Time Series Wrap-up

04/26/2022

- Is there variability in our data over some timescale?
  - → if not, then nothing interesting is going on.
  - → if so, then we need ways to search for and quantify that variability.

- What model do we use for the data?
  - → TEMPLATE: if we know the physics of the signal, then we fit the parameters as usual.
  - → FOURIER ANALYSIS: still a model, but more agnostic to the physics.

MODEL $$y_i(t_i) = \underbrace{Y_0}_{\text{constant}} + \underbrace{\sum_{m=1}^{M} \beta_m \sin(m\omega t_i + \phi_i)}_{\text{reconstruct any periodic function!}} + \underbrace{\varepsilon_i}_{\text{measurement noise}}$$

→ β_m values may be linked to physics or just agnostically describe variability.

... great, but how do we implement a Fourier analysis?

(i) Fourier transform → easy for evenly sampled data.

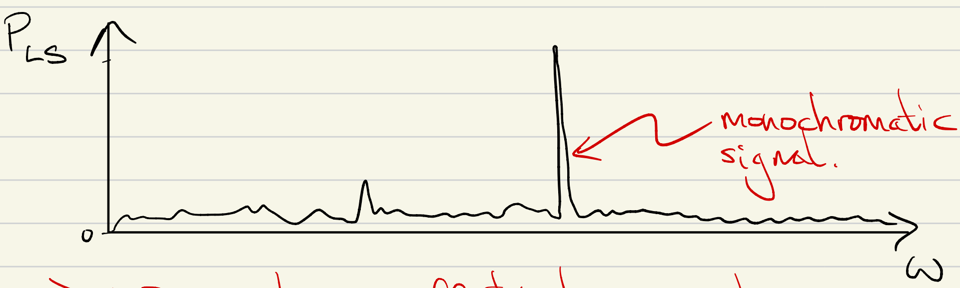
(ii) Lomb-Scargle $\underbrace{\text{Periodogram}}_{\text{power at different frequencies}}$

→ unevenly sampled data

→ heteroscedastic uncertainties.

$$P_{LS}(\omega) = \frac{1}{2}\left[\frac{\left(\sum_{j=1}^{N} y_j \sin[\omega(t_j-\tau)]\right)^2}{\sum_{j=1}^{N}\sin^2[\omega t_j]} + \frac{\left(\sum_{j=1}^{N} y_j \cos[\omega(t_j-\tau)]\right)^2}{\sum_{j=1}^{N}\cos^2[\omega t_j]}\right]$$

... where $\tau = \frac{1}{2\omega}\tan^{-1}\left(\frac{\sum_j \sin[2\omega t_j]}{\sum_j \cos[2\omega t_j]}\right)$ ensures time-shift invariance



⇒ LS periodogram effectively computes a weighted inner product of data against sines and cosines ... no need to be regular.

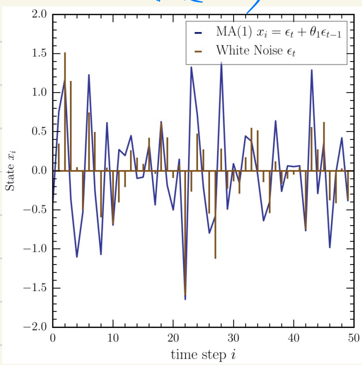
(iii) (c) ARMA Models

better for very poorly sampled data

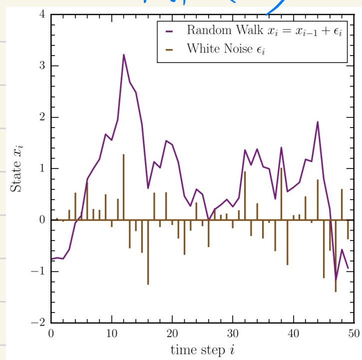
random process varies over multiple timescales!

sines + cosines may be poor basis

MA(1)



AR(1)



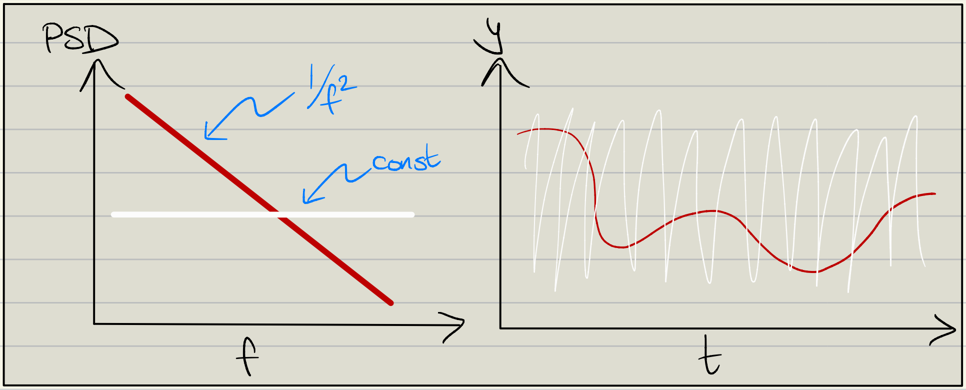
short timescale structure + long timescale structure

$$\boxed{y_i = \varepsilon_i + \sum_{j=1}^{p} a_j y_{i-j} + \sum_{j=1}^{q} b_j \varepsilon_{i-j}}$$

ARMA(p,q) Process

- ARMA assumes evenly sampled data.
  - Continuous ARMA (CARMA) extends this to unevenly sampled data.

⇒ defined by a stochastic differential equation.



AUTOCORRELATION FUNCTION ⇒ (TIME)

$$ACF(\Delta t) = \frac{\lim_{T\to\infty} \int_T g(t)\,g(t+\Delta t)\,dt}{\sigma_g^2}$$

AUTOCOVARIANCE FUNCTION ⇒ (TIME)

$$ACVF(\Delta t) = \sigma_g^2 \times ACF(\Delta t)$$

POWER SPECTRAL DENSITY ⇒ (FREQUENCY)

$$\underbrace{PSD(f) = \int_{-\infty}^{\infty} ACVF(\Delta t) \times e^{-2\pi i f \Delta t}\, d(\Delta t)}_{\text{Wiener-Khinchin Theorem}}$$