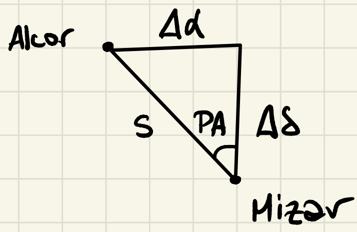
Homework 1 Solution:

1.

	α	δ	
Alcor	13:25:13.5	+54:59:16.6	(from Simbad)
Mizar	13:23:55.4	+54:55:31.3	



for small separations:

$$s = \sqrt{\Delta\alpha^2 + \Delta\delta^2}$$

$$\Delta\alpha = (\alpha_{Alcor} - \alpha_{Mizar}) \cdot \cos\delta_{avg} = 1m\,18.1s \cdot \cos(54.95^\circ)$$
$$\Delta\alpha = 78.1s \cdot 15'' \cdot \cos(54.95^\circ) = 673''$$

$$\Delta\delta = (\delta_{Alcor} - \delta_{Mizar}) = 216''$$

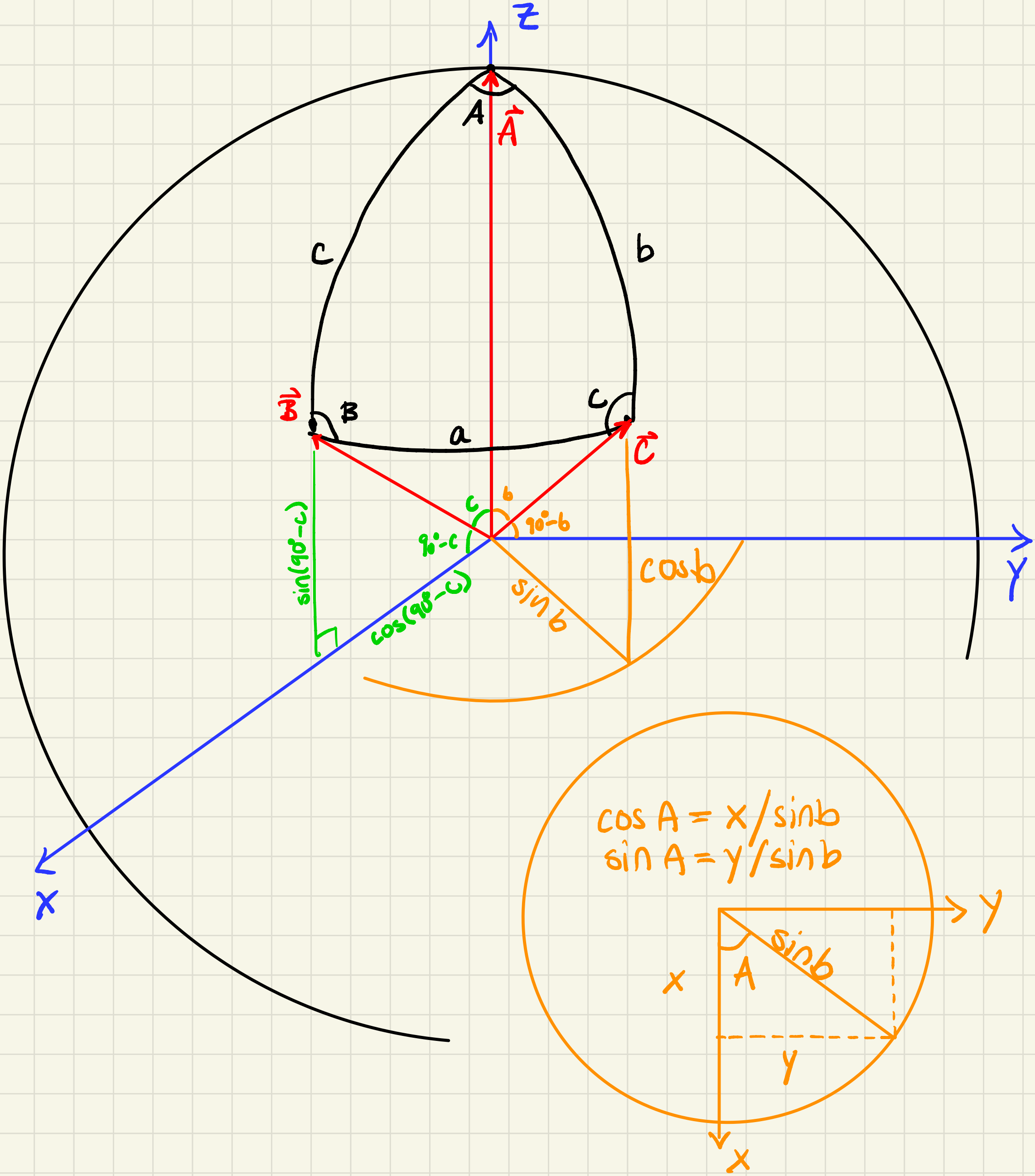
$$s = \sqrt{(673'')^2 + (216'')^2} = 707''$$

