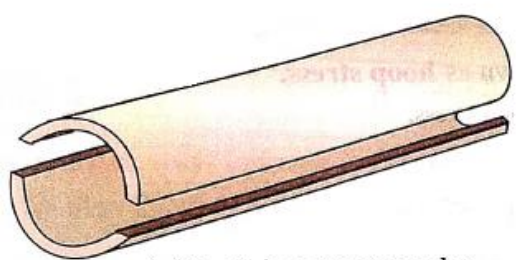
UNIT-V
THIN CYLINDERS

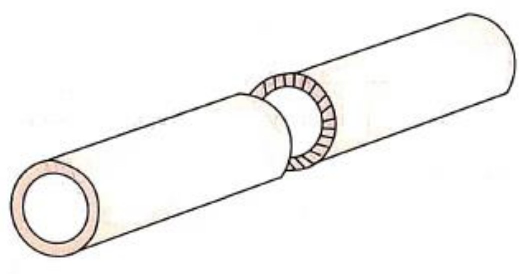
In engineering field, we daily come across vessels of cylindrical and spherical shapes consisting fluids such as tanks, boilers, compressed air receivers etc. Generally the walls of such vessels are very thin as compared to their diameters. These vessels, when empty are subjected to atmospheric pressure internally as well as externally. In such a case, the resultant pressure on the walls of the shell is zero. But whenever a vessel is subjected to internal pressure (due to steam, compressed air) its wall are subjected to tensile stresses.

In general if the thickness of the wall of a shell is less than $1/10^{th}$ to $1/15^{th}$ of its diameter is known as a thin shell.

Failure of a thin cylindrical shell due to an internal pressure: Whenever a cylindrical shell is subjected to an internal pressure. Its walls are subjected to tensile stresses. If these stresses are exceed the permissible limit, the the cylinder is likely to fail in any one of the following two ways as shown in fig.



(*a*) Split into two troughs.



(*b*) Split into two cylinders.

1. It may split up into two troughs and
2. It may split up into two cylinders.

Stresses in a thin cylindrical shell: Whenever a cylindrical shell is subjected to an internal pressure. Its walls are subjected to tensile stresses. A little consideration will show that the walls of the cylindrical shell will be subjected to the following two types of tensile stresses.

1. Circumferential stresses and
2. Longitudinal stresses

In case of thin shells, the stresses are assumed to be uniformly distributed throughout the wall thickness. However, in case of thick shells, the stresses are no longer uniformly distributed and the problem becomes complex.

The above theory also holds good, when the shell is subjected to a compressive stress

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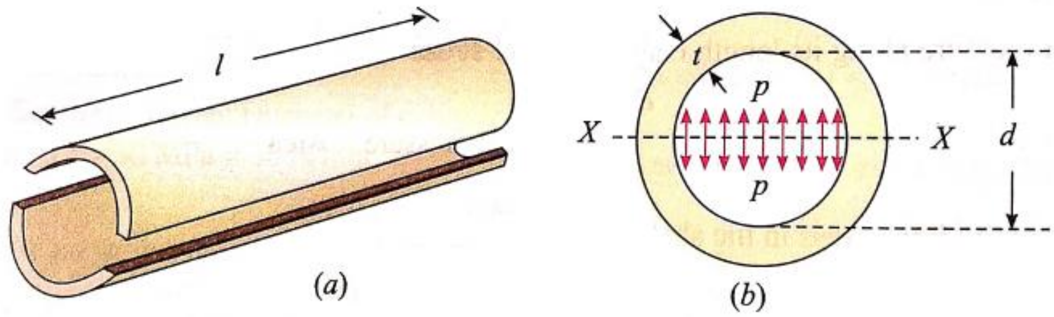
Circumferential Stress: Consider a thin cylindrical shell subjected to an internal pressure as shown in fig. We know that as a result of the internal pressure, the cylinder has a tendency to split up into two troughs as shown in fig.



