MOS

UNIT - 1

1. Define the stress.

Ans:- The Force of resistance Per unit area, offered by a body against deformation is known as stress.

$$\sigma = \frac{P}{A}$$

2. What is the relation between modulus of rigidity and modulus of elasticity

Ans:- modulus of elasticity of a material is the ratio of longitudinal stress to the longitudinal strian produced in a Body made of same material.

moduls of rigidity. is the ratio of tangential stress to shear strain.

3. Define the bulk modulus?

Bulk modulus is the measure of the decrease in volume with an increase in pressure.

4. Write down the equation of modulus of resilience?

Ans:- $U_r = \frac{\sigma^2 y}{2E}$.

Where
U_r (or) U is modulus of resilance
σ^2 is the yeild strain
E is Young's modulus

5. Define the strain energy?

strain energy is defined as the energy stored in a Body due to deformation.

What is the relation b/w bulk modulus and modulus of elasticity

$$K = \frac{Y}{3\left(1 - \frac{2}{\mu}\right)}$$

where
K is Bulk modulus
Y is young's modulus
μ is possion ratio.

Define the poisson's ratio?

Poisson's ratio is defined as the ratio of the change in the widith per unit widith of a material to the change in its length per unit length as a result of strain

state the Hooke's law?

Hooke's law states that " For relatively small deformations of an object, the displacement or size of the deformation is directly proportional to the deforming force or load."

1. A steel rod 5cm diameter and 6m long is connected to two grips and the rod is maintained at a temperature of 100°c, determine the stress and pull exerted when the temperature falls to 20°c if

i) The ends do not yield

ii) The ends yield by 0.15 cm

Take $E = 2 \times 10^5 \ N/mm^2$ and $\alpha = 12 \times 10^{-6}/^{\circ}c$

Sol:- (i) If ends do not yeild

$$E = \frac{F}{A} \times \frac{l}{\Delta l}.$$

$d = 5cm = 50mm.$

$r = 25mm$

$\Delta\theta = 100 - 20$
$= 80$

$$2 \times 10^5 = \frac{F}{\pi \times 25^2} \times \frac{\cancel{l}}{\cancel{l}\alpha\Delta\theta}$$

$$2 \times 10^5 = \frac{F}{\pi \times 25^2} \times \frac{1}{12 \times 10^{-6} \times 80}$$

$$F = 12 \times 10^{-6} \times 80 \times 25^2 \times 2 \times 10^5$$
$$= 120,000 \ N$$
$$= 120 \times 10^3 \ N.$$

$$\text{strain} = 12 \times 10^{-6} \times 80 \ (\alpha\Delta\theta).$$
$$= 96 \times 10^{-5}$$

strain exerted $= 96 \times 10^5$

pull exerted $= 120 \times 10^3$ N.

) If ends yield by 0.15cm

change in length $= l\alpha\Delta\theta - 0.15\times10^{1}$ mm

$\Rightarrow \Delta l = 6\times10^{3}\times12\times10^{-6}\times80 - 0.15\times10^{1}$ mm

$\Rightarrow \Delta l = 5.76 - 0.15\times10^{-1}$

$= 5.745$ mm $= 5.245\times10^{-3}$ m

$B = \frac{F}{A}\times\frac{l}{\Delta l}$

$2\times10^{5} = \frac{F}{\pi\times25^{2}}\times\frac{6}{5.745\times10^{-3}}$

$F = \frac{2\times10^{5}\times\pi\times25^{2}\times5.745\times10^{-3}}{6}$

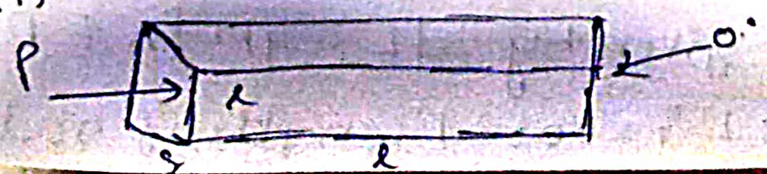
$= \frac{2256056.234}{6}$

$\boxed{\text{Pull} = 376009.370\,N}$

Stress $= \frac{5.745\times10^{-3}}{6}$ $\boxed{\text{stress} = 4.575\times10^{-4}}$

) Define volumeteric strain prove that the volumetric strain for rectangular bar subjected to an axial load P in the direction of its length is given by $\varepsilon_v = (\delta l/l)(1-2\mu)$ where μ = poisson's Ratio and $\delta l/l$ = longitudinal strain

:- The ratio of change in volume and original volume is called volumetric strain



$$\mu = \frac{\Delta l}{l} \times \frac{b}{\Delta b} = \frac{\Delta l}{l} \times \frac{h}{\Delta h}. \rightarrow (1)$$

Volume $V = lbh \rightarrow (2)$

$\Delta V = lb\Delta h + bh\Delta l + lh\Delta b \rightarrow (3)$

Volumetric strain $= \frac{\Delta V}{V} \rightarrow (4).$

From (2) (3) & (4)

$$\frac{\Delta V}{V} = \frac{lb\Delta h + bh\Delta l + lh\Delta b}{lbh}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{lb\Delta h}{lbh} + \frac{bh\Delta l}{lbh} + \frac{lh\Delta b}{lbh}$$

$$\frac{\Delta V}{V} = \frac{\Delta h}{h} + \frac{\Delta l}{l} + \frac{\Delta b}{b} \rightarrow (5)$$

From (1)

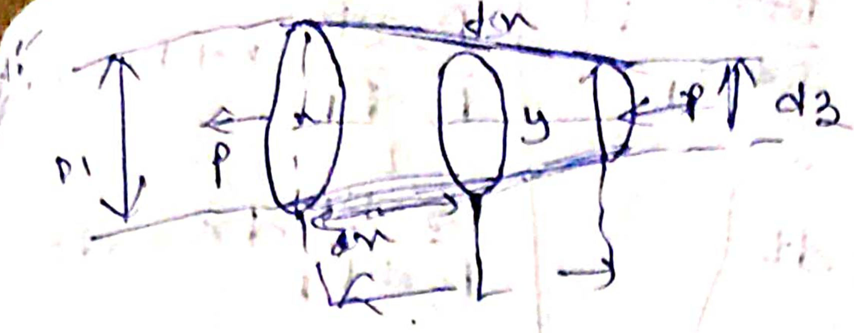
$$\frac{\Delta b}{b} = \mu \frac{\Delta l}{l} - b \;\&\; \frac{\Delta h}{h} = \mu \frac{\Delta l}{l} \rightarrow (7)$$

From (5), (6) & (7)

$$\frac{\Delta V}{V} = \mu \frac{\Delta l}{l} + \frac{\Delta l}{l} + \mu \frac{\Delta l}{l}$$

$$E_v = \frac{\Delta V}{V} = \frac{\Delta l}{l}(1+2\mu)_{4}$$

3. Prove that the total extension of a uniformly tapering rod of diameters D_1 & D_2 when the rod is subjected to an axial load P is given by $dL = 4PL/(\pi E D_1 D_2)$ where L is total length of rod.



at $x=0$ $y=D_1$

at $x=L$ $y=D_2$

$y=mx+c$

at $x=0$

$D_1=c$

at $x=L$

$D_2=mL+D_1$

$m=\frac{D_2-D_1}{E}$

$y=\frac{D_2-D_1}{L}x-D_1$

Area of cross section $=\frac{\pi\left[\frac{D_2-D_1}{2}x+D_1\right]^2}{2}$

Extension in element dx i.e., δdx

$=P/A\times\frac{dx}{E}$

$=\frac{P}{\pi\left(\frac{D_2-D_1}{L}x+D_1\right)^2}\times\frac{dx}{E}$

$$\int_0^{\delta L}\delta dx=\int_0^L\frac{P}{\pi\left(\frac{D_2-D_1}{L}x+D_1\right)^2E}dx$$

$$\delta L=\frac{P}{\pi E}\int_0^L\frac{1}{\left[\frac{D_2-D_1}{L}x+D\right]}dx$$

$$\Delta L = \frac{P}{\pi E}\int_{D_1}^{D_2} \frac{1}{t^2}\cdot \frac{L}{D_2-D_1}\, dt \qquad \frac{D_2-D_1}{L}x + D_1 = t, \quad \frac{D_2-D_1}{L}\,dx = dt$$

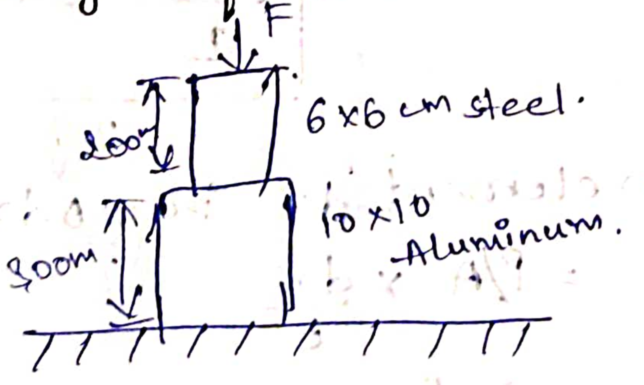
$$\cancel{\Delta L = \frac{P(D_2-D_1)}{\pi E L}\int_{D_1}^{D_2}\frac{1}{t^2}\,dt = \frac{P(D_2-D_1)}{\pi E L}\left[\frac{-1}{D_2}+\frac{1}{D_1}\right] = \frac{P(D_2-D_1)^2}{\pi E L D_1 D_2}}$$

$$\Rightarrow \Delta L = \frac{PL}{\pi E(D_2-D_1)}\int_{D_1}^{D_2}\frac{1}{t^2}\,dt$$

$$\Rightarrow \Delta L = \frac{PL}{\pi E(D_2-D_1)}\left[\frac{-1}{D_2}+\frac{1}{D_1}\right]$$

$$\Rightarrow \Delta L = \frac{PL}{\pi E(D_2-D_1)}\left(\frac{D_2-D_1}{D_2 D_1}\right) = \boxed{\frac{PL}{\pi E D_2 D_1}}\ 4$$

4. A member formed by connecting a steel Bar and an aluminium bar is shown in the figure. Assuming that the bars are prevented from buckling sideways, calculate the magnitude of force P, that will cause the total length of the member to decrease.



Given that $\Delta l_{steel} + \Delta l_{Aluminum} = 0.3\,mm \rightarrow (1)$

$E_{steel} = 2\times10^5$.

$E_{aluminum} = 6.5\times10^4$

$$E_{steel} = \frac{F}{6\times6} = \frac{20\times10^{-2}}{\Delta l_{steel}}$$

$$2\times10^5 = \frac{F}{36}\times\frac{20\times10^{-2}}{\Delta l_{steel}}$$

$$6.5\times10^{4} = \frac{F}{100} \times \frac{30\times10^{-2}}{\Delta l_{aluminium}}$$

$$\Delta l_{steel} = \frac{F}{36} \times \frac{20\times10^{-2}}{2\times10^{5}} \rightarrow (2).$$

$$\Delta l_{aluminium} = \frac{F}{100} \times \frac{30\times10^{-2}}{6.5\times10^{4}} \rightarrow (3)$$

From (1), (2) & (3):

$$\left[\frac{F}{36} \times \frac{20\times10^{-2}}{2\times10^{5}}\right] + \left[\frac{F}{100} \times \frac{30\times10^{-2}}{6.5\times10^{4}}\right] = 0.3.$$

$$F\left[\left(\frac{1}{36} \times \frac{20\times10^{-2}}{2\times10^{5}}\right) + \left(\frac{1}{100} \times \frac{30\times10^{-2}}{6.5\times10^{4}}\right)\right] = 0.3$$

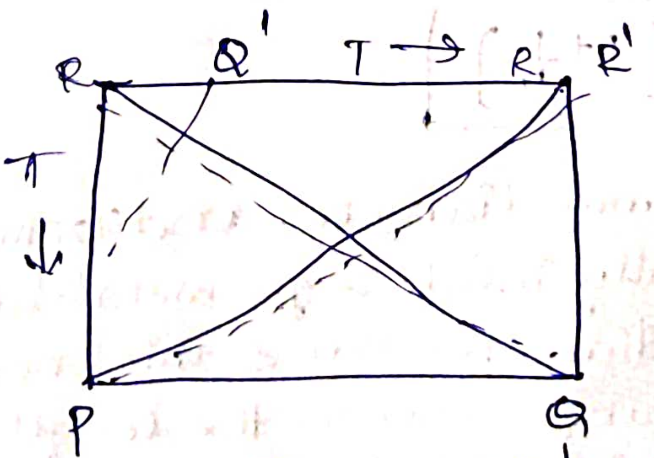
$$F\left[0.00277\times10^{-5} + 0.04615\times10^{-6}\right] = 0.3$$

$$F\left[0.00277 + 0.00461\right] = 0.3\times10^{5}$$

$$F = \frac{0.3\times10^{5}}{0.00738} = 4065040.65\,N.$$

5. Derive the relationship between modulus of elasticity and modulus of rigidity?

Sol:-



$$\text{shear strain } \phi = \frac{RR'}{RS}$$

also shear strain $= T/C$.

