Heat Exchangers

8

A heat exchanger is a device in which heat is transferred from one fluid to another. The hot fluid gets cooled, and the cold fluid is heated. The principles of heat transfer discussed so far in the earlier chapters are applied in the thermal design of a heat exchanger. Many types of heat exchangers have been developed for diverse applications in steam power plants, chemical process plants, refrigerators and air conditioners, radiators in cars, space vehicles and so on.

8.1 TYPES OF HEAT EXCHANGERS

Heat exchangers can be grouped into three broad classes:

1. Transfer type heat exchangers or recuperators,
2. Storage type heat exchangers or regenerators,
3. Direct contact type heat exchangers or mixers.

In a transfer type heat exchanger or a recuperator, the two fluids are kept separate and they do not mix as they flow through it. Heat is transferred through the separating walls. A concentric double pipe recuperator is shown in Fig. 8.1.

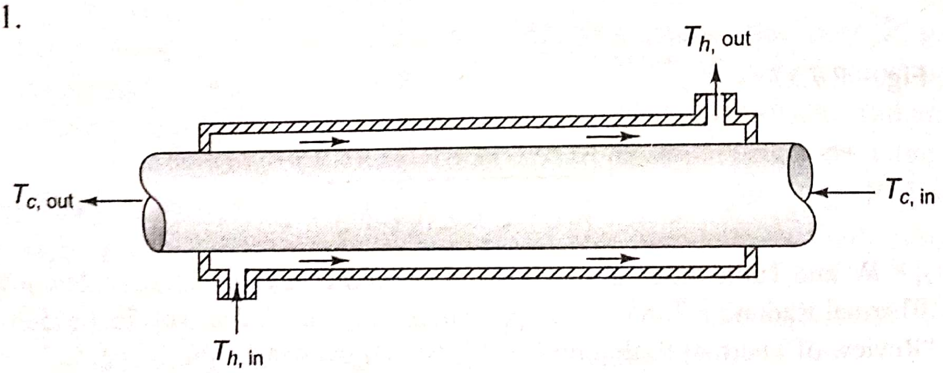

Fig. 8.1 *Concentric double pipe heat exchanger*

In a storage type heat exchanger or a regenerator, the hot and cold fluids flow alternately through a solid matrix of high heat capacity. When the hot fluid flows through the matrix in an interval of time, heat is transferred from the fluid to the matrix which stores it in the form of an increase in its internal energy. This stored energy is then transferred to the cold fluid as it flows through the matrix in the next interval of time. The matrix is thus subjected to periodic heating and cooling.

Storage type heat exchangers may have matrices which are either (i) stationary or (ii) rotating. Figure 8.2 shows a typical regenerator with a stationary matrix. During the heating period of the cycle when the hot fluid flows through the matrix, valves A and B are kept open and C and D are kept closed. During the cooling period,

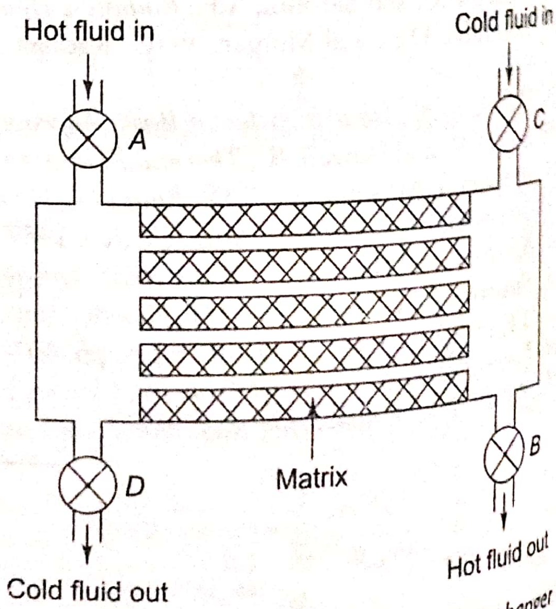

Fig. 8.2 *Single matrix storage type heat exchanger*

valves A and B are kept closed and C and D are kept open. A regenerator with a stationary matrix is used in a Stirling refrigerator, such as the Philips refrigerating machine for liquefaction of air, and in a gas turbine power plant.

A rotary regenerator has a matrix rotating at a low rpm, driven by a motor through reduction gears. The heat transfer surfaces provided in the regenerator are alternately exposed to the hot and cold fluids (Fig. 8.3). A typical application of this type of heat exchanger is found in a steam power plant for preheating of air, called Ljungström air preheater.

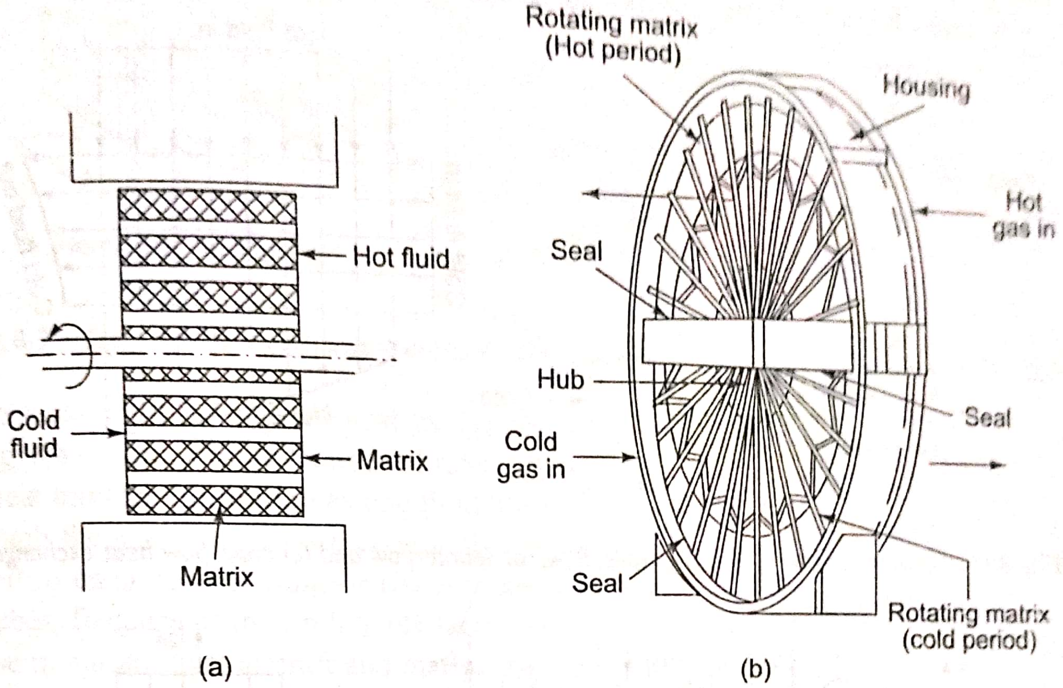

Fig. 8.3 *Rotary storage type heat exchanger*

A storage type heat exchanger provides a more compact arrangement than the transfer type with more surface area offered per unit volume. The major disadvantage is that some mixing of hot and cold fluids becomes inevitable, and it is quite difficult to seal the hot side from the cold side in the rotary regenerator. There are also more pressure drops in both the fluids.

In a direct contact heat exchanger, the two fluids mix together and transfer heat by direct contact. Open feedwater heaters, desuperheaters, cooling towers and jet condensers are examples of such heat exchangers. The heat transfer is usually accompanied by interphase mass transfer. It cannot be used for transferring heat between two gases or between two miscible liquids. A typical direct contact heat exchanger is shown in Fig. 8.4, which gives a section through a natural draught cooling tower.

