07/02

UNIT-II FRICTION

* When a body moves or tends to move over another body, a force opposing the motion develo-p at the contact surfaces. This force is called frictional force or friction.
* However, there is a limit beyond which the magnitude of force cannot increase. If the applied force is more than this limit, there will be movement of one body over the other.
This limiting value of frictional force when the motion is impending is known as "limiting friction"
* When the applied force is less than the limiting friction, the body remains at rest and such frictional force is called "static friction".
* If the applied force is more than limiting friction, the body starts moving over the othe-r body and the frictional force is called as "Dynamic friction"

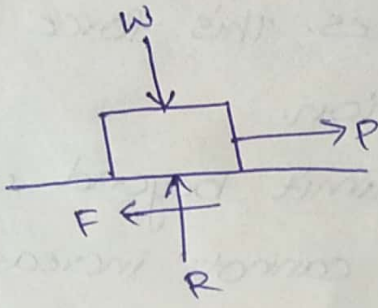
→ Dynamic friction may be classified into the following categories.

1) Sliding friction 2) Rolling friction.

1) Sliding friction: sliding friction is the friction experienced by the body when it slides over other body.

2) Rolling friction: It is a friction experienced when a body rolls over a surface.

(2m)* Coefficient of friction (μ)

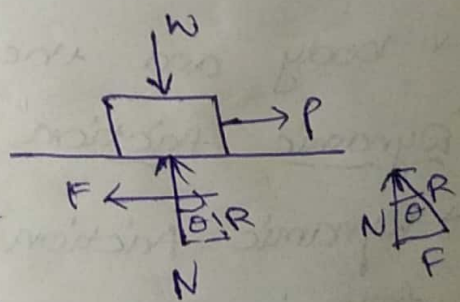


The ratio of the limiting force of friction to the normal reaction b/w 2 bodies is called coefficient of friction. It is denoted by μ.

$$\text{coefficient of friction} = \frac{\text{Limiting force of friction}}{\text{Normal reaction.}}$$

* $\boxed{\mu = \frac{F}{R}}$ or $\boxed{\mu R = F}$

Angle of friction:-



Consider the block resting on a horizontal surface & subjected to horizontal pull 'P'. Let 'F' be the frictional force developed and N be the Normal reaction. They can be combin-ed graphically to get the reaction R which acts at angle θ to the Normal reaction. This angle θ is called as angle of friction

$$\boxed{\tan\theta = \frac{F}{N}}$$ θ = angle of friction.

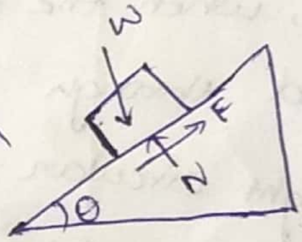
As 'p' increases F increases and hence θ also increases.

→ θ can reach the max. value α when F reaches limiting value.

$$\boxed{\tan\alpha = \frac{F}{N} = \mu}$$

α = angle of limiting friction

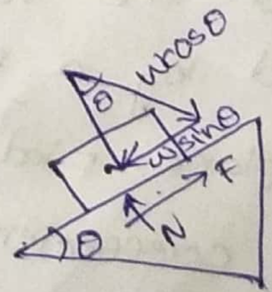
* **Angle of Repose**



consider a block of weight W resting on an inclined plane which makes an angle θ with the horizontal. when θ is small the block will rest on the plane. If 'θ' is increased gradually, a stage is reached at which the block starts sliding down the plane. The angle for which motion is impending ("to cause the motion)" is called the 'angle of repose'

* Forces Normal to the inclined plane = 0

$$\boxed{N = W\cos\theta} \quad -(1)$$



* Forces parallel to the inclined plane = 0

$$\boxed{F = W\sin\theta} \quad -(2)$$

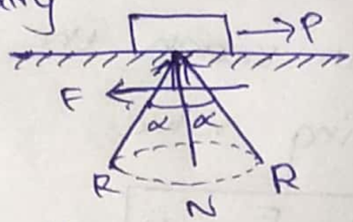
from (1) & (2)

$$\frac{F}{N} = \tan\theta$$

$$\boxed{\tan\theta = \frac{F}{N}}$$

* Cone of friction:-

When a body is having impending motion in the direction of force 'P', the frictional force will be limiting friction and the resultant reaction R will make limiting angle α with the Normal.



Thus, when the direction of force 'P' is gradually changed through 360°, the resultant 'R' generates a right circular cone with semi central angle equal to α. This inverted cone with semi central angle α is called cone of friction.

1) A block weighing 500N just starts moving down a rough inclined plane when supported by a force of 200N acting parallel to the plane in upward direction. the same block is on the verge of moving up the plane when pulled by a force of 300N acting parallel to the plane. Find the inclination of the plane and coeffecient of friction between the inclined plane and the block.

a)

Sol: Forces normal to the inclined plane = 0

$N = 500\cos\theta$ — ①

Forces parallel to the inclined plane $=0$

b)

$$F_1 + 200 - 500\sin\theta = 0 \quad (2)$$

WKT,

$$\boxed{\mu = \frac{F}{N}} \Rightarrow F = \mu N$$

$$F = \mu \times 500\cos\theta \quad (3) \text{ sub in } (2)$$

$$\mu \times 500\cos\theta + 200 - 500\sin\theta = 0$$

$$200 = 500\sin\theta - \mu 500\cos\theta \quad (4)$$

b) Forces normal to the plane $=0$

$$N = 500\cos\theta. \quad (5)$$

Forces parallel to the plane $=0$

$$F_2 + 300 - 500\sin\theta = 0 \quad (6)$$

$$F_2 = \mu \times N$$

$$\mu \times 500\cos\theta + 300 - 500\sin\theta = 0.$$

$$300 = 500\sin\theta - \mu 500\cos\theta \quad (7)$$

add $(4) + (7)$

$$200 + 300 = 500\sin\theta - \cancel{\mu 500\cos\theta} + 500\sin\theta - \cancel{\mu 500\cos\theta}$$

