

Lecture 10

Q : How do we do Bayesian parameter estimation in high dimensions?
↳ a grid is a poor choice.

- Markov Chain Monte Carlo (MCMC)

① Monte Carlo Methods

* Allow us to do numerical integration
(i.e. marginalisation?) using
RANDOM SAMPLES.

e.g. $\int g(x) f(x) dx = ?$

→ what if I had samples $\{x_i\}$ from $f(x)$?
→ simple model of $f(x)$ is a bunch of
delta functions at $\{x_i\}$.

$$\therefore f(x) \approx \frac{1}{N} \sum_{i=1}^N \delta(x - x_i)$$

$$\begin{aligned} \Rightarrow \int g(x) f(x) dx &\approx \int g(x) \frac{1}{N} \sum_{i=1}^N \delta(x - x_i) dx \\ &= \frac{1}{N} \sum_{i=1}^N \int g(x) \delta(x - x_i) dx \\ &= \frac{1}{N} \sum_{i=1}^N g(x_i) \end{aligned}$$

(2)

Markov Chains

- * Method of drawing random values, that is **MEMORYLESS** i.e. value only depends on the previous sample, NOT the entire past history.

i.e. $p(\theta_{i+1} | \theta_i, \theta_{i-1}, \theta_{i-2}, \dots) = p(\theta_{i+1} | \theta_i)$

⇒ To make sure we get a **STATIONARY DISTRIBUTION** of samples, we need **DETAILED BALANCE**.

$$p(\theta_{i+1} | \theta_i) = p(\theta_i | \theta_{i+1})$$

Metropolis - Hastings Algorithm

⇒ we propose new parameter positions to explore using a "**TRANSITION PROBABILITY**" $T(\theta_{i+1} | \theta_i)$ and "**PROPOSAL DISTRIBUTION**"

MUST HAVE

$$T(\theta_{i+1} | \theta_i) p(\theta_i) = T(\theta_i | \theta_{i+1}) p(\theta_{i+1})$$

DETAILED BALANCE

(i) Given θ_i and $T(\theta_{i+1} | \theta_i)$, propose θ_{i+1}

(ii) Compute acceptance probability.

$$\hookrightarrow p_{\text{acc}} = \frac{p(\theta_{i+1})}{p(\theta_i)} \quad \left. \begin{array}{l} \text{IF } \\ T(\theta_{i+1} | \theta_i) \\ \text{SYMMETRIC} \end{array} \right\}$$

(iii) Draw $u \in U[0, 1]$

\Rightarrow if $p_{\text{acc}} > u$:
else:
 accept θ_{i+1}
 accept θ_i

(iv) Repeat!

TRACEPLOTS

\Rightarrow the first diagnostic of your MCMC chain.

