Modeling the Volatility Index(VIX) using Time Series Analysis Models

Ruchit Vithani
Georgia Tech, Team 10

Haijiao Tao
Georgia Tech, Team 10

Dr. Nicoleta Serban*
Georgia Tech

Abstract

The project tries to model the VIX time series using various time series analysis models. We start by fitting the basic conditional mean models and observe that the residuals indicate conditional heteroskedasticity. Based on the learnings, we jointly model the conditional mean and conditional variance of the VIX time series using the joint ARIMA+GARCH models. Our order selection algorithms yield the simplest orders of ARIMA(1,1,1)+GARCH(1,1) for modeling VIX and this model performs best across all time series models. We also employ a more advanced transformer-based Neural Network model called Temporal Fusion Transformer. We find that this model outperforms the joint ARIMA+GARCH model by significant margin on the validation data. In addition to these univariate models, we also explore the predictive power of historical realized volatility of different industry classes in predicting VIX. Our findings indicate the significant predictive power of finance, services and manufacturing sectors in predicting the VIX index. However, we also observe that these multivariate models do not model conditional variance and do not yield as good performance as the joint ARIMA+GARCH model.

1 Report Organization

2 Introduction and Motivation

The study of financial markets has always been of paramount importance to investors, traders, and policymakers. One crucial aspect that captures the attention of market participants is market volatility, as it directly impacts investment decisions, risk management, and financial stability. Volatility is often considered a key barometer of market sentiment, reflecting the level of uncertainty and risk perception among market participants.

In this context, the Volatility Index, commonly known as the VIX, has emerged as a vital tool for measuring market volatility. The VIX is often referred to as the "fear gauge" as it gauges the market's expectation of future volatility. Understanding and predicting changes in the VIX can provide valuable insights into potential market movements, helping market participants make informed decisions and manage risk more effectively.

The motivation behind this time series analysis project is to develop a robust forecasting model for the VIX. Accurate forecasts of the VIX can assist investors in anticipating and preparing for shifts in market volatility, thereby enhancing their ability to make timely and strategic investment decisions. Moreover, an effective forecasting model for the VIX holds the potential to contribute to a deeper understanding of the underlying dynamics of financial markets and their inherent complexitie

*Prof. Serban is the advisor of this project

3 Background and Related Work

The literature on VIX modeling is vast. The modeling techniques encompass a wide range of time series forecasting models, such as autoregressive integrated moving average (ARIMA) and autoregressive conditional heteroskedasticity (ARCH) models. Researchers have also explored modeling techniques based on the concepts of Stochastic Calculus, commonly referred to as Stochastic Volatility Modeling.(Bergomi, 2015)

Recently, some publications have showcased the application of Neural Network models in forecasting VIX futures prices. However, none have yet employed Neural Network architectures to directly model the VIX index (Ballestra et al., 2019). In this section, we present a review of related work that has been undertaken in the modeling and forecasting of the VIX index.

The study by (Majmudar and Banerjee, 2004) focuses on utilizing various ARCH-type forecasting models. They model VIX using EGARCH, GARCH, APARCH, GJR, and IGARCH and show that EGARCH model performs best in terms of performance.

The work by (Ahoniemi, 2008) fits an ARIMA(1,1,1) model augmented with GARCH errors to forecast directional changes in the VIX index. The study also explores the impact of adding financial or macroeconomic variables, such as S&P 500 returns, and concludes that such augmentation does not significantly enhance predictive power. This study only tries to predict the directional changes instead of predicting the actual value of the time series. Our work is focused more in modeling the time series directly in the presence of some exogenous variables.

One of the more advanced time series models called ARFIMAX (ARFIMA) was utilized by (Degiannakis, 2008) and they used intraday trades data as exogenous variables to forecast VIX. They could not observe any statistically significant predictability from intraday trade data.

Thus, the prior work suggests that forecasting the VIX index is not easy. Researchers have tried using lots of exogenous variables, but most of the times, such variables have not shown to improve the performance of the forecasting.

Our Work: In this project, we expand the prior work in two dimensions. First, we compute the industry-wise realized volatilities from the historical returns of stocks and use them as exogenous variables in our models. Secondly, we employ more advanced Neural network-based models in our work to improve the performance of forecasting the VIX index. First, we produce the baseline results using standard models and then we compare the results of our new models with the baseline models.

4 VIX Background

In finance literature, two kinds of volatilities are discussed. The first kind is called **realized volatility**, which refers to the observed volatility. Sometimes, realized volatility is also referred to as historical volatility. This volatility measure is specific to each security being traded and is computed from the historical returns of that security using following equation.

$$\sigma_t^{realized} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(R_{t-i+1} - \bar{R})^2} * \sqrt{m} \tag{1}$$

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i}{n}$$

where, $\sigma_t^{realized}$ is the realized volatility at time t, n is number of historical data points considered for estimation, R_i represents the returns of the security in period i and $\sqrt{m}$ is multiplied to annualize the volatility. The value m is simply the frequency of observations in a year. For monthly returns $m = 12$, for yearly returns $m = 252$ (number of trading days in a year).

The second kind of volatility is called **implied volatility**, which as its name suggests, is implied by the market participants. What do we mean when we say "implied by market participants"? We simply mean that these volatility values are not directly computed from the historical returns as described above in the case of realized volatility. Rather, they are computed from the prices of options being traded in the market at time t. The price of options is determined by the market participants and hence, we say that the implied volatility is "implied" by the market participants.

