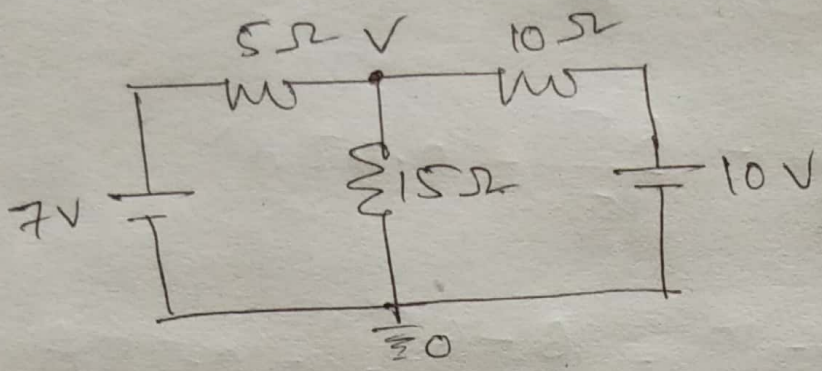
Nodal Analysis for one unknown Voltage:-

(P)



→ the point at which two or more resistors are connected that point is called node.

→ Apply KCL + ohm's law for nodal Analysis.

→ Now, Apply KCL at node, [we consider V is the higher than 7V & 10V]

$$\frac{V-7}{5} + \frac{V-10}{10} + \frac{V-0}{15} = 0$$

$$V\left[\frac{1}{5} + \frac{1}{10} + \frac{1}{15}\right] = \frac{7}{5} + 1 = \frac{12}{5}$$

$$V\left[\frac{6+3+2}{30}\right] = \frac{12}{5}$$

$$V\left[\frac{11}{30}\right] = \frac{12}{5}$$

$$V = 6.5\,V$$

$$I_{5\Omega} = \frac{V-7}{5} = \frac{7-6.5}{5} = 0.1\,A$$

$$I_{15\Omega} = \frac{V}{15} = \frac{6.5}{15} = 0.43\,A$$

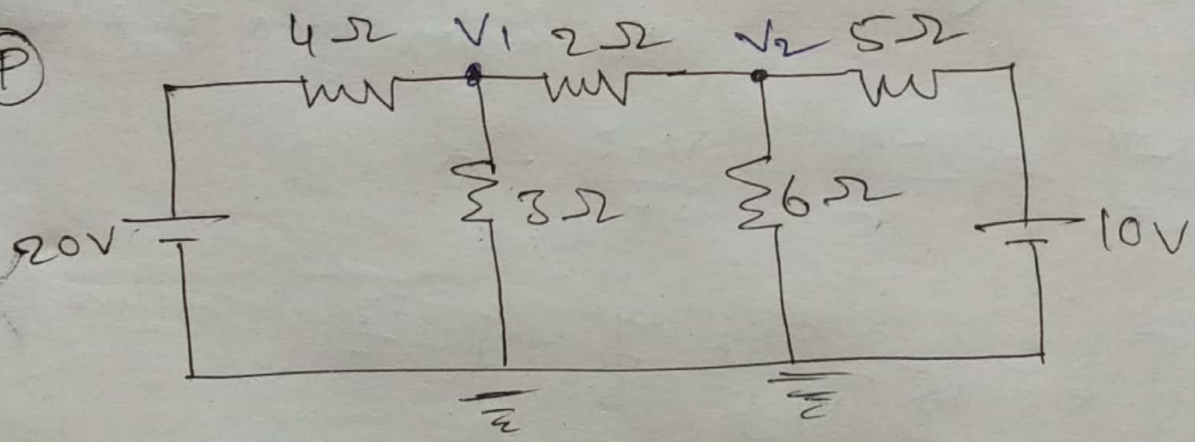
$$I_{10\Omega} = \frac{V-10}{10} = \frac{10-6.5}{10} = 0.35\,A$$

$$\therefore \quad I_{5\Omega} + I_{10\Omega} = I_{15\Omega}$$

$$0.1 + 0.35 = 0.45\,A \text{ //}$$

Nodal Analysis for two unknown Voltages:

(P)



