

Redshift

Topics: Doppler shift, cosmological redshift

Sources: Ch. 10.8, 11.9

$$1+z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}$$

Doppler Shift

Change in wavelength because source has a velocity through space.

$$1+z = \sqrt{\frac{1+\beta}{1-\beta}}, \quad \beta = \frac{v_r}{c}$$

Relativistic Doppler Shift

$$z \approx \frac{v_r}{c}$$

Non-relativistic limit

Cosmological Redshift

Change in wavelength because space is expanding as light travels through it.

$$1+z = \frac{a(t_0)}{a(t)} = \frac{1}{a(t)}$$

These effects stack:

$$1+z_{\text{cosmo}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}, \quad 1+z_{\text{Doppler}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}$$

$$\Rightarrow 1+z = (1+z_{\text{cosmo}})(1+z_{\text{Doppler}})$$

Flux and magnitude

Topics: Flux, magnitude, units

Source: Chromey Ch. 1, 10

Fluxes:

Energy flux, or just flux, is a basic physical description of the apparent brightness of a source.

This is expressed by a few different quantities (that are often confused):

I_V, I : the specific intensity / intensity in $\text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ and $\text{W m}^{-2} \text{sr}^{-1}$. aka surface brightness.

F_V, F : the flux density / flux in $\text{W m}^{-2} \text{Hz}^{-1}$ and W m^{-2} .

L_V, L : the luminosity density / luminosity in W Hz^{-1} and W .
Also called the energy flux as above.

In fact, these specific quantities can be expressed as per frequency or wavelength, i.e. f_V or f_λ .

To convert between f_V and f_λ you need to realize that 1 Angstrom worth of spectrum contains a different number of Hz depending on what part of the EM spectrum you're in:

$$c = v\lambda \Rightarrow v = \frac{c}{\lambda} \Rightarrow \frac{\delta V}{\delta \lambda} = -\frac{c}{\lambda^2}$$

e.g. A source has a flux density of $3 \times 10^{-14} \text{ erg s}^{-1} \text{cm}^{-2} \text{\AA}^{-1}$ at 5000 Å. How many $\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$ is this?

$$\frac{\delta V}{\delta \lambda} = -\frac{c}{\lambda^2} = \frac{3 \times 10^{18} \text{\AA/s}}{(5000 \text{\AA})^2} = -1.2 \times 10^{-16} \text{Hz \AA}^{-1}$$

So then:

$$f_V = f_\lambda \cdot \frac{\delta\lambda}{\delta V} = \frac{3 \times 10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}}{1.2 \times 10^6 \text{ Hz \AA}^{-1}} = 2.5 \times 10^{-25} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$$

How many $\text{W m}^{-2} \text{ Hz}^{-1}$ is this?

$$1 \text{ W} = 10^7 \text{ erg s}^{-1}$$

$$1 \text{ m}^2 = (100 \text{ cm})^2 = 10^4 \text{ cm}^2$$

$$f_V = 2.5 \times 10^{-25} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \cdot \frac{1 \text{ W}}{10^7 \text{ erg s}^{-1}} \cdot \frac{10^4 \text{ cm}^2}{1 \text{ m}^2} = 2.5 \times 10^{-28} \text{ W m}^{-2} \text{ Hz}^{-1}$$

How many Jy is this?

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$$

$$f_V = 2.5 \times 10^{-28} \text{ W m}^{-2} \text{ Hz}^{-1} \cdot \frac{1 \text{ Jy}}{10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}} = 2.5 \times 10^{-2} \text{ Jy} = 25 \text{ mJy}$$

How many $\text{ph s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}$ is this?

$$h = 6.624 \times 10^{-27} \text{ erg s}$$

$$1 \text{ ph} = E = h\nu = h c / \lambda = 6.624 \times 10^{-27} \text{ erg s} \cdot \frac{3 \times 10^{18} \text{ \AA/s}}{5000 \text{ \AA}} = 3.97 \times 10^{-12} \text{ erg}$$

$$f_\lambda = \frac{3 \times 10^{-14} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}}{3.97 \times 10^{-12} \text{ erg/ph}} = 7.5 \times 10^{-3} \text{ ph s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}$$

Solid angle (Ω)

Solid angle is a three dimensional analog of angle, it measures the part of your field of view that an object covers.

$$\Omega = \frac{A}{r^2} \text{ sr}$$

1 steradian is the unit of solid angle that a unit area subtends on the surface of a unit sphere.

There are 4π steradians.

Square degrees are another unit of solid angle.

$$\text{There are } 4\pi \text{ sr} \cdot \left(\frac{180}{\pi}\right)^2 \frac{\text{deg}^2}{\text{sr}} = 41,252.96 \text{ deg}^2$$

2.B. You observe an extended nebula with surface brightness (or specific intensity) of 1 MJy/sr. What is the flux you would measure through a circular aperture with a radius of 10 arcsec?

