STAT 206 Lab 6

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Due Tuesday, November 12, 5:00 PM

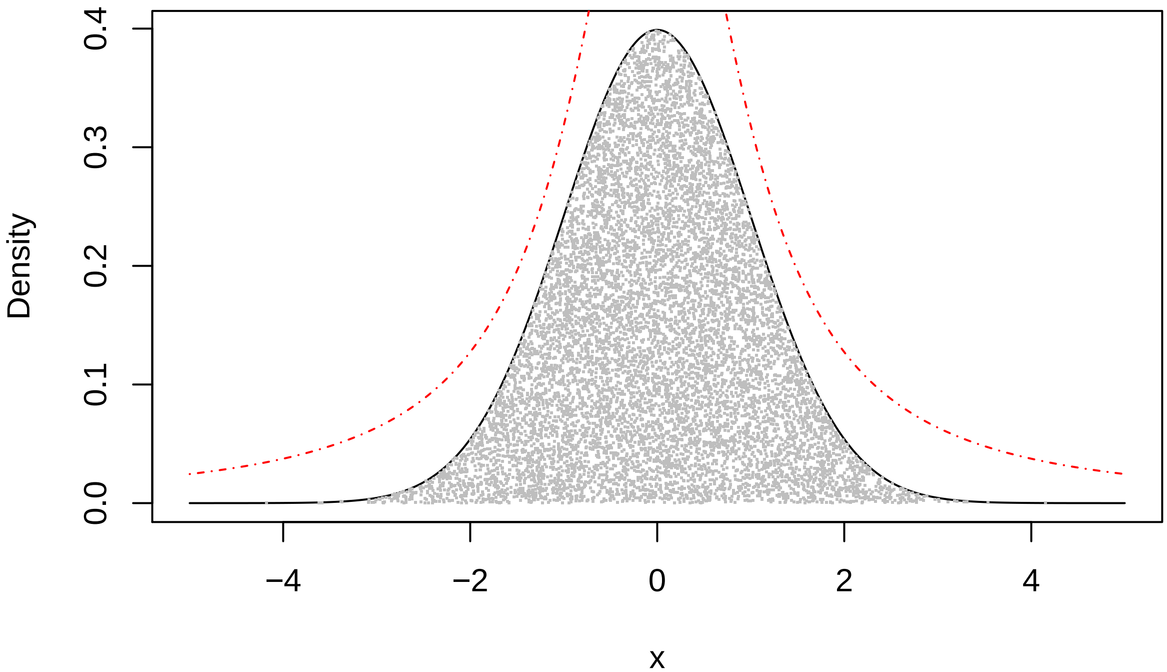
General instructions for labs: Labs must be completed as a pdf file. Give the commands to answer each question in its own code block, which will also produce plots that will be automatically embedded in the output file. Each answer must be supported by written statements as well as any code used.

