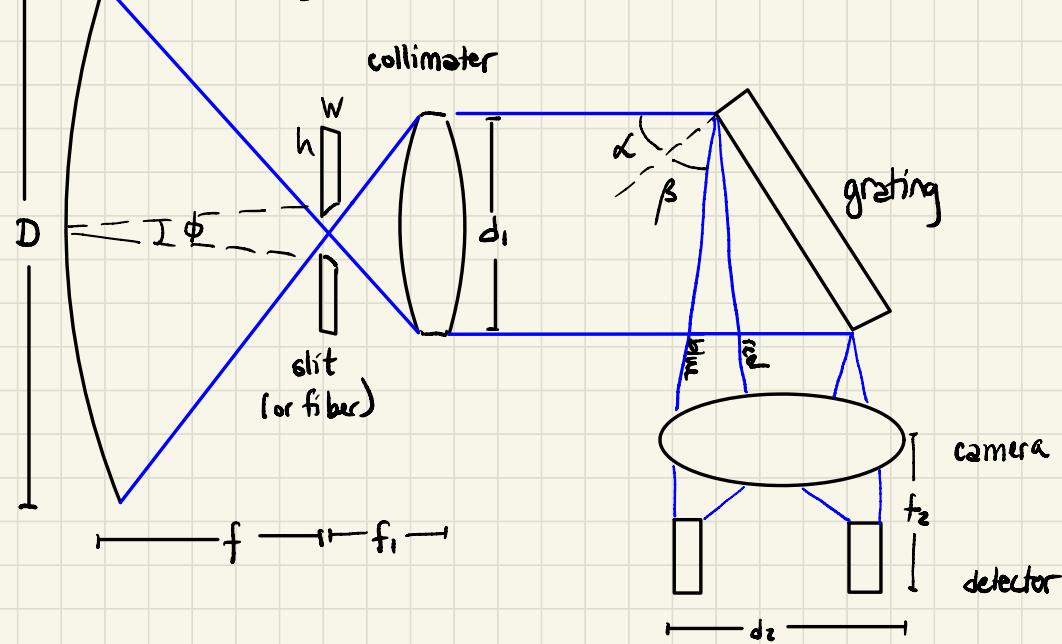


## Slit and fiber spectrometers:

- address several issues common in slitless spectrometers:  
e.g. overlapping spectra, lack of convenient wavelength calibration
- basic idea of improvement is to limit light that reaches the disperser, cost is that you lose all light that doesn't come through slit/fiber
- as long as slit is smaller than typical seeing, it limits the purity of light entering system

enhance pupil (primary)



w - physical slit width

h - physical slit height

$\phi = w/f$  - angular width of slit

$\phi' = h/f$  - angular height of slit

$w' = \text{reimaged slit width} = w \times \text{magnification} = h f_2 / f_1$

$h' = \text{reimaged slit height} = h \times \text{magnification} = w f_2 / f_1$

That's without a disperser. Adding a disperser magnifies the image of the slit.

$\alpha$  - angle of collimated rays to grating normal

$\beta$  - angle of reflected rays to grating normal

Design so:  $w' = r w f_2 / f_1$ , where  $r = \text{anamorphic mag.} = \cos \alpha / \cos \beta \approx 1$

To avoid losing light at the top/bottom of the collimator or making it bigger than needed, it needs the same focal ratio as the primary:

$$\frac{D}{f} = \frac{d_i}{f_1} \Rightarrow f_1 = \frac{d_i f}{D}$$

Then the slit dimensions can be written:

$$h' = h \left( f_2 / f_1 \right) = h \frac{D f_2}{d_i f} = \left( \frac{h}{f} \right) \frac{D f_2}{d_i} = \phi' \frac{D f_2}{d_i}$$

$$w' = w \left( f_2 / f_1 \right) = w \frac{D f_2}{d_i f} = \left( \frac{w}{f} \right) \frac{D f_2}{d_i} = \phi \frac{D f_2}{d_i}$$

where  $\phi = w/f$  is the angular size of the slit width on the sky  
and  $\phi' = h/f$  is the angular height.

Z.B. You are designing a spectrograph for use with a 5m f/12 telescope with 1" seeing. How wide should the slit width be in  $\mu\text{m}$ ?