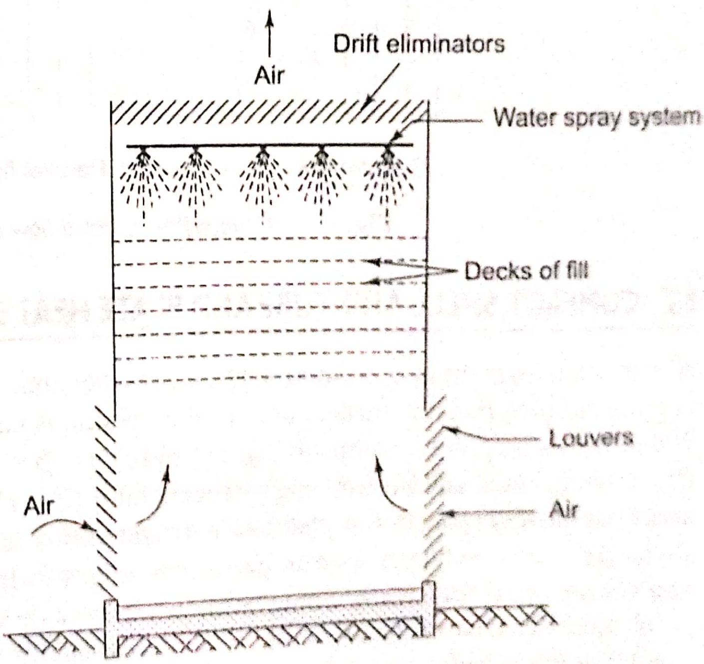

Fig. 8.4 *Direct contact heat exchanger: A natural draught cooling tower*

8.1.1 Flow Arrangements in Recuperative Heat Exchangers

There are three basic flow arrangements in recuperative heat exchangers: (1) Parallel flow, (2) Counterflow and (3) Cross flow. If both the fluids move in the same direction, it is a *parallel flow heat exchanger*. If the fluids move in opposite direction, it is a *counterflow heat exchanger*. If they flow normal to each other, it is a *cross flow heat exchanger* (Fig. 8.5). The temperatures of the two fluids usually vary from inlet to outlet of the heat exchanger, except in the case of phase change on either side when the temperature remains constant. The tubes may be in the form of coils also, as shown in Fig. 8.6.

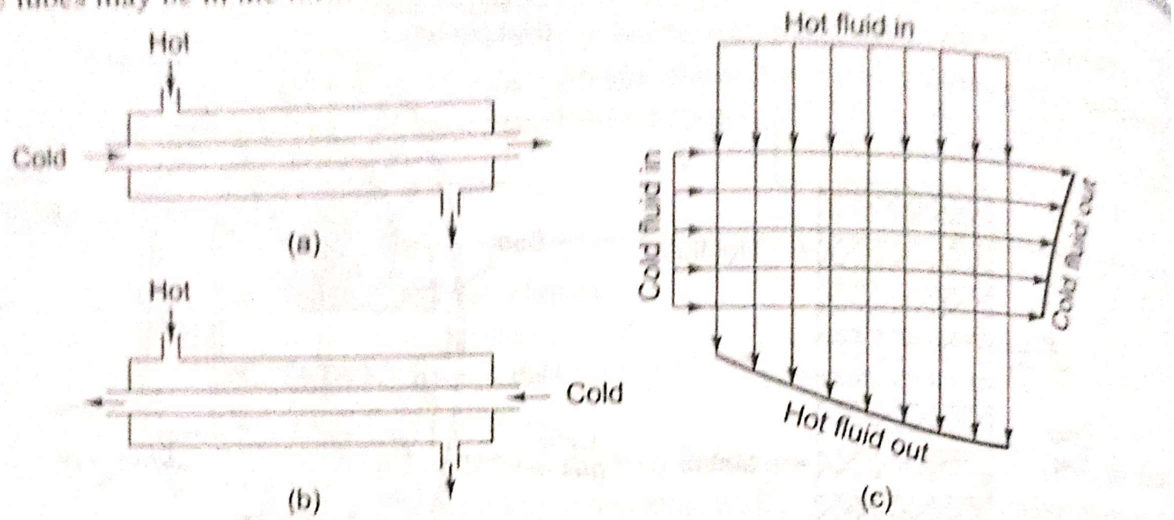

Fig. 8.5 *Schematic drawing of (a) parallel flow, (b) counterflow and (c) cross flow heat exchanger*

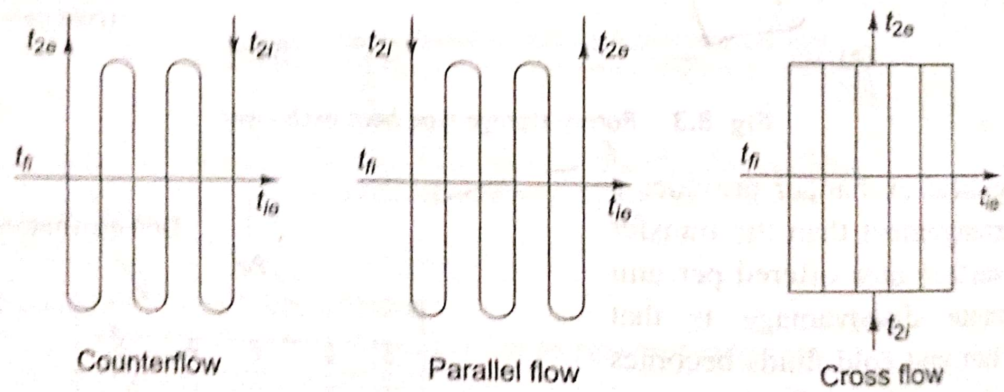

Fig. 8.6 *Counterflow, parallel flow and cross flow on tube bundles*

8.2 COMPACT, SHELL-AND-TUBE AND PLATE HEAT EXCHANGERS

A heat exchanger having a large surface area per unit volume is called a *compact heat exchanger*. The ratio of the heat transfer surface area to the volume is called the area density β. A heat exchanger with $\beta >$ 700 m^2/m^3 is said to be compact, e.g. car radiators ($\beta \approx$ 1000 m^2/m^3), ceramic regenerators in gas turbines ($\beta \approx$ 6000 m^2/m^3), and Stirling engine regenerator ($\beta \approx$ 15,000 m^2/m^3). The large surface area is obtained by attaching closely spaced thin plates or corrugated fins to the walls separating the two fluids. Compact heat exchangers are commonly used in gas-to-gas or gas-to-liquid heat transfer, with limitations on their weight and volume, with fins, if any, being used on the gas side where heat transfer coefficient is low.

In compact heat exchangers, the two fluids usually move perpendicular to each other, and such flow configuration is called *cross-flow*, as stated earlier. The cross-flow is further classified as unmixed flow and mixed flow. In Fig. 8.7(a), the cross-flow is said to be *unmixed*, since the plate fins force the fluid to flow through a particular interfin spacing and prevent it from moving in the transverse direction (i.e. flow parallel to the tubes). The cross-flow in (b) is said to be *mixed* since the fluid flow is free to move in the transverse direction.

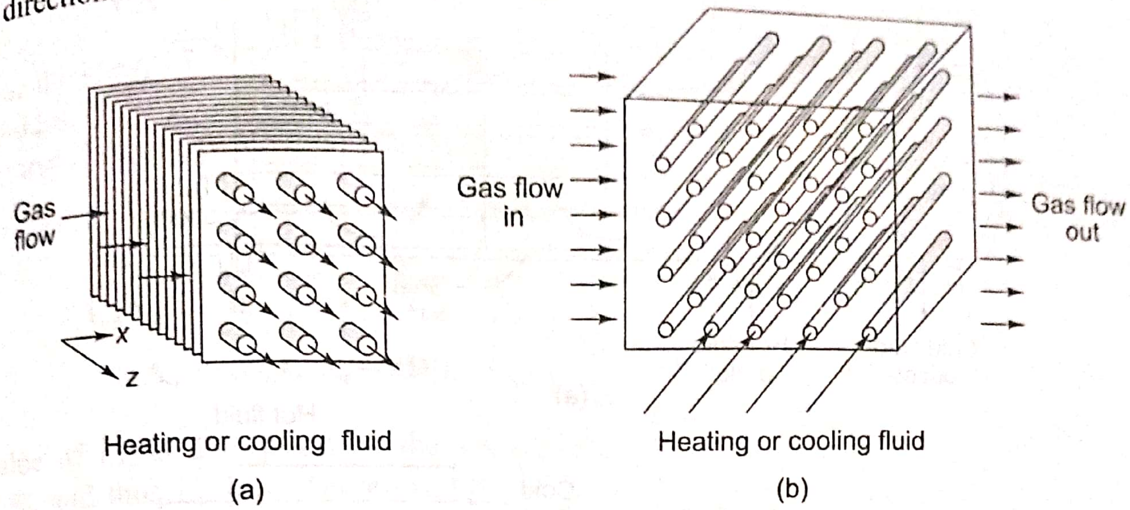

Fig. 8.7 *Cross-flow heat exchanger: (a) Fluids unmixed; (b) one fluid (gas) mixed, the other unmixed*

Perhaps the most common type of heat exchanger in industrial application is the *shell-and-tube heat exchanger* (Fig. 8.8). Here, a large number of tubes are packed inside a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer (by increasing residence time) and to maintain uniform spacing between the tubes. Because of their relatively large size and weight, shell-and-tube heat exchangers are not suitable for use in automotive, aircraft and marine applications. At both ends of the shell there are headers where the fluid accumulates before entering the tubes and after leaving them.