Let l = Length of the shell

d = diameter of the shell

t = thickness of the shell and

p = Intensity of internal pressure

Total pressure along the diameter (say X-X axis) of the shell

P = Intensity of internal pressure x Area = p x d x l

And circumferential stress in the shell

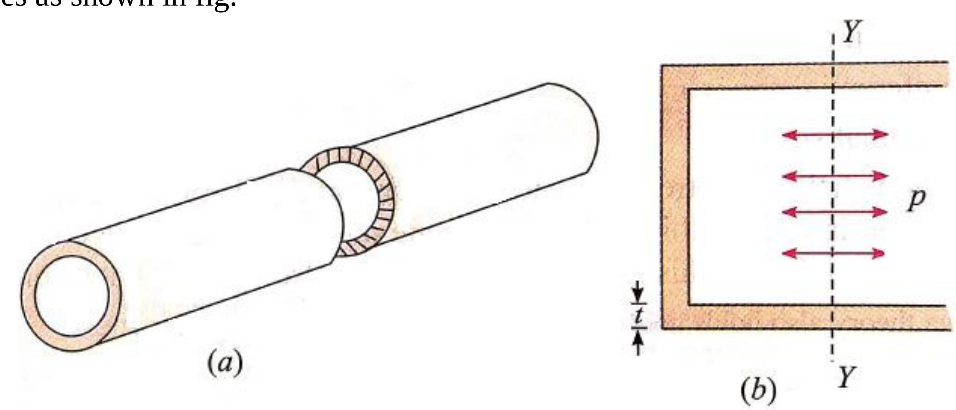
$$\sigma_c = \frac{Total\ pressure}{Resisiting\ section} = \frac{pdl}{2tl} = \frac{pd}{2t}$$

This is a tensile stress across the X-X. It is also known as **hoop stress.**

If η is the efficiency of the riveted joints of the shell, then stress

$$\sigma_c = \frac{pd}{2t\eta}$$

Longitudinal Stress: Consider a thin cylindrical shell subjected to an internal pressure as shown in fig. We know that as a result of the internal pressure, the cylinder has a tendency to split into two pieces as shown in fig.



Let l = Length of the shell

d = diameter of the shell

t = thickness of the shell and

p = Intensity of internal pressure

Total pressure along its length (say Y-Y axis) of the shell

$$P = \text{Intensity of internal pressure x Area} = p \times \frac{\pi}{4}(d)^2$$

and longitudinal stress in the shell

$$\sigma_l = \frac{Total\,pressure}{Resisiting\ section} = p \times \frac{\pi}{4}(d)^2/\pi dt = \frac{pd}{4t}$$

This is a tensile stress across the Y-Y. It may be noted that the longitudinal stress is half of the circumferential or hoop stress.

If η is the efficiency of the riveted joints of the shell, then stress

$$\sigma_l = \frac{pd}{4t\eta}$$

Problem-1: A stream boiler of 800 mm diameter is made up of 10 mm thick plates. If the boiler is subjected to an internal pressure of 2.5 MPa, find the circumferential and longitudinal stresses induced in the boiler plates.[**Ans:** 100 MPa, 50 MPa.]

Problem-2: A cylindrical shell 2 m long and 1 m internal diameter is made up of 20 mm thick plates. Find the circumferential and longitudinal stresses in the shell material, if it is subjected to an internal pressure of 5 MPa. [**Ans:** 125 MPa, 62.5 MPa]

Problem-3: A cylindrical shell of 1.3 m diameter is made up of 18 mm thick plates. Find the circumferential and longitudinal stresses induced in the boiler plates, if the boiler is subjected to an internal pressure of 2.4 MPa, Take efficiency of the joints as 70%.[**Ans:** 124 MPa, 62 MPa.]

Problem-4: A steam boiler of 1.25 m in diameter is subjected to an internal pressure of 1.6 MPa. If the steam boiler is made up of 20 mm thick plates, calculate the circumferential and longitudinal stresses. Take efficiency of the circumferential and longitudinal joints as 75% and 60% respectively. [**Ans:** 67 MPa, 42 MPa.]

Problem-5: A gas cylinder of internal diameter 40 mm is 5 mm thick, if the tensile stress in the material is not to exceed 30 MPa, find the maximum pressure which can be allowed in the cylinder.[**Ans:** 7.5 MPa.]

Design of Thin cylindrical Shells: Designing of thin cylindrical shell involves calculating the thickness (t) of a cylindrical shell for the given length (l), diameter (d), intensity of maximum internal pressure (p) and circumferential stress (σ_c). The required thickness of the shell is calculated from the relation

$$t = \frac{pd}{2\sigma c}$$

If the thickness so obtained, is not a round figure, then next higher value is provided.

