

1) $V = 5000 \frac{\text{km}}{\text{s}}$ HB line $\Rightarrow 486.135 \times 10^{-9} \text{ m}$

Assuming J, H, K filters can detect this line within FWHM centered on effective λ midpoint

Filter	Eff. λ (nm)	FWHM (nm)	Total λ range (nm)
J	1220	213	$\rightarrow 1113.5 - 1326.5$
H	1630	307	$\rightarrow 1476.5 - 1783.5$
K	2190	390	$\rightarrow 1995 - 2385$

$$1 + z_{\text{dop}} = \sqrt{\frac{1 + v/c}{1 - v/c}} \rightarrow z = \sqrt{\frac{1 + \frac{5 \times 10^6 \text{ m/s}}{3 \times 10^8 \text{ m/s}}}{1 - \frac{5 \times 10^6 \text{ m/s}}{3 \times 10^8 \text{ m/s}}}} - 1$$

$$z_{\text{dop}} = 1.68 \times 10^{-2}$$

$$1 + z = (1 + z_{\text{dop}})(1 + z_{\text{cosm}}) = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}}$$

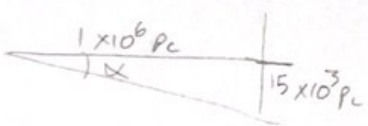
$$1 + z_{\text{cosm}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emit}}(1 + z_{\text{dop}})} \rightarrow z_{\text{cosm}} = \frac{\lambda_{\text{obs}}}{\lambda_c(1 + z_d)} - 1$$

$$z_{\text{cosm}} = \frac{\lambda_{\text{obs}}}{\lambda_c(1 + z_d)} - 1$$

J | $\lambda_{\text{obs}} = 1113.5 \text{ nm}, 1326.5 \text{ nm} \Rightarrow z_{\text{cosm}} = 1.25 - 1.68$

H | $\lambda_{\text{obs}} = 1476.5 \text{ nm}, 1783.5 \text{ nm} \Rightarrow z_{\text{cosm}} = 1.98 - 2.61$

K | $\lambda_{\text{obs}} = 1995 \text{ nm}, 2385 \text{ nm} \Rightarrow z_{\text{cosm}} = 3.04 - 3.82$

2)  $\alpha = \tan^{-1}\left(\frac{15 \times 10^3 \text{ pc}}{1 \times 10^6 \text{ pc}}\right) = 0.0150 \text{ rad}$
 $0.0150 \text{ rad} \cdot \frac{180}{\pi} = 0.859^\circ$

Total \angle size $= 2 \cdot \alpha = 0.03 \text{ rad} = 1.72^\circ$

3) 8m diam f/3 prime f/12 Nasmyth

$\rightarrow F_0 = 24 \text{ m}, 96 \text{ m}$

Plate Scale $= \frac{\theta}{F_0} \rightarrow \frac{206265}{24 \times 10^3 \text{ mm}} = 8.59''/\text{mm} \text{ OR } 2.14''/\text{mm}$

So pixels to get $0.5''/2 = 0.25''$

Pixel size: $\frac{0.25''}{8.59''/\text{mm}} = 0.0291 \text{ mm} \text{ OR } \frac{0.25''}{2.14''/\text{mm}} = 0.117 \text{ mm}$

FOV $= P \cdot N_{\text{pix}} = 0.25'' \cdot 2048 = 512'' = 8.53'$

$B = 9.5 \text{ mag}$ @ $\lambda_{\text{eff}} = 445 \text{ nm}$
 $f_{\nu}(Jy) = F_{\text{vega},0} 10^{-0.4m} = 4450 \text{ \AA}$ $f_{\nu}(Jy) = 4440$
 from notes

$$f_{\nu}(Jy) = 4440 Jy \cdot 10^{-0.4 \cdot 9.5} = 0.704 Jy$$

$$0.704 Jy \cdot \frac{10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}}{Jy} \cdot \frac{10^7 \text{ erg s}^{-1}}{W} \cdot \frac{\text{m}^2}{10^4 \text{ cm}^2} \cdot 1.5 \times 10^{11} \text{ Hz \AA}^{-1}$$

$$= 1.06 \times 10^{-12} \frac{\text{erg}}{\text{s cm}^2 \text{ \AA}}$$

$$\frac{\delta \nu}{\delta \lambda} = \frac{c}{\lambda^2} = \frac{3 \times 10^{10} \frac{\text{\AA}}{\text{s}}}{(4450 \text{ \AA})^2}$$

