Homework 2 Solution:

1. Chromey figure 10.2 shows the J, H, K bands, which cover the following wavelength range within their FWHM:

	λ	z
J:	1.15 – 1.3 μm	1.326 – 1.63
H:	1.5 – 1.8 μm	2.03 – 2.64
K:	1.9 – 2.5 μm	2.84 – 4.06

The vacuum wavelength for Hβ is $\lambda_0 = 4862.7\text{Å}$. If the gas emitting this transition is moving away at 5,000 km/s due to the bulk orbit in a binary, it will appear at:

$$\frac{\lambda_{Doppler} - \lambda_{em}}{\lambda_{em}} = \frac{v}{c} \Rightarrow \lambda_{Doppler} = \lambda_{em}\left(1 + \frac{v}{c}\right)$$

$$\lambda_{Doppler} = 4862.7\text{Å}\left(1 + \frac{5000 \text{ km/s}}{3\times10^5 \text{ km/s}}\right) = 4943.7\text{ Å}$$

This will be cosmologically redshifted to the J, H, K bands. For the blue side of J:

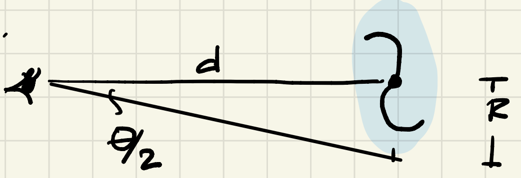
$$z_{cosmo} = \frac{1.15\times10^4\text{Å} - 4943.7\text{Å}}{4943.7\text{Å}} = 1.326$$

or:

$$1+z = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{1.15\times10^4\text{Å}}{4862.7\text{Å}} = 2.36$$

$$1+z = (1+z_{Doppler})(1+z_{cosmo}) \Rightarrow z_{cosmo} = \frac{1+z}{1+z_{Doppler}} - 1 = 1.326$$

2.



$$\tan(\theta/2) \approx \theta/2 = \frac{R}{d}$$

$$\theta = \frac{2R}{d} = \frac{30\text{kpc}}{1\text{Mpc}} = 1.72^\circ$$

3. The effective focal length is found from:

$$f\text{-ratio} = \frac{F}{D} \Rightarrow F = f\text{-ratio} \cdot D$$

The angular size of a pixel should be half of the seeing in order to Nyquist sample:

$$s = \tfrac{1}{2} \cdot 0.5'' = 0.25''$$

Prime Focus:

$$F = 3 \cdot 8\,m = 24\,m$$

$$\text{platescale} = \frac{1}{F} = \frac{206265''}{2.4 \times 10^7\,\mu m} = 0.0086\,''\mu m^{-1}$$

$$\text{pixel size} = \frac{0.25''}{0.0086\,''\mu m^{-1}} = 29.07\,\mu m$$

$$FOV = (0.25'' \cdot 2048)^2 = 512'' \times 512''$$

Nasmyth Focus:

$$F = 12 \cdot 8\,m = 96\,m$$

$$\text{platescale} = \frac{206265''}{9.6 \times 10^7\,\mu m} = 0.0021\,''\,\mu m^{-1}$$

$$\text{pixel size} = \frac{0.25''}{0.0021''\mu m^{-1}} = 119.05\,\mu m$$

$$FOV = 512'' \times 512''$$

4. $B = 9.5$

$f_{B,0} = 4440\,Jy$

$$f = f_{B,0} \cdot 10^{-0.4 m_B} = 4440\,Jy \cdot 10^{-0.4 \cdot 9.5}$$

$$f = 0.704\,Jy \cdot 10^{-23}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,Jy^{-1} = 7.04 \times 10^{-24}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}$$

At 4400Å (B band):

$$\frac{\Delta\nu}{\Delta\lambda} = \frac{c}{\lambda^2} = \frac{3 \times 10^{18}\,Å\,s^{-1}}{(4400\,Å)^2} = 1.55 \times 10^{11}\,Hz\,Å^{-1}$$

$$f = 7.04 \times 10^{-24}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1} \cdot 1.55 \times 10^{11}\,Hz\,Å^{-1} = 1.09 \times 10^{-12}\,erg\,s^{-1}\,cm^{-2}\,Å^{-1}$$

The photon energy is:

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-27}\,erg\cdot s \cdot 3\times10^{18}\,Å/s}{4400\,Å} = 4.5\times10^{-12}\,erg\ phot^{-1}$$

$$f = \frac{1.09\times10^{-12}\,erg\,s^{-1}\,cm^{-2}\,Å^{-1}}{4.5\times10^{-12}\,erg\ phot^{-1}} = 0.24\ ph\,s^{-1}\,cm^{-2}\,Å^{-1}$$

5. $V_{AB} = -2.5\log F(Jy) + 8.9 = 20$

$$F = 3.63\times10^{-5}\,Jy = 3.6\times10^{-28}\,erg\,s^{-1}\,cm^{-2}\,Å^{-1} = 9.9\times10^{-6}\,ph\,s^{-1}\,cm^{-2}\,Å^{-1}$$

The Johnson V magnitude is given by:

$$m_V = -2.5\log(f/f_{V,0}) = -2.5\log\left(\frac{3.63\times10^{-5}\,Jy}{3540\,Jy}\right)$$

$m_V = 19.97$

6. Conversions:

$1\,sr = 4.25\times10^{10}\,arcsec^2$

$1\,Jy = 10^{-23}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}$

$$\Delta\nu/\Delta\lambda = \frac{3\times10^{18}\,Å\,s^{-1}}{(5500\,Å)^2} = 9.92\times10^{10}\,Hz\,Å^{-1}$$

$$E = \frac{6.62\times10^{-27}\,erg\,s \cdot 3\times10^{18}\,Å\,s^{-1}}{5500\,Å} = 3.61\times10^{-12}\,erg\ phot^{-1}$$

$$S = 1\times10^{6}\,Jy\,sr^{-1}\cdot 10^{-23}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,Jy^{-1}\cdot\frac{1\,sr}{4.25\times10^{10}\,arcsec^2}$$

$$S = 2.35\times10^{-28}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,arcsec^{-2}\cdot 9.92\times10^{10}\,Hz\,Å^{-1}$$

$$S = 2.33\times10^{-17}\,erg\,s^{-1}\,cm^{-2}\,Å^{-1}\,arcsec^{-2}$$

$$S = 2.33\times10^{-17}\,erg\,s^{-1}\,cm^{-2}\,Å^{-1}\,arcsec^{-2}\cdot\frac{1}{3.61\times10^{-12}\,erg\ phot^{-1}}$$

$$S = 6.46\times10^{-6}\,phot\,s^{-1}\,cm^{-2}\,Å^{-1}\,arcsec^{-2}$$

$$\mu = -2.5\log\left(\frac{2.3\times10^{-5}\,Jy\,arcsec^{-2}}{3540\,Jy}\right) = 20.44\ mag\ arcsec^{-2}$$