Problem-6: A thin cylindrical shell of 400 mm diameter is to be designed for an internal pressure of 2.4 MPa. Find the suitable thickness of the shell, if the allowable circumferential stress is 50 MPa. [**Ans:** 9.6 mm say 10 mm]

Problem-7: A cylindrical shell of 500 mm diameter is required to withstand an internal pressure of 4 MPa. Find the minimum thickness of the shell, if maximum tensile strength in the plate material is 400 MPa and efficiency of the joints is 65%. Take factor of safety as 5. [**Ans:** 19.2 mm say 20 mm]

Problem-8: A pipe of 100 mm diameter is carrying a fluid under pressure of 4 MPa. What should be the minimum thickness of the pipe, if maximum circumferential stress in the pipe material is 12.5 MPa. [**Ans:** 16 mm]

Change in Dimensions of a Thin Cylindrical Shell due to an Internal Pressure: In elastic constant bodies, the lateral strain is always accompanied by a linear strain. It is thus obvious that in a thin cylindrical shell subjected to an internal pressure, its walls will also be subjected to lateral strain. The effect of the lateral strain is to cause some change in the dimensions (i.e., length and diameter) of the shell. Now consider a thin cylindrical shell subjected to an internal pressure

Let l = Length of the shell

d = diameter of the shell

t = thickness of the shell and

p = Intensity of internal pressure

We know that the circumferential stress, $\sigma c = \frac{pd}{2t}$

and longitudinal stress $\sigma_l = \frac{pd}{4t}$

Now let δ_d = Change in diameter of the shell

δ_l = Change in length of the shell and

$\frac{1}{m} = Poisson's\ ratio$

Now change in diameter and length may be found from the above equations, as usual (i.e., by multiplying the strain and the corresponding linear dimension)

$$\therefore \quad \delta d = \varepsilon_1 \cdot d = \frac{pd}{2tE}\left(1-\frac{1}{2m}\right)\times d = \frac{pd^2}{2tE}\left(1-\frac{1}{2m}\right)$$

$$\text{and} \quad \delta l = \varepsilon_2 \cdot l = \frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)\times l = \frac{pdl}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right)$$

Problem-9: A thin cylindrical drum 800 mm in diameter and 4 m long is made of 10 mm thick plates. If the drum is subjected to an internal pressure of 2.5 MPa. Determine its changes in diameter and length. Take E as 200 GPa and Poisson's ratio as 0.25. [**Ans:** 0.35 mm, 0.5 mm]

Problem-10: A cylindrical shell 3 m long has 1 m internal diameter and 15 mm metal thickness. Calculate the circumferential and longitudinal stresses, if the shell is subjected to an internal pressure of 1.5 MPa. Also calculate the changes in dimensions of the shell. Take E = 200 GPa. And Poisson's ratio = 0.3. [**Ans:** 50 MPa, 25 MPa, δd=0.21 mm, δl=0.15 mm]

Change in Volume of a Thin Cylindrical Shell due to an Internal Pressure: In elastic constant bodies, there is always an increase in in the length and diameter of a thin cylindrical shell due to an internal pressure. A little consideration will show that increase in the length and diameter of the shell will also increase its volume. Now consider a thin cylindrical shell subjected to an internal pressure

Let l = Original length of the shell
d = Original diameter of the shell
δl = Change in length due to pressure and
δd = Change in diameter due to pressure

We know that original volume,

$$V = \frac{\pi}{4} \times d^2 \times l = \left[\frac{\pi}{4}(d + \delta d)^2 \times (l \times \delta l)\right] - \frac{\pi}{4} \times d^2 \times l$$

$$= \frac{\pi}{4}(d^2 \cdot \delta l + 2dl \cdot \delta d) \quad \text{...(Neglecting small quantities)}$$

$$\therefore \quad \frac{\delta V}{V} = \frac{\frac{\pi}{4}(d^2 \cdot \delta l + 2dl \cdot \delta d)}{\frac{\pi}{4} \times d^2 \times l} = \frac{\delta l}{l} + \frac{2\delta d}{d} = \varepsilon_l + 2\varepsilon_c$$

or $\delta V = V(\varepsilon_l + 2\varepsilon_c)$

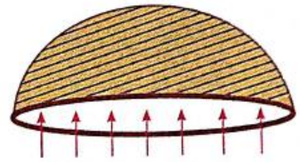
where ε_c = Circumferential strain and

ε_l = Longitudinal strain.

