Homework 3

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- I used overleaf to write these solutions.
- There are footnotes at the end of pages to indicate the source of references.
- The completed solution has been denoted by the QED symbol ($\square$)
- To solve all the problems, I used the table (figure 1) taken from the class notes.

Band	λ_{eff} (μm)	f_ν (Jy)
U	0.36	1880
B	0.44	4440
V	0.56	3540
R	0.70	2870
I	0.90	2250

Figure 1: Table of flux values in Jansky for each of the band used for solving the problems

1

Given the relation:

$$z = \frac{v}{c}$$

For a radial velocity of $v = 5,000\,\text{km/s}$ and $c \approx 3 \times 10^5\,\text{km/s}$, the redshift z is:

$$z \approx 0.0167$$

The observed wavelength λ_{obs} for an emitted wavelength λ_{em} due to redshift is:

$$\lambda_{obs} = \lambda_{em}(1 + z)$$

For the Hβ line, $\lambda_{em} \approx 4861\,\text{Å}$. Thus:

$$\lambda_{obs} \approx 4939.3\,\text{Å}$$

The typical wavelength ranges for the J, H, and K bands are: [1]

$$\begin{aligned}
\text{J-band} &: 11,000 - 14,000\,\text{Å} \\
\text{H-band} &: 15,000 - 18,000\,\text{Å} \\
\text{K-band} &: 20,000 - 24,000\,\text{Å}
\end{aligned}$$

Given this, the Hβ line does not fall into any of these windows for the given radial velocity.

To get the wavelength from the object in J-band, H-band, and K-band we need to have a greater speed. In fact, this speed should be so great that we will need to use Relativistic Doppler shift.

In case of relativistic Doppler shift, we know:

$$1 + z = \sqrt{\frac{1+\beta}{1-\beta}}$$

$$\Rightarrow \frac{(1+z)^2}{1} = \frac{1+\beta}{1-\beta}$$

By the rule of componedo-dividendo rule,

$$\Rightarrow \frac{(1+z)^2+1}{(1+z)^2-1} = \frac{2}{2 \cdot \beta}$$

$$\Rightarrow \beta = \frac{(1+z)^2-1}{(1+z)^2+1}$$

[1] https://www.andovercorp.com/products/astronomy-filters/infrared-astronomy/

As we know,

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}}$$

,

$$\Rightarrow z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$$

. So, for minimum J-band emission, we would need:

$$z = \frac{11000}{4861} - 1 = 2.262908866$$

And,

$$\beta = \frac{(1 + 2.262908866)^2 - 1}{(1 + 2.262908866)^2 + 1} = 0.828275684$$

So, for minimum H-band emission, we would need:

$$z = \frac{15000}{4861} - 1 = 3.085784818$$

And,

$$\beta = \frac{(1 + 3.085784818)^2 - 1}{(1 + 3.085784818)^2 + 1} = 0.8869650183$$

So, for minimum J-band emission, we would need:

$$z = \frac{20000}{4861} - 1 = 4.114379757$$

And,

$$\beta = \frac{(1 + 4.114379757)^2 - 1}{(1 + 4.114379757)^2 + 1} = 0.9263538382$$

All of these velocities, (0.83c, 0.89c, 0.93c) are very close to the light speed, and the black hole needs to have a radial velocity of this higher if we want to detect them in J, H, and K band. $\square$

2

Given a galaxy with a physical diameter d and a distance D, the angular size θ in radians can be approximated as:

$$\theta \approx \frac{d}{D} \tag{1}$$

For a galaxy with $d = 30\text{kpc}$ and $D = 1\text{Mpc} = 1000\text{kpc}$:

$$\theta \approx \frac{30\text{kpc}}{1000\text{kpc}} = 0.03 \text{ radians} \tag{2}$$

To convert this angle from radians to degrees:

$$\theta(\text{degrees}) = \theta(\text{radians}) \times \frac{180^\circ}{\pi} \tag{3}$$

Substituting in the value for θ from the previous equation:

$$\theta(\text{degrees}) \approx 0.03 \times \frac{180^\circ}{\pi} \approx 1.72^\circ \tag{4}$$

Thus, the galaxy would appear to be approximately 1.72° in diameter in the sky. $\square$

