UNIT-4
TESTING OF HYPOTHESIS-I

In some problems we have to conclude whether the statement concerning a parameter is true or false to estimate the value of parameter, we must test that the hypothesis about a parameter

The procedure which enables us to decide on the basis of sample resuts whether a hypothesis is true or false is called test of hypothesis

There are 2 types of hypothesis

1. Null hypothesis
2. Alternate hypothesis

1. **Null Hypothesis:-** A hypothesis of no diltance is alled a null hypothesis

The null hypothesis means there is no significant difference blw the statistic and the population parameter blw the observed difference is there due to situations in sample

The null hypothesis is denoted by "H_0"

2. **Alternative Hypothesis:-** Any hypothesis which is any complementary with null hypothesis is called alternative hypothesis and it is denoted by "H_1"

Ex:- If we want to test the null hypothesis that the population "μ" has the specified quantity μ_0 then

$H_0 : \mu = \mu_0$

Then the alternative hypothesis is

$H_1 : \mu \neq \mu_0$ (two tailed test)

$H_1 : \mu > \mu_0$ (Right tailed test)

$H_1 : \mu < \mu_0$ (Left tailed test)

Errors in sampling:- on the basis of sampling results we accept or reject after examining a sample from it so in this case we are able to commit two types of errors

* type-I error (α-error)
* type-II error (β-error)

① Type-I error:- If we reject a hypothesis when it should be accepted i.e if null hypothesis H_0 is true but it is rejected by test procedure then that error called type-I error (or) α-error

② Type-II error:- If we accept a hypothesis when it should be rejected i.e If null of hypothesis H_0 is false but it is accepted by test procedure then that error called type-II error (or) β-error

* Level of significant:-

It is denoted by "α" it is confidence with which we reject NULL hypothesis H_0. It is max probability with which we all willing to risk an error in rejecting H_0 when it is true (size of type I error in general) is 5% (or) 1%.]

Critical region:- Region corresponding to statistic 't' in sample space 'S' which leads to rejection of H_0 is called critical region (or) rejection region. i.e Region in which sample value falling is rejected is known as critical region.

In general we take critical regions 5% and 1% (or) as of normal curve

* critical values for level of significant

	1%	5%	10%
Two tailed test	$(Z_{\frac{\alpha}{2}})=2.58$	$\lvert Z_{\alpha/2}\rvert=1.96$	$\lvert Z_\alpha\rvert=1.645$
Right tailed test	$Z_\alpha=2.33$	$Z_\alpha=1.645$	$Z_\alpha=1.28$
left tailed test	$Z_\alpha=-2.33$	$Z_\alpha=-1.645$	$Z_\alpha=-1.28$

* condission (Decission):-

compare calculated value of best statistic Z with critical value (Tabulated value) Z_α at given level of significance

1. If $|Z|<Z_\alpha$, we consider that the value is no significance distance and we accept Null hypothesis H_0

2. If $|z| = z_\alpha$, then difference is significant and hence null hypothesis is rejected at given level of significant

* test of significantce of single mean [large samples]

Working rule :- Let a random sample of size $(n \geq 30)$ has sample mean $\bar{x}$ and 'μ' be population mean 'σ' be standard deviation of population Assume that population mean μ has seperate value μ_0

step 1 :- Null hypothesis :

$H_0 : \mu = \mu_0$

→ This means there is no significance difference b/w μ & μ_0

step 2 :- Alternative hypothesis :- complementary of null hypothesis

$H_1 : \mu \neq \mu_0$ (two tailed test)

$H_1 : \mu > \mu_0$ (Right tailed test)

$H_1 : \mu < \mu_0$ (left tailed test]

step 3: set the level of significance α

step 4 :- The test statistics

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}}$$

step 5 :- If "σ" is not given Replace "σ" by s.

