Lecture 10

02/17/2022

Q : How do we do Bayesian parameter estimation in high dimensions?
↳ a grid is a poor choice.

- Markov Chain Monte Carlo (MCMC)

① Monte Carlo Methods

* Allow us to do numerical integration (i.e. marginalisation!) using RANDOM SAMPLES.

e.g. $\int g(x) f(x)\, dx = ?$

→ what if I had samples $\{x_i\}$ from $f(x)$?
→ simple model of $f(x)$ is a bunch of delta functions at $\{x_i\}$.

$$\therefore \quad f(x) \approx \frac{1}{N}\sum_{i=1}^{N} \delta(x-x_i)$$

$$\Rightarrow \int g(x) f(x)\, dx \approx \int g(x) \frac{1}{N}\sum_{i=1}^{N} \delta(x-x_i)\, dx$$

$$= \frac{1}{N}\sum_{i=1}^{N} \int g(x)\delta(x-x_i)\, dx$$

$$= \frac{1}{N}\sum_{i=1}^{N} g(x_i)$$

② Markov Chains

* Method of drawing random values that is MEMORYLESS i.e. value only depends on the previous sample, NOT the entire past history.

i.e. $$p(\theta_{i+1} | \theta_i, \theta_{i-1}, \theta_{i-2}, \ldots) = p(\theta_{i+1} | \theta_i)$$

⇒ To make sure we get a STATIONARY DISTRIBUTION of samples, we need DETAILED BALANCE!

$$p(\theta_{i+1} | \theta_i) = p(\theta_i | \theta_{i+1})$$

° Metropolis-Hastings Algorithm

⇒ we propose new parameter positions to explore using a "TRANSITION PROBABILITY" or "PROPOSAL DISTRIBUTION"

$$T(\theta_{i+1} | \theta_i)$$

MUST HAVE $$\underbrace{T(\theta_{i+1} | \theta_i)\, p(\theta_i) = T(\theta_i | \theta_{i+1})\, p(\theta_{i+1})}_{\text{DETAILED BALANCE}}$$

(i) Given θ_i and $T(\theta_{i+1}|\theta_i)$, propose θ_{i+1}

(ii) Compute acceptance probability.

$$\hookrightarrow p_{acc} = \frac{p(\theta_{i+1})}{p(\theta_i)} \quad \Big\} \text{ IF } T(\theta_{i+1}|\theta_i) \text{ SYMMETRIC}$$

(iii) Draw $u \in U[0,1]$

$\Rightarrow$ if $p_{acc} > u$:
accept θ_{i+1}

else :
accept θ_i

(iv) Repeat!

$\boxed{\text{TRACEPLOTS}}$ $\Rightarrow$ the first diagnostic of your MCMC chain.