$$\frac{SS'}{ST} = \frac{T}{L}$$

$$NR' = RR' \cos 1.5'' = \frac{RA'}{\sqrt{2}}$$

$$LPR's \approx LPRS$$

Diagonal strain $= \frac{SS'}{\sqrt{2}ST \times \sqrt{2}} = \frac{SS'}{2ST}$

But $\frac{SS'}{ST} = \frac{T}{L}$

$\therefore$ diagonal strain $= \frac{T}{2C} = \frac{\sigma_A}{2C}$

Where σ_A is normal ~~at strain~~. strain due to shear stress T net strain in the diagonal direction $= \frac{\sigma_x}{e} - \frac{\sigma_x}{mE}$

$$= \frac{\sigma_x}{E}\left[1 + \frac{1}{m}\right].$$

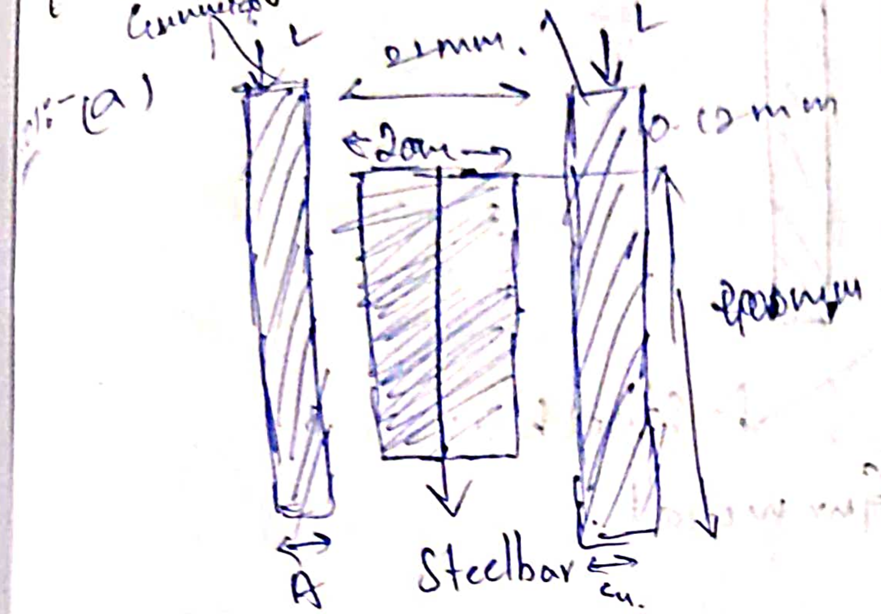
$$\Rightarrow \frac{\sigma_A}{2C} = \frac{\sigma_A}{E}\left(1 + \frac{1}{m}\right)$$

$$\Rightarrow \boxed{E = 2C\left(1 + \frac{1}{m}\right)}$$

6. A steel bar of 20mm diameter and 400mm long is placed concentrically inside a gunmetal tube. The tube has inside diameter 22m & thickness 4mm. The length of the tube exceeds the length of the steel by 0.12mm Rigid plates are placed on the compound assembly. Find (a) the load which will just make tube and bar of same length and

) the Stresses in the steel and gunmetal when a load of 50kN is appiled . E for steel = 213GN/m²

E for gunmetal = 100GN/m²



$\Delta l = 0.12$

$l = 400.12$

$E_{given\ metal} = 100 \times 10^9$

Area of cross section $= \pi \times (15^2 - 11^2)$

$= \pi \times (104)$

$= 326.72\ mm^2$

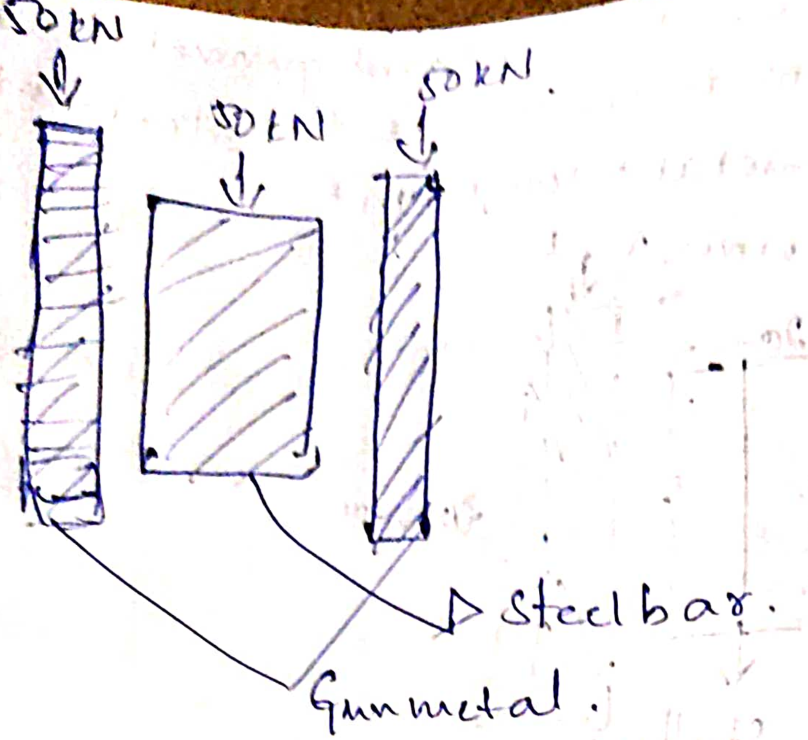
$= 326.72 \times 10^{-6}$

$$100 \times 10^9 = \frac{L}{326.72 \times 10^{-6}} \times \frac{400.12}{0.12}$$

$$L = \frac{100 \times 10^9 \times 326 \cdot 10^{-6} \times 0.12}{400.12}$$

$= 9798.660\ N.$

(b)



$$F = \frac{EA\Delta l}{l}$$

$E_{jar\ metal} = 100 \times 10^9$

$E_{steel} = 213 \times 10^9$

$$F = \frac{E_{gun\ metal} \, A_{gun\ metal} \, \Delta l_{gun\ metal}}{l_{given\ metal}} + \frac{E_{steel} \, A_{steel} \, \Delta l_{steel}}{l_{steel}}$$

$\Delta l_{steel} = \Delta l_{gun\ metal} + 0.12$

$l_{steel} = 400$

$l_{gun\ metal} = 400.12$

$A_{steel} = 10^2 \times \pi$

$= 314.15$

Subsitute values.

$$F = \frac{100 \times 10^9 \times 326.72 \times 10^{-6} \times \Delta l_{gun\ metal}}{400.12} \times \frac{213 \times 10^9 \times 314.15 \times 10^{-6} \times \Delta l_{steel}}{400}$$

$F = 50 \times 10^3$ [-ve indicates dimensions & direction]

$-50 \times 10^3 = 81655.50 \times \Delta l_{gun\ steel} + 167284.875 \times \Delta l_{steel}$.

$-50 \times 10^3 = 81655.5 \times (\Delta l_{steel} \times 0.12) + 167284.875 \times \Delta l_{steel}$

$-50 \times 10^2 = 248940.375 \times \Delta l_{steel} + 81655.50 \times 0.12$

$-5000 = 248940.375 \times \Delta l_{steel} + 4748.66$

$$\Delta l_{steel} = \frac{4798.66}{248940.375}$$

$= 0.019$ mm

$\Delta l_{gun\ metal} = 0.019 + 0.12 = 0.139$ mm

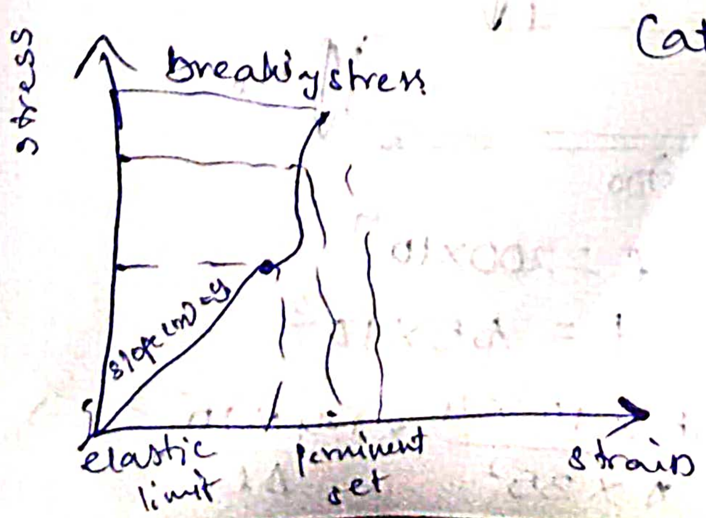
Q: Draw a typical stress-strain curve for a mild steel rod subjected to a tensile test & indicate the salient points on the curve?

Sol:- Stress vs strain curve is as follows

Stress = strain × young's modulus (y)
(in elastic limit)

strain = constant
(at permient set).

graph terminates with constant stress
(at breaking stress)

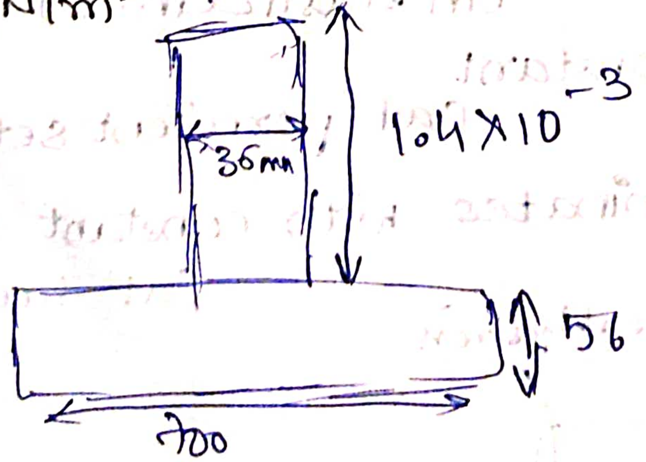


Slope of graph below it's elastic limit gives us the young's modulus of given material, as we know that the ratio of stress and strain below its elastic limit gives us young's modulus according to the hoke's law,

The body don't deform elasticity at perminant set, hence strain is constant here on reaching the value of breaking stress. the body deforms at faster rate, so the stress is constant here.

8. A Bar of mild steel has an overall length of 2.1m. The diameter up to 700 mm length is ~~56~~ 5.6 mm, the diameter of the remaining 1.4 m is 35mm. Calculate the extension of the bar due to a tensile load of 55kN. Take $E = 200 GN/m^2$

sol:-



$E = 200 \times 10^9$

$F = 55 \times 10^3$

$$200 \times 10^4 = \frac{55 \times 10^3 \times 4}{\pi \times 35^2} \times \frac{1.4 \times 10^{-3}}{\Delta l_1}$$

$$\Delta l_1 = \frac{55 \times 10^3 \times 4 \times 1.4 \times 10^{-3}}{\pi \times 35^2 \times 200 \times 10^9}$$

$$\Delta l_1 = 0.0004 \times 10^{-4}$$

$$200 \times 10^4 = \frac{55 \times 10^3 \times 4}{\pi \times (700)^2} \times \frac{56}{\Delta l_2}$$

$$\Delta l_2 = \frac{55 \times 10^3 \times 4 \times 56}{\pi \times (700)^2 \times 200 \times 10^9}$$

$$= 0.040 \times 10^{-9}$$

$$\Delta l_{net} = \Delta l_1 + \Delta l_2 = \boxed{0.0404 \times 10^{-9}}$$

UNIT-II

short answer questions;

1. what are the different types of beams?

Ans: Depending upon the end conditions there are different types of beams. the most common types are;

1. cantilever beam.
2. simply supported beam,
3. overhanging beams,
4. continuous beam, and
5. fixed (Restrained or encastred or built-in) beam

2. Define the shear force?

Ans: shear force is defined as the algebraic sum of all the forces acting on the beam on one side (left or right) of the section.

3. Define the bending moment?

Ans: bending moment is defined as the algebraic sum of all the moments of the forces on one side (left or right) of the section.

4. Define point of contra flexure.

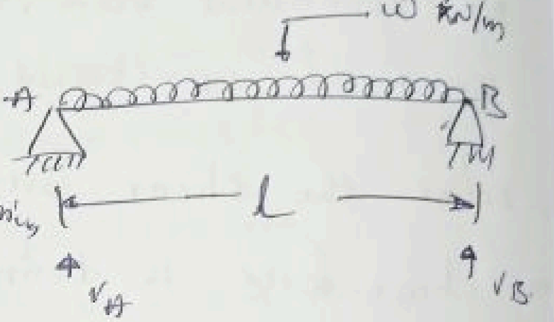
Ans: point of contra flexure is defined as a point in a beam where bending moment is zero or changes sign from positive to negative or vice-versa.

5. when will the bending moment is maximum?

when the maximum bending moment occurs in a beam,

As: the maximum bending moment occurs in a beam, when the shear force at that section is zero or changes the sign because at point of contra flexure the bending moment is zero.

6. what is maximum bending moment in a simply supported beam of span 'L' subjected to UDL load over the entire span?