Step-6 :- find critical value z_α for given level of significance α

Step-7 :- conclusion :-

a) If $|z| < z_\alpha$ Accept the null hypothesis i.e $\mu = \mu_0$

b) If $|z| > z_\alpha$ Reject the null hypothesis i.e $\mu \neq \mu_0$

Problems :-

① In Random sample of 60 workers, avg time taken by them to get work is 33.8 minutes with S.D of 6.1 min can we reject null hypothesis $\mu = 32.6$ in favour of Alternative hypothesis $\mu > 32.6$ at $\alpha = 0.025$

Given that $\mu = 32.6$

$n = 60$

$\bar{x} = 33.8$

$\sigma = S.D = 6.1$

i. Null hypothesis : let $\mu = \mu_0$

$H_0 = \mu = 32.6$

ii. Alternative hypothesis :-

$H_1 : \mu > 32.6$ (Right tailed test)

iii. level of significance :- Let $\alpha = 0.025$

iv. Test statistics :

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$z = \frac{33.8-32.6}{6.1/\sqrt{60}}$$

$$z = 1.523$$

$$|z| = 1.523$$

v. conilusion :- $|z| < z_\alpha$ $\therefore$ The null hypothesis is accepted

② According to the norms established for a mechanical aptitude test, persons who are 18 years old have an average hight of 73.2 with a S.D of 8.6 If 400 randomly selected persons of that age averaged 76.7 test the hypothesis $\mu = 73.2$ against the alternative hypothesis $\mu > 73.2$ at the 0.01 level of significants

Sol :- $\mu = 73.2$, $\sigma = 8.6$, $n = 400$, $\bar{x} = 76.7$

i. Null hypothesis :- let H_0 $\mu = \mu_0$

ii. Alternative hypothesis : Let H_1 $\mu \neq \mu_0$

But Here $H_1 : \mu > 73.2$ (Right tailed test)

iii. level of significants : Here $\alpha = 0.01$
$= 1\%$

By tabulate value at 1% level of significance is

$$z_0 = 2.33$$

iv. Test statistic :- $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$$Z = \frac{76.7-73.2}{8.6/\sqrt{400}}$$

$$Z = 3.5/0.43$$

$$Z = 8.139$$

$$|Z| = 8.139$$

v. conclusion:- Here $|Z| > Z_0$

∴ Null hypotheses is rejected

(3) A sample of 64 students have a mean weight of 70kgs can be regulated as a sample from a population mean weight 56kgs and a S.D 25kgs

Given that

$\mu = 56$; $\bar{x} = 70$; $n = 64$; $\sigma = 25$

i. Null hypothesis: let $H_0 : \mu = \mu_0$

ii. level of significants: $\alpha = 5\%$ (assumed) $Z_{\alpha/2} = 1.96$

iii. Alternative hypothesis:- $H_1 : \mu \neq \mu_0$

iv. Test statistics:- $Z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

$$= \frac{59.1-57.4}{5.2/\sqrt{40}}$$

$$Z = \frac{70-56}{25/\sqrt{64}}$$

$z = 4.43$

v. <u>conclusion</u>:- $|z| > z_{\alpha/2}$

$\therefore$ Null hypothesis is rejected

(4) An oceanographer wants to check whether the depth of the ocean in a certain region is 57.4 fathoms as had previously been recorded wt can be concluded at the level of significants $\alpha = 0.05$, if readings taken at 40 random locations in the given region yielded a mean of 59.1 fathoms with a S.D of 5.2 fathoms

Given that $n = 40$; $\sigma = 5.2$; $\mu = 57.4$; $\bar{x} = 59.1$

i. <u>Null hypotheses</u>: $H_0 : \mu = \mu_0$

ii. <u>Alternate hypothesis</u>: $H_1 : \mu \neq \mu_0$

iii. <u>level of significants</u>: Here $\alpha = 5\%$

$$z_{\alpha/2} = 1.96$$

iv. <u>Test statistic</u> $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$= \frac{59.1 - 57.4}{5.2 / \sqrt{40}}$$

$$|z| = 2.06$$

v. <u>conclusion</u>:- Here $|z| > z_{\alpha/2}$

$\therefore$ Null hypothesis is rejected

⑤ A sample of 900 members has a mean 3.4 cms and S.D 2.61 cms. Is the sample from a large population of mean 3.25 cms and S.D 2.61 cms if the population is normal and its mean is known find the 95% fiducial confidence limits of true mean