$$500 = 2\times 500\sin\theta$$

$$\cancel{500} = 10\cancel{00}\sin\theta$$

$$\sin\theta = 1/2$$

$$\theta = \sin^{-1}(1/2)$$

$$\theta = 30^\circ$$

Sub, $\theta = 30^\circ$ in eq. (3) 4

$$200 = 500\sin 30 - \mu 500\cos 30$$

$$200 = \cancel{500}^{250} \times 1/\cancel{2} - \mu 500 \frac{\sqrt{3}}{2}$$

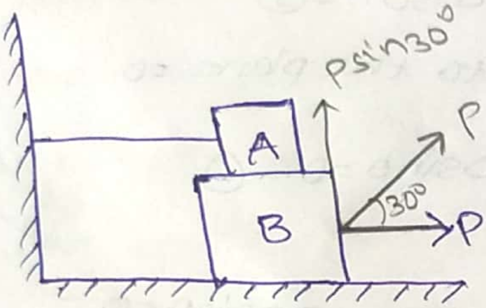
$$\mu \times 250 \times \sqrt{3} = 50 \Rightarrow \boxed{\mu = 0.115}$$

2) A block A weighing 1000N rests over block 'B' which weights 2000N as shown in fig. Block 'A' is tied to wall with a horizontal string. If the coefficient of friction b/w blocks A & B is 0.25 & b/w B & floor is 1/3. what should be the value of P to move the block

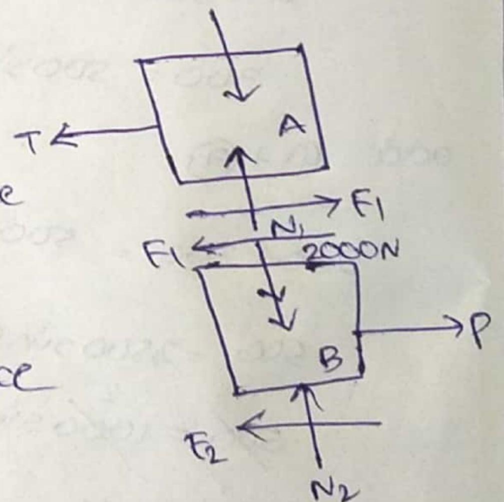
B. if a) P is horizontal

b) P acts 30° upwards to horizontal

Sol:



$\mu_1 = 0.25$ & $\mu_2 = 1/3$



A block:

Forces normal to the surface

$N_1 = 1000N$

Forces parallel to the surface

$T = F_1$

$F_1 = \mu_1 \times N_1$

$T = \mu_1 \times N_1 = 0.25 \times 1000 = 250N$

Block B :-

$N_2 = 2000 + N_1 = 2000 + 1000 = 3000$

parallel forces, $P = F_1 + F_2$

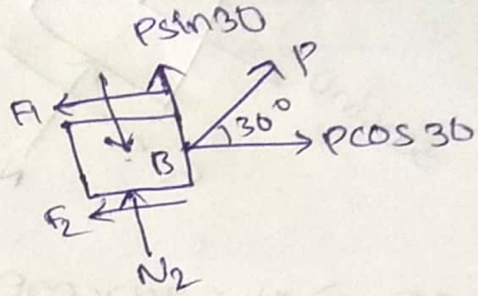
$= 250 + 1000$

$\boxed{P = 1250N}$

$F_2 = \mu_2 \times N_2$

$= \frac{1}{3} \times 3000 = 1000N$

b) when P is inclined at 30° to horizontal.



<u>A block</u>: $N_1 = 1000N$

$F_1 = 250N$

<u>B</u>

forces in vertical,

$N_2 + P\sin 30 = N_1 + 2000$

$N_2 + 0.5P = 1000 + 2000$

$N_2 = 3000 - 0.5P$

forces in horizontal,

$P\cos 30 = F_1 + F_2$

$P\cos 30 = 250 + \cancel{3000 - 0.5P}\ 1000 - \frac{1}{6}P$ — ①

$F_2 = \mu_2 N_2$

$= \frac{1}{3}(3000 - 0.5P) = 1000 - \frac{1}{6}P$

① ⇒ $P\cos 30 + \frac{1}{6}P = 1250$

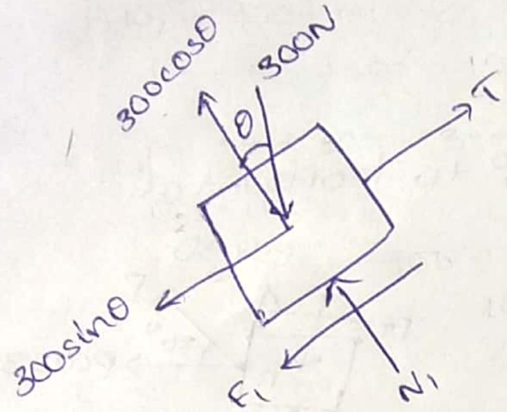
$P\frac{\sqrt{3}}{2} + \frac{1}{6}P = 1250$

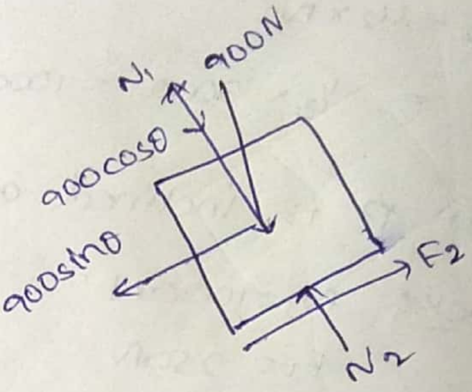
$P(1.032) = 1250$

$\boxed{P = 1211.24N}$

3) What should be the value of θ in figure that will make the motion of 900N block down the plane to impend? The coefficient of friction for all contact surface is $\frac{1}{3}$.

