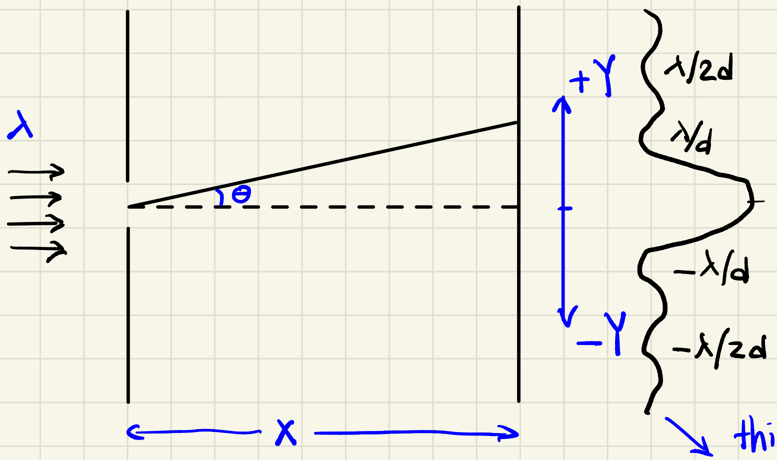
Spectrographs

Chromey Ch. 17

Diffraction:

The diffraction grating is at the heart of most optical spectrographs. Thing of a spectrograph as an extension of a two-slit interferometer.

Diffraction of a single slit:



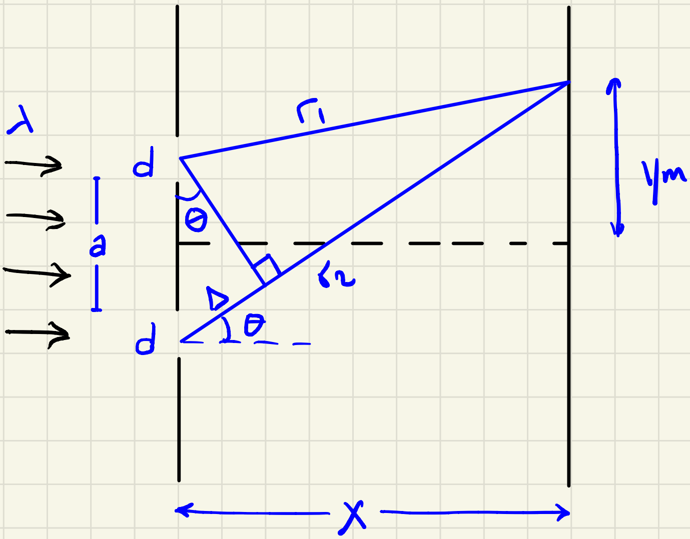
- single slit produces a series of fringes if slit width is similar to wavelength of light
- get dark fringes where light waves interfere:

$$\boxed{\sin\theta = m\frac{\lambda}{d}}$$

this is a sinc function, the Fourier Transform of a square aperture

- wide slits lead to narrow peaks and narrow slits lead to broad peaks
- fundamentally, this is why interferometers w/ long baselines obtain fine angular resolution!

Interference from multiple apertures:



- for light passing through a double slit and falling on a screen at distance x, constructive interference occurs when the path difference is an integer number of wavelengths:

$$\Delta = r_1 - r_2 = m\lambda$$

By geometry:

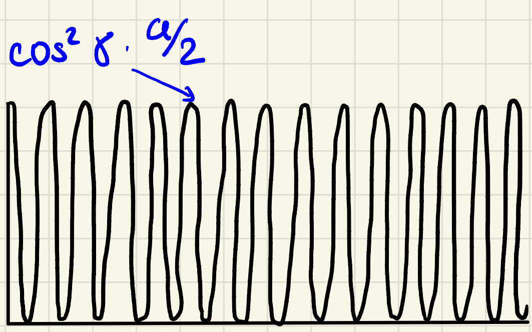
$$a\sin\theta \approx a\theta = \Delta$$

$$a\sin\theta \approx a\theta = m\lambda \quad \Longrightarrow \quad \theta = \frac{y_m}{x} = \frac{\Delta}{a} = \frac{m\lambda}{a}$$

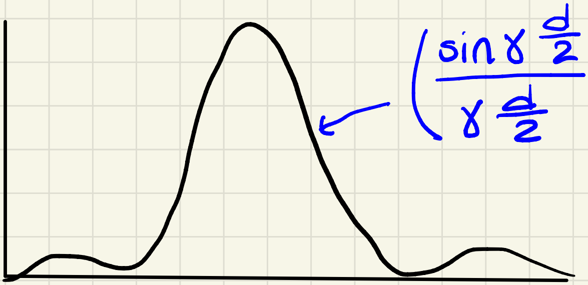
Also: $\tan\theta \approx \theta = \dfrac{y_m}{x}$

$$y_m = \frac{x}{a} m\lambda$$ this is where the mth bright fringe occurs

What does the interference pattern look like?



