Problem 1:

we know that the magnitude - flux relation,

$$m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_2}$$

If F_1 is the flux of first source,
the flux of the second source will be,
$F_2 = 1.03\, F_1$ for a 3% increase in flux.

So, $\Delta m = -2.5 \log_{10} \frac{F_1}{F_2}$

$$= -2.5 \log_{10} \frac{1}{1.03}$$

$$= -2.5 \log_{10} (0.97087)$$

$$\approx 0.03$$

So, with 3% increase in flux, magnitude will decrease by 0.03. $\square$

Problem 2:

From HW2 problem 6 we have,

$$1 \frac{MJy}{sr} = 6.40 \times 10^{-6} \text{ photons } s^{-1}\, cm^{-2}\, \mathring{A}^{-1}\, arcsec^{-2}$$

Now, the given diameter of a telescope $d = 2.3$ meters
So, the area of the light collecting surface of the telescope will be,

$$A = \pi \left(\frac{d}{2}\right)^2 \, m^2$$

$$= \pi \left(\frac{2.3}{2}\right)^2 m^2$$

$$= 4.155 \, m^2 = 4.155 \times 10^4 \, cm^2$$

We also need to consider the V filter band pass width which is around 1000 Å.

So, total photons received per unit pixel or per arcsec^2 will be,

$$6.40 \times 10^{-6} \text{ ph s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1} \text{ arcsec}^{-2} \times 4.155 \times 10^{4} \text{ cm}^2 \times 1000 \text{ Å}$$

$$\approx 265.92 \text{ ph s}^{-1} \text{ arcsec}^{-2}$$

$$= 265.92 \text{ ph s}^{-1} \text{ pixel}^{-1}$$

[As the area of one pixel is 1 arcsec^2]

$\square$

Problem 3:

Given luminosity, $L = 10^{38}$ erg s^{-1} Å^{-1}
and distance, $d = 10$ Mpc

$$= 10 \times 3.086 \times 10^{22} \text{ m}$$

$$= 3.086 \times 10^{23} \text{ m}$$

$$= 3.086 \times 10^{25} \text{ cm}$$

So, Flux, $F = \dfrac{L}{4\pi d^2} = \dfrac{10^{38} \text{ erg s}^{-1} \text{ Å}^{-1}}{3.086 \times 10^{25} \text{ cm}}$

$$= 8.356 \times 10^{-15} \text{ erg s}^{-1} \text{ Å}^{-1} \text{ cm}^{-2}$$

Energy of one photon, $E = \frac{hc}{\lambda}$

$$= \frac{6.626 \times 10^{-27} \text{ ergs} \times 3 \times 10^{10} \text{ cm/s}}{5500 \times 10^{-8} \text{ cm}}$$

$$\simeq 3.61 \times 10^{-12} \text{ erg}$$

so, We can write flux as.

$$F = \frac{8.356 \times 10^{-15} \text{ erg } s^{-1} \text{ Å}^{-1} \text{ cm}^{-2}}{3.61 \times 10^{-12} \text{ erg/ph}}$$

$$= 2.315 \times 10^{-3} \text{ ph } s^{-1} \text{ Å}^{-1} \text{ cm}^{-2}$$

We can multiply by bandwidth 1000 Å to get the flux for the total bandwidth.

$$\text{so, } F = 2.315 \times 10^{-3} \text{ ph } s^{-1} \text{ Å}^{-1} \text{ cm}^{-2} \times 1000 \text{ Å}$$

$$= 2.315 \text{ ph } s^{-1} \text{ cm}^{-2}$$

Now, given the diameter of the telescope,

$$D = 2.3 \text{ m}$$
$$= 230 \text{ cm}$$

The area of the telescope,

$$A = \pi \left(\frac{D}{2}\right)^2 = \pi \left(\frac{230}{2}\right)^2 \text{cm}^2$$
$$= 4.155 \times 10^4 \text{ cm}^2$$