Apply KCL at node (1)

$$\frac{V_1-20}{4} + \frac{V_1}{3} + \frac{V_1-V_2}{2} = 0$$

$$V_1\left[\frac{1}{4}+\frac{1}{3}+\frac{1}{2}\right] - 0.5V_2 = 5$$

$1.08V_1 - 0.5V_2 = 5$ ——— ①

Apply KCL at node ②

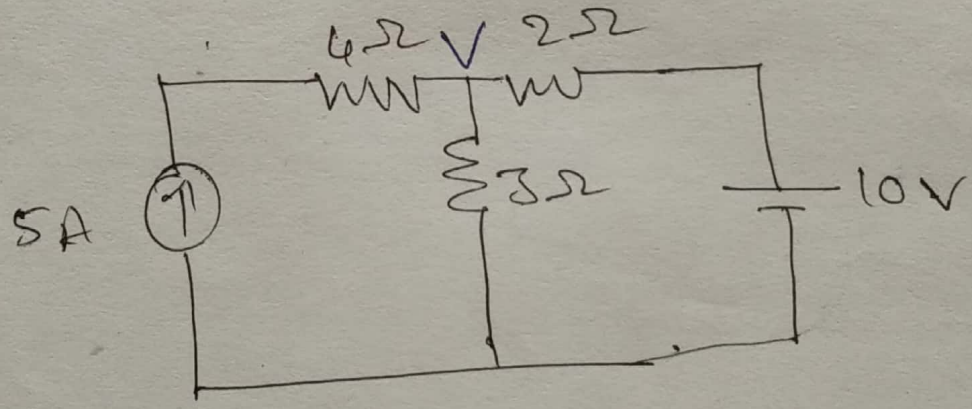
$$\frac{V_2 - V_1}{2} + \frac{V_2}{6} + \frac{V_2 - 10}{5} = 0$$

$$V_1\left[\frac{-1}{2}\right] + V_2\left[\frac{1}{2} + \frac{1}{6} + \frac{1}{5}\right] = 2 \text{ ——— } ③$$

$V_1 = 2.79\,V$, $V_2 = 3.95\,V$

Nodal Analysis when current source is given

(P)



Sol^n:- Apply KCL + ohm's law

$$\frac{V - 10}{2} + \frac{V}{3} = 5$$

$0.5V + 0.33V = 10$

$0.83V = 10$

$$V = \frac{10}{0.83} = 12V$$

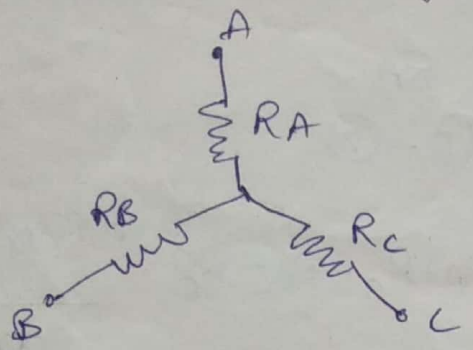
$$I_{2\Omega} = \frac{12-10}{2} = 1A$$

$$I_{3\Omega} = \frac{V}{3} = \frac{12}{3} = 4A$$

$$I_{2\Omega} + I_{3\Omega} = 1 + 4 = 5A_{\text{ans}}$$

Star – Delta transformation Technique :-

Star :-

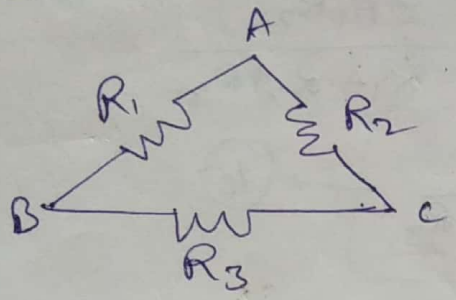


$R_{AB}(Y) = R_A + R_B$

$R_{BC}(Y) = R_B + R_C$

$R_{CA}(Y) = R_C + R_A$

Delta :-



$R_{AB}(\Delta) = R_1 // (R_2 + R_3)$

$R_{AB}(\Delta) = \dfrac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$

$R_{BC}(\Delta) = R_3 // (R_2 + R_1)$

$R_{BC}(\Delta) = \dfrac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$

$R_{CA}(\Delta) = R_2 // (R_1 + R_3)$

$= \dfrac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$

Now, Equate the Y & Δ

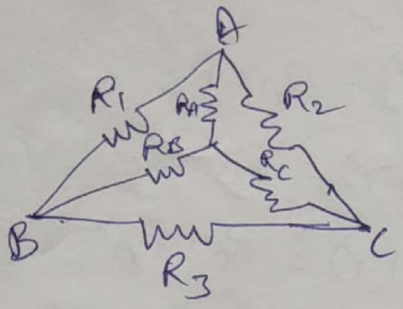
$R_{AB}(Y) = R_{AB}(\Delta)$

$R_A + R_B = \dfrac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$ — (1)