Theoretically, the prices of options are determined using the well-known Black-Scholes model (Black and Scholes, 1973). Volatility σ_t is one of the parameters of this model which determines the price of an option. The true value of σ_t is never known and this model does not determine the price of options traded in the market (Black-Scholes price is just the theoretical price, market participants determine the market price of options. Market prices are always determined from the supply-demand equilibrium). The value of implied volatility σ_t is computed by plugging in the market price of the options in the Black-Scholes equation and solving for σ_t. In other words, we ask "What value of σ_t will make the theoretical price of an option equal to the observed price of the same option in the market?". The resulting σ_t is called implied volatility.

Calculation of VIX: Once we understand the concept of implied volatility, understanding VIX is trivial. Chicago Board Options Exchange (CBOE) computes the VIX index by taking a weighted average of implied volatilities of using various options on S&P 500. The actual formula to compute the VIX value is very complicated and more interested readers should refer to (VIX White Paper) for the step-by-step calculation of VIX.

The time series of VIX index from 1990 to 2023 at daily frequency can be observed in Figure 1. One clear observation that we can make is that VIX generally spikes rapidly during the periods of stress. For example, consider the periods of 2000 dot-com bubble, 2008 Global Financial Crisis and 2020 Covid-19 pandemic. We can also see the swings in VIX suggesting the common behavior of time series with volatility clustering (High volatility is followed by periods of high volatility and low volatility is followed by the periods of low volatility).

The goal of this project is to model and accurately forecast the future values of this VIX index using various time series models and exogenous predictors.

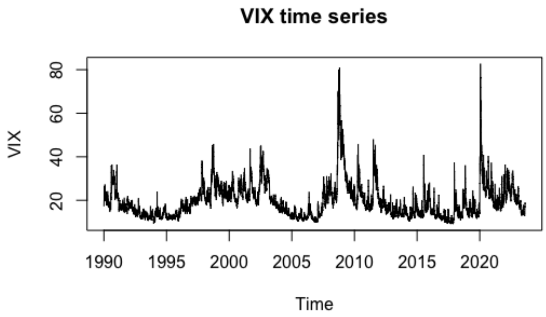

Figure 1: VIX Time Series

5 Data Description

Our project uses two primary sources of data. The VIX time series data is obtained from (CBOE), which is shown in Figure 1. In addition to VIX data, we also collect the daily stock return prices of all publicly traded companies in the US from Center for Research in Security Prices.(CRSP)[1]. CRSP dataset contains daily closing prices(PRC), outstanding shares(SHROUT), industry classification code(HSICCD), closing returns(RET) and market returns(vwretd) as relevant variables among others[2].

5.1 Data Cleaning

The VIX data does not contain any abnormal observations or missing values. However, the CRSP dataset needs to be appropriately cleaned before performing any analysis. We first replace the closing price (PRC) variable with its absolute value[3]. We also remove penny stocks from our analysis. We define penny stocks as stocks with a price less than $5. Such stocks are removed from analysis because they are generally more volatile, less liquid, and often removed from the systematic investment portfolios. We also remove stocks with abnormal returns because of events like mergers (We see spikes of returns of over 200% due to such events). This cleaning procedure is not new in Fi-

[1]CRSP data is not free to use. The students in the Quantitative Finance program get an academic license for the WRDS database which has the CRSP dataset as a part of its subscription. More information about WRDS can be found at: https://wrds-www.wharton.upenn.edu/pages/about/data-vendors/center-for-research-in-security-prices-crsp/

[2]The names mentioned in the brackets are the corresponding column names in the dataset

[3]Negative closing price indicates that the closing price was not available when the data were collected and it was computed as a midpoint of bid-ask spread

nance literature. Almost every work on CRSP data uses a similar procedure of cleaning [4].

5.2 Data Preparation

One of the main objectives of our project is to study the predictive power of realized volatilities of different industry classes on VIX. In this subsection, we provide information on how we prepare the industry-level realized volatilities using the stock returns data from CRSP.

As one can see from Data Description, the CRSP dataset does not provide the industry-level volatility data explicitly. We first group different stocks into different buckets based on their SIC code (HSICCD column in the CRSP data). The grouping is done based on ranges described in Table 3 in Appendix A. We then compute the industry-level volatility as the value-weighted average of realized volatilities of stocks in a particular industry. Value weighing scheme puts more weight on a company with a larger market capitalization. The realized volatilities of individual stocks are computed using Equation 1. We use the last three weeks of returns data ($m = 15$ trading days) for the estimation of realized volatility. The plot of industry-level realized volatilities visualized along with VIX data can be seen in Figure 5. As we can observe in Figure 5, the historical realized volatilities of all industries are highly correlated with the VIX index suggesting the possible predictive power of historical realized volatility time series on VIX.

5.3 Exploratory Data Analysis

To fit simple models like smoothing, ARMA and ARIMA, we use log transformation to stabilize the variance of the time series.