Given that

$n = 900$; $\mu = 3.25$; $\bar{x} = 3.4$; $\sigma = 2.61$

i. Null hypothesis : H_0 $\mu = \mu_0$

ii. Alternate Hypothesis : H_1 $\mu = \mu_0$

iii. level of significants : $\alpha = 5\%$ (assume

$Z_{\alpha/2} = 1.96$

iv. Test statistic : $Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$= \frac{3.4 - 3.25}{2.61 / \sqrt{900}}$

$|Z| = 1.724$

v. conclusion : $|Z| < Z_{\alpha/2}$

∴ null hypothesis is accepted

NOTE:-

The confidence limits are given by

$$\left(\bar{x} - z_\alpha \sigma/\sqrt{n},\ \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}}\right)$$

for the above problem the confident limits are

$$\left(\bar{34} - 1.96\ \frac{2.6}{\sqrt{900}},\ 3.4 + 1.96\ \frac{2.61}{\sqrt{900}}\right)$$

$$= (3.229, 3.57)$$

(6) An ambulence service claims that it takes on the avg less then 10min to reach its destination in emergency calls A sample of 36 calls has a mean of 11min and the varience of 16min test the significance at 0.05 level.

Given that $\mu = 10$, $\bar{x} = 11$, $\sigma^2 = 16$, $\alpha = 0.05$; $n = 36$

i. Nuel hypothesis :-

$$H_0\ \mu = \mu_0$$

ii. Alteonative hypothesis:-

$$H_1: \mu \neq \mu_0 \text{ [two tailed test]}$$

iii. level of significants:- $\alpha = 5\%$

$$z_{\alpha/2} = 1.96$$

iv. test significants: $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$= \frac{11-10}{4/\sqrt{36}}$

$|z| = 1.5$

v. conclusion:-

$|z| < z_\alpha$ $\therefore$ Null hypothesis is accepted

(7) mean and S.D of a population are 11795 and 14054 find n=50 95% confidence interval.

confidence interval is $\left(\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{x} + \frac{z_\alpha}{2}\frac{\sigma}{\sqrt{n}}\right)$

Here $\alpha = 5\%$ then $z_{\alpha/2} = 1.96$

$\bar{x} = 11795$ & $\sigma = 14054$

$n = 50$

Then confidence interval is

$\left(11795 - 1.96\left(\frac{14054}{\sqrt{50}}\right), 11795 + 1.96\left(\frac{14054}{\sqrt{50}}\right)\right)$

$(11795 - 3895.57, 11795 + 3895.57)$

$(7899.43, 15690.57)$

Test for Equality of two means:- (large samples)

Let $\bar{x}_1$ and $\bar{x}_2$ be the sample means of two independent large random sample size n_1 and n_2 drawn from two populations having means μ_1 & μ_2 and standard Deviations σ_1 and σ_2 To test whether the two populations means are equal

→ Let the null hypothesis be $H_0 : \mu_1 = \mu_2$

→ then alternative hypothesis $H_1 : \mu_1 \neq \mu_2$

→ where σ_1 & σ_2 are standard deviations of two populations and to test whether there is any significant difference b/w $\bar{x}_1$ and $\bar{x}_2$

we have to use the statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{S.E\ of\ (\bar{x}_1 - \bar{x}_2)} = \frac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where, $\delta = \mu_1 - \mu_2$ (= Given constant)

→ if $\delta = 0$ the two populations have same means

→ if $\delta \neq 0$ the two populations are different

under $H_0 : \mu_1 = \mu_2$

then the test statistics became

$$\boxed{z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}}$$

is approximately normally distributed with mean '0' and S.D '1'

NOTE :-

→ If the samples have been drawn from the population with common standard Deviation 'σ' then $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Hence $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

is normally distributed with mean '0' & S.D '1'

→ If 'σ' is not known we can use an estimate of σ^2 is given by $\sigma^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}$

① The means of two large samples of sizes 1000 and 2000 are 67.5 and 68 respectively can the samples been recorded as drawn from the same population of S.D 2.5 inches.