Problem-11: A cylindrical vessel 2 m long and 500 mm in diameter with 10 mm thick plates is subjected to an internal pressure of 3 MPa. Calculate the change in volume of the vessel. Take E = 200 GPa. and Poisson's ratio = 0.3 for the vessel material. [**Ans:** 185 x 10^3 mm^3]

Problem-12: A cylindrical vessel 1.8 m long 800 mm in diameter is made up of 8 mm thick plates. Find the hoop and longitudinal stresses in the vessel, when it contains fluid under pressure of 2.5 MPa. Also find the changes in length, diameter and volume of the vessel. Take E = 200 GPa. and 1/m = 0.3. [**Ans:** 125 MPa, 62.5 MPa, 1.42 mm, 0.23 mm, 1074 mm^3]

Thin Spherical Shells: Consider a thin spherical shell subjected to an internal pressure as shown in fig.



Spherical shell

Let p = Intensity of internal pressure

d = Diameter of the shell and

t = Thickness of the shell,

As a result of this internal pressure, the shell is likely to be turn away along the centre of the sphere. Therefore, total pressure acting along the centre of the sphere,

P = Intensity of internal pressure X Area

$$= p \times \frac{\pi}{4} \times d^2$$

and stress in the shell material,

$$\sigma = \frac{\text{Total pressure}}{\text{Resisting section}} = \frac{p \times \frac{\pi}{4} \times d^2}{\pi d \times t} = \frac{pd}{4t}$$

Note. If η is the efficiency of the riveted joints of the spherical shell, then stress,

$$\sigma = \frac{pd}{4t\eta}$$

Problem-13: A spherical gas vessel of 1.2 m diameter is subjected to a pressure of 1.8 MPa. Determine the stress induced in the vessel plate, if its thickness is 5 mm. [**Ans:** 108 MPa]

Problem-14: A spherical vessel of 2 m diameter is subjected to an internal pressure of 2 MPa. Find the minimum thickness of the plates required, if the maximum stress is not to exceed 100 MPa. Take efficiency of the joint is 80%. [**Ans:** 12.5 mm]

Change in Diameter and Volume of a Thin Spherical Shell due to an Internal Pressure: Consider a thin spherical shell subjected to an internal pressure

Let d = Original diameter of the shell

p = Intensity of internal pressure and

t = Thickness of the shell

we know that, the stress in a spherical shell,

$$\sigma = \frac{pd}{4t}$$

and strain in any one direction,

$$\varepsilon = \frac{\sigma}{E} - \frac{\sigma}{mE} \qquad ...(\because \sigma_1 = \sigma_2 = \sigma$$

$$= \frac{pd}{4tE} - \frac{pd}{4tEm} = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right)$$

$\therefore$ Change in diameter,

$$\delta d = \varepsilon \cdot d = \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) \times d = \frac{pd^2}{4tE}\left(1 - \frac{1}{m}\right)$$

We also know that original volume of the sphere,

$$V = \frac{\pi}{6} \times (d)^3$$

and final volume due to pressure,

$$V + \delta V = \frac{\pi}{6} \times (d + \delta d)^3$$

where $(d + \delta d)$ = Final diameter of the shell.

$\therefore$ Volumetric strain,

$$\frac{\delta V}{V} = \frac{(V + \delta V) - V}{V} = \frac{\frac{\pi}{6}(d + \delta d)^3 - \frac{\pi}{6} \times d^3}{\frac{\pi}{6} \times d^3}$$

$$= \frac{d^3 + (3d^2 \cdot \delta d) - d^3}{d^3} \qquad ...(\text{Ignoring second and higher power of } \delta$$

$$= \frac{3 \cdot \delta d}{d} = 3\varepsilon$$

and

$$\delta V = V \cdot 3\varepsilon = \frac{\pi}{6}(d)^3 \times 3 \times \frac{pd}{4tE}\left(1 - \frac{1}{m}\right) = \frac{\pi p d^4}{8tE}\left(1 - \frac{1}{m}\right)$$

Problem-15: A spherical shell of 2 m diameter is made up of 10 mm thick plates. Calculate the change in diameter and volume of the shell, when it is subjected to an internal pressure of 1.6 MPa. Take E = 200 GPa and 1/m = 0.3. [**Ans:** 0.56 mm, 3.52×10^6 mm^3]

Problem-16: A spherical container of 1 m diameter has 15 mm thick plates. Calculate the change in its diameter, if it contains a fluid under a pressure of 2 MPa. Take E= 200 MPa and μ = 0.28. [**Ans:** 0.12 mm]

Torsion of Circular Shafts

In a workshops and factories, a turning force is always applied to transmit energy by rotation. This turning force is applied either to the rim of a pulley, keyed to the shaft or at any other suitable point at some distance from the axis of the shaft. The product of this turning force and the distance between the point of application of the force and the axis of the shaft is known as **torque**, turning moment or twisting moment. Due to this torque, every cross-section of the shaft is subjected to some shear stress.

