**CS-E4820 Machine Learning: Advanced Probabilistic Methods**
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Exercise problems, round 8, Due on Tuesday, 27th March 2018, at 23:55
Please return your solutions in MyCourses as a single PDF file.

The first two questions deal with deriving the ELBO for the 'simple model', described in the PDF document simple_elbo.pdf. Before doing these exercises, familiarize yourself with the contents of the document.

**Problem 1.** *"ELBO for the simple model 1/2."*

(a) Show that the general formula for ELBO, shown in Equation (8), can be written as the sum shown in Equation (9) for the simple model.

(b) Derive the 2nd term $E_{q(\theta)}[\log p(\theta)]$ of the ELBO.
Hint: recall that $Var(X) = E(X^2) - E(X)^2$.

(c) Find out the formula for the 7th term $E_{q(\theta)}[\log q(\theta)]$ of the ELBO.
Hint: see the 6th term.

**Problem 2.** *"ELBO for the simple model 2/2."*

(a) Derive the 4th term $E_{q(\mathbf{z})q(\tau)}\left[\log p(\mathbf{x}|\mathbf{z},\theta)\right]$ of the ELBO.
Hint 1: $E(XY) = E(X)E(Y)$ if $X$ and $Y$ are assumed indendent (see how this is already used in the derivation of term 3).
Hint 2: $E\left[(X-Y)^2\right] = E\left[(X-a+a-Y)^2\right]$.

(b) Implement terms 2, 4, and 7 to the code template `ex8_1_template.py`. Verify that the ELBO increases when you run the VB algorithm.

**Problem 3.** *"Factor analysis in Edward."*

Transform the Edward code for probabilistic PCA, provided in `ex8_3_pca_demo.py` (this is based on the notebook `probabilistic_pca.ipynb, also attached`), into a probabilistic factor analysis. In detail:

(a) Modify the code that generates the simulated data set, such that each dimension has its own noise parameter.

(b) Modify the model description so that instead of a single common noise parameter, a separate parameter will be learned for each dimension. Compare the true noise parameters to the inferred ones.