Total photons collected by the telescope,

$$N_{det} = F \times A \times \text{Efficiency} \times \text{Reduction factor}$$

Efficiency is given as 50% or 0.5

We can calculate the reduction factor from the airmass,

$$\text{Reduction Factor} = 10^{-0.4 \times \text{Extinction coefficient} \times \text{airmass}}$$
$$= 10^{-0.4 \times 0.2 \text{ mag/airmass} \times 2 \text{ airmass}}$$

$$\simeq 0.692$$

so, $N_{det} = 2.315 \text{ ph s}^{-1} \text{ cm}^{-2} \cdot 4.155 \times 10^4 \text{ cm}^2 \times 0.5 \times 0.692$

$$\simeq 3.3281 \times 10^4 \text{ ph s}^{-1} \quad \square$$

Problem 4:

We know, signal to noise ratio,

$$S/N = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

Here, R_s = Rate of detection of photons from source

R_B = Rate of detection of photons from background per pixel

R_D = Rate of dark current per pixel

N_R = Number of photons per pixel of read-noise

n_{pix} = Total number of pixel

We solve the equation for t,

$$S/N = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

$$\Rightarrow (S/N)^2 = \frac{R_s^2 t^2}{R_s t + n_{pix}(R_B t + R_D t + N_R^2)}$$

$$\Rightarrow R_s^2 t^2 - (S/N)^2 \left[R_s t + n_{pix}(R_B t + R_D t + N_R^2)\right] = 0$$

$$\Rightarrow R_s^2 t^2 - (S/N)^2 \left[t(R_s + n_{pix} R_B + n_{pix} R_D) + n_{pix} N_R^2\right] = 0$$

$$\Rightarrow R_s^2 t^2 - t\left[(S/N)^2 (R_s + n_{pix} R_B + n_{pix} R_D)\right] - (S/N)^2 n_{pix} N_R^2 = 0 \quad \text{--- (1)}$$

The given values,

$$R_S = 0.2 \text{ ph } s^{-1}$$
$$R_B = 0.5 \text{ ph } s^{-1} \text{ pixel}^{-1}$$
$$R_D = 10 \text{ electrons hour}^{-1} \text{ pixel}^{-1}$$
$$= 10 \text{ eletrons hour}^{-1} \text{ pixel}^{-1} \times \frac{1 \text{ hour}}{(60 \times 60) s}$$
$$= 2.778 \times 10^{-3} \text{ electron } s^{-1} \text{ pixel}^{-1}$$
$$N_R = 5 \text{ electrons}$$
$$S/N = 100$$
$$n_{pix} = 4 \text{ pixels}$$

We can insert these values in the eauation,

$$(0.2)^2 t^2 - t\left[100^2\left(0.2 + 4.0.5 + 2.778 \times 10^{-3} \times 4\right)\right] - 100^2.4.5^2 = 0$$

$$\Rightarrow 0.04 t^2 - 22111.12 t - 10^6 = 0$$

So, $$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2.a}$$

$$= \frac{22111.12 \pm \sqrt{(22111.12)^2 + 4.0.04.10^6}}{2.0.04}$$

So, $$t = 552823.22 \text{ s}, -45.22 \text{ s}$$
$$= 9213.72 \text{ min}, -0.75 \text{ min}$$

So, we can get to 100 S/N with 9213.72 min

[as time can not be negative, -0.75 min is not possible]

□

Problem 5:

Diameter of the telescope, $D = 2.3$ m

So, the area of the light collecting surface, $A = \pi \left(\frac{D}{2}\right)^2 \, m^2$

$= \pi \left(\frac{2.3m}{2}\right)^2 m^2$

$\approx 4.155 \, m^2$

$\approx 4.155 \times 10^4 \, cm^2$

We know, the zero-magnitude of V band has a flux of 3540 Jy

So, we can use the flux-magnitude relation,

$$m - m_0 = -2.5 \log_{10} \frac{F}{F_0}$$

$$\Rightarrow \frac{m_0 - m}{2.5} = \log_{10} \frac{F}{F_0}$$

$$\Rightarrow F = F_0 \, 10^{\frac{m_0 - m}{2.5}}$$

$$= 3540 \text{ Jy} \; 10^{\frac{0-22}{2.5}}$$

$$= 5.611 \times 10^{-6} \text{ Jy}$$

Given V magnitude, $m = 22$ mag

Converting F in Jansky to f_λ in $erg\, s^{-1}\, cm^{-2}\, \mathring{A}^{-1}$,

$$f_\nu = 5.611 \times 10^{-6} \text{ Jy} \times 10^{-23} \, erg\, s^{-1}\, cm^{-2}\, Hz^{-1}\, Jy^{-1}$$