$$PA = \tan^{-1}(\Delta\alpha/\Delta\delta) = \tan^{-1}(673''/216'') = 72.2^\circ$$

2. Generally, the Law of Cosines relates the angles and sides of a triangle on the surface of a sphere.

Start by drawing a triangle on the surface of a unit sphere. Since the angles and sides are not impacted by where you put it, orient it so one corner is at the pole and another is in the xz plane.

Label the angles at each vertex A, B, C and the sides across from them a, b, c. Also draw vectors from the origin to each vertex and label them $\vec{A}$, $\vec{B}$, and $\vec{C}$.



z
y
x
A
$\vec{A}$
$\vec{B}$
B
$\vec{C}$
C
c
b
a
c
b
90°-b
90°-c
sin(90°-c)
cos(90°-c)
sin b
cos b
cos A = x/sin b
sin A = y/sin b
y
x
A
sin b
x
y

Recall the general form of the dot product of two vectors:

$$\vec{x} \cdot \vec{y} = |x||y| \cos\theta,$$

where θ is the angle between them.

Since $|\vec{B}| = |\vec{C}| = 1$

$$\cos a = \vec{B} \cdot \vec{C}$$

Write $\vec{B}$ and $\vec{C}$ in cartesian coordinates to compute the dot product.

$\vec{B}$ in cartesian coordinates:

$$(x, y, z) = (\sin c, 0, \cos c)$$

$\vec{C}$ in cartesian coordinates:

From the vertex at the pole, rotate down by c, then rotate around by A. This point has height $\cos b$ and is on a circle with radius $\sin b$.

$$(x, y, z) = (\cos A \sin b, \sin A \sin b, \cos b)$$

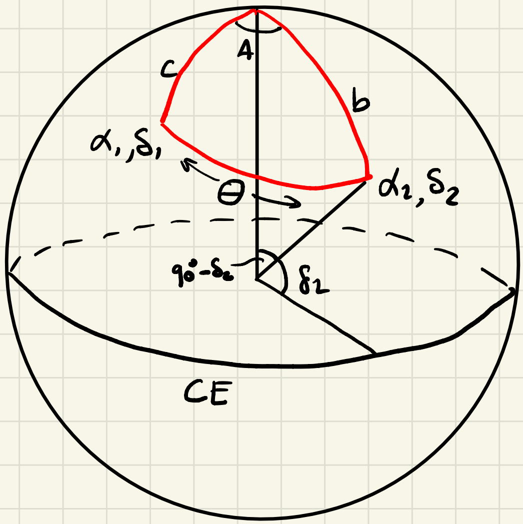
Then,

$$\cos a = \vec{B} \cdot \vec{C}$$

$$= (\sin c \,\hat{i} + 0\hat{j} + \cos c \,\hat{k}) \cdot (\cos A \sin b \,\hat{i} + \sin A \sin b \,\hat{j} + \sin b \,\hat{k})$$