double-slit interference pattern for two narrow slits



$$\left(\frac{\sin \gamma \frac{d}{2}}{\gamma \frac{d}{2}}\right)^2$$

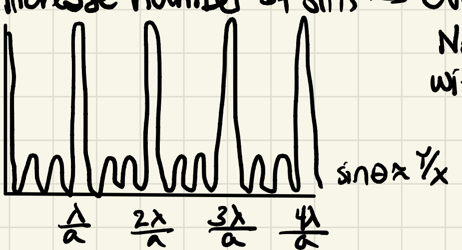
single-slit diffraction pattern

actual two-slit pattern product of single-slit diffraction and double-slit interference

$$\text{Intensity} = \underbrace{\frac{\sin^2 \gamma \frac{d}{2}}{[\gamma\, d/2]^2}}_{\text{diffraction term}} \cdot \underbrace{\left(4C^2 \cos^2 \gamma \frac{a}{2}\right)}_{\text{interference term}}, \quad \gamma = \frac{2\pi}{\lambda} \cdot \frac{Y}{X}$$

- increase slit width $\rightarrow$ diffraction pattern narrows
- increase slit separation $\rightarrow$ fringe spacing decreases
- increase wavelength $\rightarrow$ diffraction pattern and fringe spacing increase
- increase number of slits $\rightarrow$ every other fringe is suppressed

$N_{slit} - 1$ minima between suppressed maxima
with four slits, two maxima are suppressed



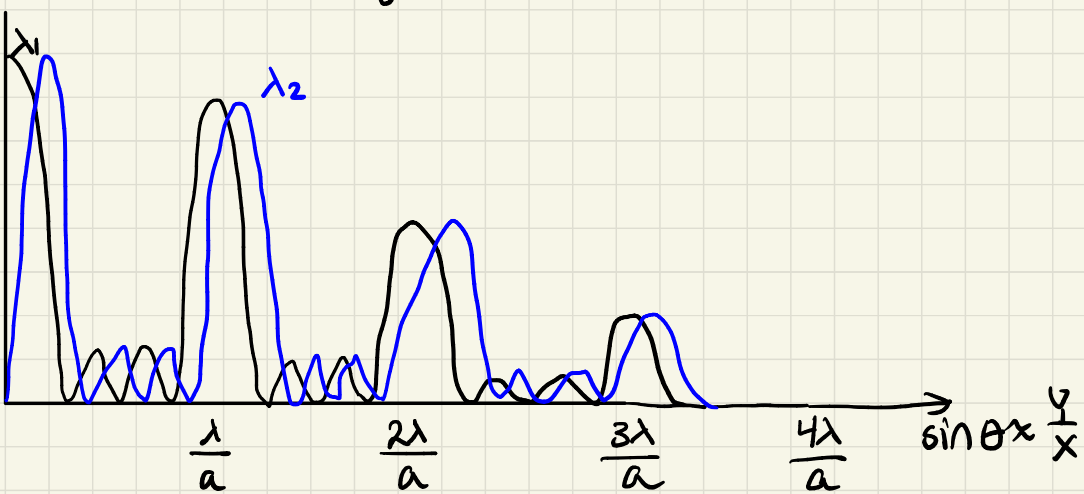
Interference pattern for 4 slits

With multiple slits, you approach the case of a diffraction grating. A diffraction grating is just a bunch of slits (grooves, really) that either transmit or reflect light (in the case of transmission or reflection gratings, respectively).

For a diffraction grating, extrapolate from 4 slits to hundreds or thousands of grooves (i.e. slits) per millimeter.

1. The intervening minima between bright maxima become many.
2. The separation of the principle maxima grows.
3. The width of the principle maxima narrows (i.e. the spectral resolving power increases).

Now imagine you have light of λ_1 and λ_2:



Extrapolate to white light, and each order (or peak) is a spectrum of light. Eventually a blue wavelength from the m+1 order will overlap a red wavelength from order m.

The "free spectral range", $\Delta\lambda_{FSR}$, is the range of wavelengths that are not blocked.