<u>Sol</u>:-





$\Sigma F_N \Rightarrow N_1 = 300\cos\theta$

$\Sigma F_P \Rightarrow T = F_1 + 300\sin\theta$

$F_1 = \mu \times N_1$

$= \frac{1}{3} \times 300\cos\theta$

$= 100\cos\theta$

$T = 100\cos\theta + 300\sin\theta$

$\Sigma F_N \Rightarrow N_2 = N_1 + 900\cos\theta$

$= 300\cos\theta + 900\cos\theta$

$= 1200\cos\theta$

$\Sigma F_P \Rightarrow F_1 + F_2 = 900\sin\theta$

$100\cos\theta + 400\cos\theta = 900\sin\theta$

$F_2 = \mu N_2$

$= \frac{1}{3} \times 1200\cos\theta$

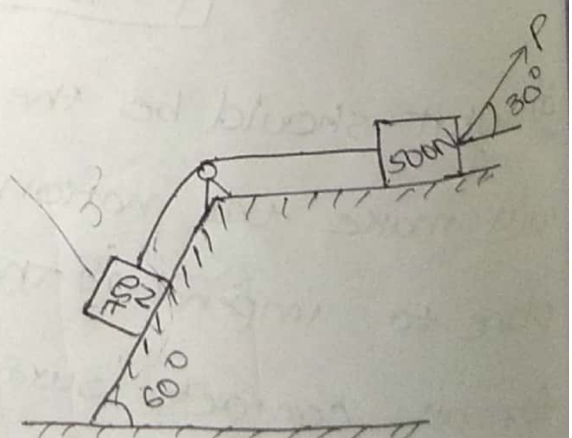
$= 400\cos\theta$

$\Rightarrow 500\cos\theta = 900\sin\theta$

$\tan\theta = 0.55$

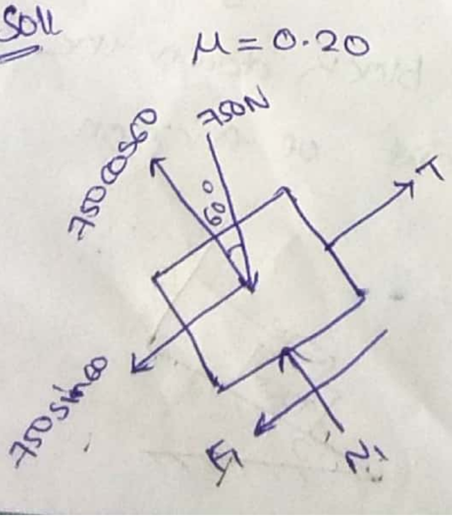
$\boxed{\theta = 29.05^\circ}$

4) What is the value of P in the system shown in figure to cause the motion of 500N block to the right side? Assume the pulley is smooth and the coefficient of friction between other contact surface is 0.20.



Sol

$\mu = 0.20$



block A :-

$\Sigma F_N \Rightarrow N_1 = 750 \cos 60$

$N_1 = 750 \times \frac{1}{2}$

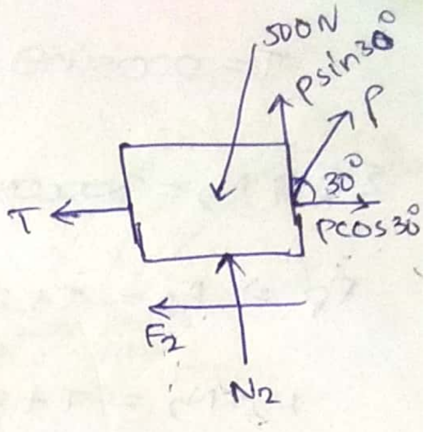
$N_1 = 0.5 \times 750 = 375N$

$F_1 = 0.20 \times 375$

$F_1 = 75N$

$T_1 = (0.2 \times 375) + 750 \sin 60$

$T_1 = 75 + 649.5 = 724.5N.$



$\Sigma F_N \Rightarrow N_2 = 500 - P\sin 30$

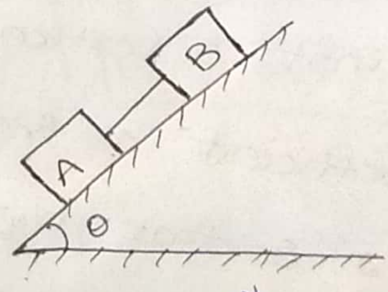
$\Sigma F_P \Rightarrow P\cos 30 = T + F_2 = 724.51 + 0.2 \times (500 - \tfrac{1}{2}P)$

$\sqrt{3}/2\, P = 724.51 + 100 - 0.1P$

$0.966P = 724.51$

$P = 853.52N$

5) A body's A & B are joinined by a cord parallel to the inclined plane as shown in the figure. under body A which weighs 200N, $\mu = 0.20$ while $\mu = 0.5$ under body B which weighs 300N. Determine the angle θ at which motion impends. What is the tension in the cord.