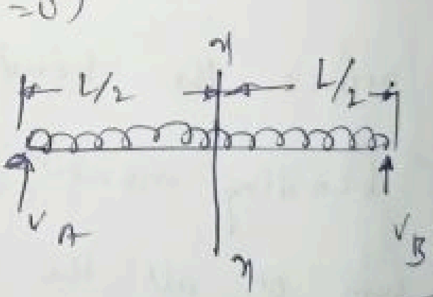
Sol. from equations of equilibrium

$V_A + V_B = wL$

from symmetry $V_A = V_B = \frac{wL}{2}$

(or take moment @ $A/B = 0$)

At section x-x;

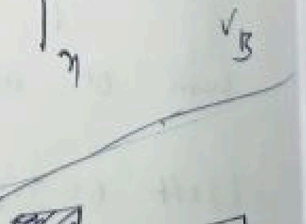


$\text{moment} = V_A(L/2) = \frac{w \times L}{2} \times \frac{L}{2}$

$= wL^2/4 - wL^2/8$

$= \frac{wL^2}{8}$;

7. Define the cantilever beam?



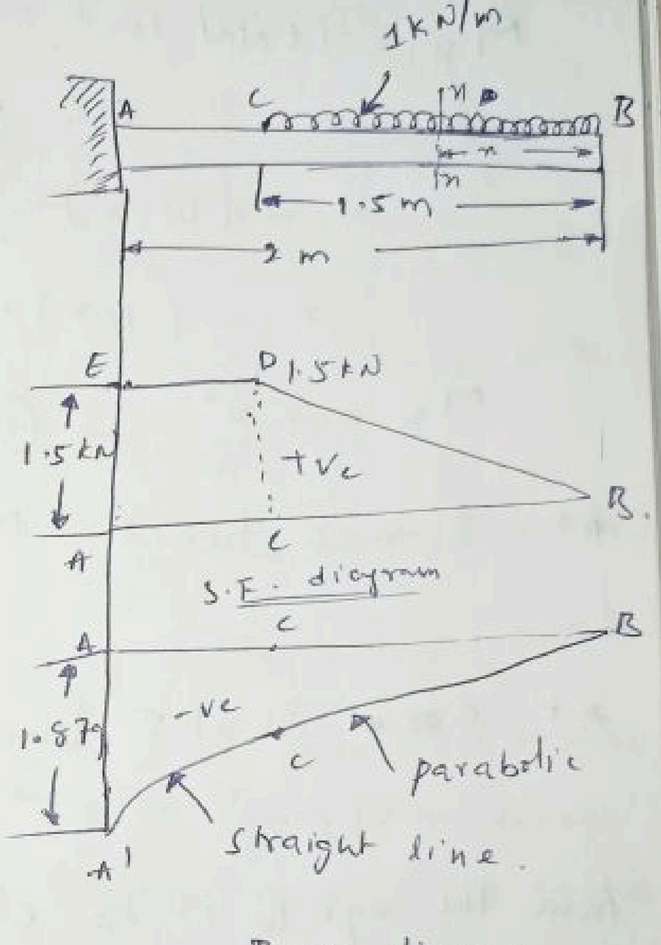
As. A cantilever beam is rigidily fixed at only one end when a load is applied to the cantilever a reaction and a resisting moment are produced at the fixed end.

PART-B.

1. A cantilever of length 2m carries a uniformly distributed load of 1kN/m run over a length of 1.5m from the free end. Draw the SF and Bm diagrams for the cantilever.

Sol. U.D.L :-

$w = 1 kN/m$



B.m diagram

Shear force diagram;

consider any section b/w C and B a distance x from the free end B. the shear force at the section B given by,

$S.F_x = wx$

$= 1 \times x$

At B, $x = 0$ hence $S.F_B = 1 \times x$

$= 1 \times 0$

$S.F_B = 0$

At C, $x = 1.5$ hence

$S.F_C = 1 \times x$

$= 1 \times 1.5$

$= 1.5 kN.$

the shear force b/n A and C remains constant those is no load b/n A and C.

Bending moment diagram;

i. The bending moment at any section b/w C and B at a distance x from the free end B is given by

M_x = -(total load on right portion) × Distance of C.G of right portion from x.

$$= -(w \cdot x) \cdot \frac{x}{2}$$

$$= -(1 \cdot x) \cdot \frac{x}{2}$$

$$M_x = -\frac{x^2}{2} \rightarrow (1)$$

At B $x=0$ hence $M_B = \frac{-x^2}{2} = -\frac{0^2}{2}$

$$M_B = 0$$

At C $x = 1.5$ hence $M_C = \frac{-x^2}{2} = -\frac{1.5^2}{2}$

$$M_C = -1.25 \text{ N-m}.$$

From the eqn (1) it is clear that the bending moment varies according to parabolic law b/n C and B

ii). The bending moment at any section b/n A and at a distance x from the free end B is obtained

total load due to u.d.l $= w \times 1.5 = 1.5$ kN ($w = 1$ kN

The load acting at a distance of $\frac{1.5}{2} = 0.75$m from the free end B (or) at a distance of $(x - 0.75)$ from any section b/n A and C.

$\therefore$ Bending moment of this load at any section b/n A and C at a distance x from free end.

$$M_x = -[\text{load due to u.d.l}) \times (x - 0.75) \quad \text{--- (1)}$$

$$= -1.5 \times (x - 0.75) \rightarrow \text{follows straight line.}$$

At C $x = 1.5\,m$, hence $M_x = -1.5\,(x - 0.75)$

$$M_C = -1.5\,(1.5 - 0.75)$$

$$M_C = -1.125\ N\text{-}m$$

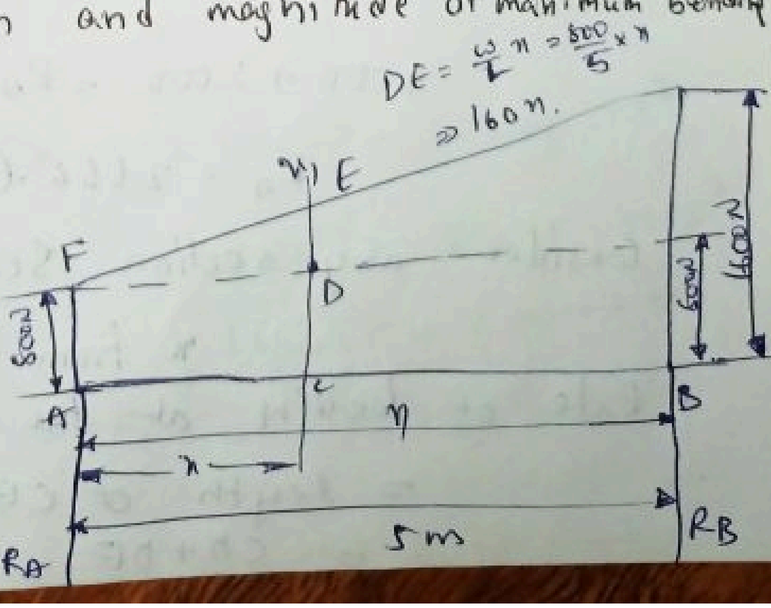
At A, $x = 2.0\,m$, hence $M_x = -1.5\,(x - 0.75)$

$$M_A = -1.5\,(2 - 0.75)$$

$$M_A = -1.875\ N\text{-}m.$$

2. A simply supported beam of length 5m carries uniformly increasing load 800 N/m run at one end to 1600 N/m run at the other end. Draw the shear force and bending moment diagrams for the beam. Also calculate the position and magnitude of maximum bending moment.

Sol: the load may be consisting of a uniformly distributed load of 800 N/m over the entire span and gradually varying load of zero at F



to 800 N/m at G

then total load on beam due to uniformly distributed load of 800 N/m $= 800 \times 5$
$= 4000 N$

then total load on beam due to triangular loading $= \frac{1}{2} \times 5 \times 800$
$= 2000 N$

Now, calculate the Reactions R_A and R_B

Taking the moments about A.

$$R_B \times 5 = 4000 \times \frac{L}{2} + 2000 \times \frac{2}{3} \times L$$

$$R_B \times 5 = 4000 \times \frac{5}{2} + 2000 \times \frac{2}{3} \times 5 \quad (\because L = 5)$$

$$R_B = \frac{4000}{2} + \frac{4000}{3}$$

$$R_B = 2000 + 1333.33$$

$$R_B = 3333.33 N$$

total load on Beam $= R_A + R_B$

$$4000 + 2000 = R_A + R_B$$

$$R_A = 2666.67 N$$

Consider any section Section $x-x$ at a distance x from A.

Rate of loading at the section $x-x$

$=$ length of CE

$=$ CD + DE

$= 800 + \frac{800}{5} \times x$

$\Rightarrow 800 + 160x$

total load on length Ax

$=$ Area of load diagram ACDEF

$=$ Area of rectangular + Area of Δ DEF

$= 800x + \frac{1}{2} \times DC \times 160x$

$= 800x + 80x^2$

Now the S.F at the section $x-x$

$S.F_x$ = RA − load on length Ax

$= RA - (800x + 80x^2)$

$S.F_x = 2666.67 - 800x - 80x^2 \rightarrow (1)$

from eqn (1) the shear force varies b/n A&B according to parabolic law.

At A $x = 0$ $S.F_x = 2666.67 - 800x - 80x^2$

$S.F_A = 2666.67 - 800 \times 0 - 80 \times 0^2$

$S.F_A = +2666.7$

At B $x = 5$ $S.F_x = 2666.7 - 800x - 80x^2$

$S.F_B = 2666.67 - 800 \times 5 - 80 \times 5^2$

$= 2666.67 - 4000 - 2000$

$S.F_B = -3333.33N$

let us find the possition of zero shear.

$$S.F_x = 2666.67 - 800x - 80x^2$$

$$0 = 2666.67 - 800x - 80x^2$$

$$x^2 + 10x - \frac{2666.67}{80} = 0$$

$$x^2 + 10x - 33.33 = 0$$

$$x^2 + 10x - 33.33 = 0$$

the above eqn is quadratic eqn.

$$x = \frac{-10 \pm \sqrt{10^2 - 4 \times 1 \times -33.33}}{2 \times 1}$$

$$x = \frac{-10 + 15.274}{2} \text{ (neglecting -ve root)}$$

$$x = 2.637.$$

<u>Bending</u> <u>moment diagram</u>;

the B.M at the section $x-x$

$$B.M_x = R_A \times x - 800 \times x \times \frac{x}{2} - \frac{1}{2} \times x \times 160 x \times \frac{x}{3}$$

$$= 2666.67x - 400x^2 - \frac{80}{3} x^3 \rightarrow (2)$$

eqn (2) the B.M b/n A and B varies cubic law

At A $x = 0$ $B.M_A = 0$

At B. $x = 5$ $B.M_B = 0$

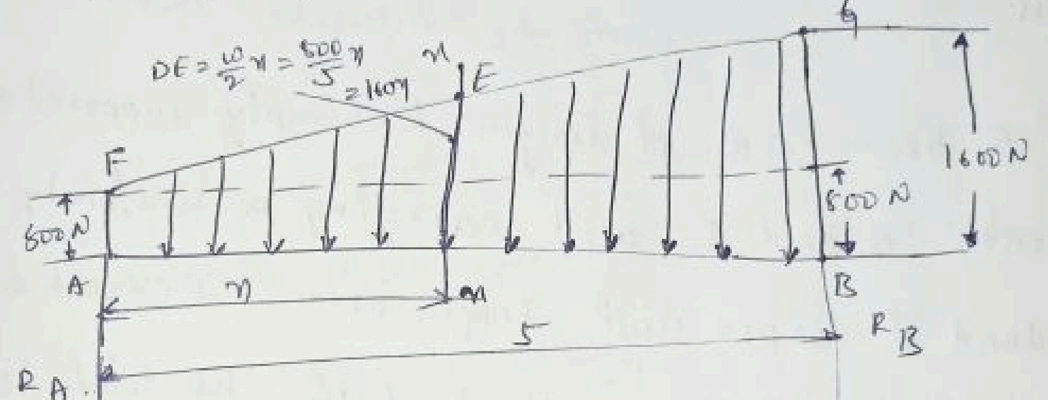
the Maximum B.M occurs where S.F is zero But S.F is zero at a distance of 2.637m from A.