Given that:

$n_1 = 1000 ; \bar{x}_1 = 67.5$

$n_2 = 2000 ; \bar{x}_2 = 68$

and $\sigma = 2.5$

i. Null hypothesis :- Let H_0 $\mu_1 = \mu_2$

ii. Alternative hypothesis : $H_1 : \mu_1 \neq \mu_2$ (two tailed test)

iii. level of significant : $\alpha = 5\%$ (assume)

$$Z_{\alpha/2} = 1.96$$

iv. Test statistic :

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{1/n_1 + 1/n_2}} = \frac{67.5 - 68}{2.5\sqrt{\frac{1}{1000} + \frac{1}{2000}}}$$

$$|Z| = 5.164$$

v. conclusion : $|Z| > Z_\alpha$

∴ The null hypothesis is rejected

(2) samples of students were drawn from two vertices and from their weights in kgms, mean & S.D's are calculated and shown below. make a large sample test to test the significance of the difference b/w the two means

	Mean	S.D	size of the sample
university A	55	10	400
university B	57	15	100

Given that $\bar{x}_1 = 55$; $\sigma_1 = 10$; $n_1 = 400$
$\bar{x}_2 = 57$; $\sigma_2 = 15$; $n_2 = 100$

i. null hypothesis :- Let $H_0 : x_1 = x_2$

ii. Alternative Hypothesis:

$H_1 : \bar{x}_1 \neq \bar{x}_2$ [two tailed test]

iii. level of significants :- $\alpha = 5\%$ (assumed)

$Z_\alpha = 1.96$

iv. test statistic:-

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{55-57}{\sqrt{\frac{10^2}{400} + \frac{15^2}{100}}}$$

$|z| = 1.265$

v. conclusion: Here $|z| < z_\alpha$

$\therefore$ The null hypothesis is accepted

(3) The Research investigator is interested in studying whether that is a significant diff in the salary of MVA credits in two metropolitan cities. A random sample of size 100 from mumbai yields an avg. income of 20,150/- Another random sample of 60 from chennai results in an avg. income of 20,250/- If the variances of both populations are given as $\sigma_1^2 = 40,000/-$ & $\sigma_2^2 = 32400/-$ respectively

Given that

$n_1 = 100$; $\bar{x}_1 = 20,150$; $\sigma_1^2 = 40,000$

$\sigma_1 = \sqrt{40,000}$

$\sigma_1 = 200$

$n_2 = 60$; $\bar{x}_2 = 20,250$; $\sigma_2^2 = 32,400$

$\sigma_2 = \sqrt{32400}$

$\sigma_2 = 180$

i. null hypothesis : let $\bar{x}_1 = \bar{x}_2$

ii. Alternative Hypothesis :- Let H_1 :

$\bar{x}_1 \neq \bar{x}_2$ (two tailed test)

iii. level of significance : let $\alpha = 5\%$

$z_\alpha = 1.96$

iv. Test statistic :

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{\sqrt{n_1}} + \frac{\sigma_2^2}{\sqrt{n_2}}}}$$

$$= \frac{20150 - 20250}{\sqrt{\frac{40000}{\sqrt{100}} + \frac{32400}{\sqrt{60}}}}$$

$$= \frac{-100}{\sqrt{4000+4182.82}}$$

$$= \frac{-100}{\sqrt{8182.82}}$$

$$= \frac{-100}{90.45}$$

$$|z| = 1.105$$

v. <u>conclusion:</u> $|z| < z_\alpha$ ∴ null hypothesis is accepted.