Sol: $W_A = 200N$, $W_B = 300N$

$\mu_A = 0.20$ $\quad \mu_B = 0.5$

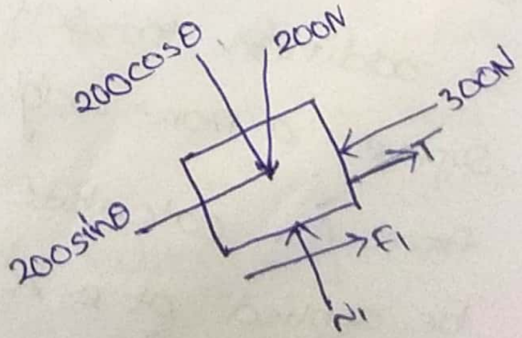
$\Sigma F \Rightarrow N_1 = 200\cos\theta$

$\Sigma F_P \Rightarrow T = 200\sin\theta - F_1$

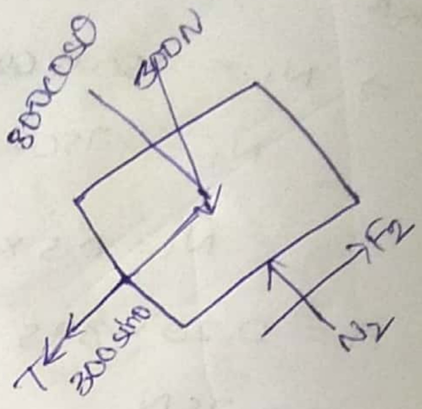
$F_1 = \mu_A \times N_1$

$= 0.20 \times 200\cos\theta$

$= 40\cos\theta.$



$T = 200\sin\theta - 40\cos\theta$ — ①



$\Sigma F_N \Rightarrow N_2 = 300\cos\theta$.

$F_p \Rightarrow F_2 = T + 300\sin\theta$

$\mu_2 \times N_2 = T + 300\sin\theta$

$0.5 \times 300\cos\theta = T + 300\sin\theta$

$150\cos\theta = T + 300\sin\theta$

$T = 150\cos\theta - 300\sin\theta$ — ②

from ① & ②

$200\sin\theta - 40\cos\theta = 150\cos\theta - 300\sin\theta$.

$500\sin\theta = 190\cos\theta$

$\tan\theta = 0.38$

$\boxed{\theta = 20.80^\circ}$

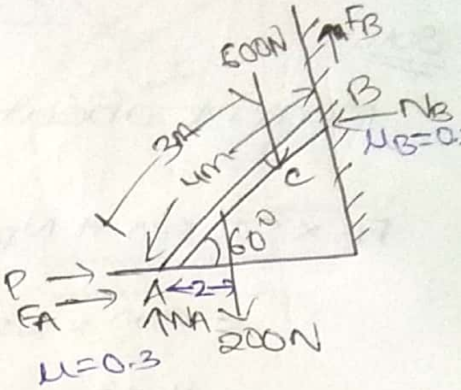
$T = 200 \times \sin(20.80) - 40\cos(20.80)$

$\boxed{T = 33.69}$

6) A ladder of length 4m weighing 200N is placed against a vertical wall as shown in figure. The coefficient of friction b/w the wall & ladder is 0.2 & that b/w the floor & ladder is 0.3. In addition to self weight the ladder has to support a man weighing 600N at a distance of 3m from A. Calculate the min. horizontal force to be applied at A to prevent slipping

Sol

Taking moment abot A,



$F_B \times 4\cos 60 + N_B \times 4 \sin 60° -$

$600 \times 3\cos 60 - 200 \times 2\cos 60 = 0.$

$F_B = \mu_B \times N_B$

$= 0.2 \times N_B$

$0.2 N_B \times \cancel{4}^{2} (\cancel{1/2}) - 200 \times \cancel{2}^{2} \, \tfrac{1}{2} + N_B \times \cancel{4}^{2} \left(\frac{\sqrt{3}}{\cancel{2}}\right) -$

$\cancel{600}^{300} \times 3 \cdot \tfrac{1}{2}$

$0.4 N_B - 400 - 3.46 N_B - 900.$

$-3.06 N_B - 1300$

$\boxed{N_B = 284.971}$

$\boxed{F_B = 56.99\,N}$

$\Sigma V = 0 \Rightarrow F_B + N_A = 600 + 200.$

$\boxed{N_A = 743.01\,N}$

$\Sigma H = 0 \Rightarrow P + F_A = N_B$

$P = 284.97 - \cancel{743}\ 222.9$

$\boxed{P = 62.07}$

$F_A = 0.3 \times 743.01$

$\boxed{F_A = 222.9}$

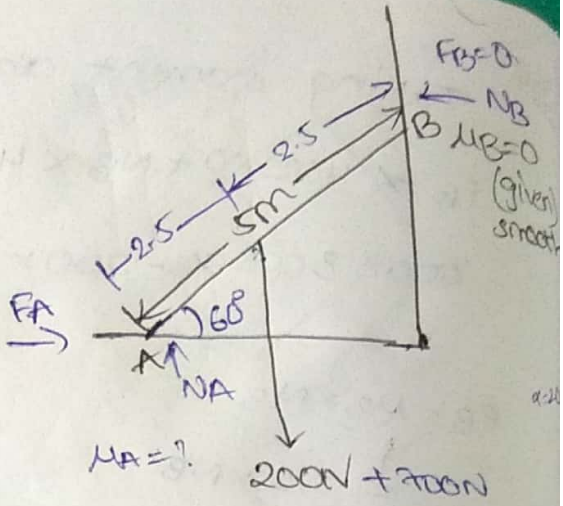
7) A ladder 5m long weighing 200N leans against a smooth vertical wall at an angle of 60° with horizontal. A man weighing 700N stands at mid height of ladder when it is about to slip. calculate the coefficient of friction b/w ladder & ground

