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Astro 8060 HW 3

1. Is error of flux = 0.03 = error of mag = 0.03?

$$\sigma_m^2 = \left(\frac{\partial m}{\partial f}\right)^2 \sigma_f^2$$

$$m_{AB} = -2.5\log[f_\nu(Jy)] + 8.9$$

$$\frac{\partial m}{\partial f} = \frac{-2.5}{f \ln 10}$$

$$\sigma_m^2 = \frac{(-2.5)^2}{(f \ln 10)^2}\sigma_f^2$$

$$\frac{\sigma_f}{f} = 0.03$$

$$\sigma_m = \frac{2.5}{f \ln 10}\sigma_f$$

$$\sigma_m = 1.08\frac{\sigma_f}{f} = 1.08 \cdot 0.03 = 0.0325!$$:)

2. 1 MJy/sr, 5500Å, V band, CCD, D = 2.3m, 1 pixel, pix = 1″², $\eta_{tot} = 1$, ph/s·pix?

$$E_{ph} = \frac{hc}{\lambda} = \frac{6.624\times10^{-27}\,erg\cdot s \cdot 3\times10^{18}\,Å/s}{5500\,Å} = 3.61\times10^{-12}\,erg$$

$$\frac{1\,MJy}{sr}\cdot\frac{sr}{4.25\times10^{10}\,arcsec^2}\cdot\frac{10^6\,Jy}{1\,MJy}\cdot\frac{10^{-26}\,W/m^2Hz}{1\,Jy}\cdot\frac{10^7\,erg/s}{1\,W}$$

$$= 2.35\times10^{-24}\,\frac{erg}{s\,m^2\,Hz\,arcsec^2}\cdot\frac{ph}{3.61\times10^{-12}\,erg}\cdot\frac{1\,m^2}{100^2\,cm^2}$$

$$= 6.51\times10^{-17}\,\frac{ph}{s\,cm^2\,Hz\,arcsec^2}$$

$$\frac{\partial\nu}{\partial\lambda} = -\frac{c}{\lambda^2} = \frac{-3\times10^{18}\,Å/s}{5500\,Å^2} = 9.91\times10^{10}\,\frac{Hz}{Å}$$

$$6.51\times10^{-17}\,\frac{ph}{s\,cm^2\,\cancel{Hz}\,arcsec^2}\cdot\frac{9.91\times10^{10}\,\cancel{Hz}}{Å} = 6.45\times10^{-6}\,\frac{ph}{s\,cm^2\,Å\,arcsec^2}$$

$$R_s = \eta_{tot}\cdot\Delta\lambda\cdot A\cdot f_\lambda$$

$$R_s = 1\cdot 880\,Å\cdot(\pi\,230\,cm)^2\cdot 6.45\times10^{-6}\,\frac{ph}{s\,cm^2\,Å\,arcsec^2}\cdot\frac{1''^2}{1\,pixel}$$

$$= \boxed{2967\,\frac{ph}{s\cdot pixel}}$$

3. $d = 10\,Mpc$, monochromatic $L = 1038\ \frac{erg}{s\,Å}$ optical,
2.3m, V filter, optics and CCD $\eta = 0.50$, $X = 2$, $\frac{ph}{s}$?

$F = \frac{L}{4\pi d^2} = \frac{1038\ erg/sÅ}{4\pi(10\,Mpc)^2} = 0.826\ \frac{erg}{s\,Å\,Mpc^2} = f_\lambda$

$\Delta\lambda = 880\,Å$ in V band

$A = \pi r^2 = \pi\left(\frac{2.3m}{2}\right)^2 = 4.15\ m^2$

$E_{ph} = \frac{hc}{\lambda} = \frac{6.624\times10^{-27}\ erg\cdot s\ (3\times10^{18}\,Å/s)}{5600\,Å}$ $\qquad 5600\,Å = \lambda_{eff_V}$

$ph = 3.54\times10^{-12}\ erg$

$f_\lambda = 0.826\ \frac{erg}{s\,Å\,Mpc^2}\cdot\frac{1^2\,Mpc^2}{(10^6)^2\,pc^2}\cdot\frac{1\,pc^2}{(3.08\times10^{16})^2\,m^2}\cdot\frac{ph}{3.54\times10^{-12}\,erg}$

$f_\lambda = 2.45\times10^{-34}\ \frac{ph}{s\,Å\,m^2}$

$\rightarrow A = K_V X = 0.2\ \frac{mag}{airmass}\cdot 2\ airmass = 0.4\ mag$ Δmag