Agenda: Accept-Reject algorithm

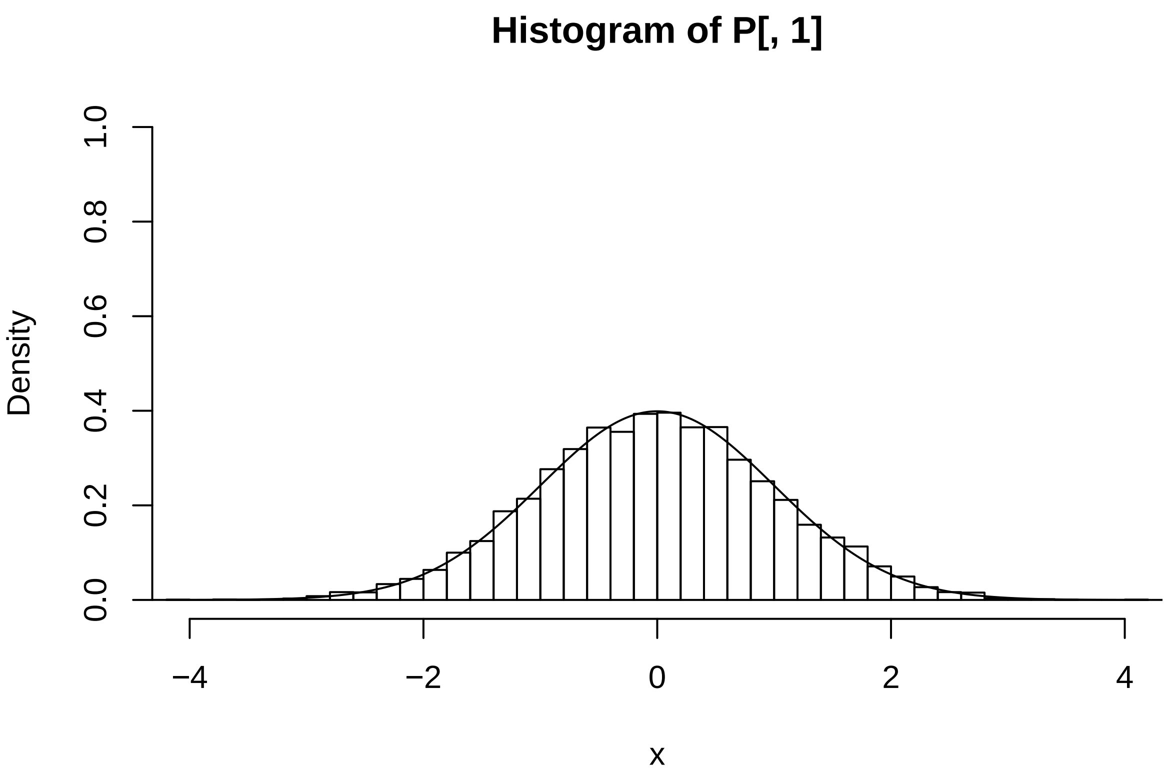
1. Write a function to simulate n $N(0,1)$ random variates using the Accept-Reject algorithm with a Cauchy candidate.

```
k <- 2
Simulate.1 <- function(N=1e+4){
  xs <- c()
  ys <- c()
  num <- 0
  test_n <- 0
  while(num < N){
    p <- runif(1)
    if(p < 0.5)
      x <- qcauchy(p)
    else
      x <- -qcauchy(1-p)
    accept.prob <- dnorm(x)/(k*dcauchy(x))
    test_n <- test_n + 1
    m <- runif(1)
    if(accept.prob>=m){
      xs <- c(xs, x)
      ys <- c(ys, m*k*dcauchy(x))
      num <- num + 1
    }
  }
  return(cbind(xs, ys, num, test_n))
}


x <- seq(-5, 5, 1e-3)
y <- k * dcauchy(x)
plot(x, dnorm(x, 0, 1), type='l', xlab='x', ylab='Density')
lines(x, y, lty=4, col=2)
P <- Simulate.1()
points(P[,1], P[,2], pch='.', col='grey')
```




```
hist(P[,1],probability=T,xlim=c(-4,4),ylim=c(0,1),xlab="x", breaks = 32)
lines(x,dnorm(x))
```



2. Simulate 1000 $N(0,1)$ random variates using your function to estimate $E[Y^3 \log(1+|Y|)]$ and $Pr(Y \in [-1,2])$. Be sure to include a Monte Carlo standard error with your estimates.

```
# Estimate expectation
Points <- Simulate.1(N=1e+3)
```

```r
X <- Points[,1]
Y <- Points[,2]
polynor.y <- function(y){
  poly_y <- y^3*log(1+abs(y))
}
Poly_y <- polynor.y(Y)
cat("Estimate of E[Y^3*log(1+|Y|) = ", mean(Poly_y), "\n")
```

```
## Estimate of E[Y^3*log(1+|Y|) =  0.001927593
```

```r
mcse <- function(y) sd(y)/sqrt(length(y))
interval <- mean(Poly_y) + c(-1,1)*1.96*mcse(Poly_y)
cat("Estimate interval of E[Y^3*log(1+|Y|)] = (", interval[1],",",interval[2], ")\n")
```

```
## Estimate interval of E[Y^3*log(1+|Y|)] = ( 0.001692986 , 0.0021622 )
```

```r
# Probability [-1,2]
judge <- ifelse(X>=-1 & X<=2, 1, 0)
Pr_y <- sum(judge)/length(X)
cat("Pr(Y in [-1,2]) = ", Pr_y, "\n")
```

```
## Pr(Y in [-1,2]) =  0.825
```