Soln

moment about A,

$F_B \times 5\cos 60 + N_B \times 5\sin 60$

$= 900 \times 2.5\cos 60.$



$N_B \times \cancel{6.12}\ 4.33 = 1125$

$N_B = 259.8\,N$

$\Sigma V = 0 \Rightarrow N_A = 200 + 700 = 900N$

$\Sigma H = 0 \Rightarrow F_A = N_B = 259.8N.$

$F_A = \mu_A \times N_A$

$259.8 = \mu_A \times 900$

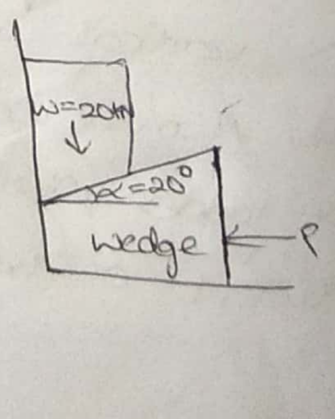
$\mu_A = 0.28$

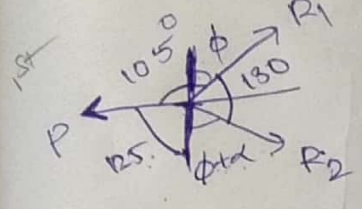
Wedge friction

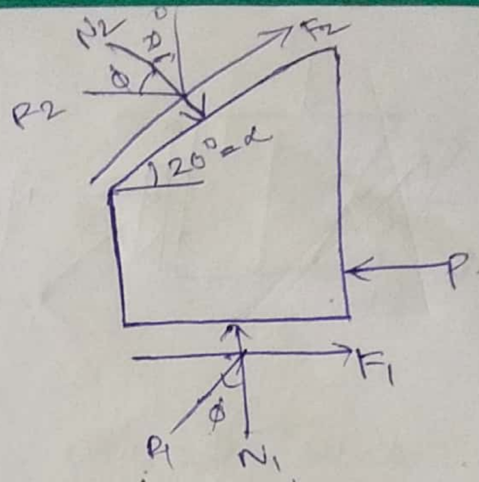
Wedge is a simple lifting machine. Generally, wedges are of metal or wood material in triangular or trapezium shape.

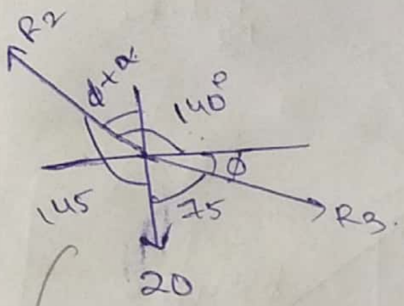
1) Determine the min. force required to move the wedge as shown in fig. The angle of friction for all contact surface is 15°.

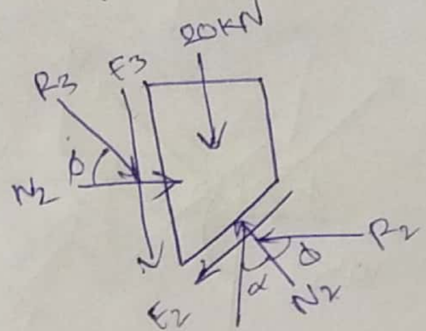
Soln $\phi = 15^0$











$$\frac{20}{\sin 140} = \frac{R_2}{\sin 75} = \frac{R_3}{\sin 145}$$

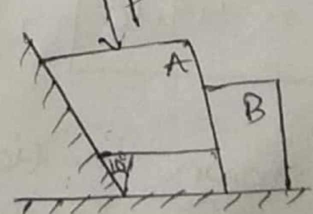
$\boxed{R_2 = 30.05\,KN}$, $\boxed{R_3 = 17.84\,KN}$

1st
$$\frac{R_1}{\sin 125} = \frac{R_2}{\sin 105} = \frac{P}{\sin 130}$$

$$\frac{R_1}{\sin 125} = 31.11 = \frac{P}{\sin 130}$$

$R_1 = 25.48\,NK$, $P = 23.83\,KN$

* A block of wt. 400N is to be moved by a force P acting on the weightless wedge A with an angle of 10°. If the coefficient of friction on all contact surfaces is 0.2. Find the value of P.



Sol:

$\mu = 0.2$

$\mu = \tan\phi$

$\phi = \tan^{-1}\mu$

$\phi = \tan^{-1}(0.2)$

$\phi = 11.30$, $\alpha = 10°$

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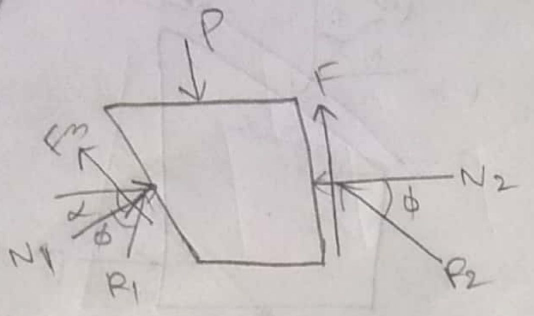
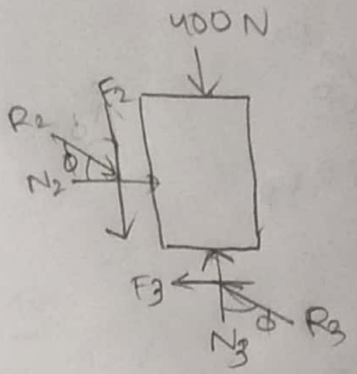
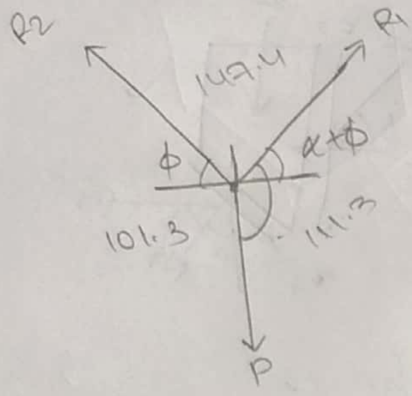
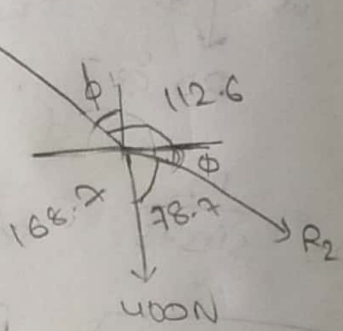




