Lecture 23

04/14/2022

Goals of Time Series Analysis

① Is the system variable over some timescale?

$\neq$ periodic, necessarily

→ NO ⇒ there's no point doing further time-series analysis

→ YES ⇒ characterize the temporal correlation i.e. model selection and parameter estimation.

⇒ predict future values of data.

e.g. $y(t) = A\sin(\omega t) + \varepsilon$

$\varepsilon \sim N(0, \sigma)$

$$V_y = \sigma^2 + \frac{1}{2}A^2$$

$$\underbrace{\chi^2_{dof} = \frac{1}{N}\sum_i \left(\frac{y_i}{\sigma}\right)^2 \sim \frac{V}{\sigma^2}}$$

if $A = 0$ ... $\chi^2_{dof} = 1$

if $A \neq 0$ ... $\chi^2_{dof} > 1$

$$\sigma_{\chi^2_{dof}} = \sqrt{2/N}$$

$$\Rightarrow P\left(\chi^2_{dof} > \underbrace{1 + 3\sqrt{2/N}}_{3\sigma \text{ deviation}}\right) \sim 10^{-3}$$

$\therefore$ Minimum detectable amplitude (as given by a 3σ tension with noise model)

$$\hookrightarrow \underbrace{A_{min} = \frac{2.9\sigma}{N^{1/4}}}$$

$\Rightarrow$ with enough data, we can dig very small signals out of large noise.

- Parameter Estimation & Model Selection

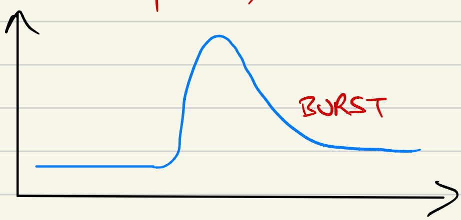
  * Similar to regression and other model fitting + comparison.
  * BUT... "x" is replaced by "t", and now the data are sequential.

$\Rightarrow$ Fourier Analysis:

$\rightarrow$ decompose a time-series into a sum of sine and cosine basis functions of varying harmonics of a base timescale.

i.e. $$y_i(t_i) = \underbrace{Y_0}_{\text{constant}} + \underbrace{\sum_{m=1}^{M} \beta_m \sin(m\omega t_i + \phi_i)}_{\text{reconstruct any periodic function!}} + \underbrace{\varepsilon_i}_{\text{measurement noise}}$$

⇒ Temporally-localized Signals



↳ can use matched filtering i.e. match a template against the data.

- Periodic Signals

  (i) Fourier transform → easy for evenly sampled data.

  (ii) Lomb-Scargle $\underbrace{\text{Periodogram}}_{\text{power at different frequencies}}$

  ↳ unevenly sampled data
  ↳ heteroscedastic uncertainties.

$$P_{LS}(\omega) = \frac{2}{N \sum_{j=1}^{N} y_j^2} \left[\left(\sum_{j=1}^{N} y_j \sin(\omega t_j) \right)^2 + \left(\sum_{j=1}^{N} y_j \cos(\omega t_j) \right)^2 \right]$$

... where $0 \leqslant P_{LS}(\omega) \leqslant 1$

$$\Rightarrow \frac{\chi^2(\omega)}{\chi_0^2} = 1 - P_{LS}(\omega)$$

↳ peak of $P(\omega)$ or $P_{LS}(\omega)$ gives the best model of noise spectrum or signal frequency.

⇒ <u>Generalized LS (GLS)</u> also fits a non-zero offset parameter.

⇒ <u>Multi-band LS</u> fits for global and per-band offsets in multiple data streams from the same source e.g. multi-band data from LSST.