(4) Two types of news cars produced in USA are tested for petrol milege one sample piece consisting of 42 cars avg 15 KMPH while the other sample consisting of 80 cars averaged 11.5kmph with populations variances as $\sigma_1^2 = 2.0$ and $\sigma_2^2 = 1.5$ respectively Test whether there is any significance difference in the petrol consuption of these two type of cars (use $\alpha = 0.01$)

Given that

$n_1 = 42$; $\bar{x}_1 = 15$; $\sigma_1^2 = 2.0$

$n_2 = 80$; $\bar{x}_2 = 11.5$, $\sigma_2^2 = 1.5$

i. <u>Null hypothesis</u>: Let H_0 $\bar{x}_1 = \bar{x}_2$

ii. <u>Alternative hypothesis</u>:

H_1: $\bar{x}_1 \neq \bar{x}_2$ (two tailed test)

iii. <u>level of significance</u>: $\alpha = 0.01 \Rightarrow 1\%$

$z_\alpha = 2.58$

iv. <u>Test statistics</u>: $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$$z = \frac{15-11.5}{\sqrt{\frac{2}{42}+\frac{1.5}{80}}}$$

$|z| = 13.779$

v. conclusion:-

Here $|z| > z_\alpha$

$\therefore$ null hypothesis is rejected

(5) A simple sample of the height of 6400 english men has a mean of 67.85 inches and a S.D of 2.56 inches while a simple sample of heights of 1600 Australians has a mean of 68.55 inches and S.D of 2.52 inches Do the Data indicate the Austrailians on the avg taller then the englishmen? (use $\alpha = 0.05$)

Given that

$n_1 = 6400$; $\bar{x}_1 = 67.85$ $\sigma_1 = 2.56$

$n_2 = 1600$; $\bar{x}_2 = 68.55$ $\sigma_2 = 2.52$

i. null hypothesis : let Ho $\bar{x}_1 = \bar{x}_2$

ii. Alternative hypothesis :-

H_1 $\bar{x}_1 \neq \bar{x}_2$ (two tailed test)

iii. level of significance : $\alpha = 0.05 = 5\%$

$z_\alpha = 1.96$

iv. Test statistics :-

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{67.85 - 68.55}{\sqrt{\frac{(2.56)^2}{6400} + \frac{(2.52)^2}{1600}}}$$

$$= -1.26$$

$$|z| = 1.26$$

v. conclusion:- $|z| < z_\alpha$

$\therefore$ null hypothesis is accepted.

⇒ Test of significant for single proportion:- (large samples)

Suppose a large random sample of size 'n' has sample proportion 'p' has members processing a certain attribute (i.e. proportion of successes. To test the hypothesis that the proportion 'P' in the population has a specified value 'p'

1. Null hypothesis:- Let H_0 $p = P$
2. Alternate hypothesis:- Let $H_1 : p \neq P$
   $[i.e\ p > P\ (or)\ p < P]$
3. level of significants:-
4. Test statistic:

$$z = \frac{p - P}{\sqrt{PQ/n}}$$

where $Q = 1 - P$

level of significance	1%	5%	10%
Two tailed test	$\|Z\| > 2.58$	$\|Z\| > 1.96$	$\|Z\| > 1.645$
Right tailed test	$Z > 2.33$	$Z > 1.645$	$Z > 1.28$
left tailed test	$Z < -2.33$	$Z < -1.645$	$Z < -1.28$

These values for rejection rule for H_0

Note:-

confidence interval for proportion P for a large sample at α level of significance is

$$\left(p - z_{\alpha/2}\sqrt{\frac{PQ}{n}},\ p + z_{\alpha/2}\sqrt{PQ/n}\right)$$

(1) In a sample of 1000 people in Karnataka 540 all rice eaters and the rest all wheat eaters can we assume that both rice and weat are equally popular in the state at 1% level of significance

