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Forecasting market turbulence using regime-switching models

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Abstract We propose an early warning system to timely forecast turbulence in the US stock market. In a first step, a Markov-switching model with two regimes (a calm market and a turbulent market) is developed. Based on the time series of the monthly returns of the S&P 500 price index, the corresponding filtered probabilities are successively estimated. In a second step, the turbulent phase of the model is further specified to distinguish between bullish and bearish trends. For comparison only, a Markov-switching model with three states (a calm market, a turbulent bullish market, and a turbulent bearish market) is examined as well. In a third step, logistic regression models are employed to forecast the filtered probabilities provided by the Markov-switching models. A major advantage of the presented modeling framework is the timely identification of the factors driving the different phases of the capital market. In a fourth step, the early warning system is applied to an asset management case study. The results show that explicit consideration of the models' signals yields better portfolio performance and lower portfolio risk compared to standard buy-and-hold and constant proportion portfolio insurance strategies.

Keywords Early warning system · Financial crisis · Logistic regression models · Markov-switching models

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1 Introduction

The financial crisis of 2007–2009 and the market turbulence in 2011 initiated a comprehensive discussion on how to best model financial markets and, in particular, how to timely forecast market downturns and other economic crises. Practitioners and academics became aware of the fact that the parameters used in their models are not constant over time and that, for example, the correlation between different markets can significantly increase during times of crises. However, the financial crisis has not been the only turbulent phase in the history of financial markets. Other famous examples are Black Monday (1987), the Gulf War aftermath (1990), the Asian financial crisis (1997), the Russian financial crisis (1998), and bursting of the dotcom bubble (2000).

Consequently, there is abundant literature on the prediction of crises and early warning systems. Duttagupta and Cashin (2008) consider banking crises using a binary classification tree model. Demirgüc-Kunt and Detragiache (2005) implement a signal extraction and a multivariate logit approach for the same purpose. Kaminsky and Reinhart (1999) use a signal extraction model to study the relation between banking and currency crises. In addition to commonly used indicators such as GDP growth and the inflation rate, Barrell et al. (2010) develop a logit model that incorporates the capital adequacy and certain liquidity ratios of banks as well as the growth in property prices to improve the quality of their early warning system. Thanks to these additional input variables, the authors are able to timely observe an increase in their crisis probability in the period prior to the sub-prime crisis. Contrary to the last approach, Davis and Karim (2008b) are not able to forecast the sub-prime crisis with the help of their multivariate logit and binary recursive tree models. By comparing the logit and the signal approach, Davis and Karim (2008a) show that a multivariate logit model is the most suitable for international early warning systems, whereas signal extraction optimizes country-specific banking crises predictions. Early warning systems for currency crises in emerging markets are studied by Kamin et al. (2001) via a probit model. Similarly, Meichle et al. (2011) apply a probit model to predict recession and expansion periods in the Swiss economy's business cycle.

Several approaches to formally define and thus subsequently identify crises are found in the literature. For example, Hamilton (1989) and Diebold et al. (1994) suggest Markov-switching models. Hamilton (1989) model is restricted to constant transition probabilities; Diebold et al. (1994) also allow for time-varying ones. With these models, the loss of information that might result from using purely binary variables or other simplifying assumptions is reduced. Other applications of Markov-switching models include Martinez-Peria (2002) and Abiad (2003), who analyze speculative attacks against the European monetary system and currency crises in Asian markets, respectively. Similarly, Maheu and McCurdy (2000) use a Markov-switching model to cluster the stock market in bull and bear periods. They extend their approach to a duration-dependent Markov-switching model to study the regimes' characteristics (means and variances) dependent on the duration. Hauptmann and Zagst (2011) use a Markov-switching approach to analyze the systemic risk of US stocks. Asset correlations in different market states are investigated by Bernhart et al. (2011). For the

S&P 500, the EuroStoxx50 and the Nikkei 225, Ernst et al. (2009) present empirical evidence that portfolio optimization based on a Markov-switching model outperforms a standard Black-Scholes framework. Other applications can be found in Areal et al. (2013), Alexander and Dimitriu (2005), and Chesnay and Jondeau (2001).

To the best of our knowledge, Chen (2009) is the one most closely related to our work. He uses a Markov-switching approach to subdivide the US stock market into two regimes. Thereafter, he studies the predictive power of several macroeconomic variables such as inflation and unemployment rate and shows that recessions can be better forecast by macroeconomic variables than by stock market movements. In this paper, we extend his approach by introducing an early warning system for US stock market recessions. To determine the number of states of a Markov-switching model we rely on the AIC and BIC criteria. As the theoretical results of Zhang and Stine (2001) imply that the autocovariance structure of weakly stationary Markov regime-switching processes can be captured by the structure of causal and invertible ARMA(p,q) processes, the number of states of a regime-switching model is in line with the order of the corresponding ARMA process with the same autocovariance structure using the AIC and the BIC criteria. In doing so, we obtain for the discrete monthly returns of the S&P 500 price index (for the period 11/1987–09/2011) two regimes: a calm market and a turbulent market. To distinguish between turbulent phases with bullish and bearish trends we similarly derive a second Markov-switching model. We also develop a single Markov regime-switching model with three states and compare the derived filtered probabilities with the corresponding ones of the first approach. The exogenous variables of the logistic model are given by publicly available capital market and macroeconomic variables.

The paper is structured as follows. The second section develops and estimates the underlying Markov-switching models with two and three states. The third section derives the logistic model used to forecast future filtered probabilities and also contains an explanation of the meaning and interactions of the deployed market data. The fourth section shows an application of the model to asset management. In the last section we summarize the main findings of the paper and make some suggestions for future research.

2 Markov-switching models

As Markov-switching models are able to incorporate observed characteristics of stock returns, including fat tails, asymmetries, autocorrelation, and volatility clustering [see, e.g., Timmermann 2000], they have become important in the economic literature. Based on empirical results, we assume at least two regimes for the monthly S&P 500 returns. Therefore, let the process $(S_t)_{t=1,\ldots,T}$ with $S_t \in \{0, 1\}$ for all $t \in \{1, \ldots, T\}$ denote the unobservable state processes with two regimes and let the process $(r_t)_{t=1,\ldots,T}$ refer to the observed monthly returns. In case of normally distributed returns with regime-dependent mean $\mu_{S_t} \in \mathbb{R}$ and regime-dependent volatility $\sigma_{S_t} > 0$, it holds for all $t \in \{1, \ldots, T\}$:

$$r_t = \mu_{S_t} + \sigma_{S_t}\epsilon_t, \;\; S_t \in \{0, 1\}, \tag{1}$$

with $\epsilon_t \sim N(0, 1)$ iid. $\forall\, t \in \{1, \ldots, T\}$. As the process of the state variable S_t is supposed to be given by a time-homogeneous Markov chain with fixed transition probabilities $p, q \in [0, 1]$, we have:

$$\begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \tag{2}$$

where

$$p := \mathbb{P}(S_t = 0 | S_{t-1} = 0)$$

and

$$q := \mathbb{P}(S_t = 1 | S_{t-1} = 1).$$

Let $(\delta, 1-\delta)$ with $\delta := \mathbb{P}(S_1 = 0) \in [0, 1]$ be the initial distribution of the Markov chain. Then, the model is completely determined by the vector:

$$\boldsymbol{\theta} = (p, q, \mu_0, \mu_1, \sigma_0, \sigma_1, \delta). \tag{3}$$

The maximum likelihood approach can be used to estimate the parameter vector $\boldsymbol{\theta}$. However, as the underlying state process $(S_t)_{t=1,\ldots,T}$ cannot be observed, the likelihood is not available in closed form. Therefore, the more sophisticated class of expectation maximization (EM) algorithms for incomplete data of Dempster et al. (1977) is applied. For Markov-switching models, such an EM algorithm has been derived by, among others, Baum et al. (1970). Due to the Gaussian setting, the optimal solution of the *M-step* of the EM algorithm can be analytically determined, resulting in reduced calculation effort[1].

