}{f_2 A}\cdot\frac{r\phi D f_2}{d_1} = \frac{r\phi D}{A d_1} = \frac{r\phi D\sigma\cos\beta}{m d_1}$$

$\mapsto A \qquad \mapsto A \qquad \mapsto 1/\phi$

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\lambda d_1 A}{r\phi D} = \frac{\lambda d_1}{r\phi D}\cdot\frac{m}{\sigma\cos\beta} = \frac{\lambda d_1}{r\phi D}\cdot\frac{\sin\beta+\sin\alpha}{\lambda\cos\beta} = \frac{\lambda d_1 f}{r w D}\,\frac{m}{\sigma\cos\beta}$$

↳ resolution, higher is better

ϕ – open the slit, resolution gets chunkier
D – bigger telescope gives worse spectral resolution
β – large angle of dispersion, better resolution
m – higher order, better resolution
d_1 – bigger collimator, better resolution

2.B. Suppose you are using an f/12 8m telescope with a 1200 ℓ/mm grating in first order. What combination of things do you need to achieve R = 4000 at 4000Å?

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\lambda d_1 f}{r \omega D} \cdot \frac{m}{\sigma \cos\beta} \qquad \text{known}$$

$$\frac{d_1}{\omega \cos\beta} = \frac{R r D \sigma}{\lambda f m} = \frac{4000 \cdot 1 \cdot \cancel{8m} \cdot \frac{1}{1200\,\ell/mm}}{4000Å \cdot 12 \cdot \cancel{8m} \cdot 1} = \frac{4000 \cdot mm}{4000 \times 10^{-7} mm \cdot 12 \cdot 1200} = 694$$

Choose d_1, ω, β. Minimize d_1, it's expensive to make big. Need $\beta < 90°$.

d_1	β	$\omega = \frac{d_1}{694 \cos\beta}$
2m (huge!)	80°	16.6mm!
1m	45°	2mm
0.3m	1°	432μm

Detailed Example of Spectrograph Design Considerations:

spectrograph design is a highly constrained problem. Each choice you make ties you into other choices.

Let's say you want to design a longslit spectrograph for an 8m telescope at the f/12 Nasmyth focus. Your goal is to do $H\alpha$ rotation curves of nearby galaxies at 25 km/s resolution. Typical seeing is 0.6 arcsec.

At $H\alpha$ $\lambda = 6563\text{Å}$, $25\,\text{km/s}$ resolution requires:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \Delta\lambda = \frac{v\lambda}{c} = \frac{25\,\text{km/s} \cdot 6563\text{Å}}{3\times10^5\,\text{km/s}} = 0.55\text{Å}$$

$$R = \frac{\lambda}{\Delta\lambda} = \frac{6563\text{Å}}{0.55\text{Å}} \approx 12000$$

1. First consider the physical slit size. At the focal plane, typical seeing is 0.6" so we probably want a physical slit width of 0.6". It's physical width will be:

$$\text{platescale} = \frac{206265''}{f} = \frac{206265''}{12\cdot 8\text{m}} = 2.15\,''/\text{mm}$$

$$w = \frac{\phi}{\text{platescale}} = \frac{0.6''}{2.15\,''/\text{mm}} = 0.27\text{mm}$$

The slit length is driven by typical galaxy size. To cover a typical 60kpc galaxy in the Virgo Cluster at 20Mpc:

$$\theta = S/L = \frac{60\text{ kpc}}{20\text{Mpc}} = 0.003\text{rad} = 620'' \approx 10\text{ arcmin}$$

$$h = \frac{\phi'}{\text{platescale}} = \frac{620''}{2.15\,''/\text{mm}} = 288\text{mm}$$

2. To Nyquist sample a star at the detector 2 pix per PSF. Modern detectors have 9-24 μm pixels. Suppose you have 24 μm pixels (best case), you want 0.6 arcsec (0.27mm = 270 μm) to project to 48 μm. So the angular platescale needed at the detector is 0.6"/48μm = 0.0125"/μm. This means you need a camera/collimator focal ratio $f_2/f_1 = w'/w = 48/270 \Rightarrow 5.5/1$. Since the f-ratio of the collimator is f/12 (to match the primary), so let's try:

$d_1 = 100\,mm$, $f_1 = 12 \cdot 100mm = 1200mm$

That requires $f_2 = f_1/5.5 = 218\,mm$.

The diameter of the collimated beam going through the grating is 100mm, so we need $d_2 > 100mm$. The f-ratio of the camera is

$$\text{f-ratio, camera} = \frac{f_2}{d_2} = \frac{218\,mm}{100mm} \Rightarrow f/2.1$$, a very fast camera!

3. What does this do to lock in our linear dispersion?

$$\text{reciprocal dispersion } (\text{Å}/\mu m) = \frac{\delta\lambda}{\delta\ell} = \frac{1}{f_2}\frac{\delta\lambda}{\delta\beta} = \frac{1}{f_2 A} = \frac{\sigma\cos\beta}{f_2 m}$$

If $\sigma = 1200\,l/mm$, $m = 2$, $\alpha = \beta = 50°$ (Littrow configuration), $f_2 = 218mm$

$$\text{reciprocal disp.} = \frac{\frac{1}{1200mm^{-1}}\cos 50°}{218\,mm \cdot 2} = 0.012\,\text{Å}/\mu m$$

With 24μm pixels, this is 0.29 Å/pix. This is slightly too big. We wanted 0.55 Å/2 = 0.275 Å/pix from the spectral resolution. But we can certainly slightly oversample, so let's continue.

4. Let's figure out what spectral resolution we're getting.

$$R = \frac{\lambda d_1 f m}{r w D \sigma \cos\beta}$$

$$= \frac{6563\times10^{-4}\,\mu m \cdot 100\times10^{3}\,\mu m \cdot (12.8\times10^{6}\,\mu m)\cdot 2}{1.270\,\mu m \cdot 8\times10^{6}\,\mu m \cdot \frac{1}{1200\times10^{-3}\,\mu m} \cdot \cos 50°}$$

$R = 10{,}890$

Not bad. What can we do to increase R?

- Could do a higher ℓ/mm grating. This only impacts reciprocal dispersion, which was slightly high anyway.

5. How big of a detector do you need to get the whole slit length?

$h = 620''$ / platescale at detector $\Rightarrow \frac{620''}{0.0125''/\mu m} = 49600\ \mu m$

With $24\,\mu m$ pixels, this is too big to fit on a 2048×2048 detector! Will need a 4096×4096 detector.