where $E_{b\lambda}$ is the monochromatic emissive power of the black surface at the wavelength, λ.

9.3 ABSORPTION, REFLECTION AND TRANSMISSION

When incident radiation (also called irradiation) impinges on a surface, three things happen: a part is reflected back, a part is transmitted through and the remainder is absorbed as shown in Fig. 9.2. If Q be the rate at which a surface receives heat and of this amount Q_ρ is reflected, Q_τ transmitted and Q_α absorbed, then by the principle of conservation of energy, the total sum must be equal to the incident radiation, *i.e.*, reflection + transmission + absorption = incident radiation.

$$Q_\rho + Q_\tau + Q_\alpha = Q \text{ or } \frac{Q_\rho}{Q} + \frac{Q_\tau}{Q} + \frac{Q_\alpha}{Q} = 1$$

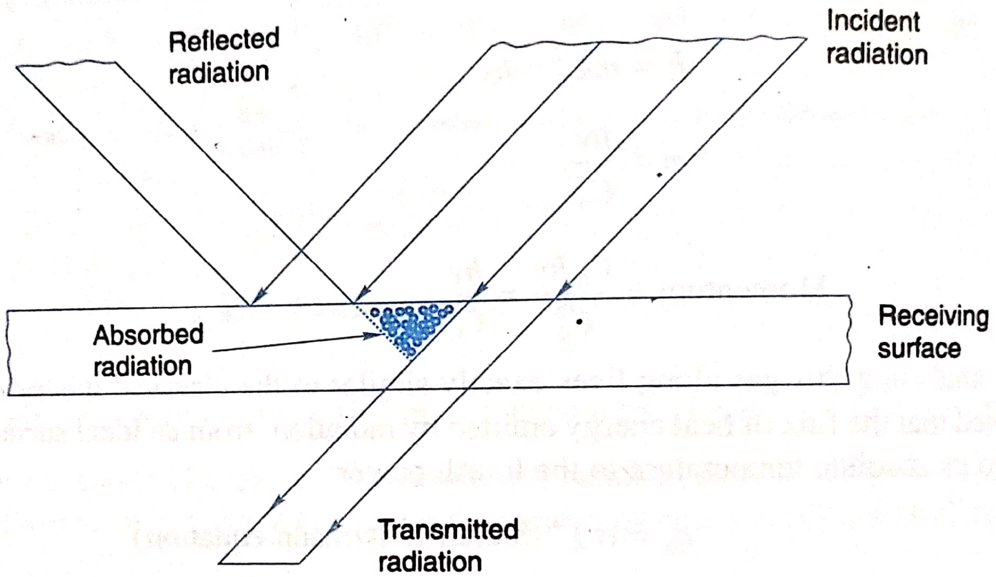

Fig. 9.2 Radiation Incident on a Surface

or

$$\rho + \tau + \alpha = 1 \tag{9.4}$$

where $\rho = \dfrac{Q_\rho}{Q}$ is the fraction of incident radiation reflected and is called the *reflectivity*. Similarly *transmissivity*, $\tau = \dfrac{Q_\tau}{Q}$ is defined as the fraction transmitted, and the absorptivity, $\alpha = \dfrac{Q_\alpha}{Q}$ is the fraction absorbed.

Equation (9.4) holds for surfaces or layers of finite thickness. The following points should be noted about ρ, τ and α:

(*i*) They are always positive and their values lie between the limits 0 and 1, *i.e.*, $0 \le \rho, \tau, \alpha \le 1$.

(*ii*) $\rho = 0$ (*i.e.*, $\tau + \alpha = 1$) represents a *non-reflecting* surface; $\rho = 1$ (*i.e.*, $\alpha = \tau = 0$) represents a *perfect reflector*, *i.e.*, it reflects all the incident radiation and does not absorb or transmit any part of it.

(iii) $\tau = 0 (i.e., \rho + \alpha = 1)$ represents an *opaque* surface;
$\tau = 1 (i.e., \rho = \alpha = 0)$ represents a *perfectly transparent* surface.