$R_B + R_C = \dfrac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3}$ — (2)

$R_C + R_A = \dfrac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$ — (3)

To get relation b/w Δ to Y



to get R_A

eqn ① + ③ − ②

$$R_A + \cancel{R_B} + R_A + \cancel{R_C} - \cancel{R_B} - \cancel{R_C} = \frac{R_1R_2 + \cancel{R_1R_3} + R_1R_2 + \cancel{R_2R_3} - \cancel{R_1R_3} - \cancel{R_2R_3}}{R_1 + R_2 + R_3}$$

$$\cancel{2}R_A = \frac{\cancel{2}R_1R_2}{R_1 + R_2 + R_3}$$

$$\therefore \boxed{R_A = \frac{R_1R_2}{R_1 + R_2 + R_3}} \quad \text{— (4)}$$

to get R_B

∴ eqn ① + ② − ③

$$\cancel{R_A} + \cancel{R_B} + R_B + \cancel{R_C} - \cancel{R_A} - \cancel{R_C} = \frac{\cancel{R_1R_2} + R_1R_3 + R_1R_3 + \cancel{R_2R_3} - \cancel{R_1R_2} - \cancel{R_2R_3}}{R_1 + R_2 + R_3}$$

$$\cancel{2}R_B = \frac{\cancel{2}R_1R_3}{R_1 + R_2 + R_3}$$

$$\boxed{R_B = \frac{R_1R_3}{R_1 + R_2 + R_3}} \quad \text{— (5)}$$

to get R_C

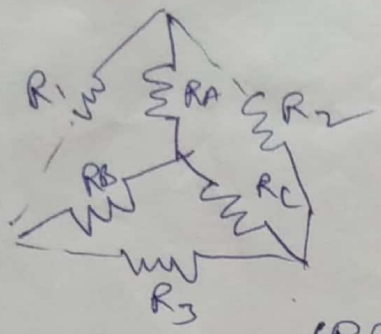
∴ eqn ② + ③ − ①

$$\cancel{R_A} + R_C + \cancel{R_B} + R_C - \cancel{R_A} - \cancel{R_B} = \frac{\cancel{R_1R_3} + R_2R_3 + \cancel{R_1R_2} + R_2R_3 - \cancel{R_1R_2} - \cancel{R_2R_3}}{R_1 + R_2 + R_3}$$

$$\cancel{2}R_C = \frac{\cancel{2}R_2R_3}{R_1 + R_2 + R_3}$$

$$\boxed{R_C = \frac{R_2R_3}{R_1 + R_2 + R_3}} \quad \text{— (6)}$$

The relation b/w Y to Δ :-



$$R_AR_B + R_BR_C + R_CR_A = \left(\frac{R_1R_2}{R_1+R_2+R_3}\right)\left(\frac{R_1R_3}{R_1+R_2+R_3}\right) + \left(\frac{R_1R_3}{R_1+R_2+R_3}\right)\left(\frac{R_2R_3}{R_1+R_2+R_3}\right) + \left(\frac{R_2R_3}{R_1+R_2+R_3}\right)\left(\frac{R_1R_2}{R_1+R_2+R_3}\right)$$

$$= \frac{R_1^2R_2R_3}{(R_1+R_2+R_3)^2} + \frac{R_1R_2R_3^2}{(R_1+R_2+R_3)^2} + \frac{R_1R_2^2R_3}{(R_1+R_2+R_3)^2}$$

$$= \frac{R_1R_2R_3\left[R_1+R_2+R_3\right]}{(R_1+R_2+R_3)^2}$$

$$R_AR_B + R_BR_C + R_CR_A = \frac{R_1R_2R_3}{R_1+R_2+R_3}$$

$$R_AR_B + R_BR_C + R_CR_A = R_1R_C$$

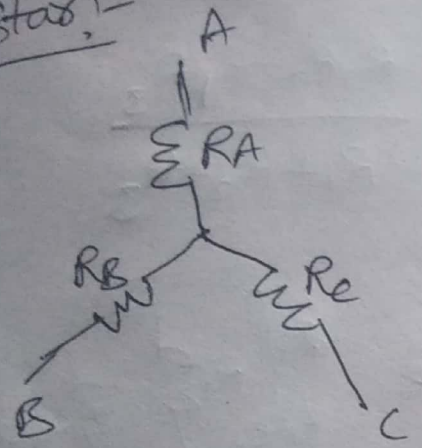
$$\therefore \quad R_1 = \frac{R_AR_B + R_BR_C + R_CR_A}{R_C}$$