For all $t \in \{1, \ldots, T\}$ let $\hat{\boldsymbol{\theta}}_t$ denote the estimate of the parameter vector $\boldsymbol{\theta}_t$ derived from $\boldsymbol{r}_t = (r_1, \ldots, r_t)$, i.e., the information available up to time t. Then, for the two states $j \in \{0, 1\}$ the filtered probabilities at time t are given by:

$$p_t^j := \mathbb{P}(S_t = j | \boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t). \tag{4}$$

Refering to the empirical results (see end of this section), let $S_t = 0$ and $S_t = 1$ denote a calm and a turbulent market, respectively. Thus, $p_t^1 := \mathbb{P}(S_t = 1 | \boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t)$ estimates the probability of a turbulent market at time t. However, it provides no information as to whether the turbulent market has a bullish or a bearish trend. Therefore, we extend the two-state Markov-switching model of Eq. (1) by introducing a third regime. For the Markov-switching model with three states $j \in \{0, 1, 2\}$ let $S_t = 0$ refer to a calm market, while $S_t = 1$ and $S_t = 2$ denote a turbulent market with mainly negative returns (bearish trend) and a turbulent market with mainly positive

[1] For comparison, we also estimated the vector $\boldsymbol{\theta}$ using the numerical optimization scheme introduced in Hamilton (2005), which is more time-consuming than the EM algorithm; the estimation results were essentially the same as those reported here.

returns (bullish trend), respectively. If $\tilde{\boldsymbol{\theta}}_t$ respresents the estimate of the parameter vector of the three-state model, the filtered probabilities of each state $j \in \{0, 1, 2\}$ at time $t \in \{1, \ldots, T\}$ are given by:

$$q_t^{3,j} := \mathbb{P}(S_t = j|\boldsymbol{r}_t; \tilde{\boldsymbol{\theta}}_t). \tag{5}$$

Even though Mittnik and Haas (2008) report good results for their three-state Markov-switching models applied to the MSCI returns of Germany and the United States, in general, estimating a Markov-switching model with three states is unstable. For more on Markov-switching models, see, e.g., Boldin (1996) and So et al. (1998). Therefore, we develop a second characterization of the market with the same three states $j \in \{0, 1, 2\}$ based on a step-by-step combination of two Markov-switching models (each with two states) as described by Eq. (1). To this end, let $\mathbb{T} = \{1, \ldots, T\}$ be the complete period under consideration. At first, for each (new) point in time $t \in \mathbb{T}$ we successively estimate the Markov-switching model defined by Eq. (1) to determine whether the market at time t is calm or turbulent. If we apply this procedure to all points in time $\mathbb{T}$, we separate the turbulent market phases, i.e., $\mathbb{T}^D = \{t \in \mathbb{T} : \mathbb{P}(S_t = 1|\boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t) > 0.5\}$ (D indicates Distress), from the calm ones, i.e., $\mathbb{T}\backslash\mathbb{T}^D$. Let $p_t^0 = \mathbb{P}(S_t = 0|\boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t)$ and $p_t^1 = \mathbb{P}(S_t = 1|\boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t)$ refer to the corresponding filtered probabilities derived from the information available up to time t. Furthermore, let $(S_t^D)_{t\in\mathbb{T}^D}$ denote the regime process restricted to the turbulent phases. Again, for the time series of the stock returns of the turbulent market, i.e., for all r_t with $t \in \mathbb{T}^D$, we successively estimate the Markov-switching model defined by Eq. (1) to separate the turbulent phases with mainly positive returns from the turbulent phases with mainly negative returns. For any point in time $t \in \mathbb{T}^D$ the filtered probabilities of the second Markov-switching model are given by $p_t^{D,0} = \mathbb{P}(S_t^D = 0|S_t = 1; \boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t)$ and $p_t^{D,1} = \mathbb{P}(S_t^D = 1|S_t = 1; \boldsymbol{r}_t; \hat{\boldsymbol{\theta}}_t)$, respectively. For reasons of clarity and comprehensibility, the estimate of the parameter vector of the second Markov-switching model has been omitted. As the estimation of the second Markov-switching model is restricted to $\mathbb{T}^D$, the distinction between mainly positive and mainly negative returns occurs only when the market has been categorized as turbulent. Furthermore, due to the preceding distinction between calm and turbulent periods, $p_t^{D,0}$ and $p_t^{D,1}$ are conditioned on the fact that the market at time t has already been characterized as turbulent, i.e., they represent conditional probabilities. Because of

$$p_t^0 = 1 - p_t^1 \quad \text{and} \quad p_t^{D,0} = 1 - p_t^{D,1} \tag{6}$$

we only consider p_t^1 and $p_t^{D,1}$ in the following and define:

$$p_t^G := p_t^0, \ p_t^Y := p_t^{D,0} \cdot p_t^1, \quad \text{and} \quad p_t^R := p_t^{D,1} \cdot p_t^1. \tag{7}$$

Due to Eqs. (6) and (7), it holds:

$$p_t^G + p_t^Y + p_t^R = 1. \tag{8}$$

For each point in time $t \in \mathbb{T}$, the probability triples $(q_t^{3,0}, q_t^{3,1}, q_t^{3,2})$ [see Eq. (5)] and $\left(p_t^G, p_t^Y, p_t^R\right)$ [see Eq. (8)] allow us to set up two traffic light systems to describe the current state of the market. Refering to Eq. (8), the exponents G, Y, and R refer to Green, Yellow and Red[2]. Before proceeding with the empirical part, we first explain why we use the sequential estimation procedure instead of an ordinary (fixed) in-sample estimation. First, the presented method covers changes in the regimes' means and variances that might occur during a longer period of time, e.g., a decade. Second, our main intent is to develop an early warning system that incorporates the latest (available) information and that supports on a monthly basis the work of, for example, asset and risk managers.

The subsequent empirical results are based on discrete monthly returns of the S&P 500 price index for the period 01/1964–09/2011. Although we have already stated that we are going to work with Markov-switching models with two and three states, the number of reasonable regimes for the considered data should be identified. For this we employ the model selection approach of Zhang and Stine (2001) combined with the AIC [see Akaike (1974)] and the BIC criteria [see, e.g., Schwarz (1978)], resulting in a lower bound that is usually identified as an estimate of the desired state dimension to avoid statistical overfitting. The results of Zhang and Stine (2001) imply that a univariate weakly stationary $r-$state Markov regime-switching autoregressive process of order m admits an ARMA(p,q) representation, i.e., there is a causal and invertible ARMA(p,q) process with the same autocovariance structure. Moreover the relation between the number of states r, the autoregressive order m, and the ARMA parameters p and q is captured by the inequalities $p \leq r \times m^2$ and $q \leq r \times m^2 - 1$. To determine the order of an $r-$state Markov regime-switching autoregressive process we first apply the AIC and BIC criteria to the whole time series of the S&P 500 returns. Both criteria prefer an AR(1) model over higher-order AR(m) models with $m = 2, \ldots, 8$. Next, we select the parameters of an ARMA(p,q) model based on the S&P 500 returns using again the AIC and BIC criteria. While the AIC suggests an ARMA(2,1) model, the BIC prefers the ARMA(1,1) model. However, referring to the preceding inequalities, both result in a Markov regime-switching autoregressive process of order 1 with at least two states since $r \geq \max\{p/m^2, (q+1)/m^2\}$. Hence, the assumed Markov-switching models with two and three states, respectively, cannot be rejected.

For the step-by-step estimation of the two Markov-switching models with two states each, the S&P returns of the period 01/1964–03/1972 are used as a training set for the first model, that is, the one that separates the calm and the turbulent phases. Thereafter, we iteratively estimate for each (new) point in time the current filtered probabilities of the two regimes using the EM algorithm of Baum et al. (1970)[3]. The training set of the second Markov-switching model separating the bullish and the bearish phases of the turbulent regime is extended to cover the period 01/1964–07/1974 so as to have a minimum of 25 data points. Then, the same sequential procedure detailed above is applied.

[2] Note that the traffic light system is just a simplification for illustrative purposes. The information we actually create, that is, the probabilities for each regime, is much richer than a simple indication (indicating color) and may be used for advanced risk management or investment strategies.

[3] As Hamilton (2005) numerical scheme provides similar results, we do not explicitly illustrate them.