Given that $n = 1000$

p = sample proportion of rice eater $= \frac{540}{1000} = 0.54$

P = population proporsion of rice eater $= 1/2 = 0.5$

$Q = 1 - P = 1 - 0.5 = 0.5$

i. Null hypothesis : Let H_0: $p = P$

ii. Alternative hypothesis:- H_1 $p \neq P$

iii. level of significants :- $\alpha = 1\%$

$|Z| > 2.58$

iv. Test statistics:-

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)\,0.25}{1000}}}$$

$|z| = 2.53$

v. <u>conclusion</u>:- $|z| > z_\alpha$

$\therefore$ null hypothesis is rejected

$\Rightarrow z_{\alpha/2} = 1.96$ (for 95%)

$\Rightarrow z_{\alpha/2} = 2.33$ (for 98%)

$\Rightarrow z_{\alpha/2} = 2.58$ (for 99%)

① A manufactural claimed that atleast 95% of the equipment which he supplied to a factory confirmed to specifications. An examination of a sample of 200 pieces of equipment reveleded that 18 per faculty Test his claim at 5% level of significance.

Given that $n = 200$

no.of pieces confirming to specification $= 200 - 18 = 182$

Let p = proportion of pieces confirming to specifications

$= \frac{182}{200} = 0.91$

$P = 95\% = \frac{95}{100} = 0.95$

$Q = 1 - P = 1 - 0.95 = 0.05$

i. <u>Null hypothesis</u>: Let H_0 $p = P$

ii. <u>Alternative hypothesis</u>:- H_1 $p \neq P$ (left tailed test)

iii. <u>level of significants</u>:- $\alpha = 5\%$ $\quad z_\alpha < 1.645$

iv. Test statistic:

$$z = \frac{p-P}{\sqrt{PQ/n}}$$

$$= \frac{0.91-0.95}{\sqrt{\frac{0.95\times0.05}{200}}}$$

$$|z| = 2.59$$

v. conclusion :- Here $|z| > z_\alpha$

Null hypothesis is rejected

② In a big city 325 men out of 600 men well found to be smokers does this information support the condition that the majority of men in the city are smokers.

Given that $n = 600$

$p = 325$

$p = \frac{325}{600} = 0.541$

P = population proportion of smokers $= 1/2 = 0.5$

$Q = 1 - P = 1 - 0.5 = 0.5$

i. Null hypothesis :- Let $H_0 : p = P$

ii. Alternative hypothesis :-

$H_1 = p > P$ (right tailed test)

iii. level of significance : Let $\alpha = 5\%$ (assume

iv. Test statistics:-

$$Z = \frac{0.54-0.5}{\sqrt{\frac{0.5\times0.5}{600}}}$$

$$|Z| = 2.05$$

v. conclusion:- $|Z| > Z_\alpha$

Null hypothesis is rejected

③ In a random sample of 125 cooldrinkers 68 said they prefer to pepsi test the null hypothesis $P > 0.5$

Given that $n = 125$

p = proportion of cool drinkers

$= 68/125 = 0.54$

$P = 0.5$

$Q = 1-0.5 = 0.5$

i. Null hypothesis:- H_0 $p = P$

ii. Alternative hypothesis:-

$H_1 = p > P$ (right tailed test)

iii. level of significance:

Let $\alpha = 5\%$

$Z_\alpha = 1.645$

iv. Test statistics

$$Z = \frac{0.54-0.5}{\sqrt{\frac{0.5\times0.5}{125}}}$$

$$|Z| = 0.90$$

v. conclusion:- $|Z| < Z_\alpha$

∴ Null hypothesis is rejected

(4) In a random variable of a sample of 160 workers exposed to a certain amount of radiation 24 experienced some i. effects. construct a 99% confidence interval for the corresponding true %

Given that $n=160$

$$P=\frac{24}{160}=0.15$$

$$Q=1-0.15=0.85$$

∴ confidence interval at 99% level of significance is

$$=\left[P-\frac{Z_\alpha}{2}\sqrt{\frac{PQ}{n}},\ P+\frac{Z_\alpha}{2}\sqrt{\frac{PQ}{n}}\right]$$

$$=\left[0.15-2.58\sqrt{\frac{0.15\times0.85}{160}},\ 0.15+2.58\sqrt{\frac{0.15\times0.85}{160}}\right]$$

$$=\left[0.15-2.58\sqrt{0.00079},\ 0.15+2.58\sqrt{0.00079}\right]$$

$$=[0.15-0.07,\ 0.15+0.07]$$

$$=[0.08,\ 0.22)$$

(5) In 80 patients are treated with an adiobetic 59 got used to the house the true population Find a 99% confidence limits of curved