The transformed $log(\text{VIX})$ and their ACF plots can be seen in Figure 6. From the ACF plots of time series, we can observe that $log(\text{VIX})$ violates the stationarity conditions. The slowly decaying ACF values indicate the presence of a trend in the time series. There is no clear seasonality in the time series, and this makes intuitive sense because volatility is just a form of unexpected shocks, and such shocks do not arrive at fixed intervals. The trends observed in the time series are temporary and non-constant suggesting that simple linear trends won't fit well on the data. Better trend-fitting models would be non-parametric moving average and spline smoothing models. The constant expectation and the constant unconditional variance assumptions of stationarity also seem to be violated. The periods of high and low variance can be clearly observed in the original and squared time series. For example, the standard deviation of movements was lower during 2016-2019, and suddenly after COVID-19, the standard deviation of movements in VIX (volatility of volatility). This difference in standard deviation can be observed in the summary statistics in Table 1.

Statistic	2014-2018	2019-2023
# Observations	1258	1204
Minimum	9.14	11.54
Maximum	40.74	82.69
1. Quartile	12.0725	16.1475
3. Quartile	16.3925	25.455
Mean	14.882774	21.687475
Median	13.73	19.9
SE Mean	0.120341	0.238957

Table 1: VIX Summary Statistics during 2014-2018 and 2019-2023

6 Modeling VIX

6.1 Baseline Models

From exploratory data analysis, we can get an idea that simple trend fitting or simple AR models would not be sufficient to model the time series because of the critical violations of the stationarity conditions. ARIMA class of models would be appropriate for fitting VIX. Based on our observation that the time series displays the characteristics of volatility clustering, the ARCH class of models also sounds reasonable.

As a baseline, we fit some of the simplest models. First, we fit the non-parametric spline smoothing model to fit the trend in the time series. In addition, we also fit the simplest versions of the ARMA class of models, namely AR(1), ARMA(1, 1), and ARIMA(1,1,1). Likewise, we also fit the simplest GARCH class of models GARCH(1,1) and ARIMA(1,1,1) + GARCH(1,1) joint modeling. These baseline models give us a rough idea of the minimum performance we want

[4] Examples include (Arnott et al., 2005), (Campbell and Hentschel, 1992) and (Ang et al., 2006)

to beat.

6.2 Customized models

The simplest models described above do not fit well on several real-world datasets in general. A more rigorous modeling approach is required to fit the data better. In this section, we explain how we customize the above models using our data.

Firstly, we customize the pure AR model by identifying the order of the AR process from PACF. We perform order selection on ARIMA to select the best order based on the Akaike Information Criterion(We use the AICc version of AIC). For ARIMA + GARCH joint modeling, we use an iterative approach to select orders of both ARIMA and GARCH.

6.3 GARCH variations

Since VIX is nothing but the volatility in financial markets, the GARCH category of models becomes extremely important. VIX index displays all the characteristics of the problems that the GARCH family of models solves. These problems include conditional heteroskedasticity, volatility clustering, and leverage property. Thus, GARCH and its variations are the most important models in this project. We fine-tune and fit EGARCH, APARCH, GJR, and IGARCH variations of GARCH family models.

6.4 Multivariate Time Series Models

As we have explained before, one of the new directions that we are working on in this project is to explore the predictive power of the historical realized volatility on the future values of the VIX index. In subsection 5.2, we explained how we are curating the industry-level realized volatility data. Using the multivariate time series analysis models, we plan to study if the historical data from stock market returns is able to improve the performance of traditional volatility models like GARCH.

We use Vector Auto Regression (VAR) models to incorporate industry-level realized volatilities as exogenous variables in predicting the VIX index. The order of VAR is selected using the AIC criterion. Since such models can include many lags of other variables that are not statistically significant predictors, we also restrict the predictors at a 0.1 significance level.

6.5 Temporal Fusion Transformer

Recently, transformer-based architectures have proven to be highly successful in a wide variety of language and vision tasks. The attention mechanism proposed in (Vaswani et al., 2017) is used as a building block of many powerful models like BERT (Devlin et al., 2018) and GPT (Radford et al., 2018). Recently, one attention-based time series forecasting model called Temporal Fusion Transformer (TFT) has been proposed in (Lim et al., 2021) which has been proven highly successful in multivariate time series forecasting tasks. In this subsection, we give a brief overview of the TFT model.

TFT is an advanced model for predicting future values in time series data. It combines transformer architecture with traditional forecasting methods to effectively capture patterns over time.

TFT's key strength lies in using a transformer architecture with attention mechanisms, originally designed for natural language processing. This allows the model to focus on different parts of the input sequence, making it adept at understanding temporal dependencies in sequential data.

The "Temporal Fusion" aspect refers to TFT's skill in merging information from different time steps. This is crucial for accurately predicting trends in time series data. TFT also stands out for its use of multi-head attention mechanisms, enabling it to simultaneously consider various aspects of the input sequence and identify diverse patterns.

An important feature of TFT is its ability to incorporate external factors (exogenous features) that may impact the time series. This holistic approach adds to the model's forecasting accuracy. TFT's flexibility extends to handling different forecasting horizons, making it suitable for various applications requiring predictions over different time spans.

7 Experimental Setup

In this section, we describe how we ran all of our experiments using the models described in section 6. Since the entire time series is quite long, some of the tasks become computationally heavy.

Thus, we subset our dataset to contain values from 2014 onwards. The time series from 2014 serves the purpose because it contains all three time periods of low, moderate, and high volatility in the financial markets. The period from 2014-2016 was a moderate volatility period. The period from 2017-2020 was a low volatility period and after COVID-19, the volatility spiked heavily. Thus, this range contains almost all environments of the financial markets.