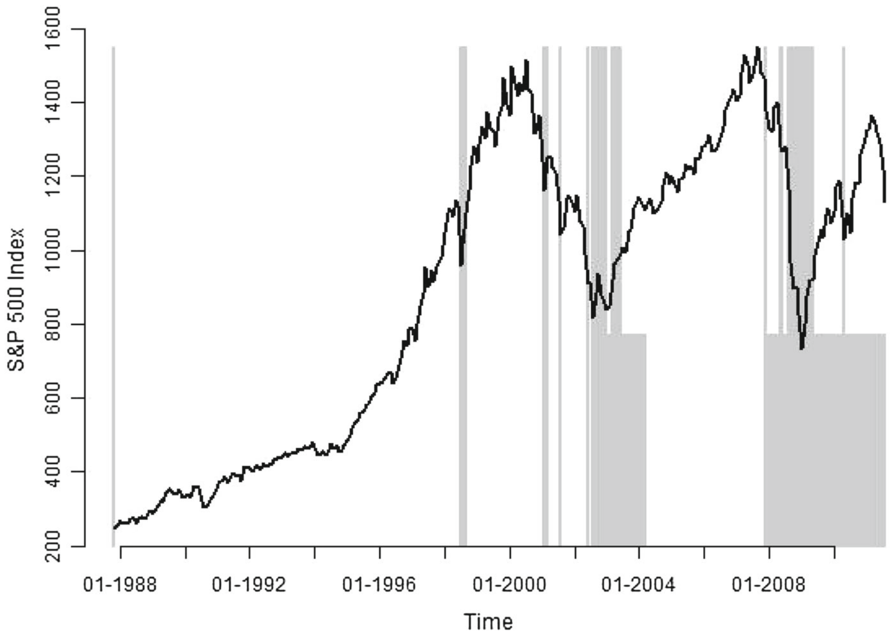

Fig. 1 S&P 500 price index separated by two two-state Markov-switching models into a calm state, a turbulent state with mainly negative returns (*high grey*) and a turbulent state with mainly positive returns (*lower grey*)

By considering the estimation results for the first two-state Markov-switching models, we observe that most of the documented economic crises in the United States are categorized as turbulent phases of the S&P 500 price index. For example, during the financial crisis, the filtered probabilities of the turbulent state are above 85 % for the period 01/2008–03/2011. This period includes the tremendous downturn in stock prices, as well as the following rally. Thereafter, the filtered probability of the turbulent regime is decreasing until August 2011, when it returns to the level of 98 %. The estimation results for the second Markov-switching model for the turbulent phase provide a good distinction between periods with mainly positive and those with mainly negative monthly returns. In the case of the financial crisis, the estimates of the second Markov-switching model indicate a change from a turbulent bearish phase towards a turbulent bullish period in June 2009. Figure 1 illustrates the markets based on the two Markov-switching models with two states each. The time series of the transition probabilities p (calm regime persists) and q (turbulent regime persists) are displayed in Fig. 2.

Figure 2 shows that the filtered probabilities are changing over time. This underlines the advantages of the sequential estimation method compared to a (fixed) in-sample estimation. Another important finding discernable from Fig. 2 is the increasing stability of the states. As both probabilities, i.e., p and q [see Eq. (2)], are more or less close to 1, a change of the current regime happens less often, as demonstrated by the events of the 1990s. Finally, using the market characterization derived from the EM algorithm we determine for each regime the annualized mean and the annualized standard

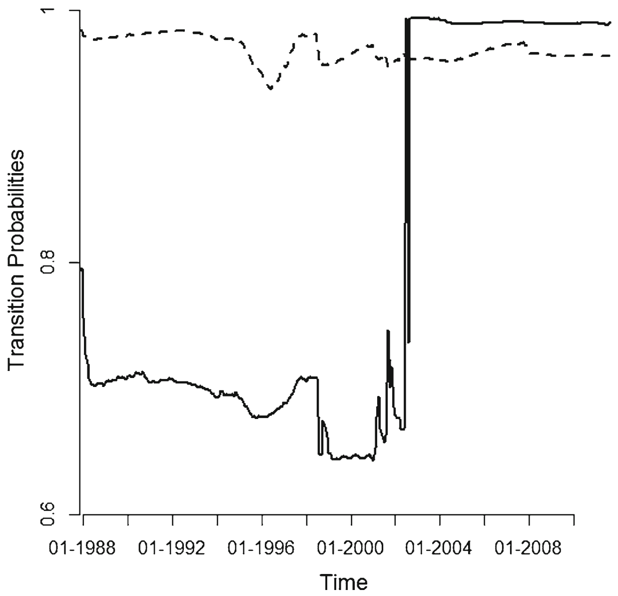

Fig. 2 Successively estimated transition probabilities provided by the EM algorithm. The *solid line* shows the probability p to remain in the calm regime. The *dashed line* shows the probability q to remain in the turbulent regime

Table 1 Sample characteristics of the monthly returns of the S&P 500 price index in the three states derived from the two two-state Markov-switching models based on the time period from 11/1987 to 09/2011

	Calm	Turbulent	Turbulent positive	Turbulent negative
Mean (ann.)	0.1050	−0.0292	0.1278	−0.1989
Standard deviation (ann.)	0.1339	0.2044	0.1536	0.2218

deviation of the included S&P 500 returns (see Table 1). A comparison of the parameters shows that the turbulent phase is more volatile (and also riskier) than the calm one. Furthermore, due to the crises that occurred during this period, the mean of the turbulent phase is negative. The further separation of the turbulent regime into phases having mainly positive or mainly negative returns proves that the turbulent regimes include the crises as well as bullish periods. However, due to the high volatility, the second Markov-switching model is not able to precisely separate all bullish periods from the bearish ones. Hence, the turbulent bullish phase also includes negative returns and vice versa. The parameters of the turbulent phase with mainly negative returns, i.e., low mean and high volatility, are as expected. However, due to the included negative returns, the mean of the turbulent phase with mainly positive returns does not exceed the mean of the calm phase by much. Even though we are not able to reject the statistics testing the equality of the parameters (mean and variance, respectively) of the calm and the turbulent bullish phase, we do not intend to merge these to simplify replication of the subsequent results due to differences in their derivation. The calm

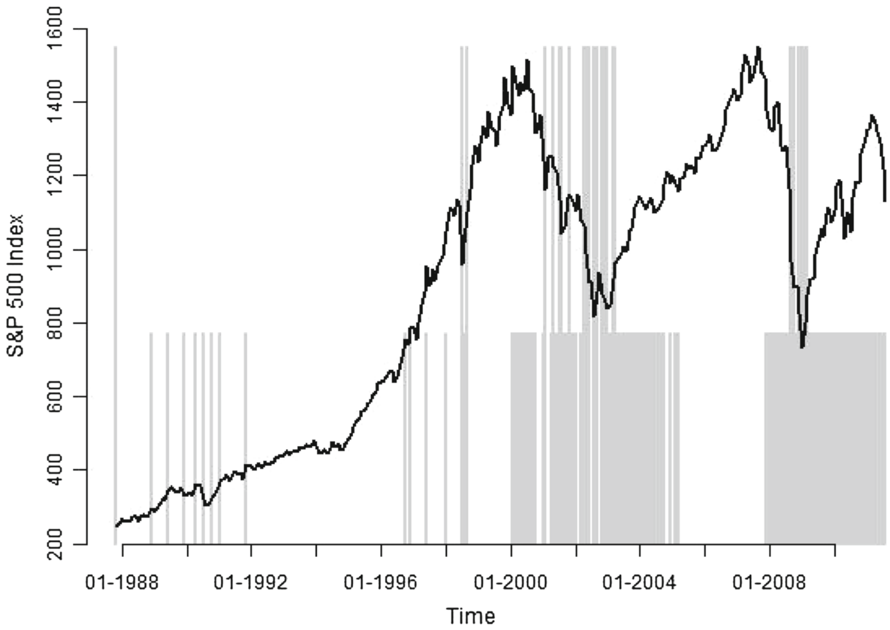

Fig. 3 S&P 500 Index; separated by a three-state Markov-switching model into a calm state, a turbulent state with mainly negative returns (*high grey*) and a turbulent state with mainly positive returns (*lower grey*)

Table 2 Sample characteristics of the monthly returns of the S&P 500 price index in three states derived from a single three-state Markov-switching model based on the time period from 11/1987 to 09/2011

	Calm	Turbulent positive	Turbulent negative
Mean (ann.)	0.1231	0.0911	−0.3035
Standard deviation (ann.)	0.1094	0.1844	0.2022

phase is based on only a single Markov-switching model, while the turbulent bullish regime represents a successive combination of two models.

