

Lecture 24

Stochastic Processes

- ↳ random in time
- ↳ exhibit variability over certain timescales, or with a certain spectrum.
- ↳ characterized only **STATISTICALLY**, not **DETERMINISTICALLY**.

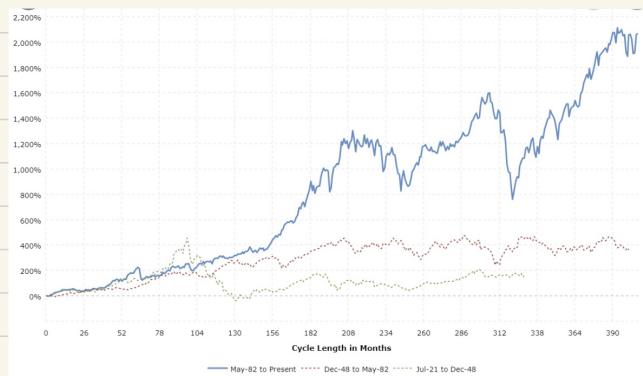
⇒ There are different flavors of randomness in time-series, and we need a way to characterize that.

Auto-regressive Models

— processes are not periodic, but do retain MEMORY.

- ↳ next value depends on the one before,
ie $y_i = \phi_1 y_{i-1} + \dots$ [ϕ = LAG COEFFICIENT]

EXAMPLE ⇒ random walk ... $y_i = y_{i-1} + \xi_i$ (noise)



* STOCK MARKET shows overall growth on long timescales
⇒ $\phi_i > 1$.

Generally \Rightarrow

$$y_i = \sum_{j=1}^p a_j y_{i-j} + \varepsilon_i$$

AR(p) Process

- * Higher "p" AR processes retain longer-term memory, creating longer timescale structure.

- Moving Average Models

\hookrightarrow similar to AR models, except MA models depend on perturbations at previous times, NOT values.

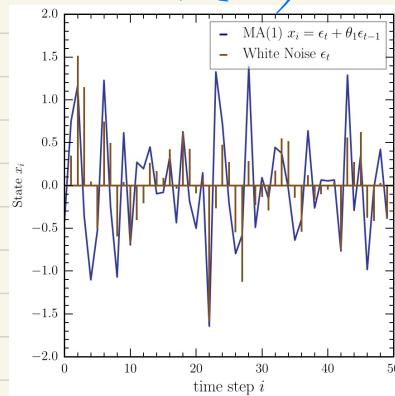
$$y_i = \varepsilon_i + \sum_{j=1}^q b_j \varepsilon_{i-j}$$

MA(q) Process

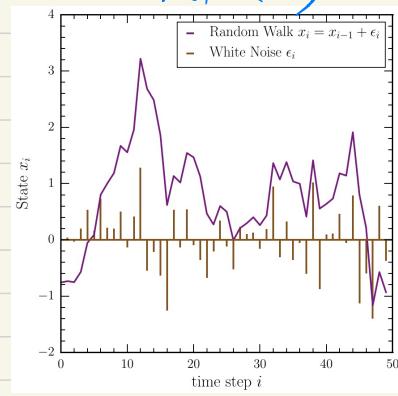
MA \Rightarrow a shock/impulse affects current value and "q" values into the future.

AR \Rightarrow a shock/impulse affects ALL future values.

MA(1)



AR(1)



short timescale structure + long timescale structure

$$y_i = \varepsilon_i + \sum_{j=1}^p a_j y_{i-j} + \sum_{j=1}^q b_j \varepsilon_{i-j}$$

ARMA(p, q) Process

↳ ARMA assumes evenly sampled data.

↳ Continuous ARMA (CARMA) extends this to unevenly sampled data.

⇒ defined by a stochastic differential equation.

find Δt at which ACF falls below a threshold
 \Rightarrow auto-correlation length!

- ACF, ACVF, SF, PSD

AUTOCORRELATION
FUNCTION
 \Rightarrow (TIME)

$$\text{ACF}(\Delta t) = \frac{\lim_{T \rightarrow \infty} \int_T g(t)g(t+\Delta t) dt}{\sigma_g^2}$$

AUTOCOVARIANCE
FUNCTION
 \Rightarrow (TIME)

$$\text{ACVF}(\Delta t) = \sigma_g^2 \times \text{ACF}(\Delta t)$$

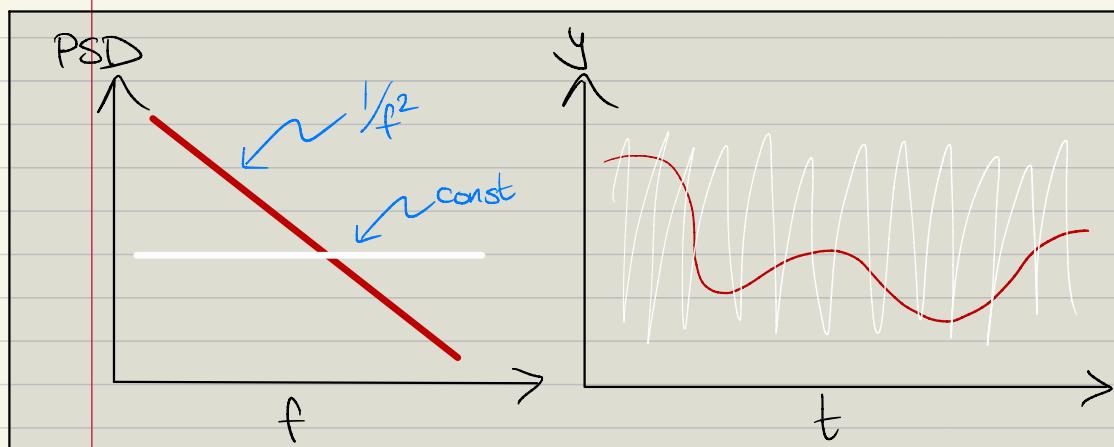
STRUCTURE
FUNCTION
 \Rightarrow (TIME)

$$\text{SF}(\Delta t) = \sigma_g [4 - \text{ACF}(\Delta t)]^{1/2}$$

POWER SPECTRAL
DENSITY
 \Rightarrow (FREQUENCY)

$$\text{PSD}(f) = \int_{-\infty}^{\infty} \text{ACVF}(\Delta t) \times e^{-2\pi f \Delta t} d(\Delta t)$$

Wieners - Khinchin Theorem



Fin...