$\frac{F_2}{F_1} = 10^{-0.4A} \Rightarrow F_2 = F_1\cdot10^{-0.4A}$

$F_{atmo} = 2.45\times10^{-34}\ \frac{ph}{s\,Å\,m^2}\cdot10^{-0.4(0.4)} = 1.69\times10^{-34}\ \frac{ph}{s\,Å\,m^2}$

$R_s = \eta\,\Delta\lambda\,A\,f_\lambda = 0.50\cdot 880\,\cancel{Å}\cdot4.15\,m^2\cdot1.69\times10^{-34}\ \frac{ph}{s\,Å\,m^2}$

$= \boxed{3.10\times10^{-31}\ ph/s}$

4 point source, $R_S = 0.2 \frac{ph}{s}$, $R_B = 0.5 \frac{ph}{s\,pix}$, $R_D = 10 \frac{e^-}{hr\,pix}$,
$N_R = 5e^-$, $\frac{S}{N} = 100$, PSF = 4 pix = n_{pix}, # 1min exposures?

$R_D = 10 \frac{e^-}{hr\,pix} \cdot \frac{hr}{3600\,s} = 0.0027 \frac{e^-}{s\,pix}$

$t = \{[(S/N)^2 (R_S + n_{pix} R_B + n_{pix} R_D)] \pm \sqrt{(S/N)^4 (R_S + n_{pix} R_B + n_{pix} R_D)^2 - 4(S/N)^2 n_{pix} N_R^2 R_S^2}\} / 2R_S^2$

$t = \{[(100)^2 (0.2 \frac{ph}{s} + 4pix\, 0.5 \frac{ph}{s\,pix} + 4pix\, 0.0027 \frac{e^-}{s\,pix})] \pm \sqrt{(100)^4 (0.2 \frac{ph}{s} + 4pix\, 0.5 \frac{ph}{s\,pix} + 4pix\, 0.0027 \frac{e^-}{s\,pix})^2 - 4(100)^2\, 4pix\, (5e^-)^2 (0.2 \frac{ph}{s})^2}\} / 2(0.2 \frac{ph}{s})^2$

$t = \{[22111.11] \pm \sqrt{4.88 \times 10^8 - 160000}\} / 0.08$

$t = (22111.11 \pm 22107.49) / 0.08$

$t = 45.22\,s$, $552732.54\,s$

$\div 60$ round up $\rightarrow$ t = 1 - 1min exp., 9213 - 1min exp.

1 - 1min exp. doesn't seem to make sense w/ a R_S this low $\rightarrow$ maybe we don't use $-\sqrt{}$ of equation $\rightarrow$ $\boxed{9213 \text{ 1min exposures}}$

5. WIRO, 2.3m = D, f/2.1, 13.5 μm pix, X = 1 airmass,
QE = 0.90, $\eta = 0.70$, $\frac{S}{N} = 100$, V = 22 mag,
$\mu_V = 20 \frac{mag}{arcsec^2}$, seeing = 1.1", $N_R = 4.5 \frac{e^-}{pix}$, $R_D = 0 \frac{e^-}{s\,pix}$,
$\mu_V = 22 \frac{mag}{arcsec^2}$, detector, source, background limited? t?

$t = \{[(S/N)^2 (R_S + n_{pix} R_B + n_{pix} R_D)] \pm \sqrt{(S/N)^4 (R_S + n_{pix} R_B + n_{pix} R_D)^2 - 4(S/N)^2 n_{pix} N_e^2 R_S^2}\} / 2R_S^2$

need R_S, n_{pix}, R_B to plug in

$R_S = \eta\, \Delta\lambda\, A\, f_\lambda$

$\eta = 0.9 \cdot 0.7 \cdot \eta_{atmo} = 0.9 \cdot 0.7 \cdot 2.5^{-K_V X} = 0.9 \cdot 0.7 \cdot 2.5^{-0.2(1)} = 0.524$

$\Delta\lambda = 880 Å$ in V band $\quad A = \pi r^2 = \pi (\frac{2.3m}{2})^2 = 4.15 m^2$

$f_\nu = f_\nu(Jy)\,10^{0.4(m_{v,0} - V_i)} = 3540\ Jy \cdot 10^{0.4(-22)}$

$= 5.61 \times 10^{-6}\ Jy$

$5.61\times10^{-6}\ \cancel{Jy} \cdot \frac{10^{-26}\ W/m^2 Hz}{1\ \cancel{Jy}} \cdot \frac{10^7\ erg/s}{1\ W} \cdot \frac{1\ m^2}{100^2\ cm^2} = 5.61\times10^{-29}\ \frac{erg}{s\,cm^2 Hz}$