For the Markov-switching model with three states we extend the original training set to the period 01/1964–10/1987 so as to ensure that all three states are properly estimated. Figure 3 and Table 2 show market characterization and the parameters of the observed regimes, respectively. In general, the filtered probabilities resulting from the three-state model estimation are more volatile than the ones from the two Markov-switching models with two states each. Consequently, in the framework of the three-state Markov-switching model regime changes happen more often (compare Figs. 1, 3). A comparison of Tables 1 and 2 shows that the parameters of the turbulent phase with mainly negative returns derived from the three-state Markov-switching model are smaller than the corresponding ones of the combined two-state approach. Hence, the three-state approach seems to perform better in terms of identifying turbulent bearish phases. Again, if we compare for both models the parameters of the calm phase, the three-state model outperforms the linked two-state models. Specifically, for the three-state Markov-switching model the mean of the calm period is larger and the corresponding standard deviation is smaller than is the case the two Markov-switching models with two states each. However, for the turbulent bullish phase, we see the opposite result, that is, the three-state approach is dominated by the two-state construction. A major drawback of the Markov-switching model with three states is that the mean of the turbulent phase with mainly positive returns is smaller than the mean of the calm phase. This implies that the turbulent bullish phase of the three-state model includes more negative and highly volatile returns than the turbulent bullish state of the linked Markov-switching models with two states each. Consider, for example, January, June and September of 2008. The three-state model classifies these as turbulent periods with mainly positive returns, even though strongly negative returns were observed. In contrast, the two Markov-switching models with two states each categorize these months as turbulent phases with mainly negative returns.

3 Forecasting framework

3.1 Econometric models

Using the estimated filtered probabilities (see Sect. 2), we develop two early warning systems denoted by *Model 1* and *Model 2*. Each of these consists of two submodels. The first submodel of *Model 1*, which we call *Model 1.1*, is built to model the filtered probabilities of the turbulent regime p_t^1 [see Eq. (6)] of the first two-state Markov-switching model. The second submodel of *Model 1* is called *Model 1.2* and covers the filtered probabilities of the turbulent state with mainly negative returns $p_t^{D,1}$ [see Eq. (6)] derived from the second two-state Markov-switching model. Refering to Eq. (5), the submodels of *Model 2* (denoted by *Model 2.1* and *Model 2.2*) are built to model the filtered probabilities of the turbulent regime $p_t^{3,0} = 1 - q_t^{3,0}$ and of the turbulent regime with mainly negative returns $p_t^{3,1} = q_t^{3,1}$ of the three-state Markov-switching model, respectively. Before we choose a suitable set of explanatory variables and risk factors for the modelling of the filtered probabilities p_t^1, $p_t^{D,1}$, $p_t^{3,0}$ as well as $p_t^{3,1}$, we transform the response variable with the logistic function to end up with an ordinary linear regression. While Demirgüc-Kunt and Detragiache (1998) use a logistic model to link a binary banking crisis dummy variable with a vector of explanatory variables, our crisis indicator is a real number. Thus, the transformed response y_t^j with $t \in \mathbb{T}$ for $j \in \{1, (3,0), (3,1)\}$ and $t \in \mathbb{T}^D$ for $j = (D, 1)$ is given by:

$$y_t^j = \ln\left(\frac{p_t^j}{1 - p_t^j}\right). \tag{9}$$

For all admissible t, j let $\boldsymbol{\beta}^{\boldsymbol{j}}$ and $\boldsymbol{x}_{\boldsymbol{t}}^{\boldsymbol{j}}$ refer to the vector of coefficients and the vector of explanatory variables, respectively. Then, the relation between the transformed response and the set of covariates is given by:

$$y_{t+1}^j = \boldsymbol{\beta}^{\boldsymbol{j}\prime} \boldsymbol{x}_{\boldsymbol{t}}^{\boldsymbol{j}} + \epsilon_t^j. \tag{10}$$

Due to the logistic-transformation of the filtered probabilities the coefficients reflect a change in $\ln(p^j_{t+1}/(1-p^j_{t+1}))$. As mentioned in Sect. 2, both Markov-switching models are successively estimated for each new point in time. To capture the time-dependency possibly inherent in the error terms of Eq. (10), the regression models are extended by time series models for the error terms. Instead of using a pure AR(p) process as done for the choice of the order of the Markov-switching models (see Sect. 2), better out-of-sample prediction results have been achieved using ARMA(p,q) processes for the error terms. Finally, we get:

$$y^j_{t+1} = \ln\left(\frac{p^j_{t+1}}{1-p^j_{t+1}}\right) = \boldsymbol{\beta}^{\boldsymbol{j}\prime}\boldsymbol{x}^{\boldsymbol{j}}_{\boldsymbol{t}} + \sum_{l=1}^{p}\phi_l \cdot \epsilon^j_{t-l} + \sum_{k=1}^{q}\theta_k \cdot \delta^j_{t-k} + \delta^j_t, \qquad (11)$$

with ϕ_l, $l = 1, \ldots, p$, and θ_k, $k = 1, \ldots, q$, denoting the coefficients of the ARMA(p,q) process, ϵ^j_t, $t \in \mathbb{T}$, being the error terms of the regression models, and δ^j_t denoting the error terms of the ARMA(p,q) process[4].

3.2 Risk factors and model estimation

Based on the data set listed in Appendix A, different combinations and transformations (e.g., interpolation, percentage change, absolute change compared to the previous month, etc.) of publicly available macroeconomic indicators were tested during derivation of the models underlying the early warning systems. All time series are shifted to their date of publication[5]. Calibration of the regression model is based on the period 11/1987–09/2011; the period 11/1987–06/2004 is used for a fixed in-sample regression. For each new point in time of the period 07/2004–09/2011, the coefficients are sequentially re-estimated. By applying the commonly used AIC criterion [cf. Akaike (1974) and the Bonferroni correction see, e.g., Shao (2003)] to the significance level $\alpha = 0.05$, a model with six covariates denoted by $\boldsymbol{x}^{\boldsymbol{1}}_{\boldsymbol{t}}$ is chosen to model p^1_{t+1}, that is, the probability that the market will be turbulent in the next month (see Sect. 3.1). After the inclusion of interaction effects between these variables (represented by their bivariate products as additional factors), a model with six covariates and four interaction effects was discovered to fit the filtered probabilities best. Note that we are currently talking about *Model 1.1*. On closer inspection, *Model 1.1* contains two term spreads based on the US Treasury Constant Maturity Rate (the 10-year 3-month spread and the five-year 3-month spread) as explaining factors. Furthermore, the corporate bond spread between Moody's Seasoned Baa Corporate Bond Yield and Moody's Seasoned Aaa Corporate Bond Yield is part of the covariates. The OECD Composite Leading Indicator for the OECD countries and the six major non-member economies (Brazil, China, India, Indonesia, Russia, and South Africa) is the fourth covariate used in the model. Finally, the USD 3-month LIBOR rate and the annualized volatility of the last

[4] For further details, see, e.g., Hamilton (1994) or Brockwell and Davis (1991).

[5] For example, the OECD CLI Index of December is published in February. Thus the data set of December is shifted to February in our estimation database.

Table 3 Summary of the estimated regression *Model 1.1*

	Estimate	Std. error	*p* value
(Intercept)	−10.46	1.24	0.000000
Termspread 10Y-3M	3.16	0.59	0.000000
Termspread 5Y-3M	−3.42	0.70	0.000002
OECD CLI	−1.89	0.45	0.000036
Corporate bond spread	10.50	1.12	0.000000
LIBOR	0.99	0.26	0.000214
Volatility	0.07	0.01	0.000000
Termspread 10Y-3M * Termspread 5Y-3M	−0.27	0.11	0.017710
Termspread 5Y-3M * OECD CLI	0.67	0.18	0.000320
OECD CLI * corporate bond spread	1.66	0.27	0.000000
Corporate bond spread * LIBOR	−1.59	0.24	0.000000

Table 4 Summary of the estimated regression *Model 1.2*

	Estimate	Std. error	*p* value
(Intercept)	−1.59	2.15	0.4636
Termspread 5Y-3M	−1.20	0.86	0.1705
OECD CLI	0.41	0.37	0.2664
LIBOR	1.53	0.61	0.0142
Volatility	0.07	0.04	0.0862
Termspread 5Y-3M * LIBOR	0.59	0.22	0.0103
OECD CLI * LIBOR	−1.21	0.25	0.0000
Volatility * LIBOR	−0.03	0.01	0.0598

22 trading days of the S&P 500 price index are included. The four interaction effects are given by the product of the two term spreads, the term spread 5Y-3M and the OECD CLI, the OECD CLI and the corporate bond spread, as well as the corporate bond spread and the LIBOR. These interactions model the variable influences of the covariates depending on the underlying economic situation. Table 3 reports the fitted regression coefficients of *Model 1.1*, their standard errors, and p-values. The adjusted R^2 of *Model 1.1* is 78.92 %.