$\alpha = 10$, $\phi = 11.30$





$$\frac{400}{\sin(112.6)} = \frac{R_2}{\sin(168.7)} = \frac{R_3}{\sin(78.7)}$$

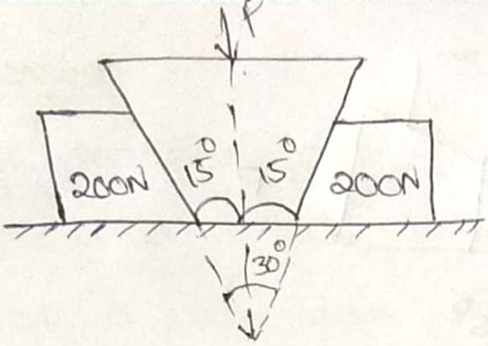
$\boxed{R_2 = 84.89}$, $\boxed{R_3 = 424.8}$

$$\frac{P}{0.538} = \frac{R_1}{0.98} = \frac{R_2}{0.93}$$

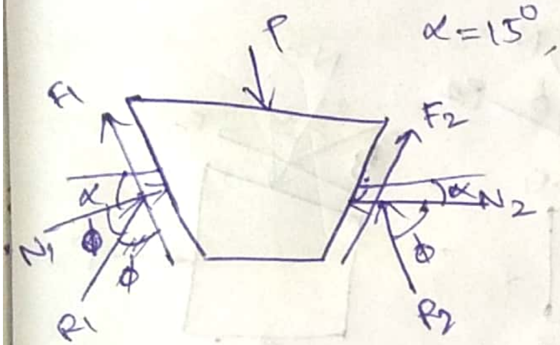
$$\frac{P}{0.538} = \frac{R_1}{0.98} = \frac{84.89}{0.93}$$

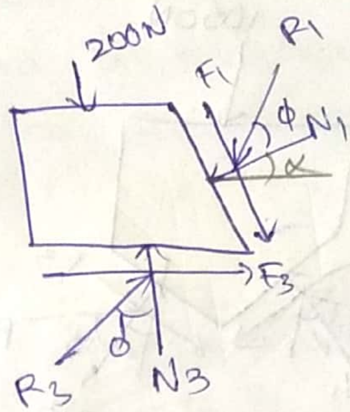
$\boxed{P = 49.10}$, $\boxed{R_1 = 89.4}$

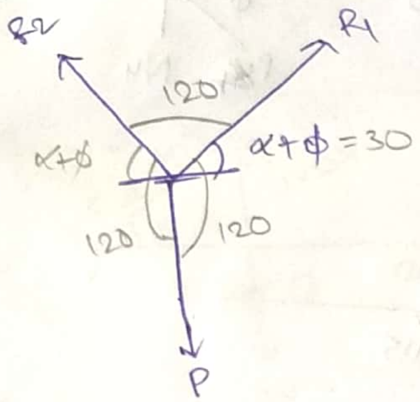
* As shown in figure two blocks each weighing 200N are to be pushed apart by a 30° wedge. The angle of friction is 15° for all the contact surfaces. What 'P' value is required to start movement of the block?

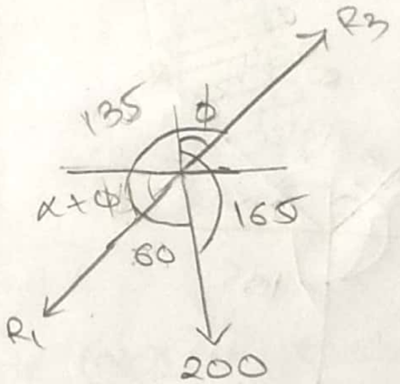


$\alpha = 15^0, \quad \phi = 15^0$









$$\frac{200}{0.707} = \frac{R_1}{0.258} = \frac{R_3}{0.866}$$

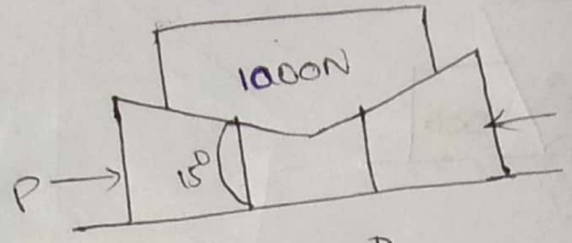
$R_3 = 244.9, \quad R_1 = 73.2$

$$\frac{P}{\cancel{0.866}} = \frac{R_1}{\cancel{0.866}} = \frac{R_2}{\cancel{0.866}}$$

