# ASSIGNMENT 1 – COMP4418

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*Question 1 :*

1. LOGICAL INFERENCE

(a) Prove whether or not the following inferences hold using a suitable semantic method ($\models$)

(i) $p \wedge (q \vee r) \models (p \wedge q) \vee (p \wedge r)$

| $p$ | $q$ | $r$ | $(q \vee r)$ | $p \wedge (q \vee r)$ | $(p \wedge q)$ | $(p \wedge r)$ | $(p \wedge q) \vee (p \wedge r)$ |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| T | T | F | T | T | T | F | T |
| T | F | T | T | T | F | T | T |
| T | F | F | F | F | F | F | F |
| F | T | T | T | F | F | F | F |
| F | T | F | T | F | F | F | F |
| F | F | T | T | F | F | F | F |
| F | F | F | F | F | F | F | F |

The truth tables for $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are equivalent ie $p \wedge (q \vee r)$ is true iff $(p \wedge q) \vee (p \wedge r)$ is true.

$\therefore$ Inference is valid.

(ii) $\models p \rightarrow (q \rightarrow p)$

| $p$ | $q$ | $q \rightarrow p$ | $p \rightarrow (q \rightarrow p)$ |
|---|---|---|---|
| T | T | T | T |
| T | F | T | T |
| F | T | F | T |
| F | F | T | T |
| | | | This is a tautology. |

$\models p \rightarrow (q \rightarrow p)$ is a tautology.

$\therefore$ The inference is valid.

(III) $\exists x \forall y$ Likes$(x, y) \models \forall x \exists y$ Likes$(x, y)$

Let

$KB = \{a \in x\ (\exists x), y, x, b \in y\ (\exists y)\}$

and

$\alpha : \exists x \forall y$ Likes$(x, y) \models \forall x \exists y$ Likes$(x, y)$

① $KB \models \alpha$ for some interpretation $I$

a is a person who belongs to the set of all people (x)

y = set of all apples.

There exists a person 'a' who belongs to the set of people (x) such that 'a' likes apples

eg: $\exists x \forall y$ Likes (John, Apple)

There exists some John that likes all apples.

$\forall x \exists y$ Likes$(x, y)$

For all persons $x$, such that there exists an apple 'b' $\in y$ (set of all apples)

eg: $\forall x \exists y$ Likes (John, Apple)

For all John, there exists some apple that is like by John.

$T \models T$

$\therefore KB \models \alpha$

② Assume $KB \not\models \alpha$ for interpretation $I$

$\exists x \forall y$ Likes (John, Apple) $\not\models \forall x \exists y$ Likes (John, Apple)

There exists no John that likes no apple and there exists no apple that is liked by John.

$[\ ] \models [\ ]$

Similarly
for ~~some~~ all John there exists no apple that he likes and for all apples there exists some John that dislikes them.

$$[\,] \models [\,]$$

This contradicts our interpretation that $KB \not\models \alpha$

Therefore $KB \models \alpha$.

(iv) $\sim p \rightarrow \sim q, p \rightarrow q \models p \leftrightarrow q$

| $p$ | $q$ | $p \rightarrow q$ | $\sim p$ | $\sim q$ | $\sim p \rightarrow \sim q$ | $p \leftrightarrow q$ |
|---|---|---|---|---|---|---|
| T | T | T | F | F | T | T |
| T | F | F | F | T | T | F |
| F | T | T | T | F | F | F |
| F | F | T | T | T | T | T. |

$\sim p \rightarrow \sim q$ and $p \rightarrow q$ is true wherever $p \leftrightarrow q$ is true

$\therefore$ The inference is valid.

(v) $\forall x . P(x) \rightarrow Q(x), \forall x\, Q(x) \rightarrow R(x),$
$\sim R(a) \models \sim P(a)$

$KB = \{P(x) \rightarrow Q(x), Q(x) \rightarrow R(x), \sim R(a)\}$

For the interpretation $I \models \sim R(a)$

$I \models \sim R(a)$
$\models \sim R(a) \rightarrow \sim Q(a)$
$\models Q(f(a)) \rightarrow R(g(a))$ (in KB)
$\models \sim Q(a) \rightarrow \sim P(a)$
$\models P(f(a)) \rightarrow Q(g(a))$ (in KB)

$\therefore$ $KB \models \alpha$ for $I$.

because $\sim Q(a), P(a) \rightarrow Q(a) \models \sim P(a)$

case (ii)

Let us assume $KB \not\models \alpha$ for $I$.

