Branch & Bound.

- It is a state space method in which all children of a node are generated before expanding any of its children.
- Back tracking is also a state space search method but it is useful subset or permutation problem, it is not good for optimisation problem
- B & B is used for optimisation problem.

Terminologies for SST.

1] Live node - A node which has been generated and all of whose children have not been yet generated is called a live node.

2] E-node (node to be expanded) - It is a live node whose children are currently been generated

3] Dead node :- It is a node which cannot be expanded further or all of whose children has been generated. There are two approaches of solving SST.

1] Backtracking - Let r be the current E-node & its child is generated. Now c will become E-node & subtree is explore. After that it will return back to parent r. Now parent r will become E-node & next child is generated. (DFS)

2] B & B - The E node remains E node until it is dead. There are 3 ways to solve branch and bound problems (BFS)

1] FIFO B & B

2] LIFO B & B

3] LC ← (Least cost search B & B)

LIFO & FIFO B & B - The next live node becomes E node depending on order of generation of that node. Selection rule for next E node is blind (i.e in order of generation)

FIFO B & B :- Nodes are processed in same order of generation. Queue data structure is used.

LIFO B & B :- Nodes are processed in reverse order of generation. Stack data structure is used

→ Selection rule for next E-node doesnot give preference to a node that has very good chance of getting answer ~~corre~~ node quickly

LC B & B :- It directs the search to the part of the tree which is most likely to contain the ans. node. For this the ranking function is required.

Rank is assigned based on additional computational efforts required to reach to an answer node from current E-node. This requires intelligence

Q. 15-puzzle problem :-

1	3	4	15
2	$\boxed{\ }$	5	12
7	6	11	14
8	9	10	13

initial arrangement

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	$\boxed{\ }$

goal arrangement

- Objective of problem is transfor the initial arrangement of tiles to goal arrangement.
- The legal moves are only adjacent tiles can be moved to an empty spot
- As per initial arrangement the tiles 2, 3, 5, 6 can only be moved to the empty space.
- Each move creates a new arrangement of tiles & that will be a state in SST.
- We will solve this problem LC B&B.

The cost of every node is calculated as

$C(x) = f(x) + g(x)$

$f(x) \Rightarrow$ cost of reaching node x from root node

$C(x) \Rightarrow$ node no. $g(x) \Rightarrow$ estimate of additional effort required to reach to ans node from current node

In this problem $f(x) \Rightarrow$ no of moves so far.
$g(x) \Rightarrow$ no. of non-blank tiles not according to the goal position.

Assumption :- One move of tile in any direction, the cost is 1

TSP

$$
C = \begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & \infty & 20 & 30 & \underline{10} & 11 \\
2 & 15 & \infty & 16 & 4 & 2 \\
3 & 3 & 5 & \infty & 2 & 4 \\
4 & 19 & 6 & 18 & \infty & 3 \\
5 & 16 & 4 & 7 & 16 & \infty
\end{array}
\quad
\begin{array}{c}
10 \\ 2 \\ 2 \\ 3 \\ 4
\end{array}
\quad = 21
$$

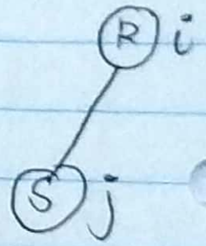
In this problem, every node in SST will have a reduced cost matrix associated. A row/column is said to be reduced if an only if it contains atleast one zero & all remaining entries are non-negative. A matrix is reduced if an only if every row and every column is reduced. For the root node the original cost matrix is reduced by obtaining row min & col min - the value is 25, it says that the tour is of atleast value 25 (i.e its a lower bound on tour)

$$C = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 10 & 20 & 0 & 1 \\ 2 & 13 & \infty & 14 & 2 & 0 \\ 3 & 1 & 3 & \infty & 0 & 2 \\ 4 & 16 & 3 & 15 & \infty & 0 \\ 5 & 12 & 0 & 3 & 12 & \infty \end{array} \qquad C = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & 10 & 17 & 0 & 1 \\ 2 & 12 & \infty & 11 & 2 & 0 \\ 3 & 0 & 3 & \infty & 0 & 2 \\ 4 & 15 & 3 & 12 & \infty & 0 \\ 5 & 11 & 0 & 0 & 12 & \infty \end{array}$$

$1+3=4$ $\qquad$ $21+4 = \underline{\underline{25}}$

Reduced cost matrix associated with node 1

Since the tour is starting at city 1, at level 0 city 1 is visited.



$\langle i,j \rangle$ path is selected, note down

1] A(i,j) since the cost of this path is included in the tour for node S

2] Mark all entries i to k (s,1) as ∞, this will ~~prevent~~ exclude all other paths than j.