Forecasting: The last 30 days of the VIX time series are reserved as a test set. We fit all our models on the remaining time series and generate 1 day-ahead rolling predictions on the test set. Comparison across all the models is done by using performance on this test dataset.

Evaluation Metric: We use Mean Absolute Error (MAE) as a means of evaluation and comparison for various models. This is the most widely used evaluation metric for financial time series forecasting tasks. The reason we do not use measures like mean squared error is that we do not want to overweight bigger errors in our evaluation. The distance from the actual observation is a more sensible metric to compare the performance across various models.

Residual Analysis and Diagnosis: Every model produces a time series of errors or residuals after fitting it on the training data. Residuals are another tool to debug and diagnose the problems with the models. Almost all models make explicit assumptions about the behavior of the residuals. The examples include the independence of the residuals and the normality assumption. We conduct such residual analysis and diagnosis to judge which models are a better fit for our data.

8 Results and Analysis

Two of our baseline models, namely ARIMA(1,1,1) and ARIMA(1,1,1) + GARCH(1,1) perform much better than our expectations. They perform even better than some of the more fine-tuned models as well. The best-performing model in all of our experiments is the Temporal Fusion Transformer, which significantly outperforms the best time-performing time series model by a significant margin. In this section, we discuss the performances of our models, the goodness of fit of various models, and the analysis of residuals from various models.

8.1 Model Performances

Table 2 lists the performances across all the models that we have fitted.

Baseline Models: The spline smoothing model is the simplest form of the model that we fit on our data and its performance is worse among all other models with the MAE of 2.8879 on the test dataset. The order of the pure AR model was identified from the PACF plot (see Figure 7). The performances of AR(5), ARIMA(1,1,1), and ARIMA(1,1,1)+GARCH(1,1) joint modeling are quite similar to each other. In modeling of the conditional mean, our results are similar to those of (Ahoniemi, 2008) who find ARIMA(1,1,1) to be the best-performing model. Although ARIMA(1,1,1) performance is quite good, the model fit is not good. The residuals of ARIMA(1,1,1) do show some serial autocorrelation as shown in Figure 2. We can observe that although the ACF plot of residuals corresponds to that of white noise, this is not the case in squared residuals. This suggests that there may be conditional heteroskedasticity present in the ARIMA residuals, which can be modeled using GARCH models.

Adding GARCH to model conditional variance indeed solves this problem. But before fitting the joint ARIMA + GARCH model, we fit the GARCH model along our time series. As expected, the performance of a pure GARCH model on the time series is not up to par. The reason for the poor performance of the GARCH model is that it can model the conditional variance very well, but we still need to estimate the conditional mean using some good mean model, otherwise, we can not expect the GARCH model to perform better. The MAE of the GARCH(1,1) model on our time series is 1.4839, a little bit better than the simple spline trend fitting. Even though the performance of GARCH(1,1) is not as good, this model is able model the heteroskedasticity in the time series. Figure 4 shows that all four moments of the ACF plots of the residuals do not contain any significant correlations, suggesting the goodness of fit of the GARCH model. This motivates us to fit the ARIMA(1,1,1) to model the conditional mean and GARCH(1,1) to model the conditional variance.

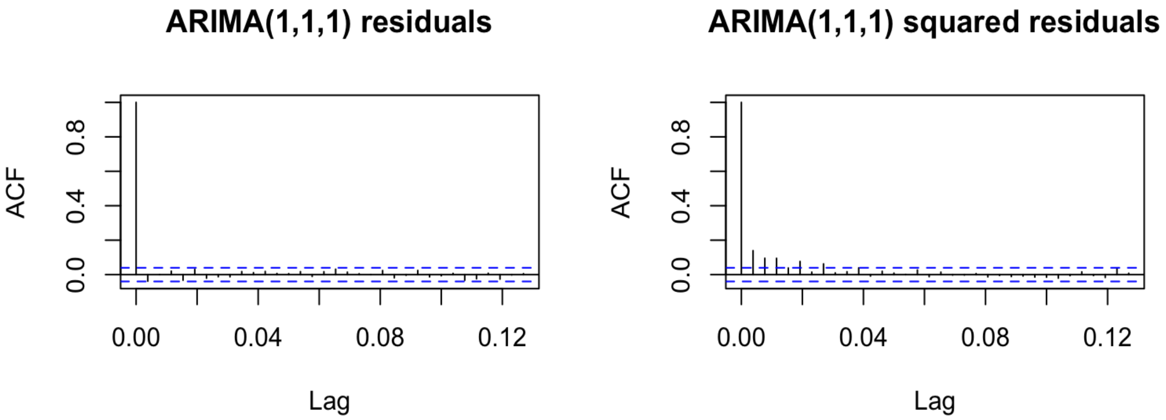

Figure 2: ARIMA(1,1,1) residuals suggesting heteroskedasticity from the ACF of squared residuals

