Homework 3 Solution:

1.

$$m - m_0 = -\frac{5}{2}\log(f/f_0) = -5/2\log f + 5/2\log f_0$$

$$m = -5/2\log f + 5/2\log f_0 + m_0$$

$$\sigma_m = |dm/df|\sigma_f$$

$$= \left|d/df\left[5/2\log f + 5/2\log f_0 - m_0\right]\right|$$

$$\sigma_m = \frac{5}{2\cdot\ln(10)}\cdot\sigma_f/f = 1.08\,\sigma_f/f \approx \sigma_f/f$$

For $\sigma_f/f = 0.03$, $\sigma_m = 0.03$ mag

2.

$R = S\cdot A_{telescope}\cdot A_{pixel}\cdot\Delta\lambda_{bandpass}$, assume $\Delta\lambda = 1500\,\text{Å}$

Source A had a surface brightness of 1 MJy per steradian at 5500 Å. This gave:

$$R_s = 6.46\times10^{-6}\,\text{ph s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\text{arcsec}^{-2}$$

$$R = 6.46\times10^{-6}\,\text{ph s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\text{arcsec}^{-2}\,\pi(1/2\cdot230\,\text{cm})^2\cdot(1\,\text{arcsec})^2\cdot(1500\,\text{Å})$$

$$R = 4\times10^{2}\,\text{ph s}^{-1}$$

3.

$$F = \frac{L}{4\pi d^2} = \frac{1038\ \text{erg s}^{-1}\text{Å}^{-1}}{4\pi\cdot(10\times10^{6}\,\text{pc}\cdot3.08\times10^{18}\,\text{cm pc}^{-1})^2} = 8.71\times10^{-50}\,\text{erg s}^{-1}\text{cm}^{-2}\text{Å}^{-1}$$

$$= 8.71\times10^{-50}\,\text{erg s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\cdot\frac{1}{3.61\times10^{-12}\,\text{erg ph}^{-1}} = 2.4\times10^{-38}\,\text{ph s}^{-1}\text{cm}^{-2}\text{Å}^{-1}$$

The efficiency is: $\eta = \eta_{telescope} \cdot \eta_{sky}$

η_{sky} from extinction at 2 airmass assuming typical V-band extinction of $K = 0.2$ mag/airmass:

$$A = X \cdot K = 2\,\text{airmass} \cdot 0.2\,\text{mag/airmass} = 0.4\,\text{mag}$$

If F_1 and m_1 are the flux and magnitude before passing through the atmosphere, and F_2 and m_2 are the attenuated values, then $F_2/F_1 = \eta_{sky}$ is effectively an extra efficiency.

$$A = m_2 - m_1 = -2.5 \log\left(F_2/F_1\right)$$

$$F_2/F_1 = 10^{-0.4 \cdot A} \approx \frac{1}{2.5^A}$$

$$\eta_{sky} = 2.5^{-0.4} = 0.69$$

$$\eta = \eta_{telescope} \cdot \eta_{sky} = 0.5 \cdot 0.69 = 0.34$$

$$R = F \cdot A_{telescope} \cdot \Delta\lambda_{bandpass} \cdot \eta$$

$$= 2.4\times10^{-38}\,\text{ph}\,s^{-1}\,cm^{-2}\,\text{Å}^{-1} \cdot \pi \cdot \left(1/2 \cdot 230\,cm\right)^2 \cdot (1500\,\text{Å}) \cdot 0.34$$

$R = 5.1\times10^{-31}\,\text{ph}\,s^{-1}$ or $R = 3.39\times10^{-31}\,\text{ph}\,s^{-1}$ for $\Delta\lambda = 1000\,\text{Å}$

Note that there was a typo in this problem, the monochromatic luminosity should have been 10^{38} erg/s. In that case:

$$F_\lambda = 8.39\times10^{-15}\,\text{erg}\,s^{-1}\,cm^{-2}\,\text{Å}^{-1}$$

$$= 2.3\times10^{-3}\,\text{ph}\,s^{-1}\,cm^{-2}\,\text{Å}^{-1}$$

Then $R = 4.8\times10^{4}\,\text{ph}\,s^{-1}$ for $\Delta\lambda = 1500\,\text{Å}$.

or $R = 3.4\times10^{4}\,\text{ph}\,s^{-1}$ for $\Delta\lambda = 1000\,\text{Å}$.

