CodingLab7

June 19, 2022

Neural Data Science

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Summer term 2022

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1 Coding Lab 7

[71]:
```
import seaborn as sns
import matplotlib.pyplot as plt
import matplotlib as mpl
import numpy as np
import scipy.optimize as opt
import scipy.io as io
import scipy as sp

mpl.rc("savefig", dpi=72)

import itertools

sns.set_style('whitegrid')
%matplotlib inline
```

1.1 Task 1: Fit RF on simulated data

We will start with toy data generated from an LNP model neuron to make sure everything works right. The model LNP neuron consists of one Gaussian linear filter, an exponential nonlinearity and a Poisson spike count generator. We look at it in discrete time with time bins of width δt. The model is:

$$c_t \sim Poisson(r_t) r_t = \exp(w^T s_t) \cdot \Delta t \cdot R$$

Here, c_t is the spike count in time window t of length Δt, s_t is the stimulus and w is the receptive field of the neuron. The receptive field variable `w` is 15×15 pixels and normalized to $||w|| = 1$. A stimulus frame is a 15 × 15 pixel image, for which we use uncorrelated checkerboard noise. R can be used to bring the firing rate into the right regime (e.g. by setting $R = 50$).

For computational ease, we reformat the stimulus and the receptive field in a 225 by 1 array. The function `sampleLNP` can be used to generate data from this model. It returns a spike count vector `c` with samples from the model (dimensions: 1 by nT = $T/\Delta t$), a stimulus matrix `s` (dimensions: 225 × nT) and the mean firing rate `r` (dimensions: nT × 1).

Here we assume that the receptive field influences the spike count instantaneously just as in the above equations. Implement a Maximum Likelihood approach to fit the receptive field.

To this end simplify and implement the log-likelihood function $L(w)$ and its gradient $\frac{L(w)}{dw}$ with respect to w (`logLikLnp`). The log-likelihood of the model is

$$L(w) = \log \prod_t \frac{r_t^{c_t}}{c_t!} \exp(-r_t).$$

Plot the true receptive field, a stimulus frame, the spike counts and the estimated receptive field.

1.1.1 Calculations

Simplify the log likelihood analytically and derive the analytical solution for the gradient. (2 pts)

See also: How to use Latex in Jupyter notebook.

$$L(w) = \log \prod_t \frac{r_t^{c_t}}{c_t!} \exp(-r_t) = \sum_t \log \frac{r_t^{c_t}}{c_t!} \exp(-r_t) = \sum_t \log r_t^{c_t} - \log c_t! + \log \exp(-r_t) = \sum_t c_t \log r_t - \log c_t! - r_t$$

Because $c_t!$ does not depend on w, we can move it to an additive constant. Using $r_t = \exp(w^T s_t) dtR$ we obtain:

$$L(w) = \sum_t c_t (w^T s_t + dtR) - \exp(w^T s_t) dtR + const_1. = \sum_t c_t w^T s_t - \exp(w^T s_t) dtR + const_2.$$

Note that s_t denotes a vector and c_t a scalar, in slight abuse of notation.

For the gradient:

$$dL(w)/dw = \sum_t c_t s_t - s_t \exp(w^T s_t) dtR = \sum_t (c_t - \exp(w^T s_t) dtR) s_t$$

This is interesting and makes intuitive sense: for the gradient, each stimulus frame is weighted by the difference between the observed and predicted count.

1.1.2 Generate data

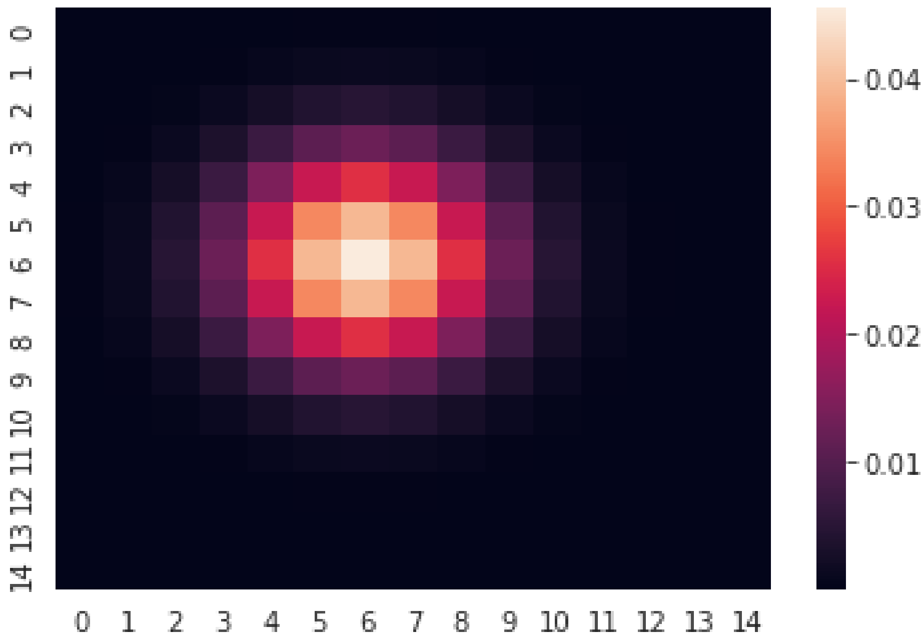
[72]:
```
def gen_gauss_rf(D, width, center=(0,0)):
    # Generate gaussian blop (image)
```

```
    sz = (D-1)/2
    x, y = sp.mgrid[-sz: sz + 1, -sz: sz + 1]
    x = x + center[0]
    y = y + center[1]
    w = np.exp(- (x ** 2/width + y ** 2 / width))
    w = w / np.sum(w.flatten())

    return w

w = gen_gauss_rf(15, 7, (1,1))
sns.heatmap(w)
```