(iv) $\alpha = 0 (i.e., \rho + \tau = 1)$ represents a *non-absorbing* surface (also called a *white* surface);
$\alpha = 1 (i.e., \rho = \tau = 0)$ represents a *perfectly absorbing* surface (also called a *black* surface, if it is diffuse).

The definitions of ρ, τ and α given above are with respect to the total values of Q, integrated with respect to the area of the surface, the solid angle in the hemispherical space above it and all the wavelengths of the spectrum.

We can also define monochromatic and directional values of ρ, τ and α by taking the corresponding values of Q_ρ, Q_τ and Q_α

$$\rho_\lambda = \frac{Q_{\rho_\lambda}}{Q_\lambda}; \tau_\lambda = \frac{Q_{\tau_\lambda}}{Q_\lambda}; \alpha_\lambda = \frac{Q_{\alpha_\lambda}}{Q_\lambda} \tag{9.5}$$

where Q_λ now represents the total heat flux per unit area received at the point at that wavelength and ρ_λ is the monochromatic reflectivity or the fraction of incident energy in the wavelength range λ to $\lambda + d\lambda$, which is reflected.

Since

$$Q_\rho = \int_0^\infty (Q_{\rho_\lambda}) d\lambda = \int_0^\infty \rho_\lambda Q_\lambda \, d\lambda \tag{9.6}$$

$$\therefore \quad \rho = \frac{Q_\rho}{Q} = \frac{1}{Q} \int_0^\infty \rho_\lambda Q_\lambda \, d\lambda \tag{9.7}$$

We can derive equations similar to Eqn. (9.7) for monochromatic absorptivity, α_λ, and monochromatic transmissivity, τ_λ, as well.

Solids generally do not transmit unless the material is of very thin section. Metals absorb radiation within a fraction of a micrometre, and insulators within a fraction of a millimetre. Glasses and liquids absorb most of the radiation within a millimetre. Solids and liquids are therefore generally considered as opaque.

Gases such as hydrogen, oxygen and nitrogen (and their mixtures such as air) have a transmissivity of practically unity. The radiation transfer through air is estimated using the relationships for radiation through a vacuum.

Two types of reflection phenomena—*specular* and *diffuse* are observed when radiation strikes a solid surface. Specular reflection occurs from a surface such as a mirror, which is very smooth. An image of the source of radiation is projected; the angle of reflection is equal to the angle of incidence. Diffuse reflection occurs when the surface is rough and the reflection from the surface occurs practically indiscriminately in all diffractions. These two types of reflection are shown in Fig. 9.3. No actual body is perfectly specular or diffuse but it is often useful to approximate to one of these ideal surfaces.

