

# Lecture 2

01/16/2024

ASTROSTATISTICS  $\Rightarrow$  extracting knowledge from astronomical data.

"knowledge" = summary of data behavior  
"data"  $\cong$  result of measurements.

- Goal of data mining and statistical inference

estimate  $h(x)$  i.e. the generating distribution from which "x" is drawn.

$h(x)$   $\longleftarrow$  population distribution  
(e.g. source redshift distribution).

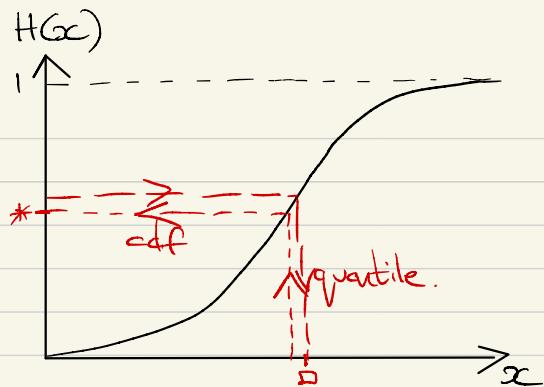
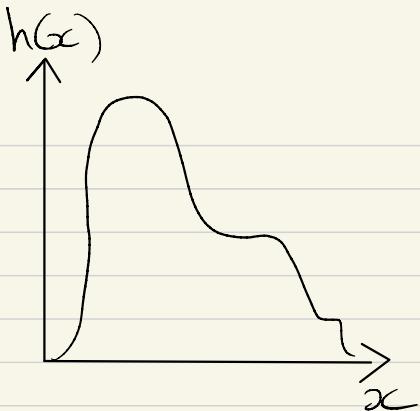
- pdf = probability density function

$\Rightarrow$  probability of value between  $x$  and  $x+dx$  =  $h(x) dx$ .

- cdf = cumulative distribution function

$$H(x) = \int_{-\infty}^x h(x') dx'$$

$$H^{-1}(x) = \text{inverse of cdf} \rightarrow \text{quantile function}$$

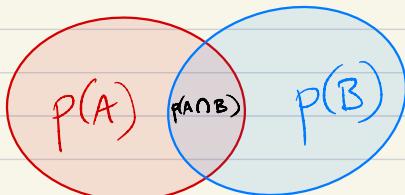


- Probability

$p(A)$  = probability of  $A$   
 $\equiv$  probability density at  $A$ .

KOLMOGOROV AXIOMS

- ①  $p(A) \geq 0 \quad \forall A$
  - ②  $p(\Omega) = 1$ , where  $\Omega$  is set of all outcomes
  - ③  $p\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} p(A_i)$  for independent events!
- union



$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$

NOTE   
 $\cup$  = union / "OR"  
 $\cap$  = intersection / "AND".

$$* p(A) + p(\bar{A}) = 1 \quad \left. \begin{array}{l} \bar{A} = \text{"NOT"} A \end{array} \right\}$$

$$* p(A \cap B) = p(A|B)p(B) \\ = p(B|A)p(A)$$

NOTE " | " = "GIVEN"

$$P(A) = \sum_i P(A \cap B_i) = \sum_i P(A|B_i)P(B_i)$$

"LAW OF TOTAL PROBABILITY"

--- if  $B_i$  are independent.

## Conditional Probability & Bayes' Rule

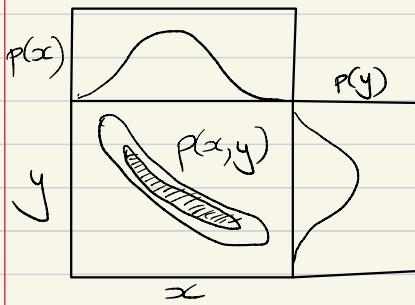
\* for DEPENDENT RV's  $p(x,y) = p(x|y)p(y)$  (I)

$$= p(y|x)p(x)$$

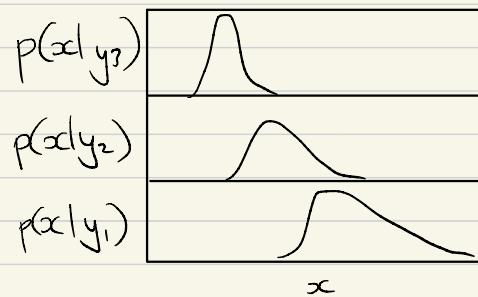
\* MARGINAL PROBABILITY  $p(x) = \int p(x|y)dy$  (II)

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LAW OF TOTAL PROBABILITY (AGAIN)  $\Rightarrow p(x) = \int p(x|y)p(y)dy$  (III)



JOINT + MARGINAL PROBS.



CONDITIONAL PROBABILITIES  
(Slices through  $p(x,y)$ ).

Bayes' Theorem (combine I and II)

$$p(y|x) = \frac{p(x|y)p(y)}{\int p(x|y)p(y)dy}$$

## Transformation of Random Variables

- \*  $x$  is a random variable.
- \*  $f(x)$  is also a rv  $\wedge f(\cdot)$

$\Rightarrow$  Use conservation of probability intervals.

$$\text{i.e. } y = f(x)$$

$$\therefore p(x)dx = p(y)dy$$

$$\Leftrightarrow p(y) = \left| \frac{dy}{dx} \right| p(x)$$

**EXAMPLE**  $y = e^x \Rightarrow p(x) = \text{Unit}(x) = \text{constant}$

$$\therefore \frac{dy}{dx} = e^x = y$$

and  $p(y) = \frac{\text{const.}}{y}$