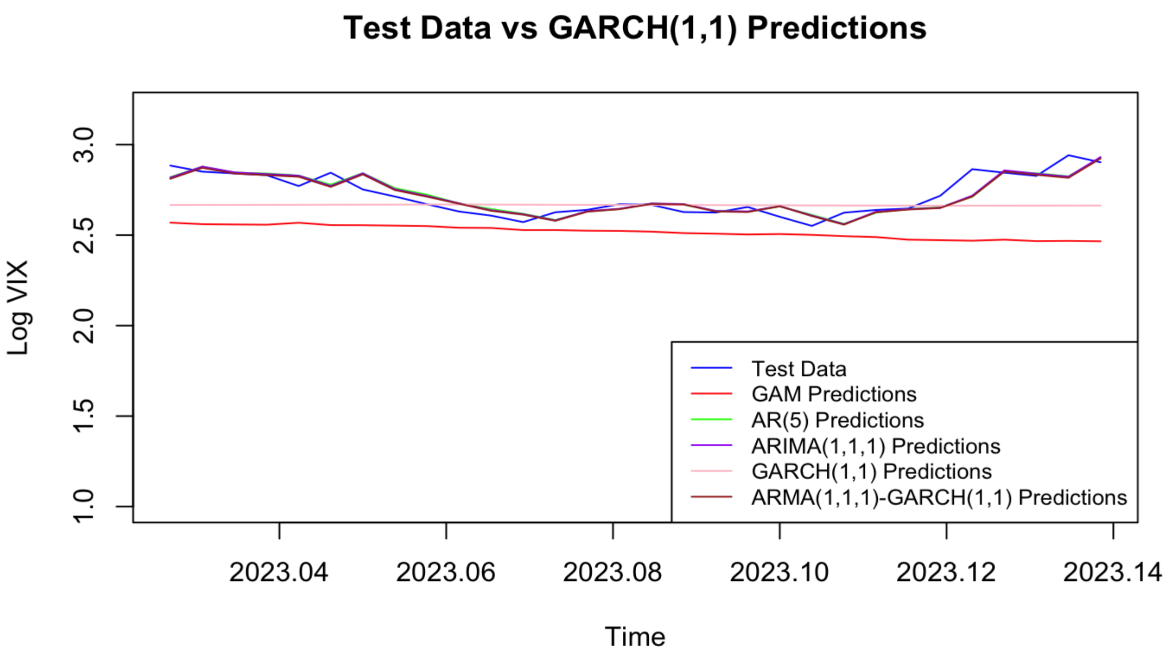

Figure 3: Predictions of baseline models on test set

The joint ARIMA+GARCH model performs better than any other GARCH-type model on our dataset. This result is also similar to (Ahoniemi, 2008), but different than (Majmudar and Banerjee, 2004) in which eGARCH was the best performer. The reason for the differences in the performance could be due the the change in the underlying distribution of the time series. Some models may work better for a particular period and the market environment, while the other models may work well at other times. One should always try out different models to understand which model is better fitting the current situation in the markets. All predictions from our baseline models can be visualized in Figure 3

Model	MAE
GAM	2.8879
AR(5)	0.6606
ARIMA(1,1,1)	0.6569
GARCH(1,1)	1.4839
ARIMA(1,1,1)+GARCH(1,1)	**0.6350**
ARIMA(1,1,2)	0.6666
ARIMAX	0.9780
eGARCH(1,1)-ARIMA(1,1,1)	0.6595
iGARCH(1,1)-ARIMA(1,1,1)	0.6507
gjrGARCH(1,1)-ARIMA(1,1)	0.6502
apARCH(1,1)-ARIMA(1,1)	0.6581
TFT	**0.1610**

Table 2: Mean Absolute Error (MAE) for Different Models

Customized Models: We perform order selection on different ARIMA and GARCH-type models. For ARIMA models, we do order selection up to orders $p = 5$ and $q = 5$. For GARCH models, we perform order selection for the max orders of $m = 3$ and $n = 3$. The best models are selected based on the AIC criterion and to our surprise, we find that the more parsimonious models are selected by the order selection algorithm. The order selection algorithm gives different orders only for the ARIMA model for which the selected orders are ARIMA(1,1,2). The performance of this model on the test set is still not as good as the ARIMA(1,1,1) or the joint ARIMA+GARCH model. The reason for a more parsimonious model being selected could lie behind the bias-variance tradeoff. Although the more complex model may better fit the training data, it may result in overfitting and poor performance on the test set predictions. The only model for which we receive different orders, ARIMA(1,1,2) gives a very comparable MAE of 0.6666 on the test set as can be observed in the results Table 2.

GARCH variations Although some researchers have found the other variations of GARCH to perform better, our results were directly opposite to those studies. We find that all of eGARCH, iGARCH, gjrGARCH and apARCH perform comparably to each other with an MAE of around 0.65, however, the performance is still not as good as GARCH(1,1) + ARIMA(1,1,1). This suggests that the simplest GARCH(1,1) is an ideal model for VIX modeling. The performances of GARCH variations are also listed in Table 2.

Multivariate Time Series Modeling A vast literature on VIX has failed to find a strong exogenous predictor for the VIX index. It is difficult to find such predictors because VIX is not a naturally occurring phenomenon. It is a human-engineered entity that depends heavily on the behaviors of the market participants, which is very stochastic in nature. This makes it very difficult to find systematic factors that can accurately explain the VIX. In this project, we explore if the preexisting tension can predict future tension in the markets. That is, can the realized volatility of different industries robustly predict the movements in the VIX index?

There is a high degree of correlation observed between VIX and the industry-wise realized volatilities as can be seen in Figure 8. We especially see that the finance, services and manufacturing sectors have the highest correlation with VIX.