[72]: <AxesSubplot:>



[73]:
```
def sample_lnp(w, nT, dt, R, v):
    '''Generate samples from an instantaneous LNP model neuron with
    receptive field kernel w.

    Parameters
    ----------

    w: np.array, (Dx * Dy, )
        (flattened) receptive field kernel.

    nT: int
```

```
        number of time steps

    dt: float
        duration of a frame in s

    R: float
        rate parameter

    v: float
        variance of the stimulus ensemble


    Returns
    -------

    c: np.array, (nT, )
        sampled spike counts in time bins

    r: np.array, (nT, )
        mean rate in time bins

    s: np.array, (Dx * Dy, nT)
        stimulus frames used

    Note
    ----

    See equations in task description above for a precise definition
    of the individual parameters.

    '''

    np.random.seed(42)

    # sampled spike counts array
    c = np.empty(0)
    # stimulus frames
    s = np.random.binomial(v,.5, size=(w.shape[0], nT))
    # mean rate
    r = np.exp(w.T@s)*dt*R


    # Poisson spike counts
    for i in r:
        c = np.append(c,np.random.poisson(lam=i))
    # ------------------------------------------------
    # Generate samples from an instantaneous LNP model
```

```
    # neuron with receptive field kernel w. (0.5 pts)
    # ------------------------------------------------

    return c, r, s

    ### variance of stimulus ensemble should be used, but cannot
    ### find any information about it ??
```

[74]:
```
D = 15      # number of pixels in one dimension,
            # the simulated RF here is a square
nT = 1000   # number of time bins
dt = 0.1    # frame rate, 0.1s per bin.
R = 50      # firing rate in Hz
v = 5       # stimulus variance

w = gen_gauss_rf(D,7,(1,1))
w = w.flatten()

c, r, s = sample_lnp(w, nT, dt, R, v)

print(r)
```

```
[ 45.16079112  61.2444825   57.92825726  70.57315883  51.03841263
  69.28122586  51.57503587  47.98142672  73.67549152  60.80166229
  36.19769933  61.61781181  50.51765486  75.82653885  68.42280142
  70.03707651  51.07818213  70.69517195  58.70229449  59.15217314
  51.99673135  58.65478548  69.03614416  67.70544604  62.0756533
  59.16600587  79.74341343  50.55251172  68.75874643  78.07916502
  75.18731366  63.34370384  74.03894968  51.30507619  51.987317
  51.00122665  46.40165604  71.58532174  56.29757445  64.10962769
  61.81287874  71.13143504  49.01688991  64.29248362  59.12257075
  61.78789427  56.73926546  66.57981907  61.95823976  88.04126598
  56.08286162  55.20293708  54.45155882  54.99377411  57.55592285
  50.17627773  40.35399913  45.76817337  72.81049238  55.16615834
  52.68164192  69.45792936  63.71786851  56.86658759  57.47433634
  68.96741661  48.79396381  50.511963    48.80983008  64.25739363
  70.6370607   73.87194704  61.14001364  64.4296986   81.58549585
  50.19563472  57.95015105  55.72805231  49.64874937  66.19899653
  63.23429517  74.04497319  60.41887163  71.32415429  76.23221357
  62.3816191   62.03771979  57.04705658  47.27700007  66.61813326
  61.97130376  46.86639261  87.87377696  64.64565502  58.56288509
  60.93311865  41.68000473  57.40290521  65.70154031  64.37529238
  54.41237409  46.88480874  61.97145198  67.17114875  54.68042318
  71.13028538  64.41631753  55.08248283  61.6020784   43.0292685
  61.41667511  44.79555967  68.1843978   63.32965658  56.30640152
  61.18748738  57.19761595  55.99533904  61.22806996  82.19799243
  63.29214525  68.86407314  56.60873813  65.06789888  56.1151765
  59.59620702  70.62038289  48.41531628  42.01391558  64.55741919
```