$1\ ph = \frac{hc}{\lambda} = \frac{6.624\times10^{-27}\ erg\cdot s \cdot 3\times10^{18}\ Å/s}{5600\ Å} = 3.54\times10^{-12}\ erg$

$\frac{\partial\nu}{\partial\lambda} = -\frac{c}{\lambda^2} = -\frac{3\times10^{18}\ Å/s}{(5600\ Å)^2} = -9.56\times10^{10}\ \frac{Hz}{Å}$

$5.61\times10^{-29}\ \frac{erg}{s\,cm^2 Hz} \cdot \frac{ph}{3.54\times10^{-12}\ erg} \cdot \frac{9.56\times10^{10}\ Hz}{Å} = 1.51\times10^{-6}\ \frac{ph}{s\,cm^2 Å}$

$4.15\,m^2 \cdot \frac{100^2\ cm^2}{1\,m^2} = 4.15\times10^4\ cm^2$

$R_S = 0.524 \cdot 880\,Å \cdot 4.15\times10^4\ cm^2 \cdot 1.51\times10^{-6}\ \frac{ph}{s\,cm^2 Å}$

$R_S = 28.97\ \frac{ph}{s}$

$R = \frac{f}{D} = 2.1 \Rightarrow f = D(2.1) = 2.3\,m(2.1) = 4.83\,m$

$S = \frac{206265''}{4.83\times10^6\ \mu m} = 0.0427\ \frac{''}{\mu m} \cdot 13.5\ \frac{\mu m}{pix} = 0.576\ \frac{''}{pix}$

$(0.576''/pix)^2 = 0.332\ \square''$ in a pixel (area)

$1.1'' \cdot \frac{pix}{0.576''} = 1.908\ pix$ across source

this is what source looks like on detector $\rightarrow$ basically $n_{pix} = 4$ pixels

$f_\nu = f_\nu(Jy)\,10^{0.4(m_{v,0} - M_V)} = 3540\ Jy \cdot 10^{0.4(-20)}$

$= 3.54\times10^{-5}\ Jy$

$3.54\times10^{-5}\ \cancel{Jy} \cdot \frac{10^{-26}\ W/m^2 Hz}{1\ \cancel{Jy}} \cdot \frac{10^7\ erg/s}{W} \cdot \frac{ph}{3.54\times10^{-12}\ erg} \cdot \frac{9.56\times10^{10}\ Hz}{Å}$

$= 0.0956\ \frac{ph}{s\,m^2 Å}$

$f_{\lambda B_{Full}} = 0.524 \cdot 880\,Å \cdot 4.15\,m^2 \cdot 0.0956\ \frac{ph}{s\,m^2 Å} = 182.94\ \frac{ph}{s}$

$M_V = 182.94\ \frac{ph}{s\ arcsec^2} \cdot \frac{0.332\ arcsec^2}{pix} = 60.737\ \frac{ph}{s\ pix} = R_{B_{Full}}$

M_{VBNew} = 22 mag R_S calculated above adjusted to per pixel

$M_{VBNew} = 28.97\ \frac{ph}{s\ arcsec^2} \cdot \frac{0.332\ arcsec^2}{pix} = 9.61\ \frac{ph}{s\ pix} = R_{BNew}$

$$t_{Full} = \left\{\left[(100)^2\left(28.97\,\tfrac{ph}{s} + 4pix\,60.737\,\tfrac{ph}{s\,pix} + 4pix\,0\,\tfrac{ph}{s\,pix}\right)\right] \pm \sqrt{(100)^4\left(28.97\,\tfrac{ph}{s} + 4pix\,60.737\,\tfrac{ph}{s\,pix} + 4pix\,0\,\tfrac{ph}{s\,pix}\right)^2 - 4(100)^2\,4pix\left(\tfrac{4.5e^-}{pix}\right)^2\left(28.97\,\tfrac{ph}{s}\right)^2}\right\} \Big/ 2\left(28.97\,\tfrac{ph}{s}\right)^2$$

$$t_{Full} = \left(2719206 \pm \sqrt{7.39\times10^{12} - 2.719\times10^{9}}\right)/3357.04$$