3

Given a telescope of diameter $D = 8000$ mm (8 meters), the image scale S in arcseconds per millimeter is given by:

$$S = 206265 \times \frac{1}{f} = 206265 \times \frac{1}{R \cdot D}$$

Here, R is the focal ratio and D is the diamter of the telescope

We calculate platescale for each of the cases,

For Prime focus (f/3):

$$S_{\text{prime}} = 206265 \times \frac{1}{3 \times 8 \times 1000} = 8.594 \text{ arcsec/mm}$$

For Nasmyth focus (f/12):

$$S_{\text{nasmyth}} = 206265 \times \frac{1}{12 \times 8 \times 1000} = 2.149 \text{ arcsec/mm}$$

To Nyquist sample the 0.5 arcsec FWHM seeing, the sampling should be at least every 0.25 arcsec.

Now we calculate the pixel size,

Pixel size at Prime focus:

$$\text{Pixel size} = \frac{0.25 \text{ arcsec}}{8.594 \text{ arcsec/mm}} = 29.09 \text{ microns}$$

Pixel size at Nasmyth focus:

$$\text{Pixel size} = \frac{0.25 \text{ arcsec}}{30.94 \text{ arcsec/mm}} = 116.33 \text{ microns}$$

Now we can calculate the field of view (FOV),

$$\text{FOV} = 0.25\,\text{arcsec} \times 2048 = 512'' = 8.533'$$

So,

- For Prime focus, pixels should be 29.09 microns.
- For Nasmyth focus, pixels should be 116.33 microns
- The FOV would be approximately 8.533 arcminute. □

4

Given a star with B magnitude $B = 9.5$.

Converting B magnitude to Flux in Jy:

The zero point flux density for the B band is 4440 Jy. So, we using the magnitude flux relation we can have:

$$f_\nu = 4440 \times 10^{0.4 \times 9.5} = 7.037^{-1}\text{Jy} = 0.7037\text{Jy}$$

Converting f_ν in Jy to f_λ in $erg\, s^{-1}\, cm^{-2}\, \text{Å}^{-1}$: Using:

$$f_\lambda = f_\nu \times \frac{c}{\lambda^2}$$

We have, $f_\nu = 7.037 \times 10^{-1} \times 10^{-23}$ erg s^{-1} cm^{-2} Hz^{-1} $= 7.037 \times 10^{-24}$ erg s^{-1} cm^{-2} Hz^{-1}

So,

$$f_\lambda = 7.037 \times 10^{-1} \times 10^{-23} \times \frac{3 \times 10^{18}}{(4400)^2} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

$$= 1.09 \times 10^{-12} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

Using Planck's constant $h \approx 6.626 \times 10^{-27}$ erg s and speed of light $c \approx 3 \times 10^{10}$ cm/s, for the B-band $\lambda \approx 4400$ Å:

$$E \approx \frac{6.626 \times 10^{-27} \text{ erg s} \times 3 \times 10^{10} \text{ cm/s}}{4400 \times 10^{-8}}$$

$$\implies E \approx 4.51 \times 10^{-12} \text{ erg/photon}$$

Flux in terms of photons:

$$F_{B,\text{photon}} \approx \frac{1.09 \times 10^{-12} \text{ erg s}^{-1}\text{cm}^{-2} \text{ Å}^{-1}}{4.51 \times 10^{-12} \text{ erg/photon}}$$

$$\implies F_{B,\text{photon}} \approx 0.2411 \text{ photons s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

So, - The flux in erg s^{-1} cm^{-2} Å^{-1} is approximately 1.09×10^{-12}. - The flux in photons s^{-1} cm^{-2} Å^{-1} is approximately 0.2411. - The flux in Jy is approximately 0.7037. □

5

Given a star with an AB magnitude $m_{AB} = 20$ at $5500\AA$.