$KB \models \sim I$

$\sim Q(a) \rightarrow \sim P(a)$

which is the same as $P(a) \rightarrow Q(a)$

which exists in the KB.

This contradicts our assumption

$KB \not\models \alpha$ for $I$.

Therefore the Interpretation is valid.

(b) Prove whether or not the following inferences hold syntactically using resolution ($\vdash$)

(i) $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

LHS: $p \wedge (q \vee r)$ (already in CNF)

RHS: $(p \wedge q) \vee (p \wedge r)$

$\Rightarrow$ neg $((p \wedge q) \vee (p \wedge r))$

$\Rightarrow (\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$

PROOF

| | | |
|---|---|---|
| 1. | $p$ | HYPOTHESIS |
| 2. | $(q \vee r)$ | HYPOTHESIS |
| 3. | $(\sim p \vee \sim q)$ | NEGATION OF CONCLUSION |
| 4. | $(\sim p \vee \sim r)$ | NEGATION OF CONCLUSION |
| 5. | $\sim q$ | (1,3) |
| 6. | $r$ | (2,5) |
| 7. | $\sim p$ | (4,6) |
| 8. | $\square$ | (1,7) |

(ii) $\vdash p \rightarrow (q \rightarrow p)$

RHS: $p \rightarrow (q \rightarrow p)$

$\Rightarrow \sim p \vee (q \rightarrow p)$

$\Rightarrow \sim p \vee (\sim q \vee p)$

$\Rightarrow \sim p \vee (\sim q \vee p)$

$\Rightarrow neg(\sim p \vee (\sim q \vee p))$

$\Rightarrow p \wedge \sim(\sim q \vee p)$

$\Rightarrow p \wedge (q \wedge \sim p)$

PROOF

| | | |
|---|---|---|
| 1. | $p$ | NEGATION OF CONCLUSION |
| 2. | $q$ | NEGATION OF CONCLUSION |
| 3. | $\sim p$ | NEGATION OF CONCLUSION |
| 4. | $\square$ | (1,3) |
| 5. | $\square$ | (2,4) |

(iii) $\exists x \forall y \, Likes(x,y) \vdash \forall x \exists y \, Likes(x,y)$

LHS: $\exists x \forall y \, Likes(x,y)$

$CNF(\exists x \forall y \, Likes(x,y)$

$\Rightarrow \forall y \, Likes(x,y)$

$\Rightarrow Likes(f(y), y)$

RHS: $\forall x \exists y \, Likes(x,y)$

neg(RHS): $\sim(\forall x \exists y \, Likes(x,y))$

$= \exists x \forall y \sim Likes(x,y)$

$= \forall y \sim Likes(x,y)$

$= \sim Likes(f(y), y)$

Likes $(f(y), y)$ $\qquad$ $\sim$Likes $(f(y), y)$

$\{f(y)/c\}$ $\qquad$ $\{f(y)/c\}$

$\square$ $\qquad$ RESOLUTION.

(iv) $\sim p \rightarrow \sim q,\ p \rightarrow q \ \vdash\ p \leftrightarrow q$

<u>LHS:</u> $\sim p \rightarrow \sim q$ $\qquad$ | $\quad p \rightarrow q$

(CNF) $\Rightarrow \sim(\sim p) \vee \sim q$ $\qquad$ | $\quad \Rightarrow \sim p \vee q$

$\Rightarrow p \vee \sim q$

<u>RHS:</u> $p \leftrightarrow q$

$\Rightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

$\Rightarrow (\sim p \vee q) \wedge (\sim q \vee p)$

$\Rightarrow \sim(\sim p \vee q) \vee \sim(\sim q \vee p)$ (NEGATION)

$\Rightarrow (p \wedge \sim q) \vee (q \wedge \sim p)$

$\Rightarrow (p \vee q) \wedge (\sim q \vee q) \wedge (p \vee \sim p) \wedge (\sim q \vee \sim p)$

$\Rightarrow (p \vee q) \wedge T \wedge T \wedge (\sim p \vee \sim q)$

$\Rightarrow (p \vee q) \wedge (\sim p \vee \sim q)$ $\quad$ (CNF)

<u>PROOF:</u>

| | | |
|---|---|---|
| 1. | $p \vee \sim q$ | HYPOTHESIS |
| 2. | $\sim p \vee q$ | HYPOTHESIS |
| 3. | $p \vee q$ | NEGATION OF CONCLUSION |
| 4. | $(\sim p \vee \sim q)$ | NEGATION OF CONCLUSION |
| 5. | $p$ | (1, 3) |
| 6. | $\sim p$ | (2, 4) |
| 7. | $\square$ | (5, 6) |

(v) $\forall x\, P(x) \rightarrow Q(x),\ \forall x\, Q(x) \rightarrow R(x),$
$\sim R(a) \vdash \sim P(a)$

CNF $(\forall x\, P(x) \rightarrow Q(x))$

$\Rightarrow P(f(x)) \rightarrow Q(g(x))$ SKOLEMISATION.

$\Rightarrow \sim P(f(x)) \vee Q(g(x))$

CNF $(\forall x\, Q(x) \rightarrow R(x))$

$\Rightarrow Q(g(x)) \rightarrow R(h(x))$ SKOLEMIZATION.

$\Rightarrow \sim Q(g(x)) \vee R(h(x))$

CNF $(\sim R(a)) \Rightarrow \sim R(h(a))$

PROOF

1. $\sim P(f(x)) \vee Q(g(x))$ HYPOTHESIS
2. $\sim Q(g(x)) \vee R(h(x))$ HYPOTHESIS
3. $\sim R(h(a))$ HYPOTHESIS
4. $P(f(a))$ NEGATION OF CONCLUSION $\{a \mid f(a)\}$
5. $\sim P(f(a)) \vee Q(g(a))$ $\{x \mid a\}$
6. $\sim Q(g(a)) \vee R(h(a))$ $\{x \mid a\}$
7. $\sim Q(g(a))$ (3,6)
8. $\sim P(f(a))$ (5,7)
9. $\square$ (4,8)

$\therefore$ The inference is valid.

*Question 2:*

QUESTION 2 : LOGIC PUZZLE.

(i) Assumptions

Male (Ivor). Female (Doris).
Male (Rodney). Female (Edwina).
Male (Patrick). Female (Rosemary).

The facts stated in the question can be represented in FOL as follows:
The facts also forms the fundamental knowledge in the knowledge base.

$KB = \{\exists x\, Opposite(Ivor, x) \wedge Profession(x, Photographer)$
$\exists y\, Opposite(Patrick, y) \wedge Profession(y, Airhostess)$
$Above(Ivor, Patrick)$
$\exists z\, Above(Edwina, z) \wedge Profession(z, Doctor)$
$Lives(Doris, 2)$
$\exists a\, Lives(a, 4) \wedge Profession(a, Store\ Detective)$
$\exists b\, Profession(b, Clerk), Lives(Rodney, P)$
$\exists q\, Lives(Rosemary, q),$
$Profession(Patrick, Lawyer)\}$

$\alpha : \forall x \exists y \exists z\, Lives(x, y) \wedge Profession(z)$

To prove this logic puzzle we must establish that there exists some interpretation I such that $KB \models \alpha$.

(ii) There are multiple interpretations. that can be derived from the given Knowledge Base.

(i) Ivor can live in Flat 1, 3 or 4.
(ii) Patrick can live in Flat 3, 5 or 6
(iii) Edwina can live in Flat 3, 4, 5 or 6.
(iv) Rodney or Rosemary can live in any flat but Flat 2 because Doris live there.

The chances that 'I' can semantically prove the puzzle is higher using Ivor or Patrick. (Only three scenarios are logically possible).

But considering 'Patrick' has more FOL statements where he is a parameter, we consider scenarios with Patrick.
The three interpretations that are to be considered are:

$I_1 \models$ Lives (Patrick, 3)
$I_2 \models$ Lives (Patrick, 5)
$I_3 \models$ Lives (Patrick, 6)

Case (i): $I_1 \models$ Lives (Patrick, 3)
$\models$ Above (Ivor, Patrick)
$\models$ Lives (Ivor, ~~2~~ 1)
$\models$ Opposite (Patrick, y) $\wedge$ Profession (y, Airhostess)
$\models$ CONTRADICTION

According to KB the person living in Flat 4 must be a store detective
The same person cannot have two professions.
$\therefore$ $I_1 \not\models$ Lives (Patrick, 3)