$$\cos a = \sin b \sin c \cos A + \cos b \cos c$$

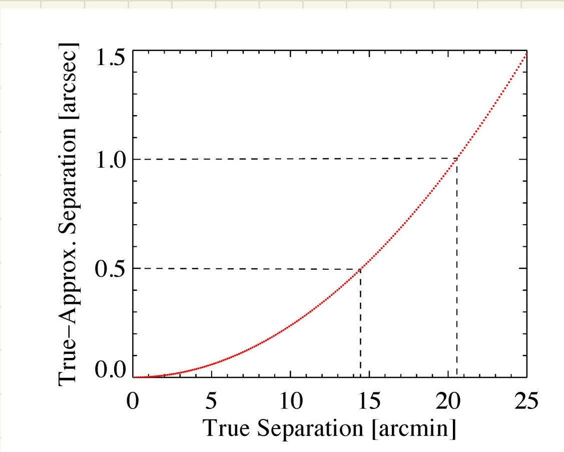
In terms of RA and declination:



$$\cos\theta = \cos(90° - \delta_2)\cos(90° - \delta_1) + \sin(90° - \delta_1)\sin(90° - \delta_2)\overbrace{\cos(\alpha_2 - \alpha_1)}^{\cos A}$$

$$\cos\theta = \sin\delta_2 \sin\delta_1 + \cos\delta_1 \cos\delta_2 \cos\Delta\alpha$$

Pythagorean approximation is good for arcsecond scale separations. By separations of 15' it is off by 0.5" and by 20' it is off by 1".



3.

Location	Constellation	l	b	RA	dec
Galactic center	Sagittarius	0	0	17:45	−28°
Galactic anti-center	Auriga	180	0	05:45	+28°
Galactic north pole	Coma	—	90	12:51	+27°
Galactic south pole	Sculptor	—	−90	00:51	−27°
Location(s) where ecliptic crosses GP	Ophiuchus/	6	0	17:51	−23°
	Taurus	186	0	05:56	+23°

4. Sep 1 is about 5 month after the vernal equinox so $RA_\odot = 0h + 5 \times 24h/12m = 10h$. Dec $\odot = 0°$ when $RA_\odot = 12h$ so assume $Dec_\odot \sim 0°$. Assume 12h night 6pm-6am.

At midnight, 22h RA is on the meridian → LST = 22h. Over the course of the night, objects move E→W.

HA is negative before crossing the meridian, so it goes from −6h to 6h over the course of the night.

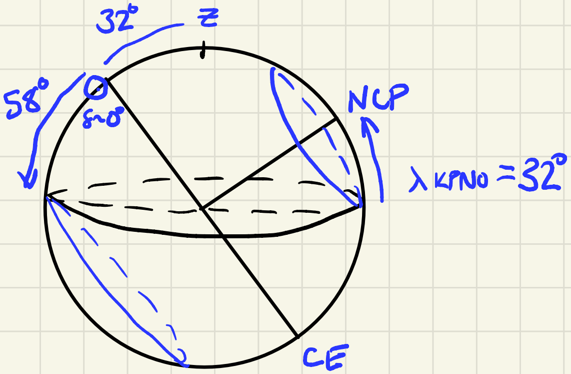
$$LST = RA_{obj} + HA_{obj} \quad \text{and} \quad LST = RA_\odot + HA_\odot$$

$$\text{local time (LT)} = LST + 12h$$

$$\Rightarrow LT = RA_{obj} + HA_{obj} - RA_\odot + 12h$$

Name	RA	dec	transit	rise	set	Z_{min}	X_{min}
UM269	00:43	00:51	02:43	20:43	08:43	31°	1.16
PDS898	15:37	34:32	17:37	11:37	23:37	2.5°	1.00
PKS2203-015	22:06	−21:19	00:06	18:06	06:06	53°	1.66