Similarly $$R_2 = \frac{R_AR_B + R_BR_C + R_CR_A}{R_B}$$

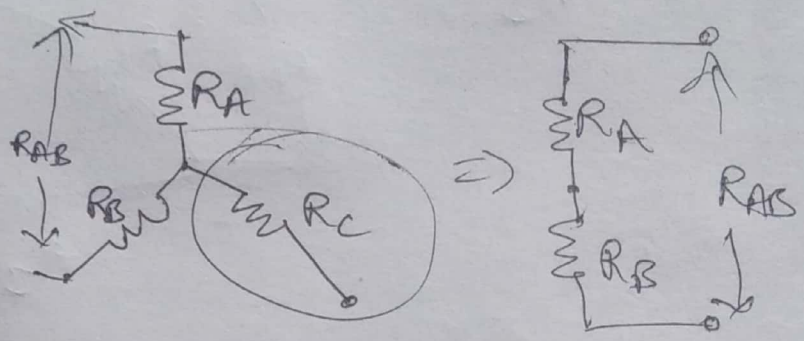
$$R_3 = \frac{R_AR_B + R_BR_C + R_CR_A}{R_A}$$

Star - delta transformation technique

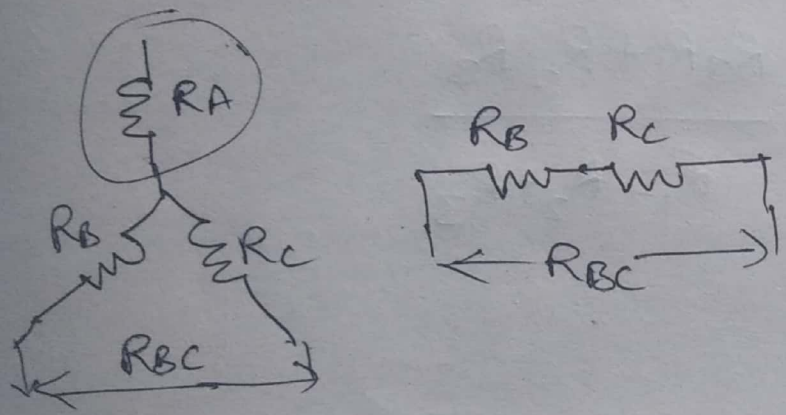
Star :-



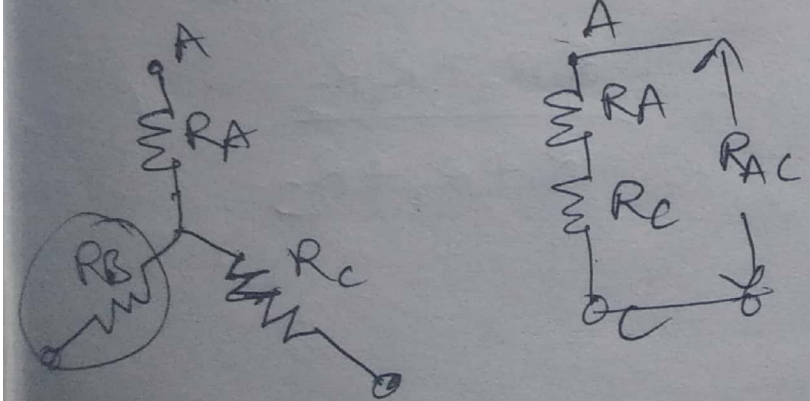
$$R_{AB}(Y) = R_A + R_B$$



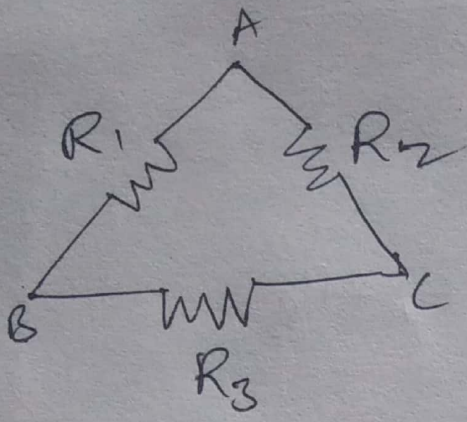
$$R_{BC}(Y) = R_B + R_C$$



$$R_{CA}(Y) = R_C + R_A$$

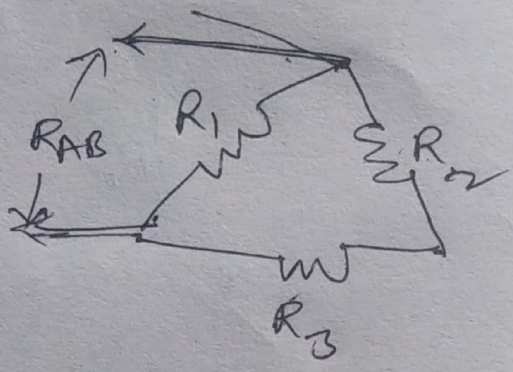


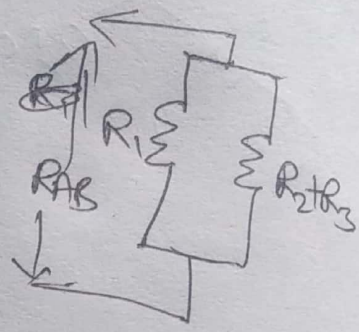
Delta :-



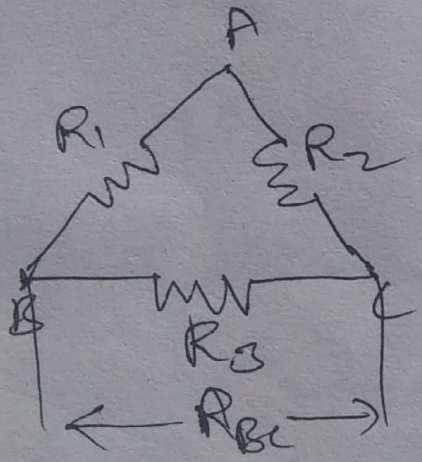
$$R_{AB}(\Delta) = R_1 // (R_2 + R_3)$$

$$= \frac{R_1R_2 + R_1R_3}{R_1 + R_2 + R_3}$$