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53.57696933	60.73879305	63.63328045	55.78699977	62.6114615
72.21731139	55.02458437	59.78700493	64.13734431	65.18191357
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84.44376235	70.02643533	90.24175627	56.98363662	59.4475462
54.49748741	48.08843157	43.19932669	67.54024099	60.09086606
70.42574603	56.96035563	60.91972082	64.94342732	60.59504128
64.17726555	65.27623182	69.64077535	45.75734243	52.39273135
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57.88887411	47.20931181	57.62296663	88.68999187	53.95437234
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```
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 68.34476269  53.79225003  53.59539158  68.8648707   44.6305188
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 76.86600508  52.89559321  71.06311976  70.69530915  47.16104073
 57.70619923  54.00515216  70.13379886  69.95551058  61.77482636
 56.68679634  53.55279301  59.9103346   72.1763101   42.02253602
 66.9682372   66.26138158  52.59582735  53.99500645  66.24954331
 82.11329909  60.30266523  55.85380784  44.11848611  61.69678255
 75.24103348  53.92751841  62.84507565  49.17936256  52.9221562
 67.96199304  80.5335813   65.5903656   71.45035478  49.85273191
 60.34320571  53.28354496  45.38882376  66.14970379  54.79864225
 62.7927989   56.17333188  60.60721678  61.60652394  68.57911984
 75.16812931  65.32476633  63.42748546  55.08563758  75.48795093
 68.41728658  57.87423659  61.76619416  61.61211353  70.11977009
 57.73117603  55.19491717  70.60646526  98.70918857  54.92074294
 53.64424853  69.63803834  55.48078047  64.62749403  63.17567559
 56.87259707  66.73762793  60.26666417  83.61176555  62.80350723
 67.00782073  67.98014269  52.29927303  62.70882125  57.22994511
 62.04673159  69.53487734  62.33837778  44.15173983  66.41555065]
```
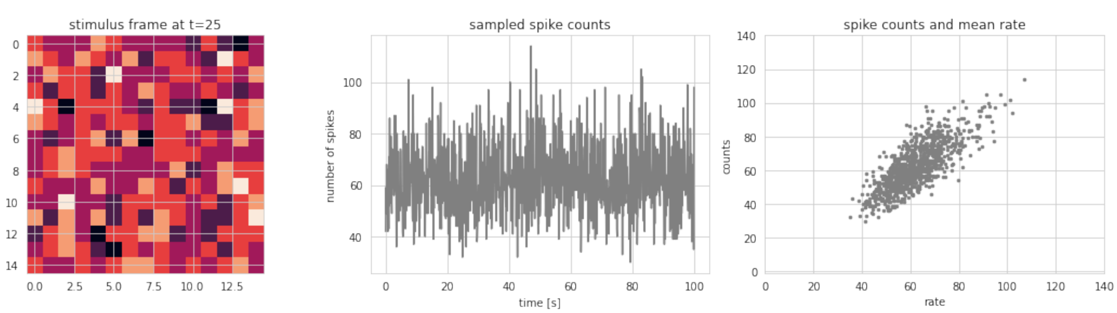

Plot the responses of the cell.

[75]:
```python
# insert your code here

# --------------
# Plot (0.5 pts)
# --------------
t = np.arange(0,nT) * dt
#plt.figure(figsize=(15,4))

fig, axs = plt.subplots(1, 3,figsize=(15,4))
# -----------------------------------------------
# (1) one example frame from `s`;
# -----------------------------------------------

axs[0].imshow(s[:,25].reshape((D,D)))
axs[0].set_title('stimulus frame at t=25')
```

```
# ------------------------------------------------
# (2) the simulated response `c`;
# ------------------------------------------------
axs[1].plot(t, c, color='grey')

axs[1].set_ylabel('number of spikes')
axs[1].set_xlabel('time [s]')
axs[2].set_ylim(0,140)
axs[1].set_xlim(-5,105)
axs[1].set_title('sampled spike counts')



# ------------------------------------------------
# (3) a scatter plot of `r` and `c`;
# ------------------------------------------------

axs[2].scatter(r,c,s=6,color = 'grey')
axs[2].set_ylabel('counts')
axs[2].set_xlabel('rate')
axs[2].set_xlim(0,140)
axs[2].set_ylim(-1,140)
axs[2].set_title('spike counts and mean rate')




plt.tight_layout()
plt.show()
```



1.1.3 Implementation

Before you run your optimizer, make sure the gradient is correct. The helper function `check_grad` in `scipy.optimize` can help you do that. This package also has suitable functions for optimization. If you generate a large number of samples, the fitted receptive field will look more similar to the true receptive field. With more samples, the optimization takes longer, however.

[76]:
```python
def negloglike_lnp(x, c, s, dt=0.1, R=50):
    '''Implements the negative (!) log-likelihood of the LNP model and its
    gradient with respect to the receptive field w.

    Parameters
    ----------

    x: np.array, (Dx * Dy, )
      current receptive field

    c: np.array, (nT, )
      spike counts

    s: np.array, (Dx * Dy, nT)
      stimulus matrix


    Returns
    -------

    f: float
      function value of the negative log likelihood at x

    df: np.array, (Dx * Dy, )
      gradient of the negative log likelihood with respect to x
    '''

    p_1 = np.sum(c*np.log(dt*R))
    p_2 = np.sum(np.log(sp.special.factorial(c)))
    # use neg loglikelihood
    f = -c@(x.T@s)
    f += -(p_1+p_2)
    f += np.sum(np.exp(x.T@s)*dt*R)
  # Compute gradient
    d_1 = c@s.T
    d_2 = dt*R*np.exp(x.T@s)
    df = -(d_1 - d_2@s.T)
    # ------------------------------------------------
    # Implement the negative log-likelihood of the LNP
    # and its gradient with respect to the receptive
    # field `w` using the simplified equantions you
    # calculated earlier. (0.5 pts)
    # ------------------------------------------------
```

```
    return f, df
```

Fit receptive field maximizing the log likelihood

[77]:
```python
f, df = negloglike_lnp(w, c, s)