3. What was the acceptance rate of the Accept-Reject algorithm? Is this close to the theoretical acceptance rate?

```r
acce_rate <- Points[1,3]/Points[1,4]
cat("The acceptance rate of Accept-Reject algorithm is ", acce_rate, "\n")
```

```
## The acceptance rate of Accept-Reject algorithm is  0.5032713
```

```r
# The theoretical acceptance rate is 1/k.
if(abs(acce_rate-1/k)<0.05){
  print("It closes to the theoretical acceptance rate.")
}else{
  print("There are some difference between the acceptance rate of Accept-Reject algorithm
        and the theoretical one.")
}
```

```
## [1] "It closes to the theoretical acceptance rate."
```

4. Write a function that continues simulation until the sample size is large enough so that your Monte Carlo error is less than $\epsilon = 0.01$ for estimating a general statistic `stat` (which will be a function). You function should also return the observed acceptance rate of the Accept-Reject algorithm and a Monte Carlo standard error.

```r
Simulate.4 <- function(Points){
  xs <- Points[,1]
  ys <- Points[,2]
  num <- Points[1,3]
  test_n <- Points[1,4]
  error <- 1
  while(error > 0.01){
    p <- runif(1)
    if(p < 0.5)
      x <- qcauchy(p)
    else
      x <- -qcauchy(1-p)
    accept.prob <- dnorm(x)/(k*dcauchy(x))
    test_n <- test_n + 1
    m <- runif(1)
    if(accept.prob>=m){
      xs <- c(xs, x)
      ys <- c(ys, m*k*dcauchy(x))
      num <- num + 1
      error <- sd(ys)/sqrt(length(ys))
    }
  }
  acce_rate <- num/test_n
  return(cbind(xs, ys, acce_rate, error))
}
Points_4 <- Simulate.4(Points)
cat("The acceptance rate of Accept-Reject algorithm is ", Points_4[1,3], "\n")
```

```
## The acceptance rate of Accept-Reject algorithm is  0.5030151
```

```r
cat("Monte Carlo standard error is ", Points_4[1,4], "\n")
```

```
## Monte Carlo standard error is  0.003195937
```

5. Use your function to estimate $E[Y^3 \log(1+|Y|)]$ and $Pr(Y \in [-1,2])$ with $\epsilon = 0.01$. Report your estimates along with confidence intervals based on the Monte Carlo standard error. What was the acceptance rate?

```r
X <- Points[,1]
Y <- Points[,2]
Poly_y <- polynor.y(Y)
cat("Estimate of E[Y^3*log(1+|Y|) = ", mean(Poly_y), "\n")
```

```
## Estimate of E[Y^3*log(1+|Y|) =  0.001927593
```

```r
interval <- mean(Poly_y) + c(-1,1)*1.96*mcse(Poly_y)
cat("Estimate interval of E[Y^3*log(1+|Y|)] = (", interval[1],",",interval[2], ")\n")
```

```
## Estimate interval of E[Y^3*log(1+|Y|)] = ( 0.001692986 , 0.0021622 )
```

```r
# Probability [-1,2]
judge <- ifelse(X>=-1 & X<=2, 1, 0)
Pr_y <- sum(judge)/length(X)
cat("Pr(Y in [-1,2]) = ", Pr_y, "\n")
```

```
## Pr(Y in [-1,2]) =  0.825
```