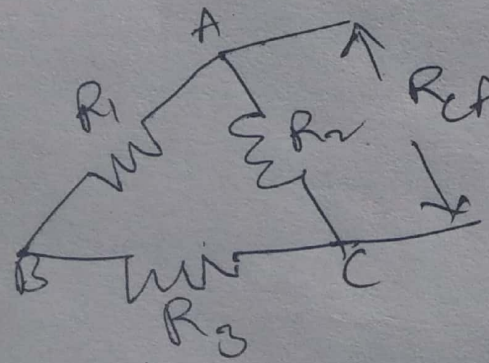


$$R_{BC}(\Delta) = R_3 // (R_1 + R_2)$$



$$= \frac{R_3R_1 + R_2R_3}{R_1 + R_2 + R_3}$$

$$R_{CA}(\Delta) = R_2 // (R_1 + R_3)$$



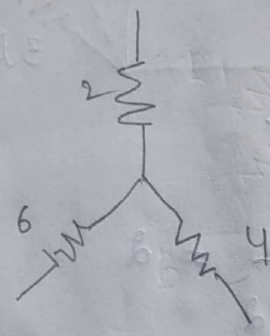
$$= \frac{R_1R_2 + R_2R_3}{R_1 + R_2 + R_3}$$

→ If $R_1 = R_2 = R_3 = R$,

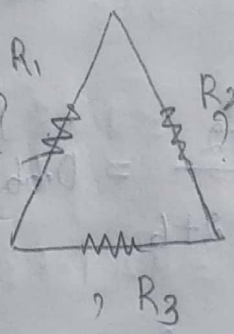
$$R_A = \frac{R}{3R} = \frac{R}{3} \Rightarrow \boxed{R_A = \frac{R}{3}}$$

Problem

1.



$\Rightarrow$



Y to Δ

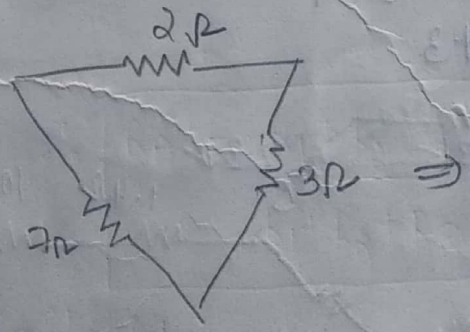
Ans:

$$R_1 = 6 + 2 + \frac{6 \times 2}{4} = 11$$

$$R_2 = 2 + 4 + \frac{2 \times 4}{6} = 7.3$$

$$R_3 = 6 + 4 + \frac{6 \times 4}{2} = 22$$

2.