$m\lambda_m = (m+1)\lambda_{m+1}$ → this is the issue

Suppose your detector responds to light up to wavelength λ_{max}. You record light with λ_{max} in order m. The spectrum from order $m+1$ will overlap it and at the same position on the detector, deposits light with $m\lambda_{max}/(m+1)$. For example, 8000Å light in order 1 is contaminated by 4000Å light from order 2. You can use an order blocking filter that blocks all light shorter than $m\lambda_{max}/(m+1)$. The free spectral range, or the wavelengths that are not blocked, are:

$$\Delta\lambda_{FSR} = \lambda_{max} - \frac{m}{m+1}\lambda_{max} = \frac{\lambda_{max}}{m+1}$$

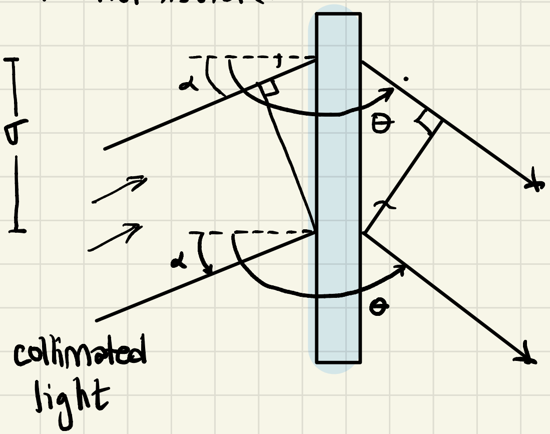
The free spectral range becomes small when high orders are used.

Angular Dispersion and the Grating Equation:

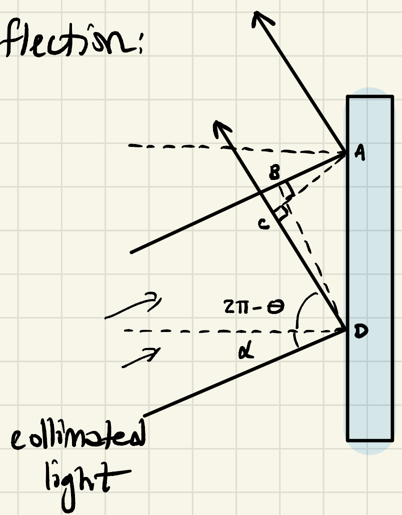
For the transmission grating below, the apertures or "slits" are transmissive grooves. For a reflection grating, they are reflective scratches.

Wave interference means that light of wavelength λ will only diffract without suppression at certain angles.

Transmission:



Reflection:



Light rays hit the grating at angle λ and leave at angle θ. After leaving the grating, their paths differ by

$$\Delta\tau = \overline{AB} - \overline{CD}$$

$$\overline{AB} = \sigma \sin\alpha$$
$$\overline{CD} = \sigma \sin(2\pi - \theta) = -\sigma \sin\theta$$

There will be constructive interference if this path difference is an integer number of wavelengths: $\Delta\tau = m\lambda$

$$m\lambda = \sigma \sin\alpha - (-\sigma \sin\theta)$$

$$\frac{m\lambda}{\sigma} = \sin\alpha \pm \sin\theta$$

Grating Equation
(reflection and transmission)
$+$ $-$

m - the order
σ - groove spacing of the grating
α - incident angle
θ - transmitted/reflected angle

The angular dispersion describes the change in transmitted angle per change in wavelength. Obtain by differentiating the grating equation:

$$\lambda = \sigma/m\,(\sin\alpha + \sin\theta)$$

$$\frac{d\lambda}{d\theta} = \frac{d}{d\theta}\left[\frac{\sigma}{m}\left(\sin\alpha + \sin\theta\right)\right]$$
$$= \frac{\sigma}{m}\cos\theta$$

$$\frac{d\theta}{d\lambda} = \frac{m}{\sigma\cos\theta} = \frac{\sin\alpha + \sin\theta}{\lambda\cos\theta}$$

don't confuse this with spectral resolution!

- Since $\cos\theta$ changes slowly, the change in angle for a grating is relatively constant with wavelength.
- This is also where you run into trouble with overlapping orders.
- Reflection gratings: α and θ have the same sign if they are on the same side of the grating normal. $m=0$ occurs when $\alpha=-\theta$. In this case, there is no dispersion, and the efficiency is maximum.
- Transmission gratings: α and θ have the same sign when they are on opposite sides of the grating normal. $m=0$ occurs when $\alpha=\theta$.

