Hw 3

1.) equation for error propagation

$$\sigma_m^2 = \left(\frac{\partial m}{\partial f}\right)^2 \sigma_f \qquad \text{where } f = \text{flux},\ m = \text{magnitude}$$

AND $m_{AB} = -2.5 \log[f_\nu(Jy)] + 8.9$

$$\frac{\partial m}{\partial f} = \frac{-2.5}{f \ln 10}$$

So,

$$\sigma_m^2 = \frac{(-2.5)^2}{(f \ln 10)^2} \sigma_f^2 \rightarrow \sigma_m = \frac{2.5}{\ln 10} \frac{\sigma_f}{f}$$

$$\frac{\sigma_f}{f} = 0.03 \qquad \text{So, } \sigma_m = 1.08 \frac{\sigma_f}{f} = 1.08 \cdot 0.03 = 0.0325$$

$\boxed{\text{So, } \sigma_m = 0.0325 \approx 0.03}$

2.) $R_s = \eta_{total} A \Delta\lambda f_\lambda$

Since $V_{band} \rightarrow \Delta\lambda \approx 880\,Å$

$r = 1.15m$

Pixel

$\eta_{total} = 1 \qquad A = \pi r^2 \qquad f_\lambda = 6.46\times10^{-6}\ phot/s/cm^2/Å/arcsec^2 \cdot 1\,arcsec^2$

$$R_s = \pi (115cm)^2 \cdot 880\,Å \cdot 6.46\times10^{-6}\ phot/s/cm^2/Å$$

$\boxed{R_s = 236.2\ ph/s}$

3.) $d = 10mpc \rightarrow 3.086\times10^{25}\ cm$

$L = 10^{38}\ erg/s/Å$

2.3m telescope

$\eta = 0.5$

$\rightarrow 2$ air masses

$R_s = \eta_{total} \cdot A \cdot \Delta\lambda \cdot f_\lambda$

$L = 4\pi d^2 \cdot f \qquad f = \frac{L}{4\pi d^2} \Rightarrow \frac{10^{38}\ erg/s/Å}{4\pi (3.086\times10^{25} cm)^2}$

$= 8.67\times10^{-50}\ erg/s/cm^2/Å$

V-band effective $\lambda \Rightarrow 5600$ Å

So $f_\lambda = 2.44 \times 10^{-38}$ ph/s/cm²/Å

$$\eta_{atmo} = (2.5^{X \cdot k})^{-1} = (2.5^{2 \cdot 0.2})^{-1} = 0.69$$

$X = 2 \rightarrow$ 2 air masses

$k = 0.2 \rightarrow$ for V band

$$\eta_{total} = 0.69 \times 0.5 = 0.345$$

$$R_S = 0.345 \times \pi (115\,cm)^2 \times 880\,Å \times 2.44 \times 10^{-38}\ ph/s/cm^2/Å = \boxed{3.07 \times 10^{-31}\ ph/s}$$

4.) $R_S = 0.2$ ph/s Want $\frac{S}{N} = 100$

$R_B = 0.5$ ph/s/pix

$R_D = 10 e^-/hr/pix \rightarrow \frac{10}{3600\,s} = 0.00277\, e^-/s/pix$

$N_R = 5e^-$

$n_{pix} = 4$

Using $$\frac{S}{N} = \frac{R_S t}{\sqrt{R_S t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

$$t = \frac{(\frac{S}{N})^2 (R_S + n_{pix} R_B + n_{pix} R_D) \pm \sqrt{(S/N)^4 (R_S + n_{pix} R_B + n_{pix} R_D)^2 - 4(\frac{S}{N})^2 n_{pix} N_R^2 R_S^2}}{2R_S^2}$$

$$= \frac{(100)^2 (0.2 + 4 \cdot 0.5 + 4 \cdot 0.00277) \pm \sqrt{(100)^4 (0.2 + 4 \cdot 0.5 + 4 \cdot 0.00277)^2 - 4(100)^2 4 \cdot 5^2 \cdot 0.2^2}}{2(0.2)^2}$$

$t = 552693$ Seconds $\frac{552693}{60} = 9211.55$

So, if we have 60s exposure time

$\boxed{\text{we need 9222 exposures}}$

5.) $d = 2.3m$ need $R_S = 28.9 ph/s$

$f/2.1$ R_B

pixelsize = 13.5 microns n_{pix}

$X = 1$ η

$Q = 0.9$

$\eta = 0.7$

$S/N = 100$

V_{mag} source = 22

$\mu_V = 20 \frac{mag}{arcsec^2}$ => moon

1.1" seeing

$R_D = 0$

$N_R = 4.5 e^-/pix$

So,

$R_S = \eta \cdot A \cdot \Delta\lambda \cdot f_\lambda$

$A = \pi (115cm)^2 = 41547 cm^2$

$\Delta\lambda = 880$ Å

$\eta_{tot} = 0.7 \cdot 0.9 \cdot \eta_{atmo} = .524$

$\eta_{atmo} = (2.5^{XK})^{-1} = (2.5^{0.2})^{-1} = 0.83$

$f_V = f_{V_{jansky}} 10^{-0.4(m)}$

$3540 \cdot 10^{0.4(-22)} = 5.61 \times 10^{-6}$ Jy => 1.511×10^{-6} ph/s/cm²/Å

$R_S = 0.524 \cdot 41547 \cdot 880 \cdot 1.511 \times 10^{-6} = 28.9 ph/s$

To get n_{pix},

$S = \frac{206265''}{f}$ $R = \frac{f}{D} = 2.1 \Rightarrow f = D(2.1) = 2.3(2.1) = 4.83 m$