$$= 5.611 \times 10^{-29} \, erg\, s^{-1}\, cm^{-2}\, Hz^{-1}$$

Now converting flux f_ν to flux f_λ,

$$f_\lambda = f_\nu \times \frac{c}{\lambda^2}$$

So, $$f_\lambda = 5.611 \times 10^{-29} \times \frac{3 \times 10^{18}}{(5600)^2} \text{ erg } s^{-1} cm^{-2} \mathring{A}^{-1}$$

$$\approx 5.368 \times 10^{-18} \text{ erg } s^{-1} cm^{-2} \mathring{A}^{-1}$$

Now, energy of a photon, $E = \dfrac{hc}{\lambda}$

$$= \frac{6.626 \times 10^{-27} \times 3 \times 10^{10}}{5600 \times 10^{-8}} \text{ erg}$$

$$= 3.55 \times 10^{-12} \text{ erg}$$

So, flux in terms of photons,

$$f_{\lambda, ph} = \frac{f_\lambda}{E} = \frac{5.368 \times 10^{-18} \text{ erg } s^{-1} cm^{-2} \mathring{A}^{-1}}{3.55 \times 10^{-12} \text{ erg/photon}}$$

$$= 1.512 \times 10^{-6} \text{ ph } s^{-1} cm^{-2} \mathring{A}^{-1}$$

Total bandwidth for V band, 1000 $\mathring{A}$

So, the flux for the total bandwidth,

$$f_{\lambda, ph V} = 1.512 \times 10^{-6} \text{ ph } s^{-1} cm^{-2} \mathring{A}^{-1} \times 1000 \mathring{A}$$

$$= 1.512 \times 10^{-3} \text{ ph } s^{-1} cm^{-2}$$

We can find out the reduction due to airmass which will be,

$$\text{Reduction} = 10^{-0.4 \times \text{Extinction Coeff.} \times \text{Airmass}}$$

$$= 10^{-0.4 \times 0.2\ \text{mag}/\text{airmass} \times 1}$$

$$\approx 0.8318$$

So, the required flux after reduction due to airmass,

$$1.512 \times 10^{-3}\ \text{ph}\ s^{-1}\ cm^{-2} \times 0.8318$$

$$\simeq 1.2577 \times 10^{-3}\ \text{ph}\ s^{-1}\ cm^{-2}$$

So, the rate of detection of photon from the source can be found by multiplying this flux value with the total area of the telescope and the efficiencies.

Given quantum efficiency is 0.90 and other efficiency will be 0.70.

$$\text{So,} \quad R_s = 1.2577 \times 10^{-3}\ \text{ph}\ s^{-1} cm^{-2} \times 4.155 \times 10^{4}\ cm^{2} \times 0.50 \times 0.70$$

$$= 18.29\ \text{ph}\ s^{-1}$$

Focal length, $f = R \cdot D = 2.1 \times 2.3\,m = 4.83\,m$
$= 4.83 \times 10^3\,mm$

Platescale, $s = \frac{206265}{f} = \frac{206265 \text{ arcsec}}{4.83 \times 10^3\,mm}$

$= 42.705$ arcsec mm^{-1}

So, one pixel subtends.

$13.5\,\mu m \cdot 10^{-3}\,mm/\mu m \cdot 42.705$ arcsec$/mm$

$= 0.5765$ arcsec

With $1.1''$ seeing, images will have a FWHM of ~ 2 pixels

So, most of the light will be distributed over 4 pixels.

