

Noise: (Howell Ch 4.4)

- for N uncorrelated incoming photons, the uncertainty is \sqrt{N}

let

N_s - total number of photons detected from source

n_{pix} - number of pixels over which light is spread

N_B - total number of photons/pixel from background light

N_D - total number of photons/pixel from dark current

detector noise

N_R - total number of photons (electrons)/pixel of read noise

- dark current - electrons have thermal energy and sometimes it causes them to "jump out of their buckets" and be read as a detection, detectors are cooled to reduce this. You have to correct for dark current and account for it in noise estimates.
- read noise - the noise associated with reading out the array, direct trade off between read time and noise
 - common to use subarray to reduce time

signal-to-noise :

The signal, S , is just N_s . The noise is the sum, added in quadrature, of all noise sources. Using Poisson statistics, these are:

$$\sigma_s = \sqrt{N_s}$$

uncertainty on counts from star (source noise)

$$\sigma_B = \sqrt{n_{pix} N_B}$$

uncertainty due to counts in background (background noise)

$$\sigma_D = \sqrt{n_{pix} N_D}$$

uncertainty due to dark current (detector noise)

$$\sigma_R = \sqrt{n_{pix} N_R^2}$$

uncertainty due to read noise (detector noise)

N_s

$$S/N = \sqrt{N_s + n_{pix} (N_B + N_D + N_R^2)}$$

not Poisson noise,
detection of individual
photons, goes as N

To compute the S/N as a function of integration time, you need to know the rate of detection of photons from the source, R_s , the rate of detection of photons from the background per pixel, R_B , and the rate of dark current per pixel, R_D . (Note that the definition of rate assumes that quantum efficiencies have already been accounted for.) Then,

$$\frac{S}{N} = \frac{R_s t}{\sqrt{R_s t + n_{pix} (R_B t + R_D t + N_B^2)}}$$

Object Limited Case:

Ideally, you'd like to be source noise limited, but it's not always possible for faint sources or bright background.

$$\frac{S}{N} = \frac{R_s t}{\sqrt{R_s t}} = \sqrt{R_s t}$$

How does S/N scale with important properties?

Integration time (t):	$S \propto t$	$N \propto \sqrt{t}$	$\Rightarrow S/N \propto \sqrt{t}$
Telescope Aperture (A):	$S \propto A^2$	$N \propto \sqrt{A^2}$	$\Rightarrow S/N \propto A$
Source flux (R_s):	$S \propto R_s$	$N \propto \sqrt{R_s}$	$\Rightarrow S/N \propto \sqrt{R_s}$
Quantum Efficiency (Q):	$S \propto Q$	$N \propto \sqrt{Q}$	$\Rightarrow S/N \propto \sqrt{Q}$
Source Distance (D):	$S \propto D^{-2}$	$N \propto \sqrt{D^{-2}}$	$\Rightarrow S/N \propto 1/D$
Bandwidth (B):	$S \propto B$	$N \propto \sqrt{B}$	$\Rightarrow S/N \propto \sqrt{B}$
Extraction Aperture (W):	$S \propto W$	$N \propto W$	\Rightarrow unaffected (point)
Extraction Aperture (W):	$S \propto W^2$	$N \propto \sqrt{W^2}$	$\Rightarrow S/N \propto W$ (extended)

Detector Noise limited case:

$$S/N = \frac{R_{st}}{\sqrt{N_{pix} (R_{st} + N_e^2)}}$$

Int. time, read noise limited (t): $S \propto t$

$$N \propto \sqrt{N_{pix} N_e^2}$$

$$S/N \propto t$$

Int. time, dark noise limited (t): $S \propto t$

$$N \propto \sqrt{N_{pix} R_{st}}$$

$$S/N \propto \sqrt{t}$$

Telescope Aperture (A): $S \propto A^2$

$$N \propto A$$

$$S/N \propto A^2$$

Source flux (R_s): $S \propto R_s$

$$N \propto R_s$$

$$S/N \propto R_s$$

Quantum Efficiency (Q): $S \propto Q$

$$N \propto Q$$

$$S/N \propto Q$$

Source Distance (B): $S \propto D^{-2}$

$$N \propto D$$

$$S/N \propto D^{-2}$$

Bandwidth (B): $S \propto B$

$$N \propto B$$

$$S/N \propto B$$

Extraction Aperture, point (w): $S \propto w$

$$N \propto \sqrt{N_{pix}} \propto \sqrt{w^2}$$

$$S/N \propto w^{-1}$$

Extraction Aperture, ext (w): $S \propto w^2$

$$N \propto \sqrt{w^2}$$

$$S/N \propto w$$

Background limited case:

$$\frac{S}{N} = \frac{R_{st}}{\sqrt{N_{pix} (R_{st})}}$$

Integration time (t): $S \propto t$

$$N \propto \sqrt{N_{pix} R_{st}}$$

$$S/N \propto \sqrt{t}$$

Telescope Aperture (A): $S \propto A^2$

$$N \propto \sqrt{A^2}$$

$$S/N \propto A$$

Source flux (R_s): $S \propto R_s$

$$N \propto \sqrt{N_{pix} R_{st}}$$

$$S/N \propto R_s$$

Quantum Efficiency (Q): $S \propto Q$

$$N \propto \sqrt{Q}$$

$$S/N \propto \sqrt{Q}$$

Source Distance (B): $S \propto D^{-2}$

$$N \propto \sqrt{N_{pix} R_{st}}$$

$$S/N \propto D^{-2}$$

Bandwidth (B): $S \propto B$

$$N \propto \sqrt{B}$$

$$S/N \propto \sqrt{B}$$

Ext. Aperture, point (w): $S \propto w$

$$N \propto \sqrt{w^2}$$

$$S/N \propto w^{-1}$$

Ext. Aperture, ext (w): $S \propto w^2$

$$N \propto \sqrt{w^2}$$

$$S/N \propto w$$

The rate of photons is related to the flux by:
 $\xrightarrow{\text{aperture area}}$

$$R = N_{\text{total}} \cdot \Delta \lambda \cdot A \cdot f_\lambda \quad \rightarrow \text{usually in } \text{phs}^{-1} \text{cm}^{-2} \text{\AA}^{-1}$$