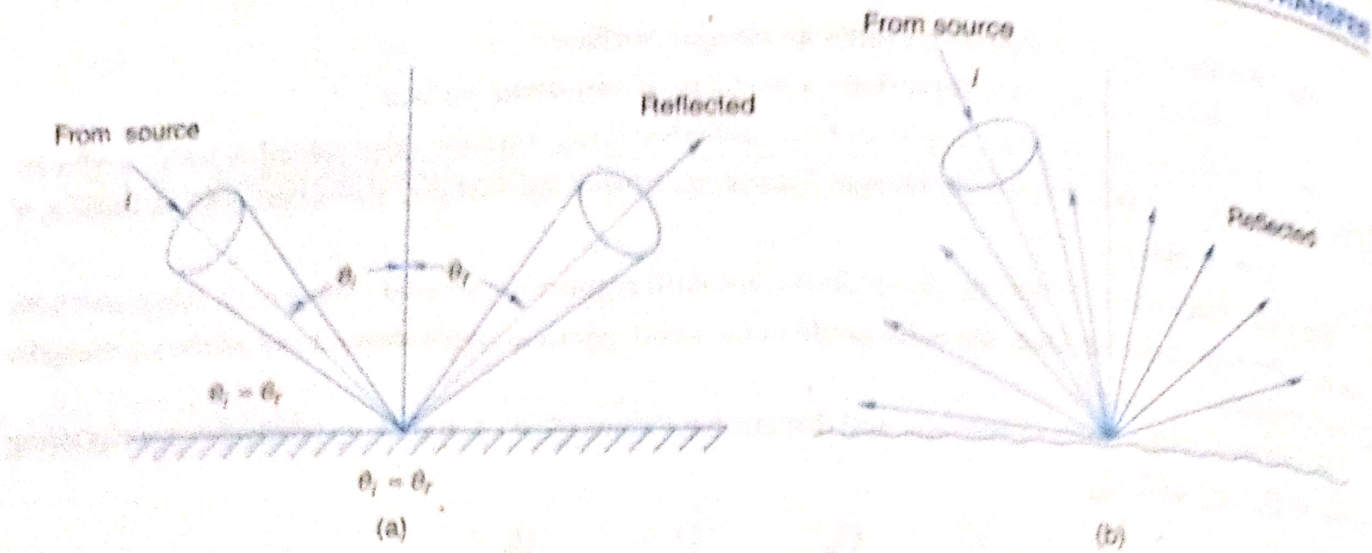

Fig. 9.3 Reflection: (*a*) Specular, (*b*) Diffuse

9.4 CONCEPT OF A BLACK BODY

A black body is an ideal body that absorbs all incident energy and reflects or transmits none. This is true of radiation for all wavelengths and for all angles of incidence for a black body, therefore, $\rho = 0$, $\tau = 0$ and $\alpha = 1$. We call it black because the materials which obey this law appear black to the eye. It should be noted that surfaces which are nearly black for radiation purposes, are not necessarily black to visible light because the visible wavelength range is only a small part of the thermal radiation range. Snow and ice or white paper are quite bright to the eye but are nearly radiation black with an absorptivity of 0.97–0.98.

No actual body is perfectly black; the concept of a black body is an idealisation with which the radiation characteristics of real bodies can be conveniently compared. A black body plays a role in thermal radiation similar to the idealized Carnot cycle in thermodynamics with which real cycles are compared.

A black body is regarded as a perfect absorber of incident radiation. A black body condition can be approached in practice by forming a cavity in a material as shown in Fig. 9.4. Radiation passing through the hole into the cavity is repeatedly absorbed and reflected at the cavity walls until it is all absorbed. The cavity itself may be made of a shiny material but will appear black when one looks in through the opening.

A black body is a perfect emitter. This is a fact which can be proved as follows. Consider a black body at a uniform temperature, placed inside an arbitrarily shaped, perfectly insulated enclosure composed of another black body whose temperature is also uniform but different from that of the former (Fig. 9.5). The black body and the enclosure will reach a common equilibrium temperature after a period of time (due to heat transfer) when the black body will radiate as much energy as it absorbs; if this were not true, its temperature would change—a direct violation of the second law of thermodynamics. Because the black body is by definition absorbing the maximum possible radiation from the enclosure at each wavelength and from each direction, it must also be emitting the maximum total amount of radiation. An interesting point to note here is that a black body continues to emit radiation even when it is in thermal equilibrium with its surroundings.