In the case of *Model 1.2* the AIC criterion and the Bonferroni correction suggest for the filtered probabilities of the turbulent regime with mainly negative returns a model consisting of the term spread 5Y-3M, the OECD CLI, the LIBOR, and the volatility as covariates. The interaction effects are given by the term spread 5Y-3M, the OECD CLI, and the volatility multiplied by the LIBOR rate. Table 4 summarizes the fitted *Model 1.2*. The missing term spread 10Y-3M, the missing corporate bond spread, and a different set of interactions simplify the distinction between *Model 1.2* and *Model 1.1*. Again, the impact of the variables from Table 4 on the probability depends on the value of the other regressor variables due to the interaction effects. The adjusted R^2 of *Model 1.2* is 65.93 %.

The fitted coefficients, the standard errors, and p-values of *Models 2.1* and *2.2* are summarized in Tables 5 and 6, respectively. Again, these models are chosen by

Table 5 Summary of the estimated regression *Model 2.1*

	Estimate	Std. error	p value
(Intercept)	−1.86	1.80	0.3029
Termspread 10Y-3M	−1.81	1.33	0.1732
Termspread 5Y-3M	7.85	1.77	0.0000
LIBOR	−0.09	0.27	0.7418
Volatility	0.18	0.02	0.0000
Termspread 10Y-3M * Termspread 5Y-3M	−0.65	0.28	0.0209
Termspread 10Y-3M * LIBOR	−0.65	0.12	0.0000

Table 6 Summary of the estimated regression *Model 2.2*

	Estimate	Std. error	p value
(Intercept)	−6.62	0.75	0.000000
Termspread 10Y-3M	0.84	0.52	0.109330
Termspread 5Y-3M	1.27	0.82	0.122461
OECD CLI	−3.80	0.54	0.000000
Corporate bond spread	−1.10	0.39	0.004931
LIBOR	0.32	0.11	0.002412
Volatility	0.11	0.01	0.000000
Termspread 10Y-3M * Termspread 5Y-3M	−0.27	0.11	0.017191
Termspread 5Y-3M * LIBOR	−0.25	0.06	0.000146
OECD CLI * LIBOR	0.30	0.07	0.000036
Termspread 10Y-3M * OECD CLI	0.98	0.16	0.000000

applying the AIC criterion and the Bonferroni correction. The missing OECD CLI, the missing corporate bond spread and the different set of interactions simplify the distinction between *Model 2.1* and *Model 1.1*. In contrast to *Model 1.2*, *Model 2.2* includes the whole set of variables of *Model 1.1*. However, the interaction variables of both models do not coincide. The adjusted R^2 of *Model 2.1* is 37.00 % and of *Model 2.2* is 54.35 %. These are significantly worse statistics than the corresponding ones for *Model 1.1* and *Model 1.2*. A comparison with the filtered probabilities derived from the two two-state Markov-switching models shows that the higher volatility inherent in the filtered probabilities of the three-state Markov-switching model might be a reason for the low adjusted R^2. However, smoothing filtered probabilities does not yield better results.

To summarize, the covariates of *Model 1* and *Model 2* contain measures of the slope of the yield curve with predictive power [see, e.g., Ang et al. (2006)], a benchmark for the global short-term interest rates, and a measure for the economic situation. The excellent forecasting ability of corporate bond spreads was previously reported in Zhang (2002). The data sets incorporate indicators calculated from stock market data along with indicators reflecting evolution of the world economy. Hence, the sets of covariates cover sentiment, liquidity, leading economic indicators and market confidence.

3.3 Prediction results

In this section we focus on the performance of the 1-month-ahead out-of-sample forecasts. The root mean squared error (RMSE) is used to measure the quality of out-of-sample prediction results. Let n be the number of out-of-sample forecasts and let $i \in \{1, \ldots, n\}$ be the i-th point in time. Furthermore, let y_i and $\hat{y}_i,\ i \in \{1, \ldots, n\}$ refer to the (afterward) measured and the (before) forecast filtered probabilities, respectively. In the case of June 2004, the filtered probabilities $\hat{y}_i$ forecast using *Model 1* or *Model 2* only incorporate information that is available up to the end of May 2004. In contrast, the filtered probabilities y_i are measured at the end of June 2004 using the EM algorithm (see Sect. 2). Depending on the model under consideration (i.e., *Model 1.1*, *Model 1.2*, *Model 2.1*, or *Model 2.2*) y_i (respectively $\hat{y}_i$) have to be replaced by the measured (respectively filtered) versions of p_t^1, $p_t^{D,1}$, $p_t^{3,0}$ or $p_t^{3,1}$. For the general expressions y_i and $\hat{y}_i$ the RMSE is given by:

$$\text{RMSE} = \sqrt{\sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{n}}.$$

The statistic calculates the root of the mean squared error of the prediction results at the considered n data points. As mentioned in Sect. 3.2, the out-of-sample period is 07/2004–09/2011. After all ARMA(p,q) processes with $p = 1, \ldots, 3$ and $q = 1, \ldots, 3$ have been tested in terms of the AIC and the BIC criteria, an ARMA(1,0) process for the error terms of *Model 1.1* is chosen to cover autocorrelations. The out-of-sample results of Model 1.1 are reliable, with an RMSE of 0.1294. *Model 1.2* separates the phases with mainly positive and mainly negative returns only if the market has been characterized as turbulent by *Model 1.1*. Therefore, due to missing points in time (whenever the market is classified as calm) no AR(m) process is applied to explain the residuals in Model 1.2. The out-of-sample results of Model 1.2 are solid and have an RMSE of 0.1781. As the submodels of *Model 2* have time-dependent error terms, an ARMA (1,1) process is fitted on the error terms of *Model 2.1*, while *Model 2.2* is extended by an ARMA (3,2) process. A comparison with *Model 1* shows that the estimation of the submodels of *Model 2* is much harder. There are several reasons for this. First, the estimation process of a three-state Markov-switching model is very unstable compared to the estimation of a two-state Markov-switching model resulting in highly volatile filtered probabilities. Second, the volatile filtered probabilities make it difficult to find regression models that can account for the high variability inherent in the data. As a consequence, the RMSE of *Model 2.1*, which is 0.2214, is clearly worse than the RMSE of *Model 1.1*. This implies that the distinction between calm and turbulent periods derived from *Model 2* does not work as well as the one based on *Model 1*. The RMSE of *Model 2.2* is 0.1350. This ratio is clearly better than the RMSE of *Model 1.2*. However, as mentioned in Sect. 2, the three-state Markov-switching model classifies more periods with negative and highly volatile returns as belonging to the turbulent phase with mainly positive returns. Hence, the better RMSE of *Model 2.2* includes no information about the usefulness of *Model 2* compared to *Model 1*. A detailed analysis shows that *Model 2* is not able to predict, for example,

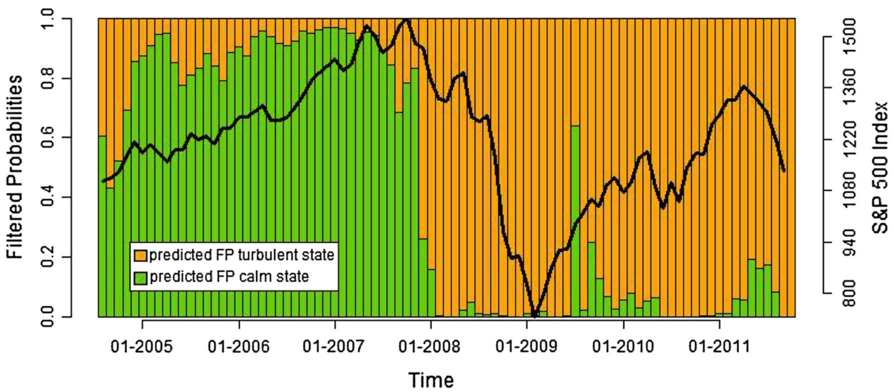

Fig. 4 One-month-ahead prediction results of *Model 1.1*. The S&P 500 price index is shaded with the forecasted filtered probabilities for being in the calm (*green*) or in the turbulent state (*orange*)

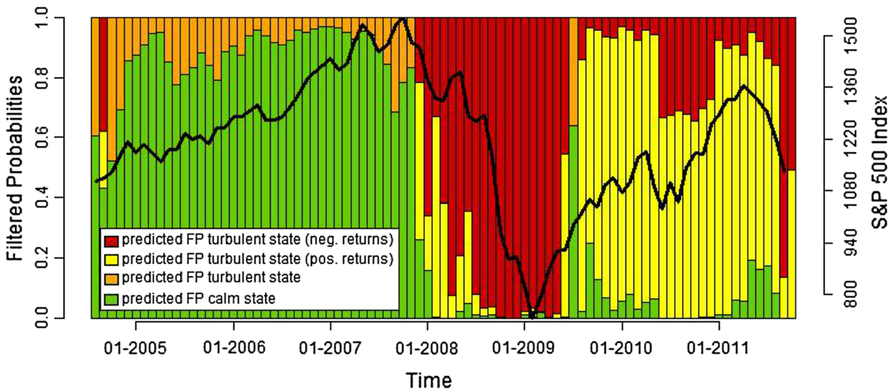

Fig. 5 One-step-ahead prediction using *Model 1.1* combined with *Model 1.2*. The S&P 500 price index is shaded with the predicted filtered probabilities. If a turbulent phase is forecasted, the predicted filtered probabilities of *Model 1.1* (calm: *green*; turbulent: *orange*) are replaced by the additional information derived from *Model 1.2* (calm: *green*; turbulent with mainly positive returns: *yellow*; turbulent with mainly negative returns: *red*)

the economic downturn between January and September 2008. Thus *Model 1* appears to be more reliable as an early warning system and we therefore concentrate on Model 1 in the following sections.