Torsion refers to twisting of a straight member under the action of a turning moment or torque which tends to produce a rotation about the longitudinal axis. Steering rods, propeller shafts, axels, drive shafts of automobiles are some example of members subjected to torsion.

Pure Torsion: A shaft of circular section is said to be in pure torsion when it is subjected to equal and opposite end couples whose axes coincide with the axis of the shaft. Since all sections of the shaft are identical and are subjected to the same torque we say the shaft is in pure torsion.

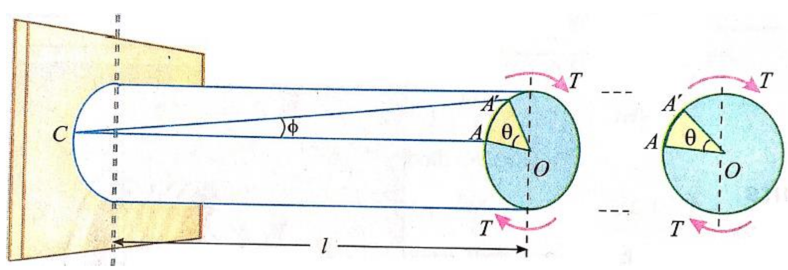
Assumptions in the theory of pure torsion:

The theory of pure torsion is based on the following assumptions.

1. The material of the shaft is uniform throughout
2. The twist along the shaft is uniform.
3. The shaft is of uniform circular section throughout.
4. Cross-sections of the shaft, which are plane before twist remain plane after twist
5. All radii which are straight before twist remain straight after twist.

Theory of pure torsion (Torsional stresses and strains):

Consider a circular shaft fixed at one end and subjected to a torque at the other end as shown in fig.



Let T = Torque in N-mm

L = Length of the shaft in mm and

R = Radius of the circular shaft in mm

As a result of this torque, every cross-section of the shaft will be subjected to shear stresses. Let the line CA on the surface of the shaft be deformed to CA’ and OA to OA’ as shown in fig.

Let $\angle$ACA’ = Φ in degrees

$\angle$AOA’ = θ in radians

τ = Shear stress induced at the surface and

C = Modulus of rigidity, also known as torsional rigidity of the shaft material

We know that shear strain = Deformation per unit length

$$= \frac{AA'}{l} = tan\Phi$$

$$= \Phi \qquad \text{(}\Phi\text{ being very small, tan }\Phi = \Phi\text{)}$$

We also know that the arc AA' = R.θ

$$\Phi = \frac{AA'}{l} = \frac{R.\theta}{l} \qquad \text{(i)}$$

If τ is the intensity of shear stress on the outermost layer and C the modulus of rigidity of the shaft, then

$$\Phi = \frac{\tau}{C} \qquad \text{(ii)}$$

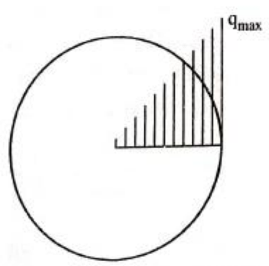
From equations (i) and (ii), we find that

$$\frac{\tau}{C} = \frac{R.\theta}{l} \quad \text{or} \quad \frac{\tau}{R} = \frac{C.\theta}{l}$$

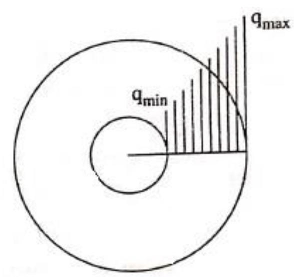
If τ_x be the intensity of shear stress, on any layer at a distance x from the centre of the shaft, then

$$\frac{\tau x}{x} = \frac{\tau}{R} = \frac{C.\theta}{l}$$

Since C, θ and l are constant, it follows that at any section of the shaft, the shear stress intensity at any point is proportional to the distance of the point from the axis of the shaft. Hence the shear stress is maximum at the surface and shear stress is zero at the axis of the solid shaft. Fig. shows the shear stress distribution for a solid shaft and a hollow shaft.



Shear stress distribution in a solid circular shaft



Shear stress distribution in a hollow circular shaft

Problem-1: A circular shaft of 50 mm diameter is required to transmit torque from one shaft to another. Find the safe torque, which the shaft can transmit, if the shear stress is not to exceed 40 MPa.