<u>case (ii)</u> : $I_2$ $\models$ Lives (Patrick, 5)
$\models$ Above (Iras, Patrick)
$\models$ Lives (Iras, 3)
$\models$ $\exists x$ Opposite(Iras, $x$) $\wedge$ Profession($x$, Photographer)
$\models$ CONTRADICTION

Lives($x$, 4) contradicts the KB because $x$ is a photographer but the resident of Flat 4 must be a Store detective

<u>case (iii)</u> : $I_3$ $\models$ Lives (Patrick, 6)

$I_3$ $\models$ Lives (Patrick, 6)
$\models$ Above (Iras, Patrick)
$\models$ Lives (Iras, 4)
$\models$ Lives (Iras, 4) $\wedge$ Profession (Iras, Store Detective)
$\models$ $\sim$ Lives (Z, 3) $\wedge$ Profession (Z, Doctor)
$\models$ $\sim$ Lives (Z, 5) $\wedge$ Profession (Z, Doctor)
$\models$ $\sim$ Lives (Edwina, 1)
$\models$ Lives (Edwina, 3) $\wedge$ Profession (Edwina, Photographer)
$\models$ Lives (Doris, 2) $\wedge$ Profession (Doris, Clerk)
$\models$ Lives (Rosemary, 5)
$\models$ Profession (Rosemary, Airhostess)
$\models$ Lives (Rodney, 1) $\wedge$ Profession (Rodney, Doctor)
$\models$ Lives (Doris, 2) $\wedge$ Profession (Doris, Clerk)

KB $\models$ $\alpha$ for $I_3$

Assuming Ivor lives in either flat 1, 3 or 4.

$I_5 \models$ Lives (Ivor, 1)
$\models$ Above (Ivor, Patrick)
$\models$ Lives (Patrick, 3)
$\models$ Opposite (Patrick, y) $\wedge$ Profession (y, Air hostess)
$\models$ CONTRADICTION.

$I_4 \models$ Lives (Ivor, 3)
$\models$ ~~Aliasyivar~~ Above (Ivor, Patrick)
$\models$ Lives (Patrick, 5)
$\models$ Opposite (Ivor, x) $\wedge$ Profession (x, Photographer)
$\models$ CONTRADICTION (Lives (4, a)) has to be a detective

$I_6 \models$ Lives (Ivor, 4)
$\models$ similar to interpretation $I_3$
Establishes $KB \models \alpha$ for $I_6$.

$I_7 \models$ Lives (Edwina, 4)
$\models$ Profession (Store Detective) $\wedge$ Lives (Edwina, 4)
$\models$ ~ Opposite (Ivor, x)
$\models$ ~ Opposite (Patrick, y)
$\models$ ~ Lives (Ivor, 3)
$\models$ ~ Lives (Ivor, 5)
$\models$ Lives (Ivor, 1)
$\models$ CONTRADICTION.

$I_8 \models$ Lives (Edwina, 5)
$\models$ not enough facts

$I_9 \models$ Lives (Edwina, 6)
$\models$ not enough facts

$I_{10} \models$ Lives (Edwina, 3) $\models I_3$ and $I_6$
$KB \models \alpha$ for $I_{10}$

Such assumptions can be made for Rodrey or Rosemary but the facts will not be sufficient. Thus the derivations would be inconclusive.

From $I_3$, $I_6$ and $I_{10}$

$$KB \models \alpha$$

Therefore the interpretations are valid

SOLUTION :

| | |
|---|---|
| FLAT 5<br>ROSEMARY<br>AIRHOSTESS | FLAT 6<br>PATRICK<br>LAWYER |
| FLAT 3<br>EDWINA<br>PHOTOGRAPHERS | FLAT 4<br>IVOR<br>STORE DETECTIVE |
| FLAT 1<br>RODNEY<br>DOCTOR | FLAT 2<br>DORIS<br>CLERK |

(iii) Yes, It is possible the name and situation of the resident of each flat. (for interpretation $I_3$, $I_6$ and $I_{10}$).

*Question 3 :*

Implementation:

The basic idea of 'The Automated Theorem Prover' is to evaluate the validity of a logical sequence and be able to produce a relevant proof if the sequence is valid.
The input is evaluated using string manipulation and separated into the LHS and RHS.
The LHS and RHS are evaluated using a Logical_Checker() which does a basic validation to ensure that the input is valid.
The Rule_Matcher() is the main function of the program and evaluates what sub expression should be evaluated next. The function evaluates all operators outside the brackets before evaluating the expressions inside. The sequence is evaluated from left to right.
The left and right expressions are passed as parameters to the Rule_Matcher which calls the next rule to be applied. The function Rule P1 – P6b call the Rule_Matcher from within until Rule P1 is derived.
The Rules and intermediate steps are stored in two global lists called Rules and Sequences respectively.
To prevent the same expressions from being evaluated, a visited list keeps track of all the evaluated expressions.
The functions use string matching and manipulation to remove characters. The recursive calls ensure the expression to be evaluated until Rule P1, where the program terminates and prints the output.