\hookrightarrow bandpass, can also be ΔV for f_V

\hookrightarrow total efficiency, can do $\Omega_{\text{atmo}} = (2.5 \times 10^{-1})^{-1}$

Z.B. You get 1 ph/s from a star whose PSF is 2 pix FWHM, meaning the light is spread over ~16 pixels. The CCD read noise is 4e-/pix and the sky brightness is 0.5 ph/s/pix and the dark current is 0.1 ph/s/pix. What exposure time is required to get S/N = 50?

$$R_{st}$$

$$\frac{S}{N} = \sqrt{R_{st} + N_{pix} (R_{st} + R_{dt} + N_e^2)}$$

try: $t = 26,600$ gives $S/N = 50.0715$

Z.B. For a 25th magnitude star in the R band, how long would be required to make a 10 σ detection with an f/10 2m telescope and a total system efficiency of 50%? Readnoise is 5 electrons, dark current is 36 electrons per hour, and sky background brightness is 25 mag/arcsec² using a CCD of 13 μ m pixels and 1" seeing. The R zero point is 2870 Jy.

Translate the magnitude into flux units:

$$f_V = 2870 \text{ Jy} \cdot 10^{-0.4 \cdot 25} = 2.87 \times 10^{-7} \text{ Jy} \cdot 10^{-25} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ Jy}^{-1}$$

$$= 2.87 \times 10^{-33} \text{ W m}^{-2} \text{ Hz}^{-1} = 2.87 \times 10^{-30} \text{ erg cm}^{-2} \text{ Hz}^{-1} \text{ s}^{-1}$$

$$\nu f_V = \lambda f_\lambda \Rightarrow f_\lambda = \frac{c}{\lambda^2} f_V = \frac{3 \times 10^{18} \text{ Å/s}}{(7000 \text{ Å})^2} \cdot 2.87 \times 10^{-30} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$$

$$f_\lambda = 1.75 \times 10^{-19} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

$$L_{ph} = \frac{6.626 \times 10^{-27} \text{ erg s} \cdot 3 \times 10^{18} \text{ Å/s}}{7000 \text{ Å}} = 2.8 \times 10^{-12} \text{ erg}$$

$$f_\lambda = \frac{1.75 \times 10^{-17} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}}{2.8 \times 10^{-12} \text{ erg ph}^{-1}} = 6.25 \times 10^{-8} \text{ ph s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

Multiply by the collecting area, bandpass, and total efficiency:

$$R_s = Q \cdot \Delta\lambda_{\text{bandpass}} \cdot A_{\text{aperture}} \cdot f_\lambda [\text{ph s}^{-1} \text{cm}^{-2} \text{\AA}^{-1}]$$
$$= 0.5 \cdot 1000 \text{\AA} \cdot \pi (1 \text{m} \cdot 100 \text{cm m}^{-1})^2 \cdot 6.25 \times 10^{-6} \text{ ph s}^{-1} \text{cm}^{-2} \text{\AA}^{-1}$$

$$R_s = 0.98 \text{ ph s}^{-1}$$

Doing the same calculation for the sky background gives:

$$R_B = 0.98 \text{ ph s}^{-1} \text{arcsec}^{-2}$$

The platescale is:

$$f = R \cdot D = 10 \cdot 2 \text{m} = 20 \text{m} \cdot 1000 \text{mm m}^{-1} = 20,000 \text{mm}$$
$$s = \frac{206265}{f} = \frac{206265}{20000 \text{mm}} = 10.3 \text{ arcsec mm}^{-1}$$

So one pixel subtends:

$$\text{pix} = 13 \mu\text{m} \cdot 10^{-3} \text{mm}/\mu\text{m} \cdot 10.3 \text{ arcsec mm}^{-1} = 0.13 \text{ arcsec}$$

With 1" seeing, star images will have a FWHM of ~ 7 pixels, so most of the light will be distributed over $N_{\text{pix}} = 49$ pixels.

Now calculate the background rate in one pixel. A pixel subtends:

$$(0.13 \text{ arcsec})^2 = 0.0169 \text{ arcsec}^2$$

so

$$R_B = 0.98 \text{ ph s}^{-1} \text{arcsec}^{-2} \cdot 0.0169 \frac{\text{arcsec}^2}{\text{pix}} = 0.02 \text{ ph s}^{-1} \text{pix}^{-1}$$

Convert the dark current:

$$R_D = 36 \text{ e}^- \text{ hr}^{-1} \cdot 60 \text{ s m}^{-1} \cdot 60 \text{ m hr}^{-1} = 0.01 \text{ e}^- \text{ s}^{-1}$$

Then:

$$\frac{S}{N} = \sqrt{\frac{0.98 \text{ ph s}^{-1} t}{0.98 \text{ ph s}^{-1} t + 49 \text{ pix} (0.02 \text{ ph s}^{-1} \text{ pix}^{-1} t + 0.01 \text{ e}^- \text{ s}^{-1}) + 5 \text{ e}^{-2}}}$$

By iteration, $t = 525 \text{ s}$ gives $S/N = 10.27$