Solution: Given: Diameter of shaft (D) = 50 mm and maximum shear stress (τ) = 40 MPa = 40 N/mm^2.

We know that the safe torque, which the shaft can transmit,

$$T = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 40 \times (50)^3 \text{ N-mm}$$

$$= 0.982 \times 10^6 \text{ N-mm} = \mathbf{0.982\ kN\text{-}m} \qquad \textbf{Ans.}$$

Problem-2: *A solid steel shaft is to transmit a torque of 10 kN-m. If the shearing stress is not to exceed 45 MPa, find the minimum diameter of the shaft.*

SOLUTION. Given: Torque (T) = 10 kN-m = 10×10^6 N-mm and maximum shearing stress (τ) = 45 MPa = 45 N/mm^2.

Let D = Minimum diameter of the shaft in mm.

We know that torque transmitted by the shaft (T),

$$10 \times 10^6 = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 45 \times D^3 = 8.836\, D^3$$

$$\therefore \quad D^3 = \frac{10 \times 10^6}{8.836} = 1.132 \times 10^6$$

or $\quad D = 1.04 \times 10^2 = \mathbf{104\ mm}$ **Ans.**

Problem-3: *A hollow shaft of external and internal diameter of 80 mm and 50 mm is required to transmit torque from one end to the other. What is the safe torque it can transmit, if the allowable shear stress is 45 MPa ?*

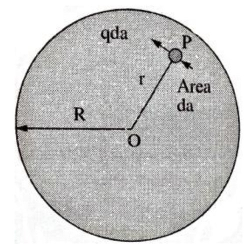
SOLUTION. Given: External diameter (D) = 80 mm; Internal diameter (d) = 50 mm and allowable shear stress (τ) = 45 MPa = 45 N/mm^2.

We know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times 45 \times \left[\frac{(80)^4 - (50)^4}{80}\right] \text{ N-mm}$$

$$= 3.83 \times 10^6 \text{ N-mm} = \mathbf{3.83\ kN\text{-}m} \quad \textbf{Ans.}$$

Torsional Moment of Resistance: Fig. shows the section of the shaft of radius R subjected to pure torsion. Let f_s be the maximum shear stress which occurs at the surface.



Consider an elemental area da at a distance r from the axis of the shaft.

Shear stress offered by the elemental area = q = $\frac{r}{R} fs$

$\therefore$ Shear resistance offered by the elemental area q.da = $\frac{r}{R} fs.da$

$\therefore$ Moment of resistance offered by the elemental area = $\frac{r}{R} fs.da.r = \frac{fs}{R} da.r2$

$\therefore$ Total moment of resistance offered by the cross section of the shaft = T = $\frac{fs}{R} \Sigma da.r2$

But Σda . r^2 represents the moment of inertia of the shaft section about the axis of the shaft. i.e., the quantity Σda . r^2 is the polar moment of inertia I_p of the section of the shaft.

$$\therefore \ T = \frac{fs}{R}.Ip \qquad \therefore \ \frac{T}{Ip} = \frac{fs}{R}$$

$$\text{But} \quad \frac{fs}{R} = \frac{C\theta}{l} \qquad \therefore \frac{T}{Ip} = \frac{fs}{R} = \frac{C\theta}{l}$$

Polar Moment of Inertia: The moment of inertia of a plane area, with respect to an axis perpendicular to the plane of the figure, is called polar moment of inertia with respect to the point, where the axis intersects the plane. In a circular plane, this point is always the centre of the circle.

Solid shaft: Radius R, Diameter D

$$I_p = \frac{\pi R4}{2} = \frac{\pi D4}{32}$$

Hollow shaft: Outer radius R_1, Inner radius R_2

Outer diameter D_1, Inner diameter D_2

$$Ip = \frac{\pi}{2}[R_1^4 - R_2^4] = \frac{\pi}{32}[D_1^4 - D_2^4]$$

Polar Section Modulus: Let T be the torsional moment of resistance of the section of a shaft of radius R and I_p the polar moment of inertia of the shaft section.

The shear stress intensity q at any point on the section distance r from the axis of the shaft is given by $q = \frac{T}{Ip}.r$

The maximum shear stress f_s occurs at the greatest radius R

$$fs = \frac{T}{Ip}.R$$

Or $T = fs.\frac{Ip}{R}$ or $T = f_s . Z_p$

Where $Z_p = \frac{Ip}{R} = \frac{Polar\ moment\ of\ inertia\ of\ the\ shaft\ section}{Maximum\ radius}$

This ratio is called the polar modulus of the shaft section. The greatest twisting moment which a given shaft section can resist = maximum permissible shear stress x polar modulus.