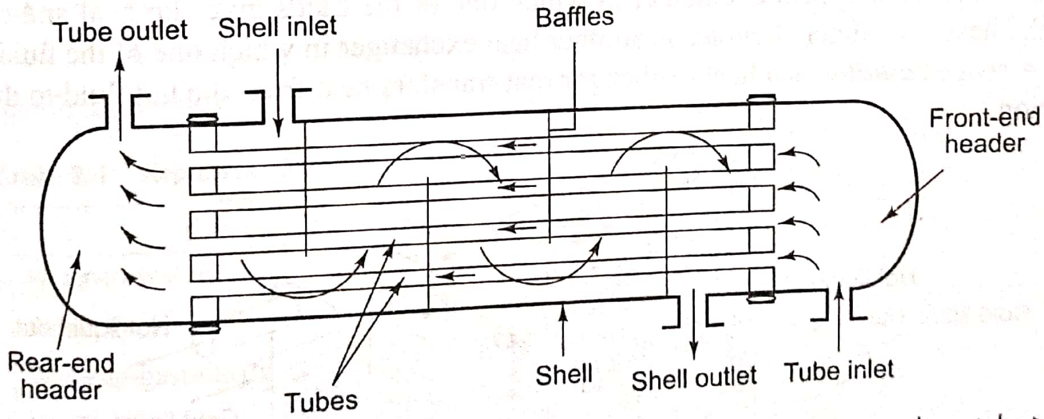

Fig. 8.8 *Schematic of a shell-and-tube heat exchanger (one-shell pass and one-tube pass)*

Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, are called *one-shell pass and two-tube pass* heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a *two-shell pass and four-tube pass* heat exchanger (Fig. 8.9).

An innovative type of heat exchanger which has found widespread use is the *plate heat exchanger*, which consists of a series of plates with corrugated flow passages (Fig. 8.10). The hot and cold fluids flow in alternate passages, and thus each cold fluid stream is surrounded by two hot fluid streams, resulting in very effective heat transfer. The heat transfer capacity can be enhanced by simply adding more plates in series. They are well suited for *liquid-to-liquid heat transfer* applications, provided that the hot and cold fluid streams are at about the same pressure.

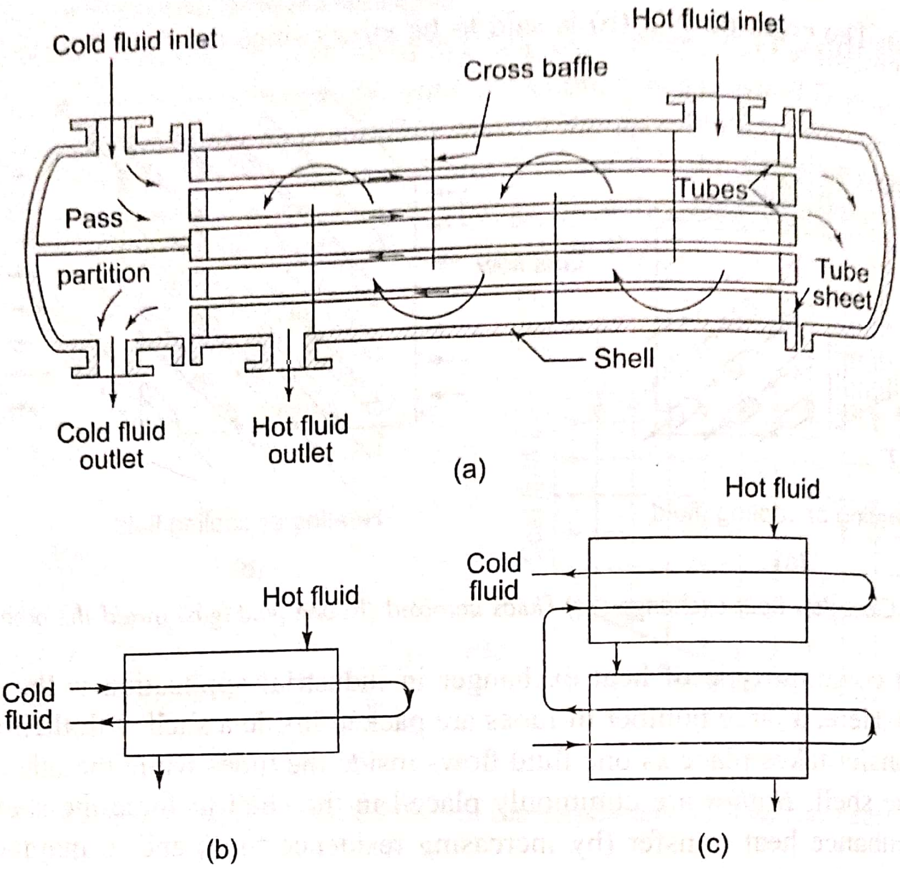

Fig. 8.9 *Multiple pass heat exchangers: (a), (b) One-shell pass and two-tube pass; (c) two-shell pass, four-tube pass*

Heat exchangers are often given specific names to reflect the specific application for which they are us For example, a *condenser* is a heat exchanger in which one of the fluids gives up heat and condenses a flows through the heat exchanger. A *boiler* is another heat exchanger in which one of the fluids absorbs and vaporises. A *space radiator* is a heat exchanger that transfers heat from the hot fluid to the surround space by radiation.

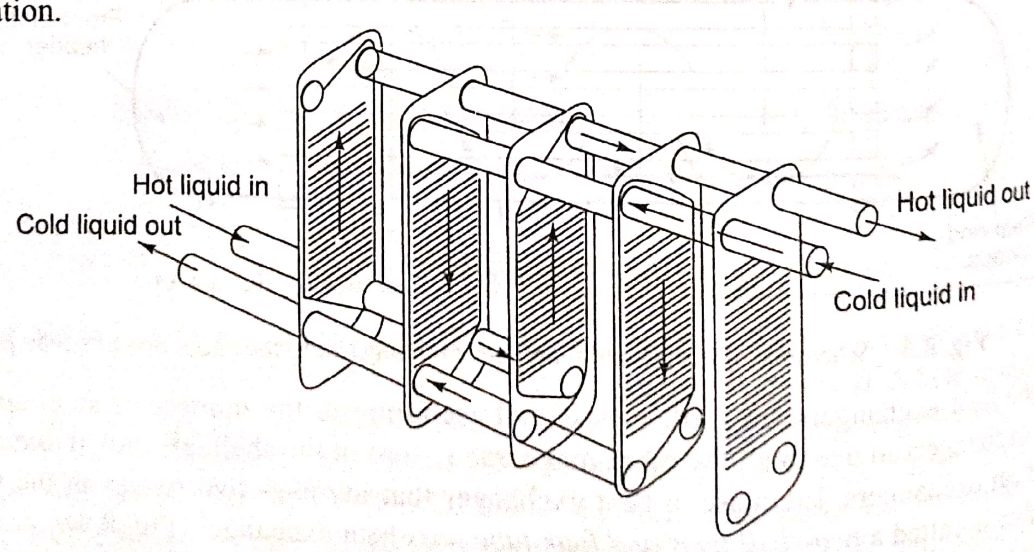

Fig. 8.10 *Plate heat exchanger*

8.3 OVERALL HEAT TRANSFER COEFFICIENT AND FOULING FACTOR

If T_h and T_c represent the bulk mean temperatures of the two fluids on either side of the plane wall, then

$$Q = UA\,\Delta T = UA\,(T_h - T_c) \quad (8$$

$$\frac{1}{UA} = \sum R = \frac{1}{h_1 A} + \frac{x_w}{k_w A} + \frac{1}{h_2 A} \tag{8.2}$$

where

x_w being the thickness of the wall, k_w the thermal conductivity and h_1 and h_2 the heat transfer coefficients on the two sides.

For heat transfer through a cylindrical wall,

$$Q = U_0 A_0 (T_h - T_c) = U_0 A_0 \Delta T \tag{8.3}$$

$$\frac{1}{U_0 A_0} = \frac{1}{h_i A_i} + \frac{x_w}{k_w \; A_{l\text{-}m}} + \frac{1}{h_0 A_0} \tag{8.4}$$

where,

$$A_0 = \pi D_0 L, A_i = \pi D_i L, A_{lm} = \frac{A_0 - A_i}{\ln (A_0/A_i)}$$

The value of U_0 is dominated by the smaller value of convection coefficient. If $h_i << h_0$ we have $1/h_i >> 1/h_0$, and thus $U_0 \approx h_i$. Therefore, the smaller heat transfer coefficient creates a bottleneck in the path of heat flow, and seriously impedes heat transfer. This situation arises when one of the fluids is a gas and the other a liquid. Since $h_{gas} << h_{liquid}$, fins are provided on the gas side to compensate for low h and enhance UA.

The range of values of overall heat transfer coefficient in different heat exchangers is given in Table 8.1. It may be noted that it varies from about 10 W/m^2 K for gas-to-gas heat exchangers to about 10,000 W/m^2 K for phase-change ones. When the tube is finned on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes

$$A = A_{total} = A_{fin} + A_{unfinned}$$

where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the area of unfinned portion of the tube surface. The effective surface area A can be estimated from

$$A = A_{unfinned} + \eta_{fin} A_{fin}$$

where η_{fin} is the fin efficiency.

Table 8.1 *Representative values of the overall heat transfer coefficients in heat exchangers*

Type of heat exchanger	U[W/(m^{2}K)]
Water-to-water	850 – 1700
Water-to-oil	100 – 350
Water-to-gasoline or kerosene	300 – 1000
Feedwater heaters	1000 – 8500
Steam-to-light fuel oil	200 – 400
Steam-to-heavy fuel oil	50 – 200
Steam condenser	1000 – 6000
Freon condenser (water cooled)	300 – 1000
Ammonia condenser (water cooled)	800 – 1400
Alcohol condensers (water cooled)	250 – 700
Gas-to-gas	10 – 40
Water-to-air in finned tubes (water in tubes)	30 – 60
	400 – 850
Steam-to-air in finned tubes (steam in tubes)	30 – 300
	400 – 4000

Equation (8.4) holds for clean surfaces on both sides of the cylindrical wall. After a period of operation scales are formed on the surfaces which offer additional resistances to heat transfer, so that

$$\frac{1}{U_0' A_0} = \frac{1}{h_{s_i} A_i} + \frac{1}{h_i A_i} + \frac{x_w}{k_w A_{lm}} + \frac{1}{h_{s_0} A_0} + \frac{1}{h_0 A_0}$$

where h_{s_i} and h_{s_0} are the scale heat transfer coefficients and U_0' is the overall heat transfer coefficient with scaled surfaces. The reciprocal of the scale heat transfer coefficient is called the *fouling factor* R_f. Fouling factors which reduce the performance of heat exchangers can be determined experimentally by estimating U_0 values for clean and scaled surfaces, so that

$$R_f = \frac{1}{U_0'} - \frac{1}{U_0}$$

Table 8.2 gives the fouling factors for certain applications.

Table 8.2 *Fouling factors*

Fluid	*Fouling factor R_f (m^2 K/W)*
Sea water	0.000172
Treated boiler feed water	0.000172
Well water	0.00036
Fuel oil	0.0009
Quenching oil	0.0007
Diesel exhaust gas	0.0018
Refrigerant vapours	0.000344
Brine	0.000172
Steam, alcohol	0.00009

The most common type of fouling is the precipitation of solid deposits in a fluid on the heat transfer surfaces. When water is hard, scales are formed, and hence water requires to be treated. Corrosion and chemical fouling by chemical reactions and biofouling due to growth of algae, and deposit of ash particles in the flue gases on air preheater surfaces are some other forms of fouling.

8.4 PARALLEL FLOW HEAT EXCHANGER

An insulated double pipe parallel flow heat exchanger along with the temperature profiles is shown in Fig. 8.11. The temperatures of the fluids vary from point to point as heat is transferred from the hot to the cold fluid.

Let $\dot{m}_h$ = mass flow rate of the hot fluid, kg/s

$\dot{m}_c$ = mass flow rate of the cold fluid, kg/s

c_h = specific heat of the hot fluid, kJ/kg K

c_c = specific heat of the cold fluid, kJ/kg K

T_{h_1} = inlet temperature of the hot fluid, °C

T_{h_2} = exit temperature of the cold fluid, °C

T_{c_1} = inlet temperature of the cold fluid, °C

T_{c_2} = exit temperature of the cold fluid, °C

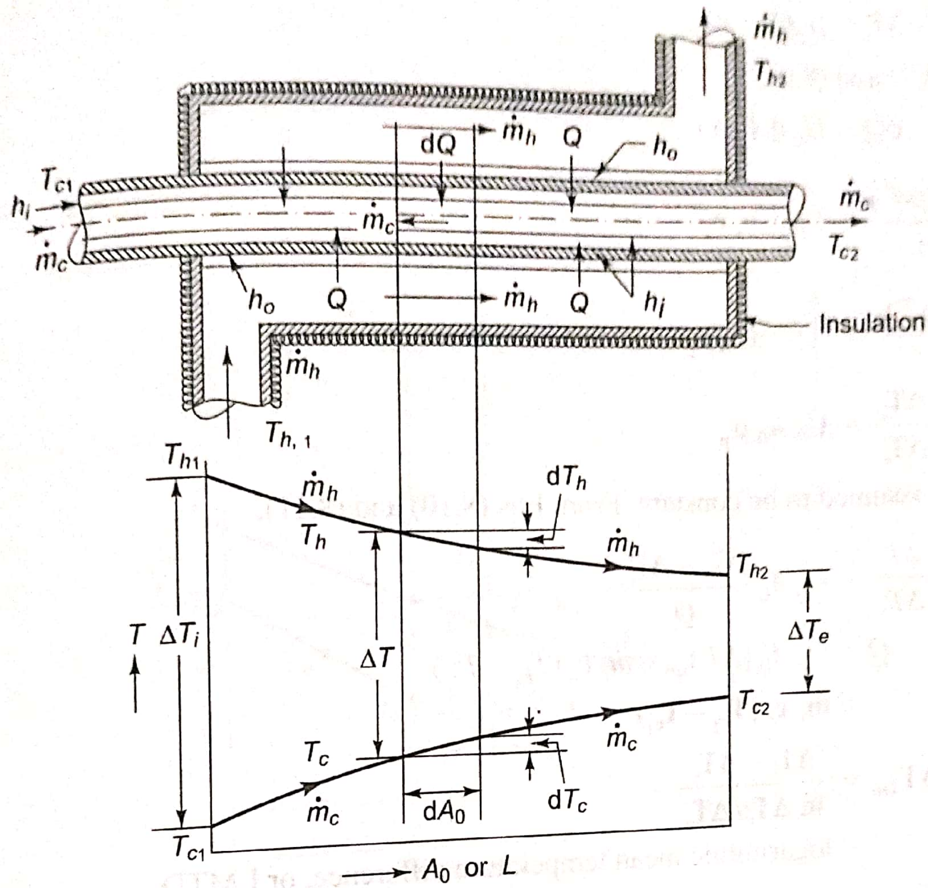

Fig. 8.11 *Parallel flow heat exchanger*

Let us consider a differential surface area dA_0 ($=\pi D_0\, dx$) of the heat exchanger where dQ amount of heat is transferred, the hot fluid temperature decreases by dT_h and the cold fluid temperature increases by dT_c. By writing an energy balance,

$$dQ = -\dot{m}_h c_h\, dT_h = \dot{m}_c c_c\, dT_c = U_0\, dA_0\, \Delta T \tag{8.7}$$

where $\Delta T = T_h - T_c$, T_h and T_c being the mean temperatures of the hot and cold fluids at that section respectively. This temperature difference ΔT between the two fluids changes from ΔT_i at inlet to ΔT_e at exit of the heat exchanger.