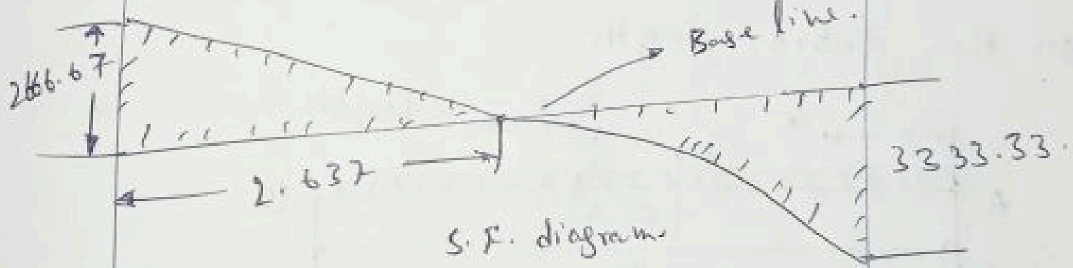
$$x = 2.637$$

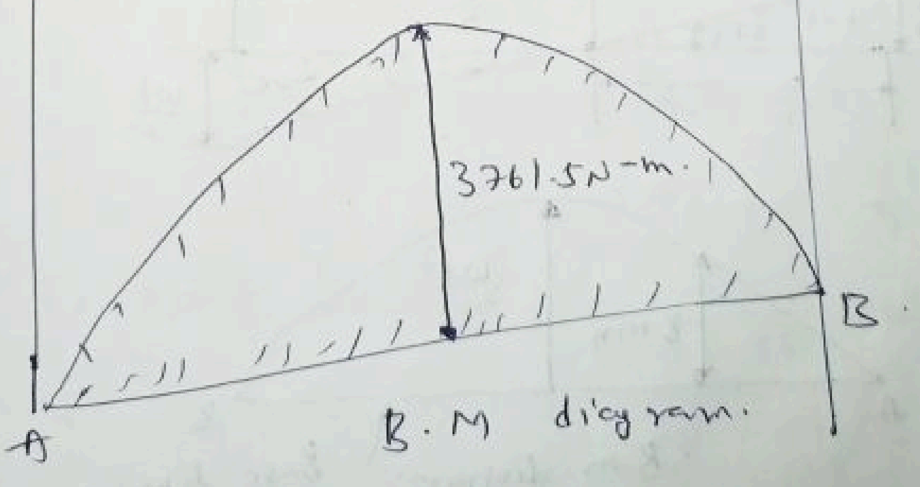
$$\therefore \text{Max. B.M} = 2666.67x - 400x^2 - \frac{80}{3}x^3$$

$$= 2666.67 \times 2.637 - 400 \times (2.63)^2 - \frac{80}{3} \times (2.63)^3$$

$$\therefore \text{Max B.M} = 3761.5 \ \text{N-m}$$



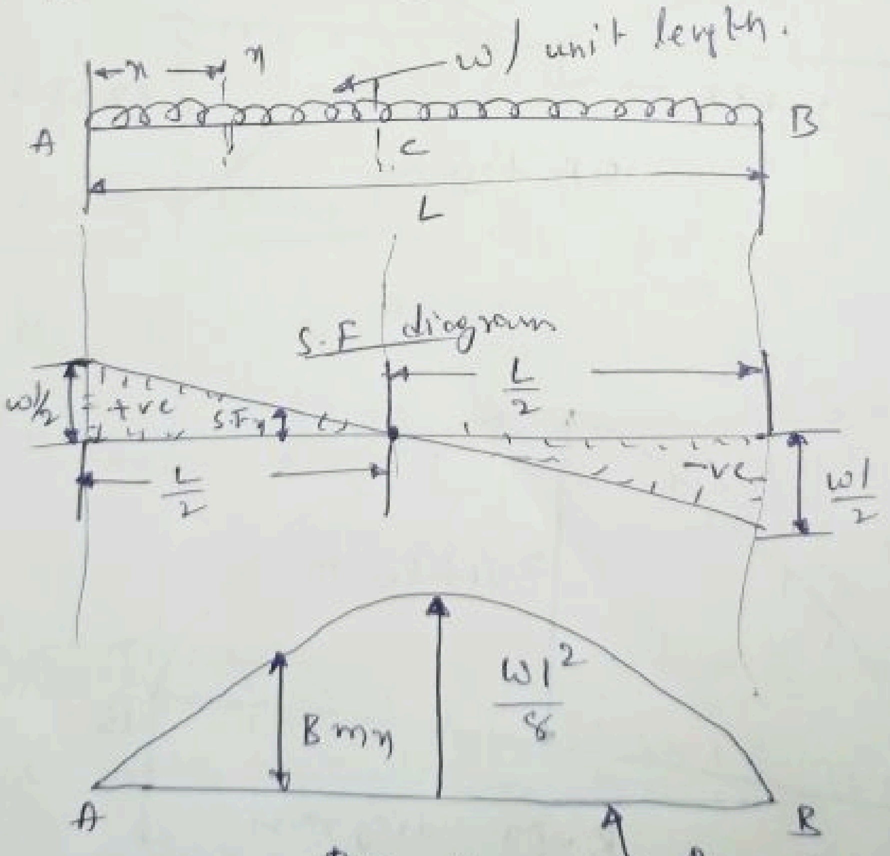




3. Draw the shear force and bending moment diagram for a simply supported beam carrying a uniformly distributed load.

Ans: S.F & B.M diagram for a simply supported beam carrying a uniformly distributed load.

A beam AB of length L simply supported at the ends A and B and carrying a uniformly distributed load of w per unit length over the equal and their magnitude will be half the total load on the entire length.



let R_A = Reaction at A

R_B = Reaction at B.

diagrams
niformly
carrying
d at the
y distributed
and
load

$\therefore R_A = R_B = \frac{wL}{2}$

consider any section x at a distance x from the left end A.

the shear force at the section is given by

$$S.F_x = +R_A - w.x$$

$$S.F_x = +\frac{wl}{2} - w.x \longrightarrow (1)$$

from eqn (1), it is clear that the shear force varies according to straight line law.

the values of shear force at different points.

At A, $x=0$ hence $S.F_x = +\frac{wl}{2} - wx$

$$S.F_A = +\frac{wl}{2} - w.0$$

$$S.F_A = \frac{+wl}{2}$$

At B, $x = L$ hence $S.F_x = +\frac{wl}{2} - w.x$

$$S.F_B = +\frac{wl}{2} - wL$$

$$S.F_B = -\frac{wl}{2}$$

At C, $x = \frac{L}{2}$ hence $S.F_x = +\frac{wl}{2} - w.x$

$$S.F_C = +\frac{wl}{2} - \frac{wL}{2}$$

$$S.F_C = 0$$

the bending moment at the section x at a distance x from the left end A is given by

$$B.M_x = +R_A \cdot x - w \cdot x \cdot \frac{x}{2} \qquad R_A = \frac{wL}{2}$$

$$= \frac{wL}{2} \cdot x - \frac{wx^2}{2} \rightarrow (2).$$

from eq^n (2). It is clear that B.M varies according to parabolic law.

the value of B.M at different points are

At A, $x = 0$ hence $B.M_x = \frac{wL}{2}x - \frac{wx^2}{2}$

$$B.M_A = \frac{wL}{2} \cdot 0 - w\frac{0^2}{2}$$

$$B.M_A = 0$$

At B, $x = L$ hence $B.M_x = \frac{wL}{2} \cdot x - \frac{wx^2}{2}$

$$B.M_B = \frac{wL}{2} \cdot L - \frac{wL^2}{2}$$

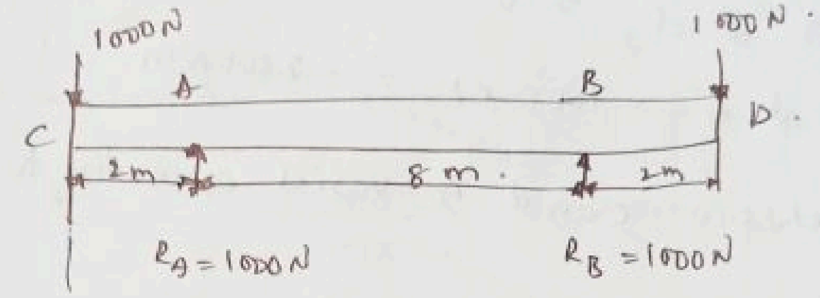
$$B.M_B = 0$$

At C, $x = \frac{L}{2}$ hence $B.M_x = \frac{wL}{2}x - \frac{wx^2}{2}$

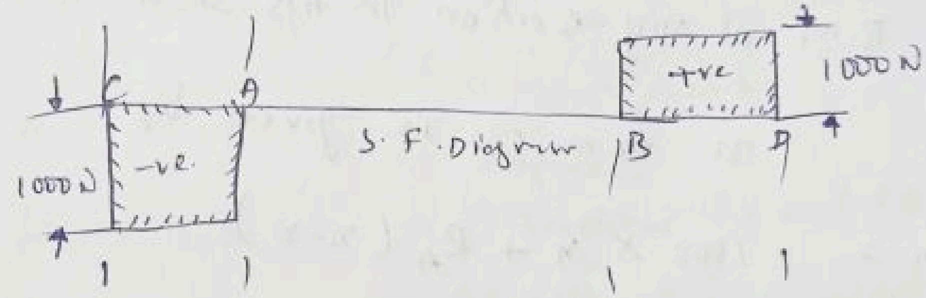
$$B.M_C = \frac{wL}{2} \cdot \frac{L}{2} - \frac{wL^2}{2}$$

$$B.M_C = +\frac{wL^2}{8}$$

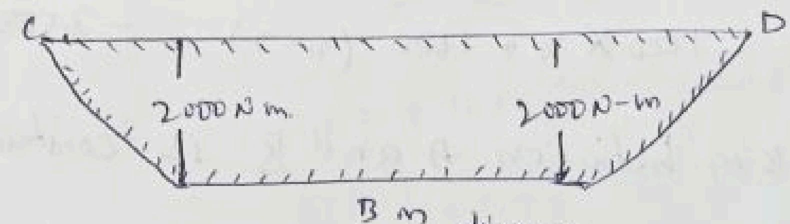
A beam of length 12m is simply supported at two supports which are 8m apart, with an overhang of 2m on each side as shown in figure. the beam carries a point load of 1000 N at each end. Draw the shear force and bending moment diagrams.



b)



c)



B m diagram.

As the loading on the beam is symmetrical. hence reactions R_A and R_B will be equal and their magnitude will be half of the total load

$$R_A = R_B = (1000 + 1000)/2 = 1000\ N$$

S. F at C = -1000 N

S.F remains constant (i.e., = 1000 N) between C and A

S.F at A = $1000 + R_A = -1000 + 1000 = 0$

S.F. remains constant (i.e., = 0) between A and

S.F at B = $0 + 1000 = + 1000 N$.

S.F remains constant (i.e, 1000 N) between B and

B.M. Diagram

B.M at C = 0

B.M at A = $-1000 \times 2 = -2000 Nm$.

B.M between C and A varies according to straight line law.

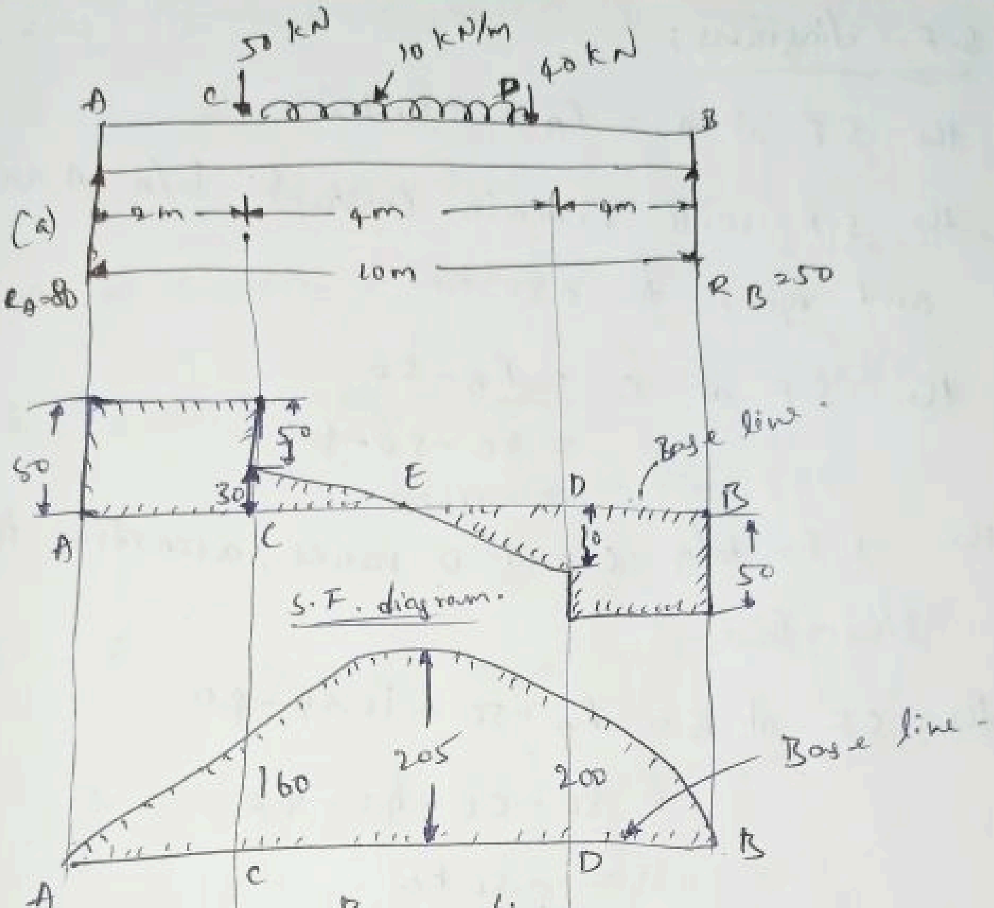
The B.M at any section in AB at a distance of x from C is given by

$$M_x = -1000 \times x + R_A (x-2)$$

$$= -1000 \times x + 1000 (x-2) = -2000 Nm.$$

Hence B.M between A and B is constant and equal to -2000 Nm.

5. A simply supported beam of length 10m carries a uniformly distributed load and two point loads as shown in figure. Draw the shear force and bending moment diagram for the beam. Also calculate the maximum bending moment.



B.M diagram.

Ans; first calculate the reactions R_A and R_B.

Taking moments of all forces about A

$$R_B \times 10 = 50\times 2 + 10\times 4\left(2+\frac{4}{2}\right) + 40(2+4)$$

$$= 100 + 160 + 240$$

$$= 500$$

$$R_B = \frac{500}{10}$$

$$R_B = 50\text{ kN}$$

total load on beam $= R_A + R_B$

$$50 + 10\times 4 + 40 = R_A + 50$$

$$R_A = 80\text{ kN}$$

S.F diagrams;

the S.F at A = R_A = +80 kN

the S.F will remain constant b/n A and C

and equal to +80 kN.

the S.F at C = $R_A - 50$

$= 80 - 50 - 40$

$= -10$ kN

the S.F b/n C and D varies, according to st[illegible]

line law.

the S.F at B = $R_A - 50 - 10 \times 4 - 40$

$= 80 - 50 - 40 - 40$

$= -50$ kN

the S.F remains constant b/n D and B and

is equal to −50 kN

the shear force is zero at point E b/n C

and D.

let the distance of E from point A is x.