$\Rightarrow$

Δ to Y

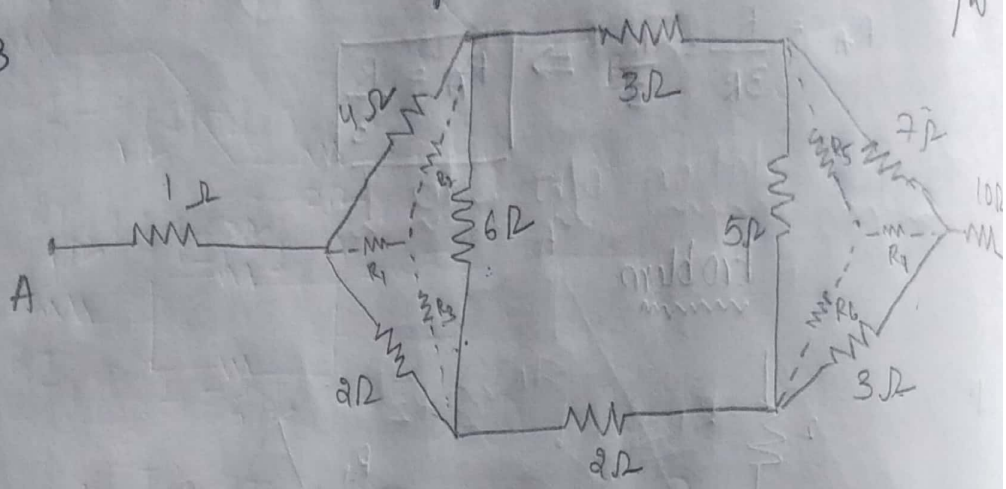
Ans:

$$R_1 = \frac{7 \times 2}{7 + 2 + 3} = 1.1$$

$$R_2 = \frac{2 \times 3}{2 + 3 + 7} = 0.5\Omega$$

$$R_3 = \frac{7 \times 3}{2 + 3 + 7} = 1.7$$

3. For the circuit shown Fig find the resistance b/w A & B



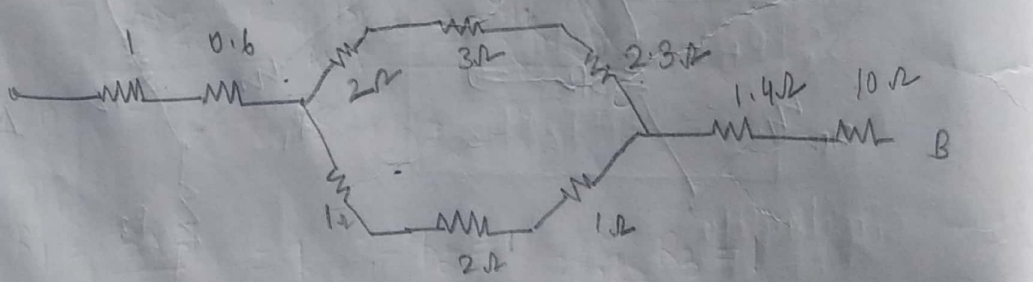
Ans:

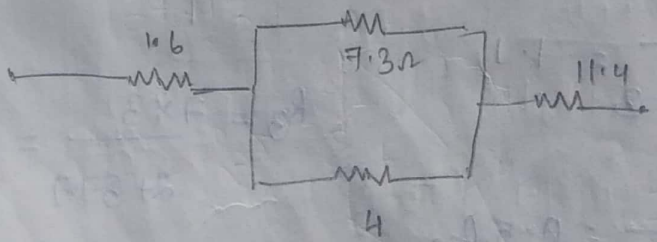
$$R_1 = \frac{4 \times 2}{4+2+6} = 0.66; \quad R_2 = \frac{6 \times 4}{6+4+2} = 2$$

$$R_3 = \frac{6 \times 2}{6+2+4} = 1$$

$$R_4 = \frac{7 \times 3}{7+3+5} = 1.4 \; ; \quad R_5 = \frac{5 \times 7}{5+7+3} = 2.33$$