$S = \frac{206265''}{4.83 \times 10^6 \mu m} = 0.0427 ''/\mu m$

need later

$\frac{0.0427''}{\mu m} \cdot \frac{13.5 \mu m}{pix} = (0.576\ ''/pix)^2 = 0.322 \frac{arcsec^2}{\text{square pix} \Rightarrow \text{basically pix}}$

$\Rightarrow \frac{1.1 \text{ arcsec}}{0.576 \frac{arcsec}{pix}} = 1.9 \Rightarrow$ across disk, Nyquist Sampling $\Rightarrow n_{pix} = 4$

for R_B

$R_B = \underset{\substack{\downarrow \\ \text{only need} \\ 0.7-0.9}}{\eta} \cdot \underset{\text{same}}{A} \cdot \underset{\text{same}}{\Delta\lambda} \cdot f_\lambda$

we have $20 \frac{mag}{arcsec^2}$

$f_\nu = 3540 \cdot 10^{-0.4(20)} = 3.54 \times 10^{-5}\ Jy/arcsec^2$

$= 9.54 \times 10^{-6}\ ph/s/cm^2/Å/arcsec^2 \cdot 0.322 \frac{arcsec^2}{pix}$

$f_\lambda = 3.07 \times 10^{-6}\ ph/s/cm^2/Å/pix$

$R_B = 70\ ph/s/pix$

$$t = \frac{\left(\frac{S}{N}\right)^2 (R_S + n_{pix} R_B + n_{pix} R_D) \pm \sqrt{\left(\frac{S}{N}\right)^4 (R_S + n_{pix} R_B + n_{pix} R_D)^2 - 4\left(\frac{S}{N}\right)^2 n_{pix} N_R^2 R_S^2}}{2 R_S^2}$$

$\boxed{t = 3698.2 \text{ seconds}}$ to reach S/N of 100

Now for $\mu_V = 22 \frac{mag}{arcsec^2} \Rightarrow$ we get $1.511 \times 10^{-6}\ ph/s/cm^2/Å/arcsec^2 \cdot 0.322 \frac{arcsec^2}{pix}$

$f_\lambda = 4.865 \times 10^{-7}\ ph/s/cm^2/Å/pix$

$R_B = 11.2$

So $\boxed{t = 881.3 \text{ seconds}}$ for $\mu_V = 22 \frac{mag}{arcsec^2}$

for first scenario:

object limited case:

$$\frac{S}{N} = \sqrt{R_S t}$$

$$\frac{(S/N)^2}{R_S} = t \quad \Rightarrow \quad \frac{100^2}{28.9} = 346 \text{ seconds}$$

background limiting:

$$\frac{S}{N} = \frac{R_S t}{\sqrt{n_{pix}(R_B t)}}$$

$$n_{pix} \cdot R_B t = \frac{R_S^2 t^2}{\left(\frac{S}{N}\right)^2}$$

$$\frac{n_{pix} \cdot R_B \cdot \left(\frac{S}{N}\right)^2}{R_S^2} = t = 3352 \text{ seconds} \rightarrow \boxed{\text{we are background limited}}$$

second scenario:

$$t = \frac{n_{pix} \cdot R_B \cdot \left(\frac{S}{N}\right)^2}{R_S^2} = 536.4 \rightarrow \text{still background limited}$$

6.) At Keck $Q = 0.8$ 10m

$S/N = 50$

$t = 10 \text{ mins} = 600s$

$\Delta\lambda = 50$ Å

At WIRO $Q = 0.95$ 2.3m

$S/N = 50$

$\Delta\lambda = 800$ Å

$t = ?$

$$t = \frac{(S/N)^2}{R_S} = \frac{50^2}{\eta_{total} \cdot \Delta\lambda \cdot A \cdot f_\lambda}$$

f_λ constant

for Keck: $\frac{50^2}{0.8(50)(\pi \cdot 500^2) f_\lambda} = 600$

$f_\lambda = 1.32 \times 10^{-7}$

for WIRO:

$R_S = 0.95 \cdot 800 \text{ Å} \cdot \pi (115)^2 \cdot 1.327 \times 10^{-7} = 4.584 \text{ ph/s}$

$t = \frac{50^2}{4.584} = \boxed{545 \text{ seconds}}$

Assume $\eta_{tot} = QE$ } source noise limited

Seeing is the same