Now the shear force at E = $R_A - 50 - 10(x-2)$

$= 80 - 50 - 10x + 20$

$E = 50 - 10x$

But shear force at E = 0

$0 = 50 - 10x$

$x = 5$ m

Bending Moment diagram:-

B.M at A = 0

B.M at B = 0

B.M at C $M_C = R_A \times 2 = 80 \times 2 = 160$ kN-m

B.M at D $M_D = R_A \times 6 - 50 \times 4 - 10 \times \cancel{4} \times \frac{4}{\cancel{2}}$

$= 80 \times 6 - 200 - 80$

$= 200$ kN-m.

At E, $x = 5$m and hence B.m at E

B, $M_E = R_A \times 5 - 50(5-2) - 10(5-2)\left(\frac{5-2}{2}\right)$

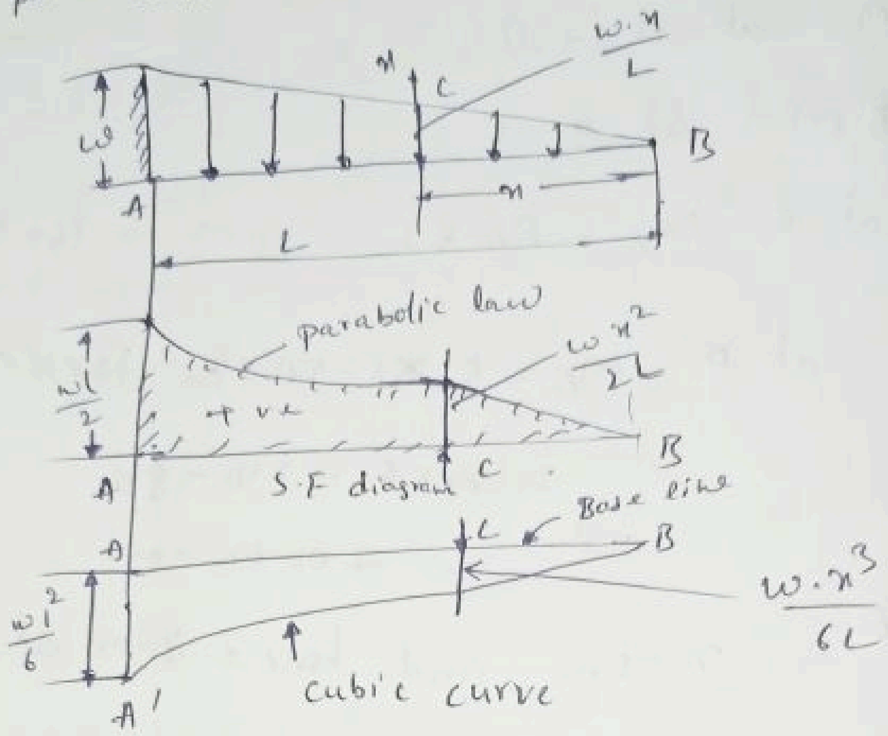
$= 80 \times 5 - 150 - 45$

$= 205$ kN-m.

the B.M b/n C and D varies according to parabolic law reaching a maximum value at E. the B.M b/n A and C and also b/n B and D varies according to linear law.

6. Draw the shear force and bending moment diagrams for a cantilever beam carrying a uniformly varying load.

Sol: A Cantilever of length L fixed at A and carrying a gradually varying load from zero at the free end to w per unit length at the fixed end.



B.M diagram

Take a section x at a distance x from the free end B.

let $S.F_x$ = shear force at the section x,

$B.M_x$ = Bending moment at the section x.

find the rate of loading at the section x. the rate of loading is zero at B and is w per metre run at A. this means that rate of loading for a length L is w per unit length.

hence rate of loading for a length of x will be $\frac{w}{L} \times x$ per unit length.

the shear force and the section x at a distance x from free end is given by.

$S.F_x$ = ~~Area~~ total load on the cantilever for a length x from the free end B.

$S.F_x$ = Area of triangle BCx

$= \frac{1}{2} \times$ base $\times$ height.

$= \frac{1}{2} \times x \times \frac{wx}{L}$

$S.F_x = \frac{wx^2}{2L}$ $\Rightarrow$ follows parabolic law.

At B, $x = 0$ hence $S.F_x = \frac{w.x^2}{2L}$

$S.F_B = \frac{w.0^2}{2L}$

$S.F_B = 0.$

At A, $x = L$ hence $S.F_x = \frac{w.x^2}{2L}$

$S.F_A = \frac{w.L^2}{2L}$

$S.F_A = \frac{wL}{2}$

The bending moment at the section x at a distance x from the free end is given by.

M_x = $-$(total load for a length x) $\times$ Distance of the load from x

= $-$(Area of triangle BCx) $\times$ Distance of C.G of the triangle from x.

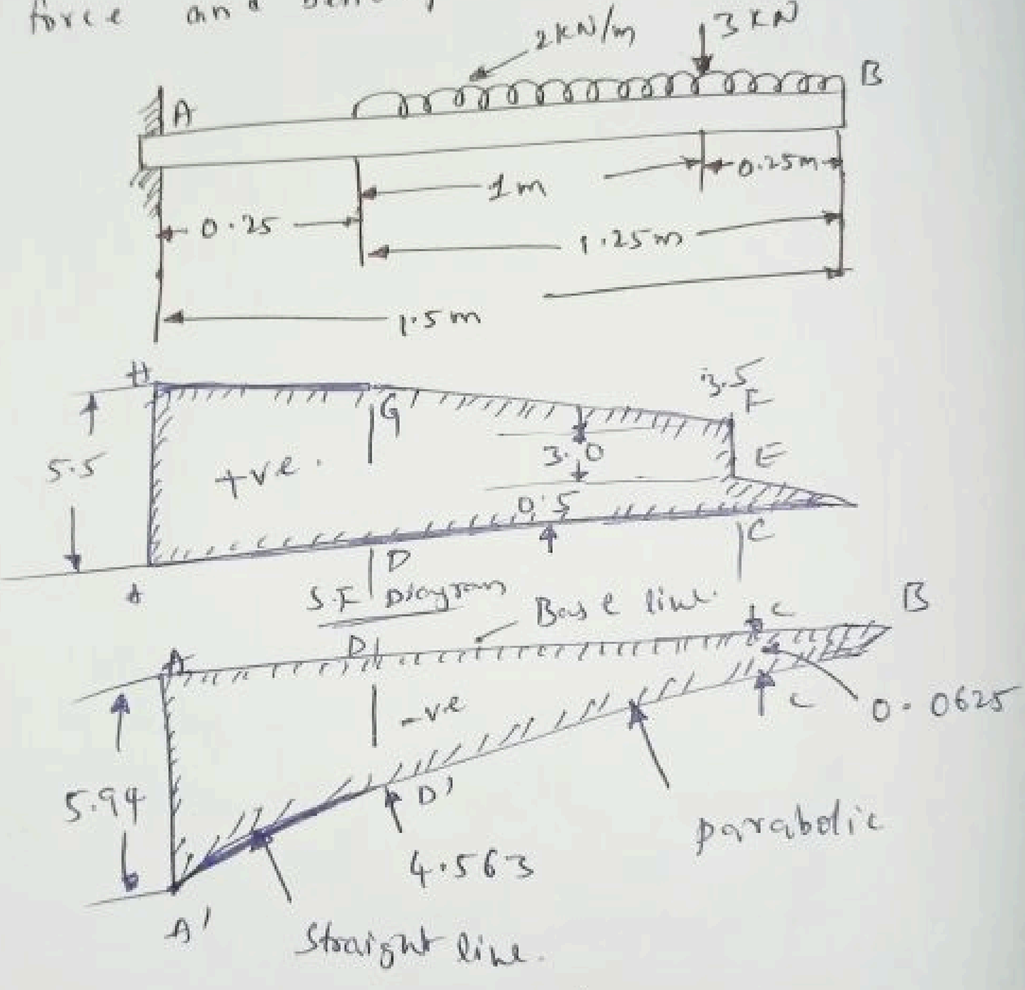
$= -\left(\frac{wx^2}{2L}\right) \times \frac{x}{3}$

$M_x = -\frac{wx^3}{6L}$ $\rightarrow$ follows the cubic law

At B, $x = 0$ hence $M_B = \frac{-10 \times 0^3}{6L} = 0$

At A, $x = L$ hence $M_A = -wL^3/6L = \frac{-wL^2}{6}$.

8. A cantilever 1.5m long is loaded with a uniformly distributed load of 2 kN/m run over a length of 1.25m from the free end. It is also carries a point load of 3kN at a distance of 0.25m from the free end. Draw the shear force and bending moment diagrams of the cantilever.



B.m diagram

Calculation for SFD;

SF at point B = 0

S.F just right to C = 2×0.25 = 0.50kN

S.F at point C = 0.50+3 = 3.50 kN

S.F just to D = 3.5+ 2×1 = 5.5kN

SF at point D = 5.5 kN

SF at point D = 5.5 kN

S.F just right to A = 5.5 kN

calculation for BMD;

BM at point B = 0

BM at point C $= -2 \times 0.25 \times \frac{0.25}{2}$

$= -0.0625$ kN-m

BM at point D $= -2 \times 1.25 \times \frac{1.25}{2} - 3 \times 1$

$= -4.5625$ kN-m

BM at point A $= -2 \times 1.25 \times \left[\frac{1.25}{2} + 0.25\right] - 3 \times \left[1 + 0.25\right]$

$= -5.9375$ kN-m.

UNIT - III

1. Write down the relation b/n maximum shear stress and average shear stress for circular section?

→ the maximum shear stress will be 1.5 times the average shear stress. for circular section beam, the shear stress distribution has a parabolic variation the shear stress is maximum when y=0, at the neutral axis. the maximum shear stress will be 4/3 times of average shear stress.

2. What is the shape of shear stress distribution for rectangular section?

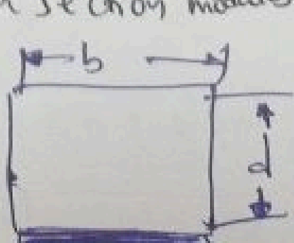
→ for rectangular section beam, the shear stress distribution is parabolic and maximum shear stress is at neutral axis of the section.

3. Define the neutral axis?

→ the line of intersection of neutral layer with beam cross-section is called neutral axis

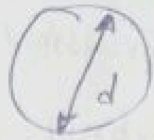
4. What is the formula of a section modulus of rectangular section?

Sol:- $Z = \frac{bd^2}{6}$



5. what is the formula of section modulus of circular section?

Ans:
$$Z = \frac{\pi d^3}{32}$$



6. write the any two assumptions in the theory of simple bending?

Ans; 1. the stresses do not exceed the limit of proportionality, and the moduli of elasticity in tension and compression are the same.

2. Transfer sections which are plane before bending remain plane after bending.

7. write down the equation of bending moment?

Ans: $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$. ; where M = BM, N mm

I = second moment of area (M.I)

σ = stress

y = distance of outer most layer from N.A

E = modulus of rigidity

8. Define the section modulus?

Ans: the ratio between moment of inertia and the distance between outer most layer and neutral axis is called section modulus, and denoted by Z, section modulus $Z = \frac{I}{y}$

where, I = moment of inertia

y = distance of outer most layer from NA.

long answer questions;

1. Derive an expression of section modulus of the following sections.

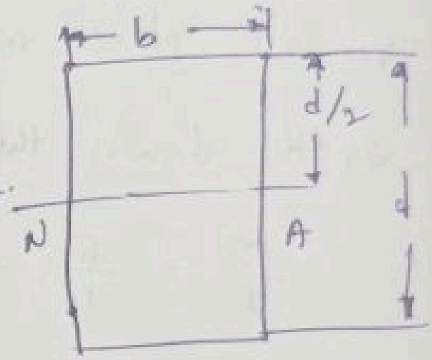
a. Rectangular section.

b. Hollow rectangular section

c. circular section D. Hollow circular section

Ans; (a) Rectangular section;

moment of inertia of a rectangular section about an axis through its C.G (or) through N.A.



$$I = \frac{bd^3}{12}$$

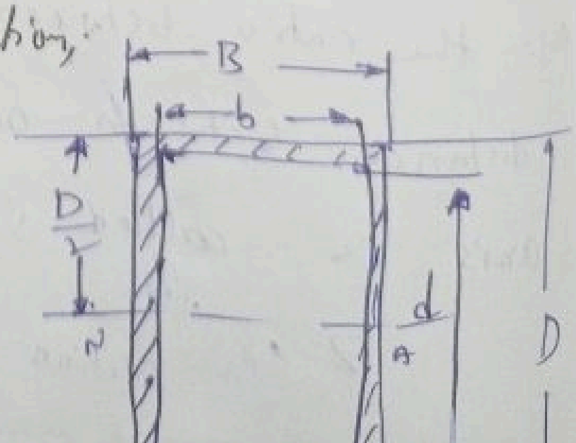
Distance of outer most layer from N.A.

$$y_{max} = d/2$$

$\therefore$ section modulus $Z = \frac{I}{y_{max}}$.

$$Z = \frac{\frac{bd^3}{12}}{d/2} \qquad Z = \frac{bd^3}{12} \times \frac{2}{d} \qquad \boxed{\therefore Z = \frac{bd^2}{6}}$$

b) Hollow rectangular section;



