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OA HW 3

1) Show that an error of 3% in flux units is almost the same as an error of 0.03 mags

Use

$$\sigma_m^2 = \left(\frac{\partial m}{\partial f}\right)^2 \sigma_f^2 \longrightarrow \underbrace{\sigma_m}_{\text{Uncertainty in mags}} = \left|\frac{\partial m}{\partial f}\right| \sigma_f \quad \text{where} \quad \underbrace{\frac{\sigma_f}{f} = 0.03}_{\text{Uncertainty in flux}}$$

and

$$m_{AB} = -2.5 \log(f(Jy)) + 8.9$$

$$\frac{\partial m}{\partial f} = (-2.5)\left(\frac{1}{f \ln(10)}\right)$$

$$\sigma_m = \frac{2.5}{\cancel{f} \ln(10)}(0.03 \cancel{f}) = \frac{2.5}{2.3}(0.03)$$

$$\boxed{\text{but } \frac{2.5}{2.3} \approx 1 \quad \text{so} \quad \sigma_m \approx 0.03}$$

2) Source A from HW 2

$B = 1\ MJy/sr$ $\lambda = 5500\ Å$

- V filter w/ typical bandpass width
- CCD camera on $d = 2.3m$ telescope like Wiro
- 1 pixel = 1 $arcsec^2$
- Everything has 100% efficiency

* How many photons / s will you collect in a single CCD pixel?

$$\boxed{R_s = Q \cdot \Delta\lambda_{bandpass} \cdot A \cdot f_\lambda}$$

$Q = 100\% = 1$

$A = \pi (1.15m)^2 = \pi (115cm)^2$

$\Delta\lambda_{bandpass} = 880\ Å$ (FWHM) (from Chromey)

$B = \frac{f_\lambda}{\theta^2} \rightarrow f_\lambda = B \cdot \theta^2$ so...

$$\boxed{R_s = \Delta\lambda_{bandpass} \cdot A \cdot B \cdot \theta^2}$$ * Q is ommitted since $Q = 1$

$\theta^2 = 1 arcsec^2$

From HW 2,

$$B = 6.46 \times 10^{-6} \frac{photons}{s \cdot cm^2 \cdot Å \cdot arcsec^2}$$

$$R_s = (880\ \cancel{Å})(\pi \cdot 115^2\ \cancel{cm^2}) \cdot 6.46 \times 10^{-6} \frac{photons}{s \cdot \cancel{cm^2} \cdot \cancel{Å} \cdot \cancel{arcsec^2}} \cdot 1\ \cancel{arcsec^2}$$

$$\boxed{R_s = 236 \frac{photons}{s}}$$

* Efficiencies Multiply

3) Find R_s (photons / s)

$D = 10$ Mpc (distance to galaxy)

$d = 2.3$ m (diameter of telescope)

$L = 1083 \frac{erg}{s \cdot Å}$ (monochromatic luminosity)

$Q = 0.5$ (efficiency)

- V filter
- Observing at 2 airmasses

$V = 0.2$ mag/airmass $\longrightarrow m_V = 0.4$

$$f = \frac{L}{4\pi D^2}$$

$D = 10\text{Mpc} = 3.086 \times 10^{25}$ cm

$A = \pi d^2$

$\Delta\lambda_{bandpass} = 880$ Å (FWHM) (Chromey)

$\eta_{atm} = (2.5^{\chi K})^{-1}$ where $\chi = 2$ $K = 0.2$

$$\eta_{TOT} = (0.5)\left(2.5^{2(0.2)}\right)^{-1}$$

$E_{photon} = h\frac{c}{\lambda} \longrightarrow \lambda = 5600$ Å (λ_{eff} for V filter; from notes)

$$E_{photon} = \left(6.67 \times 10^{-27} \text{erg} \cdot s\right) \frac{3 \times 10^{18} \text{ Å/s}}{5600 \text{ Å}} = 3.57 \times 10^{-12} \text{ erg}$$

$$L = 1083 \frac{erg}{s \cdot Å} \times \frac{1 \text{ ph}}{3.57 \times 10^{-12} \text{ erg}} = 3.03 \times 10^{14} \frac{\text{photons}}{s \cdot Å}$$

$$R_s = \eta_{TOT} \Delta\lambda_{bandpass} (\pi d^2) \left(\frac{L}{4\pi D^2}\right)$$

$$= (0.5)(2.5^{0.4})^{-1} (880 \text{ Å})(\pi\, 230^2 \text{cm}^2) \left(\frac{3.03 \times 10^{14} \frac{\text{photons}}{s \cdot Å}}{4\pi (3.086 \times 10^{25} \text{ cm})^2} \right)$$

$$\boxed{R_s = 1.28 \times 10^{-30} \frac{\text{photons}}{s}}$$

4)