For rate of detection of photon from background, we have the surface brightness,

$\mu_{v_f} = 20$ mag arcsec^{-2} [For full moon]

$\mu_{v_n} = 22$ mag arcsec^{-2} [For new moon]

We already know from before, 22 mag in V bands correspond to 1.512×10^{-3} ph $s^{-1}\,cm^{-2}$

So, $\mu_{v_n} = 1.512 \times 10^{-3}$ ph $s^{-1}\,cm^{-2}$ arcsec^{-2}

Made with Goodnotes

If we express it in pixel^{-1}:

we know, 1 pixel length subtends 0.5765 arcsec

so, 1 pixel area will subtend $(0.5765 \text{ arcsec})^2$

$= 0.3324 \text{ arcsec}^2$

so if we want to write the surface brightness per pixel,

$N_{vn} = 1.512 \times 10^{-3} \text{ ph s}^{-1} \text{ cm}^{-2} \text{ arcsec}^{-2} \times 0.3324 \text{ arcsec}^2 \text{ pixel}^{-1}$

$= 5.0259 \times 10^{-4} \text{ ph s}^{-1} \text{ cm}^{-2} \text{ pixel}^{-1}$

For the total telescope area,

$R_{Bn} = N_{vn} \times A = 5.0259 \times 10^{-4} \text{ ph s}^{-1} \text{ cm}^{-2} \text{ pixel}^{-1} \times 4.155 \times 10^{4} \text{ cm}^2$

$\approx 20.88 \text{ ph s}^{-1} \text{ pixel}^{-1}$

For full moon we can follow the following process:

So, we can use the flux-magnitude relation.

$$m - m_0 = -2.5 \log_{10} \frac{F}{F_0}$$

$$\Rightarrow \frac{m_0 - m}{2.5} = \log_{10} \frac{F}{F_0}$$

$$\Rightarrow F = F_0 \, 10^{\frac{m_0 - m}{2.5}}$$

$$= 3540 \text{ Jy } 10^{\frac{0-20}{2.5}}$$

$$= 3.54 \times 10^{-5} \text{ Jy}$$

Converting F in Jansky to f_λ in erg s^{-1} cm^{-2} $\mathring{A}^{-1}$,

$$f_\nu = 3.54 \times 10^{-5} \text{ Jy} \times 10^{-23} \text{ erg } s^{-1} cm^{-2} Hz^{-1} Jy^{-1}$$

$$= 3.54 \times 10^{-28} \text{ erg } s^{-1} cm^{-2} Hz^{-1}$$

Now converting flux f_ν to flux f_λ,

$$f_\lambda = f_\nu \times \frac{c}{\lambda^2}$$

So, $$f_\lambda = 3.54 \times 10^{-28} \times \frac{3 \times 10^{18}}{(5600)^2} \text{ erg } s^{-1} cm^{-2} \mathring{A}^{-1}$$

$$\simeq 3.386 \times 10^{-17} \text{ erg } s^{-1} cm^{-2} \mathring{A}^{-1}$$

Now, energy of a photon, $E = \frac{hc}{\lambda}$

$$= \frac{6.626 \times 10^{-27} \times 3 \times 10^{10}}{5600 \times 10^{-8}} \text{ erg}$$

$$= 3.55 \times 10^{-12} \text{ erg}$$

So, flux in terms of photons,

$$f_{\lambda,ph} = \frac{f_\lambda}{E} = \frac{3.386 \times 10^{-17} \text{ erg } s^{-1} cm^{-2} A^{o-1}}{3.55 \times 10^{-12} \text{ erg/photon}}$$

$$= 9.539 \times 10^{-6} \text{ ph } s^{-1} cm^{-2} A^{o-1}$$

Total bandwidth for V band, $1000 A^o$

So, the flux for the total bandwidth,

$$f_{\lambda,ph\,V} = 9.539 \times 10^{-6} \text{ ph } s^{-1} cm^{-2} A^{o-1} \times 1000 A^o$$

$$= 9.539 \times 10^{-3} \text{ ph } s^{-1} cm^{-2}$$

So $N_{VF} = 9.539 \times 10^{-3} \text{ ph } s^{-1} cm^{-2} \text{ arcsec}^{-2}$

$$= 9.539 \times 10^{-3} \text{ ph } s^{-1} cm^{-2} \text{ arcsec}^{-2} \times 0.3324 \text{ arcsec}^2 \text{ pixel}^{-1}$$

$$= 3.171 \times 10^{-3} \text{ ph } s^{-1} cm^{-2} \text{ pixel}^{-1}$$

Now we can calculate the background rate by multiplying the value with the total pixel area.