# function derivative to be checked
f = lambda w: negloglike_lnp(w, c, s)[0]
# gradient to be checked
df = lambda w: negloglike_lnp(w, c, s)[1]
# check if gradient is correct
print(opt.check_grad(f,df,w))

# Estimate receptife field rf
random_rf = np.random.randint(2,size=(D*D))/1000
estimated_rf = opt.minimize(negloglike_lnp,random_rf,args=(c,s),jac=True).x
# ------------------------------------------
# Estimate the receptive field by maximizing
# the log-likelihood (or more commonly,
# minimizing the negative log-likelihood).
#
# Tips: use scipy.optimize.minimize(). (1 pt)
# ------------------------------------------
```

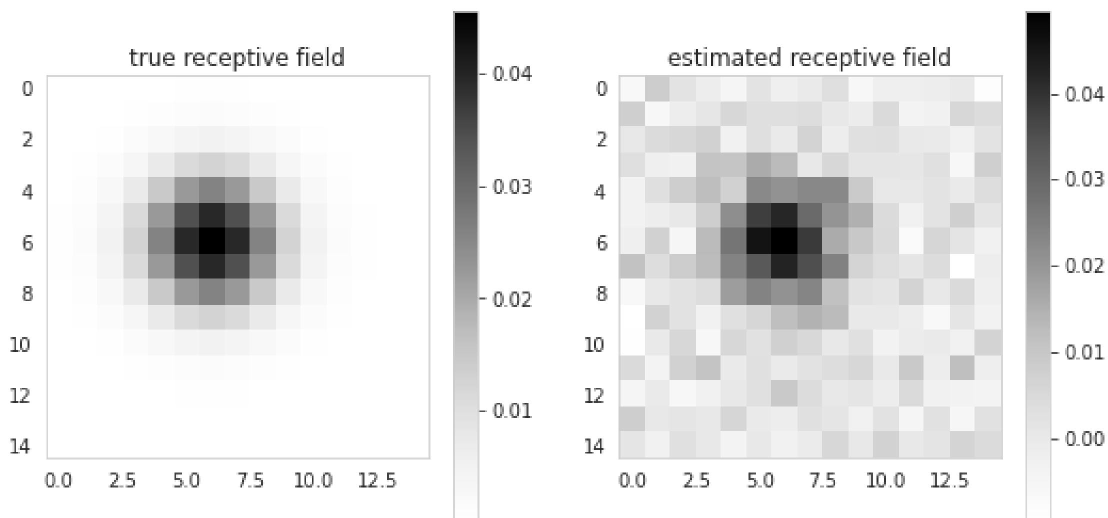
```
0.0266871430454665
```

[78]:
```python
fig, axs = plt.subplots(1,2,figsize=(10,5))


p1=axs[0].imshow(w.reshape((D,D)), cmap='Greys')
axs[0].grid(False)
axs[0].set_title('true receptive field')

p2=axs[1].imshow(estimated_rf.reshape((D,D)), cmap='Greys')
axs[1].grid(False)
axs[1].set_title('estimated receptive field')


plt.tight_layout
plt.colorbar(p1,ax=axs[0])
plt.colorbar(p2,ax=axs[1])

# ------------------------------------
# Plot the ground truth and estimated
# `w` side by side. (0.5 pts)
# ------------------------------------
```

```
/home/v/.local/lib/python3.7/site-packages/ipykernel_launcher.py:14:
MatplotlibDeprecationWarning: Auto-removal of grids by pcolor() and pcolormesh()
```

```
is deprecated since 3.5 and will be removed two minor releases later; please
call grid(False) first.

/home/v/.local/lib/python3.7/site-packages/ipykernel_launcher.py:15:
MatplotlibDeprecationWarning: Auto-removal of grids by pcolor() and pcolormesh()
is deprecated since 3.5 and will be removed two minor releases later; please
call grid(False) first.
  from ipykernel import kernelapp as app
```

[78]:
```
<matplotlib.colorbar.Colorbar at 0x7fb2b3eb36d8>
```



1.2 Task 2: Apply to real neuron

Download the dataset for this task from Ilias (`nda_ex_6_data.mat`). It contains a stimulus matrix (`s`) in the same format you used before and the spike times. In addition, there is an array called `trigger` which contains the times at which the stimulus frames were swapped.

- Generate an array of spike counts at the same temporal resolution as the stimulus frames
- Fit the receptive field with time lags of 0 to 4 frames. Fit them one lag at a time (the ML fit is very sensitive to the number of parameters estimated and will not produce good results if you fit the full space-time receptive field for more than two time lags at once).
- Plot the resulting filters

Grading: 2 pts

[79]:
```
var = io.loadmat('./data/nda_ex_6_data.mat')

# t contains the spike times of the neuron
t = var['DN_spiketimes'].flatten()
```

```
# trigger contains the times at which the stimulus flipped
trigger = var['DN_triggertimes'].flatten()

