Lecture 2

01/20/22

ASTROSTATISTICS $\Rightarrow$ extracting knowledge from astronomical data.

"Knowledge" = summary of data behavior
"data" $\equiv$ result of measurements.

- $\boxed{\text{Goal of data mining and statistical inference}}$

estimate $h(x)$ i.e. the generating distribution from which "x" is drawn.

$h(x) \longmapsto$ population distribution (e.g. source redshift distribution).

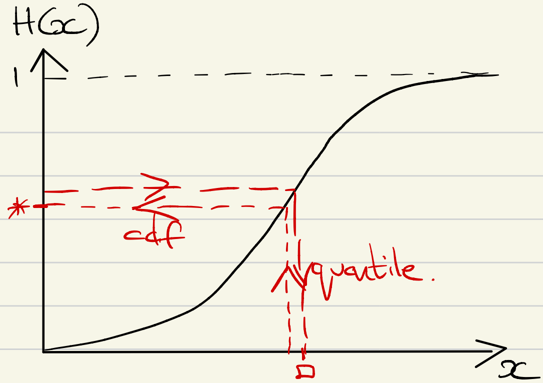
- pdf = probability density function

$\Rightarrow$ probability of value between x and $x+dx$ $= h(x)\,dx$.

- cdf = cumulative distribution function

$$H(x) = \int_{-\infty}^{x} h(x')\,dx'$$

$H^{-1}(x)$ = inverse of cdf
$\hookrightarrow$ quantile function



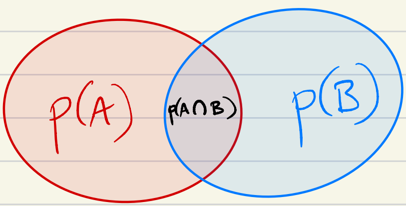
- **Probability**

$p(A)$ = probability of A
OR probability density at A.

$\boxed{\text{KOLMOGOROV AXIOMS}}$

1. $p(A) \geq 0 \quad \forall A$
2. $p(\Omega) = 1$, where Ω is set of all outcomes
3. $p\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} p(A_i)$ for independent events!

union



$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

$\boxed{\text{NOTE}}$ $\cup$ = union / "OR"
$\cap$ = intersection / "AND".

$$* \quad p(A) + p(\bar{A}) = 1 \quad \Big\} \ \bar{A} = \text{"NOT" } A$$

$$* \quad p(A \cap B) = p(A|B)\,p(B) = p(B|A)\,p(A)$$

$\boxed{\text{NOTE}}$ " $|$ " = "GIVEN"

$$\boxed{p(A) = \sum_i p(A \cap B_i) = \sum_i p(A|B_i)\,p(B_i)}$$

"LAW OF TOTAL PROBABILITY"

--- if B_i are independent.

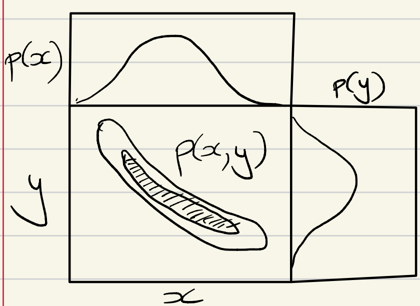
Conditional Probability & Bayes' Rule

* for DEPENDENT RV's $\quad p(x,y) = p(x|y)\,p(y) = p(y|x)\,p(x)$ I

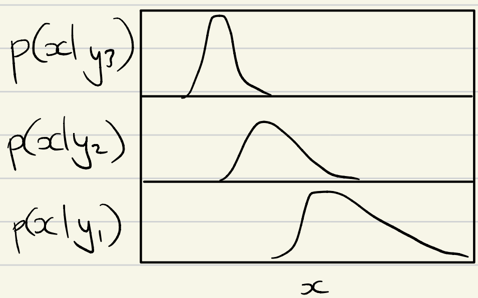
* MARGINAL PROBABILITY $\quad p(x) = \int p(x,y)\,dy$ II

LAW OF TOTAL PROBABILITY (AGAIN) $\Rightarrow$ $p(x) = \int p(x|y)\,p(y)\,dy$

III



JOINT + MARGIANAL PROBS.



CONDITIONAL PROBABILITIES (slices through $p(x,y)$).

Bayes' Theorem (combine I and II)

$$p(y|x) = \frac{p(x|y)\,p(y)}{\int p(x|y)\,p(y)\,dy}$$

Transformation of Random Variables

* x is a random variable.
* $f(x)$ is ALSO a rv $\forall f(\cdot)$

$\Rightarrow$ Use conservation of probability intervals.

i.e. $$y = f(x)$$

$$\therefore \quad \boxed{p(x)\,dx = p(y)\,dy}$$

$$\text{so} \quad p(y) = \left|\frac{dy}{dx}\right|^{-1} p(x).$$

EXAMPLE $\quad y = e^x \Rightarrow p(x) = \text{Unif}(x)$
$= \text{constant}$

$$\therefore \quad \frac{dy}{dx} = e^x = y$$

and $$p(y) = \frac{\text{const.}}{y}$$