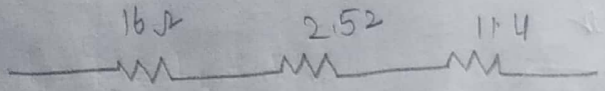
$$R_6 = \frac{5 \times 3}{5+7+3} = 1$$





7.3 & 4 are parallel

$$R_a = \frac{7.3 \times 4}{7.3+4} = 2.59$$

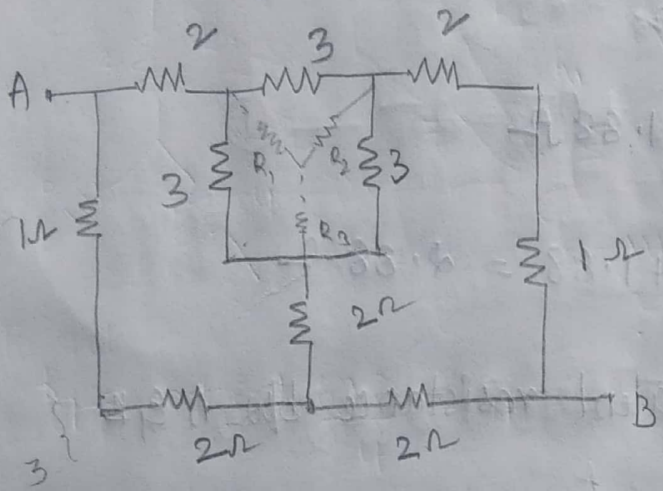


$R_{resultant} = 16 + 2.5 + 11.4$

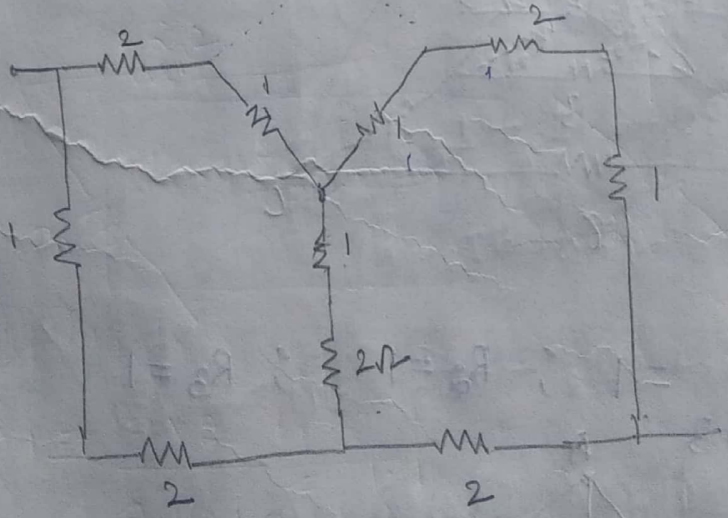
$= 15.59\Omega$

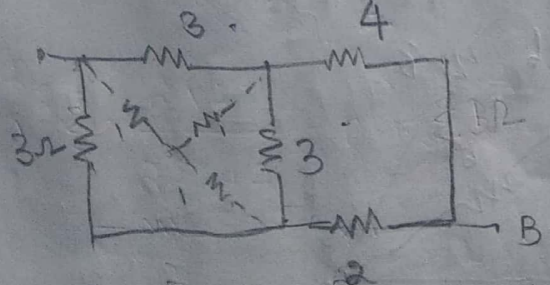
4. For circuit shown in Fig

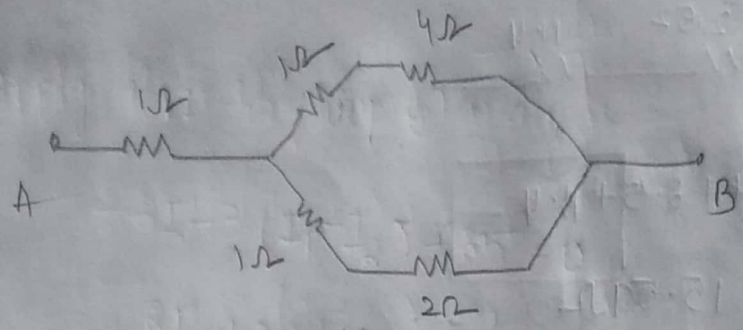
Ans:

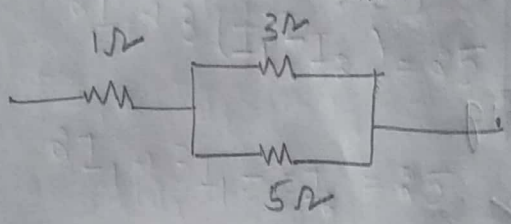


$R_1 = \frac{3\times3}{3+3+3} = \frac{9}{9} = 1$; $R_2 = \frac{3\times3}{3+3+3} = 1$; $R_3 = \frac{3\times3}{3+3+3} = 1$