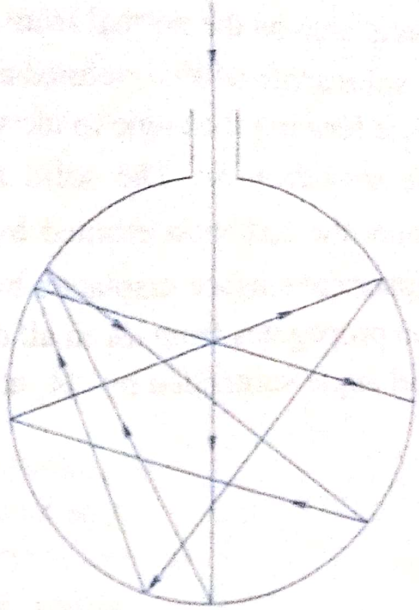

Fig. 9.4 Cavity Acting Like a Black Surface

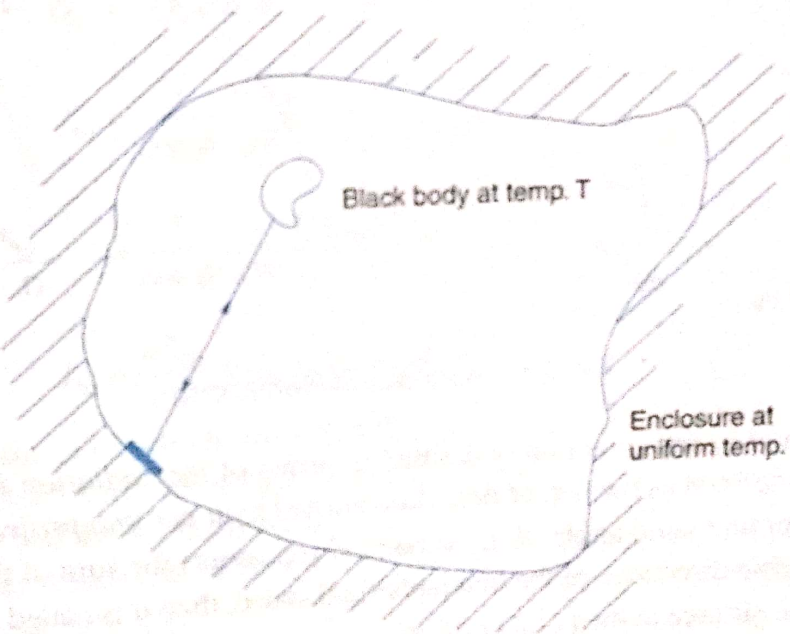

Fig. 9.5 Black Body in an Isothermal Enclosure

The total radiation emitted by a black body is a function of temperature only. If the temperature of the enclosure in Fig. 9.5 is now changed to a different uniform value, the black body will adjust its temperature until it is in thermal equilibrium with the enclosure. Then it will absorb and emit the maximum possible energy characteristic of its new temperature. Obviously, the rate of absorption and emission increases with the rise in temperature.

9.5 INTENSITY OF RADIATION

A black body emitting radiation in all directions can be visualised by considering an elementary area, dA, of Fig. 9.6; the radiation will be wholly intercepted by the hemisphere of radius, R, in whose base dA

lies. If dA_1 is a corresponding elemental area on the normal from dA to the hemisphere, the solid angle it subtends at dA, dW, is dA_1/R^2. The solid angle (dW) is defined as the ratio of the area of the surface of a sphere enclosed by the conical surface forming the angle to the square of the radius of the sphere, i.e., $dW = dA_1/R^2$. The unit used is the *steradian Sr*. The solid angle of a hemisphere surrounding dA is $2\pi R^2/R^2 = 2\pi$ Sr. Consider now the radiation emitted by dA which reaches a point P on an elementary area, dA_p, on the hemispherical surface displaced by an angle ϕ from the normal to the surface. Obviously, although radiation propagates from dA in all directions over the entire hemisphere, only a part which falls within the solid angle subtended by dA_p at dA will reach dA_p.

