CS-E4820 Machine Learning: Advanced Probabilistic Methods
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Exercise problems, round 2, due on Tuesday, 5th February 2018, at 23:55
Please return your solutions in MyCourses as a single PDF file.

Problem 1. *"Computing conditional probability."*

Consider the Bayesian network in Figure 1 which represents Mr Holmes' burglary worries as given in the figure: (B)urglar, (A)larm, (W)atson, Mrs (G)ibbon. All variables are binary with states $\{\mathsf{tr}, \mathsf{fa}\}$.

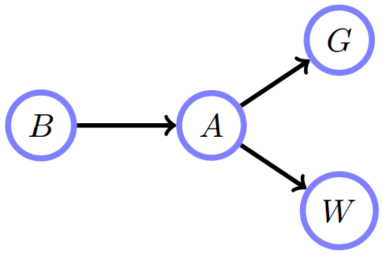

Figure 1: Bayesian network for Mr Holmes' burglary worries in Problem 1.

The probabilities are

$$
\begin{aligned}
p(B=\mathsf{tr}) &= 0.01 \\
p(A=\mathsf{tr}|B=\mathsf{tr}) &= 0.99 & \qquad p(A=\mathsf{tr}|B=\mathsf{fa}) &= 0.05 \\
p(W=\mathsf{tr}|A=\mathsf{tr}) &= 0.90 & \qquad p(W=\mathsf{tr}|A=\mathsf{fa}) &= 0.5 \\
p(G=\mathsf{tr}|A=\mathsf{tr}) &= 0.7 & \qquad p(G=\mathsf{tr}|A=\mathsf{fa}) &= 0.2.
\end{aligned}
$$

Compute the conditional probabilities

(a) $p(B=\mathsf{tr}|W=\mathsf{tr})$

(b) $p(B=\mathsf{tr}|W=\mathsf{tr}, G=\mathsf{fa})$

Problem 2. *"Conditional independence from Bayesian network."*

Based on the Bayesian network in Figure 2, which of the following conditional independence statements follow? For each statement, give a "true/false" answer; for the false statements, also mention a path between the two nodes that is not blocked. (see Barber: Bayesian Reasoning and Machine Learning, ch. 3.3.4)

(a) $A \perp\!\!\!\perp B \mid C$ **(c)** $C \perp\!\!\!\perp E \mid B, D$ **(e)** $B \perp\!\!\!\perp F \mid A, C$

(b) $A \perp\!\!\!\perp B \mid \emptyset$ **(d)** $C \perp\!\!\!\perp D \mid A, B$ **(f)** $A \perp\!\!\!\perp E \mid D, F$

Furthermore, find a Bayesian network that is *Markov equivalent* to the network in Figure 2. (see Barber: Bayesian Reasoning and Machine Learning, ch. 3.3.6)

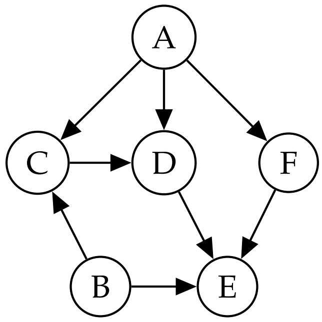

Figure 2: Bayesian network for Problem 2.

Problem 3. *"Burden of specification."*

Consider a distribution of five binary variables x_i.

(a) What is the number of parameters needed to define the distribution $p(x_1, x_2, x_3, x_4, x_5)$ if no assumptions are made, i.e. p is an arbitrary distribution.

(b) How about if the Bayesian network in Figure 3 is assumed, i.e. p factorizes as implied by the graph.

(c) And how about if, additionally to (b), we assume that the conditional distributions are shared, i.e. $p(x_{i+1} \mid x_i) = p(x_i \mid x_{i-1})$, $i = 2, 3, 4$?

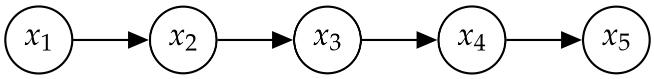

Figure 3: Model for Problem 3.

Problem 4. *"Medical diagnosis."*

Let's have the following notation:

Notation	Explanation
$A = 1$	A person has brain cancer
$B = 1$	A person has a high blood calcium level
$C = 1$	A person has a brain tumor
$D = 1$	A person has seizures that cause unconsciousness
$E = 1$	A person has severe headaches

An expert have told us the following information about the relationships between variables:

> Probability of severe headaches $P(E = 1)$ depends only on the fact whether a person has a brain tumor (C) or not. On the other hand, if one knows the blood calcium level (B) and whether the person has a tumor or not (C), one can specify the probability of unconsciousness seizures $P(D = 1)$.

In this case, the probability of D doesn't depend on the presence of the headaches (E) or (directly) on the fact whether the person has brain cancer or not (A). The probability of a brain tumor (C) depends directly only on the fact, whether the person has brain cancer or not (A).

Construct a DAG that represents (exactly) the conditional independencies specified by the expert.