CS-E4820 Machine Learning: Advanced Probabilistic Methods
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Exercise problems, round 1, due on Tuesday, 29th January 2019, at 23:55
Please return your solutions in MyCourses as a single PDF file.

Problem 1. *"Coins."*

There are two bent coins (c_1 and c_2) with different properties, and your objective is to guess which coin was used (i.e. the value of random variable $C \in \{c_1, c_2\}$), after learning whether the result of the coin toss (i.e. the random variable $X \in \{\text{h}, \text{t}\}$) was heads or tails.

As prior knowledge, we know the probability of each coin resulting in tails: $p(X = \text{t} \mid C = c_1) = \theta_1$ and $p(X = \text{t} \mid C = c_2) = \theta_2$. In addition, the prior probability for using coin c_1 is known: $p(C = c_1) = \pi_1$.

Give the posterior probability of coin c_1 being used for the toss, $p(C = c_1 \mid X)$, in terms of θ_1, θ_2 and π_1, for both $X = \text{t}$ and $X = \text{h}$.

Furthermore, plot[1] the posterior probability of coin c_1, $p(C = c_1 \mid X = \text{t})$, as a function of θ_1, when we have $\theta_2 = 0.5$ and $\pi_1 = 0.5$.

Problem 2. *"False positive paradox."*

Consider a hypothetical lie detector that is "fairly reliable", in the sense that it will correctly detect 98% of all lies, and also classify as true 98% of all statements that are actually true. This lie detector is being used in an attempt to detect academic dishonesty, by asking "did you cheat?" from all students participating in an exam of a machine learning course. (This example is still hypothetical.)

For the purposes of this question, assume as prior knowledge that there are 300 students taking the exam, and a single student has chosen to cheat. We will further assume that all students deny having cheated. If the detector now flags a particular student X as a cheater, how likely is it that X has, in fact, cheated in the exam?

Problem 3. *"Markov blanket definition."*

Consider the Bayesian network in Figure 1. What is the Markov blanket of each variable? (see Barber: Bayesian Reasoning and Machine Learning, ch. 2.1, Definition 2.5)

[1] Use the code provided in the file ex1_prob1_coins_template.py

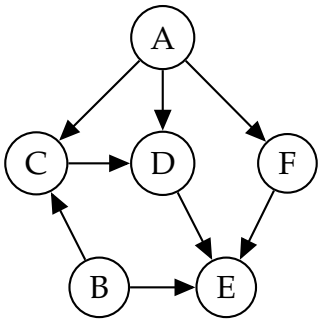

Figure 1: Bayesian network for Problem 3.