$h = 6.624 \times 10^{-27} \text{ erg} \cdot \text{s}$ $E = h \nu = \frac{hc}{\lambda} = 6.624 \times 10^{-27} \text{ erg} \cdot \frac{3 \times 10^{10} \frac{\text{\AA}}{\text{s}}}{4450 \text{ \AA}}$
 $E = 4.47 \times 10^{-12} \text{ erg}$

$$1.06 \times 10^{-12} \frac{\text{erg}}{\text{s cm}^2 \text{ \AA}} \cdot \frac{\text{Photon}}{4.47 \times 10^{-12} \text{ erg}} \Rightarrow \frac{2.38 \times 10^{-1} \text{ photon}}{\text{s cm}^2 \text{ \AA}}$$

5) $m_{AB} = 20$ @ 5500 \AA
 $f_{\nu}(Jy) = 3631 Jy 10^{-0.4 m_{AB}} \rightarrow 3631 Jy 10^{-0.4 \cdot 20} = 3.63 \times 10^{-5} Jy$
 $f_{\nu}(Jy) = F_{\text{vega},\lambda} 10^{-0.4m}$ $5500 \text{ \AA} = 550 \text{ nm} \Rightarrow \nu$
 $f_{\nu} \cdot F_{\text{vega},\lambda} = 10^{-0.4m} \rightarrow \log(f_{\nu} \cdot F_{\text{vega},\lambda}) = -0.4m$
 $m = \frac{\log(f_{\nu} \cdot F_{\text{vega},\lambda})}{-0.4} = \frac{\log(3.63 \times 10^{-5} Jy \cdot 3540 Jy)}{-0.4} = 2.23$

$$3.63 \times 10^{-5} Jy \cdot \frac{10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}}{Jy} \cdot \frac{10^7 \text{ erg s}^{-1}}{W} \cdot \frac{\text{m}^2}{10^4 \text{ cm}^2} \cdot \frac{3 \times 10^{10} \frac{\text{\AA}}{\text{s}}}{(5500 \text{ \AA})^2} \cdot \frac{\text{Photon}}{6.624 \times 10^{-27} \text{ erg} \cdot \frac{3 \times 10^{10} \frac{\text{\AA}}{\text{s}}}{5500 \text{ \AA}}}$$

$$= 9.96 \times 10^{-6} \frac{\text{photon}}{\text{cm}^2 \text{ s \AA}}$$

$$1 \frac{\text{MJy}}{\text{sr}} @ 5500 \text{ \AA}$$

$$1 \text{ sr} = 1 \text{ rad}^2 = 206265''^2$$

$$\frac{1 \text{ MJy}}{\text{sr}} \cdot \frac{\text{sr}}{(206265'')^2} \cdot \frac{10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}}{\text{Jy}} \cdot \frac{10^7 \text{ erg s}^{-1}}{\text{W}} \cdot \frac{\text{m}^2}{10^4 \text{ cm}^2}$$

$$= \frac{2.35 \times 10^{-38} \text{ erg}}{5 \text{ cm}^2 \text{ Hz (as)}^2}$$

$$\frac{2.35 \times 10^{-38} \text{ erg}}{5 \text{ cm}^2 \text{ Hz (as)}^2} \cdot \frac{3 \times 10^{18} \frac{\text{R}}{\text{s}}}{(5500 \text{ \AA})^2} = \frac{2.33 \times 10^{-17} \text{ erg}}{5 \text{ cm}^2 \text{ \AA (as)}^2}$$

$$\frac{2.33 \times 10^{-17} \text{ erg}}{5 \text{ cm}^2 \text{ \AA (as)}^2} \cdot \frac{\text{Photon}}{6.624 \times 10^{-27} \text{ erg} \cdot \text{s}} \cdot \frac{5500 \text{ \AA}}{3 \times 10^{18} \frac{\text{R}}{\text{s}}} = \frac{6.45 \times 10^{-6} \text{ Photons}}{5 \text{ cm}^2 \text{ \AA (as)}^2}$$

7) Imagine several bucket brigades trying to measure the amount of rainfall from a storm. They stand holding empty buckets under the storm for an amount of time that allows the buckets in the heaviest downpour to almost fill up. They then use umbrellas to block more rain from collecting before they pass their buckets down the line to the brigade on the end of the grid. This last brigade is oriented perpendicularly so they can pass each bucket one at a time, to the person who measures each bucket's water. In the stronger storms, they don't have to collect water very long to get enough water for measurement. In weak storms, they wait for longer. They also often stand under an artificial waterfall, so they can see if anyone who is supposed to be in the grid is absent, resulting in no collection bucket. They will also calibrate by checking if anyone has a trick bucket that is always full of water, distorting measurements. When they pass the buckets down the brigade in the measurement reading process, sometimes rain falls in or spills out leading to some more noise.