3.4 Early warning system

The estimation results of the first step of the early warning system based on the two-state Markov-switching model are shown in Fig. 4. The early warning system based on *Model 1.1* was able to forecast the market turbulence and, hence, the beginning of the financial crisis before the end of 2007.

Figure 5 includes the advanced distinction between turbulent regimes provided by *Model 1.2*. As before, the extended early warning system indicates the upcoming

Table 7 Variables causing changes in the filtered probabilities

	Responsible variables *Model 1.1*		Responsible variables *Model 1.2*	
08/2007	*S&P 500 volatility* *LIBOR*			⇝
11/2007–01/2008	*Corporate bond spread* *Term structure* *S&P 500 volatility*			⇝
		02/2008–03/2008	*S&P 500 volatility* *OECD CLI* *Term structure*	↓
		05/2009–06/2009	**OECD CLI** **LIBOR** **Term structure**	↑
		08/2011	*Term structure* *S&P 500 volatility*	↓

Bold variables indicate falling filtered probabilities, italic variables rising ones. The first column covers *Model 1.1*, i.e., the probability of turbulence. The second column covers *Model 1.2*, i.e., the probability of mainly negative returns given turbulence. The last column classifies the predicted stock index movements: ⇝ turbulence, ↓ turbulent downswings and ↑ turbulent upswings

financial crisis and the highly volatile markets in following years. Due to *Model 1.2*, the early warning system provides further information on upcoming bull and bear markets. For the in-sample results of all regression models and the out-of-sample results of *Model 2*, see Appendix B. Given the forecast probabilities of *Model 1*, we analyze the quality of the early warning system by first assuming that all months with a negative return of the S&P 500 price index are categorized as a turbulent phase with mainly negative returns, whereas all the other months should be either calm periods or turbulent phases with mainly positive returns (best-case scenario). This analysis shows that 62.79 % of the months are classified correctly. A more detailed examination reveals that eight out of the ten worst, and all five of the worst, months, measured in terms of the performance of the S&P 500, are correctly identified as turbulent time periods with mainly negative returns.

Finally, we perform a detailed analysis to verify whether the early warning system reacts appropriate to specific changes in the input variables. For instance, we were able to identify significant changes in the turbulence probability and to trace these back to changes in the underlying macroeconomic variables. Table 7 shows which variables are responsible for the movement in turbulence probability during specific crisis periods. Table 7 reveals that the prediction of the increasing turbulence starting in 2007 was first driven by the high volatility of the S&P 500 returns and the rising LIBOR rates. Afterward, the forecast was dominated by the very high corporate bond spreads and the increasing convexity of the term structure caused by a jump at the long end. Due to the increasing volatility, the declining OECD CLI, and the declining term spread 5Y-3M in 02–03/2008, the early warning system's forecast shifted to the turbulent state with mainly negative returns. An increasing OECD CLI in 05/2009 changed the model's perspective to forecast turbulent periods with mainly positive returns until 07/2011. In 08/2011, the decreasing term spread 5Y-3M and the high

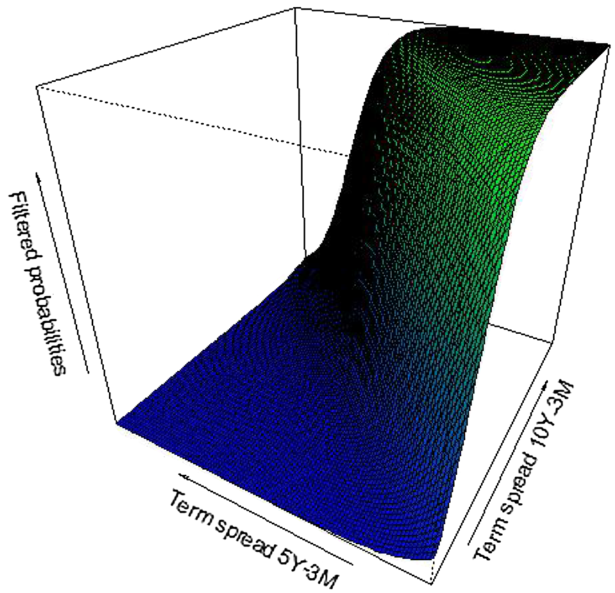

Fig. 6 Filtered probability depending on the term spreads

volatility again caused a change in the model's forecast for 09/2011 toward a turbulent phase with mainly negative returns.

3.5 Economic interpretation of regression results

Due to the interactions between the regressor variables in *Model 1.1*, the interpretation of the results in Table 3 is not straightforward. The model accounts for the fact that changes in key economic numbers have to be interpreted in the context of the whole economic situation. Consequently, to interpret one key number other key numbers have to be taken into account. The only variable that is not part of any interaction, i.e., the volatility of the S&P 500 returns, can be interpreted easily. The positive coefficient shows that an increasing volatility results in an increasing probability of turbulence. Analyzing the influence of the term spreads is more complicated as the two covariates are also part of two interaction effects, with one of these even consisting of the product of both. To get a feeling for how the yield curve affects the model, Fig. 6 shows the filtered probabilities for different values of the term spreads, while the remaining variables are kept unchanged.

In general, an increase in the term spread 5Y-3M reduces the probability of turbulence, whereas an increasing term spread 10Y-3M increases this probability. Thus, the more convex the term structure, e.g., in times of expected strengthening of monetary policies due to inflation risks, the higher is the probability of turbulence. In contrast, a concave yield curve leads to a low probability of turbulence. In an environment of successfully deployed monetary policy with expected healthy growth, the increasing OECD CLI leads to a decreased probability of turbulence. However, high corporate bond spreads or a high term spread 5Y-3M might lead to the reverse influence of the OECD CLI index due to the modeled interactions. Basically, this is what happens in an overheated economy, turbulence increases in step with increased economic uncertainty. Rising corporate bond spreads accounting for the enlarged default risk along with a low (respectively normal) interest rate level (measured by the LIBOR rate) enhance the probability of turbulence. In contrast, if the LIBOR rate is very high or the OECD CLI is very low, the model could increase the probability of turbulence in the event of falling corporate bond spreads, which is a likely scenario when there is either deep recession or recovery and stock market prices are already at a low level. Interpretation of LIBOR's impact depends on corporate bond spreads. In the case of a constant low spread (see, e.g., the economic conditions at the end of 2007), a rising LIBOR leads to increasing turbulence probabilities, indicating a bubble in the corporate investment sector. On the other hand, assuming a normal or higher level of corporate bond spreads, an increasing LIBOR reduces the probability of turbulence, denoting a prophylactic and sustainable strengthening of monetary policies. Given a low LIBOR rate, which is the case in most of the turbulent time periods, a decrease in the term spread 5Y-3M increases the probability of negative returns. This is either caused by an increase at the short end of the term structure correlating with an increasing LIBOR rate or by a decrease in the medium-term interest rate. The first case describes an environment where assets with less risk (e.g., government bonds) become more attractive, enhancing the incentive for investors to move their capital away from risky assets. The second case is an environment of high uncertainty where the prices of assets with less risk and a medium-term time to maturity rise. In both cases there is a high probability of capital movements toward assets with less risk, i.e., a so-called flight to quality. The influence caused by the OECD CLI depends to a certain extent on the level of the LIBOR. A rising OECD CLI together with a moderate or high LIBOR rate leads to a decrease in the probability of a turbulent phase with mainly negative returns, possibly indicating an upswing. But, in an environment of extremely low LIBOR rates (see, e.g., the period 08/2010–07/2011) this influence can change, implying that the growth has been too quick. In an environment of turbulent markets, which already have low LIBOR rates, rising volatility leads to an increase in the probability of negative returns.

