Convex Optimization: Assignment 3

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Problem 1

Answer: a.

According to the illustration, we have the following formalism of the optimization problem,

$$\min \sum_{t=0}^{T-1} \phi(a_t)$$
$$0 \leq a_t \leq a_{max},\ \forall t \in [0, T-1]$$
$$v_0 = a_0 - 9.8,\ v_t = v_{t-1} + a_t - 9.8,\ \forall t \in [1, T-1]$$
$$y_0 = y_{init},\ y_t = y_{t-1} + v_{t-1},\ \forall t \in [1, T]$$
$$l_t \leq y_t \leq h_t,\ \forall t \in [0, T].$$

We then reduce some unnecessary variables. It is clear to see that we have $v_t = \sum_{i=0}^{t} a_i - 9.8(t+1)$ for all $t \in [0, T-1]$, and that $y_t = \sum_{j=0}^{t-1} v_j + y_{init} = \sum_{j=0}^{t-1}(\sum_{i=0}^{j} a_i - 9.8(j+1)) + y_{init} = \sum_{j=0}^{t-1}\sum_{i=0}^{j} a_i - 9.8(t+1)t/2 + y_{init}$. Therefore, we can remove the equalities by substituting these intermediate variables.

$$\min \sum_{t=0}^{T-1} \phi(a_t)$$
$$0 \leq a_t \leq a_{max},\ \forall t \in [0, T-1]$$
$$l_t \leq \sum_{j=0}^{t-1}\sum_{i=0}^{j} a_i - \frac{9.8(t+1)t}{2} + y_{init} \leq h_t,\ \forall t \in [0, T].$$

It is clear to see that all constraints are linear. As for the objective function, since it can be constructed from summation over convex functions, it is clearly convex.

b. Since both the objective function and constraints are differentiable and convex, each subgradient is unique.

$$g_0(\mathbf{a}) = (1 + 2a_0 + 3a_0^2, \cdots, 1 + 2a_{T-1} + 3a_{T-1}^2)^T$$
$$g_{1,t,l}(\mathbf{a}) = -e_t,\ g_{1,t,r}(\mathbf{a}) = e_t,\ \forall t \in [0, T-1]$$
$$g_{2,t,l}(\mathbf{a}) = -\sum_{i=0}^{t-1}(t-i)e_i,\ g_{2,t,r}(\mathbf{a}) = \sum_{i=0}^{t-1}(t-i)e_i,\ \forall t \in [0, T],$$

where $g_{i,t,l}$ represents the left-hand side inequality of the ith set of constraints for time t and $i \in \{1, 2\}$, and e_t is the unit vector in dimentison t.

c. According to the execution of the optimization, we find the optimal value of energy for the $T = 2$ case is 589.28. The optimal value for $T = 32$ case is 39619.23. We also note there are some iterations in which lower bounds are larger than the final optimal value, and we think this is due to the infeasibility.