Now,
$$\Delta T = T_h - T_c$$
or,
$$d(\Delta T) = dT_h - dT_c \tag{8.8}$$

From Eqs (8.7) and (8.8),

$$d(\Delta T) = -\frac{dQ}{\dot{m}_h c_h} - \frac{dQ}{\dot{m}_c c_c}$$

$$= -dQ\left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right) \tag{8.9}$$

$$= -dQ\,\mu_p$$

where

$$\mu_p = \frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}$$

By integrating Eq. (8.9) from inlet to exit, μ_p being constant,

$$\Delta T_i - \Delta T_e = \mu_p Q$$ (8.10)

Again, from Eqs (8.7) and (8.9),

$$dQ = U_0\, dA_0\, \Delta T$$

$$-\frac{d(\Delta T)}{\mu_p} = U_0\, dA_0\, \Delta T$$

$$-\int_i^e \frac{d(\Delta T)}{\Delta T} = \int_i^e U_0\, dA_0\, \mu_p$$

or, $$\ln \frac{\Delta T_i}{\Delta T_e} = U_0 A_0 \mu_p$$ (8.11)

where U_0 has been assumed to be constant. From Eqs (8.10) and (8.11),

$$\ln \frac{\Delta T_i}{\Delta T_e} = U_0 A_0 \frac{\Delta T_i - \Delta T_e}{Q}$$

$\therefore$ $$Q = U_0 A_0 (\Delta T)_{lm} = \dot{m}_c c_h (T_{h_1} - T_{h_2})$$

$$= \dot{m}_c c_c (T_{c_2} - T_{c_1})$$ (8.12)

where, $$\Delta T_{lm} = \frac{\Delta T_i - \Delta T_e}{\ln \Delta T_i / \Delta T_e}$$

= logarithmic mean temperature difference, or LMTD

and $$\frac{1}{U_0 A_0} = \frac{1}{h_i A_i} + \frac{x_w}{k_w A_{lm}} + \frac{1}{h_0 A_0}$$

Here, $\Delta T_i = T_{h_1} - T_{c_1}$ and $\Delta T_e = T_{h_2} - T_{c_2}$

8.5 COUNTERFLOW HEAT EXCHANGER

Figure 8.12 shows an insulated double pipe counterflow heat exchanger along with the temperature profiles. The energy balance for the differential surface area dA_0 ($\pi D_0\, dx$) gives

$$dQ = -\dot{m}_h c_h\, dT_h = -\dot{m}_c c_c\, dT_c = U_0\, dA_0\, \Delta T$$ (8.13)

where both the hot and cold fluids undergo temperature decreases dT_h and dT_c (both being negative) in flowing the distance dx.

Since, $$\Delta T = T_h - T_c$$

$$d(\Delta T) = dT_h - dT_c = -\frac{dQ}{\dot{m}_h c_h} + \frac{dQ}{\dot{m}_c c_c}$$

$$= -dQ\left(\frac{1}{\dot{m}_h c_h} - \frac{1}{\dot{m}_c c_c}\right)$$

$$= -dQ\, \mu_c$$ (8.14)

where $$\mu_c = \frac{1}{\dot{m}_h c_h} - \frac{1}{\dot{m}_c c_c}$$ (8.15)

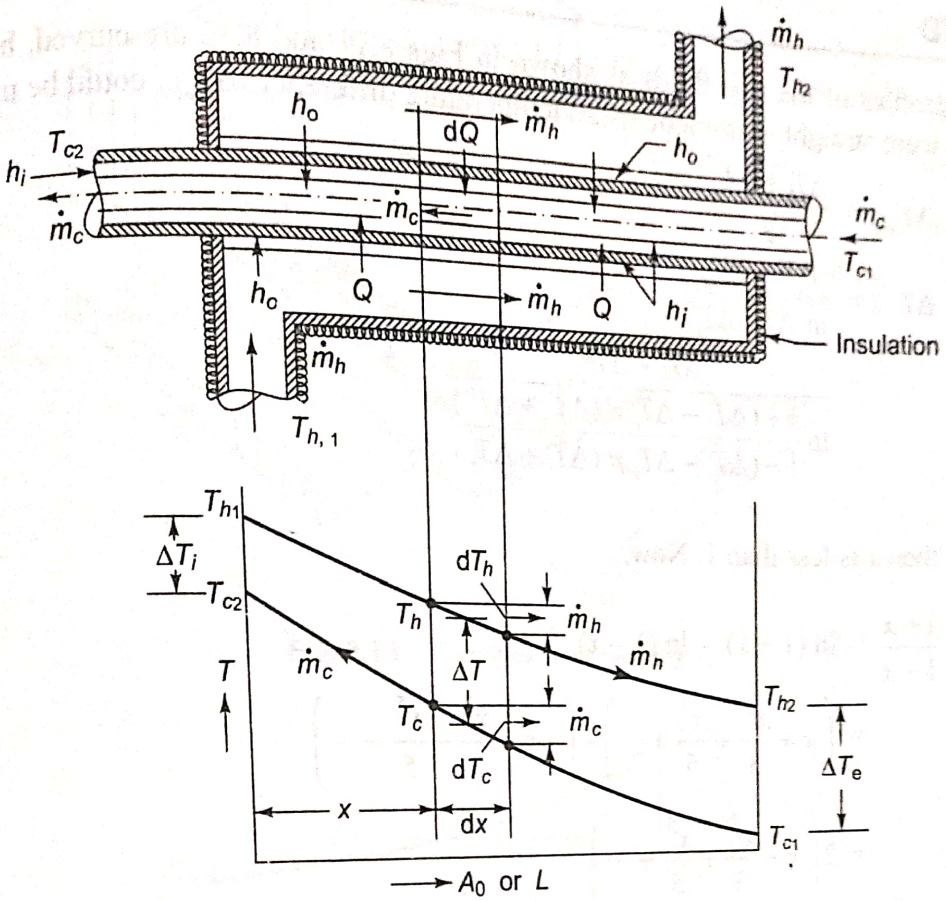

Fig. 8.12 *Counterflow heat exchanger*

Since μ_c is constant, by integrating over the whole surface,

$$\Delta T_i - \Delta T_e = \mu_c Q \tag{8.16}$$

Again, $$dQ = U_0\, dA_0\, \Delta T$$

Using Eq. (8.14),

$$-\frac{d(\Delta T)}{\mu_c} = U_0\, dA_0\, \Delta T$$

or, $$\int_i^e -\frac{d(\Delta T)}{\Delta T} = \int_i^e U_0\, dA_0\, \mu_c$$

With U_0 being assumed constant,

$$\ln \frac{\Delta T_i}{\Delta T_e} = U_0 A_0 \mu_c \tag{8.17}$$

From Eqs (8.16) and (8.17),

$$Q = U_0 A_0 (\Delta T)_{lm} \tag{8.18}$$

$$= \dot m_h c_h (T_{h_1} - T_{h_2})$$

$$= \dot m_c c_c (T_{c_2} - T_{c_1}) \tag{8.18a}$$

where $$T_{1m} = \frac{\Delta T_i - \Delta T_e}{\ln \Delta T_i / \Delta T_e} = \text{LMTD}$$

$$\Delta T_i = T_{h_1} - T_{c_2} \text{ and } \Delta T_e = T_{h_2} - T_{c_1}$$

8.6 USE OF LMTD

The temperature profiles of the two fluids as shown in Figs 8.11 and 8.12 are curved, having logarithmic variations. If they were straight, arithmetic mean temperature difference, ΔT_{am}, could be used, where

$$\Delta T_{\text{am}} = \frac{\Delta T_i + \Delta T_e}{2}$$

Now,
$$\Delta T_{\text{lm}} = \frac{\Delta T_i - \Delta T_e}{\ln \Delta T_i / \Delta T_e}$$

$$= \frac{\Delta T_i - \Delta T_e}{\ln \dfrac{1 + (\Delta T_i - \Delta T_e)/(\Delta T_i + \Delta T_e)}{1 - (\Delta T_i - \Delta T_e)/(\Delta T_i + \Delta T_e)}}$$

If $\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e} = x$, then x is less than 1. Now,

$$\ln \frac{1+x}{1-x} = \ln(1+x) - \ln(1-x)$$

$$= \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right) - \left(-x - \frac{x^3}{3} - \frac{x^5}{5} - \cdots\right)$$

$$= 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\right)$$

$$\Delta T_{\text{lm}} = \frac{\Delta T_i - \Delta T_e}{2\left[\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e} + \dfrac{1}{3}\left(\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e}\right)^3 + \dfrac{1}{5}\left(\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e}\right)^5 + \cdots\right]}$$

$$= \frac{\Delta T_i - \Delta T_e}{2\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e}\left[1 + \dfrac{1}{3}\left(\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e}\right)^2 + \cdots\right]}$$

$$= \frac{\Delta T_{\text{am}}}{1 + \dfrac{1}{3}\left(\dfrac{\Delta T_i - \Delta T_e}{\Delta T_i + \Delta T_e}\right)^2 + \cdots} \quad (8.19)$$

If $\Delta T_i \lessgtr \Delta T_e$,

$$\Delta T_{\text{lm}} < \Delta T_{\text{am}} \quad (8.20)$$

LMTD is thus less than the arithmetic mean temperature difference. It is always safer for the designer to use LMTD so as to provide larger heating surfaces for a certain amount of heat transfer.

If
$$\Delta T_i = \Delta T_e, \; (\Delta T)_{\text{lm}} = (\Delta T)_{\text{am}}$$

8.7 CROSS-FLOW HEAT EXCHANGER

A cross-flow single pass heat exchanger with plate fins and both the fluids unmixed is shown in Fig. 8.13. Since the outlet temperatures of the two fluids are not uniform over the entire cross-section (Fig. 8.14), the calculation of the mean temperature difference is considerably more difficult.