4 Applications in asset management

To verify the economic importance of the early warning system based on the two Markov-switching models (each with two states) we investigate two simple (dynamic) out-of-sample portfolio strategies. We consider the period 08/2004–09/2011 and, for the sake of simplicity, assume that there are no transaction costs. Whenever the early warning system predicts a turbulent month with mainly negative returns, *Strategy A* invests in the risk-free asset represented by the 1-month Treasury Constant Maturity Rate. For any other forecast, *Strategy A* invests in the stock market in form of the S&P 500 price index. *Strategy B* is set up as follows: if a calm period is forecast, the fraction invested in the risk-free asset equals the predicted turbulence probability, i.e., p^1. If turbulent regime is forecast, the fraction invested in the risk-free asset equals the

Table 8 Comparison of different investment strategies based on the period 08/2004–09/2011

	Buy-and-hold			*Strategy A*	*Strategy B*	CPPI ($m = 1$)	CPPI ($m = 3$)
	Risk-free	Stock market	50–50				
Terminal value	115.44	102.70	111.42	172.58	146.72	113.09	102.64
Return (p.a.)	0.0202	0.0037	0.0152	0.0791	0.0549	0.0173	0.0036
Volatility (p.a.)	0.0055	0.1580	0.0792	0.1016	0.0849	0.0263	0.0688
mod. Sharpe ratio (p.m.)[a]	–	−0.0067	−0.0067	0.1741	0.1261	−0.0280	−0.0594
Omega measure Ω (p.m.)[b]	–	0.9820	0.9820	1.6411	1.4000	0.9288	0.8499
95 %-VaR (p.m.)[c]	$1.9 \cdot 10^{-5}$	−0.0847	−0.0419	−0.0423	−0.0350	−0.0123	−0.0386
95 %-CVaR (p.m.)[d]	$1.5 \cdot 10^{-5}$	−0.1084	−0.0539	−0.0568	−0.0465	−0.0169	−0.0559
Maximum drawdown[e]	$8.3 \cdot 10^{-6}$	−0.5256	−0.2969	−0.1314	−0.1103	−0.0946	−0.2762

[a] The modified Sharpe ratio measures the expected excess return of a strategy in terms of the 1-month US Treasury Constant Maturity Rate divided by the standard deviation of the excess returns
[b] The Omega measure denotes the ratio between the expected upside and the expected downside of the excess returns, where the upside (downside) is defined as the positive (negative) excess return
[c] For a given time horizon and a fixed confidence level α the value at risk of the return R represents the return, that is not exceeded during the considered period of time with the specified probability $1 - \alpha$: $VaR_\alpha(R) = \sup\{r : \mathbb{P}(R < r) \leq 1 - \alpha\}$
[d] For a given time horizon and a fixed confidence level α the conditional value at risk or expected shortfall of the return R represents the expected return conditioned on fact that the return is worse than the corresponding value at risk: $CVaR_\alpha(R) = \mathbb{E}[R | R \leq VaR_\alpha(R)]$
[e] The Maximum drawdown of a strategy is the worst return, which could have been realized in the whole period under consideration

predicted probability of turbulence with mainly negative returns, i.e., $p^R = p^{D,1} \cdot p^1$. The remaining value of the portfolio is invested in the S&P500 price index. The final payoffs, the annualized returns, the annualized volatilities and a set of different performance measures for an investment of \$100 in *Strategy A* and *Strategy B* are shown in Table 8. For comparison, Table 8 also includes the same characteristics for the static buy-and-hold strategies[6]. Assuming that the \$100 is completely invested in the risk-free asset, is only invested in stocks or is equally invested in both assets, i.e., 50 % in the risk-free asset and 50 % in the stocks. *Strategy A* and *Strategy B* are dynamic investment strategies. Therefore, Table 8 also provides the same key figures for two constant proportion portfolio insurance (CPPI) strategies[7]. With multipliers $m = 1$

[6] In case of the buy-and-hold strategy of a 100 % investment in the stock market and a buy-and-hold strategy consisting of a 50 % investment in the stock market and a 50 % investment in the risk-free asset the equality of the Sharpe ratio and the Omega measure, respectively, follows from the fact that the benchmark yield is given by the return of the risk-free asset.

[7] In general, a CPPI strategy is as follows. First, a minimum repayment at maturity (referred to as "floor") must be specified. For any point in time before maturity the difference between the current value of the portfolio and the discounted floor (denoted as "cushion") is determined. As long as the cushion is positive, the cushion multiplied by the factor m ("exposure") is invested in the risky assets. The remaining value of the portfolio is invested in the risk-free asset. Hence, the multiplier m indicates the investor's risk appetite. If the value of the discounted floor exceeds the current value of the portfolio (cushion is negative) at any

and $m = 3$. Both CPPI strategies have a floor of \$95. Table 8 shows that the risk-free investment yields a terminal value of 115.44 (2.02 % p.a.) with an annualized volatility of 0.56 %. The buy-and-hold strategies earn at most 1.52 % p.a.; the CPPI strategies generate at most a return of 1.73 %. Not one of these strategies does as well as *Strategy A* and *Strategy B*, which realize profits of 7.91 and 5.49 %, respectively. Furthermore, in addition to enhanced returns, the early warning system can decrease the inherent volatility. Compared to a pure investment in stocks, this leads to a significant increase in the monthly modified Sharpe ratio from -0.0067 (pure investment in stocks) to 0.1741 (*Strategy A*) and 0.1261 (*Strategy B*). If we test the difference in the modified Sharpe ratios using the statistics of Jobson and Korkie (1981), *Strategy A* compared to all other significantly outperforms other strategies at a 10 % confidence level[8]. Furthermore, based on the Omega measure, *Strategy A* is also significantly better than the buy-and-hold strategies[9]. Advanced risk measures like the (conditional) value at risk and the maximum drawdown underline the superiority of investment strategies relying on the proposed early warning system compared to ordinary buy-and-hold strategies.

5 Conclusion

We set up two early warning systems to forecast turbulence in the US stock market. The first approach consists of two properly linked Markov-switching models with two states each, while the second one is a single Markov-switching model with three states. The two-step Markov-switching model at first separates the calm phases of the market from the turbulent ones using the monthly returns of the S&P 500 price index. In a second step, this model is further specified to distinguish between turbulent bull and bear markets. The estimation includes the derivation of the (conditional) filtered probabilities for a turbulent market (with mainly negative returns). A comparison of the early warning system based on the step-by-step combination of two Markov-switching models (each with two states) and the early warning system derived from the Markov-switching model with three states shows the superiority of the first approach. The relation between the filtered probabilities provided by the Markov-switching models and additional exogenous variables that are publicly available is modeled using a logistic function. As a consequence, the resulting forecasts allow identifying the factors driving the market in times of crises. Furthermore, they allow timely indication of potential upcoming market turbulence. To demonstrate the advantages of this early warning system in the asset and risk management framework, a case study illustrates that trading strategies incorporating the system's signals yield better portfolio performance together with lower portfolio risk compared to standard buy-and-hold strategies.

Footnote 7 continued
point in time, the risky investments are removed and the whole wealth is invested in the risk-free asset for the remaining time to maturity. For more information on CPPI strategies, see, e.g., Black and Perold (1992).

[8] The test statistics of Jobson and Korkie (1981) for the modified Sharpe ratio provide: *Strategy A* vs. both buy-and-hold strategies 1.826 (significant 10 %); *Strategy A* vs. *Strategy B* 1.808 (significant 10 %); *Strategy B* vs. both buy-and-hold strategies 1.489.

[9] The test statistics of Schmid and Schmidt (2008) for the Omega measure: *Strategy A* vs. both buy-and-hold strategies 1.659 (significant 10 %); *Strategy A* vs. *Strategy B* 1.551; *Strategy B* vs. both buy-and-hold strategies 1.431.

Future research could focus on dynamic and other advanced trading strategies using the signals of the early warning systems presented in this paper.

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Appendix A: Underlying data set

In the following we give an overview of all data that were used directly or after some transformations (e.g., absolute/relative change of the same variable over time, spread between two different variables, etc.). We also provide the data sources so as to simplify the verification of the results set out in Sect. 3.2.