# contains the stimulus movie with black and white pixels
s = var['DN_stim']
s = s.reshape((300,1500)) # the shape of each frame is (20, 15)
s = s[:,1:len(trigger)]
```

Create vector of spike counts

[80]:
```
c,b = np.histogram(t,bins=trigger)
# ------------------------------------------
# Bin the spike counts at the same temporal
# resolution as the stimulus (0.5 pts)
# ------------------------------------------
```

Fit receptive field for each frame separately

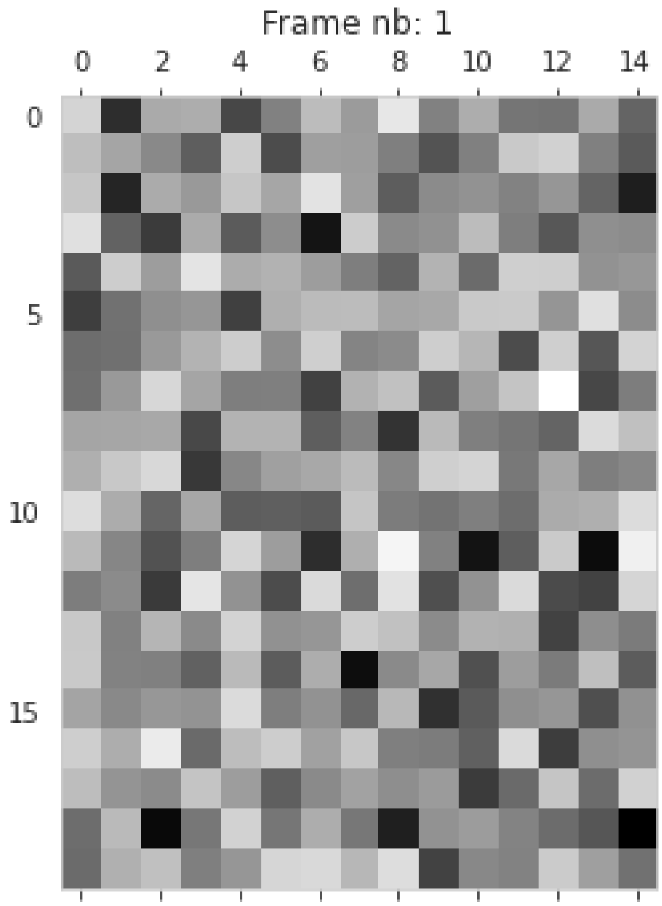
[81]:
```
nT = 1000  # number of time bins
lags = np.arange(5) # time lags of 0-5 frames
res = s.shape[0] # same as frame shape
D1 = 20 # number of pixels in one dimension
D2 = 15 # number of pixels in one dimension


random_rf = (np.random.uniform(0,1,size=res)/nT).flatten()
estim_rf_lags = np.zeros((len(lags),res))
# Use one lag at a time
for l in lags:
  estim_rf_lags[l,:] = opt.minimize(negloglike_lnp,random_rf,args=(c[l:],s[:,:
 ↪len(c)-l]),jac=True).x


# ------------------------------------------
# Fit the receptive field with time lags of
# 0 to 4 frames separately (1 pt)
#
# The final receptive field (`w_hat`) should
# be in the shape of (Dx * Dy, 5)
# ------------------------------------------
```

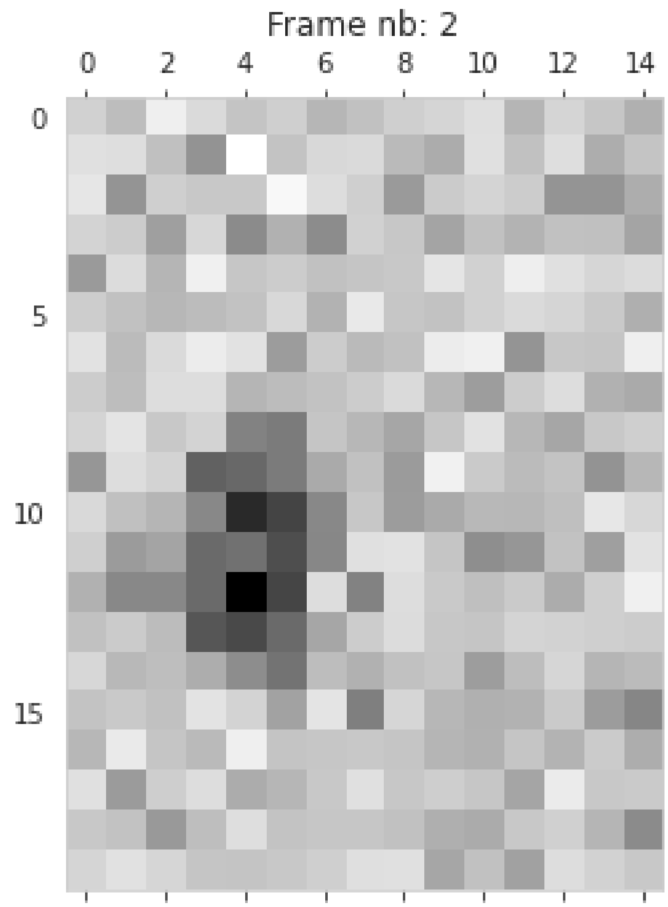
Plot the frames one by one

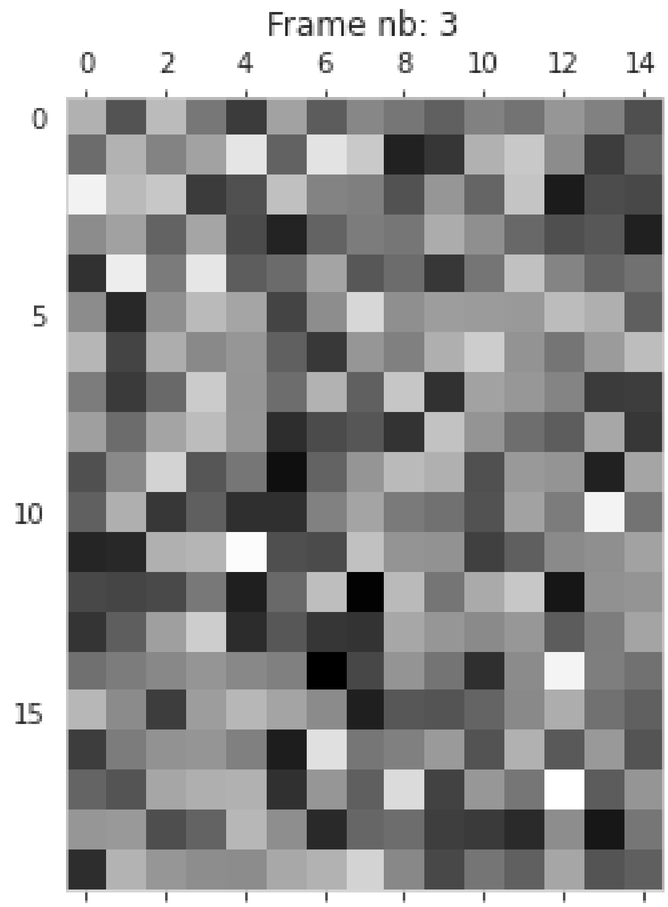
[82]:
```
matfig = plt.figure(figsize=(10,4))