We first select the VAR model using the HQ criterion. We find that the best selected order is 5. However, on fitting the VAR model using order 5, we find that the residuals reject the Null hypothesis in the normality test, heteroskedasticity test, and serial correlation test. This indicates that the residual process has heteroskedasticity, does not follow a multivariate normal distribution, and has serial correlations suggesting a poor goodness of fit. Thus, for this time series, we conclude that modeling conditional heteroskedasticity is extremely important to achieve the goodness of fit.

On restricting the selected VAR model to keep coefficients at a significance level of 0.1, we find that some lagged variables are statistically significant in predicting the VIX index. Figure 9 lists

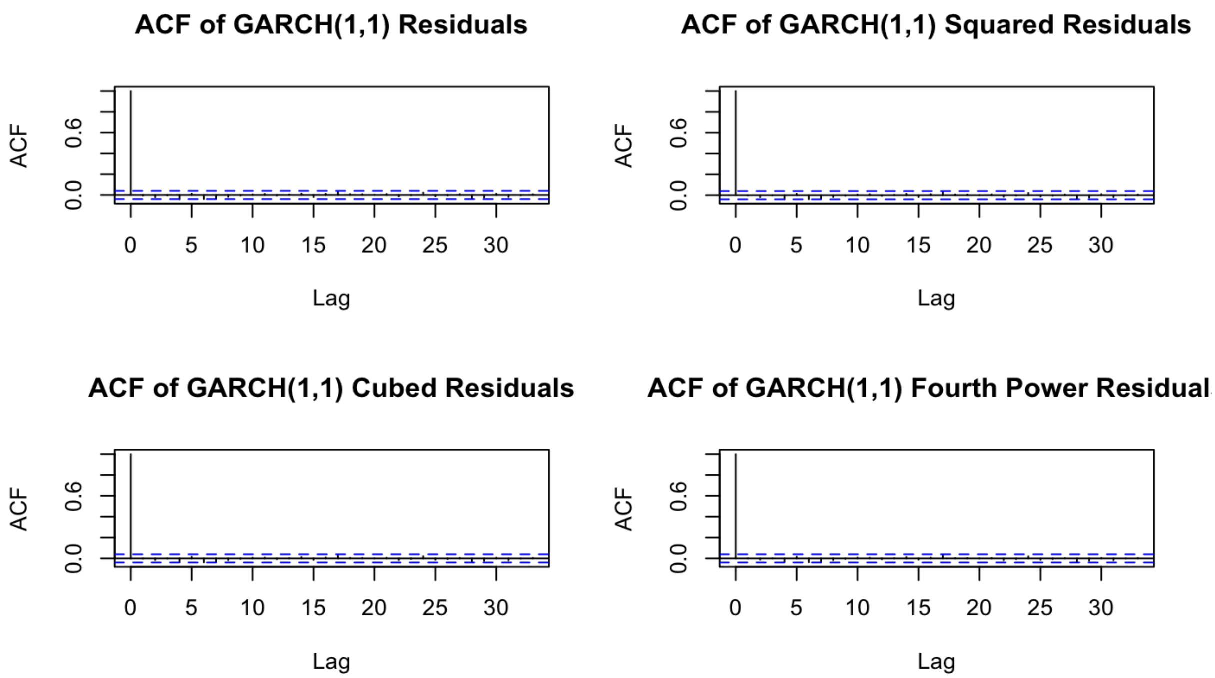

Figure 4: GARCH(1,1) residuals moments suggesting no heteroskedasticity in the residuals

the statistically significant lagged predictors for the VIX index.

Temporal Neural Network This model performs the best on the validation set with an MAE of around 0.16 (Table 2). The exceptional performance of the Temporal Fusion Transformer (TFT) model on our dataset can be attributed to its inherent strengths in capturing complex temporal dependencies and patterns within time series data. The TFT excels in several key aspects that contribute to its remarkable predictive accuracy.

While the TFT model performs very good, it also has attached costs to it. Firstly, the number of parameters of this model is huge, and thus, it significantly reduces the training time. This model was slowest to train even when the GPU resources were provided to the ARIMA models which were run on CPU.

9 Conclusions and Future Work

In this project, we try to model the VIX data using time series analysis models. We show the importance of fitting both conditional mean and conditional variance in financial time series in order to achieve goodness of fit and stable estimates of the coefficients. We also show that fitting only conditional mean produces the residuals that exhibit serial correlation indicating the improvement that can be done to achieve better predictions. We also try other exogenous variables to predict the VIX index but find that such variables unnecessarily make the model more complex and result in relatively poor performance compared to more simple models.

The simple models perform better in our case because the VIX index satisfies the assumptions made by such models. These assumptions include volatility clustering and the leverage property. We learn that if the data satisfies all the assumptions of a particular model, then that model can do better than all other models no matter how good the other models are on other tasks.

We also try more complicated Neural Network based models to compare traditional models with the newly proposed models. We find that the TFT model fits really well on the data and produces the minimum MAE score.

Future Questions: We also provide some future directions that can be explored in this area. VIX is highly driven by unexpected shocks in the financial markets. We should explore the tools of Natural Language Processing to see if we can incorporate the news events immediately into our models to forecast the movement in the VIX index. Based on our exploratory data analysis, we also suggest taking into account variables like trading volume, and macroeconomic data into account to predict the VIX index.

References

Katja Ahoniemi. 2008. Modeling and forecasting the vix index. *Available at SSRN 1033812*.

Andrew Ang, Robert J Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2006. The cross-section of volatility and expected returns. *The journal of finance*, 61(1):259–299.