Balance on Current Account
`http://research.stlouisfed.org/fred2/series/BOPBCA`
Belgischer Frühindikator
`Datastream (BG000183Q)`
Capacity Utilization Rate for Total Industry
`http://research.stlouisfed.org/fred2/series/TCU`
Civilian Unemployment Rate
`http:research.stlouisfed.org/fred2/search?st=Civilian +Unemployment+Rate`
Consumer Price Index (CPI) for All Urban Consumers (All Items)
`http://research.stlouisfed.org/fred2/series/CPIAUCSL`
JPM EMBI+ Composite - Total Return Index
`Datastream(JPMPTOT)`
Foreign Exchange Rate EURO - USD
`http://www.global-view.com/forex-trading-tools/ forex-history/`
Effective Federal Funds Rate
`http://research.stlouisfed.org/fred2/series/FEDFUNDS`
Gross Domestic Product (GDP)
`http://research.stlouisfed.org/fred2/series/GDP`
U.S. High Yield Master II Effective Yield
`http://research.stlouisfed.org/fred2/series/AAA`
Moody's Seasoned Aaa Corporate Bond Yield
`http://research.stlouisfed.org/fred2/series/BAMLH0A0HY M2EY`
Ifo Weltwirtschaftsklima Index
`http://www.cesifo-group.de/portal/page/portal/ifoHome/ a-winfo/d6zeitreihen/15reihen/_reihenwes`
Industrial Production Index
`http://research.stlouisfed.org/fred2/series/INDPRO`

3-Month US-D LIBOR Interest Rate
http://www.global-rates.com/interest-rates/libor/american-dollar/american-dollar.aspx
1-Month and 3-Month LIBOR-OIS Spread
Reuters
Board of Governors Monetary Base, Adjusted for Changes in Reserve Requirements
http://research.stlouisfed.org/fred2/series/BOGAMBSL
M1 Money Stock
http://research.stlouisfed.org/fred2/series/M1SL
M2 Money Stock
http://research.stlouisfed.org/fred2/series/M2SL
Nonfarm Payroll Employment Indicator
http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/data-files/EMPLOY/
OECD Composite Leading Indicator (OECD + Major 6 NME)
http://stats.oecd.org/index.aspx?datasetcode=MEI_CLI
Personal Saving Rate
http://research.stlouisfed.org/fred2/series/PSAVERT
ISM Manufacturing: PMI Composite Index
http://research.stlouisfed.org/fred2/series/NAPM
Producer Price Index: All Commodities
http://research.stlouisfed.org/fred2/series/PPIACO
Put-Call Ratios
http://www.cboe.com/data/putcallratio.aspx
Spot Oil Price: West Texas Intermediate
http://research.stlouisfed.org/fred2/series/OILPRICE
Moody's Seasoned Aaa Corporate Bond Yield
http://research.stlouisfed.org/fred2/series/AAA
Moody's Seasoned Baa Corporate Bond Yield
http://research.stlouisfed.org/fred2/series/BAA
S & P500 Price Index
http://research.stlouisfed.org/fred2/series/SP500
Treasury Bill Eurodollar Difference
http://www.federalreserve.gov/releases/h15/data.htm
3-Month Treasury Constant Maturity Rate
http://research.stlouisfed.org/fred2/series/GS3M
5-Year Treasury Constant Maturity Rate
http://research.stlouisfed.org/fred2/series/GS5
10-Year Treasury Constant Maturity Rate
http://research.stlouisfed.org/fred2/series/GS10
Treasury International Capital System (Cross-Border Financial Flows)
http://www.treasury.gov/resource-center/data-chart-center/tic/Pages/ticsec2.aspx
Federal Government Debt: Total Public Debt
http://research.stlouisfed.org/fred2/series/GFDEBTN

Trade Weighted Exchange Index: Major Currencies
http://research.stlouisfed.org/fred2/series/TWEXMMTH
University of Michigan: Consumer Sentiment
http://research.stlouisfed.org/fred2/series/UMCSENT
Datastream (USUMCONSH)
CBOE Volatility Index
http://www.cboe.com/micro/vix/historical.aspx

Appendix B: Illustrations

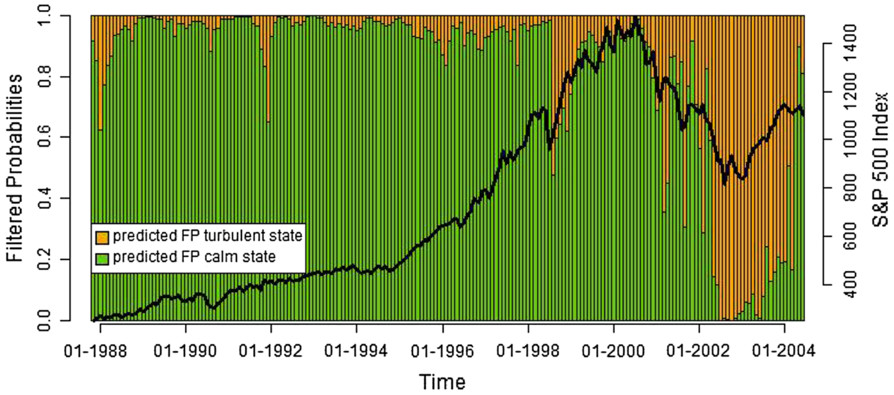

Fig. 7 In-sample test of regression *Model 1.1*

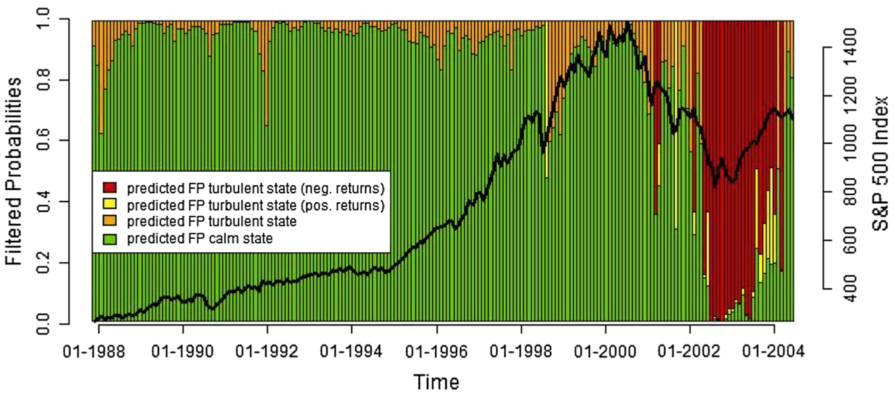

Fig. 8 In-sample test of regression *Model 1.2*

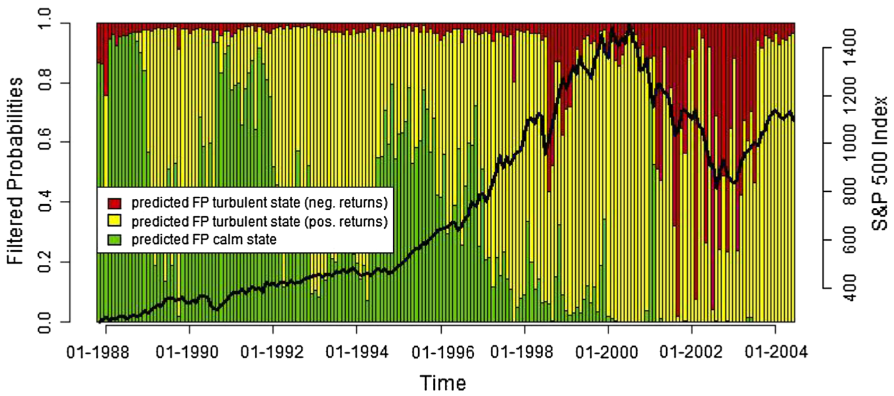

Fig. 9 In-sample test of regression *Model 2*

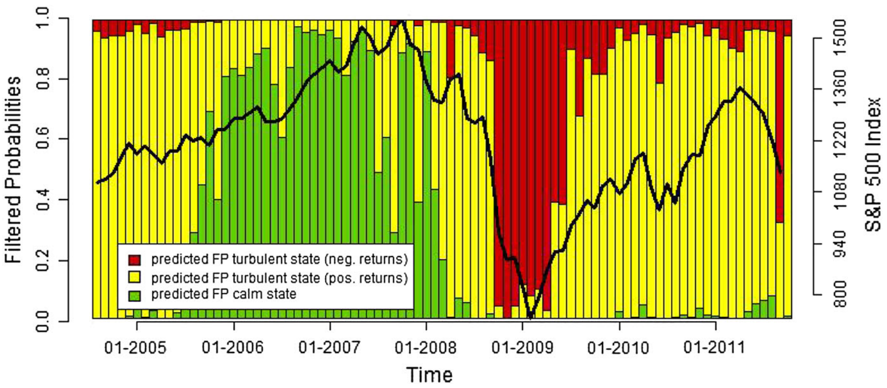

Fig. 10 Out-of-sample test of regression *Model 2*

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