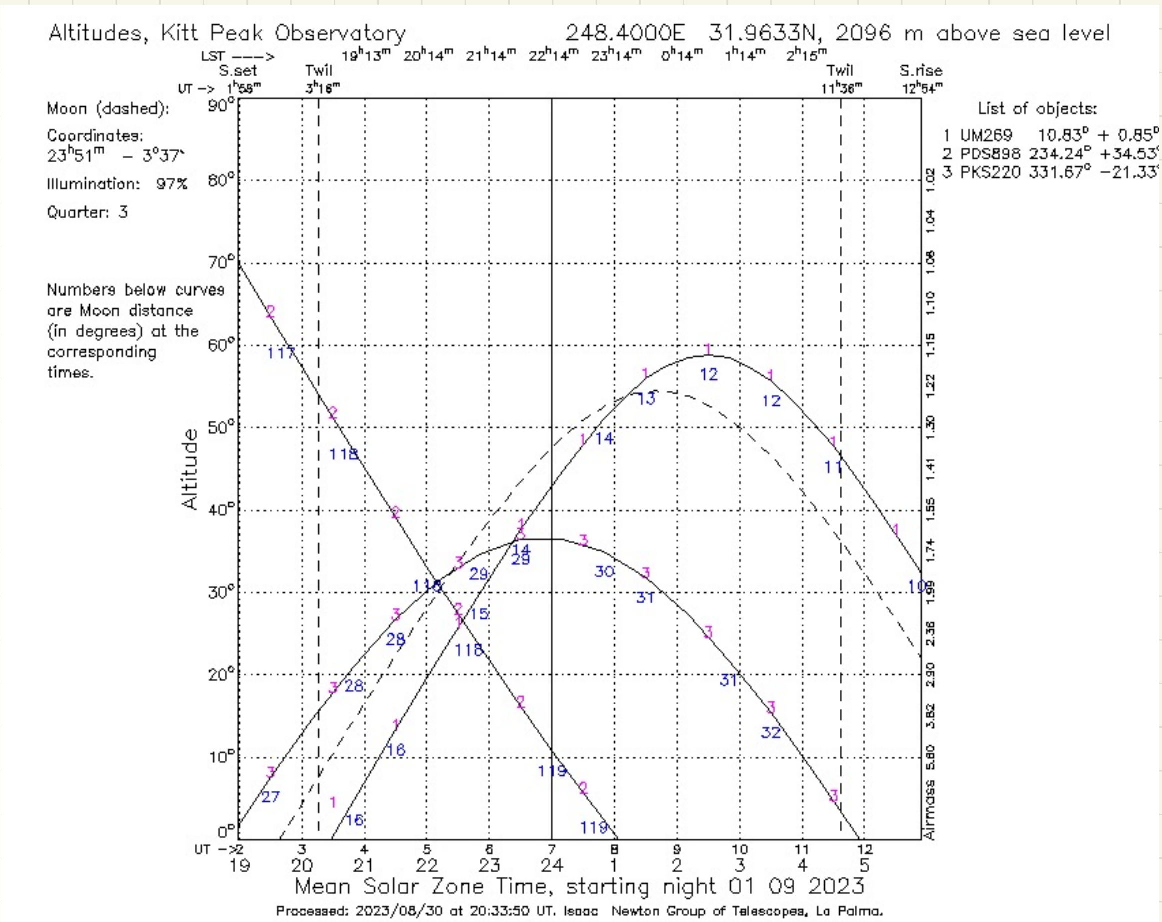
↳ up for shorter period due to southern declination



From KPNO, can see declinations of $-58°$ to $+90°$, so all objects will rise.

Circumpolar objects will have $\delta > 90° - \lambda_{obs} = 90° - 32° = 58°$. None of these in the list.

5.



6. varies

7. varies