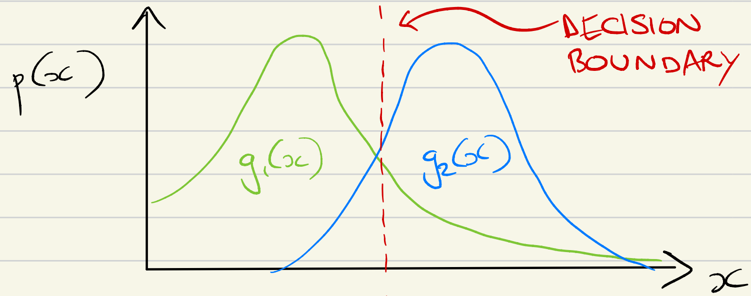
# Lecture 19

03/29/2022

## Generative vs. Discriminative Classifiers

* Estimate PDF for each class.
* Assess probability of new sample belonging to each class

* Don't care about full class PDFs.
* Find a decision boundary that best separates classes.




⇒ classify new object X

① model 1 is better fit than model 2
→ generative.

② X lies on one side of the decision boundary.
→ discriminative.

## Assessing Classification

Type I Error $\Rightarrow$ false positive.
Type II Error $\Rightarrow$ false negative.

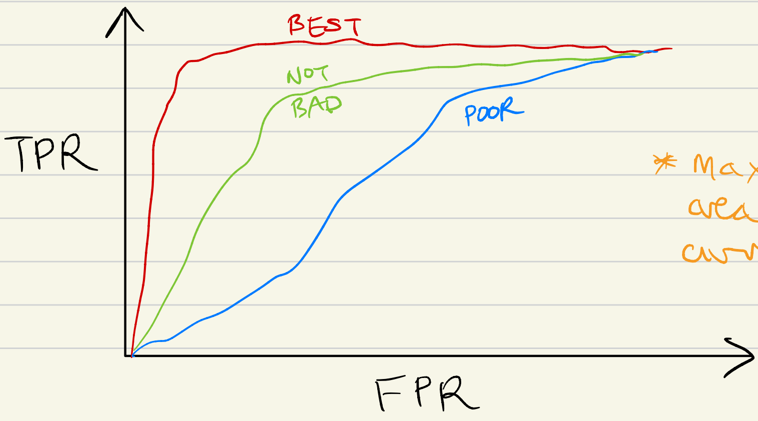
$$\underset{\text{(TPR/sensitivity/recall)}}{\text{Completeness}} = \frac{TP}{TP + FN}$$

$$\underset{\text{(false discovery rate)}}{\text{Contamination}} = \frac{FP}{TP + FP}$$

$$\text{Efficiency} = 1 - \text{Contamination}$$
$$= \text{Precision}$$

## Comparing Performance

$\Rightarrow$ we want to maximize the TPR at a given FPR ---- ROC curve.




* maximize the area under this curve.

## Gaussian Approximations to Generative Classification

$$p(y_k|x_i) = \frac{p(x_i|y_k)\,p(y_k)}{\sum_i p(x_i|y_k)\,p(y_k)}$$

### (i) Naive Bayes

→ ignore feature covariances in multi-D data.

$$\hat{y} = \arg\max_{y_k} \frac{\prod_i p(x^i|y_k)\,p(y_k)}{\sum_\ell \prod_i p(x^i|y_k)\,p(y_k)}$$

### (ii) Linear Discriminant Analysis (LDA)

$$g_k(\vec{x}) = \vec{x}^T\Sigma^{-1}\vec{\mu}_k - \frac{1}{2}\vec{\mu}_k^T\Sigma^{-1}\vec{\mu}_k + \ln p(y=y_k)$$

→ Gaussian Bayes classifier i.e. all classes modeled with multi-variate Gaussian PDFs.

→ All class covariances are EQUAL.

↳ gives fast linear dependence of class posteriors on data.

### (iii) Quadratic Discriminant Analysis (QDA)

$$g_k(\vec{x}) = -\frac{1}{2}\ln\det(\Sigma_k) - \frac{1}{2}(\vec{x}-\vec{\mu}_k)^T\Sigma^{-1}(\vec{x}-\vec{\mu}_k) + \ln p(y=y_k)$$

→ relax the requirement that all class covariances are equal.

### (iv) GMM Bayes Classifier

→ relax the assumption that class PDFs are Gaussian --- use a mixture model.

- K-nearest neighbors Classification

  - Nearest neighbor $\Rightarrow$ if $x'$ is close to $x$, then assign $x'$ to $x$'s known class.

  - K-nearest neighbor $\Rightarrow$ average the assigned category over the K-nearest neighbors.