Output:

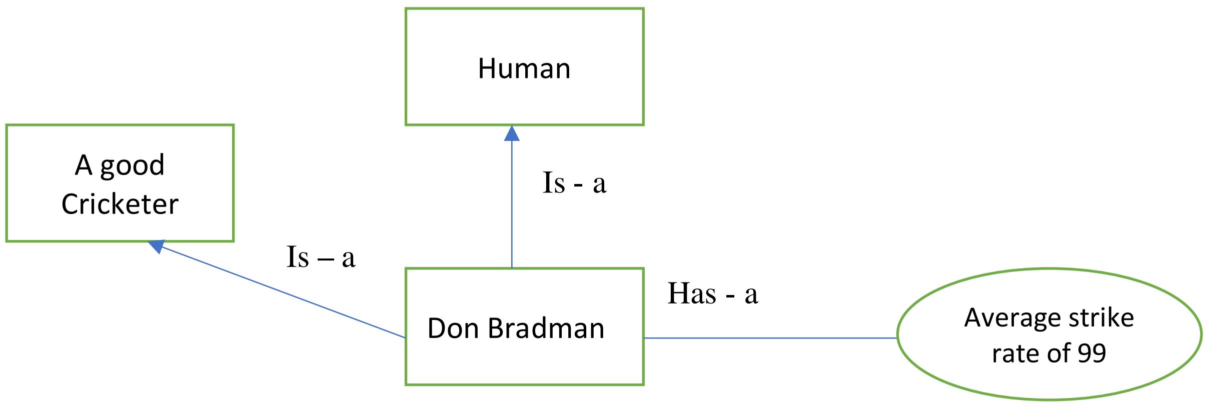
```
Vandhanas-MacBook-Pro:Desktop vandhanavisakamurthy$ python3 assign1q3.py '[p and q] seq [r]'
false
Vandhanas-MacBook-Pro:Desktop vandhanavisakamurthy$ python3 assign1q3.py '[] seq [(neg p) or p]'
true
1 . [  p ] seq [  p ]                                          Rule P1
2 . [  ] seq [ neg p , p ]                                     Rule P2a
3 . [] seq [(neg p) or p]                                      Rule P4a
QED.
Vandhanas-MacBook-Pro:Desktop vandhanavisakamurthy$ python3 assign1q3.py '[neg(p or q)] seq [neg p]'
true
1 . [ , p ] seq [ p , q ,  ]                                   Rule P1
2 . [  p ] seq [  p or q ]                                     Rule P4a
3 . [ neg(p or q) p ] seq [  ]                                 Rule P2b
4 . [  ] seq [ p , q ]                                         Rule P2a
5 . [ , p ] seq [ p , q ,  ]                                   Rule P1
6 . [  p ] seq [  p or q ]                                     Rule P4a
7 . [  ] seq [ neg p p or q ]                                  Rule P1
8 . [neg(p or q)] seq [neg p]                                  Rule P4a
QED.
Vandhanas-MacBook-Pro:Desktop vandhanavisakamurthy$
```

*Question 4 :*
*Select a method for knowledge representation and reasoning that we have not covered in lectures and provide one example that explains: (i) how the method represents knowledge; (ii) describes how inference works for reasoning with that knowledge representation*

(i). The chosen method for Knowledge Representation and Reasoning is Semantic Networks. Semantic Networks could be simple or partitioned based on the knowledge representation schema.
The basic idea of Semantic Networks is to express knowledge or facts as nodes and establish relationship between various nodes through "links". This nodal link representation of facts in a system allows the network of nodes to constantly evolve and expand. This allows a system to gather new knowledge and learn. The experiences and environment of the system contributes towards the system's learning process.