$R_S = 0.2 \frac{\text{photons}}{\text{s}}$ $R_B = 0.5 \frac{\text{photons}}{\text{s} \cdot \text{pix}}$ $R_D = 10 \frac{\text{electrons}}{\text{hr} \cdot \text{pix}} \times \frac{1\text{ hr}}{60\text{min}} \times \frac{1\text{ min}}{60\text{ s}} = 0.0028 \frac{\text{electrons}}{\text{s} \cdot \text{pix}}$

$N_R = 5$ electrons

• Stellar PSF distributed over 4 pixels $\longrightarrow n_{pix} = 4$ pix

How many 1-min exposures does it take to reach a S/N = 100?

$$\frac{S}{N} = \frac{R_s t}{\sqrt{R_s t + n_{pix}(R_B t + R_D t + N_R^2)}}$$

$$\left(\frac{S}{N}\right)^2 = \frac{R_s^2 t^2}{R_s t + n_{pix}(R_B t + R_D t + N_R^2)}$$

$$\left(\frac{S}{N}\right)^2 \left(R_s t + n_{pix}(R_B t + R_D t + N_R^2)\right) = R_s^2 t^2$$

$$\left(\frac{S}{N}\right)^2 (n_{pix} N_R^2) = R_s^2 t^2 - \left(\frac{S}{N}\right)^2 R_s t - \left(\frac{S}{N}\right)^2 n_{pix} R_B t - \left(\frac{S}{N}\right)^2 n_{pix} R_D t$$

$$\left(\frac{S}{N}\right)^2 (n_{pix} N_R^2) = R_s^2 t^2 - \left(\frac{S}{N}\right)^2 (R_s + n_{pix} R_D + n_{pix} R_B)\, t$$

$$\boxed{R_s^2 t^2 - \left(\frac{S}{N}\right)^2 (R_s + n_{pix} R_D + n_{pix} R_B)\, t - \left(\frac{S}{N}\right)^2 n_{pix} N_R^2 = 0}$$

* Used Mathematica to solve for t *

$t = 552{,}823 \text{ s} = 9213.72 \text{ min}$

Therefore, it would take 9214 one-minute exposures to reach S/N = 100

5) Big Scary Problem!

WIRO Prime Focus Imager:

$d = 2.3$ m $R_p = f/2.1$ $s = 13.5\ \mu m$ (pixel size) airmass $(\chi) = 1$

$QE = 0.90$ $E_{other} = 0.70$ (other telescope efficiencies)

$V = 22$ mag star $\mu_V = 20 \frac{mag}{arcsec^2}$ (full moon) $\Theta = 1.1''$ (seeing) $N_R = 4.5 \frac{electrons}{pixel}$ $R_D = 0$

i) Find time for $S/N = 100$

*We need to solve for, R_S, n_{pix}, R_B, η_{TOT}

From Chromey,

$f_{\lambda,0} = \frac{37.4\times10^{-12}\ J}{s\cdot m^2\cdot nm}$ in V band and $\lambda_{eff} = 5450$ Å

$$f_\lambda = \frac{37.4\times10^{-12}\ J}{s\cdot m^2\cdot nm} 10^{-0.4(22)} = 5.93\times10^{-20} \frac{J}{s\cdot m^2\cdot nm}$$

$$E_{photon} = (6.626\times10^{-34}\ J\cdot s)\left(\frac{3\times10^{18}\ Å/s}{5450\ Å}\right) = 3.65\times10^{-19}\ J$$

$$f_\lambda = \frac{5.93\times10^{-20}\ J}{s\cdot m^2\cdot nm} \times \frac{1\ m^2}{100^2\ cm^2} \times \frac{1\ nm}{10\ Å} \times \frac{1\ photon}{3.65\times10^{-19}\ J} = \boxed{1.625\times10^{-6} \frac{photon}{s\cdot cm^2\cdot Å}}$$

$$\eta_{TOT} = (0.9)\left(2.5^{0.2}\right)^{-1}(0.7) \quad \text{where } 0.2 = k \text{ for V band}$$

$$\boxed{\eta_{TOT} = 0.52}$$

$$R_S = \eta_{TOT}\,\Delta\lambda_{bandpass}\,A\,f_\lambda = (0.52)(880\ Å)(\pi 230^2\ cm^2)\left(1.65\times10^{-6}\frac{photon}{s\cdot cm^2\cdot Å}\right) = \boxed{\overset{R_S}{123.6 \frac{photons}{s}}}$$

$$d = 2.3\ m = 2.3\times10^6\ \mu m$$

$$S = \frac{206265''}{(2.3\times10^6\ \mu m)(2.1)} = 0.0427 \frac{arcsec}{\mu m}$$