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}[BD^3 - bd^3$$

$$y_{max} = \frac{D}{2}$$

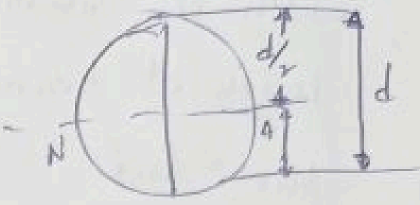
$$Z = I/y_{max}$$

$$z = \frac{\frac{1}{12}\left[BD^3 - bd^3\right]}{D/2}$$

$$\boxed{Z = \frac{1}{6D}\left[BD^3 - bd^3\right]}$$

3. <u>circular section;</u>

for circular section;



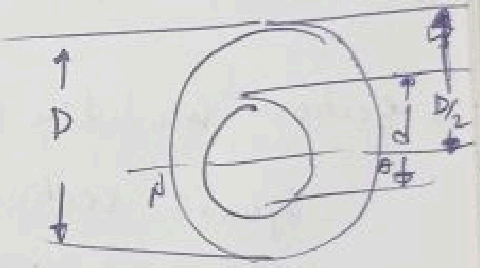
$$I = \frac{\pi d^4}{64}$$

$$y_{max} = \frac{d}{2} \qquad \therefore z = \frac{I}{y_{max}}$$

$$z = \frac{\frac{\pi d^4}{64}}{d/2} \Rightarrow Z = \frac{\pi d^4}{64} \times \frac{2}{d}$$

$$\boxed{Z = \frac{\pi d^3}{32}}$$

4. Hollow circular section;



$$I = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$y_{max} = \frac{D}{2}$$

$$Z = I/y_{max} = \frac{\frac{\pi}{64}\left(D^4 - d^4\right)}{D/2}$$

$$\boxed{Z = \frac{\pi}{32D}\left(D^4 - d^4\right)}$$

2. A square beam 20 mm x 20 mm in section and 2m long is supported at its ends. the beam fails when a point load of 400 N is applied at the centre of the beam. what uniformly distributed load per metre length will break a cantilever of the same

material 40mm wide, 60mm deep and 3m long.

Ans: Depth of beam $d = 20mm$

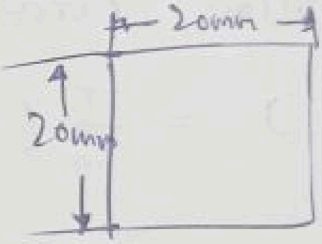
width of beam $b = 20mm$.

length of beam $L = 2m$

point load $W = 400N$

In this problem, the maximum stress for the simply supported beam is to be calculated first. As the material of the cantilever is same as that of simply supported beam, hence maximum stress for the cantilever will also be same as that of simply supported beam

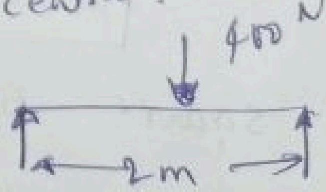
→ section modulus for the square section



$$Z = \frac{bd^2}{6} = \frac{b \times b^2}{6} \quad \boxed{d = b}$$

$$= \frac{b^3}{6} \qquad \therefore \frac{20^3}{6} = \frac{4000}{3} mm^3 = Z$$

maximum bending moment for simply supported beam carrying a point load at the centre.



$$M = \frac{WL}{4} = \frac{400 \times 2}{4}$$

$$= 200 \; N\text{-}m$$

$$M = 200 \times 10^3 \; N\text{-}mm$$

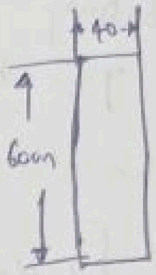
now, let us consider cantilever beam carrying u.d. load

$\sigma_{max} = 150 N/mm^2$

width of cantilever $b = 40mm$

depth of cantilever $d = 60mm$

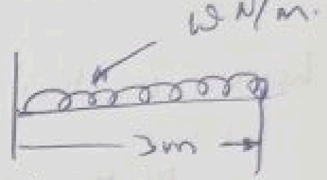
length of cantilever $L = 3m$



section modulus of rectangular section

$$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6}$$

$$Z = 2400 mm^3$$



maximum bending moment

$$M = \frac{wL^2}{2} = \frac{w \times 3^2}{2} = 4.5w\ N\text{-}m$$

$$= 4.5 \times w \times 10^3\ N\text{-}mm$$

$$M = 4500w\ N\text{-}mm$$

$$M = \sigma_{max} \times Z$$

$$4500w = 150 \times 2400$$

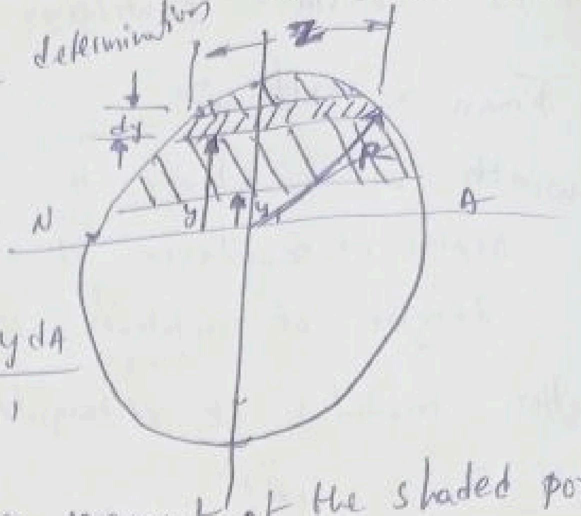
$$w = \frac{150 \times 2400}{4500} = 800\ N/m$$

4. Draw and explain shear stress distribution across circular section.

A. Let us find the shear stress distribution ~~exerted~~ in beam of circular cross-section. in a beam of circular cross-section

the value of Z width depends on y.

Using the expressions for the determination of shear stresses for any arbitrary shape or a arbitrary section.



$$\tau = \frac{FA\bar{y}}{ZI} = \frac{FA\int y\,dA}{ZI}$$

where $\int y\,dA$ is the Area moment of the shaded portion or the first moment of area.

here in this case "dA" is to be found out using the pythagoras theorem.

$$\left(\frac{Z}{2}\right)^2 + y^2 = R^2$$

$$\left(\frac{Z}{2}\right)^2 + y^2 = R^2 \Rightarrow \left(\frac{Z}{2}\right)^2 = R^2 - y^2 \text{ (or) } \frac{Z}{2} = \sqrt{R^2-y^2}$$

$$Z = 2\sqrt{R^2-y^2}$$

$$dA = Z\,dy = 2\sqrt{R^2-y^2}\,.\,dy$$

N.A for a circular cross-section $= \dfrac{\pi R^4}{4}$

hence,

$$\tau = \frac{FA\bar{y}}{ZI} = \frac{F}{\frac{\pi R^4}{4}\, 2\sqrt{R^2-y^2}} \int_{y_1}^{R} 2y\sqrt{R^2-y^2}\,dy$$

$$= \frac{4F}{\pi R^4 \sqrt{R^2-y^2}} \int_{y_1}^{R} y\sqrt{R^2-y^2}\,dy.$$ where R = Radius of the circle

the integration yields the final result to be

$$\tau = \frac{4F(R^2-y_1^2)}{3\pi R^4}$$

again this is a parabolic distribution of shear stress, having a maximum value when $y_1 = 0$

$$\tau_{max}\ m/y_1 = 0 = \frac{4F}{3\pi R^2}$$

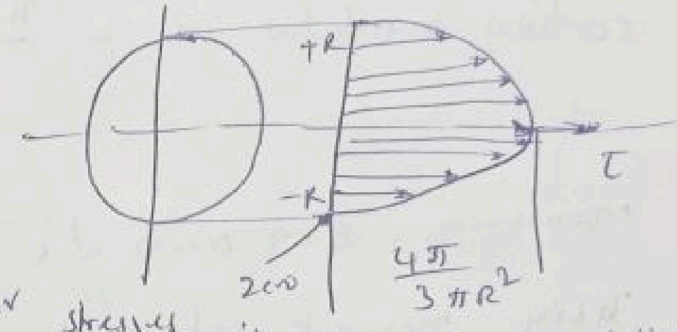
obviously at the ends of the diameter the have of $y_1 = \pm R$

thus $\tau = 0$. So, this again in parabolic distribution; maximum at the neutral axis also.

$$\tau_{avg} \text{ or } \tau_{avg} = \frac{F}{A} = \frac{F}{A e^2}$$

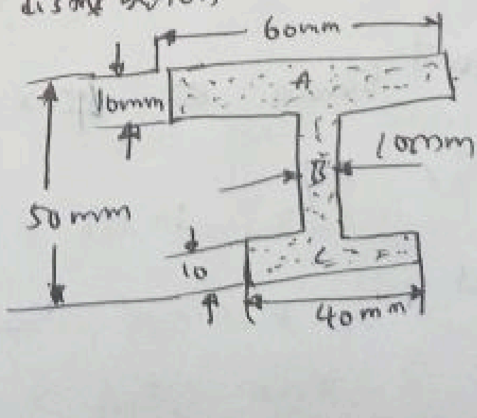
hence

$$\tau_{max}\ m = \frac{4}{3}\tau_{avg}.$$



the distribution of shear stresses is shown, which indicates a parabolic distribution.

5. calculate the on the top and bottom of the section as shown in fig. when the bending moment is 300N-m. Draw the shear stress distribution

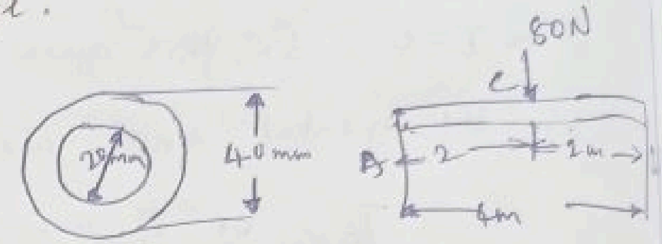


6. Calculate the maximum stress induced in a cast iron pipe of external diameter 40mm and of internal diameter 20mm and of length 4m. when the pipe is supported at its ends and carries a point load of 80N at its centre.

Sol.

find

stress, σ = ?



section modulus, $Z = \frac{\pi(D^4-d^4)}{32D} = \frac{\pi(40^2-20^2)}{32\times40}$

$= 2.945 mm^3$.

Maximum BM occurs at C

Taking moment about A,

$R_b \times 4$ = point load (w) $\times 2$

$\therefore R_b \times 4 = 80 \times 2$

$\therefore R_b = \frac{160}{4} = 40$ Nm.

~~and Bending~~ $M = \sigma . Z$

Now, M $\therefore M = \frac{wl^2}{8}$

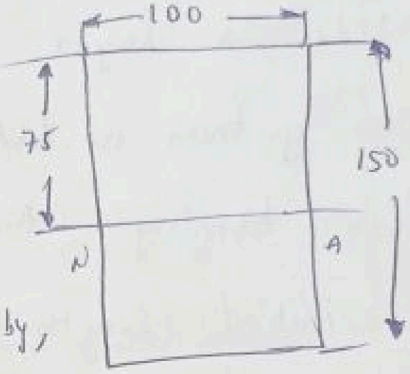
$= \frac{80\times2^2}{8} = 40$.

$40 = \sigma . Z$

$\Rightarrow$ $\sigma = \frac{40}{Z} = \frac{40}{2.945}$

$\therefore \sigma_{max} = 13.58 N/mm^2$.

7. A rectangular beam 100mm wide and 150mm deep is subjected to a shear force of 30kN. Determine the average shear stress and maximum shear stress.



Sol: Given;

width $b = 100 mm$

Depth, $d = 150mm$

i) Average shear stress is given by,

$$\tau_{avg} = \frac{F}{Area} = \frac{30,000}{b \times d}$$

$$= \frac{30,000}{100 \times 150} \Rightarrow 2 N/mm^2;$$

ii) maximum shear stress is given by equation.

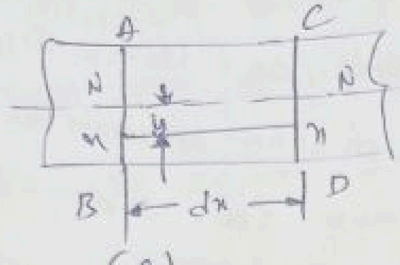
$$\tau_{max} = 1.5 \times \tau_{avg}$$

$$= 1.5 \times 2 = 3 N/mm^2,$$

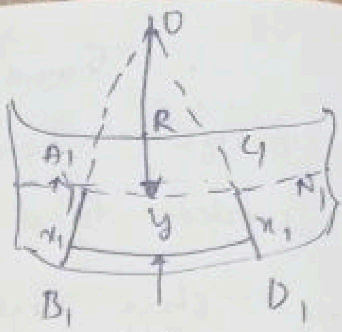
8. Derive bending equation $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$

Let AB and CD represent two transverse and parallel sections of a beam (fig a) and A_1B_1 and C_1D_1 be their position after bending (b). After bending the planes A_1B_1 and C_1D_1 subtend an angle θ at O and R be the radius of curvature.