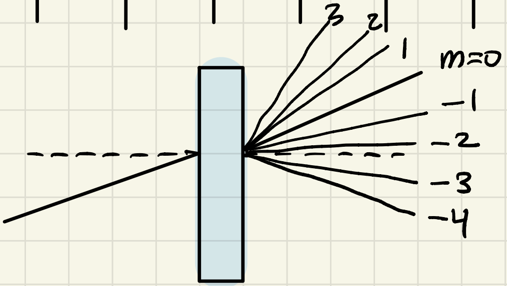
z.B. For a 600 ℓ/mm grating used in 2nd order, the grating angle of incidence is 20°. What is the angle of reflection for Hβ?

$$\theta = \sin^{-1}\left\{\frac{m\lambda}{\sigma} - \sin\alpha\right\} = \sin^{-1}\left\{\frac{2\cdot 4863\times10^{-7}\,mm}{600^{-1}\,mm} - \sin 20°\right\}$$

$\theta = 14°$

z.B. For $\alpha = 25°$ and a 300 ℓ/mm transmission grating, find the angular positions for Hβ, including symmetric negative orders.

m	-4	-3	-2	-1	0	1	2	3	4
θ	-9	-1	8	16	25	35	45	59	NaN



Other kinds of optical dispersion elements:

- prisms – often used, but usually not as the primary disperser
  they are heavy, dispersion very nonlinear with λ

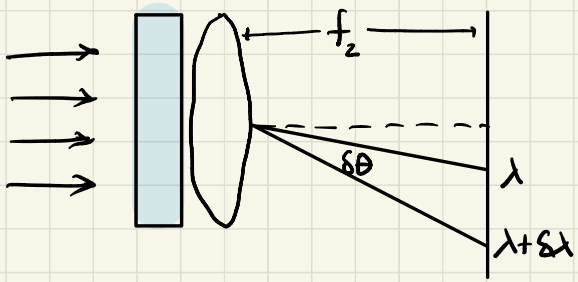
- blazed gratings – normal gratings are inefficient because most light goes to $m=0$ where there is no dispersion. Also, the grating blocks much of the light. Blazed gratings adust phase to avoid these issues, e.g. by giving the groove edge an angle.

- echelles – to get high angular dispersion with a normal grating, you need a high order, which puts the outgoing rays nearly perpendicular to the grating. Echelle gratings instead have very coarse σ, typically 10–100 l/mm and operate in high order 25-150. This gives them a very small free spectral range. To avoid this, seperate orders with another optical element onto different parts of the chip.

- grisms – a prism placed in front of the telescope must be the same size as the telescope aperture – expensive! If you place it in the focal plane, it can be smaller, but is subjected to any optical aberrations. A grating on the side of a prism, or grism, alleviates this issue.

Linear dispersion and cameras:

If you place a camera after a grating, then the linear dispersion (i.e. number of pixels per Ångstrom or mm per Ångstrom) depends on the focal length of the camera.



$$\text{linear dispersion} = \frac{\delta l}{\delta\lambda} = f_2\frac{\delta\theta}{\delta\lambda} = f_2\frac{m}{\sigma\cos\theta}$$

Recall how with telescopes, the platescale was 1/focal length? The same principle is at work here. The resulting dispersion is technically dimensionless, but can be expressed in whatever units you want, e.g. mm/Å or pix/Å.

The reciprocal dispersion, commonly Å/pix or Å/μm, is:

$$\text{reciprocal dispersion} = \frac{\delta\lambda}{\delta l} = \frac{\delta\lambda}{f_2\,\delta\theta}$$

z.B. For WIROspec 2400 l/mm grating and 120 mm camera, the linear dispersion for first order and $\theta = 32°$ is:

$$\frac{\delta l}{\delta\lambda} = f_2\frac{m}{\sigma\cos\theta} = 120\,\text{mm}\cdot\frac{1}{2400^{-1}\,\text{mm}\cdot\cos 32°} = 339603\cdot 10^{-4}\,\mu\text{m}/\text{Å}$$

$$\delta l/\delta\lambda = 33.9\ \mu\text{m}/\text{Å}$$

z.B. What is the linear dispersion in $\mu m\,Å^{-1}$ and pix $Å^{-1}$, for a 600 l/mm grating in second order used with a 150 mm camera and a CCD having 13.5 μm pixels if $\theta = 20°$?

$$\frac{\delta l}{\delta \lambda} = f_2 \frac{m}{\sigma \cos\theta} = 150\,mm \frac{2}{600^{-1}\,mm \cdot \cos 20°} = 95776 \cdot 10^{-4}\,\mu m/Å$$

$$\delta l/\delta \lambda = 19.1\ \mu m\,Å^{-1} \cdot 1\,pix/13.5\,\mu m = 1.4\ pix\,Å^{-1}$$