ARFIMA. Arfima - autoregressive fractionally integrated moving average.

Robert D Arnott, Jason Hsu, and Philip Moore. 2005. Fundamental indexation. *Financial Analysts Journal*, 61(2):83–99.

Luca Vincenzo Ballestra, Andrea Guizzardi, and Fabio Palladini. 2019. Forecasting and trading on the vix futures market: A neural network approach based on open to close returns and coincident indicators. *International Journal of Forecasting*, 35(4):1250–1262.

Lorenzo Bergomi. 2015. *Stochastic volatility modeling*. CRC press.

Fischer Black and Myron Scholes. 1973. The pricing of options and corporate liabilities. *Journal of political economy*, 81(3):637–654.

John Y Campbell and Ludger Hentschel. 1992. No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of financial Economics*, 31(3):281–318.

CBOE. Chicago board options exchange.

CRSP. Center for research in security prices.

Stavros Antonios Degiannakis. 2008. Forecasting vix. *Journal of Money, Investment and Banking*, (4).

Jacob Devlin, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova. 2018. Bert: Pre-training of deep bidirectional transformers for language understanding. *arXiv preprint arXiv:1810.04805*.

Bryan Lim, Sercan Ö Arık, Nicolas Loeff, and Tomas Pfister. 2021. Temporal fusion transformers for interpretable multi-horizon time series forecasting. *International Journal of Forecasting*, 37(4):1748–1764.

Utkarsh Majmudar and Arnab Banerjee. 2004. Vix forecasting.

Alec Radford, Karthik Narasimhan, Tim Salimans, Ilya Sutskever, et al. 2018. Improving language understanding by generative pre-training.

SEC. Sec.

Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. 2017. Attention is all you need. *Advances in neural information processing systems*, 30.

VIX White Paper. Vix white paper.

A Appendix

SIC codes are released by the SEC to group companies into different sectors or industry classes. For more information on industry classification, visit (SEC)

SIC CODE	Industries
1–999	Agriculture, Forestry and Fishing
1000–1499	Mining
1500–1799	Construction
2000–3999	Manufacturing
4000–4999	Transportation and other Utilities
5000–5199	Wholesale Trade
5200–5999	Retail Trade
6000–6799	Finance, Insurance and Real Estate
7000–8999	Services
9000–9999	Public Administration