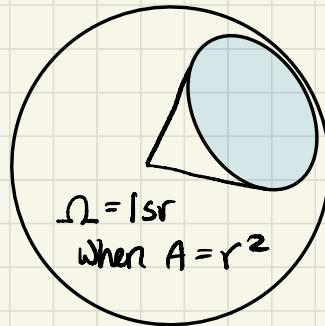
Aperture area is:

$$A = \pi r^2 = \pi \cdot (10 \text{ arcsec})^2 = 314 \text{ arcsec}^2$$

$$f_v = I_v \cdot A = 1 \frac{\text{MJy}}{\text{sr}} \cdot 314 \text{ arcsec}^2 \cdot \frac{1 \text{ sr}}{(60''/\cdot 60'/\cdot 360^\circ / 2\pi \text{ rad})^2}$$
$$f_v = 1 \frac{\text{MJy}}{\text{sr}} \cdot 314 \text{ arcsec}^2 \cdot \frac{1 \text{ sr}}{4 \cdot 2.5 \times 10^{10} \text{ arcsec}^2} = 7.38 \text{ mJy}$$

But! If the surface brightness is measured in mJy arcsec^{-2} you can't just multiply by an area. Similarly, surface brightness is not additive when expressed as magnitudes per area.

$$M = m + 2.5 \log \Theta$$



Magnitudes

A (backwards) way to describe brightness. It is actually a way to compare the brightness of two sources.

$$M_2 - M_1 = -2.5 \log \left(\frac{F_2}{F_1} \right)$$

$$\frac{F_2}{F_1} = 10^{0.4(M_2 - M_1)}$$

A common reference system is based on Vega, which has a magnitude of 0 (by design) in the Johnson-Cousins filters (i.e. UBVRI). Vega is bright, so it is now more common to refer to a group of Landolt standard stars. But in Vega magnitudes:

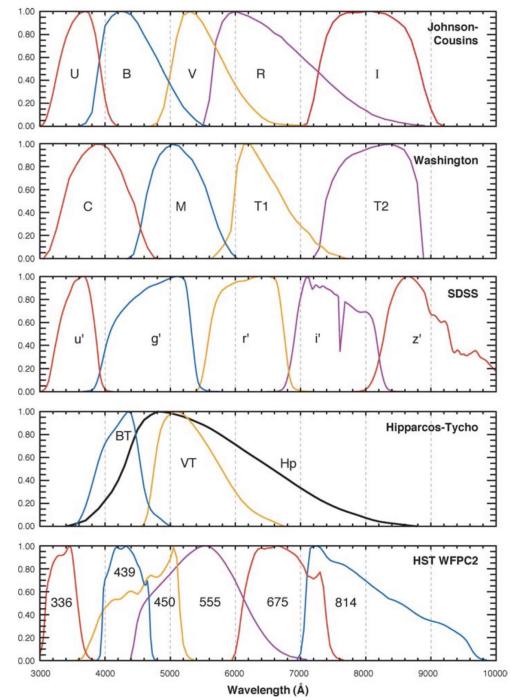
$$f_V(J_y) = F_{\text{Vega},0} 10^{-0.4M}$$

Band	$\lambda_{\text{eff}} (\mu\text{m})$	$f_V(J_y)$
U	0.36	1880
B	0.44	4440
V	0.56	3540
R	0.70	2870
I	0.90	2250

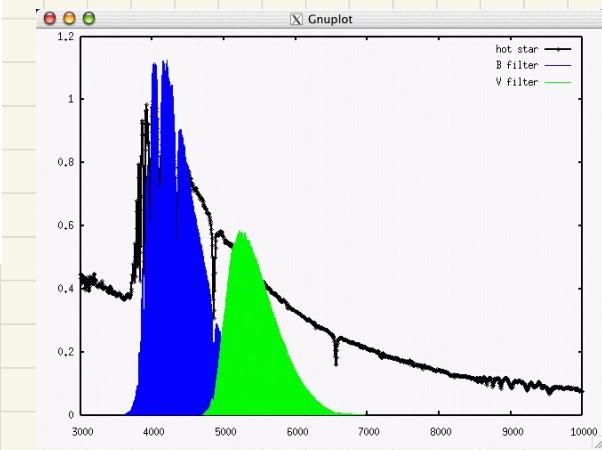
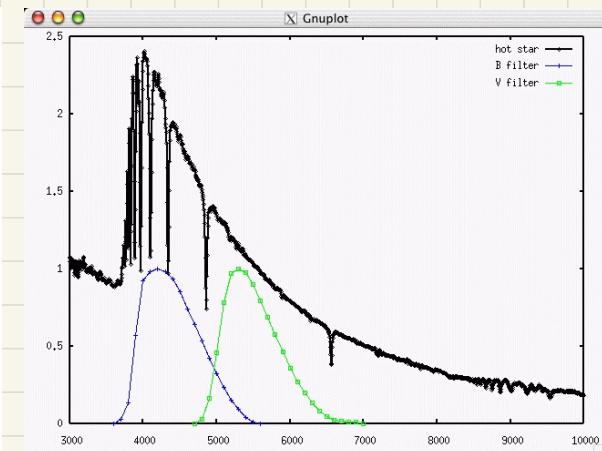
Another system attempts to address the fact that Vega's magnitude is not actually 0 in all these filters:

$$M_{AB} = -2.5 \log [f_V(J_y)] + 8.9$$

$$f_V(J_y) = 3631 J_y 10^{-0.4M_{AB}}$$



Bessell (2005)



spiff.rit.edu/classes/phys446/lectures/color/color.html

$$\lambda_{x,+}$$

$$F_x = \int_{\lambda_{x,-}}^{\lambda_{x,+}} R_x(\lambda) F_\lambda d\lambda \quad \text{flux in a band}$$

- can convert fluxes in band to a magnitude
- A "color" is the difference between magnitudes in two bands.

Luminosity

$$F = \frac{L}{4\pi d^2}$$

The absolute magnitude is the magnitude a source would have at 10pc.

$$m - M = 5 \log \left(\frac{d}{10 \text{ pc}} \right)$$

$$\hookrightarrow m - M = 5 \log d - 5$$

distance modulus

↓
dist. in pc

$$(m - M)_\lambda = (m - M)_0 + A_\lambda$$

↓
extinction
in magnitudes