Given that $n=80$

$$P=\frac{59}{80}=0.7375$$

$$Q=1-0.7375=0.2625$$

∴ confidence interval at 99% level of significance

is $\left[P-\frac{Z_\alpha}{2}\sqrt{\frac{PQ}{n}},\ P+\frac{Z_\alpha}{2}\sqrt{\frac{PQ}{n}}\right]$

$$=[0.7375-2.58\sqrt{0.00241})$$

$= [0.7375 - 0.1263, 0.7375 + 0.1263]$

$= (0.611, 0.863)$

(6) 20 people were attacked by a discaase and and only 18 survived will you reject the hypothesis that the survival rata. if attacked by this discase is 85% in favour of the hypothesis that is more at 5% level.

Given data

$n = 20$

$p = x/n = \frac{18}{20} = 0.9$

$P = \frac{85}{100} = 0.85$

$Q = 1 - 0.85 = 0.15$

i. Null hypothesis : $H_0 \quad p = P$

ii. Alternative hypothesis : $H_1 \quad p > P$ (right tailed test)

iii. level of significant $\alpha = 5\%$

$Z_\alpha = 1.645$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.9 - 0.85}{\sqrt{\frac{0.85 \times 0.15}{20}}}$$

$$= \frac{0.05}{\sqrt{0.0063}}$$

$|Z| = 0.632$

v. conclusion :- $|Z| < Z_\alpha$

∴ Null hypothesis is accepted

* Test for equality of two proportions (large samples)

let p_1 and p_2 be the proportions in two large random samples of sizes n_1 and n_2 drawn from two populations having proportions P_1 and P_2

latest whether the two populations P_1 and P_2 are equal

i. Null hypothesis is: $H_0 : P_1 = P_2$

ii. Alternative hypothesis: $H_1 : P_1 \neq P_2$

iii. level of significance:- Let us assume 'α'

iv. Test statistic:-

$$Z = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{x_1 + x_2}{n_1 + n_2}$

and $Q = 1 - P$

* Rejection rule for $H_0 : P_1 = P_2$

i. If $|z| > 1.96$ then reject H_0 at 5% level of significance

ii. If $|z| > 2.58$ then reject H_0 at 1% level of significance

iii. If $|z| > 1.645$ then reject H_0 at 10% level of significance

① Random sample of 400 men and 600 women well asked whether they would like to have my over near there residence. 200 men and 325 women well infavour of the proposal. Test the hypothe that proportions of men and women in favour of the proposal all same at 5% level

Given that

$n_1 = 400$, $n_2 = 600$

sample sizes are

let p_1 = proportion of men $= \frac{200}{400} = 0.5$

and p_2 = proportion of women $= \frac{325}{600} = 0.54$

$$P = \frac{(400)(0.5) + 600(0.54)}{400+600}$$

$P = 0.524$

$q = 1 - 0.524 = 0.476$

i. Null hypothesis: H_0 is $P_1 = P_2$

ii. Alternative Hypothesis:- H_1 $P_1 \neq P_2$ (two tailed test)

iii. level of significance : $\alpha = 5\%$

$Z_\alpha = 1.96$

iv. Test statistic:-

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.5 - 0.54}{\sqrt{0.524 \times 0.476}\sqrt{\frac{1}{400} + \frac{1}{600}}}$$

$|Z| = 1.29$

v. conclusion:- $|Z| < Z_\alpha$

Null hypothesis is accepted

(2) A manufactured of electronic equipment subjects samples of two completing brands of transistors to an accelarated per foormance. test if 45 of 180 transistors of the first kind and 34 of 120 transistors of the second kind fail the test, what can be conclude at the level of significance $\alpha=0.05$ about the difference b/w the corresponding sample proportions

