

Lecture 23

Goals of Time Series Analysis

① Is the system variable over some timescale?
 \curvearrowleft ≠ periodic, necessarily



→ NO \Rightarrow there's no point doing further time-series analysis
→ YES \Rightarrow characterize the temporal correlation i.e. model selection and parameter estimation.
 \Rightarrow predict future values of data.

e.g. $y(t) = A \sin(\omega t) + \varepsilon$

$$(\varepsilon \sim N(0, \sigma^2))$$

$$V_y = \sigma^2 + \frac{1}{2} A^2$$

$$\chi^2_{\text{dof}} = \frac{1}{N} \sum_j \left(\frac{y_j}{\sigma} \right)^2 \sim V$$

$$\underbrace{\quad}_{\text{if } A=0 \dots} \chi^2_{\text{dof}} = 1$$

$$\text{if } A \neq 0 \dots \chi^2_{\text{dof}} > 1$$

$$\sigma_{\chi^2_{\text{dof}}} = \sqrt{2/N}$$

$\Rightarrow P(\chi^2_{\text{dof}} > 1 + 3\sqrt{2/N}) \sim 10^{-3}$

$\underbrace{\phantom{P(\chi^2_{\text{dof}} > 1 + 3\sqrt{2/N}) \sim 10^{-3}}}_{3\sigma \text{ deviation}}$

- Minimum detectable amplitude (as given by a 3σ tension with noise model)

$$\hookrightarrow A_{\min} = \frac{2.9\sigma}{N^{1/4}}$$

$\underbrace{\phantom{A_{\min} = \frac{2.9\sigma}{N^{1/4}}}}_{\rightarrow \text{with enough data, we can dig very small signals out of large noise.}}$

Parameter Estimation & Model Selection

- * Similar to regression and other model fitting + comparison.
- * But... "x" is replaced by "t" and now the data are sequential.

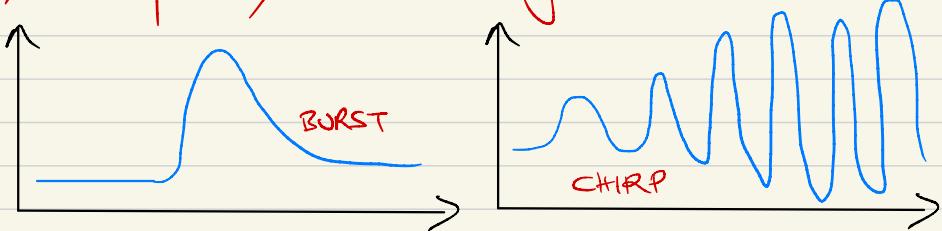
Fourier Analysis:

\rightarrow decompose a time-series into a sum of sine and cosine basis functions of varying harmonics of a base timescale.

$$\text{i.e. } y_i(t_i) = Y_0 + \sum_{m=1}^M \beta_m \sin(m\omega t_i + \phi_i) + \varepsilon_i$$

↗ constant ↗ reconstruct any periodic function! ↗ measurement noise

Temporal-localized Signals



↳ can use matched filtering i.e. match a template against the data.

- Periodic Signals

(i) Fourier transform → easy for evenly sampled data.

(ii) Lomb-Scargle Periodogram

↳ power at different frequencies
↳ unevenly sampled data
↳ heteroscedastic uncertainties.

$$P_{LS}(\omega) = \frac{2}{N \sum_{j=1}^n y_j^2} \left[\left(\sum_{j=1}^n y_j \sin(\omega t_j) \right)^2 + \left(\sum_{j=1}^n y_j \cos(\omega t_j) \right)^2 \right]$$

--- where $0 \leq P_{LS}(\omega) \leq 1$

$$\Rightarrow \frac{\chi^2(\omega)}{\chi^2_0} = 1 - P_{LS}(\omega)$$

↳ peak of $P(\omega)$ or $P_{LS}(\omega)$ gives the best model of noise spectrum or signal frequency.

⇒ Generalized LS (GLS) also fits a non-zero offset parameter.

⇒ Multi-band LS fits for global and per-band offsets in multiple data streams from the same source e.g. multi-band data from LSST.