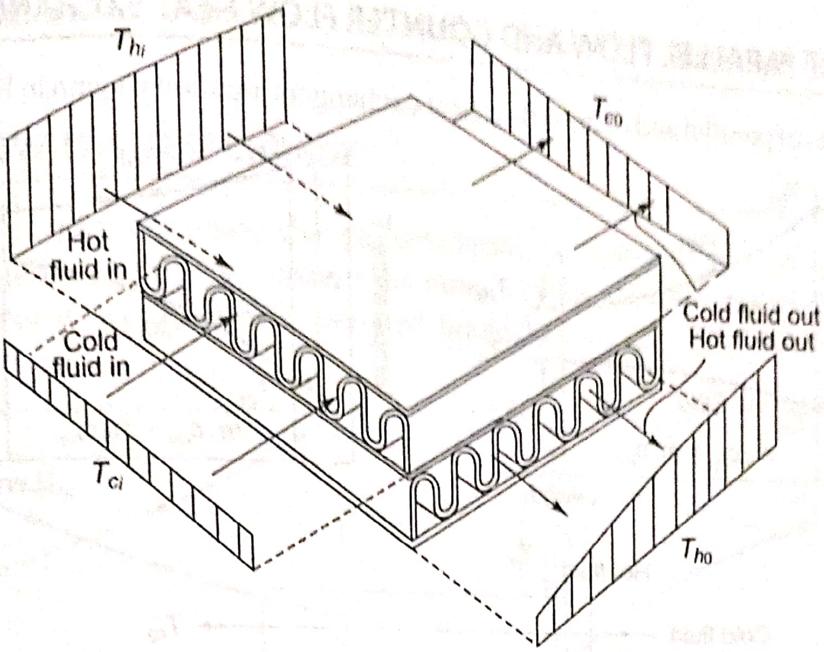

Fig. 8.13 *Cross-flow heat exchanger*

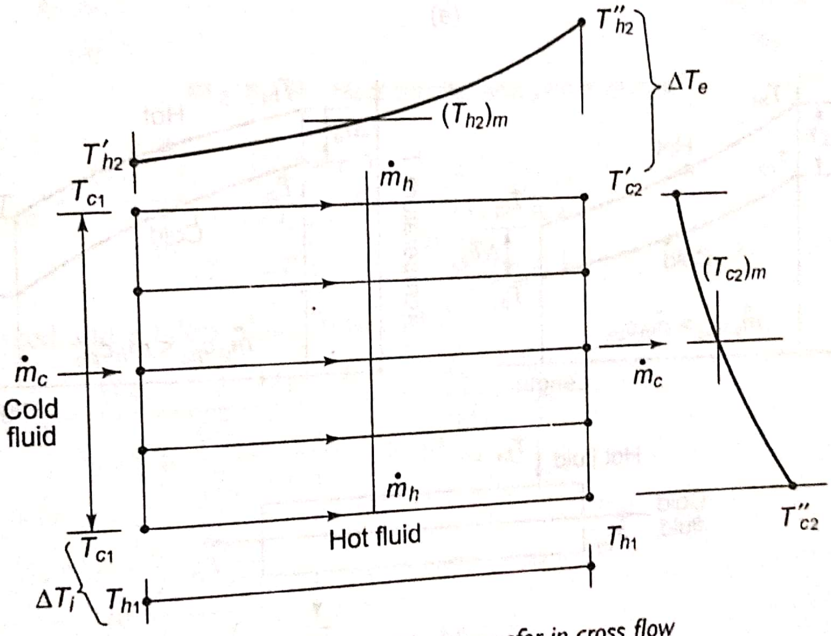

Fig. 8.14 *Overall heat transfer in cross flow*

Such a calculation was carried out by Nusselt [1]. If $(T_{h2})_m$ and $(T_{c2})_m$ represent the average hot fluid and cold fluid temperatures at exit respectively, then

$$\Delta T_e = (T_{h_2})_m - (T_{c_2})_m$$

whereas

$$\Delta T_i = T_{h_1} - T_{c_1}$$

as shown in Fig. 8.14. Then,

$$\Delta T_{\text{lm}} = \frac{\Delta T_i - \Delta T_e}{\ln \Delta T_i / \Delta T_e} \tag{8.21}$$

and

$$Q = U_0 A_0 \, \Delta T_{\text{lm}} = \dot{m}_h c_h \, [T_{h_1} - (T_{h_2})_m]$$
$$= \dot{m}_c c_c \, [(T_{c_2})_m - T_{c_1}]$$

8.8 COMPARISON OF PARALLEL FLOW AND COUNTER FLOW HEAT EXCHANGERS

The temperature profiles of parallel and counterflow heat exchangers are shown again in Fig. 8.15 for comparison.

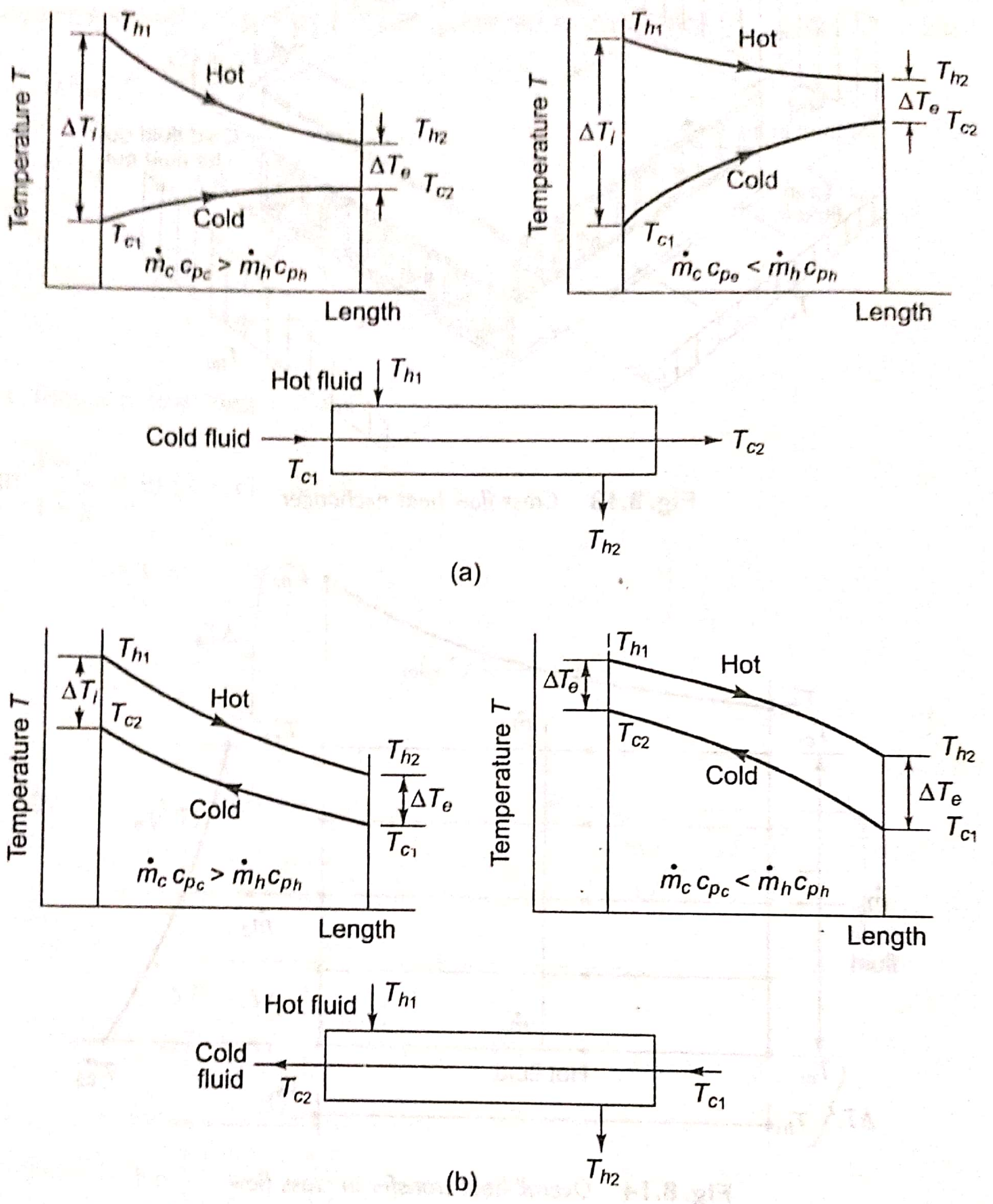

Fig. 8.15 *Comparison of parallel flow and counterflow heat exchangers*

For the same inlet and exit temperatures of the two fluids, it is found that $(\Delta T)_{lm}$ for counterflow is always greater than that for parallel flow. Since

$$Q = U_0 A_0 (\Delta T)_{lm}$$

for the same heat transfer Q and the same overall heat transfer coefficient U_0, the surface area required for counterflow operation is always less than that for parallel flow operation.