$= 13.58 N/mm^2$



(a)



(b)

consider a layer (xx) in a beam at distance y from y from neutral layer 'N N' as in fig a. After bending this layer elongates to x_1x_1.

initial length of xx = length of NN

$$= R.\theta$$

After bending length of xx $= (R+y)\theta$

Strain in the layer xx $= \dfrac{(R+y)\theta - R\theta}{R\theta} = \dfrac{y}{R}$

the beam is stressed within the elastic limit

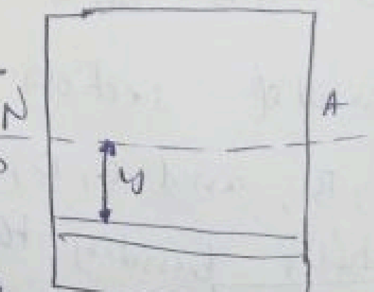
$\therefore$ stress, σ = strain x young's modulus.

$$\sigma = \frac{y}{R} \cdot E$$

or $\dfrac{\sigma}{y} = \dfrac{E}{R}$. (E & R are constant)

fig c. shows a cross-section of beam.

consider a layer at distance 'y' from NA.



force on this layer = stress x area

$$= \frac{E}{R} \cdot y \cdot dA$$

fig. c

$\therefore dA$ = cross-sectional area of layer the moment of this force about NA $= \frac{E}{R} y \cdot dA \cdot y$

$$= \frac{E}{R} y^2 \cdot dA$$

total moment (moment of resistance),

$$M = \frac{E}{R} \int y^2 . dA \qquad (E \,\&\, R \text{ are constant})$$

$$\therefore \quad M = \frac{E}{R} \cdot I$$

or $\frac{M}{I} = \frac{E}{R}$ But $\frac{E}{R} = \frac{\sigma}{y}$

Thus, a general bending equation can be written as,

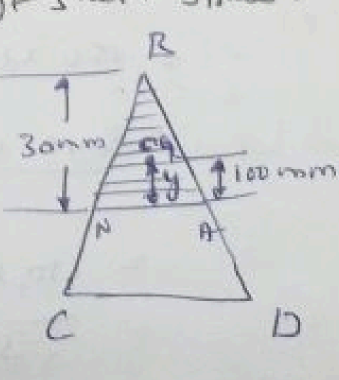
$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

3. A beam of cross-section of an isosceles triangle is subjected to a shear force of 30kN at a section where base width 150mm and height 450mm. Determine.

a) horizontal shear stress at a neutral axis

b) the distance from the top of the beam where shear stress is maximum.

c) what is the maximum value of shear stress.



Sol. Given ;

Shear force at the section, $f = 30 kN = 30{,}000 N$

Base width, CD = 150 mm

height, h = 450mm.

i). horizontal shear stress at the neutral axis.

→ The neutral axis of the triangle is at a distance $\frac{h}{3}$ from base or $\frac{2h}{3}$ from the apex.

∴ Neutral axis from B = $\frac{2 \times 450}{3}$ = 300mm.

→ The width of the section at neutral axis is obtained from similar triangles BCD and BNA A1.

$$\frac{NA}{CD} = \frac{300}{450} \quad \therefore NA = \frac{300}{450} \times CD$$

$$= \frac{300}{450} \times 150 = 100mm.$$

Shear stress at any section is given by equation

$$\tau = F \times \frac{A \times \bar{y}}{I \times b}$$

where.
τ = shear stress
F = shear force
A = Area above axis (shaded region)

$$A \Rightarrow \frac{NA \times 300}{2} = \frac{100 \times 300}{2} = 15000mm^2$$

$\bar{y}$ = Distance of the C.G of the area A from NA.

$$= \frac{1}{3} \times 300 = 100 \text{ mm}.$$

I = M.O.I of the total section about NA

$$= \frac{\text{Base width} \times \text{height}^3}{36} \quad \left[\frac{B \times h^3}{36}\right. \text{ where B = Base width}$$

$$= \frac{150 \times 450^3}{36} mm^4$$

[b = Actual width of section = 100mm

Now, Substitute

$$\tau = 30,000 \times \frac{15000 \times 100}{\frac{150 \times 450^3}{36} \times 100} = 1.18 N/mm^2$$

ii) the distance from the [...]

maximum;

→ Let the shear stre[...]

at the section EF [...]

n from the top of [...]

In fig. 8.3 (b) [...]

similar triangl[...]

$\frac{BF}{CD}$ [...]

∴ EF = [...]

The shear str[...]

as ;

where F = [...]

A = Area of s[...]

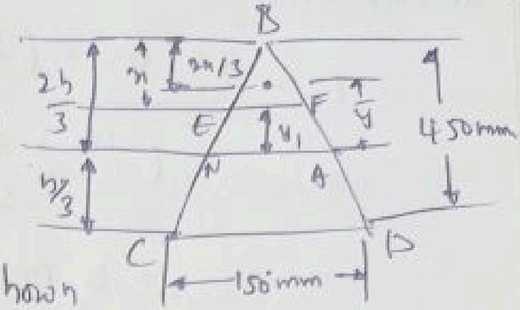
$= \frac{BF \times h}{2}$ [...]

$\bar{y}$ = Distanc[...]

$= \frac{2h}{3}$ [...]

S.F is to be obtained = NA = 100

(ii) the distance from the top of the beam where shear stress is maximum;



→ let the shear stress is maximum at the section EF at a distance x from the top of the beam as shown in fig. 8.3 (b). the distance EF is obtained from similar triangles BEF and BCD as

$$\frac{BF}{CD} = \frac{x}{450}$$

$$\therefore \quad EF = \frac{x}{450} \times CD = \frac{x}{450} \times 150 = \frac{x}{3}.$$

the shear stress at the section EF is given by eqution as;

$$\tau = F \times \frac{A \times \bar{y}}{I \times b} \quad \ldots \text{(ii)}$$

where $F = 30,000\ N$

A = Area of section above EF i.e., Area of shaded triangle BEF.

$$= \frac{BF \times x}{2} = \frac{x}{3} \times \frac{x}{2} = \frac{x^2}{6}.$$

$\bar{y}$ = Distance of C.G of the Area A from neutral axis

$$= \frac{2h}{3} - \frac{2x}{3} = \frac{2 \times 450}{3} - \frac{2x}{3} = \left(300 - \frac{2x}{3}\right)$$

I = M.O.I of ΔBCD about neutral axis

$$= \frac{150 \times 450^3}{36}\ mm^4$$

b = width of section EF $= \frac{x}{3}$

substituting these values in equation (ii), we get

$$\tau = \frac{30,000 \times \left(\frac{x^2}{6}\right) \times \left(300 - \frac{2x}{3}\right)}{\left(\frac{150 \times 450^3}{36}\right) \times \frac{x}{3}} = 0.0000395x\left(300 - \frac{2x}{3}\right)$$

$$= 0.0000395\left(300x - \frac{2x^2}{3}\right) \quad \ldots \text{(iii)}$$

for maximum shear stress $\frac{d\tau}{dx} = 0$

or $\quad 300 - \frac{2}{3} \times 2x = 0$ or $300 = \frac{4x}{3}$

$$x = \frac{300 \times 3}{4} = 225 \text{ mm.}$$

iii) <u>Value of maximum shear stress;</u>

→ the value of maximum shear stress will be obtained by substituting $x = 225$ mm. in eq (iii)

∴ maximum shear stress $= 0.0000395\left(300 \times 225 - \frac{2}{3} \times 225^2\right)$

$$= 1.333 \text{ N/mm}^2$$

PART A (EACH QUESTION CARRIES 1 MARK)

UNIT-IV

1) What do you mean by principal plane?

ANS: The planes which have no shear stress are known as principal planes.

2) What is the use of Mohr's circle?

ANS: It is used to find out the normal, tangential, resultant and principal stresses and their planes.

3) What is the radius of Mohr's circle?

ANS: Radius of Mohr's circle is equal to the maximum shear stress.

4) What are the planes along which the greatest shear stresses occurs?

ANS: Greatest shear stress occurs at the planes which is inclined at 45? to its normal.

5) What is mean by position of principal planes?

ANS: The planes on which shear stress is zero are known as principal planes. The position of principal is obtained by equating the tangential stress to zero.

6) What is another name of maximum principle stress theory?
ANS: Rakine's theory
7) The maximum tangential stress $\sigma_t = (\sigma_x \sin 2\theta)/2$ is maximum if, θ is equal to?
ANS: 45°
8) What is another name of maximum shear stress theory?
ANS: Guests theory
9) In Mohr's circle method, compressive direct stress is represented on?
ANS: Negative x-axis
10) Which theory gives satisfactory results for brittle materials?
ANS: Maximum principle stress theory
11) The angle between normal stress and tangential stress is known as angle of?
ANS: Obliquity
12) Which theory gives satisfactory results for ductile materials?
ANS: Maximum shear stress theory
13) Which of the following formulae is used to calculate tangential stress, when a member is subjected to stress in mutually perpendicular axis and accompanied by a shear stress?
ANS: $[(\sigma_x - \sigma_y)/2]\sin\theta - \tau\cos 2\theta$
14) When a component is subjected to axial stress the normal stress is?

ANS: $\sigma_n = \sigma_x \cos^2\theta$

15) What do you mean by theories failure?

ANS: Theories of failure are those theories which help us to determine the safe dimensions of a machine component when it is subjected to combined stresses due to various loads acting on it during its functionality.

PART B (EACH QUESTION CARRIES 12 MARKS)

UNIT-IV

1) A M.S. shaft having yield stress as 232 MPa is subjected to the following stresses.
   σ_x = 120 MPa, σ_y= − 60 MPa and τ_{xy} = 36 MPa. Find the factor of safety using:
   i. Rankine's theory of failure,
   ii. Guest's theory of failure

Given data: Yield stress, $\sigma_{ys} = 232$ MPa

$\sigma_x = 120$ MPa, $\sigma_y = -60$ MPa and $\tau_{xy} = 36$ MPa.

According to Rankine's theory or maximum normal stress theory of failure

$$\sigma_e = \frac{1}{2}\left[(\sigma_x + \sigma_y) + \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}\right]$$

$$\sigma_e = \frac{1}{2}\left[(120 - 60) + \sqrt{[120 - (-60)]^2 + 4(36)^2}\right] = 126.93 \text{ MPa}$$

$$FOS = \frac{\sigma_{ys}}{\sigma_e} = \frac{232}{126.93} = 1.828$$

(ii) According to Guest's theory or max shear stress theory of failure

$$\tau_e = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

or

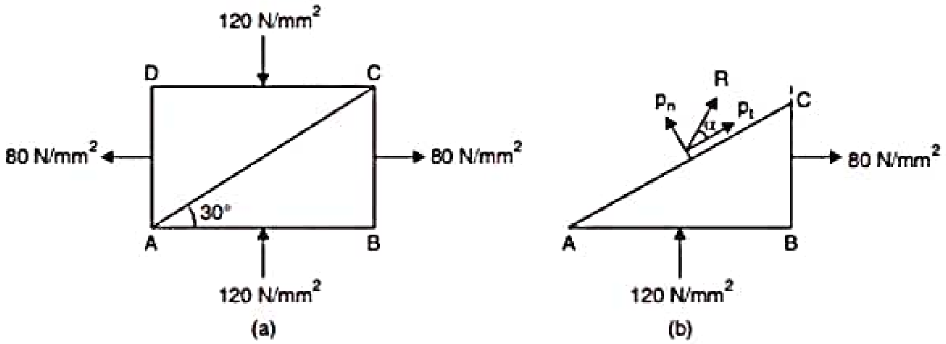
$$\sigma_e = \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \sqrt{[120 - (-60)]^4 + 4(36)^2}$$

$$\sigma_e = 193.87 \text{ MPa}$$

$$\therefore \quad FOS = \frac{\sigma_{ys}}{\sigma_e} = \frac{232}{193.87} = 1.197$$

2)

[redacted] *The direct stresses at a point in the strained material are 120 N/mm² compressive and 80 N/mm² tensile as shown in Fig. 11.9. There is no shear stress. Find the normal and tangential stresses on the plane AC. Also find the resultant stress on AC.*



[redacted]

Solution: The plane AC makes 30° (anticlockwise) to the plane of p_x (y-axis). Hence $\theta = 30°$.

$$p_x = 80 \text{ N/mm}^2 \quad p_y = -120 \text{ N/mm}^2 \quad q = 0$$

$$\therefore \qquad p_n = \frac{p_x + p_y}{2} + \frac{p_x - p_y}{2} \cos 2\theta + q \sin 2\theta$$

$$= \frac{80 - 120}{2} + \frac{80 - (-120)}{2} \cos (2 \times 30) + 0$$

$$= -20 + 100 \cos 60$$

Thus $\quad \mathbf{p_n = 30 \text{ N/mm}^2}$

$$p_t = \frac{p_x - p_y}{2} \sin 2\theta - q \cos 2\theta$$

$$= \frac{80 - (-120)}{2} \sin (2 \times 30) - 0$$

Thus $\quad \mathbf{p_t = 86.6 \text{ N/mm}^2.}$

The resultant of p_n and p_t is given by

$$p = \sqrt{p_n^2 + p_t^2} = \sqrt{30^2 + 86.6^2}$$

$$\mathbf{p = 91.65 \text{ N/mm}^2}$$

Angle made by the resultant stress with p_t is given by $\tan \alpha = \dfrac{p_n}{p_t} = \dfrac{30}{86.6}$

$$\therefore \qquad \alpha = 19.1°$$ [redacted].

3)

[redacted] *The state of stress at a point in a strained material is as shown in Fig. 11.10. Determine*

(i) the direction of principal planes

(ii) the magnitude of principal stresses and

(iii) the magnitude of maximum shear stress.