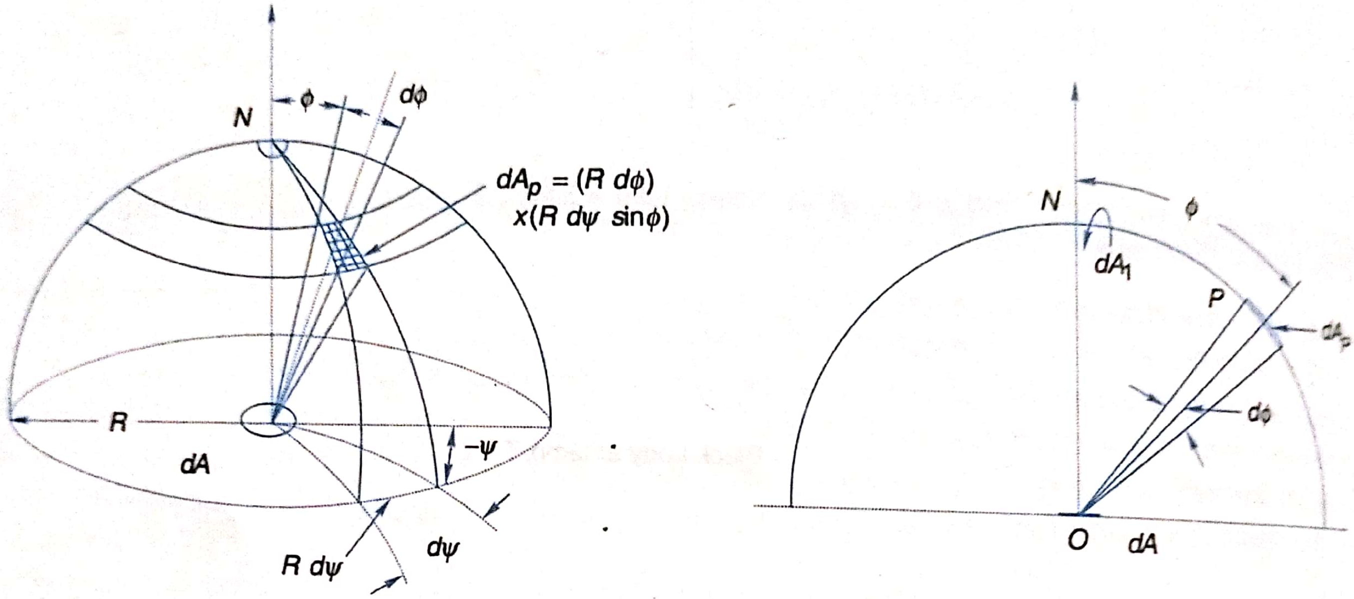

Fig. 9.6 Intensity of Radiation

The radiation emitted in any direction is defined in terms of the *radiation intensity*. The radiation intensity of a surface is defined as the rate of heat flux emitted by *it per unit projected area* normal to the direction of radiation per unit solid angle. If E_b at point O represents the sum of the total energy per unit area radiated in all possible directions of the hemispherical space, then it is called the total *hemispherical* or *emissive power* of the surface at the point O. If $dE_{b\phi}$ is the emissive power due to the energy radiated in solid angle, dW, in a particular direction ϕ, then the radiation intensity of the surface at O in that direction is

$$I_{b\phi} = \frac{dQ_b}{dA\cos\phi\, dW} = \frac{1}{\cos\phi}\left(\frac{dE_{b\phi}}{dW}\right) \tag{9.8}$$

The reason for $\cos\phi$ coming into the above equation is that E_b is defined per unit area whereas $I_{b\phi}$ is defined per unit projected area. Equation (9.8) can be written in the integral form as

$$E_b = \int I_{b\phi}\cos\phi\, dW \tag{9.9}$$

There are two types of intensities. (1) *The spectral radiation intensity*, $I_{b\lambda}$, refers to radiation in an interval $d\lambda$ around a single wavelength propagating in a given direction at a position r. It represents the amount of energy streaming through a unit area projected normal to a given direction per unit time per

$$C_3 = 0.289 \times 10^{-2}\ mK.$$

Alternately the value of $\lambda_{max} \cdot T$ can be found at the peak of the distribution curve given in Fig. 9.8 in which we also see that for a given value of λT, the ratio of $\frac{E_{b\lambda}}{T^5}$ is the same for all temperatures.

Another form of Wien's law, therefore, is :

$$\frac{E_{b\lambda\,max}}{T^5} = \text{const. or } E_{b\lambda\,max} = C_4 T^5 \tag{9.24}$$

where $C_4 = 1.307 \times 10^{-5}\ \text{W/m}^2\text{K}^5$.

9.6.5 Stefan-Boltzmann Law

It follows from the definition of total radiation intensity that

$$I_b = \int_0^{\infty} I_{b\lambda}\, d\lambda$$

This integral can be evaluated by substituting the value of $I_{b\lambda}$ from Eqn. (9.17)

$$I_b = \int_0^{\infty} \frac{2C_1}{\lambda^5\left[\exp\left(C_2/\lambda T\right) - 1\right]}\, d\lambda$$

Now let $C_2/\lambda T = y$

Then
$$I_b = \frac{2C_1\, T^4}{C_2^4} \int_0^{\infty} \frac{y^3}{e^y - 1}\, dy \tag{9.25}$$

which can be evaluated as

$$\frac{2C_1 T^4}{C_2^4} \int_0^{\infty} y^3 \left\{e^{-y} + e^{-2y} + e^{-3y} + \ldots\right\} dy$$