Converting AB magnitude to Flux Density f_ν in Jy:

$$f_\nu = 3631 \times 10^{-0.4\times 20}$$
$$\implies f_\nu \approx 3.631 \times 10^{-5} \text{ Jy}$$

Converting f_ν to Johnson V magnitude:

Using the relation:

$$m_V = -2.5 \log_{10}\left(\frac{f_\nu}{f_{\nu,0}}\right)$$

where $f_{\nu,0} \approx 3540$ Jy:

$$m_V \approx -2.5 \log_{10}\left(\frac{3.631 \times 10^{-5}}{3540}\right)$$
$$\implies m_V \approx 19.97$$

Converting f_ν in Jy to f_λ in erg s^{-1} cm^{-2} $\AA^{-1}$:

Using the relation:

$$f_\lambda = f_\nu \times \frac{c}{\lambda^2}$$

and $f_\nu \approx 3.631 \times 10^{-5} \times 10^{-23}$ erg s^{-1} cm^{-2} Hz^{-1}:

$$f_\lambda \approx 3.631 \times 10^{-5} \times 10^{-23} \times \frac{3 \times 10^{18}}{5500^2} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$
$$\implies f_\lambda \approx 3.6 \times 10^{-17} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

Converting f_λ in erg s^{-1} cm^{-2} $\AA^{-1}$ to photons s^{-1} cm^{-2} $\AA^{-1}$:

Using the energy of a photon:

$$E = \frac{hc}{\lambda}$$
$$E \approx \frac{6.626 \times 10^{-27} \times 3 \times 10^{10}}{5500 \times 10^{-8}}$$
$$\implies E \approx 3.61 \times 10^{-12} \text{ erg/photon}$$

Flux in terms of photons:

$$f_{\lambda,\text{photon}} = \frac{f_\lambda}{E}$$
$$f_{\lambda,\text{photon}} \approx \frac{3.60 \times 10^{-17}}{3.61 \times 10^{-12}}$$
$$\implies f_{\lambda,\text{photon}} \approx 9.97 \times 10^{-6} \text{ photons s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1}$$

So,

- The Johnson V magnitude is approximately $m_V \approx 19.97$.
- The flux in photons $s^{-1}cm^{-2}\AA^{-1}$ is approximately 9.97×10^{-6}. $\square$

6

Firstly,

$$1\,\text{steradian} = 1\text{rad}^2 = (\frac{180}{\pi})^2\text{deg}^2 = (\frac{180 \cdot 60 \cdot 60}{\pi})^2\text{arcsec}^2 = 4.255{\times}10^{10}\text{arcsec}^2$$

Given: Surface brightness of 1 MJy per steradian at 5500 $\AA$
Conversion to $erg\, s^{-1}\; cm^{-2}\; Hz^{-1}\; arcsec^{-2}$:

$$1\,\text{MJy/steradian} = \frac{10^{-23} \times 10^6\,\text{erg s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}}{4.255 \times 10^{10}\,\text{arcsec}^2}$$
$$\Rightarrow 2.35 \times 10^{-28}\,\text{erg s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}\text{arcsec}^{-2} \quad \Box$$

Conversion to erg s^{-1} cm^{-2} Å^{-1} arcsec^{-2}: We know that,

$$f_\lambda = f_\lambda \times \frac{c}{\lambda^2}$$

$$f_\lambda = 2.35 \times 10^{-28} \times \frac{3 \times 10^{18}}{5500^2}\text{erg s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\text{arcsec}^{-2}$$
$$\Rightarrow 2.31 \times 10^{-17}\,\text{erg s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\text{arcsec}^{-2} \quad \Box$$

Conversion to photons s^{-1} cm^{-2} Å^{-1} arcsec^{-2}:
From the solution of the previous problems we know that the energy of one photon is 3.61×10^{-12} erg s^{-1}. So, the flux in photons s^{-1} cm^{-2} Å^{-1} arcsec^{-2} will be,

$$f_{\lambda,\text{photon}} = \frac{2.31 \times 10^{-17}}{3.61 \times 10^{-12}}\text{photons s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\text{arcsec}^{-2}$$
$$\Rightarrow 6.40 \times 10^{-6}\,\text{photons s}^{-1}\text{cm}^{-2}\text{Å}^{-1}\text{arcsec}^{-2} \quad \Box$$

Conversion to mag arcsec^{-2}:
5500 $\AA$ means it is approximately in V band, which has a zero-magnitude f_ν of 3540 Jy. In case of the astronomical source we have, the surface brightness in Jy arsec^{-2} will be,

$$1\frac{\text{MJy}}{\text{steradian}} = \frac{10^6}{4.255 \times 10^{10}} = 2.35 \times 10^{-5}\frac{\text{Jy}}{\text{arcsec}^2}$$

So, the brightness in magnitude per arcsec^2 will be,

$$m = -2.5\log_{10}\left(\frac{2.35 \times 10^{-5}}{3540}\right)$$
$$\Rightarrow m \approx 20.44\,\text{mag arcsec}^{-2} \quad \Box$$

7

A very easy explanation of what is CCD and its function:

In Astronomical Observations we use a very well known instrument called "Telescope." Telescopes are used more like a light collector. For observation and further research purposes, we need these light collected by the light collector (telescope) to be processed into electrical signal so that we can read them in the computer. One of the devices that help these light transform to electrical signals is CCD or Charge Couple Device.