[redacted]

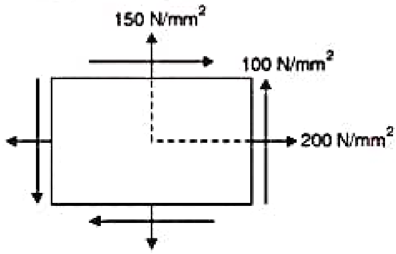


Fig. [redacted]

Solution: $p_x = 200$ N/mm² $\quad p_y = 150$ N/mm² $\quad q = 100$ N/mm²

Let the principal plane make anticlockwise angle θ with the plane of p_x *i.e.* with y-axis. Then

$$\tan 2\theta = \frac{2q}{p_x - p_y} = \frac{2 \times 100}{200 - 150} = 4$$

$$\therefore \quad 2\theta = 75.96 \quad \text{and} \quad 75.96 + 180$$

$$\therefore \quad \boldsymbol{\theta = 37.98^\circ \quad \text{and} \quad 127.98^\circ}$$

$$p_1 = \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= \frac{200 + 150}{2} + \sqrt{\left(\frac{200 - 150}{2}\right)^2 + 100^2}$$

$$= 175 + 103.08$$

$$\boldsymbol{p_1 = 278.08 \text{ N/mm}^2}$$

and

$$p_2 = \frac{p_x + p_y}{2} - \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$\therefore \quad p_2 = 175 - 103.08 = \boldsymbol{71.92 \text{ N/mm}^2}.$$

$$q_{max} = \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

i.e.,

$$\boldsymbol{q_{max} = 103.08 \text{ N/mm}^2}$$

4)

[redacted]. The state of stress in a material stressed to two-dimensional state of stress is as shown in Fig. [redacted]. Determine principal stresses and maximum shear stress and the planes on which they act.

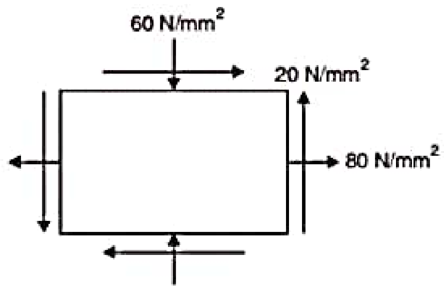


Fig. [redacted]

Solution:

$$p_{1,2} = \frac{p_x + p_y}{2} \pm \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

In this problem,

$$p_x = 80 \text{ N/mm}^2 \quad p_y = -60 \text{ N/mm}^2 \qquad q = 20 \text{ N/mm}^2.$$

$\therefore$
$$p_{1,2} = \frac{80 + (-60)}{2} \pm \sqrt{\left(\frac{80 - (-60)}{2}\right)^2 + 20^2}$$

$$= 10 \pm \sqrt{70^2 + 20^2}$$

$$= 10 \pm 72.8$$

$\therefore$
$$\mathbf{p_1 = 82.8 \text{ N/mm}^2}$$

and $p_2 = -\ 62.8\ \text{N/mm}^2$

$$q_{max} = \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= 72.8\ \text{N/mm}^2$$

Let θ be the inclination of principal stress to the plane of p_x. Then,

$$\tan 2\theta = \frac{2q}{p_x - p_y} = \frac{2 \times 20}{80 - 60} = 2$$

$\therefore$ $2\theta = 63.44°$ or $63.44 + 180$

$\therefore$ $\theta = 31.72°$ or $121.72°$

Planes of maximum shear make 45° to the above planes

$\therefore$ $\theta' = 15.86°$ and $60.86°$

5) A SAE 1045 steel rod (σ_{ys} = 309.9 MPa) of 80 mm diameter is subjected to a bending moment of 3 kNm and torque T. Taking Factor of safety as 2.5, find the maximum value of torque T that can be safely carried by rod according to:

(i) Maximum normal stress theory

(ii) Maximum shear stress theory

Given data: Material SAE 1045.

Yield stress, σ_{ys} = 309.9 MPa

FOS = 2.5 diameter d = 80 mm

$$\therefore \quad \text{Allowable stress, } \sigma_e = \frac{\sigma_{ys}}{\text{FOS}} = \frac{309.9}{2.5} = 123.96 \text{ MPa}$$

Bending moment, M_b = 3 kNm = 3×10^6 N-mm.

$$\therefore \quad \text{Bending stress, } \sigma = \frac{M_b \cdot C}{I} = \frac{3 \times 10^6 (80/2)}{(\pi/64 \times 80^4)} = 59.68 \text{ MPa} = \sigma_x$$

Torque, $M_t = T$

$$\therefore \quad \text{Shear stress, } \tau = \frac{M_t \cdot r}{J} = \frac{T \cdot (80/2)}{(\pi/32 \times 80)^4} = (9.95 \times 10^{-6}) \text{ MPa}$$

$$\therefore \quad \tau = \tau_{xy} = (9.95\ 10^{-6})\ T$$

(σ_y = 0, not given)

(i) According to maximum normal stress theory

$$\sigma_e = \frac{1}{2}\left[\sigma_x + \sqrt{\sigma_x^2 + 4\tau_{xy}^2}\right]$$

$$123.96 = \frac{1}{2}\left[59.68 + \sqrt{59.68^2 + 4(9.95 \times 10^{-6} T)^2}\right]$$

$\therefore$ Torque, $T = 8.971 \times 10^6$ N-mm = 8.971 kNm

(ii) According to maximum shear stress theory

$$\tau_e = \frac{1}{2}\sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

Assuming, $\tau_e = 0.5\ \sigma_e = 0.5 \times 123.96 = 61.98$ MPa

$$61.98 = \frac{1}{2}\sqrt{59.68^2 + 4(9.95 \times 10^{-6} T)^2}$$

Torque, $T = 5.46 \times 10^6$ N-mm = 5.46 kNm.

PART A (EACH QUESTION CARRIES 1 MARK)

UNIT-V

1) List out the modes of failure in thin cylindrical shell due to an internal pressure?

ANS: Circumferential or hoop stress and Longitudinal stress

2) What is torsion equation

ANS: $\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$

3) Define the shaft?

ANS: The shafts are the machine elements which are used to transmit power in machines.

4) What is formula of polar moment of inertia of hollow shaft?

ANS: $J=\Pi(D^4-d^4)/32$

5) What is formula of polar moment of inertia of solid shaft?

ANS: $J=\Pi D^4/32$

6) What is the equation of torque transmitted by shaft?

ANS: $T = \frac{\pi}{16}\tau D^3$

7) What is power transmitted by shaft?

ANS: If T is the applied Torque and ω is the angular velocity of the shaft

$$P=T\,\omega$$

8) What is the formula of longitudinal stress?

ANS: $\sigma_t = Pd/4t$

9) Define the angle of twist of shaft?

ANS: If a shaft of length L is subjected to a constant twisting moment T along its length, than the angle through which one end of the bar will twist relative to the other is known is the angle of twist.

10) Define the torsional stiffness?

ANS: The tensional stiffness k is defined as the torque per radius twist

11) Define the Hoop stress?

Ans: This is the stress which is set up in resisting the bursting effect of the applied pressure and can be most conveniently treated by considering the equilibrium of the cylinder.

12) Define the torsion?

ANS: Consider a shaft rigidly clamped at one end and twisted at the other end by a torque T = F.d applied in a plane perpendicular to the axis of the bar such a shaft is said to be in torsion

13) What is formula for maximum bending stress in shafts?

ANS: $\sigma_{max}=32M/\ \Pi D^3$

14) Define the modulus of elasticity in shear?

ANS: The ratio of the shear stress to the shear strain is called the modulus of elasticity in shear OR Modulus of Rigidity

15) What is the formula of circumferential stress?

ANS: $\sigma_c = Pd/2t$

PART B (EACH QUESTION CARRIES 12 MARKS)

UNIT-V

1)
A cylinder is 150 mm mean diameter and 750 mm long with a wall 2 mm thick. It has an internal pressure 0.8 MPa greater than the outside pressure. Calculate the following.

i. The circumferential strain.
ii. The longitudinal strain.
iii. The change in cross sectional area.
iv. The change in length.
iv. The change in volume.

Take $E = 200$ GPa and $\nu = 0.25$

SOLUTION

$\sigma_c = pD/2t = 30$ MPa $\quad\quad$ $\sigma_L = pD/4t = 15$ MPa

$\varepsilon_D = \Delta D/D = (pD/4tE)(2 - \nu) = 131.25\ \mu\varepsilon$

$\Delta D = 150 \times 131.25 \times 10^{-6} = 0.0196$ mm $\quad$ $D_2 = 150.0196$ mm

$A_1 = \pi \times 150^2/4 = 17671.1\ mm^2$ $\quad\quad$ $A_2 = \pi \times 150.0196^2/4 = 17676.1\ mm^2$

Change in area = $4.618\ mm^2$

$\varepsilon_L = \Delta L/L_1 = (pD/4tE)(1 - 2\nu) = 37.5\mu\varepsilon$
$\Delta L = 750 \times 37.5 \times 10^{-6} = 0.0281$ mm

Original volume = A_1L_1 = 13 253 600 mm^3
Final volume = A_2L_2 = 13 257 600 mm^3
Change in volume = 4000 mm^3
Check the last answer from equation 3.4
$\varepsilon_v = (pD/4tE)(5 - 4\nu) = 300 \times 10^{-6}$
Change in volume = $V_1 \times \varepsilon_v$ = 13 253 600 $\times 300 \times 10^{-6}$ = 4000 mm^3

2)

A shaft is made from tube. The ratio of the inside diameter to the outside diameter is 0.6. The material must not experience a shear stress greater than 500 kPa. The shaft must transmit 1.5 MW of mechanical power at 1500 rev/min. Calculate the shaft diameters.

SOLUTION

The important quantities are $P = 1.5 \times 10^6$ Watts, $\tau = 500 \times 10^3$ Pa, N = 1500 rev/min and d = 0.6D.

$N = 1500 \text{ rev/min} = 1500/60 = 25 \text{ rev/s}$ $\qquad P = 2\pi N T$

$$\text{hence } T = \frac{P}{2\pi N} = \frac{1.5 \times 10^6}{2\pi \times 25} = 9549.3 \text{ Nm}$$

$$J = \frac{\pi(D^4 - d^4)}{32} = \frac{\pi\{D^4 - (0.6D)^4\}}{32} = \frac{\pi\{D^4 - 0.36D^4\}}{32} = 0.08545D^4$$

$$\frac{T}{J} = \frac{\tau}{R} = \frac{2\tau}{D} \text{ hence } \frac{9549.3}{0.08545D^4} = \frac{2 \times 500 \times 10^3}{D} = \frac{9549.3}{0.08545 \times 2 \times 500 \times 10^3} = \frac{D^4}{D} = D^3$$

$$D^3 = 0.11175 \quad D = \sqrt[3]{0.11175} = 0.4816 \text{ m} = 481.6 \text{ mm} \qquad d = 0.6D = 289 \text{ mm}$$

3)

A shaft 50 mm diameter and 0.7 m long is subjected to a torque of 1200 Nm. Calculate the shear stress and the angle of twist. Take G = 90 GPa.

SOLUTION

Important values to use are D = 0.05 m, L = 0.7 m, T = 1200 Nm, $G = 90 \times 10^9$ Pa

$$J = \frac{\pi D^4}{32} = \frac{\pi \times 0.05^4}{32} = 613.59 \times 10^{-9} \text{ m}^4$$

$$\tau_{max} = \frac{TR}{J} = \frac{1200 \times 0.025}{613.59 \times 10^{-9}} = 48.89 \times 10^6 \text{ Pa or } 48.89 \text{ MPa}$$

$$\theta = \frac{TL}{J} = \frac{1200 \times 0.7}{90 \times 10^9 \times 613.59 \times 10^{-9}} = 0.0152 \text{ radian}$$

$$\text{Alternately } \theta = \frac{\tau L}{GR} = \frac{48.89 \times 10^6 \times 0.7}{90 \times 10^9 \times 0.025} = 0.0152 \text{ radian}$$

$$\text{Converting to degrees } \theta = 0.0152 \times \frac{180}{\pi} = 0.871°$$

4)

A sphere is 120 mm mean diameter with a wall 1 mm thick. The pressure outside is 1 MPa more than the pressure inside. Calculate the change in volume.

Take E = 205 GPa and ν= 0.26

SOLUTION

$\varepsilon_V = 3(\sigma/E)(1-\nu) = -324.87\mu\varepsilon$

(note the sphere shrinks hence the negative sign)

Original volume = $\pi D^3/6 = 904778 \text{ mm}^3$

Change in volume = $-904778 \times 324.87 \times 10^{-6} = -294 \text{ mm}^3$

5)
A shaft 40 mm diameter is made from steel and the maximum allowable shear stress for the material is 50 MPa. Calculate the maximum torque that can be safely transmitted. Take G = 90 GPa.

SOLUTION

Important values to use are:
D = 0.04 m, R = 0.02 m, $\tau = 50 \times 10^6$ Pa and $G = 90 \times 10^9$ Pa

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{r}$$

$$J = \frac{\pi D^4}{32} = \frac{\pi \times 0.04^4}{32} = 251.32 \times 10^{-9} \text{ m}^4$$

The complete torsion equation is $\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$ Rearrange and ignore the middle term.

$$T = \frac{\tau_{max} J}{R} = \frac{50 \times 10^6 \times 251.32 \times 10^{-9}}{0.02} = 628.3 \text{ Nm}$$