$$t_{Full} = 2719206 \pm 2718706 / 3357.04$$

$$t_{Full} = 0.148\,s,\ \boxed{1619.85\,s}$$

$$t_{New} = \left\{\left[(100)^2\left(28.97\,\tfrac{ph}{s} + 4pix\,9.61\,\tfrac{ph}{s\,pix} + 4pix\,0\,\tfrac{ph}{s\,px}\right)\right] \pm \sqrt{(100)^4\left(28.97\,\tfrac{ph}{s} + 4pix\,9.61\,\tfrac{ph}{s\,pix} + 4pix\,0\,\tfrac{ph}{s\,pix}\right)^2 - 4(100)^2\,4pix\left(\tfrac{4.5e}{pix}\right)^2\left(28.97\,\tfrac{ph}{s}\right)^2}\right\} \Big/ 2\left(28.97\,\tfrac{ph}{s}\right)^2$$

$$t_{New} = \left(674100 \pm \sqrt{4.54\times10^{11} - 2.719\times10^{9}}\right)/3357.04$$

$$t_{New} = 674100 \pm 672080 / 3357.04$$

$$t_{New} = 0.601\,s,\ \boxed{401.00\,s}$$

much faster (~ x4) with new Moon!

detector limited $\rightarrow \frac{S}{N} = \frac{R_s t}{\sqrt{n_{pix}(R_B t + N_R^2)}} \Rightarrow t = \frac{(S/N)\sqrt{n_{pix} N_R^2}}{R_s}$

$$t = \frac{100\sqrt{4_{pix}\cdot 4.5\,e^-/pix}}{28.97\,ph/s} = 14.64\,s$$

source limited $\rightarrow \frac{S}{N} = \sqrt{R_s t} \Rightarrow t = \frac{(S/N)^2}{R_s}$

$$t = \frac{100^2}{28.97\,ph/s} = 345.18\,s$$

background limited $\rightarrow \frac{S}{N} = \frac{R_s t}{\sqrt{n_{pix}(R_B t)}} \Rightarrow \left(\frac{S}{N}\right)^2(n_{pix} R_B \cancel{t}) = R_s^2 t^{\cancel{2}}$

$$t = \frac{(S/N)^2(n_{pix} R_B)}{R_s^2}$$

Full $\rightarrow$ $t = \frac{100^2(4pix\cdot 60.737\,ph/s\,pix)}{(28.97\,ph/s)^2} = 2894.78\,s$

New $\rightarrow$ $t = \frac{100^2(4pix\cdot 9.61\,ph/s\,pix)}{(28.97\,ph/s)^2} = 458.02\,s$

t_d and t_s << t_{Full} and t_{New} $\therefore$ not detector or source limited

$t_{bFull} > t_{Full}$ and $t_{bNew} > t_{New}$ $\rightarrow$ $\boxed{\text{background limited}}$

6. Keck, QE = 80%, S/N = 50, t = 10m, $\Delta\lambda$ = 50Å, WIRO, S/N = 50, QE = 95%, broadband V filter, source limited, t?

object limited $S/N = \frac{R_s t}{\sqrt{R_s t}} = \sqrt{R_s t} \Rightarrow R_s = (S/N)^2/t$

$R_s = QE \cdot \Delta\lambda \cdot A \cdot f_\lambda \Rightarrow f_\lambda = R_s / QE\,\Delta\lambda\, A$ = constant

$D_{Keck} = 10m$, $D_{WIRO} = 2.3m$, $\Delta\lambda_V = 880$ Å (all from internet)

Keck $\rightarrow R_s = (S/N)^2/t = 50^2/(10m \cdot \frac{60s}{m}) = 4.16$ Hz

$$\frac{R_{s\,Keck}}{QE_{Keck}\,\Delta\lambda_{Keck}\,A_{Keck}} = \frac{R_{s\,WIRO}}{QE_{WIRO}\,\Delta\lambda_{WIRO}\,A_{WIRO}}$$

$R_{s\,WIRO} = (R_{s\,Keck}\, QE_{WIRO}\,\Delta\lambda_{WIRO}\,A_{WIRO}) / (QE_{Keck}\,\Delta\lambda_{Keck}\,A_{Keck})$

$R_{s\,WIRO} = (4.16\,\frac{1}{s} \cdot 0.95 \cdot 880\text{Å} \cdot \pi(2.3m)^2) / (0.8 \cdot 50\text{Å} \cdot \pi(10m)^2)$

$R_{s\,WIRO} = 4.60$ Hz

WIRO $\rightarrow t = (S/N)^2/R_s = 50^2/4.60\,Hz = \boxed{542.6\,s}$