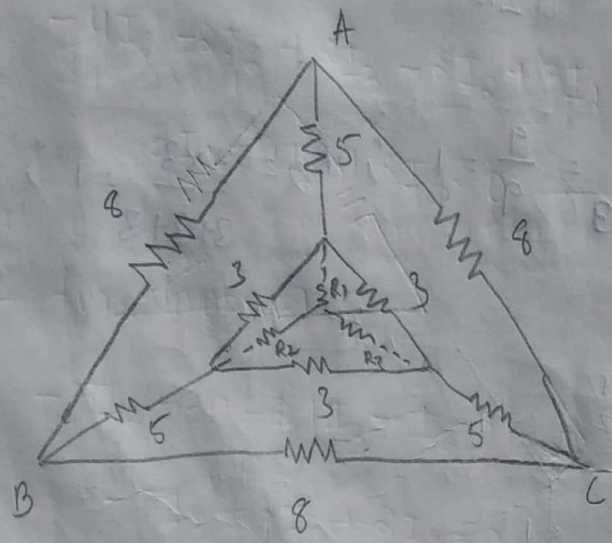


$R_{\parallel} = \frac{1}{5} + \frac{1}{3} = 1.88\,\Omega$

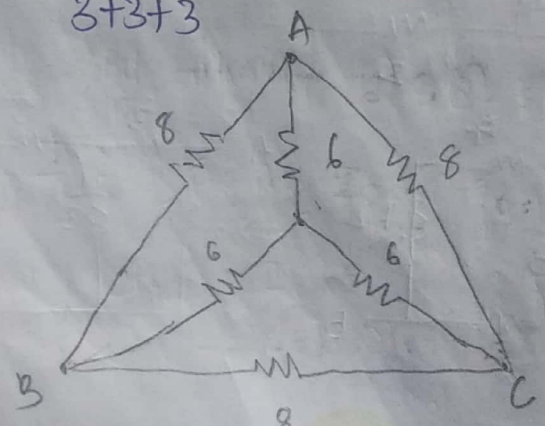
$R_{resultant} = 1 + 1.88 = 2.88\,\Omega$

5. Obtain the equivalent resistance b/w A & B

Ans

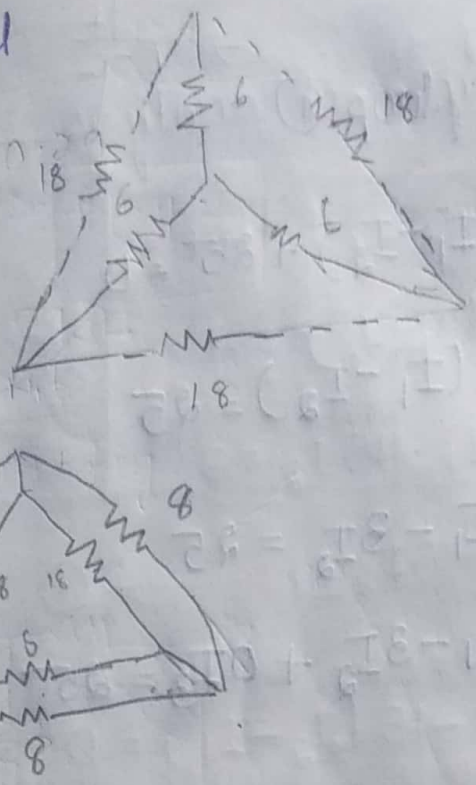


$R_1 = \frac{3\times3}{3+3+3} = 1$; $R_2 = 1$; $R_3 = 1$

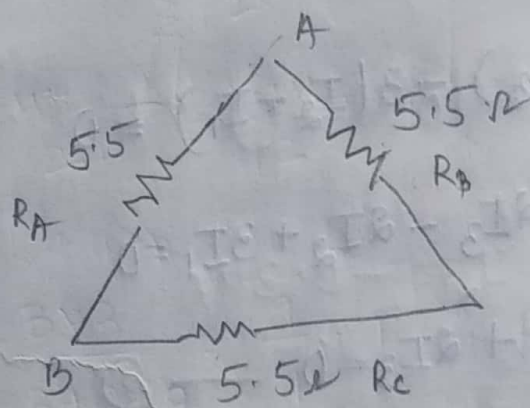


8 & 6 are in parallel

$R = \frac{1}{8} + \frac{1}{6}$



8 & 18 are in ||el so $\Rightarrow \frac{18 \times 8}{18+8} = 5.5\,\Omega$



$R_{AB} = R_A \,||\, (R_B + R_C)$

$= \frac{5.5(5.5+5.5)}{5.5+5.5+5.5} = \frac{5.5 \times 11}{16.5} = 3.7\,\Omega$

$R_{BC} = 3.7\,\Omega$; $R_{CA} = 3.7\,\Omega$