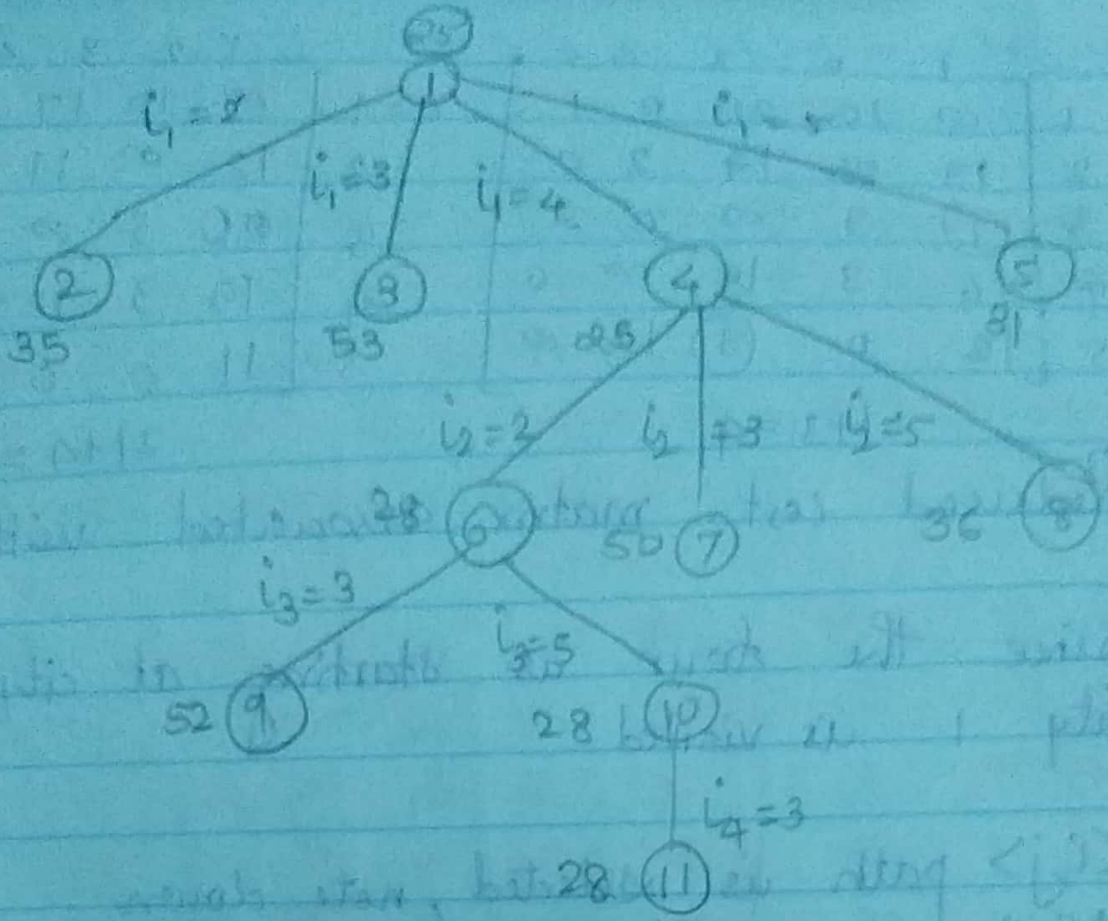
3] Mark all entries k to j as ~~inf~~ ∞. Mark entry C(i,1) as ∞ this will exclude the path back to original city (not for a leaf node).

Obtaining A'

2] Reduce matrix A' → RC (Reduced cost)

3] Cost of S, C(S) = C(R) + RC + A(i,j)

Scanned with CamScanner



Edge selected $\langle 1,2 \rangle$

i] Mark $A(1,2) = 10$

mark $A(2,1) = \infty$

$C(2) = C(1) + 0 + 10 = 25 + 0 + 10 = 35$

$$A' = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}$$

Egde selected $\langle 1,3 \rangle$

ii] Mark $A(1,3) = 17$

$A(3,1) = \infty$

$C(3) = C(r) + 11 + 17$

$= 25 + 11 + 17$

$= \underline{\underline{53}}$

$$A' = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ \underline{\underline{11}} & 0 & \infty & 12 & \infty \end{bmatrix}$$

$$A' = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & \infty \end{bmatrix}$$

Edge selected <1,4>

iii] Mark $A(1,4) = 0$

mark $A(4,1) = \infty$

$C(4) = C(1) + RC + A(1,4)$

$= 25 + 0 + 0$

$= 25$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

Edge selected <1,5>

iv] Mark $A(1,5) = 1$

Mark $A(5,1) = \infty$

$C(5) = C(1) + RC + A(1,5)$

$= 25 + 5 + 1$

$= 31$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

~~v] Mark A(1,4,2)~~

Edge selected <4,2>

v] Mark $A(4,2) = 3$

Mark $A(2,4) = \infty$

$C(2) = C(4) + RC + A(4,2)$

$= 25 + 0 + 3$

$= 28$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

25
+11
36
+3
39

Edge selected <4,3>

Mark $A(4,3) = 12$

Mark $A(3,4) = \infty$

$C(3) = C(4) + RC + A(4,3)$

$= 25 + 12 + 12$

$= 50$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & \infty \\ 0 & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ 0 & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$

24
25

Edge selected <4,5>

Mark $(4,5) = 0$

Mark $(5,4) = \infty$

$C(5) = 25 + 2 + 0 = 27$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix}$$