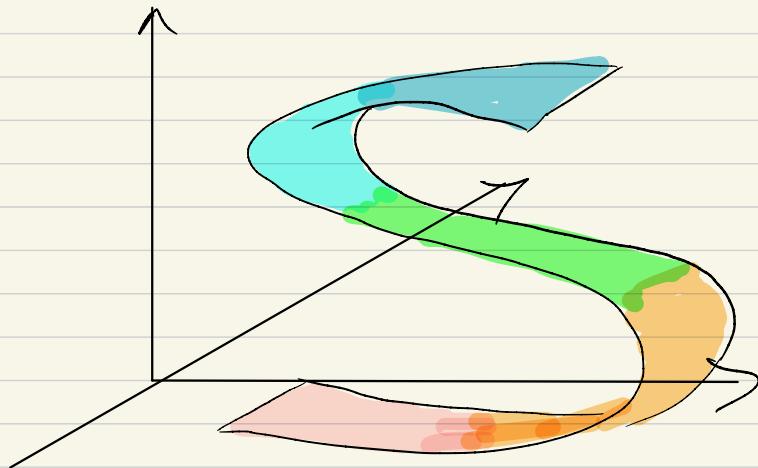


# Lecture 16

03/17/2022

- Non-linear Dimensional Reduction  
( "Manifold Learning" ).
  - ⇒ PCA, NMF, and ICA are great at first trying dimensional reduction
  - ⇒ BUT there's no reason why the physics generating our data (and their features) should be LINEAR ... **MANY PROBLEMS ARE NON-LINEAR!**



- \* Imagine we have data drawn from this 2DV S-shaped manifold embedded in 3D.  
↳ colors denote target labels

\* PCA will not be able to linearly separate.

~~why~~ BUT manifold-learning techniques can learn the geometry of the data, and UNWRAP this.

~~+~~

- Locally Linear Embedding (LLE)

- \* Embed high-D space in lower-D space.
- \* Preserve geometry of local "neighborhoods"

①

Define local geometry

don't allow  
 $W = I$

→ minimize  $\sum_i (w) = \|X - wX\|^2$   
to find  $w$ .  
→  $w$  is weights matrix that constructs a point from " $k$ " neighbors.

②

Embed in lower-D space.

→  $\sum_i (y) = \|Y - Wy\|^2$

→  $Y$  has lower dimension than  $X$ , but weights matrix  $W$  is fixed.

$X = N \times K$

matrix

$Y = N \times d$   
( $d \ll K$ )

- IsoMetric Mapping (IsoMap)

- \* IsoMap actually computes distances between points along geodesics of the manifold's geometry.  
*(not Euclidean distances.)*
- \* IsoMap then finds a lower-D embedding that preserves the geodesic distances between points  $\Rightarrow$  structure is maintained

- t-distributed Stochastic Neighbor Embedding (t-SNE)

- \* t-SNE preserves only local similarities, not global properties (which PCA tries).
  - ① Measure the similarity between nearby points in high-dimensional space.  
 $\hookrightarrow$  evaluate the probability of each point under a multi-variate Gaussian centered on each point...  $p_{ij}$
  - ② Find a lower-D mapping that preserves  $p_{ij}$ .  
 $\hookrightarrow$  dissimilar points are far apart on the lower-D embedding.