In a parallel flow heat exchanger, $T_{h_2} > T_{c_2}$, i.e. the hot fluid cannot be cooled to a temperature lower than the cold fluid temperature. In a counterflow heat exchanger, T_{h_2} can be less than T_{c_2} or T_{c_2} can be higher than T_{h_2}, i.e. the hot fluid can be cooled below T_{c_2} or the cold fluid can be heated above T_{h_2}.

Counterflow heat exchangers are, therefore, normally more common in industrial practice. For a given rate of heat flow and at given initial and final temperatures, while the smallest heating surface is required

$$= \frac{1}{a}\left[\ln x - b\frac{\ln(a+bx)}{b}\right]_{x_1}^{x_2}$$

$$= \frac{1}{a}\left[\ln\frac{x}{a+bx}\right]_{x_1}^{x_2} = \frac{1}{a}\left[\ln\frac{\Delta T}{a+b\Delta T}\right]_{\Delta T_i}^{\Delta T_e}$$

$$= \frac{1}{a}\ln\frac{\Delta T_e(a+b\Delta T_i)}{(a+b\Delta T_e)\Delta T_i} \tag{8.27}$$

Let $U_i = a + b\Delta T_i$ and $U_e = a + b\Delta T_e$, so that,

$$a = \frac{U_i\Delta T_e - U_e\Delta T_i}{\Delta T_e - \Delta T_i}$$

$$b = \frac{U_i - U_e}{\Delta T_i - \Delta T_e}$$

From Eqs (8.26) and (8.27),

$$\frac{\Delta T_e - \Delta T_i}{U_i\Delta T_e - U_e\Delta T_i}\ln\frac{\Delta T_e U_i}{\Delta T_i U_e} = \frac{\Delta T_e - \Delta T_i}{Q}A_0$$

$$\mathbf{Q} = \mathbf{A_0}\,\frac{\mathbf{U_i\Delta T_e - U_e\Delta T_i}}{\ln\dfrac{\mathbf{U_i\Delta T_e}}{\mathbf{U_e\Delta T_i}}} \tag{8.28}$$

This equation would then replace Eqs (8.18) and (8.18a).

8.12 EFFECTIVENESS—NTU METHOD

In heat exchanger calculations, 10 quantities are involved, viz. $\dot{m}_h$, c_h, $\dot{m}_c$, c_c, T_{h_1}, T_{h_2}, T_{c_1}, T_{c_2}, U_0 and A_0.

For designing the heat exchanger, $\dot{m}_h$, c_h, $\dot{m}_c$, c_c, T_{h_1}, T_{c_1}, T_{h_2} (or T_{c_2}) and U_0 are given, and from the equations

$$Q = \dot{m}_h c_h (T_{h_1} - T_{h_2}) = \dot{m}_c c_c (T_{c_2} - T_{c_1})$$
$$= U_0 A_0 (\Delta T)_{\text{lm}} \tag{8.18}$$

T_{c_2} (or T_{h_2}), ΔT_{lm} and then A_0 are estimated.

But for a given heat exchanger with specified flow rates and inlet fluid temperatures, both of which may be different from the values with which the heat exchanger was designed, one is often required to know the exit temperatures of the fluids. In this case, $\dot{m}_h$, c_h, $\dot{m}_c$, c_c, T_{h_1}, T_{c_1}, U_0 and A_0 are known, and T_{c_2} and T_{h_2} have to be estimated. To obtain the solution of this problem by the LMTD method, a *trial-and-error approach* has to be attempted.

Let us first assume a value for T_{h_2}. Using Eq. (8.18), we find Q, T_{c_2} and ΔT_{lm}. Then we estimate $Q' = U_0 A_0 \Delta T_{\text{lm}}$. If $Q = Q'$, the assumption of T_{h_2} was correct. If Q' is different from Q, a fresh value of T_{h_2} is assumed and the calculations are repeated till the condition $Q = Q'$ is achieved.

The method is quite tedious and can be avoided by following an alternative direct method called the *effectiveness-NTU method*, as discussed below.

The term effectiveness ε of a heat exchanger is defined as

Effectiveness, $\varepsilon = \dfrac{\text{Actual rate of heat transfer}}{\text{Maximum possible rate of heat transfer}}$

or, $$\varepsilon = \frac{Q}{Q_{max}} = \frac{\dot{m}_h c_h (T_{h_1} - T_{h_2})}{(\dot{m}c)_s (T_{h_1} - T_{c_1})}$$

$$= \frac{\dot{m}_c c_c (T_{c_2} - T_{c_1})}{(\dot{m}c)_s (T_{h_1} - T_{c_1})} \tag{8.29}$$

where subscript "s" denotes the smaller of the two heat capacity rates $\dot{m}_h c_h$ and $\dot{m}_c c_c$, or C_{min}. The maximum possible heat transfer depends on one of the fluids undergoing the maximum possible change in temperature and that will be the fluid which will have the minimum value of heat capacity rate.

If we allow the fluid with the larger value of $\dot{m}_c$ (or C_{max}) go through the maximum temperature difference, then by energy balance,

$$(\dot{m}c)_{max} (T_{h_1} - T_{c_1}) = \dot{m}_c c_c (T_{c_2} - T_{c_1})$$

Therefore $(T_{c_2} - T_{c_1})$ becomes greater than $(T_{h_1} - T_{c_1})$, which is impossible.

Since, $Q = \dot{m}_h c_h (T_{h_1} - T_{h_2}) = \dot{m} c_c (T_{c_2} - T_{c_1})$,

(i) if $\dot{m}_h c_h < \dot{m}_c c_c$ (Fig. 8.22)

$$(T_{h_1} - T_{h_2}) > (T_{c_2} - T_{c_1})$$

$$\varepsilon = \frac{\dot{m}_h c_h (T_{h_1} - T_{h_2})}{\dot{m}_h c_h (T_{h_1} - T_{c_1})} = \frac{T_{h_1} - T_{h_2}}{T_{h_1} - T_{c_1}} \tag{8.30}$$

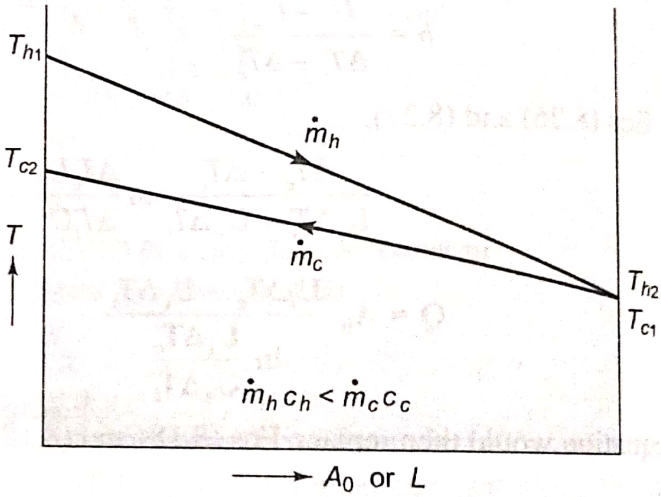

Fig. 8.22 *For $\dot{m}_h c_h < \dot{m}_c c_c$, the hot fluid is the reference fluid for heat exchanger effectiveness*

(ii) if $\dot{m}_c c_c < \dot{m}_h c_h$ (Fig. 8.23)

$$(T_{c_2} - T_{c_1}) > (T_{h_1} - T_{h_2})$$

$$\varepsilon = \frac{\dot{m}_c c_c (T_{c_2} - T_{c_1})}{\dot{m}_c c_c (T_{h_1} - T_{c_1})} = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}} \tag{8.31}$$

Thus, the effectiveness can also be defined as

$$\varepsilon = \frac{(\Delta T)_1}{(\Delta T)_{maximum}} \tag{8.32}$$

where $(\Delta T)_1$ is the larger of the two temperature differences $(T_{h_1} - T_{h_2})$ and $(T_{c_2} - T_{c_1})$, and $(\Delta T)_{max}$ is the maximum possible temperature rise or fall, which is $(T_{h_1} - T_{c_1})$.