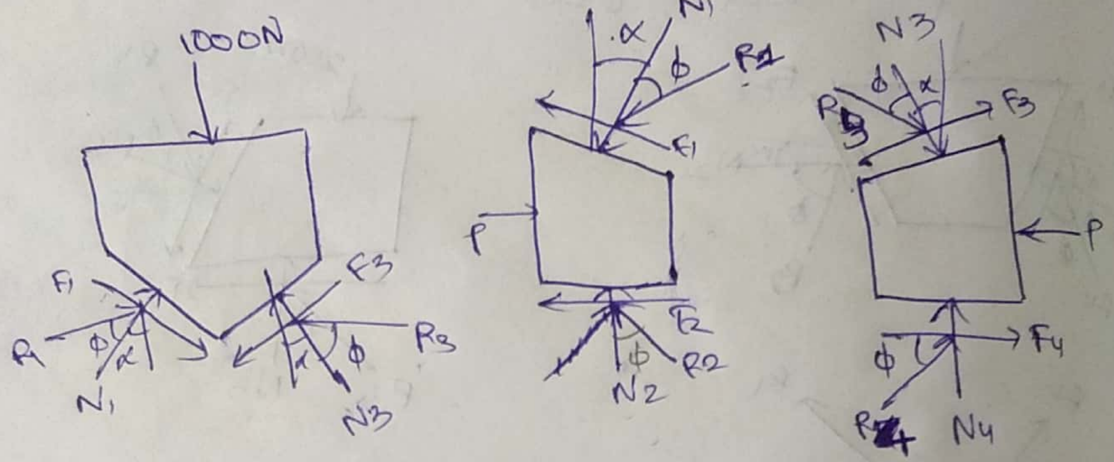
$\boxed{P = 73.2}$ $\boxed{R_2 = 73.2}$

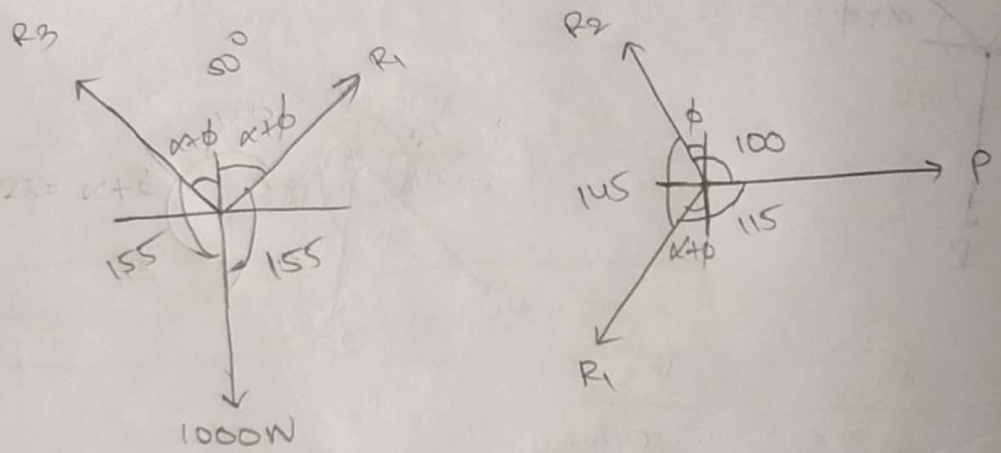
* what force 'p' must be applied to the weightle-ss wedges shown in fig. to start them under the 1000N block

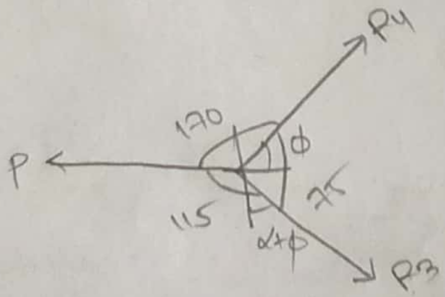
i) the angle of friction for all contact surfaces is 10°.



SOL $\phi = 10^\circ \quad \alpha = 15^\circ$







$$\frac{1000}{0.766} = \frac{R_1}{0.422} = \frac{R_3}{0.422}$$

$R_1 = 550.9N$, $R_3 = 550.9N$

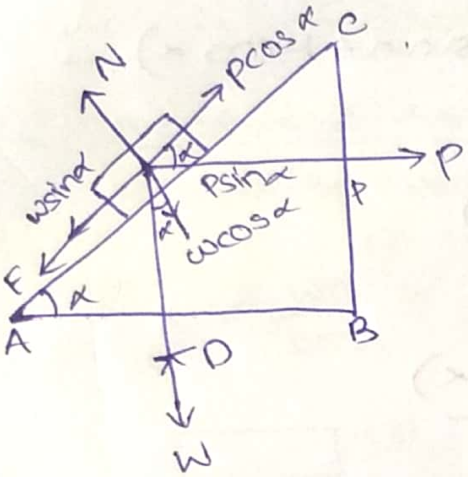
$$\frac{P}{0.573} = \frac{R_1}{0.984} = \frac{R_2}{0.906}$$

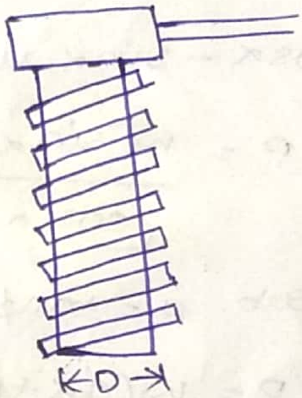
$\boxed{P = 320.7}$

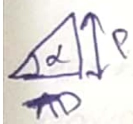
$\boxed{R_2 = 507.2}$

* Screw friction

A very large masses can be raised by pushing them up inclined plane have a shallow gradient. The screw of a jack can be considered as an inclined plane wrapped around a cylindrical core. The load resting on the head of the screw is regarded in the same way as the block on inclined plane.







The height of the plane (BC) is the distance moved axially in one revolution of the screw in its nut called "pitch".
The base of the plane AB is the circumference of the thread at the mean radius πD.

where D = mean thread diameter.