for i in lags:
  plt.matshow(estim_rf_lags[i,:].reshape((D1,D2)), cmap='Greys')
```

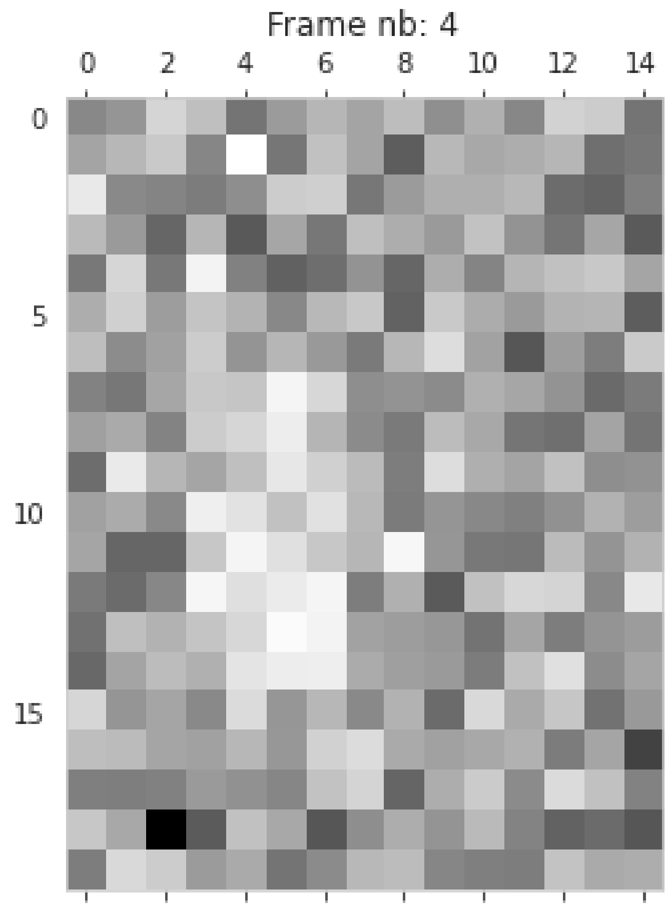
```
  plt.grid(False)
  plt.title("Frame nb: " + str(i+1))
  #plt.tight_layout()
# --------------------------------------------
# Plot all 5 frames of the fitted RFs (0.5 pt)
# --------------------------------------------
```

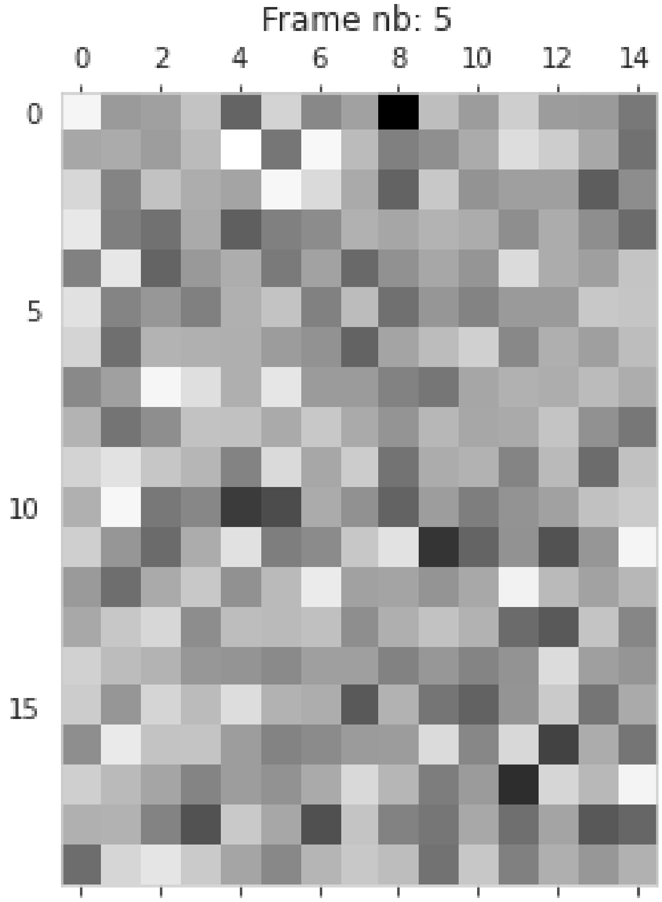
```
<Figure size 720x288 with 0 Axes>
```




Frame nb: 2
0
2
4
6
8
10
12
14
0
5
10
15


Frame nb: 3
0
2
4
6
8
10
12
14
0
5
10
15


Frame nb: 4
0
2
4
6
8
10
12
14
0
5
10
15



1.3 Task 3: Separate space/time components

The receptive field of the neuron can be decomposed into a spatial and a temporal component. Because of the