Table 3: SIC Codes and Corresponding Industries

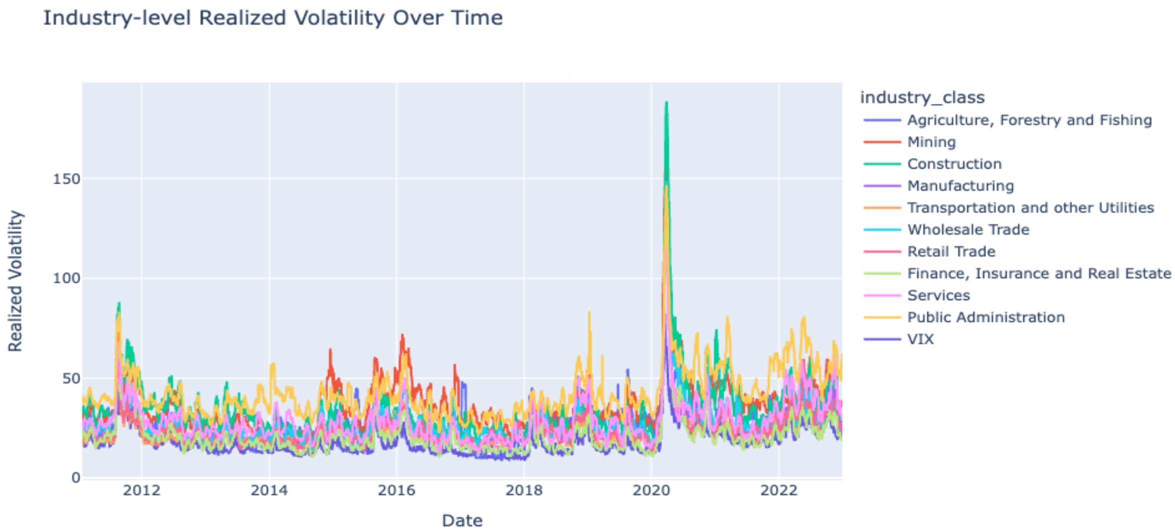

Figure 5: Industry-level Realized Volatility Over Time

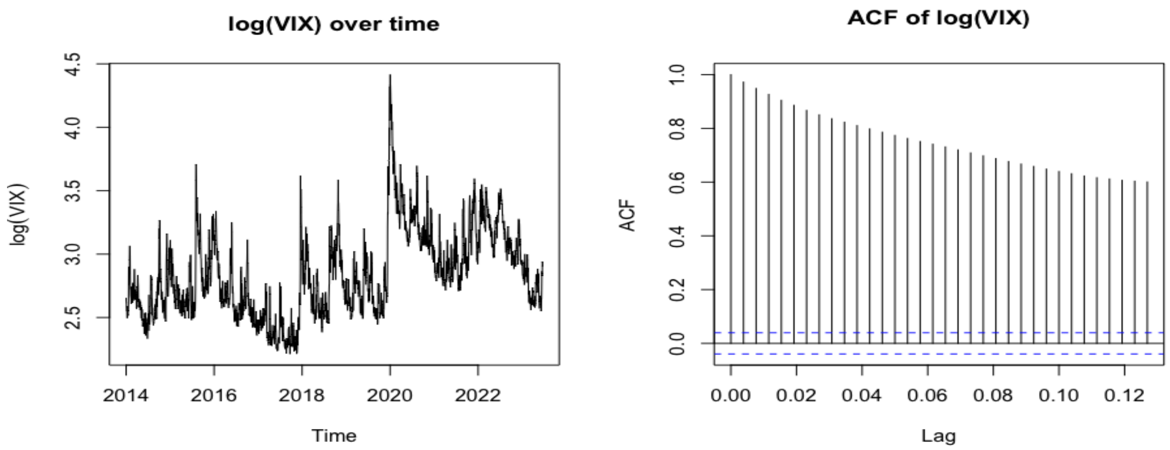

Figure 6: Transformations of VIX

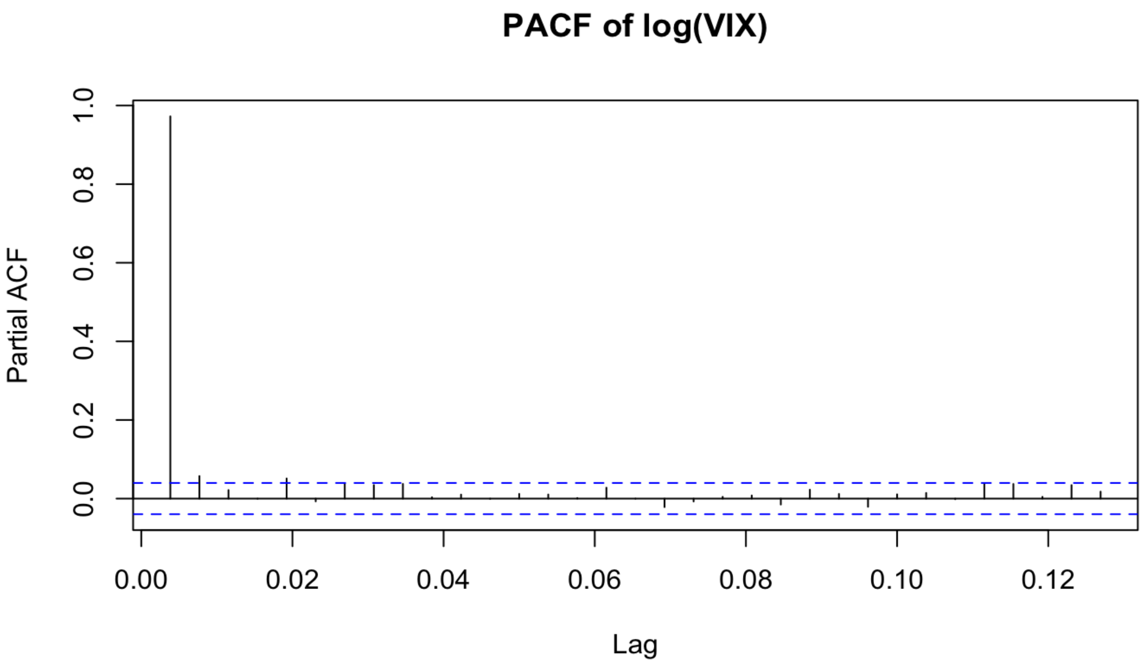

Figure 7: PACF of log(VIX)

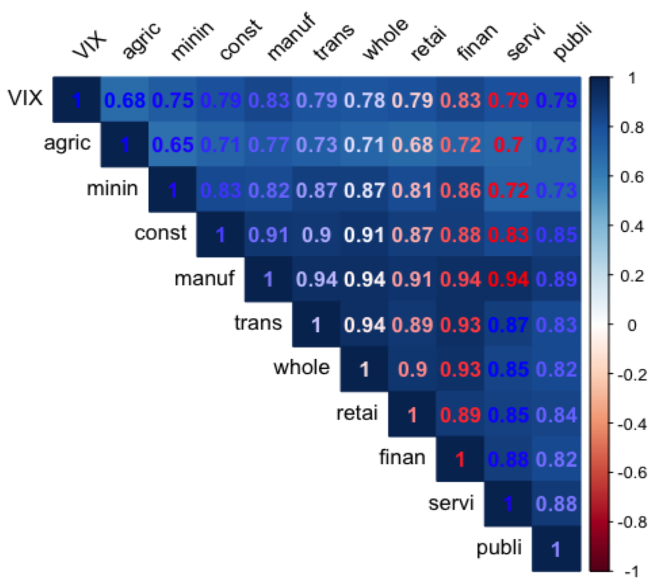

Figure 8: Correlation plot of industry-wise realized volatility and the VIX index

```
Call:
lm(formula = y ~ -1 + ., data = datares)

Coefficients:
    VIX.l1    minin.l1    trans.l1    servi.l1      VIX.l2    agric.l2    trans.l2    finan.l2    servi.l2
 8.985e-01   2.791e-02  -1.548e-01   1.448e-01   4.406e-02   1.330e-02   1.843e-01  -1.177e-01  -1.494e-01
  publi.l2    finan.l3    whole.l4       trend
 3.065e-02   1.083e-01  -4.761e-02   1.015e-05
```

Figure 9: Significant Coefficients from Restricted VAR