Hence for a shaft of a given material the magnitude of the polar modulus is a measure of its strength in resisting torsion.

Problem-4: *Calculate the maximum torque that a shaft of 125 mm diameter can transmit, if the maximum angle of twist is 1° in a length of 1.5 m. Take C = 70 GPa.*

SOLUTION. Given: Diameter of shaft (D) = 125 mm ; Angle of twist (θ) = $1° = \frac{\pi}{180}$ rad ; Length of the shaft (l) = 1.5 m = 1.5×10^3 mm and modulus of rigidity (C) = 70 GPa = 70×10^3 N/mm^2.

Let T = Maximum torque the shaft can transmit.

We know that polar moment of inertia of a solid circular shaft,

$$J = \frac{\pi}{32} \times (D)^4 = \frac{\pi}{32}(125)^4 = 24.0 \times 10^6 \text{ mm}^4$$

and relation for torque transmitted by the shaft,

$$\frac{T}{J} = \frac{C \cdot \theta}{l}$$

$$\frac{T}{24.0 \times 10^6} = \frac{(70 \times 10^3)\pi/180}{1.5 \times 10^3} = 0.814$$

$$\therefore \quad T = 0.814 \times (24.0 \times 10^6) = 19.5 \times 10^6 \text{ N-mm}$$

$$= \mathbf{19.5\ kN\text{-}m} \quad \textbf{Ans.}$$

Problem-5: A solid shaft of 120 mm diameter is required to transmit 200 kW at 100 r.p.m. If the angle of twists not to exceed 2^0, find the length of the shaft. Take modulus of rigidity for the shaft material as 90 GPa.

Solution: Given: Diameter of shaft (D) = 120 mm, Power (P) = 200 kW, Speed shaft (N) = 100 r.p.m. Angle of twist = 20. 2π/180 rad. And modulus of rigidity = 90 GPa = 90x 10^3 N/mm^2.

Let T = Torque transmitted by the shaft, and

l = Length of the shaft.

We know that power transmitted by the shaft (P),

$$200 = \frac{2\pi NT}{60} = \frac{2\pi \times 100 \times T}{60} = 10.5T$$

$$\therefore \quad T = \frac{200}{10.5} = 19 \text{ kN-m} = 19 \times 10^{o} \text{ N-mm}$$

We also know that polar moment of inertia of a solid shaft,

$$J = \frac{\pi}{32} \times (D)^4 = \frac{\pi}{32} \times (120)^4 = 0.4 \times 10^6 \text{ mm}^4$$

and relation for the length of the shaft,

$$\frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{or} \quad \frac{19 \times 10^6}{20.4 \times 10^5} = \frac{(90 \times 10^3) \times (2\pi/180)}{l}$$

$$0.931 = \frac{3.14 \times 10^3}{l}$$

$$l = \frac{(3.14 \times 10^3)}{0.931} = 3.37 \times 10^3 = \mathbf{3.37 \text{ m}} \quad \textbf{Ans.}$$

Problem-6: A solid shaft is subjected to a torque of 1.6 kN-m. Find the necessary diameter of the shaft, if the allowable shear stress is 60 MPa. The allowable twist is 1^0 for every 20 diameters length of the shaft. Take C = 80 GPa.

Solution: Given: Torque(T) = 1.6 kN-m = 1.6 x 10^6 N-mm. Allowable shear stress = (τ) = 60 MPa = 60 N/mm^2. Angle of twist = 1^0 = π/180 rad. Length of shaft (l) = 20 D and modulus of rigidity C = 80 GPa. 80 x10^3 N/mm^2.

First of all. Let us find out the value of diameter of the shaft for its strength and stiffness.

