Problem 3

a) $i = 2$

$i < 2$

$i \cdot 2$

$n = 4$

$i = 2$

$i = 4$

$n = 16$

$i = 2$

$i = 4$

$i = 16$

$n = 2$

~~$i = 2$~~

$n = 4$: 1 iter

$\log(4) = 2 \rightarrow \log(2) = 1$

$\log(2) = 1 \rightarrow \log(1) = 0$

$\log(16) = 4 \rightarrow \log(4) = 2$

$\therefore \Theta(\log \log n)$

b) $\sum_{i=1}^{n} (\Theta(1) \cdot i^3)$

b) $\sum_{i=1}^{n} \left(\Theta(1) + \sum_{k=0}^{i^3} \Theta(1) \right)$

$i = \sqrt{n}$; 4 ; 8 ; 12 ; 16

iter (k)	4	2
i =	16	4
n	16	4

$i = k\sqrt{n}$

$i = k(2)$ $\quad i = k\sqrt{n}$

Stop when

$n = i$ $\quad n = k\sqrt{n}$

$k = \sqrt{n}$

$\Theta(n) + \sum_{k=1}^{\sqrt{n}} \Theta(i^3)$ $\rightarrow$ $\Theta(n) + \sum_{k=1}^{\sqrt{n}} \Theta(k^3)\sqrt{n}^3$

$\rightarrow n^{3/2} \sum_{k=1} \Theta(k^3) \rightarrow \sqrt{n}^3 \cdot \Theta(\sqrt{n})$ $\quad \Theta(n^{7/4})$

c)

$$\sum_{i=1}^{n} \sum_{k=1}^{n} \left(\Theta(1) + \sum_{m=1}^{i} \Theta(1) \right)$$

$\hookrightarrow \Theta(n^2) + \Theta\left(\sum_{m=1}^{n} \Theta(1) \right)$

K	3.iter	4.iter
m =	4	8
n	4	8

$$\Theta(n^2) + \sum_{K=1}^{\log(2n)} \Theta(n)$$

m=1, m=2, m=4

$K = \log(2n)$

$\Theta(n^2) + \Theta(n \log 2n)$

$\therefore \Theta(n^2)$

d) size = 10

$k = \log(2n)$

$\Theta(n^2) + \Theta(n \log 2n)$

$\therefore \Theta(n^2)$

d) size = 10

$$\sum_{i=0}^{n-1} \left(\Theta(1) + O\left(\sum_{j=0}^{size-1} \right) \right)$$

if i == size // worst case: runs once

runs once:

↳ $\sum_{j=0}^{size-1}$

newsize = 15

j = 0, j < 10, j++

size = 15

// depends on n

$\therefore \Theta(n)$