Given that

$n_1=180$; $n_2=120$

P_1 = proportion of first kind $= 45/180 = 0.25$

P_2 = proportion of second kind $= 34/120 = 0.28$

$$P = \frac{(180)(0.25)+(120)(0.28)}{180+120}$$

$P = 0.262$

$q = 1-0.262 = 0.738$

i. null hypothesis :- $H_0 : P_1 = P_2$

ii. Alternative hypothesis :- $H_1 : P_1 \neq P_2$ (two tailed test)

iii. level of significance :- $\alpha = 5\%$. $Z_\alpha = 1.96$

iv. Test statistic :-

$$Z = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$Z = \frac{0.25-0.28}{\sqrt{(0.262)(0.738)\left(\frac{1}{180}+\frac{1}{120}\right)}}$$

$|Z| = 0.6$

v. conclusion :- $|Z| < Z_\alpha$

$\therefore$ null hypothesis is accepted

(3) On the basis their scores, 200 condiats of a civil service examination are devided into two groups, the upper 30% and remaining 70%. consider the 1st quesion of the examination Among the first two, 40 head the correct answer, where as among the first two, 40 head the correct answer, ~~correct~~ correct answer. on the basis of these result can one condute that the 1st Ques is not true at discriminaling ability of the type being examined

Given that $n_1 = 60$ $n_2 = 140$

$\downarrow$ $\downarrow$

$\left(\frac{30}{100} \times 200 = 60\right)$ $\left(\frac{70}{100} \times 200 = 140\right)$

$x_1 = 40$ $\therefore x_2 = 80$

consider $p_1 = \frac{x_1}{n_1} = \frac{40}{60} = 2/3 = 0.667$

$p_2 = x_2/n_2 = \frac{80}{140} = 0.571$

$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{120}{200} = 0.6$

$q = 1 - p = 1 - 0.6 = 0.4$

i. Null hypothesis : $H_0 : P = P$

ii. Alternative hypothesis :- $H_1 : P_1 \neq P_2$ (two tailed test)

iii. level of significance : $\alpha = 5\%$

$Z\alpha = 1.96$

iv. Test statistics :- $Z = \frac{P_1 - P_2}{\sqrt{pq(1/n_1 + 1/n_2)}} = \frac{0.667 - 0.571}{\sqrt{0.6 \times 0.4 (\frac{1}{60} + 1/140)}}$

$|z| = 1.27$

Q. A company wanted to introduce a new plan of work and survey was conducted for this purpose out of sample of 500 worker in one group 62% favourable the new plant and another group of sample of 400 worker 41% were against for the new plan If there any significance difference b/w the two groups in there attitude towards the new plan at 5%.

Given that

$n_1 = 500 \quad n_2 = 400$

$x_1 = \frac{62}{100} \times 500 \qquad x_2 = \frac{41}{100} \times 400$

$= 310 \qquad = 164 \Rightarrow 400 - 164$

$= 236$

$P_1 = \frac{x_1}{n_1} = \frac{310}{500} = 0.62 \qquad P_2 = \frac{x_2}{n_2} = \frac{236}{400}$

$= 0.59$

$P = \frac{x_1 + x_2}{n_1 + n_2} = \frac{310 + 236}{550 + 400} = 0.6066$

i. null hypothesis :- $H_0 : P_1 = P_2$

ii. Alternative hypothesis :- $H_1 : P_1 \neq P_2$ (two tailed test)

iii. level of significance :- $\alpha = 5\%$

$Z_\alpha = 1.96$

iv. Test statistics :-

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(1/n_1 + 1/n_2\right)}}$$

$$Z = \frac{0.62 - 0.59}{\sqrt{0.6066 \times 0.3934 \left(\frac{1}{500} + \frac{1}{400}\right)}}$$

$|z| = 0.9160$

v. conclusion:- $|z| < z_\alpha$

$\therefore$ null hypothesis is accepted