$$\frac{0.0427\ arcsec}{\mu m} \times \frac{13.5\ \mu m}{pixel} = 0.577 \frac{arcsec}{pixel} \longrightarrow \text{background rate in 1 pixel} = (0.577\,arcsec)^2$$

$$\text{Pixels across disk} = \frac{1.1''}{0.577 \frac{arcsec}{pixel}} = 1.91 \text{ pixels} \longrightarrow n_{pix} = 1.91^2\ pix$$

$$\eta_B = (0.9)(0.7) = 0.63$$

$$\mu_V = \frac{20\ mag}{arcsec^2} \longrightarrow \frac{20\,mag}{arcsec^2} \times \frac{0.577^2\,arcsec^2}{1\ pix} = 6.65\ mag/pix$$

$$f_\lambda = \frac{37.4\times10^{-12}\ \text{J}}{\text{s}\cdot\text{m}^2\cdot\text{nm}}\ 10^{-0.4(6.65)} = \frac{8.20\times10^{-14}\ \text{J}}{\text{s}\cdot\text{m}^2\cdot\text{nm}} \times \frac{1\ \text{m}^2}{100^2\ \text{cm}} \times \frac{1\ \text{nm}}{10\ \text{Å}} \times \frac{1\ \text{photon}}{3.65\times10^{-19}\text{J}}$$

$$= 2.25\ \frac{\text{photons}}{\text{s}\cdot\text{cm}^2\cdot\text{Å}}\ \text{per pixel} \quad \text{so...}$$

$$R_B = (0.63)(880\text{Å})(\pi 230^2\text{cm}^2)\left(2.25\ \frac{\text{Photons}}{\text{s}\cdot\text{cm}^2\cdot\text{Å}\cdot\text{pixel}}\right) = \boxed{2.07\times10^8\ \frac{\text{photons}}{\text{s}\cdot\text{pix}}}$$

This is clearly not right but I can't come up with anything better so I'm rolling with it.

From Problem 4:

$$R_S^2t^2 - \left(\frac{S}{N}\right)^2(R_S + n_{pix}R_D + n_{pix}R_B)t - \left(\frac{S}{N}\right)^2 n_{pix}N_R^2 = 0$$

$$\boxed{t = 4.9\times10^8\ \text{s} \approx 15\ \text{years}}$$

From Kitt Peak Manual:

$$R_B = 4.2\ \frac{\text{electrons}}{\text{s}\cdot\text{pixel}} \quad (\text{for V band @ New Moon})$$

Using this R_B, $t = 91.5$ s which is significantly shorter than 15 years. I am background limited because R_D + N_R are small enough to ignore, so I can't be detector limited and R_B is big enough to impact the S/N equation, especially in the first part of the problem.

6) Keck Imager:

QE = 0.80 S/N = 50 t = 10 mins $\Delta\lambda_{bandpass}$ = 50 Å

How long will it take to get S/N = 50 with QE = 0.95?

- Noise-limited case
- V-filter
- State any other assumptions you make

$$\frac{S}{N} = \frac{R_s t}{\sqrt{n_{pix}(R_D t + N_R^2)}}$$ Noise-limited case

Assumptions:

1) n_{pix} = 4 pixels (similar to npix from #5)
2) N_R = 4.5 electrons/pixel Same as #5
3) $R_D \approx 0.0028 \frac{\text{electrons}}{\text{s} \cdot \text{pix}}$ (from problem 4)
4) Other efficiencies = 70%
5) observing @ 1 airmass $\longrightarrow$ $X = 1$
6) $k = 0.2$ mag/airmass
7) 2.3 m telescope
8) $f_\lambda = 1.625 \times 10^{-6} \frac{\text{photon}}{\text{s} \cdot \text{cm}^2 \cdot \text{Å}}$ (Same as problem 5)

$$\eta_{TOT} = (0.95)(0.7)(2.5^{0.2})^{-1}$$

$$R_s = (0.95)(0.7)(2.5^{0.2})^{-1}(880\,\text{Å})(\pi 230^2\,\text{cm}^2)\left(1.625\times10^{-6} \frac{\text{photon}}{\text{s}\cdot\text{cm}^2\cdot\text{Å}}\right)$$

$$(S/N)^2\left(n_{pix} R_D t + n_{pix} N_R^2\right) = R_s^2 t^2$$

$$R_s^2 t^2 - (S/N)^2 n_{pix} R_D t - \left(\frac{S}{N}\right)^2 n_{pix} N_R^2 = 0$$

* Used Mathematica to solve for t *

$$\boxed{t = 3.42\,\text{s}}$$