Lecture 15

- The Curse of Dimensionality

* Imagine you want a car, and you're very picky

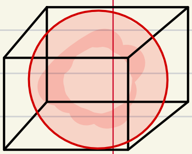
↳ for each choice, there's only 50% chance of finding a car at the dealership.

$$\Rightarrow P_{car} = P_{red} \times P_{\text{good mileage}} \times P_{\text{leather seats}} \times P_{\text{sunroof}}$$
$$= (0.5)^4$$
$$= 0.0625 \Big\} \text{ only } \sim 6\% \text{ !}$$

* Another way of seeing this ...

↳ imagine designing a suite of simulations where you want to cover the full space of initial conditions over many dimensions.

⇒ don't distribute over unit hypersphere of initial conditions!



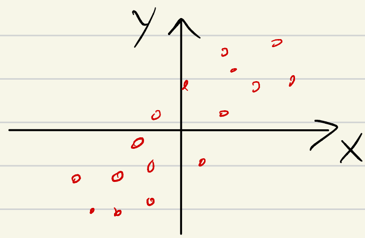
$$f_D = \frac{V_D(r)}{(2r)^D} = \frac{\pi^{D/2}}{D.2^{D-1}\Gamma(D/2)}$$

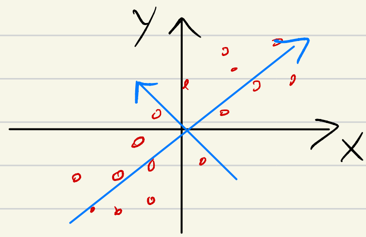
$$\longmapsto 0 \text{ as } D \uparrow$$

PHYSICALLY ---- PCA gives you the directions along which data varies, and ranks those directions by their eigen-value.

- **Principal Component Analysis (PCA)**
  - ↳ find linear combinations of features that describe directions of maximal variance.

e.g.





X, Y are measured features

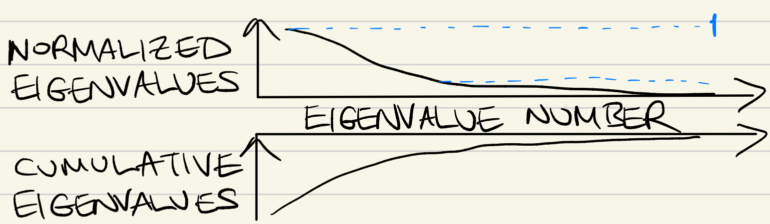
New axes are linear combinations

* EIGEN-ANALYSIS ⇒ principal components are eigen-vectors.

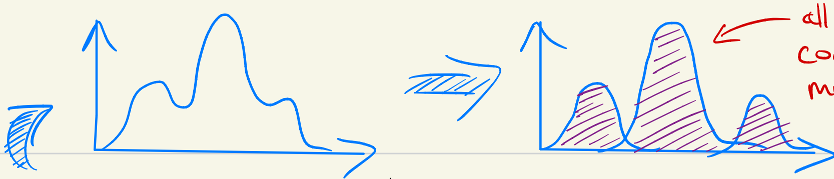
* Great for taking data with huge numbers of features, and finding the most important combinations of features.

↳ DATA COMPRESSION

* Find a new (smaller) basis that better describes the data.



↳ SCREE PLOTS tell you how many important principal components there are.



∘ Non-negative Matrix Factorization (NMF)

↳ what if you know that your basis functions and co-efficients have to be positive?

e.g. amplitudes of GMMs to describe a PDF.
→ PDFs have to be > 0.

⇒ X has all +ve elements

↳ $X = WY$

minimize $\|X - WY\|^2$
to find +ve basis matrix, W, and +ve coeffients, Y.

∘ Independent Component Analysis (ICA).

↳ separate signal components that are statistically-independent.
e.g. isolating different voices at a party.

* ICA relies on the CENTRAL LIMIT THEOREM
i.e adding a bunch of independent random signals should produce noise with GAUSSIAN STATISTICS.

↳ ∴ let's separate out the signals to be as individually non-Gaussian as possible.