Set the slit width equal to typical seeing so the system is not seeing limited.

$$f = D \cdot \text{f ratio} = 5\text{m} \cdot 12 = 60\text{m}$$

$$\text{platescale} = \frac{206265''}{f} = \frac{206265''}{60 \times 10^6 \mu\text{m}} = 0.003 \frac{''}{\mu\text{m}}$$
$$w = 1''. \left( 0.003 \frac{''}{\mu\text{m}} \right)^{-1} = 333 \mu\text{m}$$

Let's say the above system is used with a detector with  $24 \mu\text{m}$  pixels. What camera-collimator focal length ratio is needed to Nyquist sample a monochromatic image of the slit?

$$w' = 2 \text{ pix} = 2 \cdot 24 \mu\text{m} = 48 \mu\text{m}$$

$$w' = f_2/f_1 w, w = 333 \mu\text{m}$$

$$f_2/f_1 = w'/w = 48 \mu\text{m} / 333 \mu\text{m} = 0.14$$

What happens if you want high dispersion?

Recall, linear dispersion depends only on camera focal length,  $f_2$ .

$$\frac{dl}{d\lambda} = \frac{f_2 m}{dcos\beta}$$

High dispersion means big  $f_2$  means big optics.

## Spectral resolution:

Spectral resolution requires matching  $\omega'$  to the pixel size at the detector. For Nyquist sampling,  $\omega'$  should cover two pixels. For two spectral features to be resolved, they must be separated by at least  $\omega'$ . Thus, the slit size determines spectral resolution.

Since  $\omega' = \omega (f_2/f_1) = \phi D f_2/d_1$ , fixed  $\omega'$  and  $\phi$  implies that  $D f_2/d_1 = \text{constant} = \left(\frac{d_1}{d_2}\right) D f_2/d_1 = D \frac{f_2}{d_2} = D \text{ (camera f\#)}$

So, for a larger telescope,  $D$ , the ratio  $f_2/d_2$  must get small, implying a short focal length cameras (i.e. fast,  $f/2$  = fast,  $f/8$  = slow)

The limit of spectral resolution can be obtained by considering a slit of width  $\omega$  illuminated by light of wavelength  $\lambda$  to  $\lambda + \Delta\lambda$ . The slit image at each wavelength  $\omega'$  and the centers of the images are separated by

$$\Delta l = f_2 A \Delta\lambda$$

→ angular dispersion

The limit of spectral resolution  $\Delta\lambda$  is then the difference for which  $\Delta l = \omega'$ .

$$\Delta\lambda = \underbrace{\frac{1}{f_2 A}}_{\text{linear dispersion}} \Delta l = \frac{1}{f_2 A} \omega' = \frac{1}{f_2 A} \cdot \frac{r \phi D f_2}{d_1} = \frac{r \phi D}{A d_1} = \frac{r \phi D d \cos\beta}{m d_1}$$

linear dispersion  $\frac{d\lambda}{dl}$

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\lambda d_1 A}{r \phi D} = \frac{\lambda d_1}{r \phi D} \cdot \frac{m}{d \cos\beta} = \frac{\lambda d_1}{r \phi d} \cdot \frac{\sin\beta + \sin\alpha}{\lambda \cos\beta} = \frac{\lambda d_1 f}{r \omega D} \frac{m}{d \cos\beta}$$

→ resolution, higher is better

- $\phi$  - open the slit, resolution gets chunkier
- $D$  - bigger telescope gives worse spectral resolution
- $\beta$  - large angle of dispersion, better resolution
- $m$  - higher order, better resolution
- $d_1$  - bigger collimator, better resolution

z.B. Suppose you are using an  $f/12$  8m telescope with a 1200  $\text{line/mm}$  grating in first order. What combination of things do you need to achieve  $R = 4000$  at  $4000\text{\AA}$ ?

$$R = \frac{\lambda}{\Delta\lambda} = \frac{\lambda d_1 f}{w \cos\beta} \cdot \frac{m}{d \cos\beta} \quad \text{known}$$

$$\frac{d_1}{w \cos\beta} = \frac{R r D d}{\lambda f m} = \frac{4000 \cdot 1 \cdot 8\text{m} \cdot \frac{1}{1200 \text{line/mm}}}{4000\text{\AA} \cdot 12 \cdot 8\text{m} \cdot 1} = \frac{4000 \cdot \text{mm}}{4000 \times 10^{-7} \text{mm} \cdot 12 \cdot 1200} = 694$$

Choose  $d_1, w, \beta$ . Minimize  $d_1$ , it's expensive to make big. Need  $\beta < 90^\circ$ .

$d_1$	$\beta$	$w = \frac{d_1}{694 \cos\beta}$
2m (huge!)	$80^\circ$	$16.6 \text{mm}$ !
1m	$45^\circ$	$2 \text{mm}$
0.3m	$1^\circ$	$432 \mu\text{m}$

## Detailed Example of Spectrograph Design Considerations:

spectrograph design is a highly constrained problem. Each choice you make ties you into other choices.

Let's say you want to design a longslit spectrograph for an 8m telescope at the f/12 Nasmyth focus. Your goal is to do H $\alpha$  rotation curves of nearby galaxies at 25 km/s resolution. Typical seeing is 0.6 arcsec.