1. *Diameter for strength*

We know that torque transmitted by the shaft (T),

$$1.6 \times 10^6 = \frac{\pi}{16} \times \tau \times D_1^3 = \frac{\pi}{16} \times 60 \times D_1^3 = 11.78\, D_1^3$$

$$\therefore \quad D_1^3 = \frac{1.6 \times 10^6}{11.78} = 0.136 \times 10^6 \text{ mm}^3$$

$$\text{or} \quad D_1 = 0.514 \times 10^2 = 51.4 \text{ mm} \quad \ldots(i)$$

2. *Diameter for stiffness*

We know that polar moment of inertia of a solid circular shaft,

$$J = \frac{\pi}{32} \times (D_2)^4 = 0.098\, D_2^4$$

and relation for the diameter,

$$\frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{or} \quad \frac{1.6 \times 10^6}{0.098\, D_2^4} = \frac{(80 \times 10^3) \times (\pi/180)}{20 D_2}$$

$$\therefore \quad D_2^3 = \frac{(1.6 \times 10^6) \times 20}{0.098 \times (80 \times 10^3) \times (\pi/180)} = 234 \times 10^3 \text{ mm}^3$$

or

$$D_2 = 6.16 \times 10^1 = 61.6 \text{ mm} \quad ...(ii)$$

We shall provide a shaft of diameter of 61.6 mm (*i.e.*, greater of the two values). **Ans.**

Power Transmitted by a Shaft: The main purpose of the shaft is to transmit power from one shaft to another shaft in factories and workshops. Now consider a rotating shaft, which transmits power from one of its ends to another.

Let N = No. of revolutions per minute and

T = Average Torque in kN-m

Work done per minute = Force x Distance = T x 2πN = 2πNT

Work done per second = $\frac{2\pi NT}{60}\ kN - m$

Power transmitted = Work done in kN-m per second

$= \frac{2\pi NT}{60}\ kW$

Problem-7: A circular shaft of 60 mm diameter is running at 150 r.p.m. If the shear stress is not to exceed 50 MPa, find the power which can be transmitted by the shaft.

Solution- Given: Diameter of the shaft (D) = 60 mm. Speed of the shaft (N) = 150 r.p.m. and maximum shear stress (τ) = 50 MPa = 50 N/mm^2.

We know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times D^3 = \frac{\pi}{16} \times 50 \times (60)^3 \text{ N-mm}$$

$$= 2.12 \times 10^6 \text{ N-mm} = 2.12 \text{ kN-m}$$

and power which can be transmitted by the shaft,

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 150 \times 2.12}{60} = \mathbf{33.3\ kW} \quad \textbf{Ans.}$$

Problem-8: A hollow shaft of external and internal diameters as 100 mm and 40 mm is transmitting power at 120 r.p.m. Find the power the shaft can transmit, if the shear stress is not to exceed 50 MPa

Solution. Given: External diameter (D) = 100 mm; Internal diameter (d) = 40 mm ; Speed of the shaft (N) = 120 r.p.m. and allowable shear stress (τ) = 50 MPa = 50 N/mm^2.

We know that torque the shaft can transmit,

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times 50 \times \left[\frac{(100)^4 - (40)^4}{100}\right] \text{ N-mm}$$

$$= 9.56 \times 10^6 \text{ N-mm} = 9.56 \text{ kN-m}$$

and power the shaft can transmit,

$$P = \frac{2\pi NT}{60} = \frac{2\pi \times 120 \times 9.56}{60} = \mathbf{120\ kW} \qquad \textbf{Ans.}$$

Problem-9: A hollow shaft is to transmit 200 kW at 80 r.p.m. If the shear stress is not to exceed 60 MPa and internal diameter is 0.6 of the external diameter, find the diameters of the shaft. **Solution-** Given: Power (P) = 200 kW. Speed of the shaft (N) = 80 r.p.m. Maximum shear stress (τ) = 60 MPa = 60 N/mm$^{2.}$ And internal diameter of the shaft (d) = 0.6D (where D is the external diameter in mm).

We know that torque transmitted by the shaft,

$$T = \frac{\pi}{16} \times \tau \times \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times 60 \times \left[\frac{D^4 - (0.6D)^4}{D}\right] \text{ N-mm}$$

$$= 10.3\, D^3 \text{ N-mm} = 10.3 \times 10^{-6}\, D^3 \text{ kN-m} \qquad ...(i)$$

We also know that power transmitted by the shaft (P),

$$200 = \frac{2\pi NT}{60} = \frac{2\pi \times 80 \times (10.3 \times 10^{-6} D^3)}{60} = 86.3 \times 10^{-6}\, D^3$$

$$\therefore \qquad D^3 = \frac{200}{(86.3 \times 10^{-6})} = 2.32 \times 10^6 \text{ mm}^3$$

$$\text{or} \qquad D = 1.32 \times 10^2 = \mathbf{132\ mm} \qquad \textbf{Ans.}$$

$$\text{and} \qquad d = 0.6\,D = 0.6 \times 132 = \mathbf{79.2\ mm} \qquad \textbf{Ans.}$$