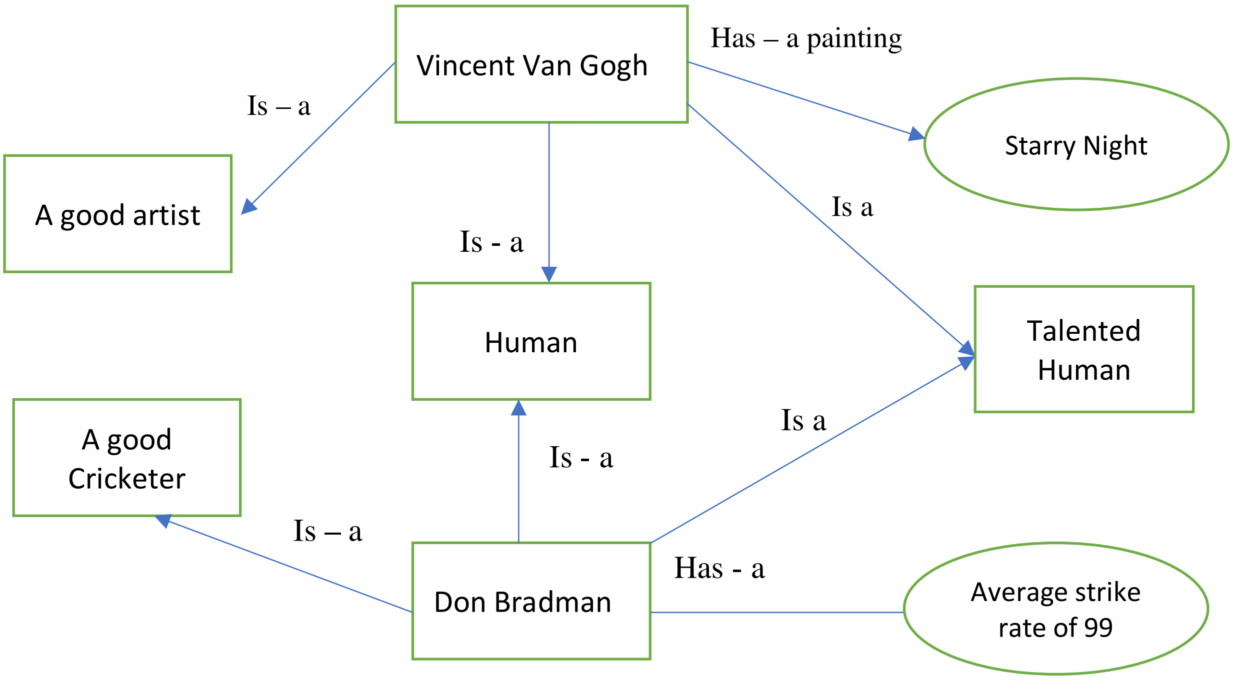
a. Every node is connected to another node in the network through a "is-a" relationship.
b. Every instance of the node inherits the properties of "has-a" relationship.
c. The Semantic Network can be easily translated into object-oriented problem statements that can be solved using object oriented programming.

Let us consider the following example :




Here, Human is a node that represents a class. Every class has properties and these properties can be inherited by various instances of this class. Don Bradman is an instance of the class Human. Don

Bradman is a good cricketer and has a strike rate of 99 is a semantic network established by is-a and has-a relationships. The network can keep expanding with more is-a relationship based instances.




Therefore a semantic network can constantly grow and establish facts and connections across various nodes. The more the number of links, the denser the network is presumed to be. This paves way for complex semantic networks or partitioned semantic networks.

(ii).

Inferences can be proved using Semantic Networks using either path based inferences or node based inferences. It is believed that using Semantic Networks can help validate inferences of First Order Logic. In fact it allows people to establish facts using a series of complex layers. In fact, a combination of path based and node based inferences helps establish logic and complex assumptions.

Is Vincent Van Gogh a talented human being ?
Vincent Van Gogh -> is a human being.
Vincent Van Gogh -> is a good artist.
He has a painting called Starry Night.
Therefore, he is a talented human being. (This was arrived using a combination of path and node based inferences).

Is Don Bradman a good cricketer ?
Don Bradman -> is a human being.
Don Bradman -> has an average strike rate of 99/100.
Therefore, he is a good cricketer.

It helps establish a detailed and structured reasoning for a given inference within the domain established.

REFERENCES :


John F. Sowa, Alexander Borgida (1991). *Principles of Semantic Networks: Explorations in the Representation of Knowledge.*
Stuart C. Shapiro . *Path Based and Node Based Inferences in Semantic Networks.*