The *heat capacity ratio R* is defined as

$$R = \frac{(\dot{m}c)_s}{(\dot{m}c)_l} = \frac{C_{min}}{C_{max}} \tag{8.33}$$

where the subscripts "s" and "l" refer to the smaller and the larger of the two values of $\dot{m}_h c_h$ and $\dot{m}_c c_c$. If $\dot{m}_h c_h < \dot{m}_c c_c$,

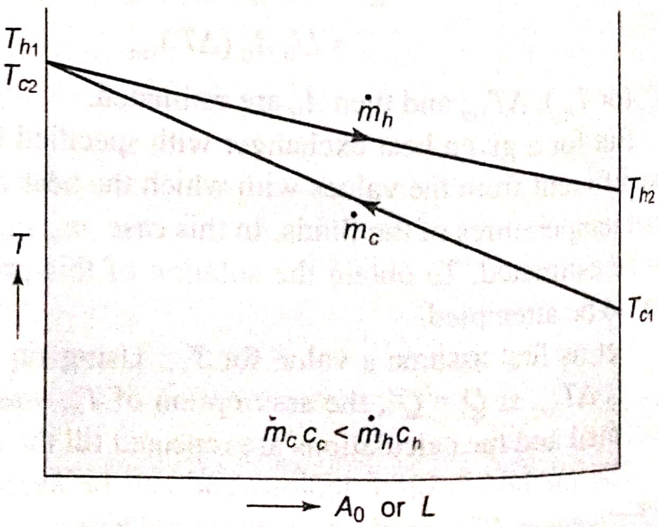

Fig. 8.23 *For $\dot{m}_c c_c < \dot{m}_h c_h$, the cold fluid is the reference fluid for heat exchanger effectiveness*

$$R = \frac{\dot{m}_h c_h}{\dot{m}_c c_c} = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{h_2}} \tag{8.33a}$$

The values of both ε and R vary between 0 and 1.

8.12.1 Parallel Flow Arrangement

From Eq. (8.11),

$$\ln \frac{\Delta T_i}{\Delta T_e} = U_0 A_0 \mu_p$$

$$\frac{\Delta T_e}{\Delta T_i} = e^{-U_0 A_0 \mu_p}$$

or,

$$1 - \frac{\Delta T_e}{\Delta T_i} = 1 - e^{-U_0 A_0 \mu_p} \tag{8.34}$$

For a parallel flow heat exchanger (Fig. 8.24),

$$1 - \frac{T_{h_2} - T_{c_2}}{T_{h_1} - T_{c_1}} = 1 - e^{-U_0 A_0 \mu_p}$$

or,

$$\frac{T_{h_1} - T_{c_1} - T_{h_2} + T_{c_2}}{T_{h_1} - T_{c_1}} = 1 - e^{-U_0 A_0 \mu_p} \tag{8.35}$$

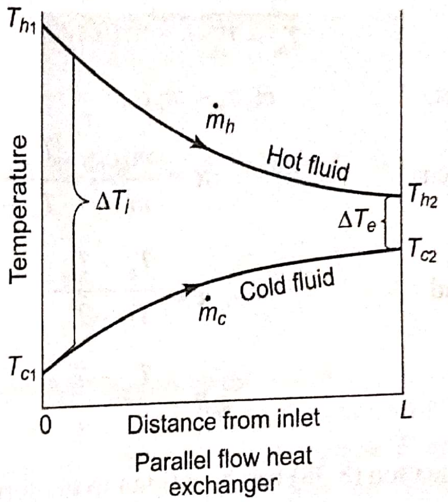

Fig. 8.24 *Temperature profiles in a parallel flow heat exchanger*

Let $\dot{m}_h c_h < \dot{m}_c c_c$,

or, $(T_{h_1} - T_{h_2}) > (T_{c_2} - T_{c_1})$

From Eqs (8.30) and (8.32),

$$\varepsilon = \frac{T_{h_1} - T_{h_2}}{T_{h_1} - T_{c_1}}$$

and from Eq. 8.33,

$$R = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{h_2}} = \frac{\dot{m}_h c_h}{\dot{m}_c c_c}$$

Equation (8.35) can be written in the following form,

$$\frac{(T_{h_1} - T_{h_2}) + (T_{c_2} - T_{c_1})}{(T_{h_1} - T_{h_2})/\varepsilon} = 1 - e^{-U_0 A_0 \mu_p}$$

or,

$$\varepsilon\left(1 + \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{h_2}}\right) = 1 - \exp\left[-U_0 A_0 \left(\frac{1}{\dot{m}_h c_h} + \frac{1}{\dot{m}_c c_c}\right)\right]$$

or,

$$\varepsilon(1 + R) = 1 - \exp\left[-\frac{U_0 A_0}{\dot{m}_h c_h}\left(1 + \frac{\dot{m}_h c_h}{\dot{m}_c c_c}\right)\right] \tag{8.36}$$

$$\boldsymbol{\varepsilon_{pf} = \frac{1 - \exp\left[-NTU\,(1 + R)\right]}{1 + R}}$$

where NTU denotes the *number of transfer units* defined by

$$\mathrm{NTU} = \frac{\mathrm{U_0 A_0}}{(\dot{m}\mathrm{c})_s} = \frac{\mathrm{U_0 A_0}}{\mathrm{C_{min}}} \tag{8.37}$$

$(\dot{m}c)_s$ or $C_{\min}$ being equal to $\dot{m}_h c_h$ in this case. NTU gives a measure of the size of the heat exchanger.

8.12.2 Counterflow Arrangement

From Eq. (8.17),

$$\ln \frac{\Delta T_i}{\Delta T_e} = U_0 A_0 \mu_c$$

or,

$$\frac{\Delta T_e}{\Delta T_i} = e^{-U_0 A_0 \mu_c}$$

For a counterflow heat exchanger, (Fig. 8.25)

$$\frac{T_{h_2} - T_{c_1}}{T_{h_1} - T_{c_2}} = e^{-U_0 A_0 \mu_c} \tag{8.38}$$

Let,

$$\dot{m}_h c_h < \dot{m}_c c_c$$

then

$$R = \frac{\dot{m}_h c_h}{\dot{m}_c c_c} = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{h_2}}$$

and

$$\varepsilon = \frac{T_{h_1} - T_{h_2}}{T_{h_1} - T_{c_1}}$$

$$\varepsilon R = \frac{T_{c_2} - T_{c_1}}{T_{h_1} - T_{c_1}} \tag{8.39}$$

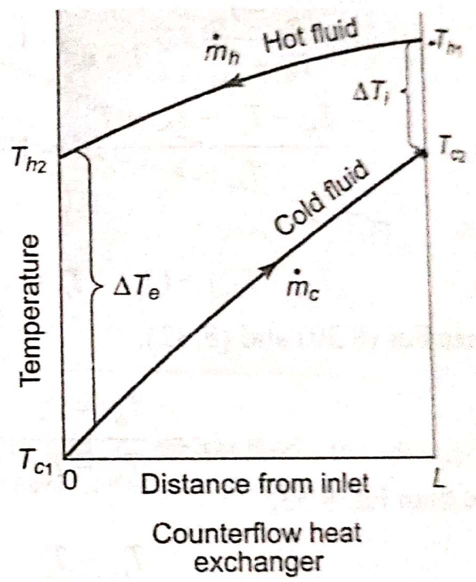

Fig. 8.25 *Temperature profiles in a counter flow heat exchanger*

Equation (8.38) can be written in the form

$$\frac{(T_{h_1} - T_{c_1}) - (T_{h_1} - T_{h_2})}{(T_{h_1} - T_{c_1}) - (T_{c_2} - T_{c_1})} = \exp\left[-U_0 A_0 \left(\frac{1}{\dot{m}_h c_h} - \frac{1}{\dot{m}_c c_c}\right)\right]$$

or,

$$\frac{1 - \dfrac{(T_{h_1} - T_{h_2})}{(T_{h_1} - T_{c_1})}}{1 - \dfrac{(T_{c_2} - T_{c_1})}{(T_{h_1} - T_{c_1})}} = \exp\left[-\frac{U_0 A_0}{\dot{m}_h c_h}\left(1 - \frac{\dot{m}_h c_h}{\dot{m}_c c_c}\right)\right]$$

or,

$$\frac{1 - \varepsilon}{1 - \varepsilon R} = \exp\left[-\mathrm{NTU}\,(1 - R)\right] \tag{8.40}$$

Let the RHS of Eq. (8.40) be equal to K.

$$\frac{1 - \varepsilon}{1 - \varepsilon R} = K$$

or,

$$K - \varepsilon R K = 1 - \varepsilon$$

$$\varepsilon(1 - RK) = 1 - K$$

$$\varepsilon = \frac{1 - K}{1 - RK}$$