Now
$$\int_0^{\infty} y^3 e^{-ny}\, dy = \frac{3!}{n^{(3+1)}} = \frac{3!}{n^4}$$

$\therefore$
$$I_b = \frac{2C_1 T^4}{C_2^4} \left\{\frac{3!}{1^4} + \frac{3!}{2^4} + \frac{3!}{3^4} + \ldots\right\} = \frac{2C_1 T^4}{C_2^4} \left(\frac{6\pi^4}{90}\right)$$

$$I_b = \frac{2C_1 T^4}{C_2^4} \cdot \frac{\pi^4}{15} = \frac{\sigma}{\pi} \cdot T^4 \tag{9.26}$$

where the constant
$$\sigma = \frac{2C_1 T^5}{15 C_2^4} = 0.567 \times 10^{-7}\ \text{W/m}^2\text{K}^4$$

$$\tau_1 = 0 \quad \text{for} \quad \lambda_0 = 0 \quad \text{to} \quad \lambda_1 = 0.4\mu$$
$$\tau_2 = 0.8 \quad \text{for} \quad \lambda_1 = 0.4\mu \quad \text{to} \quad \lambda_2 = 3.0\mu$$
$$\tau_3 = 0 \quad \text{for} \quad \lambda_2 > 3.0\mu$$

$$\lambda_0 T = 0$$
$$\lambda_1 . T = 0.4 \times 5555 = 2222.0\ \mu\text{K}$$
$$\lambda_2 . T = 3 \times 5555 = 16665.0\ \mu\text{K}$$

From Table 9.2

$$F_{0-\lambda_0 T} = 0$$
$$F_{0-\lambda_1 T} = 0.10503$$
$$F_{0-\lambda_2 T} = 0.97644$$

$$\therefore \quad \bar{\tau} = \tau_1\left(F_{0-\lambda_1 T} - F_{0-\lambda_0 T}\right) + \tau_2\left(F_{0-\lambda_2 \cdot T} - F_{0-\lambda_1 \cdot T}\right) + \tau_3\left(F_{0-\lambda_3 \cdot T} - F_{0-\lambda_2 \cdot T}\right)$$
$$= 0.8\ (0.97644 - 0.10503) = 0.70.$$

Example 9.4

It is observed that the intensity of the radiation emitted by the sun is maximum at a wavelength of 0.5μ. Assuming the sun to be a black body, estimate its surface temperature and emissive power.

Solution

According to Wien's Displacement Law (Eqn. 9.23)

$$\lambda_{max} \cdot T = C_3 = 0.289 \times 10^{-2}\ \text{mK}$$

$$\therefore \quad T = \frac{0.289 \times 10^{-2}}{\lambda} = \frac{0.289 \times 10^{-2}}{0.50 \times 10^{-6}} = 5780\ \text{K}$$

Then using Stefan-Boltzmann Law (Eqn. 9.27)

$$(E_b)_{\text{sun}} = \sigma T^4 = 0.567 \times 10^{-7} \times (5780)^4$$
$$= 63.3\ \text{MW/m}^2.$$

9.7 RADIATION FROM NON-BLACK SURFACES—EMISSIVITY

The concept of a black body is an idealization which serves as a standard for real body performance. Most surfaces encountered in engineering applications do not behave like black bodies.

The *emissivity* of a surface is a measure of how it emits radiant energy in comparison with a black surface at the same temperature. The emissive power of an actual, surface is expressed as a proportion of E_b as

$$E \propto E_b$$
$$E = \varepsilon\ E_b$$

or emissivity,
$$\varepsilon = \frac{E}{E_b} \tag{9.31}$$

Thus emissivity of a surface is the ratio of the emissive power of the surface to the emissive power of a black surface at the same temperature. This then gives,

$$E = \varepsilon \, \sigma T^4 \tag{9.32}$$

The emissivity defined in Eqn. (9.31) is also called the *total* emissivity because it represents the integrated behaviour of the material over all wavelengths. In reality, the emissivity of a material varies with temperature and the wavelength of the radiation.

The *monochromatic emissivity of* a surface is the ratio of the monochromatic emissive power of the surface to the monochromatic emissive power of a black surface at the same temperature and wavelength.