4. Modify the S/N equation for multiple exposures, Ne, because read noise is incurred with every exposure:

$$\frac{S}{N} = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + R_D t + N_e N_R^2)}}$$

$R_s = 0.2 \text{ ph s}^{-1}$
$R_B = 0.5 \text{ ph s}^{-1} \text{pix}^{-1}$
$R_D = 10 e^- \text{hr}^{-1} \text{pix}^{-1} = 0.0028 e^- s^{-1} \text{pix}^{-1}$
$N_R = 5 e^- \text{pix}^{-1}$
$n_{pix} = 4 \text{ pix}$

$$N_e = t(\text{min})/1\text{min} = \frac{t(s)}{60s}$$

$$\frac{S}{N} = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + R_D t + \frac{t}{60} \cdot N_R^2)}}$$

$t = 9.7 \times 10^5 s = 16,167 \text{ min}$ gives $S/N = 100.03$

$\Rightarrow N_e = 16,167$

5. $N_R = 4.5 e^- \text{pix}^{-1}$
$R_D \approx 0 \text{ ph. s}^{-1} \text{pix}^{-1}$
$D = 2.3 m$
f-ratio $= f/2.1$
pixel size $= 13.5 \mu m \text{ pix}^{-1}$
$V_{new moon} = 22 \text{ mag arcsec}^{-2}$
$V_{full moon} = 20 \text{ mag arcsec}^{-2}$
$\Delta\lambda_{bandpass} = 1000 Å$

$\eta_{telescope} = 0.7$
$\eta_{QE} = 0.9$
airmass $= 1$
$m_V = 22$ mag
$\lambda_V = 5500 Å$
seeing $= 1.1$ arcsec

$$\frac{S}{N} = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

$$R_s = \eta \cdot \Delta\lambda_{bandpass} \cdot A_{telescope} \cdot f_{obs} \left(ph\ s^{-1}\ cm^{-2}\ \text{Å}^{-1}\right)$$

$$f_{obs} = \frac{\Delta\nu}{\Delta\lambda} \cdot \frac{f_\nu}{E_{photon}}$$

$$f_\nu = 3540\,Jy \cdot 10^{-23}\ erg\ s^{-1}\ cm^{-2}\ Hz^{-1} \cdot 10^{-0.4 \cdot 22mag}$$

$$= 5.6\times10^{-29}\ erg\ s^{-1}\ cm^{-2}\ Hz^{-1}$$

$$f_{obs} = \frac{3\times10^{18}\,\text{Å}\,s^{-1}}{(5500\text{Å})^2} \cdot \left(\frac{6.62\times10^{-27}\,ergs \cdot 3\times10^{18}\,\text{Å}\,s^{-1}}{5500\text{Å}}\right)^{-1} \cdot 5.6\times10^{-29}\ erg\ s^{-1}\ cm^{-2}\ Hz^{-1}$$

$$f_{obs} = 1.53\times10^{-6}\ ph\ s^{-1}\ cm^{-2}\ \text{Å}^{-1}$$

$$\eta = \eta_{telescope} \cdot \eta_{QE} \cdot \eta_{atmo} = \eta_{telescope} \cdot \eta_{QE} \cdot \frac{1}{2.5^{x \cdot K}}$$

$$\eta = 0.7 \cdot 0.9 \cdot 2.5^{-1 \cdot 0.2mag} = 0.52$$

$$R_s = 0.52 \cdot 1000\text{Å} \cdot \pi\left(\tfrac{1}{2} \cdot 230cm\right)^2 \cdot 1.53\times10^{-6}\ ph\ s^{-1}\ cm^{-2}\ \text{Å}^{-1}$$