way we computed them, both are independent and the resulting spatio-temporal component is thus called separable. As discussed in the lecture, you can use singular-value decomposition to separate these two:

$$W = u_1 s_1 v_1^T$$

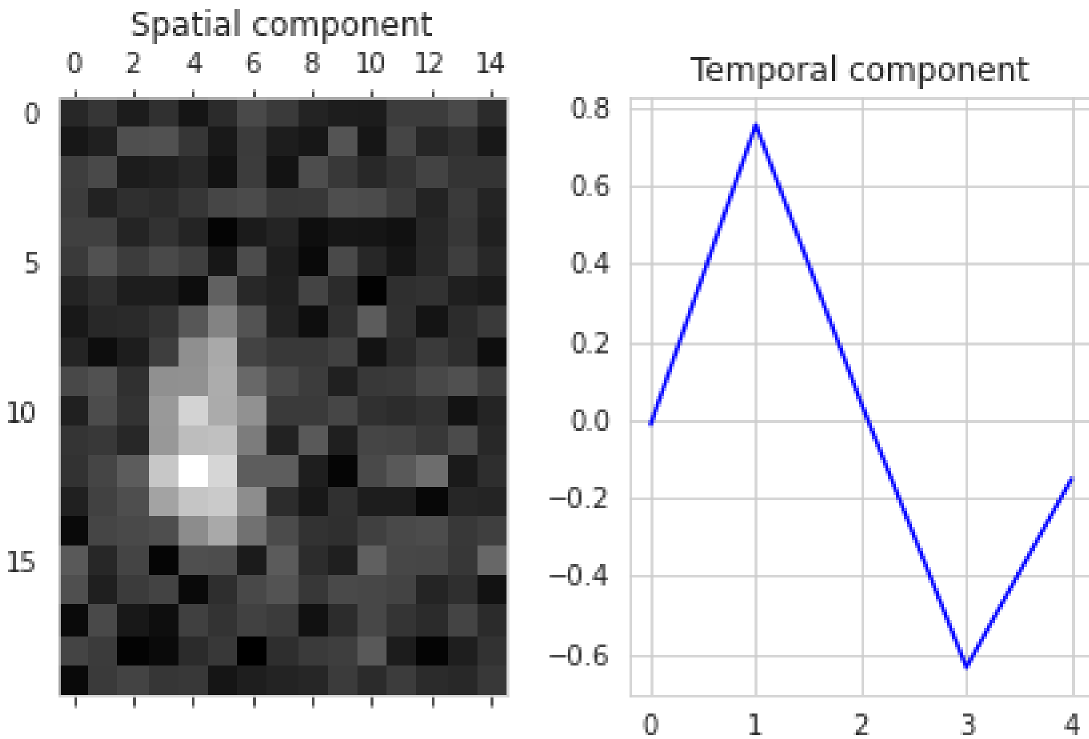
Here u_1 and v_1 are the singular vectors belonging to the 1st singular value s_1 and provide a long rank approximation of W, the array with all receptive fields. It is important that the mean is subtracted before computing the SVD.

Plot the first temporal component and the first spatial component. You can use a Python implementation of SVD. The results can look a bit puzzling, because the sign of the components is arbitrary.

Grading: 1 pts

[83]:

```
W = estim_rf_lags
# substract mean
W = W - np.mean(estim_rf_lags, axis=0)

# compute svd
u,_,v = np.linalg.svd(W)

# temporal component
u_temp = u[:,0]
# spatial component
v1 = v[0,:]
v_spatial = v1.reshape(D1,D2)

fig, axs = plt.subplots(1,2, figsize=(6,4), squeeze=True)

axs[0].matshow(v_spatial, cmap='gray')
axs[0].set_title("Spatial component")
axs[0].grid(False)

axs[1].plot(u_temp, color='blue')
axs[1].set_title("Temporal component")



plt.tight_layout()
plt.show()

# --------------------------------------------
# Apply SVD to the fitted receptive field,
# you can use either numpy or sklearn (0.5 pt)
# --------------------------------------------

# --------------------------------------------
# Plot the spatial and temporal components (0.5 pt)
# --------------------------------------------
```



1.4 Task 4: Regularized receptive field

As you can see, maximum likelihood estimation of linear receptive fields can be quite noisy, if little data is available.

To improve on this, one can regularize the receptive field vector and a term to the cost function

$$C(w) = L(w) + \alpha||w||_p^2$$

Here, the p indicates which norm of w is used: for $p = 2$, this is shrinks all coefficient equally to zero; for $p = 1$, it favors sparse solutions, a penality also known as lasso. Because the 1-norm is not smooth at zero, it is not as straightforward to implement "by hand".

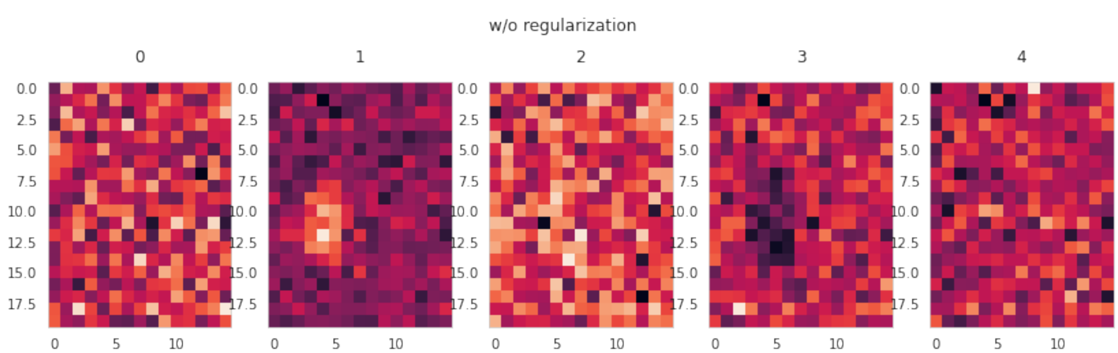
Use a toolbox with an implementation of the lasso-penalization and fit the receptive field. Possibly, you will have to try different values of the regularization parameter α. Plot your estimates from above and the lasso-estimates. How do they differ? What happens when you increase or decrease $alpha$?

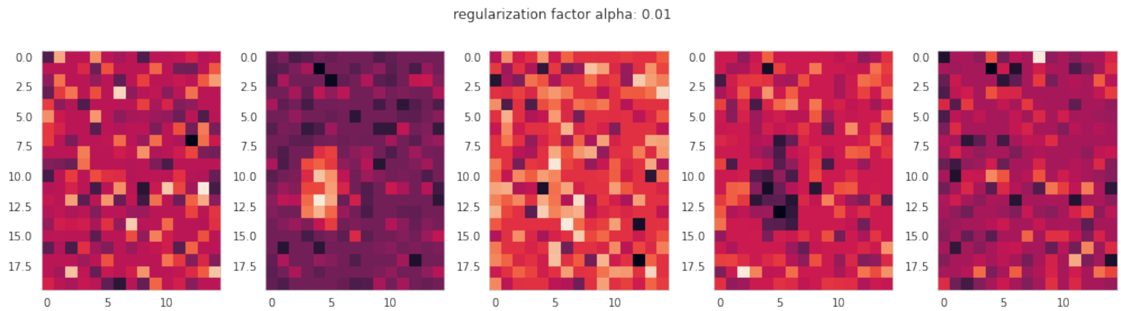
If you want to keep the Poisson noise model, you can use the implementation in `pyglmnet`. Otherwise, you can also resort to the linear model from `sklearn` which assumes Gaussian noise (which in my hands was much faster).

Grading: 2 pts

[87]:

```python
from sklearn import linear_model

alpha = 0.01
fig,axs = plt.subplots(1,len(lags),figsize=(14,4))
fig.suptitle('w/o regularization')

for i,l in enumerate(lags):
  colors = axs[i].imshow(estim_rf_lags[l,:].reshape((D1,D2)))
  axs[i].set_title(str(l), pad = 15)
  axs[i].grid(False)


fig,ax = plt.subplots(1,len(lags),figsize=(14,4))
fig.suptitle('regularization factor alpha: '+str(alpha))

for i,l in enumerate(lags):
  model = linear_model.Lasso(alpha=alpha, selection="random")
  model.fit(s.T,c)

  lasso_rf_lags = np.zeros((len(lags),res))
  model.fit(s[:,:len(c)-l].T,c[l:])

  lasso_rf_lags[l,:] = model.coef_
  ax[i].grid(False)

  colors = ax[i].imshow(lasso_rf_lags[l,:].reshape((D1,D2)))
  plt.tight_layout()



# ------------------------------------------
# Fit the receptive field with time lags of
# 0 to 4 frames separately (the same as before)
# with sklern or pyglmnet (1 pt)
# ------------------------------------------
```





For alpha=0.01 the lasso estimation fits the estimated filter quite good. This does not noticably change with slightly decreasing values for alpha. With a larger value for alpha the estimation gets less accurate. If alpha is chosen too small, the result gets worst.

[85]:
```
# insert your code here


# Included in the winodw above


# --------------------------------------------------
# Plot all 5 frames of the fitted RFs, compare them
# with the ones without regularization (0.5 pt)
# --------------------------------------------------
```