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}} \tag{9.33}$$

Normal total emissivity, ε_n, is the ratio of the normal component of the total emissive power of a surface, E_n to the normal component of the total emissive power of a black body $(E_b)_n$ at the same temperature.

$$\varepsilon_n = E_n/(E_b)_n \tag{9.34}$$

A *gray* body is defined such that the monochromatic emissivity, ε_λ, of the body is independent of wavelength.

The emissive power of the body is related to the monochromatic emissivity by

$$E = \int_0^\infty \varepsilon_\lambda \, E_{b\lambda} \cdot d\lambda$$

and also by Eqn. (9.3),

$$E_b = \int_0^\infty E_{b\lambda} \, d\lambda = \sigma T^4$$

so that

$$\varepsilon = \frac{E}{E_b} = \frac{\int_0^\infty \varepsilon_\lambda \, E_{b\lambda} . d\lambda}{\sigma T^4} \tag{9.35}$$

for a gray body $(\varepsilon_\lambda = \text{const.})$ Eqn. (9.35) reduces to

$$\varepsilon = \varepsilon_\lambda = \text{constant.}$$

The emissivity of a surface is a function of its nature and characteristics, and is independent of the wavelength or the nature of the impinging radiation waves. Thus it is essentially a surface property. The absorptivity of a surface is not a surface property because of its dependence on the nature of the incident radiation. To illustrate this point let us consider a substance whose absorptivity is known to be 0.4 in the wavelength band 0.2 to 4μ. If this substance is receiving radiation from another body emitting in the range 5 to 100μ only, the substance will be unable to absorb any of the radiation falling on it. On the other hand, if the other body happens to be a black body emitting radiation at all wavelengths, then 40% of the incident radiation of wavelength band 0.2 to 4μ will be absorbed. This is the reason why the absorptivity of a substances at a given temperature is measured when the substance is in thermal equilibrium with a black body at the same temperature. The absorptivity thus measured is usually called the equilibrium absorptivity of the substance.

minator of the above formula can be taken to be

$$\lambda^5 \exp (C_2/\lambda T)$$

ting $E_{b\lambda}$ with E_λ

$$\frac{1}{T} = \frac{1}{T_b} - \frac{\lambda}{C_2} \ln \frac{1}{\varepsilon_\lambda}$$

$$T = \frac{1}{\frac{1}{T_b} - \frac{\lambda}{C_2} \ln \frac{1}{\varepsilon_\lambda}}$$

$$= \frac{1}{\frac{1}{1673} - \frac{0.65 \times 10^{-6}}{1.439 \times 10^{-2}} \ln \frac{1}{0.6}} = 1740 \text{ K}$$

The temperature of the body = 1740 – 273 = 1467°C.

9.8 KIRCHHOFF'S LAW

Kirchhoff's law establishes a relationship between the emissive power of a surface to its absorptivity. Suppose that a small body is placed inside a large evacuated enclosure, the walls of which are maintained at a temperature, T. Heat will be exchanged between the body and the enclosure until equilibrium is established *i.e.*, when both the body and the walls of the enclosure have reached the same temperature. Then the body will emit as much energy as it absorbs. If E is the emissive power of the body, α its absorptivity, and G the rate at which the energy falls from the walls on the body, then by energy balance in the equilibrium state,

$$\alpha G = E$$

or

$$\frac{E}{\alpha} = G \tag{9.36}$$

The irradiation, G, on the small body is a function of temperature, T, and the geometrical arrangement of the body and the enclosure. If the body is quite small in comparison with the enclosure, its effect upon the irradiation field of the enclosure may be considered to be negligible. In that case, no matter what kind of body it is, G will remain the same at temperature, T.

Thus

$$\frac{E}{\alpha} = f(T), \text{ for all bodies} \tag{9.37}$$

This relation is known as *Kirchhoff's law*. It states that, at thermal equilibrium, the ratio of the total emissive power to its absorptivity is the same for all bodies. Since α must be less than unity and $\frac{E}{\alpha} = f(T)$, at a given temperature, the emissive power will be the greatest for a body with $\alpha = 1$ that is when $E = E_b$. Since the total emissivity is defined as $\varepsilon = E/E_b$ then Kirchhoff's law gives