At H $\alpha$   $\lambda = 6563\text{\AA}$ , 25 km/s resolution requires:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \Delta\lambda = \frac{v\lambda}{c} = \frac{25 \text{ km/s} \cdot 6563\text{\AA}}{3 \times 10^5 \text{ km/s}} = 0.55\text{\AA}$$
$$R = \frac{\lambda}{\Delta\lambda} = \frac{6563\text{\AA}}{0.55\text{\AA}} \approx 12000$$

1. First consider the physical slit size. At the focal plane, typical seeing is 0.6" so we probably want a physical slit width of 0.6". Its physical width will be:

$$\text{platescale} = \frac{206265''}{f} = \frac{206265''}{12.8\text{m}} = 2.15''/\text{mm}$$
$$w = \frac{\phi}{\text{platescale}} = \frac{0.6''}{2.15''/\text{mm}} = 0.27\text{mm}$$

The slit length is driven by typical galaxy size. To cover a typical 60 kpc galaxy in the Virgo Cluster at 20 Mpc:

$$\Theta = \frac{\phi}{L} = \frac{60 \text{ kpc}}{20 \text{ Mpc}} = 0.003 \text{ rad} = 620'' \approx 10 \text{ arcmin}$$
$$h = \frac{\phi'}{\text{platescale}} = \frac{620''}{2.15''/\text{mm}} = 288 \text{ mm}$$

2. To Nyquist sample a star at the detector 2 pix per PSF.  
 Modern detectors have 9-24  $\mu\text{m}$  pixels. Suppose you have 24  $\mu\text{m}$  pixels (best case), you want 0.6 arcsec ( $0.27\text{mm} = 270\mu\text{m}$ ) to project to 48  $\mu\text{m}$ . So the angular plate scale needed at the detector is  $0.6''/48\mu\text{m} = 0.0125''/\mu\text{m}$ . This means you need a camera/collimator focal ratio  $f_2/f_1 = w/w = 48/270 \Rightarrow 5.5/1$ . Since the f-ratio of the collimator is  $f/12$  (to match the primary), so let's try:

$$d_1 = 100\text{mm}, \quad f_1 = 12 \cdot 100\text{mm} = 1200\text{mm}$$

$$\text{That requires } f_2 = f_1/5.5 = 218\text{mm}.$$

The diameter of the collimated beam going through the grating is 100mm, so we need  $d_2 > 100\text{mm}$ . The f-ratio of the camera is

$$\text{f-ratio, camera} = \frac{f_2}{d_2} = \frac{218\text{mm}}{100\text{mm}} \Rightarrow f/2.1, \text{ a very fast camera!}$$

3. What does this do to lock in our linear dispersion?

$$\text{reciprocal dispersion } (\text{\AA}/\mu\text{m}) = \frac{\delta\lambda}{\delta\lambda} = \frac{1}{f_2} \frac{\delta\lambda}{\delta\beta} = \frac{1}{f_2 A} = \frac{d \cos\beta}{f_2 m}$$

If  $d = 1200\text{mm}$ ,  $m = 2$ ,  $d = \beta = 50^\circ$  (Littrow configuration),  $f_2 = 218\text{mm}$

$$\text{reciprocal disp.} = \frac{1}{218\text{mm} \cdot 2} = 0.012 \text{\AA}/\mu\text{m}$$

With 24  $\mu\text{m}$  pixels, this is  $0.29 \text{\AA}/\text{pix}$ . This is slightly too big. We wanted  $0.55 \text{\AA}/2 = 0.275 \text{\AA}/\text{pix}$  from the spectral resolution. But we can certainly slightly oversample, so let's continue.

4. Let's figure out what spectral resolution we're getting.

$$R = \frac{\lambda d f m}{r w D d \cos \beta}$$

$$= \frac{6563 \times 10^{-4} \mu\text{m} \cdot 100 \times 10^3 \mu\text{m} \cdot (12.8 \times 10^6 \mu\text{m}) \cdot 2}{1.270 \mu\text{m} \cdot 8 \times 10^6 \mu\text{m} \cdot 1200 \times 10^{-3} \mu\text{m} \cdot \cos 50^\circ}$$

$$R = 10,890$$

Not bad. What can we do to increase  $R$ ?

- Could do a higher  $\text{L/mm}$  grating. This only impacts reciprocal dispersion, which was slightly high anyway.

5. How big of a detector do you need to get the whole slit length?

$$h = 620'' / \text{pixelscale at detector} \Rightarrow \frac{620''}{0.0125''/\mu\text{m}} = 49600 \mu\text{m}$$

With  $24 \mu\text{m}$  pixels, this is too big to fit on a  $2048 \times 2048$  detector! Will need a  $4096 \times 4096$  detector.