Now, let's dive deeper into the fundamental physics of the light detection process of CCD.

The CCD is made of a material called silicon. Silicon is a very special substance because of its atomic structure. It's designed in such a way that when a photon (a tiny particle of light) hits it, it releases an electron. This process is known as the photoelectric effect. The more light (or photons) that hits a particular spot on the CCD, the more electrons get released. Think of these electrons as tiny buckets of charge. Each bucket fills up in proportion to the amount of light it receives.

Now, how do we turn these buckets of charge into an image? This is where the "Charge Coupled" part of CCD comes into play. Starting at one corner of the grid, the CCD shifts all the buckets of charge, row by row, column by column, similar to how we'd read a book: one word at a time, from left to right, top to bottom. As each bucket reaches the end of a row, its charge is measured very precisely. This measurement is then converted into a digital value, which will eventually determine the brightness and color of each pixel in the final image.

As the CCD finishes reading the entire grid, it sends all these digital values to the computer, which then compiles them into an image. So, the photo we see on the screen is a translation of the charge from millions of tiny buckets, which themselves were filled by the light from the stars!

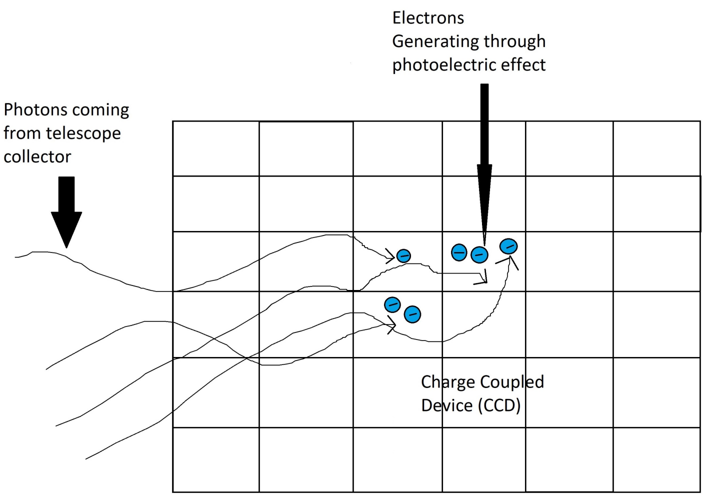

Figure 2: Charge Coupled Device

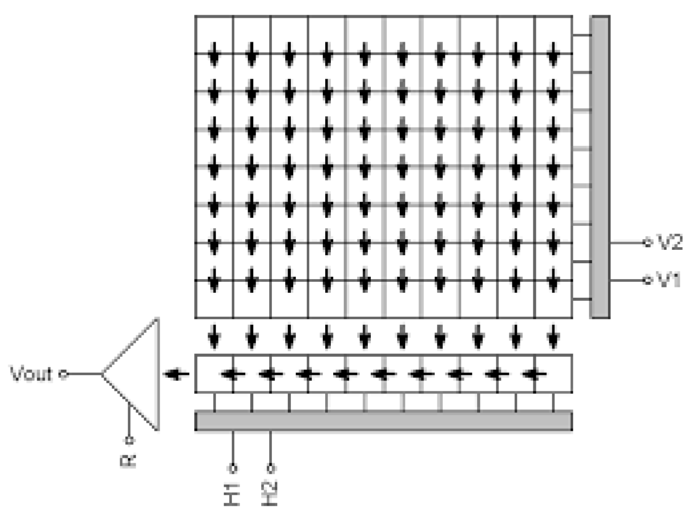

Figure 3: How charges of one row is counted at a time in CCDs.
Source: https://www.gxccd.com/art?id=303&lang=409