$$R_s = 33\ ph\ s^{-1}$$

Repeating the above calculation for full and new moon magnitudes but without extinction losses for sky brightness (i.e. $\eta = \eta_{telescope} \cdot \eta_{QE}$):

$$R_{B,full} = 254\ ph\ s^{-1}\ arcsec^{-2}$$

$$R_{B,new} = 40\ ph\ s^{-1}\ arcsec^{-2}$$

$F = \text{f-ratio} \cdot D = 2.1 \cdot 2.3\,m = 4.83\,m$

$$\text{platescale} = \frac{206265\,\text{arcsec}}{4.83 \times 10^6\,\mu m} = 0.0427\,\frac{\text{arcsec}}{\mu m} \cdot 13.5\,\frac{\mu m}{\text{pix}} = 0.577\,\frac{\text{arcsec}}{\text{pix}}$$

$$\text{FWHM} = \frac{1.1\,\text{arcsec seeing}}{0.577\,\text{arcsec pix}^{-1}} = 1.91\,\text{pix}$$

For a FWHM of ~2pix, most of the light is distributed over 4pixels, i.e. $n_{pix} \approx 4\,\text{pix}$.

Now convert background rates:

$$1\,\text{pixel} = (0.577\,\text{arcsec})^2 = 0.33\,\text{arcsec}^2 \Rightarrow 0.33\,\frac{\text{arcsec}^2}{\text{pix}}$$

$$R_{B,full} = 254\,\text{ph}\,s^{-1}\,\text{arcsec}^{-2} \cdot 0.33\,\frac{\text{arcsec}^2}{\text{pix}} = 84\,\text{ph}\,s^{-1}\,\text{pix}^{-1}$$

$$R_{B,new} = 13.2\,\text{ph}\,s^{-1}\,\text{pix}^{-1}$$

Full moon: $t = 3390\,s = 56.5\,\text{min}$ gives $S/N = 100.02$

New moon: $t = 789\,s = 13.15\,\text{min}$ gives $S/N = 100.01$

Remove sources of noise from the calculation to see what is limiting.

Assume source noise limited:

$$S/N = \sqrt{R_s t} \Rightarrow t = \frac{(S/N)^2}{R_s} = \frac{100^2}{33\,\text{ph}\,s^{-1}} = 300\,s$$

This is much shorter than both of the exposure times from the full S/N equation. Since the detector noise is obviously small, both cases are background limited.

6.

$t_{keck} = 10\,min = 600s$
$QE_{keck} = 0.8$
$S/N_{keck} = 50$
$\Delta\lambda_{keck} = 50Å$
$D_{keck} = 10m$

$QE_{WIRO} = 0.95$
$S/N_{WIRO} = 50$
$\Delta\lambda_{WIRO} = 1000Å$
$D_{WIRO} = 2.3m$

Assume the same seeing at each site

a. Source noise limited:

$S/N = \sqrt{R_s t}$, since $R_s \sim Q\,\Delta\lambda \cdot D^2$, $S/N \sim \sqrt{Q \cdot \Delta\lambda \cdot D^2 \cdot t}$

$$\frac{(S/N)_{keck}}{(S/N)_{WIRO}} = \frac{\sqrt{Q_{keck}}}{\sqrt{Q_{WIRO}}} \cdot \frac{\sqrt{\Delta\lambda_{keck}}}{\sqrt{\Delta\lambda_{WIRO}}} \cdot \frac{D_{keck}}{D_{WIRO}} \frac{\sqrt{t_{keck}}}{\sqrt{t_{WIRO}}} = 1$$

$$t_{WIRO} = \frac{Q_{keck} \cdot \Delta\lambda_{keck} \cdot D_{keck}^2}{Q_{WIRO}\,\Delta\lambda_{WIRO}\,D^2_{WIRO}}\, t_{keck}$$

$$= \frac{0.8 \cdot 50Å \cdot (10m)^2}{0.95 \cdot 1000Å\,(2.3m)^2} \cdot 600s$$

$t_{WIRO} = 478\,s$