α = angle of plane

$$\boxed{\therefore \tan\alpha = \frac{p \text{ (span)}}{\pi D}}$$

$$** \boxed{\tan\alpha = \frac{np}{\pi D}}$$

where n = no. of starts

* The load is raised or lifted by a screw jack

$\Sigma F_P \Rightarrow P\cos\alpha = F + W\sin\alpha \rightarrow (1)$

$\Sigma F_N \Rightarrow W\cos\alpha + P\sin\alpha = N \rightarrow (2)$

WKT,

$F = \mu N$

$F = \mu(W\cos\alpha + P\sin\alpha)$

from (1)

$P\cos\alpha = \mu(W\cos\alpha + P\sin\alpha) + W\sin\alpha$

$P\cos\alpha - \mu P\sin\alpha = W\sin\alpha + \mu W\cos\alpha$

$P(\cos\alpha - \sin\alpha.\mu) = W(\sin\alpha + \mu\cos\alpha)$

$$P = \frac{W(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)}$$

By sub. $\mu = \tan\phi$

$$P = \frac{W(s\alpha + \tan\phi\cos\alpha)}{(c\alpha - t\phi s\alpha)}$$

Now, multiply with '$\cos\phi$' in numerator & deno-minator

$$P = \frac{W(\sin\alpha\cos\phi + \sin\phi\cos\alpha)}{(\cos\alpha\cos\phi - \sin\phi\sin\alpha)}$$

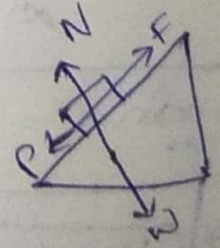
$$P = \frac{W\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$$

$\therefore \boxed{P = W\tan(\alpha+\phi)}$

load is lowered by a screw jack:

$\boxed{P = W\tan(\alpha-\phi)}$ where $(\alpha > \phi)$

$\boxed{P = W\tan(\phi-\alpha)}$ when $(\phi > \alpha)$



Efficiency of screw jack :-

$$\eta = \frac{\text{effort required without friction}}{\text{effort required with friction.}}$$

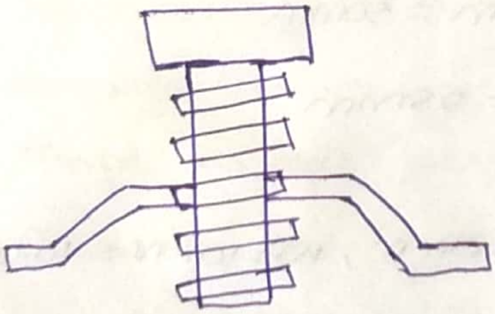
$$\boxed{\eta = \frac{w\tan\alpha}{w\tan(\alpha+\phi)}}$$ when load is raised

$$\boxed{\eta = \frac{\tan(\alpha-\phi)}{\tan\alpha}}$$ when load is lowered.

when lev is considered :-

$$\eta = \frac{\text{mechanical Advantage}}{\text{velocity Ratio.}}$$

$$\eta = \frac{W/P}{2\pi l/p} = \frac{Wp}{2\pi lP}$$



* A screw (guage) Jack has a t of 10mm pitch. what effort applied at the handle. 400mm long will be required to lift a load of 2kN. If the efficiency at this load is 45%.

Given,

efficiency of load = 45% = 0.45

W = 2kN = 2000 N

p = 10mm , l = 400mm

$$\eta = \frac{Wp}{2\pi lP} = \frac{2\times10\times1000}{2\times\frac{22}{7}\times400\times P}$$

$$P = \frac{2\times10\times1000}{2\times\frac{22}{7}\times400\times0.45}$$

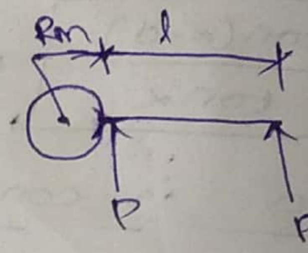
$P = 17.69N$

Torque :-

The twisting force that tends to cause rotation.

→ Torque required to rotate the screw against the load,

$$\boxed{T = P\times R_m = P'\times l}$$



* A screw & jack has mean diameter of 50mm and pitch 10mm. If the coefficient of friction b/w its screw & nut is 0.15. Find the effort required at the end of 700mm long handle to raise load of 10kN.

Sol Mean diameter $D_m = 50mm$

$R_m = 25mm$

pitch (P) = 10mm

$\mu = 0.15$, $l = 700mm$, $W = 10kN = 10,000N$

$\mu = \tan\phi$

$0.15 = \tan\phi$

$\phi = 8.53$

$$\tan\alpha = \frac{P}{\pi D}$$

$$\tan\alpha = \frac{10}{3.14\times50}$$

$\tan\alpha = 0.06$

$\alpha = 3.63$

$P = W\tan(\alpha+\phi)$

$= 10kN \tan(3.63 + 8.53)$

$= \cancel{2154}\ 10000\times\tan(12.16)$

$\boxed{P = 2156.5N}$

$P\times R_m = P'\times l$

$2156.5 \times 25 = P' \times 700$

$\boxed{P' = 77N}$

* centroid & center of Gravity :- 3-UNIT

Any rigid body is made up of a large no. of particles & each particle is attracted towards the earth.

-1 the force of attraction which is proportional to the mass of particle acts vertically downwards & is known as weight of the body.

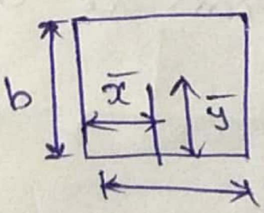
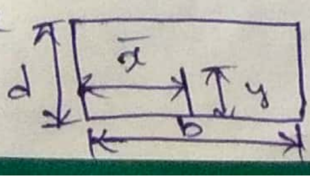
centroid :-

centroid (or) center of area is the point where the whole area of a plane fig. is Assumed to be concentrated.

center of gravity :-

center of gravity of a body is defined as the point through which resultant of the gravitational force weight acts for any orientation of body. Generally the term centroid is used for geometrical figures which have only areas but no mass & the term center of gravity is used when referring solid bodies having mass.

* centroids of plane geometrical shapes :-

	Area	$\bar{x}$	$\bar{y}$
1. square 	b^2	$b/2$	$b/2$
2 Rectangle 	bd	$b/2$	$d/2$