Lecture 24

04/19/2022

- ## Stochastic Processes

  - random in time
  - exhibit variability over certain timescales, or with a certain spectrum.
  - characterized only STATISTICALLY, not DETERMINISTICALLY.

  ⇒ There are different flavors of randomness in time-series, and we need a way to characterize that.

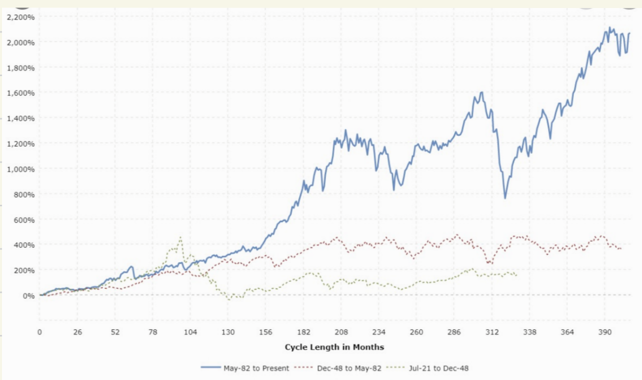
- ## Auto-regressive Models

  — processes are not periodic, but do retain MEMORY.

  - next value depends on the one before.

  i.e. $y_i = \phi_i \, y_{i-1} + \ldots$ ["ϕ" = LAG COEFFICIENT]

EXAMPLE ⇒ random walk ... $y_i = y_{i-1} + \varepsilon_i$ (noise)



* STOCK MARKET shows overall growth on long timescales

⇒ $\phi_i > 1$.

Generally $\Rightarrow$ $$\boxed{y_i = \sum_{j=1}^{p} a_j y_{i-j} + \varepsilon_i}$$

AR(p) Process

* Higher "p" AR processes retain longer-term memory, creating longer timescale structure.

Moving Average Models

↳ similar to AR models, except MA models depend on perturbations at previous times, NOT values.

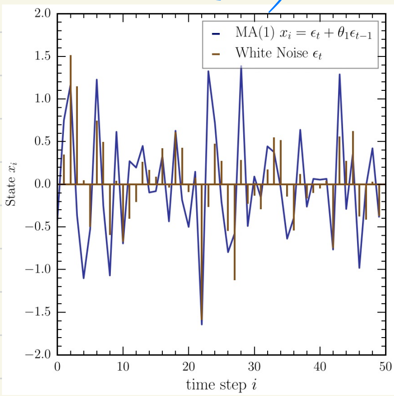
$$\boxed{y_i = \varepsilon_i + \sum_{j=1}^{q} b_j \varepsilon_{i-j}}$$

MA(q) Process

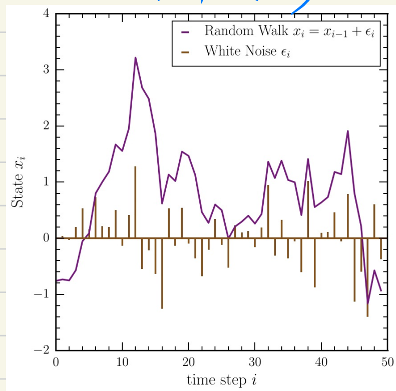
MA $\Rightarrow$ a shock/impulse affects current value and "q" values into the future.

AR $\Rightarrow$ a shock/impulse affects ALL future values.

MA(1)



AR(1)



short timescale structure + long timescale structure

$$\underbrace{\boxed{y_i = \varepsilon_i + \sum_{j=1}^{p} a_j y_{i-j} + \sum_{j=1}^{q} b_j \varepsilon_{i-j}}}_{\text{ARMA}(p,q) \text{ Process}}$$

↳ ARMA assumes evenly sampled data.

↳ Continuous ARMA (CARMA) extends this to unevenly sampled data.

⇒ defined by a stochastic differential equation.

find Δt at which ACF falls below a threshold $\Rightarrow$ auto-correlation length!

ACF, ACVF, SF, PSD

AUTOCORRELATION FUNCTION $\Rightarrow$ (TIME)

$$ACF(\Delta t) = \frac{\lim_{T \to \infty} \int_T g(t)\, g(t+\Delta t)\, dt}{\sigma_g^{2}}$$

AUTOCOVARIANCE FUNCTION $\Rightarrow$ (TIME)

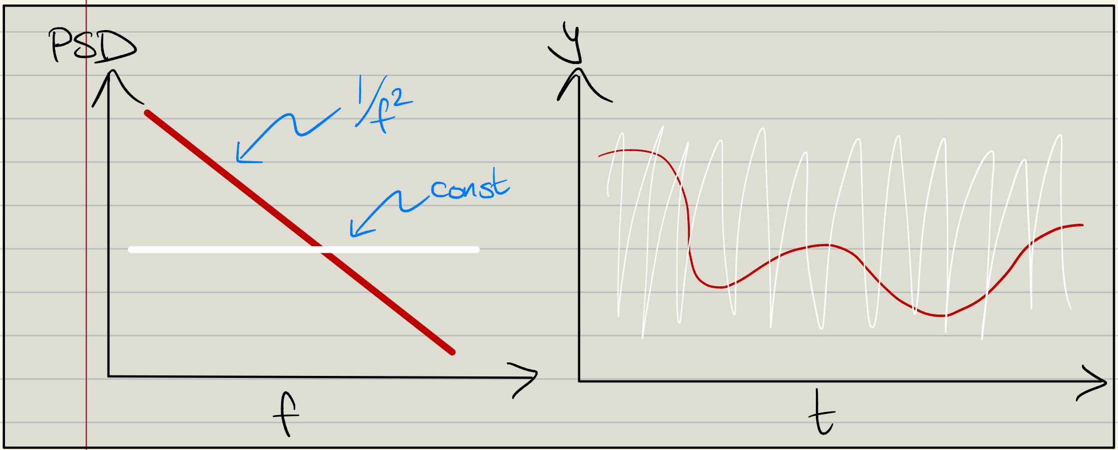
$$ACVF(\Delta t) = \sigma_g^{2} \times ACF(\Delta t)$$

STRUCTURE FUNCTION $\Rightarrow$ (TIME)

$$SF(\Delta t) = \sigma_g \left[1 - ACF(\Delta t) \right]^{1/2}$$

POWER SPECTRAL DENSITY $\Rightarrow$ (FREQUENCY)

$$\underbrace{PSD(f) = \int_{-\infty}^{\infty} ACVF(\Delta t) \times e^{-2\pi i f \Delta t}\, d(\Delta t)}_{\text{Wiener-Khinchin Theorem}}$$



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