So, $R_{BF} = 3.171 \times 10^{-3}$ ph s^{-1} cm^{-2} $pixel^{-1}$
$\times 4.155 \times 10^{4}$ cm^{2}

≈ 131.75 ph s^{-1} $pixel^{-1}$

Given read noise, $N_R = 4.5$ electrons $pixel^{-1}$

Calculating time for Full moon:

$$R_s^2 t^2 - t\left[(S/N)^2 (R_s + n_{pix} R_B + n_{pix} R_D)\right] - (S/N)^2 n_{pix} N_R^2 = 0$$

[Equation ① from problem 4]

Putting values in the equation,

$$18.29^2 t^2 - t\left[(100)^2 (18.29 + 4 \cdot 131.75 + 4 \cdot 0)\right] - (100)^2 \cdot 4 \cdot (4.5)^2 = 0$$

$$\Rightarrow 334.5241\, t^2 - 5452900\, t - 810000 = 0$$

$$\text{So, } t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5452900 \pm \sqrt{(5452900)^2 - 4 \cdot 334.5241 \cdot (-810000)}}{2 \times 334.5241}$$

So, $t = 16300.62\,s, -0.149\,s$
$= 271.68$ min, -0.002 min

As negative time doesn't make any sense, we can say the required time is, 271.68 min

Calculate the time for new-moon:

$$R_S^2 t^2 - t\left[(S/N)^2 \left(R_S + n_{pix} R_B + n_{pix} R_D\right)\right] - (S/N)^2 n_{pix} N_R^2 = 0$$

[Equation ① from problem 4]

Putting values in the equation,

$$18.29^2 t^2 - t\left[(100)^2 (18.29 + 4 \cdot 20.88 + 4 \cdot 0)\right] - (100)^2 \cdot 4 \cdot (4.5)^2 = 0$$

$$\Rightarrow 334.5241\, t^2 - 1018100\, t - 810000 = 0$$

$$\text{So, } t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1018100 \pm \sqrt{(1018100)^2 + 4 \cdot 334.5241 \times 810000}}{2 \cdot 334.5241}$$

So, $t = 3044.22\, s, \; -0.795\, s$

$= 50.74\, min, \; -0.013\, min$

Neglecting the negative values, required time will be 50.74 min

So, we can see the time required for observation changes significantly when the rate of detection of photon from the background (R_B) changes. So, we can say that the observation is background limited.

Another thing to mention the object we're trying to observe is not that much brighter than the background. This also proves the fact that our observation in this case was background limited.

$\square$

Problem 6:

Given, at Keck,

Signal to noise, $S/N = 50$

Exposure time, $t = 10$ mins

Efficiency, $QE = 80\% = 0.8$

Bandwidth, $B = 50\,\mathring{A}$

At WIRO,

$S/N = 50$

$t = ?$

$QE = 95\% = 0.95$

$B = 1000\,\mathring{A}$ (for V filter we assumed)

We know that,

$$(S/N) \propto \sqrt{t \times QE \times B}$$

So,

$$\frac{(S/N)_{KECK}}{(S/N)_{WIRO}} = \frac{\sqrt{t_{KECK} \times QE_{KECK} \times B_{KECK}}}{\sqrt{t_{WIRO} \times QE_{WIRO} \times B_{WIRO}}}$$

$$\Rightarrow \frac{50}{50} = \frac{\sqrt{10 \times 0.8 \times 50}}{\sqrt{t_{WIRO} \times 0.95 \times 1000}}$$

$$\Rightarrow t_{WIRO} = \frac{10 \times 0.8 \times 50}{0.95 \times 1000} \text{ mins} = 0.42 \text{ mins}$$

Assumptions made:

- I neglected all the other noise sources.
- I assumed the telescope's collecting areas and the overall transmission efficiency is same for them
- The stellar objects brightness doesn't change.
- The S/N scales only with the square root of exposure time and the QE of the integer.