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CS-E4820 – Machine Learning: Advanced Probabilistic Methods
Homework Assignment 2

Problem 1. "Computing conditional probability"

Given the graph in Figure 1, we have the overall probability of this network as below:

p (a, b, g, w) = p (b) * p(a | b)*p(g | a) * p(w | a)

Since all variables are binary with states {tr,fa}, we can compute:

- p (fa | tr) = 1 – p(tr | tr)
- p (fa | fa) = 1 – p(tr | fa)

From given information, these values in red are computed:

A	B	P (A | B)
tr	tr	0.99
tr	fa	0.05
fa	tr	0.01
fa	fa	0.95

W	A	P (W | A)
tr	tr	0.90
tr	fa	0.50
fa	tr	0.10
fa	fa	0.50

G	A	P (G | A)
tr	tr	0.7
tr	fa	0.2
fa	tr	0.3
fa	fa	0.8

And: p(B = fa) = 1 – p(B=tr) = 0.99

a) $p\,(B = tr|W = tr) = \frac{p\,(B=tr,W=tr)}{p(W=tr)} = \frac{\sum_a \sum_g p\,(B=tr,W=tr,A=a,G=g)}{\sum_a \sum_b \sum_g p\,(B=b,W=tr,A=a,G=g)} =$

$\frac{\sum_a \sum_g p\,(B=tr)*p(A=a|B=tr)*p(G=g|A=a)*p(W=tr|A=a)}{\sum_a \sum_b \sum_g p\,(B=b)*p(A=a|B=b)*p(G=g|A=a)*p(W=tr|A=a)} =$

$\frac{0.01*(0.99*0.7*0.9+0.99*0.9*0.3+0.01*0.5*0.2+0.01*0.5*0.8)}{0.01*(0.99*0.7*0.9+0.99*0.9*0.3+0.01*0.5*0.2+0.01*0.5*0.8)+0.99*(0.05*0.7*0.9+0.05*0.9*0.3+0.95*0.5*0.2+0.95*0.5*0.8)}$

= 0.0171

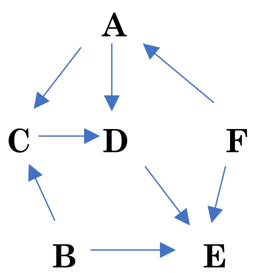
b) $p\,(B = tr|W = tr, G = fa) = \frac{p\,(B=tr,W=tr,G=fa)}{p(W=tr,G=fa)} = \frac{\sum_a p(B=tr,W=tr,A=a,G=fa)}{\sum_a \sum_b p(B=b,W=tr,A=a,G=fa)} =$
$\frac{\sum_a p(B=tr)*p(A=a|B=tr)*p(G=fa|A=a)*p(W=tr|A=a)}{\sum_a \sum_b p(B=b)*p(A=a|B=b)*p(G=fa|A=a)*p(W=tr|A=a)} =$
$\frac{0.01*(0.99*0.9*0.3+0.01*0.5*0.8)}{0.01*(0.99*0.9*0.3+0.01*0.5*0.8)+\ 0.99*(0.05*0.9*0.3+0.95*0.5*0.8)} = \mathbf{0.06916}$

Problem 2. "Conditional Independence from Bayesian network"

We need to find out if the path between nodes are blocked or not. If it is blocked, then it is conditionally independent given the conditioning set.

a) False, since C is a collider between A & B. A path between two nodes that are not blocked is A → C ← B
b) True, because there is no path between A & B directly.
c) False, the path C – A - F - E is not blocked.
d) False. Path C – D is not blocked.
e) True. All paths are blocked.
f) False. Path A – C – D – E is not blocked because D is a collider and in the conditioning set.

A Markov equivalent network is:



Problem 3. "Burden of specification".
Having a set of binary variables {x1, x2, x3, x4, x5}
We know that:

a) If every variable has k states, so the number of parameters needed to construct the network if there is no assumption, is k^5, because they might be connected in an arbitrary fashion. However, we just need to know $k^5 - 1$ parameters, since the sum of probabilities of all parameters is 1, so that we do not need to know the last parameter.

In this case, k = 2, so the number of parameters to know is **31**.

b) p(x1,x2,x3,x4,x5) = p(x1) * p(x2|x1) * p(x3|x2)*p(x4|x3)*p(x5|x4)
p(x1) takes 2 parameters.
For each $p(x_i|x_j)$: Conditioning on each value of x_j, we can define $p(x_i|x_j)$ by just knowing 1 parameter. Since there are 2 values of x_j, we also just need to know 2 parameters for each component $p(x_i|x_j)$ of the above formula.

Applying the same logic of no need to know the last parameter, the number of parameters to know is 2 + 2 x 4 – 1 = **9**.

c) We have p(x2|x1) = p(x3|x2) = p(x4|x3) = p(x5|x4)

$p(x1,x2,x3,x4,x5) = p(x1) * p(x2 | x1)^4$
Therefore, we need to know $2 + 2 - 1 =$ **3** parameters.

Problem 4. "Medical diagnosis"

Derived from the question, we can have the directed graph as below: