

# INDUSTRIAL ENGINEERING AND MANAGEMENT

PRAVIN KUMAR

# **Industrial Engineering and Management**

**Pravin Kumar**

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Chennai • Delhi

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# Preface

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*Industrial Engineering and Management* is a core subject for mechanical and production engineering students. It is also taught to other engineering branches as an open elective subject. This book is also very useful regarding production and operations management, statistical quality control, and total quality management for undergraduate students of engineering and post-graduate students of management. It has been observed that very few good books are available in industrial engineering, which fulfills the complete requirements of engineering students. Many books are available on the same title, but they are diverted from their focus and some of the contents described are not related to industrial engineering. Even contents related to industrial engineering are not properly addressed and also there is a lack of flow among the contents.

To remove these discrepancies, I planned to compile a book focused on core knowledge of industrial engineering and operations management. The main purpose of the book is to fulfill the requirements of engineering students. I have taught the same for last 15 years. In this book, the views of many faculty as well as students have been incorporated. The topics have been explained in depth with the suitable examples. Presentation of the topics is user friendly.

Simple language, proper sequence, useful content, and aesthetic design are the strengths of this book. These are very helpful for the better understanding of the readers. At the end of the each chapter, multiple-choice questions, review questions, and unsolved numerical problems are given. The solution of the unsolved problems and the power point presentation of each chapter are available online. The reader of this book can access the website easily.

The content of this book are broadly divided into three parts—industrial engineering, operations research and operations management. In industrial engineering, production systems, productivity, plant location and layout, forecasting, inventory control, manufacturing systems, material handling systems, value engineering, production planning and control, cost accounting and depreciation, and work study have been discussed. In operations research, only those topics are covered, which are required for better understanding of industrial engineering such as linear programming, transportation problems, assignment problems, sequencing of jobs, replacement analysis, queuing theory, and decision making. In operations management, only those topics are covered that are required for shop floor/production management, e.g., aggregate planning, material requirement planning, enterprise resource planning, project management, principles of management, total quality management, statistical quality control, ISO 9000 & 14000, and supply chain management. I hope this book will be very useful for the readers and fulfill their requirements.

**Pravin Kumar**

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When I conceptualized the idea to write a book on Industrial Engineering and Management, I was working as an Associate Professor in Indian Institute of Information Technology, Allahabad (on lien). I am thankful to the faculty of IIITA, Prof. Anurika Vaish, Vrijendra, Abhishek Vaish, Ranjit Singh, Saurabh Mishra, Shashikant Roy, and Prof. M.D. Tiwari (Vice Chancellor, Barkatulla University, Bhopal) for their cooperation and valuable suggestions.

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**Pravin Kumar**

## About the Author

---



Pravin Kumar obtained his Ph.D. from IIT-Delhi; M.Tech. from IT BHU (Now IIT BHU), Varanasi; and B.Sc. Engg. (Mechanical) from B.C.E. Bhagalpur. Presently, he is working as an Assistant Professor in Department of Mechanical Engineering, Delhi Technological University (Formerly Delhi College of Engineering). He has more than 15 years of teaching and research experience. He has worked in various engineering colleges and business schools such as IIIT-Allahabad, Asia-Pacific Institute of Management Delhi, Delhi School of Management, Career Institute of Technology and Management, Faridabad. His areas of expertise are Industrial Engineering, Operations Management, Quality Management, Operations Research, Supply Chain Management, and Production Engineering. He has developed many curriculums related to Production and Operations Management at Delhi Technological University, IIIT-Allahabad, Career Institute of Technology and Management. He is a life member of Indian Institution of Industrial Engineering, System Society of India, and Society of Operations Management, India. He has published more than 30 research papers in International Journals and Conferences and also authored books on Engineering Economy and Mechanical Engineering.

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# CHAPTER ▶ 1

## Industrial Engineering and Production Systems

### 1.1 INTRODUCTION

Industrial engineering determines the most effective ways to use the basic factors of production such as men, machines, materials, information, and energy to make a product or a service. These factors of production form the link between management goals and operational performance. Industrial engineering deals with increasing productivity through the management of men, methods and technology.

The American Institute of Industrial Engineering (AIIE, 1955) has defined the term 'industrial engineering' as given below (Maynard 1963):

Industrial Engineering is concerned with the design, improvement, and installation of integrated systems of men, machines, materials, and energy. It draws upon specialized knowledge and skills in the mathematical, physical and social sciences together with the principles and method of engineering analysis and design to specify, predict, and evaluate the results to be obtained from such systems.

A number of definitions have been given for industrial engineering. One more widely used and accepted definition of industrial engineering is given below:

Industrial engineering may be defined as the art of utilizing scientific principles, psychological data, and physiological information for designing, improving, and integrating industrial, management, and human operating procedures. (Nadler 1955)

Industrial management is closely related to industrial engineering and concerned with the techniques to develop, improve, implement and evaluate the integrated systems of men, materials, money, machines, methods, knowledge, information and energy. It includes the knowledge of various fields to increase the efficiency and effectiveness of an industry. The origin of industrial management has been industrial engineering. It is a process of planning, organizing, directing, controlling and managing the activities of any industry/organization. It organizes and transforms the inputs using various resources of the organization into value-added products in a controlled and an effective manner.

## 1.2 HISTORICAL DEVELOPMENT OF CONCEPTS IN INDUSTRIAL ENGINEERING AND MANAGEMENT

The evolution of industrial engineering has been defined in five different stages. These stages are mentioned below as:

- Phase 1: Pre-Industrial Revolution Era
- Phase 2: Industrial Revolution Era
- Phase 3: Scientific Management Phase
- Phase 4: Operations Research and Quantitative Management Phase
- Phase 5: Automation and Computer-Integrated Manufacturing (Modern Management)

### 1.2.1 Pre-industrial Revolution Era

Prior to the Industrial Revolution in the early 1800s, there was focus on only manual operated manufacturing activities. There was no factory concept; mostly, handicraft and agriculture products, etc. were dominated in the trade. Three major developments in this era are given below as:

- 1774: James Watt developed the steam engine.
- 1776: Adam Smith wrote *The Wealth of Nations* and advocated the concept of division of labour, skill development, specialization, etc. (Smith 1776).
- 1798: Concept of interchangeability of parts was developed by Whitney and was used in manufacturing of musket (Hatfield 2013).

### 1.2.2 Industrial Revolution

Industrial engineering emerged as a profession during the Industrial Revolution. This was due to the requirement of technically qualified and skilled people, who were needed to plan, organize and control the manufacturing processes. After the industrial revolution, Taylor and Gilbreth (Frank B. Gilbreth and his wife, Lillian Gilbreth) contributed a lot to the field of industrial engineering and later these contributions were known as the base of Scientific Management.

### 1.2.3 Scientific Management

Following contributions, as mentioned in chronological order, form a major part of Scientific Management:

- 1910: F. W. Taylor's Scientific Management
- 1911: Gilbreth's Motion Study
- 1913: Gantt's Scheduling Chart
- 1917: Harris Inventory Control
- 1924: Shewart's Statistical Control Chart
- 1927–33: Elton Mayo's Motivation Theory
- 1932: Babbage Wage payment and Time Study
- 1933: Barnes Work Study

Fredrick Winslow Taylor (Popularly known as F. W. Taylor) is considered the Father of Scientific Management. His ideas influenced by Adam Smith's book *The Wealth of Nations*, published in 1776; Thomas Malthus's *Population Theory*, published in 1798; David Ricardo's *Principles of Political Economy and Taxation*, published in 1817; John Stuart Mill's *Principles of Political Economy*, published in 1848; and Charles W. Babbage's book on the *Economy of Machinery and Manufacturers*, published in 1832.

Taylor's four principles are enumerated as follows (Taylor 1911):

1. Replace working by 'rule of thumb,' use the scientific method to study a work and determine the most efficient way to perform specific tasks.
2. Job specialization, i.e. rather than simply assign workers to do any job, match worker's capability and motivation to their jobs, and train them to work at maximum efficiency.
3. Monitor worker performance, and provide instructions and supervision to ensure that they are using the most efficient ways of working.
4. Allocate the work between managers and workers so that the managers spend their time on management, allowing the workers to perform their tasks efficiently.

Second major contributions in the field of scientific management are from Gilbreth family (Frank B. Gilbreth and his wife, Lillian Gilbreth). Frank B. Gilbreth focused on identification, analysis, measurement and setting standards for the fundamental motions, which were required to accomplish a job. His contributions were appreciated to set the standard time and method to perform a task. Lillian Gilbreth worked on a human relation aspect of engineering.

Another major contribution also came from Henry L. Gantt. He provided the concept of planning and scheduling the activities on a graphical chart, widely known as Gantt Chart. This is very helpful in reviewing the progress and updating the schedule of work.

The major development of industrial engineering was emerged during the period 1920–1940. In 1924, W. A. Shewhart developed the concept of Statistical Control Chart to measure the quality. During this period, concepts of inventory control, incentive plans, material handling, plant layout, etc. were evolved. Ralph M. Barnes worked on motion study for his doctoral work.

## **1.2.4 Operations Research and Quantitative Phase**

During World War II, concepts of Operations Research were developed and used to optimize the resources allocated in the war. During this phase, the concept of linear programming (LP) was developed by Dantiz. Some of the major developments observed during this phase are mentioned below as:

- 1956: First NC machine was developed.
- 1961: First time Robot was used.
- 1965: Flexible automation was used.

## **1.2.5 Automation and Computer-integrated Manufacturing Phase**

During this phase, most of the automation and computer-integrated manufacturing concepts were implemented in the industries. Concepts of lean manufacturing and Just-In-Time (JIT) were developed in automobile industries (Toyota manufacturing system).

### 1.2.6 Factory of the Future

Factory of the future will be highly automated. Robots will be used for various operations such as material handling, loading and unloading of jobs, welding, painting, etc. Most of the manual work will be eliminated.

## 1.3 PRODUCTION SYSTEMS

Quantitatively, production is concerned with quantity produced. But production implies the activity of producing goods and/or services. It is concerned with the transformation of inputs into the required outputs. In other words, production is a value-addition process through which raw material is converted into finished goods. At each stage of the production process, some values are added. Some examples of production are producing furniture, mobile phone, computer, car, etc.

A production system consists of inputs, i.e. raw materials, conversion subsystems, i.e. man and machine, control system, i.e. quality control and reliability, and outputs, i.e. finished products. All these components are interrelated to each other as shown in Figure 1.1. There are a number of other subcomponents of production systems that can be studied in detail in Chapter 10 'Production Planning and Control' in this book.

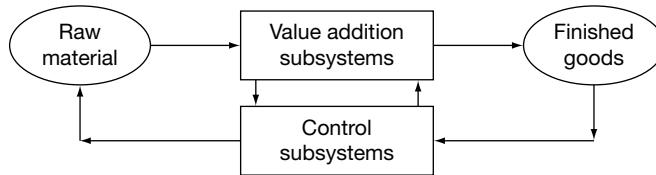


Figure 1-1: Production system of an organization

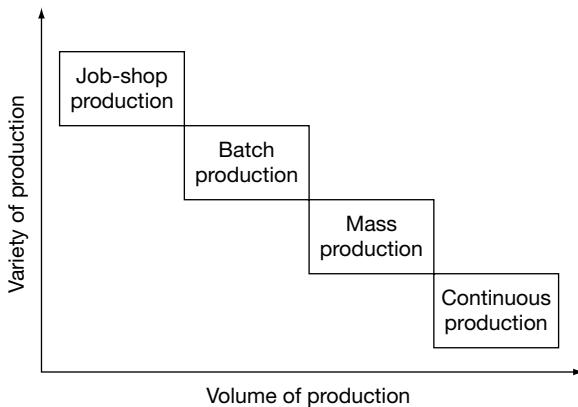
### Type of Production Systems

On the bases of volume and variety of products, production systems can be classified as job-shop production, batch production, mass production and continuous production. The first three production systems can be grouped as discrete types of production. These production systems are shown in Figure 1.2.

In discrete production systems, the production set-up is changed regularly to accommodate the production of different products of different designs and specifications, for example, auto, electronics, textile industries, etc. In continuous production systems, same product is produced continuously in the same sequence or operations, for example, petro-refinery, chemical and power plants. In discrete type of production systems, the products may be produced in different shifts, but in continuous type of production system, production continues to 24 hours without shifts.

### 1.3.1 Job-shop Production

Job-shop production is characterized by manufacturing of a large variety of products in small quantities that are designed and produced as per specifications given by customers. The main



**Figure 1-2:** Types of production systems

feature of this production system is highly flexible. A job-shop comprises general-purpose machines arranged in different departments. The process layout is most suitable for this type of production system. Each job requires unique technical requirements and processing on machines in a certain sequence.

### ***Characteristics of Job-shop Production***

1. Machines and methods employed are generic type as product changes are quite frequent.
2. Planning and control systems are flexible enough to deal with the frequent changes in product designs.
3. Manpower is skilled enough (cross-functional) to deal with changing work conditions.
4. Schedules are actually not fixed or predetermined in this system as no definite data is available on the product.
5. In-process inventory is usually high as accurate plans and schedules do not exist.
6. The product cost is normally high because of high material and labour costs.
7. Grouping of the machines is done on a functional basis (i.e. lathe section, milling section, etc.)
8. This system is highly flexible as the management has to manufacture varying product types.
9. Material handling systems are also highly flexible to meet changing product requirements.

### ***Advantages of Job-shop Production***

1. Most suitable for production of a variety of products due to the use of general-purpose machines.
2. Opportunities for learning multiple skills and getting varied exposure to the workers.
3. The full potential and skill of operators can be utilized.
4. Importance to creativity and innovative ideas.

### ***Limitations of Job-shop Production***

1. Higher cost due to low volume of production and lack of economies of scale.
2. Higher inventory cost due to higher level of inventory at all levels.
3. A complicated production planning is used.
4. Unnecessary movement of men and materials cannot be avoided in the shop due to application of functional layout.

### **1.3.2 Batch Production**

Batch production is a type of production in which the job passes through the functional departments in batches, and each batch may have a different routing. Batch production is characterized by the manufacture and stocking of a limited number of products at regular intervals, awaiting sales.

#### ***Characteristics of Batch Production***

1. Shorter production runs are used.
2. Flexible manufacturing is most suitable.
3. Plant and machinery set-up is used for production of items in a batch and a change in set-up may be required for processing the next batch.
4. Manufacturing lead time and cost of production are lower as compared to job-shop production due to higher volume.
5. As the final product is standard compared to job-shop production and manufactured in batches, economy of scale can be achieved up to some extent.
6. Machines are grouped on a functional basis similar to the job-shop manufacturing.
7. Semi-automatic and special-purpose automatic machines are generally used to take advantage of the similarity among the products.
8. Labours are multi-skilled and work upon different product batches.
9. In-process inventory is usually high owing to the type of layout and material handling policies adopted.
10. Semi-automatic material handling systems are most appropriate in conjunction with the semi-automatic machines.
11. Normally, production planning and control is difficult due to the odd size and the non-repetitive nature of order.

#### ***Advantages of Batch Production***

1. There is better utilization of plant and machinery compared to job-shop production.
2. Batch production promotes functional specialization.
3. Cost per unit is lower as compared to job-shop production.
4. Investment is lower in plant and machinery.
5. It is flexible enough to accommodate and process a number of products.
6. Job satisfaction exists for operators.

### ***Limitations of Batch Production***

1. Material handling is complex because of irregular and zigzag flows.
2. Production planning and control becomes complicated.
3. Work-in-process inventory is higher compared to mass/continuous production.
4. Higher set-up costs due to frequent changes in the set-up.

### **1.3.3 Mass Production**

Manufacture of discrete components or assemblies in a very large volume is called mass production. Machines are arranged in a line according to the sequence of operations on the product in product layout. Product and process standardization exists and all outputs follow the same path.

#### ***Characteristics of Mass Production***

1. Product and process sequences are standardized.
2. Special-purpose machines having higher production rate are used.
3. Production volume is large.
4. Production cycle time is shorter compared to job-shop and batch production systems.
5. In-process inventory is low.
6. Flow of materials, components and parts is continuous and without any backtracking.
7. Material handling can be completely automatic.

#### ***Advantages of Mass Production***

1. It has high production rate with reduced cycle time.
2. Less skilled operators may be employed.
3. Low in-process inventory is used.
4. Manufacturing cost per unit is low.

#### ***Limitations of Mass Production***

1. Breakdown of one machine stops the entire production line.
2. The line layout needs major change with changes in the product design, that is, the layout is less flexible.
3. Higher investment in production facilities is required.
4. The cycle time is determined by the slowest operation.

### **1.3.4 Continuous Production**

Production facilities for continuous production are arranged as per a predetermined sequence of production operations from the first operation to the finished product. The items are made to flow in a sequence of operations through material handling devices such as conveyors, transfer devices, etc. A highly rigid type of machine layout is used for continuous production.

### ***Characteristics of Continuous Production***

1. Dedicated plant and equipment is employed.
2. Material handling is fully automated.
3. The production process follows a predetermined sequence of operations.
4. Component materials cannot be readily identified with the final product.
5. Planning and scheduling are a routine action.

### ***Advantages of Continuous Production***

1. Product and process sequences are highly standardized.
2. The production rate is very high with reduced cycle time.
3. Capacity utilization is higher than the other production systems due to line balancing.
4. Manpower is not required for material handling, as it is completely automated.
5. A person with limited skills can be used on the production line.
6. The unit cost is lower due to the high volume of production.

### ***Limitations of Continuous Production***

1. Flexibility to accommodate and process a number of products does not exist.
2. Very high investment is required for setting flow lines.
3. Product differentiation is limited.

## **1.4 SELECTION OF PRODUCTION SYSTEMS**

Any manufacturing system cannot be an ideal system for production of a product considering all the factors simultaneously. The choice of the system depends on various factors, but basic factors that influence the selection of production system are specification of the final product and cost-effective production process. Other factors which determine the choice of the production system are given below as:

***Effect of volume/variety:*** One of the major considerations in the selection of production system is the volume/variety of the products. High product variety requires highly skilled labour, general-purpose machines, detailed production planning and control system. On the other hand, low product variety (i.e. one or few products produced in large volumes) enables the use of semi-skilled labour, highly automated production processes using special-purpose machines and simple production planning and control systems. The relationship between volume and variety of products is already shown in Figure 1.2.

***Capacity of the plant:*** The projected sales volume is a major influencing factor in determining whether the firm should go in for discrete/intermittent or continuous process. Fixed costs are high for continuous process and low for discrete process while variable costs are more in the discrete process and less for continuous process. Discrete process therefore will be cheaper to install and operate at low volumes and continuous process will be economical to use at high volume.

***Flexibility:*** Flexibility implies the ability of the company to meet the changes required in the market regarding product design and volume. If more varieties are to be manufactured,

the manufacturing facilities will have to be generalized depending upon the volume. Greater commonalities demand discrete manufacturing, which results in high inventories, large manufacturing lead times and elaborate planning and control.

**Lead time:** The lead time, more appropriately used in production system is delivery lead time expected by the customers. It is another major influencing factor in the selection of a production system. As a general rule, faster deliveries are expected by customers. The product, therefore, may require to be produced to stock using principles of batch production/mass production. If customers are ready to wait for the product, then the product may be produced to meet the order only.

**Efficiency:** Efficiency measures the speed and the cost of the transformation process. Efficiency will be higher for the products which are produced in mass. But for mass production of a product, greater demands are required. Therefore, depending upon the demand, product variety is to be considered and the process which gives the best efficiency in terms of machine and manpower utilization will have to be selected.

## 1.5 PRODUCTIVITY

Production and productivity are two different terms having different meanings. Higher production does not mean higher productivity, and vice versa. Production is related to the activity of producing goods or services. It is a process of converting inputs into some useful, value-added products/services. Productivity is concerned with how effectively the resources are utilized to increase the output of production. The productivity can be improved by increasing the output for same inputs or keeping constant output for decreased amount of inputs or increasing the output in greater proportion than the increase in inputs. Productivity may be calculated using the following formula:

$$\text{Productivity} = \text{Output}/\text{Input}$$

Productivity relates the efficient utilization of input resources for producing goods or services. Production is a measure of the output or volume produced. The emphasis is only on volume of production and not on how well the inputs or resources are utilized. In contrast, productivity emphasizes only on the ratio of the output produced to the inputs used. Productivity may be divided into two categories: partial productivity and total productivity.

### 1.5.1 Partial Productivity

Partial productivity is the ratio of the total output and individual input in the case of multifactor productivity (MFP) (Solow 1957). This term is used to measure the productivity of an individual input such as manpower, capital invested and energy utilized in production. Partial productivity is defined on the basis of the class of the input being considered. For example, if the labour was increased by 18 per cent during the last financial year, its effect on the increased output is represented by partial productivity. Similarly, partial productivity of capital, material and other inputs may be defined. The various components of partial productivity and their uses are shown in Table 1.1.

**Table 1-1:** Different forms of partial productivity

<b>Partial productivity</b>	<b>Formula</b>
1. Labour productivity	Output/Labour input
2. Material productivity	Output/Material input
3. Capital productivity	Output/Capital input
4. Energy productivity	Output/Energy input
5. Advertising and media planning productivity	Output/Advertising and media planning input
6. Other expense productivity	Output/Other expense input

### **Advantages of Partial Productivity Measure**

1. It is a good diagnostic measure to identify areas where improvements are required.
2. It is easy to calculate because it is independent of other inputs.
3. The management finds it easy to understand and pinpoint the logic for its improvement.
4. It is easy to benchmark (compare) with other industries.
5. Data may be easily generated for it.

### **Limitations of Partial Productivity Measure**

1. It can be misleading if used out of context.
2. It does not represent the overall effect of the system performance since it is concerned with the contribution of a specific input only and not all the resources.
3. Focused areas of improvements are difficult to identify. Therefore, sometimes wrong areas of management control may be identified for improvement.
4. It gives a myopic view of the performance of production systems. This means, only limited factors, which affect the output or performance, are considered.
5. It misses the holistic (or totality) approach.

**Example 1.1:** A mobile phone manufacturing company is producing 44,000 mobile phones per month by employing 200 workers in 8-hour shift. The company gets an additional order to supply 6000 mobile phones. The management has decided to employ additional workers. What will be production and productivity levels when the number of additional workers employed is: (a) 20 (b) 25 and (c) 30.

#### **Solution:**

$$\text{Present production} = 44,000 \text{ mobile phones}$$

$$\begin{aligned} \text{Present productivity (of labour)} &= \frac{\text{Present production (i.e., output)}}{\text{Total worker hours (i.e., input)}} \\ &= \frac{44,000 \text{ components}}{(200 \text{ workers})(8 \text{ hours})(30 \text{ days of the month})} \\ &= 44,000/48,000 = 0.916 \text{ mobile phone/man-hour} \end{aligned}$$

*With increased order*

- (a) When additional 20 workers are hired

$$\text{Production} = 44,000 + 6000 = 50,000 \text{ mobile phones}$$

$$\begin{aligned}\text{Productivity (of labour)} &= \text{Increased total production}/\text{Total man-hours} \\ &= 50,000/(200 + 20) (8) (30)\end{aligned}$$

$$= 0.946 \text{ mobile phones/man-hour}$$

- (b) When additional 25 workers are hired

$$\text{Production} = 44,000 + 6000 = 50,000 \text{ mobile phones}$$

$$\begin{aligned}\text{Productivity (of labour)} &= 50,000/(200 + 25) (8) (30) \\ &= 0.925 \text{ mobile phones/man-hour}\end{aligned}$$

- (c) When additional 30 workers are hired

$$\text{Production} = 44,000 + 6000 = 50,000 \text{ mobile phones}$$

$$\begin{aligned}\text{Productivity (of labour)} &= 50,000/(200 + 30) (8) (30) \\ &= 0.905 \text{ mobile phones/man-hour}\end{aligned}$$

In this example, it is clear that production has increased by 6000 units. Therefore,

$$\text{Increase in production} = (50,000 - 44,000)/44,000 \times 100 = 13.6 \text{ per cent}$$

### 1.5.2 Total Factor Productivity

TFP is the ratio of net output to the sum of associated labour and capital inputs. Net output means total output minus intermediate goods and services purchased. Notice that the denominator of this ratio is made up of only the labour and capital input factors.

$$\begin{aligned}\text{Total factor productivity (TFP)} &= \frac{\text{Net output}}{\text{Total factor input}} \\ &= \frac{\text{Total output} - \text{Materials and services purchased}}{(\text{Labour} + \text{Capital}) \text{ Inputs}} \\ \text{Total productivity} &= \frac{\text{Total output}}{\text{Total input}}\end{aligned}$$

*Advantage of TFP*

- (a) It is relatively easy to compare data from company records.
- (b) Industrialist prefers this as it is easy to compare in cross-industrial context.

*Limitations of TFP*

- (a) Many important inputs, such as material, energy, etc., are ignored.
- (b) The net output does not reflect the efficiency of the production system in a proper way.

**Example 1.2:** The data for output produced and inputs consumed for a particular type of a manufacturing organization are given below in constant money value. Find out the partial, total factor and total productivity values.

Output = Rs 3000.00

Labour input = Rs 600.00

Material input = Rs 300.00

Capital input = Rs 800.00

Energy input = Rs 150.00

Other expenses input = Rs 75.00

**Solution:**

Partial productivities

$$\text{Labour productivity} = \frac{\text{Output}}{\text{Labour input}} = \frac{3000}{600} = 5$$

$$\text{Material productivity} = \frac{\text{Output}}{\text{Material input}} = \frac{3000}{300} = 10$$

$$\text{Capital productivity} = \frac{\text{Output}}{\text{Capital input}} = \frac{3000}{800} = 3.75$$

$$\text{Energy productivity} = \frac{\text{Output}}{\text{Energy input}} = \frac{3000}{150} = 20$$

$$\text{Other expenses productivity} = \frac{\text{Output}}{\text{Other expenses input}} = \frac{3000}{75} = 40$$

$$\begin{aligned}\text{Total factor productivity (TFP)} &= \frac{\text{Net output}}{\text{Total factor input}} \\ &= \frac{\text{Total output} - \text{Materials and services purchased}}{(\text{Labour} + \text{Capital}) \text{ inputs}} \\ &= \frac{3000 - (300 + 150 + 75)}{600 + 800} = 1.76\end{aligned}$$

$$\text{Total productivity} = \frac{\text{Output}}{\text{Total input}} = \frac{3000}{600 + 300 + 800 + 150 + 75} = 1.55$$

**Example 1.3:** Table 1.2 gives the comparative study of several items of a motherboard for the years 2013 and 2014. Compute the changes in all productivity indices.

**Table 1-2:** Comparative study of productivity for the years 2013 and 2014

Items	2013	2014
Number of output at the rate of Rs 5000 per unit	10,000	16,000
Direct labour cost (Rs)	32,000	60,000

(Continued)

**Table 1-2:** (Continued)

Items	2013	2014
Capital depreciation (Rs)	8000	11,000
Capital book value (Rs)	32,000	45,000
Total indirect cost (Rs)	48,000	56,000
Energy used @ Rs 4/kW (in kW)	5000	8000
Raw materials used (Rs)	32,000	36,000
Services of consultant hired (Rs)	20,000	25,000

**Solution:**

(a) Direct labour productivity index =  $\frac{16,000/60,000}{10,000/32,000} \times 100$   
 $= 85.33$  per cent

(b) Capital depreciation productivity index =  $\frac{16,000/11,000}{10,000/8000} \times 100$   
 $= 109.09$  per cent

(c) Capital book value productivity index =  $\frac{16,000/45,000}{10,000/32,000} \times 100$   
 $= 113.77$  per cent

(d) Total indirect cost productivity index =  $\frac{16,000/56,000}{10,000/48,000} \times 100$   
 $= 137.14$  per cent

(e) Energy productivity index =  $\frac{16,000/8000}{10,000/5000} \times 100 = 100$  per cent

(f) Raw material productivity index =  $\frac{16,000/36,000}{10,000/32,000} \times 100$   
 $= 142.22$  per cent

(g) Consultant productivity index =  $\frac{16,000/25,000}{10,000/20,000} \times 100$   
 $= 128$  per cent

**Example 1.4:** Using the information given in Table 1.3, calculate the index for the following:

- (a) Direct labour productivity
- (b) Capital depreciation productivity
- (c) Capital book value productivity
- (d) Direct cost productivity
- (e) Total cost productivity
- (f) Energy productivity

**Table 1-3:** Comparative study of productivity for the years 2012 and 2013

Item	2012	2013
Number of outputs (all of one kind) (in Rs) (10,000 per unit)	250	300
Direct labour cost (in Rs)	50,000	60,000
Capital depreciation (in Rs)	4000	5000
Capital book value (in Rs)	16,000	24,000
Total indirect cost (in Rs)	40,000	44,000
Energy used (@ Rs. 4 per watt) (in kW)	700	2400
Raw material used (@ Rs 1000 per ton) (in tonnes)	12	16

**Solution:**

Calculation of productivity index

$$(a) \text{ Direct labour productivity index} = \frac{300}{60,000} \times \frac{50,000}{250} \times 100 = 100 \text{ per cent}$$

$$(b) \text{ Capital depreciation productivity index} = \frac{300}{5000} \times \frac{4000}{250} \times 100 \\ = 96.0 \text{ per cent}$$

$$(c) \text{ Capital book value productivity index} = \frac{300}{24,000} \times \frac{16,000}{250} \times 100 = 80 \text{ per cent}$$

$$(d) \text{ Direct cost productivity index} = \frac{300}{44,000} \times \frac{40,000}{250} \times 100 = 109.01 \text{ per cent}$$

$$(e) \text{ Energy used productivity index} = \frac{300}{2400 \times 4} \times \frac{700 \times 4}{250} \times 100 = 35 \text{ per cent}$$

$$(f) \text{ Raw material productivity index} = \frac{300}{16,000} \times \frac{12,000}{250} \times 100 = 90 \text{ per cent}$$

**1.5.3 Efficiency**

It is the ratio of output to standard output expected. Therefore, efficiency indicates a measure of how well the resources are utilized to accomplish a target or result. Efficiency may be calculated using the following formula:

$$\text{Efficiency} = \text{Output}/\text{Standard output}$$

Here standard output means output without loss, e.g. for a student 100 (full marks) is the standard output and the marks obtained by him is simply output.

### 1.5.4 Effectiveness and Productivity Index

The term ‘effectiveness’ is a measure of the degree of accomplishment or achievement of an objective (target). For example, a man rushes to the market to buy some medicines for a patient. He could go by a car, or by a bicycle or on foot. The cost and time are different for different modes of travel. Now suppose by the time the medicine is brought the patient dies, then the effort is not effective.

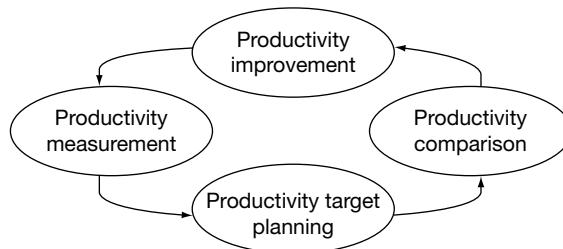
Effectiveness represents the degree of success in accomplishing objectives. Therefore, effectiveness indicates a measure of how well a set of targets or results are accomplished. Productivity is the integration of both efficiency and effectiveness. It indicates a combined effect of resource utilization (i.e. efficiency) and performance (i.e. effectiveness). The combined effect of efficiency and effectiveness is used in defining a term called *productivity index*:

$$\text{Productivity index} = \frac{\text{Performance achieved}}{\text{Input resources consumed}} = \frac{\text{Productivity in current year}}{\text{Productivity in base year}}$$

$$= \frac{\text{Effectiveness}}{\text{Efficiency}}$$

### 1.5.5 Productivity Cycle

The productivity cycle consists of four phases: productivity target planning, productivity comparison, productivity improvement and productivity measurement (see Fig. 1.3). In the first phase, a target of productivity is to be fixed considering various factors such as availability of resources and the production demand. In the second phase, productivity is compared with the productivity of competitor firms or the productivity of other sections in the same firm. In the third phase, the opportunities and scope of productivity improvement are highlighted and used for improvement. Finally, in the fourth phase, productivity is measured in other terms like efficiency. Efficiency represents the system’s ability to produce very close to standard output, i.e. output/standard output. But it is also used frequently as output/input, mathematically.



**Figure 1-3:** Productivity cycle

### 1.5.6 Factors Influencing the Productivity

There are various factors that influence productivity of an organization, such as man, machine, materials, space, energy, time and finance. Man is one of the important parts of the

production system. The number of employees, their skills and motivation affect the productivity of the system. Machines play an important role in improving the productivity. To improve the machine's availability, proper maintenance is required. Similarly, the third important component of production system is material's cost and quality. A high-quality material at low cost increases the productivity of the system. The time consumed in different processes such as inspection, maintenance, production affects the productivity of the system. In a similar way, proper utilization of space, energy saving and effective use of money increases the productivity.

*Man:* The productivity of man depends on the following processes:

1. Selection of an employee
2. The training given to employees
3. Number of personnel required for a job
4. Provision of incentive for workers

*Machine:* The productivity of a machine depends on the following factors:

1. Number of machines employed
2. Replacement policy for existing machines
3. Maintenance plans to avoid machine breakdown

*Material:* The following factors affect the productivity of a material:

1. Right quality
2. Right quantity
3. Substitutes for the existing material
4. Inspection and quality control programmes
5. Cost of material procurement and handling

*Time:* It affects the productivity in the following ways:

1. Inspection time for raw material
2. Inspection time for finished products
3. Production time
4. The time required to repair and maintenance work

*Space:* Utilization of space affects the productivity in the following ways:

1. Plant layout
2. The total area covered for production work
3. Location of different departments and shops

*Energy:* Use of energy affects the productivity in the following ways:

1. Energy-saving schemes
2. Use of renewable energy sources
3. Use of solar energy

*Finance:* Availability and efficient use of financial facilities affect the productivity.

### 1.5.7 Reasons for Lower Productivity

There are various reasons of poor productivity. Some of them are mentioned as follows:

1. Poor production planning and control
2. Low motivation of people
3. Lack of coordination
4. Unavailability of right tools, material and human force
5. Poor product design
6. Lack of standardization
7. Poor working environment
8. Non-standard methods of working
9. No accountability for loss of production
10. Government rules and regulations
11. Old age of plant and equipments
12. Weak R & D.

### 1.5.8 Ways to Improve Productivity

The productivity of any system can be improved either by proper use of resources or by effective utilization of the system or its processes. Some action plans for productivity improvement are listed below:

#### *Machine*

1. Manual labour is replaced by machines
2. Reliable machines
3. Automation.

#### *Management*

1. Motivated workforce
2. Better planning and coordination
3. Effective control over the system.

#### *Process*

1. Computerization of the system
2. Use of Management Information System (MIS)
3. Improvement in scheduling
4. Better material flow
5. Fast and accurate retrieval of parts.

#### *Work design*

1. Improved job design
2. Better work method
3. On-job training.

*Work environment*

1. Better lighting and illumination
2. Better ventilation
3. Safe workplace
4. Total quality management (TQM).

*Programme*

1. Quality circle
2. Suggestion scheme
3. Incentive scheme
4. Revise pay or policy.

*Technology*

1. Acquiring new technology such as Electro-Chemical Machining (ECM), etc.
2. Acquiring automated assembly line, for example, Surface-Mounting Technology (SMT) for printed circuit board assembly unit.
3. Acquiring computer-controlled machines, such as Computer Numerical Control (CNC) or Direct Numerical Control (DNC).
4. Using Automated Guided Vehicle (AGV) for material transportation.

*Manufacturing strategy*

1. Changing the manufacturing system from functional to a cellular layout if it is a batch production unit.
2. Adopting stockless production strategy and JIT framework in the production unit.
3. Keeping the workplace clean and environment-friendly (also termed as green-production system).
4. Opting for total change in the process/product or strategy if the system is not working properly (also known as Business Process Re-engineering or BPR).

*External environment*

1. Better political stability
2. Boosting economy and purchasing capacity of buyers
3. Globalization and open market economy

### **1.5.9 The Technology used to Improve Productivity**

1. Technology-based techniques: Computer-Aided Design (CAD), Computer-Aided Design and Drafting (CADD), Computer-Aided Engineering (CAE), Computer-Aided Process Planning (CAPP), Computer-Aided Quality Control (CAQC), Computer-Aided Instruction (CAI), Computer-Aided Manufacturing (CAM), robotics, Group Technology (GT) and Total Productive Maintenance (TPM).
2. Product-based techniques: Reliability, simplification, standardization, diversification and Research and Development (R & D).

3. Material-based techniques: Material Requirement Planning (MRP), Economic Batch Quantity (EBQ), Economic Order Quantity (EOQ), JIT and material handling.
4. Task-based techniques: Work simplification, work measurement, time study, method study, job analysis, job evaluation, merit rating, job safety and production scheduling.
5. Employee-based techniques: Incentive scheme, management by objective, job enlargement, job enrichment, recognition and punishment, Total Quality Management (TQM) and zero defects.

### **1.5.10 Guidelines for Productivity Measurement Systems**

Productivity measurement is directly related to the productivity improvement programme. A good productivity measurement should have the following characteristics.

1. It should be simple in calculation, meaningful, easy to understand and use, and provide the status of productivity in the organization.
2. It should be accurate enough to present a realistic assessment as perfect accuracy is an unreasonable expectation.
3. It should help in identifying the areas of low productivity so that productivity efforts can be applied to that area for improvement.
4. It should provide indices and information for comparison of performance for different periods.
5. It should provide indices and information for comparison of performance with other similar organizations/operations.
6. It should provide the information on interrelationship of different subsystems.
7. It should incorporate both tangible and intangible outputs and inputs to the system.
8. The productivity measurement system should be hierarchical in nature; the productivity at lower levels gives productivity of subsystem and the productivity of subsystems translates into overall productivity of the system.
9. It should facilitate to devise a reward or an incentive scheme for the workers.
10. It should lead to the participation and involvement of employees of various levels.
11. It should be economical and administratively easy to run the productivity measurement system.
12. It should be independent from the changes in monetary values and external disruptions.



### **SUMMARY**

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We have discussed basic concepts of industrial engineering and about the various phases of its conceptual development. Four types of production systems, their characteristics, advantages and disadvantages have been discussed in detail. These production systems are job-order production, batch size production, mass production and continuous production. Total productivity and partial productivity have been explained with the help of some numerical illustrations. Finally, factors affecting the productivity, the reason for poor productivity, methods to improve productivity have been outlined.

# MULTIPLE-CHOICE QUESTIONS

## Answers

1. (d)      2. (d)      3. (c)      4. (c)      5. (c)      6. (a)      7. (d)      8. (d)      9. (a)  
10. (b)     11. (d)     12. (a)     13. (c)     14. (a)     15. (c)

## **REVIEW QUESTIONS**

1. Define the term ‘industrial engineering’. How it differs from industrial management?
  2. Discuss the various phases of conceptual development of industrial engineering.
  3. What do you mean by a production system? How do you classify the production system based on volume of items produced?

4. Write the characteristics of discrete and continuous production systems.
5. Compare the characteristics of job-order production and mass production systems.
6. Write the advantages and disadvantages of batch production.
7. Write the advantages and disadvantages of continuous production.
8. What do you mean by productivity? How it differs from efficiency and effectiveness?
9. Write the importance of partial productivity.
10. What are the factors affecting the productivity of a production system?
11. Mention the reasons for poor productivity.
12. What are the methods used to improve the productivity?
13. Explain the working of productivity cycle.
14. Write the characteristics of a good productivity measurement process.

## EXERCISES

1. A company has generated productivity data for the two years (Table 1.4). Using measures of input and output in rupees, compare the total profit and productivity achieved for the two halves of the year. How does Y2 productivity compare with Y1 productivity? Use partial factor productivity to identify what might be done to improve productivity and profitability during the next year.

**Table 1-4:** Half-yearly productivity data

	<b>Y<sub>1</sub></b>	<b>Y<sub>2</sub></b>
Unit selling price	Rs 25	Rs 27
Total units sold	12,000	10,500
Labour hours	10,000	8,750
Labour cost/hr	Rs 10	Rs 10
Material usage (kg)	8000	6500
Material cost/kg	Rs 25.00	Rs 28.50
Other costs	Rs 22,000	Rs 18,500

2. XYZ Manufacturing uses two measures of productivity: (a) total sales/total inputs, (b) total sales/total labour inputs. Given data for the last three years (Table 1.5), calculate the productivity ratios. How would you interpret the results?

**Table 1-5:** XYZ Productivity data (in millions of rupees)

	<b>2012</b>	<b>2013</b>	<b>2014</b>
Sales	115	135	128
Materials	65	74	70
Labour	24	30	25
Overhead	7	10	9

3. A fast-food company produces 40,000 burgers each week. The equipment costs Rs 8000 and will remain productive for three years. The annual labour cost is Rs 10,000.
- (a) What is the productivity measured in units of output per rupee of input over a 2-year period?
- (b) The management has the option of Rs 12,000 for an equipment, with an operating life of five years. It would reduce labour costs to Rs 6000 per year. Should the management purchase this equipment (using productivity arguments alone)?



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# CHAPTER ▶ 2

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## Facility Location and Layout

### 2.1 INTRODUCTION

Plant location and layout is pre-production planning in which various factors are considered to fix up the location of the new facility and design the layout of the facilities in that location. In operations management, the word facility location and layout is frequently used in place of plant location and layout. In facility location and layout, many other facilities in addition to the manufacturing plant are considered such as warehouse, distribution centre, retail or service centre, etc. Facility location is a multi-attributes decision-making problem. A number of attributes are considered to determine the new location for the new facility. Facility layout is a problem related to the arrangement of departments or any facility inside the plant. In addition, we will study about group technology (GT) and flexible manufacturing system (FMS) and their use in facility layout.

### 2.2 FACILITY LOCATION

Facility location is a common issue for both new and existing businesses. In the global business scenario, facility location draws an especial attention of academia as well as practitioners. In global supply chain management, facility location in terms of offshore manufacturing or global marketing plays a key role. The area of study of facility location has become wider. The criteria that influence the decision of location of manufacturing plant have been discussed below:

*Proximity to market:* The new plant location is always preferred to closeness to market so that the transportation cost and distribution time can be minimized. Also, proximity to market gives better feedback of customer to produce products of better quality. The volume of production is directly related to the demand of the customer; therefore, the plant should be located very near to the market.

*Business climate:* It includes the presence of similar businesses, companies in the same industry, and other multinational companies. It also considers pro-business government legislation and local government intervention to facilitate businesses locating via provision of subsidies, tax abatements and other support.

*Total costs:* The objective is to minimize the total cost. The costs due to a particular region may include inbound logistics costs and outbound logistics costs. Land, construction, labour, taxes and energy costs make up the regional costs.

*Infrastructure:* Rail, road, air and sea port connectivity are vital for a business. Energy (Power) and telecommunication requirements must be fulfilled. In addition, the local government's initiatives to invest in upgrading infrastructure to the levels required may be an incentive to select a specific location.

*Availability of labour:* Skilled labour at low cost is another factor for facility location because labour cost is a major part of total production cost. This is the reason due to which many multinational automobile and electronics manufacturing companies prefer China and India as the best location for manufacturing.

*Availability of suppliers:* A quality supplier base makes a given location more preferable. The proximity of suppliers' plants also supports the concept of zero inventory, i.e. the inventory is managed by suppliers or vendors not by the manufacturer. This concept is also known as Vendor-Managed Inventory (VMI).

*Availability of raw material:* Availability of raw material is an important factor for plant location, for example, for thermal power plant coal and water should be easily available. Similarly, for Iron and Steel Company, iron ore should be easily available.

*Free trade zones:* Free trade zone is a closed facility into which foreign goods can be bought without custom duties. There are a large number of free trade zones in the United States. Such specialized locations also exist in other countries. Manufacturers in free trade zones can use imported components in the final product and delay payment of customs duties until the product is shipped into the host country.

*Political risk:* The uncertain geopolitical relationships among the countries present both opportunities and challenges. Political risks in both the country of location and the host country impinge location decisions.

*Trading blocs:* Various trading blocs have agreement among the member countries. As per agreement, firms locate or relocate within a bloc to take advantage of new market opportunities or lower total costs afforded by the trading agreement.

**Facility location methods:** The following methods are used for plant location:

1. Factor-rating method
2. Transportation method for linear programming
3. Centroid method
4. Cost-volume-profit ratio/Break-even analysis

## 2.2.1 Factor-rating Method

In this method, all the important factors for plant location are considered and they are assigned with some weights based on their importance. The proposed locations are rated against the factors and finally weighted average ratings are calculated for those locations. The location having highest overall rating is selected for plant location.

**Example 2.1:** Three plant locations X, Y and Z are given with rating with respect to certain factors. The weights to the factors are also mentioned as shown in Table 2.1. Find the best plant location among X, Y and Z.

**Table 2-1:** Factor rating of facility locations

Location	Factors		
	Availability of skilled labour ( $W_1 = 0.4$ )	Availability of raw materials ( $W_2 = 0.35$ )	Proximity to the markets ( $W_3 = 0.25$ )
X	70	60	55
Y	60	45	90
Z	55	95	50

### Solution:

At first, find the weighted scores of plant locations, then find the sum of weighted score for each location as shown in Table 2.2. The location having highest score is the best option for plant location.

**Table 2-2:** Weighted rating of facility locations

Location	Factors			Total
	Availability of skilled labour ( $W_1 = 0.4$ )	Availability of raw materials ( $W_2 = 0.35$ )	Proximity to the markets ( $W_3 = 0.25$ )	
X	$70 \times 0.4 = 28$	$60 \times 0.35 = 21$	$55 \times 0.25 = 13.75$	62.75
Y	$60 \times 0.4 = 24$	$45 \times 0.35 = 15.75$	$90 \times 0.25 = 22.5$	62.25
Z	$55 \times 0.4 = 22$	$95 \times 0.35 = 33.25$	$50 \times 0.25 = 12.5$	67.75

Location Z has the highest score as 67.75. Thus, Z is the best location.

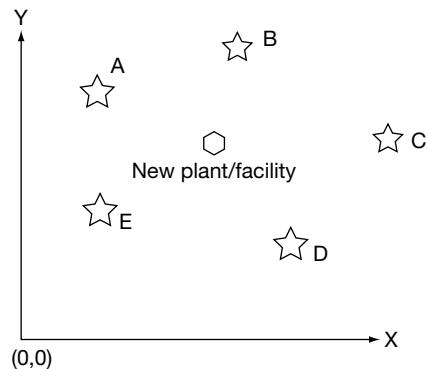
## 2.3 TRANSPORTATION METHOD

This method is based on minimization of transportation cost considering the source and supply constraints. This method is generally used for existing facilities, i.e. from which source to which destination products are to be supplied. This method is discussed in detail in ‘Transportation and Assignment’ chapter.

## 2.4 CENTROIDAL METHOD

This method is generally used to locate the warehouses or distribution centre so that maximum number of markets can be covered easily. The facility is located very near to the potential markets. The distance of various markets and corresponding volume of demand are considered

to locate the new facility so that the total transportation cost can be minimized. Let us consider five different markets A, B, C, D and E are located at different location as shown in Figure 2.1 and the demands of these markets are  $V_A$ ,  $V_B$ ,  $V_C$ ,  $V_D$ , and  $V_E$ , respectively.



**Figure 2-1:** Centroidal method of facility location

The coordinates of the location of the new facility are given as:

$$\bar{X} = \frac{\sum(X_i \cdot V_i)}{\sum V_i}; \quad \bar{Y} = \frac{\sum(Y_i \cdot V_i)}{\sum V_i}$$

where  $i$  represents the different markets A, B, ..., E and so on.

$V_i$  is the demand from  $i^{\text{th}}$  market.

**Example 2.2:** ABC manufacturing company plans to build a warehouse to serve its distribution centres in Kolkata (West Bengal), Faridabad (Haryana), Nagpur (Maharashtra) and Raipur (Chhattisgarh). The number of units to be shipped monthly from Company's warehouse to the distribution centres are shown in Table 2.3:

**Table 2-3:** Volume of demand and coordinates of markets

<b>Location</b>	<b>Annual shipping volume (<math>V_i</math>)</b>	<b>Coordinates (km)</b>	
		$X_i$	$Y_i$
Kolkata	7500	280	170
Faridabad	12,500	180	140
Nagpur	25,000	130	100
Raipur	5000	120	60

Find the appropriate location of the warehouse.

**Solution:****Table 2-4:** Weighted values of markets' coordinates

Location	Annual shipping volume ( $V_i$ )	Coordinates (km)		Weighted value of	
		$X_i$	$Y_i$	$V_i X_i$	$V_i Y_i$
Kolkata	7500	280	170	21,00,000	12,75,000
Faridabad	12,500	180	140	22,50,000	17,50,000
Nagpur	25,000	130	100	32,50,000	25,00,000
Raipur	5000	120	60	6,00,000	3,00,000
Total ( $\Sigma$ )	50,000	710	470	82,00,000	58,25,000

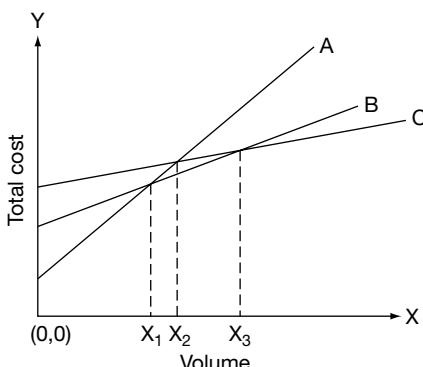
$$\bar{X} = \frac{\sum V_i X_i}{\sum V_i} = \frac{82,00,000}{50,000} = 164$$

$$\bar{Y} = \frac{\sum V_i Y_i}{\sum V_i} = \frac{58,25,000}{50,000} = 116.5$$

The location of warehouse should be at (164, 116.5) km from the manufacturing plant.

#### 2.4.1 Cost–Volume–Profit Ratio/Break-Even Analysis

In this method, it is assumed that fixed cost and variable cost vary from location to location. Thus, the total cost is a function of volume. It is decided that which location involves minimum cost for a fixed volume of production. For example, three locations A, B and C are shown in Figure 2.2 with a total cost and volume. It is observed from Figure 2.2 that for less than or equal to volume  $X_1$ , location A is profitable; between  $X_1$  and  $X_3$  volume location B is profitable; and for more than  $X_3$  volume location C is profitable.



**Figure 2-2:** Cost–volume–profit ratio for plant location

**Example 2.3:** An operations manager narrowed the search for a new facility location to four communities. The annual fixed costs (land, property taxes, insurance, equipment and buildings) and the variable costs (labour, materials, transportation and variable overhead) are shown in Table 2.5:

- Plot the total cost curves for all the communities on a single graph. Identify on the graph the approximate range over which each community provides the lowest cost.
- Using break-even analysis, calculate the break-even quantities over the relevant ranges. If the expected demand is 12,000 units per year, what is the best location?

**Table 2-5:** Fixed and variable costs for the different locations

Location	Fixed costs per year (Rs)	Variable costs per unit (Rs)
A	7,50,000	300
B	15,00,000	150
C	25,00,000	125
D	30,00,000	150

**Solution:**

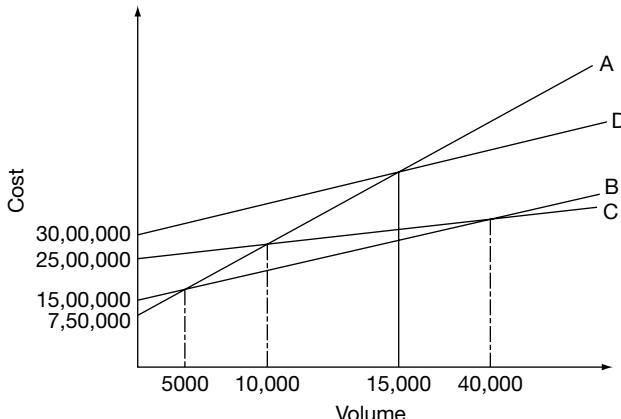
- The graphs of total costs for all the locations are shown in Figure 2.3.

$$TC_A = 7,50,000 + 300X$$

$$TC_B = 15,00,000 + 150X$$

$$TC_C = 25,00,000 + 125X$$

$$TC_D = 30,00,000 + 150X$$



**Figure 2-3:** Break-even analysis of locations A, B, C and D

- (b) From the graph shown in Figure 2.3:

Below 5000 units: A is the best location

Between 5000 and 40,000 units: B is the best location

Above 40,000 units: C is the best location

For 12,000 units: B is the best location.

## 2.5 FACILITY/PLANT LAYOUT

Plant layout is a planning concerned with the arrangement of departments, work groups within the departments, workstations, machines and stores within a production facility. The objective is to ensure a smooth work flow in the plant or a particular traffic pattern. The selection of particular pattern of layout depends on following facts:

1. Objectives of the layout, space availability, and the distance to be travelled between elements in the layout.
2. Volume of product or service to be produced.
3. Number of operations and amount of flow required between the elements in the layout.
4. Space requirement for the elements in the layout.

### ***Objectives of Plant Layout***

The objectives of plant layout are to:

1. utilize the available space effectively.
2. ensure smooth flow of materials.
3. improve the productivity.
4. reduce material-handling cost.
5. provide safety.
6. utilize labour effectively.
7. provide flexibility in operation.
8. provide easy supervision and control.
9. provide easy maintenance.
10. facilitate coordination and face-to-face communication.
11. reduce manufacturing cycle time or customer service time.

### ***Classification of Plant Layout***

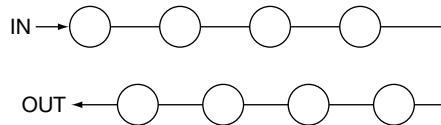
Plant layout may be classified into four classes as given below:

1. Product or line or flow shop layout
2. Process or functional layout
3. Group or hybrid or mixed layout
4. Fixed position layout

The classification of layout depends on the nature of production systems. For mass or continuous production, product layout is the most suitable; for job-shop production, process layout is the most suitable for batch production or cellular manufacturing, group layout is the most suitable; and for assembly of large-size product, fixed position layout is the most suitable.

## 2.5.1 Product Layout

In product layout, the machines are arranged according to the sequence of operations on a product. This is suitable for only mass or continuous production due to the requirement of high productivity. This layout is used in refinery, thermal power plant, chemical or fertilizer industry, etc. All the machines are arranged in a line as shown in Figure 2.4.



**Figure 2-4:** Machines arrangement in product layout

There are many advantages and disadvantages of product layout as discussed below:

### **Advantages**

1. It reduces material-handling time and hence the material-handling cost.
2. It increases the productivity due to ease of automation of the processes.
3. It reduces the manufacturing time.
4. It simplifies production planning and control systems.
5. It simplifies tasks, enabling unskilled workers to learn a task quickly.
6. It is more suitable to make to stock.

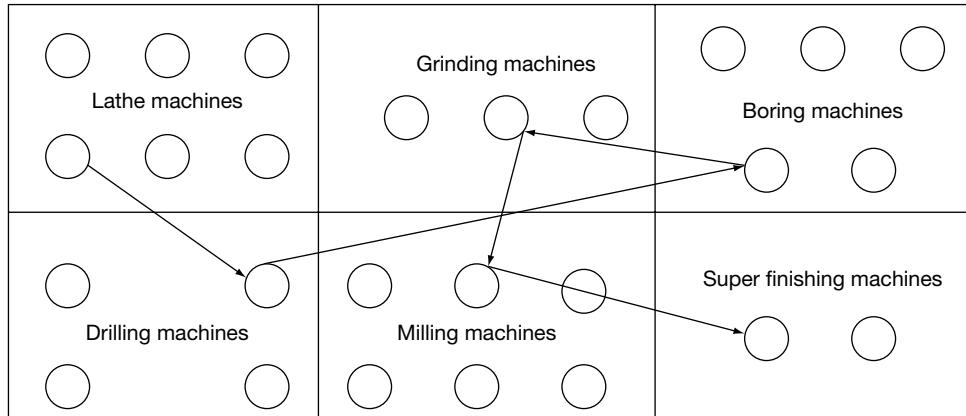
### **Disadvantages**

1. It has poor flexibility in accommodation of production of new product.
2. Special-purpose equipment and duplication is required to minimize the time of completion of the operations.
3. A breakdown of one machine or absence of one operator may stop the entire line of production.
4. To prevent the production breakdown, there is requirement of high work-in-process inventory.
5. Workers may become bored by the endless repetition of similar jobs.
6. It is less suitable to make to order.

## 2.5.2 Process Layout

The process layout is a layout in which similar machines are arranged according to their nature or functions of operations not on sequence of operations, for example, all lathe machines are arranged at one place, all the milling machines are arranged at another place and so on. A part being worked travels according to the established sequence of operations, from area to area, where the required machines are located for each operation. This type of layout is normally used in a service organization, for example, hospitals, where areas are dedicated to particular types of

medical care or facility. A schematic diagram of process layout is shown in Figure 2.5 in which the flow of material is zigzag.



**Figure 2-5:** Machines arrangement in process layout

There are following advantages and disadvantages of process layout:

### **Advantages**

1. It is more flexible compared to product layout as equipment and personnel can be used for any product.
2. Investment required is less as duplication is not necessary unless volume is large.
3. Supervisors and workers get exposure of cross-functional skills.
4. Changing work nature creates interest among the workers and makes work more satisfying for the people who prefer variety.
5. This layout is more suitable for make to order or smaller volume of production.

### **Disadvantages**

1. It requires highly skilled workers.
2. Backtracking and long movements in the handling of materials minimize the efficiency and increase the material-handling time and cost.
3. Waiting time is more in process layout.
4. Production planning and control is complex in comparison to product layout.
5. Workers' wages are higher due to involvement of highly skilled workers.
6. Due to frequent changes in the nature of the job, productivity becomes low.

### **2.5.3 Difference between Product Layout and Process Layout**

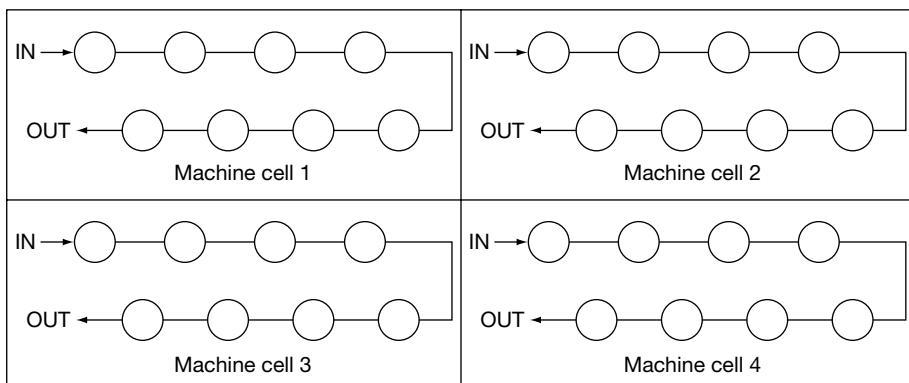
The differences between product and process layouts are shown in Table 2.6.

**Table 2-6:** Differences between product and process layouts

<b>Product layout</b>	<b>Process layout</b>
There is sequential arrangement of machines as per operations.	Functional arrangement of machines as per nature of operations of machines.
This is used for mass and continuous type of production.	This is used for job-shop and batch production.
Products produced are standard and made to stock.	The product produced is general and made to order.
This is used for stable and high demand products.	This is used for the variable demand product.
Special-purpose machines are used.	General-purpose machines are used.
Limited or semi-skilled people may be employed.	High and cross-functional skilled people are employed.
Materials and men move on a fixed path.	Materials and men move on a variable path.
This layout is highly efficient and productive.	This layout is highly flexible.

## 2.5.4 Group Layout

Group layout is based on the principle of Group Technology (GT). This layout is used where product mix is used. All the products are broken into different part families based on similarity in manufacturing and design. Separate manufacturing cells are planned for each part family and in each cell machines are arranged as per sequence of operations of the part of that part family. Thus, the cells look like a process layout, but the machines are arranged in those cells on the basis of product layout. This is the reason, this layout is also known as mixed or hybrid or cellular layout or inter-cell layout (Drira et al. 2007; Hamann and Vernadat 1992). A schematic view of group layout is shown in Figure 2.6.

**Figure 2-6:** Machine cells in group layout

The advantages and disadvantages of group layout are enumerated as follows:

### **Advantages**

1. The control required is reduced.
2. Material handling is reduced due to the formation of special cell for the individual part family.
3. Set-up time is reduced due to use of the automation process to search the suitable part family for new products.
4. In-process inventory is reduced.
5. Due to mixed properties of product and process layout, operator's expertise increases.
6. Due to requirement of better coordination among the different cells, human relations improve.

### **Disadvantages**

1. Shop flexibility is reduced.
2. Machine utilization is also reduced compared to product layout.
3. Higher skilled person in each cell is required.
4. The cell concept leads to unbalanced workload on machines.

## **2.5.5 Fixed Position Layout**

In this layout, machines and workers move to the site and products remain in a fixed position for its entire manufacturing period. This layout is used for bulky and fragile products such as planes, ships, railway wagon, etc. This is generally used for assembly shop.

The advantages and disadvantages of fixed position layout are as follows:

### **Advantages**

1. It reduces the movement of work items and minimizes damage or cost of handling.
2. There is no problem like rearrangement of a machine or reassignment of jobs to workers.

### **Disadvantages**

1. Skilled and versatile workers are required because same workers are involved in more than one operation.
2. The poor availability of necessary combination of skills may increase the wage.
3. Movement of people and equipment to and from the work site may be expensive.
4. Utilization of equipment may be low because the equipment may be left at a location where it will be needed again in the next few days rather than moved to another location where it would be productive.

## **2.6 SYSTEMATIC LAYOUT PLANNING**

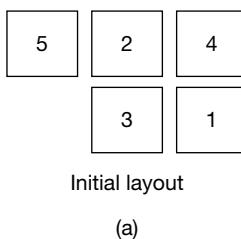
The systematic layout planning (SLP) is a tool used to arrange a workplace in a plant by locating two areas with high frequency close to each other. The process permits the quickest material flow in processing the product at the lowest cost and least amount of handling. Thus, SLP is based on

the frequency of material movement between two departments. The distance can be minimized for moving raw materials and the problem about the useless area can be solved using SLP. In SLP, the relationship of each activity in closeness area is developed to make the relationship of each activity in the graph from – to – chart as shown in Figure 2.8, and the closeness value is defined as A = absolutely necessary, E = especially important, I = important, O = ordinary closeness, U = unimportant, and X = undesirable. The coding of closeness values and their weights are shown in Figure 2.7.

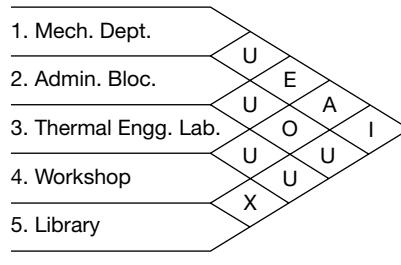
Value	Closeness	Code	Weights
A	Absolutely necessary	=====	16
E	Especially important	=====	8
I	Important	=====	4
O	Ordinary closeness	----	2
U	Unimportant		0
X	Undesirable	^VVV	80

**Figure 2-7:** Closeness values and their weights

For example, an initial layout of a mechanical engineering department and its concerned facilities are shown in Figure 2.8(a); closeness values between each department are shown in Figure 2.8(b); initial relationship diagram based on closeness values is shown in Figure 2.8(c); and final layout is shown in Figure 2.8(d). In the final layout, the facilities having high closeness values are kept close to each other. In Figure 2.8(d), facility 1 has a highest closeness value with facility 4 and then facility 3, therefore, facilities 4 and 3 are very close to facility 1. The relationship between facilities 4 and 5 is undesirable; therefore they are far apart from each other. Similarly, the relationship between 1 and 5 is simply important and relationships between facilities 3 and 5 and between facilities 2 and 5 are unimportant.

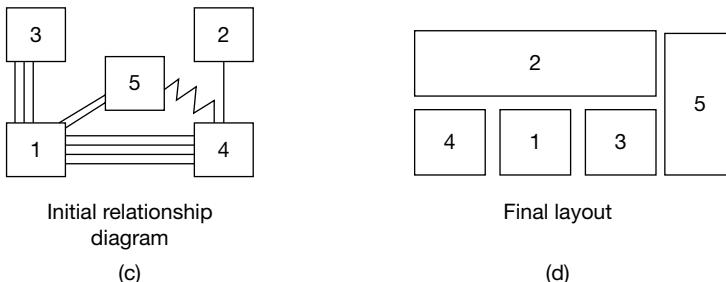


(a)



(b)

**Figure 2-8:** Systematic layout planning



**Figure 2-8:** (Continued)

## 2.7 BLOCK DIAGRAM

A block diagramming is a method to visualize the amount of movement that occurs between departments. Each block represents one department of a facility. Blocks can be moved around in order to minimize the distance travelled between them. Block diagramming is not performed frequently. The resulting layouts are usually implemented for the long range of time. Proper designs can lead to increased efficiency within a company. An example of block diagramming is demonstrated below:

**Step 1:** Find the area needed for the departments as shown in Table 2.7.

**Table 2-7:** Departments and area needed

Departments	1	2	3	4	5	6	Total
Area needed (m <sup>2</sup> )	100	80	80	120	80	80	540

**Step 2:** Start with an initial layout as shown in Figure 2.9.

**Step 3:** Find the amount of movements between the departments as shown in Table 2.8

2	4	3
6	5	1

**Figure 2-9:** Initial layout

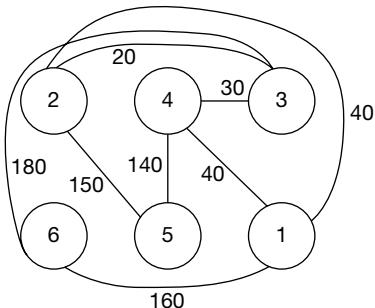
**Table 2-8:** Frequency of interdepartmental movement

<b>Department</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>
1	—	40		40		160
2		—	20		150	
3			—	30		180
4				—	140	
5					—	
6						—

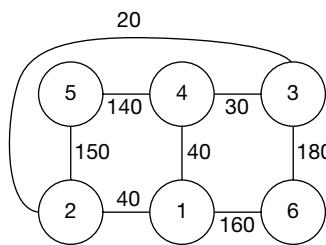
**Step 4:** Show initial traffic as shown in Figure 2.10.

**Step 5:** Rearrange the blocks; the blocks with high interaction should be close to each other as shown in Figure 2.11.

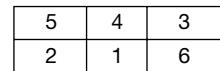
**Step 6:** Finalize the layout of blocks as shown in Figure 2.12.



**Figure 2-10:** Initial traffic



**Figure 2-11:** Blocks rearrangement



**Figure 2-12:** Final layout of blocks

Beyond these methods, Singh and Sharma (2006) reviewed different approaches for facility layout problems, such as application of computers in plant layout.

## 2.8 ASSEMBLY LINE BALANCING

Line balancing is the distribution of load on different workstations to minimize the idle time. All the different activities of a task or an assembly has different processing times that lead to idle time on the workstations having shorter processing time. Line balancing is concerned with the way assigning of activities so that the idle time can be minimized.

The work to be performed at a workstation is equal to the sum of the tasks assigned to that workstation. The assembly line balancing problem is related to assigning all the tasks to be performed in a series of workstations so that each workstation has no more than can be done in the workstation cycle time, and so that the idle time across all workstations is minimized. The problem is complicated due to precedence relationship among the work elements or tasks. There are certain rules of line balancing as given below:

1. Identify the cycle time and determine the minimum number of workstations using the following formulas:

$$\text{Cycle time, } C = \frac{\text{Production time per day}}{\text{Total number of units required per day}}$$

$$\text{Theoretical number of workstations, } N_t = \frac{\text{Sum of task time (T)}}{\text{Cycle time (C)}}$$

2. Tasks are assigned to work stations moving left to right through the precedence diagram.

3. Before each assignment, use the following criteria to determine which tasks are eligible to be assigned to a workstation
  - (a) All proceeding tasks should be arranged in the sequence.
  - (b) The task time does not exceed the time remaining at the workstation, if no task is eligible, move to the next workstation.
4. After each task assignment, determine the time remaining at the current workstation by subtracting the sum of task times already assigned to it from the cycle time.
5. Break ties, if any, using the following rules:
  - (a) Assign the task with the longest task time.
  - (b) Assign the task with the greatest number of followers.
  - (c) If there is still a tie, choose one task arbitrarily.
6. Continue until all tasks have been assigned to work stations.
7. Compute appropriate measures (per cent idle time and efficiency) for the set of assignments.

$$\text{Efficiency} = \frac{\text{Sum of task time (T)}}{\text{Actual number of work stations (N}_a\text{)} \times \text{Cycle time (C)}}$$

8. If efficiency is unsatisfactory, rebalance using a different decision rule.

**Example 2.4:** The following tasks must be performed on an assembly line in the sequence and times specified in Table 2.9;

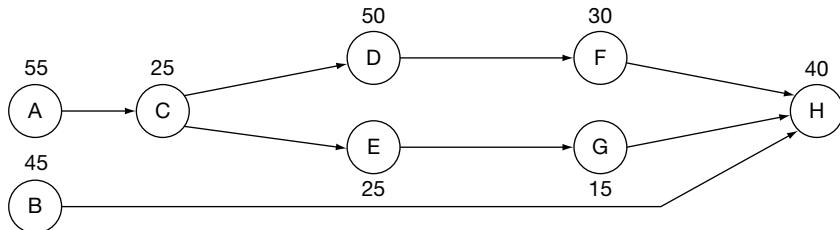
**Table 2-9:** Tasks with the predecessors and completion time

Task	Task time (in seconds)	Predecessor
A	55	—
B	45	—
C	25	A
D	50	C
E	25	C
F	30	D
G	15	E
H	40	B, F, G

- (a) Draw the schematic precedence diagram.
- (b) What is the theoretical minimum number of stations required to meet a forecast demand of 500 units per 8-hour day?
- (c) Use the longest-task-time rule and balance the line in the minimum number of stations to produce 500 units per day.
- (d) Find the efficiency.

**Solution:**

- (a) Precedence diagram (Network diagram) as per sequence of operations is shown in Figure 2.13.



**Figure 2-13:** Network or Precedence diagram

$$\begin{aligned}
 (b) \quad \text{Cycle time, } C &= \frac{\text{Production time per day}}{\text{Total number of units required per day}} \\
 &= \frac{8 \times 60 \times 60}{500} = 57.6 \text{ seconds per unit} \\
 \text{Theoretical number of workstations, } N_t &= \frac{\text{Sum of task time (T)}}{\text{Cycle time (C)}} \\
 &= \frac{285}{57.6} = 4.94 \approx 5
 \end{aligned}$$

- (c) To assign the task on the work stations, at first assign the task of largest activity time considering the sequence of operations. The sequence of operations should not be disturbed. Suppose two tasks have same activity time than see the total number of followers (activities) and assign the activity first on the workstation having largest number of followers. Here, followers mean the activities linked to the said activity till the end and to be completed. The assignment of tasks on the five workstations and idle time on the workstations are shown below as:

Work Station 1:	Task A = 57.6 – 55 = 2.6	(Idle time)
Work Station 2:	Task B = 57.6 – 45 = 12.6	(Idle time)
Work Station 3:	Task C = 57.6 – 25 = 32.6	
	Task E = 32.6 – 25 = 7.6	(Idle time)
Work Station 4:	Task D = 57.6 – 50 = 7.6	
Work Station 5:	Task F = 57.6 – 30 = 27.6	
	Task G = 27.6 – 15 = 12.6	(Idle time)
Work Station 6:	Task H = 57.6 – 40 = 17.6	(Idle time)

$$\begin{aligned}
 (d) \quad \text{Efficiency} &= \frac{\text{Sum of task time (T)}}{\text{Actual number of work stations } (N_a) \times \text{Cycle time (C)}} \\
 &= \frac{285}{6 \times 57.6} = 82.46\%
 \end{aligned}$$

## 2.9 GROUP TECHNOLOGY

Group Technology (GT) is a manufacturing philosophy in which similar parts are identified and grouped together to take advantage of their similarities in manufacturing and design. It provides an integration of the design and manufacturing activities and helps to improve the productivity in batch manufacturing industries.

GT implies the notion of recognizing and exploiting similarities in three different ways: (a) by performing the like activities together, (b) by standardizing the similar tasks and (c) by efficiently storing and retrieving the information about recurring problems.

### 2.9.1 Part Family

The part family is a collection of parts which are similar either because of geometric shape and size or because of similar processing steps required in their manufacture. By grouping work-parts into families, we may use group layout instead of process-type layout. In process-type layout, workpiece moves between sections, sometimes visits a section several times. This causes a significant amount of material handling, large in-process inventory, more set-ups, long manufacturing lead times and high cost. In group layout, we have a production shop with machines arranged in cells. Each cell is organized to specialize in the manufacture of a particular part family. Cells can be designed to form the production flow lines with conveyors transporting work-parts between machines in the cell.

Different GT approaches have been developed to decompose a large manufacturing system into smaller and manageable systems based on similarities of design attributes and part features.

There are three main approaches for grouping parts into families and implementing GT in a plant. These three approaches are mentioned below as:

1. Visual inspection method
2. Classification and coding by examination of design and production data
3. Production flow analysis

*Visual inspection method:* It is less sophisticated and the least expensive method. It involves arranging a set of parts into part families by visually inspecting the physical characteristics of the parts or their photographs. Although this method is the least accurate one among the three, this method is very popular.

*Parts classification and coding:* This is a time-consuming and complicated method. A classification and coding system should be custom-engineered for a given company or an industry.

*Production flow analysis:* In production flow analysis, the parts or products are classified on the basis of production flow, i.e. sequence of operations on different workstations.

### 2.9.2 Types of Classification and Coding Systems

Parts classification systems fall broadly into one of the following two categories:

1. Systems based on part design attributes
2. Systems based on part manufacturing attributes

*Part design attributes:* There are various design attributes which form the basis for part classification and coding. These attributes are basic external and internal shapes, length/diameter ratio, material type, part function, major dimensions, minor dimensions, tolerances and surface finish.

*Part manufacturing attributes:* The different manufacturing attributes for part classification and coding are major process, minor operations, major dimension, length/diameter ratio, surface finish, machine tool, operation sequence, production time, batch size, annual production, fixtures needed and cutting tools.

### Type of Codes

*Monocode or hierarchical code:* In this code structure, the interpretation of each succeeding symbol depends on the value of the preceding symbols.

*Polycode or chain code:* Code symbols are independent of each other. Each digit in a specific location of the code describes a unique property of the work piece.

*Mixed code:* It has the advantages of both mono- and polycodes. Most coding systems use this code structure.

### 2.9.3 Opitz Classification System

It was developed at the Technical University of Aachen in Germany. It is best known and widely used classification system and can be applied to machined parts, non-machined parts and purchased parts. It requires both design and manufacturing information. The following digit sequence is used:

12345      6789      ABCD

*Form code:* It includes the first five digits and describes the primary design attributes of the part.

*Supplementary code:* It indicates some of the attributes that is used in manufacturing (dimensions, work material, starting raw piece shape and accuracy).

*Secondary code:* It identifies the production operation type and sequence and can be designed by the firm to serve its own particular needs.

- The complete coding system is too complex.
- There is an entire book on this system written by Opitz.

### 2.9.4 Problems in Implementation of GT

The problems that have prevented the widespread application of GT are given below:

1. The problem of identifying part families among the many components produced by a plant.
2. The expense of parts classification and coding.
3. Rearranging the machines in the plant into the appropriate machine cells.
4. The general resistance when changeover to a new system is decided.

## 2.9.5 Advantages of GT

Advantages of GT layout regarding different aspects are as given below:

### ***Engineering Design***

1. Reduction in new parts design.
2. Reduction in the number of drawings through standardization.
3. Reduction of drafting effort in new shop drawings.
4. Reduction of number of similar parts, easy retrieval of similar functional parts and identification of substitute parts.

### ***Layout Planning***

1. Reduction in production floor space required.
2. Reduced material-handling effort.

### ***Equipment, Tools, Jigs and Fixtures***

1. Standardization of equipment.
2. Reduced number of tools, pallets, jigs and fixtures.
3. Significant reduction in costs of releasing new parts.

### ***Process Planning***

1. Reduction in set-up time and production time.
2. Improved machine loading and shortened production cycles.
3. Reduction in number of machining operations and numerical control (NC) programming time.

### ***Production Control***

1. Reduced work-in-process inventory.
2. Easy identification of bottlenecks.
3. Improved material flow and reduced warehousing cost.
4. Faster response to schedule changes.
5. Improved usage of jigs and fixtures, pallets, tools, material handling and manufacturing equipment.

### ***Quality Control***

1. Reduction in number of defects leading to reduced inspection effort.
2. Reduced scrap generation.
3. Better output quality.

### ***Purchasing***

1. Coding of purchased parts leading to standardized rules for purchasing.
2. Reduced number of parts and raw materials.
3. Economies in purchasing because of accurate knowledge of raw material requirements.
4. Simplified vendor evaluation procedures leading to just-in-time purchasing.

### **Customer Satisfaction**

1. Accurate and faster cost estimates.
2. Efficient spare parts management, leading to better customer services.

### **Employee Satisfaction**

1. The workers in a cell observe the parts from raw material state to the finished part state. They visualize their contribution to the firm, and are more satisfied.
2. Work part quality is more easily traced in GT, and so the workers are more responsible for the quality of work they accomplish.

## **2.10 CELLULAR MANUFACTURING**

Cellular manufacturing is an application of GT in manufacturing in which all or a portion of a firm's manufacturing systems have been converted into cells. A manufacturing cell is a cluster of machines or processes located in close proximity and dedicated to the manufacture of a family of parts.

### **Objective of Cellular Manufacturing**

Primary objectives in implementing a cellular manufacturing system are to reduce the following:

1. Set-up times,
2. Flow times,
3. Inventory,
4. Market response time.

### **2.10.1 Machine Cell Design**

Machine cells can be classified as follows:

1. *Single machine cell*: It consists of one machine, supporting fixtures and tooling. One or more part families with one basic type of process (such as milling) can be processed.
2. *Group machine cell with manual handling*: This type of cell is often organized into a U-shaped layout. It includes more than one machine to process one or more part families. Material handling is performed by the human operators who run the cell.
3. *Group machine cell with semi-integrated handling*: A mechanized handling system, such as a conveyor, is used to move parts between machines in the cell. If the parts made in the cell have identical routings, in-line layout is selected. If the routings vary, loop layout is more appropriate.
4. *Flexible manufacturing system*: This is highly automated machine cell. It combines automated processing stations with a fully integrated handling system.

**Structural issues in cell design:** There are following structural issues in cell design:

1. Selection of part families and grouping of parts into families.
2. Selection of machine and process populations and grouping of these into cells.

3. Selection of tools, fixtures and pallets.
4. Selection of material-handling equipments.
5. Choice of equipment layout.

***Operational issues in cell design:*** The operational issues in cell design are stated as follows:

1. Detailed design of jobs.
2. Organization of supervisory and support personnel around the cellular structure
3. Formulation of maintenance and inspection policies.
4. Procedures' design for production planning, scheduling, control and acquisition of related software and hardware.
5. Modification of cost control and reward systems.
6. Outline of procedures for interfacing with the remaining manufacturing system.

***Evaluation of structural issues:*** The following points should be strictly adhered to during the evaluation of structural issues:

1. Equipment and tooling investment (low)
2. Equipment relocation cost (low)
3. Inter- and intra-cell material-handling costs (low)
4. Floor space requirements (low)
5. The extent to which parts are completed in a cell (high)
6. Flexibility (high)

***Evaluation of operational issues:*** The following points should be kept in mind during the evaluation of operational issues:

1. Equipment utilization (high)
2. Work-in-process inventory (low)
3. Queue lengths at each workstation (short)
4. Job throughput time (short)
5. Job lateness (low)

## 2.10.2 Best Machine Arrangement

The important factors to determine the type of machine cell and the best arrangement of equipment in the cell include the following:

***Volume of work to be done in the cell:*** It includes the number of parts per year and the amount of work required per part. Number of machines in the cell, total cost of operating the cell and the amount of investment needed are required.

***Variations in process routings of the parts:*** It determines the type of workflows such as straight line or U-shape or loop flows.

***Part size, shape, weight and other physical attributes:*** It determines the size and type of material handling and processing equipment that can be used.



## SUMMARY

In this chapter, we have discussed about the factors for plant location, methods of plant layout, objectives of plant layout, GT, cellular manufacturing and FMS. Plant location is a crucial decision-making process in which a number of factors concerning technical, social, political, economic, etc. are considered. Plant layout plays an important role in increasing the productivity of an organization. GT and cellular manufacturing are used to provide the flexibility, improve productivity, minimize cycle time, etc.

## MULTIPLE-CHOICE QUESTIONS

- Which of the following is the advantage of process layout?
    - Equipment used is generic
    - Low unit cost
    - Labour specialization
    - Low material handling cost
  - Locating all the milling machines in one work centre, lathes in another work centre, and grinding machines in yet another work centre represents what type of layout?
    - fixed-position layout
    - product layout
    - process layout
    - group layout
  - A common goal in designing product layouts is
    - minimizing the number of workers
    - minimizing idle time
    - minimizing material handling costs
    - maximizing the production rate
  - An assembly line consists of 4 tasks with times of 12, 15, 10, and 18 min. The cycle time for the line is 20 min. The theoretical minimum number of workstations for this situation is
    - 1
    - 2
    - 3
    - 4
  - Another name for a product layout is
    - line layout
    - functional layout
    - mixed-model layout
    - group technology layout
  - Another name for a process layout is
    - line layout
    - functional layout
    - mixed-model layout
    - group technology layout
  - Which of the following is an advantage of group layout?
    - It is easier to balance the flow of work through a cell
    - It requires less training of workers
    - It has reduced work-in-process inventory
    - It requires less capital investment



16. Systematic layout planning is concerned with
  - (a) Sequence of operation
  - (b) Nature of machines
  - (c) Group technology
  - (d) Frequency of material movements between two departments

## Answers

1. (a)      2. (c)      3. (d)      4. (c)      5. (a)      6. (b)      7. (c)      8. (c)      9. (d)  
10. (c)     11. (c)     12. (a)     13. (a)     14. (c)     15. (b)     16. (d)

## REVIEW QUESTIONS

1. Discuss the importance of plant location decision mentioning the factors influencing the plant location.
2. What are the mathematical techniques used to evaluate the plant locations? Discuss cost-volume-profit relationship used to evaluate the plant locations.
3. What are the objectives of plant layout?
4. Discuss the advantages and disadvantages of product layout.
5. Discuss the advantages and disadvantages of process layout.
6. Discuss the advantages and disadvantages of group layout.
7. Discuss the advantages and disadvantages of fixed-position layout.
8. Explain the process of systematic layout planning.
9. Explain the use of the block diagram in the rearrangement of departments or facilities.
10. Differentiate the product layout and process layout.
11. What do you understand by line balancing? Discuss the steps used to balance the production line.
12. What do you understand by group technology? Enumerate the advantages and disadvantages of group technology.
13. Discuss the classification and coding systems used in group technology.
14. What is a flexible manufacturing system? What are its components? Why is an FMS capable of producing a wide range of lot sizes?

## EXERCISES

1. In a university, a mechanical workshop has four shops, each dedicated to specific problems: foundry shop (shop A), machine shop (shop B), forging shop (shop C), and fabrication shop (shop D). The workshop is 1000 feet long and 250 feet wide. Each room is 250 feet by 250 feet. The present location of rooms is A, B, C, D, that is, a straight line. The load summary shows the number of contacts that each superintendent in a shop has with other superintendents in the other shops. Assume that all controllers are equal in this value.

Load summary: AB = 15, AC = 25, AD = 35, BC = 20, BD = 15, CD = 25.

- (a) Evaluate this layout according to the material handling cost method.
- (b) Improve the layout by exchanging functions within rooms. Show your amount of improvement.

2. An assembly line is to operate 8 hours per day with a desired output of 240 units per day. Table 2.10 contains information on this product's task times and precedence relationships:

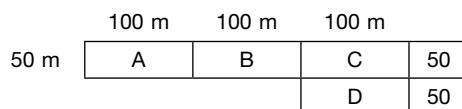
**Table 2-10:** Predecessor and completion times of various tasks

Task	Task time (Seconds)	Immediate predecessor
A	50	—
B	70	A
C	10	A
D	40	A
E	80	B, C
F	20	C, D
G	20	E, F
H	50	G

- (a) Draw the precedence diagram.
  - (b) What is the workstation cycle time?
  - (c) Balance the assembly line using the longest task time.
  - (d) What is the efficiency of the line balance?
3. An initial solution has been given to the following process layout problem (Table 2.11). Given the flows described and a cost of Rs 5.00 per unit per metre, compute the total cost for the layout. Each location is **100 m** long and **50 m** wide as shown in the Figure 2.14. Use the centres of departments for distances and measure distance using metropolitan-rectilinear distance.

**Table 2-11:** Frequency of inter-departmental materials' movement

		Departments			
		A	B	C	D
Department	A	0	15	30	60
	B		0	15	10
	C			0	20
	D				0



**Figure 2-14:** Initial layout



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# CHAPTER ▶ 3

## Forecasting

### 3.1 INTRODUCTION

Forecasting is a method to use the past experience and estimate the future. This is a very critical planning tool. It can be used for sales forecasting, demand forecasting and technology forecasting. In production engineering, demand estimation is an important part of production planning. There are two types of demands: dependent and independent demands. The quantity of dependent demand is estimated by the demand of complete or end product. But for the estimation of independent demand, various qualitative and quantitative models of forecasting are used.

Forecasting is the initial phase of production planning in which the quantity of product required in the near future is estimated and production schedule is prepared accordingly. The term ‘forecasting’ can be defined in many ways. Some of the definitions given by the researchers are as follows:

‘Forecasting is predicting, projecting, or estimating some future event and condition which is outside the organization’s control and provide for basis of managerial planning’. (Golden et al. 1994)

‘Forecasting is generally used to predict or describe what will happen given a set of circumstances or assumptions’. (Waddell 1994)

‘Forecasting is a projection into the future of expected demand, given a standard set of environmental conditions’. (Mentzer and Moon 2005)

In the long-term forecasting, the demand of the product, change in design in the product (technology forecasting) and demand of new product are estimated; and expansion of plant or requirement of additional resources is estimated accordingly. Thus, forecasting is a tool to estimate quantitative changing in demand in future. This may be based on trends of past data on actual demand or experts’ opinion depends upon the type of forecasting.

#### 3.1.1 Characteristics of Forecasts

Forecasts have the following characteristics.

1. Forecasts are normally wrong since it is an approximation of demand estimates in the future.
2. A good forecast has more than a single number, as it includes
  - (a) A mean value and standard deviation.
  - (b) An accuracy range (high and low).

3. Aggregated forecasts are usually more accurate because it can adjust the variation in actual demand of individual products.
4. Accuracy erodes as we go further into the future because the increase in time horizon increases the uncertainty in demand pattern.
5. Forecasts should not be used to the exclusion of known information.

### 3.1.2 Forecasting Horizons

Forecasting can be classified in terms of time span they will cover in the future. The basic types of time horizon forecasts are long term, medium term and short term (Korpela and Tuominen 1996). The ranges of these three forecasting horizons are: long-range forecasting (for more than 2 years), medium-range forecasting (for 3 months to 2 years) and short-range forecasting (for less than 3 months). Long-range forecasting is normally used for higher investments such as expansion factory capacity, location of the new plant/facilities, launching of the new product, etc. Medium-range forecasting is used for production planning, sales planning, increasing department capacities, etc. Short-range forecasting is used to schedule the production, work assignment, sequencing, etc. The time span and examples of these forecasting horizons are described in Table 3.1.

**Table 3-1:** Forecasting horizons and their examples

Forecast horizon	Time span	Item being forecast
Long range (More than 2 years)	Years	<ul style="list-style-type: none"> <li>• Product lines</li> <li>• Factory capacities</li> <li>• Planning for new products</li> <li>• Capital expenditures</li> <li>• Facility location or expansion</li> <li>• R &amp; D</li> </ul>
Medium range (3 months to 2 years)	Months	<ul style="list-style-type: none"> <li>• Product mix</li> <li>• Department capacities</li> <li>• Sales planning</li> <li>• Production planning and budgeting</li> <li>• Specific product quantities</li> <li>• Machine capacities</li> <li>• Planning</li> </ul>
Short range (Less than 3 months)	Weeks	<ul style="list-style-type: none"> <li>• Purchasing</li> <li>• Scheduling</li> <li>• Workforce levels</li> <li>• Production levels</li> <li>• Job assignments</li> </ul>

### 3.1.3 Steps of Forecasting

To forecast demand, the following essential steps need to be taken:

1. *Determination of objectives of forecasting:* The objectives of forecasting should be clear. Without focused objective, forecasting is meaningless.

2. *Selection of the items to be forecast:* The forecast for one item cannot be used for the other item. Therefore, the item must be selected for which forecast is to be made.
3. *Determination of the time horizon of the forecast:* The nature of the product decides the time horizon of its demand forecast. A long-range forecast is more error-prone as compared to a short-range forecast. Therefore, the horizon or a range of forecasting is an important issue.
4. *Selection of the suitable forecasting model:* There are a number of forecasting models. The suitability of these models is to be checked for the forecast of a particular product. The choice should be of a forecasting model that would give a more accurate estimation of demand.
5. *Collection of the data:* Future is predicted on the basis of the past data. Therefore, the relevant past data should be collected from various sources such as annual reports, magazines, journals, etc.
6. *Validation of the forecasting model:* The selected forecasting model is to be validated using the collected data.
7. *Forecasting the demand:* After data collection and model validation, forecast is made for a specific time in the future.
8. *Implementation of the results:* Finally, the results are implemented to fulfil the objectives. Here, results mean the quantity/volume of product forecasting. The entire production systems are changed/adjusted to meet the expected demand in future.

When carrying out demand forecasts, one often confronts with the problem of the inappropriate selection of a forecast method. It should be noted that in every actual forecast situation methods have their advantages and disadvantages; hence, it is important to define and analyse forecast method selection criteria (Pilinkiene 2008). In order to select the appropriate method, several criteria should be considered such as the degree of forecast accuracy, time span, the amount of necessary initial data, forecast costs, result implementation and applicability level (Pilinkiene 2008).

## 3.2 FORECASTING METHODS

Forecasting methods are divided into two categories: *qualitative forecasting* and *quantitative forecasting*.

### 3.2.1 Qualitative Forecasting

Qualitative forecasting is usually based on judgements about causal factors that underlie the demand of particular products or services. Qualitative forecasting can be used to formulate forecasts for new products for which there are no historical data. Qualitative techniques provide the means to adjust the forecasts using the experience and judgement of people knowledgeable about the product being forecast and the environment affecting the forecast. In other words, one can say that qualitative forecasting emphasizes predicting the future. Some qualitative methods of forecasting are described below as:

1. *Executive committee consensus:* This method of qualitative forecasting is based on the consensus of the committee of executives formed for making forecasts about specific items. Senior managers draw upon their collective wisdom to map out future events. They conduct discussions in open meetings. However, these discussions may subject to the drawbacks of group think and personality dominance.
2. *Delphi method:* In the Delphi method, at least two rounds of forecasts are obtained independently from a small group of experts. This group can be between 5 and 20 experienced and suitable experts and poll them for their forecasts and reasons (Armstrong et al. 2005). The experts never actually meet and typically do not know who the other panel members are (Wisniewski 2006). After each round, the experts' forecasts summed up and reported back to the experts. The cycle can go on from a second to a third round and so on if appropriate (Lancaster and Lomas 1985). Generally the Delphi method is used to produce a narrow range of forecasts rather than a single view of the future (Wisniewski 2006).  
In this method, the forecasts of a panel of experts are elicited through a sequence of questionnaires. Each questionnaire builds on the previous questionnaire in a series of 'rounds.' The experts answer the questionnaires independent of each other. After each round, a facilitator provides an anonymous summary of the experts' forecasts from the previous round as well as the reasons they provided for their judgements. The experts are encouraged to revise their earlier answers in the light of the replies of other members of their panel. It is believed that the group will eventually reach a consensus. The process is stopped after a number of rounds or when a consensus is reached.
3. *Survey of sales force:* The survey of the sales force method is a sales forecasting technique. It predicts future sales by analysing the opinions of salespeople as a group. The sales people continually interact with customers, and from this interaction they usually develop a knack for predicting future sales. The survey of the sales force method is considered a valuable management tool and is commonly used in business and industry throughout the world. This method can be further improved by providing the sales people with sufficient time to forecast and by offering incentives for accurate forecasts. Companies can make their salespeople better forecasters by training them to interpret effectively their interactions with customers.
4. *Expert evaluation technique:* Expert evaluations are made using the experience of people, such as executives, salespeople, marketing people, distributors or outside experts, who are familiar with a product line or a group of products, to generate sales forecasts. The advantage of soliciting contributions from more than one person for making expert evaluations is that it can offset biases introduced into a forecast that produced by one person.

### ***Advantages of Qualitative Forecasting***

There are following advantages of qualitative forecasting:

1. Qualitative forecasting techniques have the ability to predict changes in sales patterns.
2. Qualitative forecasting techniques allow decision-makers to incorporate rich data sources consisting of their intuition, experience and expert judgement.

### **Disadvantages of Qualitative Forecasting**

There are following disadvantages of qualitative forecasting:

1. In a qualitative method, the ability to forecast accurately suffers due to the following reasons:
  - (a) Forecasters only consider readily available and/or recently perceived information.
  - (b) The forecasters are unable to process large amounts of complex information.
  - (c) The forecasters are overconfident about their ability to forecast accurately.
  - (d) Political factors within organizations as well as political factors between organizations may influence the forecasting accuracy.
  - (e) The forecasters tend to infer relationships or patterns in a data when there are no patterns.
  - (f) Anchoring on forecasts implies that forecasters get influenced by initial forecasts (e.g. those generated by quantitative methods) when making qualitative forecasts.
2. The future ability to forecast accurately may be reduced when a forecaster tries to justify, rather than understand a forecast that proves inaccurate information.
3. Qualitative forecasting techniques lead to inconsistencies in judgement due to moods and/or emotions of forecasters as well as due to the repetitive decision-making inherent in generating multiple individual product forecasts.
4. Qualitative forecasting techniques are expensive and time-intensive.

### **3.2.2 Quantitative Forecasting**

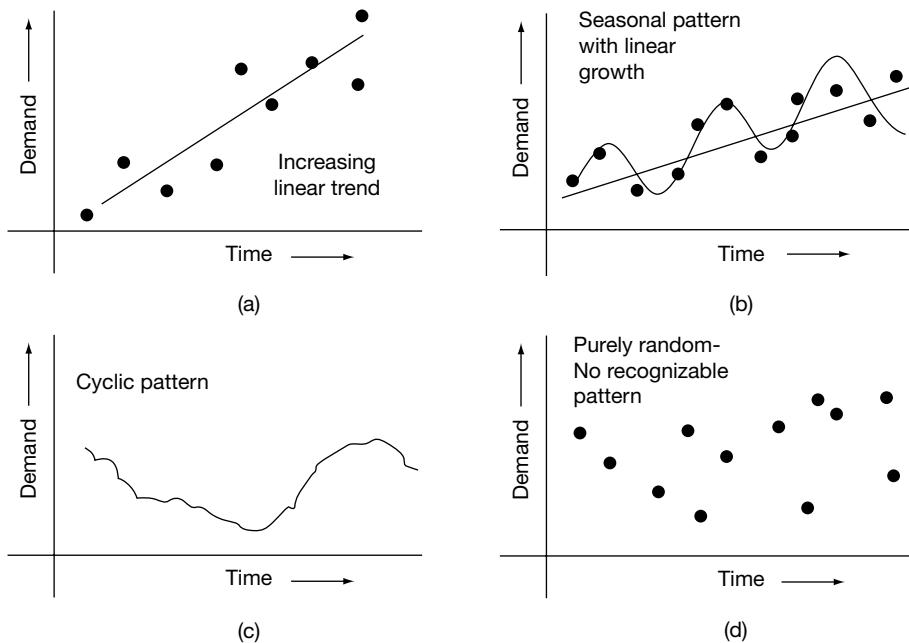
Quantitative forecasting is based on the assumption that the ‘forces’ that generated the past demand will also generate the future demand, i.e., history will tend to repeat itself. Analysis of the past demand pattern provides a good basis for forecasting the future demand. Majority of quantitative approaches fall in the category of time-series analysis. Quantitative methods of forecasting are divided into two categories: (a) time-series methods and (b) causal methods.

## **3.3 TIME-SERIES FORECASTING**

The time-series forecasting methods are based on the analysis of historical data. Time series may be defined as a set of observations measured at successive times or over successive periods. In the time-series methods, the assumption is that the past patterns in the data can be used to forecast future data points. Here, future data point means the points on the projection of the line using extrapolation. The patterns of demand variations of a time series are shown in Figure 3.1.

*Trends* are noted by an upward or downward sloping line. There is a linear relationship between time and demand. The demand in the future can be estimated by extending the straight line or extrapolation. The linear trend is shown in Figure 3.1(a).

*Seasonality* is a data pattern that repeats itself over the period of one year or less. It leads rise or fall in demand among the seasons. In a particular season, demand may increase while in other season demand may decrease. Thus, the demand is dependent on season or quarter of a year. The seasonal variation in demand is shown in Figure 3.1(b).



**Figure 3-1:** (a) Linear variation, (b) seasonal variation, (c) cyclic variation and (d) random variation

**Cycle** is a data pattern that may cover several years before it repeats itself. Cyclic nature in demand variation shows the same style in increasing or decreasing in demand in future. If we draw a linear line through the mean values of the actual demands, we can observe that the upper and lower variation in demands follow the same pattern throughout the straight line as shown in Figure 3.1(c).

**Random fluctuation (noise)** results from random variation or unexplained causes. There are a number of causes of random fluctuation in demand, for example, anticipation of increasing the price or shortages of product in near future increases the demand. Random fluctuations in demand are shown in Figure 3.1(d).

The following methods are for time-series forecasting:

- Naïve method
- Simple moving average (SMA) method
- Weighted moving average (WMA) method
- Exponential smoothing method

#### **(A) Naive Method**

The naive forecasting model is a special case of the moving average forecasting model where the number of periods used for smoothing is 1. Therefore, in the naive method, the forecast for a period,  $t$ , is simply the observed value of the previous period,  $t - 1$ . Due to the simplistic nature

of the naive forecasting model, it can only be used to forecast up to one period in the future. It is not at all useful as a medium-/long-range forecasting tool.

The naive method is used partly for completeness and partly for its simplicity. It is unlikely that any one will want to use this model directly because it can be only used for the just next period and also, the forecast for the next period is same as the previous actual demand, i.e. there is no change is considered in forecasting the demand. Instead, consider using either the moving average model or the more general WMA model with a higher (i.e. greater than 1) number of periods, and possibly a different set of weights.

### **(B) Simple Moving Average Method**

The SMA method uses the average value of actual demand for some recent periods. The value of averaging period depends on the nature of variation in the actual demands for last some periods and accuracy of forecasting. Using this method, we can forecast the demand only for the next period in the future, but to know the forecasting errors we have to find the forecast value based on the actual demands for the past periods, also. Suppose  $t$  represents the current period and we want to forecast for the period  $t + 1$ . Specifically, the forecast for period  $t + 1$  can be calculated at the end of period  $t$  (after the actual demand for period  $t$  is known) as

$$\begin{aligned} F_{t+1} &= \frac{1}{n}(D_t + D_{t-1} + \dots + D_{t+n-1}) \\ \Rightarrow F_{t+1} &= \frac{1}{n} \sum_{i=t-n+1}^t D_i \end{aligned}$$

where  $D$  indicates the demand and  $F$  indicates the forecast;  $t$  is the time period;  $n$  is the number of the averaging period.

The assumptions in making the forecast using the SMA method are as follows:

1. All  $n$  past observations are treated equally.
2. Observations older than  $n$  are not included at all.
3. It requires that  $n$  past observations be retained.

**Example 3.1:** The monthly demands for an office furniture (in units) are given in Table 3.2. Forecast the demand using 3-period and 5-period SMA for the 12<sup>th</sup> month. Also, show the variation in forecasting graphically.

**Table 3-2:** Monthly demands for furniture for eleven months

Month ( $t$ )	1	2	3	4	5	6	7	8	9	10	11	12
Demand ( $D_t$ )	600	628	670	735	809	870	800	708	842	870	739	—

#### **Solution:**

We use the formula

$$F_{t+1} = \frac{1}{n}(D_t + D_{t-1} + \dots + D_{t+n-1}) = \frac{1}{n} \sum_{i=t-n+1}^t D_i$$

*For 3-period moving average method*

Here, the average value of last three periods is used as forecast for the fourth period as shown in the formula below:

$$F_4 = \frac{1}{3}(D_1 + D_2 + D_3) = \frac{1}{3}(600 + 628 + 670) = 632.66 \approx 633$$

Similarly,  $F_5, F_6, \dots, F_{12}$  can be calculated as:

$$F_5 = \frac{1}{3}(D_2 + D_3 + D_4) = 678$$

$$F_6 = \frac{1}{3}(D_3 + D_4 + D_5) = 738$$

.....

$$F_{12} = \frac{1}{3}(D_9 + D_{10} + D_{11}) = 817$$

*For 5-period moving average method*

Here, the average value of last five periods is used as forecast for the sixth period as shown in the formula below:

$$\begin{aligned} F_6 &= \frac{1}{5}(D_1 + D_2 + D_3 + D_4 + D_5) \\ &= \frac{1}{5}(600 + 628 + 670 + 735 + 809) = 688.4 \approx 689 \end{aligned}$$

Similarly,  $F_7, F_8, \dots, F_{12}$  can be calculated as:

$$F_7 = \frac{1}{5}(D_2 + D_3 + D_4 + D_5 + D_6) = 743$$

$$F_8 = \frac{1}{5}(D_3 + D_4 + D_5 + D_6 + D_7) = 777$$

.....

$$F_{12} = \frac{1}{5}(D_7 + D_8 + D_9 + D_{10} + D_{11}) = 792$$

The forecasting of the demand for the 12 months is shown in Table 3.3 using 3-period simple average and 5-period SMA.

**Table 3-3:** Forecasting of furniture demand using simple moving average (SMA)

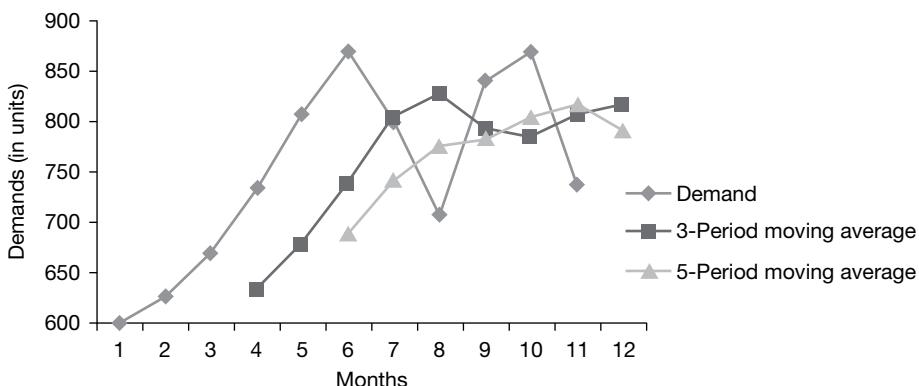
Month	Demand	3-Period moving average	5-Period moving average
1	600	—	—
2	628	—	—

(Continued)

**Table 3-3:** Forecasting of furniture demand using simple moving average (*Continued*)

Month	Demand	3-Period moving average	5-Period moving average
3	670	—	—
4	735	633	—
5	809	678	—
6	870	738	689
7	800	805	743
8	708	827	777
9	842	793	785
10	870	784	806
11	739	807	818
12	—	817	792

It can be observed from the graph in Figure 3.2 that the variation in actual demand is very high and it is difficult to forecast using any trend line. Thus, the moving average method is suitable for this kind of data. The 5-period moving average graph is smoother than the 3-period moving average graph, but the limitation is forecasting error. A larger periods of averaging show large variation from the actual demand curve as shown in Figure 3.2. Three-period averaging is closer the actual demand capered to five-period averaging.

**Figure 3-2:** Variation in furniture demand with time

### (C) Weighted Moving Average Method

The WMA method is very similar to SMA method, but in the former method different weights are provided for the periods. The largest weight is provided with most recent period and the weights are decreasing as we move to the previous period in the past. A WMA is a moving average where each historical demand may be weighted as:

$$F_{t+1} = W_1 D_t + W_2 D_{t-1} + \dots + W_n D_{t+1-n}$$

where  $n$  is the total number of periods in the average,  $W_t$  is the weight applied to period  $t$ 's demand,  $W_1 > W_2 > \dots > W_n$ , sum of all the weights = 1, Forecast  $F_{t+1}$  = forecast for period  $t + 1$ .

The assumptions in making the forecast using the SMA method are as follows:

1. Adjustments in the moving average to more closely reflect fluctuations in the data.
2. Weights are assigned to the most recent data.
3. Requires some trial and error to determine precise weights.

**Example 3.2:** Using the data shown in Table 3.3, forecast the demand for the periods with the three-period WMA method. The most recent data should be given 50 per cent weightage, second year past data, 30 per cent, and third year past data, 20 per cent. Also, compare the result with the result of three-period SMA graphically.

**Solution:**

$$F_{t+1} = W_1 D_t + W_2 D_{t-1} + \dots + W_n D_{t+1-n}$$

$$F_4 = W_1 D_3 + W_2 D_2 + W_3 D_1 = 0.5 \times 670 + 0.3 \times 628 + 0.2 \times 600 = 643.4 \approx 644$$

Similarly,  $F_5, F_6, \dots, F_{12}$  can be calculated as:

$$F_5 = W_1 D_4 + W_2 D_3 + W_3 D_2 = 695$$

$$F_6 = W_1 D_5 + W_2 D_4 + W_3 D_3 = 759$$

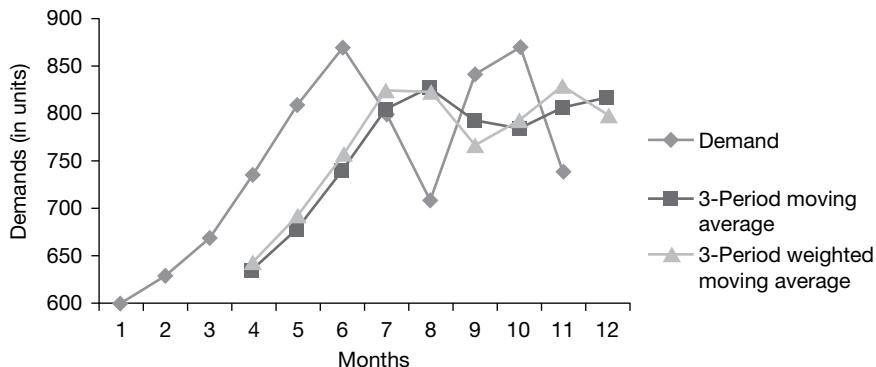
.....

$$F_{12} = W_1 D_{11} + W_2 D_{10} + W_3 D_9 = 799$$

The forecasting of the demand for the 12<sup>th</sup> month is shown in Table 3.4.

**Table 3-4:** Forecasting of furniture demand using weighted moving average (WMA) method

Month	Demand	3-Period WMA
1	600	—
2	628	—
3	670	—
4	735	644
5	809	695
6	870	759
7	800	825
8	708	823
9	842	768
10	870	794
11	739	830
12	—	799



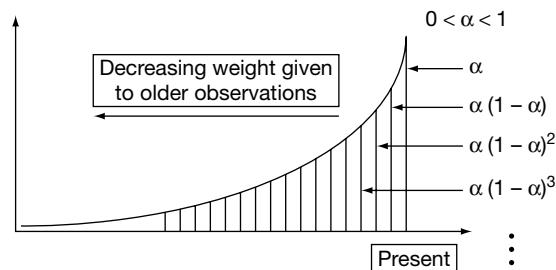
**Figure 3-3:** Variation in furniture demand using 3-period SMA and WMA methods

In Figure 3.3, we observe that the WMA forecast closer to the actual demand compared to the SMA method, i.e. the forecasts from WMA pursuing the actual demand closely since the recent period is given more weightage.

#### (D) Exponential Smoothing Method

Exponential smoothing is the most popular and cost effective of the statistical methods. It is based on the principle that the latest data should be weighted more heavily and ‘smoothes’ out cyclical variations to forecast the trend (Armstrong et al. 2005). It assumes that the data gets older, becomes less relevant and should be given less weight. In order to calculate the forecasting, the old average, the actual new demand and a weighting factor are needed.

Exponential smoothing gives greater weight to demand in more recent periods, and less weight to demand in earlier periods as shown in Figure 3.4. It is a sophisticated WMA method that calculates the average of a time series by giving recent demands more weight than earlier demands. In this method, both the actual demands and forecast of the past demands are used in the calculation of forecasting, i.e. all the past data are used to estimate the demand in the future. Here,  $\alpha$  is a smoothening factor, its value is decreasing as we move towards the past. Figure 3.4 shows the decreasing pattern for the value of  $\alpha$  as we move towards the past; this decrease follows an exponential pattern. Thus, this method is known as exponential smoothing method.



**Figure 3-4:** Decreasing weight to the older observations

The forecast for the period  $t + 1$  is equal to the actual demand for the period  $t$  plus the  $\alpha$  times of the difference of the actual and forecasting value for the period  $t$ . Here,  $\alpha$  tries to smoothen the variation in the previous period actual and forecasting values of the demand.

The forecast for  $t^{\text{th}}$  period can be given as

$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t = F_t + \alpha(D_t - F_t)$$

Here,  $\alpha$  is smoothening factor.

**Example 3.3:** Using the data shown in Table 3.3, forecast the demand for the periods using the exponential smoothing method ( $\alpha = 0.3$  and  $\alpha = 0.5$ ). Also, compare the results graphically.

**Solution:**

Using the formula

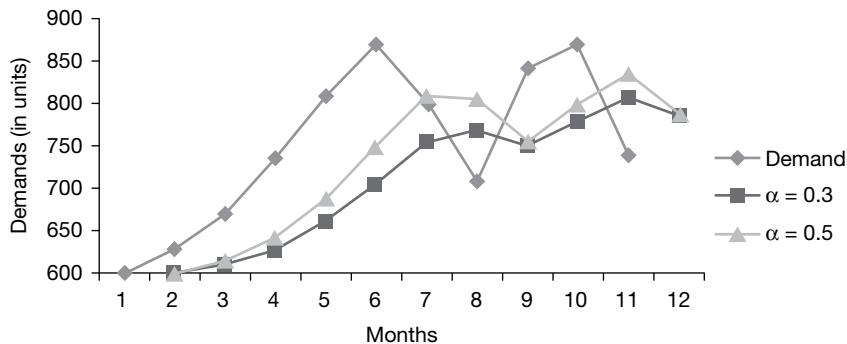
$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t = F_t + \alpha(D_t - F_t)$$

The demand forecast data is prepared (see Table 3.5).

**Table 3-5:** Furniture demand forecasts using exponential smoothing

Month	Demand	Forecast, $F_t$	
		$\alpha = 0.3$	$\alpha = 0.5$
1	600	—	—
2	628	$600 + 0.3(600 - 600) = 600$	600
3	670	$600 + 0.3(628 - 600) = 608.4$	614
4	735	$608.4 + 0.3(670 - 608.4) = 626.8$	642
5	809	$626.8 + 0.3(735 - 626.8) = 659.3$	688.5
6	870	$659.3 + 0.3(809 - 659.3) = 704.2$	748.7
7	800	$704.2 + 0.3(870 - 704.2) = 753.9$	809.3
8	708	$753.9 + 0.3(800 - 753.9) = 767.7$	804.6
9	842	$767.7 + 0.3(708 - 767.7) = 749.8$	756.3
10	870	$749.8 + 0.3(842 - 749.8) = 777.4$	799.1
11	739	$777.4 + 0.3(870 - 777.4) = 805.2$	834.5
12	—	$805.2 + 0.3(739 - 805.2) = 785.3$	786.2

For smaller values of  $\alpha$ , we get a smoother curve. In Figure 3.5, it can be observed that the curve for  $\alpha = 0.3$ , is smoother than that of the curve for  $\alpha = 0.5$ . For smooth curve, forecasting is easy, but the forecasting error may be large because there is a large deviation of the forecast of the actual value. This type of forecasting model is used for the demand fluctuates continuously and there is requirement of smoothening the curve for ease of forecasting.



**Figure 3-5:** Variation in forecasting using exponential smoothing factor 0.3 and 0.5

### Features of Exponential Smoothing Method

Some of the features of exponential smoothing method are enumerated as follows.

1. The emphasis given to the most recent demand levels can be adjusted by changing the smoothing parameter.
2. Exponential smoothing is simple and requires minimal data.
3. Larger  $\alpha$  values emphasize recent levels of demand and result in forecasts more responsive to changes in the underlying average.
4. Smaller  $\alpha$  value treats past demand more uniformly and result in more stable forecasts.
5. When the underlying average is changing, results will lag actual changes.
6. The new forecast is the weighted sum of old forecast and actual demand.
7. Only two values, i.e. actual demand and forecast for just previous period ( $D_t$  and  $F_t$ ), are required, compared with actual demands for  $n$  period in moving average method.
8. Parameter  $\alpha$  is determined empirically (whatever works best). But, the rule of thumb can be used as:  $\alpha < 0.5$ .
9. Typically,  $\alpha = 0.2$  or  $\alpha = 0.3$  works well.

### Adjusted Exponential Smoothing

Double exponential smoothing is also called exponential smoothing with trend. If trend exists, single exponential smoothing may need adjustment. There is a need to add a second smoothing constant to account for the trend. It is similar to a single exponential smoothing. The basic idea is to introduce a trend estimator that changes over time. If the underlying trend changes, overshoots may happen. Issues to choose two smoothing rates,  $\alpha$  and  $\beta$  are very important.

The following points may help in choosing the value of  $\alpha$  and  $\beta$ :

- $\beta$  close to 1 means quicker responses to trend changes, but may over-respond to random fluctuations.
- $\alpha$  close to 1 means quicker responses to level changes, but again may over-respond to random fluctuations.

Adjusted forecasting is given by

$$AF_{t+1} = F_{t+1} + T_{t+1}$$

Here,  $T$  is an exponentially smoothed trend factor given by

$$T_{t+1} = \beta(F_{t+1} - F_t) + (1 - \beta)T_t$$

where  $T_t$  is the last period trend factor and  $\beta$  is a smoothing constant for trend. The formula for the trend factor reflects a weighted measure of the increase or decrease between the next forecast,  $F_{t+1}$ , and the current forecast,  $F_t$ .  $\beta$  is a value between 0.0 and 1.0 and reflects the weight given to the most recent trend data. It is usually determined and subjectively based on the judgement of the forecasters. High  $\beta$  reflects that trend changes more than in the case of low  $\beta$ . It is common for  $\beta$  to equal  $\alpha$  in this method.

**Example 3.4:** Using the data shown in Table 3.6, forecast the demand for the periods with adjusted exponential smoothing method ( $\alpha = 0.5$  and  $\beta = 0.3$ ). Also, compare the result graphically with simple exponential smoothing ( $\alpha = 0.5$ ).

#### Solution:

Adjusted forecast for period 3:

We have

$$\begin{aligned} T_3 &= \beta(F_3 - F_2) + (1 - \beta)T_2 \\ &= (0.3)(614 - 600) + (0.7)(0) = 4.2 \end{aligned}$$

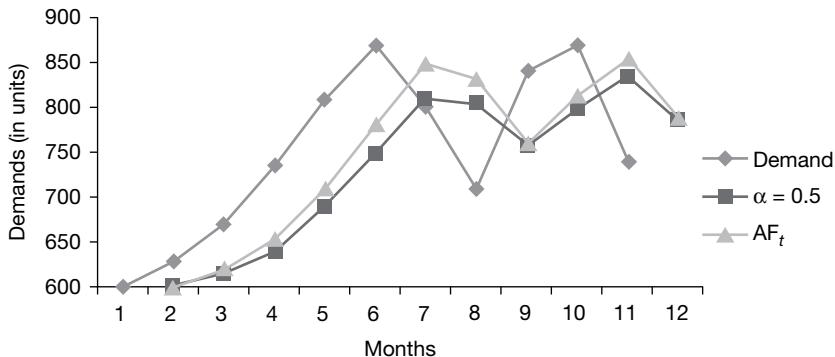
Therefore,

$$AF_3 = F_3 + T_3 = 614 + 4.2 = 618.2$$

**Table 3-6:** Adjusted exponential smoothing forecasts

Month	1	2	3	4	5	6	7	8	9	10	11	12
Demand	600	628	670	735	809	870	800	708	842	870	739	-
$\alpha = 0.5$	600	614	642	688.5	748.7	809.3	804.6	756.3	799.1	834.5	786.7	
$AF_t$	600	618.2	653.34	710.38	782.07	850.84	832.26	761.17	815.34	856.48	787.7	
$T_t$	0	4.2	11.34	21.88	33.37	41.54	27.66	4.87	16.24	21.98	1.04	

Single exponential smoothing forecasting with  $\alpha = 0.5$ , adjusted exponential smoothing (double exponential smoothing), and actual demand are shown in Figure 3.6.  $AF_t$  shows the curve with trend adjustment and  $F_t$  shows the curve for exponential smoothing without trend adjustment.



**Figure 3-6:** Variation in demand forecasts using adjusted exponential forecast

### (E) Simple Linear Regression Method

This is a mathematical technique that relates to one variable, that is, the independent variable, with another variable called the dependent in the form of a linear equation. The linear regression equation is

$$y = a + b \cdot x$$

where  $y$  is the dependent variable,  $a$  is the intercept,  $b$  is the slope of the line and  $x$  is the independent variable. The variables  $a$  and  $b$  are given by

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2}$$

where  $a$  is a constant,  $b$  is a coefficient of variable  $x$ , and  $\bar{x}$  is the mean value of  $x$ ,  $\bar{y}$  is the mean value of  $y$ ,  $x$  is time,  $y$  is the demand, and  $n$  is the period for which data is analysed using linear regression methods.

Coefficient of correlation shows the strength of correlation between two variables. The linear regression model of forecasting is only used when the value of the coefficient of correlation ( $r$ ) is high, i.e. near to 1. Coefficient of correlation ( $r$ ) in a linear regression equation is the measure of the strength of the relationship between the independent (time) and dependent variable (demand). It is given by

$$r = \frac{\sum_{t=1}^n (x_t - \bar{x})(y_t - \bar{y})}{\sqrt{\sum_{t=1}^n (x_t - \bar{x})^2} \sqrt{\sum_{t=1}^n (y_t - \bar{y})^2}}$$

The values of  $r$  vary between  $-1$  and  $+1$  with a value of  $+1$ , indicating a strong linear relationship between the variables and  $-1$  showing the strong reciprocal relationship. Coefficient of

determination ( $r$ ) is computed by squaring  $r$ . This is an indication of the percentage of variation in the dependent variable as a result of the behaviour of the independent variable.

**Example 3.5:** The weekly demands of a motorcycle by a retailer are shown in Table 3.7. Find an equation of the regression line and estimate the demand for the 14<sup>th</sup> week.

**Table 3-7:** Weekly demand of the car

Week (X):	1	2	3	4	5	6	7	8	9	10	11	12
Demand (Y):	420	450	460	420	500	550	480	520	610	570	600	590

**Solution:**

We use the formulas

$$a = \bar{y} - b\bar{x}$$

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2}$$

For the calculations, see Table 3.8.

**Table 3-8:** Actual demands and forecasts of the motorcycle

Week (X)	Demand (Y)	xy	x <sup>2</sup>	Forecast
1	420	420	1	419.4
2	450	900	4	436.63
3	460	1380	9	453.86
4	420	1680	16	471.09
5	500	2500	25	488.32
6	550	3300	36	505.55
7	480	3360	49	522.78
8	520	4160	64	540.01
9	610	5490	81	557.24
10	570	5700	100	574.47
11	600	6600	121	591.17
12	590	7080	144	608.93
$\Sigma x = 78$	$\Sigma y = 6170$	$\Sigma xy = 42,570$	$\Sigma x^2 = 650$	

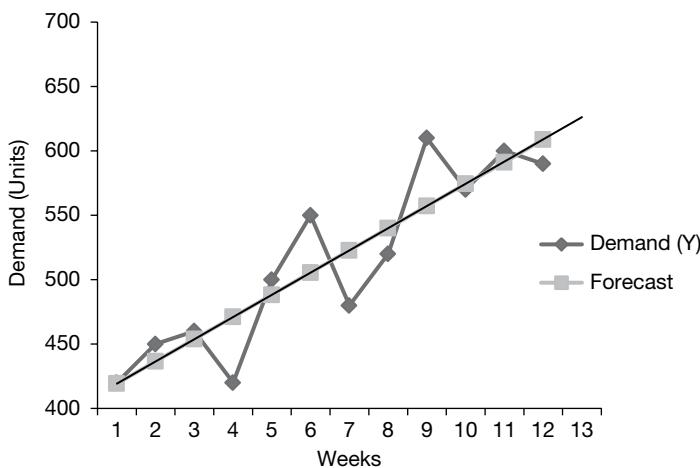
Substituting the values from Table 3.8, we get

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2} = \frac{42570 - 12 \times (6170/12) \times (78/12)}{650 - 12 \times (78/12)^2} = 17.23$$

$$a = \bar{y} - b\bar{x} = 514.1667 - 17.23 \times 6.5 = 402.17$$

Therefore, the regression equation is

$$\begin{aligned}y &= 402.17 + 17.23x \\y_{14} &= 402.17 + 17.23 \times 14 \\&= 643.39\end{aligned}$$



**Figure 3-7:** Actual demands and forecasts of the motorcycle using linear regression

In Figure 3.7, we can observe the linear nature of variation in actual demand. But, the variation is not exactly linear in nature. A straight line can be drawn covering the maximum points. The above variation shows the strong correlation between weeks and demand. The relationship will be exactly a straight line when the value of  $r$  becomes 1.

#### (F) Causal Method of Forecasting

Causal forecasting methods are based on a known or perceived relationship between the factor to be forecasted and other external or internal factors. The following models are used as causal forecasting methods:

1. *Regression:* A mathematical equation relates a dependent variable to one or more independent variables that are believed to influence the dependent variable.
2. *Econometric model:* This model comprises a system of interdependent regression equations that describe some sector of economic activity.
3. *Input-output model:* This model describes the flows from one sector of the economy to another, and so predicts the inputs required to produce outputs in another sector.
4. *Simulation modelling:* Simulation modelling leads to mathematical and computer simulation. The actual system or real world is simulated and a model is formulated which is finally used to correlate the demand variation with the variation in other factors affecting the demand such as income of consumer, price of the product, technological change in product design, etc.

### 3.3.1 Seasonal Adjustment

A seasonal pattern is a repetitive increase or decrease in demand. There are several methods for reflecting seasonal patterns in time-series forecasts. A seasonal factor is used to adjust for seasonal patterns. Seasonal factors are multiplied by the normal forecast to get seasonally adjusted forecasts.

One method for determining seasonal factors is to divide the demand for each seasonal pattern by the total annual demand:

$$S_i = \sum D_i / \sum \sum D_{ij}$$

where  $i$  shows the specific season or quarter of a year;  $j$  shows the number of years  $S_i$  is seasonal adjustment factor;  $\sum D_i$  shows the summation of demand in  $i^{\text{th}}$  quarter of the past years; and  $\sum \sum D_{ij}$  shows the total demand in the past years. This results in a seasonal factor between 0.0 and 1.0 that can be applied to any time-series method. This equation reflects that a factor of total annual demand is assigned to each season. The seasonal factors are then multiplied by the annual forecast of the demand to yield a forecast for each season/quarter.

**Example 3.6:** The quarterly demands of a raincoat from a shop for the years 2012, 2013 and 2014 are given in Table 3.9. Forecast the demand for the year 2015 quarterly.

**Table 3-9:** Quarterly demands of the machine

Year	Demand				Total
	Quarter 1	Quarter 2	Quarter 3	Quarter 4	
2012	176	136	113	225	650
2013	191	153	125	232	701
2014	203	156	131	246	736
Total	570	445	369	703	2087

**Solution:**

We have

$$S_1 = \sum D_1 / \sum \sum D_{ij} = 570 / 2087 = 0.273$$

$$S_2 = \sum D_2 / \sum \sum D_{ij} = 445 / 2087 = 0.213$$

$$S_3 = \sum D_3 / \sum \sum D_{ij} = 369 / 2087 = 0.176$$

$$S_4 = \sum D_4 / \sum \sum D_{ij} = 703 / 2087 = 0.336$$

Multiply the forecasted demand for entire year by seasonal factors to determine quarterly demand.

Forecast for the entire year (trend line for data in Table 3.9):

$$b = \frac{\sum xy - n(\bar{y})(\bar{x})}{\sum x^2 - n(\bar{x})^2} = \frac{4260 - 3 \times (695.66) \times (2)}{14 - 3 \times (2)^2} = 43.02$$

So,

$$a = \bar{y} - b\bar{x} = 695.66 - 43.02 \times 2 = 609.62$$

The regression equation is

$$\begin{aligned} y &= 609.62 + 43.02x \\ &= 609.62 + 43.02 \times 4 \\ &= 781.7 \end{aligned}$$

Seasonally adjusted forecasts:

$$\begin{aligned} SF_1 &= (S_1)(F_5) = (0.273)(781.7) = 213.4 \\ SF_2 &= (S_2)(F_5) = (0.213)(781.7) = 166.5 \\ SF_3 &= (S_3)(F_5) = (0.176)(781.7) = 137.5 \\ SF_4 &= (S_4)(F_5) = (0.336)(781.7) = 262.6 \end{aligned}$$

### **Multiplicative Seasonal Method**

To deal with seasonal effects in forecasting, two major parts of forecasting are to be completed, as described below:

1. A forecast for the entire period (i.e. year) must be made using whatever forecasting technique is appropriate.
2. This forecast will be developed to reflect the seasonal effects in each period (i.e. month or quarter).

The multiplicative seasonal method adjusts a given forecast by multiplying the forecast by a seasonal factor. The following steps are used to calculate the seasonal adjusted forecasting:

**Step 1:** Calculate the average demand  $D_{avg,t}$  per period for each year ( $t$ ) from the past data by dividing the total demand for the year ( $\sum D_p$ ) by the number of periods ( $p$ ) in that year ( $t$ ).

**Step 2:** Divide the actual demand  $D_{p,t}$  for each period ( $p$ ) by the average demand  $D_{avg,t}$  per period (calculated in Step 1) to get a seasonal factor  $f_{p,t}$  for each period; repeat for each year of data.

**Step 3:** Calculate the average seasonal factor  $f_p$  for each period by summing all the seasonal factors  $\sum f_{p,t}$  for that period and dividing by the number of seasonal factors.

**Step 4:** Determine the forecast for a given period in a future year by multiplying the average seasonal factor  $f_p$  by the forecasted demand in that future year.

**Example 3.7:** Solve Example 3.6 using the multiplicative method of seasonal adjustment.

**Solution:**

**Table 3-10:** Actual quarterly demand of a machine for the years 2012, 2013 and 2014

Year	Demand				Total	Avg
	Quarter 1	Quarter 2	Quarter 3	Quarter 4		
2012	176	136	113	225	650	162.5
2013	191	153	125	232	701	175.25
2014	203	156	131	246	736	184.0

**Table 3-11:** Seasonal factor for forecasting of the machine for the years 2012, 2013 and 2014

Year	Demand			
	Quarter 1	Quarter 2	Quarter 3	Quarter 4
2012	176/162.5 = 1.08	136/162.5 = 0.821	113/162.5 = 0.695	225/162.5 = 1.38
2013	191/175.25 = 1.09	153/175.25 = 0.873	125/175.25 = 0.713	232/175.25 = 1.32
2014	203/184 = 1.1	156/184 = 0.847	131/184 = 0.711	246/184 = 1.33
<b>Avg. seasonal factor</b>	<b>1.09</b>	<b>0.847</b>	<b>0.706</b>	<b>1.34</b>

Annual forecast for the year 2015 is predicted to be 782 units.

The average forecast per quarter is  $782/4 = 195.5$  units.

Now,

$$\text{Quarterly forecast} = \text{Avg. forecast} \times \text{Seasonal factor}$$

Therefore,

$$\begin{aligned} Q_1 &: 1.09(195.5) = 213.09 \\ Q_2 &: 0.847(195.5) = 165.58 \\ Q_3 &: 0.706(195.5) = 138.02 \\ Q_4 &: 1.34(195.5) = 261.97 \end{aligned}$$

### 3.4 FORECASTING PERFORMANCE MEASUREMENT

Forecasting performance can be measured through the following terms:

Mean absolute deviation: Mean absolute deviation (MAD) is the average deviation of forecasting of actual demand. It can be calculated as:

$$\text{MAD} = \frac{1}{n} \sum_{t=1}^n |D_t - F_t|$$

where  $D_t$  is the actual demand for the period  $t$  and  $F_t$  is the forecast for the period  $t$ , and  $n$  is the total number of periods.

**Mean absolute percentage error (MAPE):** It is very similar to MAD, but it is shown in percentage. The MAPE can be calculated as:

$$\text{MAPE} = \frac{100}{n} \sum_{t=1}^n \left| \frac{D_t - F_t}{D_t} \right|$$

where  $D_t$  is the actual demand for the period  $t$  and  $F_t$  is the forecast for the period  $t$ , and  $n$  is the total number of periods.

**Mean square error:** Mean square error (MSE) is used to show the small deviation at larger scale by squaring the deviation. It can be calculated as:

$$\text{MSE} = \frac{1}{n} \sum_{t=1}^n (D_t - F_t)^2$$

where  $D_t$  is the actual demand for the period  $t$  and  $F_t$  is the forecast for the period  $t$ , and  $n$  is the total number of periods.

### Tracking Signal

Tracking signal monitors any forecasts that have been made in comparison with actual, and warn when there are unexpected departures of the outcomes from the forecasts. The tracking signal is a simple indicator that forecast bias is present in the forecast model. It is most often used when the validity of the forecasting model might be in doubt.

$$\begin{aligned} \text{Tracking signal} &= \frac{\sum_{t=1}^n (\text{Actual demand} - \text{Forecast demand})_t}{\text{MAD}} \\ &= \frac{\sum_{t=1}^n (D_t - F_t)}{\text{MAD}} \end{aligned}$$

where  $D_t$  is the actual demand for the period  $t$  and  $F_t$  is forecasting for the period  $t$ , and  $n$  is the total number of periods.

**Example 3.8:** Using Table 3.12, find MAD, MAPE and MSE.

**Table 3-12:** Forecast errors

Week (X)	Demand (D)	Forecast (F)	$ D_t - F_t $	$ (D_t - F_t)/D_t  \times 100$	$(D_t - F_t)^2$
1	420	419.4	0.6	0.142	0.36
2	450	436.63	13.37	2.971	178.75
3	460	453.86	6.14	1.334	37.69
4	420	471.09	51.09	12.164	2610.18

(Continued)

**Table 3-12:** (Continued)

<b>Week (X)</b>	<b>Demand (D)</b>	<b>Forecast (F)</b>	$ D_t - F_t $	$ D_t - F_t /D_t \times 100$	$(D_t - F_t)^2$
5	500	488.32	11.68	2.336	136.42
6	550	505.55	44.45	8.081	1975.80
7	480	522.78	42.78	8.912	1830.12
8	520	540.01	20.01	3.848	400.40
9	610	557.24	52.76	8.649	2783.61
10	570	574.47	4.47	0.784	19.98
11	600	591.17	8.83	1.471	77.96
12	590	608.93	18.93	3.208	358.34
			$\Sigma = 275.11$	$\Sigma = 53.86$	$\Sigma = 10409.61$

**Solution:**

$$MAD = \frac{1}{n} \sum_{t=1}^n |D_t - F_t| = \frac{275.11}{12} = 22.92$$

$$MAPE = \frac{100}{n} \sum_{t=1}^n \left| \frac{D_t - F_t}{D_t} \right| = \frac{53.86}{12} = 4.48 \text{ per cent}$$

$$MSE = \frac{1}{n} \sum_{t=1}^n (D_t - F_t)^2 = \frac{10409.61}{12} = 867.46$$

**SUMMARY**

In this chapter, we have studied about forecasting, and its various models. Forecasting is an important part of production planning which affects the various industrial or production activities as discussed in purposes of forecasting for different time horizons. Forecasting is always wrong, but its accuracy depends on the time horizon and analysis of the past data and present market scenario. We have discussed about both the qualitative and quantitative forecasting. Qualitative forecasting is normally used for the new product whose past data are not available and quantitative forecasting involves the past data assuming that the past will be repeated in future.

**MULTIPLE-CHOICE QUESTIONS**

1. In trend-adjusted exponential smoothing, the trend-adjusted forecast consists of
  - (a) the old forecast adjusted by a trend factor
  - (b) the old forecast and a smoothed trend factor
  - (c) an exponentially smoothed forecast and a smoothed trend factor
  - (d) an exponentially smoothed forecast and an estimated trend value

2. The primary method for associative forecasting is
  - (a) Delphi method
  - (b) executive consensus
  - (c) regression analysis
  - (d) exponential smoothing
3. The mean absolute deviation (MAD) is used for
  - (a) estimating the trend line
  - (b) eliminating the forecast errors
  - (c) measuring the forecast accuracy
  - (d) Seasonal adjustment
4. Customer service levels can be improved by
  - (a) sampling plan
  - (b) control charting
  - (c) short-term forecast accuracy
  - (d) customer selection
5. In business, forecasts are the basis for
  - (a) production planning
  - (b) budgeting
  - (c) sales planning
  - (d) all of the above
6. The two basic classifications of forecasting are
  - (a) mathematical and statistical
  - (b) judgemental and qualitative
  - (c) historical and associative
  - (d) qualitative and quantitative
7. Which of the following is not a type of judgemental forecasting?
  - (a) executive opinions
  - (b) sales force opinions
  - (c) the Delphi method
  - (d) time series analysis
8. A series of questionnaire is used in
  - (a) expert opinion
  - (b) sales force opinion
  - (c) time series analysis
  - (d) the Delphi method
9. In which of the following forecasting techniques, the last period actual demand is used as the forecasting for the current period?
  - (a) Naïve method
  - (b) Moving average method
  - (c) Exponential smoothing method
  - (d) Regression method
10. Gradual and long-term movement in time series data is called
  - (a) cycles
  - (b) seasonal variation
  - (c) trend
  - (d) None of these
11. The basic difference between seasonality and cycles is
  - (a) the duration of the repeating patterns in seasonality is longer than cycles
  - (b) the duration of the repeating patterns in seasonality is shorter than cycles the magnitude of the variation
  - (c) the duration of the repeating patterns in seasonality may be longer or shorter than the cycles depends on the product
  - (d) none of these
12. Smoothing constant in the Naïve method is
  - (a) 0.2 to 0.5
  - (b) 0.5 to 0.7
  - (c) 1
  - (d) None of these

13. Moving average forecasting techniques is used for
  - (a) immediately reflect changing patterns in the data
  - (b) lead changes in the data
  - (c) smoothening the variations in the data
  - (d) operate independently of recent data.
14. Which of the following forecasting techniques is based on the previous forecast plus a fixed fraction of the forecast error?
  - (a) naive forecast
  - (b) simple moving average forecast
  - (c) exponentially smoothed forecast
  - (d) regression method
15. In an exponential smoothing method,
  - (a) all the past data have equal weights
  - (b) recent data has lesser weight than the past data
  - (c) recent data has more weight than the past data
  - (d) none of these

## Answers

1. (c)      2. (c)      3. (c)      4. (c)      5. (d)      6. (d)      7. (d)      8. (d)      9. (a)  
10. (c)     11. (b)     12. (c)     13. (c)     14. (c)     15. (c)

## REVIEW QUESTIONS

1. What is the objective of forecasting? Discuss the various steps involved in demand forecasting.
2. Explain the Delphi method of forecasting. How is it different from executive opinion method?
3. How does the weighted moving average method overcome the limitations of the moving average method?
4. How does the weighted average method differ from the exponential smoothing method regarding the weight assignment to the recent data?
5. What do you mean by seasonal variations in demand? How do you account the seasonality in forecasting problems?
6. What are the techniques used to find the forecasting errors? Explain in detail.
7. What are the advantages of regression method of forecasting over simple moving average method of forecasting?

## EXERCISES

1. A company finds the relationships between the demand and economic index for the past 10 years as shown in Table 3.13. (a) Determine the coefficient of correlation between these two variables and (b) Determine the equation of the line of best fit. (c) Find the demand corresponding to economic index 130.

**Table 3-13:** Demands and corresponding economics Indices for last 10 years

S. no.	Demand	Economic index	S. no.	Demand	Economic index
1.	420	106	6.	540	111
2.	380	103	7.	720	122
3.	460	108	8.	280	100
4.	300	101	9.	180	92
5.	240	97	10.	400	105

2. The quarterly demands of three years 2012, 2013 and 2014 are given below (Table 3.14). Calculate the quarterly demands for the year 2015.

**Table 3-14:** Quarterly demand

Year	Quarter I	Quarter II	Quarter III	Quarter IV
2012	250	400	470	340
2013	310	400	550	390
2014	420	520	600	480

3. The monthly demands for office furniture (in units) are given in Table 3.15. Forecast the demand using 3-period and 5-period simple moving average for the 12th month. Also, show the variation in forecasting graphically.

**Table 3-15:** Monthly demands for furniture for 11 months

Month ( $t$ )	1	2	3	4	5	6	7	8	9	10	11	12
Demand ( $D_t$ )	670	698	740	805	879	940	870	778	912	940	809	—

4. Using the data shown in Table 3.15, forecast the demand for the periods with the three-period weighted moving average method. The most recent data should be given 50 per cent weightage, second year past data 30 per cent, and third year past data 20 per cent. Also, compare the result with the result of three-period simple moving average graphically.  
 5. Using the data shown in Table 3.15, forecast the demand for the periods using the exponential smoothing method ( $\alpha = 0.3$  and  $\alpha = 0.5$ ). Also, compare the results graphically.  
 6. Using the data shown in Table 3.15, forecast the demand for the periods with adjusted exponential smoothing method ( $\alpha = 0.5$  and  $\beta = 0.3$ ). Also, compare the result graphically with simple exponential smoothing ( $\alpha = 0.5$ ).  
 7. The weekly demands of a motorcycle by a retailer are shown in Table 3.16. Find an equation of the regression line and estimate the demand for the 14th week.

**Table 3-16:** Weekly demand of the motor cycle

Week (X)	1	2	3	4	5	6	7	8	9	10	11	12
Demand (Y)	500	530	540	500	580	630	560	600	690	650	680	670



## REFERENCES AND FURTHER READINGS

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# CHAPTER ▶ 4

## Aggregate Planning

### 4.1 INTRODUCTION

Aggregate production planning is a planning in which general levels of employment and output are planned to balance supply and demand typically for periods of one year or less. The term ‘aggregate’ implies for groups of products, or product types (i.e., product ‘families’) rather than for specific or individual products. It is intermediate-range capacity planning, used to establish employment levels, output rates, inventory levels, subcontracting and backorders for products that are aggregated, i.e. grouped or brought together. It is not specifically focused on individual products, but deals with the products in the aggregate.

Aggregate planning is essentially a broader approach to planning. Planners generally try to avoid focusing on individual products or services unless, of course, the organization has only one major product or service. Instead, they focus on overall, or aggregate, capacity. Aggregate planning is closely related to other corporate decisions involving, for example, budgeting, personnel and marketing. Most budgets are based on assumptions about aggregate output, personnel levels, inventory levels, purchasing levels, etc. An aggregate plan should thus be the basis for initial budget development and for budget revisions as conditions warrant (Pan and Kleiner 1995).

Aggregate planning identifies the best way to utilize the limited resources of a firm to meet the fluctuating demand. It simultaneously establishes optimal production, inventory and release levels over a given finite planning horizon to meet the total demand (Buffa and Taubert 1972; Hax and Candea 1984). The first production planning research was conducted by Hax and Meal (1975) and aimed to find feasible solutions to planning. Axsater (1985) divided the planning into two parts: aggregate planning and detailed planning. Unlike detailed planning, aggregate planning is performed at higher level. Aggregate planning does not create restrictions for detailed planning while at the same time considers long-term constraints.

Objectives of aggregate planning are to

- (a) minimize costs of production/maximize profits
- (b) maximize customer service
- (c) minimize inventory investments
- (d) minimize changes in production rates
- (e) minimize changes in workforce levels
- (f) maximize utilization of plant and equipment

## 4.2 AGGREGATE PLANNING STRATEGIES

The aggregate planning problem can be clarified by a discussion of the various decision options available. These will be divided into two types:

1. The options, influencing demand; and
2. The options, influencing supply.

Demand can be modified or influenced in several ways as mentioned below:

- Pricing: Price reduction leads to an increase in the demand and increase in price leads to a decrease in the demand subject to type of goods.
- Advertising and promotion: Advertising may increase the demand by offering discounts, additional offer, free post-sales service, etc.
- Back orders: It can be achieved by short selling, sell now and deliver later.
- New demand: New demand can be created by alternative use of the product.

There are also a large number of variables available to modify supply through aggregate planning (Stevenson 1986). These include: hiring and layoff of employees, using overtime and undertime, using part-time or temporary labour, carrying inventory, subcontracting, and making cooperative arrangements.

On the basis of the above discussion, we observe that there are two strategies to meet the fluctuating demand in the market. These strategies are: level strategies and chase strategies. In the case of level strategy, production is uniform whereas in case of chase strategy, production chases the demand by fluctuating the workforce utilization. Organizations must compare workforce fluctuation costs with inventory costs to decide which strategy to use. Level strategy is used when inventory costs are low as compared to the costs of fluctuating the workforce and when efficient production is the primary goal. When inventory costs are high as compared to workforce fluctuation costs, chase strategy is used, although it is less efficient for production. Apart from the production strategies such as level and chase strategies, we also have planning strategies. There are three major strategies associated with aggregate planning (Chase and Aquilano 1985):

- Production variations due to hiring, firing, overtime or undertime,
- Permitting inventory levels to vary, and
- Subcontracting

The demand of a product may continuously fluctuate with the market. The management has to respond to the changes in demand by absorbing the demand fluctuations by carrying or allowing inventory, adjusting the workforce through hiring and firing, and adjusting the production rate through overtime and undertime, combining these alternatives. Each of these alternatives is associated with some costs. The main purpose of aggregate planning is to minimize the all costs over the planning horizon.

For example, forecasts of the expected demand of a product for the next six months and the production days available during these months of planning horizon are given in Table 4.1. Also, the costs associated with various production factors are shown in Table 4.2. Now, we want to find the total production costs if we select different strategies of aggregate planning.

**Table 4-1:** Expected demands and the production days for the next six months

Month	Expected demands	Production days	Demand per day (computed)
Jan	1200	22	54
Feb	1000	15	67
Mar	1100	21	53
Apr	1400	25	56
May	1200	22	54
June	1600	20	80
<b>Total</b>	<b>7500</b>	<b>125</b>	

**Table 4-2:** Costs associated with various production factors

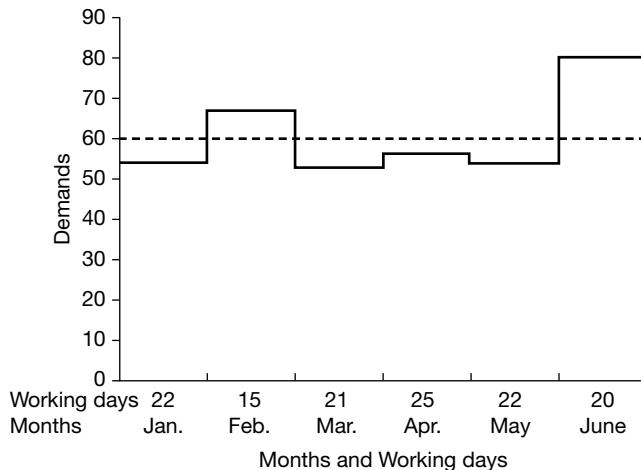
Cost information	
Inventory carrying cost	Rs 4 per unit per month
Subcontracting cost per unit	Rs 8 per unit
Average pay rate	Rs 5 per hour (Rs 40 per day)
Overtime pay rate	Rs 8 per hour (above 8 hours per day)
Labour-hours to produce a unit	1.8 hours per unit
Cost of increasing daily production rate (hiring and training)	Rs 320 per unit
Cost of decreasing daily production rate (layoffs)	Rs 540 per unit

#### **A. Constant Workforce Level and Carrying the Inventory**

$$\begin{aligned} \text{Average total production} &= \frac{\text{Total expected demand during planning horizon}}{\text{Total number of working days available during planning horizon}} \\ &= \frac{7500}{125} = 60 \text{ units per day} \end{aligned}$$

In this case, we maintain the constant production level equal to average production required per day. In Figure 4.1, it can be observed that the constant production level is 60 units per day, while the expected demand is continuously fluctuating from one month to another. Finally, at the end of the planning horizon, the cumulative inventory becomes zero.

Table 4.3 shows the status of changes in inventory in each month and ending inventory, i.e. cumulative inventory at the end of each month. We observe that at the end of the sixth month, the total inventory becomes zero that means the demand in peak period is adjusted by the inventory of the slack period. But, the total inventory carried over one month to another is equal to 1000 units and in this case, total production costs will be equal to the normal production cost plus inventory carrying cost of 1000 units in a month.



**Figure 4-1:** Constant production level equal to average expected demand per day

**Table 4-3:** Monthly change in inventory and cumulative or ending inventory

Month	Production days	Production at 60 units per day	Demand forecast	Monthly inventory change	Ending inventory (Cumulative)
Jan	22	1320	1200	+120	120
Feb	15	900	1000	-100	20
Mar	21	1260	1100	+160	180
Apr	25	1500	1400	+100	280
May	22	1320	1200	+120	400
June	20	1200	1600	-420	0
<b>Total inventory for one month = 1000</b>					

Labour-hours required to produce 60 units per day =  $60 \times 1.8 = 108$

$$\text{Number of labours required per day} = \frac{108}{8} = 13.5$$

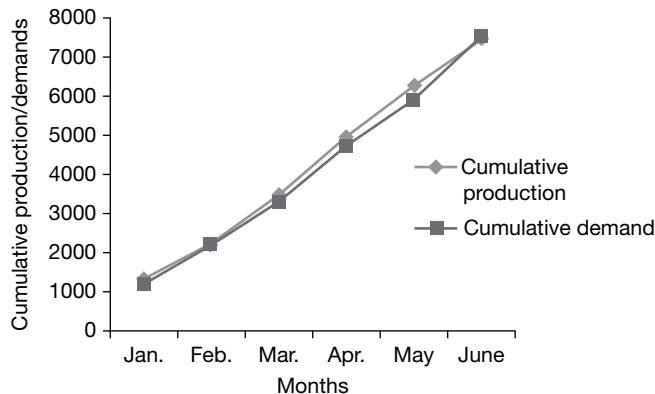
$$\text{Labour cost per day} = 13.5 \times 5 \times 8 = \text{Rs } 540$$

$$\text{Total labour cost for 125 working days} = 125 \times 540 = \text{Rs } 67,500$$

$$\text{Inventory carrying cost} = 1000 \times 4 = \text{Rs } 4000$$

$$\begin{aligned}\text{Total production cost} &= \text{Regular-time labour cost} + \text{Inventory carrying cost} \\ &= \text{Rs } 67,500 + \text{Rs } 4000 = \text{Rs } 71,500\end{aligned}$$

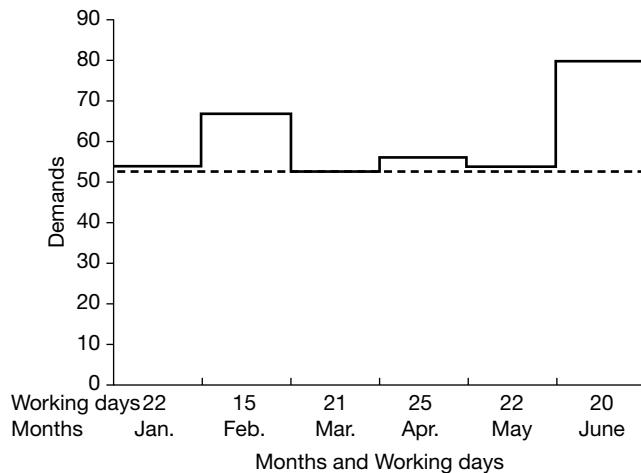
Figure 4.2 shows the cumulative production and cumulative demand and how they become zero at the end of the planning horizon.



**Figure 4-2:** Cumulative production and cumulative demand

### B. Subcontracting as a Strategy to Meet the Fluctuating Demand

In the case of adopting subcontracting as a strategy to meet the fluctuating demand, the production level remains constant equal to the minimum production level required, and any extra demand occurs during the planning horizon is fulfilled by subcontracting. Figure 4.3 shows the constant production level at minimum level equal to 53 units per day and the extra demands that are fulfilled by subcontracting.



**Figure 4-3:** Meeting fluctuating demand by subcontracting

$$\text{Regular-time labour cost for 125 working days} = \frac{53 \times 1.8}{8} \times 40 = \text{Rs } 59,625$$

$$\text{Subcontracting units} = 7500 - 53 \times 125 = 875 \text{ units}$$

Subcontracting cost =  $875 \times 8 = \text{Rs } 7000$

$$\begin{aligned}\text{Total production cost} &= \text{Regular time labour cost} + \text{Subcontracting cost} \\ &= \text{Rs } 59,625 + \text{Rs } 7000 = \text{Rs } 66,625\end{aligned}$$

### **C. Overtime as a Strategy to Meet the Fluctuating Demand**

In the case of overtime production strategy, we maintain the production level at the minimum rate required per day or the initial workforce available and the extra unit requirement is fulfilled by overtime production. Here, we maintain the production rate of 53 units per day and if the demand increases that amount is fulfilled by overtime production.

$$\text{Regular-time labour cost for 125 working days} = \frac{53 \times 1.8}{8} \times 40 = \text{Rs } 59,625$$

$$\text{Overtime units} = 7500 - 53 \times 125 = 875 \text{ units}$$

$$\text{Overtime cost} = 875 \times 1.8 \times 8 = \text{Rs } 12,600$$

$$\begin{aligned}\text{Total production cost} &= \text{Regular time labour cost} + \text{Overtime cost} \\ &= \text{Rs } 59,625 + \text{Rs } 12,600 = \text{Rs } 72,225\end{aligned}$$

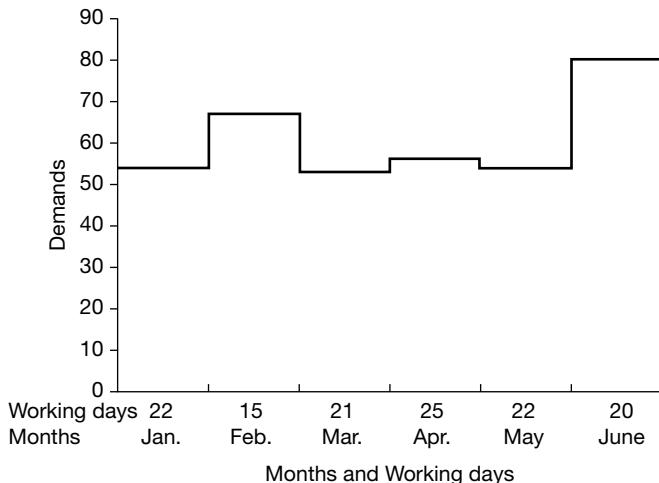
### **D. Hiring and Firing as a Strategy to Meet the Fluctuating Demand**

In the case of hiring and firing, we have to produce the products as per the demand of the market. Daily production rate fluctuates continuously. The cost of hiring and firing depends on the increase and decrease in the daily production rate in the specific month, respectively. The total cost of production will be equal to the regular time production cost plus hiring and firing costs of the labour (Rs 89,580) as given in Table 4.4.

**Table 4-4:** Hiring and firing costs with regular-time labour cost

Month	Forecast (units)	Daily production rate	Basic production cost (demand $\times$ 1.8 hrs/unit $\times$ Rs 5/hr)	Extra cost of increasing production (hiring cost)	Extra cost of decreasing production (layoff cost)	Total cost
Jan	1200	54	Rs 10,800	—	—	Rs 10,800
Feb	1000	67	Rs 9000	Rs 4160 (= 13 $\times$ Rs 320)	—	Rs 13,160
Mar	1100	53	Rs 9900	—	Rs 7560 (= 14 $\times$ Rs 540)	Rs 17,460
Apr	1400	56	Rs 12,600	Rs 960 (= 3 $\times$ Rs 320)	—	Rs 13,560
May	1200	54	Rs 10,800	—	Rs 1080 (= 2 $\times$ Rs 540)	Rs 11,880
June	1600	80	Rs 14,400	Rs 8320 (= 26 $\times$ Rs 320)	—	Rs 22,720
	Total		Rs 67,500	Rs 13,440	Rs 8640	<b>Rs 89,580</b>

Table 4.4 shows the hiring and firing costs of labours with regular-time production cost, and Figure 4.4 shows the aggregate planning with hiring and firing of labour. The total cost of production using hiring and firing of labours as a strategy will be Rs 89,580.



**Figure 4-4:** Aggregate planning with hiring and firing

*If we compare all the three strategies of aggregate planning, we find that with the given data, subcontracting option with a total production cost of Rs 66,625 is the most suitable option.*

## 4.3 MIXED STRATEGY

The model of mixed strategy is developed on the basis that all the four (discussed in previous section) strategies are used optimally at the same time. For this model, we use linear programming model and liner decision rule model.

### 4.3.1 Linear Programming for Aggregate Planning

If all the cost parameters in aggregate planning are linear, we can apply the linear programming as a tool to solve the problem. Suppose following notations are given for the different cost parameters:

$c_H$  = Cost of hiring one worker,

$c_F$  = Cost of firing one worker,

$c_i$  = Cost of one unit in inventory for unit period,

$c_R$  = Cost of producing one unit on regular time,

$c_O$  = Incremental cost of producing one unit on overtime,

$c_U$  = Idle cost per unit of production,

$c_S$  = Cost of subcontract one unit of production,

$n_t$  = Number of production days in period  $t$ ,

$K$  = Number of aggregate units produced by one worker in one day,

$I_O$  = Initial inventory on hand at the start of the planning horizon,

$W_O$  = Initial workforce at the start of the planning horizon,

$D_t$  = Forecast of demand in time  $t$ .

The cost parameters may be time dependent. They may be useful for modelling changes in the costs of hiring or firing due to shortages in the labour pool, or changes in the costs of production or shortage due to poor or disrupted supply of resources, or change in interest rates. There are some useful problem variables which are given below as:

$W_t$  = Workforce level in period  $t$ ,

$P_t$  = Production level in period  $t$ ,

$I_t$  = Inventory level in period  $t$ ,

$H_t$  = Number of workers hired in period  $t$ ,

$F_t$  = Number of workers fired in period  $t$ ,

$O_t$  = Overtime production in units,

$U_t$  = Worker idle time in units,

$S_t$  = Number of units subcontracted from outside.

Suppose the total time of production is  $T$ ; now the total costs involved in production during time  $T$  will be:

$$\begin{aligned} & c_H H_t \text{ (i.e., Hiring cost)} + c_F F_t \text{ (i.e., Firing cost)} + c_I I_t \text{ (i.e., Inventory carrying cost)} \\ & + c_R P_t \text{ (i.e., Regular time production cost)} + c_o O_t \text{ (i.e., Overtime production cost)} \\ & + c_U U_t \text{ (i.e., Idle production cost)} + c_s S_t \text{ (i.e., subcontracting production cost)} \end{aligned}$$

The objective of linear programming is to minimize the total production costs, i.e. the objective function is:

$$\text{Minimize } Z = \sum_{t=1}^T (c_H H_t + c_F F_t + c_I I_t + c_R P_t + c_o O_t + c_U U_t + c_s S_t)$$

Subject to constraints:

$$W_t = W_{t-1} + H_t - F_t; \quad 1 \leq t \leq T; \quad \text{Workforce constraints}$$

$$I_t = I_{t-1} + P_t + S_t - D_t; \quad 1 \leq t \leq T; \quad \text{Inventory constraints}$$

$$P_t = Kn_t W_t + O_t - U_t; \quad 1 \leq t \leq T; \quad \text{Production level constraints}$$

**Case I:** No hiring and firing, no subcontracting, no overtime production, only constant workforce and inventory are allowed

$$\begin{aligned} \text{Total cost} &= c_I I_t \text{ (i.e., Inventory carrying cost)} + c_R P_t \text{ (i.e., Regular time production cost)} \\ &\quad + c_U U_t \text{ (i.e., Idle production cost)} \end{aligned}$$

$$\text{Thus, Minimize } Z = \sum_{t=1}^T (c_I I_t + c_R P_t + c_U U_t)$$

Subject to the constraints:

$$\begin{array}{lll} W_t = W_{t-1}; & 1 \leq t \leq T; & \text{Workforce constraints} \\ I_t = I_{t-1} + P_t - D_t; & 1 \leq t \leq T; & \text{Inventory constraints} \\ P_t = Kn_t W_t - U_t; & 1 \leq t \leq T; & \text{Production level constraints} \end{array}$$

**Case II:** Only constant workforce and overtime production are allowed.

$$\begin{aligned} \text{Total cost} &= c_I I_t \text{ (i.e., Inventory carrying cost)} + c_R P_t \text{ (i.e., Regular time production cost)} \\ &\quad + c_o O_t \text{ (i.e., Overtime production cost)} + c_U U_t \text{ (i.e., Idle production cost)} \end{aligned}$$

The objective of linear programming is to minimize the total production costs, i.e. the objective function is:

$$\text{Minimize } Z = \sum_{t=1}^T (c_I I_t + c_R P_t + c_o O_t + c_U U_t)$$

Subject to the constraints

$$\begin{array}{lll} W_t = W_{t-1}; & 1 \leq t \leq T; & \text{Workforce constraints} \\ I_t = I_{t-1} + P_t - D_t; & 1 \leq t \leq T; & \text{Inventory constraints} \\ P_t = Kn_t W_t + O_t - U_t; & 1 \leq t \leq T; & \text{Production level constraints} \end{array}$$

**Case III:** Only constant workforce and subcontracting are allowed.

$$\begin{aligned} \text{Total cost} &= c_I I_t \text{ (i.e., Inventory carrying cost)} + c_R P_t \text{ (i.e., Regular time production cost)} \\ &\quad + c_U U_t \text{ (i.e., Idle production cost)} + c_s S_t \text{ (i.e., subcontracting production cost)} \end{aligned}$$

The objective of linear programming is to minimize the total production costs, i.e. the objective function is:

$$\text{Minimize } Z = \sum_{t=1}^T (c_R P_t + c_U U_t + c_s S_t)$$

Subject to the constraints

$$\begin{array}{lll} W_t = W_{t-1}; & 1 \leq t \leq T; & \text{Workforce constraints} \\ I_t = I_{t-1} + P_t + S_t - D_t; & 1 \leq t \leq T; & \text{Inventory constraints} \\ P_t = Kn_t W_t - U_t; & 1 \leq t \leq T; & \text{Production level constraints} \end{array}$$

**Case IV:** Only hiring and firing are allowed.

$$\begin{aligned} \text{Total cost} &= c_H H_t \text{ (i.e., Hiring cost)} + c_F F_t \text{ (i.e., Firing cost)} \\ &\quad + c_R P_t \text{ (i.e., Regular time production cost)} + c_U U_t \text{ (i.e., Idle production cost)} \end{aligned}$$

The objective of linear programming is to minimize the total production costs, i.e., the objective function is:

$$\text{Minimize } Z = \sum_{t=1}^T (c_H H_t + c_F F_t + c_R P_t + c_U U_t)$$

Subject to constraints:

$$\begin{array}{lll} W_t = W_{t-1} + H_t - F_t; & 1 \leq t \leq T; & \text{Workforce constraints} \\ I_t = I_{t-1} + P_t - D_t; & 1 \leq t \leq T; & \text{Inventory constraints} \\ P_t = K n_t W_t - U_t; & 1 \leq t \leq T; & \text{Production level constraints} \end{array}$$

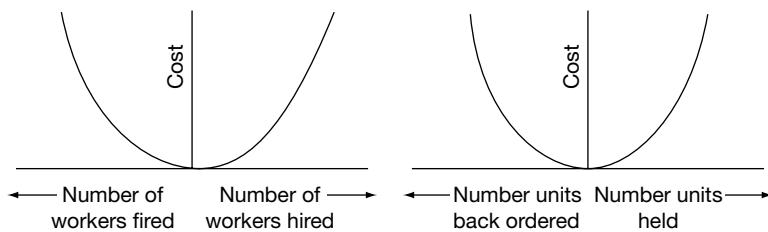
#### 4.3.2 The Linear Decision Rule

The second approach to solve the aggregate programming problem is linear decision rule (LDR). This rule was suggested by Holt et al. (1956). The costs involved in this model are inventory costs, cost of changing production level, number of workers, etc. These costs are represented by quadratic functions. The total cost over the period  $T$  can be given as:

$$\text{Minimize } Z = \sum_{t=1}^T \left[ c_1 W_t + c_2 (W_t - W_{t-1})^2 + c_3 (P_t - K n_t W_t)^2 + c_4 P_t + (I_t - c_6)^2 \right]$$

Subject to the constraints:

$$I_t = I_{t-1} + P_t - D_t; \quad \text{for } 1 \leq t \leq T$$



**Figure 4-5:** LDR for hiring and firing and backordering

The value of the constants  $c_1, c_2, c_3, \dots, c_6$  must be determined for the specific applications. The cost functions as quadratic functions (as shown in Figure 4.5) have some advantages over the linear programming approach. As quadratic functions are differentiable, the standard rules of calculus for maxima and minima can be applied to find the optimal solution.



## SUMMARY

In this chapter, we have discussed the various strategies to meet the fluctuating demand in the intermediate range of planning. Level strategies are used to level the inventory as per demand, but in chase strategies, the fluctuating demand is chased by various strategies such as overtime production, subcontracting, changing the workforce level, etc. Also, we have discussed the linear programming and LDR as a tool to formulate the mixed strategy.

## MULTIPLE-CHOICE QUESTIONS

1. Aggregate planning is capacity planning for
  - (a) the long range
  - (b) the intermediate range
  - (c) the short range
  - (d) typically one to three months
2. A chase demand strategy is used when
  - (a) material cost is high
  - (b) labour cost is high
  - (c) inventory costs are high
  - (d) all of the above
3. Which of the following aggregate planning method is an optimal solution?
  - (a) linear programming
  - (b) search decision rule
  - (c) trial and error with a spreadsheet
  - (d) none of these
4. Which of the following statements regarding aggregate planning for services is NOT true?
  - (a) Services cannot be inventoried
  - (b) Demand is difficult to predict
  - (c) Capacity is easy to predict
  - (d) Labour is a big constraint
5. Which of the following statements is NOT true for level production strategy of aggregate planning?
  - (a) Level production strategy sets production rate constant
  - (b) The main costs of level production involve hiring and firing
  - (c) Level production strategy uses inventory to absorb variations in demand
  - (d) All of the above
6. Level strategy is used when
  - (a) inventory costs are low as compared to the costs of fluctuating the workforce
  - (b) efficient production is the primary goal
  - (c) costs of subcontracting are high
  - (d) all of the above
7. Overtime as a strategy to meet the fluctuating demand is used when
  - (a) overtime cost is less than the inventory cost
  - (b) subcontracting cost is less than the inventory cost
  - (c) inventory cost is less than the cost of hiring and firing of the workers
  - (d) none of these

8. Hiring and firing as a strategy to meet the fluctuating demand is used under
  - (a) a level production strategy
  - (b) a chase demand strategy
  - (c) both
  - (d) none of these
9. Linear programming is used for
  - (a) only hiring and firing strategy
  - (b) only constant workforce strategy
  - (c) only overtime production strategy
  - (d) all the above and mixed strategy
10. Aggregate planning
  - (a) is used for only single product
  - (b) may be used for a group of products
  - (c) is based on long range forecasting
  - (d) none of these
11. Aggregate planning is used for
  - (a) the best way to utilize the limited resources of a firm
  - (b) minimizing the overall production cost
  - (c) meeting the fluctuating demand in the market
  - (d) all of the above
12. Which of the following strategies is used to adjust the capacity to match demand?
  - (a) using part-time labourers
  - (b) changing price
  - (c) backordering
  - (d) none of these
13. Which of the following strategies is known for lower employee morale?
  - (a) hiring and firing strategy
  - (b) constant workforce strategy
  - (c) overtime production strategy
  - (d) subcontracting strategy
14. Which of the following strategies is used to manipulate the demand of the product?
  - (a) hiring and firing strategy
  - (b) constant workforce strategy
  - (c) overtime production strategy
  - (d) pricing strategy
15. Which of the following strategies is a demand option?
  - (a) changing workforce level
  - (b) changing inventory level
  - (c) changing production level
  - (d) changing price level

## Answers

1. (b)      2. (c)      3. (a)      4. (c)      5. (b)      6. (d)      7. (a)      8. (b)      9. (d)  
10. (b)     11. (d)     12. (a)     13. (a)     14. (d)     15. (d)

## REVIEW QUESTIONS

1. What do you mean by aggregate planning? How does it differ from long-term planning?
2. How does the chase strategy differ from level strategy?
3. What are the actions taken for chasing the fluctuating demand?

4. What are the actions taken for levelling the production to meet the fluctuating demand?
5. What is the method to influence the fluctuating demand in the markets?
6. What are the limitations of hiring and firing options of workers?
7. In which situations subcontracting option is considered as the most suitable action?

## EXERCISES

1. Forecasts of the expected demand of a product for the next six months and the production days available during these months of planning horizon are given in Table 4.5. Also, the costs associated with various production factors are shown in Table 4.6. Find the production costs under the following strategies.
  - (a) Constant workforce level and carrying the inventory
  - (b) Subcontracting as a strategy to meet the fluctuating demand
  - (c) Overtime as a strategy to meet the fluctuating demand
  - (d) Hiring and firing as a strategy to meet the fluctuating demand

**Table 4-5:** Product demand and available production days

Month	Expected demands	Production days
Jan	1000	20
Feb	800	13
Mar	900	19
Apr	1200	23
May	1000	20
June	1400	18
	7300	113

**Table 4-6:** Various costs involved in production

Cost information	
Inventory carrying cost	Rs 5 per unit per month
Subcontracting cost per unit	Rs 8 per unit
Average pay rate	Rs 5 per hour (Rs 40 per day)
Overtime pay rate	Rs 10 per hour (above 8 hours per day)
Labour-hours to produce a unit	2 hours per unit
Cost of increasing daily production rate (hiring and training)	Rs 300 per unit
Cost of decreasing daily production rate (layoffs)	Rs 500 per unit



## REFERENCES AND FURTHER READINGS

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# CHAPTER ▶ 5

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## Capacity Planning: MRP, MRP II and ERP

### 5.1 INTRODUCTION

Capacity planning is the process of determining the production capacity needed by an organization to meet changing demands for its products. In the context of capacity planning, the capacity is the maximum amount of work that an organization is capable of doing in a given period of time. There are three basic steps of capacity planning: determination of service level or production level requirements, analysis of the current capacity of the organization and planning for the capacity required in future.

The required production level of product and services can be determined by finding the workloads and identifying service level for each workload. The current capacity of the systems can be analysed by measuring the production level and comparing with the target or objectives, measuring the overall resource usage, measuring resource usage by workload and identifying the components of response time. The future plan consists of future processing requirements and plan for future system configuration and specifications.

*Capacity requirement planning:* The capacity requirement planning (CRP) is a solution to meet the target of manufacturing organizations with discrete production systems. It allows the maintenance of resources, tools, products, production plans and their interdependencies. CRP supports the decision-making process with maximum user-friendliness, interconnectivity, portability and flexibility. CRP first assesses the schedule of production that has been planned by the company, and then analyses the company's actual production capacity.

In this chapter, we will study about the material requirement planning (MRP), manufacturing resource planning (MRP II) and enterprise resource planning (ERP).

### 5.2 MATERIALS REQUIREMENT PLANNING

MRP was first introduced and used by IBM in 1970 and essentially substitutes excessive inventories for better information systems. The MRP system was initiated in the 1960s and was spearheaded by a team of IBM innovators comprising Joe Orlicky, George Plossl and Ollie Wright, who sought to create a structured methodology for planning and scheduling materials for complex manufactured products. MRP schedules the production of jobs through the factory and eliminates the excessive work-in-process inventory levels required to compensate for job-scheduling problems that arise in decoupled cost or operation centres. MRP releases work orders for parts based upon a master production schedule and the current number and location of parts

within the plant. MRP is a system which maximizes the efficiency in the timing of raw materials orders through to the manufacture and assembly of the final product.

MRP is a computer-based inventory management system designed to assist the managers in scheduling and placing orders for items of dependent demand. Dependent demand items are raw materials, component parts and subassemblies. For example, in a plant that manufactures motorcycle, dependent demand inventory items might include rims, tires, seats and bike chains, gearbox, engine blocks, electrical items, handle assembly items, break, clutches, etc.

MRP has been expanded to include information feedback loops so that production personnel could change and update the inputs into the system as needed. The next generation of MRP, known as manufacturing resources planning or MRP II, also incorporated marketing, finance, accounting, engineering and human resources aspects in the planning process. A related concept that expands on MRP is ERP (Enterprise Resource Planning), which uses computer technology to link the various functional areas across an entire business enterprise.

MRP begins with a schedule for finished goods that is converted into a schedule of requirements for the subassemblies, the component parts and the raw materials needed to produce the final product within the established schedule. MRP is designed to answer three questions: 'what is needed? How much is needed? when is it needed?' The functions of MRP are to:

- create schedules and identify the specific parts and materials required to produce an end item,
- determine exact number of components needed, and
- determine the dates when orders for those materials should be released, based on lead times.

The entire MRP can be divided into three parts: inputs to MRP, processes and outputs of MRP as shown in Figure 5.1.

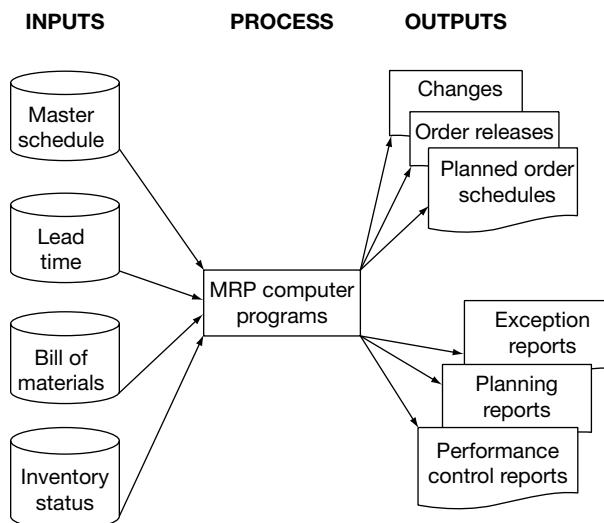


Figure 5-1: MRP Structures

### 5.2.1 MRP Inputs

The main sources of MRP inputs are a master schedule, lead times, a bill of materials and an inventory record with present status. The bill of materials is a listing of all the raw materials, component parts, subassemblies and assemblies required to produce one unit of a specific finished product. The bill of materials is arranged in a hierarchy as shown in Figure 5.3, so that managers can see what materials are needed to complete each level of production. MRP uses the bill of materials to determine the quantity of each component that is needed to produce a certain number of finished products. From this quantity, the system subtracts the quantity of that item already in inventory to determine order requirements.

The master schedule outlines the planned production activities of the plant. Based on the forecasting and the orders received from the customers/markets, it states the quantity of each product that will be manufactured and the time frame in which they will be manufactured. The master schedule divides the planning horizon into a number of weeks. The schedule covers a time frame long enough to produce the final product. This total production time consists of the sum of the lead times of all the related fabrication and assembly operations of the components and subcomponents. Master schedules are generated according to demand and without regard to capacity.

The inventory records file provides an accounting of how much inventory is already on hand or on order, and thus should be subtracted from the material requirements. The inventory records file is used to track information on the status of each item by time period. This includes gross requirements, scheduled receipts and the expected amount on hand. It includes other details for each item as well, like the supplier, the lead time and the lot size.

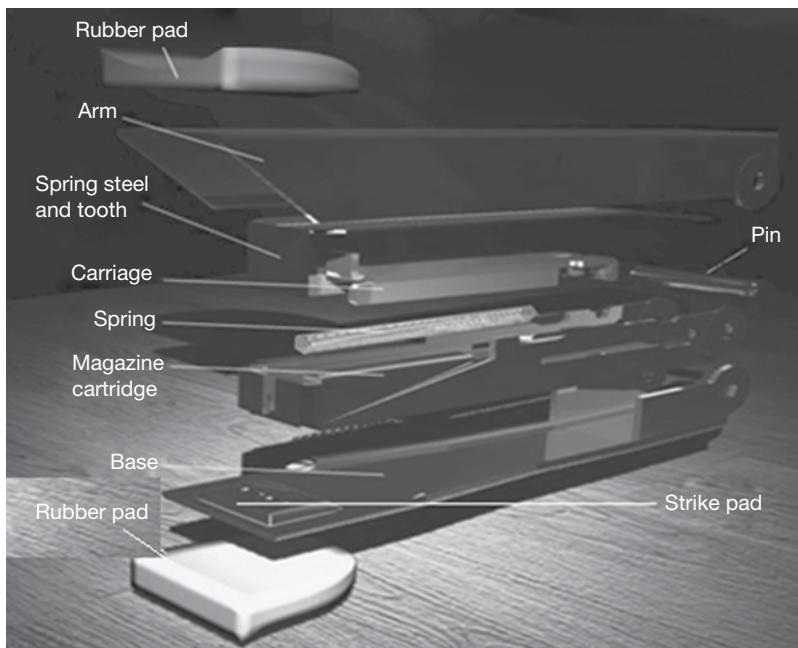
### 5.2.2 MRP Processing and Outputs

The MRP system determines the net requirements for raw materials, component parts and subassemblies based on the information gathered from the bill of materials, master schedule and inventory records file. It determines the gross material requirements, then subtracts out the inventory on hand and adds back in the safety stock in order to compute the net requirements. The main outputs from MRP include three primary reports and three secondary reports. The primary reports consist of planned order schedules, order releases and changes to planned orders, which might include cancellations or revisions of the orders. The secondary reports generated by MRP include performance control reports (which are used to track problems like missed delivery dates and stock outs in order to evaluate system performance), planning reports (which can be used in forecasting future inventory requirements); and exception reports (which call managers' attention to major problems).

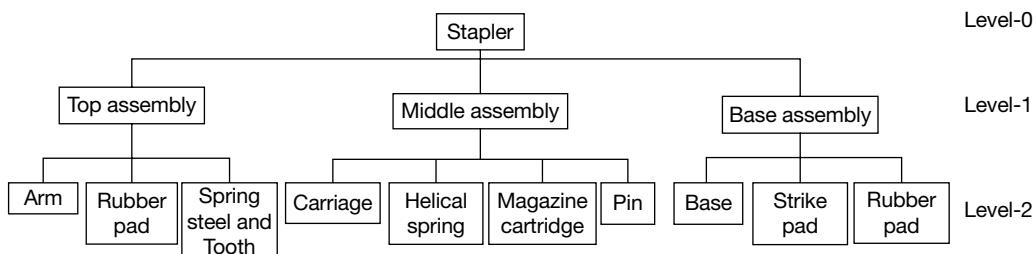
The MRP process involves the following steps:

- Determine the gross requirements of a finished product.
- Determine the net requirements and when orders will be released for fabrication or subassembly.
- Net requirements = Total requirements – Available inventory.
- Net requirements = (Gross requirements + Allocations) – (On hand) – Scheduled receipts.

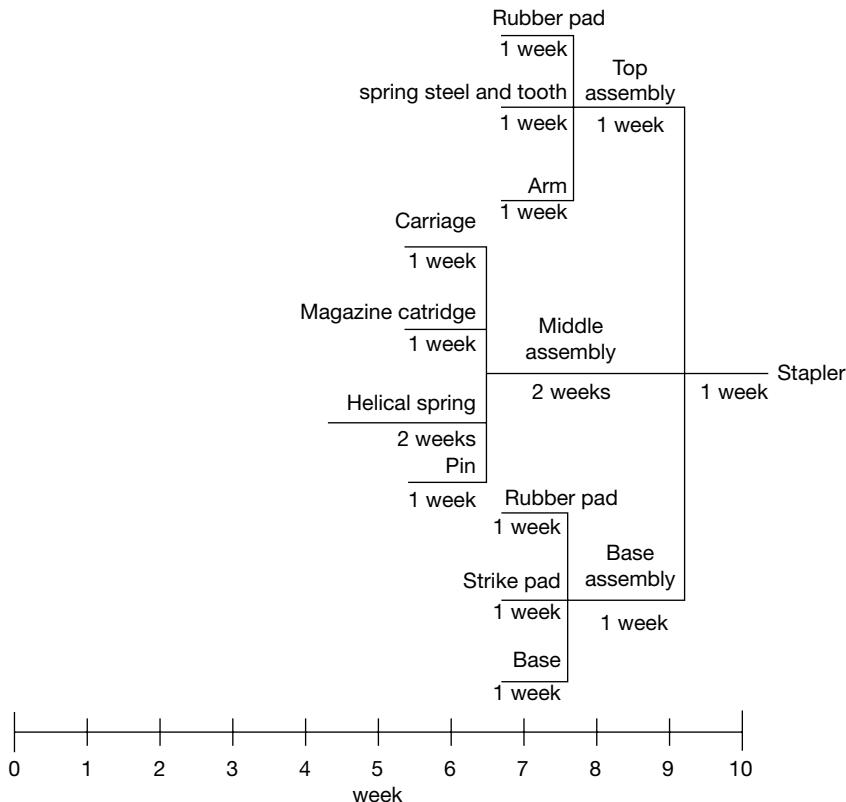
**Example 5.1:** A firm manager receives an order of 500 staplers. The order is to be delivered at the end of 10<sup>th</sup> week from now. He had 200 staplers in the inventory at the time of the order receipts. The product components and its structure are shown in Figures 5.2 and 5.3, respectively. The lead time to manufacture the components is shown in Figure 5.4. Using MRP, determine the schedules for the orders to be released for the manufacturing and assemblies of the components so that the 500 staplers can be delivered on time.



**Figure 5-2:** Stapler with its components



**Figure 5-3:** Product structure (Bill of materials) of staplers



**Figure 5-4:** Lead times of the various components of the stapler

### Solution:

The order release schedules and the amount of the components at various stages of the manufacturing are shown in Tables 5.1 to 5.14.

**Table 5-1:** MRP schedules for the stapler

**Table 5-2:** MRP schedules for the top assembly of the stapler

Item: Top assembly	Lead time: 1 week									
	Week									
	1	2	3	4	5	6	7	8	9	10
Gross requirements									300	
Scheduled receipts									0	
Projected on-hand inventory									0	
Net requirements									300	
Planned order receipts									300	
Planned order releases								300		

**Table 5-3:** MRP schedules for the rubber pad (top assembly) of the stapler

**Table 5-4:** MRP schedules for the spring steel and tooth (top assembly) of the stapler

Item: Spring steel and tooth	Lead time: 1 week									
	Week									
	1	2	3	4	5	6	7	8	9	10
Gross requirements								300		
Scheduled receipts								0		
Projected on-hand inventory								0		
Net requirements								300		
Planned order receipts								300		
Planned order releases								300		

**Table 5-5:** MRP schedules for the arm (top assembly) of the stapler

Item: Arm	Lead time: 1 week									
	Week									
	1	2	3	4	5	6	7	8	9	10
Gross requirements								300		
Scheduled receipts								0		
Projected on-hand inventory								0		
Net requirements								300		
Planned order receipts								300		
Planned order releases								300		

**Table 5-6:** MRP schedules for the middle of the stapler

**Table 5-7:** MRP schedules for the carriage (middle assembly) of the stapler

**Table 5-8:** MRP schedules for the magazine cartridge (middle assembly) of the stapler

**Table 5-9:** MRP schedules for the helical spring (middle assembly) of the stapler

**Table 5-10:** MRP schedules for the pin (middle assembly) of the stapler

**Table 5-11:** MRP schedules for the base assembly of the stapler

**Table 5-12:** MRP schedules for the rubber pad (base assembly) of the stapler

Item: Rubber pad	Lead time: 1 week									
	Week									
	1	2	3	4	5	6	7	8	9	10
Gross requirements								300		
Scheduled receipts								0		
Projected on-hand inventory								0		
Net requirements								300		
Planned order receipts								300		
Planned order releases								300		

**Table 5-13:** MRP schedules for the strike pad (base assembly) of the stapler

Item: Strike pad	Lead time: 1 week									
	Week									
	1	2	3	4	5	6	7	8	9	10
Gross requirements								300		
Scheduled receipts								0		
Projected on-hand inventory								0		
Net requirements								300		
Planned order receipts								300		
Planned order releases								300		

**Table 5-14:** MRP schedules for the base of the stapler

Item: Base	Lead time: 1 week									
	Week									
	1	2	3	4	5	6	7	8	9	10
Gross requirements									300	
Scheduled receipts								0		
Projected on-hand inventory								0		
Net requirements								300		
Planned order receipts								300		
Planned order releases								300		

### 5.2.3 Benefits and Limitations of MRP

The following are benefits of MRP:

1. Increased customer service and satisfaction.
2. Improved utilization of facilities and personnel.
3. Better inventory planning and scheduling.
4. Faster response to market changes and shifts.
5. Reduced inventory levels without reducing customer services.

The following are limitations of MRP:

1. Delays in scheduled receipts.
2. Changes in planned order sizes because of capacity constraints.
3. Changes in gross requirements which dictate changes in lot sizes at sub-component levels.
4. Unavailability of raw materials for one sub-component may negate the need for a fellow subcomponent as both must be ready for the parent production.
5. Utilization of the same parts at different levels, indicating the need to restructure the bill of materials.

### 5.3 MRP II

In the 1980s, MRP technology was expanded to create a new approach called manufacturing resource planning, or MRP II. ‘The techniques developed in MRP to provide valid production schedules proved so successful that organizations became aware that with valid schedules other resources could be better planned and controlled,’ Gordon Minty noted in his book *Production Planning and Controlling*. ‘The areas of marketing, finance, and personnel were affected by the improvement in customer delivery commitments, cash flow projections, and personnel management projections.’

Minty says that ‘MRP II does not replace MRP, and also it is not an improved version of MRP. Rather, it represents an effort to expand the scope of production resource planning and to involve other functional areas of the firm in the planning process,’ such as marketing, finance, engineering, purchasing and human resources. MRP II differs from MRP in that all of these functional areas have input into the master production schedule. From that point, MRP is used to generate material requirements and helps production managers plan the capacity. MRP II systems often include simulation capabilities, so managers can evaluate various options. MRP II integrates many areas of the manufacturing enterprise into a single entity for planning and control purposes, from board level to operative and from five-year plan to individual shop-floor operation. It builds on closed-loop MRP by adopting the feedback principle, but extending it to additional areas of the enterprise, primarily manufacturing-related.

MRP II is defined by the American Production and Inventory Control Society (APICS) as a method for the effective planning of all the resources of a manufacturing company (Higgins, LeRoy and Tierney 1996). It is a new generation of the MRP system, which is a set of techniques that uses bills of material, inventory, data and a master production schedule to calculate the requirements for materials in a manufacturing company.

### **5.3.1 Characteristics of MRP II**

The characteristics of MRP II can be summarized as follows (Higgins, LeRoy and Tierney 1996):

1. The operation and financial system are the same.
2. It has simulation capabilities that enable predictions to be made beforehand.
3. It involves every facet of the business from planning to execution.
4. MRP II offers a systematic method for planning and procuring materials to support production.
5. MRP II is not a proprietary software system and can thus take many forms.
6. It is almost impossible to visualize an MRP II system that does not use a computer, but an MRP II system can be based on either purchased / licensed or in-house software.
7. Almost every MRP II system is modular in construction.
8. Characteristic basic modules in an MRP II system are master production scheduling (MPS), item master data, bill of materials (BOM), production resources data, inventories and orders, purchasing management, MRP, shop-floor control (SFC), capacity planning or CRP, standard costing, cost reporting/management, distribution resource planning (DRP).
9. Some ancillary systems in MRP II are business planning, lot traceability, contract management, tool management, engineering change control, configuration management, shop-floor data collection, sales analysis and forecasting, Finite Capacity Scheduling (FCS), general ledger, accounts payable (purchase ledger), accounts receivable (sales ledger), sales order management, Distribution Requirements Planning (DRP), warehouse management, project management, technical records, estimating, Computer-Aided Design/Computer-Aided Manufacturing (CAD/CAM), Computer-Aided Process Planning (CAPP).

### 5.3.2 Advantages of Using MRP II

The following are advantages of MRP II:

1. Better control of inventories
2. Improved scheduling
3. Productive relationships with suppliers
4. Improved design control
5. Better quality and quality control
6. Reduced working capital for inventory
7. Improved cash flow through quicker deliveries
8. Accurate inventory records
9. Timely and valid cost and profitability information

## 5.4 ENTERPRISE RESOURCE PLANNING

ERP is a management tool used for planning and monitoring all of the resources of a manufacturing company, including the functions of manufacturing, marketing, finance and engineering (Wight 1993). ERP represents the application of the latest IT to the MRP II system. This is recognized as being an effective management system (Ormsby et al. 1990) that has an excellent planning and scheduling capability offering significant gains in productivity, dramatic increases in customer service, much higher inventory turns and greater reduction in material costs.

ERP systems as comprehensive package software solutions that seek to integrate the complete range of business processes and functions in order to present a holistic view of the business from a single information and IT architecture (Gable 1998). Thus, ERP is seen as 'an integrated, multi-dimensional system for all functions, based on a business model for planning, control, and global (resource) optimization of the entire supply chain, by using state of the art Information Systems/Information Technology (IS/IT) that supplies value added services to all internal and external parties' (Slooten and Yap 1999). The main ERP Critical Success Factors (CSFs) fall under one of four main categories, namely, commitment from top management (Devenport 1998), Business Process Reengineering (BPR), the IT infrastructure (Summer 1999), and deploying change management.

*Top management commitment:* Management must be a part of ERP implementation and it has been clearly demonstrated that for IT projects to succeed, top management support is critical.

*Business process re-engineering:* The concept of BPR was first introduced by Hammer in 1990. BPR has been defined as a fundamental redesign of business processes to achieve dramatic improvements in critical areas such as cost, quality, service and speed (Hammer 1990). BPR began as a technique to help the organizations fundamentally rethink how they do their work in order to dramatically improve customer service, cut operational costs, and become world-class competitors. It has become a popular management tool for dealing with rapid technological and business change in today's competitive environment (Hamid 2004). With the rise of e-commerce, enterprise systems, customer relationship management and other technology-enabled new business practices, businesses now face major changes in much shorter time periods. The

challenges of the new Internet economy may offer an opportunity to apply the lessons learned from a decade of BPR efforts, which likewise sought ways to manage major change (Alan et al. 2003). Implementing an ERP system involves reengineering the existing business processes to the best business process standard (Gibson et al. 1999).

*IT Infrastructure:* Adequate hardware and networking infrastructure are required for ERP application. An ERP system relies on its operation on sophisticated information technology infrastructure. In addition to this infrastructure, clearly, the software configuration has a critical influence on the implementation process and outcome (Holland and Light 1999).

*Change management:* One of the main obstacles facing ERP implementation is resistance to change. ‘About half of ERP projects fail to achieve results because managers underestimate the efforts involved in managing change’ (Pawlowski et al. 1999). To successfully implement ERP, the way organizations do business will need to change and the ways people do their jobs will need to change too (Koch et al. 1999). Thus, change management is essential for preparing a company for the introduction of an ERP system, and its successful implementation.

### **5.4.1 Features of ERP**

ERP has the following features:

1. It provides multi-platform, multi-facility, multi-mode manufacturing, multi-currency, multi-lingual facilities.
2. It supports strategic, tactical and operational planning and execution activities.
3. It covers all functional areas like manufacturing, selling and distribution, payables, receivables, inventory, accounts, human resources, purchases, etc.
4. It performs core activities and increases customer service, thereby improving the corporate image.
5. It bridges the information gap across organizations.
6. It provides complete integration of systems across the departments and companies under the same management.
7. It allows the automatic introduction of the latest technologies like Electronic Fund Transfer (EFT), Electronic Data Interchange (EDI), Internet, Video conferencing, E-commerce, etc.
8. It eliminates the problems like material shortages, productivity enhancements, customer service, cash management, inventory problems, quality problems, on time delivery, etc.
9. It provides intelligent business tools like decision support system, executive information system, data mining, etc.

### **5.4.2 Benefits of ERP**

The areawise benefits of ERP are shown below as:

*Operational benefits:* Cost reduction, cycle time reduction, productivity improvement, quality improvement, customer service improvement, etc.

*Managerial benefits:* Better resource management, improved decision-making and planning, and performance improvement.

*Strategic benefits:* Support for business growth, support for business alliance, building business innovations, building cost leadership, generating product differentiation, building external linkages, enabling e-governance, sustaining competitiveness, etc.

*IT infrastructure benefits:* Building business flexibility for current and future changes, IT cost reduction, increased IT infrastructure capability, etc.

*Organizational benefits:* Changing work patterns, facilitating organizational learning, empowerment, building a common vision, shifting work focus, increased employee morale and satisfaction.



## SUMMARY

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In this chapter, we have learnt about the capacity planning. We have discussed about MRP, manufacturing resource planning, ERP and BPR. The main focus of the chapter was to minimize the cost of the product by minimizing the inventory cost, improving the quality and providing the product on time to the customer. Finally, we introduced the management tool, ERP, which is used to integrate all the resources available in the enterprise.

## MULTIPLE-CHOICE QUESTIONS

1. MRP means
  - (a) Maximum retail price
  - (b) Material requirement planning
  - (c) Manufacturing resource planning
  - (d) Money resource planning
2. MRP is used for
  - (a) Dependent demand
  - (b) Independent demand
  - (c) Reserve stock
  - (d) Spare parts
3. Which of the following is NOT an input required for MRP?
  - (a) Bill of material
  - (b) Master production schedule
  - (c) Inventory on hand
  - (d) Financial status of the company
4. The hierarchy of components requires to produce a final product is represented by
  - (a) Bill of material
  - (b) Components directory
  - (c) Lead time
  - (d) Master file

5. MRP process involves the following steps
  - (a) Net requirements = (Gross requirements + Allocations) – (On hand) – Scheduled receipts
  - (b) Net requirements = (Gross requirements + Allocations) + (On hand) – Scheduled receipts
  - (c) Net requirements = (Gross requirements + Allocations) – (On hand) + Scheduled receipts
  - (d) None of these
6. MRP II
  - (a) does not replace MRP
  - (b) is not an improved version of MRP
  - (c) represents an effort to expand the scope of production resource planning and to involve other functional areas of the firm in the planning process
  - (d) all the above
7. ERP is a management tool used for planning
  - (a) manufacturing activities
  - (b) marketing activities
  - (c) financial activities
  - (d) all the business activities
8. BPR (Business Process Reengineering)
  - (a) is a tool used to redesign the business processes to achieve dramatic improvements in critical areas such as cost, quality, service and speed
  - (b) is a tool of ERP
  - (c) both is correct
  - (d) none of these
9. The demand for a given item is known as dependent if
  - (a) The item has many children
  - (b) There is a deep bill of materials
  - (c) The finished products are mostly services
  - (d) The item belongs to a clearly identifiable parent
10. The master production schedule represents the
  - (a) Financial requirements for the production
  - (b) Starting time components manufacturing
  - (c) Finishing time of components manufacturing
  - (d) Starting and finishing time of different products
11. The MPS calls for 120 units of product A. There are 20 units of product A on hand. One unit of A requires 3 units of B and 5 units of C. There are 50 units of B and 130 units of C on hand. The net requirement of B is
  - (a) 200
  - (b) 250
  - (c) 300
  - (d) 350

## Answers

1. (b)    2. (a)    3. (d)    4. (a)    5. (a)    6. (d)    7. (d)    8. (c)    9. (d)  
10. (d)   11. (b)   12. (d)   13. (b)   14. (c)   15. (d)

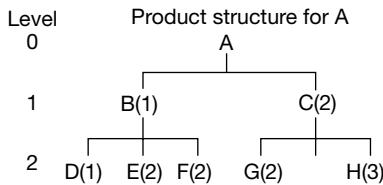
## **REVIEW QUESTIONS**

1. What do you mean by capacity resource planning? Discuss its importance in production management.
  2. What is MRP? Discuss the various inputs required for MRP. What are the outputs of MRP?
  3. How does the MRP differ from MRP-II?
  4. What is ERP? What are the advantages of ERP?
  5. Define BPR and discuss its utilization in ERP.
  6. What are the influencing factors for the success of ERP implementation?

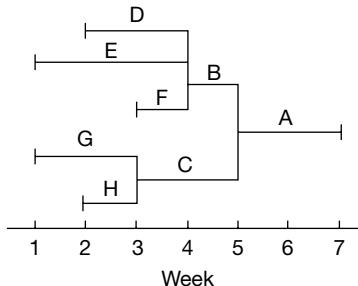
## **EXERCISES**

1. A firm manager receives an order of 200 units of a product A. The order is to be delivered at the end of 7<sup>th</sup> week from now. He had 50 units of product A in the inventory at the time of the order receipts. The product structure is shown in Figure 5.5. The lead times to manufacture the components

are shown in Figure 5.6. Using MRP, determine the schedules for the orders to be released for the manufacturing and assemblies of the components so that the 200 units of the product A can be delivered on time.



**Figure 5-5:** Product structure of A



**Figure 5-6:** Lead time for the components

2. End item X is composed of three subassemblies: A, B and C. A is assembled using 3 Ds and 4 Es; B is made of 2 Fs and 2 Gs; and C is made of 3 Hs. On-hand inventories are 20 Bs, 40 Ds and 200 Es. Scheduled receipts are 10 as at the start of week 3, 30 as at the start of week 6, and 200 Cs at the start of week 3. One hundred Xs will be shipped at the start of week 6, and another 100 at the start of week 7. Lead times are two weeks for subassemblies and one week for components D, E and F. Final assembly of X requires one week. Include an extra 10 per cent scrap allowance in each planned order of D. The minimum order size for E is 200 units. Develop each of the following:
  - (a) A product structure tree.
  - (b) An assembly time chart.
  - (c) A master schedule for X.
  - (d) A material requirements plan for A, D and E using lot-for-lot ordering.



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# CHAPTER ▶ 6

## Inventory Control

### 6.1 INTRODUCTION

Inventory is a stock of items kept on hand to meet the fluctuating demand. A certain level of inventory is maintained that will help meet the anticipated demand. If demand is not known with certainty, safety stocks are kept on hand. Additional stocks are sometimes used to meet seasonal or cyclical demand. Sometimes, large amounts of items are purchased and stored to take advantage of discounts.

In an organization, normally, four types of inventory are used as discussed below:

1. *Raw materials and parts*: These include all raw materials, components and assemblies used in the manufacture of a product.
2. *Consumables and spares*: These may include materials required for maintenance and day-to-day operations.
3. *Work-in-progress inventory*: These are items under various stages of production not yet converted as finished goods.
4. *Finished products*: These are finished goods not yet sold or put into use.

#### 6.1.1 Objectives of Inventory Control

The following are the objectives of inventory control:

1. To keep the investment in inventory to the minimum.
2. To minimize idle time by avoiding stock-outs and shortages.
3. To avoid carrying cost.
4. To improve customer satisfaction level with lesser inventory.
5. To avoid obsolescence of inventory.

#### 6.1.2 Functions of Inventory

The following are the functions of inventory:

1. To meet anticipated demand: The demand cannot be estimated with full accuracy. There is always chance of increasing or decreasing the demand due to various factors. To meet this uncertainty in demand variation or to meet the anticipated demand, inventory is required.

2. To smooth production requirements: There may be a sudden breakdown of the machine involved in production. In this situation, an inventory is required at each workstation to continue the production without any disruption.
3. To decouple operations: In case of product mix or multi-product production, we try to postpone the product differentiation towards the end of the production process, so that the demand of a customized product can be fulfilled very easily. Initially, a generic product is produced and at the end of production; these products are stored in generic form and finally customized as per demand. The point of customization or differentiation is known as decoupling point.
4. To protect against stock-outs: A safety or reserve stock is required to meet the shortages or delay in the replenishment of inventory. In case of shortages, production is disrupted or demand of the customer remains unfulfilled which leads to poor customer satisfaction level. Thus, inventory is required to protect against stock-out.
5. To help hedge against price increases: The price of a product may increase in near future. In this case, inventory is required to help hedge against price increases.
6. To take advantage of quantity discounts: Inventory provides economies of scale. If we produce or purchase the items in bulk or batch, there may be provision of quantity discount.

### 6.1.3 Understocking and Overstocking

Understocking of inventory results in cost incurred when an item is out of stock. It includes the cost of lost production during the period of stock-out and the extra cost per unit which might have to be paid for an emergency purchase. Also, sometimes the company may incur an intangible cost like opportunity cost and poor reputation in the market due to understocking.

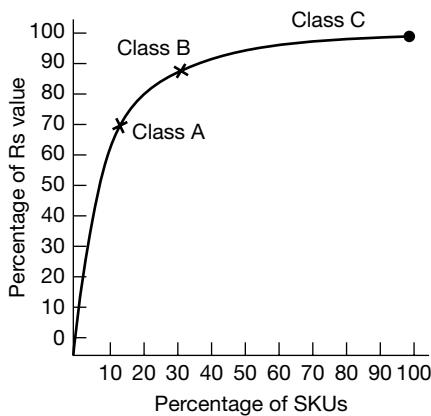
Overstocking results in the cost incurred due to obsolescence and pilferage, and the opportunity cost in the form of capital invested in inventory that could be invested for some other purposes.

## 6.2 CLASSIFICATIONS OF INVENTORY

The methods used to analyse the items in inventory are discussed in the following subsections.

### ***Always Better Control Analysis***

As is evident from the above heading, 'ABC' connotes 'Always Better Control.' The basis of analysing the annual consumption cost of the inventory items is the principle 'Vital Few – Trivial Many,' and the criterion used here is the money spent, not the quantity consumed. Figure 6.1 shows the value and corresponding units in inventory. A-class items are very important and have value up to 70 per cent, but their inventory used is up to 10 per cent. B-class items are of moderate importance and have a value up to 20 per cent and also their inventory used is up to 20 per cent. C-class items are of lower importance and have a value up to 10 per cent, but inventory up to 70 per cent.



**Figure 6-1:** ABC analysis

### **Vital, Essential and Desirable Analysis**

The inventory items, when subjected to analysis based on their criticality, can be classified into vital, essential and desirable (VED) items. This analysis is termed as VED analysis.

1. *Vital*: Items without which production or a system comes to a standstill. The non-availability of these items cannot be tolerated.
2. *Essential*: Items whose non-availability can be tolerated for 2–3 days because similar or alternative items are available.
3. *Desirable*: Items whose non-availability can be tolerated for a long period.

Although the proportion of vital, essential and desirable items varies from company to company depending on the type and quantity of workload, on an average vital items are 10 per cent, essential items are 40 per cent and desirable items are 50 per cent of the total inventory available.

### **Fast, Slow-Moving and Non-moving Analysis**

Fast, slow-moving and non-moving (FSN) analysis is based on the rate of consumption of inventory items. The items can be classified as fast-moving, slow-moving and non-moving items. An understanding of the movement of items helps in keeping proper levels of their inventory by deciding a rational policy or reordering. Here, rational policy is concerned with decisions about reorder point and order size as per consumption of the items. This method is based on the fact that some stock items have a much higher annual usage than others.

### **Scarce, Difficult, or Easy Analysis**

The classification of inventory items is based on their availability that could be scarce, difficult or easy (SDE). The importance and storage of an item are decided on the basis of difficulty or ease of its availability.

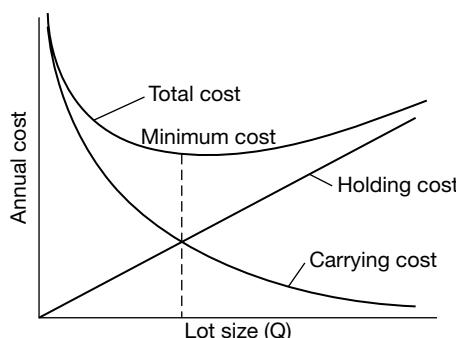
1.  $S$  refers to scarce items, especially imported ones and those which are very much in short supply.
2.  $D$  refers to difficult items which are procurable in the market, but not easily available. For example, items which have to come from far off cities or there is not much competition for them in the market or their good quality supplies are difficult to get or procure.
3.  $E$  refers to easy items. These are mostly local items which are easily available.

### 6.3 INVENTORY COSTS

The following costs are involved in keeping inventory:

1. *Inventory carrying costs*: Costs of holding items in storage. Important features of these costs are as follows:
  - (a) These costs vary with the level of inventory and sometimes with the length of time the inventory is held.
  - (b) These include facility operating costs, record-keeping, interest, etc.
  - (c) These are assigned on per unit basis per time period, or as percentage of average inventory value (usually estimated as 10 to 40 per cent).
2. *Ordering costs*: Costs of replenishing the stock of inventory. Important features of these costs are as follows:
  - (a) These costs are expressed as rupees per order, independent of the order size.
  - (b) These costs vary with the number of orders made.
  - (c) These costs include purchase orders, shipping, handling, inspection, etc.
3. *Shortage (stock-out) costs*: Costs associated with insufficient inventory. Important features of these costs are as follows:
  - (a) These costs result in permanent loss of sales and profits for items not on hand.
  - (b) Sometimes, penalties are involved; if the customer is internal, work delays could result, i.e., if the consumer is ready to bear the small delay the supplier may be imposed with penalty due to shortages to minimize the loss.

The relationship between the ordering cost, inventory carrying cost and total cost is shown in Figure 6.2.



**Figure 6-2:** Optimization of inventory costs

## 6.4 CONTINUOUS AND PERIODIC INVENTORY REVIEW SYSTEMS

The inventory review system is concerned with the volume of items in stock and its replenishment. There are two types of review systems: continuous review and periodic review. In a continuous review system, an order is placed for the same constant amount when inventory decreases to a specified level. In a periodic review system, an order is placed for a variable amount over a specified period of time.

In the continuous review system, the following process is followed.

1. The inventory level is reviewed continuously.
2. Whenever inventory decreases to a predetermined level, the *reorder point*, an order is placed for a *fixed amount* to replenish the stock. The fixed amount is termed as the *economic order quantity* (EOQ), whose magnitude is set at a level that *minimizes the total inventory carrying, ordering and shortage costs*.

*Because of continual reviewing*, the management is always aware of the status of the inventory level and critical parts, but this system is relatively expensive to maintain.

In the periodic review system, the following process is followed:

1. Inventory on hand is counted at specific time intervals and an order placed that brings inventory up to a specified level. Inventory is not monitored between counts; therefore, the system is less costly to track and keep account of inventory. The system results in less direct control of the management and thus generally higher levels of inventory to guard against stock-outs.
2. A new order quantity each time an order is placed. It is used in smaller retail stores, drug stores, grocery stores and offices.

## 6.5 ECONOMIC ORDER QUANTITY

The EOQ and formula were originally presented by Ford Whitman Harris (1913) in his paper 'Factory, The Magazine of Management'. The EOQ, or the economic lot size, is the quantity ordered when inventory decreases to the reorder point. The amount is determined using the EOQ model. The purpose of the EOQ model is to determine the optimal order size that will minimize the total inventory costs.

### ***Assumptions of the EOQ Model***

The following are the assumptions of the EOQ model:

1. The demand rate is constant and is known with certainty.
2. No constraints are placed on the size of each lot.
3. The only two relevant costs are the inventory holding cost and the fixed cost per lot for ordering or set-up.
4. Decisions for one item can be made independently of decisions for other items.
5. The lead time is constant and is known with certainty.

In this section, we will discuss the following four EOQ inventory models:

1. Basic EOQ model
2. EOQ model without instantaneous receipt
3. EOQ model with shortages
4. EOQ model with quantity discount

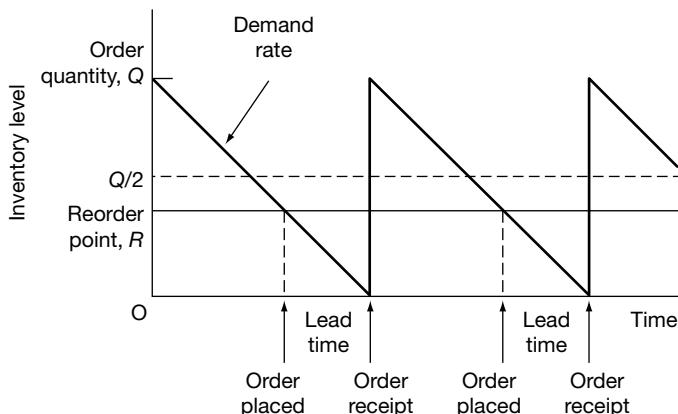
### 6.5.1 Basic EOQ Model

The following are the assumptions of the basic EOQ model:

1. Demand is known with certainty and is relatively constant over time.
2. No shortages are allowed.
3. Lead time for the receipt of orders is constant.
4. The entire order quantity is received at once and instantaneously.

In Figure 6.3,  $Q$  is maximum inventory level. At the beginning of the cycle, the inventory level is  $Q$  and at the end of the cycle it becomes zero. Therefore, the average inventory level is  $Q/2$ . The reorder point depends on the consumption rate and lead time. At reorder point, the inventory should be able to meet the need during the lead time of replenishment/supply, i.e. reorder point is equal to lead time multiplied by consumption rate. Lead time is the time interval between the point of order placement and order receipt. In the basic model of EOQ, the supply is instantaneous demand is constant throughout the year.

The inventory carrying cost is usually expressed on a per unit basis of time, traditionally one year. The inventory carrying cost is calculated by multiplying inventory cost per unit per year to average inventory size. Ordering cost per year can be calculated by multiplying the ordering cost per order to the number of orders per year. The derivation of expression for EOQ ( $Q^*$ ) can be given below as:



**Figure 6-3:** The basic EOQ model

Inventory carrying cost per unit per year =  $C_c$

$$\text{Average inventory} = \frac{Q}{2}$$

$$\text{Annual inventory carrying cost} = \frac{C_c Q}{2}$$

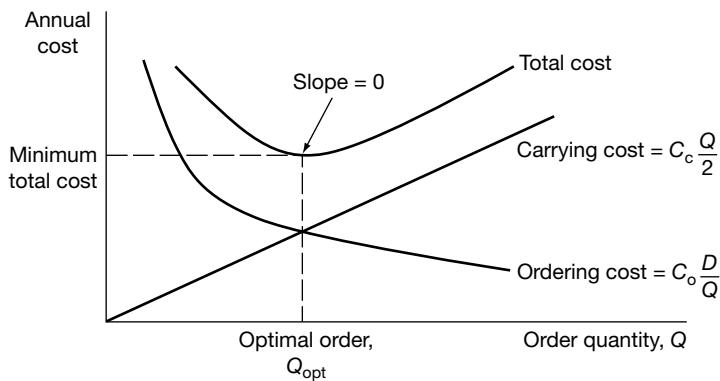
Total annual ordering cost equals cost per order ( $C_o$ ) times number of orders per year. Now number of orders per year with known and constant demand is given by  $D/Q$ , where  $D$  is the annual demand and  $Q$  is the order size. Also, we have

$$\text{Annual ordering cost} = C_o \times \frac{D}{Q}$$

$$\text{Therefore, total inventory cost, } TIC = C_o \times \frac{D}{Q} + C_c \times \frac{Q}{2}$$

Differentiating  $TIC$  w.r.t.  $Q$ , we get minimum  $TC$  as:

$$\begin{aligned}\frac{dTIC}{dQ} &= 0 \\ Q^* &= \sqrt{\frac{2C_o D}{C_c}}\end{aligned}$$



**Figure 6-4:** Optimization of total inventory costs in basic EOQ model

Inventory carrying cost increases as the order size increases since it is calculated per unit of item per unit time. But, the ordering cost decreases due to increase in order size because of less number of orders. The total variable cost is the summation of carrying costs and ordering cost as shown in Figure 6.4. At an optimum order size, the slope of total cost becomes zero and total variable costs become minimum.

**Example 6.1:** A manufacturer deals exclusively with the production of brake shoes. Currently, the company is trying to decide on an inventory and reorder policy for the brake shoes. A brake shoe costs the company Rs 75 each, and demand is about 5000 brake shoes per year distributed fairly evenly throughout the year. Reordering costs are Rs 800 per order and carrying costs are figured at 20 per cent of the cost of the item. The company is open 300 days a year (6 days a week and closed two weeks in August). The lead time is 40 working days. Find the EOQ, the reorder point, the number of orders per year and total variable costs.

**Solution:**

We have  $C_i = \text{Rs } 75$ ,  $D = 5000$  units,  $C_o = \text{Rs } 800$ ,  $C_c = 0.2 \times C_i = 0.2 \times 75 = \text{Rs } 15$ ,  $L = 40$  days. Now the EOQ is

EOQ is

$$Q^* = \sqrt{\frac{2C_o D}{C_c}} = \sqrt{\frac{2 \times 800 \times 5000}{15}} = 730.29 = 731 \text{ unit of brake shoes}$$

Now,  $L = 40$  days and daily demand  $d = 5000/300 = 16.67$ ; therefore, the reorder point is

$$r = (16.67)(40) = 666.66 \approx 667 \text{ units}$$

The company should reorder 731 brake shoes when his inventory position reaches 667. Number of reorder times per year is

$$(5000/731) = 6.83$$

or after

$$(300/6.83) = 43.92 \approx 44 \text{ working days}$$

So

$$\text{Total costs} = (\text{Carrying cost}) + (\text{Ordering cost})$$

$$\begin{aligned} TC &= [C_c(Q/2)] + [C_o(D/Q)] \\ &= [0.2(75)(Q/2)] + [800(5000/Q)] \\ &= 15Q + (40,00,000/Q) \end{aligned}$$

Therefore,

$$TC = 15(731) + (40,00,000/731) = \text{Rs } 16,436.95.$$

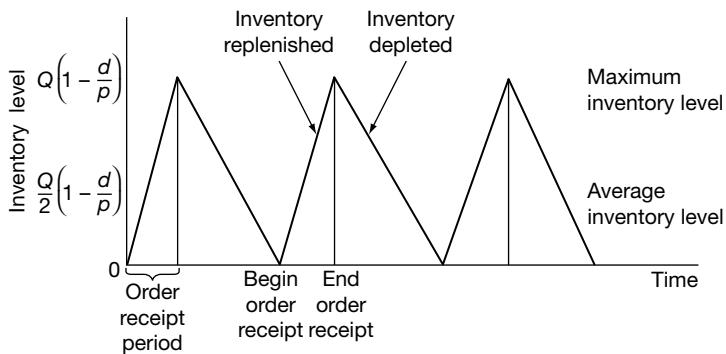
## 6.5.2 EOQ Model without Instantaneous Receipt

The second major paper was presented by Taft (1918) who developed the production lot size model. This model is an extension of the simple EOQ model incorporating the production rate. Camp (1922) was the first to present a general formula to determine the production order quantity, such that the total cost per unit for setting up plus interest on stores investment would be a minimum. The following are the assumptions of the EOQ model with instantaneous receipt:

### 6.5.3 Demand Occurs at a Constant Rate of $D$ Items Per Year

1. The production rate is  $P$  items per year (and  $P > D$ ).
2. Set-up cost: Rs  $C_o$  per run.
3. Carrying costs: Rs  $C_c$  per item in inventory per year.
4. Purchase cost per unit is constant (no quantity discount).
5. Set-up time (lead time) is constant.
6. Planned shortages are not permitted.

In this model, the inventory replenishment is not instantaneous. Generally, this type of inventory model is used in production of an item. Inventory replenishment is a continual process; similarly, the inventory depletion is also a continual process. Both the processes occur simultaneously, but the rate of inventory replenishment (i.e.  $p$ ) is greater than inventory depletion (i.e.  $d$ ) as shown in Figure 6.5. The fraction of total production stored in stock is  $(p-d)/p$  or  $(1 - d/p)$ . The maximum level of inventory stored in stock is  $Q(1 - d/p)$  and average inventory level is  $Q(1 - d/p)/2$ . If  $p$  is the daily rate at which the order is received over time and  $d$  is the daily rate at which inventory is demanded, then



**Figure 6-5:** Graphical representation of the EOQ model without instantaneous receipt

$$\text{Maximum inventory level} = Q \left(1 - \frac{d}{p}\right)$$

$$\text{Average inventory level} = \frac{Q}{2} \left(1 - \frac{d}{p}\right)$$

$$\text{Total ordering cost} = C_o \frac{D}{Q}$$

$$\text{Total carrying cost} = C_c \frac{Q}{2} \left(1 - \frac{d}{p}\right)$$

$$\text{Total Inventory cost, TIC} = C_o \frac{D}{Q} + C_c \frac{Q}{2} \left(1 - \frac{d}{p}\right)$$

Differentiating  $TIC$  w.r.t.  $Q$ , we get minimum  $TC$  as:

$$\frac{dTIC}{dQ} = 0 = -C_o \frac{D}{Q^2} + \frac{C_c}{2} \left( 1 - \frac{d}{p} \right)$$

or,

$$Q^* = \sqrt{\frac{2C_o D}{C_c \left( 1 - \frac{d}{p} \right)}}$$

Therefore,

$$\text{Optimal order size } Q^* = \sqrt{\frac{2C_o D}{C_c (1 - d/p)}}$$

**Example 6.2:** A cement manufacturing company has been using production runs of 1,00,000 packets, 10 times per year to meet the demand of 10,00,000 packets annually. The set-up cost is Rs 5000 per run and the carrying cost is estimated at 10 per cent of the manufacturing cost of Rs 100 per packet. The production capacity of the machine is 5,00,000 packets per month. The factory is open 365 days per year. Calculate the optimal production lot size; the number of production runs per year; the total annual variable cost; and the difference between existing total annual variable cost and optimal annual variable costs; the order time between production runs; the maximum inventory; and machine utilization.

### Solution:

We have  $D = 1,00,000$ ;  $C_o = \text{Rs } 5,000$ ;  $C_c = \text{Rs } 10$ ;  $d = 1,00,000$  per year,  $p = 12 \times 5,00,000 = 6,00,000$  per year

The optimal production lot size is given by

$$\begin{aligned} Q^* &= \sqrt{2DC_o / [(1 - d/p)C_c]} \\ &= \sqrt{2(1,00,000)(5,000) / [(10)(1 - 1/6)]} \\ &= 34,641 \text{ packets} \end{aligned}$$

The number of production runs per year is

$$D/Q^* = 28.86 \text{ times per year}$$

Also,

$$\text{Optimal } TC = 5000(10,00,000/34641) + 10 \times \frac{34641}{2} \times \left( 1 - \frac{10,00,000}{60,00,000} \right) = \text{Rs } 288,675.13$$

$$\text{Current } TC = 5000(10,00,000/1,00,000) + 10 \times \frac{1,00,000}{2} \times \left( 1 - \frac{10,00,000}{60,00,000} \right) = \text{Rs } 466,666.66$$

Therefore, the difference is

$$466,666.66 - 288,675.13 = \text{Rs } 177,991.53$$

There are 28.86 cycles per year. Thus, each cycle lasts

$$(365/28.86) = 12.64 \text{ days}$$

The time to produce 34,641 packets per run is

$$(34,641/6,000,000)365 = 2.10 \text{ days}$$

Thus, the machine is idle for

$$(12.64 - 2.10) = 10.54 \text{ days between each run}$$

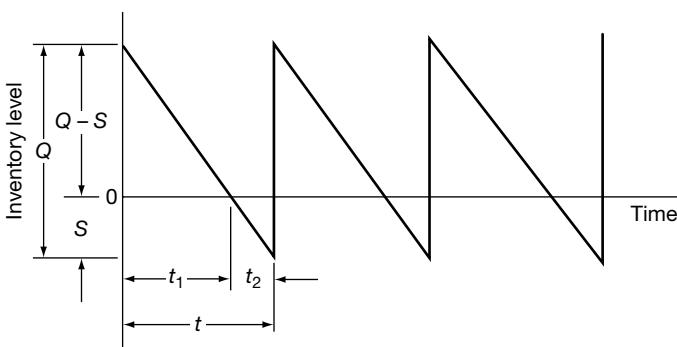
#### 6.5.4 EOQ Model with Shortages

The assumptions of the EOQ model with shortages are as follows:

1. Demand occurs at a constant rate of  $D$  items/year.
2. Ordering cost: Rs  $C_o$  per order.
3. Holding cost: Rs  $C_h$  per item in inventory per year.
4. Backorder cost: Rs  $C_b$  per item backordered per year.
5. Purchase cost per unit is constant (no quantity discount).
6. Set-up time (lead time) is constant.
7. Planned shortages are permitted (backordered demand units are withdrawn from a replenishment order when it is delivered).

In this model,  $Q$  is the maximum size of inventory;  $S$  is the inventory required or to be consumed during the shortages period  $t_2$ ;  $Q - S$  is the inventory available for the period  $t_1$ . Here,  $t_2$  indicates the shortage period. Total cycle time is  $t$  as shown in Figure 6.6. Inventory exists only for  $t_1$  period and for rest  $t_2$  period, there is a shortage of items.

$$\text{Time period in days} = \frac{\text{Total units over time period}}{\text{Demand in units per day}}$$



**Figure 6-6:** Graphical representation of the EOQ model with shortages

$$\begin{aligned}
 \text{Average inventory level} &= \frac{\{( \text{Average inventory level over } t_1 ) \times t_1 \\
 &\quad + (\text{Average inventory level over } t_2) \times t_2\}}{t} \\
 &= \frac{\frac{(Q-S)}{2} t_1 + 0 \cdot t_2}{t} \\
 &= \frac{\frac{(Q-S)}{2} \cdot \frac{(Q-S)}{D} + 0 \cdot t_2}{\frac{Q}{D}} \\
 &= \frac{(Q-S)^2}{2Q}
 \end{aligned}$$

$$\begin{aligned}
 \text{Average shortage level} &= \frac{\{( \text{Average shortage level over } t_1 ) \times t_1 \\
 &\quad + (\text{Average shortage level over } t_2) \times t_2\}}{t} \\
 &= \frac{0 \cdot t_1 + \frac{(S)}{2} \cdot t_2}{t} \\
 &= \frac{0 + \frac{S}{2} \cdot \frac{S}{D}}{\frac{Q}{D}} \\
 &= \frac{(S)^2}{2Q}
 \end{aligned}$$

$$\text{Total shortage cost} = C_s \frac{S^2}{2Q}$$

$$\text{Total carrying cost} = C_c \frac{(Q-S)^2}{2Q}$$

$$\text{Total ordering cost} = C_o \frac{D}{Q}$$

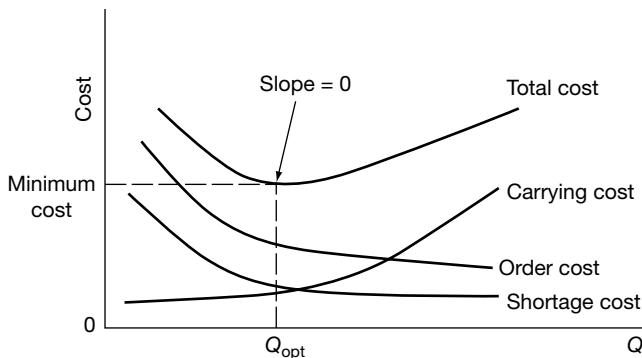
Total inventory cost = Ordering cost + Carrying cost + Shortage cost

$$= C_s \frac{S^2}{2Q} + C_c \frac{(Q-S)^2}{2Q} + C_o \frac{D}{Q}$$

Differentiating the total inventory cost w.r.t.  $Q$ , we get

$$\text{Optimal order quantity, } Q^* = \sqrt{\frac{2C_o D}{C_c} \left( \frac{C_s + C_c}{C_s} \right)}$$

$$\text{Shortage level, } S^* = Q^* \left( \frac{C_c}{C_c + C_s} \right)$$



**Figure 6-7:** Optimization of the total inventory costs in the EOQ model with shortages

In Figure 6.7, there are three variable costs: carrying cost, ordering cost and shortages cost. These costs vary with order size,  $Q$ . Ordering cost and shortages cost decrease with increase in order size, but carrying cost increases with increase in order size. Total cost is the summation of these three costs and becomes minimum when its slope equals to zero. At zero slope of total cost, the inventory size is known as optimal or EOQ.

**Example 6.3:** A car retailer has a monthly demand of 12 cars. Each car costs Rs 8,50,000. There is also a Rs 10,000 order cost (independent of the number of cars ordered). The retailer has an annual carrying cost at the rate of 5 per cent on each car. It takes two weeks to obtain cars after they are ordered. For each week, if one car is out of the market, the retailer loses Rs 4000 profit. (a) Find the EOQ and the optimal order policy. (b) How many days after receiving an order does the retailer run with inventory? (c) How long is the retailer without any inventory per cycle?

#### Solution:

We have

$$D = 12 \times 12 = 144 \text{ cars; } C_o = \text{Rs } 10,000;$$

$$C_c = 0.05(8,50,000) = \text{Rs } 42,500;$$

$$C_b = 4000 \times 52 = \text{Rs } 2,08,000$$

Now,

$$(a) Q^* = \sqrt{2DC_o/C_c} \times \sqrt{(C_c + C_b)/C_b} \\ = \sqrt{2(144)(10,000)/42,500} \times \sqrt{(42,500 + 2,08,000)/2,08,000} = 9.03$$

$$S^* = Q^*[C_c/(C_c + C_b)] \\ = 9.03[42,500/(42,500 + 2,08,000)] \\ = 1.53$$

Demand is 12 cars per month or 3 cars per week. Since lead time is 2 weeks, lead time demand is 6. Since the optimal policy is to order 9 cars and one car is out of market each week. The order should be placed when there are 5 cars remaining in inventory.

$$(b) \text{ Inventory exists for } C_b/(C_c + C_b) = 2,08,000/(42,500 + 2,08,000) \\ = 0.83 \text{ of the order cycle}$$

Note  $(Q^* - S^*)/Q^* = 0.83$  also, before  $Q^*$  and  $S^*$  are rounded. An order cycle is

$$Q^*/D = 0.0627 \text{ years} = 22.88 \text{ days}$$

Thus, the retailer has inventory  $(0.83)(22.88) = 19$  days after receiving an order.

(c) Service is out of stock for approximately  $22.88 - 19 = 3.88$  days.

### 6.5.5 EOQ Model with Quantity Discount

The EOQ model with quantity discount is applicable when a supplier offers a lower purchase cost for an item ordered in large quantities. In this model, variable costs are annual carrying, ordering and purchase costs. For the optimal order quantity, the annual carrying and ordering costs are not necessarily equal.

The following are the assumptions of the EOQ model with discount:

1. Demand occurs at a constant rate of  $D$  items/year.
2. Ordering cost is Rs  $C_o$  per order.
3. Holding or carrying cost is Rs  $C_c = \text{Rs } C_I I$  per item in inventory per year. (Note that the holding cost is based on the cost of the item,  $C_i$ )
4. The purchase cost is Rs  $C_1$  per item if the quantity ordered is between 0 and  $x_1$ , Rs  $C_2$  if the ordered quantity is between  $x_1$  and  $x_2$ , and so on.
5. Delivery time (lead time) is constant.
6. Planned shortages are not permitted.

$$\begin{aligned} \text{Total annual cost} &= \text{Carrying} + \text{Ordering} + \text{Purchase} \\ &= [(1/2)Q^* C_c] + [DC_o/Q^*] + DC \end{aligned}$$

**Example 6.4:** An electronic shop carries film for the camera. The film costs the shop Rs 80 per unit and is sold for Rs 85. The shop has the film shelf life of 18 months. The shop sales are 21 films per week, and its annual inventory carrying cost rate is 20 per cent. It costs the shop Rs 20 to place an order. The film manufacturing company offers a 7 per cent discount on orders of 400 films or more, a 10 per cent discount for 900 films or more and a 15 per cent discount for 2000 films or more. Determine the shop's optimal quantity.

**Solution:**

We have  $D = 21(52) = 1092$ ;  $C_c = 0.20 (C_i)$ ;  $C_o = 20$ .

- (a) For  $C_4 = 0.85(80) = \text{Rs } 68$

To receive a 15 per cent discount, the shop must order at least 2000 films. Unfortunately, the batteries' shelf life is 18 months. The demand in 18 months is

$$78 \times 21 = 1638 \text{ films}$$

If he orders 2000 films, he would have to scrap 372 of them. This would cost more than the 15 per cent discount would save.

- (b) For  $C_3 = 0.90(80) = \text{Rs } 72$

$$\begin{aligned} Q_3^* &= \sqrt{2DC_o/C_c} \\ &= \sqrt{2(1092)(20)/[0.20(72)]} \\ &= 55 \text{ films (not feasible)} \end{aligned}$$

The most economical, feasible quantity for  $C_3$  is 900 films. Since the carrying cost is a function of material cost and equals to Rs 72 only when order size is 900 films.

- (c) For  $C_2 = 0.93(80) = \text{Rs } 74.4$

$$\begin{aligned} Q_2^* &= \sqrt{2DC_o/C_c} \\ &= \sqrt{2(1092)(20)/[0.20(74.4)]} \\ &= 54.18 \text{ films (not feasible)} \end{aligned}$$

The most economical, feasible quantity for  $C_2$  is 400 films. Since the carrying cost is a function of material cost and equals to Rs 74.4 only when order size is 400 films.

- (d) For  $C_1 = 1.00(80) = \text{Rs } 80$

$$\begin{aligned} Q_1^* &= \sqrt{2DC_o/C_c} \\ &= \sqrt{2(1092)(20)/0.2(80)} \\ &= 52.24 \text{ films (feasible)} \end{aligned}$$

We know that

$$TC_i = (1/2)(Q_i^* C_c) + (DC_o/Q_i^*) + DC_i$$

Therefore, we have

$$TC_3 = (1/2)(900)(72) + ((1092)(20)/(900)) + (1092)(14.4) = \text{Rs } 48,149.06$$

$$TC_2 = (1/2)(400)(74.4) + ((1092)(20)/(400)) + (1092)(14.88) = \text{Rs } 31,183.56$$

$$TC_1 = (1/2)(53)(80) + ((1092)(20)/53) + (1092)(16) = \text{Rs } 20,004.07$$

Comparing the total costs for 53, 400 and 900 films, the lowest total annual cost is Rs 20,004.07. The shop should order 53 films at a time.

## 6.6 REORDER POINT

The reorder point is the inventory level at which a new order is placed. The order must be placed while there is enough stock in place to cover demand during lead time

$$R = dL$$

where  $d$  is the demand rate per time period and  $L$  is the lead time. The inventory level might be depleted at a slower or faster rate during lead time. When demand is uncertain, safety stock is added as a hedge against stock-out.

The service level is the probability that the amount of inventory on hand is sufficient to meet demand during lead time (i.e., the probability of a stock-out will not occur). The higher the probability of existing inventory results in more likelihood of meeting the customer demand. A service level of 95 per cent means there is a 0.95 probability that demand will be met and 0.5 is the probability of a stock-out.

### 6.6.1 Reorder Point for Variable Demand

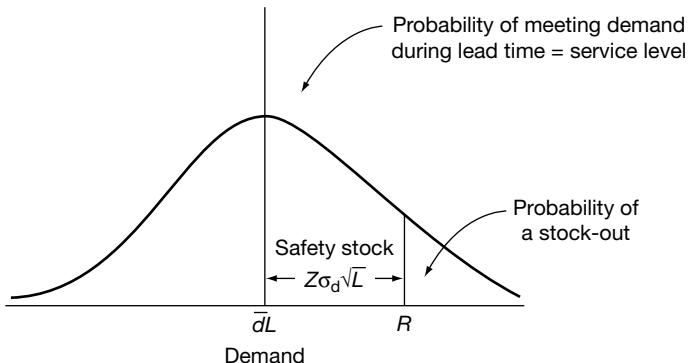
There are three situations for determining the reorder point: reorder point with variable demand, reorder point for variable lead time, and reorder point for both variable demand and lead time.

If the daily demand is a variable and lead time of supply is a constant, the following expression is used to find the reorder point. The first term of the reorder point, i.e.  $\bar{d}L$  represents the average inventory required for the lead-time period and the second term, i.e.  $Z\sigma_d\sqrt{L}$  represents the safety stock.

$$R = \bar{d}L + Z\sigma_d\sqrt{L}$$

where  $R$  is the reorder point,  $\bar{d}$  is the average daily demand,  $L$  is the lead time,  $\sigma_d$  is the standard deviation of daily demand,  $Z$  is the number of standard deviations corresponding to service level probability, and  $Z\sigma_d\sqrt{L}$  is the safety stock.

In Figure 6.8, probability of meeting demand and probability of a stock-out are shown on the standard normal distribution curve. The area lies in the left side of reorder point,  $R$  shows the probability of meeting the demand and right side of the reorder point,  $R$  shows the probability of a stock-out. The area between the mean value, i.e.  $dL$  and  $R$  shows the probability of meeting the demand from the safety stock.



**Figure 6-8:** Probability distribution for stock-out

**Example 6.5:** A furniture house wants a reorder point with a 95 per cent service level and a 5 per cent stock-out probability. The average demand per day is 75 chairs with a lead time of 5 days. The standard deviation is 6 chairs per day. Find the reorder point.

**Solution:**

For a 95 per cent service level,  $Z = 1.65$ . So, the reorder point is

$$\begin{aligned} R &= \bar{d} \times L + Z\sigma_d\sqrt{L} \\ &= 75(5) + (1.65)(6)\sqrt{5} \\ &= 397.13 \approx 398 \text{ chairs} \end{aligned}$$

## 6.6.2 Reorder Point for Variable Lead Time

When lead time is variable and daily demand is constant, following expression is used to determine the reorder point:

$$R = d\bar{L} + Zd\sigma_L$$

where  $d$  is the constant daily demand,  $\bar{L}$  is the average lead time,  $\sigma_L$  is the standard deviation of lead time,  $d\sigma_L$  is the standard deviation of demand during lead time, and  $Zd\sigma_L$  is the safety stock.

**Example 6.6:** A cloth merchant has a fixed demand of 45 jackets per day. The average lead time is 8 days and the deviation in lead time is 2 days. Calculate the reorder point with 95 per cent service level.

**Solution:**

We have demand rate,  $d = 45$  jackets per day, average lead time,  $\bar{L} = 8$  days, deviation in lead time,  $\sigma = 2$  days,  $Z$  corresponding to 95 per cent service level is 1.65 then,

$$\begin{aligned} R &= d\bar{L} + Zd\sigma \\ &= 45 \times 8 + 1.65 \times 45 \times 2 \\ &= 508.5 \\ &= 509 \text{ jackets} \end{aligned}$$

### 6.6.3 Reorder Point for Variable Demand and Variable Lead Time

When daily demand and lead time both are variables, the reorder point can be determined by the following expression:

$$R = \bar{d}\bar{L} + Z\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$$

where  $\bar{d}$  is the average demand and  $\bar{L}$  is the variable lead time,  $\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$  is the standard deviation during the lead time  $Z\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2}$  is the safety stock.

**Example 6.7:** A cloth merchant has an average demand of 45 jackets per day with a standard deviation of 6 jackets per day. The average lead time is 8 days and the deviation in lead time is 2 days. Calculate the reorder point with 95 per cent service level.

**Solution:**

$$\begin{aligned} R &= \bar{d}\bar{L} + Z\sqrt{(\sigma_d)^2\bar{L} + (\sigma_L)^2\bar{d}^2} \\ &= (45)(8) + (1.65)\sqrt{(6)^2(8) + (2)^2(45)^2} \\ &= 511.11 \approx 512 \text{ jackets} \end{aligned}$$

### 6.7 ORDER QUANTITY FOR VARIABLE DEMAND

In this chapter, we have studied to determine the order size, i.e. EOQ only when demand is constant. Here, we will discuss the determination order size when demand is variable. Due to variability in demand, we use safety stock in determination of order size as shown in following expression:

$$Q = \bar{d}(t_b + L) + Z\sigma_d\sqrt{t_b + L} - I$$

where  $\bar{d}$  is the average demand rate,  $t_b$  is the fixed time between orders,  $L$  is the lead time,  $\sigma_d$  is the standard deviation of demand,  $Z\sigma_d\sqrt{t_b + L}$  is the safety stock, and  $I$  is the inventory in stock.

**Example 6.8:**

A small shop has an average demand of 45 jackets per day and the standard deviation in average demand is 6 jackets. The fixed time between orders is 16 days and the lead time is 8 days. Inventory in stock is 40 jackets. For 95 per cent of service level what should be the order quantity?

**Solution:**

$$\begin{aligned}Q &= \bar{d}(t_b + L) + Z\sigma_d \sqrt{t_b + L} - I \\&= 45(16 + 8) + 1.65 \times 6 \times \sqrt{16 + 8} - 40 \\&= 1088.49 \text{ jackets}\end{aligned}$$

**SUMMARY**

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In this chapter, we have discussed about the importance of inventory and classification of items in inventory based on their value, availability and consumption rate. In addition to these discussions, various inventory models have been explained with numerical illustrations. Also, reorder point has been explained with variable demand, variable lead time and variable demand and lead time both. Finally, the order size for variable demand has been illustrated in this text.

**MULTIPLE-CHOICE QUESTIONS**

1. Which of the following DOES NOT belong to the assumption of economic batch quantity (EBQ)?
  - (a) Only two or more items are involved
  - (b) Annual demand is known
  - (c) Usage rate is constant
  - (d) Usage occurs continually
2. Inventory carrying costs are influenced by
  - (a) only ordering
  - (b) only holding carrying cost per unit.
  - (c) both ordering and holding
  - (d) only production cost
3. Which of the following statements concerning the basic EOQ model is true?
  - (a) a decrease in demand will increase the EOQ value
  - (b) the annual holding cost is less than the annual ordering cost for a smaller-order quantity compared to EOQ
  - (c) an increase in holding cost will increase the EOQ value
  - (d) as annual ordering costs increase, so do annual carrying costs
4. ABC analysis deals with
  - (a) ordering cost
  - (b) flow of materials
  - (c) ordering schedule of the materials
  - (d) the cost involved with the materials

5. Which of the following are differences between periodic inventory and continuous inventory systems?
  - (a) Continuous inventory systems are time-phased while periodic inventory systems are not
  - (b) Periodic inventory systems have regular order times while continuous inventory systems have irregular order times
  - (c) Periodic inventory systems have regular order quantities while continuous inventory systems have irregular order quantities
  - (d) none of the above
6. A fixed time between orders and a variable order quantity are characteristics of a
  - (a) continuous inventory system
  - (b) two-bin system
  - (c) periodic inventory system
  - (d) MRP system
7. The function of inventory is
  - (a) to meet the fluctuating demand of the market
  - (b) to make smooth production systems
  - (c) to help hedge against price increase
  - (d) all of the above
8. VED analysis of inventory deals with
  - (a) utility of the materials
  - (b) cost of the materials
  - (c) availability of the material
  - (d) consumption of the material
9. FSN analysis of inventory deals with
  - (a) utility of the materials
  - (b) cost of the materials
  - (c) availability of the material
  - (d) consumption of the material
10. SDE analysis of inventory deals with
  - (a) utility of the materials
  - (b) cost of the materials
  - (c) availability of the material
  - (d) consumption of the material
11. Which of the following is NOT the part of assumptions of basic EOQ model?
  - (a) Demand is known with certainty and is relatively constant over time.
  - (b) Shortages are allowed.
  - (c) Lead time for the receipt of orders is constant.
  - (d) The entire order quantity is received at once and instantaneously.
12. Which of the following EOQ model assumes that material cost is the variable cost?
  - (a) Basic EOQ model with instantaneous receipt.
  - (b) EOQ model without instantaneous receipt.
  - (c) EOQ model with shortages.
  - (d) EOQ model with quantity discount.
13. To determine the reorder point, which of the following information is required?
  - (a) Demand rate
  - (b) Lead time
  - (c) Safety stock
  - (d) All the above

## Answers

1. (a)      2. (c)      3. (b)      4. (d)      5. (b)      6. (c)      7. (d)      8. (a)      9. (d)  
10. (c)     11. (b)     12. (d)     13. (d)     14. (b)     15. (b)

# **REVIEW QUESTIONS**

1. What are the various types of inventory maintained in a manufacturing organization?
  2. What are the objectives of inventory?
  3. Discuss the functions of inventory.
  4. Write short notes on the following
    - (a) ABC analysis
    - (b) VED analysis
    - (c) FSN analysis
    - (d) SDE analysis
  5. Derive an expression for economic order quantity subject to instantaneous supply, no shortage, and without any discount.
  6. Discuss the reorder point for variable demand and variable lead time.

## **EXERCISES**

1. A company ABC is planning to stock a product A. The company has developed the following information:

Annual usage = 6000 units

Cost of the product = Rs 350 /unit

Ordering cost = Rs 50 per order

Carrying cost = 25 per cent of the materials cost per unit per year.

(a) Determine the optimal number of units per order

(b) Find the optimal number of orders/year

(c) Find the annual total inventory cost

2. A furniture house decides to use the EOQ model to determine the order size of furniture from a carpenter. The inventory manager observes that the annual demand of the furniture is 500 units for last two years and expected to be same in the coming years. He has estimated that preparation of an order and other variable costs associated with each order are about Rs 250, and it costs him about 20 per cent per year to hold items in stock. His cost for the furniture is Rs 1000.
  - (a) How many furniture should be ordered each time?
  - (b) How many orders would there be?
  - (c) Determine the approximate length of cycle time.
  - (d) Calculate the minimum total inventory cost.
3. An auto component manufacturer supplies headlight assembly to car dealers. The annual demand is approximately 6000 headlight assemblies. The supplier pays Rs 2000 for each headlight assembly and estimates that the annual holding cost is 15 per cent of the headlight assemblies' value. It costs approximately Rs 500 to place an order. The supplier currently orders 500 headlights per month.
  - (a) Determine the total inventory costs for the current order quantity.
  - (b) Determine the economic order quantity (EOQ).
  - (c) How many orders will be placed per year using the EOQ?
  - (d) Determine the total inventory costs for the EOQ.
4. In Exercise 3, upon closer inspection, the supplier determines that the demand for headlight assembly is normally distributed with mean 4 headlight assemblies per day and standard deviation 3 headlight assemblies per day. (The supplier plant is open 300 days per year.) It usually takes about 4 days to receive an order from the factory.
  - (a) What is the standard deviation of usage during the lead time?
  - (b) Determine the reorder point needed to achieve a service level of 95 per cent.
  - (c) What is the safety stock? What is the holding cost associated with this safety stock?
  - (d) How would your analysis change if the service level changed to 98 per cent?



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# CHAPTER ▶ 7

## Product Design and Development

### 7.1 INTRODUCTION TO ENGINEERING DESIGN

Design may be defined as the iterative decision-making activity to create the plans by which resources are converted, preferably optimally, into systems, processes or devices to perform the desired functions and to meet human needs.

Simply design can be defined as to formulate a plan to produce a product for the satisfaction of human needs. The concept of design may be applied in a number of fields such as:

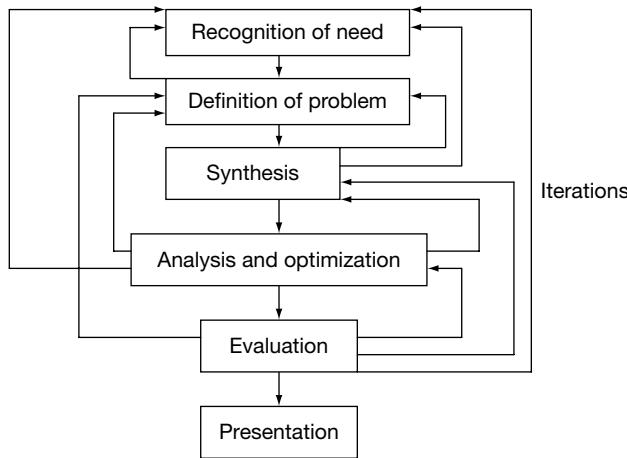
1. Clothing design
2. Building design
3. Highway design
4. Computer-aided design
5. Ship design
6. Machine design
7. Engineering design
8. Process design.

#### 7.1.1 Engineering Design Process

The sequence of operation of the design process is shown in Figure 7.1. The process begins with the recognition of a need and a decision to do something about it. After many iterations, the process ends with the presentation of the plans for the satisfying the need (Kumar and Ramaswami 2006).

*Recognition and identification of need:* Recognition of the need is a creative and sense-related phase of design. This is the feeling of requirement, sense, uneasiness and test of people.

*Defining the problem:* It must include all the specifications for the thing that is to be designed. The specifications are the input and output quantities, the characteristics, and dimensions of these quantities. The specification includes the cost, the volume of production, the expected life, the operating conditions, maximum range, expected variations in the variables and dimensions, and weight limitations.



**Figure 7-1:** Engineering design process

*Synthesis:* The design synthesis is defined as the process of creating or selecting configuration, materials, shapes and dimensions for a product. After defining the problem, synthesis is the next steps in the design process. Synthesis always takes place with both analysis and optimization, because the system under design must be analysed to determine whether the performance complies with the specifications. The analysis may reveal that the system is not an optimal one. If the design fails either or both of these tests, the synthesis procedure must begin again.

*Analysis and optimization:* Design is an iterative process in which we proceed through several steps, evaluate and results, and then return to an earlier phase of the procedure repeatedly. Thus, we may synthesize several components of a system, analyse and optimize them, and return to synthesis to see that what effect this has on the remaining parts of the system, which will admit some form of mathematical analysis.

*Evaluation:* Evaluation is the final proof of a successful design and usually involves the testing of a prototype in the laboratory. Here, design is checked to know that the design really satisfies the needs. Is it reliable? Will it compete successfully with other existing products? Is it economical to manufacture and to use? Is it easily maintained? Is it profitable for sale?

*Presentation/Communication:* Presentation is a way to communicate the design to others. It is a selling job. There are three methods of communication: written, oral and graphic forms. These methods are used by an engineer to explain the design to the designated persons.

### 7.1.2 Feasible Design and Optimal Design

The feasibility of the design means to check the possibility to manufacture or conversion of design into physical form, to check the ability of customers to pay for the design, and finally the effect on the environment due to manufacturing and use of the designed product. The economic

worth of the design is very important which mean the society should be willing to pay equal and more than the cost. Financial feasibility looks upon a product as a project and verifies that the net present value of all cash flows during product life cycle exceed the initial investment. An optimal design is one which is best out of a number of feasible designs. There are following design optimization methods:

*Optimization by evolution:* Most of the design is optimized at the time of technological evolution. The survival of the design and the resulting product is based on the natural selection of user acceptance.

*Optimization by intuition:* Intuition means to do without knowing why one does it. The gift of seems to be closely related to unconscious mind. The history of technology is full of examples of engineers who used intuition to make major advances. Although the knowledge and the tools available today are so much more powerful, there is no question that intuition continues to play an important role in technological development.

*Optimization by trial-and-error modelling:* This refers to the usual situation in modern engineering design where it is recognized that the first feasible design is not necessarily the best. Therefore, the design model is exercised for a few iterations in the hope of finding an improved design. However, this mode of operation is not true optimization. Such a design should not be called an optimal design.

*Optimization by mathematical modelling:* This is the area of current active development in which mathematically based strategies are used to search for an optimum design.

### 7.1.3 Type of Design at Evolution Stage

The different types of designs at evolution stage are discussed below as:

*Adaptive design:* It requires only minor modification usually in size alone. The level of creativity needed is negligible because the adaptation of existing design is essential. Several authors like Martin S. Ray called it an adaptive design even if the changes are more, but the original design concept is the same.

*Variant design:* This design approach is followed by companies who wish to serve product variety to satisfy varying customer's tastes and which involves varying the size of arrangement of certain aspects of the chosen system, the functions and the solution principle remaining unchanged.

*Creative design:* The most demanding design effort and creativity is needed when a totally new product is to be designed. This type of the design is known as creative design. Creative design means a design produced or created by brainstorming.

*Conceptual design:* Conceptual design is that part of the design process in which, by the identification of the essential problems through abstraction, by the establishment of functional structures and by the search for appropriate solution principles and their combination, the basic solution path is laid down through the elaboration of a solution concept.

### 7.1.4 Product Specification

The term ‘product design’ is used in its narrow sense to refer to the detailed design phase, which constitutes the specification of design parameters, the determination of precedence relations in the assembly, and the detail design of the components (including material and process selection). These decisions generally result in geometric models of assemblies and components, a bill of materials and control systems for production (Krishnan and Ulrich 2001).

Product specification means the technical description of the product, which includes all the engineering characteristics required to perform the functions of the product satisfactorily. Product specification may be in terms of units of weight, mass, dimensions, energy, current, voltage, frequency, etc.

The final product specification is the last stage of the design process. After a number of iterations of analysis, optimization and synthesis, the accurate product specifications are decided and then designs are presented or communicated. There may be two phases of product specification:

- Product specification after recognition of needs, i.e., target specification.
- Product specification after design evaluation. This is the final product specification.

The methods used to establish the target specifications: There are many methods, which are used to convert the customer needs into engineering design by producing the target specification for the product/services. Quality function deployment is one of the best methods to transform the customer requirement into engineering design. For the detail study of Quality Function Deployment see Chapter 22.

## 7.2 PRODUCT CONCEPT AND CONCEPT SELECTION

### 7.2.1 Product Concept

Production concept is a description of methodology to make a product and its working principle. This is a phase that realizes the needs after identification and analysis of customer needs. We can say how the identified need of customer can be satisfied. Product concept consists of sketch or drawing of product with technical description. In the process of concept generation, a complex problem is broken into smaller/simpler problems and solutions for these smaller problems are searched internally and externally. The following steps should be followed for concept generation processes:

*Understanding of problem:* A concept generation team should have a better understanding of the problem. Then, the problem is broken into smaller subproblems. After breaking of the problem, critical subproblems are found and focused first.

*External search:* In this step, the existing solutions for the problem and subproblems are found from external sources with the help of experts, patents, literature and benchmarking.

*Internal search:* In this step, the knowledge of individual team members is used to solve the problem and subproblems.

*Systematic exploration:* All the information gathered from external or internal sources is explored systematically. Since, there are fragments of solutions or information that must be organized and analysed in a systematic way.

Feedback of the team about the results: In this step, the feedback about the confidence, satisfaction and concept of team member that they have used to find the solution, is monitored and recorded. There may be errors during the concept generation process since it is a very general phenomenon, which must be considered. For example, consideration of only one or two solutions alternatives, often proposed by the most members, involvement of only one or two persons seriously in the process, resulting in lack of confidence and commitment in the team, failure to consider entire categories of the solutions of the solutions, and ineffective and insufficient integration of promising partial solutions.

## 7.2.2 Concept Selection

Concept selection is a process of shortlisting the concept by comparing the many concepts on the basis of the customer's need, relative strength and weakness. There are two phases of concept selection:

- Screening of concepts.
- Scoring or rating of concepts.

The basic rules for concept screening and scoring are as follows:

1. All the solutions should be related to same product design specifications.
2. All the concepts must be evaluated for the same criteria.
3. The concepts should be compared with a predetermined reference or datum using '+', '-' and 'S' symbol for more, less and same, respectively, as the reference.
4. Scores achieved by individual concepts should be examined and verified for need satisfactions.
5. If no one concept emerges strongly, then it means the criterion is vague or ambiguous.
6. If same concept emerges continually, change the reference and check the emergence again.
7. Finalize the best concept.

## 7.2.3 Types of Product Development

A product is something that can be marketed to customers because it provides satisfaction to their needs. Products can be goods or services or a combination of both.

*New-to-the-company product:* It is a good or a service that is new to the company, but has been sold by a competitor in the past.

*Improvement in an existing product:* It is an enhancement of a product already on the market.

*Extension to an existing product line:* It is a new product developed as a variation of an already existing product.

## 7.2.4 Brainstorming

Brainstorming is an idea generation process to solve a problem in an optimum way. In this method, the team members assemble and, with the help of a facilitator, quickly generate several ideas. In order to encourage the flow of ideas, the facilitator specifically discourages the evaluation of the ideas until all ideas have been recorded. Proponents of brainstorming are enthusiastic about the number of ideas generated. The concept has great intuitive appeal if several people get together, exchange ideas freely, and build on them, there should be a significant synergy.

Recent studies have shown that the process may not be effective as claimed. Several groups were studied as brainstorming groups and as groups of individuals and were asked to generate ideas on their own. Invariably, the groups of individuals generated more useful and better ideas. In addition, although brainstorming generates ideas, it is not always effective in providing coherent structure to them. Two enhancements of traditional brainstorming try to overcome these drawbacks:

- The affinity diagram or structured brainstorming.
- Electronic brainstorming.

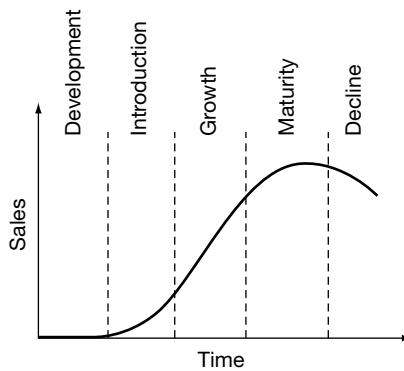
The affinity diagram is one of the seven new tools for quality improvement programme in which ideas are grouped on the basis of affinity storming all the idea generated are evaluated by each other by the team member on-line with the help of computer networked.

*Brainstorming in a design process:* In a design of a product or process, the following steps for brainstorming are used:

1. Select a group of people (around 5 to 6) to produce ideas.
2. State the problem clearly.
3. Give some fixed time (approximately 10 min) to each member to write down on a piece of paper as many solution ideas as possible.
4. Each member reads one idea in turn, and those listening speak out any encouraged to combine ideas. Every idea which is mentioned is noted down.
5. Rule out judgement as to whether an idea is practical or not. If some shortcomings are apparent in an idea, others may suggest improvements, but may not point out the shortcomings.
6. Encourage humour to relax the members. A relaxed mind is the best for brainstorming. It also helps to build up mutual trust which is essential to the success of the session.
7. At the end try to find a logical classification of all the ideas produced.

## 7.3 PRODUCT LIFE CYCLE

A new product passes through a sequence of phases from introduction to growth, maturity and decline. This sequence is known as the product life cycle and is associated with changes in the market situation, thus affecting the marketing strategy and the marketing mix. The product sales and time can be plotted as a function of the life-cycle phase as shown in Figure 7.2. The product life cycle depicts sales across time and contains the familiar introductory, growth, maturity and



**Figure 7-2:** Product life cycle

decline stages. Likewise, diffusion of innovation models also consists of the same four stages, although not all four stages are modelled in all empirical studies (Mahajan and Muller 1979; Spears and Germain 1995).

### **Introduction Phase**

In the introduction phase, the firm seeks to build product awareness and develop a market for the product. The impact on the marketing mix is as follows:

1. Product branding and quality level are established and intellectual property protection such as patents and trademarks is obtained.
2. Pricing may be low penetration pricing to build market share rapidly, or high skim pricing to recover development costs.
3. Distribution is selective until consumers show acceptance of the product.
4. Promotion is aimed at innovators and early adopters. Marketing communications seeks to build product awareness and to educate potential consumers about the product.

### **Growth Phase**

In the growth phase, the firm seeks to build brand preference and increase market share. The following strategies are followed in this phase:

1. Product quality is maintained and additional features and support services may be added.
2. Pricing is maintained as the firm enjoys increasing demand with little competition.
3. Distribution channels are added as demand increases and customers accept the product.
4. Promotion is aimed at a broader audience.

### **Maturity Phase**

At maturity, the strong growth in sales diminishes. The competition may appear with similar products. The primary objective at this point is to defend market share while maximizing profit.

The following strategies are followed in this phase:

1. Product features may be enhanced to differentiate the product from that of competitors.
2. Pricing may be lower because of the new competition.
3. The distribution becomes more intensive and incentives may be offered to encourage preference over competing products.
4. Promotion emphasizes product differentiation.

### **Decline Phase**

As sales decline, the firm has several options:

1. Maintain the product, possibly rejuvenating it by adding new features and finding new uses.
2. Harvest the product—reduce costs and continue to offer it, possibly to a loyal niche segment.
3. Discontinue the product, liquidating remaining inventory or selling it to another firm that is willing to continue the product.

## **7.4 MORPHOLOGY OF DESIGN**

Morphology of design is the study of a product throughout its entire life cycle. Figure 7.3(a) shows the nature of sales and profit during pre-market and market phases of product life cycle. The various sub-phases of product life cycle as shown in Figure 7.3(b) are discussed as follows:

*Need analysis:* The first step of product design is need analysis, i.e. to recognize and analyze the needs of customers and to determine the aim or purpose of product design.

*Feasibility:* The second step is to check the feasibility of the solutions of the problem. There may be four types of feasibility checks: social, economic, market and environment.

*Preliminary design:* After the feasibility study, preliminary design of the product is prepared which is to be synthesized and analysed properly.

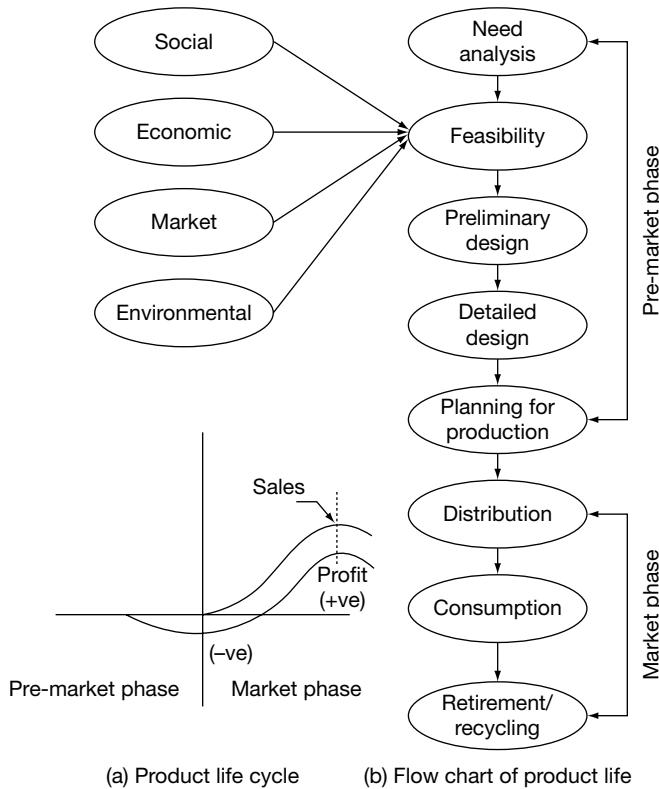
*Detailed design:* After analysis of the preliminary design, a complete or detailed design is to be prepared. This is the final design of the product.

*Planning for production:* After finalizing the detailed design, a complete schedule of production is made. The purchase order of raw material is placed and production is started.

*Distribution:* The product is distributed to the distributors and retailers in this phase.

*Consumption:* This is a very important phase. In this phase, real testing of design is done by the feedback from the customers.

*Retirement/Recycling:* At the end of the life of the product, it is retired and disassembled for recycling if it is possible.

**Figure 7-3:** Morphology of design

## 7.5 STANDARDIZATION, SIMPLIFICATION, DIFFERENTIATION AND DIVERSIFICATION

### 7.5.1 Standardization

Standardization is the process of establishing a technical standard, which could be a standard specification, standard test method, standard definition, standard procedure, etc. Product standardization is a technique in engineering design that aims to reduce the number of different parts within a product.

Benefits of product standardization are lower supply chain costs, less variety of suppliers, less supplier in numbers, less stock-keeping units (SKU), more economies of scale, less variety of production operations, faster product design, re-utilization of standardized parts across a product family, and re-utilization of standardized parts across a product generations.

As companies move towards standardized practices to gain efficiencies by producing fewer errors, less rework, and higher quality products in less time. The difficulty is creating the right

balance between standardization, flexibility and time. Too much standardization can reduce flexibility in the design environment and take too much time. Too little standardization allows errors and inefficiencies back into the design process.

### **7.5.2 Simplification**

It is the process of reducing variety of a product by limiting product range, design and/or type of material. Simplification offers boost to standardization. The marginal difference in size or specification does not add much in attributes, which may be termed as variety. Therefore, simplification is needed in product development. Simplification provides better customer service due to limited variety, better after-sales planning, and reduced volume. It is also helpful in reducing inventory level and complex material planning. It is helpful in focusing effort on limited parts and therefore lesser cost may be anticipated. It is helpful in better product quality due to concerted efforts on a limited product range. The combination of simplification and standardization leads to specialization. Limited but focused product variety is helpful for a company to specialize in a particular area.

### **7.5.3 Differentiation**

Differentiation is the process of distinguishing a product from other manufacturers already existing in the market. The aim of differentiation is to make it more appealing to the target market. It is the way to show that the product is different from another in terms of value. The areas of differentiation may include quality, price, functions, design, characteristics, advertising and availability.

### **7.5.4 Diversification**

Product diversification is the modification of a product or service to reach a more expansive target market. Unlike product differentiation, this is not about highlighting anything, rather it is about finding a new section of the market to attract. The downside to product diversification is that any failures will also associate themselves with the original brand. Alternatively, the brand could be so successful, it could suppress the other brands and make it unattractive.

Differentiation is less risky compared to diversification because it is an amendment of a pre-existing and an already established product or service; therefore, there is the guarantee that it is going to have interest among target customers. With diversification there is the risk of too little interest or too much interest, and with the too little interest possibility that could mean a loss of capital.

## **7.6 INTERCHANGEABILITY AND MODULAR DESIGN**

### **7.6.1 Interchangeability**

Interchangeability is the ability of an item to be used in place of another without any alteration or changes. Interchangeable items are so similar in function and physical characteristics that they are considered equivalent in performance and durability. Each is capable of replacing the others without causing a need for alteration or adjustment to fulfil the same requirement.

Interchangeability refers to a condition in which existing two or more items with characteristics, making them equivalent in performance and durability, making them fully exchangeable. In mechanical engineering, interchangeability is the ability to select components for assembly at random and fit them together within proper tolerances.

## 7.6.2 Modular Design

For designing a system synthetically, the system could be designed by two broad ways. The first way is to design the complete system using the known theories, and use the system, as it is designed, in the real conditions. The other way is to design the different components of the system separately, and test each component in separate conditions. Modular design is an approach that subdivides a system into smaller parts, i.e. modules that can be independently designed and then used in different systems to drive multiple functionalities.

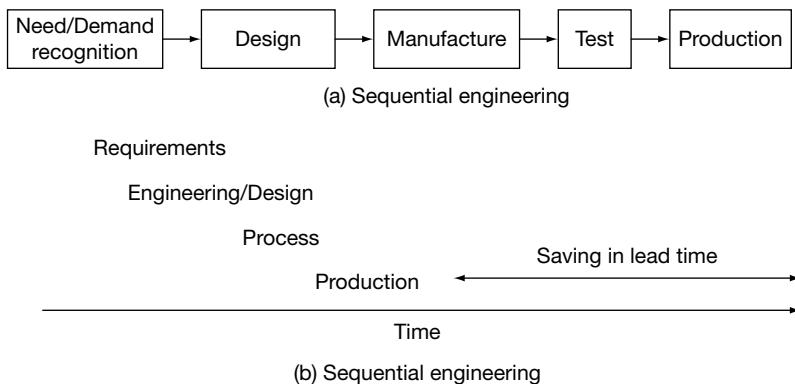
Modularity is a method of organizing complex products and process efficiently (Baldwin and Clark 1997) by decomposing complex tasks into simpler modules so they can be managed independently (Mikkola 2001). Modularity in design has been researched to reduce design process complexity (Fujita 2002; Ulrich and Eppinger 1995). Modularity in design can be, therefore, defined as choosing the design boundaries of a product and of its components, i.e. on how to divide a system into modules, so that the design features and tasks are interdependent within and independent across modules (Huang and Kusiak 1998). Ulrich (1995) analysed the structures of design, in terms of product structure, physical functions, etc. and distinguished them into modular architecture and architecture integral. A modular system can be characterized by the following:

1. Functional partitioning into discrete scalable, reusable modules consisting of isolated, self-contained functional elements.
2. Rigorous use of well-defined modular interfaces, including object-oriented descriptions of module functionality.
3. Ease of change to achieve technology transparency and, to the extent possible, make use of industry standards for key interfaces.

In addition to cost reduction and flexibility in design, modularity offers other benefits such as augmentation (adding new solution by merely plugging in a new module), and exclusion. Examples of modular systems are cars, computers and water purifiers (RO systems). Modular design is an attempt to combine the advantages of standardization and customization. A downside to modularity is that the modular systems are not optimized for performance. This is usually due to the cost of putting up interfaces between modules.

## 7.7 CONCURRENT ENGINEERING

The ‘term’ concurrent engineering (CE), also called simultaneous engineering, was first coined in the United States in 1989. It means a way of work where various engineering activities in the product development, production process development and field support development are integrated and performed as much as possible in parallel rather than in sequence as shown in Figure 7.4(a) and (b).



**Figure 7-4:** Sequential and concurrent engineering: a conceptual diagram

CE is a systematic/management technique used for product development, which provides an integrated approach to the design of products and their related processes from concept to disposal. Various elements of product life cycle such as manufacturability, assemblability, testability, serviceability, reliability, quality, cost, disposability, user requirements, etc. are incorporated during the product design and development phase. Integration is usually accomplished through computerized technical tools and cross-functional teams consisting internal (e.g. manufacturing, assembly, R & D, service, process planning) and external (e.g. customers, suppliers) members. The basic CE has two elements: improved process and closer cooperation.

### 7.7.1 Sequential Versus CE

The marketing department of an organization identifies the need of a product, expected performance and the viable price range from the potential customers. This department passes this information to the design department that works on the design specifications and detailed design. The design department makes a design, which is usually best from the viewpoint of design department only. This design is passed to the manufacturing department to develop the manufacturing processes necessary to produce the design. If any changes or corrections are required, then the design is passed back to the designers for necessary modifications or corrections. Only when the manufacturing department is fully satisfied with the design, it passes the design to the next department in the progression as shown in Figure 7.5. This traditional sequential approach to the product design is more expensive and increases the product development lead time that organizations cannot afford these days because of decreasing product life cycle.

In CE, once the needs of the customers are identified, a multifunctional team is formed that will consider all the aspects of the product life cycle such as manufacturability, assemblability,



**Figure 7-5:** Sequential engineering process

testability, reliability, serviceability, cost, user requirements, disposability at the time of the design itself as shown in Figure 7.6. There is a close integration between the various departments during the design leading to a design that requires hardly any correction or modification. This way the product can be brought to the market quickly at a low cost and with an improved quality.

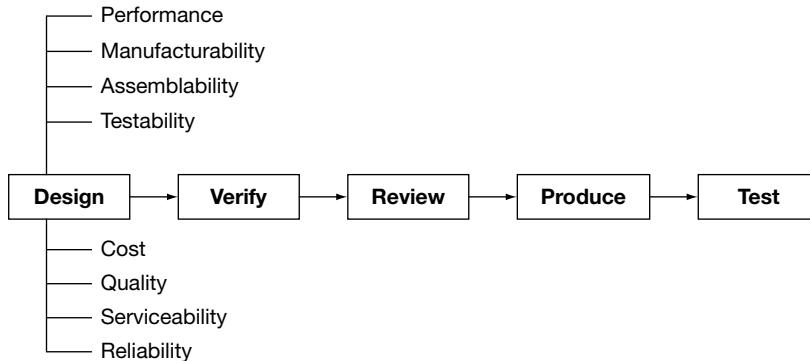


Figure 7-6: Concurrent engineering process

## 7.7.2 Factors Influencing the Need of CE

There are many factors that influence the need of CE, are discussed below as:

*Lead time:* One of the prime motivations for CE approach is the desire to decrease or shorten the product development time or lead time. It is fully recognized that addressing all the problems of product life cycle in the design phase shortens the product lead time. These days, for some products, average product lifetime is less than the average product development time. Therefore, for a company to survive and remain competitive, it has to decrease the product development time maintaining the high quality and low cost.

*New manufacturing methods/technology push:* New manufacturing processes are being developed continuously. Newer methods may bring down the costs, production time and even may improve the quality. But, such knowledge is often with the production engineer and not the design engineer. Therefore, to make the optimum use of newer manufacturing methods, close cooperation between design, production, and R & D departments is essential.

*More demanding customers:* Nowadays, customers are becoming increasingly more demanding. Low cost and good quality, they take for granted and then they demand products, which are more closely targeted to their needs. Companies, hence, have to be not only effective, but also innovative to fulfil the demand of customized products.

*Increased competition:* With the opening of the global markets, the competition has increased manifolds. To survive in the market, an organization has to develop and introduce innovative products well ahead of the competitors. Any delay in the introduction of the product in time causes losses to the profits of the organization. Because of the short lifetime of the products, no time is available to the companies for re-engineering or modifications and the only alternative

left is to look forward to the philosophy of ‘right first time’. This can be possible by involving the people from manufacturing, finance, sales, services and specialist vendors, etc. at the time of the product design itself.

### 7.7.3 Benefits of CE

Concurrent engineering has been the focus of many industrial organizations for new product development due to the ability of the cross-functional team to reduce the total time to design and manufacture or time to market. This reduction in the time to market is a major source of competitive advantage in the manufacturing environment we have today. Use of CE should improve the performance of the organization in general, but certain human and organizational characteristics may affect the degree to which improvement is felt in the organization. Typical benefits of CE are given below as:

- Reduction in time to market
- Improved product quality
- Increased customer satisfaction
- Reduced cost of production
- Reduction in engineering change requests
- Increased return on investment
- Increased level of group productivity
- Interdepartmental cooperation
- Increased employee satisfaction
- Reduction in development cycle time
- Increase in sales
- Reduction in life cycle costs
- Design rationalization
- Better communication
- Increased flexibility to accommodate changes
- Decreased occurrence of obsolescence
- Better use of scarce technical resources
- Other benefits: reduced lead time for creating bid proposals, reduced product development costs, parts reduction, lower inventories, fewer rework orders, and less scrap.

## 7.8 ECONOMIC CONSIDERATIONS IN PRODUCT DESIGN

The consideration of cost plays an important role in the design decision process that we could easily spend as much time in studying the cost factor as in the study of the entire subject of design. Here, a few general approaches and simple rules are introduced below as:

*Standard sizes:* The use of standard or stock size is a first principle of cost reduction. Although great many sizes are usually listed in catalogues, they are not all readily available. Some sizes are used so infrequently that they are not stocked. A rush order for such sizes may mean more expense and delay. There are many purchased parts in a complete product; therefore, the parts selection must be based on case of availability in the market.

*Large tolerances:* Tolerance is most significant among the effects of design specification on costs. Tolerances in design influence the probability of the end product in many ways; close tolerance may necessitate additional steps in processing or even render a part completely impractical to produce economically. Tolerances cover dimensional variation and surface-roughness range and also the variation in mechanical properties resulting from heat treatment and other processing operations. Since parts having large tolerances can often be produced by machines with higher production rate, labour costs will be smaller than if skilled workers were required. Also, there will be less rejection and less time-consuming in assemblies.

*Breakeven point:* Sometimes it happens that, when two or more design approaches are compared to cost, the choice between the two depends upon a set of condition such as the quantity of production, the speed of the assembly lines, or some other condition. There then occurs a point corresponding to equal cost which is called breakeven point.

## 7.9 AESTHETIC CONSIDERATIONS IN DESIGN

Today's market is very competitive and there are a number of products in the market, having the same qualities of efficiency, durability and cost. In this situation, the customer is attracted towards the most appealing products. The external appearance is an important feature, which gives grace and lustre to the product and dominates the market. The growing realization of the need of aesthetic considerations in product design has given rise to a separate discipline, known as industrial design. The role of an industrial designer is to create new forms and shapes which are aesthetically pleasing.

The external appearance of the product does not depend upon only two factors, form and colour. It is a cumulative effect of a number of factors such as rigidity and resilience, tolerances and surface finish, noise, etc.

## 7.10 ERGONOMIC CONSIDERATIONS IN DESIGN

Due to fast-growing technology, the system design has become very complex and the overall performance of the product or system depends on the performance of components. For example, the performance of a computer depends on the performance of RAM, hard disk, microprocessor, monitor and motherboard, etc. If any one of these components fails, computer becomes idle. Thus, each part must interact with each other properly for smooth functioning of the computer. A designer should consider the following points to accomplish an overall performance of a system:

1. Reliability level
2. Safety
3. Convenience
4. Adaptability under various conditions
5. Ease of maintenance
6. Cost.



## SUMMARY

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In this chapter, the process of product design with steps has been discussed in detail. Various phases of product life cycle have been discussed and explained. Standardization, simplification, differentiation and the concept of interchangeability have been introduced. The concepts of sequential engineering and CE have also been discussed. The design and development concepts may be used for new as well as existing products.

## MULTIPLE-CHOICE QUESTIONS

1. Need recognition can be done by
  - (a) Modelling and simulation
  - (b) Customer survey
  - (c) Product testing
  - (d) All the above
2. Which of the following is not the part of engineering design process?
  - (a) Need recognition
  - (b) Problem formulation
  - (c) Synthesis
  - (d) Pareto chart
3. Product specification means
  - (a) details of the customers using the product for long time
  - (b) the detail of technical description expected by the customer
  - (c) the technical description of the product which helps to perform the functions of the product satisfactorily
  - (d) details of product's cost and price
4. Brain storming is used to
  - (a) increase the creativity of the people
  - (b) generate of the ideas for product development
  - (c) sort out the ideas for product development
  - (d) both (b) and (c)
5. Sales are the highest in which of the following phase of product life cycle?
  - (a) Introduction phase
  - (b) Growth phase
  - (c) Maturity phase
  - (d) Declining phase
6. Price of the product is reduced and some additional offer in made in which of the following phase of product life cycle?
  - (a) Introduction phase
  - (b) Growth phase
  - (c) Maturity phase
  - (d) Declining phase
7. Morphology of design is
  - (a) the study of a product throughout its entire life cycle
  - (b) treatment at the end of the use of the product
  - (c) rejection of the design due to poor performance of the product
  - (d) none of these

8. Standardization in product designing means
  - (a) Minimization of variation in specification of the product components
  - (b) Improve the utilization of parts across a product family
  - (c) Improving the quality and reducing the rework of the parts
  - (d) All the above
9. Simplification means
  - (a) Reducing the variety
  - (b) Boosting to standardization
  - (c) Better after sales planning and reducing the inventory
  - (d) All the above
10. Interchangeability means
  - (a) ability of an item to be used in place of another without any change
  - (b) differentiate the product from another available in the market
  - (c) diversify the products to expand the market
  - (d) none of these
11. Modular design means
  - (a) design the different components of the system separately
  - (b) design the different components of the system as in the integrated form
  - (c) different modules of the design of the same components
  - (d) none of these
12. Concurrent engineering means
  - (a) Using the current technology in product design
  - (b) Using the current technology in product manufacturing
  - (c) various engineering activities in the product development run in parallel
  - (d) all the above
13. Sequential engineering means
  - (a) Using the current technology in product design
  - (b) Using the current technology in product manufacturing
  - (c) the different engineering activities in the product development run in sequence
  - (d) all the above
14. Benefit of concurrent engineering is
  - (a) reduction in time to market
  - (b) reduced cost of production
  - (c) increased level of group productivity
  - (d) all the above
15. Ergonomic considerations in design include
  - (a) Safety
  - (b) Convenience
  - (c) Adaptability under various conditions
  - (d) All the above

## Answers

1. (b)      2. (d)      3. (c)      4. (d)      5. (c)      6. (d)      7. (a)      8. (d)      9. (d)  
10. (a)     11. (a)     12. (c)     13. (c)     14. (d)     15. (d)

## REVIEW QUESTIONS

1. What do you mean by the term ‘engineering designs’? What are the steps used in engineering design? Discuss them.
2. What do you mean by feasible design and optimal design?
3. Explain the importance of brain storming in product design.
4. What do you mean by product life cycle? Explain all the phases of product life cycle.
5. Discuss in detail about morphology of design.
6. Explain the terms: standardization, simplification, differentiation and diversification.
7. Explain interchangeability and modular design.
8. Differentiate concurrent engineering and sequential engineering. Write the advantages of concurrent engineering over the sequential engineering.
9. Write a short note on aesthetic considerations in engineering design.
10. Write a short note on ergonomic considerations in engineering design.



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# CHAPTER ▶ 8

## Manufacturing Systems

### 8.1 INTRODUCTION

Manufacturing systems consist of all the processes, technology and resources required for converting the raw materials into finished goods. It consists of all the resources required to transform the material from its raw form to finished form. The resources involved in this transformation process may be men, materials, money, machines, management, energy, etc. A manufacturing system's components may be broadly categorized as:

- (a) Production machines, tools, jigs, fixtures, etc.
- (b) Material handling systems
- (c) Computer systems
- (d) Human resources

**Production machines, tools, fixtures and other related hardware:** In manufacturing systems, the term 'workstation' refers to a location in the factory where some well-defined operations are accomplished by men or/and machines. The machines used in manufacturing or assembly can be classified as:

- Manually operated machines
- Semi-automated machines
- Fully automated machines

*Manually operated machines:* In manually operated machines, the machines only provide the power for operations, but the control of operations is totally with operators. An operator is required to be at machine continuously during operation.

*Semi-automated:* A semi-automated machine performs a portion of the work cycle under some form of program/automated control, and a worker tends to the machine for the remainder of the cycle. Typical worker tasks include loading and unloading parts.

*Fully automated:* A fully automated machine operates for extended periods (longer than one work cycle) without involvement of worker in the operations.

#### **Material Handling System**

The main objectives of a material handling system are to load the job at each workstation, position the job at each workstation, unload the job at each workstation, transport the job between stations

in multi-station manufacturing systems and store the finished jobs into the storage. Material handling systems are discussed in detail in Chapter 9.

### **Computer Systems**

A computer system is required to control the functions of the machines and to participate in the overall coordination and management of the manufacturing system. The objectives of a computer system are to give instructions to workers, transform part programs to machine language, control the material handling system, schedule production, monitor safety measures, control the quality of job to be produced, etc.

### **Human Workers**

Two types of labours are involved in a manufacturing system: direct labours and indirect labours. Direct labours perform some or all of the value-added work that is accomplished on the job. They directly add the value to the job by performing manual work on it or by controlling the machines that perform the work. While indirect labours manage or support manufacturing activities without directly involvement in job processing. All the administrators, marketing personals, financial managers, human resource managers are indirect labours.

## **8.2 FLEXIBLE MANUFACTURING SYSTEM**

Flexible manufacturing system (FMS) is a highly automated group technology (GT) machine cell, consisting of a group of processing stations (usually computer numerical control (CNC) machine tools), interconnected by an automated material handling and storage system, and controlled by an integrated computer system. It can process different part style in the non-batch mode. It can accommodate changes in production schedules and new part designs. It can respond to equipment malfunction and machine breakdowns.

### **8.2.1 FMS Components**

Usually, FMS consists of processing workstations, material handling devices and control software. These components of FMS are discussed below as:

1. *Processing stations (numerical control (NC) machine tools, CNC, direct numerical control (DNC))*: These are the programmable workstations and able to accommodate the changes in the design of the product without losing the time. New product design can be easily accommodated for processing on these work centres.
2. Automated material handling and storage system (AMHSS)
  - Automated guided vehicles (AGVs)
  - Conveyors
  - Automated storage and retrieval systems (AS/RS)
  - Industrial robots

These are the material handling devices used to transfer the components from one work station to another. This is also used for loading and unloading of the components on the work centres.

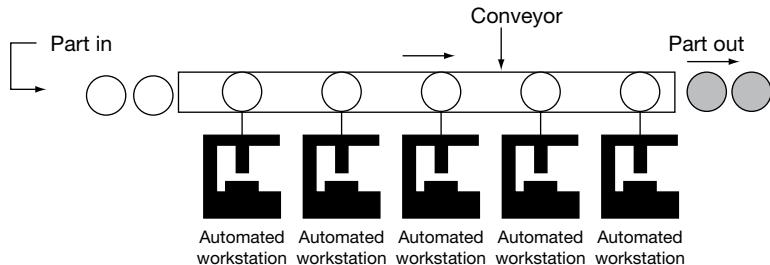
3. *Control software:* Control software is the computer program which is used to control the entire operation in the systems.
4. *Human workers:* Human workers are used to control the entire activity, including alterations in the computer programs.

*FMS workstations:* FMS workstations consist of the following facilities:

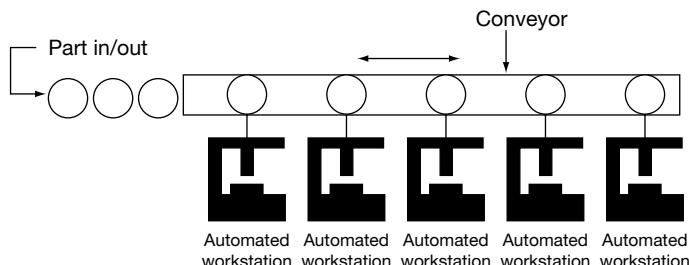
1. Machining centres
2. Head changers
3. Head indexers
4. Milling modules
5. Turning modules
6. Assembly workstations
7. Inspection stations
8. Sheet metal processing machines
9. Forging stations

### 8.2.2 FMS Layout Configurations

1. *In-line:* There are two types of arrangement in in-line layout, in the first type parts move in only one direction. Parts enter from one side and out from the other side after being processed on various workstations as shown in Figure 8.1. But, in the second type of the arrangement, parts may flow in both the directions as shown in Figure 8.2.

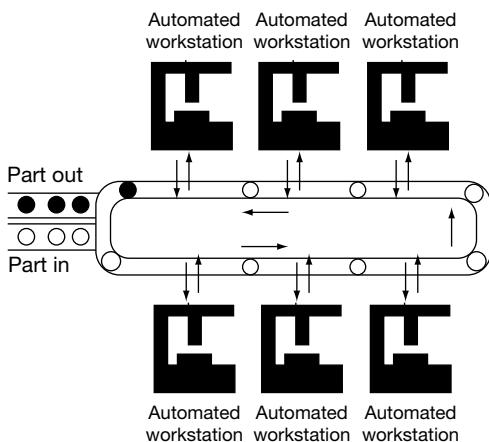


**Figure 8-1:** In-line arrangement for unidirectional flow of the parts

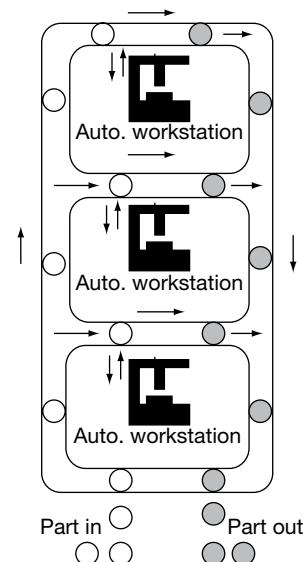


**Figure 8-2:** In-line arrangement for flow of the parts in both directions

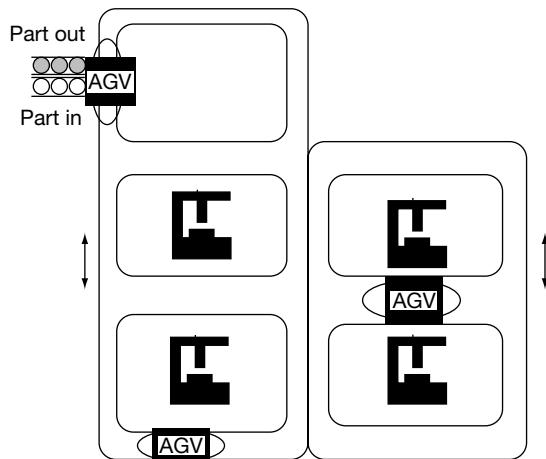
2. *Loop*: In the loop arrangement, the parts enter at one point in the loop and they move in the loop as the machines are arranged in the loop for the processing and finally, it leaves the loop after complete processing as shown in Figure 8.3.
3. *Ladder*: Workstations are arranged in the ladder form and parts move in the same fashion as shown in Figure 8.4.
4. *Open field*: In the open-field arrangement, the layout of the workstations is slightly complex and parts are transferred using AGVs. The path of movement is already programmed. The arrangement is shown in Figure 8.5.



**Figure 8-3:** Loop arrangement of workstations

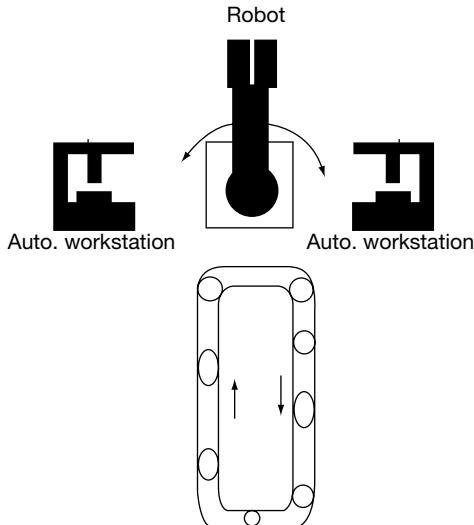


**Figure 8-4:** Ladder arrangement of workstations



**Figure 8-5:** Open-field arrangement of workstations

5. *Robot-centred cell:* In robot-centred cell, parts are loaded and unloaded and also transferred from one workstation to another with the help of robot as shown in Figure 8.6.



**Figure 8-6:** Robot-centred cell

### 8.2.3 Functions of Computer in FMS

The following are functions of computer in FMS:

1. Control of each workstation
2. Distribution of control instruction to workstations
3. Production control
4. Traffic control
5. Shuttle control
6. Work handling system monitoring
7. Tool control
8. System performance and monitoring

### 8.2.4 FMS Data Files

FMS data files consist of the following files.

*Part program file:* It consists of part programs for each workstation.

*Routing file:* It consists of a number of alternate route sheets.

*Part production file:* It consists of production rates for various machines, in-process inventory, and inspection required and so on.

*Pallet reference file:* It consists of the record of the part which requires pallet fixtures.

*Station tool file:* It consists of the tool records for different workstations.

*Tool life file:* It keeps the record of the tool life for the different cutting tools.

## 8.2.5 FMS Planning

FMS planning is influenced by the following factors:

1. The volume of the work to be produced by the system.
2. Variations in process routings.
3. Physical characteristics of the working parts.
4. Part families defined according to product commonality.
5. FMS manpower requirement.
6. Appropriate production volume range.
7. Minimum number of machines per FMS.
8. Minimum normal tolerance at work in an FMS.

## 8.2.6 FMS Benefits

The following are benefits of FMS:

1. Higher machine utilization.
2. Reduced work in progress.
3. Lower manufacturing lead time.
4. Greater flexibility in production scheduling.
5. Higher labour productivity.

## 8.3 CAD/CAM

Computer-aided design and computer-aided manufacturing (CAD/CAM) involves the use of the computer to accomplish certain functions in design and manufacturing. CAD is concerned with the use of the computer to support the design functions; and CAM is concerned with the computer to support manufacturing functions. The combination of CAD and CAM in the symbolic form is represented as CAD/CAM to integrate design and manufacturing functions in a firm.

### 8.3.1 Computer-aided Design

CAD can be defined as any design activity that involves the effective use of computer to create, modify and document an engineering design. There are four phases of CAD:

- Synthesis (Geometric modelling).
- Analysis and optimization (Engineering analysis).
- Evaluation (Design review and evaluation).
- Presentation (Automated drafting).

Geometric modelling is concerned with the mathematical description of the geometry of an object. The mathematical description, called a model, is fed into computer memory. The image is displayed on a graphics terminal to perform certain operations on the model. There are various types of geometric models in CAD: wire frame and solid models, coloured and animation, two dimensional and three dimensional.

The second phase is engineering analysis, which includes the stress-strain calculations, heat transfer analysis, dynamic simulation and optimization. The CAD system increases the designer's analysis ability.

The third phase is design evaluation and review procedures. Some CAD features which are helpful evaluating and reviewing a proposed design include:

- (a) Automatic dimensioning routines.
- (b) Interference checking routings.
- (c) Kinematics routines.

The fourth phase where CAD is useful in the design process is presentation and documentation. The CAD system can be used as automated drafting machines to prepare highly accurate engineering drawing quickly. It is estimated that a CAD system increases productivity in the drafting function by five times the manual preparation of the drawing.

*Objectives of CAD system:* The objectives of CAD system are to increase the productivity of the designer, improve the quality of the design, improve the design documentation, create a manufacturing database.

### **8.3.2 Computer-aided Manufacturing**

CAM is mainly used for manufacturing planning and manufacturing control. In manufacturing planning computer is used indirectly to provide information for the effective planning and management of production activities. The computer is used for following planning activities:

1. Cost estimating.
2. Computer-aided process planning (CAPP).
3. Computer-assisted NC part programming.
4. Development of work standards.
5. Computer-aided line balancing.
6. Production and inventory planning.

*Manufacturing control:* It is concerned with managing and controlling the physical operations in the factory to implement the manufacturing plans. Mainly three types of controls are required in manufacturing as discussed below:

*Shop-floor control:* It is concerned with the problem of monitoring the progress of processing, assembling, and inspection of the products in the factory.

*Inventory control:* It is concerned with the demand fulfilment and also to reduce the inventory to eliminate the wastage and extra money investment. Thus, the optimum inventory size is maintained based on accurate demand forecasting.

*Quality control:* The purpose of the quality control is to assure that the quality of the product and its components meet the standards specified by the product designer. To accomplish its mission, quality control depends on the inspection activities performed in the factory at various times throughout the manufacture of the product.

### **8.3.3 Computer-aided Process Planning**

Process planning is concerned with the preparation of rout sheets which list the sequence of operations and work centres required to produce the product and its components. CAPP is available today to prepare these route sheets. The route sheet typically lists the production operations, machine cells or workstations where each operation is performed, fixtures and tooling required, and the standard time for each task. There are two approaches of CAPP systems:

1. Retrieval or Variant CAPP systems.
2. Generative CAPP system.

*Retrieval CAPP systems:* Retrieval CAPP systems are based on GT and part classification and coding system. In this method, a search is made of the part family file to determine if a standard route sheet exists for the given part code. If the file contains a process plan for the part, it is retrieved and displayed for the user. The standard process plan is examined to determine whether any modifications are required. It might be possible that the new part has the same code number, there are minor differences in the process required to make the part. The user edits the standard plan accordingly. Thus, the capacity to alter an existing process plan that gives the retrieval system its other name: variant CAPP system.

If the file does not contain a standard process plan for the given code number, then the user may search the computer file for a similar or related code number for which a standard route sheet does exist. Either by editing an existing process plan, or by starting from scratch, the user writes the route sheet for the new part. This route sheet becomes the standard process plan for the new part code number.

*Generative CAPP system:* A generative CAPP system creates the process plan based on logical procedures a human planner would use. The design of generative CAPP is based on the artificial expert system. An artificial expert system is a computer program that is capable of solving complex problems that normally require a human being who has years of education and experience. There are several ingredients in this system. First, technical knowledge of manufacturing and the logic that is used by successful process planners must be captured and coded into a computer program. The second ingredient is a computer-compatible description of the part like geometric model of CAD system of the part and GT code number of the part that defines the part feature in detail. The third ingredient is the capability to apply the process knowledge and planning logic contained in the knowledge base to a given part description.

#### **Advantages of CAPP**

There are following advantages of CAPP:

1. Process rationalization and standardization which results in a low cost of production with high quality of product.
2. Increased productivity of process planners.

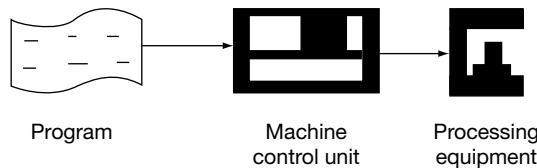
3. Reduced lead time for process planning.
4. Improved legibility.
5. Incorporation of other application program.

### **8.3.4 Numerical Control**

Numerical control (NC) can be defined as a form of programmable automation in which the machining process is controlled by numbers, letters and symbols. NC technology has been applied to a wide variety of operations, but principal application is in machining operations. An operational NC system consists of the following three basic components:

- (a) Program of instruction.
- (b) Controller unit.
- (c) Machine tools.

The program of instruction consists of details of the sequence of operations in symbolic, numeric, or alphanumeric form on any medium like tape, which can be interpreted by the controller unit. The controller unit consists of the electronics and hardware that read and interpret the program of instructions and convert it into mechanical actions of the machine tool. The typical elements of the conventional NC controller unit include the tape reader, a data buffer, signal output channels to the machine tool, feedback channels from the machine tool, and the sequence control to coordinate the overall operation of the forging element. Machine tool is the part of NC system which performs useful work. It also includes the cutting tools, work fixtures and other auxiliary equipment needed in the machining operation. The three components of NC system are shown in Figure 8.7.



**Figure 8-7: NC components**

#### ***Limitations/drawbacks of the Conventional NC System***

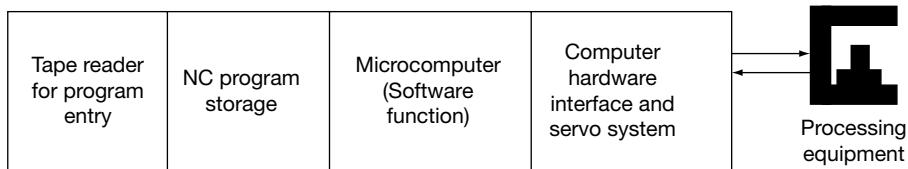
There are following limitations/drawbacks of the conventional NC system:

- Part programming mistakes in punched tape are common.
- Short life of punch tape due to wear and tear.
- Less reliable tape reader component.
- Less flexible hardwired controller unit.
- Non-optimal speed and feed.

### **8.3.5 Computer Numerical Control**

The structure of a CNC system is almost similar to NC system, but the way of using the program is different. In a conventional NC system, the punched tape is cycled through the tape reader for

each work part in the batch. The machine control unit reads in a block of instructions on the tape, executing that block before proceeding to the next block. In CNC, the entire program is entered once and stored in computer memory. The machining cycle for each part is controlled by the program contained in memory rather than from the tape itself. The general configuration of CNC system is shown in Figure 8.8.



**Figure 8-8:** Components of CNC systems

The following features are associated with CNC system:

1. Storage of more than one program.
2. Use of floppy/discs.
3. Program editing on the machine tool site.
4. Fixed cycles and programming subroutines.
5. Interpolation.
6. Positioning features for set-up.
7. Cutter length compensation.
8. Diagnostics.
9. Communication Interface.

### ***Advantages of CNC System***

There are following advantages of CNC system:

1. The part program tape and tape reader are used only once to enter the program into the memory.
2. Tape can be edited at the machine site.
3. Greater flexibility.
4. Metric conversion.
5. Compatible with total manufacturing information system.

### **8.3.6 Comparison of NC and CNC Machines**

NC machines offered a reliable way of producing machine parts using pre-programmed commands. These commands consisted of alphanumeric characters defined by the RS233 IEEE code. These characters were coded on punch paper tape in formats specifically planned for a certain machine tool. These programs (punched tape) would then read into the NC control using a paper tape reader. If during testing a program error were detected, the paper tape would have to be edited. This process meant duplicating a tape up to the incorrect character(s), retying the correct characters,

and then continuing with the duplication process. This was a time consuming process. During the running of NC programs, if a tool would begin to wear causing part dimensions to approach tolerance limits, the operator would have to stop and adjust the tool(s) to compensate for this wear.

In CNC machines, data for the control are still coded using either RS232 or the newer, more acceptable RSxyz ASCII code (American Standard Code for Information Interchange). Entire CNC programs may be loaded into the memory of the CNC control enabling the programmer or machine operator to edit the programs at the machine. If program changes are required, many CNC machines have built-in paper tape punch machines that allow for the generation of a new tape at the control. Tool wear is handled by adjusting program data in memory or calling in from a tool register one of several pre-programmed tool offsets.

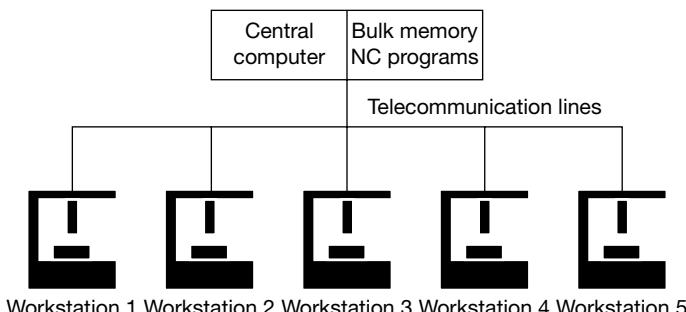
As the implementation of microprocessors expanded, OEM's (Original Equipment Manufacturer's) of NC/CNC machines began using them in the construction of controls. By the late 1970s, nearly all NC/CNC manufacturers were using microprocessors (computers) in their controls. Today the phrase NC is commonly used when referring to CNC machines because the need to differentiate the two no longer exists. Today, ALL modern NC machines are in fact CNC machines.

### **8.3.7 Direct Numerical Control**

Direct numerical control (DNC) can be defined as a manufacturing system in which a number of machines are controlled by computer through direct connection and in real time. The tape reader is omitted in the DNC, thus relieving the system of its least reliable component. Instead of using the tape reader, the part program is transmitted to the machine tool directly from the computer memory. In principle, one computer can be used to control up to 256 machine tools. When the machine needs control commands, they are communicated to it immediately.

The DNC system consists of following four components as shown in Figure 8.9.

1. Central computer.
2. Bulk memory, storage of NC part programs.
3. Telecommunication lines.
4. Machine tools.



**Figure 8-9:** General configuration of DNC

### **Advantages of DNC System**

There are following advantages of DNC system:

- (a) Time saving due to control of more than one machine by a single computer.
- (b) Greater computational capability for such functions as circular interpolation.
- (c) The remote computer location is a safe environment.
- (d) Elimination of tapes and tape reader at the machine for improved reliability.
- (e) Elimination of hardware controller unit on some systems.
- (f) Programs stored as cutter location data can be post processed for whatever suitable machine is assigned to process the job.

## **8.4 LEAN MANUFACTURING**

Lean manufacturing is a management philosophy coined by the Toyota Motor Company. The concept of lean manufacturing is derived from the Toyota Production System (TPS). Lean manufacturing is a philosophy that considers the expenditure of resources for any non-value-added process is a waste and it has to be eliminated. Lean manufacturing incorporates a number of management tools, such as just in time, total quality management, total productive maintenance, Kaizen, etc. Lean production is interchangeably used with the TPS; the main aim of lean production is to reduce the seven categories of wastes: defects, overproduction, transportation, waiting, inventory, motion and processing. Some of the major lean tools applied in the manufacturing industry are identified as in their paper on a lean supply chain, the 15 tools identified by them were: planning, just-in-time (JIT)/pull systems, Kaizen, Kanban, Poka-Yoke, research and development (R & D), safety practices, six sigma, statistical process control (SPC), total productive maintenance (TPM), total quality management (TQM), TPS, value stream mapping (VSM) and 5S.

### **8.4.1 The 3 M's of Lean (Muda, Mura and Muri)**

#### **Muda**

Lean manufacturing is a Japanese method focused on 3 M's. These M's are: muda (waste), mura (inconsistency) and muri (unreasonableness). Muda specifically focuses on activities to be eliminated. There are different categories of wastes in manufacturing industry. Waste is broadly defined as anything that adds cost to the product without adding value to it. Generally, muda (or waste) can be grouped into the following categories:

1. *Overproduction and early production:* Overproduction may cause waste because it captures resources too early and retains the value that is added to the product until it is used or sold. The production of a product well before its use means there is chance to obsolete the product even if a perfectly good product is produced. Producing a product simply to keep a production resource busy is a wrong practice and that must be avoided.
2. *Delays:* Various delays, such as delay in supply of raw materials or components result in the underutilization of resources and increased delivery time. Raw materials and component parts should be completed at the right time. Too early is not good, but late is even worse.

3. *Movement and transport:* Unnecessary movement and transportation must be avoided. Material handling is required in all the manufacturing operations, when possible, the handling should be integrated with the process; and the transport distances should be minimized.
4. *Poor process design:* A poorly designed process results in the misuse/waste of manufacturing resources. A single process may not be the perfect process. Process improvements are made continuously with new efficiencies embedded within the process. Continuous process improvement is very important part of lean manufacturing.
5. *Over inventory:* Over inventory increases the inventory costs. It is common practice for a manufacturer to store a supplier's product on the production site. The supplier, right up until the time that they are drawn from inventory, owns the materials. In many ways this is advantageous to both the user and supplier. The supplier stores his material off-site, and the user does need to commit capital to a large 'safety stock' of material.
6. *Inefficient performance of a process:* Insufficient (or poor) process performance may result in the over utilization of manufacturing resources. There is no single optimal process that can always be better than the other process. Many processes operate far below the desired efficiency. Continuous process improvement is necessary for a manufacturing firm to remain competitive. Excess material and human movements must be avoided in the process of waste elimination.
7. *Making defective items:* Poor quality is never desirable. The production of defective items always results in labour and material waste. Furthermore, the cost of rework increases the price of the product. A process should be run as fast as possible without sacrificing acceptable quality.

Thus, waste in any form must be eliminated from the systems. Waste always results in inefficiency and high costs.

### ***Mura***

The second 'M' is used for mura or inconsistency. Inconsistency is responsible for the variability in manufacturing parameters. Mura can be observed in all manufacturing activities. Due to lack of consistency, there may be variation in dimensions and specifications of the product, which may be treated as scrap or may require rework.

### ***Muri***

The final 'M' is used for muri or unreasonableness. Muri applies to a variety of manufacturing and management activities. An example of being unreasonable by blaming someone for problems rather than looking at resolution of problems. It is unreasonable to blame rather than mitigate issues. To develop a new culture for eliminating the blaming practice, following points must be considered as the management tool:

1. Problems are recognized as opportunities.
2. It is okay to make legitimate mistakes.
3. Problems are exposed because of increased trust.
4. People are not problems – they are problem-solvers.
5. Emphasis is placed on finding solutions instead of 'who did it'.

Much of lean manufacturing is applying ‘common sense’ to manufacturing environments. In implementing Lean, 5 S’s are frequently used to assist in the organization of manufacturing. The 5 S’s concepts are discussed in detail in Chapter 12 of this book.

### **8.4.2 Toyota Production Systems**

Toyota is a renowned company in automobile manufacturing industry and also, for the world-famous TPS (Toyota Production System, i.e., Lean manufacturing). Founder of Toyota motors, Kiichiro Toyoda was forced to develop the most cost-efficient strategies in order to keep the company competitive. Taiichi Ohno, principal inventor of the concept Toyota Production Systems (TPS), has been quoted a very useful sentence as ‘Having no problem is the biggest problem of all.’

Following are the principles of lean manufacturing.

1. To cut lead time, cut out all the activities that do not add value.
2. A line must stop if there is a problem.
3. Deal with defects only when they occur, and less staff will be needed.
4. Ask yourself ‘Why?’ five times.
5. Train people to follow rules and standards as if second nature.
6. Find where a part is made cheaply and use that price as a benchmark.
7. Develop people who can come up with unique ideas.

*Jidoka:* The principle of Jidoka requires the production line to stop when something goes wrong. If something goes wrong at station 1, the operator presses a button on and on a display board of all workstations. All work stops, and everyone pitches in to solve the problem. The line has stopped here because of a specific reason.

### **8.4.3 Evolution of Toyota Motor and Concept of Lean Manufacturing**

Taiichi Ohno, considered to be the creator of the Toyota Production System and the Father of the Kanban System, joined the Toyoda Automatic Loom Works after graduating from the Nogoya Technical High School in 1932. Early in his career, he expanded upon the JIT concepts developed by Kiichiro Toyoda to reduce waste, and started experimenting with and developing methodologies to produce needed components and subassemblies in a timely manner to support final assembly. During the chaos of World War II, the Loom Works was converted into a Motors Works and Taiichi Ohno made the transition to car and truck parts production. The war resulted in the levelling of all Toyoda Group Works production facilities, but under the management of Eiji Toyoda, the plants were gradually rebuilt and Taiichi Ohno played a major role in establishing the JIT principles and methodologies developed in the Loom manufacturing processes.

At the reconstructed Toyoda Group Automotive Operations, Taiichi Ohno managed the machining operations under severe conditions of material shortages as a result of the war. Gradually he developed improved methods of supporting the assembly operations. Two other

gentlemen who helped shape the Toyota Production System were Shigeo Shingo, a quality consultant hired by Toyota, who assisted in the implementation of quality initiatives; and Edward Deming who brought statistical process control to Japan.

#### **8.4.4 The Concept of the JIT System**

The JIT concept states ‘nothing is to be produced until it is required.’ According to the JIT system, the finished goods are assembled just before they are sold, the sub-assemblies are made just before the products are assembled and the components are fabricated just before the sub-assemblies are made. Here, the work-in-progress inventory is always kept at a low level, thus reducing the production lead times. The firms should achieve and maintain high performance levels in all their operational areas to facilitate the smooth flow of materials in the JIT Systems.

The JIT System involves the active participation, involvement, and cooperation of all its employees. The JIT manufacturing system is based on the concept of continuous improvement, which includes the two mutually supporting components of people involvement and total quality control.

*People involvement:* The human resources management plays an important role in the implementation of the JIT manufacturing system. The successful implementation of a JIT program requires teamwork, discipline and supplier involvement.

*Team work:* Team work involves activities like suggestion programs, brainstorming and quality circle programs which enable employees to actively participate. Suggestion and brainstorming programs are conducted to encourage the employees to their ideas on how to improve a process. In quality circles, people working in similar types of operations meet at regular intervals and discuss ways of improving the quality of their processes.

*Supplier involvement:* Firms can involve suppliers in design review and to suggest new designs and methods for improving product quality or productivity. JIT firms make strategic relationship with their suppliers instead of inviting competitive bids from a set of suppliers. The JIT firm can share its production plans and schedule with its suppliers so that they can plan their business and capacity requirements beforehand. The suppliers should devote their schedules to the JIT firm’s needs as they contribute to the improvement of the firm’s manufacturing operations. The maintenance of linear production schedules requires the identification and elimination of production bottlenecks, a balance in the production system, and a reduction in setup time.

*Total quality control:* A firm can produce high-quality products only through the combined efforts of all the departments including the purchase department, quality control department, and personnel department. The concept of ‘immediate customer’ helps the firms achieve the required quality levels.

*Concept of immediate customer:* A JIT uses the concept of ‘immediate customer’ where each worker in the firm considers the next worker who continues the production process as the customer. Therefore, it is the responsibility of the worker to ensure that the product is processed to meet specifications and quality requirements before passing it on to the next worker. Only

items of acceptable quality are delivered to the immediate customer. In case a worker delivers a defective item or an improperly finished item to his/her immediate customer, the worker who identifies the defect is authorized to stop the process and take necessary actions thereafter.

### **8.4.5 Objective of JIT**

JIT Manufacturing tries to smooth the flow of materials from the suppliers to the customers, thereby increasing the speed of the manufacturing process. The objective of JIT is to change the manufacturing system gradually rather than drastically including the following points:

1. To be more responsive to customers,
2. To have better communication among departments and suppliers,
3. To be more flexible,
4. To achieve better quality,
5. To reduce product cost.

### **8.4.6 Advantages of JIT System**

The advantages of JIT System to firms are given below as:

1. Increased utilization of machinery and equipment,
2. Reduced investment in inventory,
3. Improvement in the quality of product or service,
4. Reduction in space requirements of the firm,
5. Reduction in production cycle time,
6. Zero inventory storage and maintenance costs,
7. Closer relationship with suppliers,
8. Reduced formal paper work, and
9. Higher involvement of employees as they are responsible for producing good-quality goods.

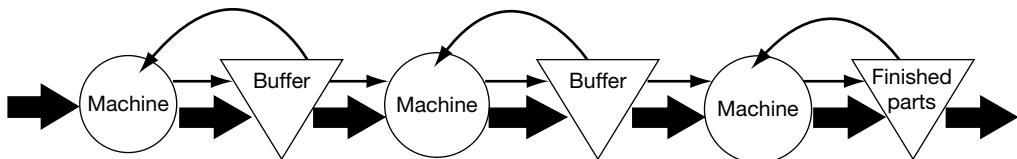
The advantages of JIT Systems to suppliers are long-term guaranteed contract for supply of materials, steady and continuous demand for their materials, less expenditure on promotional activities, and timely payment of materials supplied.

### **8.4.7 Kanban Control in Lean Manufacturing**

A *lean* manufacturing system is one that meets high throughput or service demands with very little inventory. Despite its significant success, Kanban control is *not* a perfect mechanism to control a lean system. Kanban control uses the levels of buffer inventories in the system to regulate production. When a buffer reaches its preset maximum level, the upstream machine is told to stop producing that part type.

This is often implemented by circulating cards, the *kanbans*, between a machine and the downstream buffer. The machine must have a card before it can start an operation. It can then pick raw materials out of its upstream (input) buffer, perform the operation, attach the card to the finished part, and put it in the downstream (output) buffer. The number of cards circulating

determines the buffer size, since once all cards are attached to parts in the buffer, no more parts can be made. When the machine picks up the raw materials to perform an operation, it also detaches the card that was attached to the material. The card is then circulated back upstream to signal the next upstream machine to do another operation. This way, a demand for a unit of finished goods percolates up the supply chain.



**Figure 8-10:** Kanban control

In Figure 8.10, movement of parts shown by thick arrow, circulation of Kanban by thin arrow. Machines are shown as circles and buffers as triangles. The last buffer is the finished parts inventory. Kanban control ensures that parts are not made except in response to a demand. The analogy is to a supermarket; only the goods that have been sold are restocked on the shelves. However, it has a major drawback as it uses the parts themselves as carriers of information. A machine is told to stop production when its output buffer is full. This requires that a number of parts sit in the buffer without serving any apparent purpose but to block the upstream machine. That's not quite right, though. The parts waiting in a buffer do serve a purpose; they act as a *buffer inventory*, partially decoupling the operation of downstream machines from any interruptions of upstream production. If a machine fails, the machine downstream of it, can continue production by consuming the parts that are already in the buffer. With luck, the upstream machine will be repaired before the buffer is empty, and the failure will not affect the downstream machine (or the customer at the downstream end of the chain).

### **Benefits of Kanban System**

Some common benefits realized by warehouse, shipping, and logistics managers using Kanban are given below as:

- Lowers overhead costs
- Standardizes production goals
- Increases efficiency
- Reduces obsolete inventory
- Gives work area personnel more control
- Improves flow
- Prevents over production
- Provides manager's progress reports
- Improves responsiveness to changes in demand
- Improves teamwork
- Kanban can reduce inventories by 75 per cent in some industries

## 8.5 AGILE MANUFACTURING

Agility is a tool to respond rapidly to continuous and unpredicted changes in the market driven by customer designed products and services. The concept of flexible and more responsive manufacturing led to the new concept of agile manufacturing system (AMS) (Gunasekaran and Yusuf 2002). The concept of agile manufacturing was first time launched in 1991 at Lehigh University. Agile manufacturing is a combination of flexible and responsive manufacturing systems. Flexible manufacturing shows the ability to change the entire systems as per changes in customer requirements. Agility should not be confused with leanness. Lean manufacturing is focused on the elimination of waste and large inventories. Lean leads to high productivity and quality, but it does not necessarily imply being responsive. But, lean manufacturing reduces the cost of production by eliminating the waste. There is equal importance of cost and responsiveness for AMS.

### 8.5.1 Variables of AMS

There are many variables of agile manufacturing; some important of them are: (i) manufacturing flexibility; (ii) strategic production planning; (iii) concurrent engineering; (iv) automation in manufacturing; (v) rapid prototyping (RP); (vi) integrated information system; (vii) multi-functional workforce; (viii) rapid partnership and supplier development (ix) virtual enterprise formation; (x) product decision; (xi) market information system; (xii) product mix; (xiii) product postponement and; (xiv) AMS. The brief discussions about these enablers are given below:

**Manufacturing flexibility:** Manufacturing flexibility can be defined as the ability of manufacturing system to adapt its capabilities to produce quality products timely and cost effective manner to respond to changing product characteristics, material supply, and demand, or to employ technological process enhancements. Avittathur and Swamidass (2007) worked on the manufacturing flexibility from concept development to supplier buyer integrations.

**Strategic production planning:** There are a number of uncertainties related to production activities such as fluctuation in demand, the introduction of new products or customization of existing products. To respond the demand uncertainties, a manufacturer should have multiple options such as outsourcing, subcontracting, overtime production, daily wages workers, expansion of plants, etc. To introduce new products into the market within short notice, there is requirement of generic machines and multi-skilled personals with advanced technologies and strong research and development department. For customization of existing products, the decoupling point in the manufacturing or assembly should be shifted to last stage.

**Concurrent engineering:** Concurrent engineering also called simultaneous engineering was coined in the United States in 1989. It means a way of work where the various engineering activities in the product development, process development and field support development are integrated and performed as much as possible in parallel rather than in sequential order. Multifunctional team and emerging techniques such as CAD/CAM, computer-aided engineering (CAE), CAPP, design for manufacturing (DFM), design for assembly (DFA) and quality function deployment (QFD) play an important role in concurrent engineering.

**Automation in manufacturing:** Automation is an integration of knowledge of various fields such as mechanical, electronics, computer and information technology. It is very helpful to reduce production lead time, improve quality and lower the manufacturing cost. Some of the important technologies which have been frequently used are FMS, GT, cellular manufacturing, computer-aided quality control (CAQC), CNC, DNC, AGV, AS/RS, etc.

**Rapid prototyping:** Rapid prototyping (RP) is a combination of technology, which converts directly a design file into a prototype, which is used in analysis during the design development phase with the least time. RP is able to produce a prototype in shorter lead time with less material wastage. This valuable benefit of the technology makes it worth for agile manufacturing. Prototyping is a process of analysis of product design before putting it into real use. RP has been used in the production of three-dimensional object by stereo-lithography.

**Integrated information system:** Information system's flexibility can be defined as the ability of an organization's collective information system to adapt and support changing requirements of the business functions such as product development, sourcing, manufacturing and logistics, and other strategic goals. Sethi and King (1994) developed a concept called CAPITA (competitive advantage provided by and information technology application), which measures the firm's competitive benefits gained through the use of a single information technology application (Kumar et al. 2007).

**Multi-functional workforce:** The need for frequent changing in roles and responsibilities shows the requirement of multi-functional workforce so that during new demand or in changing circumstances existing workforce can be employed in any work due to time constraints. It is considered as people flexibility, and its components are employee skill flexibility, skills replaceability and employee skills utilization.

**Rapid partnership and supplier development:** A single manufacturing organization cannot be an expert of all the business activities such as supply chain, government policy, trade regulations and legislation, legal formalities and competitor's action. Thus, strategic alliances have become a need for manufacturers in today's business environment. Rapid partnership with these service providers has become an influencing factor for agility (Gunasekaran 1998). Integrated-sourcing flexibility in supply chain management improves the organization's ability to deliver products and services in a timely and effective manner. Sourcing flexibility is the ability to change sourcing decisions such as the number of suppliers per part and delivery schedule. Sourcing flexibility facilitates a faster response when there is uncertainty; therefore, it has a positive impact on manufacturing flexibility. Sourcing flexibility may support the arms-length relationship with suppliers, but strategic partnership with flexibility is more beneficial to cover various uncertainties in the supply chain. Manufacturers always try to find the reliable suppliers and establish strategic relationships with them.

**Virtual enterprise formation:** Virtual enterprise is a temporary integration of organizations on the basis of core competencies having a focus on speed to market, cost reduction and quality. An organization may not be successful alone to respond rapidly to changing market. Therefore, the temporary alliance based on core competencies can help to achieve improved flexibility and responsiveness of organizations. Three important factors for partnership are: pre-qualifying partners, evaluation of product design and capability of potential partners, selecting the optimal number of partners for manufacturing of a particular product.

**Product decision:** Product decision is based on market information, availability of resources, product mix and delayed product differentiation. In push-based system, production decision is based on long-term forecasting while pull-based system is demand driven (Simchi-Levi et al. 2002). There should be strong information channel to know the changing market behaviour.

**Market information system:** The market information system is a base for AMS. Agility means dancing to the tune of the market. Therefore, the strong market information system is required to know the changing customer needs and its behaviour with competitor's strategies. It is also very helpful to forecast the demand and to customize the product according to customer requirements.

**Product mix:** The product mix is a set of similar or dissimilar products of various configurations and specifications provided by same manufacturer. This may help to satisfy the diversified demand of customers. The product mix is very helpful to design and produce a new or customized product with short lead time. For example, Maruti-Suzuki started to produce Maruti-800 model at beginning and now producing a new model of car in every 2–4 years according to requirement of Indian market.

**Product postponement:** Product postponement is a delay in time when a product gets its identity. Using a high level of part commonality at early stages of the manufacturing process can delay the product differentiation. For example, in an apparel industry, traditionally, the manufacture of clothing starts with the dyeing of the yarn followed by the knitting of the garment. This process may result in out of stock of desired colour and over inventory of unpopular colour. In a market characterized by very short product life cycles, this mismatch of inventory and customer demand cannot be corrected using a traditional manufacturing approach. The typical result is the end-of-season markdown. Benetton uses the bleached yarn and delay dyeing until information on the preferred colours became available through EDI to reduce the inventory size.



## SUMMARY

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In this chapter, we have discussed about the manufacturing systems, including the concepts of FMS, lean manufacturing, and agile manufacturing. Flexible manufacturing is a type of programmable automation of manufacturing systems in which the changes required may be easily accommodated. Lean manufacturing is focused on the elimination of different types of wastes. The agile manufacturing is focused on the rapid response to the changes required as per requirement of the market. We have also discussed about the CAD/CAM, NC, CNC and DNC in brief under the topic FMS.

## MULTIPLE-CHOICE QUESTIONS

1. In a manufacturing system, computer system is required
  - (a) to control the functions of the machines
  - (b) to participate in the overall coordination
  - (c) to perform the management of the manufacturing system
  - (d) all the above

2. Which of the following is NOT a part of flexible manufacturing system?
  - (a) a highly automated group technology machine cell
  - (b) a group of processing stations
  - (c) forecasting
  - (d) an integrated computer system
3. Which of the following is NOT a part of FMS workstations?
  - (a) Moulding machines
  - (b) Machining centres
  - (c) Head changers
  - (d) Milling modules
4. Part program file consists of
  - (a) alternate route sheets
  - (b) part programs for each workstations
  - (c) tool records
  - (d) record of tool lives
5. Part production file consists of
  - (a) production rates for various machines
  - (b) in-process inventory
  - (c) inspection required
  - (d) all the above
6. Geometric modelling is concerned with
  - (a) the mathematical description of the geometry of an object
  - (b) the stress-strain calculations
  - (c) interference checking routings
  - (d) the documentation
7. CAM is mainly used for
  - (a) Computer-aided process planning
  - (b) Computer-assisted NC part programming
  - (c) Computer-aided line balancing
  - (d) All the above
8. CAPP is used for
  - (a) preparation of route sheet
  - (b) determining the sequence of operations
  - (c) both (a) and (b)
  - (d) none of these
9. The main difference between retrieval and generative CAPP is
  - (a) Retrieval CAPP system is based on part family and automated search of route sheet whereas generative CAPP is based on artificial expert that a human planner would use to generate route sheet.
  - (b) generative CAPP is based on part family and automated search of route sheet whereas retrieval CAPP system is based on logical procedures a human planner would use to generate route sheet.

- (c) retrieval CAPP is based on the determination of routesheet by an engineer whereas generative CAPP is based on part family and automated search of route sheet.

(d) none of these

10. DNC system

  - (a) can control more than one machines simultaneously
  - (b) generate the program automatically
  - (c) is used to discharge the product after finishing the operations
  - (d) dispose off the scrap produced

11. Lean manufacturing is concerned with the minimization of

  - (a) inventory
  - (b) delays
  - (c) scrapes
  - (d) all the above

12. The inventor of Toyota production system was

  - (a) Taiichi Ohno
  - (b) Kiichito Toyoda
  - (c) Eiji Toyoda
  - (d) Deming

13. Just-In-Time concepts was developed by

  - (a) Taiichi Ohno
  - (b) Kiichito Toyoda
  - (c) Eiji Toyoda
  - (d) Deming

14. Who is known as the father of Kanban System?

  - (a) Taiichi Ohno
  - (b) Kiichito Toyoda
  - (c) Eiji Toyoda
  - (d) Deming

15. Agile manufacturing is concerened with

  - (a) flexibility with speed
  - (b) speed
  - (c) flexibility
  - (d) quality of product

## Answers

1. (d)    2. (c)    3. (a)    4. (b)    5. (d)    6. (a)    7. (d)    8. (c)    9. (a)  
10. (a)   11. (d)   12. (a)   13. (b)   14. (a)   15. (a)

## **REVIEW QUESTIONS**

1. Explain the term ‘manufacturing systems’.
  2. What do you mean by flexible manufacturing system (FMS)? What are the components of FMS?
  3. Explain the various types of layout configurations used in an FMS.
  4. Discuss the advantages of FMS.
  5. Write short notes on (a) CAD/CAM, (b) CAPP, (c) CNC and (d) DNC.
  6. Explain the concept of lean manufacturing and also discuss the term TPS (Toyota Production System).
  7. Write notes on Muda, Mura and Muri.
  8. Discuss the concept of just-in-time manufacturing.

9. Discuss the concepts of Kanban to minimize the inventory.
10. What do you mean by agile manufacturing? How it differs from flexible manufacturing? Discuss the variables of agile manufacturing system.



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# CHAPTER ▶ 9

## Material Handling Systems

### 9.1 INTRODUCTION

Material handling is defined as the movement, storage, control and protection of materials and products throughout the production process, distribution, consumption and disposal. It is loading and unloading of different types of materials to and from the transporting vehicles/material handling devices. Material handling systems are applied in both manufacturing and distribution systems. It is a non-value-adding activity, but involves costs (Bolz and Stocker 1951). The three basic functions of material handling systems are picking up the load, transporting the load and setting the load down (Apple 2000; Raymond 1985). The production effectiveness can be increased by having the right quantity of material at the right places at the right time (Amstead et al. 1979; James 1977; Vijayaram 2006).

One of the major objectives of work study and plant layout is to minimize material handling time and cost. Material handling does not add any value to the product. It only helps in value addition at different workstations in the plant. Poor material handling can increase the cost of production. Therefore, material handling should be as minimum as possible. Material handling is considered as an integral part of the total manufacturing system.

### 9.2 RELATIONSHIP BETWEEN MATERIAL HANDLING AND PLANT LAYOUT

Plant layout is pre-production planning. Material handling and manufacturing process equipments are established as per requirement of plant layout. The material handling systems are to be integrated with the following material handling functions:

1. Transportation to the production facility
2. Receiving, storage and retrieval
3. Transportation to the work-centres/workstations
4. Transportation between operations and staging
5. Packaging and shipping

### 9.3 FUNCTIONS OF MATERIAL HANDLING SYSTEMS

The functions of material handling are as follows:

1. To plan the plant layout, building and production centre as per the suitability of a particular type of material handling equipments.
2. To minimize the movement of materials and material transportation time.
3. To optimize the speed of material handling equipments and devices.
4. To eliminate or minimize the backtracking.
5. To prevent the accident and facilitate the safe movement of materials.
6. To minimize the damage of materials during movement, loading and unloading.
7. To minimize the material handling costs.
8. To provide training to the persons operating the material handling equipments for their efficient handling and safe working.

### 9.4 OBJECTIVES OF MATERIAL HANDLING SYSTEMS

The objectives of materials handling are:

1. To increase the space utilization by keeping the materials in racks, one above the other. The material handling generally affects space requirements. Storing various types of materials like raw materials, parts, sub-assemblies, assemblies and finished parts is facilitated by material handling devices and methods, stored at proper space.
2. To improve the operating efficiency by reducing material handling. Increasing load per movement using the proper material handling devices improves productivity and saves cost of materials handling.
3. To ensure on-time delivery of the materials at different work centres/facilities. Materials, parts and finished products must be moved from the place of storage to their destination speedily and efficiently.
4. To reduce the production cost by increased capacity, improving working conditions, improving customer services, increased equipment and space utilization and reducing cost of materials handling.

### 9.5 PRINCIPLES OF MATERIAL HANDLING SYSTEMS

Material handling is a technique, the efficiency of which is governed by the following principles:

1. The materials should be moved minimally as much as possible. The selection of production machinery and the type of the plant layout should be such that the long movement of material may be eliminated as far as possible.
2. The time of each move should be minimized. This can be attained through the shortest routes and use of mechanized or automated material handling equipment in place of manual labour.
3. The distance of each move should be minimized. This can be accomplished through the use of the shortest routes and elimination of backtracking.

4. The principle of unit load should be applied. According to this principle, the materials should be moved in lots rather than on an individual basis. The optimum number of pieces should be moved in one unit. The principle of unit load avails the economies in the form of reduced loading, labour cost, packing cost, elimination of damage and pilferage, savings in time and the effective utilization of material handling equipment.
5. The gravity feed should be used wherever possible as it is the cheapest source of motive power.
6. Re-handling and backtracking of the materials should be avoided. This can be attained through effective layout and efficient routing and scheduling.
7. The appropriate material handling equipment should be selected so that the safety, efficiency and flexibility can be maintained.
8. The design of the container, pallets, drums, etc. should entail economy in handling and reduce the material damages during the transit.
9. The material handling service must not interfere with the production flow.
10. The safety of the people must be taken care of and it should be the first preference.
11. The provision of the stand-by facilities should be made so that the sudden breakdown may not shut down the operations.
12. There should be periodical maintenance of material handling devices.
13. The material handling services should be evaluated periodically and necessary changes should be incorporated wherever it is possible.
14. The adverse effects on the environment should be minimized when selecting material handling equipment and procedures.
15. Simplify handling by eliminating, reducing or combining unnecessary movements and/or equipment.

## 9.6 TYPES OF MATERIAL HANDLING EQUIPMENTS

Material handling equipments can be classified into the following five major categories (Michael 2012):

1. *Transport equipments:* These are the equipments used to move material from one location to another (e.g. between workplaces, between a loading dock and a storage area, etc.). The major subcategories of transport equipments are conveyors, cranes and industrial trucks.
2. *Positioning equipments:* These equipments are used to handle material at a single location so that the material is in the correct position for subsequent handling, machining, transport or storage. Unlike transport equipments, positioning equipment is usually used for handling at a single workplace.
3. *Unit load formation equipments:* These equipments are used to restrict materials so that they maintain their integrity when handled a single load during transport and for storage. If materials are self-restraining (e.g. a single part or interlocking parts), then they can be formed into a unit load with no equipment.

4. *Storage equipments:* These equipments are used for holding or buffering materials over a period of time. Some storage equipments may include the transport of materials (e.g. the storage/retrieval machines of an automated storage/retrieval system).
5. *Identification and control equipments:* These equipments are used to collect and communicate the information that is used to coordinate the flow of materials within a facility and between a facility and its suppliers and customers. The identification of materials and associated control can be performed manually with no specialized equipment.

## 9.6.1 Transport Equipments

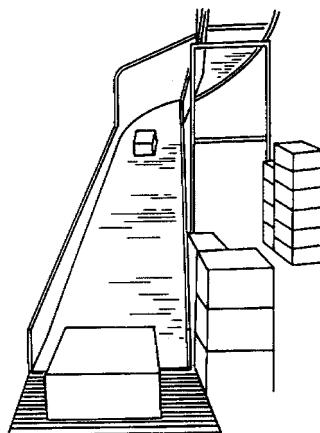
### A. Conveyors

#### Chute conveyor (Figure 9.1)

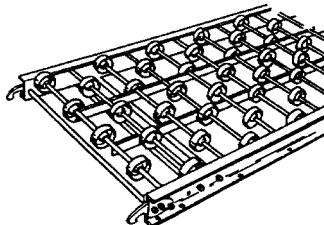
Chute conveyor is used to accumulate the materials in bulk in the shipping area. It is used to link two handling devices and convey items between floors. But it is difficult to position the materials accurately using this device.

#### Wheel conveyor (Figure 9.2)

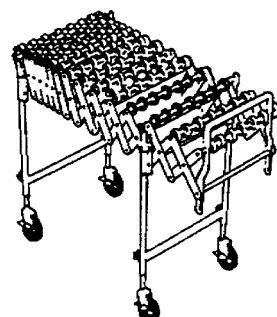
Wheel conveyor consists of a series of skate wheels mounted on a shaft or axle, where the spacing of the wheels is dependent on the load being transported. Slope for gravity movement depends on load weight. It is used for light-duty applications and different versions of the device are available.



**Figure 9-1:** Chute conveyor



**Figure 9-2:** Wheel conveyor

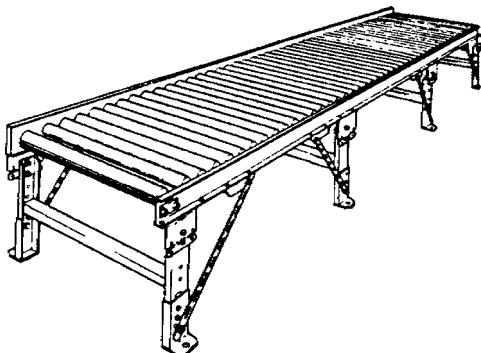


#### Roller conveyor (Figure 9.3)

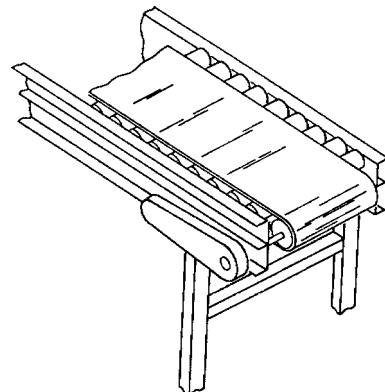
Roller conveyor is used to handle rigid materials. This may be powered or gravity fed. The roller used may be tapered to give proper orientation to the materials.

**Flat belt conveyor (Figure 9.4)**

Flat belt conveyor is used for transporting light- and medium-weight materials between operations, departments, levels and buildings, when an incline or decline is required. It provides considerable control over the orientation and placement of the load. Some operations are difficult to perform on the belt such as smooth accumulation, merging and sorting. The belt is roller- or slider bed-supported; the slider bed is used for small and irregularly shaped items.



**Figure 9-3:** Roller conveyor



**Figure 9-4:** Flat belt conveyor

**Bucket conveyor (Figure 9.5)**

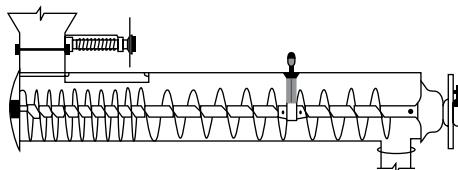
Bucket conveyor is used to move bulk materials in a vertical or an inclined path. Buckets are attached to a cable, chain or belt as shown in Figure 9.5. Buckets are automatically unloaded at the end of the conveyor run.

**Screw conveyor (Figure 9.6)**

Screw conveyor consists of a tube through which a shaft-mounted helix revolves to push loose material forward in a horizontal or an inclined direction. It is most commonly applied in the processing industry and chemical processing.



**Figure 9-5:** Bucket conveyor



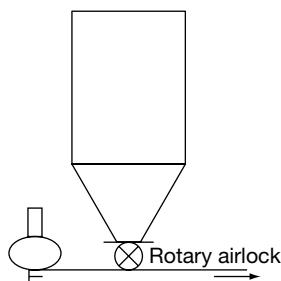
**Figure 9-6:** Screw conveyor

### Pneumatic conveyor (Figure 9.7)

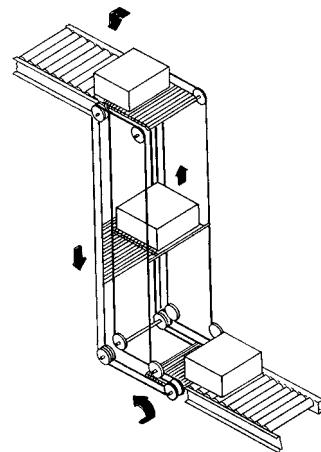
Pneumatic conveyor can be used for both bulk and unit movement of materials. In this conveyor, air pressure is used to convey materials through a system of vertical and horizontal tubes. The major advantages are that the material is completely enclosed and it is easy to implement turns and vertical moves. It moves a mixture of air and solid and pushes material from one entry point to several discharge points. Pull systems move material from several entry points to one discharge point. Push-pull systems are combinations with multiple entry and discharge points.

### Vertical lift conveyor (Figure 9.8)

Vertical lift conveyor is used for low-frequency intermittent vertical transfers. The carrier is used to raise or lower a load to different floors. The device can be manually or automatically loaded and/or controlled and can interface with horizontal conveyors.



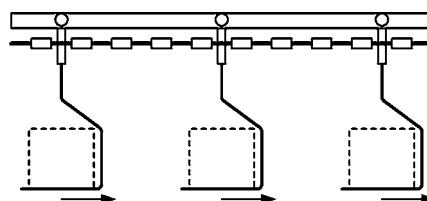
**Figure 9-7:** Pneumatic conveyor



**Figure 9-8:** Vertical lift conveyor

### Trolley conveyor (Figure 9.9)

This device uses a series of trolleys supported from or within an overhead track. Trolleys are equally spaced in a closed loop path and are suspended from a chain. Carriers are used to carry multiple units of a product. It is commonly used in processing, assembly, packaging and storage operations.



**Figure 9-9:** Trolley conveyor

## B. Cranes

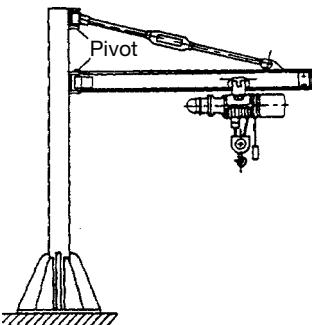
Cranes are used to move loads over different paths within a restricted area where the use of a conveyor cannot be justified due to intermittent flow of materials. They provide more flexibility in movement than conveyors, but less flexible in movement than industrial trucks. Most cranes utilize hoists for vertical movement, although manipulators can be used if the precise positioning of the load is required.

### Jib crane (Figure 9.10)

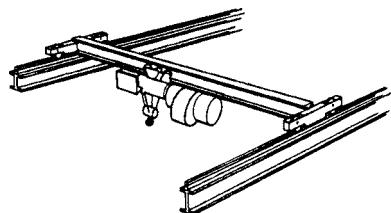
Jib crane operates like an arm in a work area, where it can function as a manipulator for positioning the jobs. A hoist is attached to the arm for lifting load. The arm can rotate 360° and the hoist can move along the arm.

### Bridge crane (Figure 9.11)

Bridge mounted on tracks that are located on opposite walls of the facility and it enables three-dimensional handling.



**Figure 9-10:** Jib crane



**Figure 9-11:** Bridge crane

### Gantry crane (Figure 9.12)

Gantry crane can be single leg, double leg or mobile types. This is similar to a bridge crane except that it is floor supported at one or both ends instead of overhead supported and is used to span a smaller portion of the work area as compared to a bridge crane. The supports can be fixed in position or they can travel on runways. It can be used outdoors when floor is supported at both ends.

### Stacker crane (Figure 9.13)

Similar to a bridge crane except that instead of a hoist, it uses a mast with forks or a platform to handle unit loads. It is equipped with fork trucks on a rail. It is used for storing and retrieving unit loads in storage racks, especially in high-rise applications in which the racks are more than 15 m height. It can be controlled by remote or by an operator in a cab on the mast.

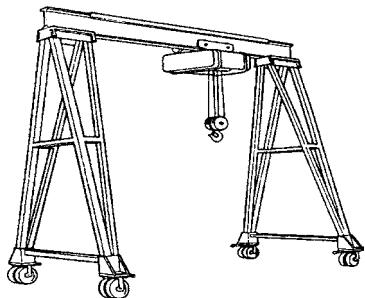


Figure 9-12: Gantry crane

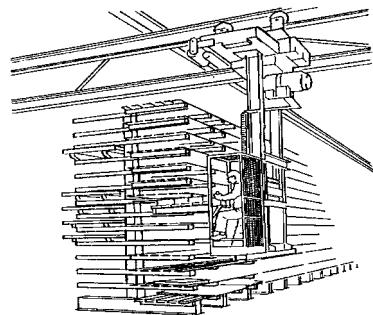


Figure 9-13: Stacker crane

### C. Industrial Trucks

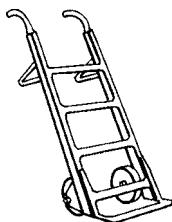
Industrial trucks are used to move materials over horizontal paths with no restrictions on the area covered. They can provide vertical movement if the truck has lifting capabilities. They are used when there is an intermittent flow volume such that the use of a conveyor cannot be justified. They provide more flexibility in movement than conveyors and cranes.

Some important characteristics of trucks are enumerated as follows:

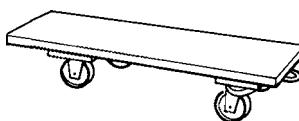
1. The trucks may or may not have forks for handling pallets, or truck may have a flat surface on which loads are placed.
2. The truck may have manual or powered vertical and/or horizontal movement capabilities.
3. For non-automated trucks, the operator can ride in the truck or the operator is required to walk with the truck during travel.
4. The truck can be used to lift loads for stacking purposes.
5. The lift truck may be designed to have a small turning radius.
6. The truck may be automated so that it can transport loads without involving an operator.

#### Hand trucks (Figure 9.14)

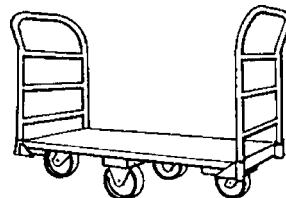
Hand truck is used to move the lighter load. Two-wheeled trucks are used to tilt the load and to move from one place to another place. Dolly is three- or multiple-wheeled hand truck with a flat platform in which, the load is used for pushing since it has no handle. Floor hand truck is four- or multiple-wheeled hand truck with handles for pushing or hitches for pulling.



(a) Two-wheeled hand truck



(b) Dolly

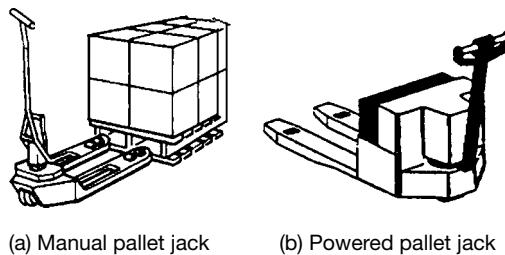


(c) Floor hand truck

Figure 9-14: Hand trucks

### Pallet jacks (Figure 9.15)

In this jack, front wheels are mounted inside the end of the forks and extend to the floor as the pallet is only lifted enough to clear the floor for subsequent travel. Reversible pallets cannot be used, double-faced non-reversible pallets cannot have deck boards where the front wheels extend to the floor, and enables only two-way entry into a four-way notched-stringer pallet because the forks cannot be inserted into the notches.



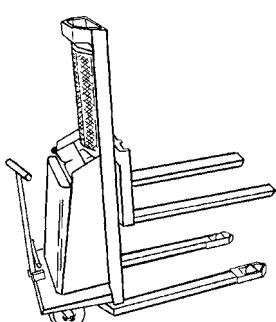
**Figure 9-15:** Pallet jacks

### Walkie stacker (Figure 9.16)

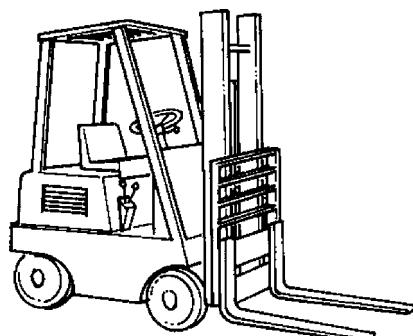
Walkie stacker is used for manual lifting and/or travel (and straddle load support). This device may be powered by an engine.

### Counterbalanced lift truck (Figure 9.17)

This is also referred to as fork lift truck. The weight of the vehicle behind the front wheels of the truck counterbalances weight of the load and front wheels act as a fulcrum or pivot point. The rated capacity used to lift the load is more than 13 ft height. It is very useful in the plant because of their flexibility: indoor/outdoor operation over a variety of different surfaces; variety of load capacities available; and a variety of attachments available.



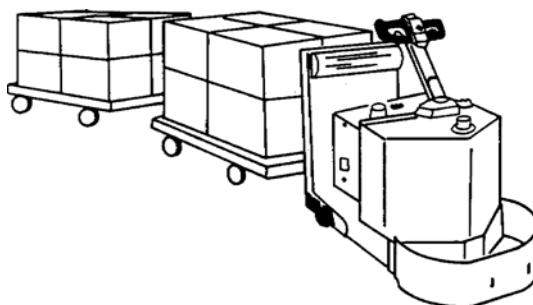
**Figure 9-16:** Walkie stacker



**Figure 9-17:** The counterbalanced lift truck

### Automatic guided vehicle (Figure 9.18)

Automatic Guided Vehicles (AGVs) do not require any operator. They are good for high labour cost, hazardous, or environmentally sensitive, low-to-medium volume, medium-to-long distance random material flow operations (transport between work cells in a flexible manufacturing system environment). Two means of guidance used for AGV systems are: *fixed path* (wire, tape, paint for guidance) and *free-ranging* (software guided), but absolute position estimates (from lasers) are needed to correct dead-reckoning error.



**Figure 9-18:** Automatic guided vehicle (AGV)

### 9.6.2 Positioning Equipments

Positioning equipments are used to give proper orientation or position of the material or job. This equipment has following advantages:

1. It raises the productivity of each worker when the frequency of handling is high.
2. It improves the product quality and limits damage to materials and equipment when the item handled is heavy or awkward to hold and damage is likely through human error or inattention, and
3. It reduces fatigue and injuries when the environment is hazardous or inaccessible.

#### Lift/Tilt/Turn table (Figure 9.19)

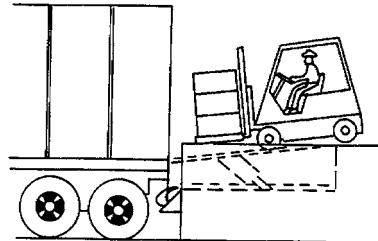
Lift table is used when positioning involves the lifting, tilting or turning off a load.

#### Dock leveller (Figure 9.20)

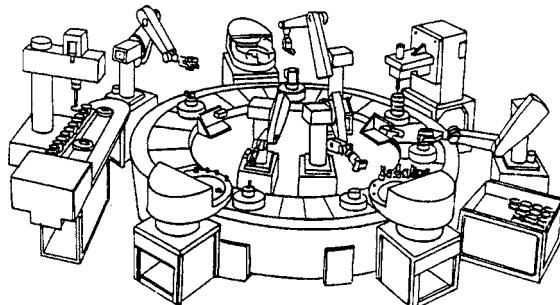
This is used at loading docks to compensate for height differences between a truck bed and the dock.

#### Rotary index table (Figure 9.21)

Rotary indexing table is used for the synchronous transfer of small parts from station to station in a single work-centre. The circular table rotates in discrete intermittent steps to move parts between stations located along its perimeter. Since each part moves between stations at the same

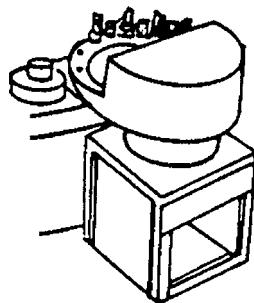
**Figure 9-19:** Lift/Tilt/Turn table**Figure 9-20:** Dock leveller

time, it is difficult to put buffers between stations. It is different from conveyors used as in-line indexing machines, where linear transfers can take place between multiple work-centres separated by long distances, since a rotary index table is restricted to circular transfers with a single compact work-centre.

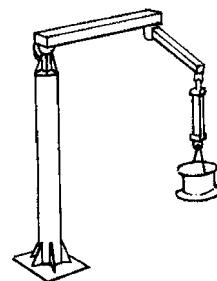
**Figure 9-21:** Rotary index table

### **Parts feeder** (Figure 9.22)

Part feeders are used for feeding and orienting small identical parts, particularly in automatic assembly operations. Motion of parts in a random pile is channelled so that each part automatically assumes a specified orientation, where the symmetries of a part define its possible orientations. Motion can be imparted through vibration, gravity, centrifugal force, tumbling or air pressure. Vibratory bowl feeder is the most versatile type of parts feeder in which the parts are dumped into a bowl and then vibrated and moved uphill along a track towards an outlet, where rejected parts fall off the track and are recycled. Parts feeders can be used to provide inspection capabilities with respect to the shape and weight of parts (e.g. the coin feeder of a vending machine).



**Figure 9-22:** Parts feeder

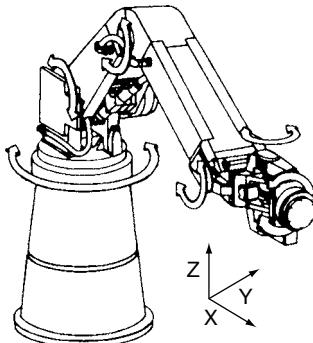


**Figure 9-23:** Manipulator

### **Manipulator (Figure 9.23)**

The manipulator is used for vertical and horizontal translation and rotation of loads. It can be powered manually, electrically or pneumatically. Manipulator's end-effector can be equipped with mechanical grippers, vacuum grippers, electromechanical grippers, or other tooling. Manipulators fill the gap between hoists and industrial robots and they can be used for a wider range of positioning tasks than hoists and are more flexible than industrial robots due to their use of manual control.

### **Industrial robot (Figure 9.24)**



**Figure 9-24:** Industrial robot

Industrial robot is used in positioning to provide variable programmed motions of loads. Intelligent industrial robots utilize sensory information for complex control actions, as opposed to simple repetitive 'pick-and-place' motions. Industrial robots are also used for parts fabrication, inspection and assembly tasks. It consists of a chain of several rigid links connected in series by revolute or prismatic joints with one end of the chain attached to a supporting base and the other end free and equipped with an end-effector. Robot's end-effector may be equipped with

mechanical grippers, vacuum grippers, electromechanical grippers, welding heads, paint spray heads or any other tools. Although similar in construction, an industrial robot is distinguished from a manipulator by the use of programmed control logic as opposite to manual control.

### **9.6.3 Unit Load Formation Equipments**

Unit load formation equipments are various types of packaging of the materials those can provide the safe movement or handling of the materials. These can be boxes, bags, cartons, pallets, skids, slip sheets, containers, etc. There are some advantages and disadvantages of unit loads as discussed below:

#### ***Advantages of Unit Loads***

- More items can be handled at the same time, thereby reducing the number of trips required and, potentially, reducing handling costs, loading and unloading times, and product damage.
- Enables the use of standardized material handling equipments.

#### ***Disadvantages of Unit Loads***

- Time spent forming and breaking down the unit load.
- Cost of containers/pallets and other load restraining materials used in the unit load
- Empty containers/pallets may need to be returned to their point of origin.

### **9.6.4 Storage Equipments**

The most common reason for storing a product allows the other elements of production to operate more efficiently on a per unit basis because the fixed costs associated with utilizing the element can be spread over more products, e.g. storing up to a truckload of product in a facility reduces the per-unit costs of shipping; and buffering or storage of WIP enables batch production which reduces the per-unit set-up costs.

Other potential reasons for storage include time bridging that allows the product to be available when it is needed; processing for some products, storage can be considered as a processing operation because the product undergoes a required change during storage; and securing, for example, nuclear waste storage. The storage equipments used in an industry are racks, frames, shelves, bins, drawers, AS/RS, etc.

#### ***Automatic Storage/Retrieval Systems***

It consists of an integrated computer-controlled system that combines the storage medium, transport mechanism, and controls with various levels of automation for fast and accurate random storage of products and materials. Storage/retrieval (S/R) machine in an automatic storage/retrieval system (AS/RS) operates in narrow aisles, serving rack slots on both sides of the aisle; it can travel in horizontal and vertical directions at the same time.

### **9.6.5 Identification and Communication Equipments**

Identification and control equipment is used to collect and communicate the information that is used to coordinate the flow of materials within a facility and between a facility and its suppliers

and customers. The major types of identification and communication equipment are bar codes, radio-frequency tag, magnetic strips, machine vision, electronic data interchange, etc.

### **Selection of Material Handling Equipments**

Selection of the material handling devices is based on the following factors:

1. Size, shape, weight and volume of the materials.
2. Types of materials to be moved.
3. Layout of the plant, production centre and other facilities.
4. Types of buildings, width of roads, corridors, pavements, floor levels, doors, height ceiling, size of rooms and storing places, single-storey buildings and multi-storey buildings.
5. Methods, direction and movement of materials.
6. Types of machines and equipments used for production.
7. Production processes and methods of manufacturing.
8. Material handling cost.
9. Cost of material handling equipments.
10. Material handling efficiency and automation needed.
11. Life of equipment.
12. Safety of equipments.
13. Maintenance of material handling equipments.
14. Ease of operation.



### **SUMMARY**

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In this chapter, we have discussed about objectives, functions and principles of material handling equipments. Different types of handling equipments are introduced. Finally, the bases for equipment selection have been explained. The main purpose of the chapter was to familiarize the readers with material handling equipments frequently used in industries.

### **MULTIPLE-CHOICE QUESTIONS**

1. Material handling is defined as
  - (a) the activities of material management
  - (b) the function of purchasing of the materials for manufacturing
  - (c) the movement, storage, control and protection of materials and products throughout the production process
  - (d) quality testing of the materials
2. Which of the following is NOT a function of material handling systems?
  - (a) to minimize the movement of materials and material transportation time.
  - (b) to optimize the speed of material handling equipments and devices.
  - (c) to eliminate or minimize the material handling cost.
  - (d) to minimize the material costs.

3. Which of the following is NOT an objective of material handling systems?
  - (a) to increase the space utilization by keeping the materials in racks, one above the other
  - (b) to improve the quality of the product to be manufactured
  - (c) to improve the operating efficiency by reducing material handling
  - (d) to ensure on-time delivery of the materials at different work centres/facilities
4. Materials handling covers the following processes except
  - (a) Storing the product
  - (b) Handling the materials
  - (c) Moving an assembly
  - (d) Surface finishing
5. Material handling process can be justified with the process characteristics
  - (a) high repeatability
  - (b) high volume
  - (c) line flows
  - (d) all the above
6. When encountering bottlenecks in a process, the AGV generally responds in the following manners
  - (a) Fixes the problems causing the bottlenecks
  - (b) Reroutes the materials to alternative workstations
  - (c) Halts the process and sounds the alarm
  - (d) All the above
7. Material handling in automobile company is done by
  - (a) Overhead crane
  - (b) Trolley
  - (c) Belt conveyor
  - (d) All the above
8. Which of the following material handling equipment has counter balance type weight?
  - (a) Gravity conveyor
  - (b) Forklift
  - (c) Overhead crane
  - (d) Robot
9. A device used for lifting or lowering an object suspended from a hook at the end of retractable chains is called
  - (a) Jib crane
  - (b) Hoist
  - (c) Portable elevator
  - (d) Chain conveyor
10. Which of the following devices is used in part orientation in an automated assembly line?
  - (a) Part feeder
  - (b) Manipulator
  - (c) Dock leveller
  - (d) Rotary indexing table
11. In a chemical industry, frequently used conveyor is
  - (a) belt conveyor
  - (b) roller conveyor
  - (c) screw conveyor
  - (d) all the above
12. Which of the following is a positioning equipment?
  - (a) belt conveyor
  - (b) trolley conveyor
  - (c) trucks
  - (d) industrial robot
13. Industrial truck is frequently used in
  - (a) Warehouses
  - (b) Manufacturing work of shop floor
  - (c) Both (a) and (b)
  - (d) None of these

14. Automatic guided vehicle can be controlled by
  - (a) a software
  - (b) sensors using wire, tape, paints
  - (c) lasers
  - (d) all the above
15. Which of the following devices is used for vertical and horizontal translation and rotation of loads?
  - (a) Manipulator
  - (b) AGV
  - (c) Lift table
  - (d) Rotary index table

## Answers

1. (c)
2. (d)
3. (b)
4. (d)
5. (d)
6. (b)
7. (a)
8. (b)
9. (b)
10. (a)
11. (c)
12. (d)
13. (c)
14. (d)
15. (a)

## REVIEW QUESTIONS

1. Define the term ‘material handling systems’ and highlight the relationship between material handling and plant layout.
2. Explain the functions of material handling systems.
3. Explain the objectives of material handling systems.
4. Discuss the principles of material handling systems.
5. What are the various types of material handling equipments used in a manufacturing industries? Explain in brief.
6. Differentiate the use of positioning equipments and the use of transport equipments.
7. Write short notes on automated guided vehicle (AGV).
8. Write short notes on automated storage (AS) and automated retrieval (AV).



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# CHAPTER ▶ 10

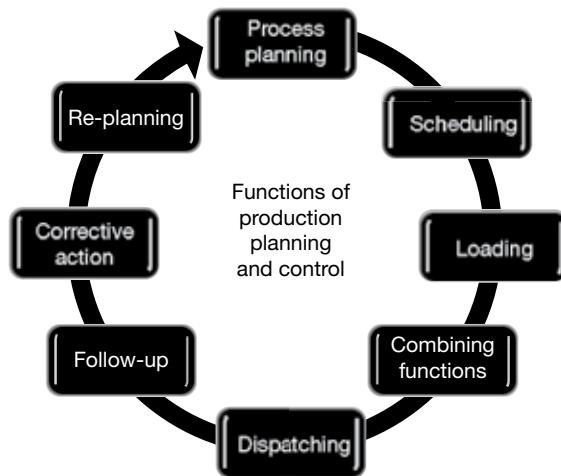
## Production Planning and Control

### 10.1 INTRODUCTION

The efficiency of a production system can be improved by manufacturing the required quantity of the product of the right quality at the right time by the cheapest method of production. Production planning and control (PPC) is a tool to coordinate all the manufacturing activities and looks after the manufacturing activities. Broadly, PPC consists of planning, routing and dispatching in the manufacturing process. Planning and control are two basic and interrelated managerial functions. They are so interrelated that they can be considered as being one function. Planning is the preparation activity while control is the post-operation function. Planning sets the objectives, goals, targets on the basis of available resources with their given constraints. Similarly, control involves assessment of the performance; such an assessment can be done effectively only when some standards are set in advance. Planning involves setting up to such standard. The controlling is made by comparing the actual performance with these present standard and deviations are ascertained and analysed.

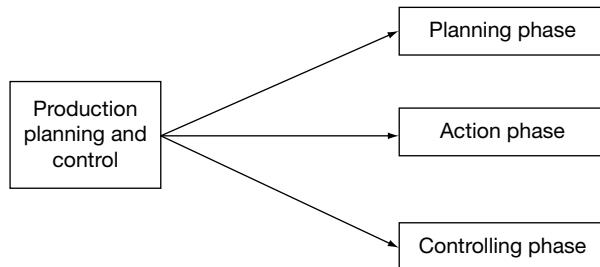
Production is an organized activity of converting raw materials into useful products, but before starting that work of actual production, production planning is done in order to anticipate possible difficulties and decide in advance as to how production should be carried out in the best and economical way. Since only the planning of production is not sufficient, hence the management takes all possible steps to see that project or plan chalked by the planning department are properly adhered to and the standards set and are attained in order to achieve it. At the same time, control over production is exercised. The aim of production control is to produce the products of the right quality, in the right quantity at the right time by using the best and least expensive methods. The major functions of PPC are shown in Figure 10.1.

Process planning is a process of routing and preparation of route sheet. Scheduling is the part of planning and concerned with the schedule when an activity will start and when the same will be finished. Loading is a prose of assigning the jobs to the machines and the personal. While it is easy to define ‘where’ as process planning, ‘how much work’ as loading, and ‘when as scheduling’ separately, but in actual operations these three functions are often combined and performed concurrently. Dispatching is the execution of the planned functions. It is action and controlling phase of the PPC. Reporting or follow-up is the activity to follow the planned activity and report it. The manufacturing activity of a plant is said to be ‘in control’ when the actual performance is within the objectives of the planned performance. Corrective action is a controlling activity. A plant in which all manufacturing activity runs on schedule in all probability is not



**Figure 10-1:** Functions of production planning and control

being scheduled to its optimum productive capacity. With an optimum schedule, manufacturing delays are the rule, not the exception. Re-planning revises routes, loads and schedules; a new plan is developed, if it is required. All the above PPC functions can be grouped into three classes: planning phase, action phase and controlling phase as shown in Figure 10.2.



**Figure 10-2:** Three phases of production planning and control

## 10.2 OBJECTIVES OF PRODUCTION PLANNING AND CONTROL (PPC)

The objectives of PPC are enumerated as follows.

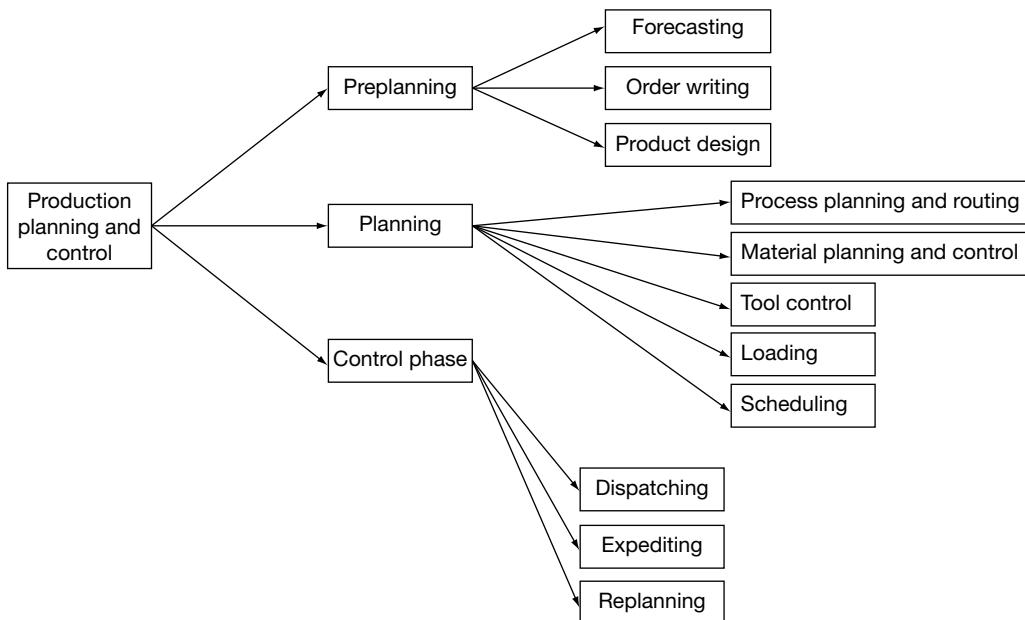
1. To utilize the resources effectively.
2. To ensure the steady flow of production.
3. To estimate the resources.
4. To ensure optimum inventory.

5. To coordinate activities of the departments.
6. To minimize wastage of raw materials.
7. To improve the labour productivity.
8. To help capture the market.
9. To provide a better work environment.
10. To facilitate quality improvement.
11. To improve the consumer satisfaction.
12. To reduce the production costs.

Now let us discuss each objective of production planning one by one.

### 10.3 PRODUCTION PLANNING

The production planning and control again can be subdivided into preplanning, planning and control phases. The activities of preplanning, planning and control may be considered to take place in a time sequence. The preplanning is completed before production. Planning takes place immediately before the start of production and control is exercised during production. The main operational functions under preplanning are forecasting, order writing and product design. Similarly, the operational functions under planning phase are process planning and routing, material planning and control, tool control, loading, and scheduling. In the control phase, the main operational functions are dispatching. These operational functions are shown in Figure 10.3.



**Figure 10-3:** Preplanning, planning and control phases

### ***Preplanning***

It is the procedure followed in developing and designing a work or production of a developing and installing a proper layout or tools. It may involve many functions of the organization and draws upon forecasting, product design, jigs and tool design, machine selection and estimating to enable the proper design to be made. In short, preplanning decides what shall be made and how it shall be made. In respective manufacturing, a large uneconomic output could be produced if preplanning is not done in an effective manner.

*Forecasting:* It is an important function before planning. It may be sales forecasting, technology forecasting or demand forecasting. It sets a target before starting the production activities, i.e. what and how much should be produced. Forecasting has already been discussed in detail in Chapter 3 in this book.

*Order writing:* It is a process to authorize one or more persons to take charge of a particular job or task.

*Product design:* It is another important process. Product design and development has already been discussed in detail in Chapter 7 in this book. Once a product is designed, it prevails for a long time, therefore various factors are to be considered before designing it. These factors are standardization, reliability, maintainability, servicing, reproducibility, sustainability, product simplification, quality commensuration with cost, product value, consumer quality and needs and tastes of consumers.

*Volume decision:* Volume decision is based on forecasting. Also, it is seen that it is decided on the basis that it is made to meet the order or made for stocking. If the manufacturer observes that demand will increase in the future, then he will make the items in stock; otherwise, he will manufacture only on receiving the order from the market or customers.

*Make or buy decision:* After designing a product, it is to be decided that what are to be made inside the plant and what are to be purchased from vendor, i.e. make or buy decision. Make or buy decision is based on the availability of resources and facility inside the plant. If it is available in sufficient amount, then it is to be manufactured in the plant. If it is not available, then it is to be observed the availability of vendors for the same components or subassembly and to be bought from the vendor.

The make decision is influenced by the following factors:

1. The lower cost of the components because the firm does not have to pay for vendor's overhead or profit.
2. Improved availability because the firm does not have to depend on vendors.
3. Better quality control of the components.
4. Availability of manufacturing resources to produce the components.
5. Maintaining the design secrecy or trade secrecy.

The buy decision is influenced by the following factors:

1. The provision of lower cost, higher quality with faster delivery by the vendor.
2. Requirement of special machine to produce that item.

3. Purchasing cost is lower than the manufacturing cost.
4. Holding the patent of manufacturing of that item by the vendor.
5. Ability of the vendor to meet the buyer's need regarding quality, quantity, price and delivery period.

### **Planning**

This stage decides where and when the product shall be made. It includes the sequencing of operations, routing and the time schedule for manufacturing. It also states procedures for material planning and supplies, machine loading and deliveries. To perform the functions properly, it will need past records of performance and to control statistic which may be obtained from pre-planning, cost control or progress.

*Routing:* Under this function, the path and sequence of operations are established. To perform these operations, the proper class of machines and personnel required are also worked out. The main aim of routing is to determine the best and cheapest sequence of operations and to ensure that this sequence is strictly followed. The main aim of the sequencing of jobs on different machines is to minimize the idle time on the machines. The details of the sequencing with the Johnson's rule has been discussed in Chapter 17 in this book.

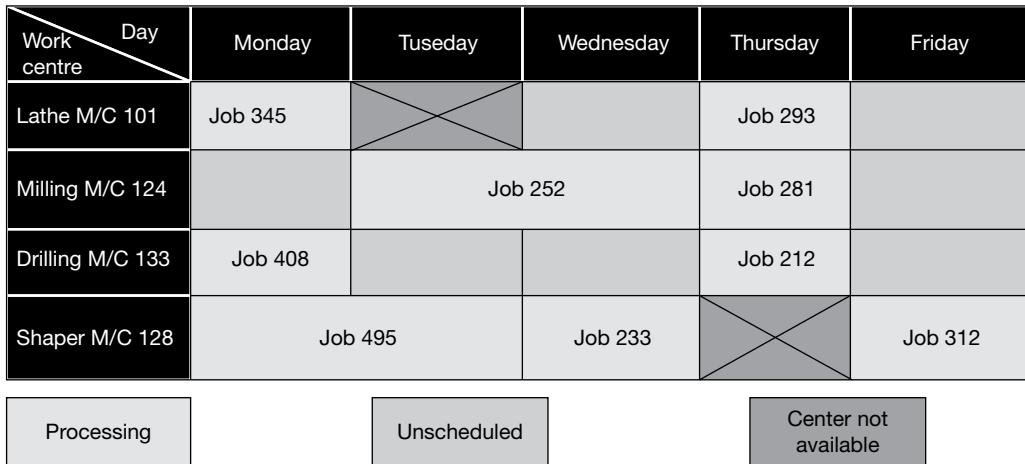
*Route sheet:* It is a document providing the information and instruction for converting the raw materials into finished products. It defines each step of the operation and the paths through which the product or job will follow during the operations. Route sheet consists of the following information:

1. The required operations and desired sequence
2. Machines or equipments to be used
3. Set-up time and operation time
4. The required tools, jigs and fixtures
5. Detailed drawing of pars, sub-assemblies and final assemblies
6. Design specifications of the product
7. Specification of raw material to be used
8. Manufacturing and operations parameters and their specifications
9. Inspection procedure and metrology tools used for inspection
10. Packing and handling instruction of the parts or sub-assemblies.

*Estimating:* Estimating is concerned with the requirement of resources required to produce an item. When production orders and detailed operation sheet are available with specification feeds, speed and use of auxiliary attachments and method, the operation time can be worked out. It may consequently result in wide scatter of operation times and unduly large fluctuations and perhaps instabilities in time schedules. Estimating manpower, machine capacity and materials required meeting the planned targets are the key activities before budgeting for resources.

*Loading:* Loading is the process of assigning specific jobs to the machines, men or work centre on relative priorities and capacity utilization. It ensures maximum utilization of resources such as men and machines and avoid bottlenecks in production. Overloading and underloadings are

avoided. The load chart can be shown using Gantt chart as shown in Figure 10.4. Using the Gantt load chart, it can be observed that when the machines are busy and are available or unscheduled. It also shows that when machines are not available due to the certain reason like a breakdown or repairing activity.



**Figure 10-4:** Gantt load chart

**Scheduling:** Scheduling provides the information that when an activity will start and when the same will be finished. It ensures that the products are completed as per the schedule. It means working out of time that should be required to perform each operation and also the time necessary to perform the entire series as routed. It mainly concerns with time element and priorities of a job. It is not an independent decision as it takes into account the following factors.

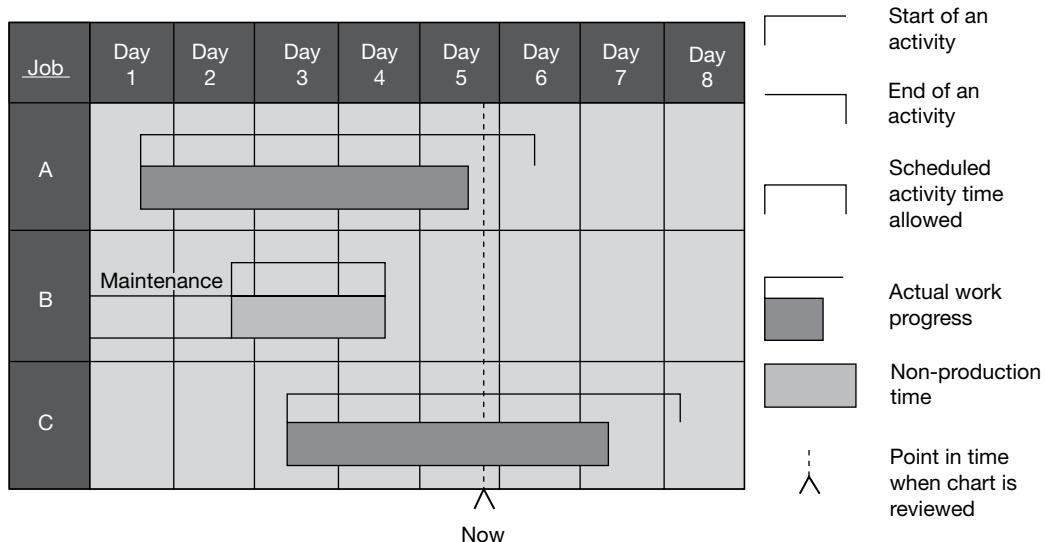
1. Physical plant facilities of the type required to process the material being scheduled.
2. Personnel who possesses the desired skills and experience to operate the equipment and perform the type of work involved.
3. Necessary materials and purchased parts.

**Forward and backward scheduling:** Forward scheduling starts just the requirements are known and produces a feasible schedule, though it may not meet the due dates. It frequently results in excessive work-in-process inventory. Backward scheduling begins from the due date and schedules the final operation first. It is produced by working backward through the processes. Resources may not be available to accomplish the schedule.

**Master schedule:** Scheduling starts with preparation of master schedule which is weekly or monthly breakdown of the production requirement for each product for a definite time period, by having this as a running record of total production requirements. The entrepreneur is in a better position to shift the production from one product to another as per the changed production requirements. A master schedule is followed by the operator schedule which fixes the total time

required to do a piece of work with a given machine or which shows the time required to do each detailed operation of a given job with a given machine or process.

Scheduling can be shown on the Gantt chart as shown in Figure 10.5. The various symbols have been used in this chart. ┌ is used to start an activity; └ is used to end an activity; ┌─┐ is used to show the scheduled activity time or the total activity time; ┌─█ is shows the actual progress of the activity; ┌─█ shows the non-production time and; ↗ shows the time at which the chart is reviewed.



**Figure 10-5:** Gantt scheduling chart

### **Production Control**

Production control determines whether resources to execute the plan have been provided and if not, takes action to correct the deficiency. This refers to the stage of ensuring that the planned action is intact carried out. Control initiates the plan at the right time using dispatching and thereafter control makes appropriate adjustments through progressing to take care of any unforeseen circumstances that might arise. It includes the measurement of actual results, comparison of the same with the planned action and feeding back information the planning stage to make any adjustments required. The pattern of control is seen in material control, machine utilization, labour control, cost control and quality control.

*Dispatching:* It is the execution of the planned functions. This authorizes to start a work which has already been planned under the above discussed planning functions that are mainly routing and scheduling. It is releasing of orders and instructions for the starting of production in accordance with the route sheets and schedule charts.

*Expediting/Follow-up/Progressing:* Expediting ensures that the work is carried out as per the plan and delivery schedules are met. Expediting includes activities such as status reporting, attending to bottlenecks or holdups in production removing the same, controlling variations or deviations from planned performance levels, following up and monitoring progress of work through all stages of production, coordinating with purchase, stores, tool room and maintenance departments and modifying the production plans and re-plan if necessary.

*Inspection and replanning:* Inspection is the process of examining an object for identification or checking it for verification of quality and quantity in any of its characteristics. It is an important tool for ascertaining and controlling the quality of a product. Inspection is an appraisal activity that compares goods or services to a standard.

If there is any gap between the existing and planned performance level, replanning is done to fill the gap and to improve the performance. Re-planning revise routes, loads and schedules; a new plan is developed. In manufacturing, this is often required. Changes in market conditions, manufacturing methods, or many other factors affecting the plant will often indicate that a new manufacturing plan is needed.

## 10.4 FACTORS AFFECTING PPC

There factors affecting PPC are as follows:

1. *Use of computers:* Today, computers are used in most of the production activities, such as automation, computer-integrated manufacturing, computer-aided design and manufacturing, computer-aided process planning, computer-aided quality control, information and communication technology, radio frequency identification, automated storage and retrieval systems, etc.
2. *Seasonal variations in the demand:* Demand for certain products is affected by seasons, for instance, umbrellas, woolen clothes and raincoats. PPC must take such changes into consideration while planning and control activities of inputs and outputs.
3. *Test marketing:* With an aggressive marketing strategy, new products are to be test-marketed in order to know the trends. This is a short-cycle operation, intermittent in nature and often upsets regular production.
4. *After sales service:* This has become an important parameter for expansion of the target markets. In supporting the after sales services, many items are returned for repair. These are unscheduled work and also overload the production line.
5. *Losses due to unpredictable factors:* Losses may occur due to accidents, fire, earthquakes, tsunami and theft of production inputs, mainly materials and components. These are unpredictable. Shortage of input or disruption in the supply of the inputs due to such factors upsets the planned production schedule in time and quantity.
6. *Losses due to predictable factors:* There are losses of inputs, due to natural engineering phenomena like production losses and changes in consumption of materials and occurrence of defective.
7. *Production of order:* There are many occasions when in the last minute prioritization of existing orders takes place due to external pressure. These changes in priority are often decided by sufficiently high level of management.

8. *Design changes:* Design changes by R & D and the engineering department forces the PPC to change the input materials and process.
9. *Rejection and replacement:* There are many occasions when sub-assemblies or finished goods are rejected during production stage or final inspection. PPC must cater for contingency plans to take care of rework without affecting scheduled quality.



## SUMMARY

In this chapter, we have discussed all the functions of PPC. The production planning is divided into three parts: preplanning, planning and control. Preplanning covers the functions performed before production of the components, and planning consists of the functions performed during production, and control is used to alter the process to meet the target of the performance level. Objectives and factors affecting the PPC are discussed in detail.

## MULTIPLE-CHOICE QUESTIONS

1. Gantt chart is used for
  - (a) Forecasting
  - (b) Production schedule
  - (c) Inventory control
  - (d) Routing
2. Routing presents
  - (a) Utilization of manpower
  - (b) Utilization of machine
  - (c) Flow of materials in plant
  - (d) Inspection of materials
3. The routing function is a production system design, which is concerned with
  - (a) Production schedule
  - (b) Loading of the machine
  - (c) Quality assurance of the product
  - (d) None of the above
4. Scheduling
  - (a) prescribes the sequence of operations to be performed
  - (b) is concerned with the starting of processes
  - (c) determine the programme for the operations
  - (d) none of the above
5. In manufacturing management, the term ‘dispatching’ is used to describe
  - (a) dispatch of sales order
  - (b) dispatch of factory mail
  - (c) dispatch of work order through shop floor
  - (d) none of the above
6. Loading is the process of
  - (a) calculating the workload on the machine
  - (b) assigning specific jobs to the machines
  - (c) balancing the load on the machines
  - (d) none of these

7. Make or buy decision is a part of
  - (a) Production planning
  - (b) Production control
  - (c) Preproduction planning
  - (d) None of these
8. Forecasting is a part of
  - (a) Production planning
  - (b) Production control
  - (c) Preproduction planning
  - (d) None of these
9. Production planning includes
  - (a) sequencing of operations
  - (b) routing
  - (c) time schedule for manufacturing
  - (d) all the above
10. Expediting ensures that,
  - (a) the production is carried out as per the plan and delivery schedules are met
  - (b) product are delivered to the customer as per schedule
  - (c) materials are purchased as per date of receipt
  - (d) none of these
11. Production planning and control (PPC)
  - (a) is tool to coordinate all the manufacturing activities
  - (b) is used to recognize the need of customers
  - (c) is used to chase the competitors in the market
  - (d) none of the above
12. Which of the following is NOT a part of production control?
  - (a) dispatching
  - (b) expediting
  - (c) replanning
  - (d) master schedule
13. Which of the following is NOT a part of preproduction planning?
  - (a) Forecasting
  - (b) Make or buy decision
  - (c) Loading
  - (d) Product design
14. Order writing is a part of
  - (a) preproduction planning
  - (b) production planning
  - (c) production control
  - (d) none of these
15. Route sheets consist of the following information
  - (a) the required operations and desired sequence
  - (b) machines, tools, jigs and fixtures or equipments to be used
  - (c) set-up time and operation time
  - (d) all the above

## **Answers**

1. (b)      2. (c)      3. (d)      4. (c)      5. (c)      6. (b)      7. (c)      8. (c)      9. (d)  
10. (a)     11. (a)     12. (d)     13. (c)     14. (a)     15. (d)

## REVIEW QUESTIONS

1. Explain the functions of production planning and control. What are the objectives of production planning and control?
2. Explain the various phases of production planning and control.
3. What is preplanning? Explain the various activities lie under the phase preplanning.
4. What do you mean by rout sheet? Explain the information required to prepare a route sheet.
5. What is scheduling? Use a Gantt chart to explain the scheduling.
6. Consider an example and show the Gantt load chart used in production planning and control.
7. What do you mean by production control? Explain it.
8. What is the difference between dispatching and loading?
9. Write short notes on (a) Expediting, (b) inspection and replanning, (c) Estimating.
10. What are the factors influencing the production planning and control?



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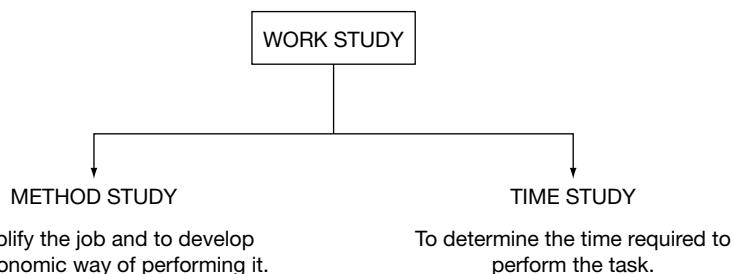
# CHAPTER ▶ 11

## Work Study and Ergonomics

### 11.1 INTRODUCTION

Work study is a process of investigation of the works done in an organization and finding the most effective way of doing the job with efficient utilization of the available resources (man, material, money and machinery) within least possible time. Work study consists of two broad studies, i.e. method study and time study as shown in Figure 11.1. Method study is concerned with the way of performing a job and time study is concerned with measuring the time to complete the job.

Work study is a generic term for techniques, particularly method study and work measurement, which are used for the examination of human work in all its contexts, and which lead systematically to an investigation of all the factors which affect the efficiency and economy of the situation being reviewed, in order to seek improvements. (Adebayo 2007; ILO 1981)



**Figure 11-1:** Classification of work study

#### **Purposes of Work Study**

The following are the purposes of conducting work study:

1. To standardize the method of doing a task.
2. To determine the standard time for doing a task, for use in the wage payment/determination.
3. To minimize the unit cost of production by selecting proper machine tools, optimum cutting parameters and proper process.

4. To minimize the materials movement and idle time of the workers and machines by proper layout of the plant.
5. To eliminate the unnecessary human motions in performing a task.
6. To utilize the facilities such as man, machine and materials most effectively.

## **11.2 METHOD OR MOTION STUDY**

Methods engineering grew out of the pioneering developments of Frank B. Gilbreth, and his wife, Lillian M. Gilbreth, who developed many of the tools of ‘motion study’ as a part of formulating a systematic approach to the analysis of work methods. Frank B. Gilbreth first became interested in methods, analysis as an outgrowth of his observations of bricklaying work.

Methods engineering is a necessary function to ensure the most efficient methods. This activity is most frequently performed by industrial engineers; however, all engineers should be concerned with work methods. It is related to the study of methods for performing a job to design effective and efficient work system in order to achieve process improvements, improved layout, better working environment and reduced fatigue. Method study is associated with the reduction of the work content of a job or operation, while work measurement (WM) is mostly concerned with the investigation and reduction of time consumption or any ineffective time associated with it. Mundel (1946) published the method engineering with several studies in his textbook *Systematic Motion and time study*. Method study and time study are interrelated to each other. To know the effectiveness of a method, one of the important parameters is time consuming by the method which can be known by time study or work measurement.

### **Steps to Conduct Method Study**

*Select:* In this step, a job or process is selected which is to be considered for method study.

*Record:* All the relevant facts/information about the process or the job is collected through data collection or direct observation.

*Examine:* All the data collected in second step are examined properly.

*Develop:* Some of the efficient and economical methods of doing that job or process are developed.

*Evaluate or Measure:* Results of different alternative methods are evaluated and the best one is selected.

*Define:* New method among the alternative is defined and presented.

*Install:* New method as standard practice is installed and persons in applying it are trained to use the new method.

*Maintain:* Control procedure is established and maintained.

### **Objectives of Method Study**

The following are the objectives of method study:

1. To improve the process of doing the work.
2. To improve the plant layout.

3. To minimize the human motions and fatigue of the operators.
4. To maximize the utility of man, machine and material.
5. To improve the overall working environment.

### ***Factors Influencing the Method Study***

In method study, an industrial engineer has to consider the following factors:

1. Motion economy factors.
2. Economic factors.
3. Technical factors.
4. Human factors.
5. Operational complexity.
6. Delays.

*Motion economy factors:* When a job is considered for method study, one has to think of whether there is a possibility of reducing the unnecessary motions. The jobs that are performed regularly, the micro-motion studies may expose such unnecessary motions. By economizing these motions, we can save human energy and utilize it for higher productivity.

*Economic factors:* Before conducting the study, one has to think is it worth to conduct motion or micro-motion study? If a work selected for investigation results in a very small economy, it is suggested that the study should not be conducted. One may have to consider the economy in removing the bottlenecks in movements and process such as movement of materials in one operation to the other, layout of machines, operational sequences, etc.

*Technical factors:* When a new method is developed and implemented, it is required to check availability of adequate technical knowledge and resources required in the method. This is a constraint for development of new methods. In most of the cases, technical factors and economic factors are interlinked.

*Human factors:* This factor is difficult to evaluate because it is associated with physiological, psychological and anatomical features of human beings. But these difficulties can be overcome to some extent through scientific methods such as ergonomics. If workers and their representatives are allowed to participate in problem formulation and implementation, the problem can be eliminated. The full and active participation from the workers' side and a due consideration to their views from the management side would reduce the difficulties and gives an impression among the workers that the new methods are for their betterment.

*Operational complexity:* When a method study is taken up for a job, most often the managers forget to take such operational complexity/flexibility into consideration. It is the industrial engineer's responsibility to bring out detailed description about the operational flexibility or complexity, so that the critical examination can be facilitated to check under these constraints.

*Delays:* There are three types of delays—unavoidable delay, avoidable delay and planned rest pauses. In unavoidable delay, operators cannot avoid the delay, for example, operator inevitably waits for the material to come or for his turn to do the job; heat-treated products have to be cooled

before an operation on a certain machine; welded joints are allowed to cool before taking for operation, etc. The avoidable delays can be avoided by proper planning. For example, a machinist need not have to wait if cleaning is done prior to his arrival. The third type of delay is planned rest pauses. These are also inevitable but somewhat flexible to change their schedules or duration in accordance with ergonomics studies. The industrial engineers who conduct method study must take these delays into consideration while developing the new method. Method study uses the symbols mentioned in Figure 11.2.

S. No.	Activity	Symbol	Remarks
1.	Operation	○	Indicates the main process, method or operation.
2.	Inspection	□	Indicates an inspection for quality and/or check for quantity.
3.	Transport	→	Indicates the movement of workers, materials or equipment from place to place.
4.	Temporary storage or delay	D	Indicates a delay in the sequence of events: for example, work waiting between consecutive operations, or any object laid aside temporarily without record until required.
5.	Permanent storage	▽	Indicates a controlled storage in which material is received into or issued from a store under some form of authorization; or an item is retained for reference purposes.

**Figure 11-2:** Symbols used in method study

### **Tools Used in Method Study**

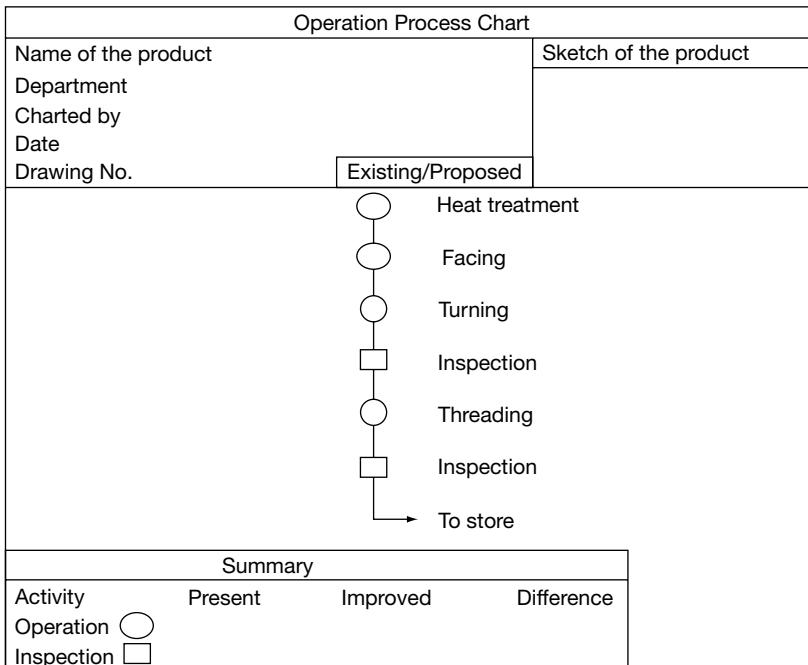
The tools used for method study may be classified under three heads as discussed below:

- Charts;
- Diagrams; and
- Photographic aids.

There are two types of charts: A-type and B-type. A-type charts indicate the sequence of events happening in the order in which events occur irrespective of time scale, whereas B-type charts indicate the sequence of happening in the order in which events occur on a time scale. Some of the charts in common use are discussed in the following subsections.

#### **11.2.1 Operation Process Chart**

Operation process chart is a graphical representation of a process containing the name of the product, assembly or sub-assembly, name of person making chart, date, a sketch of the product, sequence of operations, the name of the department, drawing number and summary. The operation process chart is shown in Figure 11.3.



**Figure 11-3:** Operation process chart

### ***Advantages of Operation Process Chart***

There are following advantages of operation process chart:

1. It represents basic activities and summarizes overall picture of an operation.
2. It visualizes the process and makes easy to understand.
3. It gives a scope for improvement of operations or their sequence by combination or rearrangement or elimination.
4. It helps in identifying the stages of inspection and thus provides an idea for better layout of the workplace.
5. It shows a way to think of improvements in material handling and reducing the material movements.

### ***Limitations of Operation Process Chart***

The following information is not given in operation process chart:

1. Distance between workstations or shops.
2. Time to travel between workstations or shops.
3. Any interference or delays between two workstations.
4. Back tracking, if any.

## 11.2.2 Flow Process Chart

The flow process chart was introduced by Frank Gilbreth to members of ASME in 1921 as the presentation *Process Charts, First Steps in Finding the One Best Way to Do Work* (Gilbreth 1921). It is an improved version of the operation process chart, which is set out sequence of flow of a product, man or machine or material by recording all events in its process, in terms of appropriate symbols. Flow process chart is shown in Figure 11.4. It has advantages over operation process chart due to having following extra information:

1. Distance between workstations or shops.
2. Time to travel between the workstations or shops.
3. Any interference during transportations and possible causes.
4. Backtracking, if any.
5. Mode of transport between two workstations.

Flow Process Chart				
Name of the product Department Charted by Date Drawing No.	Sketch of the product			
	Existing/Proposed	Distance (m)	Time (min.)	Remarks
Mild steel rod from store	○ □ → D ▽			
To heat treatment lab.	○ □ → D ▽	25	30	Hand trolley
Heat treatment	○ □ → D ▽			
To lathe machine	○ □ → D ▽	30	32	Hand trolley
Facing	○ □ → D ▽			
Turning	○ □ → D ▽			
Inspection	○ □ → D ▽			
Threading	○ □ → D ▽			
Inspection	○ □ → D ▽			
To store	○ □ → D ▽	28	29	Hand trolley
Store	○ □ → D ▽			
Summary				
Activity	Present	Improved	Difference	
Operation	○			
Inspection	□			
Transportation	→			
Delay	D			
Storage	▽			

**Figure 11-4:** Flow process chart

### 11.2.3 Two-hand Process Chart

This chart is used to study the motions of two hands of workers and bring out the effectiveness and hence the efficiency. It is defined as a process chart in the activities of a worker's both hands recorded in relationship to each other. This technique is widely used for micro-motion study. The purposes of two-hand process chart are:

1. To balance the motions of both hands and reduce fatigue.
2. To reduce or eliminate non-productive motions.
3. To shorten the duration of productive motions.
4. To train new operators in the ideal method.
5. To sell the proposed method.

A structure of two-hand process chart is shown in Figure 11.5.

Method : Existing/Proposed Operation : Operator : Start of Activity : Department :		Machine No. : Operation No. : Charted by : End of Activity : Date :		
Description Left Hand	Symbol	Symbol	Description Right Hand	
	○ → ▽ D □	○ → ▽ D □		
<b>Summary</b>				
Activity/Symbol	Existing		Proposed	Saving
	LH	RH	LH	RH
Operation (○)				
Transportation (⇒)				
Storage (▽)				
Delay (D)				
Inspection (□)				
Distance in Mtrs.				

**Figure 11-5:** Two-hand process chart

### 11.2.4 Multiple Activity Chart

In all the above charts, they represent only one of resource like a worker or an equipment or material, etc. But the relation between a man and machine or man and material, etc. is not shown. The multiple activity charts can show all these relationships in a very compact manner. This chart enables the industrial engineer to study whether any unnecessary waiting times can be eliminated. A multiple activity chart is a chart in which the activities of more than one subject (worker, machine or item of equipment) are recorded on a common time scale to show their interrelationship. Using separate vertical columns or bars to represent the activities of different operatives, machines against a common time scale, the chart shows clearly the periods of idleness on the chart on the part of any of the subjects during the process.

The multiple activity charts are prepared in one of the four ways as: one man—one machine chart, two men—one machine chart, one man—two machine chart, and two or more men—one or more machine chart. The main advantages of this chart are to use as a tool for finding the optimum number of men and machines, capacity calculations, incentive calculations, manpower planning, material planning, job scheduling, etc. For example, multiple activity charts concerning operation and a man-machine process charts are shown in Figures 11.6 and 11.7, respectively.

### 11.2.5 SIMO Chart

It is one of the recording techniques of micro-motion study and records simultaneously the different therbligs performed by different parts of the body of one or more operators simultaneously. ‘SIMO’ stands for ‘Simultaneous Motion Cycle Chart’. It is used to show the simultaneous nature of motions; more generally, a therblig chart for two-hand work with motion symbols is plotted vertically with respect to time, showing the therblig and a brief description for each activity, and individual time’s values and body-member detail. A two-hand SIMO chart is shown in Figure 11.8.

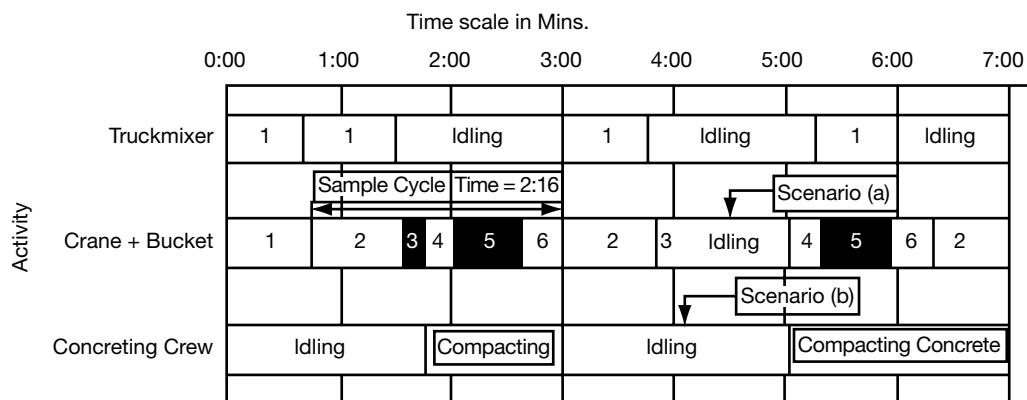


Figure 11-6: Multiple activity chart for concern operation

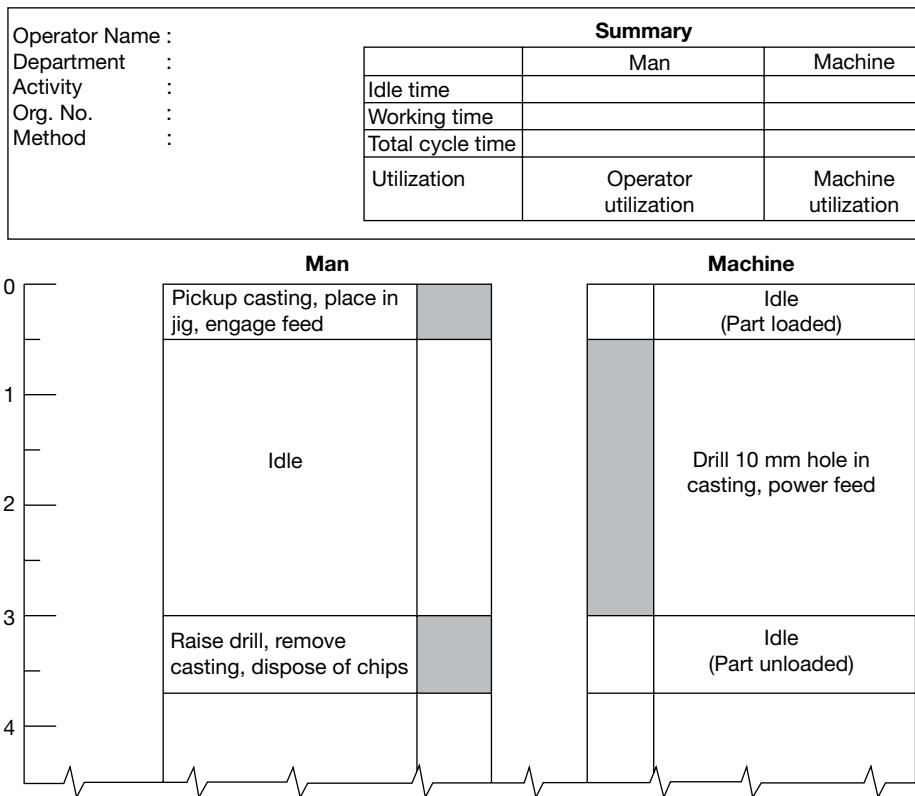


Figure 11-7: Man-machine process chart

SIMO CHART							
OPERATION :				FILM NO. :			
Method :				OPERATION NO. :			
Charted by :				Date :			
LEFT HAND	TH	AB	TIME	RIGHT HAND	TH	AB	TIME
To bolt		TE	12	To nut		TE	12
Grasp bolt		G	10	Grasp nut		G	10
Move to assembly		TL	10	Move to assembly		TL	10
Head bolt		H	10	Put nut over the bolt		H	10

Figure 11-8: SIMO Chart

The SIMO chart represents the following features:

1. The cycle time.
2. Percentage utilization of the hands or the part of the body taking operation.
3. Total working time of the part of the body.
4. Total idle time of the part of the body.
5. Total time for which two or more parts of the body are involved simultaneously.

### 11.2.6 Flow Diagram

It is an effective tool for layout of single or multiple products. It uses all the symbols used in a flow process chart, but it is better than the operation process chart and flow process chart as it is easy to draw. In this technique, a scaled drawing of the plant is used on the drawing sheet. Therefore, the actual distance between workstations and the total distance travelled by the work piece could be shown on a certain scale. A sample flow diagram is shown in Figure 11.9. A square shows the position of the machine or workstation.

### 11.2.7 String Diagram

This is very simple and easy to use. It can be used for single or multiple products. In the case of multiple products, multicolours of strings are used. In this technique, no symbol is used. It uses a drawing sheet. In place of squares (as in flow diagram), pins are used. Pin represents the workstations. The pins are connected by the strings showing the path of movement with distance. An example of string diagram is shown in Figure 11.10.

### 11.2.8 Travel Chart or Trip Frequency Chart

A travel chart is a tabular record for presenting quantitative data about movements of workers, materials or equipments between any numbers of places, workstations or departments over any given period of time. The string diagrams take rather a long time to construct and when frequent

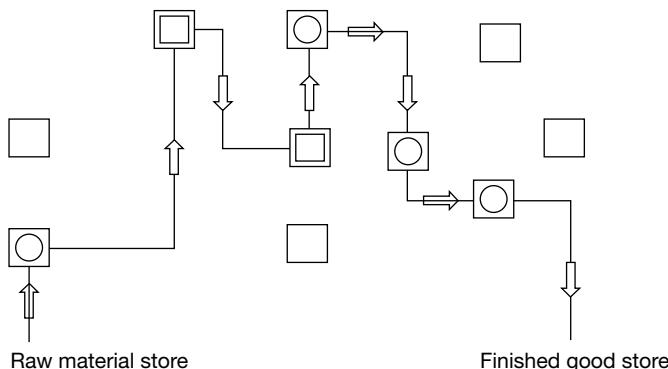


Figure 11-9: Flow diagram

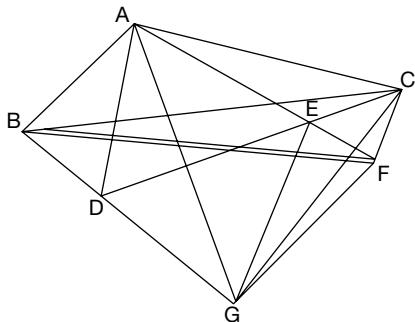


Figure 11-10: String diagram

movements along complex paths are involved, the diagram becomes very complex. Travel chart is a quicker and an easier recording technique. The travel chart is always a square and each small square represents a workstation. If, for example, there are 10 workstations, then the travel chart will have  $10 \times 10 = 100$  small squares. The squares from left to right along the top of the chart represent the places from where movement takes place; those down the left-hand edge represent the stations to which the movement is made. For example, consider a movement from station 2 to station 8. To record this, the study man enters the travel chart at the square numbered 2 along the top of the chart, runs his pencil down vertically through all the squares underneath this one until he reaches the square which is horizontally opposite the station marked 8 on the left-hand edge. This is the terminal square, and he will mark in that square to indicate his journey from station 2 to station 8 at 6 as shown in Figure 11.11. All journeys are recorded in the same way. The travel chart can also be made to indicate the weight/material moved per trip. This is the technique used in plant layout as block diagrams.

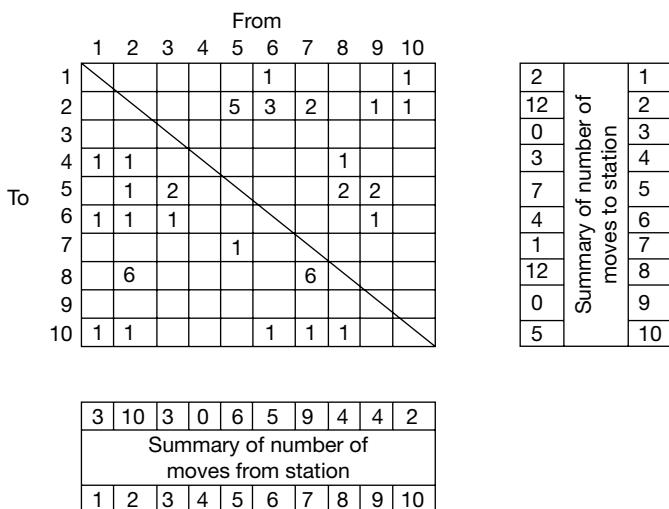


Figure 11-11: Travel chart

### 11.2.9 Photographic Aids

Still and video photography techniques are employed to record and analyse the operations and procedures to perform a task. There are different types of analysis such as memo-motion and micro-motion studies, cyclographs and chrono-cycle graphs, etc. All are very expensive methods involving special photographic equipment. Photographic aids are used for detailed investigation of very short duration, highly repetitive and high-speed operations. These aids are discussed in the following sections as micro-motion, memo-motion, cycle graph and chrono-cycle graph.

## 11.3 MICROMOTION STUDY

Micromotion study was proposed by Frank B. Gilbreth. It is a technique of recording and analysing the timing of the fundamental elements of an operation or activity with a view to achieving the best method of performing the operation. Frank B. Gilbreth considered that an operation consists of minute elements or sub-divisions. These elements may be repetitive or non-repetitive. He called these elements *THERBLIG*. Standard symbols and colours of *THERBLIG* are shown in Figure 11.12.

It is an analysis technique making use of motion pictures (videotape) taken at a constant speed. The film becomes a permanent record of both the methods being used and the time consumed in doing the work. Micromotion study provides a valuable technique for making minute analyses of those operations that are short in the cycle, contain rapid movements and involve high production over a long period of time. Thus, it is very useful in analysing operations such as the sewing of garments, assembly of small parts and a number of similar activities.

### **Purposes of Micromotion Study**

The following are the purposes of micromotion study:

1. To study the nature and path of movements for accomplishing the elements of an operation.
2. To find the most efficient way of accomplishing the elements.
3. To impact training to the operators regarding motion economy principles so that unnecessary motion or movement by the operator may be avoided.
4. To keep a permanent record of the most efficient way of doing a task for new reference.
5. To collect motion time data (MTD) for calculating synthetic time standard for different elements.

## 11.4 MEMO-MOTION STUDY

Memo-motion study was proposed by M. E. Mundel. It is a special type of micromotion study in which the motion pictures or video tape is taken at slow speeds. With this technique, photograph is also taken of the moving part of the body, as was done in micromotion study. But the speed of the movie camera is kept considerably lower. In micromotion study, the record is made by film operated at the speed of 16 frames per second or by constant speed camera at 1000 frames per minute. In memo-motion study, the photograph is taken by cameras using 50 to 100 frames per minute. The cost of operation is much less compared to that in case of micro-motion study.

Name of symbole		Therblig symbol	Color	Color symbol
Search	Sh		Black	
Select	St		Gray (light)	
Grasp	G		Lake red	
Transport empty	TE		Olive green	
Transport loaded	TL		Green	
Hold	H		Gold ochre	
Release load	RL		Carmine red	
Position	P		Blue	
Pre-position	PP		Sky-blue	
Inspect	I		Burnt ochre	
Assemble	A		Violet (heavy)	
Disassemble	DA		Violet (light)	
Use	U		Purple	
Unavoidable delay	UD		Yellow ochre	
Avoidable delay	AD		Lemon yellow	
Plan	Pn		Brown	
Rest	R		Orange	

**Figure 11-12:** Standard symbols for Therblig

Memo-motion is superior with respect to other forms of time and motion study for the following applications (Norbury 1954):

1. Single operator repetition work
2. The study of a group of men or machines
3. Team studies
4. Utilization studies
5. WM

Memo-motion study has been used to study the flow and handling of materials, crew activities, multi-person and machine relationships, stockroom activities, department store clerks and a variety of other jobs. It is particularly valuable on long-cycle jobs or jobs involving many interrelationships.

*Cycle graph:* In this method, a small electric bulb is attached to each part of the body which makes movement for carrying out operations. The path of movement is photographed by high-speed camera, such a record is called cycle-graph.

*Chronocycle graph:* It is an improvement over the previous method. In this method, a bulb is also attached to the moving part of the body, but the bulb is made slowly off and quickly on and photograph is taken. The path of the bulb, i.e. the path of movement appears dotted, the dots taking pear shape. If the movement is fast, the dots are spaced far apart, whereas they are closer if the speed is slow.

## 11.5 TIME STUDY OR WORK MEASUREMENT

It is related to time measurement of work required for a job, to arrive at the best method of work, improved planning and control. Work Measurement (WM) is a technique of establishing the proper time required in performing a job or work. Since it is concerned with measurement of time, so it is also called time study. WM is concerned with investigating, reducing and eliminating ineffective time, whatever may be the cause. It is the means of measuring the time taken in the performance of an operation or a series of operations in such a way that the ineffective time is shown up and can be separated out. The basic procedure for time study may be summarized below as:

*Select:* Select the job for study

*Divide:* Divide the operation into smaller elements.

*Measure and Record:* Measure and record the time taken by the operator for each element.

*Determine the number of observations required:* Determine the number of such readings necessary for getting the normal time (NT) for each element.

*Rate the performance:* Rate the operator's performance; determine the allowances and then finally calculate the standard time.

### **Objectives of Time Study**

There are following objectives of time study:

1. To analyse the activities for doing the job with a view to reducing or eliminating some unnecessary or repetitive activities so that human effort can be minimized.
2. To compare the time of performance by alternative methods.
3. To standardize the efficient method of performing a job.
4. To standardize the conditions for efficient performance.
5. To determine the optimum number of men and machines.
6. To determine man and the machines ratio for effective and efficient utilization of both.
7. To provide information and basis for production, planning and scheduling.
8. To balance the work of all the workers working in a group.
9. To determine the normal time for a job, to be used as a basis for wage incentive schemes for the workers.

### **Limitations of Time Study**

Some of the limitations of time study are given below as:

1. There are variations of the standard time determined by different observers. Even the same observer sets different standard time each time he is asked to conduct the time study.
2. Time study involves an element of subjectivity of the observer. Sufficient judgement has to be used by the observer in the choice of a measure of central tendency, deciding the degree of personal allowance and so on.
3. The standard time determined by time study may not be accurate because of incorrect performance rating of the operator under study.
4. Time study usually has an adverse effect on the workers. They may not show the normal behaviour pattern when they are being observed. Even the trade unions may resist stop watch time studies.

### **Difference Between Motion Study and Time Study**

The differences between the time and motion study are specified in Table 11.1.

**Table 11-1:** Difference between motion Study and time study

Basis	Motion study	Time study
Purpose	Concerned with the motions or movements of workers performing each operation on the job.	Concerned with the determination of time taken by the workers
Scope	Covers only workers.	Covers both workers and machines.
Procedure	Conducted by photographic procedures.	Conducted with the help of a stopwatch.

## **11.6 WORK MEASUREMENT TECHNIQUES**

Many work measurement techniques have been in use as are discussed in the following subsections:

### **11.6.1 Direct Time Study**

Direct time study (DTS) involves direct observation of a task using a stopwatch or other chronometric device to record the time taken to accomplish the task. The task is usually divided into work elements and each work element is measured separately. While observing the worker, the time study analyst evaluates the worker's performance/pace, and a record of this pace is attached to each work element time. This evaluation of the worker's pace is called performance rating. The observed time is multiplied by the performance rating to obtain the normal time for the element or the task.

$$\text{Normal time} = \text{Observed time} \times (1 + \text{Rating factor})$$

$$\text{Standard time} = \text{Normal time} + \text{Allowance}$$

### **Number of Readings Required**

If an activity is measured by a time measuring device or stop watch, then the question arises that how many readings should be taken. In general, we take readings for  $\pm 5$  per cent accuracy and 95 per cent confidence level. The number of readings can be calculated as:

$$N = \left[ \frac{40\sqrt{n\sum(fx^2) - [\sum(fx)]^2}}{\sum(fx)} \right]^2$$

where  $n = \sum f$  = Number of observations taken

$N$  = Number of observations required for  $\pm 5\%$  accuracy and 95% confidence level

$x$  = Value of observations

**Example 11.1:** For a particular element of a job, 20 observations (as shown in Table 11.2) were taken by an observer using a stopwatch. Check whether these number of observations are sufficient for  $\pm 5$  per cent accuracy with 95 per cent confidence level. Find the minimum number of observation required.

**Table 11-2:** Observations

Time $x$ (in min.)	0.04	0.05	0.06	0.07	0.08
Frequency	3	2	8	5	2

#### **Solution:**

The solution for this problem is shown in Table 11.3.

**Table 11-3:** Calculation table

$x$ (min)	$f$	$fx$	$x^2$	$fx^2$
0.04	3	0.12	0.0016	0.0048
0.05	2	0.10	0.0025	0.0050
0.06	8	0.48	0.0036	0.0288
0.07	5	0.35	0.0049	0.0245
0.08	2	0.16	0.0064	0.0128
Total	20	1.21	0.0190	0.0759 ( $\approx 0.076$ )

$$N = \left[ \frac{40\sqrt{n\sum(fx^2) - [\sum(fx)]^2}}{\sum(fx)} \right]^2 = \left[ \frac{40\sqrt{20 \times 0.076 - 1.21^2}}{1.21} \right]^2 = 338.77 \approx 339$$

Total number of observation is only 20, but the required number of observations is 339.

## 11.6.2 Westinghouse Rating System

The Westinghouse rating system provides the rating factor for skill, effort, conditions and consistency as shown in Table 11.4. This is used to determine the rating factor in different conditions and situations. These situations are categorized and scaled. This scaling is directly used to calculate the total rating. This rating is used to determine the normal time and hence the standard time.

**Table 11-4:** Westinghouse performance rating systems

Skill	Effort	Conditions	Consistency
Super	Excessive	Ideal	Perfect
A1 = +0.15	A1 = +0.13	A = +0.06	A = +0.04
A2 = +0.13	A2 = +0.12		
Excellent	Excellent	Excellent	Excellent
B1 = +0.11	B1 = +0.10	B = +0.04	B = +0.03
B2 = +0.08	B2 = +0.08		
Good	Good	Good	Good
C1 = +0.06	C1 = +0.05	C = +0.02	C = +0.01
C2 = +0.03	C2 = +0.02		
Average	Average	Average	Average
D = 0.00	D = 0.00	D = 0.00	D = 0.00
Fair	Fair	Fair	Fair
E1 = -0.04	E1 = -0.04	E = -0.03	E = -0.02
E2 = -0.10	E2 = -0.08		
Poor	Poor	Poor	Poor
F1 = -0.16	F1 = -0.12	F = -0.07	F = -0.04
F2 = -0.22	F2 = -0.17		

**Example 11.2:** The following observations (Table 11.5) of the actual time taken by a worker for doing a job repeatedly were taken by a time-study observer. The ratings for that worker are as follows:

*Skill: Good C1; Effort: Excellent B1; Condition: Ideal; Consistency: Good.*

Calculate the normal time.

**Table 11-5:** Observations

Observation no.	1	2	3	4	5	6	7	8	9	10
Time (min.)	0.3	0.4	0.7	0.4	0.5	0.6	0.5	0.6	0.6	0.4

**Solution:**

$$\begin{aligned} \text{Average time} &= \frac{0.3 + 0.4 + 0.7 + 0.4 + 0.5 + 0.6 + 0.5 + 0.6 + 0.6 + 0.4}{10} \\ &= \frac{5.0}{10} = 0.5 \text{ min} \end{aligned}$$

$$\text{Rating factor} = 0.06 + 0.1 + 0.06 + 0.01 = 0.19$$

$$\text{Normal time} = \text{Observed time} (1 + \text{Rating factor}) = 0.5(1 + 0.19) = 0.595 \text{ min.}$$

### 11.6.3 Synthetic Rating

Morrow introduced the system of synthetic rating. An observer records the actual time of performance for an element. Performance times for such elements have been standardized, which are known as ‘Predetermined Motion Time Standard (PMTS)’ values. The PMTS value of the elements from such tables is noted. The ratio of PMTS value of the element taken from tables to average actual observed time for the same element gives the rating factor.

$$R = \frac{P}{A}$$

where  $R$  is performance rating factor,  $P$  is PMTS value for the element in minutes (from tables), and  $A$  is the average actual time observed for the same element in minutes.

**Example 11.3:** The average actual times for the five elements of a task were measured and shown in Table 11.6. The predetermined motion times for three elements are also given. Calculate the performance rating factor.

Table 11-6:

Element no.	1	2	3	4	5
Avg. actual time, $A$ (min.)	0.12	0.14	0.22	0.34	0.12
Predetermined motion time ( $P$ )	0.14	—	0.20	—	0.10

**Solution:**

$$\text{Performance rating factor for element 1, } R = \frac{P}{A} = \frac{0.14}{0.12} = 116.67\%$$

$$\text{Performance rating factor for element 3, } R = \frac{P}{A} = \frac{0.20}{0.22} = 90.9\%$$

$$\text{Performance rating factor for element 5, } R = \frac{P}{A} = \frac{0.10}{0.12} = 83.33\%$$

$$\text{Average, } R = \frac{116.67 + 90.9 + 83.33}{3} = 96.96\%$$

### 11.6.4 Objective Rating

Objective rating was proposed by M. E. Mundel and is carried out in two steps.

*Step 1:* The speed or pace or tempo of the operator is rated against an objective pace standard. This objective pace standard is same for all the jobs irrespective of the job difficulties and its limiting effect on the pace. Mundel used the term ‘base time’ for this, time rated against pace.

$$B = P \times T$$

where  $B$  is base time,  $P$  is rated pace and  $T$  is observed time.

*Step 2:* Now, the base time obtained in step 1 is appraised by an adjustment factor, which Mundel calls job difficulty factor or job complexity or secondary adjustment. It is expressed as:

$$NT = B \times M$$

where  $NT$  is normal time,  $B$  is base time and  $M$  is job difficulty factor as given in Table 11.7.

**Table 11-7: Job difficulty factor**

S. no.	Description	Notation letter	Conditions	Per cent adjustment
1.	Amount of body used	A	Finger used loosely	0
		B	Wrists and Fingers	1
		C	Elbows, wrists and fingers	2
		D	Arms, etc.	5
		E1	Trunks, etc.	8
		E2	List with leg from floor	10
2.	Foot pedals	F	No pedals or one pedal with fulcrum under foot	0
		C	Pedal or pedals with fulcrum outside the foot	5
3.	Bi-manualness	H1	Hands help each other or alternate	0
		H2	Hands work simultaneously doing the same work	18
4.	Eye-hand coordination	I	Rough work, mainly feet	2
		J	Moderate vision	2
		K	Constant but not closed	4
		L	Watchful, fairly close	7
		M	Within 1/64"	10
5.	Handling requirements	N	Can be handled roughly	0
		O	Only roll control	2
		P	Must be controlled but may be squeezed	3
		Q	Handle carefully	4
		R	Fragile	5
6.	Weight		Identified by the actual weight for resistance	

**Example 11.4:** The observed time for an element is 0.24 min, the pace rating is 90 per cent and the sum of all secondary adjustment (for job complexity) amounts to 40 per cent, find the NT.

**Solution:**

$$\begin{aligned} \text{Normal time} &= \text{Observed time} \times \text{Pace rating} \times \text{Secondary adjustment} \\ &= 0.24 \times \frac{90}{100} \times \frac{140}{100} = 0.30 \text{ min.} \end{aligned}$$

### 11.6.5 Skill and Effort Rating

This system was introduced by Charles E. Bedaux. In this system, an observer is supposed to evaluate the work rate or speed of the worker's movement and how fast he is performing the motions, but not the movements and skill he is applying. Unlike the other methods, Bedaux introduced a unit 'B' that represents a standard minute, which is composed of

1. Work component
2. Relaxation component

The procedure follows the following steps:

1. Divide the operation into smallest measurable elements. (Smallest time is greater than or equal to 3 seconds).
2. Measure element with the help of a stopwatch having 60 divisions on its dial.
3. Take a sufficient number of observations and calculate average time.
4. Estimate the efficiency of the operator in terms of B values assuming the average worker must obtain 60 B per hour and maximum B value can be 80 B per hour. Thus, convert the observed time in terms of B's with reference to a standard of 60 B per hour values.
5. Allow the relaxation factors as shown in Table 11.8.

**Table 11-8:** Relaxation factors

Nature of work	Factor
Light work	1.1 to 1.2
Medium work	1.2 to 1.35
Heavy work	1.35 to 1.5
Very heavy work	1.5 to 3.0

6. Now calculate B values for a work element by the formula.

$$B = \frac{T_b \times V \times R_a}{60 \times 60}$$

where  $T_b$  is observed time in seconds,  $V$  is speed of work, and  $R_a$  is relaxation allowance.

7. The sum of all the values of various work elements gives B values per work piece.
8. Variable time (lost time) and setting are to be recorded regularly and special B values are to be provided accordingly.
9. Irregular times and disturbance are not to be included.
10. Purely machining times where the workman is not involved are evaluated separately and added as a method allowance, since only human effort is measured by the Bedaux system.

### 11.6.6 Physiological Evaluation of Performance Level

There is a positive relationship between the physical work and the amount of oxygen consumed. It has also been tried out to find the changes in heartbeat for various physical works. This is assumed to be the most reliable measure of muscular activity and studies are still going on by many experts in industrial engineering, bio-medical engineering and physiology.

*Allowances:* There are following types of allowance that are provided in the calculation of standard time:

*Personal allowance:* It includes restroom breaks, phone calls, water fountain stops, cigarette breaks (5 per cent typical), etc.

*Fatigue allowance:* Rest allowance to overcome fatigue due to work-related stress and conditions (5 per cent or more).

*Delay allowance:* Machine breakdowns, foreman instructions (5 per cent typical).

*Contingency allowance:* Additional allowance due to a problem with the task (e.g. raw material problem). It cannot be greater than 5 per cent.

*Policy allowance:* These allowances are intended to cover special work situations that are usually associated with a wage incentive system.

*Training allowance:* For workers whose responsibilities include teaching other new workers in their jobs.

*Learning allowance:* For workers who are learning a new task or new employees who are just beginning to work.

**Example 11.5:** An operator was kept under observation for 20 days. He was found working on 900 occasions and abstaining including idle time was 100. He produced 250 jobs during these days. The observation per day was for 5 hours only and the total number of observations was 1000. Take 110 as performance rating for operator and 20 per cent as allowance. Calculate the standard time.

**Solution:**

$$\begin{aligned}\text{Observed time} &= \text{Total working time} \times \frac{\text{Percentage of working time}}{100} \\ &= 20 \text{ days} \times 5 \text{ hours} \times 60 \text{ minutes} \times \frac{90}{100} = 5400 \text{ min}\end{aligned}$$

$$\text{Normal time per unit of job} = \text{Observed time} \times \text{Rating factor}$$

$$= \frac{5400 \text{ min}}{250} \times \frac{110}{100} = 23.76 \text{ min}$$

$$\text{Standard time} = \text{Normal time} + \text{Allowance} = 23.76 \times 1.2 = 28.52 \text{ min.}$$

## 11.7 PREDETERMINED MOTION TIME SYSTEM

It is a work measurement technique. PMTS is the Maynard Operation Sequence Technique (MOST), which was first released in 1972 (Zandin 2003). Manual operations are divided into fundamental elements and time to accomplish each element is tabulated. To get the standard time of any operation, the operation is divided into fundamental elements. The time of performance for each element is taken directly from the database. Therefore, before doing any operation, its standard time could be known. This is the difference with the time study where standard time is finalized after an operation is carried out. The common PMTS systems are mentioned as follows:

*Work factor system (WFS):* In WFS, the time for an operation is calculated on the basis of the use of some variables such as body member, distance moved, manual control, weight or resistance involved, etc.

*Method-time-measurement (MTM)*: This is a technique of method analysis and additionally it does the work of time study by providing time values for each motion.

*Basic motion time study (BMTS)*: This system was developed by Ralph Presgrave, G. B. Bailey and other members of the staff of J. D. Woods and Gordon Ltd. of Toronto, Canada, and was first used in 1950. The basic motion is defined as a complete movement of a body member, such as saying hand moves from rest and again comes to rest.

## 11.8 PRINCIPLE OF MOTION ECONOMY

The principles of motion economy can be classified into three groups (Kanawaty 1992; Meyers and Stewart 2002):

1. Principles related to the use of the *human body*;
2. Principles related to the arrangement of the *workplace*; and
3. Principles related to the *design of tools and equipment*.

Every work involves some or all of the THERBLIG. Each THERBLIG consumes time. To reduce the time taken for performing any work, some principles have been propounded by Barnes which are known as principles of motion economy.

### **Use of Human Body**

1. Both the hands should start and finish their motions at the same time.
2. Both the hands should not be made idle simultaneously, except while at rest.
3. Arms' motions should be symmetrical and simultaneous.
4. As far as possible, arrangements should be made for minimum movements of hands and body.
5. Straight-line movements should be availed in the presence of continuous curved movements.
6. Free swinging movements should be preferred over controlled and restricted movements.
7. Work should be arranged to give the operator repetitive nature of motion, i.e. rhythm in motion.
8. Work should be arranged to give minimum eye movement so that THERBLIG 'search' and 'find' take minimum time.
9. Momentum should be used for helping the operator, but the momentum should be minimized if it is to be overcome by the muscular effort of the operator.

### **Arrangement of the Workplace**

1. For all tools and workpieces, definite places should be fixed up.
2. To reduce searching time, tools and workplaces should be prepositioned.
3. The workpiece should be delivered at the nearest to the workstation by gravity feed, or bins, or containers or transfer machines.
4. Tools, materials and controlling levers should be located within easy reach of both the hands of the operators.
5. Tools and workpiece should be arranged according to the sequence of the operations.

6. Arrangement should be made for automatic disposal of the finished goods.
7. Adequate light, a comfortable chair, convenient height of the workpiece should be provided.
8. The workplace and the workpieces should have colour contrast.

### **Design of Tools and Equipments**

1. As far as possible, jig, fixture or foot-operated device should be used instead of hands for 'holding' the workpiece.
2. Tools should be combined, if possible.
3. If fingers are used, load to each finger should be given according to its capacity.
4. Handles of levers, cranes or large screw drivers should be made sufficiently large so that there is maximum contact with the hands.
5. The levers, cross bars, hand wheel, etc. should be so arranged that the operators can use them with the least change in their body position.

## **11.9 WORK SAMPLING**

Work sampling is a method to find the percentage of the day of worker is working and the percentage of the day he is idle. The working and idle time of a worker are used to fix up performance rate, and establish the standard time for an operation. The following types of sampling are used in an industry:

*Systematic sampling:* If the observations are taken at a fixed interval of time, it is called systematic sampling.

*Random sampling:* If the observations are taking random spaced time, it is called random sampling.

*Stratified sampling:* If observations are taken at a random spaced time after a finite number of regularly spaced time intervals, it is called stratified sampling.

### **Procedure for Work Sampling**

Five steps are involved in making a work-sampling study:

- (a) Identify the activity or activities that are to be considered for the study. For example, determine the percentage of time that equipment is working, remain idle, or under repair.
- (b) Estimate the proportion of time of the activity of interest to the total time (e.g. that the equipment is working 80 per cent of the time). These estimates can be made from the analyst's knowledge, past data, reliable guesses of others, or a pilot work-sampling study.
- (c) State the desired accuracy in the study results.
- (d) Determine the specific times when each observation is to be made.
- (e) After two or three intervals during the study period, recompute the required sample size by using the data collected thus far.
- (f) Adjust the number of observations, if appropriate.

### **Advantages of Work Sampling**

The following are the advantages of work sampling over the direct time measurement systems:

1. The activities that are difficult to measure using time study can be easily measured by work-sampling process.
2. The men or machines can be studied in groups with work sampling.
3. Since the observations are taken over a period of days or weeks, therefore, day-to-day variation cannot affect the result appreciably.
4. The chances of getting misleading results are less since the operators are not under direct observation.
5. The study is less time-consuming and less tiring for the observers.
6. No time measuring device is required.
7. The calculation is easy and less time-consuming.

### **Limitations of Work Sampling**

The following are limitations of work sampling:

1. Study of an individual machine or operator is not economical.
2. The operator may change the work pattern in the presence of an observer.
3. This study presents an average result and it cannot give information regarding individual activities.
4. The approach of statistical analysis creates confusion among the workers.
5. Minimum number of observations, randomness in observations, instantaneous observations are essential and must be focused on the accuracy of the results.

### **Minimum Number of Observations**

The number of observations in work sampling affects the accuracy of prediction. For any desired level of accuracy, sufficient number of observations is required. Suppose the desired level of accuracy is given as  $\pm e$  per cent and the corresponding value of  $z$  for the given confidence level is known, the number of observations can be calculated as:

$$z \times \sigma = e \text{ (error)} \Rightarrow \sigma = \frac{e}{z}$$

$$N = \frac{pq}{\sigma^2} = \left( \frac{z}{e} \right)^2 pq$$

where  $z$  is the corresponding value of confidence level,  $\sigma$  is standard deviation,  $p$  is the percentage occurrence of an activity or delay being measured, expressed as a percentage of total number of observations as a decimal,  $q$  is the percentage of non-occurrence of an activity,  $e$  is accuracy in fraction, and  $N$  is the number of observations.

**Example 11.6:** Calculate the required number of observations necessary for an accuracy of  $\pm 3$  per cent and confidence level of 98 per cent, if the total number of observations of machines working is 3000 and idle 1000.

**Solution:**

$$p = 1000/4000 = 0.25; q = 3000/4000 = 0.75;$$

$z$  corresponding to 95% confidence level = 1.65

$$N = \frac{pq}{\sigma^2} = \left( \frac{z}{e} \right)^2 pq = \left( \frac{1.65}{0.03} \right)^2 \times 0.25 \times 0.75 \\ = 567.187 \cong 568 \text{ observations}$$

## 11.10 JOB DESIGN

Job design is a function of specifying the work activities of an individual or group in an organizational setting. The objective of job design is to develop jobs that meet the requirements of the organization and satisfy the jobholder's personal and individual requirements. This is a complex function to plan and make the strategies that who does what, where, why and how to do a job.

### **Objectives of Job Design**

The objectives of job design are:

1. To establish organizational chart.
2. To develop work assignments or jobs that meet both organizational and the worker needs.
3. To develop job descriptions of the works.
4. To motivate human resource on the job at work place through implementation of job design components and relative worth of a job.

### **Approaches to Job Design**

*Mechanistic approach:* In this approach, jobs that are simple, routine, repetitive tasks carried out every day such as assembly jobs, packing processes are specialized. Generally, workers are fixed at one specific work location, station, area, such as assembly operators, machine operator, and data entry clerk, crane/tractor operator, etc.

*Motivational approach:* In this approach, five core dimensions of job such as skills variety, job identity, task significance, task autonomy and feedback are used. Skills variety is a job that requires workers to use a variety of skills and talents. Job identity allows the worker to perceive the job as a whole, from start to finish. Task significance shows that it is important and has an impact on the organization and society and the job is meaningful. Task autonomy provides freedom, independence, discretion, responsibility and accountability to perform the job. Feedback is monitoring activities that a job should provide clear, timely information on performance, roles and functions, status and problems.

The three ways of job expansion are job enrichment, job rotation and job enlargement as discussed below:

Job enrichment is a process of vertical expansion of job where higher levels of tasks are added, increase in job responsibilities and job depth, increase in work autonomy and basic managerial skills, and preparation of a staff for a potential promotion.

Job rotation is a process of worker to be sent to learn and work in another operation on a scheduled basis, to set up a periodic work rotation schedule, giving exposure to other jobs to acquire new skills, knowledge, adds motivation and challenges to job and worker.

Job enlargement is a process of horizontal expansion of job where varied tasks of similar activities are added, the increase in job scope, combines various tasks at a similar level into a job, and allowing workers to perform a whole unit of work.

*Biological approach:* This approach considers ergonomics factors to design a job that fits the worker's physiological nature, and not to fit the worker to the job.

*Perceptual approach:* This approach considers mental factors and demands that do not exceed the mental capabilities of the worker.

## **11.11 JOB RATING OR EVALUATION**

It is a process of assessment of the relative worth of a job qualitatively and quantitatively. It is used to differentiate the wages for the workers employed in different types of jobs. This is essential for wage differentials. The jobs are rated on the basis of difficulty to perform, complexity to understand, the responsibility involved, education and training required, and working conditions and environment.

## **11.12 MERIT RATING**

In merit rating, a man is rated based on his performance. There are various criteria used to rate a man; these criteria are attendance, co-operation with another person, dependability, cautiousness during working, leadership ability, appearance, initiative, judgement, versatility, working knowledge and general awareness. Employee rating is used to keep a record of the relative value of each employee, justify and determine the wage differential for the same job, have better information for promotion, transfer, or layoff, and point out the weakness of the employees for self-development.

### **11.12.1 Wage and Wage Incentives**

Wage is the remuneration given to the worker for the work performed. A wage incentive is the amount or facility in terms of monetary given to workers to motivate them for higher production and productivity. Wage incentives may be in the form of financial incentive, leave with pay, free medical facility, free housing, free transportation from residence to workplace and workplace to residence, recreational facilities, leave travel concession, free education to children and higher educational facility for the employee, etc.

### **11.12.2 Wage Differentials**

The differences in wages among the employee having same/different qualification and skill are known as wage differentials. The reasons for wage differentials are enumerated as follows:

1. Difference in marginal productivity of the workers.
2. Difference in qualification, experience and training of the workers.

3. Difference in skills and specialties of the workers.
4. Difference in the difficulties and risk involved in the work.
5. Difference in responsibilities required for completion of the job.
6. Availability of the worker.

### 11.12.3 Wage Incentive Plans

There are a number of wage incentive plans. These plans vary from industry to industry, company to company and department to department. Some of the basic wage incentive plans are discussed in the following subsections:

*Flat day rate:* This is a simple wage determination based on work-hour. It is hardly an incentive plan. The daily earning can be determined using the following formula:

$$E = R \times T$$

where  $E$  is daily earning,  $R$  is wage in Rs per hour, and  $T$  is time or hours per day.

*Piece rate system:* In this system, wage is determined on the basis of pieces produced not on the time. Different types of piece rate plans commonly used to determine the wages are straight piece rate system, a piece rate system with a guaranteed minimum, Taylor's differential piece rate system and Meerick's differential piece rate system.

*Straight piece rate system:* In this system, the wage is directly proportional to the number of pieces produced and it can be determined as:

$$E = R \times N$$

where  $E$  is daily earning,  $R$  is wage in Rs per piece, and  $N$  is number of pieces produced in a day.

*Piece rate system with guaranteed minimum:* To remove the shortcomings of the straight piece rate system, a minimum wage is fixed based on an hourly basis and rest is based on the number of pieces produced in a day. The wage can be determined using the formula:

$$E = R_1 \times T + R_2 \times N$$

where  $E$  is daily earning,  $R_1$  is wage in Rs per hour,  $R_2$  is wage in Rs per piece,  $T$  is the number of hours in a day, and  $N$  is the number of pieces produced in a day.

**Example 11.7:** Suppose a department follows a piece rate system of incentive with guaranteed minimum wage. The hourly rate is Rs 15 only. The piece rate for a particular type job is Rs 10. An average worker can make only 10 jobs a day (8 hours a day). Calculate an average worker's daily earning.

#### **Solution:**

Given  $R_1 = \text{Rs } 15$ ;  $R_2 = \text{Rs } 10$ ;  $T = 8$  hours;  $N = 10$  jobs a day.

$$E = R_1 \times T + R_2 \times N = 15 \times 8 + 10 \times 10 = \text{Rs } 220 \text{ per day.}$$

*Taylor's differential piece rate system:* Taylor determined the standard production rate. If the number of pieces produced by the worker is less than the standard production rate, the wage paid will be based on Rs  $R_1$  (smaller value) per piece and if the number of pieces produced is equal or greater than the standard value, then the wage paid will be based on Rs  $R_h$  (higher value) per piece.

**Example 11.8:** The management decides to implement Taylor's differential piece rate system for a plant. The standard rate of production is fixed at 8 units per hour. The lower and higher wage rates are Rs 0.40 and 0.50, respectively, for each job. Calculate the daily wage of a worker if the production rates are 6 units per hour and 10 units per hour.

**Solution:**

$$\text{Standard rate of production (daily)} = 8 \times 8 = 64 \text{ units}$$

$$\begin{aligned}\text{The daily wage when production rate is 6 units per hour} &= 6 \times 8 \times 0.40 \\ &= \text{Rs } 19.2\end{aligned}$$

$$\begin{aligned}\text{The daily wage when production rate is 10 units per hour} &= 10 \times 8 \times 0.5 \\ &= \text{Rs } 40\end{aligned}$$

*Meerick's differential piece rate system:* This is similar to Taylor's piece rate system. A standard production rate is decided for different types of jobs. Three wage rates say  $R_1$ ,  $R_2$  and  $R_3$  are fixed for each type jobs.  $R_1$  is the lowest and  $R_3$  is the highest wage rate. If the production rate is up to 83 per cent of the standard production rate, the worker is paid at the rate of  $R_1$ . For the production rate between 83 and 100 per cent of the standard production rate, the worker is paid at the rate of  $R_2$  and for the production rate over 100 per cent of the standard production rate, the worker is paid at the rate of  $R_3$ .

A commonly used relationship between  $R_1$ ,  $R_2$  and  $R_3$  is given as:

$$R_2 = 1.08R_1 \quad \text{and} \quad R_3 = 1.2R_1$$

**Example 11.9:** The standard output rate of a particular product is 12 per day. The minimum wage rate is Rs 2 per product. Calculate the daily earning if the production rates are 9, 11 and 13 per day.

**Solution:**

$$R_1 = \text{Rs } 2.00; R_2 = \text{Rs } 2.00 \times 1.08 = \text{Rs } 2.16; R_3 = \text{Rs } 2.00 \times 1.2 = \text{Rs } 2.4$$

$$\text{The standard output rate} = 12 \text{ per day}$$

$$83 \text{ per cent of the standard rate} = 0.83 \times 12 = \text{Rs } 9.96$$

$$\begin{aligned}\text{Daily earning for production rate 9 products per day} &= \text{Rs } 2 \times 9 \\ &= \text{Rs } 18 \text{ per day}\end{aligned}$$

$$\begin{aligned}\text{Daily earning for production rate 11 products per day} &= \text{Rs } 2.16 \times 11 \\ &= \text{Rs } 23.76 \text{ per day}\end{aligned}$$

$$\begin{aligned}\text{Daily earning for production rate 13 products per day} &= \text{Rs } 2.4 \times 13 \\ &= \text{Rs } 31.2 \text{ per day}\end{aligned}$$

**Halsey Plan:** In this plan, the minimum wage is guaranteed. If a worker takes less time than the standard time to complete a job, then he is paid an extra amount for the time saved. The worker is not paid for the total time saved, but for the fraction of the time saved. This fraction may vary from 33 per cent to 75 per cent, but the commonly used fraction is 50 per cent. The earning can be determined as:

$$E = R \times T_a + \frac{R}{2} (T_s - T_a)$$

where  $E$  is earning,  $R$  is the hourly rate of wage,  $T_a$  is actual time taken to complete the task, and  $T_s$  is standard time to complete the task.

**Example 11.10:** A manager sets a target for the worker to complete his job in 6 hours. The workers are promised to pay incentive according to Halsey 50-50 plan. The hourly wage rate is Rs 5. The worker could complete the task in 5 hours only. Calculate the total earning and hourly wage rate of the worker.

**Solution:**

$$T_s = 6 \text{ hours}; T_a = 5 \text{ hours}; R = \text{Rs } 5$$

$$E = R \times T_a + \frac{R}{2} (T_s - T_a) = 5 \times 5 + \frac{5}{2} (6 - 5) = \text{Rs } 27.5 \text{ for five hours}$$

$$\text{Hourly earning} = \frac{\text{Rs } 27.5}{5} = \text{Rs } 5.5$$

Therefore, daily earning =  $\text{Rs } 5.5 \times 8 = \text{Rs } 44$

**Rowan Plan:** According to this plan, the wage can be determined as:

$$E = R \times T_a + \left( \frac{T_s - T_a}{T_s} \right) \times R \times T_a$$

where  $E$  is earning,  $R$  is the hourly rate of wage,  $T_a$  is actual time taken to complete the task, and  $T_s$  is the standard time to complete the task.

**Example 11.11:** Using the information given in Example 2.9, calculate the daily earning of the worker under Rowan Plan.

**Solution:**

$$T_s = 6 \text{ hours}; T_a = 5 \text{ hours}; R = \text{Rs } 5$$

$$E = R \times T_a + \left( \frac{T_s - T_a}{T_s} \right) \times R \times T_a = 5 \times 5 + \frac{1}{6} \times 5 \times 5 \\ = \text{Rs } 29.16 \text{ for five hours}$$

$$\text{Hourly earning} = \frac{\text{Rs } 29.16}{5} = \text{Rs } 5.83$$

Therefore, daily earning =  $\text{Rs } 5.83 \times 8 = \text{Rs } 46.64$

**Bedaux Plan:** According to this plan, the wage can be determined as:

$$E = R \times T_a \quad \text{for } T_a = T_s$$

$$E = R \times T_a + 0.75(T_s - T_a) \quad \text{for } T_a < T_s$$

where  $E$  is earning,  $R$  is the hourly rate of wage,  $T_a$  is actual time taken to complete the task, and  $T_s$  is the standard time to complete the task.

**Example 11.12:** Using the information given in Example 2.9, calculate the daily earning of the worker under Bedaux plan.

**Solution:**

$$T_s = 6 \text{ hours}; T_a = 5 \text{ hours}; R = \text{Rs } 5$$

$$E = R \times T_a + 0.75(T_s - T_a) \quad \text{for } T_a < T_s$$

$$= 5 \times 5 + 0.75 \times 1 = \text{Rs } 25.75 \text{ for five hours}$$

$$\text{Hourly earning} = \frac{\text{Rs } 25.75}{5} = \text{Rs } 5.15$$

Therefore, daily earning =  $\text{Rs } 5.15 \times 8 = \text{Rs } 41.20$

### **Group Incentives**

Group incentives are given to the workers working in a group and when the determination of individual performance is difficult.

## **11.13 Ergonomics**

Ergonomics is the scientific study of the relationship between man and his work environment.

—(Murrell 1971)

Ergonomics is the application of human biological science in conjunction with engineering science to the worker and his working environment so as to obtain maximum satisfaction for the worker which at the same time enhances productivity.

—I.L.O (Helander 1995)

Ergonomics discovers and applies information about human behavior, abilities, limitations and other characteristics to the design of tools, machines, systems, tasks, jobs, and environments for safe, comfortable, and efficient human use.

—(Sanders and McCormick 1992)

Since human being is a complex creature and in this world everything revolves around him, thus ergonomics is essentially an interdisciplinary activity taking into consideration large number of areas and science which contributes in this such as: anatomy and physiology, human psychology, experimental psychology, and industrial medicine, medical science, design engineering, product development, architecture and town planning, personnel management, etc.

### ***Areas of Application***

Ergonomics has wide industrial applications; some of them are mentioned as follows:

1. Design of machines, equipments, plants and factories.
2. Design of roads, houses, dams, schools, offices.
3. Design of furniture, seats, tables, and auditorium.
4. Design of working conditions and environment.
5. Measurement of human work load and capacities of work
6. Study of human characteristics such as habits and ways of working and health and safety aspects.
7. Selection, training and development of people/workforce.
8. Development of anthropometrical data and its application.
9. Biomedical and biotechnology.
10. Human resource planning and management.
11. Study of psychological environment.

### ***Man–Machine Environment System***

It is defined as an operating combination of one or more men with one or more machines/physical equipments interfacing to bring about, from the given input to some desired output (objective) with the given/existing constraints and environments. Or, a man with some common working machines/tools/equipments/devices working within environment/constraints to generate desired output/objective.

### ***Manufacturing and Ergonomics***

Ergonomics is applied in manufacturing in all the three steps: input, value addition and output. Four-Ms (man, machine, material and money) are the inputs, two-Ms (method and management) are the value addition and product and services are the output of the manufacturing systems. Ergonomics is applied to all the variables of the systems.

### ***Role of Ergonomics in Manufacturing***

Ergonomics plays an important role in the following areas of manufacturing:

1. Product design and development.
2. Material selection, purchase, storage and value addition.
3. Efficient and effective design of methods and procedure.
4. Machines, equipment design and uses.
5. Factory layout and workplace design.
6. Material handling, equipment design, selection and material handling system.
7. Quality inspection and control.
8. Warehousing and distribution.
9. Performance measures and tests.
10. Cost calculation.

**Murrell's Ergonomics Check List (Murrell 1971)**

1. What role is man expected to play?
2. How will the equipment fit the man?
3. Can operator sits/stands while operation?
4. Is the equipment operated partly fully?
5. Is the operator man/woman?
6. What information does operator need to do the job?
7. What controls will be needed/type, operating body part?
8. What physical work operator is required to do? Is it per his/her capability?
9. What form of communication will take place between different components of the system?
10. What ambient conditions likely to be?
11. What are the physical and mental demands?
12. What are the expected maintenance requirements?

**SUMMARY**

In this chapter, three major topics have been covered; these topics are work study, wage determination and ergonomics. Work study is an important management technique to improve the efficiency and economy of an organization. Various tools and techniques related to method study, time study, work sampling have been discussed in detail. Job rating and merit rating have been introduced that are used in the determination of wage incentive and wage differential. Finally, ergonomics has been introduced and shown the men-machine relationship.

**MULTIPLE-CHOICE QUESTIONS**

1. Work study is concerned with
  - (a) Improving the present method and finding the standard time
  - (b) Improving the production schedules
  - (c) Motivating the workers
  - (d) Meeting the production target
2. The symbol 'O' in work study is used for
 

(a) operation	(b) inspection
(c) delay	(d) storage
3. The symbol  $\square$  in work study is used for
 

(a) operation	(b) inspection
(c) delay	(d) storage
4. The symbol  $\nabla$  in work study is used for
 

(a) operation	(b) inspection
(c) delay	(d) storage

5. The symbol  $\Rightarrow$  in work study is used for
  - (a) operation
  - (b) inspection
  - (c) delay
  - (d) transport
6. The symbol  $\blacktriangleright$  in work study is used for
  - (a) operation
  - (b) inspection
  - (c) delay
  - (d) transport
7. Micro-motion study is
  - (a) Stage-wise analysis of motion study
  - (b) Complete detail of motion study
  - (c) Subdivision of an operation into therblig
  - (d) None of these
8. Standard time is equal to
  - (a) Normal time + Allowance
  - (b) Observed time + Allowance
  - (c) Observed time  $\times$  rating factor
  - (d) Normal time  $\times$  rating factor
9. Job evaluation is a method of
  - (a) Finding the relative worth of the job
  - (b) Finding the contribution of job
  - (c) Finding the workers performance
  - (d) All the above
10. The percentage of idle time of a worker can be found by
  - (a) Time study
  - (b) Method study
  - (c) Stopwatch
  - (d) Work sampling
11. Which of the following is effective in reducing the job monotony and demotivating the worker?
  - (a) Job enlargement
  - (b) Job rotation
  - (c) Job expansion
  - (d) All the above
12. In a Halsey wage incentive plan
  - (a) A worker is paid as per efficiency
  - (b) A worker is ensured of the minimum payment
  - (c) A worker is always in profit
  - (d) All the above
13. Which of the following represents a group incentive plan?
  - (a) Halsey plan
  - (b) Rowan Plan
  - (c) Bedaux plan
  - (d) Lincoln Plan

14. Performance rating is equal to
  - (a) Observed performance + Normal performance
  - (b) Observed performance – Normal performance
  - (c) Observed performance × Normal performance
  - (d) Observed performance / Normal performance
15. PMTS includes
  - (a) WFS
  - (b) MTM
  - (c) BMTS
  - (d) all the above

## Answers

1. (a)      2. (a)      3. (b)      4. (d)      5. (d)      6. (c)      7. (c)      8. (a)      9. (a)  
10. (d)     11. (d)     12. (b)     13. (d)     14. (d)     15. (d)

## REVIEW QUESTIONS

1. Define the term ‘work study’. Explain the purposes of using work study.
2. What is method study? What are the steps of conducting method study?
3. Discuss the objectives of method study.
4. What are the factors influencing the method study? Discuss them.
5. Differentiate between operation process chart and flow process chart mentioning their advantages and disadvantages.
6. Write short notes on (a) two-hands chart, (b) multiple activity chart, and (c) SIMO chart.
7. Differentiate between flow diagram and string diagram.
8. Explain the working of travel chart.
9. What do you mean by cycle graph and chronocycle graph?
10. What is difference between micro-motion and memo-motion study.
11. What do you mean by time study? What are the objectives of conducting a time study?
12. What is difference between method study and time study?
13. What are the various types of allowances used in calculation of standard time?
14. Write short notes on (a) PMTS, (b) WFS, (c) MTM, and (d) BMTS.
15. Explain the principles of motion economy.
16. What do you mean by work sampling? Mention its advantages and disadvantages.
17. What do you mean by job design? What are the objectives of job design?
18. Discuss the various job incentive plans.
19. Write short notes on merit rating and job rating.
20. Explain the use of ergonomics in manufacturing industry.

## EXERCISES

- For a particular element of a job 50 observations (as shown in Table 11.9) were taken by an observer using a stopwatch. Check whether these number of observations are sufficient for  $\pm 5$  per cent accuracy with 95 per cent confidence level. Find the minimum number of observations required.

**Table 11-9:** Observations

Time x (in min)	0.05	0.06	0.07	0.08	0.09
Frequency	8	5	20	12	5

- An operator was kept under observation for 10 days. He was found working on 400 occasions and abstaining including idle time was 100. He produced 120 jobs during these days. The observation per day was for 8 hours only and the total number of observations was 500. Take 120 as performance rating for operator and 15 per cent as allowance. Calculate the standard time.
- Calculate the required number of observations necessary for an accuracy of  $\pm 2$  per cent and confidence level of 96 per cent, if the total number of observations of machines working is 4500 and machines idle is 500.
- A manager sets a target for the worker to complete his job in 5 hours. The workers are promised to pay incentive according to Halsey 50-50 plan. The hourly wage rate is Rs 8. The worker could complete the task in 4 hours only. Calculate the total earning and hourly wage rate of the worker.
- The management decides to implement Taylor's differential piece rate system for a plant. The standard rate of production is fixed at 10 per hour. The lower and higher wage rates are Rs 0.40 and 0.50, respectively, for each job. Calculate the daily wage of a worker if the production rates are 8 and 12 per hour.



## REFERENCES AND FURTHER READINGS

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# CHAPTER ▶ 12

## Reliability and Maintenance Engineering

### 12.1 INTRODUCTION

Reliability is the probability with which an item or system will remain functioning under stated operational and environmental conditions for a specified period of time. Quantitatively, reliability is the probability of success. There are two types of reliability, predicted reliability and assessed reliability, which are very likely to differ. The predicted reliability is the estimated quantity based on the known reliabilities of individual components. The assessed reliability is arrived at by conducting reliability tests. A failure is said to occur with a product when the same stops to function. Failure rate is the probability that a new item will fail only after a duration of time  $t$  but before  $t + \delta t$  and is designated by  $\lambda(t)$ . When the expected duration of service,  $t$  is taken as a parameter, the quantity  $R(t)$  is called quantitative reliability, and is related to the failure rate by the following relation.

$$\lambda(t) = -\frac{dR(t)/dt}{R(t)}$$

where  $R(t) = p(\tau > t)$

Here,  $\tau$  is the service life of the product without failure and  $p$  is the probability of service life greater than the given time  $t$ .

Integrating and substituting the boundary condition that at the beginning of time,  $t = 0$ , the reliability is equal to 1, and further assuming that the failure rate is independent of time, we obtain the following exponential relation for reliability.

$$R(t) = e^{-\lambda t}$$

where  $\lambda$  is the failure rate.

Reliability engineering is concerned with four key elements of this definition:

1. Reliability is a probability. This means that failure is treated as a random phenomenon: it is a recurring event and the likelihood of failures varies over time according to the given probability function.
2. Reliability is predicted on ‘intended function’. It means an operation without failure.
3. Reliability applies to a specified period of time. In practical terms, this means that a system has a specified chance that it will operate without failure before time  $t$ . Reliability engineering ensures that components and materials will meet the requirements during the specified time.

- Reliability is restricted to operation under stated conditions. This constraint is necessary because it is impossible to design a system for unlimited conditions. The operating environment must be addressed during design and testing.

The objectives of reliability engineering, in order of priority to minimize costs and generating reliable products, are:

- To apply engineering knowledge and specialist techniques to prevent or to reduce the likelihood or frequency of failures.
- To identify and correct the causes of failures that do occur, despite the efforts to prevent them.
- To determine ways of coping with failures that do occur, if their causes have not been corrected.
- To apply methods for estimating the likely reliability of new designs, and for analysing reliability data.

## 12.2 RELIABILITY CURVES

The continuous distributions used in reliability are exponential, normal and Weibull. The frequency distributions of these probability distributions are shown in Figure 12.1. These are functions of time.

Reliability can be calculated as,

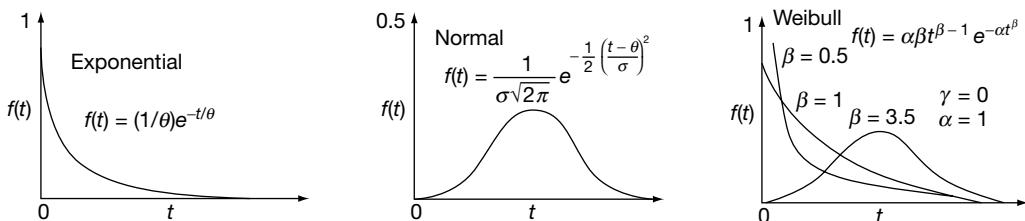
$$R_t = 1 - \int_0^t f(t) dt.$$

The reliability curves for various distributions such as exponential normal and Weibull distribution, respectively, are shown in Figure 12.2.

### **Relationship Among $\lambda(t)$ , $f(t)$ , $F(t)$ and $R(t)$**

After defining the failure rate, probability density function (PDF), cumulative density function (CDF) and reliability, it is important to establish a relationship among them to find the individual values. The failure rate is denoted by,  $\lambda(t)$  at time  $t$  is expressed as :

$$\begin{aligned} \lambda(t) &= \frac{f(t)}{1-F(t)} = \frac{f(t)}{R(t)}; \\ \lambda(t) &= -\frac{d}{dt}(\ln R(t)) \end{aligned} \quad (12.1)$$



**Figure 12-1:** Frequency distribution as a function of time

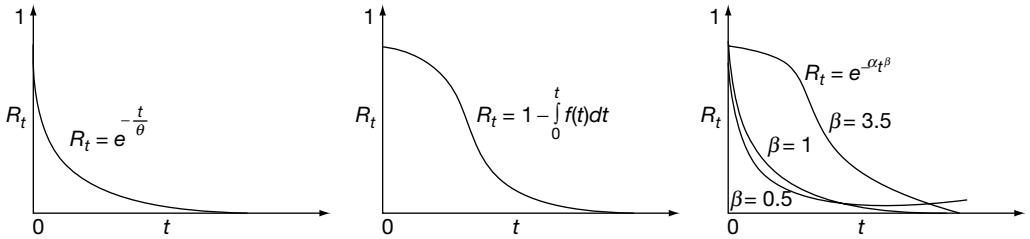


Figure 12-2: Reliability as a function of time

The PDF,  $f(t)$  at time  $t$  is expressed as:

$$\begin{aligned} f(t) &= \frac{d}{dt} F(t) = -\frac{d}{dt} R(t) \\ f(t) &= \lambda(t) e^{-\int_0^t \lambda(s) ds} \end{aligned} \quad (12.2)$$

The CDF,  $F(t)$  for time  $t$  is expressed as:

$$\begin{aligned} F(t) &= \int_0^t f(t) dt; \quad F(t) = 1 - R(t) \\ F(t) &= 1 - e^{-\int_0^t \lambda(s) ds} \end{aligned} \quad (12.3)$$

The reliability,  $R(t)$  for time  $t$  is expressed as:

$$\begin{aligned} R(t) &= 1 - F(t); \quad R(t) = \int_t^\infty f(t) dt \\ R(t) &= e^{-\int_0^t \lambda(s) ds} \end{aligned} \quad (12.4)$$

**Failure Rate Curve:** Failure rate curves for exponential, normal and Weibull distributions, respectively, are shown in Figure 12.3.

$$\text{Failure rate, } \lambda = \frac{f(t)}{R(t)}$$

where  $\lambda$  is the failure rate and  $\theta$  is the mean time to failure (MTTF) or mean life.

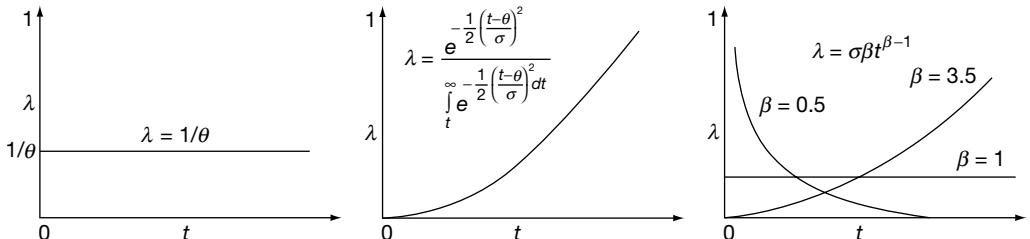


Figure 12-3: Failure rate as a function of time

## 12.3 FAILURE PATTERN

Product breakdowns often tend to follow a Poisson distribution, i.e. failures occur infrequently during the normal life of a product. Equipment manufacturers have found that the failure during the very early and late stages of product life often differs from the experienced during the normal operating life. This difference in failure rates is depicted in the form of the ‘bathtub’ curve as shown in Figure 12.4. Early failures (may be due to improper assembly or damage in shipment) may tend to follow a negative exponential pattern. During the typical operating lifetime, failure occurs on a rare-event basis, often described by a Poisson distribution. As components wear out and fail, the products may follow a pattern described by a normal distribution.

The bathtub curve represents the failure rate of a population of statistically identical items and it is the same as the life cycle of the product involving the debugging phase, chance-failure phase and wear-out phase. For a constant failure rate (CFR) of a product that is **irreparable**, the MTTF is defined as:

$$\text{MTTF} = \int_0^{\infty} R(t) dt = \frac{1}{\lambda}$$

If an item is **reparable**, then the same quantity is termed as mean time between failures (MTBF).

$$\text{MTBF} = \frac{\text{Operating time}}{\text{Number of failures}} = \frac{\text{Total time} - \text{Non-operating time}}{\text{Number of failures}} = \frac{\text{TT} - \text{NOT}}{F}$$

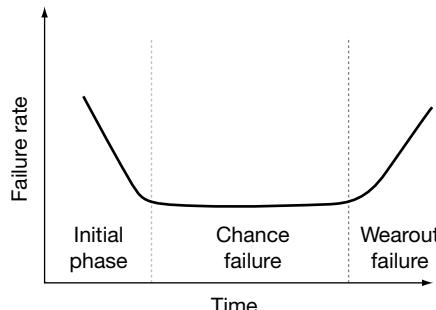
The reliability analysis is conducted in two stages, first at the design stage and then at the production stage.

Failure rate may be expressed in the following two ways:

1. Percentage failure (FR per cent).
2. Number of failures per given operation time ( $\text{FR}_n$ ).

$$\text{FR\%} = \frac{\text{Number of failures}}{\text{Number tested}}; \quad \text{FR}_n = \frac{\text{Number of failures}}{\text{Operating time}} = \frac{F}{\text{TT} - \text{NOT}}$$

where FR% is percentage failure rate;  $\text{FR}_n$  is number of failure per given operation time; F is number of failures; TT is total time; NOT is non-operating time.



**Figure 12-4:** Bath-tub failure curve

**Example 12.1:** Twenty machines have been operated for 100 hours. One machine fails in 60 hours and another fails in 70 hours. What is the MTBF and what will be reliability at 500 hours and 900 hours?

**Solution:**

Eighteen of the machines ran for 100 hours, while one machine ran for 60 hours and another one ran for 70 hours. Thus, total running time is  $18 \times 100 + 60 + 70 = 1930$  hours.

$$\text{MTBF} = \frac{\text{Operating time}}{\text{Number of failures}} = \frac{1930}{2} = 965 \text{ hours per failure}$$

$$\text{Failure rate } (\lambda) = \frac{1}{\text{MTBF}} = \frac{1}{965} = 0.0010362 \text{ failure/hour}$$

$$R_{500} = e^{-\lambda t} = e^{-0.0010362 \times 500} = 0.595$$

$$R_{900} = e^{-\lambda t} = e^{-0.0010362 \times 900} = 0.394$$

**Mean time to failure:** The MTTF is one of the measures which characterize the failure distribution for a component or system. It is defined by

$$\text{MTTF} = E(T) = \int_0^{\infty} tf(t)dt \quad (12.5)$$

which is the mean or expected value of probability distribution defined by  $f(t)$ . It is also expressed in terms of reliability function as

$$\text{MTTF} = \int_0^{\infty} R(t)dt \quad (12.6)$$

*Variance:* The variance,  $\sigma^2$  is another measure, which is used to characterize the failure distribution. It is defined as

$$\sigma^2 = \int_0^{\infty} (t - \text{MTTF})^2 f(t)dt \quad (12.7)$$

The variance represents the average squared deviation from the MTTF. It is a measure of spread or dispersion of the failure times about the MTTF. It can also be written as

$$\sigma^2 = \int_0^{\infty} t^2 f(t)dt - (\text{MTTF})^2 \quad (12.8)$$

The square root of the variance is the standard deviation.

**Example 12.2:** The time-to-failure PDF for a system is  $f(t) = 0.02$ ;  $0 \leq t \leq 50$  days

Find

- (a) Reliability for time  $t$ ,  $R(t)$
- (b) The hazard rate function at time  $t$ ,  $\lambda(t)$
- (c) The MTTF
- (d) The standard deviation

**Solution:**

- (a) The relation between reliability function and PDF is

$$R(t) = \int_t^{\infty} f(t) dt$$

Using the given  $f(t)$  value

$$R(t) = \int_t^{\infty} (0.02) dt = \int_t^{50} (0.02) dt = 1 - 0.02t$$

Therefore,  $R(t) = 1 - 0.02t$

- (b) The relation for  $\lambda(t)$  in terms of reliability function and PDF is

$$\lambda(t) = \frac{f(t)}{R(t)}, \text{ using the given } f(t) \text{ value and the } R(t) \text{ value derived in part (a),}$$

$$\lambda(t) = \frac{0.02}{1 - 0.02t}$$

- (c) Using the relation, MTTF =  $\int_0^{\infty} R(t) dt$

$$\text{MTTF} = \int_0^{50} (1 - 0.02t) dt = \left[ t - 0.02 \frac{t^2}{2} \right]_0^{50} = 25 \text{ days}$$

- (d) Using the relation, variance,  $\sigma^2 = \int_0^{\infty} t^2 f(t) dt - (\text{MTTF})^2$

$$\sigma^2 = \int_0^{50} t^2 (0.02) dt - (25)^2 = \left[ 0.02 \frac{t^3}{3} \right]_0^{50} - 625 = 208.33$$

Therefore, standard deviation,  $\sigma = 14.4$  days.

**Example 12.3:** A component has the following liner hazard rate, where  $t$  is in years:  $\lambda(t) = 0.05t$ ,  $t \geq 0$

- (a) Find  $R(t)$  and determine the probability of a component failing within the two months of its operation.
- (b) What is the design life if a reliability of 0.96 is desired?

**Solution:**

- (a) The relation between reliability function and failure rate function is

$$R(t) = e^{-\int_0^t \lambda(t) dt}, \text{ substituting given failure rate function}$$

$$R(t) = e^{-\int_0^t (0.05t) dt} = e^{-0.025t^2}$$

$$R(t) = e^{-0.025t^2}$$

The probability of a component failing within the two months (2/12 year) of its operation,

$$\begin{aligned}\Pr\left(T \leq \frac{1}{6}\right) &= 1 - \Pr\left(T > \frac{1}{6}\right) = 1 - R\left(\frac{1}{6}\right) \\ &= 1 - e^{-0.025\left(\frac{1}{6}\right)^2} = 0.00069\end{aligned}$$

- (b) Design life if a reliability of 0.96 is desired, i.e.,  $R(t) = 0.96$

From part (a),  $R(t) = e^{-0.025t^2}$ , therefore  $0.96 = e^{-0.025t^2}$

$$t = \sqrt{\frac{-1}{0.025} \log(0.96)} = 1.277 \text{ years}$$

## 12.4 BASIC RELIABILITY MODELS

In this section, some important failure patterns based on the exponential, Weibull and normal probability distributions models are discussed.

### 12.4.1 CFR Model

The exponential probability distribution has a constant failure rate (CFR) and is called a CFR model. It is one of the most common and important distributions in reliability engineering. Failures due to completely random or chance events will follow this distribution. The failure behaviours of most of the electronic components are represented by the exponential distribution.

The expressions for  $R(t)$ ,  $F(t)$ ,  $f(t)$ , MTTF and  $\sigma^2$  are derived in the case of the CFR model as: For CFR model, let us assume  $\lambda(t) = \lambda$ ,  $t \geq 0$ ,  $\lambda > 0$ , then using the relation in Equations (12.3) and (12.4), the expressions of  $R(t)$  and  $F(t)$  are derived and given in Equations (12.9) and (12.10).

$$R(t) = e^{-\int_0^t \lambda(t) dt} = e^{-\lambda t} \quad (12.9)$$

and

$$F(t) = 1 - R(t) = 1 - e^{-\lambda t} \quad (12.10)$$

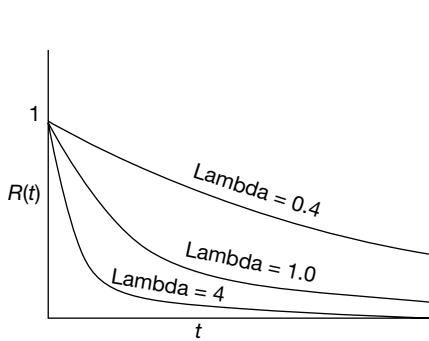
$R(t)$  and  $F(t)$  curves are shown in Figures 12.5 and 12.6, respectively, for three different values of  $\lambda$  in case of CFR model.

Using Equation (12.2), the expression for  $f(t)$  is derived as:

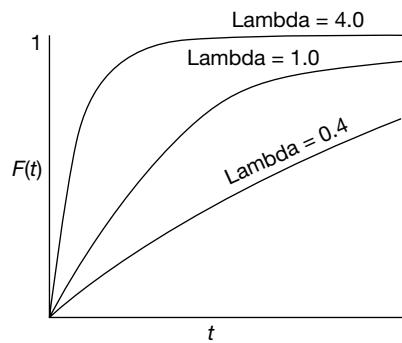
$$f(t) = -\frac{dR(t)}{dt} = -\frac{d(e^{-\lambda t})}{dt} = \lambda e^{-\lambda t} \quad (12.11)$$

where  $\lambda$  is the distribution parameter.

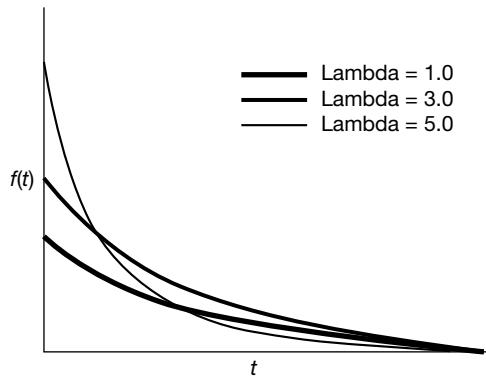
Curves for  $f(t)$  are shown in Figure 12.7 for different values of  $\lambda$ .



**Figure 12-5:** Reliability,  $R(t)$



**Figure 12-6:** Cumulative distribution function,  $F(t)$



**Figure 12-7:** Probability density function

Using Equation (12.5), the expression for MTTF is derived as

$$\text{MTTF} = \int_0^{\infty} R(t)dt = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda} \quad (12.12)$$

Using Equation (12.8), the expression for variance is derived as

$$\sigma^2 = \int_0^{\infty} t^2 \lambda e^{-\lambda t} dt - \left(\frac{1}{\lambda}\right)^2 = \frac{1}{\lambda^2} \quad (12.13)$$

An important property of the CFR model is memorylessness. It means the time to failure of a component is not dependent on how long the component has been operating. There is no ageing or wear-out effect. The probability that the component will operate for the next 1000 hours is the same regardless of whether the component is brand new and has been operated for so many hours. This property is consistent with the completely random, and independent nature of the failure process. For example, when external, random environmental stresses are the primary cause of failures, the failure or operating history of the component will not be relevant.

**Example 12.4:** A component experiences chance (CFR) failures with an MTTF of 1000 hrs. Find the following:

- (a) The reliability for a 100 hr mission
- (b) The design life for a 0.96 reliability
- (c) The median time to failure

**Solution:**

- (a) For CFR model,

$$\text{Failure rate, } \lambda = \frac{1}{\text{MTTF}} = \frac{1}{1000} \text{ per hour and } R(t) = e^{-\lambda t}$$

$$\text{Therefore, } R(100) = e^{-\left(\frac{1}{1000}\right)100} = 0.904$$

- (b)  $R(t) = e^{-\lambda t} = 0.96$ , therefore

$$\text{Design life, } t = -\frac{1}{\lambda} \log(0.96)$$

$$t = -\frac{1}{\left(\frac{1}{1000}\right)} \log(0.96) = 40.82 \text{ hours}$$

- (c) For the median life to failure,  $t_{\text{med}}$

$$R(t_{\text{med}}) = e^{-\lambda t_{\text{med}}} = 0.50$$

$$\text{median life to failure, } t_{\text{med}} = -\frac{1}{\lambda} \log(0.5)$$

$$t_{\text{med}} = -\frac{1}{\left(\frac{1}{1000}\right)} \log(0.5) = 693.14 \text{ hours}$$

### 12.4.2 Weibull Model

There are many situations in which the failure rate cannot be constant as in the case of CFR model. In such cases, the Weibull model is more versatile and appropriate which can incorporate the decreasing and increasing failure rate. The mechanical components, which fail due to the gradual wear and tear, are modelled by Weibull distribution.

The expressions for  $R(t)$ ,  $F(t)$ ,  $f(t)$ , MTTF and  $\sigma^2$  are derived in the case of the Weibull failure model as:

For Weibull model, the failure rate  $\lambda(t)$  is given by the relation

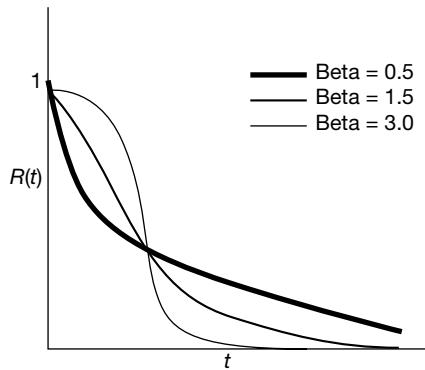
$$\lambda(t) = \frac{\beta}{\theta} \left( \frac{t}{\theta} \right)^{\beta-1}, \theta > 0, \beta > 0, t \geq 0 \quad (12.14)$$

where  $\beta$  is the shape parameter and  $\theta$  is the scale parameter. The values of the shape parameter,  $\beta$  are  $< 1$ ,  $= 1$  and  $> 1$  for decreasing, constant and increasing failure rates, respectively.

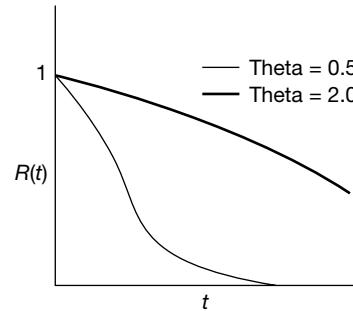
Using Equation (12.4), we have

$$R(t) = e^{-\int_0^t \lambda(\tau) d\tau} = e^{-\int_0^t \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} dt} = e^{-\left(\frac{t}{\theta}\right)^\beta} \quad (12.15)$$

The variation of  $R(t)$  with time  $t$  for different values of  $\beta$  and  $\theta$  is shown in Figures 12.8 and 12.9, respectively.



**Figure 12-8:**  $R(t)$  for different values of  $\beta$

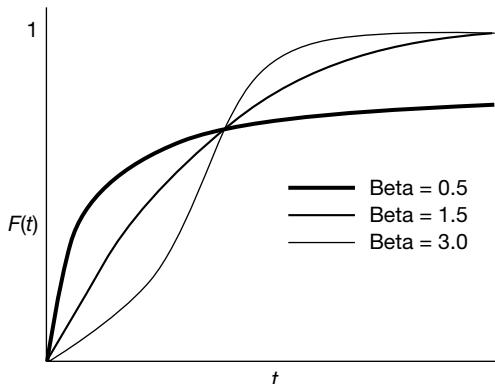


**Figure 12-9:**  $R(t)$  for different values of  $\theta$

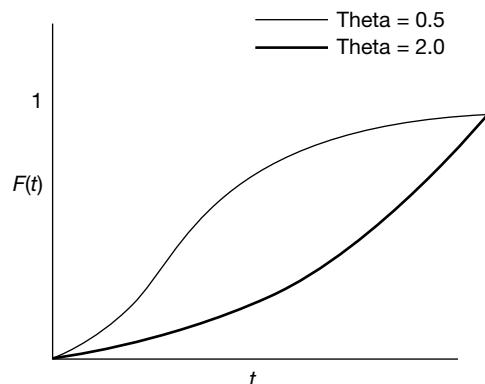
Similarly, using Equation (12.3), the expression for CDF is derived as:

$$F(t) = 1 - R(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^\beta} \quad (12.16)$$

The variation of  $F(t)$  with time  $t$  for different values of  $\beta$  and  $\theta$  is shown in Figures 12.10 and 12.11, respectively.



**Figure 12-10:**  $F(t)$  for different values of  $\beta$

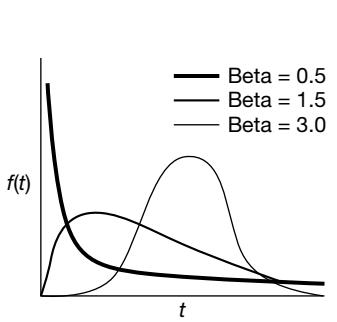


**Figure 12-11:**  $F(t)$  for different values of  $\theta$

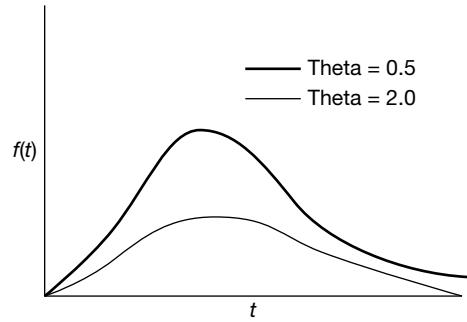
Using Equation (12.2), the expression for  $f(t)$  is derived as:

$$f(t) = -\frac{dR(t)}{dt} = -\frac{d(e^{-\left(\frac{t}{\theta}\right)^\beta})}{dt} = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^\beta} \quad (12.17)$$

The variation of  $f(t)$  with time  $t$  for different values of  $\beta$  and  $\theta$  is shown in Figures 12.12 and 12.13, respectively.



**Figure 12-12:**  $f(t)$  for different values of  $\beta$



**Figure 12-13:**  $f(t)$  for different values of  $\theta$

Using Equation (12.6), we have

$$\text{MTTF} = \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-\left(\frac{t}{\theta}\right)^\beta} dt = \theta \Gamma\left(1 + \frac{1}{\beta}\right) \quad (12.18)$$

Using Equation (12.8),  $\sigma^2 = \int_0^{\infty} t^2 f(t) dt - (\text{MTTF})^2$

$$\begin{aligned} \sigma^2 &= \int_0^{\infty} t^2 \left(\frac{\beta}{\theta}\right) \left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^\beta} dt - \left[\theta \Gamma\left(1 + \frac{1}{\beta}\right)\right]^2 \\ &= \theta^2 \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2 \right\} \end{aligned} \quad (12.19)$$

The value of gamma function can be taken from the table provided in the 'Appendix' 2 of the book.

**Example 12.5:** For a system having a Weibull failure distribution with a shape parameter of 1.5 and a scale parameter of 500 days, find the following:

- (a)  $R$  (200 days)
- (b) MTTF
- (c) median time to failure,  $t_{\text{med}}$
- (d) The design life for a reliability of 0.96
- (e) The standard deviation

**Solution:**

- (a) For Weibull failure model,

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}, \text{ where } \beta = 1.5, \theta = 500 \text{ days}$$

Therefore, for  $t = 200$  days

$$R(200) = e^{-\left(\frac{200}{500}\right)^{1.5}} = 0.7765$$

- (b) For Weibull model (having
- $t_0 = 0$
- ), the expression for MTTF is

$$\text{MTTF} = \theta \Gamma\left(1 + \frac{1}{\beta}\right), \text{ substituting the parameters values}$$

$$\text{MTTF} = 500 \Gamma\left(1 + \frac{1}{1.5}\right) = 500 \times 0.90167 = 450.835 \text{ days}$$

- (c) For median life to failure,
- $t_{\text{med}}$

$$R(t_{\text{med}}) = e^{-\left(\frac{t_d}{\theta}\right)^\beta} = 0.50$$

$$\text{Median life to failure, } t_{\text{med}} = \theta(-\log(0.5))^{\frac{1}{\beta}}$$

$$t_{\text{med}} = 500(-\log(0.5))^{1/1.5} = 301.61 \text{ days}$$

- (d)
- $R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta} = 0.96$
- , therefore

$$\text{Design life, } t = \theta(-\log(0.96))^{\frac{1}{\beta}}$$

$$t = 500(-\log(0.96))^{1/1.5} = 59.28 \text{ days}$$

- (e) For Weibull model, the expression for variance is

$$\text{Variance} = \sigma^2 = \theta^2 \left\{ \Gamma\left(1 + \frac{2}{\beta}\right) - \left( \Gamma\left(1 + \frac{1}{\beta}\right) \right)^2 \right\}$$

Substituting the parameter values,

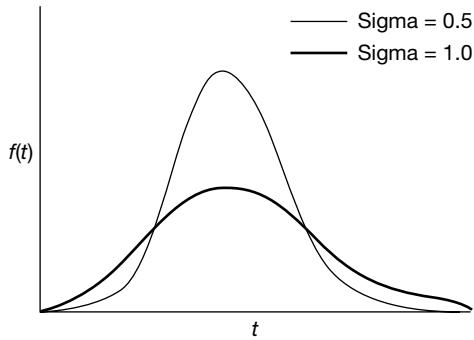
$$\sigma^2 = 500^2 \left\{ \Gamma\left(1 + \frac{2}{1.5}\right) - \left( \Gamma\left(1 + \frac{1}{1.5}\right) \right)^2 \right\}$$

$$= 500^2 \times \left\{ 1.18819 - 0.90167^2 \right\} = 93,795.30$$

Therefore, standard deviation,  $\sigma = 306.26$  days

### 12.4.3 Normal Model

The normal distribution is most widely used distribution for modelling the stochastic behaviour of systems. In reliability practices, it is used to model fatigued and weared out phenomenon of the components. The PDF,  $f(t)$  is a well-known bell-shaped curve as shown in Figure 12.14 and it is expressed as:



**Figure 12-14:** Probability density function,  $f(t)$  for different values

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2}}, \text{ where } -\infty < t < \infty \quad (12.20)$$

where parameters  $\mu$  and  $\sigma$  are the mean and standard deviation of the distribution, respectively.

$$R(t) = \int_t^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \frac{(t-\mu)^2}{\sigma^2}} dt \quad (12.21)$$

The closed-form solution of this integral cannot be obtained. It is calculated by numerical solution. First, the PDF,  $f(t)$  expression is transformed into standard normal variable,  $z$ , function using transformation,  $z = \frac{T - \mu}{\sigma}$ . After transformation,  $f(t)$  reduces to  $\phi(z)$  as  $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ .

Then, the cumulative distribution function,  $F(t)$  is derived as

$$F(t) = \Pr(T \leq t) = \Pr\left(\frac{T - \mu}{\sigma} \leq \frac{t - \mu}{\sigma}\right) = \psi(z) = \int_{-\infty}^z \phi(z) dz \quad (12.22)$$

This can be solved with the help of standard normal distribution tables.

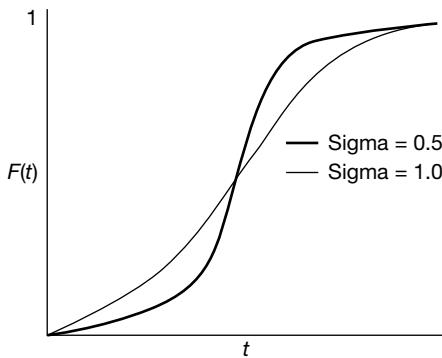
Further, the reliability expression reduces to

$$R(t) = \Pr(T > t) = \Pr\left(\frac{T - \mu}{\sigma} > \frac{t - \mu}{\sigma}\right) = 1 - \psi(z) = 1 - \int_{-\infty}^z \phi(z) dz \quad (12.23)$$

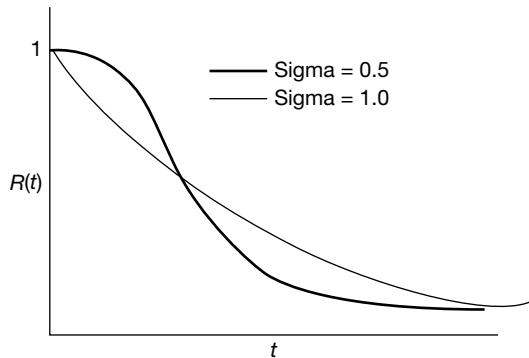
The variation of  $R(t)$  and  $F(t)$  with time  $t$  for different values of  $\sigma$  is shown in Figures 12.15 and 12.16, respectively.

## 12.5 EVALUATION OF SYSTEM RELIABILITY ( $R_s$ )

Most products are made up of a number of components. The reliability of each component and the configuration of the systems consisting of these components determine the system reliability or in other words the reliability of the product. Although the product design, manufacture and



**Figure 12-15:** Reliability,  $R(t)$  for different values of  $\sigma$



**Figure 12-16:** Cumulative distribution function,  $F(t)$  for different values of  $\sigma$

maintenance influence reliability, improving reliability is largely the domain of design. One common approach for increasing the reliability of the system is through the redundancy in design, which is usually achieved by placing components in parallel. As long as one component operates, the system will work.

### 12.5.1 System Components in Series

When the components of a system are in series as shown in Figure 12.17, for working of the system, each component must operate. Assuming that the components operate independently of each other and there are  $n$  components in series with the reliability of the  $i^{\text{th}}$  component being denoted as  $R_i$ , the system reliability is given by,

$$R_s = R_1 \times R_2 \times R_3 \times \dots \times R_i \times \dots \times R_n = \prod_{i=1}^n R_i$$

For higher the value of  $n$ , reliability of the system will be lower. Even if each individual component is over designed with higher reliability, the effect is nullified by the number of components in series. Since manufacturing limitations and resource limitations restrict the maximum reliability of any given component, any effort to redesign those results in lesser number of components in the series is a viable alternative.

When the exponential model is used, the system reliability in terms of the failure rates of individual components is given by



**Figure 12-17:** Components in series connection

$$R_s = e^{-\lambda_1 t} \times e^{-\lambda_2 t} \times e^{-\lambda_3 t} \times \dots \times e^{-\lambda_i t} \times \dots \times e^{-\lambda_n t} = e^{-\left(\sum_{i=1}^n \lambda_i\right)t} \quad (12.24)$$

Thus, the failure rate of the system (equivalent failure rate) with an exponential model for reliability (chance-failure phase of bathtub curve) is

$$\lambda_s = \sum_{i=1}^n \lambda_i$$

If each component that fails is replaced immediately by another component that has the same failure rate, the MTTF of the system is given by

$$\text{MTTF} = \frac{1}{\sum_{i=1}^n \lambda_i}$$

If all components in the series have an identical failure rate, say  $\lambda$ , the MTTF of the system will be a special case of the above.

$$\text{MTTF} = \frac{1}{n\lambda}$$

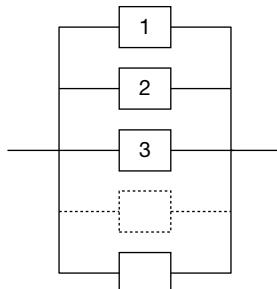
### 12.5.2 System Components in Parallel

The components in parallel (as shown in Figure 12.18) are said to be redundant. The system operates as long as at least one of the components operates and the system fails only if all the components fail. Redundant components improve the system reliability and the braking mechanism with dual subsystems in automobiles is an example.

The equations for calculating the system reliability, failure rate and MTTF for a system with components in parallel are as follows.

For exponential failure model,

$$R_s = 1 - \prod_{i=1}^n \left(1 - e^{-\lambda_i t}\right)$$



**Figure 12-18:** Components in parallel connection

$$\begin{aligned}
 R_s &= 1 - \{(1 - R_1) \times (1 - R_2) \times (1 - R_3) \times \dots \times (1 - R_i) \times \dots \times (1 - R_n)\} \\
 &= 1 - \prod_{i=1}^n (1 - R_i)
 \end{aligned} \tag{12.25}$$

In the special case of all components having the same failure rate,

$$R_s = 1 - \left(1 - e^{-\lambda t}\right)^n$$

and the MTTF is

$$\text{MTTF} = \frac{1}{\lambda} \left( 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$$

**Example 12.6:** Which system, (i) or (ii), has the highest reliability at the end of 90 operations hours?

- (i) Two CFR components in parallel, each having an MTTF of 900 hrs.
- (ii) A Weibull component with a shape parameter 2 and a characteristic life of 10,000 hrs in series with a CFR component with a failure rate of 0.00005 per hour.

#### Solution:

- (i) Reliability for a system with two CFR components in parallel, each having an MTTF of 900 hrs.

Component reliability in CFR model is given by  $R(t) = e^{-\lambda t}$ , where

$$\text{Failure rate, } \lambda = \frac{1}{\text{MTTF}} = \frac{1}{900} \text{ per hour}$$

$$\text{Therefore, } R(100) = e^{-\lambda t} = e^{-\left(\frac{1}{900}\right)90} = 0.9048.$$

System reliability in parallel configuration

$$R(t) = 1 - (1 - R_1(t))(1 - R_2(t)),$$

Since both components are identical,  $R_1(90) = R_2(90) = 0.9048$

$$R(90) = 1 - (1 - 0.9048)(1 - 0.9048) = 0.9909$$

- (ii) Reliability for a system with a Weibull component with a shape parameter 2 and a characteristic life of 10,000 hrs in the series with a CFR component with a failure rate of 0.00005.

$$\text{Weibull component reliability, } R_1(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}, \text{ where } \beta = 2, \theta = 10,000 \text{ hrs}$$

$$R_1(90) = e^{-\left(\frac{90}{10000}\right)^2} = 0.9999$$

Exponential component reliability,  $R_2(t) = e^{-\lambda t}$ , where  $\lambda = 0.00005$  per hour

$$R_2(90) = e^{-(0.00005)(90)} = 0.9955$$

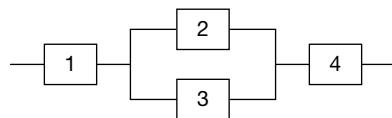
System reliability in series configuration is given by  $R(t) = R_1(t)R_2(t)$ ,  
Therefore,

$$R(90) = 0.9999 \times 0.9955 = 0.9954$$

So, system reliability is higher in system (ii).

### 12.5.3 System Components in Series and Parallel Mixed

A system may be called complex system, if some components are in series and others are in parallel (as shown in Figure 12.19). However, the calculation of system reliability of complex systems is complicated because by using a reduction process where each sub-system that is in parallel is represented by the equivalent series element, the system reliability can be finally found by the equations pertaining to the system with components in series.



**Figure 12-19:** Mixed connection (series and parallel)

$$R_s = R_1 \times \{1 - (1 - R_2)(1 - R_3)\} \times R_4$$

**Example 12.7:** In a system, 10 components, each of reliability factor 0.95, are in series. What is the overall reliability of the system? If these components are arranged in parallel connection and have reliability factor of 0.3 each, find the reliability of the systems.

#### Solution:

In series connection:

$$R_s = R_1 \times R_2 \times R_3 \times \dots \times R_i \times \dots \times R_n = \prod_{i=1}^n R_i = (0.95)^{10} = 59.87\%$$

In parallel connection:

$$\begin{aligned} R_s &= 1 - \{(1 - R_1) \times (1 - R_2) \times (1 - R_3) \times \dots \times (1 - R_i) \times \dots \times (1 - R_n)\} \\ &= 1 - \prod_{i=1}^n (1 - R_i) = 1 - (1 - 0.3)^{10} = 97.17\% \end{aligned}$$

**Example 12.8:** In a system, there are four components in parallel followed by three components in series. The components in parallel have a reliability of 0.7 each and those in series have a reliability of 0.8 each. Determine the reliability of the entire system.

**Solution:**

The overall reliability of the components in parallel connection is

$$R_I = 1 - (1 - R_p)^4 = 1 - (1 - 0.7)^4 = 0.99$$

The overall reliability of the components in series connection is

$$R_{II} = R_s^3 = (0.8)^3 = 0.512$$

Reliability of the system,  $R_s = R_I \times R_{II} = 0.9919 \times 0.512 = 0.5078 = 50.78\%$

**Example 12.9:** The length of time that a particular piece of equipment operates before failure is a random variable with cumulative distribution function.

$$F(t) = 1 - e^{-0.043t^{2.6}}$$

What is the probability that the equipment operates for more than 5 years without experiencing failure?

**Solution:** The probability that the equipment operates for more than  $t$  years without experiencing failure.

$$\begin{aligned} R &= 1 - \text{Probability that it will fail within period } t \\ &= 1 - F(t) = 1 - (1 - e^{-0.043t^{2.6}}) = e^{-0.043t^{2.6}} \\ R_s &= e^{-0.043(5)^{2.6}} = 0.0593 = 5.93\% \end{aligned}$$

**Example 12.10:** The MTTF of engine, gearbox and tyre system of a vehicle are 280, 3200 and 140 hours, respectively. Determine the reliability of the vehicle at 30 hours of use. Determine the time at which the reliability is 90 per cent.

**Solution:** We know that  $\text{MTTF} = \frac{1}{\lambda}$  and  $R = e^{-\lambda t}$

$\text{MTTF}_{\text{Engine}} = 280$  hours,  $\text{MTTF}_{\text{Gearbox}} = 3200$  hours, and  $\text{MTTF}_{\text{Tyresystem}} = 140$  hours

Case I:

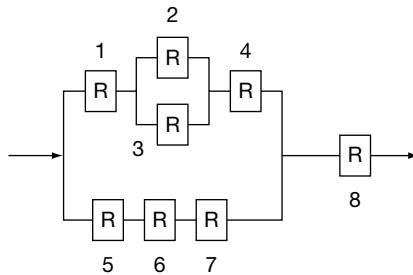
$$R_s = R_1 \times R_2 \times R_3 = e^{-30/280} \times e^{-30/3200} \times e^{-30/140} = 0.7183 = 71.83\%$$

Case II:

$$R_s = e^{-t/280} \times e^{-t/3200} \times e^{-t/140} = e^{-t/0.0110} = 90 \Rightarrow t = 9.578 \text{ hours}$$

**Example 12.11:** (a) For the network shown in Figure 12.20, deduce an expression for the system reliability in terms of the component reliabilities. Assume that each component has a reliability of  $R$ .

(b) Compute the system reliability if  $R = 0.98$



**Figure 12-20:** Network of components

**Solution:**

Given,  $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = R = 0.98$

Components,  $R_2$  and  $R_3$ , are in parallel. Let us replace them by an equivalent subsystem, then the reliability of subsystem,  $R_{23}$ , is

$$R_{23} = 1 - (1 - R_2)(1 - R_3)$$

Therefore,  $R_{23} = 1 - (1 - R)^2 = 2R - R^2$

Component,  $R_1$ , subsystem,  $R_{23}$  and component,  $R_4$  are in series. Let us replace them by an equivalent subsystem,  $R_{1234}$ , then the reliability of subsystem,  $R_{1234}$ , is

$$R_{1234} = R_1 \times R_{23} \times R_4,$$

Therefore,  $R_{1234} = R \times (2R - R^2) \times R = 2R^3 - R^4$

Components,  $R_5$ ,  $R_6$  and  $R_7$  are in series. Let us replace them by an equivalent subsystem, then the reliability of the subsystem,  $R_{567}$ , is

$$R_{567} = R_5 \times R_6 \times R_7,$$

Therefore,  $R_{567} = R \times R \times R = R^3$

Subsystems,  $R_{1234}$  and subsystem,  $R_{567}$  are in parallel. Let us replace them by another equivalent subsystem,  $R_{17}$ , then the reliability of the subsystem,  $R_{17}$ , is

$$R_{17} = 1 - (1 - R_{1234})(1 - R_{567}),$$

Therefore,  $R_{17} = 1 - \{1 - (2R^3 - R^4)\} \{1 - R^3\} = 3R^3 - R^4 - 2R^6 + R^7$

Now, subsystems,  $R_{17}$  and component,  $R_8$ , are in series. Let us replace them by another equivalent subsystem,  $R_{18}$ , then reliability of subsystem,  $R_{18}$ , is

$$R_{18} = R_{17} \times R_8,$$

Therefore,  $R_{18} = 3R^3 - R^4 - 2R^6 + R^7 \times R = 3R^4 - R^5 - 2R^7 + R^8$ .

(b) Putting  $R = 0.98$

$$R_{18} = 0.977$$

## 12.6 IMPROVEMENT IN RELIABILITY OF A SYSTEM

The ways used to improve the reliability of a product or system are as follows.

1. Improve the design of components.
2. Simplify the design of the system.
3. Improve production techniques.
4. Improve quality control.
5. Test components and the system.
6. Install parallel systems.
7. Perform periodic preventive maintenance.

### 12.6.1 Redundancy

Redundancy is the duplication of critical components or functions of a system with the intention of increasing reliability of the system. Redundancy does not just imply a duplication of hardware, since it can also be implemented at the software level. However, to avoid common mode of failures, redundant elements should be realized independently from each other. From an operating point of view, redundancies are classified as active redundancy, warm and passive redundancies. From system configuration point of view, the redundancies are classified as high-level and low-level redundancy. These redundancy types are elaborated in the following subsections.

**Active redundancy:** It is also known as parallel or hot redundancy. In this case, redundant components are subjected from the beginning of the same load as operation elements. If any one element fails, then remaining redundant components share the load of failed components. Since all the components are active, the failure rate in the reserve state is the same as in the operation state.

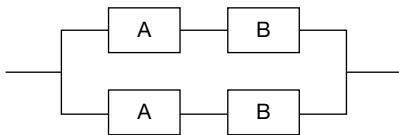
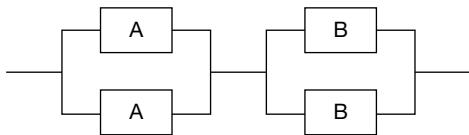
**Warm redundancy:** It is also known as lightly loaded redundancy. Redundant components are subjected to a lower load until one of the operating components fails. Load sharing is possible in this case. The failure rate in the reserve state is lower than in the operating state.

**Passive redundancy:** It is also known as standby or cold or unloaded redundancy. Redundant components are subjected to no load until one of the operating components fails. In this case, no load sharing is possible. The failure rate in the reserve state is assumed to be zero.

**High-level redundancy:** In high-level redundancy, parallel-series circuit is used. In this circuit, the system is counted as success if any one of the parallel circuit arms is functioning, which in turn is possible only if all the components of that particular arm are operating. It is a case of redundancy at the subsystem level.

A typical example of a high-level redundancy is shown in Figure 12.21. This arrangement can be converted into the simple parallel form by first evaluating the series reliability of each path and then treating them as two components in parallel.

**Low-level redundancy:** In low-level redundancy, series-parallel circuit is used. As shown in Figure 12.22, Unit A has two parallel redundant elements, Unit B has two parallel redundant

**Figure 12-21:** RBD for high-level redundancy**Figure 12-22:** RBD of low-level redundancy

elements. For Unit A to be successful, 1 or 2 must operate, for Unit B success, 3, 4 must operate for system success. It is a case of redundancy at component level.

It can be proved that low-level redundancy is more reliable than the high-level redundancy.

## 12.7 DESIGN GUIDELINES FOR RELIABILITY

1. Carry design-redesign iteration with the help of manufacturing and assembly engineers until a final design that is most acceptable and efficient evolves. Once finalized, follow the design in the subsequent steps of product life cycle.
2. Formulate detailed specifications as exactly as possible.
3. Give priority to just in time and lean manufacturing with minimum waiting times and inventory.
4. Carry out the process planning activity, based on the final design, if possible, using an efficient CAPP system, keeping the machine facilities in close reference.
5. Taking into account, while deciding the machining/forming parameters in process plan, all the environmental and current status of machine tools instead of those quoted in their catalogues. Catalogues give the specification of a new and unused machine tool, whereas the machine tools in the shop floor may have depreciated.
6. Plan for on-line inspection methods and Non-Destructive Testing (NDT), wherever appropriate and feasible.
7. Provide for condition monitoring so that the present data can help in the manufacture of future products.
8. Ensure that the design data is clearly understandable and is widely displayed on the shop floor so as to be visible to the operators.
9. Plan for future corrections based on the on-line data collected during the present activities conducted on the product during the entire product life cycle.

## 12.8 RELIABILITY TESTING

The purpose of reliability testing is to discover potential problems with the design as early as possible and, ultimately, provide confidence that the system meets its reliability requirements. Reliability testing leads to reduced unscheduled downtime, increased production opportunity/profitability and savings in repair cost and part inventory. It also facilitates the transition from ‘run to failure’ to pro-active maintenance. Some of the commonly used reliability testing techniques are vibration analysis, balancing and alignment services, ultrasonic testing infrared thermography. Reliability testing may be performed at several levels. Complex systems may be tested at component, unit, assembly, subsystem and system levels.

### **12.8.1 Component Stress Testing**

Stress testing is about simulating large workloads to see how the component performs under peak usage conditions. With component stress testing, you isolate the constituent components and services, figure out what functional and interface methods they expose. The idea here is to stress each component in isolation far beyond what the normal application would experience.

### **12.8.2 Integration Stress Testing**

After stressing each individual component, the entire system with all of its components and supporting services is subjected to stress. Integration stress testing is largely concerned about interactions with other services, processes and data structures from both internal components and other external application services. Integration testing starts with just basic functional testing. It is required to know the coded pathways and user scenarios; understand what the users are trying to do, and identify all the ways the user goes through the application.

### **12.8.3 Accelerated Reliability Testing**

Accelerated life tests are conducted on materials, components and manufacturing processes to determine their useful life in the required product application. Their purpose is not to expose defects, but to identify and quantify the failures and failure mechanisms which cause products to wear out at the end of their useful life. Because of this, accelerated life tests must last long enough to cause the samples under test to fail by wear out. The test time may typically vary from a few weeks to a few months.

Accelerated life testing employs a variety of high stress test methods to shorten the life of a product or quicken the degradation of the product's performance. The goal is to efficiently obtain performance data that, when properly analysed, yields reasonable estimates of the product's life or performance under normal operating conditions.

## **12.9 MAINTAINABILITY**

Maintenance management is the direction and organization of resources in order to control the availability and performance of industrial plant to some specified level. Maintenance plays a supporting role to keep equipments and machines operating effectively to carry out the required production processes and to maintain quality, quantity and cost standards. Some important definitions of maintenance are given below as:

Maintenance is that function of maintenance management that is concerned with the day to day problem of keeping the physical plant in good operating condition.

—Harold T Amrine, John A Ritchey

Maintenance relates to profitability through equipment output and equipment running cost. Maintenance work rises the level of equipment performance and availability, but at the same time it adds to running costs. The objective of an industrial maintenance department should be the achievement of the optimum balance between these effects.

—A Kelley, Harri S.M.J.

The ability of an item, under stated conditions of use, to be retained in, or restored to a state in which it can perform its required function(s), when maintenance is performed under stated conditions and using prescribed procedures and resources. Maintainability can be expressed in terms of 'Mean Time To Repair' (MTTR).

Maintainability criteria have two aspects.

1. The ability of an equipment to meet operational objective with a minimum expenditure of maintenance effort under operational environmental conditions in which scheduled and unscheduled maintenance is performed.
2. The probability that an item will be restored to specified conditions within a given period of time when a maintenance action is performed in accordance with prescribed procedure and resources.

It is the probability that a failed component or system will be restored or repaired to a specified condition within a period of time when maintenance is performed in accordance with prescribed procedures.

If  $T$  is the random variable representing the repair time and  $t$  is the time in which the system is repaired, then maintainability is defined as

$$M(t) = \Pr(T \leq t) \quad (12.26)$$

Let the repair time be exponentially distributed with parameter  $\mu$ , then the repair-density function is

$$g(t) = \mu e^{-\mu t} \quad (12.27)$$

Therefore,

$$M(t) = \Pr(T \leq t) = \int_0^t \mu e^{-\mu t} dt = 1 - e^{-\mu t} \quad (12.28)$$

The expected value of repair time is called the mean time to repair (MTTR) and is given by

$$\text{MTTR} = \int_0^\infty t g(t) dt \quad (12.29)$$

For an exponentially distributed repair time, this expression is simplified as

$$\text{MTTR} = \int_0^\infty t \mu e^{-\mu t} dt = \frac{1}{\mu} \quad (12.30)$$

### 12.9.1 Factors Contributing to Maintainability at Design Stage

Some of the factors which contribute to maintainability of equipment are as follows:

1. Accessibility of assemblies and components
2. Standardization
3. Monitoring facilities
4. Procedures

5. Identification
6. Safety
7. Availability of literature
8. Training

One of the unforeseen benefits of design for maintainability is that an easily maintained design will, more than likely, be easy to assemble and manufacture. The manufacturing engineer and maintainability engineer on a concurrent engineering team have common issues, such as designs that use common fasteners. This results in reduced amounts of different tool requirements for maintenance or assembly and speeds both the assembly and disassembly processes.

An important element of maintainability is testability. Testability is a design characteristic which allows the status of an item to be determined and the isolation of faults within the item to be performed in a timely manner. The benefits include lowest programming cost, lowest testing cost, longest test equipment life and highest product confidence.

### **12.9.2 Benefits of Maintainability**

The major benefits of maintainability of a system are expressed in terms of manufacturing, quality and support benefits as discussed below:

#### ***Manufacturing Benefits***

1. Fewer false starts
2. Reduced labour costs
3. Reduced scrap costs
4. Faster production
5. Higher product quality

#### ***Quality Benefits***

1. Fewer line stoppages
2. Lower inspection costs
3. Less MRB (Material Review Board) activity
4. Higher product quality

#### ***Support Benefits***

1. Fewer maintenance actions
2. Less expensive maintenance actions
3. Shorter downtimes
4. Lower inventory costs
5. Lower support equipment costs
6. Lower training costs

### 12.9.3 Human Factor Aspect in Design for Maintainability

The methodology of human factors engineering can be applied to systems design to minimize the time and effort required to perform periodic preventive maintenance as well as unscheduled maintenance. Field observation tools may be developed and field study may be conducted to ascertain the level of effort required to maintain existing systems and to identify opportunities for system improvement. The benefits of such field study aimed at design for maintainability include:

1. Greater task efficiency and reduced MTTR
2. Improved system performance and reliability
3. Increased process tool utilization
4. Fewer operational errors
5. Decreased stress in maintenance technicians
6. Reduced fatigue and incidence of cumulative trauma
7. Reduced training time and costs
8. Less reliance on maintenance manuals
9. Reduced installation and maintenance costs
10. Decreased maintenance staffing

## 12.10 DESIGN FOR MAINTAINABILITY

'Maintenance is an activity to keep the resources in working condition or restore them to operating status.' Maintenance does not mean complete prevention of failure. Instead, it tries to prevent failure and if failure occurs it restores the system very quickly and economically. To improve the productivity and proper maintenance of tools, equipments and other resources are required. There are a number of maintenance philosophies that are used for successful operation.

*Predictive maintenance:* It is the process of using data and statistical tools to determine when a piece of equipment will fail and that piece of equipment or tools are changed before occurrence of failure.

*Preventive maintenance:* It is the process of periodically performing activities such as lubrication on the equipment to keep it running. It is the routine inspection and service activities designed to detect potential failure conditions and make minor adjustments or repairs that will help prevent major operating problems.

*Breakdown maintenance:* It is the repairs, often of an emergency nature and a cost premium of facilities and equipment that have been used until they fail to operate. Breakdown maintenance is likely to be sick leave, hospitalization or other healing period during which the employee is incapable of performing at a normal level.

*Total productive maintenance:* It is based on philosophy of total quality management. Total productive maintenance (TPM) is keeping the plant and equipment at its highest productive level through co-operation of all areas of organization. The objective of TPM is to create a system in which all maintenance activities can be planned and not interfere with the production process. Surprise equipment breakdowns should not occur.

The overall goals of TPM are given below as:

1. Maintaining and improving equipment capacity.
2. Maintaining equipment for increasing the life.
3. Using support from all areas of operation.
4. Encouraging input from all employees.
5. Using teams for continuous improvement.

## 12.11 MAINTENANCE COSTS

Equipment breakdowns, idle workers and equipments, resulting in lost production time, delayed schedules and expensive emergency repairs. These downtime costs usually exceed the preventive maintenance costs for inspections, service and scheduled repairs up to an optimal point. Beyond this optimal point, an increasingly higher level of preventive maintenance is not economically justified, and the firm would be better off waiting for breakdowns to occur. Whereas the optimal level of maintenance activity (M) is easily identified on a theoretical basis, in practice this necessitates knowing a good deal about the various costs associated with both the preventive and breakdown maintenance activities. This includes the knowledge of probability of breakdowns and the amount of repair time required. Although these data are not always easily obtained, good maintenance records will provide substantial help in estimating the probability distributions of breakdown and repair times.

## 12.12 AVAILABILITY

It is the probability that a system is available for use at a given time. It is a function of reliability and maintainability. It is the operating time divided by total time, which is the available time per day minus the planned downtime.

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}; \quad \text{MTTR} = \frac{\sum(T \times F)}{\sum F}.$$

where  $T$  = Repair time,  $F$  = Failure rate

**Example 12.12:** The following data was collected for an automobile:

MTBF = 500 hrs; Mean waiting for spares = 5 hrs

Mean time for repairs = 48 hrs; Mean administrative time = 2 hrs

Compute the availability of the automobile.

**Solution:**

Total mean down time =  $5 + 48 + 2 = 55$  hrs.

Using relation,

$$\text{Availability} = \frac{\text{uptime}}{\text{uptime} + \text{downtime}}$$

$$\text{Availability} = \frac{500}{500 + 55} = 0.90$$

The automobile would be available 90 per cent of the time.

There are few other related availability measures which are defined as:

1. Point availability,  $A(t)$  at time  $t$  is defined as:
2. Average availability over the interval  $[0, T]$  is defined as:

$$A(T) = (1/T) \int_0^T A(t) dt$$

3. Mission or interval availability is defined as:

$$A_{t_2-t_1} = \left( \frac{1}{t_2 - t_1} \right) \int_{t_1}^{t_2} A(t) dt$$

4. Steady-state availability is defined as:

$$A = \lim_{T \rightarrow \infty} A(t)$$

**Example 12.13:** A plant runs in two shifts of 16 hours a day. During each shift of 8 hours, there is 2 hrs of planned downtime. If machine is stopped each day on average of 110 min for set-up and 75 min for breakdowns and repairs, find the availability of the equipment.

**Solution:**

$$\text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} = \frac{\text{Actual running time}}{\text{Planned running time}}$$

$$\begin{aligned} \text{Planned running time} &= \text{Total plant time} - \text{Planned downtime} \\ &= 16 - 2 \times 2 = 12 \text{ hours} \end{aligned}$$

$$\begin{aligned} \text{Actual running time} &= \text{Planned running time} - \text{All other downtime} \\ &= 12 \times 60 - (110 + 75) = 535 \text{ min} \end{aligned}$$

$$\text{Availability} = \frac{535}{12 \times 60} = 0.743 \Rightarrow 74.3 \text{ per cent}$$

## 12.13 SERVICEABILITY

Service may be defined as the activity, including diagnosis, maintenance, repair and anything else that affects the activity to keep the system functioning. Serviceability is a measure of the following aspects:

1. How often the system needs servicing (the less often the better the serviceability)
2. How easy it is to service (the easier to service the better the serviceability)
3. How long it takes to service (shorter the time it takes to service better the serviceability)
4. How much the service costs (the lesser it costs to service the higher the serviceability)

Serviceability aspect is important because no product is 100 per cent reliable. So, serviceability is not the same as reliability. Previously, many researchers have investigated on the aspect of serviceability. Noteworthy contributions are that of Makino who proposed the Design For Serviceability (DFS) and the work carried out at Ohio state university, which led to the method of Design Compatibility Analysis (DCA). Based on a set of compatibility information, that is good, poor and bad examples of design, DCA evaluates a candidate design with respect to its specifications and constraints, provides an overall assessment, and suggests improvements. DCA is effective in component selection, design for injection moulding, forging product and process design, and serviceability design and personal computers. The methodology leads to computer programs that incorporate current knowledge to improve future designs.

### **12.13.1 Design for Serviceability**

Not taking serviceability concerns into consideration at the design stage can result in one or more of the following problems:

1. The product cannot be repaired.
2. The cost of repair is very high.
3. Increased service and warranty expenditure.
4. Intangible effects on customer satisfaction.

### **12.13.2 Serviceability Variables**

There are a few subcategories of the aspects of serviceability:

1. Target audience
2. Factors affecting serviceability
3. Subcomponents of serviceability
4. Repair trade-offs

### **12.13.3 Target Audience**

The audiences for DFS are the inexperienced audience, the service engineer and the design engineer. The inexperienced engineer can see how different designs affect the life-cycle cost of a product by browsing through the DFS software. If the design experiences are gathered and stored in a knowledge base supported by the software, this user can learn and keep at finger tips the people's design experiences. The service engineers need a tool to answer designers' questions regarding the cost benefits of various designs and to defend their own ideas. The design engineer is enabled by the knowledge of serviceability and a related software package to use those concerns of serviceability at the early stages of design.

### **12.13.4 Major Factors Affecting Serviceability**

Following is the list of factors that affect serviceability:

1. Reliability of components and subsystems
2. Labour cost
3. Inventory cost

4. Accessibility of components to be serviced
5. Availability of necessary parts, tools and anything else needed for service
6. Mechanic training
7. Customer preferences
8. Where the product is serviced
9. Length and coverage of warranty
10. Special tool, repair equipment and replacement part production

We know that Failure Modes and Effects Analysis (FMEA) is a tool to find out the mode and criticality of failure. In the context of serviceability, a new term is defined, Service Mode Analysis (SMA) which involves steps that are to be taken to correct, or prevent the various malfunctions that a system experience in service and the issues associated.

### **12.13.5 Subcomponents of Serviceability**

Following are the subcomponents of serviceability:

1. Diagnosability
2. Maintainability
3. Repairability
  - (a) Malfunction repairability
  - (b) Crash repairability

### **12.13.6 Repair Trade-offs**

The *repair trade-off* is the ease of removal and decreased total manufacturing cost versus loss in ability to make fine repairs and the cost of a module. If only one small and inexpensive component costing less than one dollar fails, will it be necessary to remove and replace a 100 dollar module that only takes 10 min to remove? Or just replace the inexpensive component, which takes 3 hrs to access (the mechanic has to open up the modular part, which may be complicated and therefore has an elaborate disassembly procedure)?

The decision depends on the relative costs of labour and the module in question. If a component in the modular part of an automobile has a life of 1000 miles (needs replacement after that) and rest all the components in that modular part have a life of 10,000 miles, then the entire modular part may have to be replaced for every 1000 miles if the component is not separable from the module or difficult to access inside the intricacy of the modular part. Therefore, such aspects of serviceability related repair trade-off aspects should be taken into account at the design stage itself. The permanency in the connection between the components of a modular part has a tendency to increase the reliability of the part but this has to weigh against the increased module cost, should a repair be necessitated.

## **12.14 HOUSEKEEPING AND 5S CONCEPTS**

Housekeeping is a part of maintenance management that supports to maintain the workplace properly. A Japanese concept widely known as 5S concept has been used to maintain the workplace. The details of 5S concept are given below as:

1. **Seiri:** *Seiri* means sort the unneeded equipments, tools, furniture; unneeded items on walls, bulletins; items blocking aisles or stacked in corners; unneeded inventory, supplies, parts; safety hazards, those are not required at the workplace.
2. **Seiton:** *Seiton* means arrangement of equipment in proper order. Use the items and then return to its place. It deals with items not in their correct places; correct places not obvious; aisles, workstations, and equipment locations not indicated; items are not putting away immediately after use, etc.
3. **Seiso:** *Seiso* means keep the workplace and equipments clean and dustless. It is concerned with floors, walls, stairs, equipments and surfaces not cleaned; cleaning materials not easily accessible; labels, signs broken or unclean; and other cleaning problems.
4. **Seiketsu:** *Seiketsu* means standardization. It is concerned with the necessary information not visible; standards not known; checklists missing; quantities and limits not easily recognizable; items cannot be located within 30 sec.
5. **Shitsuke:** *Shitsuke* means self-discipline. It is concerned with number of workers without 5S training; number of daily 5S inspections not performed; number of personal items not stored; number of times job aids not available or up to date.



## SUMMARY

In this chapter, the reliability of a system has been explained and various frequency distributions for reliability have been discussed in detail. Series parallel and mixes connection of components and reliability calculation using these connections have been demonstrated. Maintainability and its significance in manufacturing have been explained. Similarly, the concepts of availability, serviceability and 5S have been emphasized. After completion of this chapter, a student will be able to calculate reliability of a system, mean time to failure, MTBF and availability of an item.

## MULTIPLE-CHOICE QUESTIONS

1. The probability with which an item or system will remain functioning under stated operational and environmental conditions for a specified period of time, is known as
  - (a) quality
  - (b) responsiveness
  - (c) assurance
  - (d) reliability
2. Reliability engineering is concerned with key elements
  - (a) intended function
  - (b) specified period of time
  - (c) stated conditions
  - (d) all the above
3. Which of the following is represented by Bathtub curve?
  - (a) failure rate
  - (b) reliability
  - (c) availability
  - (d) none of these
4. Mean time between failure is used for
  - (a) irrepairable items
  - (b) repairable items
  - (c) both (a) and (b)
  - (d) none of these

5. Exponential distribution is used for
  - (a) decreasing failure rate
  - (b) increasing failure rate
  - (c) constant failure rate
  - (d) none of these
6. Normal distribution is used for
  - (a) decreasing failure rate
  - (b) increasing failure rate
  - (c) stochastic behaviour of systems
  - (d) none of these
7. Weibull Distribution is used for
  - (a) decreasing failure rate
  - (b) increasing failure rate
  - (c) constant failure rate
  - (d) all the above
8. The mechanical components, which fail due to the gradual wear and tear, are best modelled by
  - (a) exponential Distribution
  - (b) normal distribution
  - (c) Weibull distribution
  - (d) none of these
9. In a system, of 10 components, each of reliability factor 0.90 are in series. What is the overall reliability of the system?
  - (a) 0.95
  - (b) 0.75
  - (c) 0.55
  - (d) 0.35
10. In a system, of 10 components, each of reliability factor 0.90 are in parallel. What is the overall reliability of the system?
  - (a) 0.99
  - (b) 0.89
  - (c) 0.79
  - (d) 0.59
11. In the case of active redundancy,
  - (a) the redundant components are subjected to a lower load until one of the operating components fails.
  - (b) the redundant components are subjected from the beginning of the same load as operation elements.
  - (c) the redundant components are subjected to no load until one of the operating components fails.
  - (d) none of these
12. In the case of warm redundancy,
  - (a) The redundant components are subjected to a lower load until one of the operating components fails.
  - (b) The redundant components are subjected from the beginning of the same load as operation elements.
  - (c) The redundant components are subjected to no load until one of the operating components fails.
  - (d) None of these
13. In the case of passive redundancy,
  - (a) the redundant components are subjected to a lower load until one of the operating components fails.
  - (b) the redundant components are subjected from the beginning of the same load as operation elements.
  - (c) the redundant components are subjected to no load until one of the operating components fails.
  - (d) none of these

14. Under predictive maintenance,
  - (a) the components are changed periodically
  - (b) the components are changed after failure
  - (c) the components are changed based on prediction of failure
  - (d) none of these
  
15. Breakdown maintenance is used when
  - (a) cost of failure is more than the cost of maintenance
  - (b) cost of failure is equal to the cost of maintenance
  - (c) cost of failure is less than the cost of maintenance
  - (d) none of these

## Answers

1. (d)      2. (d)      3. (a)      4. (b)      5. (c)      6. (c)      7. (d)      8. (c)      9. (d)  
 10. (a)     11. (b)     12. (a)     13. (c)     14. (c)     15. (c)

## REVIEW QUESTIONS

1. Define the term ‘reliability’. Discuss the objectives of reliability engineering.
2. Discuss the bathtub curve used for failure pattern.
3. What do you mean by MTBF and MTTF?
4. Explain the basic failure rate models: (a) Constant failure rate model, (b) Weibull model, (c) Normal model.
5. Derive the expressions for reliability of a systems in (a) Series (b) Parallel.
6. What are the ways to improve the reliability of a system?
7. Write short notes on various types of redundancy used to improve the reliability.
8. Discuss the methods used for reliability testing.
9. What do you mean by maintainability? Discuss the factors contributing to the maintainability at design stage.
10. Write short notes on (a) maintenance policy, (b) availability, (c) serviceability (d) 5s principles of housekeeping.

## EXERCISES

1. Fifty machines have been operated for 1000 hours. Ten machines fail in 640 hours and 20 machines fail in 800 hours. What is MTBF and what will be the reliability at 1500 and 2000 hours?
2. The time-to-failure density function (PDF) for a system is  

$$f(t) = 0.01 \quad 0 \leq t \leq 50 \text{ days}$$
 Find
  - (a) Reliability for time  $t$ ,  $R(t)$
  - (b) The hazard rate function at time  $t$ ,  $\lambda(t)$

- (c) The MTTF  
(d) The standard deviation
3. A component has the following linear hazard rate, where  $t$  is in years:  $\lambda(t) = 0.06t$ ,  $t \geq 0$
- Find  $R(t)$  and determine the probability of a component failing within the two months of its operation.
  - What is the design life if a reliability of 0.98 is desired?
4. A component experiences chance (CFR) failures with an MTTF of 900 hrs. Find the following:
- The reliability for a 50 hr mission
  - The design life for a 0.98 reliability
  - The median time to failure
5. For a system having a Weibull failure distribution with a shape parameter of 0.5 and a scale parameter of 200 days, find the following:
- $R(100 \text{ days})$
  - MTTF
  - median time to failure,  $t_{\text{med}}$
  - The design life for a reliability of 0.98
  - The standard deviation
6. In a system, there are five components in parallel followed by four components in series. The components in parallel have a reliability of 0.8 each and those in series have a reliability of 0.6 each. Determine the reliability of the entire system.



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# CHAPTER ▶ 13

## Cost Accounting and Depreciation

### 13.1 INTRODUCTION

Accounting is a method in which all the costs incurred in carrying out an activity are collected, classified and recorded. This data is then summarized and analysed to determine the selling price (SP). Cost analysis and estimation are slightly difficult due to some unforeseen such as inflation, unpredictable change in technology and the dynamic nature of markets. Cost analysis consists of accumulation, examination and manipulation of cost data for comparisons and projections. Economists and accountants interpret the term ‘cost’ in different ways, but they are related things. Economists define ‘cost’ in terms of the opportunities that are sacrificed when a choice is made. Hence, under the economist’s definition, costs are simply benefits lost. Accountants define ‘cost’ in terms of resources consumed. To distinguish between these two cost concepts, what economists do is referred to as cost estimation and what accountants do is termed as cost measurement. Various combinations of costs are historical and current costs, explicit and implicit costs, incremental and sunk costs, opportunity cost, and long-run and short-run costs. These costs are explained in the following paragraphs:

*Historical cost vs. current cost:* Cost data is known as historical if it is stored for a period of time and then used; and cost data is known as current cost when the cost is paid under prevailing market conditions. It is assumed that current cost exceeds historical cost, but it is not always true. For example, current costs of electronics items are less than their costs 10 years back. Therefore, the current cost for such items is determined by what is referred to as a replacement cost which is defined as the cost of duplication using the current technology.

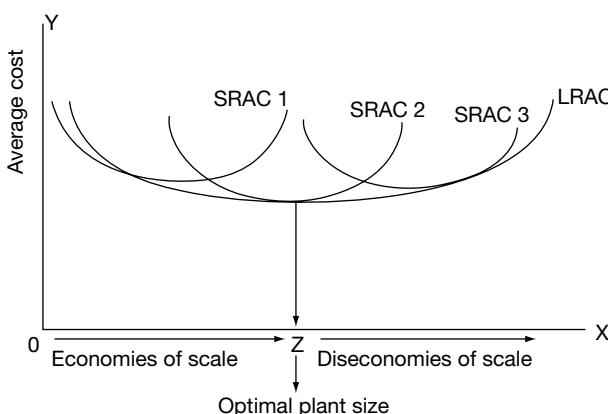
*Explicit cost vs. implicit cost:* Explicit cost is the amount of money paid by the entrepreneur for purchasing/hiring the services of various production factors to the agencies outside the organization or the persons that are not the part of his organization. This cost is in the nature of contractual payment and may include rent for land, wages to the labour, interest on capital and costs incurred on heads such as raw materials, fuel, power, etc. Implicit cost is more difficult to compute and is likely to be overlooked in decision analysis. Implicit cost is normally computed on the basis of the opportunity cost concept so as to reach an accurate estimate of total cost.

*Incremental cost vs. sunk cost:* Incremental cost refers to change in cost caused by a given managerial decision while sunk cost does not change or vary across decision alternatives. A sunk cost is an expenditure that has been incurred and cannot be recovered. As the sunk cost cannot be recovered, it is irrelevant for decision-making.

**Opportunity cost:** The opportunity cost is the value of the alternatives foregone by adopting a particular strategy or employing resources in a specific manner. It is also called *alternative cost* or *economic cost*. For example, if an asset is used for one purpose, the opportunity cost is the value of the next best purpose the asset could have been used for. The opportunity cost plays an important role in a decision-making process, but the opportunity cost is not treated as an actual cost in any financial statement.

**Long-run costs vs. short-run costs:** The long run is the hypothetical time period in which there are no fixed factors of production as to changing the output level by changing the capital stock or by entering or leaving an industry. The long-run contrasts with the short run in which some factors are variable and others are fixed, constraining entry or exit from an industry. In the short run, the average total cost (ATC) curve is U-shaped. The short-run average cost (SRAC) curve initially falls and reaches at minimum level and then starts to rise. The reason for the average cost to fall in the beginning of production is that the fixed factors of a firm remain the same. The change takes place only in the variable factors such as raw materials and labour. As the fixed cost (FC) gets distributed over the output with the expansion of production, the average cost begins to fall. When a firm fully utilizes its scale of operation, i.e. capacity, then the average cost is at its minimum level. The firm then operates to its optimum capacity. If the firm in the short run increases its level of output with the same fixed capacity, the economies of that scale of production change into diseconomies and the average cost then begins to rise sharply.

In the long-run, all costs of a firm are assumed as variable. The factors of production can be used in varying proportions to deal with an increased output. The firm having a time period long enough can build a larger scale to produce the anticipated output. The long-run average cost (LRAC) curve is also U-shaped but is flatter than the short-run curve as is illustrated in Figure 13.1. The short-run is a period of time during which one or more of a firm's inputs cannot be changed. In contrast, the long run is a period of time during which all inputs can be changed.



**Figure 13-1: Long-run and short-run average cost curves**

Economies of scale exist when LRAC declines as output expands along with the capacity due to factors such as labour specialization, better technology, commercial and financial–managerial

advantages and also through learning economies. Diseconomies of scale exist when LRAC increases as output along with the capacity enlarges. The cost increases mainly due to the administrative disadvantage of large scale when the firm size expands beyond the optimal size.

## 13.2 COST ELEMENTS

The total production costs in short-run production can be divided into two groups as: fixed cost (FC) and variable cost (VC). FC does not vary with production quantity, whereas VC varies with production quantity. In addition to these costs, some other terminologies like average variable cost (AVC), ATC and marginal cost (MC) are discussed in detail in the following paragraphs:

1. *Fixed cost (FC)*: It refers to the costs of all fixed inputs in a production process that does not change with the quantity of the output produced.
2. *Variable cost (VC)*: It refers to the cost of all variable inputs in a production process that changes with the quantity of the output produced.
3. *Total cost (TC)*: It is the sum of  $FC$  and  $VC$ ,  $TC = FC + VC$ .
4. *Average cost or average total cost (AC or ATC)*: Average cost (AC), also known as ATC, is the average cost per unit of output. It is the ratio of  $TC$  and the quantity ( $Q$ ) the firm is producing. The ATC and AVC curves are shown in Figure 13.2. Mathematically, ATC and AVC are expressed as

$$ATC = TC / Q$$

$$ATC = AFC + AVC$$

$$AFC = FC / Q$$

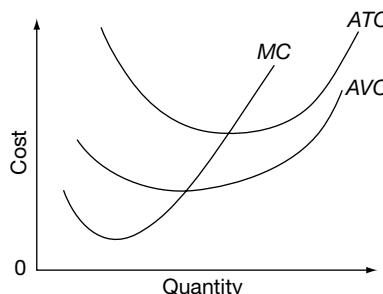
$$AVC = VC / Q$$

where  $AFC$  is average fixed cost,  $AVC$  is average variable cost,  $FC$  is fixed cost,  $VC$  is variable cost and  $Q$  is volume of output.

5. *Marginal cost (MC)*: It is the additional cost that results from increasing the output by one unit. Mathematically, it is represented as:

$$MC = \Delta TC / \Delta Q$$

where  $\Delta TC$  is change in total cost and  $\Delta Q$  is change in quantity.



**Figure 13-2: ATC and AVC curves**

### 13.3 COST ACCOUNTING

Broadly, three elements of cost are considered in the production process; these are material cost, labour cost and expenses. Material cost can be divided into two classes: direct material cost and indirect material cost. *Direct material cost* is the cost incurred on the material which is processed through various production stages, e.g., raw material. The cost of direct material includes the purchase price as well as expenses such as freight, insurance, loading and unloading charges, etc. *Indirect material cost* is the cost incurred on the material which is not an integral part of the product but used to process the direct material, e.g., lubricating oil or the coolant.

*Labour cost* can be divided into two classes: direct and indirect labour costs. *Direct labour cost* consists of wages of the workers directly involved in the manufacturing of the product, e.g. labourers working on the machine. *Indirect labour cost* consists of the wages of the workers engaged indirectly to help the production of the product. But, they are not directly engaged or responsible for the production, e.g. administrative staff.

Apart from the material and labour costs, there are several other expenditures such as cost of advertisement, depreciation charges of plant and machinery, the cost of transportation and packages, etc. These expenditures are known as *expenses*. Expenses can be divided into two classes: *direct expenses* and *indirect expenses*. *Direct expenses* are those which can be directly charged for a particular product, e.g. cost of jigs and fixture for a special job, experimental set for a special job, are the direct expenses. Indirect expenses include factory expenses, administrative expenses, selling expenses and distribution expenses.

*Prime cost:* It is also called as a direct cost and consists of direct labour, direct material and direct expenses; i.e. Prime cost = Direct material cost + Direct labour cost + Direct expenses.

*Factory cost:* It consists of prime cost and factory expenses; i.e. Factory cost = Prime cost + Factory expenses.

*Production cost:* It consists of factory cost and administrative expenses; i.e. Production cost = Factory cost + Administrative expenses.

*Total cost:* It includes production cost and selling and distribution charges; i.e. Total cost = Production cost + Selling expenses + Distribution charges.

**Example 13.1:** (a) Two mechanics are employed in a casting process for 20 jobs, each weighing 5 kg in a shift of 8 hours. They are paid at the rate of Rs 120 and Rs 100 per day. The forged material costs Rs 4.50 per kg. If the factory and administrative on costs put together are thrice the labour cost, find the cost of production per unit.

(b) A product is manufactured in batches of 500. The direct material cost is Rs 15,000, direct labour cost is Rs 20,000, and factory overheads are 40 per cent of the prime cost. If the selling expenses are 30 per cent of the factory cost, what would be the selling price of each product so that profit is 20 per cent of the total cost?

**Solution:**

- (a) Labour cost per day = Rs 120 + Rs 100 = Rs 220 per day.  
 Material cost =  $20 \times 5 \times 4.5 = \text{Rs } 450$ .  
 Factory and administrative overhead =  $3 \times \text{Labour cost} = 3 \times 220 = \text{Rs } 660$ .  
 Production cost = Labour cost + Material cost + Factory and administrative overheads  
                   = Rs 220 + Rs 450 + Rs 660 = Rs 1330 per 20 jobs  
 Production cost per unit = Rs 66.5
- (b) Direct material cost = Rs 15,000; Direct labour cost = Rs 20,000  
 Factory overhead =  $0.4 (\text{Direct material cost} + \text{direct labour cost}) = 0.4 \times 35,000$   
                   = Rs 14,000  
 Selling expenses =  $0.3 \times \text{factory cost} = 0.3 \times (\text{Direct material cost} + \text{Direct labour cost} + \text{Factory overhead})$   
                   =  $0.3 \times (\text{Rs } 15,000 + \text{Rs } 20,000 + \text{Rs } 14,000) = \text{Rs } 14,700$   
 Selling price =  $1.2 \times (\text{Direct material cost} + \text{Direct labour cost} + \text{Factory overhead} + \text{Selling expenses})$   
                   =  $1.2 \times (\text{Rs } 15,000 + \text{Rs } 20,000 + \text{Rs } 14,000 + \text{Rs } 14,700)$   
                   = Rs 76,440  
 Selling price per unit =  $\text{Rs } 76,440 / 500 = \text{Rs } 152.88$

### 13.4 COMPUTATION OF MATERIAL VARIANCES

Variation in standard costs and the actual costs is also an important part of the cost analysis. Variance is the difference between the standard cost and the actual cost. In other words, it is the difference between what the cost should have been and what the actual cost is. Material variances is the difference between the standard cost of material used for actual production and actual cost of material used. Thus, the main variance in this category is the cost variance, which is thereafter broken down into other variances. Element wise computation of variances can be described as follows:

*Material cost variance:* As mentioned earlier, this variance shows the difference between the standard cost of material consumed in actual production and the actual cost. The following formula is used for computation of this variance.

$$\text{Material cost variance} = \text{Standard cost of material consumed} - \text{Actual cost}$$

If the actual cost of material consumed is less than the standard cost of material consumed, the variance is favourable (F), otherwise it is adverse (A).

*Material price variance:* One of the reasons for the difference between the standard material cost and actual material cost is the difference between the standard price and actual prices. Material price variance measures the difference between the standard price and actual prices with reference to the actual quantity consumed. The following formula is used for the computation of material price variance:

$$\text{Material price variance} = \text{Actual quantity} [\text{Standard price} - \text{Actual price}]$$

*Material quantity variance:* This variance measures the difference between the standard quantity of material consumed in actual production and the actual quantity consumed and the same is multiplied by standard price. The following formula is used for computation of material quantity variance:

$$\text{Material quantity variance} = \text{Standard price} [\text{Standard quantity} - \text{Actual quantity}]$$

The total of price variance and quantity variance is equal to cost variance.

$$\text{Material cost variance} = \text{Material price variance} + \text{Material quantity variance}.$$

**Example 13.2:** Calculate material variances from the following data.

Standard quantity of materials for producing 1 unit of a product 'X' is 10 kg. The standard price is Rs 8 per kg. During a particular period, 100 units of 'X' were produced. Actual material consumed was 1,200 kg at a cost of Rs 7,500.

**Solution:**

$$\begin{aligned}\text{Material cost variance} &= \text{Standard cost of materials} - \text{Actual cost} \\ &= 100 \text{ units} \times 10 \text{ kg} \times \text{Rs } 8 - \text{Rs } 7,500 \\ &= \text{Rs } 8,000 - \text{Rs } 7,500 = \text{Rs } 500 \text{ [Favourable]}\end{aligned}$$

$$\begin{aligned}\text{Material price variance} &= \text{Actual quantity} \times [\text{Standard price} - \text{Actual price}] \\ &= 1,200 \times [\text{Rs } 8 - \text{Rs } 6.25] = \text{Rs } 2,100 \text{ [Favourable]}\end{aligned}$$

$$\begin{aligned}\text{Material quantity variance} &= \text{Standard price} \times [\text{Standard quantity} - \text{Actual quantity}] \\ &= \text{Rs } 8 \times [1,000 - 1,200] = -\text{Rs } 1,600 \text{ [Adverse]}\end{aligned}$$

## 13.5 BREAK-EVEN ANALYSIS

Break-even analysis (BEA) is a graphical representation of the cost–volume relationship. Break-even point (BEP) represents the level of output at which there is no profit and no loss. Profit is the difference between total revenue and total production cost:

$$\text{Total revenue} = \text{No. of units produced} \times \text{Price per unit}$$

$$\text{Total cost} = \text{No. of units produced} \times \text{Cost per unit}$$

Total cost consists of VC and FC. VC varies with the volume of production but, FC remains fixed. By definition, VCs change (in total) in response to changes in volume of production. It is also assumed that the relationship between VCs and volume of production is proportional. Examples include the costs of direct labour, raw materials and sales commissions. By definition, FCs do not change (in total) in response to changes in volume or activity. Examples include the costs of depreciation, supervisory salaries and maintenance expenses.

***Assumptions:***

*BEA* or cost-volume-profit (*CVP*) analysis has the following assumptions:

1. The changes in the level of revenues and costs arise due to the changes in the number of products/or services units produced and sold. A cost driver is any factor that affects costs, a revenue driver is any factor that affects revenue.
2. Total costs can be divided into a fixed component and a component that is variable with respect to the level of output.

*VCs* include the following:

- (a) Direct materials.
- (b) Direct labour.
- (c) Direct chargeable expenses.

Variable overheads include the following:

- (a) Variable part of factory overheads.
- (b) Administration overheads.
- (c) Selling and distribution overheads.

3. There is a linear relationship between revenue and cost. When put in a graph, the behaviour of total revenue and total cost is linear (straight line), that is,  $y = mx + c$  holds good, which is the equation of a straight line, where  $y$  is revenue and  $x$  is the cost,  $m$  is the coefficient of cost representing profit,  $c$  is a constant.
4. The unit *SP* (Selling Price), unit *VCs* and *FCs* are constant.
5. The theory of *CVP* is based upon the production of a single product. However, of late, management accountants are functioning to give a theoretical and a practical approach to multi-product *CVP* analysis.
6. The *CVP* analysis either covers a single product or assumes that the sales mix of multiple products will remain constant as the level of total units sold changes.
7. All revenues and costs can be added and compared without taking into account the time value of money.
8. The theory of *CVP* is based on the technology that remains constant.
9. The theory of price elasticity is not taken into consideration.

Margin of safety, contribution margin (*CM*) and profit volume ratio are the important terms and help in the decision-making process. These terms are discussed as follows:

*Margin of safety:* The margin of safety is the difference between the expected level of sales and a break-even sales. It may be expressed in units or rupees of sales.

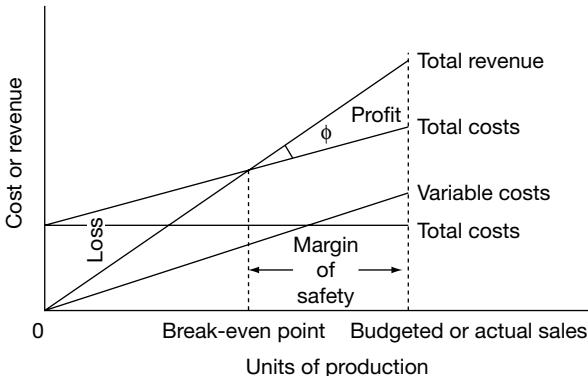
*Contribution margin:* It is the difference between the *SP* and the *VC* per unit. It measures the amount each unit sold contributes to cover *FCs* (first) and increase profit (once *FCs* are covered). The relationship is as  $SP - VC$ .

*Contribution margin ratio:* This ratio expresses the contribution of every sales rupee in covering *FCs* (first) and operating profit (second). It is calculated as follows:

$$\frac{SP - VC}{SP}$$

*Angle of incidence,  $\phi$ :* It is the angle at which the total revenue line intersects the total cost line (see in Figure 13.3).

*Profit-volume ratio:* It is the ratio of contribution and sales.



**Figure 13-3:** Break-even analysis

### Calculation of BEP

The *BEP* can be calculated in terms of physical units and in terms of sales turnover.

*BEP in terms of physical units:* The number of units required to be sold to achieve the *BEP* can be calculated using the following formula:

$$BEP = \frac{FC}{SP - VC}$$

where  $FC$  is the fixed cost,  $VC$  is the variable cost,  $SP$  is the selling price,  $C$  is the contribution per unit ( $C = SP - VC$ ). On the  $x$ -axis, *BEP* in Figure 13.3 shows the *BEP* in terms of physical units.

*BEP in terms of sales volume:* *BEP* in terms of sales volume can be calculated using the following formula:

$$BEP = \frac{FC}{SP - VC} \times SP$$

where  $FC$  is the fixed cost,  $SP$  is the selling price per unit, the  $VC$  is the variable cost per unit.

On the  $y$ -axis, *BEP* in Figure 13.3 shows the *BEP* in terms of sales volume.

**Example 13.3:** A company XYZ sold an auto component of 1,000 units of a product of with variable cost of Rs 10 per unit. Each unit contributes 20 per cent of its revenue to the company's *FCs* and profits. The company wants to reduce the product's price by 10 per cent. Calculate how many more units the company is required to sell at the 10 per cent price reduction to earn the same profit as before the price reduction.

**Solution:**

Let  $x$  be the selling price in rupees per unit of the product. Contribution to the  $FC$  and profit is  $0.20x$ . Now

$$x = 0.20x + 10$$

or

$$x = 12.5$$

The selling price is reduced by 10 per cent. Now new selling price  $= 0.9 \times 12.5 =$  Rs 11.25/unit. We have,

$$Q = \frac{FC + P}{SP - VC}; \text{ and } Q_{\text{new}} = \frac{FC + P}{SP_{\text{new}} - VC}$$

So,

$$\begin{aligned} \Rightarrow \frac{Q}{Q_{\text{new}}} &= \frac{SP_{\text{new}} - VC}{SP - VC} = \frac{11.25 - 10}{12.5 - 10} = 0.555 \\ \Rightarrow Q_{\text{new}} &= \frac{Q}{0.555} = \frac{1,000}{0.555} = 1,800 \text{ units.} \end{aligned}$$

**Example 13.4:** From the given information about a company, calculate the break-even point and the turnover required to earn a profit of Rs 50,000. Fixed overheads are Rs 2,50,000; selling price is Rs 25; and variable cost per unit is Rs 5. If the company earns a profit of Rs 50,000, express the margin of safety available to it.

**Solution:**

Contribution per unit ( $S - V$ ) = Rs  $(25 - 5) =$  Rs 20 per unit

Fixed overheads = Rs 2,50,000

$$\begin{aligned} \text{Break-even point} &= \frac{\text{Fixed overheads}}{\text{Contribution per unit}} \\ &= \frac{2,50,000}{20} \\ &= 12,500 \text{ units or sales of Rs 3,12,500} \end{aligned}$$

Turnover required to earn a profit of Rs 50,000:

$$\begin{aligned} Q &= \frac{\text{Fixed overheads} + \text{Profit}}{\text{Contribution per unit}} \\ &= \frac{\text{Rs } 2,50,000 + 50,000}{20} \\ &= 15,000 \text{ units or sales of Rs 3,75,000} \end{aligned}$$

Margin of safety:

Actual sales = 15,000 units or Rs 3,75,000

Sales at break-even point = 12,500 units or Rs 3,12,500

Margin of safety = Actual sales – Sales at BEP = 2500 units of Rs 62,500.

**Example 13.5:** The profit–volume ( $P/V$ ) ratio of a pharmaceutical company is 50 per cent and the margin of safety is 30 per cent. Find out the break-even point and the net profit if the sales volume is Rs 10,00,000.

**Solution:**

Sales = Rs 10,00,000

Margin of safety (30 per cent) = Rs 3,00,000

So,

$$BEP = \text{Sales} - \text{Margin of safety} = 10,00,000 - 3,00,000 = \text{Rs } 7,00,000$$

Now

Sales at  $BEP$  = Rs 7,00,000

So,

$$\begin{aligned}\text{Contribution at } BEP &= \text{Sales at } BEP \times P/V \text{ Ratio} \\ &= 7,00,000 \times 50 \text{ per cent} \\ &= \text{Rs } 3,50,000\end{aligned}$$

The fixed overheads are

$$\begin{aligned}\text{Fixed overheads} &= \text{Sales at } BEP - \text{Contribution at } BEP \\ &= \text{Rs } (7,00,000 - 3,50,000) \\ &= \text{Rs } 3,50,000\end{aligned}$$

Therefore, net profit if the sales volume is Rs 10,00,000 =

$$\begin{aligned}(\text{Sales} \times P/V \text{ Ratio}) - \text{Fixed cost} &= (10,00,000 \times 50 \text{ per cent}) - 3,50,000 \\ &= 5,00,000 - 3,50,000 \\ &= \text{Rs } 1,50,000\end{aligned}$$

### 13.5.1 Sales Mix BEP Calculation

Sales mix is the proportion in which two or more products are sold. For the calculation of BEP for sales mix, the following assumptions are made in addition to those already made for CVP analysis:

1. The proportion of sales mix must be predetermined.
2. The sales mix must not change within the relevant time period.

The calculation method for the BEP of sales mix is based on the contribution approach method. Since we have multiple products in sales mix, therefore it is most likely that we will be dealing with products with different CM per unit and CM ratios. This problem is overcome by calculating weighted average CM per unit and CM ratio. These are then used to calculate the BEP for sales mix.

The calculation procedure and the formulas are discussed in the following example:

**Example 13.6:** Information that is related to sales mix of product A, B and C is given in Table 13.1.

**Table 13-1:** Information related to product mix

Product	A	B	C
Sales price per unit	Rs 24	Rs 27	Rs 35
Variable cost per unit	Rs 12	Rs 15	Rs 18
Sales mix percentage	30%	25%	45%
Total fixed cost		Rs 1,80,000	
Total sales		Rs 8,00,000	

Calculate the *BEP* in units and in rupees.

**Solution:**

**Step 1:** Calculate the *CM* per unit (Table 13.2) for each product.

**Table 13-2:** Contribution margin per unit

Product	A	B	C
Sales price per unit	Rs 24	Rs 27	Rs 35
Variable cost per unit	Rs 12	Rs 15	Rs 18
CM per unit = Sales price per unit – Variable cost per unit	Rs 12	Rs 12	Rs 17

**Step 2:** Calculate the weighted-average *CM* per unit (Table 13.3) for the sales mix.

$[Product\ A\ CM\ per\ Unit \times Product\ A\ Sales\ Mix\ Percentage + Product\ B\ CM\ per\ Unit \times Product\ B\ Sales\ Mix\ Percentage + Product\ C\ CM\ per\ Unit \times Product\ C\ Sales\ Mix\ Percentage = Weighted\ Average\ Unit\ Contribution\ Margin]$

**Table 13-3:** Weighted average contribution margin per unit

Product	A	B	C
Sales price per unit	Rs 24	Rs 27	Rs 35
Variable cost per unit	Rs 12	Rs 15	Rs 18
CM per unit = Sales price per unit – Variable cost per unit	Rs 12	Rs 12	Rs 17
Sales mix percentage	30%	25%	45%
CM per unit × Sales mix percentage	Rs 3.6	Rs 3.0	Rs 7.65
Sum: Weighted average CM per unit = $\Sigma$ CM per unit × Sales mix percentage	Rs 14.25		

**Step 3:** Calculate total units of sales mix required to break-even using the formula:

$$BEP \text{ in units of sales mix} = \text{Total fixed cost} \div \text{Weighted average } CM \text{ per unit}$$

$$= Rs\ 1,80,000 \div Rs\ 14.25 = 12,632$$

**Step 4:** Calculate number units of product A, B and C at *BEP* (Table 13.4).

**Table 13-4:** Product units at break-even point

Product	A	B	C
Sales mix ratio	30%	25%	45%
Total break-even units	12,632	12,632	12,632
Product units at BEP = Sales mix ratio × Total break-even units	3790	3158	5684

**Step 5:** Calculate *BEP* in rupees (Table 13.5).

**Table 13-5:** Break-even point in rupees

Product	A	B	C
Product Units at BEP	3790	3158	5684
Price per Unit	Rs 24	Rs 27	Rs 35
Product sales in rupees = Product units at BEP × Price per unit	Rs 90,960	Rs 85,266	Rs 1,98,940
Sum: BEP in rupees = $\Sigma$ Product sales in rupees			Rs 3,75,166

## 13.6 DEPRECIATION

Depreciation is the decreasing value of physical properties or assets with the passage of time and use. It is used in accounting concept for annual deduction from a firm's before-tax income to reflect in the firm's financial statements, i.e. the effect of time and use on the value of its assets. In general, a property will be depreciable if it meets the following conditions:

1. It is used in business or held to produce income.
2. It has a determinable useful life, and the life is longer than 1 year.
3. Some parts of the property that wears out, decays, gets used up, becomes obsolete or loses value from natural causes.
4. It is not inventory, stock in trade or investment property.

Organizations also consider the effect of income tax on the financial results of an engineering project because income tax usually represents a significant cash outflow that cannot be ignored in decision-making. In this chapter, we will learn how income tax liabilities (or credits) and after-tax cash flows are determined in engineering practice. An after tax cash flow (ATCF) procedure will be used in capital investment analysis because it avoids innumerable problems associated with measuring corporate net income. This procedure helps in finding a project's profitability.

### 13.6.1 The Major Causes of Depreciation

An asset depreciates due to the following reasons:

*Wear and tear:* Wear and tear refers to a decline in the efficiency of an asset due to its constant use. When an asset loses its efficiency, its value goes down and depreciation arises. This is true in case of tangible assets like plant and machinery, building, furniture, tools and equipment used in the factory.

*Effusion of time:* The value of an asset may decrease due to the passage of time, even if it is not in use. There are some intangible fixed assets like copyright, patent right and leasehold premises which decrease its value as time elapses.

*Exhaustion:* An asset may lose its value because of exhaustion too. This is the case with wasting assets such as mines, quarries, oil wells and forest-stand. On account of continuous extraction, a stage will come where mines and oil wells get completely exhausted.

*Obsolescence:* Changes in fashion are external factors which are responsible for throwing out of assets even if those are in good condition. For example, black-and-white TVs have become obsolete with the introduction of colour TVs, the users have discarded black-and-white TVs although they are in good condition. Such a loss on account of a new invention or changed fashions is termed as obsolescence.

*Other causes:* Market value (MV) and accident of an asset are other causes of depreciation which decrease in the value of assets.

### 13.6.2 Need for the Provision of Depreciation

The need for provision for depreciation arises for the following reasons:

1. *Ascertainment of true profit or loss:* Depreciation is a loss. It must be considered in order to find out true profit/loss of a business.
2. *Ascertainment of the true cost of production:* Goods are produced with the help of plant and machinery which incurs depreciation in the process of production. This depreciation must be considered as a part of the cost of production of goods. Otherwise, the cost of production would be shown less than the true cost.
3. *True valuation of assets:* Value of assets gradually decreases on account of depreciation. If depreciation is not taken into account, the value of asset will be shown in the books at a figure higher than its true value and hence the true financial position of the business will not be disclosed through Balance Sheet.
4. *Replacement of assets:* After some time, an asset will be completely exhausted on account of use. A new asset, then is purchased which may require large sums of money. If the whole amount of profit is withdrawn from business each year without considering the loss on account of depreciation, necessary sum may not be available for buying the new assets. In such a case, the required money is to be collected by introducing fresh capital or by obtaining loan by selling some other assets.
5. *Keeping capital intact:* Capital invested in buying an asset, gradually diminishes on account of depreciation. If loss on account of depreciation is not considered in determining profit/loss at the year end, profit will be shown more. If the excess profit is withdrawn, the working capital will gradually reduce, the business will become weak and its profit earning capacity will also fall.
6. *Legal restriction:* According to Companies Act, 1956, dividend cannot be declared without charging depreciation on fixed assets. Thus, in case of joint stock companies, charging of depreciation is compulsory.

### 13.6.3 Some Important Terms Used in Depreciation

1. *Adjusted cost basis:* It denotes the original cost basis of the asset, adjusted by allowable increases or decreases. It is used to compute depreciation deductions.
2. *Cost basis:* It represents the initial cost of acquiring an asset (purchase price plus any sales taxes), including transportation expenses and other normal costs of making an asset serviceable for its intended use. This amount is also called the *unadjusted cost basis*.
3. *Book value:* Book value (*BV*) represents the amount of capital that remains invested in the property and must be recovered in the future through the accounting process. The *BV* of a property may not be an accurate measure of its *MV*. In general, *the BV of a property at the end of year k is*

$$(\text{Book value})_k = \text{Adjusted cost basis} - \sum_{j=1}^k (\text{Depreciation deduction})_j$$

4. *Market value:* Market value (*MV*) is the value of a property that is paid by a willing buyer to a willing seller, where both the buyer and the seller have equal advantage and are under no compulsion to buy or sell.
5. *Recovery period:* This period equals the number of years over which the capital invested on a property is recovered through the accounting process. In classical methods of depreciation, this period is normally the useful life of a property. In modified accelerated cost recovery system (*MACRS*), this period is the *GDS* class life for the general depreciation system (*GDS*), and it is the *ADS* class life for the alternative depreciation system (*ADS*). The *MACRS* has been explained later in the chapter.
6. *Recovery rate:* It is a percentage (in the form of a fraction) for each year of the *MACRS* recovery period that is utilized to compute an annual depreciation deduction.
7. *Salvage value:* Salvage value (*SV*) is the estimated value of a property at the end of its useful life. In other words, it is the expected selling price (*SP*) of a property when the asset can no longer be used efficiently by its owner. Under *MACRS*, the *SV* of a depreciable property is defined to be zero.
8. *Useful life:* It is the expected period over which a property can be used economically in a business. It is not how long the *property will* last but how long the owner expects to effectively use it.

### 13.6.4 Classical Depreciation Methods

The straight-line (SL), declining-balance (DB) and a unit of production methods are considered as classical methods to calculate depreciation of an equipment, asset and other depreciable things.

#### (A) SL Method

The SL method assumes that a constant amount is depreciated each year over the useful life of an asset. The difference of the cost basis and SV is equally divided throughout the life of the asset to find the value of annual depreciation. To know the total depreciation upto year *k* (say), the annual depreciation is multiplied by number of years, i.e. *k*.

Let  $N$  be the depreciable life of the asset in years;  $B$ , the cost basis;  $d_k$ , the annual depreciation deduction in year  $k$  ( $1 \leq k \leq N$ );  $BV_k$ , the  $BV$  at the end of year  $k$ ;  $SV_N$ , the estimated  $SV$  at the end of year  $N$ ;  $d_k^*$ , the cumulative depreciation through year  $k$ . Then

$$d_k = (B - SV_N) / N$$

$$d_k^* = k \cdot d_k \text{ for } 1 \leq k \leq N$$

$$BV_k = B - d_k^*$$

An estimate of the final  $SV$ , which is generally the same as the final  $BV$  at the end of year  $N$ . In some cases, the estimated  $SV_N$  may not equal an asset's actual terminal  $MV$ .

**Example 13.7:** An equipment was purchased for Rs 2,50,000 and Rs 50,000 more was spent on its installation and commissioning. The estimated residual value after 7 years was Rs 70,000.

- (a) Calculate the annual rate of depreciation.
- (b) Determine the amount of depreciation at the end of five years after the purchase of the furnace.

**Solution:**

We have a basis ( $B$ ) = Rs 2,50,000 + Rs 50,000 = Rs 3,00,000; salvage value ( $SV_7$ ) = Rs 70,000; and depreciable life ( $N$ ) = 7 years, then

- (a) Depreciation rate is given by

$$\frac{B - SV_N}{N} = \frac{3,00,000 - 70,000}{7} = \text{Rs } 30,000/\text{year}$$

- (b) Depreciation at the end of 5 years is

$$\text{Rs } 30,000 \times 5 = \text{Rs } 1,50,000$$

**Example. 13.8:** A device costing Rs 5,00,000 is required to be replaced after every 10,000 hours. If the plant operates 24 hours a day and 30 days a month, what is the depreciation charged per month for the device?

**Solution:**

We have  $B$  = Rs 5,00,000,  $N$  =  $10,000/24 = 416.66$  days = 13.88 month. Then depreciation charged per month is

$$d = \frac{B - SV_N}{N} = \frac{5,00,000}{13.88} = \text{Rs } 36,000 / \text{month}$$

**(B) Sum of Year Digit Method**

In this method, the amount of depreciation decreases from start to end of life of an asset. Following formula is used to calculate the depreciation.

$$d_k = \frac{n - (k - 1)}{\sum_{k=1}^n k} (B - S)$$

where  $k$  is the year for which depreciation is to be calculated;  $n$  is life of the asset;  $B$  is the initial cost; and  $d_k$  is the depreciation for the  $k$ .

**(C) DB Method**

In the *DB* method, it is assumed that the annual depreciation is a fixed percentage of the *BV* at the beginning of the year. The ratio of the depreciation in any one year to the *BV* at the beginning of that year is constant throughout the life of the asset. In this method,  $R = 2/N$  when a 200 per cent *DB* is used (i.e. twice the *SL* rate of  $1/N$ ), where  $N$  equals the useful life of an asset. If the 150 per cent *DB* method is used, then  $R = 1.5/N$ . The following relationships can be used for the *DB* method:

$$\begin{aligned}d_1 &= B(R) \\d_k &= B(1-R)^{k-1}(R) \\d'_k &= B[1-(1-R)^k] \\BV_k &= B(1-R)^k\end{aligned}$$

**Example 13.9:** A milling machine has a cost basis of Rs 5,00,000 and a depreciable life of 8 years. The estimated salvage value of the machine is zero at the end of the 8 year period. Use the *DB* method to calculate the annual depreciation amounts when

- (a)  $R = 2/N$
- (b)  $R = 1.5/N$
- (c) Using the *SL* method

**Solution:**

We have  $B = \text{Rs } 5,00,000$ ;  $N = 8$  years;  $S = 0$ .

- (a) Annual depreciation amount when  $R = 2/N$  is

$$d_k = B \cdot R = 5,00,000 \times 2 / 8 = \text{Rs } 1,25,000$$

- (b) Annual depreciation amount when  $R = 1.5/N$  is

$$d_k = B \cdot R = 5,00,000 \times 1.5 / 8 = \text{Rs } 93,750$$

- (c) *SL* is

$$d = \frac{B - SV_N}{N} = \frac{5,00,000}{8} = \text{Rs } 62,500$$

**Example 13.10:** A device was purchased for Rs 80,000. After the *SL* method was used to calculate depreciation for a total life of 12 years, the equipment's expected salvage value is Rs 2000. What would be the difference between the equipment's book value (after 8 years) and the book value that would have resulted if the *DB* method at the rate of 12 per cent applied for 8 years had been used?

**Solution:**

We have  $B = \text{Rs } 80,000$ ;  $SV_N = \text{Rs } 2,000$ ;  $N = 12$  years. Then

$$d_k = \frac{B - SV_N}{N} = \frac{80,000 - 2,000}{12} = \text{Rs } 6,500/\text{year}$$

$$BV_{15_{SL}} = B - d_k \times N = 80,000 - 6,500 \times 8 = \text{Rs } 28,000$$

$$BV_{15_{DB}} = B(1-R)^k = 80,000(1-0.12)^8 = \text{Rs } 28,770.76$$

Therefore the difference is

$$28,000 - 28,770.76 = -\text{Rs } 770.76$$

#### (D) DB Method Switchover to SL Method

Since the *DB* method never reaches a *BV* of zero, it is permissible to switch from this method to the *SL* method so that an asset's *BV<sub>N</sub>* will be zero (or some other determined amount, such as *SV<sub>N</sub>*). Also, this method is used in calculating the *MACRS* recovery rates.

**Example 13.11:** A device has cost basis of Rs 1,50,000 and 10-year depreciable life. The estimated *SV* of the machine is Rs 10,000 at the end of 10 years. Use the *DB* method with switchover to *SL* method (Table 13.6).

#### Solution:

$$SV_9 = \text{Rs } 10,000$$

**Table 13-6:** DB method with switchover to SL method

EOY <sub>k</sub>	(1) Beginning of year <i>BV</i> (Rs)	(2) = (1) × 2/10 200% DB method (Rs)	(3) = {(1) – SV <sub>9</sub> }/N SL Method ( <i>BV<sub>10</sub></i> = Rs 10,000) (Rs)	(4) Depreciated amount selected (Rs)
1	1,50,000	30,000	14,000	30,000
2	1,20,000	24,000	12,222.22	24,000
3	96,000	19,200	10,750	19,200
4	76,800	15,360	9,554.85	15,360
5	61,440	12,288	8,573.33	12,288
6	49,152	9,830.4	7,830.4	9,830.4
7	39,321.6	7,864.32	7,330.4	7,864.32
8	31,457.28	6,291.45	7,152.42	7,152.42 <sup>a</sup>

<sup>a</sup>Switchover occurs in Year 8.

#### (E) Units-of-production Method

All the depreciation methods discussed above are based on the elapsed time (years), that is, on the theory that a decrease in the value of property is a function of time. When the value decreases due to the use of an asset, depreciation may be expressed in terms of a unit of production. The units-of-production method is normally used in such a case. This method results in the cost basis (minus final *SV*) being allocated equally over the estimated number of units produced during the useful life of the asset. In the units-of-production method, the depreciation rate is calculated as

$$\text{Depreciation per unit of production} = \frac{B - SV_N}{N}$$

where *N* is unit of production.

**Example 13.12:** A machine has a basis of Rs 8,00,000 and is expected to have a Rs 45,000  $SV_N$  when replaced after producing 50,000 units of a product. Find its depreciation rate per unit of the product, and find its  $BV$  after production of 12,000 units of the product.

**Solution:**

We have  $B = \text{Rs } 8,00,000$ ;  $SV_N = \text{Rs } 45,000$ ; and  $N = 50,000$  units, then

The depreciation per unit of product is given by

$$\frac{8,00,000 - 45,000}{50,000} = \text{Rs } 15.1 \text{ per unit of product}$$

$$BV = \text{Rs } 8,00,000 - \text{Rs } 15.1 \text{ hours} \times 12,000 = \text{Rs } 6,18,800$$

### 13.6.5 Modified Accelerated Cost Recovery System

The MACRS is used to calculate depreciation of the most tangible depreciable property placed in service for the last two decades. MACRS consists of two systems for computing depreciation deductions: *GDS* and *ADS*. In general, *ADS* provides a longer recovery period and uses only the *SL* method of depreciation. Properties that come under any tax-exempt are depreciated under *ADS*. Any property that is qualified under *GDS* can also be depreciated under *ADS*.

Using MACRS to calculate depreciation, the data required is: the cost basis ( $B$ ), the date the property was placed in service, the property class and recovery period, the MACRS depreciation method to be used (*GDS* or *ADS*), and the time convention that applies (half year).

#### Property Class and Recovery Period

Under MACRS, tangible depreciable property is classified into asset classes. The property in each asset class is then assigned a class life, *GDS* recovery period (and property class) and *ADS* recovery period. The class life, *GDS* recovery period (and property class) and *ADS* recovery period (all in years) for different asset classes are listed in Table 13.7.

Table 13-7: MACRS class lives and recovery periods

Internal revenue service (IRS) asset classes	Asset description	ADS class life (in years)	GDS class life (in years)
00.11	Office furniture, fixtures and equipment	10	7
00.12	Information systems: computers/peripherals	6	5
00.22	Automobiles, taxis	2	5
00.241	Light general-purpose trucks	4	5
00.25	Railroad cars and locomotives	15	7
00.40	Industrial steam and electric distribution	22	3
01.11	Cotton gin assets	10	7
01.21	Cattle, breeding or dairy	7	5

(Continued)

**Table 13-7:** (Continued)

<b>Internal revenue service (IRS) asset classes</b>	<b>Asset description</b>	<b>ADS class life (in years)</b>	<b>GDS class life (in years)</b>
13.00	Offshore drilling assets	7.5	5
13.30	Petroleum refining assets	16	10
15.00	Construction assets	6	5
20.10	Manufacture of grain and grain mill products	17	10
20.20	Manufacture of yarn, thread and woven fabric	11	7
24.10	Cutting of timber	6	5
32.20	Manufacture of cement	20	15
20.1	Manufacture of motor vehicles	12	7
48.10	Telephone distribution plant	24	15
48.2	Radio and television broadcasting equipment	6	5
49.12	Electric utility nuclear production plant	20	15
49.13	Electric utility steam production plant	28	20
49.23	Natural gas production plant	14	7
50.00	Municipal waste water treatment plant	24	15
57.0	Distributive trades and services	9	5
80.00	Theme and amusement park assets	12.5	7

### Depreciation Calculation Under MACRS

The primary methods used under MACRS for calculating depreciation deductions over the recovery period of an asset are summarized as follows:

1. **GDS 3-, 5-, 7-, and 10-year personal property classes:** The 200 per cent *DB* method, which switches to the *SL* method when that method provides a greater deduction.
2. **GDS 15- and 20-year personal property classes:** The 150 per cent *DB* method, which switches to the *SL* method when that method provides a greater deduction.
3. **GDS non-residential real and residential rental property classes:** The *SL* method over fixed *GDS* recovery periods.
4. **ADS:** The *SL* method for both personal and real property over fixed ADS recovery periods.

A half-year time convention is used in MACRS depreciation calculations for tangible personal property. This means that all assets placed in service during the year are treated as if their use began in the middle of the year, and one-half year of depreciation is allowed. When an asset is disposed of, the half-year convention is also allowed. If the asset is disposed of before the full recovery period, then only half of the normal depreciation deduction can be taken for that year.

The *GDS* recovery rates ( $r_k$ ) for six personal property classes are listed in Table 13.8. The *GDS* personal property rates in Table 13.8 include the half-year convention as well as switchover from the *DB* method to the *SL* method when that method provides a greater deduction. If an asset is disposed of in year  $N + 1$ , the final *BV* of the asset will be zero. Furthermore, there are  $N + 1$  recovery rates shown for each *GDS* property class for a property class of  $N$  years. The depreciation deduction ( $d_k$ ) for an asset under *MACRS* (*GDS*) is calculated as

$$d_k = r_k \cdot B; 1 \leq k \leq N + 1$$

where  $r_k$  is the recovery rate for year  $k$  from Table 13.8.

**Table 13-8:** MACRS applicable percentage for property classes

Recovery year	3-Year property	5-Year property	7-Year property	10-Year property	15-Year property	20-Year property
1	33.33	20.00	14.29	10.00	5.00	3.750
2	44.45	32.00	24.49	18.00	9.50	7.219
3	14.81*	19.20	17.49	14.40	8.55	6.677
4	7.41	11.52*	12.49	11.52	7.70	6.177
5		11.52	8.93*	9.22	6.93	5.713
6		5.76	8.92	7.37	6.23	5.285
7			8.93	6.55*	5.90*	4.888
8			4.46	6.55	5.90	4.522
9				6.56	5.91	4.462*
10				6.55	5.90	4.461
11				3.28	5.91	4.462
12					5.90	4.461
13					5.91	4.462
14					5.90	4.461
15					5.91	4.462
16					2.95	4.461
17						4.462
18						4.461
19						4.462
20						4.461
21						2.231

All classes convert to *SL* depreciation in the optimal year, shown with an asterisk (\*).

Table 13.9 provides information regarding the principal features of *GDS* under *MACRS*. Normally, the choice would be to use *GDS* for calculating depreciation deductions.

**Table 13-9:** MACRS (GDS) property classes and primary methods for calculating depreciation deductions

GDS property class and class life	Depreciation method	Useful life	Examples of assets and equipments
3-year	200% DB with switchover to SL	Four years or less	Includes some race horses and tractor units for over-the-road use.
5-year	200% DB with switchover to SL	More than 4 years to less than 10	Includes cars and light trucks, semiconductor manufacturing equipment, qualified technological equipment, computer-based central office switching equipment, some renewable and biomass power facilities and research and development property.
7-year	200% DB with switchover to SL	10 years to less than 16	Includes single-purpose agricultural and horticultural structures and railroad track. Includes office furniture and fixtures and property not assigned to a property class.
10-year	200% DB with switchover to SL	16 years to less than 20	Includes vessels, barges, tugs and similar water transportation equipment.
15-year	150% DB with switchover to SL	20 years to less than 25	Includes sewage treatment plants, telephone distribution plants and equipment for two-way voice and data communication.
20-year	150% DB with switchover to SL	25 years or more	Excludes real property. Includes municipal sewers.
27.5-year	SL	N/A	Residential rental property.
39-year	SL	N/A	Non-residential real property.

**Table 13-10:** MACRS percentage for real property (real estate property) table

Recovery year	Month 1	Month 2	Month 3	Month 4	Month 5	Month 6	Month 7	Month 8	Month 9	Month 10	Month 11	Month 12
1	2.461	2.247	2.033	1.819	1.605	1.391	1.177	0.963	0.749	0.535	0.321	0.107
2-39	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564	2.564
40	0.107	0.321	0.535	0.749	0.963	1.177	1.391	1.605	1.819	2.033	2.247	2.461

The useful life is 39 years for non-residential real property. The depreciation is straight line using the mid-month convention. Thus a property placed in service in January would be allowed 11½ months depreciation for recovery Year 1. Table 13.10 is similar to Table 13.9 but the former is used for real estate properties. The properties can be divided into three classes: 1-year recovery period, 2–39-year recovery period, and 40-year recovery period.

**Example 13.13:** Firm ‘XYZ’ bought a timber cutting equipment 2 years ago for Rs 20,00,000. The equipment is expected to be operational for 10 more years. If its class life is 6 years, what

property class does the equipment belong to under *MACRS (GDS)*? What is the depreciation charged for the equipment 4 years after its purchase by the firm? Also find the *BV* at the end of the fourth year if the equipment is disposed off.

**Solution:**

We have  $B = \text{Rs } 20,00,000$ ; Class life = *ADS* 6 years; Service life = 12 years; See Table 13.7 for depreciation *ADS* class life 6 years, the corresponding *GDS* recovery period is 7 years property; it belongs to *IRS* class 24.10. Use percentage depreciation for 5-year property to calculate the yearly depreciation using Table 13.8. *BTCF* is cash flow before tax. We simply deduct the depreciation from a cost basis ( $B$ ).

**Table 13-11: Cash flows before tax**

<b>EOY<sub>k</sub></b>	<b>BTCF</b>	<b>Depreciation</b>
0	-20,00,000	
1	-	$20,00,000 \times 0.20 = 4,00,000$
2	-	$20,00,000 \times 0.32 = 6,40,000$
3	-	$20,00,000 \times 0.192 = 3,84,000$
4	-	$20,00,000 \times 0.1152 = 2,30,400$
		$\Sigma d_k = \text{Rs } 16,54,000$

Therefore, we have

$$BV_3 = 20,00,000 - 14,24,000 = \text{Rs } 5,76,400$$

The depreciation deduction in year four can only be

$$(0.5)(0.1152)(20,00,000) = \text{Rs } 1,15,200$$

when the equipment is disposed of prior to year 5. Thus, the *BV* at the end of year four is

$$BV_4 = BV_3 - \text{Rs } 1,15,200 = \text{Rs } (5,76,400 - 1,15,200) = \text{Rs } 4,61,200$$

**Example 13.14:** A company XYZ traded-in a used office furnitures with a *BV* of  $\text{Rs } 15,000$ . The vendor, having accepted the older furnitures as a trade-in, was willing to make a deal and offered to sell the company a new furnitures with an *MV* of  $\text{Rs } 2,00,000$  for  $\text{Rs } 1,70,000$  in cash.

- (a) What *GDS* property class the new office furnitures would fit into?
- (b) How much depreciation can be deducted each year based on this class life?

**Solution:**

To calculate the *BTCF*

- (a) The office furniture belongs to the *IRS* class 00.11 of *ADS* class life 10 years and the *GDS* recovery period of 7 years. See Table 13.7.
- (b) The cost basis of the office furnitures is  $\text{Rs } 1,85,000$  which is the sum of  $\text{Rs } 1,70,000$  cash price of the office furnitures and  $\text{Rs } 15,000$  *BV* remaining on the trade-in. The depreciation deducted in each year is shown in Table 13.12.

**Table 13-12:** Cash flows before tax

<b>EOY<sub>k</sub></b>	<b>BTCF</b>	<b>Depreciation</b>
0	-1,85,000	
1	—	$1,85,000 \times 0.1429 = 26,436.5$
2	—	$1,85,000 \times 0.2449 = 45,306.5$
3	—	$1,85,000 \times 0.1749 = 32,356.5$
4	—	$1,85,000 \times 0.1249 = 23,106.5$
5	—	$1,85,000 \times 0.0893 = 16,520.5$
6	—	$1,85,000 \times 0.0892 = 16,502$
7	—	$1,85,000 \times 0.0893 = 16,520.5$
8	—	$1,85,000 \times 0.0446 = 8251$

### 13.6.6 Taxes

Various types of taxes used in engineering economy are discussed as follows:

1. *Income tax:* It is assessed as a function of gross revenues minus allowable deductions. It is levied by the federal, state and occasionally municipal governments.
2. *Property tax:* It is assessed as a function of the value of property owned, such as land, buildings, equipment and so on, and the applicable tax rates. It is independent of the income or profit of a firm. It is levied by municipal, county or state governments. In India it is levied by state government.
3. *Sales tax:* It is assessed on the basis of purchases of goods or services and is thus independent of gross income or profit of a firm. It is levied by state, municipal or county governments.
4. *Excise tax:* It is federal tax assessed as a function of the sale of certain goods or services often considered non-necessities and is hence independent of the income or profit of a business. Although excise tax is usually charged to the manufacturer or the original provider of the goods or services, a portion of its cost is passed on to the purchaser.

### BTCF and ATCF

In BTCF, cash flows are shown without tax deduction. In after tax cash flow, the tax is deducted from the regular cash flows. The depreciation is deducted from the annual cash inflow to calculate taxable income. Annual income tax is deducted from the cash inflows to calculate the after tax cash flows.

**Example 13.15:** A new equipment costs Rs 5,00,000 and is expected to reduce net annual operating costs by Rs 50,000 per year for 10 years. It will have an *MV* of Rs 40,000 at the end of the 10<sup>th</sup> year.

- (a) Find *ATCFs* and *BTCFs* with a recovery period of 5 years.
- (b) Find the present worth of *BTCFs* and *ATCFs* using *MARR* = 10 per cent

Assume the effective income tax rate is 40 per cent.

**Solution:**(a) *ATCF and BTCF (Table 13.13)***Table 13-13:** Cash flows before tax and after tax

<b>EOY<sub>k</sub></b>	<b>(A) BTCFs</b>	<b>Depreciation Factor</b>	<b>(B) Depreciation</b>	<b>(C) = (A) – (B) Taxable Income</b>	<b>Income Tax, (D) = –t(C) at t = 40%</b>	<b>(E) = (A) + (D) ATCF</b>
0	–5,00,000					–5,00,000
1	50,000	$5,00,000 \times 0.2$	1,00,000	–50,000	20,000	70,000
2	50,000	$5,00,000 \times 0.32$	1,60,000	–1,10,000	44,000	94,000
3	50,000	$5,00,000 \times 0.192$	96,000	–46,000	18,400	68,400
4	50,000	$5,00,000 \times 0.1152$	57,600	–7,600	3,040	53,040
5	50,000	$5,00,000 \times 0.1152$	57,600	–7,600	3,040	53,040
6	50,000	$5,00,000 \times 0.0576$	28,800	21,200	–8,480	41,520
7	50,000	—	—	50,000	–20,000	30,000
8	50,000	—	—	50,000	–20,000	30,000
9	50,000	—	—	50,000	–20,000	30,000
10	50,000	—	—	50,000	–20,000	30,000
10	40,000	—	—	40,000	–16,000	24,000
<b>Total BTCFs = Rs 40,000</b>			<b>Total ATCFs = Rs 24,000</b>			

(b) *PW of BTCFs and ATCFs*

$$\begin{aligned} PW \text{ of } BTCF &= -5,00,000 + 50,000(P/A, 10\%, 10) + 40,000(P/F, 10\%, 10) \\ &= -5,00,000 + 50,000 \times 6.145 + 40,000 \times 0.3855 = -Rs. 1,77,330 \end{aligned}$$

$$\begin{aligned} PW \text{ of } ATCFs &= -5,00,000 + 70,000(P/F, 10\%, 1) + 94,000(P/F, 10\%, 2) + \\ &\quad 68,400(P/F, 10\%, 3) + 53,040(P/F, 10\%, 4) + \\ &\quad 53,040(P/F, 10\%, 5) + 41,520(P/F, 10\%, 6) + \\ &\quad 30,000 \{(P/F, 10\%, 7) + (P/F, 10\%, 8) + (P/F, 10\%, 9) + \\ &\quad (P/F, 10\%, 10)\} + 24,000(P/F, 10\%, 10) \\ &= -5,00,000 + 70,000 \times 0.9091 + 94,000 \times 0.8264 + \\ &\quad 68,400 \times 0.75123 + 53,040 \times 0.683 + 53,040 \times 0.62 + \\ &\quad 41,520 \times 0.5645 + 30,000 \{0.5132 + 0.4665 + 0.4241 + 0.3855\} \\ &\quad + 24,000 \times 0.3855 \\ &= -Rs 1,51,817.188 \end{aligned}$$

[Note: To find the interest factor used in the above formulas, please, consult the Chapter 14 and see the introduction to the Time Value of Money in Section 14.3.2 and Table 14.6.]



## SUMMARY

In this chapter, we have discussed about various cost elements and its calculation. BEA has been discussed for single product and sales mix. In the second part of this chapter, we have discussed about the method to calculate the depreciation. The cash flows has been calculated before tax and after tax. Both the approaches of depreciation classical and modified accelerated cost recovery system have been explained with the help of some numerical examples.

## MULTIPLE-CHOICE QUESTIONS

1. In a long-run production, all the costs are
  - (a) Variables
  - (b) Fixed
  - (c) May be fixed or variable, do not depend on time
  - (d) None of these
2. Prime cost is expressed as
  - (a) Direct material cost + Direct labour cost + Direct expenses
  - (b) Direct material cost + Direct labour cost + Direct expenses + factory expenses
  - (c) Factory cost + Administrative expenses
  - (d) Production cost + Selling expenses
3. Factory cost is expressed by
  - (a) Direct material cost + Direct labour cost + Direct expenses
  - (b) Direct material cost + Direct labour cost + Direct expenses + Factory expenses
  - (c) Prime cost + Administrative expenses
  - (d) Production cost + Selling expenses
4. Production cost is expressed as
  - (a) Factory cost + Administrative expenses
  - (b) Direct material cost + Direct labour cost + Direct expenses
  - (c) Prime cost + Administrative expenses
  - (d) None of these
5. Break-even analysis consists of
  - (a) Fixed cost
  - (b) Variable cost
  - (c) Fixed and variable costs
  - (d) Operations cost
6. Margin of safety is equal to
  - (a) Difference of expected level of sale and break-even sale
  - (b) Difference between selling price and variable cost per unit
  - (c) Ratio of Difference between selling price and variable cost per unit and selling price
  - (d) None of these

7. Contribution margin is equal to
  - (a) Difference of expected level of sale and break-even sale
  - (b) Difference between selling price and variable cost per unit
  - (c) Ratio of difference between selling price and variable cost per unit and selling price
  - (d) None of these
8. Contribution margin ratio is
  - (a) Difference of expected level of sale and break-even sale
  - (b) Difference between selling price and variable cost per unit
  - (c) Ratio of difference between selling price and variable cost per unit and selling price
  - (d) None of these
9. Break-even analysis shows profits when
  - (a) Revenue is greater than the total cost
  - (b) Revenue is less than the total cost
  - (c) Revenue is equal to the total cost
  - (d) None of these
10. Break-even analysis shows no profit and no loss when
  - (a) Revenue is greater than the total cost
  - (b) Revenue is less than the total cost
  - (c) Revenue is equal to the total cost
  - (d) None of these
11. Depreciation of an asset occurs due to
  - (a) Wear and tear
  - (b) Obsolescence
  - (c) Exhaustion
  - (d) All the above
12. Book value of the product is equal to
  - (a) Adjusted cost basis minus the sum of depreciations
  - (b) Customer willing to pay for the product at the end of the life
  - (c) Tax imposed on the market value of the product
  - (d) None of these
13. Market value of the product is equal to
  - (a) Adjusted cost basis minus the sum of depreciations
  - (b) Customer willing to pay for the product at the end of the life
  - (c) Tax imposed on the market value of the product
  - (d) None of these
14. A device costing Rs 1,00,000 is required to be replaced after every 10,000 hours. If the plant operates 24 hours a day and 30 days a month, the depreciation charged per month for the device in rupees will be
  - (a) 6500
  - (b) 7205
  - (c) 8000
  - (d) 8500

15. Marginal cost means
- the additional cost used for a fix profit
  - the additional cost that results from increasing the output by one unit
  - the additional cost to produce at the end of the production
  - none of these

## Answers

1. (a)      2. (a)      3. (b)      4. (a)      5. (c)      6. (a)      7. (b)      8. (c)      9. (a)  
 10. (c)     11. (d)     12. (a)     13. (b)     14. (b)     15. (b)

## REVIEW QUESTIONS

- Distinguish between average cost and marginal cost, explicit cost and implicit cost, short-run average cost and long-run average cost, and variable cost and semi-variable cost.
- (b) Diagrammatically represent the relationship between average fixed cost, average variable cost, unit cost and marginal cost.
- Write short notes on
  - Break-even volume and opportunity cost
  - Direct and indirect cost
  - Fixed of cost and margin of safety
- Differentiate between fixed and variable costs.
- Discuss the managerial implication of break-even analysis as a tool for profit planning. What are its limitations?
- Explain the methods to calculate the depreciation.
- Write a short note on Modified Accelerated Cost Recovery System.

## EXERCISES

- (a) Two technicians are engaged on milling machines for 30 jobs, each weighing 6 kg in a shift of 8 hours. They are paid at the rate of Rs 150 and Rs 120 per day. The material costs Rs 5.0 per kg. If the factory and administrative on costs put together are twice the labour cost, find the cost of production per unit.
- (b) A product is manufactured in batches of 120. The direct material cost is Rs 800, direct labour cost is Rs 1,050, and factory overheads are 40 per cent of the prime cost. If the selling expenses is 20 per cent of the factory cost, what would be the selling price of each product so that profit is 10% of the total cost?
- A company XYZ provides the following data (Table 13.14) relating to the current year:

**Table 13-14:** Data related to sales and costs of ABC Ltd

	<b>First-half of the year</b>	<b>Second-half of the year</b>
	Rs	Rs
Sales	25,000	30,000
Total cost	20,000	22,000

Assuring that there is no change in prices and variable costs, and that the fixed expenses are incurred equally in the two half-year period, calculate for the year:

- |                         |                                 |
|-------------------------|---------------------------------|
| (a) Profit-volume ratio | (b) Fixed expenses              |
| (c) Break-even sales    | (d) Percentage margin of safety |

3. Table 13.15 shows the records of ABC Ltd. As on 31<sup>st</sup> March of the year.

**Table 13-15:** Record of sales and profits for ABC Ltd

	<b>2013</b>	<b>2014</b>
	Rs (in lakhs)	Rs (in lakhs)
Sales	150	200
Profit	10	15

Calculate:

- (a) The  $P/V$  ratio and total fixed cost.
- (b) The break-even level of sales.
- (c) Sales required to earn profit of Rs 100 lakhs.
- (d) Profit or loss for sales of Rs 300 lakhs.

4. From the following data (Table 13.16), compute material vacancies:

**Table 13-16:** Quantities and prices for alpha, beta and gamma

Name of the material	<b>Standard</b>		<b>Actual</b>	
	Quantity	Price	Quantity	Price
X	5000	15	5200	18
Y	3000	27	3150	25
Z	2500	38	2250	40

5. A multi-product company provides the following costs and output data (Table 13.17) for the last year.

**Table 13-17(a):** Sales information for the products A, B and C

<b>Products</b>			
Sales mix	<u>A</u>	<u>B</u>	<u>C</u>
	50%	30%	20%
	Rs	Rs	Rs
Selling price	25	30	35
Variable cost per unit	15	20	24
Total fixed cost			Rs 1,50,000
Total sales			Rs 5,00,000

The company proposes to replace Product C by Product D. Estimated cost and output data are:

**Table 13-17(b):** Sales information for the products A, B and D

	<b>Products</b>		
Sales mix	A 50% Rs	B 30% Rs	D 20% Rs
Selling price	25	30	32
Variable cost per unit	15	20	22
Total fixed cost			Rs 1,50,000
Total sales			Rs 5,00,000

Analyse the proposed change and suggest that decision the company should take. Calculate the net profit and overall BEP to make your analysis.

6. A small organization is producing 100 shoes/day. The direct material cost is found to be Rs 1,600, direct labour cost is Rs 2,000, and factory overheads chargeable are Rs 2,500. If the selling cost is 45 per cent of factory cost, what must be the selling price of each shoe to realize a profit of 18 per cent of selling price?
7. A company produces 200 units of a product per day. Direct materials involved is Rs 18,000, direct labour Rs 12,000 and factory overheads Rs 6,000. If the profit is 20 per cent of selling price and selling overheads one 30 per cent of factory cost, calculate the selling price per unit.
8. A lathe machine has a cost basis of Rs 2,00,000 and a depreciable life of 5 years. The estimated salvage value of the machine is zero at the end of 5 years. Use the declining balance method to calculate the annual depreciation when (a)  $R = 2/N$  (b)  $R = 1.5/N$  (c) Using SL method.



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# CHAPTER ▶ 14

## Replacement Analysis and Selection among Alternatives

### 14.1 INTRODUCTION

The policy for the replacement of an equipment or machine can be made if the reliable data related to return from the equipment, maintenance cost and replacement cost are available. The equipments can be divided into two classes: first is that equipment whose efficiency remains constant and fails suddenly; and the second is that equipment whose efficiency decreases with time. The first category operates at constant efficiency for a certain time period and then deteriorates or fails suddenly, and the second category deteriorates with time resulting in an increase in operating cost including maintenance cost.

Replacement becomes important when the job performing the machine parts, equipments, etc. becomes less effective due to sudden failure or deterioration in their performance. The decrease in performance or breakdown may be gradual or sometimes sudden. The cost factor is very important in the replacement model.

The need for replacement of items is felt when,

1. The existing item or system has become inefficient or requires more maintenance.
2. The existing equipment has failed due to accident or otherwise and does not work at all.
3. The existing equipment is expected to fail shortly.
4. The existing equipment has become obsolete due to the availability of equipment with latest technology and better design.

The solution to replacement problem is nothing but arriving at the optimal decision that determines the time at which the replacement is most economical instead of continuing at an increased maintenance cost. The main objective of replacement policy is to direct the organization in many situations so that it can take the right decision.

**Types of Failures:** As the term ‘failure’ encompasses wider concept, failures can be discussed under the following two categories.

- (a) **Sudden failure:** This type of failure can be observed in the items that do not deteriorate gradually with age, but fail suddenly after some period of service. The time period between installation and failure will not be constant for any particular equipment. However, the failure pattern will follow certain frequency distribution that may be progressive, retrogressive or random in nature.

*Progressive failure:* It is said to be progressive failure, when probability of failure increases with the age of an item, for example, light bulbs, tyres, etc.

*Retrogressive failure:* Certain items will have more probability of failure in the initial years of their life and with the increase in the life of an item the chances of failure become less. That is, the ability of the item to survive in the initial years of life increases its expected life. Aircraft engines exemplify industrial equipments with this type of distribution of life span.

*Random failure:* It is said to be random failure, when constant probability of failure is associated with equipment that fails because random causes such as physical shocks that are independent of age. In the case of random failure, virtually all items fail before ageing has any effect. For example, vacuum tubes, items made of glass or mirror, fruits, vegetables, etc. may fail independent of their age.

(b) **Gradual Failure:** In this failure, the failure mechanism is progressive; as the age of an item increases, its performance deteriorates. This results in:

- Increased operating cost
- Decreased productivity of the item
- Decrease in resale value of an item (Example: mechanical items like pistons, bearing rings, tyres, etc.)

The replacement situations generally are divided into the following two types:

1. Group replacement items that fail completely, e.g., electrical bulbs, etc.
2. Replacement of capital equipment whose performance decreases with time, e.g., machine tools, vehicles in a transport organization, aeroplanes, etc.

## 14.2 REPLACEMENT OF ITEMS THAT FAIL COMPLETELY

Consider a system usually made up of a large number of components that are prone to failure with age, e.g., the failure of a resistor or condenser in television, radio, computer, etc. In some cases, the failure of a component may cause the complete failure of the system. In such cases, the cost of overall failure will be quite higher than the cost of component itself. For example, the cost of a component in an aircraft is very less, but its failure may result in total collapse of the aeroplane. When dealing with such situations, two types of replacement policies shall be considered.

1. *Individual replacement:* In this policy, an item is replaced immediately after its failure.
2. *Group replacement:* In this policy, the decision is about the age when all the items should be replaced, irrespective of whether the items have failed or not. In this policy, the items that fail before the optimal time, will be replaced individually.

### 14.2.1 Individual Replacement Policy

Under this policy, an item is immediately replaced after its failure. To determine the probability of failure (or life span of any item), mortality tables are used.

**Mortality Table:** This table is used to derive the probability distribution of the life span of equipment. Let

$N(t)$  = number of survivors at any time  $t$

$N(t-1)$  = number of survivors at any time  $t-1$

$N$  = Initial number of equipments

The probability of failure during the time period  $t$  is given by

$$p(t) = \frac{N - N(t)}{N}$$

The probability of an equipment survived to an age  $(t-1)$ , and will fail during the interval  $(t-1)$  to  $t$  can be defined as the conditional probability of failure and is given by

$$p_c(t) = \frac{N(t-1) - N(t)}{N(t-1)}$$

Probability of survival at an age  $t$  is given by

$$p_s(t) = 1 - p(t) = 1 - \frac{N - N(t)}{N} = \frac{N(t)}{N}$$

**Mortality Theorem:** A large population is subject to a given mortality law for a very long period of time. All deaths are immediately replaced by births and there are no other entries or exits. Then the age distribution ultimately becomes stable and that the number of deaths per unit time becomes constant (which is equal to the size of the total population divided by the mean age at death).

#### Assumptions:

- (i) All deaths are immediately replaced by births, and
- (ii) There are no other entries or exits.

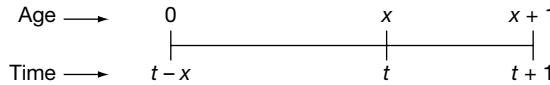
However, in reality, it is impossible to have these conditions. But, the reason for assuming the above two is that the analysis will be easier by keeping the virtual human population in mind. Such models can be applied to industrial items, where the death of a person is equivalent to the failure of an item or part and birth of a person is equivalent to replacement.

**Proof:** For convenience, let each death occurs just before some time  $t = w$ , where  $w$  is an integer and no member of the population can survive up to and beyond  $w + 1$  time units, i.e. life span of any member lies between  $t = 0$  and  $t = w$ .

Let  $f(t)$  = number of births at time  $t$ ,

$p(x)$  = probability of member will die (fail) just before age  $x + 1$ , i.e. at age  $x$ .

Now  $f(t-x)$  = the number of births at time  $(t-x)$ . The age of such newly born members who remain alive at time  $t$  will obviously be  $x$ . This can be understood from the following Figure 14.1.

**Figure 14-1:** Relation between age and time period

Hence, the expected number of deaths of such alive members at time  $t$  is  $p(x)f(t-x)$ . Therefore, the total number of deaths at time  $t$  will be

$$= \sum_{x=0}^w f(t-x)p(x), t = w, w+1, w+2, \dots$$

Also, the total number of births at time  $(t+1) = f(t+1)$ .

Since all deaths at time  $t$  are replaced immediately by births at the time  $(t+1)$ , therefore,

$$f(t+1) = \sum_{x=0}^w f(t-x)p(x), t = w, w+1, \dots \quad (14.1)$$

The difference Equation (14.1) in  $t$ , may be solved by substituting  $f(t) = A\alpha^t$ , where  $A$  is an arbitrary constant and then difference Equation (14.1) becomes

$$A\alpha^{t+1} = A \sum_{x=0}^w \alpha^{t-x} p(x) \quad (14.2)$$

On dividing by  $A\alpha^{t-w}$ ,

$$\text{we get } \alpha^{w+1} = \sum_{x=0}^w \alpha^{w-x} p(x)$$

$$\text{or, } \alpha^{w+1} - \sum_{x=0}^w \alpha^{w-x} p(x) = 0$$

$$\text{or, } \alpha^{w+1} - [\alpha^w p(0) + \alpha^{w-1} p(1) + \alpha^{w-2} p(2) + \dots + p(w)] = 0 \quad (14.3)$$

Since the sum of probabilities is zero, therefore we have

$$\sum_{x=0}^w p(x) = 1$$

$$\text{or, } 1 - \sum_{x=0}^w p(x) = 0$$

$$\text{or, } 1 - [p(0) + p(1) + \dots + p(w)] = 0 \quad (14.4)$$

Equation (14.3) is a polynomial function, hence it has  $w+1$  roots. Suppose the roots of the equation are  $\alpha_0, \alpha_1, \dots, \alpha_w$ . Putting the first root  $\alpha_0 = 1$ , Equation (14.3) will be same as Equation (14.4). Now, the solution of difference Equation (14.3) will be of the form:

$$f(t) = A_0 + A_1 \alpha_1^t + A_2 \alpha_2^t + \dots + A_w \alpha_w^t \quad (14.5)$$

where  $A_0, A_1, A_2, \dots, A_w$  are constants whose values can be determined from the age distribution at some given point in time. Further, it can be observed that the absolute value of all the remaining roots is less than unity, i.e.  $|\alpha_i| < 1$  for  $i = 1, 2, 3, \dots, w$ . Hence,  $\alpha'_1, \alpha'_2, \alpha'_3, \dots, \alpha'_w$  tend to zero as  $t \rightarrow \infty$ .

It shows that the number of deaths per unit time is constant and equal to  $A_0$ .

To show that the age distribution ultimately becomes stable:

Let  $P(x)$  = the probability of members remain alive longer than  $x$  time units.

Then,  $P(x) = 1 - p(\text{survivor will die before attaining the age } x)$

$$= 1 - [p(0) + p(1) + \dots + p(x-1)] \text{ and } P(0) = 1 \text{ (Assumption)}$$

Since the number of births and deaths has become constant, each equal to  $A_0$ , the expected number of survivors of age  $x$  is also stable at  $A_0 P(x)$ .

As the deaths are replaced immediately (i.e. the number of births is always equal to the number of deaths), the size  $N$  of the total population remains constant, i.e.,

$$N = A_0 \sum_{x=0}^w P(x) \quad (14.6)$$

$$A_0 = \frac{N}{\sum_{x=0}^w P(x)} \quad (14.7)$$

If the denominator of Equation (14.7) is equivalent to mean age at death, then the age distribution will become stable.

$$\sum_{x=0}^w P(x) = \sum_{x=0}^w P(x)\Delta(x), \quad \text{Since } \Delta(x) = (x+1) - x = 1.$$

$$\begin{aligned} \sum_{x=0}^w P(x) &= \sum_{x=0}^w P(x)\Delta(x) = [P(x).x]_0^{w+1} - \sum_{x=0}^w (x+1)\Delta p(x) \\ &= [P(w+1).(w+1) - 0] - \sum_{x=0}^w (x+1)\Delta p(x) \end{aligned} \quad (14.8)$$

But,  $P(w+1) = 1 - [p(0) + p(1) + \dots + p(w)] = 0$

And  $\Delta P(x) = P(x+1) - P(x)$

$$\begin{aligned} &= [1 - \{p(0) + p(1) + \dots + p(x)\}] - [1 - \{p(0) + p(1) + \dots + p(x-1)\}] \\ &= -p(x) \end{aligned}$$

Substituting the value of  $P(w+1)$  and  $\Delta P(x)$  in Equation (14.8), we get

$$\sum_{x=0}^w P(x) = \sum_{x=0}^w (x+1)p(x) = \text{Mean age at death}$$

Hence, from Equation (14.7), we get

$$A_0 = \frac{N}{\text{Average age at death}}$$

### 14.2.2 Group Replacement of Items that Fail Completely

There are certain items such as electric bulbs that either work or fail completely. In some cases, a system is made up of a big number of similar types of items that are increasingly prone to failure with age. While replacing such failed items, always a set-up cost will be there for replacement. The so-called set-up cost is independent of the number of items to be replaced and hence it may be advantageous to replace an entire group of items at fixed intervals. Such a policy is referred as group replacement policy and found attractive when the value of any individual item is so less and the cost of keeping records for age of individual items is not justifiable.

Group replacement policy is defined in the following ways:

- (a) Group replacement at the end of  $t^{\text{th}}$  period, if the cost of individual replacements for the  $t^{\text{th}}$  period is greater than the average cost per period through the end of  $t^{\text{th}}$  period.
- (b) Individual replacement at the end of  $t^{\text{th}}$  period, if the cost of individual replacement at the end of  $t^{\text{th}}$  period is less than the average cost per period through the end of  $t^{\text{th}}$  period.

Let,

$N_t$  = number of units failing during time  $t$

$N$  = Total number of units in the system

$C(t)$  = Cost of group replacement after time period  $t$

$C_1$  = Cost of replacing an individual item

$C_2$  = Cost of replacing an item in a group

Then  $C(t) = C_1 [N_1 + N_2 + \dots + N_{t-1}] + C_2 N$

Therefore, the average cost per unit period will be

$$F(t) = \frac{C(t)}{t} = \frac{C_1 [N_1 + N_2 + \dots + N_{t-1}] + C_2 N}{t} \quad (14.9)$$

Now, in order to determine the replacement age  $t$ , the average cost per unit period should be minimum. The condition for minimum of  $F(t)$  is given as:

$$\Delta F(t-1) < 0 < \Delta F(t) \quad (14.10)$$

$$\begin{aligned} \text{Now, } F(t) - F(t+1) &= \frac{C(t+1)}{t+1} - \frac{C(t)}{t} = \frac{C(t) + C_1 N_t}{t+1} - \frac{C(t)}{t} \\ &= \frac{t C_1 N_t - C(t)}{t(t+1)} = \frac{C_1 N_t - C(t)/t}{t+1} \end{aligned} \quad (14.11)$$

which must be greater than zero for minimum  $F(t)$ ,

$$\text{i.e. } C_1 N_t > C(t)/t \quad (14.12)$$

$$\text{Similarly, from } \Delta F(t-1) < 0, \quad C_1 N_{t-1} < C(t)/t \quad (14.13)$$

Thus, from Equations (14.12) and (14.13), the group replacement policy is completely established.

**Example 14.1:** A computer consists of 5000 resistors. When any resistor fails, it is replaced. The cost of individual replacement of the resistor is Rs 10 per resistor but if all the resistors are replaced simultaneously, the cost becomes Rs 2 per resistor. The probabilities of failure  $P(t)$  during the month  $t$  are given as shown in Table 14.1.

**Table 14-1:** Probability of failure of the resistors

$t$	0	1	2	3	4	5	6
$p(t)$	—	0.02	0.08	0.3	0.4	0.1	0.1

Find the optimal time of replacement of the resistors.

**Solution:** Let  $N_i$  be the number of resistors replaced at the end of  $i^{\text{th}}$  month. Hence,

$$N_0 = \text{Number of resistors in the beginning} = 5000$$

$N_1 = N_0 p_1 = 5000 \times 0.02 = 100$  resistors are replaced by the end of first month.

$$\begin{aligned} N_2 &= N_0 p_2 + N_1 p_1 = 5000 \times 0.08 + 100 \times 0.02 \\ &= 402 \text{ resistors are replaced by the end of the second month.} \end{aligned}$$

$$\begin{aligned} N_3 &= N_0 p_3 + N_1 p_2 + N_2 p_1 = 5000 \times 0.3 + 100 \times 0.08 + 402 \times 0.02 \\ &= 1516 \text{ resistors are replaced by the end of the third month.} \end{aligned}$$

$$\begin{aligned} N_4 &= N_0 p_4 + N_1 p_3 + N_2 p_2 + N_3 p_1 = 5000 \times 0.4 + 100 \times 0.3 + 402 \times 0.08 + 1516 \times 0.02 \\ &= 2092.48 \approx 2093 \\ &= 2093 \text{ resistors are replaced by the end of the fourth month.} \end{aligned}$$

$$\begin{aligned} N_5 &= N_0 p_5 + N_1 p_4 + N_2 p_3 + N_3 p_2 + N_4 p_1 \\ &= 5000 \times 0.1 + 100 \times 0.4 + 402 \times 0.3 + 1516 \times 0.08 + 2093 \times 0.02 \\ &= 823.74 \approx 824 \text{ resistors are replaced by the end of the fifth month.} \end{aligned}$$

$$\begin{aligned} N_6 &= N_0 p_6 + N_1 p_5 + N_2 p_4 + N_3 p_3 + N_4 p_2 + N_5 p_1 \\ &= 5000 \times 0.1 + 100 \times 0.1 + 402 \times 0.4 + 1516 \times 0.3 + 2093 \times 0.08 + 824 \times 0.02 \\ &= 1309.52 \approx 1310 \text{ resistors are replaced by the end of the sixth month.} \end{aligned}$$

$$\text{Expected life} = 1 \times 0.02 + 2 \times 0.08 + 3 \times 0.3 + 4 \times 0.4 + 5 \times 0.1 + 6 \times 0.1 = 3.78 \text{ months}$$

$$\text{Average number of failures per month} = \frac{5000}{3.78} = 1322.75 \approx 1323$$

Hence, the total cost of individual replacement at the rate of Rs 10 per resistor will be  $1323 \times 10 = \text{Rs } 13,230$ . The cost of replacement of all the resistors at the same time can be calculated as shown in Table 14.2.

**Table 14-2:** Average cost per month

End of the month	Total cost of group replacement	Total cost at the end of the month (Rs)	Average cost per month
1	$N_1 \times 10 + N_0 \times 2$	11,000	11,000
2	$(N_1 + N_2) \times 10 + N_0 \times 2$	15,020	<b>7510</b>
3	$(N_1 + N_2 + N_3) \times 10 + N_0 \times 2$	30,180	10,060
4	$(N_1 + N_2 + N_3 + N_4) \times 10 + N_0 \times 2$	51,110	12,777.5
5	$(N_1 + N_2 + N_3 + N_4 + N_5) \times 10 + N_0 \times 2$	59,350	11,870
6	$(N_1 + N_2 + N_3 + N_4 + N_5 + N_6) \times 10 + N_0 \times 2$	72,450.0	12,075

The average cost of replacement at the end of the second month is minimum for group replacement. Thus, all the resistors should be replaced at the end of the second month.

**Example 14.2:** Table 14.3 shows that the mortality rates have been observed for a certain type of electric bulbs (1000 bulbs):

**Table 14-3:** Mortality rate of the electric bulbs

Week	1	2	3	4	5
Percentage failing at the end of the week	10	25	50	80	100

Each bulb costs Rs 10 to replace an individual bulb on failure. If all bulbs were replaced at the same time in the group, it would cost Rs 2 per bulb. It is under proposal to replace all bulbs at fixed intervals of time, regardless of the bulbs have burnt out; and also it is to continue replacing immediately burnt out bulbs. Determine the time interval at which all the bulbs should be replaced.

**Solution:**

Let  $p_i$  = the probability that a new light bulb fails during the  $i^{\text{th}}$  week of its life.

Thus  $p_1$  = the probability of failure in 1<sup>st</sup> week =  $10/100 = 0.10$

$p_2$  = the probability of failure in 2<sup>nd</sup> week =  $(25 - 10)/100 = 0.15$

$p_3$  = the probability of failure in 3<sup>rd</sup> week =  $(50 - 25)/100 = 0.25$

$p_4$  = the probability of failure in 4<sup>th</sup> week =  $(80 - 50)/100 = 0.3$

$p_5$  = the probability of failure in 5<sup>th</sup> week =  $(100 - 80)/100 = 0.2$

Since the sum of all the above probabilities is unity, the further probabilities  $p_6, p_7, p_8$ , so on, will be zero. Thus, all light bulbs are sure to burn out by the 5<sup>th</sup> week. Furthermore, it is assumed that bulbs that fail during a week are replaced just before the end of that week.

Let  $N_i$  = the number of replacements made at the end of the  $i^{\text{th}}$  week.

$$N_0 = 1000 \text{ electric bulbs.}$$

$$N_1 = N_0 p_1 = 1000 \times 0.1 = 100 \text{ bulbs are replaced by the end of the first week.}$$

$$N_2 = N_0 p_2 + N_1 p_1 = 1000 \times 0.15 + 100 \times 0.1 = 160$$

= 160 bulbs are replaced by the end of the second week.

$$N_3 = N_0 p_3 + N_1 p_2 + N_2 p_1 = 1000 \times 0.25 + 100 \times 0.15 + 160 \times 0.1$$

= 281 bulbs are replaced by the end of the third week.

$$N_4 = N_0 p_4 + N_1 p_3 + N_2 p_2 + N_3 p_1$$

$$= 1000 \times 0.3 + 100 \times 0.25 + 160 \times 0.15 + 281 \times 0.1 = 377$$

= 377 bulbs are replaced by the end of the fourth week.

$$N_5 = N_0 p_5 + N_1 p_4 + N_2 p_3 + N_3 p_2 + N_4 p_1$$

$$= 1000 \times 0.2 + 100 \times 0.3 + 160 \times 0.25 + 281 \times 0.15 + 377 \times 0.1$$

= 350 bulbs are replaced by the end of the fifth week.

It has been found that expected number of bulbs failing in each week increases until the 4<sup>th</sup> week and then decreases.

### Individual replacement:

$$\text{The mean age of the bulb} = 1 \times p_1 + 2 \times p_2 + 3 \times p_3 + 4 \times p_4 + 5 \times p_5$$

$$= 1 \times 0.1 + 2 \times 0.15 + 3 \times 0.25 + 4 \times 0.3 + 5 \times 0.2 = 3.35 \text{ weeks.}$$

The average number of failures in each week in steady state =  $1000/3.35 = 299$

The cost of replacing the bulbs individually =  $10 \times 299 = \text{Rs } 2990$ .

### Group replacement:

The replacement of all 1000 bulbs at the same time in bulk costs Rs 4 per bulb and replacement of an individual bulb on failure costs Rs 10. Costs of replacement of all bulbs simultaneously are calculated in Table 14.4.

**Table 14-4:** Group replacement

End of week	Total cost of group replacement	Total cost at the end of the week	Average cost per week (Rs)
1	$N_1 \times 10 + N_0 \times 2$	3000	3000
2	$(N_1 + N_2) \times 10 + N_0 \times 2$	4600	<b>2300</b>
3	$(N_1 + N_2 + N_3) \times 10 + N_0 \times 2$	7410	2470
4	$(N_1 + N_2 + N_3 + N_4) \times 10 + N_0 \times 2$	11,180	2795
5	$(N_1 + N_2 + N_3 + N_4 + N_5) \times 10 + N_0 \times 2$	14,680	2936

Table 14.4 shows that the average cost per month at the end of the second month is minimum for group replacement. Therefore, all the bulbs should be replaced at the end of the second month.

### 14.3 REPLACEMENT OF ITEMS THAT DETERIORATE

To find the optimal period to replace an item can be explained by considering an example of a machine which is to be replaced by a new one. The main aim is to compare the running cost of the machine and the saving capital costs. There are two types of the costs associated with the machine or equipment:

1. The running costs and
2. The capital cost of the vehicle.

These costs can be summarized as an average cost per month. It can be observed that the average monthly cost will go on decreasing, with increase in time. However, there will be an age at which the rate of increase in running cost is considerably higher than the savings in average capital costs. Thus, at this age, it is justifiable to replace the equipment.

#### 14.3.1 Replacement Policy for the Items whose Maintenance Cost Increases with Time, and Money Value does not Change with Time, i.e., Constant (No Interest, No Taxes on the Income)

The maintenance cost of a machine is given as function increasing with time and machine's scrap value is constant. The equipment is replaced under the following criteria:

- (a) When time is a continuous variable and the maintenance cost is equal to the average annual cost.
- (b) When time is a discrete variable and the maintenance cost in the  $(n + 1)^{\text{th}}$  year becomes greater than the average annual cost in the  $n^{\text{th}}$  year.

##### When Time $t$ is a Continuous Variable

Let  $t$  be a continuous variable.

$R_t$  = Maintenance cost at time  $t$

$C$  = the capital cost of the item

$S$  = the scrap value of the item

Then the annual cost of the item at any time  $t = R_t + C - S$ . Here,  $C - S$  is depreciation of the equipment.

The maintenance cost incurred in  $n$  years of time span =  $\int_0^n R_t dt$

The total cost in the time span of  $n$  years =  $P(n) = \int_0^n R_t dt + C - S$

$$\text{Now, average total cost (per year)} = F(n) = \frac{P(n)}{n} = \frac{1}{n} \int_0^n R_t dt + C - S \quad (14.14)$$

To minimize  $F(n)$ , differentiate Equation (14.14) with respect to  $n$  and equate to zero.

$$\frac{dF(n)}{dn} = \frac{1}{n} R_n + \left[ -\frac{1}{n^2} \right] \int_0^n R_t dt - \frac{C - S}{n^2} = 0$$

$$\text{or, } R_n = \frac{1}{n} \int_0^n R_t dt - \frac{C - S}{n} = \frac{P(n)}{n} \quad (14.15)$$

Hence, the maintenance cost at time  $n$  = average cost in time  $n$ .

### When Time 't' is a Discrete Variable

Since time is measured in discrete units, the cost Equation (14.14) can be written as

$$F(n) = \frac{P(n)}{n} = \frac{1}{n} \sum_{t=1}^n \frac{R_t}{n} + C - S \quad (14.16)$$

By using finite differences,  $F(n)$  will be minimum if the following relationship is satisfied.

$$\Delta F(n-1) < 0 < \Delta F(n) \quad (14.17)$$

Now, differentiating (14.17) under the summation sign by definition of first difference,

$$\begin{aligned} \Delta F(n) &= F(n+1) - F(n) \\ &= \left[ \sum_{t=1}^n \frac{R_t}{n+1} + \frac{C-S}{n+1} \right] - \left[ \sum_{t=1}^n \frac{R_t}{n} + \frac{C-S}{n} \right] \\ &= \left[ \frac{R_{n+1}}{n+1} + \sum_{t=1}^n \frac{R_t}{n+1} \right] - \sum_{t=1}^n \frac{R_t}{n} + (C-S) \left[ \frac{1}{n+1} - \frac{1}{n} \right] \\ &= \frac{R_{n+1}}{n+1} - \sum_{t=1}^n R_t \left[ \frac{1}{n+1} - \frac{1}{n} \right] + (C-S) \left[ \frac{1}{n+1} - \frac{1}{n} \right] \\ &= \frac{R_{n+1}}{n+1} - \sum_{t=1}^n \frac{R_t}{n(n+1)} - \frac{C-S}{n(n+1)} > ; \text{ for minimum } F(n) \end{aligned}$$

or, 
$$\frac{R_{n+1}}{n+1} > \sum_{t=1}^n \frac{R_t}{n(n+1)} - \frac{C-S}{n(n+1)}$$

or, 
$$R_{n+1} > \sum_{t=1}^n \frac{R_t}{n} - \frac{C-S}{n}$$

or, 
$$R_{n+1} > \frac{P(n)}{n}$$

Similarly, it can be shown that  $R_n < \frac{P(n)}{n}$ , by virtue of  $\Delta F(n-1) < 0$

Hence, 
$$R_{n+1} > \frac{P(n)}{n} > R_n.$$

**Example 14.3:** A machine costs Rs 50,000 and the operating costs are estimated at Rs 1000 for the first year increasing by Rs 8000 per year in the second and subsequent years. The resale value of the machine is Rs 5000. When the machine should be replaced?

**Solution:** Given,  $C = \text{Rs } 50,000$ ;  $S = \text{Rs } 5,000$ . The calculation of average running cost of the machine is shown in Table 14.5.

**Table 14-5:** Average running cost

Year of service <i>n</i>	Running cost (Rs), $R(n)$	Cumulative running cost (Rs). $\Sigma R(n)$	Depreciation cost (Rs), $C - S$	Total cost (Rs), $TC$	Average cost (Rs) $ATC_n$
1	1000	1000	45,000	46,000	46,000
2	9000	10,000	45,000	55,000	27,500
3	17,000	27,000	45,000	72,000	<b>24,000</b>
4	25,000	52,000	45,000	97,000	24,250
5	33,000	85,000	45,000	130,000	26,000

The machine should be replaced at the end of the third month because of the minimum average cost.

### 14.3.2 Time Value of Money

Replacement policy for the items whose maintenance cost increases with time and money value changes with constant rate (Considering interest and tax): In this case, we should have a basic understanding of the time value of money, which is described under the following sections.

#### Simple Interest

When the interest charged on a principal is directly proportional to the principal, the interest rate and the number of interest periods, the interest is said to be simple interest. Simple interest may be calculated using the following formula:

$$I = (P)(N)(i)$$

where  $I$  is interest,  $P$  is principal,  $N$  is the number of interest periods (years) and  $i$  is the interest rate. Total amount at the end of the period ( $N$ ) is

$$F = P + (P)(N)(i) = P[1 + (N)(i)]$$

### **Compound Interest**

When the interest charged for a period is based on the remaining principal amount plus the interest accumulated up to the beginning of that period, the interest is said to be compound interest. The amount at the end of period  $N$  may be calculated using the following formula:

$$F = P(1+i)^N$$

where  $F$  is the total amount of money at the end of period  $N$ ,  $P$  is the principal amount,  $i$  is the interest rate compounded annually and  $N$  is number of years. This formula may also be written in the factor form as

$$F = P(1+i)^N = P(F/P, i\%, N)$$

where

$$(1+i)^N = (F/P, i\%, N)$$

### **Uniform Series Compound Interest Formula**

Uniform series compound interest formula is used to calculate the compound interest for periodic investment of equal amount for a certain time period uniformly. It is also known as a **sinking fund**. The other equivalent terms such as future worth ( $FW$ ) or present worth ( $PW$ ) of the investment may also be calculated.

The uniform series compound interest is used in repayment of house or car loan. EMI is paid according to a uniform payment series for the fixed period of time. The amount of each future annuity payment required to dissipate a given present value when the interest rate and number of payments are known is termed **capital recovery**.

Let  $A$  be the end-of-year payment in a uniform payment series for  $N$  years at an interest rate of  $i$  per cent compounded per year. The future value ( $F$ ) of the annual payment may be calculated as

$$\begin{aligned} F &= A(1+i)^{N-1} + A(1+i)^{N-2} + \cdots + A(1+i) + A \\ &= A[(1+i)^{N-1} + (1+i)^{N-2} + \cdots + (1+i) + 1] \end{aligned}$$

or,

$$F = A \left[ \frac{(1+i)^N - 1}{(1+i) - 1} \right] = A \left[ \frac{(1+i)^N - 1}{i} \right] = A(F/A, i \text{ per cent}, N)$$

where  $\left[ \frac{(1+i)^N - 1}{i} \right] = (F/A, i \text{ per cent}, N)$

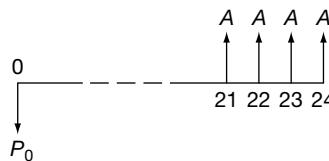
Similarly, the present value ( $P$ ) of the annual payment may be calculated as

$$P = \frac{F}{(1+i)^N} = \frac{A \left[ \frac{(1+i)^N - 1}{i} \right]}{(1+i)^N} = A \left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] = A(P/A, i \text{ per cent}, N),$$

where  $\left[ \frac{(1+i)^N - 1}{i(1+i)^N} \right] = (P/A, i \text{ per cent}, N)$

**Example 14.4:** A father wants to see his son grow up to become an engineer. He plans to invest a certain sum towards his son's education upon his birth. He needs to withdraw Rs 100,000 each year from the 21<sup>st</sup> year to the 24<sup>th</sup> year of his son's age. How much should he invest now if the rate of interest is 8 per cent compounded annually?

**Solution:** In this problem, first, the future equivalent of four equities is found as  $F_{24}$  as shown in Figure 14.2 and then the present equivalent of that future equivalent is found as  $P_0$ .



**Figure 14-2:** Cash flow diagrams

$$F_{24} = A \left[ \frac{(1+i)^N - 1}{i} \right] = 1,00,000 \left[ \frac{(1+0.08)^4 - 1}{0.08} \right] = 1,00,000 \times 4.506 = \text{Rs } 4,50,600$$

$$P_0 = \frac{F_{24}}{(1+i)^N} = \frac{4,50,600}{(1+0.08)^{24}} = 4,50,600 \times 0.1577 = \text{Rs } 71,059.62.$$

### Relationships Among Compound Interest Factors

Relationships among various compound interest factors are shown in Table 14.6.

**Table 14-6:** Relationships among compound interest factors

To find	Given	Factor to be multiplied	Factor name	Factor symbol
Single cash flow				
$F$	$P$	$(1 + i)^N$	Single payment compound amount	$(F/P, i \text{ per cent}, N)$
$P$	$F$	$\frac{1}{(1+i)^N}$	Single payment $PW$	$(P/F, i \text{ per cent}, N)$

(Continued)

**Table 14-6: Continued**

To find	Given	Factor to be multiplied	Factor name	Factor symbol
Annuities or uniform series				
$F$	$A$	$\frac{(1+i)^N - 1}{i}$	Uniform series compound amount	$(F/A, i \text{ per cent}, N)$
$P$	$A$	$\frac{(1+i)^N - 1}{i(1+i)^N}$	Uniform series $PW$	$(P/A, i \text{ per cent}, N)$
$A$	$F$	$\frac{i}{(1+i)^N - 1}$	Sinking fund	$(A/F, i \text{ per cent}, N)$
$A$	$P$	$\frac{i(1+i)^N}{(1+i)^N - 1}$	Capital recovery	$(A/P, i \text{ per cent}, N)$

### **PW Analysis**

$PW$  is a present value without interest.  $PW$  can be calculated for future amount and/or uniform series of payment.

**Present worth ( $P$ ) of the future worth ( $F$ ) for the period  $N$  at the interest rate  $i$  per cent:** Following formula can be used to find the  $PW$  of the future value ( $F$ ):

$$P = \frac{F}{(1+i)^N} = F(P/F, i \text{ per cent}, N)$$

**Example 14.5:** Mr. X wants his savings to yield Rs 5,00,000 at the end of 5 years. How much should he deposit today if the rate of interest is fixed at 10 per cent compounded annually?

**Solution:** Given  $F = \text{Rs } 500,000$ ;  $i = 10$  per cent compounded annually;  $N = 5$  years. The present value of the future amount can be found as:

$$P = \frac{F}{(1+i)^N} = \frac{5,00,000}{(1+0.1)^5} = 5,00,000 \times 0.6209 = \text{Rs } 3,10,460.66$$

**Present worth ( $P$ ) of the annuity ( $A$ ) for the period  $N$  at the interest rate  $i$  per cent:** To know that how much money should be invested at present to get a fixed amount at the end of every year for the fixed period, we have to find the present value of given annuity using the following formula:

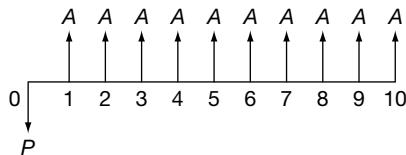
$$P = A \left\{ \frac{(1+i)^N - 1}{i(1+i)^N} \right\} = A(P/A, i \text{ per cent}, N)$$

**Example 14.6:** Mr. John is retiring in this running year. He wants to invest some money so that he can earn Rs 50,000 per year for the next 10 years. If the rate of interest is 10 per cent compounded annually, what is the amount of the current investment?

**Solution:**

Given  $A = \text{Rs } 50,000$ ;  $N = 10$  years;  $i = 10$  per cent;  $P = ?$

The cash flow diagram is given in Figure 14.3.



**Figure 14-3:** Cash flow diagram

$$P = A \left\{ \frac{(1+i)^N - 1}{i(1+i)^N} \right\} = 50,000 \left\{ \frac{(1+0.1)^{10} - 1}{0.1(1+0.1)^{10}} \right\} = 50,000 \times 6.145 = \text{Rs } 3,07,250.$$

**FW Analysis**

**FW** is the future value of the money with interest for a fixed period. The future value can be found for present investment as well as for uniform series payments.

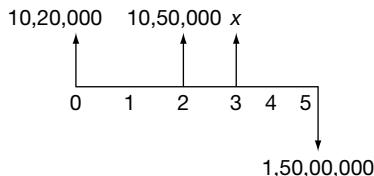
**Future worth ( $F$ ) of the present value ( $P$ ) for the period  $N$  at the interest rate  $i$  per cent:** If an amount  $P$  is invested at present and  $i$  is the interest rate, the amount will grow in the future at the end of the period  $N$  as:

$$F = P(1+i)^N = P(F/P, i \text{ per cent}, N)$$

**Example 14.7:** Mr. Rakesh is planning to purchase a house in Delhi city 5 years from now. The home will cost Rs 1,50,00,000 at that time. He plans to make three deposits at the interest rate of 12 per cent in the following ways: Rs 10,20,000 today; Rs 10,50,000 two years from now, and Rs  $X$  three years from now. How much Rakesh needs to invest in year three to ensure that he has the necessary funds to buy the vacation home at the end of year five?

**Solution:**

The cash flow diagram is shown in Figure 14.4.



**Figure 14-4:** Cash flow diagram

The future value of Rs 10,20,000; Rs 10,50,000; and Rs X should be equal to Rs 1,50,00,000.

$$10,20,000(1+0.12)^5 + 10,50,000(1+0.12)^3 + X(1+0.12)^2 = 1,50,00,000$$

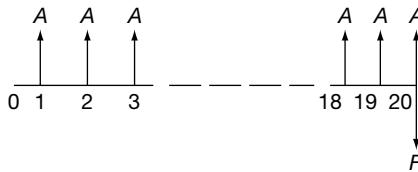
$$\text{or, } X = \frac{1,50,00,000 - 10,20,000 \times 1.762 - 10,50,000 \times 1.405}{1.254} = \text{Rs } 93,52,081.34.$$

**Future worth ( $F$ ) of the Annuity ( $A$ ) for the period  $N$  at the interest rate  $i$  per cent:** If an annuity  $A$  is invested at the end of each year till  $N$  years,  $i$  is the interest rate, the amount will grow at the end of period  $N$  as:

$$F = A \left[ \frac{(1+i)^N - 1}{i} \right] = A(F/A, i \text{ per cent}, N)$$

**Example 14.8:** Mr. Sonu deposits Rs 25,000 per year at the end of the year for 20 years. What will he get at the end of 20 years if the interest rate is 8 per cent?

**Solution:** The cash flow diagram of the problem is shown in Figure 14.5.



**Figure 14-5:** Cash flow diagram

$$F = A \left[ \frac{(1+i)^N - 1}{i} \right] = 25,000 \left[ \frac{(1+0.08)^{20} - 1}{0.08} \right] = 25,000 \times 45.762 = \text{Rs } 11,44,050.$$

### Annual Cash Flow Analysis

Annual cash flow is the annual receipt of the present investment and/or the annual deposits to get some fixed amount in future. A person may wish to invest now to get an equal assured amount at the end of each year for certain periods. Similarly, he may wish to deposit an equal amount at the end of each year to get some assured amount at the end of the investment.

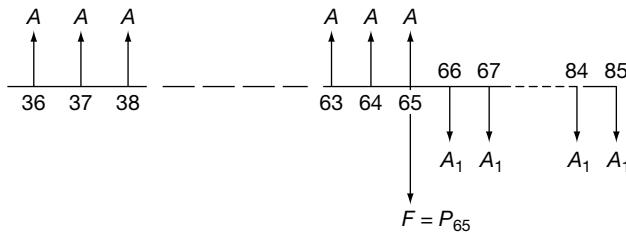
**Annuity ( $A$ ) of the future value ( $F$ ) for the period  $N$  at the interest rate  $i$  per cent:** This is just the reverse of  $FW$  of annuity. In this case, annuity is calculated to get assured sum at the end of the given period. The following formula is used to calculate annuity from the future value:

$$A = F \left[ \frac{i}{(1+i)^N - 1} \right] = F(A/F, i\%, N)$$

**Example 14.9:** A 35-year-old professor is planning to ensure an annual income of Rs 2,00,000 after his retirement at the age of 65, till he reaches 85. He plans to make annual deposits into his

account till he turns 65. If the interest rate is a constant 10 per cent during the period, how much should he deposit annually into his account to generate the required income after retirement?

**Solution:** In this problem, the future value of 30 equities (from the end of 36<sup>th</sup> year to the end of 65<sup>th</sup> year) at the end of 65<sup>th</sup> year should be equal to the present value of 20 annual receipts (from the end of 66<sup>th</sup> year to 85<sup>th</sup> year) at the end of 65<sup>th</sup> year or at the beginning of 66<sup>th</sup> year as shown in Figure 14.6.



**Figure 14-6:** Cash flow diagram

Here,  $P_{65}$  is the present equivalence of annual receipt from the end of 66<sup>th</sup> year to the end of 85<sup>th</sup> year;  $A_1$  is the annual receipt,  $i$  is interest rate compounded yearly; and  $N$  is the total receipt period.  $P_{65}$  can be calculated as:

$$P_{65} = A_1 \left\{ \frac{(1+i)^N - 1}{i(1+i)^N} \right\} = 2,00,000 \left\{ \frac{(1+0.1)^{20} - 1}{0.1(1+0.1)^{20}} \right\} = \text{Rs } 17,02,800.$$

Now, the present value of the total receipt at the end of 65<sup>th</sup> year or at the beginning of 66<sup>st</sup> year should be equal to a  $FW$  of total deposits at the end of 65<sup>th</sup> year.

$$F = P_{65} = A \left\{ \frac{(1+i)^N - 1}{i} \right\}$$

or,

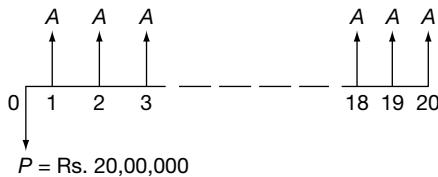
$$A = P_{65} \left\{ \frac{i}{(1+i)^N - 1} \right\} = 17,02,800 \left\{ \frac{0.1}{(1+0.1)^{30} - 1} \right\} = \text{Rs } 10,353.024.$$

**Annuity ( $A$ ) from the present value ( $P$ ) for the period  $N$  at the interest rate  $i$  per cent:**  
Annuity from present value can be calculated using the following formula:

$$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] = P(A/P, i \text{ per cent}, N)$$

**Example 14.10:** Mr. Smith has borrowed Rs 2,000,000 to finance his house in Greater Noida. The annual rate of interest is 12 per cent and the loan instalments are paid annually. If the mortgage is on a 20-year schedule, what is the annual payment Smith has to make?

**Solution:** The flow diagram of the problem is shown in Figure 14.7.



**Figure 14-7:** Cash flow diagram

Let  $A$  be the annual amount of loan disbursement;  $P$  be the present value of the loan;  $i$  be the interest rate; and  $N$  be the number of periods. The value of  $A$  can be calculated as:

$$A = P \left[ \frac{i(1+i)^N}{(1+i)^N - 1} \right] = 20,00,000 \left[ \frac{0.12(1+0.12)^{20}}{(1+0.12)^{20} - 1} \right] = 20,00,000 \times 0.1339 = \text{Rs } 2,67,800.$$

### Rate of Return Analysis

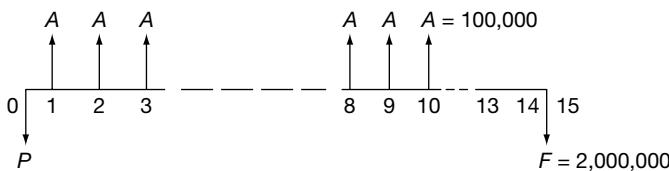
Rate of return is concerned with the interest rate on an investment. The decision of investment in terms of amount and period depends on the interest rate.

**Interest rate ( $i$ ) from the principal ( $P$ ), future value ( $F$ ) and period ( $N$ ):** If principal, future value and periods are given, the interest rate can be calculated. But the calculation to find the interest rate becomes complex due to the non-linear nature of the equation. Therefore, an interpolation method is used to find the value of  $i$ . Interpolation is a method in which it is assumed that for a small interval of periods the nature of the graph is linear.

**Example 14.11:** Mr. Prince wants to purchase a small shop after his retirement, which will cost Rs 20,00,000 in 15 years. He is going to put aside Rs 1,00,000 each year for 10 years towards this. At what interest rate should he invest his money to have enough funds to purchase the house after 15 years?

**Solution:**

The cash flow diagram is shown in Figure 14.8.



**Figure 14-8:** Cash flow diagram

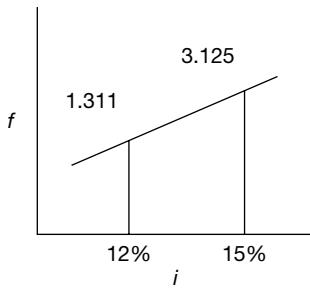
Here, at first present value of 10 equal investments,  $P$  is found and then the future value of  $P$  at the end of 15<sup>th</sup> year is equated to Rs 2,000,000. The calculation is as shown below:

$$P = A \left\{ \frac{(1+i)^N - 1}{i(1+i)^N} \right\} = 100,000(P/A, i \text{ per cent}, 10)$$

$$F = 2,000,000 = P(1+i)^{15}$$

$$\begin{aligned} \text{or, } 2,000,000 &= 100,000 \left\{ \frac{(1+i)^{10} - 1}{i(1+i)^{10}} \right\} (1+i)^{15} \\ 20 &= \left\{ \frac{(1+i)^{10} - 1}{i} \right\} (1+i)^5 \\ (1+i)^{15} - (1+i)^5 - 20i &= f(\text{say}) \end{aligned}$$

At  $i = 15$  per cent,  $f = 3.125$  and at  $i = 12$  per cent,  $f = 1.311$  (Figure 14.9).



**Figure 14-9:** Interpolation of the value of  $i$  corresponding to  $f = 0$ .

Using interpolation and extrapolation, we get

$$\frac{12-i}{1.311} = \frac{15-12}{3.125-1.311} \Rightarrow i = 9.83 \text{ per cent}$$

**Periods from the principal ( $P$ ), future value ( $F$ ) and interest rate ( $i$ ):** In this section, we will discuss about the period of investment, i.e. how much time is required to grow an amount to a fixed future value at a given interest rate?

**Example 14.12:** Mr. X is currently investing money in a bank account which has a nominal 10 per cent interest compounded annually. If he invests Rs 20,000 today, how many years will it take for his account to grow to Rs 100,000?

**Solution:** Given the present value,  $P = \text{Rs } 20,000$ ;  $I = 10$  per cent compounded annually; and  $F = \text{Rs } 100,000$ ; we want to calculate time in the number of years required to grow the money equals to  $F$ .

$$F = P(1+i)^N = P(F/P, i\%, N)$$

$$1,00,000 = 20,000(1+0.1)^N$$

$$N = \frac{\log 5}{\log 1.1} = 16.88 \text{ years.}$$

### **Normal and Effective Interest Rate**

Effective interest rate per year,  $i_a$ , is the annual interest rate taking into account the effect of any compounding during the year. Mathematically, it is given by

$$i_a = (1+i)^m - 1 = \left(1 + \frac{r}{m}\right)^m - 1$$

where  $r$  is the nominal interest rate per interest period (usually 1 year),  $i$  is the interest rate per interest period,  $i_a$  is the effective interest rate per year and  $m$  is the the number of compounding sub-periods per time period.

The above expression may be expressed as:

$$i_a = \left(1 + \frac{r}{C \times K}\right)^K - 1$$

where  $K$  is the effective period in a year and  $C$  is the number of compounding in an effective period.

For continuous compounding,

$$i_a = e^r - 1$$

**Example 14.13:** A bank charges an interest rate of 1 per cent per month on the unpaid balance of all accounts of a credit card. The annual interest rate, they claim, is 12 (1 per cent) = 12 per cent. What is the effective rate of interest per year being charged by the bank?

**Solution:** Nominal interest rate is 12 per cent per year compounded monthly; the effective interest rate per year can be calculated using the following expression.

$$i_a = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047 \text{ or } 10.47 \text{ per cent per year.}$$

**Example 14.14:** Mr. Rocky is planning to retire in 15 years. He can deposit money at 10 per cent compounded quarterly. What deposit should he make at the end of each quarter until he retires so that he can make a withdrawal of Rs 60,000 semiannually over five years after his retirement? Assume that his first withdrawal occurs at the end of six months after his retirement.

**Solution:** Given  $i = 10$  per cent compounded quarterly;  $N_1 = 60$ ;  $N_2 = 10$ ;  $A_2 = \text{Rs } 60,000$ ;  $A_1 = ?$

10 per cent interest rate compounded quarterly therefore the quarterly effective interest rate for the first 15 years can be calculated as:

$$i_a = \left(1 + \frac{0.10}{4 \times 1}\right)^1 - 1 = 2.5\% \text{ Quarterly}$$

Similarly, the semiannually effective interest rate for 5 years after retirement can be calculated as:

$$i_a = \left(1 + \frac{0.10}{2 \times 2}\right)^2 - 1 = 5.06 \text{ per cent semiannually}$$

Quarterly interest rate =  $0.10/4 = 0.025 = 2.5$  per cent

$$A_1(F/A, 2.5 \text{ per cent}, 60) = 60,000(P/A, 5.06 \text{ per cent}, 10)$$

$$A_1 = \frac{60,000(P/A, 5.06 \text{ per cent}, 10)}{(F/A, 2.5 \text{ per cent}, 60)} = \frac{60,000 \times \left\{ \frac{1.0506^{10} - 1}{0.0506(1.0506)^{10}} \right\}}{\left\{ \frac{1.025^{60} - 1}{0.025} \right\}} = \text{Rs } 3,396.95 \text{ per quarter.}$$

**Example 14.15:** How many years will it take an investment to triple if the interest rate is 10 per cent compounded

- (a) quarterly?      (b) monthly?      (c) continuously?

**Solution:** Given  $I = 10$  per cent compounded as per the given conditions in (a), (b) and (c). The effective interest rate can be calculated as:

$$(a) \quad i_a = \left(1 + \frac{0.1}{4}\right)^4 - 1 = 0.1038$$

$$3P = P(1+0.1038)^N$$

$$N = \frac{\log 3}{\log 1.1038} = 11.124 \text{ years.}$$

$$(b) \quad i_a = \left(1 + \frac{0.1}{12}\right)^{12} - 1 = 0.1047$$

$$3P = P(1+0.1047)^N$$

$$N = \frac{\log 3}{\log 1.1047} = 11.033 \text{ years.}$$

$$(c) \quad i_a = e^{0.1} - 1 = 0.1051$$

$$3P = P(1+0.1051)^N$$

$$N = \frac{\log 3}{\log 1.1051} = 10.99 \text{ years.}$$

**Example 14.16:** Find the effective interest rate per payment period for an interest rate of 10 per cent compounded monthly for each of the given payment schedule:

- (a) monthly
- (b) quarterly
- (c) semiannually
- (d) annually

**Solution:**

$$(a) i_a = \left(1 + \frac{r}{C \times K}\right)^K - 1 = \left(1 + \frac{0.1}{12 \times 1}\right)^1 - 1 = 0.833 \text{ per cent.}$$

$$(b) i_a = \left(1 + \frac{0.1}{4 \times 3}\right)^3 - 1 = 2.52 \text{ per cent.}$$

$$(c) i_a = \left(1 + \frac{0.1}{2 \times 6}\right)^6 - 1 = 5.1 \text{ per cent.}$$

$$(d) i_a = \left(1 + \frac{0.1}{1 \times 12}\right)^{12} - 1 = 10.47 \text{ per cent.}$$

### 14.3.3 Replacement of an Asset with Changing Value of Money with Time

#### Lives of Assets

Life of an asset can be defined in different ways. The different types of lives are defined during the life span of an asset that is discussed below as:

1. *Economic life:* It is the period in years in which an asset results in the minimum equivalent uniform annual cost (EUAC) of operation. The economic life of an asset coincides with the period between the date of its acquisition and the date of its abandonment.
2. *Ownership life:* It is the period between the date of acquisition and the date of disposal of an asset by its owner. An asset may have different types of use by the owner during this period.
3. *Physical life:* It is the period between the original acquisition and final disposal of an asset over a succession of its owners.
4. *Useful life:* It is the period in years during which an asset is kept in productive service. It is an estimate of how long an asset is expected to be used in a trade or business to produce income.

#### Economic Life of the Challenger

The economic life of an asset is usually shorter than its useful or physical life and it minimizes the EUAC of owning and operating the asset, i.e., after completion of the economic life of an asset, the maintenance cost increases. It is essential to know a challenger's economic life. Here,

challenger means the new asset or the asset which is used to replace the old one or existing one (defender). The new and existing assets should be compared for their economic (optimum) lives. The economic data regarding challengers are updated regularly, and replacement studies are repeated to ensure an ongoing evaluation of improvement opportunities.

### **Economic Life of the Defender**

Generally, the economic life of the defender is considered one year. Thus, care must be taken when comparing a defender asset with a challenger asset because different lives may be involved in the analysis. The defender should be kept longer than its apparent economic life till the time its marginal cost is less than the minimum *EUAC* of the challenger over its economic life.

## **14.4 REPLACEMENT OF THE DEFENDER**

Sometimes the useful lives of the defender and the challenger(s) are not known and cannot be reasonably estimated. In this case, it is important to know the economic life, minimum *EUAC* and total year-by-year (marginal) costs for both the best challenger and the defender so that they can be compared on the basis of their economic lives and the costs most favourable to each.

The estimated initial capital investment as well as the annual expense and market value (*MV*) estimates may be used to determine the *PW* through year *k*, using the following equation:

$$PW_k(i\%) = I - MV_k(P/F, i\%, k) + \sum_{j=1}^k E_j(P/F, i\%, j) \quad (14.18)$$

where  $PW_k$  is the present worth for the year *k*,  $I$  is the initial investment for the asset,  $MV_k$  is the *MV* of the asset at the end of year *k*, and  $E_j$  is the annual expenses up to the year *k*.

*PW* of an asset is the sum of the initial investment at time zero adjusted by the present value of the *MV* of the asset and the *PW* of the annual expenses. The total marginal cost for each year *k*, given by  $TC_k$ , is calculated using Equation (14.19) by finding the increase in the *PW* of total costs from year *k* – 1 to year *k* and then determining the equivalent worth of this increase at the end of year *k*. That is,

$$TC_k = (PW_k - PW_{k-1})(F/P, i \text{ per cent}, k) \quad (14.19)$$

This relationship can be simplified using Equation (14.20).

$$TC_k(i\%) = (1+i)MV_{k-1} - MV_k + E_k \quad (14.20)$$

Equation (14.20) is the sum of the loss in *MV* during the year of service, the opportunity cost of capital invested in the asset at the beginning of year *k* and the annual expenses incurred in year *k*, given by ( $E_k$ ). These total marginal (year-by-year) costs, according to Equation (14.20), are then used to find the *EUAC* of each year prior to and including the year *k*. The minimum  $EUAC_k$  value during the useful life of an asset identifies its economic life,  $N_C^*$ . The  $EUAC_k$  can be calculated as

$$EUAC_k = \left[ \sum_{j=1}^k TC_j(P/F, i \text{ per cent}, j) \right] (A/P, i \text{ per cent}, k)$$

**Example 14.17:** A company is considering the replacement of a machine required in its assembly line. A planning horizon of 10 years is to be used in the replacement study. The old machine (defender) has a current *MV* of Rs 25,00,000. If the defender is retained, it is anticipated to have annual operating and maintenance costs of Rs 8,00,000. It will have a zero *MV* at the end of 10 additional years of service. The new machine (challenger) will cost Rs 50,00,000 and will have operating and maintenance costs of Rs 1,00,000. It will have an *MV* of Rs 5,00,000 at the end of the planning horizon. Determine the preferred alternative using a *PW* comparison and a *MARR* (before tax) of 12 per cent per year.

**Solution:**

We have

Defender's current *MV* = Rs 25,00,000; *AE* = Rs 8,00,000

Challenger's purchase price = Rs 50,00,000; *AE* = Rs 1,00,000

*MV* at the end of 10 years = Rs 5,00,000

Then *PW* of the defender is given by

$$\begin{aligned} & -\text{Rs } 25,00,000 - 8,00,000 (P/A, 18 \text{ per cent}, 10) \\ &= -\text{Rs } 25,00,000 - 8,00,000 \left( \frac{(1+0.18)^{10} - 1}{0.18(1+0.18)^{10}} \right) \\ &= -\text{Rs } 60,95,272.85. \end{aligned}$$

The *PW* of the challenger is

$$\begin{aligned} & -\text{Rs } 50,00,000 - 1,00,000 (P/A, 18 \text{ per cent}, 10) + 5,00,000 (P/F, 18 \text{ per cent}, 10) \\ &= -\text{Rs } 50,00,000 - 1,00,000 \left( \frac{(1+0.18)^{10} - 1}{0.18(1+0.18)^{10}} \right) + 5,00,000 \times \frac{1}{(1+0.18)^{10}} \\ &= -\text{Rs } 50,00,000 - 1,00,000 \times 4.494 + 5,00,000 \times 0.191 \\ &= -\text{Rs } 53,53,900 \end{aligned}$$

*PW* of challenger is more than the defender. Hence, challenger is the best option.

**Example 14.18:** A company has two options concerning a machine whose present *MV* is estimated at Rs 1,00,000. The first option is to retain the machine for its remaining service life of 5 years and then replace it immediately with a new machine whose initial cost, life, *MV* and running costs are Rs 60,000, 6 years, Rs 20,000 and Rs 15,000 per year, respectively. It is estimated that the old machine's annual running cost is Rs 20,000 and it will have an *MV* of Rs 40,000 at the end of its life. The second option consists of having no machine and subcontracting the desired service at a cost of Rs 60,000 per year. Using a study period of 5 years and a *MARR* of 10 per cent, determine which option is preferable.

**Solution:** We have

$$\begin{aligned}
 AW \text{ of option 1} &= [-1,00,000 - 20,000(P/A, 10 \text{ per cent}, 5) \\
 &\quad + 40,000(P/F, 10 \text{ per cent}, 5) \\
 &\quad - 60,000(P/F, 10 \text{ per cent}, 5) \\
 &\quad - 15,000(P/A, 10 \text{ per cent}, 6)(P/F, 10 \text{ per cent}, 5) \\
 &\quad + 20,000(P/F, 10 \text{ per cent}, 11)](A/P, 10 \text{ per cent}, 11) \\
 &= \left[ -1,00,000 - 20,000 \left\{ \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} \right\} + 40,000 \times \frac{1}{(1+0.1)^5} - 60,000 \times \frac{1}{(1+0.1)^5} \right. \\
 &\quad \left. - 15,000 \left\{ \frac{(1+0.1)^6 - 1}{0.1(1+0.1)^6} \right\} \left\{ \frac{1}{(1+0.1)^5} \right\} + 20,000 \frac{1}{(1+0.1)^{11}} \right] \left\{ \frac{0.1(1+0.1)^{11}}{(1+0.1)^{11} - 1} \right\} \\
 &= - \text{Rs } 26,427.38
 \end{aligned}$$

$$AW \text{ of option 2} = -\text{Rs } 60,000$$

Hence, the first option is better due to least negative *AW*.

**Example 14.19:** A management is considering the replacement of an existing equipment. The data for the analysis of the challenger are: initial investment = Rs 1,00,000; there is no annual maintenance cost for the first four years, but in years 5 and 6, it will be Rs 25,000 and Rs 28,000, respectively, and in the year 7 increasing by Rs 5,000 and each year thereafter. The salvage value is zero at all times and *MARR* is 10 per cent per year. What is the economic life of the equipment?

**Solution:**

We have

$$P = \text{Rs } 1,00,000; AE_5 = \text{Rs } 25,000; AE_6 = \text{Rs } 28,000; AE_7 = \text{Rs } 33,000; AE_8 = \text{Rs } 38,000.$$

Then

$$\begin{aligned}
 EUAC_5 &= [\text{Rs } 1,00,000 + 25,000(P/F, 10 \text{ per cent}, 5)][A/P, 10 \text{ per cent}, 5] \\
 &= [\text{Rs } 1,00,000 + 25,000 \times 0.62][0.264] = \text{Rs } 30,492.0
 \end{aligned}$$

$$\begin{aligned}
 EUAC_6 &= [\text{Rs } 1,00,000 + 25,000(P/F, 10 \text{ per cent}, 5) + 28,000(P/F, 10 \text{ per cent}, 6)] \\
 &\quad [A/P, 10 \text{ per cent}, 6] \\
 &= [\text{Rs } 1,00,000 + 25,000 \times 0.62 + 28,000 \times 0.564][0.229] = \text{Rs } 30,065.86
 \end{aligned}$$

$$\begin{aligned}
 EUAC_7 &= [\text{Rs } 1,00,000 + 25,000(P/F, 10 \text{ per cent}, 5) + 28,000(P/F, 10 \text{ per cent}, 6) \\
 &\quad + 33,000(P/F, 10 \text{ per cent}, 7)] \times [A/P, 10 \text{ per cent}, 7] \\
 &= [\text{Rs } 1,00,000 + 25,000 \times 0.62 + 28,000 \times 0.564 + 33,000 \times 0.513] \times [0.205] \\
 &= \text{Rs } 30,385.31
 \end{aligned}$$

$$\begin{aligned}
 EUAC_8 &= [\text{Rs } 1,00,000 + 25,000(P/F, 10 \text{ per cent}, 5) + 28,000(P/F, 10 \text{ per cent}, 6) \\
 &\quad + 33,000(P/F, 10 \text{ per cent}, 7) + 38,000(P/F, 10 \text{ per cent}, 8)](A/P, 10 \text{ per cent}, 8) \\
 &= [\text{Rs } 1,00,000 + 25,000 \times 0.62 + 28,000 \times 0.564 + 33,000 \times 0.513 \\
 &\quad + 38,000 \times 0.466](0.187) = \text{Rs } 31,028.72
 \end{aligned}$$

Since  $EUAC_6$  is minimum, therefore the economic life of the machine is 6 years.

**Example 14.20:** The installed cost of a small generator has remained relatively constant at Rs 80,000. Records of operation and maintenance expenses indicate the average expenses per year as a function of the age of the machine. The  $MVs$  of the machine are also reasonably well known as a function of age. Using the information given in Table 14.7, determine the best lifetime to replace the machine if  $MARR$  is 10 per cent per year.

**Table 14-7:** Data on the small generator

Year	1	2	3	4	5	6
<b>Operations and maintenance expenses</b>	Rs 25,000	Rs 30,000	Rs 40,000	Rs 45,000	Rs 50,000	Rs 55,000
<b>MV</b>	Rs 60,000	Rs 50,000	Rs 45,000	Rs 40,000	Rs 35,000	Rs 30,000

#### Solution:

The solution can be summarized as shown in Table 14.8.

**Table 14-8:** Calculation of  $EUAC$

EOY	<b><math>MV</math> at the end of year</b>	<b>Loss in <math>MV</math> during the Year <math>k(A)</math></b>	<b>Cost of capital = 10 per cent of beginning of <math>MV (B)</math></b>	<b>Annual expenses (C)</b>	<b>Total marginal cost (<math>A + B + C</math>)</b>	<b><math>EUAC^*</math></b>
0	80,000					
1	60,000	20,000	8000	25,000	53,000	53,000
2	50,000	10,000	6000	30,000	46,000	21,904.76
3	45,000	5000	5000	40,000	50,000	15,105.74
4	40,000	5000	4500	45,000	54,500	11,743.15
5	35,000	5000	4000	50,000	59,000	9,664.05
6	30,000	5000	3500	55,000	63,500	<b>8,230.06</b>

$$^* EUAC_k = \left[ \sum_{j=1}^k TC_j (P/F, i \text{ per cent}, j) \right] (A/P, i \text{ per cent}, k)$$

$EUAC$  is minimum in year 6, thus the machine should be replaced at the end of the sixth year.

### 14.4.1 Selection Among Alternatives

In this section, selection among the alternatives is discussed with the techniques to evaluate the alternatives. Generally, the alternatives to be considered require investment of different amounts of capital, and their annual revenues and costs may vary. Sometimes, the alternatives have different useful lives. Several methods have been used to evaluate the alternatives. These methods are present worth ( $PW$ ), annual worth ( $AW$ ), future worth ( $FW$ ), internal rate of return ( $IRR$ ), and external rate of return ( $ERR$ ). The alternative having minimum investment of capital and producing satisfactory functional results will be chosen unless the incremental capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits. Incremental is meant for differential investment and differential benefits at the same interest rate. The incremental rate of return should be larger than the  $MARR$  (minimum attractive rate of return). It is assumed that all mutually exclusive alternatives selected for analysis meet the functional requirements established for an engineering project. However, differences among the alternatives may occur in many forms. These forms may be operational performance factors (such as output capacity, speed, thrust, heat dissipation rate, reliability, fuel efficiency and set-up time), quality factors, useful life, the capital investment required, revenue changes and annual expenses or cost savings.

#### **Study Period**

The study period is the planning horizon on the time scale over which mutually exclusive alternatives are compared. The useful lives of the alternatives relative to the selected study period may involve the following conditions:

1. Useful lives are same for all alternatives and are equal to the study period.
2. The alternatives have unequal useful lives, and the useful life of at least one alternative does not match the study period.

*Unequal lives* of the alternatives complicate their analysis and comparison. To conduct engineering economy analyses in such cases, we compare mutually exclusive alternatives over the same period of time. The repeatability assumption and the co-terminated assumption are used for these comparisons. The repeatability assumption involves the following two main conditions:

1. The study period over which the alternatives are being compared is either indefinitely long or equal to a common multiple of the lives of the alternatives.
2. The economic consequences that are projected to happen in an alternative's initial useful life span will also happen in all succeeding life spans.

**Example 14.21:** A manager has to select one project from three projects. All the projects are mutually exclusive. Each project's capital investment and annual operating expenses are shown in Table 14.9. Using the desired  $MARR$  of 12 per cent per year, and for the same analysis period, determine which project should be adopted on the basis of the  $PW$  method. Confirm your selection using the annual worth ( $AW$ ) and  $FW$  methods.

**Table 14-9:** Details of three project proposals

	<b>Project 1</b>	<b>Project 2</b>	<b>Project 3</b>
Capital investment	Rs 8,00,000	Rs 6,00,000	Rs 5,00,000
Project life	6 years	6 years	6 years
Annual revenue	Rs 4,00,000	Rs 3,00,000	Rs 2,00,000
Annual expenses	Rs 2,00,000	Rs 1,50,000	Rs 1,00,000
Market value	Rs 1,00,000	Rs 80,000	Rs 50,000

**Solution:****Present worth method**

MARR = 12 per cent,  $N = 6$  years.

$$\begin{aligned}
 PW \text{ for } P_1 &= -P + (R - E)(P/A, i \text{ per cent}, N) + SV(P/F, i \text{ per cent}, N) \\
 &= -8,00,000 + 2,00,000(P/A, 12 \text{ per cent}, 6) + 100,000(P/F, 12 \text{ per cent}, 6) \\
 &= -8,00,000 + 2,00,000 \times 4.11 + 100,000 \times 0.506 \\
 &= \text{Rs } 72,600
 \end{aligned}$$

$$\begin{aligned}
 PW \text{ for } P_2 &= -P + (R - E)(P/A, i \text{ per cent}, N) + SV(P/F, i \text{ per cent}, N) \\
 &= -6,00,000 + 150,000(P/A, 12 \text{ per cent}, 6) + 80,000(P/F, 12 \text{ per cent}, 6) \\
 &= -6,00,000 + 1,50,000 \times 4.11 + 80,000 \times 0.506 \\
 &= \text{Rs } 59,980
 \end{aligned}$$

$$\begin{aligned}
 PW \text{ for } P_3 &= -P + (R - E)(P/A, i \text{ per cent}, N) + SV(P/F, i \text{ per cent}, N) \\
 &= -5,00,000 + 1,00,000(P/A, 15 \text{ per cent}, 6) + 50,000(P/F, 15 \text{ per cent}, 6) \\
 &= -5,00,000 + 1,00,000 \times 4.11 + 50,000 \times 0.506 \\
 &= \text{Rs } 63,700
 \end{aligned}$$

Since  $PW_1 > PW_2 > PW_3$ , project 1 is preferred.

**Annual worth method**

$$\begin{aligned}
 AW \text{ for } P_1 &= -P(A/P, i \text{ per cent}, N) + (R - E) + SV(A/F, i \text{ per cent}, N) \\
 &= -8,00,000(A/P, 12 \text{ per cent}, 6) + 2,00,000 + 100,000(A/F, 12 \text{ per cent}, 6) \\
 &= -8,00,000 \times 0.243 + 2,00,000 + 1,00,000 \times 0.123 \\
 &= \text{Rs } 17,900
 \end{aligned}$$

$$\begin{aligned}
 AW \text{ for } P_2 &= -P(A/P, i \text{ per cent}, N) + (R - E) + SV(A/F, i \text{ per cent}, N) \\
 &= -6,00,000(A/P, 12 \text{ per cent}, 6) + 1,50,000 + 80,000(A/F, 12 \text{ per cent}, 6) \\
 &= -6,00,000 \times 0.243 + 1,50,000 + 80,000 \times 0.123 \\
 &= \text{Rs } 14,040
 \end{aligned}$$

$$\begin{aligned}
 AW \text{ for } P_3 &= -P(A/P, i \text{ per cent}, N) + (R - E) + SV(A/F, i \text{ per cent}, N) \\
 &= -5,00,000(A/P, 12 \text{ per cent}, 6) + 1,00,000 + 50,000(A/F, 12 \text{ per cent}, 6) \\
 &= -5,00,000 \times 0.243 + 1,00,000 + 50,000 \times 0.123 \\
 &= -\text{Rs } 15,350
 \end{aligned}$$

Since  $AW_1 > AW_2 > AW_3$ , project 1 is preferred.

### **Future worth method**

$$\begin{aligned}
 FW \text{ for } P_1 &= -P(F/P, i \text{ per cent}, N) + (R - E)(F/A, i \text{ per cent}, N) + SV \\
 &= -8,00,000(F/P, 12 \text{ per cent}, 6) + 2,00,000(F/A, 12 \text{ per cent}, 6) + 1,00,000 \\
 &= -8,00,000 \times 1.974 + 2,00,000 \times 8.11 + 1,00,000 \\
 &= \text{Rs } 1,42,800
 \end{aligned}$$

$$\begin{aligned}
 FW \text{ for } P_2 &= -P(F/P, i \text{ per cent}, N) + (R - E)(F/A, i \text{ per cent}, N) + SV \\
 &= -6,00,000(F/P, 12 \text{ per cent}, 6) + 1,50,000(F/A, 12 \text{ per cent}, 6) + 80,000 \\
 &= -6,00,000 \times 1.974 + 1,50,000 \times 8.11 + 80,000 \\
 &= \text{Rs } 1,12,100
 \end{aligned}$$

$$\begin{aligned}
 FW \text{ for } P_3 &= -P(F/P, i \text{ per cent}, N) + (R - E)(F/A, i \text{ per cent}, N) + SV \\
 &= -5,00,000(F/P, 12 \text{ per cent}, 6) + 1,00,000(F/A, 12 \text{ per cent}, 6) + 50,000 \\
 &= -5,00,000 \times 1.974 + 1,00,000 \times 8.11 + 50,000 \\
 &= -\text{Rs } 1,26,000
 \end{aligned}$$

Since  $FW_1 > FW_2 > FW_3$ , project 2 is preferred.

### **Conditions for PW Comparison:**

PW comparison can be used under the following conditions:

1. Cash flows are known.
2. Cash flows are in constant value rupees.
3. The interest rate is known.
4. Comparisons are made in cash flows before tax.
5. Comparisons do not include intangible considerations.
6. Comparisons do not include consideration of the availability of funds to implement alternatives.

### **Rate of Return Methods**

To compare the alternatives, convert all relevant cash flows into equivalent PW and find the rate of interest for which PW equivalence of cash inflows become equal to cash outflows; this interest rate is known as *IRR*. Similarly, convert all relevant cash flows into equivalent FW and find the

rate of interest for which  $FW$  equivalence of cash inflows become equal to cash outflows; this interest rate is called  $IRR$ . Consider the general case of two alternatives  $A$  and  $B$ . If  $MARR < IRR(A) < IRR(B)$ , then alternative  $B$  will be desirable for investment. For an incremental internal rate of return, if  $MARR < IRR$  for  $\Delta(B - A)$ , then  $B$  will be desirable for investment. Similar concepts are used for external rate of return, i.e.  $MARR < ERR(A) < ERR(B)$ , and  $MARR < ERR$  for  $\Delta(B - A)$ , to invest in alternative  $B$ .

**Example 14.22:** Two mutually exclusive machines are considered for purchase. Information relevant for comparing the machines is summarized in Table 14.10. Use the internal rate of return ( $IRR$ ) method to determine the better generator to be purchased. The  $MARR$  is 10 per cent per year.

**Table 14-10:** Details of two generators

	<b>Machine A</b>	<b>Machine B</b>
Capital investment	Rs 80,000	Rs 60,000
$MV$ at the end of service life	Rs 60,000	Rs 50,000
Annual fuel and maintenance expenses	Rs 3,000	Rs 4,000
Annual return	Rs 10,000	Rs 8,000
Service life	5 years	5 years

**Solution:** At first, find the value of  $IRR$  for both the generators by equating the  $PW$  of cash outflows and cash inflows, i.e.  $PW = 0$ . Select the generator for which  $IRR$  is greater than  $MARR$  and larger between the alternatives.

Machine A :

$$PW = 0 = -80,000 + 7000(P/A, i, 5) + 60,000(P/F, i, 5)$$

$$\text{At } i = 10 \text{ per cent, } PW = -80,000 + 7000 \times 3.791 + 60,000 \times 0.620 = -\text{Rs } 16,263$$

$$\text{At } i = 12 \text{ per cent, } PW = -80,000 + 7000 \times 3.605 + 60,000 \times 0.5674 = -\text{Rs } 20,721$$

Using interpolation,  $IRR = 2.7$  per cent  $< MRR$ , not justified.

Machine B:

$$PW = 0 = -60,000 + 4,000(P/A, i, 5) + 50,000(P/F, i, 5)$$

$$\text{At } i = 10 \text{ per cent, } PW = -60,000 + 4000 \times 3.791 + 50,000 \times 0.620 = -\text{Rs } 13,836$$

$$\text{At } i = 12 \text{ per cent, } PW = -60,000 + 4000 \times 3.605 + 50,000 \times 0.5674 = -\text{Rs } 17,210$$

Using interpolation,  $IRR = 1.798$  per cent  $< MRR$ , not justified.

None of the machines has an  $IRR$  more than the  $MRR$ . Therefore, no machine is to be selected. If, it is compared to each other neglecting the  $IRR$ , then Machine A is better than B.

**Example 14.23:** A person has identified three mutually exclusive investment alternatives. The expected annual income from the alternatives is shown in Table 14.11. The life of all the three

alternatives is estimated to be 10 years with zero *MV*. The *MARR* is 10 per cent. Find the alternative that should be selected by using the following:

- (a) *IRR* on incremental investment.
- (b) *PW* on incremental investment.

**Table 14-11:** Income from three investment alternatives

	<b>Alternative A</b>	<b>Alternative B</b>	<b>Alternative C</b>
Investment	Rs 10,000	Rs 15,000	Rs 20,000
Annual net income	Rs 3000	Rs 5000	Rs 6000

**Solution:**

- (a) *IRR* on incremental investment:

$$\begin{aligned} PW \text{ of } \Delta(B - A) &= -(15,000 - 10,000) + (5000 - 3,000)(P/A, i \text{ per cent}, 10) \\ 0 &= -5000 + 2000(P/A, i \text{ per cent}, 10) \end{aligned}$$

At  $i = 10$  per cent,

$$PW \Delta(B - A) = -5000 + 2000 \times 6.145 = 7290$$

At  $i = 12$  per cent,

$$PW \Delta(B - A) = -5000 + 2000 \times 5.65 = 6300$$

Using interpolation, at  $i = 24.72$  per cent,  $PW\Delta = 0$ ,

$$IRR = 24.72 \text{ per cent} > 10 \text{ per cent (MARR)}$$

which is justified and *A* is eliminated. Now

$$\begin{aligned} PW \text{ of } \Delta(C - B) &= -(20,000 - 15,000) + (6000 - 5000)(P/A, i \text{ per cent}, 10) \\ 0 &= -5000 + 1000(P/A, i \text{ per cent}, 10) \end{aligned}$$

At  $i = 10$  per cent,

$$PW \Delta(C - B) = -5000 + 1000 \times 6.145 = 1145$$

At  $i = 12$  per cent

$$PW \Delta(C - B) = -5000 + 1000 \times 5.65 = 650$$

Using interpolation, at  $i = 14.62$  per cent,  $PW\Delta = 0$ ,

$$IRR = 14.62 \text{ per cent} > 10 \text{ per cent (MARR)}$$

Hence, *C* should be selected.

- (b) *PW* at *MARR* (12 per cent) for incremental investment:

$$\begin{aligned} PW \text{ of } \Delta(B - A) &= -(15,000 - 10,000) + (5000 - 3000)(P/A, 10 \text{ per cent}, 10) \\ &= -5000 + 2000 \times 6.145 \\ &= Rs 7290 > 0 \end{aligned}$$

*B* is superior than *A*. Also

$$\begin{aligned} PW \text{ of } \Delta(C - B) &= -(20,000 - 15,000) + (6000 - 5000)(P/A, 10 \text{ per cent}, 10) \\ &= -5000 + 1000 \times 6.145 \\ &= \text{Rs } 1145 > 0 \end{aligned}$$

Thus, *C* is superior than *B* and so, *C* should be selected.

**Example 14.24:** For two mutually exclusive alternatives, *X* and *Y*, associated costs are given in Table 14.12. They have useful lives of 4 and 6 years, respectively. If *MARR* = 10 per cent per year, show which alternative is more desirable by using equivalent present worth methods. Use the repeatability assumption.

**Table 14-12:** Details of two alternatives

	Alternative X	Alternative Y
Capital investment	Rs 30,000	Rs 45,000
Annual cash inflow	Rs 10,000	Rs 12,000
Useful life	4 years	6 years
<i>MV</i> at the end of useful life	0	0

**Solution:** In this example, study period is unequal. Therefore, a common multiplicative study period may be used for comparison, i.e. 12 years of the evaluation period.

$$\begin{aligned} PW \text{ of } X &= -30,000[1 + (P/F, 10 \text{ per cent}, 4) + (P/F, 10 \text{ per cent}, 8)] \\ &\quad + 10,000(P/A, 10 \text{ per cent}, 12) \\ &= -30,000[1 + 0.683 + 0.466] + 10,000 \times 6.814 \\ &= \text{Rs } 3670 \end{aligned}$$

$$\begin{aligned} PW \text{ of } Y &= -45,000[1 + (P/F, 10 \text{ per cent}, 6)] + 12,000(P/A, 10 \text{ per cent}, 12) \\ &= -45,000[1 + 0.564] + 12,000 \times 6.814 \\ &= \text{Rs } 11,388 \end{aligned}$$

*PW* of *Y* is greater than *PW* of *X*. Therefore, alternative *Y* is preferred.

**Example 14.25:** A purchase committee has recommended two generators *A* and *B*, mutually exclusive, to supply the electricity. Their cost data are given in Table 14.13. The MARR is 8 per cent per year.

- (a) Determine which generator should be selected if the repeatability assumption applies.
- (b) Determine which generator should be selected if the analysis period is 7 years, the repeatability assumption does not apply and the machine can be leased for Rs 12,000 per year after the useful life of either machine.

**Table 14-13:** Details of two generators

	<b>Generator X</b>	<b>Generator Y</b>
Capital investment	Rs 1,00,000	Rs 80,000
Annual expenses	Rs 50,000	Rs 60,000
Useful life	4 years	6 years
<i>MV</i> at the end of useful life	Rs 20,000	Rs 15,000

**Solution:**

(a) We have

$$\begin{aligned}
 PW \text{ of generator } X &= -1,00,000[1+(P/F, 8 \text{ per cent}, 4)+(P/F, 8 \text{ per cent}, 8)] \\
 &\quad -50,000(P/A, 8 \text{ per cent}, 12)+20,000[(P/F, 8 \text{ per cent}, 4) \\
 &\quad +(P/F, 8 \text{ per cent}, 8)+(P/F, 8 \text{ per cent}, 12)] \\
 &= -1,00,000[1+0.735+0.540]-50,000\times 7.536 \\
 &\quad +20,000[0.735+0.540+0.397] \\
 &= -\text{Rs } 5,70,860
 \end{aligned}$$

$$\begin{aligned}
 PW \text{ of generator } Y &= -80,000[1+(P/F, 8 \text{ per cent}, 6)]-60,000(P/A, 8 \text{ per cent}, 12) \\
 &\quad +15,000[(P/F, 8 \text{ per cent}, 6)+(P/F, 8 \text{ per cent}, 12)] \\
 &= -80,000[1+0.630]-60,000\times 7.536+15,000[0.630+0.397] \\
 &= -\text{Rs } 5,67,155
 \end{aligned}$$

Since  $PW$  of generator  $X > PW$  of generator  $Y$ , generator A should be preferred. However,  $PW$  for both the generators are negative.

(b) We have

$$\begin{aligned}
 PW \text{ of generator } X &= -1,00,000-50,000(P/A, 8 \text{ per cent}, 4) \\
 &\quad +20,000(P/F, 8 \text{ per cent}, 4) \\
 &\quad +12,000(P/A, 8 \text{ per cent}, 3)(P/F, 8 \text{ per cent}, 4) \\
 &= -1,00,000-50,000\times 3.312+20,000 \\
 &\quad \times 0.735+12,000\times 2.577\times 0.735 \\
 &= -\text{Rs } 2,28,170.86
 \end{aligned}$$

$$\begin{aligned}
 PW \text{ of generator } Y &= -80,000-60,000(P/A, 8 \text{ per cent}, 6) \\
 &\quad +15,000(P/F, 8 \text{ per cent}, 6) \\
 &\quad +12,000(P/A, 8 \text{ per cent}, 1)(P/F, 8 \text{ per cent}, 6) \\
 &= -80,000-60,000\times 4.623+15,000\times 0.630 \\
 &\quad +12,000\times 0.926\times 0.630 \\
 &= -\text{Rs } 3,40,929.44
 \end{aligned}$$

Since  $PW$  of generator  $X > PW$  of generator  $Y$ , generator  $X$  should be preferred. However,  $PWs$  for both the generators are negative.



## SUMMARY

In this chapter, we have discussed the problems related to the replacement model subject to continuous deterioration and sudden failure. We have also discussed the problem of continuous deterioration when the money value changes with time. The time value of money is discussed in detail in this chapter. In the last section of this chapter, we learnt about the selection among the alternatives and evaluation of alternatives using various techniques.

## MULTIPLE-CHOICE QUESTIONS

1. An equipment is replaced with a new one, when
  - (a) the equipment has become inefficient or requires more maintenance.
  - (b) the equipment has failed due to accident or otherwise and does not work at all.
  - (c) the equipment is expected to fail shortly.
  - (d) all the above
2. Progressive failure is the failure
  - (a) whose probability increases with the age of an item.
  - (b) whose probability of failure in the initial years of the life is more
  - (c) whose probability is random
  - (d) all the above
3. Retrogressive failure is the failure
  - (a) whose probability increases with the age of an item.
  - (b) whose probability of failure in the initial years of the life is more
  - (c) whose probability is random
  - (d) all the above
4. Gradual failure is due to
  - (a) increased operating cost
  - (b) decreased productivity of the item
  - (c) decrease in resale value of an item
  - (d) all the above
5. Which of the following is the assumption of the Mortality theorem?
  - (a) death of everybody is certain
  - (b) every death is replaced by the new birth
  - (c) mortality can be avoided forever by proper maintenance
  - (d) all the above
6. For an item that deteriorates gradually is replaced, when
  - (a) average monthly cost will go on decreasing, with increase in time.
  - (b) the rate of increase in running cost is considerably higher than the savings in average capital costs.

- (c) the item becomes unattractive
  - (d) all the above
7. Economic life of an asset can be defined as
- (a) the period in years in which the asset results in the minimum equivalent uniform annual cost (EUAC) of operation.
  - (b) the period between the date of acquisition and the date of disposal of the asset by its owner.
  - (c) the period between the original acquisition and final disposal of the asset over a succession of its owners.
  - (d) the period in years during which the asset is kept in productive service.
8. Ownership life of an asset can be defined as
- (a) the period in years in which the asset results in the minimum equivalent uniform annual cost (EUAC) of an operation.
  - (b) the period between the date of acquisition and the date of disposal of the asset by its owner.
  - (c) the period between the original acquisition and final disposal of the asset over a succession of its owners.
  - (d) the period in years during which the asset is kept in productive service.
9. Physical life of an asset can be defined as
- (a) the period in years in which the asset results in the minimum equivalent uniform annual cost (EUAC) of operation.
  - (b) the period between the date of acquisition and the date of disposal of the asset by its owner.
  - (c) the period between the original acquisition and final disposal of the asset over a succession of its owners.
  - (d) the period in years during which the asset is kept in productive service.
10. Useful life of an asset can be defined as
- (a) the period in years in which the asset results in the minimum equivalent uniform annual cost (EUAC) of operation.
  - (b) the period between the date of acquisition and the date of disposal of the asset by its owner.
  - (c) the period between the original acquisition and final disposal of the asset over a succession of its owners.
  - (d) the period in years during which the asset is kept in productive service.
11. Unequal lives of the alternatives can be compared using repeatability assumptions under the following condition
- (a) The study period over which the alternatives are being compared is either indefinitely long or equal to a common multiple of the lives of the alternatives.
  - (b) The economic consequences that are projected to happen in an alternative's initial useful life span will also happen in all succeeding life spans.
  - (c) Both (a) and (b)
  - (d) None of these

12. The condition for using present worth comparison
  - (a) Cash flows are known.
  - (b) Cash flows are in constant value rupees.
  - (c) The interest rate is known.
  - (d) All the above
13. If  $MARR < IRR (A) < IRR (B)$ , then
  - (a) alternative A will be desirable for the investment
  - (b) alternative B will be desirable for the investment
  - (c) alternatives A and B, both can be considered for the investment
  - (d) none of them is desirable for the investment
14. If  $MARR > IRR (A) > IRR (B)$ , then
  - (a) alternative A will be desirable for the investment
  - (b) alternative B will be desirable for the investment
  - (c) alternatives A and B both can be considered for the investment
  - (d) none of them is desirable for the investment
15. For an incremental internal rate of return, if  $MARR < IRR \text{ for } \Delta(B - A)$ , then
  - (a) alternative A will be desirable for the investment
  - (b) alternative B will be desirable for the investment
  - (c) alternatives A and B both can be considered for the investment
  - (d) none of them is desirable for the investment

## Answers

1. (d)      2. (a)      3. (b)      4. (d)      5. (b)      6. (b)      7. (a)      8. (b)      9. (c)  
10. (d)     11. (c)     12. (d)     13. (b)     14. (d)     15. (b)

## REVIEW QUESTIONS

1. What do you mean by replacement of equipment? In which conditions the replacement of equipment is felt?
2. What are the types of failures? Explain them.
3. What is the difference between individual and group replacement policies?
4. Discuss the mortality theorem and its applications.
5. Discuss the importance of time value of money in equipment replacement policy.
6. Discuss the terms: economic life, ownership life, useful life and physical life.
7. Explain the method of comparison between the alternatives having unequal lives.

## EXERCISES

1. A TV consists of 1000 resistors. When any resistor fails, it is replaced. The cost of individual replacement of the resistor is Rs 5 per resistor but if all the resistors are replaced at a time, the cost becomes Rs 2 per resistor. The probability of failure  $P(t)$  during the month  $t$  is given as shown in Table 14.14.

**Table 14-14:** Probability of failure of the resistors

<b><math>t</math></b>	0	1	2	3	4	5	6
<b><math>p(t)</math></b>	—	0.03	0.06	0.3	0.4	0.2	0.01

Find the optimal time of replacement of the resistors.

2. Table 14.15 shows the mortality rates have been observed for a certain type of electric bulbs (500 bulbs):

**Table 14-15:** Mortality rate of the electric bulbs

<b>Week</b>	1	2	3	4	5
<b>Percentage failing at the end of the week</b>	25	40	50	80	100

Each bulb costs Rs 8 to replace an individual bulb on failure. If all bulbs were replaced at the same time in the group, it would cost Rs 3 per bulb. It is under proposal to replace all bulbs at fixed intervals of time, whether or not the bulbs have burnt out; and also it is to continue replacing immediately burnt out bulbs. Determine the time interval at which all the bulbs should be replaced.

3. A machine costs Rs 80,000 and the operating costs are estimated at Rs 2000 for the first year increasing by Rs 5000 per year in the second and subsequent years. The resale value of the machine is Rs 4000. When the machine should be replaced?
4. Mr Sharma is planning to purchase a house in Gurgaon 5 years from now. The home will cost Rs 2,00,00,000 at that time. He plans to make three deposits at the interest rate of 12 per cent in the following ways: Rs 20,00,000 today; Rs 25,00,000 two years from now, and Rs  $X$  three years from now. How much he needs to invest in year three to ensure that he has the necessary funds to buy the vacation home at the end of year five?
5. Find the effective interest rate per payment period for an interest rate of 8 per cent compounded monthly for each of the given payment schedule:
- |                  |               |
|------------------|---------------|
| (a) monthly      | (b) quarterly |
| (c) semiannually | (d) annually  |
6. A company has two options concerning a machine whose present market value ( $MV$ ) is estimated at Rs 1,20,000. The first option is to retain the machine for its remaining service life of 8 years and then replace it immediately with a new machine whose initial cost, life, market value and running costs are Rs 80,000, 4 years, Rs 25,000 and Rs 20,000 per year, respectively. It is estimated that the old machine's annual running cost is Rs 25,000 and it will have an MV of Rs 50,000 at the end of its life. The second option consists of having no machine and subcontracting the desired service at a cost of Rs 80,000 per year. Using a study period of 12 years and an MARR of 10 per cent, determine which option is preferable.

7. Two mutually exclusive machines are considered for purchase. Information relevant for comparing the machines is summarized in Table 14.16. Use the internal rate of return (IRR) method to determine the better generator to be purchased. The *MARR* is 12 per cent per year.

**Table 14-16:** Details of two generators

	<b>Machine A</b>	<b>Machine B</b>
Capital investment	Rs 100,000	Rs 80,000
Market value at the end of service life	Rs 80,000	Rs 70,000
Annual fuel and maintenance expenses	Rs 5000	Rs 6000
Annual return	Rs 12,000	Rs 10,000
Service life	5 years	5 years

8. For two mutually exclusive alternatives, *X* and *Y*, associated costs are given in Table 14.17. They have useful lives of 4 and 6 years, respectively. If *MARR* = 8 per cent per year, show which alternative is more desirable by using equivalent present worth methods. Use the repeatability assumption.

**Table 14-17:** Details of two alternatives

	<b>Alternative X</b>	<b>Alternative Y</b>
Capital investment	Rs 25,000	Rs 40,000
Annual cash inflow	Rs 8000	Rs 10,000
Useful life	4 years	6 years
Market value at the end of useful life	0	0



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# CHAPTER ▶ 15

## Value Engineering

### 15.1 INTRODUCTION

Value engineering is a tool used for cost reduction and value improvement of a product/process. It improves the effectiveness of the system that has been conventionally performed in producing a product. It is also known as value analysis. Value analysis questions and probes the various purposes, design, method of manufacture and other issues related to the product with a view to pinpointing the unnecessary costs, which can be eliminated without adversely affecting the quality, efficiency, safety and other essential features of the product.

Value engineering may be defined as a technique that yields in value improvement by deciding the essential function of a product and the accomplishment of this function at minimum cost without any degradation in its quality. The value of a product is the relationship between its function and cost. The value of a product can be improved by minimizing the cost for same function or improving the function of the product for the same cost or increasing the function in higher proportion than the increment in the cost of the product. Numerically, the value can be expressed as: Value = Function/Cost.

The value can be measured in terms of cost paid by the consumer for the required functions or services of the product. This means that cost is reduced without compromising with the quality and functions of the product. Thus, the value analysis is a technique which builds ‘value’ into a product. Value can also be defined in terms of a combination of quality, efficiency, price and service which ensures the ultimate economy and satisfaction of the purchaser.

### 15.2 TYPES OF VALUE

Broadly, value is classified into the following categories:

1. *Esteem value*: It is defined as the qualities, properties and attractiveness for a product/service to make its ownership desirable.
2. *Use value*: It is defined as the qualities and properties needed to accomplish a service, product or work.
3. *Cost value*: It is defined as the total cost of material, labour, overhead and services to produce an item (or to deliver a service).
4. *Exchange value*: It is defined as the property and qualities of a product (or service) which facilitate its exchange (or trade) for something else that is needed.

## 15.3 SEVEN PHASES OF VALUE ANALYSIS

There are seven phases in which the value analysis of a product/service is accomplished. These seven phases are discussed below as:

### 15.3.1 General Phase

In this phase, the project for value analysis is selected and the objective of value analysis is determined. It is performed in the following steps:

1. Identify and select the project.
2. Establish priorities.
3. Plan a specific project.
4. Constitute a team for value analysis.
5. Prepare terms of reference for the selected project.
6. Fix responsibility for data collection.

### 15.3.2 Information Phase

In this phase, all the necessary information related to the project is gathered. It consists of the following steps:

1. Collection of facts
2. Determination of costs
3. Fixation of cost of specifications and requirements.

### 15.3.3 Function Phase

In this phase, the functions of the product are analysed and classified as primary and secondary functions. The unnecessary functions may be eliminated and necessary or primary functions may be improved depending on requirements. It consists of the following steps:

1. Finding the area of value analysis and the functions that the product actually performs.
2. Establishing the relationship between these functions to the cost and worth of providing the product.
3. Determination of the poor value functions and justification for the value analysis effort.
4. Fixing the reference points using which the cost of alternatives can be compared.
5. Formulation of a minimum cost for a psychological incentive to the consumer.
6. Motivation among product development team for value analysis.

### 15.3.4 Creation Phase

This is also known as *innovation phase*. In this phase, different ideas are generated to solve a problem and compared with each other. It consists of the following steps:

1. Idea generation or brainstorming.
2. Determination of problem solution by:
  - (a) Evaluating all the ideas generated by the team
  - (b) Making a fair comparison
  - (c) Determining interrelationships
  - (d) Anticipating objections, if any
  - (e) Determining and developing place and time for appropriate action.

### **15.3.5 Evaluation Phase**

In this phase, the most promising ideas are evaluated in the following steps:

1. Selecting the most promising ideas generated in the creation phase for further analysis.
2. Screening the ideas by eliciting answers to the following questions:
  - (a) Will the idea work?
  - (b) Is it cheaper than the present design?
  - (c) Is it feasible to implement?
  - (d) Will it satisfy the user requirements?
3. Finding out the most suitable proposal and obtain its cost.

### **15.3.6 Investigation Phase**

This is an implementation phase in which the proposal is investigated and converted into a work plan. It consists of the following steps:

1. Bringing the partially developed or selected untried ideas to fruition and to find out their feasibility and limitations.
2. Preparation of a work plan for converting the selected ideas into tangible proposals.

### **15.3.7 Recommendation Phase**

In this phase, the final proposal is recommended to the management with benefit and limitations of the proposal. It consists of the following steps:

1. Preparation and submission of the planned proposals along with benefits and limitations to the management.
2. Review of the submitted proposals, if not acceptable to the management.

## **15.4 APPLICATION OF VALUE ANALYSIS**

Value analysis can be applied to everything like materials, methods, processes, services, etc. to bring about economics. For applying value analysis, one should naturally start with the items where the maximum annual saving can be achieved. It implies that items whose total annual consumption in rupees is high should receive top priority in the application of value analysis.

Similarly, scarce materials, imported materials, or those difficult to obtain should also receive the attention of the value analyst. Bearing this in mind, value analysis can be systematically applied to categories of items, such as those listed below, to bring about substantial cost reduction.

1. Capital goods such as plant, equipment, machinery, tools and appliances.
2. Raw and semi-processed material, including fuel.
3. Subcontracted parts, components, sub-assemblies, etc.
4. Purchased parts, components, sub-assemblies, etc.
5. Maintenance, repairs and operational items.
6. Furnishing items such as paints, oils, varnishes, etc.
7. Packing materials and packaging.
8. Printing and stationery items.
9. Miscellaneous items of regular consumption.
10. Power, water supply, compressed air, steam and other utilities (services).
11. Materials handling and transportation costs.

## 15.5 ADVANTAGES OF VALUE ANALYSIS

The following advantages are associated with value analysis:

1. Production price is reduced at the same quality of product or the improved quality product can be provided at the same or reduced price.
2. Management effectiveness can be measured in terms of cost saving.
3. It helps in generating new ideas for the R & D department.
4. It creates cost and quality consciousness among the manpower.
5. It creates innovative/creative ideas among the employees.
6. It dilutes resistance to change and accelerates the process of change implementation.

## 15.6 VALUE ANALYSIS OF A GRAPHITE PENCIL BY MATRIX METHOD

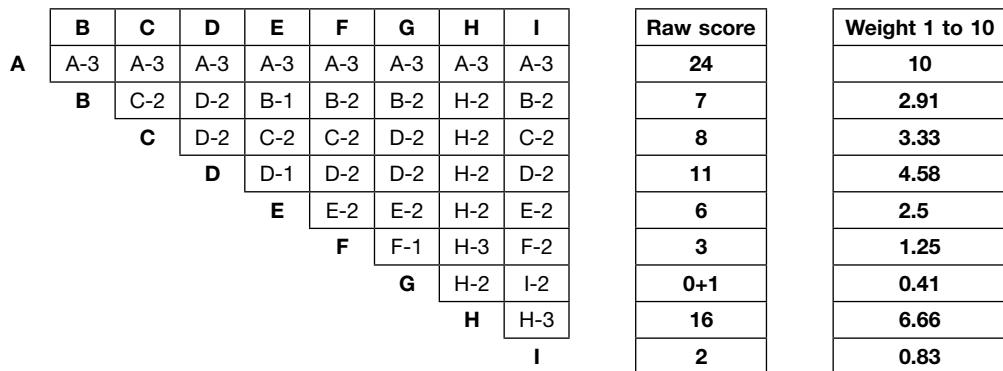
Matrix method of value analysis is used to compare the functions of various components of a product according to their importance. The product is broken into smaller elements and functions and costs of each element are determined. In a matrix form, these functions are compared on some numerical scale, e.g. 5-point rating scale. Finally, weights of these functions are calculated on a 10-point rating scale. An example of value analysis of a graphite pencil by matrix method is given in Figure 15.1. A pencil worth Rs 3 is considered for value analysis. The parts used to make a pencil are wood, lead, adhesive, paint and embossing. Now, value of each part of the pencil is approximated as mentioned below:

Lead	50 paise
Wood	150 paise
Paint	40 paise
Adhesive	30 paise
Embossing	30 paise
Total	Rs 3.00

The functions of each part can be defined in the following way:

Lead	—	Marking (A)
Wood	—	Griping (B)
		Sharpening (C)
		Protect lead (D)
		Protect hand (E)
Paint	—	Smoothness (F)
		Aesthetic (G)
Adhesive	—	Bonding (H)
Embossing	—	Branding the company name (I)

Now, the ranking through weightage is done using the matrix shown in Figure 15.1.



**Figure 15-1:** Combined matrix for value analysis of a pencil

The results of the pairwise comparisons (A to B, A to C, and so on) are calculated and entered to give a relative importance factor, or weight, for each criterion (Figure 15.1). The procedure for this is to enter the letter of the criterion judged to be most important at each intersection on the weighting matrix. Add to that letter a number (1 to 3) signifying how much more important is that criterion. If both criteria at an intersection are equally important, enter a zero.

For example, the first matrix entry in Figure 15.1, A-3 is at the intersection of row A and column B. This indicates that criterion A is more important than criterion B by a factor of 3 (on a scale of 1 to 3). The raw score shown in the rectangle on the right is the sum of all the A's (24), B's (7) and so forth. If any raw score is zero, then enter the raw score for that criterion, an amount equal to one-half of the lowest value shown for any of the other criteria. In this case, raw score for 'G' will be half of the raw score of 'I', i.e., 1. The weight is determined by rescaling the raw scores into a 0 to 10 scale. The criterion with the highest raw score is given a weight of 10 while the others are proportioned to that highest score. For example, the weight of criterion A, to make a mark, in Figure 15.1 is 10. The value can be found by dividing the weight of the functions of the part calculated from the matrix by the cost of the corresponding part.

## 15.7 FUNCTION ANALYSIS SYSTEM TECHNIQUE

Function analysis system technique (FAST) was invented by Charles Bytheway in 1964. FAST permits the inclusion of the people with different technical backgrounds to effectively communicate and resolve issues that require multidisciplinary considerations. FAST builds upon value analysis by linking the simply expressed, verb–noun functions to describe complex systems. It describes the item or system under study and causes the team to think through the functions that the item or system performs, forming the basis for a wide variety of subsequent approaches and analysis techniques. It contributes significantly to the most important phase of value engineering, i.e. function analysis.

The FAST diagram or model has been considered as an excellent communications tool. It allows multidisciplinary team members to contribute and communicate with one another while addressing the problem objectively without biased or preconceived conclusions. In this model, there is no right or wrong model or result. The problem should be structured until the product development team members are satisfied that the real problem is identified. After producing the problem statement, the single most important output of the multidisciplinary team engaged in developing a FAST model is consensus. Since the team has been charged with the responsibility of resolving the assigned problem, it is their interpretation of the FAST model that reflects the problem statement. The team members must discuss and reconfigure the FAST model until consensus is reached and all participating team members are satisfied that their concerns are expressed in the model. Once consensus has been achieved, the FAST model is complete and the team can move on to the next creative phase. There are usually two types of FAST diagrams, the technical FAST diagram and the customer FAST diagram. A technical FAST diagram is used to understand the technical aspects of a specific portion of a total product. A customer FAST diagram focuses on the aspects of a product that the customer cares about and does not look into the technicalities, mechanics or physics of the product. A customer FAST diagram is usually applied to a total product.



### SUMMARY

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In this chapter, we have discussed about the values of product and/or services. Various types of values have been highlighted and steps for value analysis have been discussed in detail. Finally, an example of value analysis of graphite pencil using matrix method has been illustrated. Value analysis is a technique with immense potential. If employed systematically, it can achieve great economies and increased efficiency.

### MULTIPLE-CHOICE QUESTIONS

1. Value analysis is used to
  - (a) improve the worth of the product for which a customer makes payment
  - (b) reduce the cost for the same worth
  - (c) improve the quality for the same cost
  - (d) all the above



## Answers

1. (d)    2. (d)    3. (c)    4. (c)    5. (d)    6. (d)    7. (d)    8. (a)    9. (d)  
10. (d)   11. (d)   12. (c)   13. (d)   14. (c)   15. (a)   16. (c)   17. (b)   18. (a)

## REVIEW QUESTIONS

1. Explain the term ‘value engineering’.
2. How do you classify the value of a product? Explain each type of the value.
3. Discuss the seven phases of value analysis.
4. Collect the data for a bicycle and do the value analysis.
5. Write a note on function analysis system technique (FAST).
6. Define value. How does it differ from quality?
7. What types of questionnaires are asked in value analysis? Discuss in detail.
8. What do you mean by FAST? Explain in detail.

## EXERCISES

1. Explain the value analysis of a FOUNTAIN PEN using matrix method.



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# CHAPTER ▶ 16

## Linear Programming and Transportation Problem

### 16.1 INTRODUCTION TO LINEAR PROGRAMMING

Linear programming, developed by Dantiz, is a technique to optimize the objective under various constraints. In a linear programming problem (LPP), the nature of the relationship between variables and constraints is linear. This is a very useful technique in industrial and engineering applications to optimize the cost or profit under limited resources. Here, optimization means minimization or maximization; it depends on the objective function. If the objective function is a cost function, then it is to be minimized. If the objective function is a profit function, then it is to be maximized. There are following properties of LPP:

1. The relationship between variables and constraints is linear.
2. The model has an objective function.
3. The model has structural constraints.
4. The model has non-negativity constraint.

#### ***Basic Assumptions of LPP***

The following are some important assumptions made in formulating an LPP model:

1. All the situations, such as resources, course of action, etc., are considered as deterministic in nature.
2. The relationship between variables and resources is linear in nature.
3. The decision variables are continuous.
4. LPP is a static model, i.e. it is a single-stage decision problem.

#### **16.1.1 Structure of Linear Programming Model**

The objective function may be written as:

$$\text{Maximize/Minimize } Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$$

(Where  $C_1, \dots, C_n$  are coefficients of variable; and  $X_1, \dots, X_n$  are decision variables.)

Subject to

$$\left. \begin{array}{l} a_{11}X_1 + a_{12}X_2 + \dots + a_{1n}X_n (\leq, =, \geq) b_1 \\ a_{21}X_1 + a_{22}X_2 + \dots + a_{2n}X_n (\leq, =, \geq) b_2 \\ \dots \dots \dots \dots \dots \dots \dots \\ a_{m1}X_1 + a_{m2}X_2 + \dots + a_{mn}X_n (\leq, =, \geq) b_m \\ X_1, X_2, \dots, X_n \geq 0 \text{ (Non-negativity of the variables)} \end{array} \right\} \text{constraints}$$

**Example 16.1:** There are two suppliers A and B. Supplier A can produce maximum 120 components per day and Supplier B can produce maximum 80 components per day. Buyer X consumes 40 components per day and Buyer Y consumes 160 components per day. The transportation costs of Supplier A to Buyer X and Y are Rs 40 and Rs 60 per component, respectively. Similarly, the transportation costs of Supplier B to Buyer X and Y are Rs 25 and Rs 80 per component, respectively. Formulate the LPP to minimize the transportation cost and solve using graphical method.

**Solution:**

#### Formulation of LP Model

Suppose the amount of the components sold by Supplier A to Buyers X and Y are  $A_X$  and  $A_Y$ , respectively; and the amounts sold by Supplier B to Buyers  $B_X$  and  $B_Y$ , respectively.

Total transportation cost  $Z$  will be the sum of the product of transportation cost and amount to be transported, i.e.  $C_{AX}A_X + C_{AY}A_Y + C_{BX}B_X + C_{BY}B_Y$ .

Subject to the constraints

$$\left. \begin{array}{l} A_X + A_Y = b_A \\ B_X + B_Y = b_B \\ A_X + B_X = d_X \\ A_Y + B_Y = d_Y \end{array} \right\} \begin{array}{l} \text{Supply constraints} \\ \text{Demand constraints} \end{array}$$

Substituting the values from Example 16.1, we get

$$\text{Min } Z = 40A_X + 60A_Y + 25B_X + 80B_Y \} \text{ Objective function}$$

Subject to

$$\left. \begin{array}{l} A_X + A_Y = 120 \\ B_X + B_Y = 80 \\ A_X + B_X = 40 \\ A_Y + B_Y = 160 \end{array} \right\} \begin{array}{l} \text{Supply constraints} \\ \text{Demand constraints} \end{array}$$

$$A_X, A_Y, B_X, B_Y \geq 0 \} \text{ Non-negativity constraint of decision variables.}$$

This problem can be solved by simplex technique. The minimum value of transportation cost will be Rs 11,400.00. The simplex technique to solve the LPP will be discussed in Section 16.2.

## 16.2 SIMPLEX TECHNIQUE

The simplex technique can be explained in a better way using an example.

### Example 16.2:

$$\text{Maximize } Z = 5X_1 + 4X_2 + 7X_3$$

$$\text{Subject to } X_1 + 3X_2 + 2X_3 \leq 22$$

$$3X_1 + 2X_2 + X_3 \leq 26$$

$$X_1 + 2X_2 + X_3 \leq 18$$

$$X_1, X_2, X_3 \geq 0$$

**Step 1:** All the inequality constraints should be converted into the equality constraints by adding slack variables (for  $\leq$ ) and artificial variables (for  $\geq$ ). The constraint with inequality  $\geq$  will be discussed in Big-M method or two-phase method of simplex technique. After forming the equality constraints, the form of LPP will be

$$\text{Maximize } Z = 5X_1 + 4X_2 + 7X_3 + 0S_1 + 0S_2 + 0S_3$$

$$\text{Subject to } X_1 + 3X_2 + 2X_3 + S_1 = 22$$

$$3X_1 + 2X_2 + X_3 + S_2 = 26$$

$$X_1 + 2X_2 + X_3 + S_3 = 18$$

$$X_1, X_2, X_3, S_1, S_2, S_3 \geq 0$$

**Step 2:** Formulate the initial simplex table as shown in Table 16.1, which is self-explanatory as all the elements of the table have been mentioned in detail.

### Step 3:

- (a) Identify key column and key row. *The column having maximum positive value in the case of maximization and minimum negative value in the case of minimization in the row of  $c_j - z_j$  is the key column.* In Table 16.2, column of  $X_3$  is the key column, since the value of  $c_j - z_j$  is 7, which is maximum positive value among all the elements in the row of  $c_j - z_j$ .
- (b) Now find the ratio of elements of solution column and the elements of key column which is shown as rightmost column of Table 16.2. The minimum positive value of the ratio will be the key row. In Table 16.2, the row with  $S_1$  variable is the key row as it has least ratio, i.e. 11.
- (c) The intersection element of the key column and key row is the key number. In Table 16.2, key number is 2 which is highlighted with grey colour.

Table 16-1: Initial simplex table

Constant in the right side of the constraints

		$C_j$	5	4	7	0	0	0
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	$S_3$
0	$S_1$	22	1	3	2	1	0	0
0	$S_2$	26	3	2	1	0	1	0
0	$S_3$	18	1	2	1	0	0	1
	$Z_j$	0	0	0	0	0	0	0
	$C_j - Z_j$	5	4	7	0	0	0	0

Coefficients of basic variables in objective function

Value of objective function

Difference between  $c_j$  and  $Z_j$ . In the case of maximization, all the value of this row must be negative for optimal solution; and in the case of minimization, all the value of this row must be positive for optimal solution

Summation of product of corresponding column elements and elements of  $CB_i$ , such as summation of  $CB_1 \times X_1$ ,  $CB_1 \times X_2$  and so on.

Coefficient of variables in constraints

**Iteration 1****Table 16-2:** First iteration table

		<b>C<sub>j</sub></b>	5	4	7	0	0	0	<b>Ratio</b>
<b>CB<sub>i</sub></b>	<b>Basic variables</b>	<b>Solution</b>	<b>X<sub>1</sub></b>	<b>X<sub>2</sub></b>	<b>X<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>	
0	S <sub>1</sub>	22	1	3	2	1	0	0	22/2 = 11
0	S <sub>2</sub>	26	3	2	1	0	1	0	26/1 = 26
0	S <sub>3</sub>	18	1	2	1	0	0	1	18/1 = 18
	<i>z<sub>j</sub></i>	0	0	0	0	0	0	0	
	c <sub>j</sub> -z <sub>j</sub>		5	4	7	0	0	0	

**Iteration 2**

In Table 16.2, divide all the elements of key row by key number and replace the basic variable of this row, i.e. S<sub>1</sub> with the decision variable of the key column, i.e. X<sub>3</sub>. In other rows, the new element will be substituted as

$$\text{New element} = \text{old element} - \frac{(\text{corresponding old element in key row} \times \text{corresponding element of the row in key column})}{\text{Key number}}$$

After calculation of all the new values, form a new table with new elements as shown in Table 16.3.

**Table 16-3:** Second iteration table

		<b>C<sub>j</sub></b>	5	4	7	0	0	0
<b>CB<sub>i</sub></b>	<b>Basic variables</b>	<b>Solution</b>	<b>X<sub>1</sub></b>	<b>X<sub>2</sub></b>	<b>X<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>
7	X <sub>3</sub>	11	0.5	1.5	1	0.5	0	0
0	S <sub>2</sub>	15	2.5	0.5	0	-0.5	1	0
0	S <sub>3</sub>	7	0.5	0.5	0	-0.5	0	1
	<i>z<sub>j</sub></i>	77	3.5	10.5	7	3.5	0	0
	c <sub>j</sub> -z <sub>j</sub>		1.5	-6.5	0	-3.5	0	0

**Iteration 3**

Now repeat the same process to find the key column, key row and key element of the Table 16.3. Replace all the elements with new elements as discussed in Step 3 and form a new table with new elements as shown in Table 16.4. The iteration is repeated again and again till all the values in c<sub>j</sub>-z<sub>j</sub> row become negative or zero for maximization and positive and zero for minimization.

**Table 16-4:** Third iteration table

		<b>C<sub>j</sub></b>	<b>5</b>	<b>4</b>	<b>7</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>CB<sub>i</sub></b>	<b>Basic variables</b>	<b>Solution</b>	<b>X<sub>1</sub></b>	<b>X<sub>2</sub></b>	<b>X<sub>3</sub></b>	<b>S<sub>1</sub></b>	<b>S<sub>2</sub></b>	<b>S<sub>3</sub></b>
7	X <sub>3</sub>	8	0	1.4	1	0.6	-0.2	0
5	X <sub>1</sub>	6	1	0.2	0	-0.2	0.4	0
0	S <sub>3</sub>	4	0	0.4	0	-0.4	-0.2	1
	<i>c<sub>j</sub>-z<sub>j</sub></i>		86	5	10.8	7	3.2	0.6
	<i>c<sub>j</sub>-z<sub>j</sub></i>			0	-6.8	0	-3.2	-0.6

In Table 16.4, all the values in *c<sub>j</sub>-z<sub>j</sub>* row are either negative or zero. Thus, optimality is reached. No further iteration is required. The optimal value of the objective function is 86. The values are  $X_1 = 6$ ,  $X_2 = 0$  and  $X_3 = 8$ .

### 16.3 BIG-M METHOD

*Big-M method is used when for  $\geq$  or/and = type of constraint.*

#### Example 16.3:

$$\text{Maximize } Z = 3X_1 + 2X_2 + 3X_3$$

Subject to

$$2X_1 + X_2 + X_3 \leq 23$$

$$X_1 + 4X_2 + 2X_3 \geq 8$$

$$X_1, X_2, X_3 \geq 0$$

**Step 1:** Convert all the inequality constraints into equality constraints as shown below:

$$\text{Maximize } Z = 3X_1 + 2X_2 + 3X_3 + 0S_1 + 0S_2 - MR_1$$

Subject to

$$2X_1 + X_2 + X_3 + S_1 = 23$$

$$X_1 + 4X_2 + 2X_3 - S_2 + R_1 = 8$$

$$X_1, X_2, X_3, S_1, S_2, R_1 \geq 0$$

**Note:** (a) In the case of minimization, the sign of artificial variable (*R*) in objective function should be positive and in the case of maximization the sign of artificial variable (*R*) should be negative in the objective function.

(b) For the constraint  $\geq$  type, subtract a slack variable *S* and add an artificial variable *R* to convert into equality constraint; and for the constraint having '=' sign, only add an artificial variable *R* and do not subtract a slack variable *S* to convert into equality constraint.

**Step 2:** Make an initial simplex table as discussed in Example 16.2. Table 16.5 shows the initial simplex table.

**Table 16-5:** Initial simplex table

		$C_j$	3	2	3	0	0	-M
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	$R_1$
0	$S_1$	2	2	1	1	1	0	0
-M	$R_1$	8	3	4	2	0	-1	1
	$Z_j$	-8M	-3M	-4M	-2M	0	M	-M
	$C_j - Z_j$		$3 + 3M$	$2 + 4M$	$3 + 2M$	0	-M	0

In the case of Big-M method, the value of  $M$  is considered a very large value. In the column of basic variable, for the constraint  $\leq$  type, slack ( $S$ ) is inserted, but for the constraint  $\geq$  and  $=$  type only artificial variable ( $R$ ) is inserted. The rest of the process is similar to the process of Example 16.5.

### Iteration 1

In Table 16.6, column of  $X_2$  variable is the key column as it has highest  $(2 + 4m) c_j - z_j$  value and row of  $S_1$  is the key row as it has the lowest ratio (2). The key number is 1. Now produce the next table with new elements, considering 1 as key number using the following formula:

(corresponding old element in key row)

$$\text{New element} = \text{old element} - \frac{\times \text{corresponding element of the row in key column}}{\text{Key number}}$$

**Table 16-6:** First iteration table

		$C_j$	3	2	3	0	0	-M	Ratio
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	$R_1$	
0	$S_1$	2	2	1	1	1	0	0	2/1 = 2
-M	$R_1$	8	3	4	2	0	-1	1	8/2 = 4
	$Z_j$	-8M	-3M	-4M	-2M	0	M	-M	
	$C_j - Z_j$		$3 + 3M$	$2 + 4M$	$3 + 2M$	0	-M	0	

### Iteration 2

The solution in Table 16.7 is optimal as all the  $c_j - z_j$  values are negative and zero. The value of objective is 4. Value of decision variables are  $X_1 = 0$ ,  $X_2 = 2$ ,  $X_3 = 0$ .

**Table 16-7:** Second iteration table

		$C_j$	3	2	3	0	0	-M
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$X_3$	$S_1$	$S_2$	$R_1$
2	$X_2$	2	2	1	1	1	0	0
-M	$R_1$	0	-5	0	-2	-4	-1	1
	$Z_j$	4	$4 + 5M$	2	$2 + 2M$	$2 + 4M$	M	-M
	$C_j - Z_j$		$-5M - 1$	0	$1 - 2M$	$-2 - 4M$	-M	0

### 16.3.1 Case of Minimization

**Example 16.4:**

Minimize  $Z = 3X_1 + 2X_2$

Subject to

$$X_1 + 2X_2 \leq 12$$

$$X_1 + X_2 = 55$$

$$X_1 + 2X_2 \geq 10$$

$$X_1, X_2 \geq 0$$

**Step 1:** Convert all the inequality constraints into equality constraints as shown below:

Minimize  $Z = 3X_1 + 2X_2 + 0S_1 + MR_1 + 0S_2 + MR_2$

Subject to

$$X_1 + 2X_2 + S_1 = 12$$

$$X_1 + X_2 + R_1 = 55$$

$$X_1 + 2X_2 - S_2 + R_2 = 10$$

$$X_1, X_2, S_1, S_2, R_1, R_2 \geq 0$$

**Note:** In the case of minimization, the sign of artificial variable ( $R$ ) in the objective function should be positive and in the case of maximization the sign of artificial variable ( $R$ ) should be negative in the objective function.

**Step 2:** Make an initial simplex table as shown in Table 16.8.

**Table 16-8:** Initial simplex table

$CB_i$	Basic variables	$C_j$	3	2	0	$M$	0	$M$
		$X_1$	$X_2$	$S_1$	$R_1$	$S_2$	$R_2$	
0	$S_1$	12	1	2	1	0	0	0
$M$	$R_1$	5	1	1	0	1	0	0
$M$	$R_2$	10	5	2	0	0	-1	1
	$Z_j$	15M	6M	3M	0	M	-M	M
	$C_j - Z_j$	3 - 6M	2 - 3M	0	0	M	M	0

This is a case of minimization; therefore, the column with minimum  $c_j - z_j$  value ( $X_1$  column in Table 16.8) will be key column. For optimality, all values of the  $c_j - z_j$  must be positive or zero.

**Iteration 1****Table 16-9:** First iteration table

		$C_j$	3	2	0	$M$	0	$M$	Ratio
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$S_1$	$R_1$	$S_2$	$R_2$	
0	$S_1$	12	1	2	1	0	0	0	12/1 = 12
$M$	$R_1$	5	1	1	0	1	0	0	5/1 = 5
$M$	$R_2$	10	5	2	0	0	-1	1	10/5 = 2
	$Z_j$	$15M$	$6M$	$3M$	0	$M$	$-M$	$M$	
	$C_j - Z_j$		$3 - 6M$	$2 - 3M$	0	0	$M$	0	

The minimum  $c_j - z_j$  value is  $3 - 6M$ , therefore, the column with  $X_1$  variable is the Key column and row with  $R_2$  variable is the key row as it has least ratio value equal to 2. Find the new value of the rows and column for the next iteration using the following formula:

$$\text{New element} = \text{old element} - \frac{(\text{corresponding old element in key row}) \times (\text{corresponding element of the row in key column})}{\text{Key number}}$$

**Iteration 2****Table 16-10:** Second iteration table

		$C_j$	3	2	0	$M$	0	$M$	Ratio
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$S_1$	$R_1$	$S_2$	$R_2$	
0	$S_1$	10	0	$8/5$	1	0	$1/5$	$-1/5$	$10 \times 5/8 = 6.25$
$M$	$R_1$	3	0	$3/5$	0	1	$1/5$	$-1/5$	$3 \times 5/3 = 5$
3	$X_1$	2	1	$2/5$	0	0	$-1/5$	$1/5$	$2 \times 5/2 = 5$
	$Z_j$	$6 + 3M$	3	$6/5 + 3/5M$	0	$M$	$-3/5 + 1/5M$	$3/5 - 1/5M$	
	$C_j - Z_j$		0	$4/5 - 3/5M$	0	0	$3/5 - 1/5M$	$6/5M - 3/5$	

The column of  $X_2$  variable has negative  $c_j - z_j$  value, therefore next iteration is required and the solution is not optimal. In the case of minimization, all  $c_j - z_j$  must be positive or zero.

**Iteration 3****Table 16-11:** Third iteration table

<b><math>CB_i</math></b>	<b>Basic variables</b>	<b>Solution</b>	<b><math>C_j</math></b>	<b>3</b>	<b>2</b>	<b>0</b>	<b><math>M</math></b>	<b>0</b>	<b><math>M</math></b>
<b><math>CB_i</math></b>	<b>Basic variables</b>	<b>Solution</b>	<b><math>X_1</math></b>	<b><math>X_2</math></b>	<b><math>S_1</math></b>	<b><math>R_1</math></b>	<b><math>S_2</math></b>	<b><math>R_2</math></b>	
0	$S_1$	2	0	0	1	-8/3	-1/3	1/3	
2	$X_2$	5	0	1	0	5/3	1/3	-1/3	
3	$X_1$	0	1	0	0	-2/3	-1/3	1/3	
	$Z_j$	10	3	2	0	4/3	-1/3	1/3	
	$C_j - Z_j$		0	0	0	$M - 4/3$	1/3	$M - 1/3$	

All  $C_j - Z_j$  are positive (which is required in the case of minimization). Hence, the solution is optimal.  $X_1 = 0$ ,  $X_2 = 5$ ,  $Z = 10$ .

## 16.4 TWO-PHASE METHOD

Two-phase method is substitute for Big-M method. The entire solution is divided into two phases: the first phase is concerned with only artificial variable and the second phase without an artificial variable. A case of maximization deals with an example as given below:

**Example 16.5:**

$$\text{Maximize } Z = X_1 + 3X_2$$

Subject to

$$3X_1 + X_2 \geq 6$$

$$X_1 + 3X_2 = 18$$

$$X_1, X_2 \geq 0$$

**Phase 1**

**Step 1:** Convert the inequality constraints into equality constraints.

$$\text{Maximize } Z = -R_1 - R_2$$

Subject to

$$3X_1 + X_2 - S_1 + R_1 \geq 6$$

$$X_1 + 3X_2 + R_2 = 18$$

$$X_1, X_2, S_1, R_1, R_2 \geq 0$$

**Note:** In the case of minimization, the sign of artificial variable ( $R$ ) in the objective function should be positive and in the case of maximization the sign of artificial variable ( $R$ ) should be negative in the objective function.

**Step 2:** Make an initial simplex table as shown in Table 16.12.

**Table 16-12:** Initial simplex table

		$C_j$	0	0	1	0	1
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$R_1$	$S_1$	$R_2$
-1	$R_1$	6	3	1	1	-1	0
-1	$R_2$	18	1	3	0	0	1
	$z_j$	-24	-4	-4	-1	1	-1
	$c_j - z_j$		4	4	2	-1	2

### Iteration 1

In Table 16.13, there are two key columns we can consider any one arbitrarily. But, in column with variable  $X_2$ , the ratio is same for both the rows, i.e. 6. Therefore, column with variable  $X_1$  is considered as a key column. Using the earlier process to find the new elements, produce the new table for the next iteration.

**Table 16-13:** First iteration table

		$C_j$	0	0	1	0	1	Ratio
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$R_1$	$S_1$	$R_2$	
-1	$R_1$	6	3	1	1	-1	0	6/3 = 2
-1	$R_2$	18	1	3	0	0	1	18/1 = 18
	$z_j$	-24	-4	-4	-1	1	-1	
	$c_j - z_j$		4	4	2	-1	2	

### Iteration 2

In Table 16.14, column with  $X_2$  variable is the key column and row with variable  $R_2$  is the key row. The iteration will be continued till all  $c_j - z_j$  elements become negative or zero.

**Table 16-14:** Second iteration table

		$C_j$	0	0	1	0	1	Ratio
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$R_1$	$S_1$	$R_2$	
0	$X_1$	2	1	1/3	1/3	-1/3	0	2 × 3 = 6
-1	$R_2$	16	0	8/3	-1/3	1/3	1	16 × 3/8 = 6
	$z_j$	-16	0	-8/3	1/3	-1/3	-1	
	$c_j - z_j$		0	8/3	2/3	1/3	2	

**Iteration 3****Table 16.15:** Third iteration table

		$C_j$	0	0	1	0	1
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$R_1$	$S_1$	$R_2$
0	$X_1$	0	1	0	3/8	-3/8	-3/8
0	$X_2$	6	0	1	-1/8	1/8	3/8
	$Z_j$	0	0	0	0	0	0
	$c_j - z_j$		0	0	1	0	1

In the column of basic variables, all the artificial variables are replaced with decision variable. Therefore, optimality is reached and second phase of optimization is required.

**Phase 2**

In phase-2, artificial variables are removed and other variables are taken into account.

**Iteration 4****Table 16.16:** Fourth iteration table

		$C_j$	1	3	0
$CB_i$	Basic variables	Solution	$X_1$	$X_2$	$S_1$
1	$X_1$	0	1	0	-3/8
3	$X_2$	6	0	1	1/8
	$Z_j$	18	1	3	0
	$c_j - z_j$		0	0	0

Now all the  $c_j - z_j$  elements are zero. Therefore, optimality is reached. The value of objective function is 18 and value of decision variables  $X_1$  and  $X_2$  are 0 and 6, respectively.

**16.5 DUALITY**

The dual form of the LPP is used to reduce the number of constraints. If the number of constraints are more than the number of variables, then in this situation, simplex techniques consumes more time; therefore, primal form of LPP is converted into a dual form of LPP. The generalized form of LPP is shown below:

The objective function may be written as:

$$\text{Maximize/Minimize } Z = C_1 X_1 + C_2 X_2 + \dots + C_n X_n$$

$$\begin{aligned} \text{Subject to} \quad & a_{11} X_1 + a_{12} X_2 + \dots + a_{1n} X_n (\leq, =, \geq) b_1 \\ & a_{21} X_1 + a_{22} X_2 + \dots + a_{2n} X_n (\leq, =, \geq) b_2 \\ & \dots \\ & a_{m1} X_1 + a_{m2} X_2 + \dots + a_{mn} X_n (\leq, =, \geq) b_m \\ & X_1, X_2, \dots, X_n \geq 0 \text{ (Non-negativity of the variables)} \end{aligned} \quad \left. \right\} \text{constraints}$$

The dual form of LPP can be formulated as

$$\begin{array}{l} \text{Min/Max } Z = \begin{array}{|c|c|c|c|c|} \hline & C_1 & X_1 + & C_2 & X_2 + \dots + & C_n & X_n \\ \hline a_{11} & X_1 + & a_{12} & X_2 + \dots + & a_{2n} & X_n (\leq, =, \geq) & b_1 \\ \hline a_{21} & X_1 + & a_{22} & X_2 + \dots + & a_{2n} & X_n (\leq, =, \geq) & b_2 \\ \hline \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \hline a_{m1} & X_1 + & a_{m2} & X_2 + \dots + & a_{mn} & X_n (\leq, =, \geq) & b_m \\ \hline \end{array} \longrightarrow Y_1 \\ Y_2 \\ \vdots \\ Y_m \\ X_1, X_2, \dots, X_n \geq 0 \text{ (Non-negativity of the variables)} \end{array}$$

The corresponding dual equation may be written as:

$$\text{Min/Max } Z = b_1 Y_1 + b_2 Y_2 + \dots + b_m Y_m$$

Subject to

$$\begin{array}{l} a_{11} Y_1 + a_{21} Y_2 + \dots + a_{m1} Y_m (\geq, \leq) C_1 \\ a_{12} Y_1 + a_{22} Y_2 + \dots + a_{m2} Y_2 (\geq, \leq) C_2 \\ \cdots \\ \cdots \\ a_{1n} Y_1 + a_{2n} Y_2 + \dots + a_{nn} Y_n (\geq, \leq) C_n \\ Y_1, Y_2, \dots, Y_m \geq 0 \end{array}$$

Some guidelines to convert primal into dual form of LLP are shown in Table 16.17.

**Table 16-17:** Guidelines for conversion of primal LPP into dual LPP

Type of problem	Objective function	Constraint type	Nature of variables
Primal	Maximize	$\leq$	Restricted in sign
Dual	Minimize	$\geq$	Restricted in sign
Primal	Minimize	$\geq$	Restricted in sign
Dual	Maximize	$\leq$	Restricted in sign
Primal	Maximize	$=$	Restricted in sign
Dual	Minimize	$\geq$	Unrestricted in sign
Primal	Minimize	$=$	Restricted in sign
Dual	Maximize	$\leq$	Unrestricted in sign
Primal	Maximize	$\leq$	Unrestricted in sign
Dual	Minimize	$=$	Restricted in sign
Primal	Minimize	$\geq$	Unrestricted in sign
Dual	Maximize	$=$	Restricted in sign

**Example 16.6:** Convert the following primal LPP into dual form of LPP.

$$\text{Minimize } X = 25X_1 + 30X_2 + 40X_3$$

$$\begin{aligned} \text{Subject to} \quad & 2X_1 + 4X_2 + 6X_3 \geq 4 \\ & 4X_1 + 9X_2 + 8X_3 \geq 10 \\ & 8X_1 + 8X_2 + 2X_3 \geq 25 \\ & X_1, X_2 \text{ and } X_3 \geq 0 \end{aligned}$$

**Solution:**

$$\text{Maximize } Z = 4Y_1 + 10Y_2 + 25Y_3$$

$$\begin{aligned} \text{Subject to} \quad & 2Y_1 + 4Y_2 + 8Y_3 \leq 25 \\ & 4Y_1 + 9Y_2 + 8Y_3 \leq 30 \\ & 6Y_1 + 8Y_2 + 2Y_3 \leq 40 \\ & Y_1, Y_2, \text{ and } Y_3 \geq 0 \end{aligned}$$

**Example 16.7:** Convert the following primal LPP into dual form of LPP.

$$\text{Maximize } X = 25X_1 + 30X_2 + 40X_3$$

$$\begin{aligned} \text{Subject to} \quad & 2X_1 + 4X_2 + 6X_3 = 4 \\ & 4X_1 + 9X_2 + 8X_3 = 10 \\ & 8X_1 + 8X_2 + 2X_3 = 25 \\ & X_1, X_2 \text{ and } X_3 \geq 0 \end{aligned}$$

**Solution:**

$$\text{Minimize } Z = 4Y_1 + 10Y_2 + 25Y_3$$

$$\begin{aligned} \text{Subject to} \quad & 2Y_1 + 4Y_2 + 8Y_3 \geq 25 \\ & 4Y_1 + 9Y_2 + 8Y_3 \geq 30 \\ & 6Y_1 + 8Y_2 + 2Y_3 \geq 40 \\ & Y_1, Y_2 \text{ and } Y_3 \text{ are unrestricted in sign.} \end{aligned}$$

**Example 16.8:** Convert the following primal LPP into dual form of LPP.

$$\text{Minimize } X = 25X_1 + 30X_2 + 40X_3$$

$$\begin{aligned} \text{Subject to} \quad & 2X_1 + 4X_2 + 6X_3 \geq 4 \\ & 4X_1 + 9X_2 + 8X_3 \leq 10 \\ & 8X_1 + 8X_2 + 2X_3 \geq 25 \\ & X_1, X_2 \text{ and } X_3 \geq 0 \end{aligned}$$

**Solution:**

Convert the  $\leq$  into  $\geq$

$$\text{Minimize } X = 25X_1 + 30X_2 + 40X_3$$

$$\text{Subject to } 2X_1 + 4X_2 + 6X_3 \geq 4$$

$$-4X_1 - 9X_2 - 8X_3 \geq -10$$

$$8X_1 + 8X_2 + 2X_3 \geq 25$$

$$X_1, X_2 \text{ and } X_3 \geq 0$$

Dual form of LPP

$$\text{Maximize } Z = 4Y_1 - 10Y_2 + 25Y_3$$

$$\text{Subject to } 2Y_1 - 4Y_2 + 8Y_3 \leq 25$$

$$4Y_1 - 9Y_2 + 8Y_3 \leq 30$$

$$6Y_1 - 8Y_2 + 2Y_3 \leq 40$$

$Y_1, Y_2$  and  $Y_3$  are unrestricted in sign.

**Example 16.9:** Convert the following primal LPP into dual form of LPP.

$$\text{Maximize } X = 25X_1 + 30X_2 + 40X_3$$

$$\text{Subject to } 2X_1 + 4X_2 + 6X_3 \geq 4$$

$$4X_1 + 9X_2 + 8X_3 \leq 10$$

$$8X_1 + 8X_2 + 2X_3 = 25$$

$$X_1, X_2 \text{ and } X_3 \geq 0$$

**Solution:**

Modified form of primal LPP

$$\text{Maximize } X = 25X_1 + 30X_2 + 40X_3$$

$$\text{Subject to } 2X_1 + 4X_2 + 6X_3 \geq 4$$

$$4X_1 + 9X_2 + 8X_3 \leq 10$$

$$8X_1 + 8X_2 + 2X_3 \leq 25$$

$$8X_1 + 8X_2 + 2X_3 \geq 25$$

$$X_1, X_2 \text{ and } X_3 \geq 0$$

Convert  $\geq$  into  $\leq$

$$\text{Maximize } X = 25X_1 + 30X_2 + 40X_3$$

$$\text{Subject to } -2X_1 - 4X_2 - 6X_3 \leq -4$$

$$4X_1 + 9X_2 + 8X_3 \leq 10$$

$$8X_1 + 8X_2 + 2X_3 \leq 25$$

$$-8X_1 - 8X_2 - 2X_3 \leq -25$$

$$X_1, X_2 \text{ and } X_3 \geq 0$$

Dual form of LPP

$$\text{Minimize } Z = -4Y_1 + 10Y_2 + 25Y'_3 - 25Y''_3$$

$$\text{Subject to } -2Y_1 + 4Y_2 + 8Y'_3 - 8Y''_3 \geq 25$$

$$-4Y_1 + 9Y_2 + 8Y''_3 - 8Y'_3 \geq 30$$

$$-6Y_1 + 8Y_2 + 2Y''_3 - 2Y'_3 \geq 40$$

$$Y_1, Y_2, Y'_3 \text{ and } Y''_3 \geq 0$$

Putting  $Y'_3 - Y''_3 = Y_3$ , we get

$$\text{Minimize } Z = -4Y_1 + 10Y_2 + 25Y_3$$

$$\text{Subject to } -2Y_1 + 4Y_2 + 8Y_3 \geq 25$$

$$-4Y_1 + 9Y_2 + 8Y_3 \geq 30$$

$$-6Y_1 + 8Y_2 + 2Y_3 \geq 40$$

$Y_1, Y_2 \geq 0$  and  $Y_3$  are unrestricted in sign.

## 16.6 GRAPHICAL METHOD

Graphical method is used to solve the linear programming with two decision variables because the solution can be shown in only one plan, i.e.  $X-Y$  plane.

### Example 16.10:

$$\text{Minimize } 2X_1 + X_2$$

Subject to

$$X_1 + X_2 \leq 7$$

$$3X_1 + 2X_2 \geq 18$$

$$2X_1 + X_2 \leq 12$$

$$X_1, X_2 \geq 0$$

### Solution:

The graphs of the constraints are shown in Figure 16.1. The optimum values of decision variables are  $X_1 = 4$  and  $X_2 = 3$ . The minimum value of the objective function is 11.

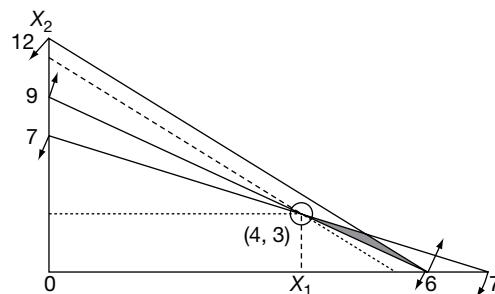


Figure 16-1: Optimal solution

**Example 16.11:**

Maximize  $3X_1 + 2X_2$

Subject to

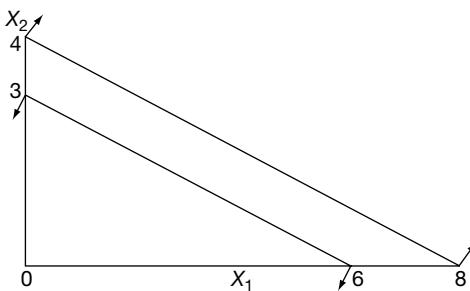
$$X_1 + 2X_2 \geq 8$$

$$3X_1 + 6X_2 \leq 18$$

$$X_1, X_2 \geq 0$$

**Solution:**

The graphs of the constraints are shown in Figure 16.2. The solution is infeasible.



**Figure 16-2:** Infeasible solution

**Example 16.12:**

Maximize  $5X_1 + 3X_2$

Subject to

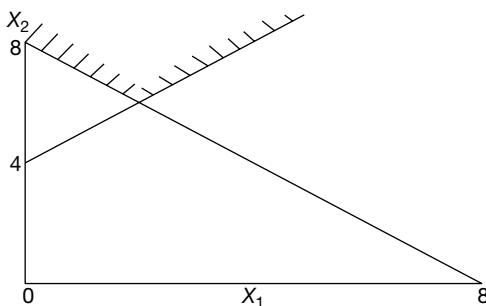
$$X_1 + X_2 \geq 8$$

$$-X_1 + X_2 \geq 4$$

$$X_1, X_2 \geq 0$$

**Solution:**

The graphs of the constraints are shown in Figure 16.3. The solution is unbounded.



**Figure 16-3:** Unbounded solution

## 16.7 INTRODUCTION TO TRANSPORTATION PROBLEM

Transportation is a type of LPP in which the transportation cost is minimized. In transportation problem, there are different sources from which the materials are supplied to some fixed destinations at minimum cost we have already taken an example of transportation in Example 16.1 to formulate a LPP. The LPP form of transportation problem may be expressed as

$$\text{Maximize } Z = \sum_{j=1}^n \sum_{i=1}^m C_{ij} X_{ij}$$

Subject to

$$\sum_{j=1}^n X_{ij} = b_i; \quad \text{Supply constrain}$$

$$\sum_{i=1}^m X_{ij} = d_j; \quad \text{Demand constrain}$$

$$X_{ij} \geq 0$$

where  $i = 1, 2, \dots, m$  are the number of sources, and

$j = 1, 2, \dots, n$  are the number of destinations.

## 16.8 TABULAR METHOD TO FIND THE BASIC FEASIBLE SOLUTION OF TRANSPORTATION MODEL

Three methods are used to find the basic feasible solution of the transportation problem. The solution may be optimal or may not be optimal. After finding the basic feasible solution, it is checked for optimality using MODI (modified distribution method) or U-V method. These three methods to find the basic feasible solution are given below as:

1. North-west corner method
2. Least cost method
3. Vogel's approximation method

**Example 16.13:** Show a tabular form of transportation problem and explain each cell of the table with their significance.

### Solution:

A tabular form of a transportation problem is shown in Table 16.18.

In Table 16.18, the values shown in the different rows and column are the transportation cost per unit of the item. For an example, the transportation cost per unit item from source 'A' to destination 'J' is Rs 7. The last column shows the supply constraints or limits of different sources.

For example, Source 'A' can supply maximum 400 items at a time to different destinations. Similarly, last row shows the demand constraints of the different destinations or buyers. For example, destination 'I' has demand equal to 600 items.

**Table 16-18:** Transportation table

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
B	11	23	17	18	21	600
C	13	9	19	17	15	925
D	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

### 16.8.1 North-West Corner Method

**Example 16.14:** Allocate the items in transportation problem shown in Table 16.18 using the north-west corner method.

#### Solution:

- At first see the problem and ensure that the problem is balanced transportation problem, i.e. total supply is equal to total demand.
- Find the north-west cell of the table. In Table 16.18, cell A-I is the north-west cell.
- Find the least value between demand and supply which can be assigned to the cell A-I. Here, supply (400) is less than the demand (600); therefore, assign 400 to the cell A-I.
- Now the Source A has already been exhausted. But, rest demand of 200 items of destination I is to be fulfilled. Therefore, remove the row having source A. The resulting transportation table is shown in Table 16.19.

Find the next northwest corner of Table 16.19, i.e. cell B-I and assign the minimum amount of supply and demand. The rest of the 200 demand of destination I is to be fulfilled. Thus, assign 200 items to cell B-I from source B and remove the first column, since the demand of destination 'I' has been fulfilled. The result is shown in Table 16.20.

Repeat the same process again and assign 400 items to the cell B-J from source B; since, source B has 400 items in its stock. Now, source B has already been exhausted. Remove the second row. The result is shown in Table 16.21.

**Table 16-19:** Result after deleting the first row

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
	<b>400</b>					
<b>B</b>	11	23	17	18	21	600
<b>C</b>	13	9	19	17	15	925
<b>D</b>	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

**Table 16-20:** Result after the removal of the first column

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
	<b>400</b>					
<b>B</b>	11	23	17	18	21	600
<b>C</b>	13	9	19	17	15	925
<b>D</b>	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

**Table 16-21:** Result after deleting the second row

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
	<b>400</b>					
<b>B</b>	11	23	17	18	21	600
<b>C</b>	13	9	19	17	15	925
<b>D</b>	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

Now, the rest of the demand of 100 items of destination J is to be fulfilled from the source C. Remove the second column after assigning 100 items to the cell C-J. The result is shown in Table 16.22.

**Table 16-22:** Result after the removal of the second column

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
	<b>400</b>					
<b>B</b>	11	23	17	18	21	600
	<b>200</b>	<b>400</b>				
<b>C</b>	13	9	19	17	15	925
		<b>100</b>				
<b>D</b>	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

Now, source C has 825 items in its stock and destination K has demand of 750 items. Therefore, assign 750 items to Cell C-K and remove the third column as the demand of destination K has already been fulfilled. The result is shown in Table 16.23.

**Table 16-23:** Result after the removal of the third column

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
	<b>400</b>					
<b>B</b>	11	23	17	18	21	600
	<b>200</b>	<b>400</b>				
<b>C</b>	13	9	19	17	15	925
		<b>100</b>	<b>750</b>			
<b>D</b>	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

Now, source C has 75 items in its stock. Therefore, assign 75 items to the Cell C-L and remove the third row. The result is shown in Table 16.24.

Now, the rest demand of 275 items of destination L is to be fulfilled by the source D. Assign 275 items to the Cell D-L and remove the fourth column. The result is shown in Table 16.25.

Now, D has 200 items in its stock and destination M has also demand of 200 items. Assign 200 items to the Cell D-M from stock D and remove the fourth row and fifth column. The result is shown in Table 16.26.

**Table 16-24:** Result after the removal of the third row

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	200	400				
C	13	9	19	17	15	925
	100	750	75			
D	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

**Table 16-25:** Result after the removal of the fourth column

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	200	400				
C	13	9	19	17	15	925
	100	750	75			
D	19	27	25	13	23	475
				275		
<b>Demand</b>	600	500	750	350	200	2400

**Table 16-26:** Result after the removal of the fourth row and the fifth column

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	200	400				
C	13	9	19	17	15	925
	100	750	75			
D	19	27	25	13	23	475
				275	200	
<b>Demand</b>	600	500	750	350	200	2400

**Step 2:** Check the number of allocated cell, the number of allocated cell should not be less than  $m + n - 1$ . Here,  $m$  is number of rows and  $n$  is number of columns. In this case,  $m = 4$  and  $n = 5$ . The allocated cell is equal to 8 (i.e.  $4 + 5 - 1 = 8$ ). Thus, this is basic feasible solution.

$$\begin{aligned}\text{Transportation cost} &= 400 \times 15 + 200 \times 11 + 400 \times 23 + 100 \times 9 + 750 \times 19 + 75 \times 17 + 275 \\ &\quad \times 13 + 200 \times 23 \\ &= \text{Rs } 42,000\end{aligned}$$

### 16.8.2 Least Cost Method

**Example 16.15:** Consider the transportation Table 16.18. Allocate the items using the least cost method.

**Solution:**

The same example as shown in Example 16.13 can be solved by the least cost method. In this method, at first, find the cell having least cost and assign it first with minimum of demand and supply. Remove the row or column exhausted completely. Again search the next least cost cell among the remaining cells and repeat the process unless fulfil all the demands of the destinations with sources.

In Table 16.27, Cell A-J has the least cost as Rs 7; therefore, assign the cell first with 400 items as minimum is supply among supply and demand for the Cell A-J. Now, source A is already exhausted. Remove the first row. The result is shown in Table 16.27.

**Table 16-27:** Result after the removal of the first row

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
C	13	9	19	17	15	925
D	19	27	25	13	23	475
<b>Demand</b>	600	500	750	350	200	2400

Find the least cost cell among the remaining unallocated cells. Cell C-J is the least cost cell. Assign the cell C-J with 100 items to fulfil the demand of destination J because this is less than the supply (925). Remove the second column. Remove the first row. The result is shown in Table 16.28.

**Table 16-28:** Result after the removal of the second column

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	600					
C	13	9	19	17	15	925
	925					
D	19	27	25	13	23	475
	475					
<b>Demand</b>	600	500	750	350	200	2400

After the removal of the first row and second column, the least cost cell is B-I. For the cell B-I, demand and supply are equal (600). Therefore, assign 600 items to the cell B-I. Demand of the destination I is fulfilled; and also, source B has already been exhausted. Remove the first column and second row. The result is shown in Table 16.29.

**Table 16-29:** Result after the removal of the second row and the first column

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	600					
C	13	9	19	17	15	925
	925					
D	19	27	25	13	23	475
	475					
<b>Demand</b>	600	500	750	350	200	2400

After the removal of the second row and first column, the least cost cell is D-L. For the cell D-L, demand is less than the supply ( $350 < 475$ ). Therefore, assign 350 items to the cell D-L. Demand of the destination L is fulfilled and source D has 125 items in its stock. Remove the fourth column. The result is shown in Table 16.30.

After the removal of the fourth column, the least cost cell is C-M. For the cell C-M, demand is less than the supply ( $200 < 625$ ). Therefore, assign 200 items to the cell C-M. Demand of the destination M is fulfilled and source C has 625 items in its stock. Remove the fifth column. The result is shown in Table 16.31.

After the removal of the fifth column, the least cost cell is C-K. For the cell C-K, demand is more than the supply ( $750 > 625$ ). Therefore, assign 625 items to the cell C-K. Now, the source C has already been exhausted. Remove the third row. The result is shown in Table 16.32.

**Table 16-30:** Result after the removal of the fourth column

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	600					
C	13	9	19	17	15	925
	100					
D	19	27	25	13	23	475
	350					
<b>Demand</b>	600	500	750	350	200	2400

**Table 16-31:** Result after the removal of the fifth column

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	600					
C	13	9	19	17	15	925
	100					
D	19	27	25	13	23	475
	350					
<b>Demand</b>	600	500	750	350	200	2400

**Table 16-32:** Result after the removal of the third row

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	400					
B	11	23	17	18	21	600
	600					
C	13	9	19	17	15	925
	100					
D	19	27	25	13	23	475
	350					
<b>Demand</b>	600	500	750	350	200	2400

After the removal of the third row, only one cell D-K is left. For the cell D-K, demand is equal to the supply (125). Therefore, assign 125 items to the cell D-K. Now, the source D has already been exhausted. Remove the fourth row and third column. The result is shown in Table 16.33.

**Table 16-33:** Result after the removal of the fourth row and the third column

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
		400				
<b>B</b>	11	23	17	18	21	600
	600					
<b>C</b>	13	9	19	17	15	925
	100	625		200		
<b>D</b>	19	27	25	13	23	475
		125	350			
<b>Demand</b>	600	500	750	350	200	2400

Transportation cost =  $400 \times 7 + 600 \times 11 + 100 \times 9 + 625 \times 19 + 200 \times 15 + 125 \times 25 + 350 \times 13 = \text{Rs } 32,850$ .

Here, the number of allocated cells (7) is less than the  $m + n - 1 = 8$ , thus optimality is to be checked. This is a case of the degeneracy, which is discussed in the later part of this chapter.

### 16.8.3 Vogel's Approximation Method

**Example 16.16:** Consider the Transportation Table 16.18, and allocate the items using Vogel's approximation method.

#### Solution:

The same transportation problem as shown in Table 16.18 can be solved by Vogel's approximation method. In this method, at first, find the penalty for each row and each column. To find the penalty for a row, calculate the difference of the minimum and second minimum of the row. For example, the first row has minimum as Rs 7 and the second minimum as Rs 15 the penalty for the row will be Rs 8. Similarly, find the penalty for the each row and each column. The result is shown in Table 16.34.

Select the row or column with maximum penalty. If there is a tie between two rows or two columns, select arbitrarily anyone with the cell having minimum transportation cost. In Table 16.34, the first row has a maximum penalty equal to Rs 8. Thus, select the first row. Now find the cell having minimum cost in the first row and assign them first with a minimum of the demand and supply. In Table 16.34, the cell A-J has the minimum cost (Rs 7); assign the cell A-J with 400 items because supply is less than the demand ( $400 < 500$ ). Now, the source A has already been exhausted and demand of 100 items of destination J is to be fulfilled. Remove the first row. The result is shown in Table 16.35.

**Table 16-34:** Initial transportation table with row and column penalties

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
B	11	23	17	18	21	600	6
C	13	9	19	17	15	925	4
D	19	27	25	13	23	475	6
<b>Demand</b>	600	500	750	350	200	2400	
<b>Penalty</b>	2	2	2	4	0		

**Table 16-35:** Result after the removal of the first row

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
C	13	9	19	17	15	925	4
D	19	27	25	13	23	475	6
<b>Demand</b>	600	500	750	350	200	2400	
<b>Penalty</b>	2	2	2	4	0		

After the removal of the first row, repeat the same process to find the penalties for remaining rows and columns. The second column has the maximum penalty of Rs 14. In the second column, Cell C-J has the minimum cost (Rs 9). Assign the cell C-J with the rest of the demand of destination J, i.e. 100 items, which is less than the supply from C (925). Now remove the second column because the demand of destination J has been fulfilled. The result is shown in Table 16.36.

After the removal of the second column, find the penalties for remaining rows and columns. Fourth row has the maximum penalty of Rs 10. In the fourth row, Cell D-L has the minimum cost (Rs 13). Assign the cell D-L with the minimum of demand and supply, i.e. 350 items, which is less than the supply from D (475). Now, remove the fourth column because the demand of destination L has been fulfilled and the source D has 125 more items in its stock. The result is shown in Table 16.37.

**Table 16-36:** Result after the removal of the second column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
C	13	9	19	17	15	925	4
		100					
D	19	27	25	13	23	475	6
<b>Demand</b>	600	500	750	350	200	2400	
<b>Penalty</b>	2	14	2	4	6		

**Table 16-37:** Result after the removal of the fourth column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
	600						
C	13	9	19	17	15	925	2
		100					
D	19	27	25	13	23	475	10
				350			
<b>Demand</b>	600	500	750	350	200	2400	
<b>Penalty</b>	2	14	6	4	8		

After the removal of the fourth column, find the penalties for remaining rows and columns. Fifth column has the maximum penalty of Rs 8. In the third row, Cell C-N has the minimum cost (Rs 15). Assign the cell C-M with the minimum of demand and supply, i.e. 200 items, which is less than the Supply from the source C (825). Now, remove the fifth column because the demand of the destination is already fulfilled and source C has 625 items in its stock. The result is shown in Table 16.38.

After the removal of the fifth column, only two cells are unallocated. Now assign the Cell C-K with remaining stock of C, i.e. 625 items and assign the cell D-K with the remaining demand of the destination K, i.e. 125 items. Now the demand of the destination K has been fulfilled and the Sources C and D have already been exhausted. Remove the third column and the third and fourth rows. The result is shown in Table 16.39.

**Table 16-38:** Result after the removal of the fifth column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
	600						
C	13	9	19	17	15	925	4
		100			200		
D	19	27	25	13	23	475	2
			350				
<b>Demand</b>	600	500	750	350	200	2400	
<b>Penalty</b>	2	14	6	4	8		

**Table 16-39:** Result after the removal of the fifth column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
	600						
C	13	9	19	17	15	925	4
		100	625		200		
D	19	27	25	13	23	475	2
			125	350			
<b>Demand</b>	600	500	750	350	200	2400	
<b>Penalty</b>	2	14	6	4	8		

Transportation cost =  $400 \times 7 + 600 \times 11 + 100 \times 9 + 625 \times 19 + 200 \times 15 + 125 \times 25 + 350 \times 13 = \text{Rs } 32,850$ .

Here, the number of allocated cells (7) is less than the  $m + n - 1 = 8$ , thus optimality is to be checked. This is case of the degeneracy, which is discussed in the later part of this chapter.

## 16.9 TEST OF OPTIMALITY USING MODI OR U-V METHOD

The basic feasible solution can be calculated by any method: north-west corner method or least cost method or Vogel's approximation method. After finding the basic feasible solution, optimality of the solution is to be checked whether it is an optimal solution. To check the optimality, use the following steps:

- Step 1:** Assign all the rows and columns with  $U_i$  and  $V_j$ , where  $i = 1, 2, \dots, m$  is the number of rows and  $j = 1, 2, \dots, n$  is the number of columns.
- Step 2:** Assign any one  $U$  or  $V$  equal to zero (say  $U_1 = 0$ ).
- Step 3:** Calculate all the values of  $U_i$  and  $V_j$ . For allocated cell,  $C_{ij} = U_i + V_j$ .
- Step 4:** Find the penalties for all the unallocated cells using the formula  $P_{ij} = U_i + V_j - C_{ij}$ .
- Step 5:** Check the penalties for the entire unallocated cell. It must be negative.
- Step 6:** If any one or more than one unallocated cells have positive penalties. Mark the cell having highest positive penalty.
- Step 7:** Start to form a loop from that marked cell with the allocated cells. The loop can turn only on the allocated cell, thus corner of the loop can be only on the allocated cell.
- Step 8:** Start to mark + and – signs alternatively at each corner or turning point of the loop. The starting point of the loop should be marked with + sign.
- Step 9:** See the allocated valued for the entire cell marked with – sign on the loop corner and find the numerically lowest value. Subtract this value from the entire – signed cell on the corner of the loop and add to the + signed cell on the corner of the loop.
- Step 10:** Go to step 3 to check the optimality. If all the unallocated cell has – ve penalty then the solution is optimal otherwise continue the same process.

**Example 16.17:** Test the optimality and find the optimal solution of the solution shown in Table 16.39.

### Solution:

Considering the basic feasible solution from Example 16.16, this is reproduced in Table 16.40.

**Table 16-40:** Solution from Vogel's approximation method

Sources	Destinations					Supply
	I	J	K	L	M	
A	15	7	21	19	15	400
	<b>400</b>					
B	11	23	17	18	21	600
	<b>600</b>					
C	13	9	19	17	15	925
	<b>100</b>					
D	19	27	25	13	23	475
	<b>125</b>					
<b>Demand</b>	600	500	750	350	200	2400

Total number of rows,  $m = 4$ ; total number of columns,  $n = 5$ . To satisfy the condition of degeneracy, the number of allocated cells should be greater than or equal to  $m + n - 1 = 8$ . But, in Table 16.40, number of allocated cells is only 7. Thus, there is a case of degeneracy.

*Removal of Degeneracy:* The degeneracy can be removed by assigning the remaining unallocated cell with negligible amount, i.e.  $\varepsilon$ . The cell which has the minimum cost and also does not form a closed loop with the allocated cells can be assigned with  $\varepsilon$ . In Table 16.40, the cell C-I can be allocated with  $\varepsilon$  because this does not form a closed loop with allocated cells. After allocating the cell C-I with  $\varepsilon$ , the result is shown in Table 16.41.

**Table 16-41:** Result after allocating  $\varepsilon$  to a non-allocated cell

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
	<b>400</b>					
<b>B</b>	11	23	17	18	21	600
	<b>600</b>					
<b>C</b>	13	9	19	17	15	925
	$\varepsilon$	<b>100</b>		<b>625</b>	<b>200</b>	
<b>D</b>	19	27	25	13	23	475
	<b>125</b>					
<b>Demand</b>	600	500	750	350	200	2400

Now, the number of allocated cells is 8 and there is no case of degeneracy. Now, the optimality is to be checked. Optimality using MODI method can be checked.

In Table 16.41, after allocation of the cell C-I with  $\varepsilon$ , we can observe that all the unallocated cells have negative or zero penalty. Thus, the solution is optimal as shown in Table 16.42.

**Table 16-42:** Optimal table

	Sources	Destinations					Supply
		$V_1 = 11$	$V_2 = 7$	$V_3 = 17$	$V_4 = 5$	$V_5 = 13$	
$U_1 = 0$	<b>A</b>	15	7	21	19	15	400
		-ve	<b>400</b>		-ve	-ve	
$U_2 = 0$	<b>B</b>	11	23	17	18	21	600
		-ve	0	-ve	-ve	-ve	
$U_3 = 2$	<b>C</b>	13	9	19	17	15	925
		$\varepsilon$	<b>100</b>		<b>625</b>	<b>200</b>	
$U_4 = 8$	<b>D</b>	19	27	25	13	23	475
		0	-ve	<b>125</b>	<b>350</b>	-ve	
	<b>Demand</b>	600	500	750	350	200	2400

### 16.9.1 Unbalanced Transportation Problem

If total supply is equal to total demand, this is known as a balanced transportation problem. If the total supply is less than or greater than the total demand, this is known as unbalanced transportation problem. The unbalanced transportation problem can be converted into balanced transportation problem in the following ways:

- (a) If the supply is greater than the demand, add a dummy column (destination or demand).
- (b) If the demand is greater than the supply, add a dummy row (source or supply).

**Example 16.18:** Find the optimal solution of the transportation problem shown in Table 16.43 using Vogel's approximation method.

**Table 16-43:** Unbalanced transportation problem

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
B	11	23	17	18	21	600	6
C	13	9	19	17	15	800	4
D	19	27	25	13	23	400	6
Demand	600	500	750	350	200	2400/ 2200	
Penalty	11	7	17	13	15		

#### Solution:

In this case, supply is less than the demand; therefore, one dummy row (source) is to be added. After addition of a dummy row, the result is presented in Table 16.44.

Now use any one method (say Vogel's approximation method) to find the basic feasible solution. The method of finding the penalty has already been discussed in the previous problem. The first iteration is shown in Table 16.45.

After the removal of the fifth row and the fifth column, new penalty is calculated for each row and column and the new cell A-J is allocated with 400 items. The first row is removed because source A has already been exhausted. After the removal of the first row, the result is shown in Table 16.46.

After the removal of the first row, new penalty is calculated for each row and column and the new cell C-J is allocated with 100 items. Now, the demand of destination D has been fulfilled. The second column row is removed. After the removal of the second column, the result is shown in Table 16.47.

**Table 16-44:** Result after addition of dummy row

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
B	11	23	17	18	21	600	6
C	13	9	19	17	15	800	4
D	19	27	25	13	23	400	6
Dummy	0	0	0	0	0	200	0
Demand	600	500	750	350	200		
Penalty	11	7	17	13	15		

**Table 16-45:** Result after the removal of the fifth row and fifth column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
B	11	23	17	18	21	600	6
C	13	9	19	17	15	800	4
D	19	27	25	13	23	400	6
Dummy	0	0	0	0	0	200	0
Demand	600	500	750	350	200		
Penalty	11	7	17	13	15		

**Table 16-46:** Result after the removal of the first row

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
	400						
B	11	23	17	18	21	600	6
C	13	9	19	17	15	800	4
D	19	27	25	13	23	400	6
Dummy	0	0	0	0	0	200	0
Demand	600	500	750	350	200	2400	
Penalty	2	2	2	4	15		

**Table 16-47:** Result after the removal of the second column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
	400						
B	11	23	17	18	21	600	6
C	13	9	19	17	15	800	4
	100						
D	19	27	25	13	23	400	6
Dummy	0	0	0	0	0	200	0
Demand	600	500	750	350	200	2400	
Penalty	2	14	2	4	15		

After the removal of the second column, the new penalty is calculated for each row and column and the new cell D-L is allocated with 350 items. Now, the demand of destination L has been fulfilled. The fourth column row is removed. After the removal of the fourth column, the result is shown in Table 16.48.

**Table 16-48:** Result after the removal of the fourth column

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
	600						
C	13	9	19	17	15	800	4
		100					
D	19	27	25	13	23	400	12
			350				
Dummy	0	0	0	0	0	200	0
Demand	600	500	750	350	200	2400	
Penalty	2	14	6	4	15		

After the removal of the fourth column, only two cells are unallocated. Now, assign the Cell C-K with the remaining stock of C, i.e. 700 items and assign the cell D-K with the remaining demand of the destination K, i.e. 50 items. Now, the demand of the destination K has been fulfilled and the Sources C and D have already been exhausted. Remove the third column and the third and fourth rows. The result is shown in Table 16.49.

**Table 16-49:** Result after the removal of third column and third and fourth rows

Sources	Destinations					Supply	Penalty
	I	J	K	L	M		
A	15	7	21	19	15	400	8
		400					
B	11	23	17	18	21	600	6
	600						
C	13	9	19	17	15	800	4
		100	700				
D	19	27	25	13	23	400	12
			50	350			
Dummy	0	0	0	0	0	200	0
Demand	600	500	750	350	200	2400	
Penalty	2	14	6	4	15		

$$\text{Transportation cost} = 400 \times 7 + 600 \times 11 + 100 \times 9 + 700 \times 19 + 50 \times 25 + 350 \times 13 = \text{Rs } 29,400$$

### Test of Optimality

This is the case of degeneracy because the number of occupied cells (7) is less than  $m + n - 1$  (i.e. 9). To remove the degeneracy assign, the cell C-I and dummy-K with  $\epsilon$ , since, they do not form any loop with the occupied cell. After assigning the cell with  $\epsilon$ , the result is shown in Table 16.50.

**Table 16-50:** Result after assigning  $\epsilon$

Sources	Destinations					Supply
	I	J	K	L	M	
<b>A</b>	15	7	21	19	15	400
<b>B</b>	11	23	17	18	21	600
<b>C</b>	13	9	19	17	15	800
<b>D</b>	19	27	25	13	23	400
<b>Dummy</b>	0	0	0	0	0	200
<b>Demand</b>	600	500	750	350	200	2400

Find the values of  $U_i$  and  $V_j$  for all the rows and columns, respectively. To calculate  $U_i$  and  $V_j$ , use the formula  $U_i + V_j = C_{ij}$  for all the allocated cell considering  $U_i = 0$ . Now, find the penalties for all the unallocated cells using the formula  $P_{ij} = U_i + V_j - C_{ij}$ . The cell C-M has positive penalty (2), therefore, a loop should be started from this point with the occupied cells. The loop is C-M, C-K, Dummy- $\epsilon$ , Dummy-M. Assign + sign to cell C-M, - sign to C-K, + sign to Dummy- $\epsilon$ , - sign to Dummy-M. The result after forming the loop is shown in Table 16.51.

**Table 16-51:** Result after forming a loop

Sources	Destinations					Supply
	$V_1 = 11$	$V_2 = 7$	$V_3 = 17$	$V_4 = 5$	$V_5 = 17$	
$U_1 = 0$	15	7	21	19	15	400
	-ve		-ve	-ve		
$U_2 = 0$	11	23	17	18	21	600
		-ve	0	-ve	-ve	
$U_3 = 2$	13	9	19	17	+ 15	800
	$\epsilon$				2	
$U_4 = 8$	19	27	25	13	23	400
	0	-ve	50	350	0	
$U_5 = -17$	0	0	0	0	0	200
	-ve	-ve	$\epsilon$	-ve	200	
<b>Demand</b>	600	500	750	350	200	2400
<b>Penalty</b>	2	14	6	4	15	

Find the minimum allocation among the -sign cells on the loop corner and add to the +sign cells on the loop corner and subtract from the -sign cells on the loop corners. Now, recalculate the values of  $U$  and  $V$  for each row and column, respectively; and find the penalties for all the unallocated cells. It is observed that all the penalties are now -ve or zero. Thus, optimality is reached. The optimum allocated is shown in Table 16.52.

**Table 16-52:** Optimum allocation table

		Destinations					Supply
		$V_1 = 11$	$V_2 = 7$	$V_3 = 17$	$V_4 = 5$	$V_5 = 13$	
Sources	I	J	K	L	M		
	A -ve	15 <b>400</b>	7 -ve	21 -ve	19 -ve	15 -ve	400
$U_2 = 0$	B 600	11 -ve	23 0	17 -ve	18 -ve	21 -ve	600
	C 0	13 <b>100</b>	9 500	19 500	17 -	15 <b>200</b>	800
$U_4 = 8$	D 0	19 -ve	27 50	25 350	13 -	23 -ve	400
	Dummy -ve	0 -ve	0 <b>200</b>	0 -ve	0 -ve	0 -ve	200
<b>Demand</b>		600	500	750	350	200	2400
<b>Penalty</b>		2	14	6	4	15	

Transportation cost =  $400 \times 7 + 600 \times 11 + 100 \times 9 + 500 \times 19 + 50 \times 25 + 350 \times 13 + 200 \times 15 = \text{Rs } 28,600$

**Example 16.19:** Consider the problem shown in Table 16.53. Find the basic feasible solution using the north-west corner method and test for the optimality.

**Table 16-53:** Transportation table (Unbalanced transportation:  
supply > demand)

		Destinations					Supply
		I	J	K	L	M	
Sources	A	15	7	21	19	15	400
	B	11	23	17	18	21	
C	13	9	19	17	15		925
	D	19	27	25	13	23	
<b>Demand</b>		450	500	350	250	500	2050/2400

**Solution:**

Using the north-west corner method as discussed earlier, a basic feasible solution is found, which is shown in Table 16.54.

**Table 16-54:** Basic feasible solution using the north-west corner method

Sources	Destinations						Supply
	I	J	K	L	M	Dummy	
A 400	15	7	21	19	15	0	400
B 50	11	23	17	18	21	0	600
		500	50				
C 300	13	9	19	17	15	0	925
D 250	19	27	25	13	23	0	475
					125	350	
<b>Demand</b>	450	500	350	250	500	350	2400

$$\begin{aligned}
 \text{Transportation cost} &= 400 \times 15 + 50 \times 11 + 500 \times 23 + 50 \times 17 + 300 \times 19 + 250 \times 17 + 375 \times 15 \\
 &\quad + 125 \times 23 \\
 &= \text{Rs } 37,350
 \end{aligned}$$

**Optimality Test using MODI Method**

Find the values of  $U$  and  $V$  for each row and column, respectively, considering  $U_1 = 0$ . Now, find the penalty for all the unallocated cells. We observe that cell A-J has a maximum positive penalty (20). Start a loop from the cell A-J to A-I, B-I, B-J. Assign + sign to starting loop cell, i.e. A-J, – sign to A-I, + sign to B-I, and – sign to B-J. The result is shown in Table 16.55.

**Table 16-55:** Result after forming a first loop

Sources	Destinations						Supply	
	$V_1 = 15$	$V_2 = 27$	$V_3 = 21$	$V_4 = 19$	$V_5 = 17$	$V_6 = -6$		
$U_1 = 0$ A 400	–	15	+	7	21	19	15	400
				(20)	0	0	2	
$U_2 = -4$ B 50	+	11		23	17	18	21	600
			500	–	–ve	–ve	–ve	
$U_3 = -2$ C 0		13		9	19	17	15	375
		0		16	300	250	375	
$U_4 = 6$ D 2		19		27	25	13	23	350
		2		6	1	12	125	
<b>Demand</b>	450	500	350	250	500	350	2400	

Subtract the minimum value of allocation among the –ve marked cell and subtract the value from the allocation of –ve marked cell on the loop corner and add the same to the +ve marked cell on the loop corner. Again, find the new value of U and V for all the rows and columns, respectively. Find the penalty for each unallocated cell. It is observed that Cell C-J has maximum positive penalty (16). Now, form a loop C-J, C-K, B-K, B-J; and repeat the process of adding and subtracting the lowest-marked corner cell allocation. The new table is shown in Table 16.56.

**Table 16-56:** Second loop formation

		Destinations						Supply
		$V_1 = -5$	$V_2 = 7$	$V_3 = 1$	$V_4 = -1$	$V_5 = -3$	$V_6 = -26$	
Sources	I	J	K	L	M	Dummy		
$U_1 = 0$	A	15	7	21	19	15	0	400
		–ve	400	–ve	–ve	–ve	–ve	
$U_2 = 16$	B	11	23	17	18	21	0	600
		450	100	50	–ve	–ve	–ve	
$U_3 = 18$	C	13	9	19	17	15	0	925
		0	(16)	300	250	375	–ve	
$U_4 = 26$	D	19	27	25	13	23	0	475
		2	6	2	12	125	350	
<b>Demand</b>		450	500	350	250	500	350	2400

Again, find the new values of  $U$  and  $V$  for all the rows and columns, respectively. Find the penalty for each unallocated cell. It is observed that Cell D-L has the maximum positive penalty (12). Now form a loop D-L, D-M, C-M, C-L; and repeat the process of adding and subtracting the lowest-marked corner cell allocation. The new table is shown in Table 16.57.

**Table 16-57:** Third loop formation

		Destinations						Supply
		$V_1 = 11$	$V_2 = 7$	$V_3 = 17$	$V_4 = 15$	$V_5 = 13$	$V_6 = -10$	
Sources	I	J	K	L	M	Dummy		
$U_1 = 0$	A	15	7	21	19	15	0	400
		–ve	400	–ve	–ve	–ve	–ve	
$U_2 = 0$	B	11	23	17	18	21	0	600
		450	–ve	150	–ve	–ve	–ve	
$U_3 = 2$	C	13	9	19	17	15	0	925
		0	100	200	250	375	–ve	
$U_4 = 10$	D	19	27	25	13	23	0	475
		2	–ve	2	(12)	125	350	
<b>Demand</b>		450	500	350	250	500	350	2400

Again, find the new values of  $U$  and  $V$  for all the rows and columns, respectively. Find the penalty for each unallocated cell. It is observed that Cell C-Dummy has the maximum positive penalty (4). Now form a loop D-Dummy, C-Dummy, C-L, D-L, and repeat the process of adding and subtracting the lowest-marked corner cell allocation. The new table is shown in Table 16.58.

**Table 16-58:** Fourth loop formation

	Sources	Destinations						Supply
		$V_1 = 11$	$V_2 = 7$	$V_3 = 17$	$V_4 = 15$	$V_5 = 13$	$V_6 = 2$	
$U_1 = 0$	A		15	7	21	19	15	0
		–ve		400	–ve	–ve	–ve	2
$U_2 = 0$	B		11	23	17	18	21	0
		450	–ve	150	–ve	–ve	–ve	2
$U_3 = 2$	C		13	9	19	17	15	0
		–ve		100	200	125	500	(4) +
$U_4 = -2$	D		19	27	25	13	23	0
		–ve	–ve	–ve	125	–ve	360	–
<b>Demand</b>		450	500	350	250	500	350	2400

Again, find the new values of  $U$  and  $V$  for all the rows and columns, respectively. Find the penalty for each unallocated cell. It is observed that all the unallocated cells have –ve penalty. Thus the solution is optimal. The optimal table is shown in Table 16.59.

**Table 16-59:** Optimal table

	Sources	Destinations						Supply
		$V_1 = 11$	$V_2 = 7$	$V_3 = 17$	$V_4 = 11$	$V_5 = 13$	$V_6 = -2$	
$U_1 = 0$	A		15	7	21	19	15	0
		–ve		400	–ve	–ve	–ve	400
$U_2 = 0$	B		11	23	17	18	21	0
		450	–ve	150	–ve	–ve	–ve	600
$U_3 = 2$	C		13	9	19	17	15	0
		–ve		100	200	–ve	500	125
$U_4 = 2$	D		19	27	25	13	23	0
		–ve	–ve	–ve	250	–ve	225	475
<b>Demand</b>		450	500	350	250	500	350	2400

$$\begin{aligned} \text{Transportation cost} &= 400 \times 7 + 450 \times 11 + 150 \times 17 + 100 \times 9 + 200 \times 19 + 500 \times 15 + 250 \times 13 \\ &= \text{Rs } 25,750 \end{aligned}$$



## SUMMARY

In this chapter, we learnt about the formation of LPP and its solution using simplex method and graphical method. In addition to this, we learnt about the transportation problems and its conversion into LPP. Also, we discussed about the methods to find the basic feasible solutions of the transportation problem and its optimal solution using modified distribution method.

## MULTIPLE-CHOICE QUESTIONS

1. The transportation model is used to evaluate location alternatives minimizes total
  - (a) sources
  - (b) destinations
  - (c) capacity
  - (d) shipping costs
2. Linear programming as an optimization tool was developed by
  - (a) F. W. Taylor
  - (b) Dantiz
  - (c) Vogel
  - (d) Taguchi
3. Which of the following is NOT a property of linear programming?
  - (a) the relationship between variables and constraints is linear
  - (b) the model has an objective function
  - (c) the model has structural constraints
  - (d) the model may have negativity constraint
4. Graphical method to solve the linear programming problem can be applied for
  - (a) two-variable problems
  - (b) three-variable problems
  - (c) four-variable problems
  - (d) for any variable problems
5. The basic assumption of an LPP is
  - (a) all the situations, such as resources, course of action, etc. are considered as deterministic in nature
  - (b) the relationship between variables and resources is linear in nature
  - (c) LPP is a static model, i.e. it is a single-stage decision problem
  - (d) all the above
6. Big-M method is used for the linear programming problem having the constraints of the nature
  - (a) less and equal to the constant
  - (b) greater and equal to the constant
  - (c) mix constraints of less and equal and greater and equal to the constant
  - (d) both (b) and (c)
7. Which of the following statement is correct?
  - (a) slack variable is used to convert the constraints less and equal to constant into equality constraints
  - (b) artificial variable is used to convert the constraints greater and equal to constant into equality constraints
  - (c) artificial variable is used in Big-M method
  - (d) all the above
8. Dual form of the linear programming problem is mostly used
  - (a) to reduce the number of constraints, if the number of constraints is less than the number of variables
  - (b) to reduce the number of constraints, if the number of constraints is more than the number of variables
  - (c) to reduce the number of constraints, If the number of constraints equals the number of variables
  - (d) none of these

9. Which of the following is the limitation of transportation problem?

  - (a) It can be solved only by linear programming
  - (b) Only transportation cost can be optimized
  - (c) Unbalanced transportation problem cannot be solved
  - (d) all the above

10. Which of the following is a method to find the basic feasible solution?

  - (a) Least cost method
  - (b) Vogel's approximation method
  - (c) North-West corner method
  - (d) All the above

11. If supply is more than the demand, then

  - (a) a dummy supply or row is to be added
  - (b) a dummy demand or column is to be added
  - (c) one row is randomly is to be reduced
  - (d) one column is randomly is to be reduced

12. If there are  $m$  rows and  $n$  columns, then the number of allocated cells should be equal to

  - (a)  $m - n + 1$
  - (b)  $m - n - 1$
  - (c)  $m + n - 1$
  - (d)  $m + n + 1$

13. In a modified distribution method, if the solution is not optimal, the loop should start from

  - (a) highest positive penalty cell and form a loop with allocated cells
  - (b) any unallocated cell and form a loop with allocated cells
  - (c) most negative penalty cell and form a loop with allocated cells
  - (d) any one of the above

14. If the transportation problem is used for maximization, then

  - (a) all the cost factors should be subtracted from the highest cost factor and then follow the same rule as in the case of minimization
  - (b) the minimum cost factor should be subtracted from all the cost factors and then follow the same rule as in the case of maximization
  - (c) any of (a) and (b)
  - (d) none of them

15. In Vogel's approximation method, the opportunity cost associated with a row is determined by

  - (a) the difference between the smallest cost and the next smallest cost in the row
  - (b) the difference between the smallest unused cost and the next smallest unused cost in the row
  - (c) the difference between the smallest cost and next smallest unused cost in the row
  - (d) none of the above

## Answers

1. (d)      2. (b)      3. (d)      4. (a)      5. (d)      6. (d)      7. (d)      8. (b)      9. (b)  
10. (d)     11. (b)     12. (c)     13. (a)     14. (a)     15. (a)

## REVIEW QUESTIONS

1. Write the basic assumptions of linear programming problems.
2. Write short notes on basic variable, slack variable and artificial variable.
3. What do you mean by transportation problem? Explain the methods used to find the basic feasible solution.
4. Discuss the way to remove the degeneracy from the transportation problem.

## EXERCISES

1. A company XYZ produces two products air conditioners and refrigerators. The two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 air conditioners and 35 refrigerators because of the limited facilities. The company employs a total of 60 workers in the production of these two products. An air conditioner requires 2 man-week of labour while a refrigerator requires 1 man-week of labour. The air conditioner contributes a profit of Rs 600 and a refrigerator a profit of Rs 400. What should be the weekly production of air conditioners and refrigerators so that the total profit can be maximized.

2. Using the graphical technique, maximize  $Z = 2x_1 + x_2$

Subject to     $x_1 + x_2 \geq 1$   
                   $x_1 + 2x_2 \leq 10$   
                   $x_2 \leq 4$   
                   $x_1, x_2 \geq 0$

3. Using the graphical technique, maximize  $Z = 2x_1 + 3x_2$

Subject to     $x_1 + x_2 \leq 4$   
                   $6x_1 + 2x_2 \geq 8$   
                   $x_1 + 5x_2 \geq 4$   
                   $3 \geq x_1 \geq 0$   
                   $3 \geq x_2 \geq 0$

4. Maximize  $Z = 3x_1 + 2x_2 + 5x_3$

Subject to     $x_1 + 2x_2 + 3x_3 \leq 8$   
                   $3x_1 + 2x_2 + 6x_3 \leq 12$   
                   $2x_1 + 3x_2 + 4x_3 \leq 12$   
                   $x_1, x_2, x_3 \geq 0$

5. Using Big-M method or, 2-phase, maximize  $Z = 3x_1 + 2x_2 + 3x_3$

Subject to     $2x_1 + x_2 + x_3 \leq 2$   
                   $3x_1 + 4x_2 + 2x_3 \geq 8$   
                   $x_1, x_2, x_3 \geq 0$

6. Using Big-M method or, 2-phase, minimize  $Z = 12x_1 + 20x_2$

Subject to     $6x_1 + 8x_2 \geq 100$   
                   $7x_1 + 12x_2 \geq 120$   
                   $x_1, x_2 \geq 0$

7. Find the basic feasible solution of the transportation problem shown in Table 16.60, using
- North-West corner method,
  - Least cost method
  - Vogel's approximation method
  - Check the optimality using modified distribution method

**Table 16-60:** Transportation problem

Sources	Destinations					Supply
	I	J	K	L	M	
A	12	4	18	16	12	300
B	8	20	14	15	18	500
C	10	6	16	14	12	825
D	16	24	22	10	20	375
Demand	500	400	650	250	200	2000

8. Find the optimal solution of the transportation problem shown in Table 16.61.

Sources	Destinations					Supply
	I	J	K	L	M	
A	12	4	18	16	12	300
B	8	20	14	15	18	500
C	10	6	16	14	12	800
D	16	24	22	10	20	300
Demand	500	400	650	250	200	2000/1900



## **REFERENCES AND FURTHER READINGS**

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## Assignment and Sequencing Models

### 17.1 INTRODUCTION TO ASSIGNMENT PROBLEM

Assignment problem is very similar to the transportation problem. The only difference is that in the case of transportation problem one source can supply to a number of destinations, but in assignment problem one job is allocated to only one machine or one machine is used for only one job. The main objective of the assignment problem is to minimize the time or cost of manufacturing of the products by allocating one job to one machine or one machine to one job or one destination to one origin or one origin to one destination only. It is a minimization model. If we want to maximize the objective function, then all the elements of the matrix are to be subtracted from the highest element in the matrix or to multiply the entire matrix by  $-1$  and continue with the same procedure. For solving the assignment problem, we use Hungarian method.

#### 17.1.1 Hungarian Method

An assignment problem can be solved by the Hungarian method using the following steps:

Step 1: Prepare a cost matrix.

Step 2: Ensure that the matrix is square by the addition of dummy rows/columns if necessary.

Step 3: Find the minimum value in each row and subtract it from all the entries in the corresponding row.

Step 4: Find the minimum value in each column after step 3 and subtract it from all the entries in the corresponding column.

Step 5: Draw a minimum number of lines to cover all the zeros of the matrix.

- Scan all the rows from top to bottom. If there is exactly only one zero in a row, mark a square around that zero entry and draw a vertical line passing through that zero, otherwise skip that row.
- Scan all the columns from the first to last column. If there is exactly only one zero in a column, mark a square around that zero entry and draw a vertical line passing through that zero, otherwise skip that row.

Step 6: Check if the number of squares marked is equal to the number of rows of the matrix, treat the solution as marked by the squares as the optimal solution. If the number of squares marked is not equal to the number of rows of the matrix, go to Step 7.

Step 7: Identify the minimum value of the undeleted cell values. Make a new matrix following the following sub-steps:

- Copy as it is the entries on the line to the new matrix, but, do not copy the values on the intersection of the lines.
- Add the minimum value identified in the undeleted matrix to the values at the intersection in the previous matrix and make them the values for the intersection points in the new matrix.
- Subtract the minimum undeleted cell values from all the undeleted cell values and then copy them to the corresponding positions of the next matrix.

Step 8: Repeat the process go to Step 5 and find the optimal solution in several iterations.

**Note:** Sometimes, it is observed that the number of iterations are endless when the number of zeros in the applicable rows as well as column is more than one. Under such a condition, one should mark squares on diagonally opposite cells having zeros. That means multiple optimal solutions exist.

**Example 17.1:** Allocate the jobs on the machines shown in the cost matrix in Table 17.1.

Table 17-1: Cost matrix

Job	Machines				
	1	2	3	4	5
1	8	10	13	10	6
2	5	14	12	12	9
3	11	12	5	7	7
4	10	8	9	11	8
5	6	11	13	9	13

**Solutions:** The matrix is balanced; therefore, there is no need to add a dummy row or dummy column. Find the row minimum as shown in Table 17.2. Subtract the row minimum from each element of the corresponding row. The resultant table is shown in Table 17.3.

Table 17-2: Cost matrix with row minimum

Job	Machines					Row minimum
	1	2	3	4	5	
1	8	10	13	10	6	6
2	5	14	12	12	9	5
3	11	12	5	7	7	5
4	10	8	9	11	8	8
5	6	11	13	9	13	6

Now, find the column minimum as shown in Table 17.3. Subtract the column minimum from each element of the corresponding column as shown in Table 17.4.

**Table 17-3:** Cost matrix with column minimum

Job	Machines				
	1	2	3	4	5
1	2	4	7	4	0
2	0	9	7	7	4
3	6	7	0	2	2
4	2	0	1	3	0
5	0	5	7	3	7
<b>Column minimum</b>	0	0	0	2	0

Mark single zero in row with square and mark vertical line through that zero. Scan all the rows from top to bottom. If there is exactly only one zero in a row, mark a square around that zero entry and draw a vertical line passing through that zero. Scan all the columns from the first to the last column. If there is exactly only one zero in a column, mark a square around that zero entry and draw a vertical line passing through that zero. The first iteration is shown in Table 17.4.

**Table 17-4:** Table with minimum number of lines (Iteration 1)

Job	Machines				
	1	2	3	4	5
1	2	4	7	2	0
2	0	9	7	5	4
3	6	7	0	0	2
4	2	0	1	1	0
5	0	5	7	1	7

Copy as it is the entries on the line Table 17.4 to the new Table 17.5, but, do not copy the values on the intersection of the lines. Add the minimum value identified in the undeleted matrix to the values at the intersection in the previous matrix and make them the values for the intersection points in the new matrix. Subtract the minimum undeleted cell value from all the undeleted cell values (1) and then copy them to the corresponding positions of the next matrix. Second iteration is shown in Table 17.5.

**Table 17-5:** Table with minimum number of lines (Iteration 2)

Job	Machines				
	1	2	3	4	5
1	2	4	6	1	0
2	0	9	6	4	4
3	7	8	0	0	3
4	2	0	0	0	0
5	0	5	6	0	7

Now, all the rows and columns have square marked zero. The optimal solution is reached and shown in Table 17.6.

**Table 17-6:** Optimal solution

Job	Machine	Time
1	5	5
2	1	5
3	3	5
4	2	8
5	4	9

Total time = 33 hours

**Example 17.2:** Allocate the jobs on the machines shown in the cost matrix 17.7.

**Table 17-7:** Cost matrix

Job	Machines			
	1	2	3	4
1	25	34	26	33
2	38	32	27	30
3	29	36	28	36
4	34	31	28	27
5	27	44	30	35

### Solution:

In this matrix, there are 5 jobs, but, 4 machines; thus, this is a case of unbalanced assignment problem. Add a dummy machine and find the row minimum. The new cost matrix is shown in Table 17.8.

**Table 17-8:** Cost matrix with row minimum

Job	Machines				Dummy	Row minimum
	1	2	3	4		
1	25	34	26	33	0	0
2	38	32	27	30	0	0
3	29	36	28	36	0	0
4	34	31	28	27	0	0
5	27	44	30	35	0	0

Now, find the column minimum as shown in Table 17.9.

**Table 17-9:** Cost matrix with column minimum

Job	Machines				Dummy
	1	2	3	4	
1	25	34	26	33	0
2	38	32	27	30	0
3	29	36	28	36	0
4	34	31	28	27	0
5	27	44	30	35	0
<b>Column minimum</b>	25	31	26	27	0

Mark single zero in row with square and mark vertical line through that zero. Scan all the rows from top to bottom. If there is exactly only one zero in a row, mark a square around that zero entry and draw a vertical line passing through that zero. Scan all the columns from the first to the last column. If there is exactly only one zero in a column, mark a square around that zero entry and draw a vertical line passing through that zero. The first iteration is shown in Table 17.10.

**Table 17-10:** Table with minimum number of lines (first iteration)

Job	Machines				Dummy
	1	2	3	4	
1	-[0]-	3	0	6	0
2	13	1	1	3	[0]
3	4	5	2	9	0
4	-9	0	-2	-[0]	0
5	2	13	4	8	0

Copy as it is the entries on the line Table 17.10 to the new Table 17.11, but, do not copy the values on the intersection of the lines. Add the minimum value identified in the undeleted matrix to the values at the intersection in the previous matrix and make them the values for the intersection points in the new matrix. Subtract the minimum undeleted cell value from all the undeleted cell values (1) and then copy them to the corresponding positions of the next matrix. The second iteration is shown in Table 17.11.

**Table 17-11:** Table with minimum number of lines (second iteration)

Job	Machines				Dummy
	1	2	3	4	
1	-[0]-	3	0	6	1
2	-12	-[0]-	0	-2	[0]
3	3	4	1	8	0
4	-9	0	-2	-[0]	1
5	1	12	3	7	0

Copy as it is the entries on the line in Table 17.11 to the new Table 17.12, but, do not copy the values on the intersection of the lines. Add the minimum value identified in the undeleted matrix to the values at the intersection in the previous matrix and make them the values for the intersection points in the new matrix. Subtract the minimum undeleted cell value from all the undeleted cell values (1) and then copy them to the corresponding positions of the next matrix. The third iteration is shown in Table 17.12.

**Table 17-12:** Table with minimum number of lines (Third iteration)

Job	Machines				Dummy
	1	2	3	4	
1	-0	-3	0	-6	2
2	-12	0	0	-2	1
3	2	3	0	7	0
4	-9	0	2	0	2
5	0	11	2	6	0

Now, all the rows and columns has one zero marked with square; an optimal solution is reached. Table 17.13 shows the optimal solution.

**Table 17-13:** Optimal solution

Job	Machine	Time
1	1	26
2	2	32
3	3	0
4	4	27
5	5	27

Total time = 112 hours

## 17.2 SEQUENCING PROBLEM

Sequencing is the decision-making regarding the order of processing of the jobs on different machines. The main objective of the sequencing problem is to minimize the idle time of the machines. Since all the jobs do not require same processing time, therefore, the machines have to wait for the semi-finished product for next processing. In this situation, it is important to rearrange the jobs on different machines to minimize the idle time and hence the total processing times. **Johnson's rule** is frequently used to solve the sequencing problem.

## 17.2.1 Johnson's Rule

### *n-Job, 2-Machines*

**Table 17-14:** Sequencing problem

Job	M/C 1	M/C 2
1	$t_{11}$	$t_{12}$
2	$t_{21}$	$t_{22}$
3	$t_{31}$	$t_{32}$
4	$t_{41}$	$t_{42}$

1. List the job with the time for processing on M/C 1 and M/C 2 as shown in Table 17.14.
2. Examine the time for processing on M/C 1 and M/C 2 and find the minimum processing time in each column, i.e. find out  $\min(t_{i_1}, t_{i_2})$  for all the job,  $i$ , where  $i = 1, 2, \dots, n$  is the number of jobs and  $j = 1, 2$  is the number of machines.
3. Make blocks equal to the number of jobs for sequencing as shown in Table 17.15.

**Table 17-15:** Blocks of jobs for sequencing

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4. If the shortest time is at the M/C 1, place the job first in the block and if the shortest time is at the M/C 2, place the job last in the block shown in Table 17.15.
5. If there is tie in selecting the minimum of all the processing times, then there may be three situations.
  - (a) Minimum time among all the processing times is same on M/C 1 for the job  $k$  and on the M/C 2 for the job  $r$ , place the  $k^{\text{th}}$  job first and  $r^{\text{th}}$  job last.
  - (b) If the tie for the minimum occurs among processing times  $t_{i_1}$  on M/C 1 only, then select the smallest job first and then the largest job just after the first job.
  - (c) If the tie for the minimum occurs among processing times  $t_{i_2}$  on M/C 2 only, then select the smallest job last and then the largest job just before the last job.
  - (d) If the tie for the minimum occurs among processing times and time for both the jobs on M/C 1 is same and M/C 2 is same, then place any one arbitrarily first and second.

**Example 17.3:** Arrange the jobs on M/C 1 and M/C 2 using Johnson's rule (Table 17.16).

**Table 17-16:** Sequencing problem

Job	A	B	C	D	E	F	G	H	I
<b>M/C 1</b>	3	6	5	10	7	9	8	6	5
<b>M/C 2</b>	7	9	8	5	4	10	4	9	12

**Solution:**

The minimum processing time is 3 hours of job A on M/C 1; therefore, job A should be placed at the first place on the left side of the block as shown in Table 17.17. Now, delete the job A from Table 17.16 and produce new Table 17.18.

**Table 17-17:** Placement of Job A

A							
---	--	--	--	--	--	--	--

**Table 17-18:** Result after removing Job A

Job	B	C	D	E	F	G	H	I
M/C 1	6	5	10	7	9	8	6	5
M/C 2	9	8	5	4	10	4	9	12

Examine the minimum time among the rest jobs. It is observed that Jobs E and G, both have minimum time 4 hours on M/C 2. There is a tie. Now, Job E (total time =  $7 + 4 = 11$  hours) is smaller than Job G (total time =  $8 + 4 = 12$ ); thus, place Job E at the last position and Job G just before it as shown in Table 17.19.

**Table 17-19:** Placement of Jobs E and G

A							G	E
---	--	--	--	--	--	--	---	---

Remove Jobs E and G from Table 17.18 and produce new Table 17.20.

**Table 17-20:** Result after the removal of Jobs E and G

Job	B	C	D	F	H	I
M/C 1	6	5	10	9	6	5
M/C 2	9	8	5	10	9	12

Examine the minimum time among the rest jobs. It is observed that Job D has a minimum time on M/C 2, therefore, place Job D right side (last position of the remaining block). Also, Jobs C and I have minimum time 5 hours on M/C 1 and there is a tie. Now, Job C (total time =  $5 + 8 = 13$  hours) is smaller than Job I (total time =  $5 + 12 = 17$ ); thus, place Job C at first on the left side and I just after it as shown in Table 17.21. Remove Jobs C, D and I from Table 17.20 and produce new Table 17.22.

**Table 17-21:** Placement of Jobs C, D and I

A	C	I				D	G	E
---	---	---	--	--	--	---	---	---

**Table 17-22:** Result after the removal of Jobs C, D and I

Job	B	F	H
M/C 1	6	9	6
M/C 2	9	10	9

Examine the minimum time among the rest jobs. It is observed that Jobs b and H have a minimum time on M/C 1; there is a tie. But, both the jobs have same time on M/C 2 also. Therefore, we can place any one job either B or H first, i.e. left side of the block and other job just after the first can be placed as shown in Table 17.23. Remove Job B and H from Table 17.22 and produce new Table 17.24.

**Table 17-23:** Placement of Jobs B and H

A	C	I	B	H		D	G	E
OR								
A	C	I	H	B		D	G	E

**Table 17-24:** Result after the removal of Jobs B and H

Job	F
M/C 1	9
M/C 2	10

Now, there is only one Job F, which can be placed in the blank place as shown in Table 17.25.

**Table 17-25:** Placement of Job F

A	C	I	B	H	F	D	G	E
OR								
A	C	I	H	B	F	D	G	E

The entry and out times of the different jobs on M/C 1 and M/C 2 are shown in Table 17.26.

**Table 17-26:** In and out times of the jobs on M/C 1 and M/C 2

Job sequence	M/C 1		M/C 2	
	In	Out	In	Out
<b>A</b>	0	3	3	10
<b>C</b>	3	8	10	18
<b>I</b>	8	13	18	30
<b>H</b>	13	19	30	39
<b>B</b>	19	25	39	48
<b>F</b>	25	34	48	58
<b>D</b>	34	44	58	63
<b>G</b>	44	52	63	67
<b>E</b>	52	59	67	71

The jobs A to I will be finished in 71 min. Total idle time on Machine I will be 12 min, i.e. 71-59; and total idle time on Machine II will be the time elapsed in finishing the first job on Machine I, i.e. 3 min.

***n-job, m-Machines***

**Example 17.4:** Arrange the jobs on M/C 1 and M/C 2 using Johnson's rule (Table 17.27).

**Table 17-27:** Sequencing problem

Jobs	M/C 1	M/C 2	M/C 3	M/C 4	M/C 5
A	8	6	3	4	10
B	7	7	5	6	11
C	6	5	6	7	9
D	9	4	4	3	7

**Solution:**

At first, convert the problem of *n*-job, *m*-machines into *n*-job, 2-machines.

Add the processing time of the jobs from first M/C to the second last M/C and consider this as M/C X. Similarly, add the processing times of the jobs from the second M/C to the last M/C and consider it as M/C Y. Now, this is in the form of *n*-job, 2-machines as shown in Table 17.28.

**Table 17-28:** Result after conversion into *n*-job, 2-machines

Jobs	M/C X	M/C Y
A	21	23
B	25	29
C	24	27
D	20	18

The minimum processing time is 18 hours of a job D at M/C Y; therefore, Job D should be placed in the last place on the right side of the block as shown in Table 17.29. Now, delete the job D from Table 17.28 and produce new Table 17.30.

**Table 17-29:** Placement of Job D**Table 17-30:** Result after the removal of Job D

Jobs	M/C X	M/C Y
A	21	23
B	25	29
C	24	27

The minimum processing time is 21 hours of Job A on M/C 1; therefore, Job A should be placed in the first place on the left side of the block as shown in Table 17.31. Now, delete the job A from Table 17.30 and produce new Table 17.32.

**Table 17-31:** Placement of Job A

A			D
---	--	--	---

**Table 17-32:** Result after the removal of Job A

Jobs	M/C X	M/C Y
B	25	29
C	24	27

The minimum processing time is 24 hours of Job C on M/C 1; therefore, Job C should be placed at the first place on the left side of the remaining blocks as shown in Table 17.33. Now, delete Job C from Table 17.32 and produce new Table 17.34.

**Table 17-33:** Placement of Job C

A	C		D
---	---	--	---

**Table 17-34:** Result after the removal of Job A

Jobs	M/C X	M/C Y
B	25	29

Now, only one Job B is left, which can be placed in the blank block as shown in Table 17.35.

**Table 17-35:** Placement of Job B

A	C	B	D
---	---	---	---

The entry and out times of the different jobs on M/C 1 and M/C 2 are shown in Table 17.36.

**Table 17-36:** In and out times of the jobs on M/C 1, 2, 3 and 4

Jobs in sequence	M/C 1		M/C 2		M/C 3		M/C 4		M/C 5	
	In	Out								
A	0	8	8	14	14	17	17	21	21	31
C	8	14	14	19	19	25	25	32	32	41
B	14	21	21	28	28	33	33	39	41	52
D	21	30	30	34	34	38	39	42	52	59

The total job completion time is 59 min. Idle time on M/C 1 is 29 min (i.e. 59-30); on M/C 2 is 37 min (i.e. 8 + 21 - 19 + 30 - 28 + 59 - 34); on M/C 3 is 41 min (i.e. 14 + 19 - 17 + 28 - 25 + 34 - 33 + 59 - 38); on M/C 4 39 min (i.e. 17 + 25 - 21 + 33 - 32 + 59 - 42); and on M/C 5 is 22 min (i.e. 21 + 32 - 31).



## SUMMARY

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In this chapter, we have discussed the method to solve the assignment problem. Hungarian model is used to solve the assignment problem. In addition to this, we have also solved the problems related to sequencing model using Johnson's Rule. In this model, we have solved the sequencing problems related to  $n$ -jobs, two machines and  $n$ -job,  $m$ -machines.

## MULTIPLE-CHOICE QUESTIONS

1. The major difference between transportation problem and assignment problem is that
  - (a) In the case of transportation problem, one source may supply to a number of destinations, but in assignment problem one job is allocated to only one machine or one machine is used for only one job.
  - (b) In the case of transportation problem, one source can supply to only one destination, but in assignment problem one job may be allocated on a number of machines.
  - (c) Both are exactly similar problems.
  - (d) none of these
2. The assignment problem can be solved by
  - (a) MODI method
  - (b) Vogel's approximation method
  - (c) Hungarian method
  - (d) Least cost method
3. In an assignment problem,
  - (a) one agent can do parts of several tasks
  - (b) one task can be done by several agents
  - (c) each agent is assigned to its own best task
  - (d) none of these
4. To make an unbalance assignment problem balance, what is added with all entries zeros?
  - (a) Dummy row
  - (b) Dummy column
  - (c) Any one from (a) and (b)
  - (d) None of these
5. In a  $n \times n$  matrix of assignment problem, the total number of assignments will be
  - (a)  $n$
  - (b)  $2n$
  - (c)  $2n - 1$
  - (d)  $(n + 1)/2$
6. Johnson's rule to the sequencing of jobs through two work centres states that
  - (a) The job with the smallest processing time for the first process should be done first and the job with the smallest processing time for the second process should be done last.
  - (b) The job with the longest processing time for the first process should be done first and the job with the longest processing time for the second process should be done last.
  - (c) The job with the smallest processing time for the first process should be done first and the job with the longest processing time for the second process should be done last.
  - (d) The job with the longest processing time for the first process should be done first and the job with the smallest processing time for the second process should be done last.

7. If an assignment problem consists of 4 workers and 5 jobs,
  - (a) one worker will not get a job's assignment
  - (b) one worker will be assigned two jobs
  - (c) each worker will contribute work towards the fifth job
  - (d) one job will not get a worker assigned
8. The assignment method is
  - (a) method to highlight overloads in a given work centre
  - (b) a computerized method of determining appropriate tasks for an operation
  - (c) a form of linear programming for optimally assigning tasks or jobs to resources
  - (d) the same thing as the Gantt schedule chart
9. The assignment model is a special case of the
  - (a) maximum-flow model
  - (b) transportation model
  - (c) shortest-route model
  - (d) none of the above models
10. An assignment problem is a special form of transportation problem where all supply and demand values equal
  - (a) 0
  - (b) 1
  - (c) 2
  - (d) 3
11. As per Johnson's rule, for  $n$  job  $m$  machines, the  $m$  machines are converted into the group of
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
12. The main purpose of the sequencing model is to
  - (a) improve the quality of the machine
  - (b) maintain the flow of jobs
  - (c) minimize the idle times on the machines
  - (d) None of these
13. Sequencing is an input to the
  - (a) Dispatching
  - (b) Routing
  - (c) Scheduling
  - (d) None of these
14. The assignment method is appropriate in solving problems that have the following characteristics:
  - (a) There are  $n$  things to be distributed to  $n$  destinations.
  - (b) Only one criterion should be used.
  - (c) Each work-centre should be assigned only one job.
  - (d) All the above
15. There are  $n$  jobs and 4 machines, say M1, M2, M3 and M4. According to the Johnson rule, it will be converted into  $n$  job two machines as
  - (a) Machines M1, M2, M3 as one group and M2, M3, M4 as another group
  - (b) Machines M1, M2 as one group and M3, M4 as another group
  - (c) Machines M1, M3 as one group and M2, M4 as another group
  - (d) Machines M1, M4 as one group and M2, M3 as another group

## Answers

1. (a)    2. (c)    3. (c)    4. (c)    5. (a)    6. (a)    7. (d)    8. (c)    9. (b)  
10. (b)   11. (b)   12. (c)   13. (c)   14. (d)   15. (a)

## REVIEW QUESTIONS

1. Differentiate between transportation problem and assignment problem.
2. What is use of assignment problem in production engineering?
3. What are the objectives of sequencing?
4. Explain the Johnson's rule for the sequencing of the jobs on machines.
5. Explain the Hungarian method to solve the assignment method.

## EXERCISES

1. Allocate the jobs on the machines shown in the Cost matrix in Table 17.37.

**Table 17-37:** Cost matrix

Job	Machines				
	1	2	3	4	5
1	10	12	15	12	8
2	7	16	14	14	11
3	13	14	7	9	9
4	12	10	11	13	10
5	8	13	15	11	15

2. Allocate the jobs on the machines shown in the Cost matrix in Table 17.38.

**Table 17-38:** Cost matrix

Job	Machines			
	1	2	3	4
1	20	29	21	28
2	33	27	22	25
3	24	31	23	31
4	29	26	33	22
5	22	39	25	30

3. Arrange the jobs on M/C 1 and M/C 2 using Johnson's rule (Table 17.39).

**Table 17-39:** Sequencing problem

<b>Job</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>	<b>F</b>	<b>G</b>	<b>H</b>	<b>I</b>
<b>M/C 1</b>	5	8	7	12	9	11	10	8	7
<b>M/C 2</b>	9	11	10	7	6	12	6	11	14

4. Arrange the jobs on M/C 1 and M/C 2 using Johnson's rule (Table 17.40).

**Table 17-40:** Sequencing problem

<b>Jobs</b>	<b>M/C 1</b>	<b>M/C 2</b>	<b>M/C 3</b>	<b>M/C 4</b>	<b>M/C 5</b>
<b>A</b>	7	5	2	3	9
<b>B</b>	6	6	4	5	10
<b>C</b>	5	4	5	6	8
<b>D</b>	8	3	3	2	5



## REFERENCES AND FURTHER READINGS

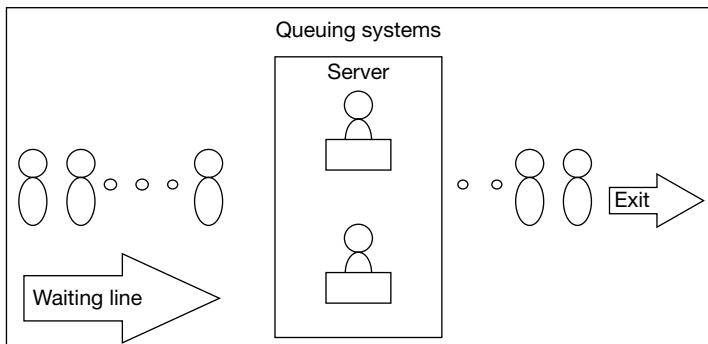
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# CHAPTER ▶ 18

## Waiting Line Theory

### 18.1 INTRODUCTION

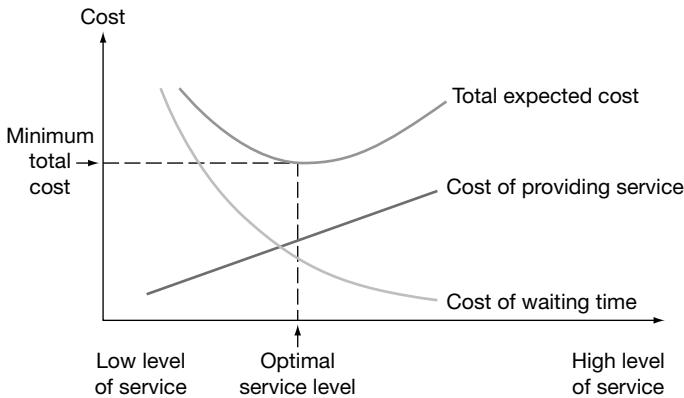
The waiting line is a daily life problem; we have to wait in a queue to get served. We have to face this problem everywhere, for example, queue at the railway ticket reservation counter, bank counter to get the demand draft, queue at the airport, traffic queue waiting for the green light, etc. Analysis of queues in terms of the length of the waiting line, average waiting time and other factors help us to understand service systems, maintenance activities and control activities. The main things in the waiting line theory are the arrival pattern of the customer and the service provided to the customer. The arrival population may be infinite and finite. The customer coming to a health centre may be infinite because we do not know that how many patients will come today to consult the doctor. But, students waiting to get corrected their assignment by the teacher are the finite population. Thus, when potential new customers for the waiting line system are affected by the number of customers already in the system, the customer population is termed as *finite*. When the number of customers waiting in line does not significantly affect the rate at which the population generates new customers, the customer population is considered as *infinite*. A basic queuing system is shown in Figure 18.1.



**Figure 18-1:** Basic queuing system

Cost optimization in waiting line theory is very important. If we increase the service level, the cost of services will increase, but the cost of waiting time will decrease. In addition to this, some servers may remain idle for some time. If we decrease the service level, the cost of waiting line

will increase, but the cost of services will decrease. Thus, we find the optimal cost in waiting line theory. The explanation of the costing system may be understood with the graph shown in Figure 18.2.



**Figure 18-2:** Cost optimization in waiting line

### The Behaviour of Arrivals

It is assumed that in most of the cases, an arriving customer is a patient customer, and they will wait in the queue until they will get served and will not switch between lines. Unfortunately, time is important for everybody and people have been known to balk or to renege. The customer decides not to enter the waiting line due to the long length of the queue, is known as *balking*. The customer enters the line, but decides to exit before being served due to long waiting time is known as *reneging*. The customer enters one line and then switches to a different line in an effort to reduce the waiting time, is known as *jockeying*.

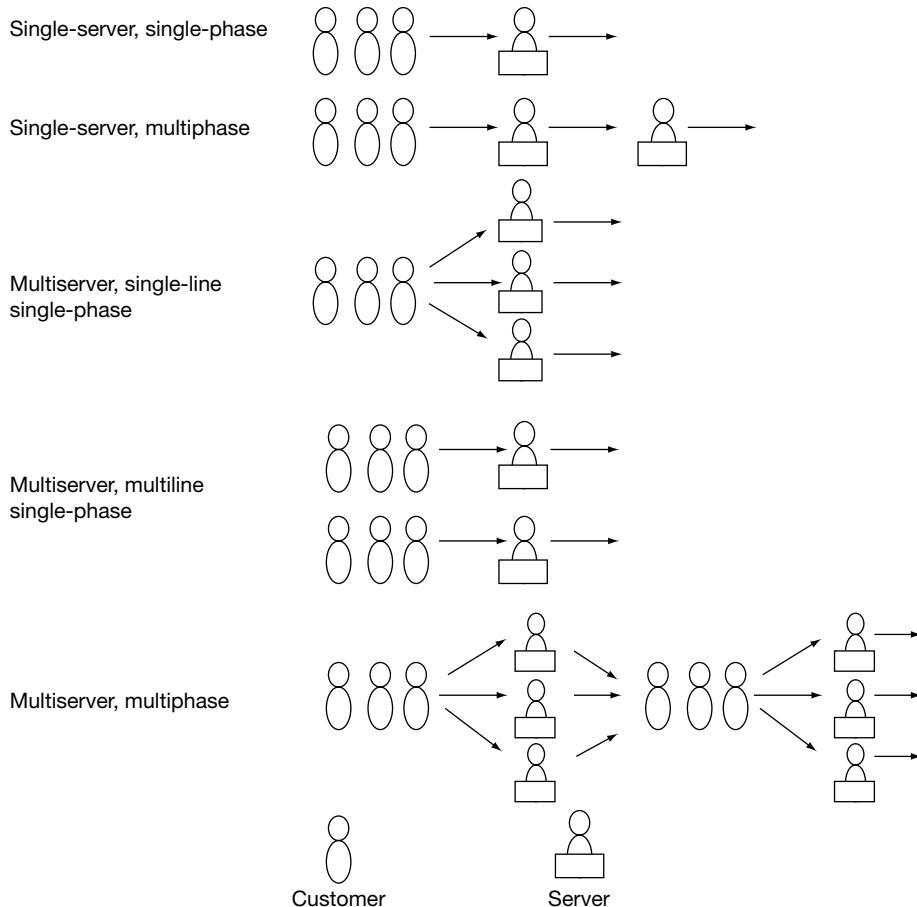
## 18.2 THE SERVICE CHARACTERISTICS

The service system is characterized by the number of waiting lines, the number of servers, the arrangement of the servers, the arrival and service patterns and the service priority rules. The different types of service characteristics are shown in Figure 18.3. Generally the service pattern used in the waiting line is first-come, first-serve (FCFS) or first-in, first-out (FIFO), but in the case of manufacturing systems, the service pattern may be based on the importance of the customer. In inventory system, we follow last-come, first-serve (LCFS) or last-in, first-out (LIFO).

**Single-channel queuing system:** A service system consists of one waiting line and one server.

**Multiple-channel queuing system:** A service system consists of one waiting line but with multiple servers.

**Single-phase system:** A system in which the customer receives service from only one station and then exits the system.

**Figure 18-3:** The server systems

**Multiphase system:** A system in which the customer receives services from several stations before exiting the system.

#### **Parameters for Measuring the Performance of Waiting Line**

The following parameters are used to measure the performance of waiting line:

1. Average time spent by the customer in the queue, i.e. waiting time.
2. Average queue length or average number of customers in the queue.
3. Average time spent by the customer in the queue as well as in service systems.
4. Average number of customers in the system.
5. The probability that the service facility will be idle.
6. Utilization factor for the system.
7. Probability of a specific number of customers in the system.

## 18.3 MATHEMATICAL DISTRIBUTIONS

To describe a waiting system, it is necessary to know the manner in which customers or the waiting units are arranged for service. Queuing formulas require an **arrival rate**, the number of units per period. A *constant* arrival distribution is periodic, with exactly the same time between successive arrivals. Usually, we assume that the arrival of customer in the system follows the Poisson distribution and for the distribution of the time between arrivals is exponentially distributed. Second, we can set any time length ( $T$ ) and try to determine how many arrivals might enter the system within  $T$ .

### Poisson Distribution

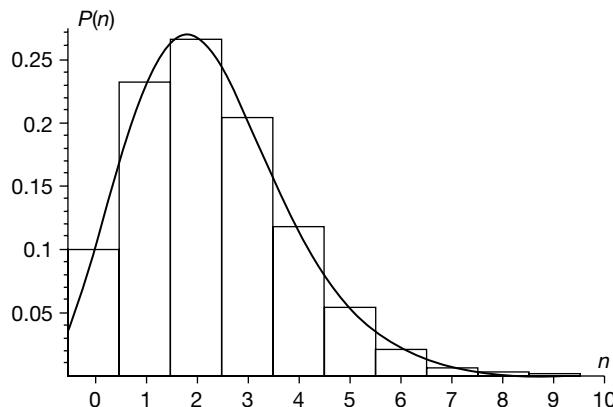


Figure 18-4: Poisson distribution

In Poisson distribution, number of arrivals during time  $t$  can be explained as shown in Figure 18.4 and is obtained by finding the probability of exactly  $n$  arrivals during the time,  $t$ . If the arrival process is random, the distribution is random; the formula can be given as:

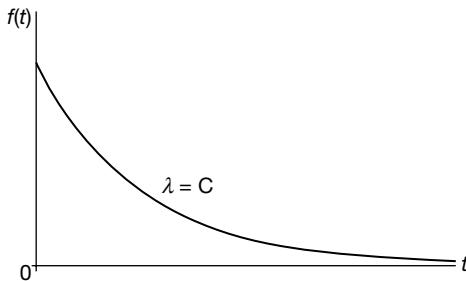
$$p(n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

### Exponential Distribution

In the first case, when arrivals at a service facility occur in a purely random fashion, a plot of the inter-arrival times yields an exponential distribution as shown in Figure 18.5. The probability function is given as:

$$f(t) = \lambda e^{-\lambda t}$$

where  $\lambda$  is the mean number of arrivals per unit time.



**Figure 18-5:** Exponential distribution

### Notations used in Queuing Models

$\lambda$  = Arrival rate.

$\mu$  = Service rate.

$\frac{1}{\mu}$  = Average service rate.

$\rho = \frac{\lambda}{\mu}$  = Ratio of arrival and service rates for single server.

$L_q$  = Length of queue or average number of customer waiting in line.

$L_s$  = Average number of customer in the system (Queue + Service).

$W_q$  = Average waiting time.

$W_s$  = Average time in the system (Queue + Service).

$n$  = Number of customers in the system.

$S$  = Number of identical service channels.

$P_n$  = Probability of exactly  $n$  customers in the system;  $P_w$  = Probability of waiting in queue.

$D$  = Probability that an arrival will wait in queue.

$F$  = Efficiency factor, Measure of the effect of having to wait in line

$H$  = Average number of customers being served.

$j = N - n$  = Population source less those in queuing system.

$N$  = Number of customers in population source.

$t$  = Average time to perform the service.

$U$  = Average time between customer service requirements.

$X$  = Service factor, or proportion of service time required.

Some of the basic models of waiting line and their properties are summarized in Table 18.1.

**Table 18-1:** Basic models of waiting line theory

Model	Layout	Service phase	Source population	Arrival pattern	Queue discipline	Service pattern	Queue length	Notation of the model
1	Single channel	Single	Infinite	Poisson	FCFS	Exponential	Unlimited	M/M/1:∞/FCFS
2	Single channel	Single	Infinite	Poisson	FCFS	Exponential	Limited	M/M/1:N/FCFS
3	Single channel	Single	Infinite	Poisson	FCFS	Constant	Unlimited	
4	Single channel	Single	Finite	Poisson	FCFS	Exponential	Limited	M/M:1:N/GD
5	Multiple channel	Single	Infinite	Poisson	FCFS	Exponential	Unlimited	M/M/S:∞/FCFS
6	Multiple channel	Single	Infinite	Poisson	FCFS	Exponential	Limited	M/M/S:N/FCFS
7	Multiple channel	Single	Finite	Poisson	FCFS	Exponential	Limited	M/M/S:N/GD
8	Single channel	Multiple	Infinite	Poisson	FCFS	Exponential	Unlimited	M/E <sub>k</sub> /1:∞/FCFS

## 18.4 WAITING LINE MODELS

### I. Single server, single phase, exponential service, infinite queue, single line, and an infinite population Model (M/M/1:∞/FCFS).

This model has the following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered infinite.
2. The capacity of the system having a customer is infinite.
3. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .
4. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an exponential distribution with an average of  $1/\mu$ .
5. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$\rho = \frac{\lambda}{\mu}; \text{ (For all situations, } \mu \text{ should always be larger than the value of } \lambda\text{.)}$$

$$L_s = \frac{\lambda}{\mu - \lambda};$$

$$L_q = \rho L_s = \frac{\lambda^2}{\mu(\mu - \lambda)};$$

$$W_s = \frac{1}{\mu - \lambda};$$

$$W_q = \rho W_s = \frac{\lambda}{\mu(\lambda - \mu)};$$

$$p_n = (1 - \rho) \rho^n$$

**Example 18.1:** The arrival rate of customers at the single window ticket booking counter of a cinema hall follows a Poisson distribution and the service time follows an exponential distribution. The arrival rate and service rate are 90 and 120 customers per hour, respectively. Find the utilization of the booking clerk, average number of customers waiting in queue, average number of customers waiting in the system, average waiting time per customer in the queue, average waiting time per customer in the system, probability of zero customers in the system and probability of five customers in the system.

**Solution:**

Given, arrival rate,  $\lambda = 90$  customers,  $\mu = 120$  customers

Utilization of booking clerk,

$$\rho = \frac{\lambda}{\mu} = \frac{90}{120} = 0.75$$

Average number of customers waiting in the queue,

$$L_q = \rho L_s = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{90^2}{120(120 - 90)} = 2.25 \text{ customers}$$

Average number of customers waiting in the system,

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{90}{120 - 90} = 3 \text{ customers}$$

Average waiting time per customer in the queue,

$$W_q = \frac{L_s}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{90}{120(120 - 90)} = 0.025 \text{ hours}$$

Average waiting time per customer in the system,

$$W_s = \frac{W_q}{\rho} = \frac{0.025}{0.75} = \frac{1}{(120 - 90)} = 0.333 \text{ hours}$$

Probability of zero customers in the system =  $(1 - \rho) \rho^0 = 1 - 0.75 = 0.25$

Probability of five customers in the system =  $(1 - \rho) \rho^5 = (1 - 0.75) 0.75^5 = 0.059$

## II. Single server, single phase, exponential service, finite queue, single line, and an infinite population model (M/M/1:N/FCFS).

This model has the following assumptions:

1. There is no balking, reneging or jockeying and customers come from a population that can be considered infinite.
2. The capacity of the system having customer is finite ( $N$ ).
3. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .
4. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an exponential distribution with an average of  $1/\mu$ .
5. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$\rho = \frac{\lambda}{\mu};$$

$$L_s = \begin{cases} \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}}; & \rho \neq 1 \text{ (i.e. } \lambda \neq \mu) \\ \frac{N}{2}; & \rho = 1 \text{ (i.e. } \lambda = \mu) \end{cases}$$

$$L_q = L_s - \frac{\lambda(1-p_N)}{\mu}$$

$$W_s = \frac{L_s}{\lambda(1-p_N)}$$

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda(1-p_N)}$$

$$p_N = p_0 \rho^N; p_0 = \frac{1-p}{1-p^{N+1}}$$

$$\lambda_{\text{eff}} = \lambda(1-p_N)$$

$$\rho_{\text{eff}} = \frac{\lambda_{\text{eff}}}{\mu}$$

**Example 18.2:** Cars arrive at a service centre with the arrival rate of 25 cars per hour and service rate of 15 cars per hour. The arrival rate follows the Poisson distribution and service time follows the exponential distribution. The number of waiting space is limited to 5 cars only. Find the utilization of service centre, average number of cars waiting in the system, the effective arrival rate of the cars, average number of cars in the queue, average waiting time in the system and average waiting time in the queue.

**Solution:**

Given, arrival rate,  $\lambda = 25$  cars,  $\mu = 15$  cars,  $N = 5$  cars.

Utilization of service centre,

$$\rho = \frac{\lambda}{\mu} = \frac{25}{15} = 1.66;$$

$$p_N = \frac{1 - \rho}{1 - \rho^{N+1}} \rho^N = \frac{1 - 1.66}{1 - 1.66^6} 1.66^5 = 0.417$$

Effective arrival rate of cars,

$$\lambda_{\text{eff}} = \lambda(1 - p_N) = 25(1 - 0.417) = 14.561$$

Average number of cars waiting in the system,

$$\begin{aligned} L_s &= \frac{\rho}{1 - \rho} - \frac{(N+1)\rho^{N+1}}{1 - \rho^{N+1}} \\ &= \frac{1.66}{1 - 1.66} - \frac{(6)1.66^6}{1 - 1.66^6} = 3.786 \text{ cars} \end{aligned}$$

Average number of cars waiting in the queue,

$$\begin{aligned} L_q &= L_s - \frac{\lambda(1 - p_N)}{\mu} \\ &= 3.786 - \frac{14.561}{15} = 2.815 \text{ cars} \end{aligned}$$

Average waiting time in the system,

$$W_s = \frac{L_s}{\lambda(1 - p_N)} = \frac{3.786}{14.561} = 0.26 \text{ hours}$$

Average waiting time in the queue,

$$W_q = W_s - \frac{1}{\mu} = \frac{L_q}{\lambda_{\text{eff}}} = \frac{2.815}{14.561} = 0.193 \text{ hours}$$

### **III. Single server, single phase, constant service time, infinite queue, single line, and infinite population model.**

This model has following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered infinite.
2. The capacity of the system having customer is infinite.
3. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .

4. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time  $1/\mu = \text{constant}$ .
5. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$\begin{aligned} L_s &= L_q + \frac{\lambda}{\mu} \\ L_q &= \frac{\left(\frac{\lambda}{\mu}\right)^2}{2\left(1 - \frac{\lambda}{\mu}\right)} = \frac{\lambda^2}{2\mu(\mu - \lambda)} \\ W_s &= W_q + \frac{1}{\mu} \\ W_q &= \frac{L_q}{\lambda} = \frac{\lambda}{2\mu(\lambda - \mu)} \end{aligned}$$

**Example 18.3:** A logistics service company deals with transportation of used auto components for reprocessing. The truck driver has to wait for 20 min for unloading the components at reprocessing plant. The cost of waiting is Rs 500 per hour. A new technology is used to unload the truck load at a constant rate of 15 trucks per hour at the cost of Rs 25 per truck. The truck arrival follows the Poisson distribution at an average rate of 12 trucks per hour. Suggest that whether the new technology should be used or not.

**Solution:**

$$\text{Waiting cost before the use of new technology} = \frac{20}{60} \times 500 = \text{Rs } 166.66$$

Under the new system,  $\lambda = 12$  trucks per hour, and  $\mu = 15$  trucks per hour

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)} = \frac{12^2}{2 \times 15(15 - 12)} = 1.6 \text{ hours}; \quad W_q = \frac{\lambda}{2\mu(\lambda - \mu)} = 0.133 \text{ hours}$$

$$\text{Waiting cost per trip with new technology} = 1.33 \times 500 = \text{Rs } 665$$

Cost of using new technology is more than the cost of waiting before using the new technology. Thus, the use of new technology is not suggested.

**IV. Single server, single phase, exponential service, infinite queue, single line, and a finite population model (M/M/1:N/GD):** This model has the following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered finite, say  $N$ .
2. The capacity of the system having customer is finite, say  $N$ .

3. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .
4. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an exponential distribution with an average of  $1/\mu$ .
5. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

Suppose that there are  $n$  customers in the system, and then the system is left to accommodate  $M-n$  more customers. Thus, the further arrival rate of the customers to the system will be  $\lambda(M-n)$  for  $s=1$ .

$$\lambda_n = \begin{cases} \lambda(N-n); & n = 1, 2, 3, \dots, N \\ 0; & n > N \end{cases}$$

$$\mu_n = \mu$$

Probability that system remains idle,

$$P_0 = \left[ \sum_{n=0}^N \frac{N!}{(N-n)!} \left( \frac{\lambda}{\mu} \right)^n \right]^{-1}$$

Probability that  $n$  customers remain in the system,

$$P_n = \frac{N!}{(N-n)!} \left( \frac{\lambda}{\mu} \right)^n P_0$$

$$L_q = N - \left( \frac{\lambda + \mu}{\lambda} \right) (1 - P_0)$$

$$L_s = L_q + (1 - P_0) = N - \frac{\mu}{\lambda} (1 - P_0)$$

$$W_q = \frac{L_q}{\lambda(N - L_s)}; W_s = W_q + \frac{1}{\mu} = \frac{L_s}{\lambda(N - L_s)}$$

**Example 18.4:** In a workshop, a technician takes care of 5 machines. The mean time to failure is 6 hours for each machine and follows the exponential distribution. The mean time to repair the machine is 1 hour and follows the exponential distribution. Find the probability that the technician will be idle, probability that 3 machines will be in the system of rectification or repair,

average number of machines waiting to be repaired, average number of machines waiting to be repaired plus being repaired, and average time of waiting machine in the queue to be repaired.

**Solution:**

Given,  $\lambda = 1/6 = 0.166$  machine per hour,  $\mu = 1$  machine per hour,  $N = 5$  machines,  $\rho = \lambda/\mu = 0.166$ .

Probability that the technician will remain idle,

$$\begin{aligned} p_0 &= \left[ \sum_{n=0}^N \frac{N!}{(N-n)!} \left( \frac{\lambda}{\mu} \right)^n \right]^{-1} \\ &= \left[ \sum_{n=0}^5 \frac{N!}{(N-n)!} \left( \frac{\lambda}{\mu} \right)^n \right]^{-1} = 0.362 \end{aligned}$$

Probability that 3 machines remain in the system,

$$\begin{aligned} p_3 &= \frac{N!}{(N-n)!} \left( \frac{\lambda}{\mu} \right)^3 p_0 \\ &= \frac{5!}{(5-3)!} (0.166)^3 \times 0.362 = 0.099 \end{aligned}$$

Average number of machines waiting to be repaired,

$$\begin{aligned} L_q &= N - \left( \frac{\lambda + \mu}{\lambda} \right) (1 - p_0) \\ &= 5 - \left( \frac{0.166 + 1}{0.166} \right) (1 - 0.362) = 0.518 \text{ machine} \end{aligned}$$

Average number of machines waiting to be repaired and under repair,

$$L_s = L_q + (1 - p_0) = N - \frac{\mu}{\lambda} (1 - p_0) = 5 - \frac{1}{0.166} (1 - 0.362) = 1.156 \text{ machines}$$

Average waiting time of the machine in the queue,

$$W_q = \frac{L_q}{\lambda(N - L_s)} = \frac{0.518}{0.166(5 - 1.156)} = 0.811 \text{ hours}$$

Average waiting time of the machine in the queue and under repair,

$$W_s = \frac{L_s}{\lambda(N - L_s)} = \frac{1.156}{0.166(5 - 1.156)} = 1.811 \text{ hours}$$

## V. Multiple server, single phase, exponential service, infinite queue, single line, and an infinite population model (M/M/s:∞/FCFS).

This model has the following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered infinite.
2. There are multiple servers (multiple channels).
3. The capacity of the system having customer is infinite.
4. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .
5. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an exponential distribution with an average of  $1/\mu$ .
6. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$\begin{aligned}
 p_0 &= \left\{ \sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s! \left[ 1 - \frac{\rho}{s} \right]} \right\}^{-1} \\
 L_s &= L_q + \frac{\lambda}{\mu} \\
 L_q &= \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda \mu}{(s\mu - \lambda)^2} \right] p_0 \\
 W_s &= W_q + \frac{1}{\mu} \\
 W_q &= \frac{L_q}{\lambda} = \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\mu}{(s\mu - \lambda)^2} \right] p_0 \\
 p(n \geq s) &= \frac{1}{s!} \left( \frac{\lambda}{\mu} \right)^s \frac{s\mu}{s\mu - \lambda} p_0
 \end{aligned}$$

**Example 18.5:** The breakdown rate of the machines in a workshop is 6 per hour and follows a Poisson distribution. Three mechanics are employed to repair the machines. The service rate of the mechanics is 4 machines per hour and also follows the Poisson distribution. Find the utilization of mechanics, probability of the mechanics remains idle, average number of the machines waiting in queue to be repaired, average number of machines in the system, the average time of the machine waiting in the queue and average time of the machine waits in the system.

**Solution:**

Given,  $\lambda = 6$  machines per hour,  $\mu = 4$  machines per hour,  $\rho = \lambda/\mu = 1.5$

Probability of the machine remaining idle,

$$\begin{aligned} p_0 &= \left\{ \sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s}{s! \left[ 1 - \frac{\rho}{s} \right]} \right\}^{-1} \\ &= \left\{ \frac{1.5^0}{0!} + \frac{1.5^1}{1!} + \frac{1.5^2}{2!} + \frac{1.5^3}{3! \left[ 1 - \frac{1.5}{3} \right]} \right\}^{-1} = 0.21 \end{aligned}$$

Average number of machines in the queue,

$$\begin{aligned} L_q &= \left[ \frac{1}{(s-1)!} \left( \frac{\lambda}{\mu} \right)^s \frac{\lambda \mu}{(s\mu - \lambda)^2} \right] p_0 \\ &= \left[ \frac{1}{(3-1)!} \left( \frac{6}{4} \right)^3 \frac{6 \times 4}{(3 \times 4 - 6)^2} \right] \times 0.21 = 0.236 \text{ machines} \end{aligned}$$

Average number of machines in the system,

$$L_s = L_q + \frac{\lambda}{\mu} = 0.236 + \frac{6}{4} = 1.736 \text{ machines}$$

Average waiting time of machines in the queue,

$$W_q = \frac{L_q}{\lambda} = \frac{0.236}{6} = 0.0393 \text{ hours}$$

$$W_s = W_q + \frac{1}{\mu} = 0.0393 + \frac{1}{4} = 0.289 \text{ hours}$$

## VI. Multiple server, single phase, exponential service, finite queue, single line, and an infinite population model (M/M/s:N/FCFS).

This model has the following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered infinite.
2. There are multiple servers (multiple channels).
3. The capacity of the system having customer is finite, say  $N$ .
4. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .
5. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an exponential distribution with an average of  $1/\mu$ .
6. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$\begin{aligned}
 p_0 &= \left\{ \sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s [1 - (\rho/s)]^{N-s+1}}{s! [1 - \rho/s]} \right\}^{-1}; \text{ for } \frac{\rho}{s} \neq 1 \\
 &= \left\{ \sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s [N-s+1]}{s!} \right\}^{-1}; \text{ for } \frac{\rho}{s} = 1 \\
 p_n &= \frac{\rho^n}{n!} p_0; \text{ for } 0 \leq n \leq s \\
 &= \frac{\rho^n}{s! s^{n-s}} p_0; \text{ for } s \leq n \leq N \\
 L_s &= L_q + \frac{\lambda}{\mu} (1 - p_N) \\
 L_q &= \frac{(\rho)^{s+1}}{(s-1)! (s-\rho)^2} \left[ 1 - \left( \frac{\rho}{s} \right)^{N-s} - \left( 1 - \frac{\rho}{s} \right) (N-s) \left( \frac{\rho}{s} \right)^{N-s} \right] p_0; \text{ for } \frac{\rho}{s} \neq 1 \\
 &= \frac{(\rho)^s}{2(s)!} [(N-s)(N-s+1)] p_0; \text{ for } \frac{\rho}{s} = 1 \\
 W_s &= \frac{L_s}{\lambda(1-p_N)} \\
 W_q &= W_s - \frac{1}{\mu} = \frac{L_q}{\lambda(1-p_N)} \\
 \lambda_{\text{eff}} &= \lambda(1-p_N) \\
 \rho_e &= \frac{\lambda_e}{\mu}
 \end{aligned}$$

**Example 18.6:** The arrival rate of cars at a service centre is 6 per day. The service centre has three parallel channels for the services. The service rate of individual channel is 4 cars per day. The arrival rate and service rate follow the Poisson distribution. At a point of time, the maximum number of cars permitted in the system is 4. Find the probability that the service centre remains idle, average number of cars in queue, average number of the cars in the system, average waiting of the car time in the queue, and average waiting time of the car in the system.

#### Solution:

Given,  $\lambda = 6$  machines per hour,  $\mu = 4$  machines per hour, number of channels = 3,  $N = 4$ ,  $\rho = \lambda/\mu = 1.5$ .

Probability of the machine remaining idle,

$$p_0 = \left\{ \sum_{n=0}^{s-1} \frac{\rho^n}{n!} + \frac{\rho^s [1 - (\rho/s)]^{N-s+1}}{s![1-\rho/s]} \right\}^{-1}; \text{ for } \frac{\rho}{s} \neq 1$$

$$= \left\{ \sum_{n=0}^2 \frac{1.5^n}{n!} + \frac{1.5^3 [1 - (1.5/3)]^{4-3+1}}{3![1-1.5/3]} \right\}^{-1} = 0.256$$

Average number of machines in the queue,

$$L_q = \frac{(\rho)^{s+1}}{(s-1)!(s-\rho)^2} \left[ 1 - \left( \frac{\rho}{s} \right)^{N-s} - \left( 1 - \frac{\rho}{s} \right) (N-s) \left( \frac{\rho}{s} \right)^{N-s} \right] p_0; \text{ for } \frac{\rho}{s} \neq 1$$

$$= \frac{(1.5)^{3+1}}{(3-1)!(3-1.5)^2} \left[ 1 - \left( \frac{1.5}{3} \right)^{4-3} - \left( 1 - \frac{1.5}{3} \right) (4-3) \left( \frac{1.5}{3} \right)^{4-3} \right] \times 0.256 = 0.072 \text{ cars}$$

$$p_N = \frac{\rho^N}{s! s^{N-s}} p_0; \text{ for } s \leq n \leq N$$

$$= \frac{1.5^4}{3! 3^{4-3}} \times 0.256 = 0.072$$

$$\lambda_{\text{eff}} = \lambda(1 - p_N) = 6(1 - 0.072) = 5.568$$

Average number of machines in the system,

$$L_s = L_q + \frac{\lambda_{\text{eff}}}{\mu} = 0.072 + \frac{5.568}{4} = 1.468 \text{ cars}$$

Average waiting time of machines in the queue,

$$W_q = \frac{L_q}{\lambda_{\text{eff}}} = \frac{0.072}{5.568} = 0.0129 \text{ hours}$$

Average waiting of machines in the system,

$$W_s = \frac{L_q}{\lambda_{\text{eff}}} = 0.263 \text{ hours}$$

## VII. Multiple server, single phase, exponential service, finite queue, single line, and finite population model (M/M/s:N/GD).

This model has the following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered finite, say  $N$ .
2. There are multiple servers (multiple channels).
3. The capacity of the system having customer is finite, say  $N$ .

4. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .
5. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an exponential distribution with an average of  $1/\mu$ .
6. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$L_s = L_q + \frac{\lambda_{\text{eff}}}{\mu}$$

$$\lambda_{\text{eff}} = \mu(s - \bar{s}); \text{ In steady-state condition, } \lambda_{\text{eff}} = \mu(N - L_s)$$

$$L_q = L_s - (s - \bar{s})$$

$$W_s = \frac{L_s}{\lambda_{\text{eff}}} = \frac{L_s}{\mu(s - \bar{s})}$$

$$W_q = \frac{L_q}{\lambda_{\text{eff}}} = \frac{L_q}{\mu(s - \bar{s})}$$

$$\rho_e = \frac{\lambda_{\text{eff}}}{\mu}$$

$$p_n = \begin{cases} \frac{N!}{n!(N-n)!} \left(\frac{\lambda}{\mu}\right)^n p_0; & 0 \leq n \leq s \\ \frac{N!}{s!s^{n-s}(N-n)!} \left(\frac{\lambda}{\mu}\right)^n p_0; & s < n \leq N \end{cases}$$

$$p_0 = \left\{ \sum_{n=0}^s \frac{N!}{n!(N-n)!} \left(\frac{\lambda}{\mu}\right)^n + \sum_{n=s+1}^N \frac{N!}{s!s^{n-s}(N-n)!} \left(\frac{\lambda}{\mu}\right)^n \right\}^{-1}$$

$$\text{or } p_0 = \left\{ \sum_{n=0}^s {}^N C_n \rho^n + \sum_{n=s+1}^N {}^N C_n \frac{n!}{s!} \cdot \frac{\rho^n}{s^{n-s}} \right\}^{-1}$$

**Example 18.7:** In a machine shop, machine breakdown rate is 3 machines per hour. The maintenance shop has two mechanics who can attend the breakdown machine individually. The service rate of each mechanic is 2 machines per hour. Initially, there are 6 working machines in the shop. Find the utilization of mechanics, probability of the mechanics remaining idle, average number of the machines waiting in the queue to be repaired, average number of machines in the system, the average time of the machine waiting in the queue and average time of the machine waits in the system.

**Solution:**

Given:  $\lambda = 3$  machines per hour;  $\mu = 2$  machines per hour;  $N = 6$  machines,  $S = 2$  machines.

$$\rho = \frac{\lambda}{\mu} = \frac{3}{2} = 1.5$$

$$\begin{aligned} p_0 &= \left[ \sum_{n=0}^s \frac{N!}{n!(N-n)!} \rho^n + \sum_{n=s+1}^N \frac{N!}{s!s^{n-s}(N-n)!} \rho^n \right]^{-1} \\ &= \left[ \sum_{n=0}^2 \frac{6!}{n!(6-n)!} \rho^n + \sum_{n=3}^6 \frac{6!}{2!2^{n-2}(6-n)!} \rho^n \right]^{-1} \\ &= 0.00103 \end{aligned}$$

$$\begin{aligned} L_q &= \sum_{n=s+1}^N (n-s)p_n \\ &= \sum_{n=s+1}^N (n-s)^N C_n (\rho)^n p_0 \\ &= \sum_{n=3}^6 (n-2)^6 C_n (1.5)^n \times 0.00103 = 1.821 \text{ machines} \end{aligned}$$

$$\bar{S} = \sum_{n=0}^s (s-n)p_n = \sum_{n=0}^2 (2-n)p_n = 2p_0 + p_1 = 0.01133$$

$$\lambda_{\text{eff}} = \mu(s - \bar{s}) = 2(2 - 0.01133) = 3.97734 \text{ machines per hour}$$

$$L_s = L_q + (s - \bar{s}) = 1.821 + (2 - 0.01133) = 3.80967 \text{ machines}$$

$$W_q = \frac{L_q}{\lambda_{\text{eff}}} = \frac{1.821}{3.97734} = 0.457 \text{ hour}$$

$$W_s = \frac{L_s}{\lambda_{\text{eff}}} = \frac{3.80967}{3.97734} = 0.957 \text{ hour}$$

### VIII. Single server, multiple phase, erlang service time distribution, infinite queue, single line, and an infinite population model ( $M/E_k/1:\infty/\text{FCFS}$ ).

This model has the following assumptions:

1. There is no balking, reneging or jockeying, and customers come from a population that can be considered infinite.
2. There are  $k$  numbers of server phases.
3. The capacity of the system having customer is infinite.
4. Customer arrivals are described by a Poisson distribution with a mean arrival rate of  $\lambda$  (lambda). This means that the time between successive customer arrivals follows an exponential distribution with an average of  $1/\lambda$ .

5. The customer service rate is described by a Poisson distribution with a mean service rate of  $\mu$  (mu). This means that the service time for one customer follows an Erlang distribution.
6. The waiting line priority rule used is first-come, first-served.

Using these assumptions, we can calculate the operating characteristics of a waiting line system using the following formulas:

$$\begin{aligned} L_s &= L_q + \frac{\lambda}{\mu} = \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} \\ L_q &= \left( \frac{k+1}{2k} \right) \frac{\lambda}{\mu-\lambda} \\ W_q &= \frac{L_q}{\lambda} = \frac{k+1}{2k} \frac{\lambda}{\mu(\lambda-\mu)} \\ W_s &= W_q + \frac{1}{\mu} = \frac{k+1}{2k} \frac{\lambda}{\mu(\lambda-\mu)} + \frac{1}{\mu} \end{aligned}$$

Expected number of phases in the system,

$$\begin{aligned} L_q(k) &= \frac{L_s(k)}{\mu} = \frac{k+1}{2} \frac{\lambda}{\mu(\mu-\lambda)} \\ L_s(k) &= \frac{k+1}{2} \frac{\lambda}{\mu-\lambda} \end{aligned}$$

**Example 18.8:** In a marriage party, the guests have to pass through three counters for the dinner. At the first counter, they get collect coupons; at the second counter, they collect the food items, and at the third counter they get the sweet dish. Server at each counter takes approximately 2 min, although the distribution of guest arrival is approximately Poisson distribution at an average rate of 8 guests per hour. Find the average time a guest spends waiting in the queue, the average time of getting the service, and the most probable time in getting the service.

#### Solution:

Given,  $\lambda = 8$  guests per hour,  $\mu = 2 \times 3 = 6$  min per guest or 10 customers per hour.

The average time the guest spends in the queue,

$$\begin{aligned} W_q &= \frac{k+1}{2k} \frac{\lambda}{\mu(\mu-\lambda)} \\ &= \frac{3+1}{2 \times 3} \times \frac{8}{10(10-8)} = 0.266 \text{ hour} \end{aligned}$$

Average time of getting the service,

$$\frac{1}{\mu} = \frac{1}{10} = 0.1 \text{ hours or } 6 \text{ min.}$$

The most probable time in getting the services

$$= \frac{k-1}{k\mu} = \frac{3-1}{3 \times 10} = 0.066 \text{ hours}$$



## SUMMARY

In this chapter, we have discussed the waiting line theory in detail. The different combinations of waiting line models have been explained with some examples, such as a single server, multiple server, single channel, multiple channel, single phase, multiple phase, limited population, unlimited population, limited population, finite queue length and infinite queue length. This chapter is very helping in decision-making regarding the provision of optimum number of service providers to control the waiting time optimally.

## MULTIPLE-CHOICE QUESTIONS

- Which of the following techniques deals with the problem of supplying sufficient facilities to production lines or individuals that require uneven service?
    - Supply-demand theory
    - PERT
    - Inventory theory
    - Queuing theory
  - What are the two sources of costs in queuing analysis?
    - arrivals and departures
    - arrivals and idleness
    - waiting customers and capacity
    - equipment breakdowns and departures
  - System performance is measured by
    - Average number of customers being refused service
    - Average time customers wait
    - System utilization
    - Both (b) and (c)
  - An M/D/3 system is a system with
    - three channel system, Poisson arrivals, and normally distributed service time
    - D channel system with exponential arrivals and Poisson service times
    - three-channel system with binomial arrival times and normally distributed service times
    - three-channel system, Poisson arrivals and constant service time
  - The utilization for a system represents
    - the average number of customers in the queue
    - the probability that the service facility is being used

- (c) the probability that no one is in the system
  - (d) the steady-state average waiting time

6. Which of the following is NOT the assumption of M/M/1 model?

  - (a) Arrivals are served on a last-in, first-served basis
  - (b) Service times follow the negative exponential probability distribution
  - (c) Arrivals follow the Poisson distribution and come from an infinite population
  - (d) None of these

7. Utility ratio is equal to

  - (a) The ratio of arrival rate of the customers and the service rate
  - (b) The sum of the arrival rate of the customers and the service rate
  - (c) The difference between arrival rate of the customers and the service rate
  - (d) None of these

8. In a single-server queuing model, the average number customers in the system is calculated by dividing the arrival rate by

  - (a) service rate
  - (b) service time
  - (c) difference of service rate and arrival rate
  - (d) sum of service rate and arrival rate

9. The major inputs for analysing a queuing system is

  - (a) the service and arrival rate
  - (b) the nature of the calling population
  - (c) the queue discipline
  - (d) all of the above

10. Which of the following item is NOT a part of the queuing system?

  - (a) arrival rate
  - (b) service facility
  - (c) waiting line
  - (d) activity flow

11. Items may be taken from a queue

  - (a) on a first-come-first-serve basis
  - (b) on a last-come-first-serve basis
  - (c) according to the due date of the item
  - (d) all of the above

12. The length of a queue

  - (a) could be finite
  - (b) could be infinite
  - (c) can constantly change
  - (d) all of the above

13. A cinema hall has a single ticket counter. Arrivals follow a Poisson distribution, while service times follow an exponential distribution. What type of queuing model is exhibited in this problem?

  - (a) M/D/1
  - (b) M/M/2
  - (c) M/G/1
  - (d) M/M/1

14. The customer who arrives at a bank, sees a long line, and leaves to return another time is

  - (a) reneging
  - (b) balking
  - (c) cropping
  - (d) jockeying

15. The customer enters the line, but decides to exit before being served due to long waiting time is known as

  - (a) reneging
  - (b) balking
  - (c) cropping
  - (d) jockeying

## Answers

1. (d)    2. (c)    3. (d)    4. (d)    5. (b)    6. (a)    7. (a)    8. (c)    9. (d)  
10. (d)   11. (a)   12. (d)   13. (d)   14. (b)   15. (a)

# **REVIEW QUESTIONS**

1. What do you mean by waiting line theory? Explain the various arrival behaviour of customers in a system.
  2. Discuss the parameters used for the performance measurement of the waiting line.
  3. Write short notes on Poisson distribution and exponential distribution. Discuss their use in waiting line theory.
  4. What do you mean by system utilization?
  5. Explain the various service patterns used in the waiting line theory.

# **EXERCISES**

1. The arrival rate of passengers at the single-window railway ticket booking counter follows a Poisson distribution and the service time follows an exponential distribution. The arrival rate and service rate are 80 customers and 100 passengers per hour, respectively. Find the utilization of the booking clerk, average number of passengers waiting in queue, average number of passengers waiting in the system, average waiting time per passengers in the queue, average waiting time per passenger in the system, probability of a zero passenger in the system and probability of five passengers in the system.
  2. Cars arrive at a service centre with the arrival rate of 20 cars per hour and service rate of 15 cars per hour. The arrival rate follows the Poisson distribution and service time follows the exponential distribution. The number of waiting space is limited to 8 cars only. Find the utilization of service centre, average number of cars waiting in the system, the effective arrival rate of the cars, average number of cars in the queue, average waiting time in the system and average waiting time in the queue.
  3. A 3PL company deals with transportation of used auto components for reprocessing. The truck driver has to wait for 30 min for unloading the components at reprocessing plant. The cost of waiting is Rs 1000 per hour. A new technology is used to unload the truck load at a constant rate of 20 trucks per hour at the cost of Rs 40 per truck. The truck arrival follows the Poisson distribution at an average rate of 15 trucks per hour. Suggest that whether the new technology should be used or not.
  4. In a workshop, a technician takes care of 8 milling machines. The mean time to failure is 10 hours for each machine and follows the exponential distribution. The mean time to repair the machine is 2 hours and follows the exponential distribution. Find the probability that the technician will be idle, probability that 4 machines will be in the system of rectification or repair, average number of machines

waiting to be repaired, average number of machines waiting to be repaired plus being repaired, and average time of waiting machine in the queue to be repaired.

5. The breakdown rate of the machines in a workshop is 5 per hour and follows a Poisson distribution. Two mechanics are employed to repair the machines. The services rate of the mechanics is 5 machines per hour and also follows the Poisson distribution. Find the utilization of mechanics, probability of the mechanics remaining idle, average number of the machines waiting in queue to be repaired, average number of machines in the system, the average time of the machine waiting in the queue and average time of the machine waiting in the system.
6. The arrival rate of customer at a cinema hall booking counter is 600 per day. The cinema hall has three parallel counters for ticket booking. The service rate of individual counter is 300 customers per day. The arrival rate and service rate follow the Poisson distribution. At a point in time, the maximum number of customers permitted in the system is 30. Find the probability that the booking clerk remaining idle, average number of customers in the queue, average number of the customers in the system, the average waiting time for customer in the queue and average waiting time of the customer in the system.



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# CHAPTER ▶ 19

## Principles of Management

### 19.1 INTRODUCTION

Management is a very broad term and defined in a number of ways. But, all the definitions focus on the effective and efficient utilization of resources to achieve the goal. Management is the process of using organizational resources to achieve the organization's goals by planning, organizing, leading and controlling. In a simple way, management can be defined as an art and science used to get the work done. Some more definitions of management are given below:

Management is the coordination of all resources through the process of planning, organizing, directing and controlling in order to attain stated objectives.

—Henry L. Sisk

Management is the art and science of organizing and directing human efforts applied to control the forces and utilize the materials of nature for the benefit of man.

—ASME

Management is the art of knowing what you want to do and then seeing that it is done in the best and cheapest way.

—F. W. Taylor

#### 19.1.1 Nature of Management

Some important characteristics of management are described below.

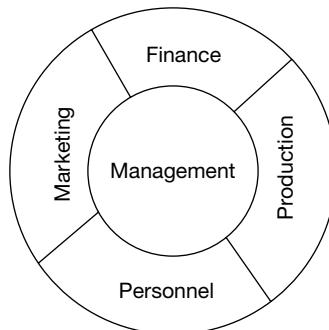
- (a) *Management is an art as well as science:* Management uses theoretical knowledge to manage the activities or task, which is known as art, and at the same time, management uses a number of mathematical/scientific tools and techniques to improve the efficiency of a system, which is a part of science. Thus, we can say that management has two faces of a coin: on one side, it is art and on the other it is science. Management has got scientific principles which constitute the elements of science and skill and talent which are the attributes of art.
- (b) *Management is goal-oriented:* The basic goal of management is to improve the efficiency and economy in the utilization of all the available resources. The effectiveness

of management is measured by the extent to which the established organizational or other goals are achieved.

- (c) *Management is a profession:* Management is used as profession and profession requires specialized knowledge and often, long intensive academic preparation. The essential features of a profession are: a well-defined body of knowledge, restricted entry, service motive, code of conduct, representative professional association.
- (d) *Management is universal:* Management is an essential part of all the activities irrespective of nature. All the different types of organizations or businesses require management regardless of size. The principles of management are applicable in all areas of organized activities. Managers at all levels and in all organizations perform the same basic functions.
- (e) *Management is an integrative force:* The importance of management lies in the coordination; it facilitates among the people in an organization. Management relates the individual goals to organizational goals. It integrates all the different resources.
- (f) *Management is multidisciplinary:* Management consists of knowledge derived from several disciplines such as engineering, sociology, psychology, economics and anthropology. Thus, it is multidisciplinary in nature.
- (g) *Management is a social process:* Human relation is the most important part of management. According to Appley, ‘Management is the development of people not the direction of things. It is the pervasiveness of human element which gives management its special character as a social process.’
- (h) *Management is a continuous process:* The cycle of management continues to operate until its objective or goal is achieved.
- (i) *Management is intangible:* Management is an intangible activity. It cannot be seen directly, but its presence can be felt everywhere in the form of results. However, the managers who perform the functions of management are very much tangible and visible.

### 19.1.2 Scope of Management

Management is applicable in marketing, finance, production and personnel area. The scope of management (Figure 19.1) may be classified into the following areas:



**Figure 19-1:** Scope of management

1. *Production management:* Production management consists of manufacturing activities to produce the right product of the right quality at the right time in the right quantity and at the right cost. It includes the following activities:
  - (a) Product design and development
  - (b) Facility location and layout
  - (c) Production planning and control
  - (d) Sourcing and outsourcing activities
  - (e) Inventory and store management
  - (f) Manufacturing
  - (g) Maintenance management
  - (h) Quality management
2. *Marketing management:* Marketing management refers to recognizing the consumer needs and fulfilling them. It involves the following activities:
  - (a) Need recognition and expectation of consumers
  - (b) Product planning and development
  - (c) Price estimation
  - (d) Selection of market channel and distribution of products
  - (e) Retailing
  - (f) Sales promotion and advertisement
3. *Financial management:* Financial management ensures the provision of the funds at the right time and at reasonable cost. It comprises the following activities:
  - (a) Fund estimation for both long- and short-term needs
  - (b) Selection of appropriate sources of funds
  - (c) Raising the required funds at the right time
  - (d) Proper utilization and allocation of funds
  - (e) Administration of earnings
4. *Personnel management:* Personnel management involves planning, organizing and controlling the selection, recruitment, compensation and integration of human resources of an organization. It consists of the following activities:
  - (a) Manpower planning
  - (b) Recruitment and selection
  - (c) Training
  - (d) Appraisal
  - (e) Promotions and transfers
  - (f) Compensation
  - (g) Employee welfare services
  - (h) Personnel records and research

## **19.2 CLASSICAL SCHOOL OF MANAGEMENT**

Classical management theory was developed during the Industrial Revolution. In those days, managers were searching the way how to train employees or deal with increased labour

dissatisfaction, so they began to test solutions. As a result, the classical management theory was developed to find the ‘one best way’ to perform and manage tasks. This school of thought is made up of two schools:

1. Classical scientific school.
2. Classical administrative school.

### **19.2.1 Classical Scientific School**

Scientific management is devoted to determine the best way to do a job using scientific methods. Frederick W. Taylor, known as the father of scientific management, was the major contributor in the field. Using his principles of scientific management, Taylor was able to define the ‘one best way’ for doing each job.

Frank B. Gilbreth and Lillian M. Gilbreth were inspired by Taylor’s work and proceeded to study and develop their own methods of scientific management. Frank and Lillian Gilbreth focused on increasing worker productivity through the reduction of wasted hand and body motions. They experimented with tools and equipment to optimize work performance and developed the micromotion study to analyse the worker’s motions and optimize performance. They labelled 17 basic motion elements called ‘therbligs’ in order to eliminate waste motions.

Therbligs are guidelines devised by Taylor and others to improve production efficiency are still used in today’s organizations. The therblig is used in the study of motion economy in the workplace. A workplace task is analysed by recording each of the therblig units for a process, with the results used for optimization of manual labour by eliminating unneeded movements. A basic motion element is one of a set of fundamental motions required for a worker to perform a manual operation or task. The set consists of 17 elements, each describing a standardized activity. The detail of therbligs has been already discussed in Chapter 11 (Work Study and Ergonomics).

#### **Taylor’s Four Basic Principles of Scientific Management**

- (a) Development of a true science of management so that the best method for performing each task could be determined.
- (b) Scientific selection of workers so that each worker could be given responsibility for the task for which he/she was best suited.
- (c) Putting a right worker in the right job is not enough; a system of financial incentives is also needed to motivate the workers.
- (d) The functional division of labour between management and workers improves the productivity and promotes friendly cooperation between management and labour.

#### **Drawback of Taylor’s Scientific Management**

The following are the drawbacks of Taylor’s scientific management:

1. Taylor was able to increase the productivity and pay in a number of instances, but workers and unions began to oppose his approach because they feared that working harder or faster would exhaust whatever work was available causing layoffs.

2. Taylor's critics objected to the speed-up condition that placed undue pressure on employees to perform faster and faster.
3. The emphasis on productivity and by extension on profitability led some managers to exploit both the workers and customers.
4. Owing to the above-listed adverse effects, more workers joined unions thus reinforcing a pattern of suspicion and mistrust that clouded labour relations for decades.

## **19.2.2 Classical Administrative School**

Scientific management concentrates on only increasing the productivity, but the classical administrative approach to management concentrates on the total organization. The emphasis is on the development of managerial principles rather than work methods. The main contributors to the classical administrative theory include Max Weber, Henri Fayol, Mary Parker Follett and Chester I. Barnard. These theorists studied the flow of information within an organization and emphasized the importance of understanding how an organization operated.

### **(A) Max Weber's Bureaucracy**

In the late 1800s, Max Weber disliked that many European organizations were being managed on a personal family basis and that employees were loyal to individual supervisors rather than the organization. He believed that organizations should be managed impersonally and that a formal organizational structure, where specific rules were followed, was important. Max Weber gave the theory of bureaucracy that is explained below.

#### ***Characteristics of Bureaucracy***

1. *A well-defined hierarchy:* All positions within a bureaucracy are structured in a way that permits the higher positions to supervise and control the lower positions. There should be a clear chain of command, control and order throughout the organization.
2. *Division of labour and specialization:* All responsibilities, specialties and expertise should be divided among the employees. One should not be a jack of all trades.
3. *Rules and regulations:* There should be a practice to follow standard rules and regulations in the whole organization.
4. *Impersonal relationships between managers and employees:* Managers should maintain an impersonal relationship with employees so that favouritism and personal prejudice do not influence their decisions.
5. *Competence:* Competence of the employees should be the basis for all decisions made in hiring, job assignments and promotions to foster an ability and merit as the primary characteristics of a bureaucratic organization, not the relationship or acquaintance with the employees.
6. *Records:* A bureaucracy needs to maintain records of all its activities.

### **(B) Henri Fayol's 14 Principles of Management**

Henri Fayol, a French mining engineer, developed 14 principles of management based on his management experiences. These principles are described as follows.

1. *Division of work*: Division of work and specialization improves the productivity with the same effort.
2. *Authority and responsibility*: Authority is the right to give orders and the power to exact obedience. Authority creates responsibility for getting the work completed.
3. *Discipline*: Obedience, respect and proper behaviour within an organization are absolutely essential.
4. *Unity of command*: An employee should receive orders from only one superior.
5. *Unity of direction*: Organizational activities must have one central authority and one plan of action.
6. *Subordination of individual interest to general interest*: The interests of one employee or group of employees are subordinate to the interests and goals of the organization.
7. *Remuneration of personnel*: Salaries – the price of services rendered by employees – should be fair and should provide satisfaction to both the employee and the employer.
8. *Centralization*: The objective of centralization is the best utilization of personnel. The degree of centralization varies according to the dynamics of each organization.
9. *Scalar chain*: A chain of authority, that is, scalar chain, exists from the highest organizational authority to the lowest ranks.
10. *Order*: An organizational order for materials and personnel is essential. Right materials and right employees are necessary for each organizational function and activity.
11. *Equity*: In organizations, equity is a combination of kindness and justice. Both equity and equality of treatment should be considered when dealing with employees.
12. *Stability of tenure of personnel*: To attain the maximum productivity of personnel, a stable workforce is needed.
13. *Initiative*: Thinking out a plan and ensuring its success is an extremely strong motivator. Zeal, energy and initiative are desired at all levels of the organizational ladder.
14. *Esprit de corps*: Teamwork is fundamentally important for an organization. Work teams and extensive face-to-face verbal communication encourage teamwork.

### (C) Mary Parker Follett

Mary Parker Follett focused her efforts on ethics, power and leadership. She encouraged employee involvement or participation in decision-making process. She stressed the importance of men rather than methods, a concept developed in scientific management before her time. As a result, she was a pioneer but was often not taken seriously by management scholars of her time. But, times change and her innovative ideas from the past suddenly take on new meanings. These innovative ideas are related to reciprocal relationships in understanding the dynamic aspects of the individual in relationship to others, the principle of integration and power sharing among the people. Her ideas on negotiation, power and employee participation were influential in the development of organizational studies.

### (D) Chester Barnard

Chester Barnard worked as president of New Jersey Bell Telephone Company. He introduced the idea of the informal organization *cliques* (exclusive groups of people) that naturally form within a company. He felt that these informal organization cliques served necessary and vital communication functions of the entire organization and that they could help the organization

accomplish its goals. Barnard felt that it was particularly important for managers to develop a sense of common purpose which encourages greatly the willingness to cooperate. He is credited with developing the acceptance theory of management, which emphasizes the willingness of employees to accept that managers have legitimate authority to act.

### 19.3 SYSTEMS APPROACH

During the 1960s, researchers began to analyse organizations from a systems perspective based on the physical sciences. The systems approach to management propounds that organizations are not self-contained, but they rely on and are affected by factors in their external environment. A system is a set of interrelated and interdependent parts arranged in a manner that produces a unified whole. The two basic types of systems are open system and closed system.

A closed system is not influenced by and does not interact with its environment. An open system interacts with its environment. Using the systems approach, managers envision an organization as a body with many interdependent parts, each of which is important for the well-being of the entire organization. Managers coordinate the work activities of various parts of the organization, realizing that decisions and actions taken in one organizational area will affect other areas.

### 19.4 CONTINGENCY APPROACH

The contingency approach to the management proposes that different organizations require different ways of managing. The contingency approach subscribes to view that the organization recognizes and responds to situational variables as they arise. The contingency approach to management assumes that there is no universal answer to all the questions related to the management because organizations, people and situations vary and change over time. Thus, the right thing to do depends on a complex variety of critical environmental and internal contingencies.

### 19.5 BEHAVIOURAL APPROACH

Organizational behaviour (OB) is the field of study that is concerned with the actions (behaviours) of people at work. The OB research has contributed much of what we know about human resources management and contemporary views of motivation, leadership, trust, teamwork and conflict management. The early advocates of the organizational behaviour approach were Robert Owen, Hugo Munsterberg, Mary Parker Follett and Chester Barnard. Their ideas served as the foundation for employee selection procedures, motivation programmes, work teams and organization–environment management techniques.

#### **Hawthorne Studies**

The Hawthorne studies (also referred to as the observer effect) were the most important contribution to the development of OB. This series of experiments were conducted from 1924 to the early 1930s at western Electric company's Hawthorne works in Cicero, Illinois, USA. Initially, these were devised as a scientific management experiment to assess the impact of changes in various physical environment variables such as illumination of light on the employee productivity. After Harvard professor Elton Mayo and his associates joined the Hawthorne studies

as consultants, other experiments were included to look at redesigning jobs, make changes in workday and workweek length, introduce rest periods and introduce individual versus group wage plans. It was observed that there is no effect of change in illumination of light on worker's productivity, significantly. The main factors of consistant productivity was manager's attention towards the workers.

The researchers concluded that social norms or group standards were key determinants of individual work behaviour. Although criticized on procedures, analyses of findings and conclusions, the Hawthorne studies stimulated interest in the human behaviour in organizational settings. In the present context, the behavioural approach assists managers in designing jobs that motivate workers, in working with employee teams and in facilitating the flow of communication within organizations.

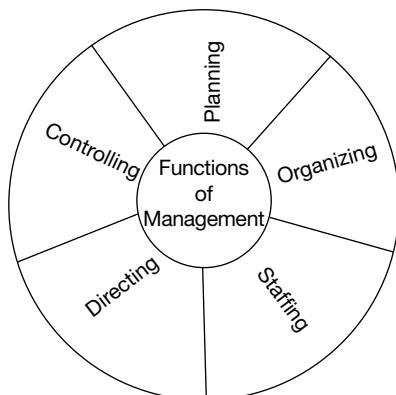
## 19.6 QUANTITATIVE APPROACH

The quantitative approach to management, also known as operations research or decision science, uses quantitative techniques to improve decision-making. This approach includes application of statistics, optimization models, information models and computer simulations. The quantitative approach originated during World War II, as mathematical and statistical solutions to military problems were developed for wartime use. Today, the quantitative approach has contributed most directly to managerial decision-making, particularly in planning and controlling. The availability of sophisticated computer software programs has made the use of quantitative techniques quite feasible for managers.

## 19.7 FUNCTIONS OF MANAGEMENT

The five major functions of management, as depicted in Figure 19.2, are as follows:

1. Planning
2. Organizing



**Figure 19-2:** Functions of management

3. Staffing
4. Directing
5. Controlling

## **Planning**

Planning is a preparation or preproduction phase. In other words, it outlines what is to be done, when it is to be done, how it is to be done and why it is to be done. Planning involves problem-solving and decision-making. Whenever there is a problem, the manager should know how to solve it and what are the alternatives? The main part of planning consists of setting goals and schedules of activities.

## **Organizing**

After planning, the management has to organize organizational activities. Managers determine what activities are required to achieve organizational objectives and how these activities should be divided among the departments and employees. The production work is given to the production department; finance department is assigned the work of arranging funds; personnel department has to recruit and train the people with required skills, and the sales department is to take care of sales. The organizing function of management involves creating departments and defining the duties and responsibilities of people in different positions within each department.

## **Staffing**

Staffing is concerned with hiring/employing of people for the various activities to be performed. The objective of staffing is to ensure the right type of people for different positions. Staffing includes the function of recruitment, selection and placement of employees.

## **Directing**

The directing function of management includes leading the subordinates, supervising their performance, communicating information and motivating them. A manager should be a good leader. He/she should be able to instruct and guide the people in the work assigned to them. He/she should keep a watch on the performance of his/her subordinates and help them out whenever they come across any difficulty. The communication, that is, the exchange of information between managers and workers, should take place in clearly understandable words and without delay. Information should flow from managers to workers and from workers to managers at the right time. Managers should also understand the needs of their subordinates and accordingly inspire them in their work.

## **Controlling**

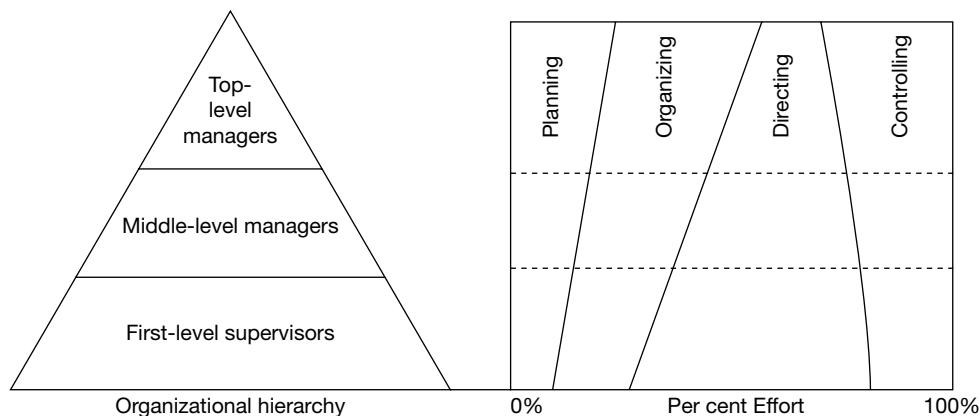
The management has to ensure that the performance of work is in accordance with the plans. The controlling means to compare the actual performance of the employee with the standard one. If differences are noticed, corrective steps are taken which may include revision of plans, improvement in the division of work and providing better guidance.

## 19.8 LEVELS OF MANAGEMENT

We have already stated how managers jointly perform the various functions. The managers at different levels perform different types of duties. Some managers decide about the objectives of the business as a whole; some managers perform functions to accomplish these objectives in different departments, like production and sales; and some managers are concerned with the supervision of day-to-day activities of workers. The various levels of management are as follows:

1. Top-level management
2. Middle-level management
3. Lower-level management (First-level supervisors)

Figure 19.3 depicts the functions, positions and relations of different levels of management.



**Figure 19.3: Levels of management**

### Top-level Management

The top-level management is generally occupied by governing body or ownership group. They are at strategic level and deals with strategy and policy-related matters. The major functions of the top-level management are as follows:

1. To make a corporate plan for the entire organization covering all areas of operations.
2. To decide upon the matters which are vital for the survival, profitability and growth of the organization, such as the introduction of a new product, shifting to a new technology and opening a new plant.
3. To decide corporate goals.
4. To decide the structure of the organization, creating various positions therein.
5. To exercise overall managerial control through the process of reviewing overall financial and operating results.
6. To make decisions regarding disposal and distribution of profits.

7. To select key officials and executives for the organization.
8. To coordinate various sub-systems of the organization.
9. To maintain liaison with outside parties having a stake in the business, such as government, trade union and trade associations.
10. To formulate basic policies and providing direction and leadership to the organization as a whole.

### **Middle-level Management**

The middle-level management exists between strategic and operational levels. They exist at the tactical level. They are responsible to find the way how to convert the strategy or policy into operations. It consists of departmental managers, deputy managers, foremen and administrative officers. These executives are mainly concerned with the overall functioning of their respective departments. They act as a link between top- and lower-level managers. The activities of middle-level managers centre on determining departmental goals and devising ways and means for accomplishing them. The main functions of middle-level managers are enumerated as follows:

1. To prepare departmental plans covering all activities of the respective departments within the basic framework of the corporate plan.
2. To establish departmental goals and to decide upon various ways and means of achieving these goals to contribute to organizational goals.
3. To perform all other managerial functions with regard to departmental activities for securing smooth functioning of the entire department.
4. To issue detailed orders and instructions to lower-level managers and coordinate the activities of various work units at lower level.
5. The middle-level managers explain and interpret policy decisions made at the top level to the lower-level managers.

### **Lower-level or Supervisory-level Management**

The lower-level management is known as supervisory or operational management. They are responsible to implement the decision taken by middle-level managers. It consists of factory supervisors, superintendents, foremen, sales supervisors and accounts officers. They are concerned with personal oversight and direction of operative employees. They directly guide and control the performance of the rank and file workers. They issue orders and instructions and guide day-to-day activities. They also represent the grievances of the workers to the higher levels of management. The supervisory management performs the following functions:

1. Planning of day-to-day work
2. Assignment of jobs and issuing orders and instructions
3. Supervising and guiding workers
4. Maintaining close personal contacts with workers to ensure discipline and teamwork
5. Evaluating operating performance
6. Sending reports and statements to higher authorities
7. Communicating the grievances and suggestions of workers to higher authorities

## 19.9 SKILLS OF MANAGER

In modern business, the management job requires several skills to manage an organization successfully. These skills expected from managers have been classified into three categories, namely, technical, human and conceptual skills.

### Technical Skills

Technical skills refer to the ability and knowledge of using the equipment, technique and procedures involved in performing various managerial tasks. First, a manager must know which skills should be employed in his/her organization and be familiar enough with their potentiality to ask discerning questions to his/her technical advisors. Second, a manager must understand the role of each skill employed and the interrelationship between different skills.

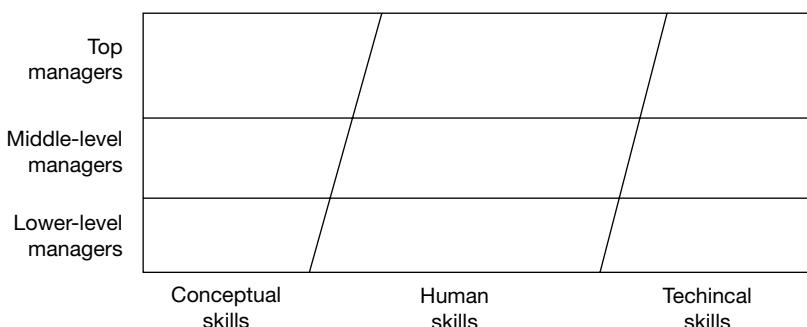
### Human Skills

Technical skills involve mastery of things, whereas human skills are concerned with understanding the people. Human skills imply the ability to work effectively with other people, both as an individual and in a group. These are required to win the cooperation of others and to build effective work teams. One should have a sense of feeling for others and a capacity to look at things from others' point of view.

### Conceptual Skills

These skills refer to the ability to visualize the entire picture or to consider a situation in its totality. Such skills help the manager to conceptualize an environment, analyse the forces working in a situation and take a broad and far-sighted view of the organization. Conceptual skills also include the competence to understand a problem in all its aspects and to use original thinking in solving the problem. Such competence is necessary for rational decision-making.

Thus, technical skills deal with jobs, human skills with persons and conceptual skills with ideas. These three types of skills are interrelated. However, the proportion and relative significance of these skills vary with the level of management as shown in Figure 19.4. Technical skills are most important at the supervisory or operating level where a close understanding of job



**Figure 19.4:** The relationship between the level of management and skills required

techniques is necessary to guide workers. As one moves up the management hierarchy, technical skills become less important. The higher-level managers deal with subordinate managers. For them, specialized technical knowledge is comparatively less important. Conceptual skills are very important for the top management in formulating long-term plans, making broad policy decisions and relating the organization to its industry and the economy.

## 19.10 MANAGERIAL ROLES

Managerial roles fit into three broad categories as described below.

### Interpersonal Role

Managers are expected to coordinate and interact with employees and provide direction to the organization. Here comes into play their interpersonal role. Three parts of the manager's interpersonal role are as follows:

1. *Figurehead role*: It symbolizes the organization and what it is trying to achieve.
2. *Leader role*: The manager is expected to train, counsel, mentor and encourage high employee performance.
3. *Liaison role*: The manager should coordinate with and link people inside and outside the organization to help achieve organizational goals.

### Informational Role

This role is associated with the tasks needed to obtain and transmit information for management of the organization. Three parts of the informational role are as follows:

1. *Monitor role*: Analysing information from both the internal and external environments.
2. *Disseminator role*: Transmitting information to influence attitudes and behaviour of employees.
3. *Spokesperson role*: Using information to positively influence the way people in and out of the organization would respond to it.

### Decisional Role

The decisional role is associated with the methods managers use to plan strategy and utilize resources to achieve organizational goals. Four parts of the decisional role are as follows:

1. *Entrepreneur role*: Deciding upon new projects or programs to initiate and invest.
2. *Disturbance handler role*: Assuming responsibility for handling an unexpected event or crisis.
3. *Resource allocator role*: Assigning resources between functions and divisions, setting budgets of lower-level managers.
4. *Negotiator role*: Seeking to negotiate solutions between other managers, unions, customers or shareholders.

## 19.11 THEORY OF MOTIVATION

Motivation is a psychological process that causes the arousal, direction and persistence of voluntary actions that are goal directed. It is the process by which a person's efforts are energized, directed and sustained towards attaining a goal. Motivation is the result of an interaction between the person and a situation; it is not a personal trait. Motivation works best when individual needs are compatible with organizational goals. There are following early theories of motivation:

- (a) Maslow's Hierarchy of Needs
- (b) MacGregor's Theories X and Y
- (c) Herzberg's Two-Factor Theory

### 19.11.1 Maslow's Hierarchy of Needs

Maslow's hierarchy of needs is shown in Figure 19.5. The needs are classified into five categories: physiological, safety, social, esteem and self-actualization. Physiological needs include basic requirements of the human body: food, water, sleep, sex, etc. Safety needs include desires of a person to be protected from physical and economic harm. Social or belongingness and love needs include a desire to give and receive affection, be in the company of others, etc. Esteem needs include self-confidence and sense of self-worth, esteem from others, self-esteem (feeling of self-confidence and self-respect). Self-actualization needs include desire for self-fulfilment.



Figure 19-5: Maslow's hierarchy of needs

### 19.11.2 MacGregor's Theories X and Y

Theory X assumes that the workers have little ambition, dislike work, avoid responsibility and require close supervision. According to the Theory X, managers believe the workers are irresponsible and resistant to change, dislike work, lack ambition and prefer to be led. Theory Y assumes that workers can exercise self-direction, desire responsibility and like to work. According to the Theory Y, managers believe workers are willing to work, imaginative and creative, self-directed, capable of self-control and accept responsibility. The main requirement is appreciation and motivation.

### 19.11.3 Herzberg's Two-Factor Theory

Job satisfaction and job dissatisfaction are created by two factors: hygiene and motivators. Hygiene or extrinsic (environmental) factors create job dissatisfaction and motivators or intrinsic (psychological) factors create job satisfaction. Hygiene factors are supervision, company policy, relationship with supervisor, working conditions, salary, relationship with peers, personal life, relationship with subordinates, status, security, etc. Motivators are achievement, recognition, work itself, responsibility, advancement, growth, etc.

## 19.12 ADMINISTRATION AND MANAGEMENT

Many researchers like Henri Fayol, William Newman, Chester Barnard, George Terry, Louis A. Allen, Koontz and O' Donnell make no distinction between management and administration. According to Newman, management or administration is 'the guidance, leadership and control of the efforts of a group of individuals towards some common goals'. According to Fayol, all undertakings require the same functions and all must observe the same principles. There is one common science which can be applied equally well in public and private affairs. Therefore, the distinction between administration and management is superfluous or academic. In actual practice, the two terms are used interchangeably. The term 'administration' is more popular in government and other public organizations while the term 'management' is more commonly used in the business world where economic performance is of primary importance. However, some basic differences between administration and management can be observed regarding nature, type of work, level of authority, influencing factors, the direction of human efforts, main function and used. These differences are summarized in Table 19.1.

**Table 19-1:** Distinction between administration and management

Points of distinction	Administration	Management
1. Nature	Thinking function	Executive function
2. Type of work	Determination of major objectives and policies	Implementation of policies
3. Levels of authority	Top-level function	Normally, middle- and lower-level function
4. Influencing factors	Influenced by public opinion and outside factors	Influenced by the organization's policy and objectives
5. Direction of human efforts	Not concerned with the direction of human efforts	Concerned with the direction of human efforts for implementation management policies
6. Main function	Planning and control	Directing and organizing
7. Skills required	Conceptual and human skills	Technical and human skills
8. Uses	Government and public sector	Business organizations
9. Illustrations	Commissioners, registrars, vice-chancellors	Managing directors, managers, supervisors



## SUMMARY

In this chapter, evolution of the concepts in management has been discussed in detail. The nature and scope of management have been explored. Various schools of management have been highlighted. Different functions, skills, roles and levels of management, theory of motivation have been discussed and explained. Also, administration and management have been differentiated briefly.

## MULTIPLE-CHOICE QUESTIONS

1. The management has following nature
  - (a) It is an art as well as science.
  - (b) It is goal-oriented.
  - (c) It is a profession.
  - (d) All the above.
2. In Wealth of Nations, Adam Smith described the breakdown of jobs into narrow and repetitive tasks, which is called as:
  - (a) assembly lines
  - (b) work denomination
  - (c) division of labour
  - (d) greatest common factor of work
3. Frederick W. Taylor advocated which of the following management principles?
  - (a) Work and responsibility should be divided almost equally between managers and workers.
  - (b) Workers should perform all work, while the management should maintain responsibility for the work performed.
  - (c) Managers should perform more work than workers, because managers are generally more skilled.
  - (d) Workers can be highly productive even if they are randomly selected for a job.
4. Frank and Lillian Gilbreth were the first researchers to utilize motion pictures to the study of
  - (a) reactions of workers in group settings
  - (b) motion of the body parts
  - (c) workers reactions to pay increases
  - (d) groups of workers in tense situations where they are assigning each other tasks
5. Which of the following phrases of F. W. Taylor is most associated with scientific management?
  - (a) management relations
  - (b) one best way
  - (c) supply and demand
  - (d) quality control
6. The primary issue that motivated Taylor to create a more scientific approach to management was
  - (a) worker efficiency
  - (b) worker effectiveness
  - (c) worker absenteeism and turnover
  - (d) workplace safety
7. Which of the following is NOT one of Fayol's principles of management?
  - (a) division of work
  - (b) unity of command
  - (c) discipline
  - (d) equality

## Answers

1. (d)      2. (c)      3. (a)      4. (b)      5. (b)      6. (a)      7. (d)      8. (b)      9. (a)  
10. (d)     11. (c)     12. (a)     13. (d)     14. (a)     15. (a)

## REVIEW QUESTIONS

1. Define the term 'management'. Discuss the nature of management.
2. What is the scope of management? Discuss in brief.
3. Explain the four basic principles of scientific management given by F. W. Taylor. What are the limitations of F. W. Taylor's principle of scientific management?
4. What do you mean by bureaucracy? Discuss its characteristics.
5. Discuss the Henri Fayol's 14 principles of management.
6. Write the contributions of Mary Parker Follet and Chester Barnard in the field of management.
7. Write short notes on (a) system approach, (b) contingency approach and (c) Hawthorne study.
8. Explain the five basic functions of management.
9. Write the functions of top-level, middle-level and lower-level management.
10. What are the various types of skills required for a manager.
11. Explain the roles of a manager.
12. Write the short notes on the following motivation theory
  - (a) Maslow's hierarchy of needs
  - (b) MacGregor's theories X and Y
  - (c) Herzberg's two-factor theory



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# CHAPTER ▶ 20

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## Firm's Ownership, Organizational Design and Structure

### 20.1 INTRODUCTION

A firm is a group of people located in any premises or area, who do work and transform raw materials into goods and services with the help of tools and sell them. The firm may be industrial, commercial or financial. Firm's size and characteristics depend on the type of ownerships. In this chapter, we will discuss about the type of a firm's or business ownership and different structures of an organization. Some major types of firm's ownerships are sole partnership, general partnership, limited partnership, limited liability partnership, corporation, joint stock company (private and public company), cooperative society, etc.

### 20.2 SOLE PROPRIETORSHIP

A sole proprietorship is the simplest form of business ownership. It is owned by a single person. The owner enjoys alone with the business's profits and losses. There is no liability protection for the owner. The owner is liable to pay all the debts. If the individual is sued and loses, the business and personal property may be seized to pay obligations.

Characteristics of sole partnership are enumerated as follows:

1. Sole proprietorships do not have perpetuity.
2. The proprietor may sell or quit the business.
3. The business does not need to be registered with the state.
4. The trademarks can be registered with the Secretary of State's office if desired.
5. A federal employer's identification number is not required as long as there are no employees.
6. The business income is taxed at the sole proprietor's individual tax rate.

The following are advantages and disadvantages of sole partnership:

#### **Advantages**

1. It is easy to start the business.
2. No documents are required to file with the state.
3. There is only one level of taxation.
4. Decision-making is easy.
5. One can discontinue his/her business at his/her will.

***Disadvantages***

1. No tax breaks are provided for company benefits.
2. The owner is liable for all debts incurred, i.e. unlimited liability.
3. The amount of capital investment one can raise is limited.

## **20.3 COOPERATIVE SOCIETY**

A business formed by a group of people to obtain services or products more effectively and economically than they could get individually is called a cooperative and jointly known as cooperative society. Members of the cooperative own finance and operate the business for mutual benefit. Cooperative society may be following types: consumer's cooperative society, producer's cooperative society, cooperative marketing society, cooperative credit society, cooperative farming society and housing cooperative society.

Characteristics of cooperative societies:

1. In cooperatives, net income paid to owners is taxed as personal income.
2. Ownership benefits can be distributed in proportion to use.
3. Control is democratic with one vote per patron.
4. There is limited return on equity capital.
5. All income is distributed to members on a participation basis.
6. Cooperative organizations do not come under state or federal income tax liability.
7. Like a sole proprietorship, partnership, or corporation, a cooperative must operate as a business with complete planning, management, and written policies.

***Advantages of Cooperative Societies***

1. There is requirement of smaller-than-usual capital investment.
2. Authority is divided among the members or owners.
3. There are possible difficulties in cancelling the membership.
4. There is an economical purchasing of supplies due to economy of scale.
5. Expenses are shared by all the members of trade shows/exhibits.

***Disadvantages of Cooperative Societies***

1. There may be some losses of independence.
2. Working capital is less than the required.
3. Decision-making is complex in comparison to sole partnership.
4. Quality control may become difficult.

## **20.4 PARTNERSHIP**

### **20.4.1 General Partnership**

A general partnership is formed by two or more persons. Similar to a sole proprietorship, the partners are not separate from the business. Unless agreements/provisions have been established, each individual is responsible for all of the partnership's debt. Also, each partner can incur debt

on behalf of the business unless other provisions have been made. Each partner shares in the losses and profits of the business. Partnerships do not have perpetuity. When a partner leaves or dies, the business ends. As with a sole proprietorship, the general partnership does not need to be registered with the Secretary of State's office. The partnership does need a Federal Employer's Identification Number even if there are no employees. Characteristics of a general partnership are very similar to sole partnership.

#### ***Advantages of General Partnership***

- (a) It is relatively easy to create.
- (b) Only one level of taxation is subjected.

#### ***Disadvantages of General Partnership***

- (a) There are no tax breaks for company benefits similar to sole partnership.
- (b) Each partner is liable for debts incurred.
- (c) In case of death or quit, business may end.

### **20.4.2 Limited Partnership**

One main difference between the general partnership and a limited partnership is that the limited partners' risk is reduced to the amount contributed to the partnership. The general partnership does not need any special registration, but a limited partnership certificate needs to be filed with the Secretary of State's office. Tax reporting and payment is carried out the same way as a general partnership. There are certain advantages and disadvantages of limited partnership as discussed below:

#### ***Advantages of Limited Partnership***

1. There is limited risk to some partners.
2. It is relatively easier to create.
3. One level of taxation is possible.

#### ***Disadvantages of Limited Partnership***

1. There is liability for debts incurred by partners.
2. The business owners are taxed as corporations under certain circumstances.
3. There are no tax breaks for company benefits.
4. There is no perpetuity of business.

### **20.4.3 Limited Liability Partnership**

Limited liability partnerships are similar to other partnerships except that the partners have limited personal liability due to negligence, malpractice and conduct of partners not under immediate and direct control of a partner. The partnership is not free from liability due to debts or acts of partners under direct control of a partner. A limited liability partnership is designed to help the professionals who operate as a partnership and desire the limited liability protection afforded.

## 20.5 CORPORATION

In a corporation, articles of incorporation are required to be filed with the state. The name of the business is followed by the word ‘incorporated’ or ‘Inc.’ to indicate that it is a corporation. A corporation is able to buy and sell property, sue and be sued, and protect its owners from liability. The owners, called shareholders, are individuals who own shares of the corporation’s stock. Corporations have perpetuity because if an owner, shareholder, dies, the business does not cease to exist. There are two types of corporations: C-corporations and S-corporations.

C-corporations must pay tax on its income. In addition, taxes must be paid on dividends paid to shareholders in a C-corporation. Because of this, earnings are taxed at two different levels. However, a C-corporation may be able to eliminate one level of taxation if it does not pay dividends or reduces its profit to zero by paying higher to its employees.

S-corporations are taxed at the individual level. As a result, there is only one level of tax instead of two. However, only certain corporations can be S-corporations. To qualify as an S-corporation, the corporation cannot have over 75 shareholders and have only one type of stock. S-corporation shareholders can only be individuals, estates, and certain trusts. Shareholders cannot be non-resident aliens. There are following advantages of a corporation:

### **Advantages**

1. Owners liability is limited to amount invested.
2. There is perpetuity of business.
3. Corporation has the ability to buy and sell assets.
4. S-corporation has one tax level.
5. Cash can be raised through sale of stock.
6. There may be one or more owners.
7. Business is own legal entity.

### **20.5.1 Limited Liability Company**

Like a corporation, articles of organization creating the Limited Liability Company (LLC) must be filed with the Secretary of State. The members (owners) of the LLC should create an operating agreement with the rules that the business will be operated. The operating agreement also states the duties and rights of each member. A LLC is a separate legal entity like a corporation. As such, it is capable of buying and selling assets, as well as to sue and be sued. In addition, an LLC is not a perpetual business. A LLC either exists for a fixed period of time or until a member leaves, dies, or is prematurely terminated. A LLC is also deemed terminated when 50 per cent or more of the total interests are exchanged within a 12-month period.

In a general partnership, all members are personally liable for the actions of the business. Limited partnerships require that at least one person must be liable. A LLC does not require anyone to be personally liable for the actions of the business except for foreign LLCs. There are two basic rights afforded to LLC members. The first is the right to receive some form of compensation. This right can usually be transferred unless the LLC agreement states otherwise.

The right to vote and take part in management is the second right of an LLC. This right is not as easily transferred as the first right.

For federal tax purposes, the Internal Revenue Service (IRS) classifies most LLCs as partnerships. However, there are some exceptions. The business will be automatically classified as a corporation by the IRS if any of the following apply: the business is organized as a corporation, the business is an association, a joint-stock company or joint-stock organization, an insurance company, a bank with a state charter that is federally insured, state owned. Publicly traded, or has certain foreign businesses. The LLC will be taxed as a partnership unless any of the following apply: there is only one member, the business is not domestic, the LLC chooses to be classified as a corporation, or is publicly traded. For state tax purposes, Kansas classifies LLCs under their federal classification.

Limited liability companies must use accrual accounting methods if any of the following apply: the LLC is classified as a corporation for tax purposes, has a C-corporation as a member, is classified as a tax shelter, is a syndicate, or offers public securities. LLC may have following advantages and disadvantages:

### ***Advantages of LLC***

1. It is a separate legal entity.
2. There is limited liability for members.
3. The company is taxed as partnership under certain circumstances.
4. Members accept profits from business and participate in the management of the business.

### ***Disadvantages of LLC***

1. The company is taxed as a corporation under certain circumstances.
2. The members must accept losses.
3. There is no perpetuity of business.

## **20.6 JOINT HINDU FAMILY BUSINESS**

It is a form of commercial organization where a business is owned by the members of a Hindu family living jointly. When the business continues from one generation to another in a Hindu family, it becomes a joint Hindu family business. A business where a person becomes a member only by virtue of birth in the Hindu family is called joint Hindu family business. The joint Hindu family business is a distinct form of business organization existing only in India. It is an enlarged form of sole trading concern. The business comes into existence by the operation of the Hindu Law. Here the head of the family manages the business. He is known as Karta and has unlimited liability.

The Hindu Women's Rights to Properties Act, 1937: The act has given the same rights to a widow as that to a male owner. She too can participate in the business and can demand partition.

Hindu Succession Act, 1956: The act extends the line of co-parcency interest to the female members born in a joint Hindu family.

## 20.6.1 Characteristics of Joint Hindu Family Business

The features of a joint Hindu family business are discussed below as:

1. *Limited capital*: The business has to depend upon the savings of the family (Karta and co-parceners). Borrowing is possible from banks, friends and relatives, but again the amount would be limited.
2. *Limited liability of co-parceners*: The liability of the co-parceners are limited to the extent of their share in the family business, i.e. their private property cannot be used to pay the debts of the firm.
3. *Unlimited liability of Karta*: The liability of the Karta is unlimited, i.e. if the assets of the business are insufficient to pay the debts of the firm, Karta's private property can be used to pay off the debts.
4. *Joint ownership*: The business is jointly owned by all the members of a Joint Hindu Family. Three successive generations can inherit the business by virtue of their birth in the family.
5. *Sole management and control of Karta*: The head of the family is known as Karta. He is the sole manager of the business. He has the right to enter into contracts on behalf of the other members. The co-parceners have no right to interfere in the activities of the Karta. But if they disapprove his activities, they can demand partition of the family business.
6. *Flexibility of operations*: This type of business offers a good deal of flexibility in business operations. The Karta can expand or contract the business, change the line of business or even closedown the business if the situation so demands.
7. *Continuity and stability*: This form of business is not dissolved due to the death, insanity or insolvency of a co-parcener or of the Karta. The business is continuous in nature and stable in existence.
8. *Business secrecy*: There is a great deal of business secrecy in the organization. The business secrets are known to the co-parceners in general and Karta in particular.
9. *Minimum government regulation*: The business is subjected to least government regulations. The government has not laid any rules and regulations over its working.
10. *Quick decision-making*: Quick decisions and prompt actions are possible in this type of business as all the powers are in the hands of Karta and he is under no obligations to consult the co-parceners.
11. *Limited managerial skills*: The Karta and the co-parceners may lack the necessary managerial skills that are required in today's competitive business world.
12. *Lack of economies of scale*: Due to the limited scale of operation, bulk buying and selling are normally not possible. Thus, the business may not be able to obtain economies of scale.
13. *Weak bargaining power*: A joint Hindu family purchases goods or raw materials on a small scale from wholesalers leading to weak bargaining power. Secondly, they may not have the skills of bargaining. Thus, they may not be able to obtain competitive terms.
14. *Close contact with customers*: Karta and the co-parceners have close contacts with the customers. Close contract with customers to help them to know their likes, dislikes, taste and preferences and to serve them accordingly.

15. *Close contact with employees:* Karta and the co-parceners have close contacts with the employees. Close contact with employees helps in avoiding frictions and conflicts and leads to better relations.
16. *No legal status:* Like sole trading concern, joint Hindu family business lacks legal status. Registration of this type of business is not compulsory. The members and the firm do not have separate legal entity.

## 20.7 JOINT STOCK COMPANY

A joint stock company is a voluntary association of persons generally formed for undertaking some big business activity. It is established by law and can be dissolved by law. The company has a separate legal existence, so that even if its members die, the company remains in existence. Its members contribute money for some common purpose. The money so contributed, constitutes the capital of the company. The capital of the company is divided into small units called shares. Since members invest their money by purchasing the shares of the company, they are known as shareholders and the capital of the company is known as share capital.

### 20.7.1 Characteristics of Joint Stock Company

There are following characteristics of joint stock company:

1. *Formation:* The formation of a joint stock company is time-consuming and it involves the preparation of several documents and compliance of several legal requirements before it starts its operation.
2. *Independency:* A company exists independent of its members. It can make contracts, purchase and sell things, employ people and conduct any lawful business in its own name. It can sue and can be sued in the court of law.
3. A shareholder cannot be held responsible for the acts of the company.
4. *Common seal:* Since a company has no physical existence, it must act through its Board of Directors. But all contracts entered by them shall have to be under the common seal of the company. This common seal is the official signature of the company. Any document with the common seal and duly signed by an officer of the company is binding on the company.
5. *Perpetual existence:* The company enjoys continuous existence. Death, lunacy, insolvency or retirement of the members does not affect the life of the company. It is created by law, and it can only be dissolved by law.
6. *Limited liability of members:* The company form of business is able to attract large number of people to invest their money in shares because it offers them the facility of limited risk and liability. A shareholder can be held liable only to the extent of the face value of the shares he holds, and if he/she has already paid it, which is normally the case, he cannot be asked to pay any further amount.
7. *Transferability of shares:* The members of the company (public company) are free to transfer the shares held by them to others as and when they like. They do not need the consent of other shareholders to transfer their shares.

8. *Membership*: To form a joint stock company, a minimum of two members are required in case it is a private limited company and seven members in case of public limited company. The maximum limit is 50 in the case of private limited company. There is no maximum limit of membership for a public limited company.
9. *Democratic management*: Since the people of different categories and areas contribute towards the capital of a company. So, it is not possible for them to look after the day-to-day management of the company. They may take part in deciding the general policies of the company, but the day-to-day affairs of the company are managed by their elected representatives, called Directors.

## 20.7.2 Types of Joint Stock Companies

There are two types of joint stock company: *private company and public company*.

Private company under the Companies Act, 1956, by ‘Private Company’, we mean a company, which has the following *features*:

1. It cannot have more than 50 members. Employees of the company are not included in this.
2. It cannot invite the public to purchase its shares and debentures through open invitation.
3. It restricts the rights of the members to sell or transfer their shares.
4. It must have a minimum paid up share capital of one lakh rupees.

The *private companies* have to follow all these conditions noted above. It is compulsory for these companies to write ‘Private Limited’ after their names. The ownership of these companies is confined only to well-known selected persons. It requires minimum of two persons to start a private limited company. Usually, whenever partnership firms are in need of more money to expand their business, they convert themselves into private Companies. It may be noted that private companies are exempted from various regulations of the Companies Act. In fact, they combine the advantages of both the company and the partnership forms of business organization.

A public company must have the following *features*:

1. It can invite the public to subscribe to its shares and debentures by open invitation.
2. A minimum of seven members is required to establish a public company. There is no limit on the maximum number of its members.
3. There is no restriction on the transfer of shares, i.e. the shareholders are free to sell their shares to the public.
4. The public company must have a minimum paid up capital of 5 lakh rupees.
5. A public company must write public limited or simply limited after its name. Reliance Industries Limited (RIL), Bajaj Auto Limited, Hindustan Lever Limited (HLL), Steel Authority of India Limited (SAIL) are some examples of public companies.

In addition to the type of companies, there may be some other types of companies like government company, statutory company, chartered company, foreign company, Indian company, multinational corporation, holding company, and subsidiary company. Let us have a brief idea about all these.

1. *Government company*: Any company in which at least 51 per cent of the capital is invested by the Government is known as Government Company. Examples: Indian Telephone Industry (ITI), Bharat Heavy Electricals Limited (BHEL), etc.
2. *Statutory company*: A company created by a special Act of Parliament or state legislature is termed as statutory company. Examples: Life Insurance Corporation of India (LIC), Securities Exchange Board of India (SEBI), etc.
3. *Chartered company*: A company created under a special charter granted by the king or queen of England. Example: East India Company.
4. *Foreign company*: A company which is incorporated in a country outside India and having business operation in India, is known as Foreign Company. Example: Citibank, G.E. Capital, Honda Motors, etc.
5. *Indian/Domestic company*: A company registered in India as per the Indian Companies Act is known as the Indian/Domestic Company. Examples: Associated Cement Company (ACC), Tata Iron and Steel Company (TISCO), etc.
6. *Multinational corporation (Company)*: A company which is registered in one country but carries on business in a number of other countries. A separate section (20.8) of multinational companies is used to discuss in detail.
7. *Holding and Subsidiary Company*: If a company controls another company, the controlling company is termed as 'Holding Company' and the company so controlled is called a 'subsidiary company'.

### **Differences between Private and Public Companies**

The following are some differences between the private and the public companies (Table 20.1):

**Table 20-1:** Comparison between private and public companies

<b>Basis of distinction</b>	<b>Private company</b>	<b>Public company</b>
1. Minimum number of members required	Two	Seven
2. Maximum number of members	50	Unlimited
3. Minimum paid-up capital	Rs 1.00 Lakh	Rs 5.00 lakh
4. Identification	Use Pvt. limited as suffix.	Use limited as suffix.
5. Share transfer	No provision for share transfer.	Share can be transferred to other's name.
6. Invitation to public to purchase share	Pvt. limited cannot give open advertisement to public to purchase share.	Public company can give open advertisement to public to purchase share.
7. Commencement of business	Private company can be immediately started just after incorporation.	Public company cannot be immediately started just after incorporation. It has to a certificate to start or commencement of business.

### 20.7.3 Merits of Joint Stock Company

Joint stock company has the following merits:

1. *Large resources:* A joint stock company can raise large financial resources because of its large number of members and it can raise funds through debentures, public deposits, loans from financial institutions without much difficulty.
2. *Limited liability:* In a joint stock company, the liability of its members is limited to the extent of shares held by them.
3. *Perpetual existence:* A company is an artificial person created by law and possesses an independent legal status. It is not affected by the death, insolvency, etc. of its members. Thus, it has a perpetual existence.
4. *Benefits of large-scale operation:* The joint stock company is the only form of business organization which can provide capital for large-scale operations. It results in large-scale production consequently leading to increase in efficiency and reduction in the cost of operation.
5. *Liquidity:* The transferability of shares acts as an added incentive to investors as the shares of a public company can be traded easily in the stock exchange. The public can buy shares when they have money to invest and convert shares into cash when they need money.
6. *Professional management:* Companies, because of the complex nature of their activities and large size of business, require professional managers at every level of the organization.
7. *Research and development:* A company generally invests a lot of money on research and development for improved processes of production, designing and innovating new products, improving quality of product, new ways of training its staff, etc.
8. *Tax benefits:* Although the companies are required to pay tax at a high rate, in effect, their tax burden is low as they enjoy many tax exemptions under the Income Tax Act.

### 20.7.4 Demerits of Joint Stock Company

In spite of several merits of a joint stock company has some demerits as discussed below:

1. *Difficulty in formation:* The formation of a company involves compliance with a number of legal formalities under the companies Act and compliance with several other rules and regulations framed by the government from time to time.
2. *Control by a group having a majority of shares:* Theoretically, a company is supposed to be managed by trained and experienced Directors. But, practically, this is not exactly true in many cases. Most of the companies are managed by the Directors belonging to the same family. Since most of the shareholders are widely dispersed, they have an indifferent attitude towards the management of the company. The shareholders holding a majority of the shares take all decisions on behalf of the company.
3. *Excessive government control:* A company is expected to comply with the provisions of several acts. Noncompliance with these, invites heavy penalty. This affects the smooth functioning of the companies.

4. *Delay in decision-making*: A company has to fulfil certain procedural formalities before making certain decisions, as they require the approval of the Board of Directors and /or the General Body of shareholders. Such formalities are time consuming and therefore, some important decisions may be delayed.
5. *Lack of secrecy*: It is difficult to maintain secrecy in many matters as they may require approval of the board of directors and/or general body whose proceedings are usually open to public.
6. *Social abuses*: A joint stock company is a large-scale business organization having huge resources. This provides a lot of power to them. Any misuse of such power creates unhealthy conditions in the society, e.g. having monopoly of a particular business, industry or product; influencing politicians and government in getting their work done.

## 20.8 MULTINATIONAL CORPORATION

A multinational corporation (MNC) is one which is registered as a company in one country but carries on business in a number of other countries by setting up factories, branches or subsidiary units. Such a company may produce goods or arrange services in one or more countries and sell these in the same or other countries. There are many MNCs running their business in India such as Hyundai Motor Company, Coca Cola Company, Sony Corporation, McDonald's Corporation, Honda, Samsung, LGEIL, etc. All these corporations generally have production, marketing and other facilities in several countries. They have set up their branches and subsidiary units in our country and also in other countries. They are controlled from the headquarters of these companies in the home country, which lay down broad policies to be pursued.

### 20.8.1 Advantages of MNC

There are following advantages of MNCs:

1. *Investment of foreign capital*: Direct investment of capital by MNC helps under-developed countries to speed up their economic development.
2. *Generation of employment*: Expansion of industrial and trading activities by MNC leads to creation of employment opportunities and raising the standard of living in host countries.
3. *Advanced technology*: With substantial resources MNC undertakes Research and Development activities which contribute to improved methods and processes of production and thus, increase the quality of products. Gradually, other countries also acquire these technologies.
4. *Growth of ancillary units*: Suppliers of materials and services and ancillary industries often grow in host countries as a result of the operation of MNC. Increase in Exports and Inflow of Foreign Exchange: Goods produced in the host countries are sometimes exported by MNC. Foreign exchange, thus earned contributes to the foreign exchange reserves of host countries.
5. *Healthy competition*: Efficient production of quality goods by MNCs prompt the domestic producers to improve their performance in order to survive in the market.

## 20.8.2 Disadvantages of MNC

There are following disadvantages of MNCs:

1. *Least priorities for development of host countries*: MNC generally invests capital in the most profitable industries and do not take into account the priorities of developing basic industries and services in the backward regions of the host country.
2. *Adverse effect on domestic enterprises*: Due to large-scale operation and technological skills, MNCs are often able to dominate the markets in host countries and tend to acquire monopoly power. Thus, many local enterprises are compelled to close down.
3. *Change in culture*: Consumer goods, which are introduced by MNCs in the host countries, do not generally conform to the local cultural norms. Thus, the consumption habits of people as regards food and dress tend to change away from their own cultural heritage.

## 20.9 DEPARTMENTALIZATION

Departmentalization is a process of grouping the activities of a Firm. Departmentalization of a firm may be based on grouping of functions to be performed, products to be produced, type of customers, geographic location of facilities, types of processing, division just like a separate entities, etc. These departmentalizations are discussed below as:

- **Functional departmentalization**: Activities are grouped on the basis of functions to be performed. Similar type of activities is performed in one department. For example, there may be human resources, IT, accounting, manufacturing, logistics, and engineering as a department in an organization. It can be used in all types of organizations.
- **Product departmentalization**: Activities are grouped on the basis of the product line. The tasks are grouped according to a specific product and all the activities related to that product are placed under one manager. Each major product area in the corporation is under the authority of a senior manager who is specialist in, and is responsible for, everything related to the product line.
- **Customer departmentalization**: Activities are grouped according to the type of customers and type of requirements and needs of the customers. The assumption is that customers in each department have a common set of problems and needs that can best be solved by specialists. This type of departmentalization may be observed in large-size banking organization or sales and purchase departments.
- **Geographic departmentalization**: If an organization's customers are geographically dispersed, it can group jobs based on geography. The departments may be zonal or regional type. This type of departmentalization may be observed in multinational companies operating in a number of countries.
- **Process departmentalization**: Activities are grouped on the basis of product or service or customer flow. Because each process requires specific skills; process departmentalization allows homogenous activities to be categorized in one group.

- **Divisional departmentalization:** This type of departmentalization is done on the basis of separate or independent division, all the divisions contribute to the corporation's profitability. This type of the design is called divisional departmentalization.

## 20.10 CLASSIFICATION OF ORGANIZATIONS

On the basis of administrative procedures, there are three basic types of organizations: line organization, line and staff organization, and functional organization. There are some other classes of organizations such as project organization, matrix organization, etc. In project organization various projects are running and the persons or the staff are especially involved in only one project. But in matrix organization, one functional staff is responsible for all the projects for that function; for example, a human resource manager is responsible for the recruitment of people for all the projects.

### 20.10.1 Line Organization

It is the simplest form of organization. In this organization, the authority flows from top to bottom, i.e. the line of command is carried out from top to bottom. This is the reason for calling this as scalar or line organization which means scalar chain of command is a part and parcel of this type of organization. There are following characteristics of line organization:

1. It is the simplest form of organization.
2. Line of authority flows from top to bottom.
3. Specialized and supportive services do not take place in this organization.
4. Unified control by the line officers can be maintained since they can independently take decisions in their areas and spheres.
5. This line of the organization always helps in bringing efficiency in communication and bringing stability to a concern.

#### **Merits of Line Organization**

1. *Simplest:* It is the most simple and oldest method of administration.
2. *Unity of Command:* In these organizations, superior-subordinate relationship is maintained and scalar chain of command flows from top to bottom.
3. *Strong discipline:* The control is unified and concentrates on one person and therefore, he can independently make decisions of his own. Unified control ensures better discipline.
4. *Fixed responsibility:* In this type of organization, every line executive has got fixed authority, power and fixed responsibility attached to every authority.
5. *Flexibility:* There is a co-ordination between the top most authority and bottom line authority. Since the authority relationships are clear, line officials are independent and can flexibly take the decision. This flexibility gives satisfaction of line executives.
6. *Quick decision:* Due to the factors of fixed responsibility and unity of command, the officials can take prompt decision.

### **Demerits of Line Organization**

1. *Over-reliance*: The line executive's decisions are implemented to the bottom. This results in over-relying on the line officials.
2. *Lack of specialization*: A line organization flows in a scalar chain from top to bottom and there is no scope for specialized functions. For example, expert advices whatever decisions are taken by line managers are implemented in the same way.
3. *Inadequate communication*: The policies and strategies which are framed by the top authority are carried out in the same way. This leaves no scope for communication from the other end. The complaints and suggestions of lower authority are not communicated back to the top authority. So there is one way communication.
4. *Lack of co-ordination*: Whatever decisions are taken by the line officials, in certain situations wrong decisions, are carried down and implemented in the same way. Therefore, the degree of effective co-ordination is less.
5. *Authority leadership*: The line officials have tendency to misuse their authority positions. This leads to autocratic leadership and monopoly in the concern.

### **20.10.2 Line and Staff Organization**

Line and staff organization is an extended form of line organization and it is more complex than line organization. According to this administrative organization, specialized and supportive activities are attached to the line of command by appointing staff supervisors and staff specialists who are attached to the line authority. The power of command always remains with the line executives, and staff supervisors guide, advise and counsel the line executives. Personal Secretary to the Managing Director is a staff official. There are following characteristics of line and staff organization:

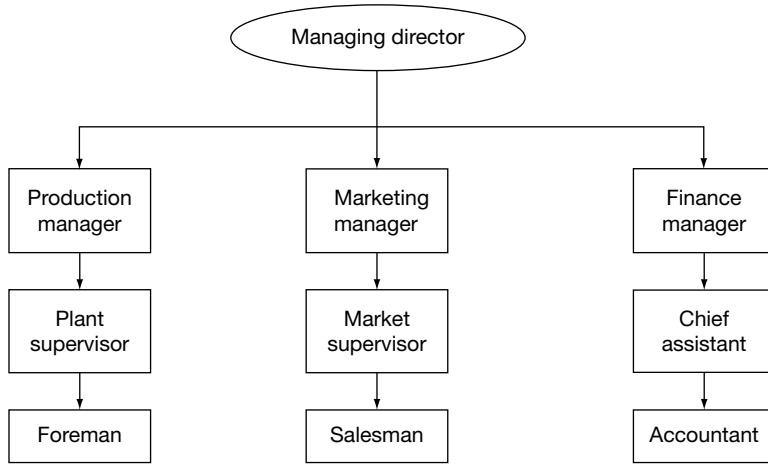
1. There are two types of staff: (a) Staff Assistants- P.A. to Managing Director, Secretary to Marketing Manager. (b) Staff Supervisor-Operation Control Manager, Quality Controller, PRO.
2. Line and staff organization is more complex than the line concern.
3. Division of work and specialization takes place in line and staff organization.
4. The whole organization is divided into different functional areas to which staff specialists are attached.
5. Efficiency can be achieved through the features of specialization.
6. There are two lines of authority which flow at one time in a concern: line authority and staff authority.
7. Power of command remains with the line executive and staff serves only as counsellors.

An example of hierarchy of line and staff organization is shown in Figure 20.1.

### **Merits of Line and Staff Organization**

There are following merits of line and staff organization:

1. *Relief to line of executives*: In a line and staff organization, the advice and counselling which is provided to the line executives divides the work between the two. The line executive can concentrate on the execution of plans and they get relieved of dividing their attention to many areas.



**Figure 20-1:** Structure of line and staff organization

2. *Expert advice:* The line and staff organization facilitates expert advice to the line executive at the time of need. The planning and investigation, which is related to different matters, can be done by the staff specialist and line officers can concentrate on execution of plans.
3. *Benefit of specialization:* Line and staff through division of the whole concern into two types of authority divides the enterprise into parts and functional areas. This way every officer or official can concentrate in its own area.
4. *Better co-ordination:* Line and staff organization through specialization is able to provide better decision-making and concentration remains in few hands. This feature helps in bringing co-ordination in work as every official is concentrating in their own area.
5. *Benefits of research and development:* Through the advice of specialized staff, the line executives get time to execute plans by taking productive decisions which are helpful for a concern. This gives a wide scope to the line executive to bring innovations and go for research work in those areas. This is possible due to the presence of staff specialists.
6. *Training:* Due to the presence of staff specialists and their expert advice serves as ground for training to line officials. Line executives can give due concentration to their decision-making. This in itself is a training ground for them.
7. *Balanced decisions:* The factor of specialization which is achieved by line staff helps in bringing co-ordination. This relationship automatically ends up the line official to take a better and balanced decision.
8. *Unity of action:* Unity of action is a result of unified control. Control and its effects take place when co-ordination is present in the concern. In the line and staff authority all the officials have got independence to make decisions. This serves as effective control in the whole enterprise.

### **Demerits of Line and Staff Organization**

1. *Lack of understanding*: In a line and staff organization, there are two authorities flowing at one time. This results in the confusion between the two. As a result, the workers are not able to understand as to who is their commanding authority. Hence the problem of understanding can be a hurdle in effective running.
2. *Lack of sound advice*: The line official gets used to the expertise advice of the staff. At times the staff specialist also provides wrong decisions which the line executive has to consider. This can affect the efficient running of the enterprise.
3. *Line and staff conflicts*: Line and staff are two authorities which are flowing at the same time. The factors of designations, status, influence sentiments which relate to their relation, can pose a distress on the minds of the employees. This leads to disturbing the co-ordination which hampers a concern's working.
4. *Costly*: In line and staff concern, the concerns have to maintain the high remuneration of staff specialist. This proves to be costly for a concern with limited finance.
5. *Assumption of authority*: The power of concern is with the line official, but the staff dislikes it as they are the one more in mental work.

### **20.10.3 Functional Organization**

In this organization, the entire organization is divided into different functional departments such as marketing, finance, production, personnel departments, etc. The functional authority remains confined to functional guidance to their departments. This helps in maintaining the quality and uniformity of performance of different functions throughout the enterprise. There are following characteristics of functional organization:

1. The entire organizational activities are divided into specific functions such as operations, finance, marketing and personal relations.
2. A complex form of administrative organization compared to the other two.
3. Three authorities exist-Line, staff and function.
4. Each functional area is put under the charge of functional specialists and he has got the authority to give all decisions regarding the function whenever the function is performed throughout the enterprise.
5. The principle of unity of command does not apply to such organization as it is present in the line organization.

### **Merits of Functional Organization**

1. *Specialization*: Better division of labour takes place, which results in specialization of function and its consequent benefit.
2. *Effective control*: Management control is simplified as the mental functions are separated from manual functions. Checks and balances keep the authority within certain limits. Specialists may be asked to judge the performance of various sections.
3. *Efficiency*: Greater efficiency is achieved because of every function performing a limited number of functions.

4. *Economy:* Specialization compiled with standardization facilitates, maximum production and economical costs.
5. *Expansion:* Expert knowledge of functional manager facilitates better control and supervision.

### ***Demerits of Functional Organization***

1. *Poor coordination:* The functional system is quite complicated to put into operation, especially when it is carried out at low levels. Therefore, co-ordination becomes difficult.
2. *Lack of command:* Disciplinary control becomes weak as a worker is commanded not by one person but a large number of people. Thus, there is no unity of command.
3. *Difficulty in fixing responsibility:* Because of multiple authorities, it is difficult to fix responsibility.
4. *Conflicts:* There may be conflicts among the supervisory staff of equal ranks. They may not agree on certain issues.
5. *Costly:* Maintenance of specialist's staff of the highest order is expensive for a concern.

## **20.11 DELEGATION OF AUTHORITY**

Delegation of authority means the division of authority and powers downwards to the subordinate. Delegation is about entrusting someone else to do parts of your job. Delegation of authority can be defined as subdivision and sub-allocation of powers to the subordinates in order to achieve effective results. There are following elements of delegation:

**Authority:** In the context of a business organization, authority can be defined as the power and the right of a person to use and allocate the resources efficiently, to take decisions and to give orders so as to achieve the organizational objectives. Authority must be well defined. All people who have the authority should know what is the scope of their authority and they shouldn't misuse it. Authority is the right to give commands, orders and get the things done. The top-level management has greater authority. Authority always flows from top to bottom. It explains how a superior gets work done from his subordinate by clearly explaining what is expected of him and how he should go about it. Authority should be accompanied with an equal amount of responsibility. Delegating the authority to someone else does not imply escaping from accountability.

**Responsibility:** It is the duty of a person to complete the task assigned to him. A person who is given the responsibility should ensure that he accomplishes the tasks assigned to him. If the tasks for which he was held responsible are not completed, then he should not give explanations or excuses. Responsibility without adequate authority leads to discontent and dissatisfaction among the person. Responsibility flows from bottom to top. The middle- and lower-level management hold more responsibility. The person held responsible for a job is answerable for it.

**Accountability:** It means giving explanations for any variance in the actual performance from the expectations set. Accountability cannot be delegated. For example, if 'A' is given a task with sufficient authority, and 'A' delegates this task to B and asks him to ensure that task is done well,

responsibility rest with ‘B’, but accountability still rests with ‘A’. The top-level management is most accountable. Being accountable means being innovative as the person will think beyond his scope of the job. Accountability can't be escaped. It arises from responsibility.

### **Differences between Authority and Responsibility**

The differences between authority and responsibility are mentioned in Table 20.2.

**Table 20-2:** Differences between authority and responsibility

<b>Authority</b>	<b>Responsibility</b>
It is the legal right of a person to command his subordinates.	It is the obligation of subordinate to perform the work assigned to him.
Authority is attached to the position of a superior in concern.	Responsibility arises out of superior-subordinate relationship in which subordinate agrees to carry out duties given to him.
Authority can be delegated by a superior to a subordinate.	Responsibility cannot be shifted and is absolute.
It flows from top to bottom.	It flows from bottom to top.



## **SUMMARY**

In this chapter, we have discussed about different types of firm's ownerships, their characteristics, merits and demerits. These firm's ownerships are sole partnership, cooperative society, partnerships, corporation, joint Hindu family business, joint stock company, multinational company, etc. Finally, we have classified the organization based on administrative structure and discussed their advantages and limitations.

## **MULTIPLE-CHOICE QUESTIONS**

1. Which of the following is NOT one of the three major categories of firm ownership?
  - (a) sole proprietorship
  - (b) partnership
  - (c) corporation
  - (d) mutual agreement
2. One of the main advantages of sole proprietorship is
  - (a) unlimited liability
  - (b) personal time commitment
  - (c) relative freedom from government regulation
  - (d) limited managerial expertise

3. One of the main disadvantages of the sole proprietorship is
  - (a) all losses is to be borne by the owner
  - (b) direct control of business
  - (c) ease of open and close
  - (d) no special taxation
4. A corporation can
  - (a) enter into contracts
  - (b) own property
  - (c) sue and be sued
  - (d) all the above
5. Cooperatives are
  - (a) either market focused
  - (b) just another form of a corporation
  - (c) going to pay both state and federal taxes
  - (d) none of these
6. In limited liability,
  - (a) all shareholders must hold a minimum of 20 shares in a company.
  - (b) all shareholders are equally responsible for all the debts of the company.
  - (c) the responsibility of shareholders for the debts of a company is limited to the number of debentures they hold in the company.
  - (d) the responsibility of shareholders for the debts of a company is limited to the amount they agreed to pay for the shares when they bought them.
7. Which of the following is an advantage of a partnership?
  - (a) The partnership usually consists of up to 50 people.
  - (b) Shares can be issued to the general public.
  - (c) The owners of such enterprises do not need to publish their accounts.
  - (d) All the above
8. Which of the following describes a holding company form of business organization?
  - (a) A company that controls more than 33 per cent of the equity of another company.
  - (b) A company that usually acts as a market leader in an industry.
  - (c) A single company that organizes its activity into a matrix format.
  - (d) A company that often exists only to hold over 50 per cent of the equity of a group of subsidiary companies.
9. A major difference between a public limited company and a private limited company is
  - (a) The public limited company has limited liability.
  - (b) The public limited company has a limited number of shareholder.
  - (c) The public limited company can sell shares on the open market.
  - (d) The public limited company has shareholders.

10. Organizational structure refers to coordination and
  - (a) division of labour
  - (b) delegation of authority
  - (c) assumption of authority
  - (d) decentralized decision-making
11. Departmentalization refers to
  - (a) organic structure
  - (b) functional structure
  - (c) line structure
  - (d) the organizational chart
12. An advantage of functional structures is that it
  - (a) encourages specialization and increases employee identity with their profession.
  - (b) permits greater specialization so that organization has expertise in each area.
  - (c) direct supervision is easier.
  - (d) all of the above.
13. Which of the following defines how job tasks are formally divided, grouped and coordinated?
  - (a) Organizational structure
  - (b) Work specialization
  - (c) Departmentalization
  - (d) Organizational behaviour
14. The basis by which jobs are grouped together is termed as:
  - (a) departmentalization
  - (b) bureaucracy
  - (c) specialization
  - (d) centralization
15. The matrix structure combines which two forms of the departmentalization?
  - (a) process and functional
  - (b) functional and product
  - (c) product and process
  - (d) none of the above

## Answers

1. (d)      2. (c)      3. (a)      4. (d)      5. (b)      6. (d)      7. (c)      8. (d)      9. (c)  
10. (a)      11. (d)      12. (d)      13. (a)      14. (a)      15. (b)

## REVIEW QUESTIONS

1. What do you mean by firm and its ownership? Explain in brief.
2. Differentiate between sole proprietorship and partnership.
3. Differentiate between cooperative society and corporations.
4. Write short notes on joint Hindu family business.
5. Discuss the characteristics of joint stock companies and differentiate between private and public company.
6. Define 'multinational corporation'.
7. Write short note on departmentalization.
8. Compare line, line and staff, and functional organizational structures.
9. What do you mean by delegation of authority? Write short notes on responsibility and accountability.
10. Differentiate between authority and responsibility.



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# CHAPTER ▶ 21

## Project Management PERT and CPM

### 21.1 INTRODUCTION

The project is any activity or work that has definite objective and consumes resources in terms of time, money, men, machine, etc. According to the Project management Institute (PMI), it can be defined as ‘a temporary endeavour undertaken to create a unique product or service’ (Project Management Institute, 2004). Here, temporary means every project has a definite end and the uniqueness of the project means, the project’s result is different from the results of other functions of the organization. A project has some more **definitions** that are discussed below as:

1. A project is a temporary process, which has a defined starting and ending time, a set of tasks and a budget. This process is carried out to accomplish a well-defined goal or an objective.
2. A project is a temporary effort of sequential activities designed to accomplish a unique purpose.
3. A project is a group of interrelated activities, constrained by time, cost and scope, designed to fulfil a unique purpose.
4. A project is an undertaking that encompasses a set of tasks or activities having a definable starting point and well-defined objectives. Usually, each task has a planned completion date and assigned resources.
5. A project is a clear set of activities with related inputs and outputs aimed to accomplish objectives and goals linked to anticipated effects and impacts in a target population.
6. A project is a group of unique, interrelated activities that are planned and executed in a certain sequence to create a unique product or service, within a specific time frame, budget and the client’s specifications.

A project has following **features**:

1. *Start and end*: A project has a definite start and end on a time scale. But, practically, it is difficult to fix the end of the project due to delay in finishing the project subject to a number of constraints.
2. *Life cycle*: A project has a life cycle with different phases, such as, conception of the project, selection of project, planning, scheduling, monitoring and controlling of the project, and finally, evaluation and termination, i.e. closing the project.

3. *Budget*: There is expenditure in commencing and starting the project. The project cost is an important part of the project management. It may vary with project completion time.
4. *Activities*: A project has a number of non-repetitive smaller activities, which consume some time for completion.
5. *Interdependencies*: Each sequential activity is interdependent on the predecessor activities finish time.
6. *Resources*: A project consumes resources in the form of time, money, labour, etc.
7. *Management principles*: A project follows a planned, organized method to meet its objectives with specific goals of quality and performance.
8. *Manager's responsibility*: A project has a manager responsible for its outcomes.
9. *Team roles and responsibility*: It is to be developed, defined, and established to perform the assigned task to those team members.
10. *Objective*: A project has fixed objective that is to be achieved within the scheduled time under the allocated resources.

**Project management** is defined as scientific approach to planning, implementation, monitoring and control of various aspects of a project such as man, machines, materials, money, time and other resources. According to the Project Management Institute (2004), 'Project Management is the application of knowledge, skills, tools and techniques to project activities in order to meet stakeholder's needs and expectations from a project'. That is, project management is an interrelated group of processes that enables the project team to achieve a successful project.

**Project scope:** Project scope defines the work that must be accomplished to produce a deliverable with specified features or functions. The deliverable can be a product, service or other result. It decides the features or functions that characterize the deliverable.

### **21.1.1 Classification of Projects**

On the basis of the reach or area covered, projects are classified into the following categories:

1. *Personal projects*: Projects of individuals such as planning for a tour and completion of a small project on an individual level.
2. *Local or neighbourhood projects*: Projects at the society or village level, such as construction of a community hall, school, etc.
3. *Organizational projects*: Projects at the organizational level such as purchasing a new machine, producing and launching a new product, etc.
4. *National projects*: Projects at the country level, such as launching a new version of a missile, building a dam and establishing a hydroelectric power plant, etc.
5. *Global projects*: Projects at the world level, such as constructing an ITER (International Thermonuclear Energy Reactor), building a tunnel between two countries for transportation, laying a petroleum pipeline between two countries, etc.

### **Advantages of Project Management**

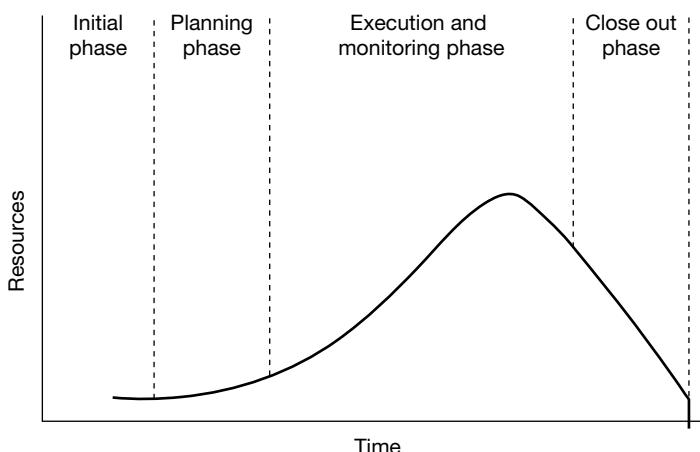
Initially, project management was considered as expensive and additional responsibility to the manager. But, it has been observed that there are a number of advantages of project management. These advantages are mentioned below:

1. Project management allows us to accomplish more work in less time, with fewer people.
2. It increases the profitability.
3. Project management provides better control of scope changes.
4. Project management makes the organization more efficient and effective through better organizational process control.
5. Project management allows us to work more closely with our customers and workers or team members.
6. Project management provides a means for solving problems.
7. Project management increases quality.
8. Project management reduces power struggles due to a team working culture.
9. Project management allows people to make good company decisions.
10. Project management increases our business opportunity.

## 21.2 PROJECT LIFE CYCLE

The different phases of project life cycle are very similar to product life cycle as discussed in Chapter 7. The main stages in the project management life cycle, depicted in Figure 21.1, are as follows.

1. *Initial phase:* The initial phase is the conceptual phase in the project life cycle. In this phase, project is conceptualized and selected on the basis of feasibility study. It essentially involves starting up the project. A project is initiated by defining its purpose and scope, justification for initiating it and the solution to be implemented. The initial phase covers tasks such as documenting a business case, conducting a feasibility study, setting terms of reference, appointing the project team and setting up a project office.
2. *Planning:* Planning is the second phase of the project life cycle. It involves the preparation of the planning documents to help the project team throughout the project



**Figure 21-1:** Project life cycle

delivery. It involves setting out the roadmap for the project by creating the following plans: project plan, resource plan, financial plan, quality plan, acceptance plan and communications plan.

3. *Execution and monitoring:* This phase involves building the deliverables and controlling the project delivery, scope, costs, quality, risks and issues. This is the phase in which the deliverables are physically built and presented to the customer for acceptance. While each deliverable is being constructed, a suite of management processes is undertaken to monitor and control the deliverables as output of the project. These processes include managing time, cost, quality, change, risks, issues, suppliers, customers and communication. Once all the deliverables have been produced and the customer has accepted the final solution, the project is ready for closure.
4. *Closeout:* It involves closing the project by releasing staff, handing over deliverables to the customer and completing a post-implementation review. The project closure involves releasing the final deliverables to the customer, handing over the project documentation to the customer terminating supplier contracts, releasing project resources and communicating project closure to all stakeholders. The last remaining step is to undertake a post-implementation review to identify the level of project success and note any lessons learned for future projects.

## **21.3 PROJECT APPRAISAL**

Project appraisal is the analysis of a proposed project to determine its merit and acceptability in accordance with established criteria. This is the final step before a project is agreed for financing. It checks whether the project is feasible against the situation on the ground in terms of the set objectives and costs are reasonable. There are six major appraisals of project as discussed in the following subsections.

### **21.3.1 Technical Appraisal**

The following factors are incorporated in a technical appraisal of a project:

1. The project must be technology proven or tested. If not, has it ever been successful elsewhere and can that success be replicated in the current context and conditions?
2. The technology/process/equipment technically must fit in the organization's existing technology/process/equipment and machinery. If not, the aspects are to be outlined that do not fit and the required measures are to be planned to implement in this regard.
3. The list of equipment and machinery to be installed must meet the cost and specifications of the equipments required by the organization.
4. The equipment capacity is to be analysed and checked that it is as per the organization's requirement.
5. The recommended equipment supplier capability must be evaluated and checked.

### 21.3.2 Financial Appraisal

The following factors are incorporated in the financial appraisal of a project:

1. Requirement of financial support to the project from various agencies.
2. Check the availability of funds to cover the expenditure required during the project life.

### 21.3.3 Economic Appraisal

The following factors are incorporated in the economic appraisal of a project:

1. Ensure that the nation and society at large will be better off as a result of the project.
2. The benefits should be greater than the project costs over the life of the investment.
3. Social cost-benefit analysis.
4. Direct economic benefits and costs in terms of shadow prices.
5. Impact of the project on the distribution of income in society.
6. Impact on the level of savings and investments in society.
7. Impact on fulfilment of national goals:
  - (a) Self-sufficiency.
  - (b) Employment.
  - (c) Social order.

### 21.3.4 Social Appraisal

The following factors are incorporated in the social appraisal of a project:

1. The effect of the project on different groups at individual, household and community levels.
2. Impact of project on women and men and their participation in various stages of the project life cycle.
3. The social benefits of the project should be greater than the social costs over the life of the investment.
4. Identification of stakeholders and finding interests.
5. Effect of the proposed project on the stakeholders.
6. The project priorities for different groups?
7. Identification of stakeholders' capacity to participate in the project.

### 21.3.5 Market Appraisal

The following factors are incorporated in a market appraisal of a project:

1. Estimation of the aggregate demand of the product or service to be produced under the project.
2. Estimation of the market share of the proposed project and service.

### 21.3.6 Ecological Appraisal

The following factors are incorporated in an ecological appraisal of a project:

1. The impact of the project on the quality of air, water, noise, vegetation and human life.
2. Estimation of the likely damage to the ecology and the cost of restoration.

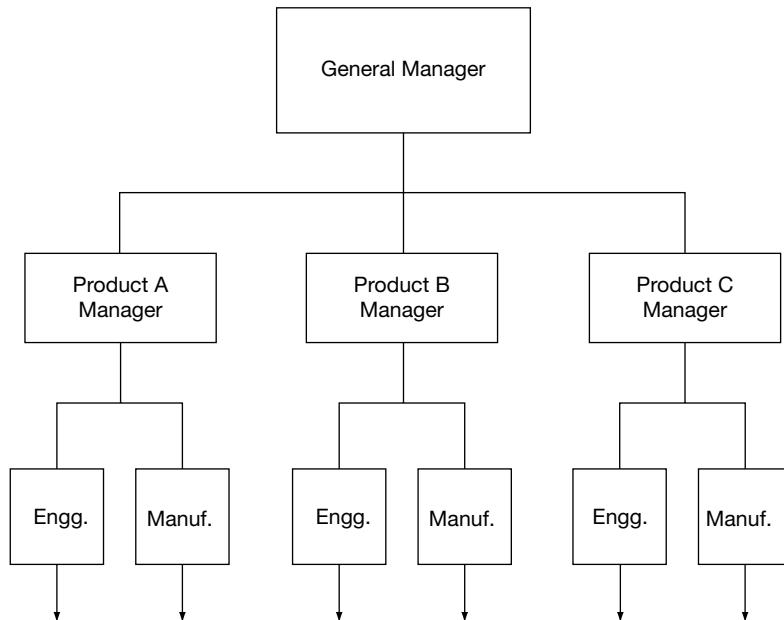
## 21.4 PROJECT STRUCTURE

Three types of project structure may be used to tie up the project with the parent firm: (a) pure project, (b) functional project, (c) matrix project. Each one of them has been discussed in the following subsections.

### 21.4.1 Pure Project

A pure project is where a self-contained team works full time on the project/product. A structure of pure project is shown in Figure 21.2. A pure project has the following advantages:

1. The project manager has full authority over the project/product.
2. Team members report to one boss.
3. It has shortened communication chains.
4. Team pride, motivation and commitment are high.



**Figure 21-2:** Structure of pure project

A pure project has the following disadvantages:

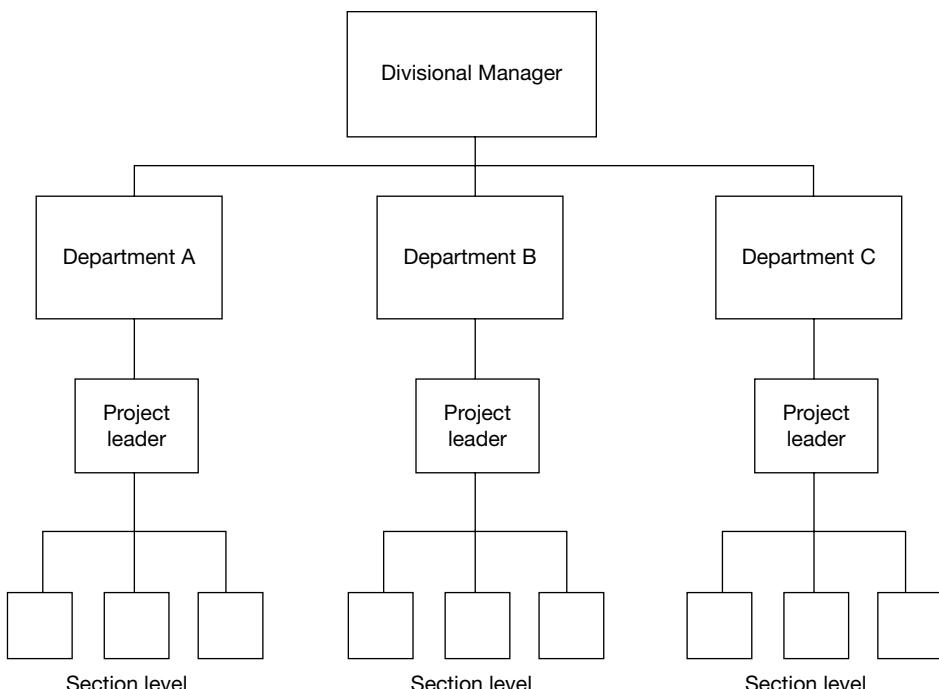
1. There is duplication of resources.
2. Organizational goals and policies are ignored.
3. There is a lack of technology transfer.
4. Team members have no functional area, i.e. 'home'. The team members are employed for the particular project only.

## 21.4.2 Functional Project

A functional project is housed within a functional division, i.e. the project belongs to a particular functional department. As shown in Figure 21.3, projects at section level belong to the different departments.

A functional project has the following advantages:

1. A team member can work on several projects.
2. Technical expertise is maintained within the functional area.
3. The functional area is a 'home' after the project is completed.
4. A critical mass of specialized knowledge creates synergistic solutions to a project's technical problem.



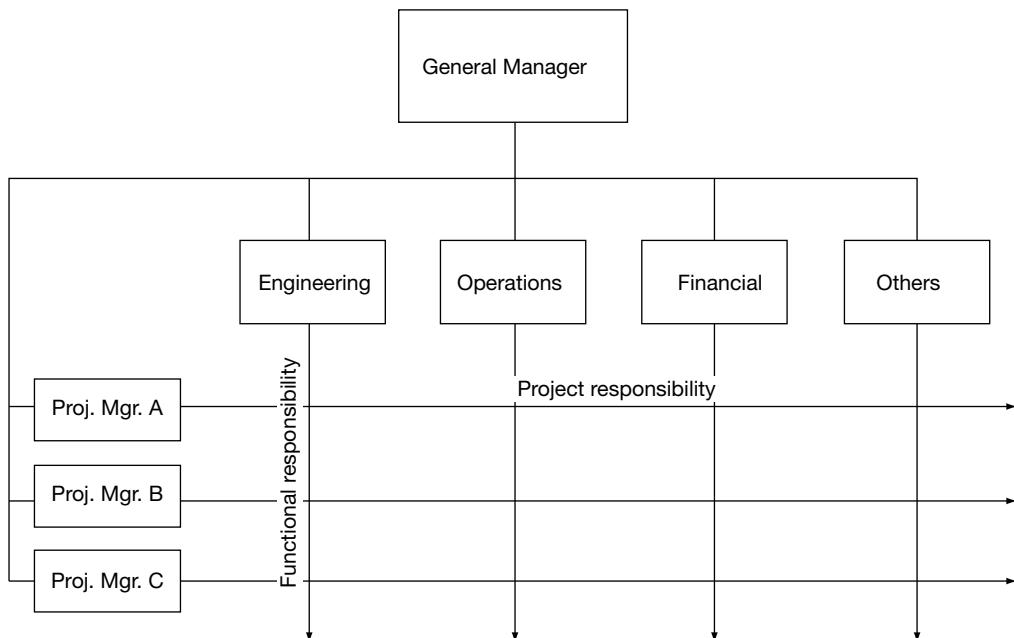
**Figure 21-3:** Structure of functional project

A functional project has the following disadvantages:

1. Aspects of the project that are not directly related to the functional area get short-changed.
2. Motivation of team members is often weak.
3. The needs of the client are responded slowly.

### 21.4.3 Matrix Project

This is the mixed form of pure and functional projects. In matrix project, the activities of the project belong to different functional area and they are completed in that respective functional department. Figure 21.4 shows that projects A, B and C pass through all the four departments. The project manager has to coordinate all the departmental heads to complete the project.



**Figure 21-4:** Structure of matrix project

A matrix project has the following advantages:

1. There is enhanced communication between functional areas.
2. Pinpointed responsibility, i.e. a project manager has the responsibility to complete the project.
3. Duplication of resources is minimized i.e. sharing of equipment and people are minimized across the projects.

4. There is functional ‘home’ for team members. After completion of the project team members return to their respective department, therefore, they are less worried about life-after-project.
5. The policies of the parent organization are followed.

A matrix project has the following disadvantages:

1. There are two bosses: project manager and functional manager.
2. The success of the project depends on project manager’s negotiating skills.
3. A project manager uses sub-optimization for success of his/her own project only and he/she may damage the other projects. Thus, there is potential for sub-optimization.

## 21.5 TERMINOLOGY USED IN PROJECT SCHEDULING

In this section, various terms used in project scheduling have been defined.

1. *Network planning model*: A network planning model is used for project scheduling and control. A project is made up of a sequence of activities that form a network diagram. The path taking the longest time through this network of activities is called the ‘critical path.’ The critical path provides a wide range of scheduling information useful in managing a project. The critical path method (CPM) helps identify the critical path(s) in a project network.
2. *Program evaluation review technique (PERT)*: This technique was developed by the US Navy. It is useful in managing those projects that can be split into the activities having uncertainty overestimation of time for completion. Beta distribution is used to find the expected time of project completion from three estimates of time – optimistic time ( $t_o$ ), most likely time ( $t_m$ ) and pessimistic time ( $t_p$ ). Beta distribution is a continuous distribution which gives the probability of project completion for various ranges of project completion time; it is used in the approximation to normal distribution:

$$\text{Expected time, } t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$\text{Standard deviation, } \sigma = \frac{t_p - t_o}{6}$$

$$\text{Variance, } \sigma^2 = \left( \frac{t_p - t_o}{6} \right)^2$$

3. *Critical path method*: It was developed by EI DuPont Company for cost and resource allocation to projects having certainties for completion of time. The path taking the longest time through this network of activities is called the ‘critical path’. There are a number of activities in a project and they are linked together sequentially, i.e., there is sequential relationships among the activities. All the activities run parallel to each other maintaining the constraints of sequence. Thus, the project completion time becomes the total time consumed by the longest path i.e. critical path.

4. *Event*: The point at which an activity is started or completed is known as an event. An event does not consume time. The event which follows another event is known as the successor event to the latter and the event which occurs before another event is known as the predecessor event to the latter.
5. *Activity*: An activity is that part of a project which consumes time. An activity connects two events. A dummy activity is the one that does not consume any time or resource.

### **Comparison between PERT and CPM**

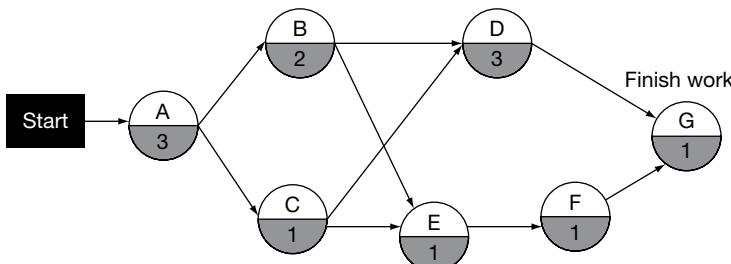
The following differences exist between PERT and CPM:

1. PERT has a probabilistic approach while CPM has deterministic approach. In PERT, there are three time estimates: optimistic time, the most likely time and pessimistic time. The mean value of these times (Using  $\beta$ -distribution) becomes the activity time, but in CPM there is only one deterministic time of an activity.
2. PERT is event-oriented while CPM is activity-oriented.
3. PERT is primarily concerned with time while CPM is concerned with time as well as cost and resource allocation.

#### **21.5.1 Network Conventions**

In the network convention, all the activities of the project are represented in a diagram, called network diagram. There are two types of conventions: activity-on-node and activity-on-arrow. This diagram is also known as a precedence diagram. Both the conventions are discussed below:

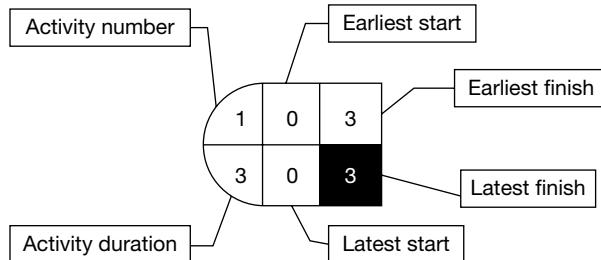
1. *Activity-on-node (AON)*: Nodes represent activities and arrows show precedence relationships. In Figure 21.5, A to G shows the activities, i.e. the numbers in the upper half of the nodes and the numbers in the lower half of the nodes represent the activity time. The arrow between the nodes shows the precedence relationship, for example, activity A should be completed before the completion of activities B and C. Similarly, activities B and C should be completed before the completion of activities D and E. Each node can represent the complete details of an activity as shown in Figure 21.6.



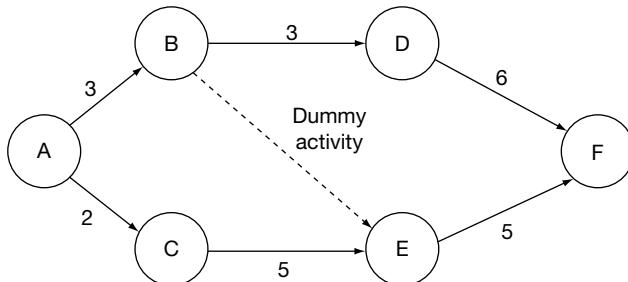
**Figure 21-5: Activity on nodes (AON)**

2. *Activity-on-arrow (AOA)*: Arrows represent activities, and nodes are events for points in time. This convention is just reverse of AON. An arrow between two nodes shows

the activity. Here, nodes are the events as shown in Figure 21.7. The dotted line shows the dummy activity which consumes zero/no time. This is used to complete the network diagram.



**Figure 21-6:** Node configuration



**Figure 21-7:** Activity on arrows

3. *Float:* Float is concerned with activity. It indicates the extent of flexibility with regard to starting an activity without affecting the project completion time. There are three types of floats as discussed below:

$$\begin{aligned} \text{Total float, } (F_T) &= [\text{Latest finish time } (T_{Lj}) \text{ for the activity } (i-j) \\ &\quad - \text{Earliest start time } (T_{Ei}) \text{ for the activity } (i-j)] \\ &\quad - \text{Activity time } (t_{i-j}) \end{aligned}$$

$$\begin{aligned} \text{Free float, } (F_F) &= [\text{Earliest finish time } (T_{Ej}) \text{ for the activity } (i-j) \\ &\quad - \text{Earliest start time } (T_{Ei}) \text{ for the activity } (i-j)] \\ &\quad - \text{Activity time } (t_{i-j}) \end{aligned}$$

$$\begin{aligned} \text{Independent float, } (F_I) &= [\text{Earliest finish time } (T_{Ej}) \text{ for the activity } (i-j) \\ &\quad - \text{Latest start time } (T_{Li}) \text{ for the activity } (i-j)] \\ &\quad - \text{Activity time } (t_{i-j}) \end{aligned}$$

where  $i$  and  $j$  indicate the events of activity starts and activity finish, respectively. For example, if an activity is indicated as B-C; here B indicates the event of starting the

activity B-C or finishing the activity A-B; similarly, event C indicates the finishing of the activity B-C or starting of activity C-D.

Independent float can be absorbed without any further planning since it affects subsequent activities. Free float does not affect subsequent activities. Total float should not be absorbed by any activity because it affects both previous and subsequent activities.

4. *Slack:* Slack concerns an event. It is the other way to judge the flexibility of an activity within the bounds imposed by head and tail events.

$$\text{Slack} = \begin{cases} T_{Li} - T_{Ei} & \text{for event } i \\ T_{Lj} - T_{Ej} & \text{for event } j \end{cases}$$

**Example 21.1:** The time estimates in weeks for the activities of a PERT network are given in Table 21.1.

**Table 21-1:** Three time estimates for the activities of a project

Activity	Optimistic time ( $t_o$ )	Most likely time ( $t_m$ )	Pessimistic time ( $t_p$ )
A-B	1	1	7
A-C	2	5	8
A-D	2	2	8
C-E	3	6	15
D-E	1	4	7
E-F	2	5	14
B-F	2	5	8

- (a) Draw the network diagram.
- (b) Calculate the earliest start (ES) and latest start (LS) for all the activities.
- (c) Determine the project completion time.
- (d) Calculate the standard deviation and variance of the project.

What is the probability that the project will be finished (i) at least 3 weeks earlier than the expected completion time, (ii) no more than 2 weeks later than expected completion time, (iii) after 2 weeks of the expected completion time, (iv) in between 15<sup>th</sup> to 20<sup>th</sup> week of the scheduled completion.

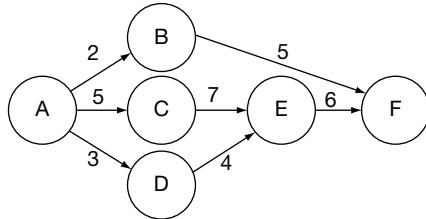
#### Solution:

Calculate the single estimated time for three times for each activity using the formula:

$$\text{Expected time, } t_e = \frac{t_o + 4t_m + t_p}{6}$$

- (a) In Figure 21.8, activities are shown on the arrows and events.
- (b) To determine the project completion time, first we should find the expected time of each activity as shown in Table 21.2 and then find the earliest start time (ES) for all the

activities as discussed in the following paragraph. On the basis of the earliest start time of the activities, the longest path, i.e. critical path is found and the earliest finish time of the last activity becomes the project completion time.



**Figure 21-8:** Network diagram for the activities shown in Table 21.1

**Table 21-2:** Expected time and variance of activities

Activity	$t_o$	$t_m$	$t_p$	$t_e = \frac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
A-B	1	1	7	2	-
<b>A-C</b>	<b>2</b>	<b>5</b>	<b>8</b>	<b>5</b>	<b>1</b>
A-D	2	2	8	3	-
<b>C-E</b>	<b>3</b>	<b>6</b>	<b>15</b>	<b>7</b>	<b>4</b>
D-E	1	4	7	4	-
<b>E-F</b>	<b>2</b>	<b>5</b>	<b>14</b>	<b>6</b>	<b>4</b>
B-F	2	5	8	5	-

The earliest start time of all the activities is calculated in the forward direction, that is, from A to F. The earliest start time of the first activity is zero. To find the earliest start time of the successor activity ( $j$ ), the activity time ( $t_{ij}$ ) is added to the earliest start time of the predecessor activity ( $i$ ). If an event has two or more predecessor activities, largest activity time is considered to calculate the earliest start time of the successor activity. Latest start time is calculated in backward direction, that is, from F to A. The latest start time of the last event is the same as the earliest start time. The activity time ( $t_{ij}$ ) is subtracted from the latest start time of the successor activity ( $j$ ) to find the latest start time of the predecessor activity ( $i$ ). In case an activity has two or more successor activities, the smallest activity time is to be subtracted from the latest start time of the corresponding successor event ( $j$ ) to find the latest start time of the predecessor event ( $i$ ); remember, only the smallest latest start time among them is to be considered as the latest start time of the predecessor event ( $i$ ) as shown in Table 21.3.

**Table 21-3:** ES and LS of activities

Events	A	B	C	D	E	F
<b>ES</b>	0	2	5	3	12	18
<b>LS</b>	0	13	5	8	12	18

- (c) A-C-E-F is a critical path and the project completion time is 18 weeks.  
 (d) To calculate the standard deviation and variance of the project, only the activities lying on the critical path are considered. In this case, we should consider the activities A-C, C-E and E-F. In Table 21.2, variances are for these activities are shown with bold letters.

$$\text{Variance} = 1 + 4 + 4 = 9$$

$$\text{Standard deviation} = \sqrt{9} = 3 \text{ weeks}$$

- (e) To determine the probability that the project will be completed within the specified time we use

$$z = \frac{x - \mu}{\sigma}$$

where  $\mu = t_p$  is the project mean time,

$s$  is the project standard deviation,

$x$  is the proposed project time,

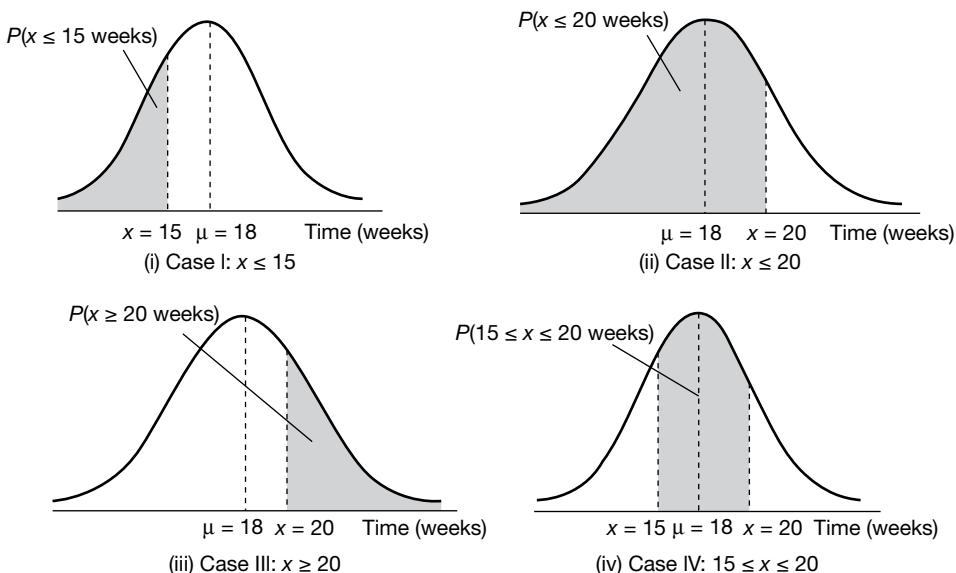
and  $z$  is the number of standard deviations.

Now, four cases arise (see Figure 21.9):

$$\text{Case I: } z = \frac{x - \mu}{\sigma} = \frac{15 - 18}{3} = -1$$

From statistical normal distribution table at  $z = -1$ ,

$$\text{Probability} = 1 - 0.8413 = 0.1587$$



**Figure 21-9:** Probability of project completion time on normal distribution curve

$$\text{Case II: } z = \frac{x - \mu}{\sigma} = \frac{20 - 18}{3} = 0.6667$$

From statistical normal distribution table at  $z = 0.6667$ ,

$$\text{Probability} = 0.7454$$

$$\text{Case III: } z = \frac{x - \mu}{\sigma} = \frac{20 - 18}{3} = 0.6667$$

From statistical normal distribution table at  $z = 0.6667$ ,

$$\text{Probability} = 1 - 0.7454 = 0.2546$$

$$\text{Case IV: } z_1 = \frac{x - \mu}{\sigma} = \frac{15 - 18}{3} = -1$$

$$z_2 = \frac{x - \mu}{\sigma} = \frac{20 - 18}{3} = 0.6667$$

From statistical normal distribution table, between  $Z_1 = -1$  and  $Z_2 = 0.6667$ , probability  $= 0.7478 - 0.1587 = 0.5891$

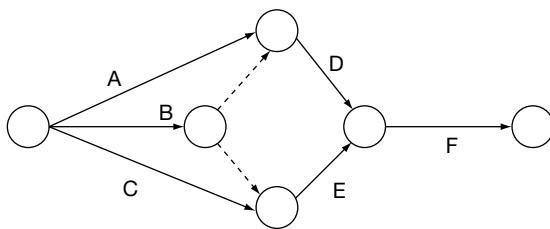
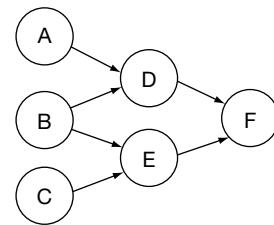
**Example 21.2:** Draw the network diagram for the activities of a project shown in Table 21.4 using AOA and AON.

**Table 21-4:** 4 Predecessors and successors of the activities of a project

Activity	Predecessor	Successor
A	-	D
B	-	D, E
C	-	E
D	A, B	F
E	B, C	F
F	D, E	-

### Solution:

To draw a network diagram, we should follow the sequence of the activities as shown in Table 21.4. Here, activities are shown on arrows and events on nodes. In Figure 21.10, we see that there is no predecessor for the activities A, B, and C and the successor activities for A and B is D and for B and C is E. Since the events D and E are the successor activities of activity B and the finishing event (node) of activity B cannot be directly joined to starting events (nodes) of activity D and E, therefore dummy activities (shown by dotted lines) are used to complete the diagram. Dummy activity consumes no time or any other resources. In the similar way activity F is successor of activities D and E.

**Figure 21-10:** Network diagram using AOA**Figure 21-11:** Network diagram using AON

When we show the activities on nodes and events on arrows, the reverse concepts of AOA is used. Arrows show the events of starting and completion of activities. Head of the arrow shows the starting of successor activity and tail shows the finishing of predecessor activity as shown in Figure 21.11. For example, the head of the arrow between activities A and D shows the starting of activity D and tail shows the finishing of activity A. The rest of the procedure to draw the network is same as discussed in the previous paragraph.

## 21.6 PROJECT CRASHING (OPTIMIZATION THROUGH CPM)

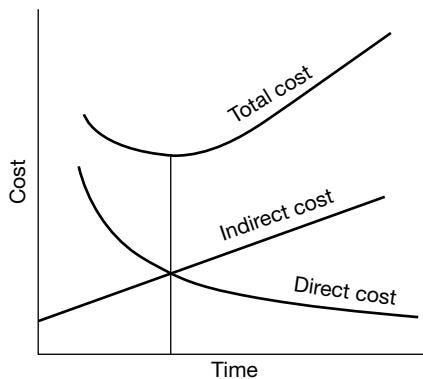
Crashing of a project means reducing the time of a critical activity that usually incurs additional direct costs. Cost-time solutions focus on reducing (crashing) activities on the critical path of a project to shorten the overall duration of the project. The reasons for crashing a project are as follows:

1. Customer requirements and contract commitments.
2. Time-to-market pressures.
3. Incentive contracts (bonuses for early completion).
4. Unforeseen delays.
5. Overhead and goodwill costs.
6. Pressure to move resources to other projects.

A project can be crashed by reducing the normal time of its critical activities. This can be achieved by increasing the resources or direct cost to perform these activities. By reducing the time of a critical activity, the indirect cost of the activity can be decreased but its direct cost will increase. The additional cost can be justified by the time-cost trade-off. The optimum total cost is shown in Figure 21.12.

Two types of costs are associated with a project as described below:

1. *Indirect costs:*
  - (a) These costs cannot be associated with any particular work package or project activity. These costs are incurred on supervision, administration, consultants and interest.
  - (b) Indirect costs vary (increase) with time. Reducing the project time directly reduces indirect costs.



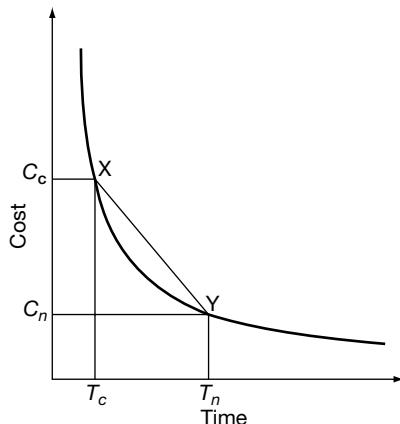
**Figure 21-12:** Project cost optimization

2. *Direct costs:*

- (a) These are normal costs that can be assigned directly to a specific work package or project activity. These costs are incurred on labour, materials, equipment and subcontractors.
- (b) Crashing activities increase direct costs.

An activity is completed in time  $T_n$  and the cost involved is  $C_n$ . Suppose the activity completion time can be compressed maximum up to  $T_c$  and the cost corresponding to the crashed time is  $C_c$ . Then the slope of the direct cost curve is (as shown in Figure 21.13).

$$\frac{C_c - C_n}{T_n - T_c} = \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}}$$



**Figure 21-13:** Direct cost corresponding to completion time

Assumptions for time–cost trade-off are as follows:

1. The cost relationship is linear.
2. Normal time assumes low-cost, efficient methods to complete the activity.
3. Crash time represents a limit – the greatest time reduction possible under realistic conditions.
4. Slope of the line XY represents a constant cost *per unit of time*.
5. All accelerations must occur within the normal and crash times.
6. By crashing the activities, the existing critical path should remain the critical path, but another critical path may exist additionally.

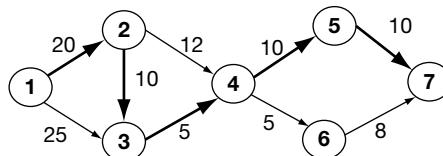
**Example 21.3:** The details of the activities of a project are given in Table 21.5. Using the given information, crash the activities step by step until all paths become critical.

**Table 21-5:** Costs of the activities of a project corresponding to normal and crash time

Activities	Normal		Crash		Cost slope = $\frac{C_c - C_n}{T_n - T_c}$
	Time in days	Cost in Rs	Time in days	Cost in Rs	
1-2	20	600	17	720	40
1-3	25	200	25	200	00
2-3	10	300	8	440	70
2-4	12	400	6	700	50
3-4	5	300	2	420	40
4-5	10	300	5	600	60
4-6	5	600	3	900	150
5-7	10	500	5	800	60
6-7	8	400	3	700	60
<b>Total = 3600</b>					

### Solution:

In Figures 21.14, 21.15, 21.16 and 21.17, crashing of activity times are shown stepwise. In Figure 21.14, a network diagram with critical paths is shown with thick arrows without crashing the activity time. After crashing, some additional critical paths may appear that are shown in following steps. Finally, at the end of crashing all the activities lie on critical paths and further crashing is not possible.



**Figure 21-14:** Network diagram using AOA with critical path

**Step 1:** Critical paths are 1-2-3-4-5-7

The activities 1-2 and 3-4 have a minimum slope (Rs 40) on the critical path. Therefore, on crashing both the activities by 3 days, the cost will be  $3600 + 40 \times 3 + 40 \times 3 = \text{Rs } 3840$  and the resulting network will be as given in Figure 21.15.

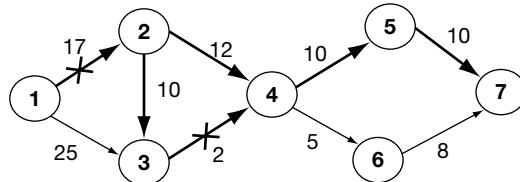


Figure 21-15: Network diagram after crashing the activities 1-2 and 3-4

**Step 2:** Critical paths are 1-2-3-4-5-7 and 1-2-4-5-7

The activities 2-3, 2-4, 4-5, and 5-7 have slopes 70, 50, 60, and 60, respectively. If the activities 2-3, 2-4, 4-5, and 5-7 are crashed by 2, 2, 5, and 2 days, respectively, then all the activities will be on critical path. The cost will be  $\text{Rs } 3840 + 2 \times 70 + 2 \times 50 + 5 \times 60 + 2 \times 60 = \text{Rs } 4500$ . The resulting network will be as given in Figure 21.16.

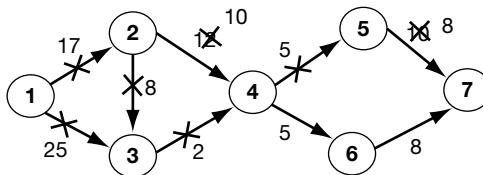


Figure 21-16: Network diagram after crashing the activities 2-3, 2-4, 4-5 and 5-7

**Step 3:** Critical paths are 1-2-3-4-5-7, 1-2-3-4-6-7, 1-2-4-5-7, 1-2-4-6-7, 1-3-4-5-7, and 1-3-4-6-7. The activity 5-7 has a minimum cost slope as Rs 60. It can be crashed by 3 days. Similarly, to maintain the critical path 6-7 can also be crashed by 3 days. The cost will be  $\text{Rs } 4500 + 3 \times 60 + 3 \times 60 = \text{Rs } 4860$  and the resulting network will be as given in Figure 21.17:

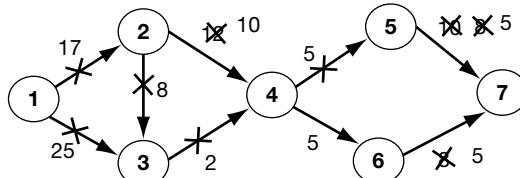


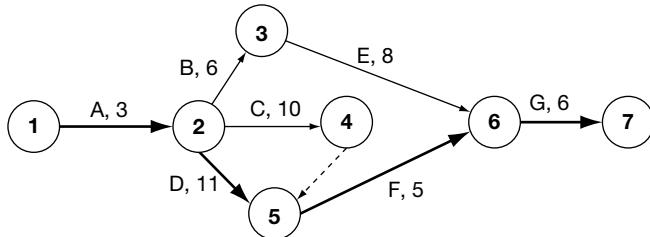
Figure 21-17: Network diagram after completion of crashing the activities

The project completion time = 37 days.

**Example 21.4:** The total normal direct cost of a project is Rs 450 and its indirect cost is Rs 400. By crashing the activities, the indirect cost decreases by Rs 50 per day. The normal and crash costs with time are given in Table 21.6 and the network diagram in Figure 21.18. Find the optimum project duration.

**Table 21-6:** Normal and crash costs and time of a project

Activity	Slope	Maximum crash time	Normal		Crash	
			Time	Cost	Time	Cost
A	20	1	3	50	2	70
B	40	2	6	80	4	160
C	30	1	10	60	9	90
D	25	4	11	50	7	150
E	30	2	8	100	6	160
F	30	1	5	40	4	70
G	00	0	6	70	6	70

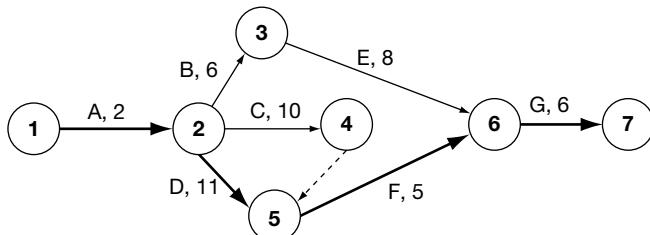


**Figure 21-18:** Network diagram with critical path

### Solution:

**Step 1:** Critical path is 1-2-5-6-7

Crashing the activity A by one day, the direct cost will be  $450 + 20 = \text{Rs } 470$  and the resulting network will be as given in Figure 21.19.



**Figure 21-19:** Network diagram after crashing the activity A by one day

**Step 2:** Critical path is 1-2-5-6-7

Crashing the activity D by one day, the direct cost will be  $470 + 25 = \text{Rs } 495$  and the resulting network will be as given in Figure 21.20.

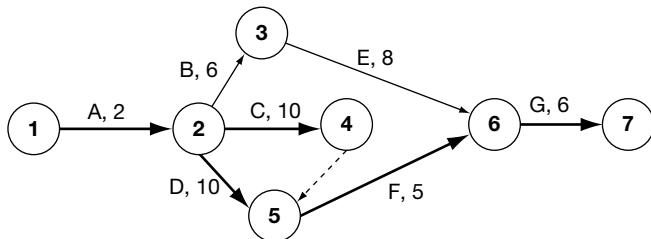


Figure 21-20: Network diagram after crashing the activity D by one day

**Step 3:** Critical path is 1-2-5-6-7 and 1-2-4-5-6-7.

Crashing the activity F by one day, the direct cost will be  $495 + 30 = \text{Rs } 525$  and the resulting network will be as given in Figure 21.21.

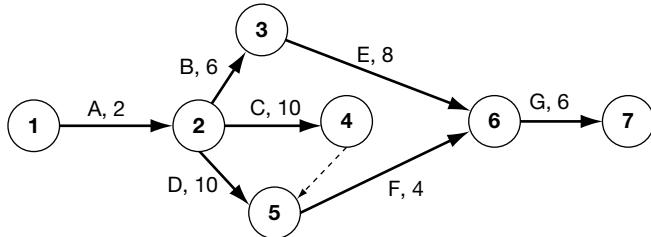


Figure 21-21: Network diagram after crashing the activity F by one day

**Step 4:** Critical path is 1-2-5-6-7, 1-2-4-5-6-7, and 1-2-3-6-7.

Crashing the activities C, D and E by one day each, the direct cost will be  $525 + 30 + 25 + 30 = \text{Rs } 610$  and the resulting network will be as given in Figure 21.22.

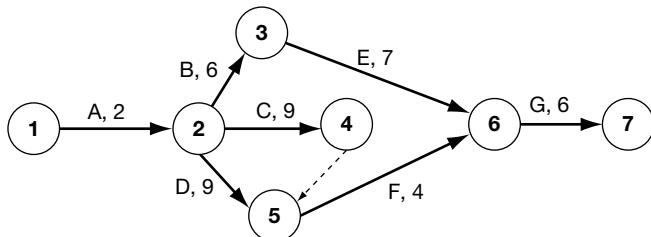


Figure 21-22: Network diagram after crashing the activities C, D and E each by one day

After crashing the activities, direct cost increases and indirect cost decrease as stated in the statement of the problem. The effect of crashing the activities by each day on the direct and indirect cost is shown in Table 21.7. We observe that the optimal time of project completion is 22 days since the total costs increase for further crashing the activities.

**Table 21-7:** Time-cost trade-off

Project duration (Days)	Direct costs (Rs)	Indirect costs (Rs)	Total costs (Rs)
25	450	400	850
24	470	350	820
23	495	300	795
<b>22</b>	<b>525</b>	<b>250</b>	<b>775</b>
21	610	200	810

Minimum project cost is Rs 775 and project duration is 22 days.

### 21.6.1 Linear Programming Approach to Solve the Project Crashing Problem (Wang and Liang, 2004)

**Objective function:** Minimize total cost

Total cost = total normal cost + total incremental direct cost of crashing + fixed indirect cost + saved total variable indirect cost due to crashing

i.e., 
$$\text{Min } Z = \sum_i \sum_j C_{D_{ij}} + \sum_i \sum_j k_{ij} Y_{ij} + [C_I + m(E_n - T_o)]$$

Subject to

$$E_i + t_{ij} - E_j \leq 0, \quad \forall i, \forall j$$

$$t_{ij} = D_{ij} - Y_{ij}, \quad \forall i, \forall j$$

$$Y_{ij} = D_{ij} - d_{ij}, \quad \forall i, \forall j$$

$$E_1 = 0$$

$$E_n - \sum_i \sum_j t_{ij} \geq 0, \text{ for each path of the network}$$

where  $(i, j)$  = activity between event,  $i$  and event  $j$

$z$  = total project costs (Rs)

$D_{ij}$  = normal time for activity  $(i, j)$  (days)

$d_{ij}$  = minimum crashed time for activity  $(i, j)$  (days)

$C_{D_{ij}}$  = normal (direct) cost of activity  $(i, j)$  (Rs)

$C_{d_{ij}}$  = minimum crashed (direct) cost of activity  $(i, j)$  (Rs)

$k_{ij}$  = incremental crashing cost of activity  $(i, j)$  (the direct cost to crash activity  $(i, j)$  by one period (Rs/day)

$t_{ij}$  = crashed duration time for activity  $(i, j)$  (days)  
 $Y_{ij}$  = crash time for activity  $(i, j)$  (days)  
 $E_i$  = earliest time for event  $i$  (days)  
 $E_j$  = earliest time for event  $j$  (days)  
 $E_0$  = project start time (days)  
 $E_n$  = project completion time (days)  
 $T_0$  = project completion time under normal conditions (days)  
 $C_I$  = fixed indirect costs under normal conditions (Rs)  
 $m$  = variable indirect cost per unit time (Rs/day)

**Example 21.5:** Solve Exercise problem 21.4 using Linear Programming.

**Solution:**

**Problem formulation**

$$\text{Minimize } Z = (450 + 20Y_{12} + 40Y_{23} + 30Y_{24} + 25Y_{25} + 30Y_{36} + 30Y_{56}) + [400 + 50(E_n - 25)]$$

Subject to

$$E_n \leq 25 \quad (1)$$

$$t_{45} = 0 \quad (2)$$

$$E_n - t_{12} - t_{24} - t_{45} - t_{56} - t_{67} \geq 0 \quad (3)$$

$$E_n - t_{12} - t_{23} - t_{36} - t_{67} \geq 0 \quad (4)$$

$$E_n - t_{12} - t_{25} - t_{56} - t_{67} \geq 0 \quad (5)$$

$$E_1 - E_2 + t_{12} \leq 0 \quad (6)$$

$$E_2 - E_5 + t_{25} \leq 0 \quad (7)$$

$$E_2 - E_4 + t_{24} \leq 0 \quad (8)$$

$$E_2 - E_3 + t_{23} \leq 0 \quad (9)$$

$$E_4 - E_5 + t_{45} \leq 0 \quad (10)$$

$$E_5 - E_6 + t_{56} \leq 0 \quad (11)$$

$$E_3 - E_6 + t_{36} \leq 0 \quad (12)$$

$$E_6 - E_7 + t_{67} \leq 0 \quad (13)$$

$$E_1 = 0 \quad (14)$$

$$Y_{12} + t_{12} = 3 \quad (15)$$

$$Y_{24} + t_{24} = 10 \quad (16)$$

$$Y_{23} + t_{23} = 6 \quad (17)$$

$$Y_{25} + t_{25} = 11 \quad (18)$$

$$Y_{36} + t_{36} = 8 \quad (19)$$

$$Y_{45} + t_{45} = 0 \quad (20)$$

$$Y_{56} + t_{56} = 5 \quad (21)$$

$$Y_{67} + t_{67} = 6 \quad (22)$$

$$Y_{12} \leq 1 \quad (23)$$

$$Y_{23} \leq 2 \quad (24)$$

$$Y_{24} \leq 1 \quad (25)$$

$$Y_{25} \leq 4 \quad (26)$$

$$Y_{36} \leq 2 \quad (27)$$

$$Y_{56} \leq 1 \quad (28)$$

$$Y_{67} \leq 0 \quad (29)$$

$$Y_{45} = 0 \quad (30)$$

### Solution:

Using simplex method or LINDO software, we can get the following values of the variables.

Total cost of the project ( $z$ ) = 775

Project completion time after crashing ( $E_n$ ) = 22 days

$$Y_{12} = 1, Y_{23} = 0, Y_{24} = 0, Y_{25} = 1, Y_{45} = 0, Y_{56} = 1, Y_{36} = 0, Y_{67} = 0.$$

$$t_{12} = 2, t_{23} = 6, t_{24} = 10, t_{25} = 10, t_{45} = 0, t_{56} = 4, t_{36} = 8, t_{67} = 6.$$

## 21.7 RESOURCE LEVELLING

Resource levelling is the process of resource allocation that ensures resource demand does not exceed resource availability. The first allocation of resources may lead to uneven resource usage or over-utilization of some resources. It may therefore be necessary to rearrange resources – this is called levelling. ‘A technique in which start and finish dates are adjusted based on resource constraints with the goal of balancing demand for resources with the available supply’ – (PMI-2004). The ideal situation is to buildup of resource usage at the beginning of the project and reduction at the end of the project. The approach to resource leveling also depends on whether resources are dedicated to a particular project or shared across several projects and whether there is a need to keep all resources fully utilized.

Generally, there are two approaches to levelling and smoothing the resources required:

- *Time-limited resource considerations:* In this case, emphasis is given on completing the project within a specified time. This time is usually determined by network analysis.

Adjustments in the timing of any activity, and the resources required at a given time, must be undertaken within the float (slack) available. Obviously there can be no adjustment of activities which are on the critical path.

- *Resource-limited resource considerations:* In this case, the project must be completed with the resources available even if this means extending the project duration. If the total resource demand exceeds the resource availability at any time, then some of the activities must be delayed until there is sufficient resource availability.



## SUMMARY

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In this chapter, we have discussed about the nature and types of projects and various types of project appraisals required before starting any project. Project management is a separate stream in management and has a broad area. In this text we have an emphasis on project scheduling and costing. PERT and CPM have been explained with some numerical illustrations. Also, project crashing to minimize the project completion time and its effect project cost has been discussed with numerical illustrations.

## MULTIPLE-CHOICE QUESTIONS

1. CPM has
  - (a) One time estimate
  - (b) Two time estimates
  - (c) Three time estimates
  - (d) None of these
2. PERT has
  - (a) One time estimate
  - (b) Two time estimate
  - (c) Three time estimates
  - (d) None of these
3. PERT is
  - (a) The event-oriented technique
  - (b) The activity-oriented technique
  - (c) Work-oriented technique
  - (d) None of these
4. CPM is
  - (a) The event-oriented technique
  - (b) The activity-oriented technique
  - (c) Work-oriented technique
  - (d) None of these
5. The project completion time in network analysis is given by time following time of final activity on the critical path
  - (a) Early start
  - (b) Early finish
  - (c) Latest start
  - (d) Latest finish

6. A dummy activity
  - (a) is represented by dotted line in the network
  - (b) does not require any time for completion
  - (c) is artificially introduced to complete the network
  - (d) all the above
7. The difference between the time available for the activity and the time required to complete an activity is known as
  - (a) Slack
  - (b) Float
  - (c) Activity time
  - (d) None of these
8. The activities on critical path have
  - (a) Zero float
  - (b) Zero slack
  - (c) Both (a) and (b)
  - (d) None of these
9. Critical path has
  - (a) Minimum time to complete the project
  - (b) Maximum time to complete the project
  - (c) Minimum cost to complete the project
  - (d) Maximum cost to complete the project
10. The least possible time to complete an activity is known as
  - (a) Normal time
  - (b) Crash time
  - (c) Standard time
  - (d) None of these
11. By crashing the activity time,
  - (a) Direct cost decreases
  - (b) Direct cost increases
  - (c) Indirect cost increases
  - (d) None of these
12. Critical path in the network joins the activities having
  - (a) Maximum activity time
  - (b) Minimum activity time
  - (c) Zero activity time
  - (d) Average activity time
13. In which type of the project, the activities of the project belong to different functional area and they are completed in that respective functional department?
  - (a) Pure project
  - (b) Functional project
  - (c) Matrix project
  - (d) None of these
14. Which of the following is the disadvantage of the pure project?
  - (a) The project manager has full authority over the project/product.
  - (b) Team members report to one boss.
  - (c) It has shortened communication chains.
  - (d) There is lack of technology transfer.

15. Which of the following is the advantage of the functional project?
- Aspects of the project that are not directly related to the functional area get short-changed.
  - Motivation of team members is often weak.
  - A team member gets the opportunity to work on the different project.
  - The needs of the client are responded slowly.

## Answers

1. (a)      2. (c)      3. (a)      4. (b)      5. (b)      6. (d)      7. (b)      8. (c)      9. (a)  
 10. (b)     11. (b)     12. (a)     13. (c)     14. (d)     15. (c)

## REVIEW QUESTIONS

- What do you mean by project? Define the term ‘project management’.
- What are the advantages of project management. Discuss the project life cycle with various phases.
- What are the various methods of project appraisal? Discuss in detail.
- Compare pure project, functional project and matrix project.
- Differentiate between PERT and CPM.
- Write short notes on float and slack.
- Write a short note on project cost optimization.

## EXERCISES

- A project has the following activities with three time estimates (in weeks) (Table 21.8). Draw the network diagram. Find out the project completion time, total float, and free float for the non-critical activities, probability to complete the project before 25 weeks, and probability to complete the project before 15 weeks, probability to complete the project between 15<sup>th</sup> and 25<sup>th</sup> week.

**Table 21-8:** Project activity details

Activity	Optimistic time	Most likely time	Pessimistic Time
A-C	1	3	5
C-D	2	4	6
C-E	3	5	7
B-F	5	6	7
D-G	5	7	9
E-G	6	8	10
F-G	7	9	11
G-H	2	3	4

2. A project has the following activities with three time estimates in days (Table 21.9).

**Table 21-9:** Project activity details

Activity	Immediate predecessor	Optimistic time	Most likely time	Pessimistic time
A	—	5	4	6
B	—	12	8	16
C	A	5	4	12
D	B	3	1	5
E	D, A	2	2	2
F	B	5	4	6
G	C, E, F	14	10	18
H	G	20	18	34

- (a) Draw a network diagram.  
 (b) What is the probability completion time does not exceed 60 days?
3. Table 21.10 gives the activity time (in days) and direct cost of Rs as:

**Table 21-10:** Cost and activity time of a project

Activity	Immediate predecessor	Normal time	Crash time	Normal cost	Crash cost
A	—	4	3	60	90
B	—	6	4	150	250
C	—	2	1	38	60
D	A	5	3	150	250
E	C	2	2	100	100
F	A	7	5	115	175
G	D, B, E	4	2	100	240

The indirect cost varies per day as Rs 25. Draw the network diagram. Determine the project duration for which the total project cost will be minimum.



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## Total Quality Management

### 22.1 INTRODUCTION

‘Quality’ is a relative term and multifaceted entity which depends on the perception of users and providers. Quality may be expressed in terms of fitness for use (user-based approach), conformance to requirements (manufacturer-based approach), degree of preference (value-based approach), and degree of excellence (transcendent approach).

Quality of conformance is concerned with how well the manufactured product confirms to the design specifications. The quality of design of a product is concerned with the tightness of the specifications for manufacturing of the product. Quality control is a tool by means of which we observe the actual performance and its comparison with some standards. If there is a deviation between the observed performance and the standard performance, then it is necessary to take corrective action.

### 22.2 DEFINITIONS OF QUALITY

There are many definitions of quality. Quality may be considered as ‘Performance to standards’ or ‘meeting the customer’s needs’ or ‘satisfying the customer.’ Some common definitions of quality are given below as:

- Conformance to specifications measures how well the product or service meets the manufacturing and design specifications given by the design engineer.
- Fitness for use gives more emphasis on how well the product performs its intended functions.
- Price is the quality of a product or service that consumers want to pay for its usefulness, i.e. quality of the product may be measured in term of price paid by customers for its performance. This is the definition that relates quality and price; it assumes that the definition of quality is price sensitive.
- Quality does not mean only to the product or service itself; it also applies to the other factors, such as process, systems, individuals and working environment. For example, the quality of a good does not only depend on quality of process and machine, but also depends on the working environment, management’s commitment, and individuals working on that machine.

- Psychological criterion is involved on the judgemental evaluation of what constitutes product or service quality. Different factors contribute to the evaluation, such as the working environment or the perceived value of the products/services. For example, a student may receive average care of a teacher in a class room, but a very friendly teacher may leave the impression of high quality by providing personal care of students.

## 22.3 DIFFERENCES BETWEEN QUALITY OF GOODS AND SERVICES

Defining quality of physical goods is often different from that of services. Goods are tangible products that can be physically seen, touched, and directly observed. Examples include TV, computer, clothes, food items, etc. Therefore, quality definition for physical goods focuses on tangible product features. The most common quality definitions for goods are conformance, which is a degree to which a product meets the stated standards. Other common definitions of quality for goods include performance, such as the speed of a car; reliability that the product will function for an expected life without failure; features that are additional to the basic functions; durability that is an expected satisfactory operational life of the product; and serviceability that how readily a product can be serviced. The relative importance of these definitions is based on the preferences of each individual customer. It has been observed that the different persons have different versions of definition of quality.

In contrast to goods, service is intangible. Usually, the product cannot be seen or touched. Rather, it is experienced. Examples include delivery of lecture, treatment of doctor, and massage at the beauty parlour. Service quality is perception based and varies from person to person. It is also based on the performance of service providers and their behaviour and courtesy. The quality parameters for services include responsiveness, reliability, promptness, empathy and tangibility.

## 22.4 DIMENSIONS OF QUALITY

Garvin gave eight dimensions of quality that are explained below as:

**Performance:** It is measurable primary characteristics of a product or service related to its functioning.

**Features:** These are added characteristics that enhance the appeal of a product or service.

**Conformance:** It is the virtue of a product meeting specifications or industry standards.

**Reliability:** It is consistency of performance over time.

**Durability:** It is a time span showing useful life of a product or service.

**Serviceability:** It is the property concerned with the process of resolution of problems and complaints.

**Aesthetics:** It is the sensory characteristics of a product or service.

**Perceived quality:** It is subjective assessment of quality based on the perception of consumer related to the product.

Parasuraman, Zeithaml and Berry proposed five dimensions of service quality as given below:

**Reliability:** It is the ability to perform the promised service dependably and accurately.

**Responsiveness:** It is the willingness to help customers and provide prompt service.

**Assurance:** It is employees' knowledge, courtesy, and their ability to inspire trust and confidence.

**Empathy:** It is the caring and individualized attention given to customers.

**Tangibles:** It is appearance of physical facilities, equipment, personnel and written material.

## 22.5 QUALITY PLANNING, ASSURANCE AND CONTROL

*Quality planning* allows the quality parameters to be incorporated into the deliverable product before its production. It is therefore planned during the development phase of the product. It may involve identifying standards or best practices. In other words, it may be defined as systematic process that translates quality policy into measurable objectives and requirements, and lays down a sequence of steps for realizing them within a specified timeframe. Design of experiment is one of the tools used for quality planning. Quality planning involves the following activities:

- (a) Identifying the customers' requirements.
- (b) Listing the project deliverables to be produced.
- (c) Setting quality criteria for these deliverables.
- (d) Defining quality standards for the deliverables.
- (e) Gaining the customers agreement with the targets set.

*Quality assurance (QA)* refers to the planned and systematic activities used in a quality system to fulfil the quality requirements for a product and service. It is the technique to compare the existing quality parameters with a standard, and to monitor of processes and an associated feedback loop for error prevention. This is contrast with quality control, which focuses on outputs. *Quality assurance* is the activity done during the actual task to ensure that the standards identified during quality planning are met. It is therefore done during the implementation phase of the project. There are a number of tools available for a manager to assure the quality of products. One is *quality audits*, a structured review of quality for improving the performance. Another is *benchmarking*, comparing methods or products with others of recognized standard quality. Thus, a benchmark is not something created in the project, but something recognized by the project and used for comparison with products or methods in the project.

*Quality control* is an activity to improve the quality of products or process by focusing on outputs such as rework decisions, acceptance decisions and adjustment of processes. It is also done during the implementation phase of the project. Rework means a product does not meet standards, cannot be delivered to the end user, and requires some additional works to be done to bring it up to standards. Acceptance means the product meets the required standards and can be delivered to the end user. Process adjustments are corrective actions taken to increase acceptance and decrease rework.

## 22.6 COSTS OF QUALITY

The reason that quality has gained a big importance is that organizations have observed the effect of poor quality in terms of high conformance and non-conformance cost. Quality affects all aspects of the organization and has dramatic cost implications. Poor quality creates dissatisfaction among customers and eventually leads to market loss. Cost of quality can be broadly divided into two categories: quality control costs (cost of conformance) and quality failure costs (cost of failure). Quality control costs again divided into two categories: prevention costs and appraisal costs. Quality failure costs can be further divided into two categories: internal failure costs and external failure costs. The cost of failure decreases, and cost of appraisal and prevention increases when quality conformance increases or move towards 100 per cent, as shown in Figure 22.1.

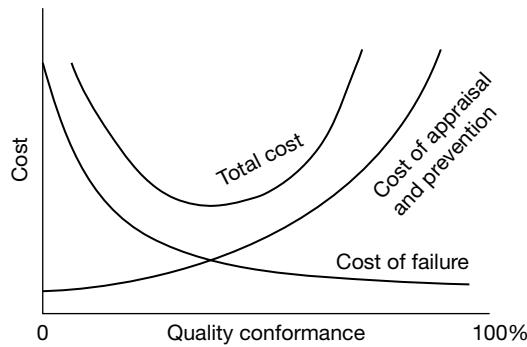


Figure 22-1: Optimum cost of quality

**Prevention costs** are the costs incurred in preventing the occurrence of poor quality of product. They include the costs involved in quality planning, developing and implementing a quality plan, product and process design, employee training, as well as maintaining records or documentation of information and data related to quality. The major prevention costs may be explained as follows:

*Quality planning and engineering:* Creation of the overall quality plan, the inspection plan, the reliability plan, the data system, all activities of the quality assurance function, the preparation of manuals and procedures used to communicate the quality plan, costs auditing the system.

*New products review:* Evaluation of new designs, preparation of tests and experimental programmes to evaluate the performance of new products.

*Product and process design:* Costs incurred during the design of the product or the selection of the production processes that are intended to improve the overall quality of the product.

*Process control:* The cost of process control techniques, such as control charts that monitor the manufacturing process in an effort to reduce variation and build quality into the product.

*Burn-in:* The cost of pre-shipment operation of the product to prevent early failures in the field.

*Training:* The cost of developing, preparing, implementing, operating and maintaining formal training programmes for quality.

*Quality data acquisition and analysis:* The cost of running the quality data system to acquire data on product and process performance includes the cost of analysing these data to identify quality problems.

**Appraisal costs** are the costs incurred in quality measurement. They include the cost of inspections, testing, and performing audits to make sure that quality standards are being met. The costs of worker time spent measuring quality and the cost of equipment used for quality appraisal are also included in this category.

*Inspection and testing of incoming materials:* Costs associated with the inspection and testing of all vendor-supplied materials, such as

- Receiving, inspection and test
- The same at the vendor's facility
- Periodic audit of the vendor's quality assurance system.

*Product inspection and testing:* The cost of checking the conformance of the product throughout its various stages of manufacturing, such as

- Final acceptance testing
- Packing and shipping checks
- Salaries of inspectors and supervisors

*Materials and services consumed:* The cost of operating a system that keeps the measuring instruments and equipment in calibration.

*Maintaining accuracy of test equipment:* The cost of operating a system that keeps the measuring instruments and equipment in calibration.

**Internal failure costs** are associated with defective products, components, and materials that fail to meet quality requirements and results in manufacturing losses. One part of the internal failure cost is rework, which is the cost of correcting the defective item. Sometimes the item is so defective that it cannot be rectified and must be scrapped. Its costs include all the material, labour, and machine cost spent in producing the defective product. Another part of internal failure costs includes the cost of machine downtime due to failures in the process.

*Scrap costs:* The net loss of labour, material and overhead (fixed costs) resulting from defective product that cannot economically be repaired or used.

*Rework:* The costs of correcting nonconforming units so that they meet specifications. Rework costs usually include additional operations or steps in the manufacturing process, thus includes extra direct labour, machine time and supply costs.

*Retest cost:* The costs of re-inspection and retesting of products that have undergone rework or other modification.

*Downtime costs:* The costs of idle production facilities that result from non-conformance to requirements.

*Downgrading cost:* The price differential between the normal selling price and a selling price that might be obtained for a product that does not meet the customer's requirements. Downgrading is common in the textile, apparel goods, electronics, and carpet industries.

**External failure costs** are associated with quality problems that occur at the customer site or plant. They occur due to shipment of defective products to customers. These costs can be particularly damaging because of poor reputation and faith between producer and customers. They include everything, such as customer complaints, product returns, repairs, warranty claims, recalls, and litigation costs resulting from product liability issues. A final component of this cost is lost sales and lost customers.

If we discuss about the trade-off between quality control cost and quality failure costs, both are reverse to each other. If quality control costs increases then failure costs will decrease and vice versa. The optimum quality can be determined using the graph in Figure 22.1.

*Complaint adjustment:* They are all the cost of investigation and adjustment of justified complaints attribute to the nonconforming product.

*Returned product and material:* They are the costs associated with the receipt, handling, and replacement of the nonconforming product or material returned from the field. Also recalls of defective product.

*Warranty charges:* They are the costs involved in service to customers under warranty contracts. They include repair and replacement cost of product or parts.

*Liability costs:* They are costs or awards incurred as a result of product liability litigation.

*Indirect costs:* In addition to the direct operating costs of external failures, there are some indirect costs such as customer dissatisfaction with the level of quality, loss of goodwill, loss of business reputation, loss of future business, and loss of market share.

## 22.7 EVOLUTION OF QUALITY CONCEPTS

Some milestones in the evolution of quality concepts are given below in chronological order:

- 1700–1900 Quality was determined by individual efforts of craftsmen.
- 1901 First standards laboratories were established in Great Britain.
- 1907–1908 AT&T started systematic inspection and testing of products and materials.
- 1919 Technical Inspection Association was formed in England; later it became the Institute of Quality Assurance.
- 1924 W. A. Shewhart introduced the concepts of control chart in Bell Laboratories.
- 1928 Acceptance sampling techniques were developed and refined by H. F. Dodge and H. G. Roming.
- 1940 The US War Department published a guide for using control charts to analyse process data.
- 1946 American Society for Quality Control (ASQC) was formed as a merger of various quality societies; International Standards Organization (ISO), was founded; Deming was invited in Japan to help occupation forces in rebuilding Japanese Industry; The Japanese Union of Scientists and Engineers (JUSE) was formed.
- 1946–1949 Deming was invited to give statistical quality control seminar to Japanese Industry.
- 1948 G. Taguchi started study and experimentation of Design of Experiment (DOE).
- 1950 K. Ishikawa introduces cause and effect diagram.

1951	A. V. Feigenbaum publishes the first edition of his book <i>Total Quality Management</i> ; JUSE started Deming Prize for significant achievement in quality control and quality methodology.
1954	Joseph M. Juran was invited by the Japanese to lecture on quality management and improvement.
1957	J. M. Juran and F. M. Gryna published <i>Quality Control Handbook</i> .
1960s	Courses in Statistical Quality Control become widespread in Industrial Engineering academic programmes; Zero Defect (ZD) programmes were introduced in some US industries.
1975–1978	Interest in quality circles begins in North America, which grows into the total quality management (TQM) movement.
1987	ISO published the first quality system standards.
1988	The Malcolm Baldrige National Quality Award is established by the US Congress; The European Foundation for Quality Management was founded which administers the European Quality Award.
1989	Motorola started a six-sigma programme.
1997	Motorola's Six-Sigma programme became popular in other industries.
1998	The American Society for Quality Control became the American Society for Quality (ASQ).
2000	ISO 9000:2000 standard was issued; Supply chain management and supplier quality became even more critical factor in business success.

## 22.8 QUALITY GURUS AND THEIR PHILOSOPHIES

The important contributions of Quality Gurus are summarized in Table 22.1:

**Table 22-1:** Quality gurus and their contributions

Quality guru	Main contributions
Walter A. Shewhart	Contributed to understanding of process variability. Developed concept of statistical control charts.
W. Edwards Deming	Stressed management's responsibility for quality. Developed '14 Points' to guide companies in quality improvement.
Joseph M. Juran	Defined quality as 'fitness for use.' Developed concept of cost of quality.
Armand V. Feigenbaum	Introduced concept of total quality control.
Philip B. Crosby	Coined the phrase 'quality is free.' Introduced concept of zero defects.
Kaoru Ishikawa	Developed cause-and-effect diagrams. Identified concept of 'internal customer.'
Genichi Taguchi	Focused on product design quality. Developed Taguchi loss function.

## 22.8.1 Walter A. Shewhart

Walter A. Shewhart was a statistician at Bell Labs during the 1920s and 1930s. He contributed quality control charts that are used to identify whether the variability in the process is due to chance cause or an assignable cause, such as poor skilled workers or miscalibrated machinery. He advocated that eliminating variability in product improves its quality. His work was recognized as the foundation for today's statistical process control, and he is often referred to as the 'Grandfather of Quality Control.' Quality control charts will be discussed in detail in Chapter 23.

## 22.8.2 W. Edwards Deming

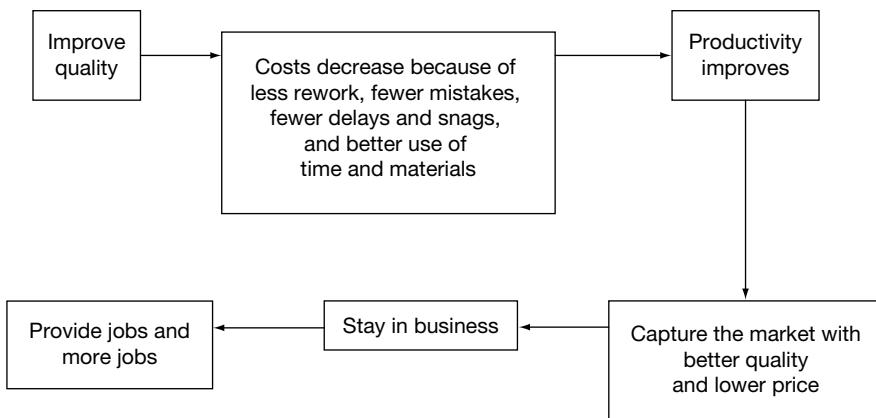
W. Edwards Deming is referred to as the 'Father of Quality Control.' He was a statistics professor at New York University in the 1940s. After World War II, he assisted many Japanese companies in improving quality. The Japanese regarded him with starting the Deming Prize in 1951, an annual award given to the firm that demonstrates outstanding quality. It was almost 30 years later that American businesses began adopting the Deming's philosophy.

### *Deming's 14 Points*

1. **Create a vision and demonstrate commitment:** An organization should define its value, mission, and vision of the future to provide long-term direction for its management and employees. According to Deming, businesses should not exist simply for profit; they are social entities and their aim is to serve their customers and employees. The responsibility lies with top management who must show commitment.
2. **Learn the new philosophy:** Historical methods of management developed by F. W. Taylor in early 20<sup>th</sup> Century, such as work measurement and observational work relationship will not work in today's global business environment. Deming experienced this problem a long time ago and sought to change in the attitudes that ignored the importance of quality improvement. A company must take a customer-driven approach based on mutual cooperation between labour and management and never-ending cycle of improvement. Everyone from boardroom to stockroom should learn the principle of quality and performance excellence to focus on customers' needs.
3. **Understand inspection:** Inspection does not add value to the product; this is only for gathering the information related to production of defective products. Deming encouraged workers to take responsibility for their work, rather than leave the problems for someone else down the production line. Simple statistical tools can be used to help the control process and eliminate mass inspection as the principal activity in quality control.
4. **Stop making decisions purely on the basis of cost:** Purchasing departments have long been driven by the objectives—cost minimization and competition among suppliers. The goal is to have a single supplier for each item to develop a long term relationship of loyalty and trust, thereby providing improved products and services. Management

previously favoured multiple suppliers for the reasons, such as providing protection against strike, natural disasters, while ignoring many hidden costs such as increased transportation costs, inventory costs, and administrative costs.

5. **Improve constantly and forever the system:** Improvements are necessary in both design and operations. Improved design of goods and services comes from understanding customer needs and continual market surveys and sources of feedback, and from understanding the manufacturing and service delivery process. When quality improves, productivity improves and costs decrease as shown in Deming chain reaction in Figure 22.2.



**Figure 22-2:** Deming chain reaction

6. **Institute training:** People are the most valuable resource of an organization. They want to do a good job, but they often does not know how. Management must take responsibility for helping them. Training does not only result in improvements in quality and productivity, but it adds to worker morale and demonstrates to workers that the company is dedicated to helping them and investigating in their future.
7. **Institute leadership:** Deming recognized that one of the biggest hurdles in improvement was a lack of leadership, not supervision. Supervision is simply overseeing and directing work; leadership means providing guidance to help employees do their jobs better with less effort.
8. **Drive out fear:** Fear is manifested in many ways: fear of reprisal, fear of failure, fear of the unknown, fear of relinquishing control, and fear of change. Workers are often afraid to report quality problems. No system can work without the mutual respect of managers and workers.
9. **Optimize the efforts of team:** Teamwork helps to break down barriers between departments and individuals. Barriers between functional areas occur when managers fear that they might lose power. Internal competition for raises and performance rating contributes to building barriers. The lack of cooperation leads to poor quality because

other departments cannot understand what their internal customers want and do not get what they need from their internal suppliers.

10. **Eliminate exhortation for the workforce:** Many early attempts to improve quality were focused on behavioural change. However, posters, slogans, and motivational programmes for zero defect, do right first time, improve productivity and quality and so on are directed at the wrong people. Motivational approaches overlook the major source of many problems. Causes of variations are stemming from the design of the system are management's problems, not the workers'.
11. **Eliminate the numerical quota's for the workforce and management by objective:** Many organizations manage by the numbers. Goals are useful, but numerical goals set for others without incorporating a method to reach the goal generate frustration and resentment. Management must understand the system and continually try to improve it, rather than focus on short-term goals.
12. **Remove barriers to pride in workmanship:** Deming believed that one of the biggest barriers to pride in workmanship is performance appraisal. Performance appraisal destroys teamwork by promoting competition for limited resources, fosters mediocrity because objectives typically are driven by numbers and what the boss wants rather than by quality, focuses on the short term and discourages risk taking, and confounds the 'people resources' with other resources.
13. **Encourage education and self-improvement for everyone:** Continue broad education for self-improvement. Organizations must invest in their people at all levels to ensure success in the long term. Developing the worth of the individual is a powerful motivation method.
14. **Take action to accomplish the transformation:** The management has to accept the primary responsibility for the never ending improvement of the process. It has to create a corporate structure to implement the philosophy. Any culture change begins with top management and includes everyone.

### 22.8.3 Joseph M. Juran

Joseph M. Juran is considered to have had the greatest impact on quality management after W. Edwards Deming. Juran originally worked in the quality programme at Western Electric. He published his book *Quality Control Handbook* in 1951, after the publication of this book, he went to Japan to work with manufacturers and teach classes on quality in 1954.

Juran did not propose a major cultural change in an organization, but rather sought to improve quality by working within the system familiar to managers. He argued that the employees at different levels in an organization speak in their own languages. According to Juran, top management speaks in the language of rupees whereas workers speak in the language of things or facilities. Therefore, the middle management must be able to speak in both the languages and translate between rupees and things.

Juran defined quality by considering both internal and external perspectives, i.e. quality is related to product performance that results in customer satisfaction and freedom from product deficiencies which avoids customer dissatisfaction.

### Juran's Quality Trilogy

Juran's trilogy for quality planning control and improvement is shown in Figure 22.3. Quality Planning starts with identifying internal and external customers with their needs, translating customer needs into specification, developing a product's feature that responds to those needs and developing the processes capable of producing the product or delivering the service. According to Juran, quality control involves determining what to be controlled, establishing units of measurement, establishing standards of performance, measuring actual performance, interpreting the difference between actual performance and standard performance, and taking action on the differences. Juran specified a detailed programme for quality improvement such as providing need for improvement, organizing support for the projects, diagnosing the causes, providing remedies for the causes, providing operating conditions, and providing control to maintain improvements.

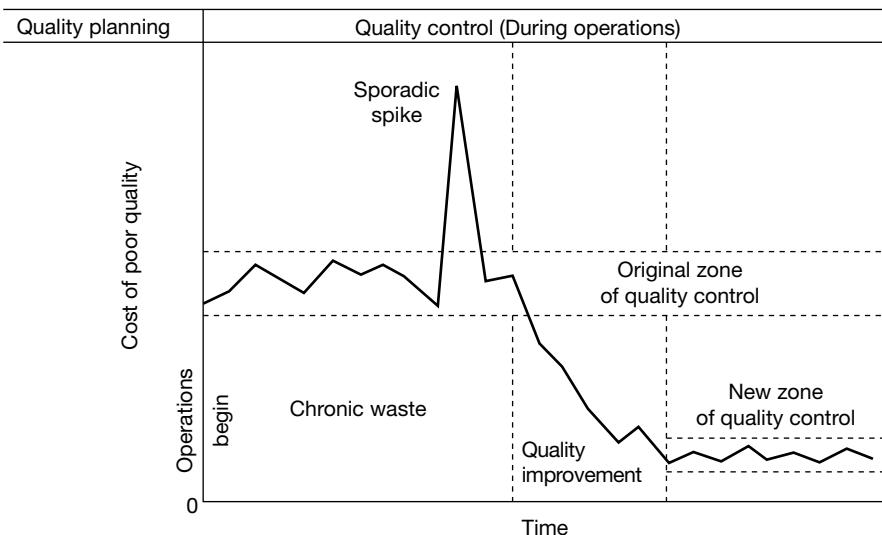


Figure 22-3: Juran's quality trilogy

### 22.8.4 Philip B. Crosby

Philip B. Crosby is another popular guru in the area of TQM. He worked as the vice president for quality at ITT (International Telephone and Telegraph). He developed the famous phrase 'Do it right the first time' and the philosophy of 'zero defects', arguing that no amount of defects should be considered acceptable. To promote his view, Crosby wrote a book titled *Quality Is Free*, which was published in 1979. He became famous for coining the phrase 'quality is free' and for pointing out the costs of quality that include not only the costs of wasted labour, equipment time, scrap, rework, and lost sales, but also organizational costs that are hard to quantify.

Similar to Deming and Juran, Crosby stressed the role of management in the quality improvement effort and the use of statistical control tools in measuring and monitoring quality. Crosby's quality management includes the following points:

- **Quality means conformance to requirements, not elegance:** Requirements must be clearly stated so that they cannot be misinterpreted. Requirements act as communication devices. Once the requirements are established, then one can take measurements to determine conformance to those requirements.
- **There is no such thing as quality problem:** Problems must be highlighted by those individuals or departments that cause them. A firm may experience accounting problems, manufacturing problems, design problems, front desk problems, and so on. In other words, quality originates in functional departments not in quality departments, therefore, the burden of responsibility of such problems falls on those functional departments.
- **There is no such thing as the economics of quality; doing the job right first time is always cheaper:** According to Crosby, economics of quality has no meaning. Quality is free. What costs money are all actions that involve not doing jobs right the first time. The Deming chain reaction sends the similar message.
- **The only performance measurement is the cost of quality, which is the expense of non-conformance:** Most of the companies spend 15 to 20 per cent of their sales rupees on quality costs. A well-run quality programme can achieve a cost of quality that is less than 2.5 per cent of sales, primarily in prevention and appraisal categories.
- **Zero defects:** Crosby found that zero defect concept was widely misunderstood and resisted. The theme of zero defect is do it right the first time. That means concentrating on preventing defects rather than just finding and fixing them.

Most of the human errors are caused by lack of attention rather than lack of knowledge. Lack of attention is created when it is assumed that error is inevitable. If we make a conscious effort to do our jobs right the first time, we will take a giant step toward eliminating waste of rework, scrap, and repair that increases cost and reduces individual opportunity.

## 22.8.5 Armand V. Feigenbaum

Feigenbaum is best known for coining the phrase 'Total Quality Control'. His book *Total Quality Control* was first published in 1951 under the title *Total Quality Control: Principles, Practices, and Administration*. His philosophy can be summarized in three steps as shown below:

- **Quality leadership:** A continuous management emphasis is grounded on sound planning rather than reaction to failure. Management must maintain a constant focus and lead the quality effort.
- **Modern quality technology:** The traditional quality department cannot resolve 80 to 90 per cent of quality problems. This task requires the integration of staff, engineers and workers involved in the process to evaluate and implement new techniques to satisfy customers in future.
- **Organizational commitments:** Continuous training and motivation of the entire workforce as well as an integration of quality in business planning indicate the importance of quality and provide the means for including it in all aspects of the firm's activities.

## 22.8.6 Genichi Taguchi

Genichi Taguchi is a Japanese quality leader well known for his work in the area of product design. He observed that as much as 80 per cent of all defective items were produced due to poor product design. Taguchi stressed efforts to improve quality at the design stage. It is much cheaper and easier to make changes during the product design stage than later during the production process.

Taguchi is well known for design of experiment. This method is an engineering approach to develop the robust design. Robust design is a design that results in products that can perform over a wide range of conditions. Taguchi's philosophy is based on the idea that it is easier to design a product that can perform over a wide range of environmental conditions than it is to control the environmental conditions.

Taguchi gave two concepts: (i) Taguchi loss function and (ii) design characteristic and noise. The loss function attempts to provide a formal process for computing the cost of deviation from the target value. The cost measured in this case is a social cost. If a product is made and it meets the target dimensions exactly, the cost is very low. This is because people buying the product will be happy that the product is precisely as specified. If the dimension of a product deviates from target, some people become unhappy and if deviation further increases, more people become unhappy as shown in Figure 22.4. Thus, the social cost increases. Taguchi termed this loss as 'social cost'. Taguchi loss function can be defined as:

$$\text{Loss-function for one piece of product, } L = k(y - m)^2$$

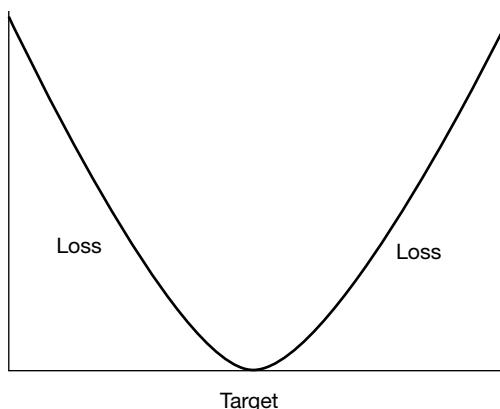
where  $L$  = Loss in rupees

$y$  = Quality characteristic

$m$  = Target value for  $y$

$k$  = Constant.

Taguchi's second contribution relates the design of products. He postulated two causes of variation in products: design characteristics and noise. Online activities such as statistical control charts to check the defects control only some of the defects. The more significant causes are outer



**Figure 22-4:** Taguchi loss function

and inner noise. Outer noise is the result of variation in operating environment and human errors and these are the factors which cannot be controlled. Inner noise is variation due to the factors such as deterioration of tools.

### 22.8.7 Kaoru Ishikawa

Kaoru Ishikawa is best known for the development of quality tools called cause-and-effect diagrams, also called fishbone or Ishikawa diagrams (Figure 22.5). These diagrams are used for quality problem-solving, and we will discuss them under the topic ‘7-Basic Tools for Quality Control’ later in this chapter. He emphasized the importance of the ‘internal customer,’ and ‘production process’. He also gave emphasis on the importance of total quality control, rather than just focusing on products and services.

Ishikawa believed that everyone in the company must have the same vision and a common goal. He emphasized that quality initiatives should be pursued at every level of the organization and that all employees should be involved. Ishikawa was a proponent of implementation of quality circles (small teams of employees) to solve quality problems.

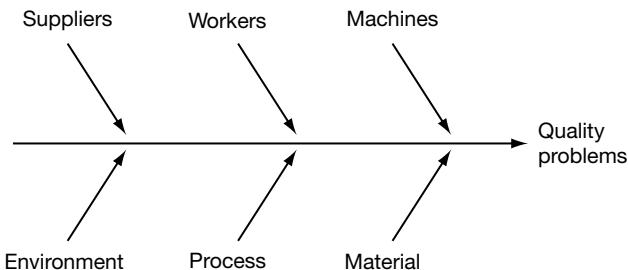


Figure 22-5: Cause-and-effect diagram

## 22.9 TOTAL QUALITY MANAGEMENT

Total quality management (TQM) is an integrated organizational effort designed to improve quality at every level. An organization can be broadly divided into four parts: products, process, individuals and systems. TQM considers the quality improvement of all the four parts not only the products or processes. TQM incorporates the philosophy of customer focus, continuous improvement, process management, use of quality tools, employee empowerment, benchmarking, and team approach.

### 22.9.1 Customer Focus

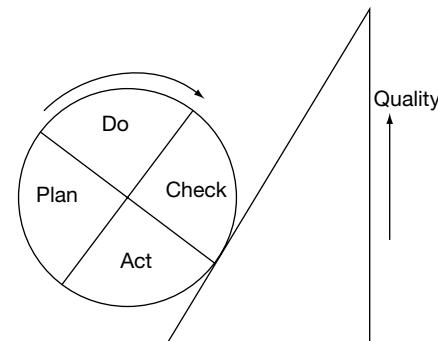
One of the main features of TQM is the organization’s focus on its customers. Quality is defined as fulfilling or exceeding customer expectations. The goal is to first identify and then meet customer needs. We can say that quality is customer-driven. However, it is not always easy to know what exactly the customer wants, because tastes and preferences change. Also, customer expectations often vary from one customer to the next.

## 22.9.2 Continuous Improvement

Another feature of the TQM philosophy is the focus on continuous improvement. Traditional systems operate on the assumption that once a company achieved a certain level of quality, it is successful and need not to further improve. Continuous improvement is a philosophy of never-ending improvement process; this is also known as *Kizen* in Japanese word. PDCA cycle is a tool of continuous improvement.

## 22.9.3 The Plan-Do-Check-Act Cycle

The plan-do-check-act (PDCA) cycle is concerned with the activities that a company needs to perform in order to incorporate continuous improvement in its operation. This cycle, shown in Figure 22.6, is also referred to as the Shewhart cycle or the Deming wheel. The circular nature of this cycle shows that continuous improvement, i.e., a never-ending process.



**Figure 22-6:** Plan-do-check-action (PDCA) cycle

**Plan:** The first step in the PDCA cycle is to plan. Managers must evaluate the current process and make plans based on existing problems. They need to document all current procedures, collect data, and identify problems. This information should then be studied and used to develop a plan for improvement as well as to evaluate the performance. Planning step involves identification of themes, data collection and analysis, and causal analysis.

**Do:** The next step in the cycle is implementing the plan. During the implementation phase, managers should document all changes made and collect data for evaluation.

**Check:** The third step is to check (study or analyse) the data collected in the previous phase. The data are evaluated to see whether the plan is achieving the goals established in the plan phase.

**Act:** The last phase of the cycle is to act on the basis of the results of the previous three phases. The best way to accomplish this is to communicate the results to other members in the company and then implement the new procedure if it has been successful. This is a cyclic process; the next step is to plan again. After we have acted, we need to continue evaluating the process, planning and repeating the cycle again.

### **22.9.4 Employee Empowerment**

Another aspect of TQM philosophy is employee empowerment to seek out the quality problems and correct them. With the old concept of quality, employees were afraid to identify problems for fear that they would be reprimanded. Often poor quality was passed on to someone else, in order to make it another's problem. The new concept of quality, TQM, provides incentives for employees to identify quality problems. Employees are appreciated for uncovering quality problems, not punished.

At present, workers are empowered to make decisions relative to quality in the production process. They are considered a vital element of the effort to achieve high quality. Their contributions are highly appreciated, and their suggestions are implemented. In order to perform this function, employees are given continual and extensive training in quality measurement tools. TQM differentiates between external and internal customers. External customers are those that purchase and use the company's goods and services. Internal customers are employees of the organization who receive goods or services from others in the company.

### **22.9.5 Benchmarking**

Another way of continuous improvement is to compare the existing practices with the best one among the companies. This is known as benchmarking. The ability to learn and study how others do things is an important part of continuous improvement. The benchmark company does not have to be in the same business, as long as it excels at something that the company doing the study wishes to emulate.

### **22.9.6 Team Approach**

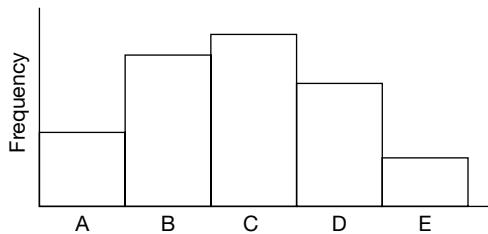
TQM is a team effort at organization level. Teamwork is highly appreciated to solve the quality problems. Using techniques such as brainstorming, group discussion, and quality control tools, teams work regularly to correct problems. The contributions of teams are considered vital to the success of the company. For this reason, companies set aside time in the working day for team meetings. Different teams consist of different degree of structure and formality, and solve different types of problems. One of the most common types of teams is the 'quality circle', a team of volunteer production employees and their supervisors whose purpose is to solve quality related problems. The circle is usually composed of eight to ten members, and decisions are made through group consensus. The teams usually meet weekly during working hours in a place designated for this purpose. They follow a preset process for analysing and solving quality problems. Open discussion is promoted, and criticism is not allowed. The functioning of quality circles is friendly and casual.

## **22.10 SEVEN BASIC TOOLS FOR QUALITY CONTROL**

Seven basic tools of quality control are histogram, Pareto diagram, check sheet, flowchart, scatter diagram, cause-and-effect diagram and control charts.

### 22.10.1 Histogram

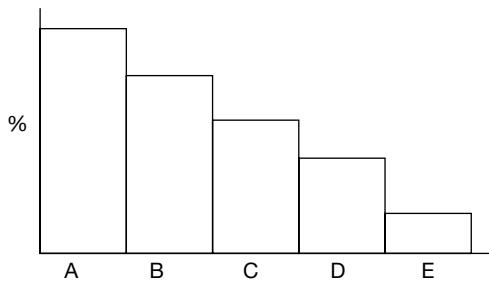
Histograms, also known as frequency distribution diagrams, are bar charts showing the distribution pattern of observations grouped in convenient class intervals and arranged in order of magnitude as shown in Figure 22.7. Histograms are useful in studying patterns of distribution and in drawing conclusions about the process based on the pattern.



**Figure 22-7:** Histogram

### 22.10.2 Pareto Diagram

The origin of the tool lies in the observation by an Italian economist Vilfredo Pareto that an 80 per cent of the wealth was in the hands of a 20 per cent people. He observed that this type of distribution pattern was common in most fields. Pareto diagram is a tool that arranges items in the order of the magnitude of their contribution as shown in Figure 22.8, thereby identifying a few items exerting maximum influence. This tool is used in SPC and quality improvement for prioritizing projects for improvement, prioritizing setting up of corrective action teams to solve problems, identifying products on which most complaints are received, identifying the nature of complaints occurring most often, identifying most frequent causes for rejections or for other similar purposes.



**Figure 22-8:** Pareto diagram

### 22.10.3 Check Sheets

Check sheets are tools for collecting data. They are designed specific to the type of data to be collected. Check sheets aid in systematic collection of data. Some examples of check sheets are

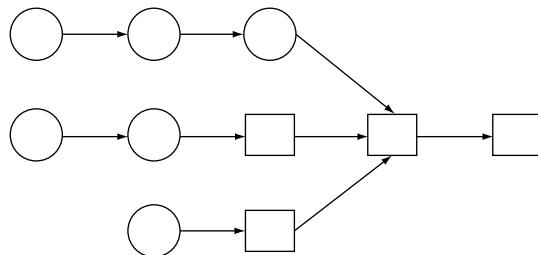
daily maintenance check sheets, attendance records, production log books, etc. Table 22.2 shows the check sheet for the defects.

**Table 22-2:** Check sheets

Defect type	No. of defects	Total
Rough surface	✓✓✓	3
Poor material	✓✓✓✓✓✓✓	7
Wrong dimensions	✓✓✓	3
Poor strength	✓✓	2

## 22.10.4 Flowchart

Flowchart demonstrates the various steps in a process by illustrating it in a clear and comprehensive manner. It identifies the areas where workflow may be blocked, or diverted. It also identifies where steps need to be added or removed to improve efficiency and create a standardized workflow. An example of the flowchart is shown in Figure 22.9.



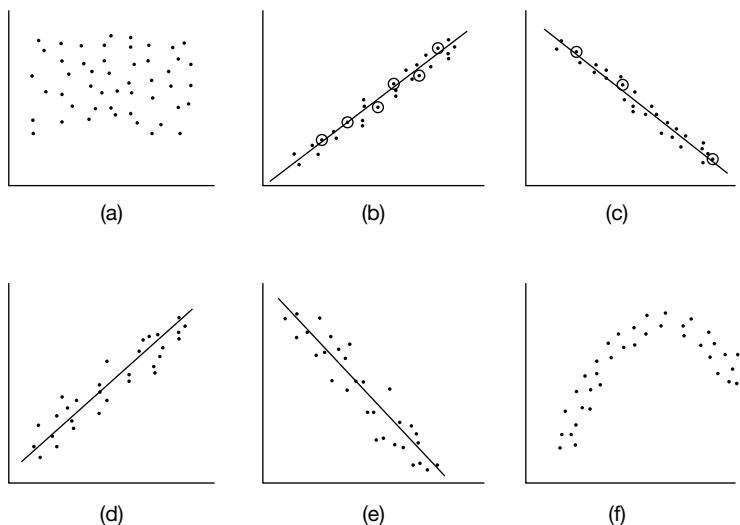
**Figure 22-9:** Flowchart

To prepare a flowchart, the process broken down into specific steps and put on paper in a flowchart. This procedure exposes some of the processes that are not working correctly. Other problems and hidden traps are often exposed when working through this process. Flowcharting also breaks the process down into its many sub-processes. Analysing each of these separately minimizes the number of factors that contribute to the variation in the process.

## 22.10.5 Scatter Diagrams

To solve a problem or analyse a situation, one needs to know the relationship between two variables. A relationship may or may not exist between two variables. If a relationship exists, it may be positive or negative; it may be strong or weak and may be simple or complex. A tool to study the relationship between two variables is known as scatter diagram. It consists of plotting a series of points representing several observations on a graph in which one variable is on the X-axis and the other variable in on the Y-axis. If more than one set of values are identical,

requiring more points at the same spot, a small circle is drawn around the original dot to indicate the second point with the same values. The way the points lie scattered in the quadrant gives a good indication of the relationship between the two variables. Let us see some common patterns seen in Scatter Diagrams and the conclusions one can draw based on these patterns. Figure 22.10 show some of the more common patterns.



**Figure 22-10:** Scatter diagrams showing various trends

## 22.10.6 Cause and Effect Diagram

A Cause-and Effect Diagram is a tool that shows systematic relationship between possible causes and their effect. It is an effective diagram to systematically generate ideas about causes for problems in a structured form. This tool was developed by Kauro Ishikawa, and therefore, it is also known as Ishikawa diagram. Fishbone-diagram is another name of cause-and-effect diagram due to the similarity in shape. The fishbone diagram organizes and displays the relationships between different causes for the effect that is being examined. The major categories of causes are put on major branches connecting to the backbone, and various sub-causes are attached to the branches. A tree-like structure shows the many facets of the problem as shown in Figure 22.11.

## 22.10.7 Control Charts

Control charts are a very useful statistical quality control tool. We will study in detail about the control charts in Chapter 23. These charts are used to evaluate whether a process operating is either under control or out of control. The control chart may be used for variables (such as weight, length, diameter, etc.) and attributes (defects and defectives).

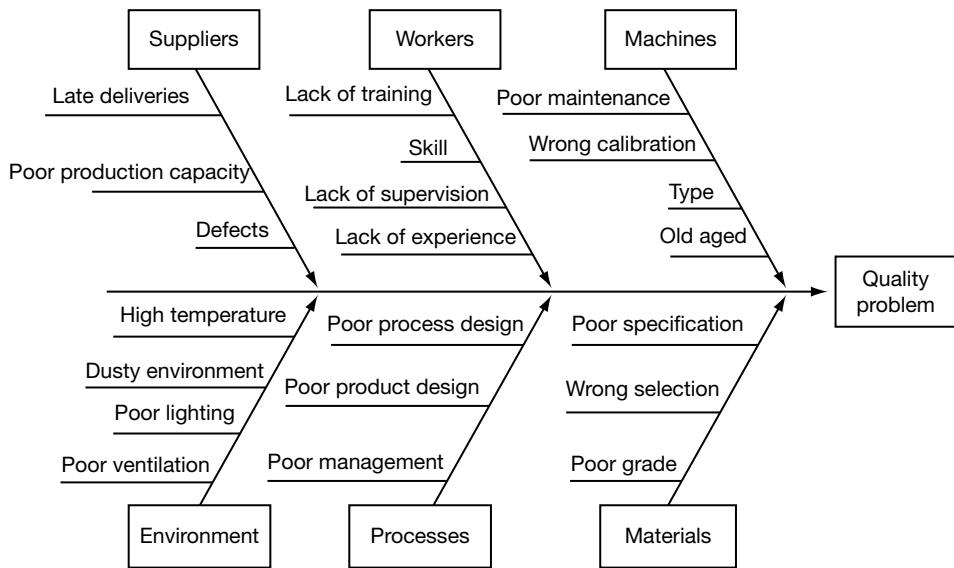


Figure 22-11: Cause and effect diagram

To evaluate whether or not a process is under control, we regularly measure the variable of interest and plot it on a control chart. The chart has a line down the centre as shown in Figure 22.12 representing the average value of the variable we are measuring. Above and below the centre line are two lines, called the upper control limit (UCL) and the lower control limit (LCL). As long as the observed values fall within the upper and LCLs, the process is in control and there is no problem with quality. When a measured observation falls outside of these limits, there is a problem with the process.

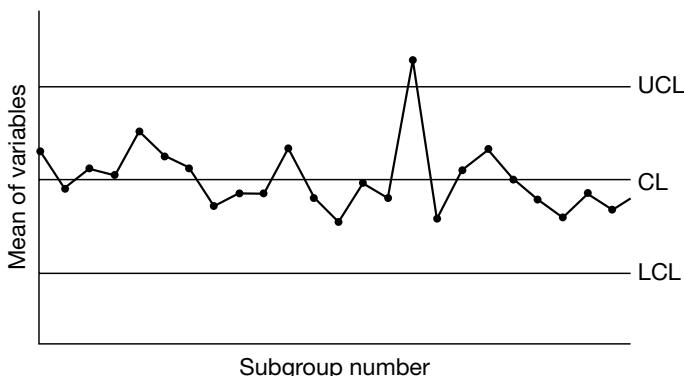


Figure 22-12: A sample of control chart

## 22.11 QUALITY FUNCTION DEPLOYMENT

Quality function deployment (QFD) is a tool to translate the customer's voice into engineering characteristics or design of a product. QFD enables us to view the relationships among the variables involved in the design of a product, such as technical versus customer requirements. In this text, an example is shown that how to incorporate the customer's voice/requirements in improving the quality of a ceiling fan. This is also known as house of quality (HOQ). The QFD of ceiling fan is shown in Figure 22.13.

QFD begins by identifying important customer requirements; these requirements are numerically scored (on a five-point rating scale) based on their importance, and scores are translated into product characteristics. Evaluations are then made of how the product is compared with its main competitors' product relative to the identified characteristics. This is shown on the right side of the house. Finally, specific goals are set to address the identified problems. The resulting matrix looks like a picture of a house and is often called the HOQ.

In Figure 22.13, 'A' shows the customer attributes that is customer voice about ceiling fan; 'B' shows the relative importance of customer attributes on 5 point rating scale; 'C' shows the engineering characteristics of ceiling fan; and 'D' shows the correlation matrix, i.e. correlation between customer attributes and engineering characteristics in terms of strong, medium and weak. 'E' shows the comparative analysis of case fan and competitors' fans; 'F' is the roof of the house showing the relationships among the engineering characteristics; and 'G' shows the absolute and relative weights of engineering characteristics. The absolute weight of engineering characteristics is calculated by summing the products of correlation strength and importance of corresponding customer attributes. For example, the absolute weight of air delivery =  $9 \times 5 + 9 \times 5 + 9 \times 2 = 108$ . The weights of various symbols are mentioned in Figure 22.13. 'H' shows the technical assessment of case fan and competitors' fans and 'I' shows the target value of engineering characteristics. On the basis of the relative weights of engineering characteristics, we can decide the priority of engineering characteristics of the product for quality improvement or design improvement. Thus, QFD is able to compare the customer assessment and technical assessment of a product with competitors' products.

## 22.12 QUALITY AWARDS

A number of quality awards have been used in various countries to promote TQM among its industries and service sectors. These awards can be broadly classified as competitive awards, non-competitive awards and awards which are a combination of the two. Competitive awards are given to the winners of the first, second and third place subject to a minimum qualification standard. The Malcolm Baldrige National Quality Award in the United States is an example of a competitive award. In non-competitive awards, the winner is the one who attains a minimum standard like pass mark in an exam. In the combination award schemes, awards are given to all who qualify the minimum requirement and special awards are given to the best. The European Foundation for Quality Management (EFQM) award scheme is an example of this type of award. Nowadays, there are a number of quality and business excellence awards all over the world. Some of the prominent award schemes are given below.

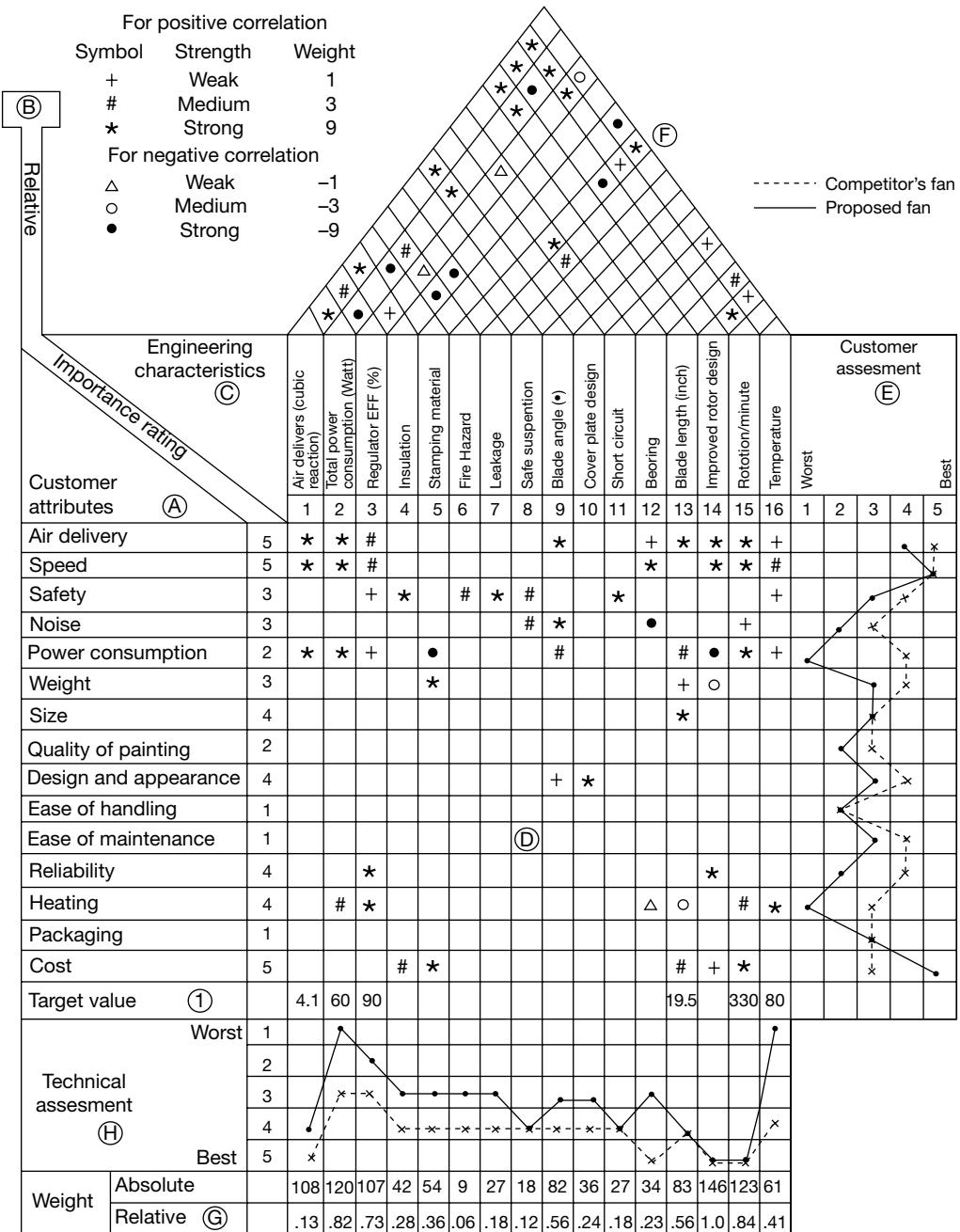


Figure 22-13: QFD for ceiling fan

Source: Das et al. (1999).

***Quality Awards in Europe***

- Austrian Quality Award
- Belgian Quality Award
- Czech Quality Award
- Danish Quality Award
- Dutch Quality Prize and Quality Award
- Estonian National Quality Award Contest
- European Quality Award
- Finnish Quality Award
- Flemish Quality Award
- French Quality Movement
- Hellenic National Quality Award Greece
- Hungarian Quality Award
- IQA National Award – Institute of Quality Assurance Awards
- Latvian Quality Award
- Lithuanian National Quality Prize
- ‘The Ludwig-Erhard-Preis’ in Germany
- Northern Ireland Quality Award
- ‘Oscar of Quality’, the National Quality Award of Serbia and Montenegro
- Polish Quality Award
- Portuguese Quality Award
- Premio Principe Felipe a la Calidad Industrial – Prince Philip
- Industrial Quality Award
- Quality Scotland Award
- ‘Joseph M. Juran’ Romanian Quality Award
- SAP Quality Award Schweiz 2004
- Slovak Republic National Quality Prize
- The Business Excellence Prize of the Republic of Slovenia
- Swedish Quality Award
- Swiss Quality Prize for Business Excellence ESPRIX
- TÜSİAD – Turkish Industrialist’s and Businessmen’s Association
- KalDer – Turkish Society for Quality – Quality Award in Turkey
- United Kingdom Business Excellence Award
- The Wales Quality Award

***Quality Awards in the Commonwealth of Independent States***

- Government Premium of the Republic of Belarus for Achievements in the Field of Quality
- Prize of the Government of Kazakhstan for Achievements in the Field of Quality
- Kyrgyz Quality Award
- Russian Quality Award of the Government
- The Ekaterina Velikaya Award for the best Russian Enterprises
- SEAL of Excellence Award for Handicraft Products in Central Asia by UNESCO-CASCA
- Ukrainian Quality Award

### **Quality Awards Schemes in North America**

- Canada Award for Excellence
- Malcolm Baldrige National Quality Award
- Washington State Quality Award

### **Quality Awards Schemes in Asia**

- Deming Prize in Japan
- Hong Kong Awards for Industry: Quality
- Philippine Quality Award
- Rajiv Gandhi National Quality Award in India
- Singapore Quality Award for Business Excellence

### **Quality Awards in Latin America**

- Premio Nacional a la Calidad – National Quality Award in Argentina
- Prêmio Nacional da Qualidade – National Quality Award in Brasil
- State of Rio de Janeiro Empresarial Quality Management Prize and Premio Qualidade Rio

### **Quality Awards in India**

- Rajiv Gandhi Quality Award
- Kirloskar Quality Award

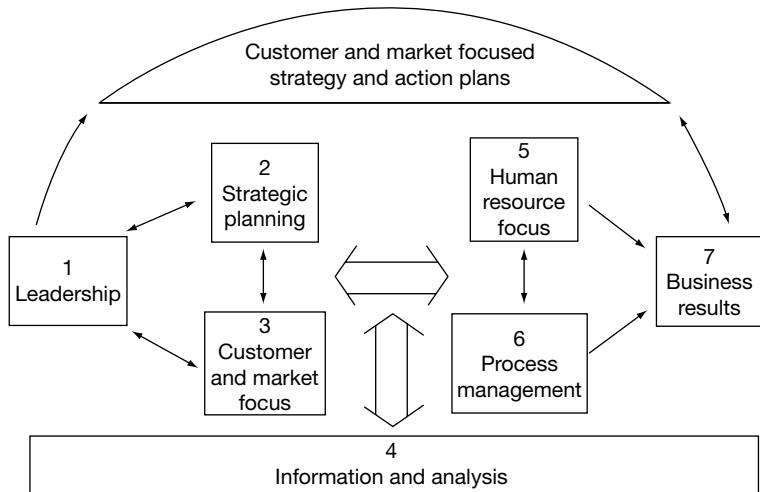
## **22.12.1 Malcolm Baldrige National Quality Award**

The Malcolm Baldrige National Quality Award (MBNQA) was established by the United States in 1987 to promote quality awareness, identify the requirements for quality excellence and share information about successful quality strategies and benefits. The National Institute of Standards and Technology (NIST) currently administers the award, with ASQ assisting with the application review process, preparation of award documents and other administrative duties.

A set of core principles for quality management were developed. These core principles are customer-driven quality, leadership, continuous improvement and learning, employee satisfaction, design quality and prevention, planning for the future, company responsibility and citizenship, and results were developed. These core principles form a framework for the basis of the MBNQA. The criteria, used to assess an applicant's performance, are divided into seven categories and provide the strategic direction for the entire system (Figure 22.14). The categories are leadership, strategic planning, customer and market focus, information and analysis, human resource focus, process management and business results. The Baldrige model is refined annually, with major improvements implemented every two years.

## **22.12.2 European Quality Award**

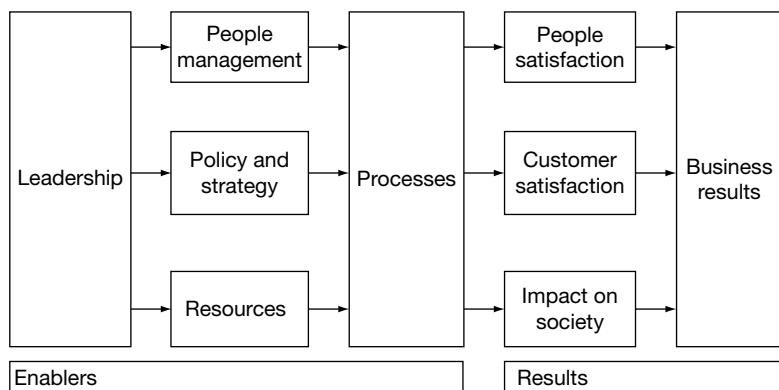
In 1988, 14 major European companies formed the EFQM with the endorsement of the European Commission. EFQM had developed the European Quality Award programme to honour outstanding European companies. Unlike other awards, the European Quality Award is a



**Figure 22-14:** Baldridge award criteria for performance excellence framework

regional programme that currently involves 16 countries: Austria, Belgium, the Czech Republic, Denmark, Germany, Hungary, Ireland, Italy, the Netherlands, Norway, Portugal, Russia, Slovenia, Spain, Turkey and the United Kingdom.

The award is similar to the MBNQA, but its criteria are comprised of enablers and results (Figure 22.15). The quality improvement enablers are leadership, people management, policy and strategy, resources and processes; and results are people satisfaction, customer satisfaction, impact on society and business results. EFQM improves its own quality model by continually analysing applicant feedback and making necessary adjustments.



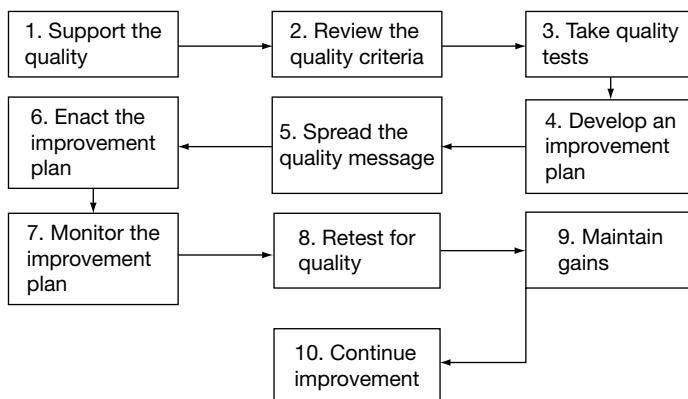
**Figure 22-15:** European quality model

### 22.12.3 Deming Prize

The Deming Prize was established in Japan by the Union of Japanese Scientists and Engineers (JUSE) in 1951. It was named in honour of W. Edwards Deming. Today, the Deming Prize honours private and public organizations for the successful implementation of quality control activities. Unlike other national or regional quality awards, the Deming Prize does not provide a model framework for organizing and prioritizing criteria. Instead, the evaluation includes 10 equally weighted points that each applicant must address. The 10 points involve the following categories: policies, organization, information, standardization, human resources, quality assurance, maintenance, improvement, effects and future plans. Expert panel members judge performance against these points.

### 22.12.4 Canadian Quality Award

The Canadian Quality Award was introduced for Business Excellence in 1984, but the programme was revised in 1989 to reflect the MBNQA concept. The resulting programme, i.e. the Canadian Quality Award was released in 1989. Canada's National Quality Institute continues to use the reward to honour the practice of continuous quality improvement in Canadian organizations. Instead of a framework linking award criteria, the Canadian Quality Award relies on a continuous improvement guide entitled 'The Roadmap to Excellence' (Figure 22.16).

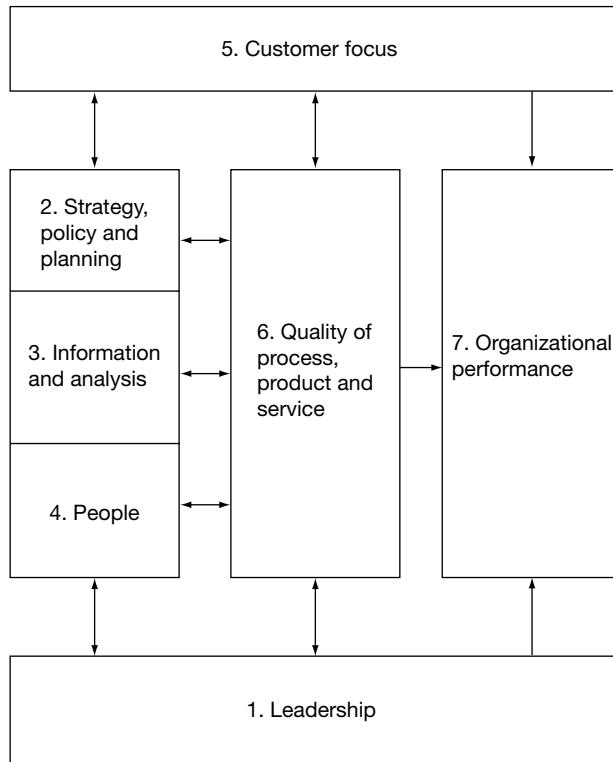


**Figure 22-16:** A roadmap to excellence for Canadian Quality Award

### 22.12.5 Australian Quality Award

The Australian Quality Award provides a model certified by the Australian Quality Council, an organization recognized by the Commonwealth Government of Australia as the top organization for quality management. The council was formed in 1993 with the merger of Enterprise Australia, the Total Quality Management Institute, the Australian Quality Awards Foundation and the Quality Society of Australia. Six additional organizations later joined the council, encouraging

quality performance in Australian industries. The goal of the award programme is to develop and deploy a comprehensive and contemporary body of quality principles and best practices. The council measures quality performance through seven categories of criteria (Figure 22.17).



**Figure 22-17:** Australian quality criteria framework

The people, information and analysis, strategy, policy, and planning categories have the greatest effect on the quality of processes. The quality of the processes, in turn, affects organizational performance. Customer focus and leadership are key elements, interacting with all the other parts of the model. Although it is similar to the MBNQA, the Australian Quality Award has an increased emphasis on the significance of multicultural management.



## SUMMARY

In this chapter, we have discussed about the quality of goods and services, various definitions of quality, and evolution of quality concepts. A number of quality gurus and their contributions in the field of quality have been outlined. Philosophy of total quality management has been

explained in brief. Different tools for quality control have been explained. The world-famous quality awards and their frameworks have been explained. QFD and its application have been demonstrated to improve the quality of a ceiling fan.

## MULTIPLE-CHOICE QUESTIONS

1. Quality may be expressed as
  - (a) Degree of satisfaction
  - (b) Conformance to the specification
  - (c) Degree of perfection
  - (d) All the above
2. Which of the following is NOT the property of physical goods?
  - (a) It can be stored
  - (b) Its quality can only be determined on the basis of perception of customer
  - (c) It can be transferred from one place to another.
  - (d) Ownership can be transferred easily
3. Which of the following is NOT the Garvin's eight dimensions of quality?

(a) Performance	(b) Features
(c) Conformance	(d) Empathy
4. What are the five dimensions of the service quality?

(a) Performance, reliability, responsiveness, assurance, empathy	(b) Reliability, responsiveness, assurance, empathy, tangibility
(c) Reliability, features, assurance, empathy, tangibility	
(d) Reliability, responsiveness, assurance, empathy, conformance	
5. The cost of training to the employee for quality improvement belongs to
  - (a) Prevention cost
  - (b) Appraisal cost
  - (c) Internal failure cost
  - (d) External failure cost
6. The cost of inspection and testing of incoming materials belongs to
  - (a) Prevention cost
  - (b) Appraisal cost
  - (c) Internal failure cost
  - (d) External failure cost
7. The concept of control chart was introduced by
  - (a) W. A. Shewhart
  - (b) H. F. Dodge
  - (c) G. Taguchi
  - (d) Joseph M. Juran
8. The cause-and-effect diagram was introduced by
  - (a) W. A. Shewhart
  - (b) H. F. Dodge
  - (c) K. Ishikawa
  - (d) Joseph M. Juran

9. "14 Points" to guide companies in quality improvement was developed by
  - (a) W. A. Shewhart
  - (b) W. E. Deming
  - (c) K. Ishikawa
  - (d) Joseph M. Juran
10. *Quality is Free* is written by
  - (a) W. A. Shewhart
  - (b) H. F. Dodge
  - (c) K. Ishikawa
  - (d) Philip B. Crosby
11. *Total Quality Control* was first published in 1951 by
  - (a) A. V. Feigenbaum
  - (b) W. A. Shewhart
  - (c) H. F. Dodge
  - (d) K. Ishikawa
12. Which of the following is the philosophy of total quality management?
  - (a) Customer focus
  - (b) Continuous improvement
  - (c) Team Approach
  - (d) All the above
13. Quality function deployment (QFD) is a tool to
  - (a) Plot the control chart
  - (b) Plot the O.C. Curve
  - (c) Plot the Pareto diagram
  - (d) Translate the customer's voice into engineering characteristics or design of a product
14. Malcolm Baldrige National Quality Award was established in
  - (a) Japan
  - (b) The United States
  - (c) Australia
  - (d) Germany
15. The concept of robust quality was given by
  - (a) W. A. Shewhart
  - (b) W. E. Deming
  - (c) K. Ishikawa
  - (d) G. Taguchi

## Answers

1. (d)      2. (b)      3. (d)      4. (b)      5. (a)      6. (b)      7. (a)      8. (c)      9. (b)  
10. (d)     11. (a)     12. (d)     13. (d)     14. (b)     15. (d)

## REVIEW QUESTIONS

1. Define the term 'quality'. What are the various perspectives to define the quality?
2. Enumerate the Garvin's eight dimensions of quality.
3. Discuss the five dimensions of service quality.
4. Write short notes on quality planning, quality assurance and quality control.
5. What do you mean by cost of quality? Define the cost of conformance and the cost of non-conformance.
6. Discuss in brief about the contribution of W. A. Shewhart, W. E. Deming, K. Ishikawa, Philip B. Crosby, G. Taguchi, A. V. Feigenbaum, and J. M. Juran, in the field of quality management.

7. Discuss the use of 7-basic tools for quality control.
8. Discuss the philosophy of TQM (total quality management).
9. Write short note of quality function deployment (QFD).
10. Write short note on Malcolm Baldrige National Quality Award.



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# CHAPTER ▶ 23

## Statistical Quality Control

### 23.1 INTRODUCTION

Control charting is one of the tools of *statistical quality control* (SQC). It is the most technically sophisticated tool of SQC. Walter A. Shewhart (1920) of the Bell Telephone Labs developed it as a statistical tool to study the manufacturing process variation for the purpose of improving the economic effectiveness of the process. These methods are based on continuous monitoring of process variation.

A typical control chart is a graphical display of a quality characteristic that has been measured or computed from a sample versus the sample number or time. The chart contains a centre line that represents the average value of the quality characteristic corresponding to the under control state. Two other horizontal lines, called the upper control limit (UCL) and the lower control limit (LCL) are also drawn. These control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them. As long as the points plot within the control limits, the process is assumed to be under control, and no action is necessary.

However, a point that plots outside of the control limits is interpreted as evidence that the process is out of control, and investigation and corrective action is required to find and eliminate the assignable causes responsible for this behaviour. The control points are connected with straight-line segments for easy visualization. Even if all the points plot inside the control limits, if they behave in a systematic or non-random manner, then this is an indication that the process is out of control.

*Assignable cause:* It is the cause which can be controlled after inspection. Most of the causes for variation are assignable cause. These causes can be avoided by proper control.

*Chance cause:* It is the cause which cannot be avoided. It may occur suddenly due to unpredictable reasons.

### 23.2 SOURCES OF VARIATIONS

There are five sources of variations: process, materials, environment, operators, and inspection. Variation due to **process** includes tool wear, way of operation, tool vibration, jigs and fixtures, etc. Variation due to **materials** includes quality characteristics of raw materials, such as tensile strength, ductility, malleability, hardness, toughness, thickness, porosity, moisture contents, etc. Variation due to the **environment** includes temperature, light, radiation, humidity, etc. Variation due to **operator** includes skill, motivation, wages, etc. Finally, variation due to **inspection** includes incorrect application of quality standards and inspection device.

### 23.2.1 Types of Variations

There are three types of variations in production of piece parts:

- (i) **Within piece variation:** This type of variation in quality parameters varies with in piece, for example, surface roughness of the different faces of the piece.
- (ii) **Piece-to-piece variation:** This type of variation in quality parameters varies from piece to piece, for example, change in surface finish due to continuous tool wear.
- (iii) **Time-to-time variation:** This type of variation in quality parameters varies due to different time of the day, for example, product produced in the morning will be better than the product produced after launch due to laziness of the worker.

## 23.3 CHART TECHNIQUES

Following steps are followed to produce a control chart:

- (i) **Select the quality characteristics:** Selection of quality characteristics is very important in control chart for variables/attributes. Only one-quality characteristics can be used at a time in control chart for the variable. But, in case of control charts for attributes, more than one quality characteristics can be considered to define that the product is defective or not. Similarly, in C-chart a no. of defects can be considered as count of defects in the products.
- (ii) **Choose the rational subgroup:** There are two ways to select the subgroup samples – product produced at one instant of time or close to that instant as possible and products produced over a period of time so that they can represent all the products. On the basis of subgroup size, there are some empirical judgement as discussed below:
  - As the subgroup size increases, the control limits become closer to central value, which makes the control chart more sensitive.
  - As the subgroup size increases, the inspection cost per subgroup increases.
  - When destructive testing is used and the item is expensive, a small subgroup size of 2 or 3 is preferred.
  - On the basis of statistical distribution, subgroup averages become normal when subgroup size equals or exceeds 4 even the subgroups are taken from non-normal population.
  - When the subgroup size exceeds from 10 or 12, the range (R-chart) should be replaced by standard deviation (S-chart).
- (iii) **Collect the data:** After deciding the subgroup size, data should be collected to prepare the control chart.
- (iv) **Determine the trial central line and control limits:** On the basis of the nature of data collected, use suitable control chart and determine the trial central line and control limits to check which subgroup is out of control.
- (v) **Establish the revised central line and control limits:** After finding the subgroup out of control, establish the revised central line and control limits by discarding those subgroups.
- (vi) **Achieve the purpose:** Finally, achieve the purpose of control chart for which it was prepared.

## 23.4 CONTROL CHARTS FOR VARIABLES

Control charts for variables monitor characteristics that can be measured and have a continuous scale, such as height, weight, volume, or width. When an item is inspected, the variable being monitored is measured and recorded. For example, if we produce bolts, thread dimension might be an important variable. We can take samples of bolts and measure their thread major and minor diameters. Two of the most commonly used control charts for variables monitor both the central tendency of the data (the mean) and the variability of the data (either the standard deviation or the range). Note that each chart monitors a different type of information. When observed values go outside the control limits, the process is assumed not to be in control. Production is stopped, and employees attempt to identify the cause of the problem and correct it.

### 23.4.1 Objectives of Variable Control Charts

There are following objectives of variable control charts:

- (i) To achieve quality improvement in products.
- (ii) To determine the process capability.
- (iii) To provide information for decision with regard to product specification.
- (iv) To provide information about the current production process.
- (v) To provide information about the decisions taken for currently produced items.

### 23.4.2 X-bar and R-chart

The X-bar (arithmetic mean) and R (range) control chart is used with variables data when subgroup or sample size is between 2 and 15; however, this is not a fixed rule. The steps for constructing this type of control chart are:

**Step 1:** Determine the data to be collected. Decide what questions about the process you plan to answer. Refer to the data collection module for information on how this is done.

**Step 2:** Collect and enter the data by subgroup. A subgroup is made up of variable data that represents a characteristic of a product produced by a process. The *sample size* relates to how large the subgroups are. Enter the individual subgroup measurements in time sequence in the portion of the data collection section of the control chart labelled *measurements* ( $X_1, X_2, X_3, \dots, X_n$ ).

**Step 3:** Calculate and enter the average for each subgroup. Use the formula below to calculate the average (mean) for each subgroup and enter it on the line labelled *Average* in the data collection section.

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

where  $\bar{X}$  is the average of the measurements within each subgroup,  
 $X_i$  is the individual measurements within a subgroup,  
 $i = 1, 2, \dots, n$  is the number of measurements within a subgroup or subgroup size, and

**Step 4:** Calculate and enter the range for each subgroup. Use the following formula to calculate the range ( $R$ ) for each subgroup. Enter the range for each subgroup on the line labelled *Range* in the data collection section.

$$\text{Range} = \text{Largest value in each subgroup} - \text{Smallest value in each subgroup}$$

$$R = X_{\max} - X_{\min}$$

**Step 5:** Calculate the grand mean of the subgroup's average. The grand mean of the subgroup's average ( $\bar{X}$ ) becomes the centre line for the control chart, i.e.,  $\bar{X}$  chart.

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_g}{g} = \frac{\sum_{j=1}^g \bar{X}_j}{g}$$

where  $\bar{\bar{X}}$  is the grand mean of subgroup's average.

$\bar{X}_j$  is the average of  $j^{\text{th}}$  subgroup, and

$g$  is the total no. of subgroups.

**Step 6:** Calculate the average of the subgroup ranges. The average of all subgroups becomes the centre line for the R-chart.

$$\bar{R} = \frac{R_1 + R_2 + \dots + R_g}{g} = \frac{\sum_{j=1}^g R_j}{g}$$

where  $R_i$  is the individual range for each subgroup

$\bar{R}$  is the average of the ranges for all subgroups

$g$  is the number of subgroups

**Step 7:** Calculate the UCL and LCL for the averages of the subgroups. At this point, the chart will look like a run chart. Now, the uniqueness of the control chart becomes evident as the control limits are calculated. Control limits define the parameters for determining whether a process is in statistical control. To find the  $\bar{X}$ -bar control limits (Figure 23.1), use the following formula:

$$\text{UCL}_{\bar{X}} = \bar{\bar{X}} + z\sigma_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$$

$$\text{LCL}_{\bar{X}} = \bar{\bar{X}} - z\sigma_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$$

$$\text{Here, } 3\sigma_{\bar{X}} = \frac{3\sigma'}{\sqrt{n}} = \frac{3}{d_2\sqrt{n}} \cdot \bar{R} = A_2\bar{R}$$

To find the R control limits (Figure 23.2), use the following formula:

$$\text{UCL}_R = \bar{R} + 3\sigma_R = \bar{R} + 3d_3\sigma = \bar{R} + 3d_3 \frac{\bar{R}}{d_2} = \left(1 + \frac{3d_3}{d_2}\right) \bar{R} = D_4\bar{R}$$

$$\text{LCL}_R = \bar{R} - 3\sigma_R = \bar{R} - 3d_3\sigma = \bar{R} - 3d_3 \frac{\bar{R}}{d_2} = \left(1 - \frac{3d_3}{d_2}\right) \bar{R} = D_3\bar{R}$$

Here,  $\sigma$  shows the standard deviations.

The value of the factor  $A_2$ ,  $D_4$ , and  $D_3$  are given in the statistical tables in the ‘Appendix’ section.

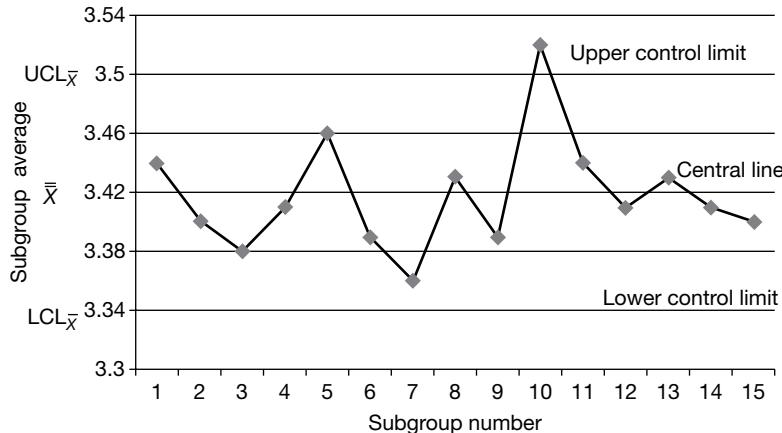


Figure 23-1: Presentation of X-bar chart

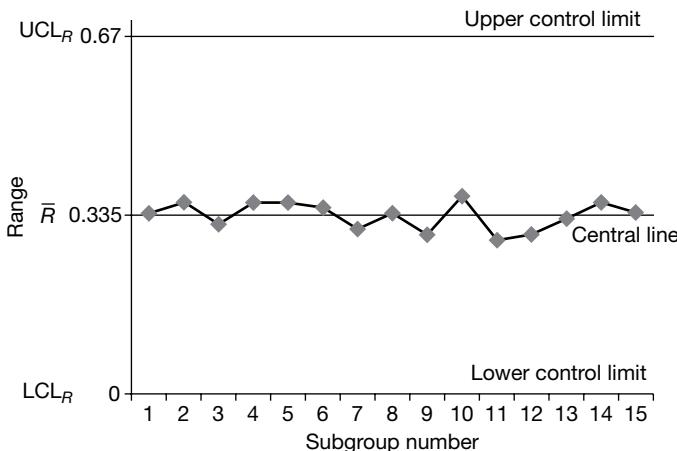


Figure 23-2: Presentation of R-chart

### Establish the Revised Control Limits

If some data in X-bar and R-charts are out of control, establish the revised control limits by discarding the subgroups out of control in the corresponding chart. To find the grand average and control limits following formula can be used.

$$X_0 = \frac{\sum_{j=1}^g \bar{X} - \bar{X}_d}{g - g_d}; \quad \bar{R}_0 = \frac{\sum_{j=1}^g R - R_d}{g - g_d}$$

where  $\bar{X}_d$  are discarded subgroup averages

$g_d$  is number of discarded subgroups

$R_d$  are discarded subgroup ranges

Using the standard value of the factors, the central lines and the  $3\sigma$  control limits for actual operations are obtained using the formula:

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 = \bar{X}_0 + A \frac{\bar{R}_0}{d_2}$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 = \bar{X}_0 - A \frac{\bar{R}_0}{d_2}$$

and

$$UCL_R = D_2\sigma_0$$

$$LCL_R = D_1\sigma_0$$

where the value of  $A$ ,  $D_1$ , and  $D_2$  are the factors which will be taken from the ‘Statistical Table Appendix’.

**Example 23.1:** A workshop produced the following data (in millimetre) regarding a product manufactured on 5<sup>th</sup> August 2014 during two shifts of six hours each as shown in Table 23.1. Find the control limits for X-bar and R-chart.

**Table 23-1:** Data regarding a product manufactured as on 5.08.2014

Subgroup no.	$X_1$	$X_2$	$X_3$	$X_4$	$\bar{X}$	$R$
1	36	41	46	37	40	10
2	34	36	34	40	36	6
3	68	59	69	64	65	10
4	44	40	38	34	39	10
5	34	43	41	42	40	9
6	46	41	41	44	43	5
7	36	38	41	33	37	8
8	51	49	52	48	50	4
9	42	36	43	47	42	11
10	39	38	38	41	39	3
11	41	37	37	37	38	4
12	47	35	40	38	40	12
13	45	42	38	39	41	7
14	43	45	50	42	45	8
15	39	29	35	33	34	10
16	34	29	40	41	36	12
17	58	28	44	38	42	30

(Continued)

**Table 23-1:** Data regarding a product manufactured as on 5.08.2014 (*Continued*)

Subgroup no.	$X_1$	$X_2$	$X_3$	$X_4$	$\bar{X}$	$R$
18	38	37	32	33	35	6
19	48	45	55	56	51	11
20	45	37	40	38	40	8
21	35	40	42	39	39	7
22	39	36	39	42	39	6
23	35	38	36	43	38	8
24	43	44	38	39	41	6

**Solution:**

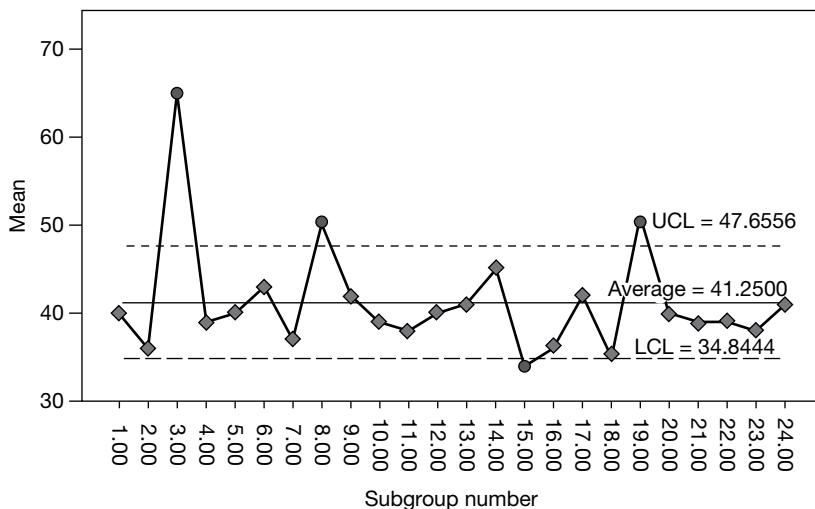
$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_g}{g} = \frac{\sum_{j=1}^g \bar{X}_j}{g} = \frac{990}{24} = 41.25; \quad \bar{R} = \frac{R_1 + R_2 + \dots + R_g}{g} = \frac{\sum_{j=1}^g R_j}{g} = \frac{211}{24} = 8.79$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + z\sigma_{\bar{X}} = \bar{\bar{X}} + 3\sigma_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R} = 41.25 + 0.729 \times 8.79 = 47.65$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - z\sigma_{\bar{X}} = \bar{\bar{X}} - 3\sigma_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R} = 41.25 - 0.729 \times 8.79 = 34.84$$

The value of  $A_2$  is taken for subgroup size,  $n = 4$  from the statistical table given in Appendix 3.

The  $\bar{X}$ -chart is shown in Figure 23.3. The 3<sup>rd</sup>, 8<sup>th</sup>, 15<sup>th</sup>, and 19<sup>th</sup> data are out of control. We have to find the revised control limits for  $\bar{X}$ .

**Figure 23-3:** X-bar chart

$$UCL_R = D_4 \bar{R} = 2.282 \times 8.79 = 20.058$$

$$LCL_R = D_3 \bar{R} = 0 \times 8.79 = 0$$

The value of  $D_4$  and  $D_3$  are taken for subgroup size,  $n = 4$  from the statistical table given in Appendix 3.

The seventeenth point is out of control in R-chart as shown in Figure 23.4; therefore, we have to draw revised control limits for R-chart.

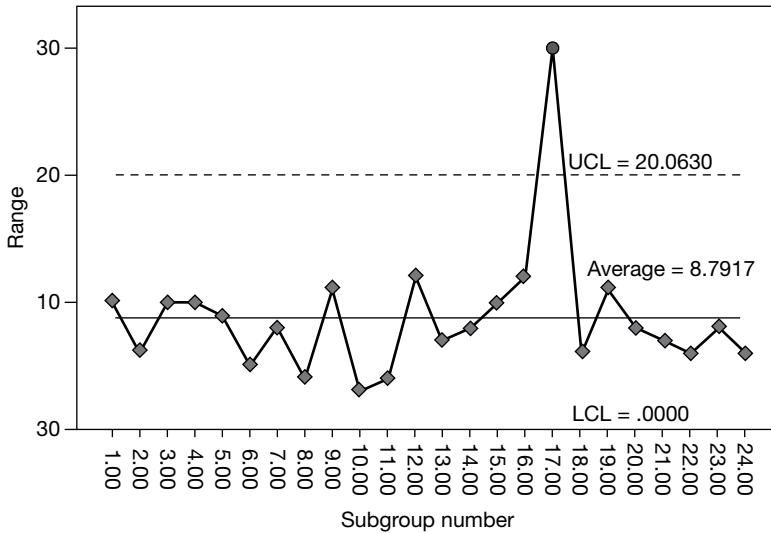


Figure 23-4: R-chart

### Revised Control Limits

$$\bar{X}_0 = \frac{\sum_{j=1}^g \bar{X} - \bar{X}_d}{g - g_d} = \frac{990 - 65 - 50 - 34 - 51}{20} = 39.5$$

$$\bar{R}_0 = \frac{\sum_{j=1}^g R - R_d}{g - g_d} = \frac{211 - 30}{23} = 7.86$$

$$UCL_{\bar{X}} = \bar{X}_0 + A\sigma_0 = \bar{X}_0 + A \frac{\bar{R}_0}{d_2} = 39.5 + 1.5 \times \frac{7.86}{2.059} = 45.226$$

$$LCL_{\bar{X}} = \bar{X}_0 - A\sigma_0 = \bar{X}_0 - A \frac{\bar{R}_0}{d_2} = 39.5 - 1.5 \times \frac{7.86}{2.059} = 33.773$$

and

$$UCL_R = D_2\sigma_0 = 4.698 \times \frac{7.86}{2.059} = 17.93$$

$$LCL_R = D_1\sigma_0 = 0 \times \frac{7.86}{2.059} = 0$$

The value of  $A$ ,  $d_2$ ,  $D_2$ , and  $D_1$  are taken for subgroup size,  $n = 4$  from the statistical table given in Appendix 3.

### 23.4.3 X-bar and S-chart

X-bar and R-charts are the most common charts for variables, but many companies use X-bar and S-charts. R-chart uses only two values in a subgroup maximum and minimum values. But, the S-chart uses all the data in subgroups. S-chart is therefore more accurate than R-chat. For subgroup size less than 10, both R-chart and S-chart show the same variation, but for the subgroup size more than 10, S-chart is preferred. Standard deviation for each subgroup can be found using the following formula:

$$S_j = \sqrt{\frac{n \sum_{i=1}^n X_i^2 - \left( \sum_{i=1}^n X_i \right)^2}{n(n-1)}}$$

where  $S_j$  is standard deviation of  $j^{th}$  subgroup.

Centre line in X-bar chart and S-chart can be found as:

$$\begin{aligned} \bar{\bar{X}} &= \frac{\sum_{j=1}^g \bar{X}_j}{g}; & \text{where } \bar{X}_j &= \frac{X_1 + X_2 + \dots + X_n}{n}; & \forall j \\ \bar{S} &= \frac{\sum_{j=1}^g S_j}{g}; & \text{where } S_j &= \sqrt{\frac{n \sum_{i=1}^n X_i^2 - \left( \sum_{i=1}^n X_i \right)^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{(n-1)}}, & \forall j \end{aligned}$$

Control limits in X-bar and S-chart can be calculated as:

$$\begin{aligned} UCL_{\bar{X}} &= \bar{\bar{X}} + A_3 \bar{S}; & LCL_{\bar{X}} &= \bar{\bar{X}} - A_3 \bar{S} \\ UCL_S &= B_4 \bar{S}; & LCL_C &= B_3 \bar{S} \end{aligned}$$

The value of  $A_3$ ,  $B_4$ , and  $B_3$  can be taken for specific subgroup size from statistical table given in Appendix 3.

#### **Establish the Revised Control Limits**

If some data in X-bar and S-charts are out of control, establish the revised control limit by discarding the subgroups out of control in the corresponding chart. To find the grand average and control limits, following formula can be used.

$$X_0 = \frac{\sum_{j=1}^g \bar{X} - \bar{X}_d}{g - g_d}; \quad \bar{S}_0 = \frac{\sum_{j=1}^g S - S_d}{g - g_d}; \quad \sigma_0 = \frac{\bar{S}_0}{c_4}$$

$$UCL_{\bar{X}} = X_0 + A\sigma_0; \quad LCL_{\bar{X}} = X_0 - A\sigma_0$$

$$UCL_S = B_6 \sigma_0; \quad LCL_S = B_5 \sigma_0$$

The value of  $A$ ,  $B_6$  and  $B_5$  can be taken for specific subgroup size from statistical table given in Appendix 3.

**Example 23.2:** Find the control limits for X-bar and S-chart from the data given in Example 23.1.

**Solution:**

Subgroup no.	X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	$\bar{X}$	s
1	36	41	46	37	40	4.546061
2	34	36	34	40	36	2.828427
3	68	59	69	64	65	4.546061
4	44	40	38	34	39	4.163332
5	34	43	41	42	40	4.082483
6	46	41	41	44	43	2.44949
7	36	38	41	33	37	3.366502
8	51	49	52	48	50	1.825742
9	42	36	43	47	42	4.546061
10	39	38	38	41	39	1.414214
11	41	37	37	37	38	2
12	47	35	40	38	40	5.09902
13	45	42	38	39	41	3.162278
14	43	45	50	42	45	3.559026
15	39	29	35	33	34	4.163332
16	34	29	40	41	36	5.597619
17	58	28	44	38	42	12.54326
18	38	37	32	33	35	2.94392
19	48	45	55	56	51	5.354126
20	45	37	40	38	40	3.559026
21	35	40	42	39	39	2.94392
22	39	36	39	42	39	2.44949
23	35	38	36	43	38	3.559026
24	43	44	38	39	41	2.94392

$$\bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \dots + \bar{X}_g}{g} = \frac{\sum_{j=1}^g \bar{X}_j}{g} = \frac{990}{24} = 41.25$$

$$\bar{S} = \frac{\sum_{j=1}^g S_j}{g} = \frac{93.624}{24} = 3.901$$

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_3 \bar{S} = 41.25 + 1.628 \times 3.901 = 47.6$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - A_3 \bar{S} = 41.25 - 1.628 \times 3.901 = 34.899$$

$$UCL_S = B_4 \bar{S} = 2.266 \times 3.901 = 8.84$$

$$LCL_S = B_3 \bar{S} = 0 \times 3.901 = 0$$

The value of  $A_3$ ,  $B_4$  and  $B_3$  are taken for subgroup size,  $n = 4$  from statistical table given in Appendix 3.

The  $X$ -bar chart considering the standard deviation in place of range is shown in Figure 23.5 and 23.6. It is observed that the data in subgroups 3<sup>rd</sup>, 8<sup>th</sup>, 15<sup>th</sup>, and 19<sup>th</sup> are out of control in X-bar chart and 17<sup>th</sup> subgroup is out of control in s-chart. Therefore, we have to find the revised control limits.

$$X_0 = \frac{\sum_{j=1}^g \bar{X} - \bar{X}_d}{g - g_d} = \frac{990 - 65 - 50 - 34 - 51}{20} = 39.5$$

$$\bar{S}_0 = \frac{\sum_{j=1}^g S - S_d}{g - g_d} = \frac{93.624 - 12.543}{23} = 3.525$$

$$\sigma_0 = \frac{\bar{S}_0}{c_4} = \frac{3.525}{0.9213} = 3.82$$

$$UCL_{\bar{X}} = X_0 + A\sigma_0 = 39.5 + 1.5 \times 3.82 = 45.23$$

$$LCL_{\bar{X}} = X_0 - A\sigma_0 = 39.5 - 1.5 \times 3.82 = 33.77$$

$$UCL_S = B_6 \sigma_0 = 2.088 \times 3.82 = 7.97$$

$$LCL_S = B_5 \sigma_0 = 0 \times 3.75 = 0$$

The value of  $A$ ,  $B_6$  and  $B_5$  are taken for subgroup size,  $n = 4$  from statistical table given in Appendix 3.

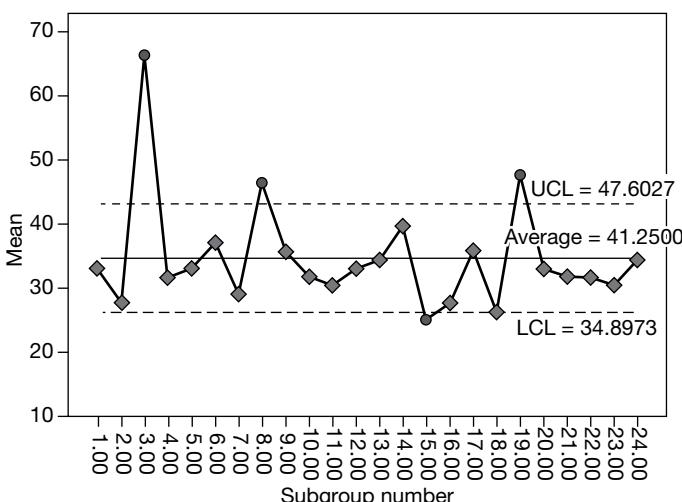


Figure 23-5: X-bar chart

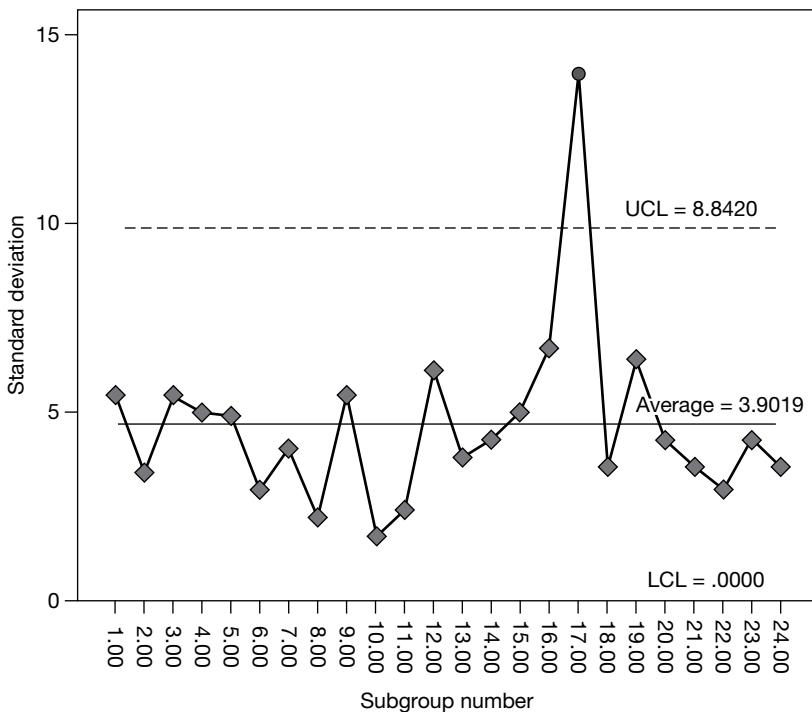


Figure 23-6: S-chart

**Note:** It has been observed that for the subgroups of size less than 12, there is no significant difference between X-bar and R-chart and X-bar and S-chart. But, for the subgroups larger than 12 X-bar and S-chart gives better result than that of X-bar and R-chart.

## 23.5 CONTROL CHART FOR TRENDS

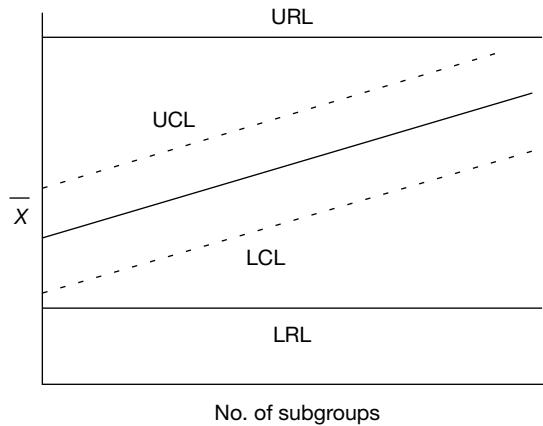
Due to die wear, the measurement gradually increases until it reaches the upper reject limit. Then die is replaced or reworked. Since the central line is on a slope, its equation must be determined. This is estimated using least-square method of plotting a line to a set of points. The equation for the trend line using the slope-intercept form is  $\bar{X} = a + bW$  as shown in Figure 23.7.

where  $W$  = Subgroup numbers represent the horizontal line

$a$  = Intercepts of vertical axis

$$a = \frac{\sum \bar{X} \sum W^2 - \sum W \sum (\bar{W} \cdot \bar{X})}{g \sum W^2 - (\sum W)^2}$$

$$b = \text{Slope of line} = \frac{g \sum (\bar{W} \cdot \bar{X}) - \sum W \sum \bar{X}}{g \sum W^2 - (\sum W)^2}$$

**Figure 23-7:** Trends chart

## 23.6 PROCESS UNDER CONTROL

The natural pattern of the variations of data on a control chart for the process under control can be given below as:

- (i) About 2/3 (67 per cent) of the points lie near the centre line.
- (ii) A few points (5 per cent) lie closer to control limits.
- (iii) The points are located back and forth across the centre line.
- (iv) Points are balanced on both sides of the centre line.
- (v) No point lies beyond the control limits.

When the process is under control, the following advantages can be observed by producer as well as consumer:

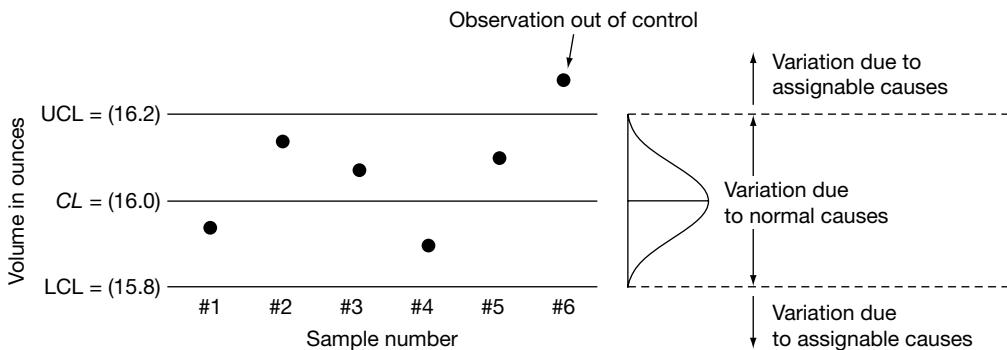
- (i) There will be fewer variations in product specifications.
- (ii) Due to the uniformity in products, only few samples are required for inspection or quality testing.
- (iii) The process capability ( $6\sigma$ ) can be easily attained. With the knowledge of process capability, a no. of decisions related to product specifications can be taken, such as amount of rework in case of insufficient tolerance, tight specifications/loose specifications, degree of quality assurance, etc.

## 23.7 PROCESS OUT OF CONTROL

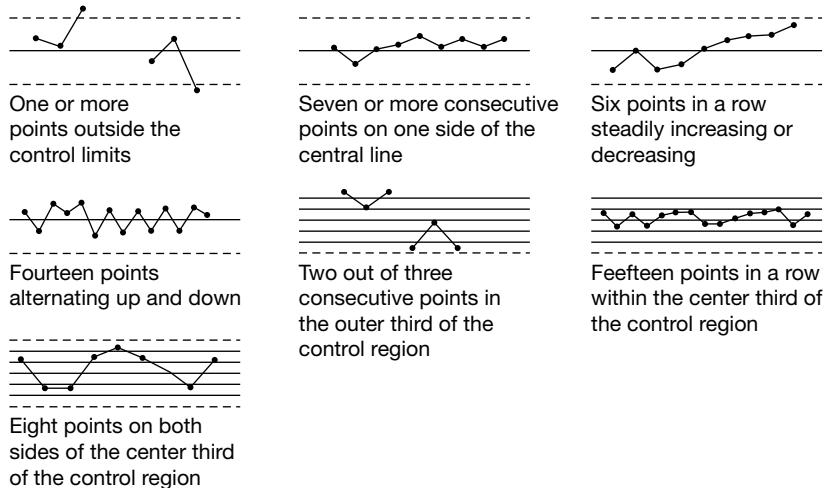
The process out of control can be shown in Figure 23.8.

### 23.7.1 Pattern of Data Out of Control

The patterns shown in Figure 23.9 of the data on control chart are assumed to be out of control.



**Figure 23-8:** Process out of control



**Figure 23-9:** Pattern of data out of control

### 23.7.2 Reasons for Process Out of Control

- Change or jump in level:** Jump in level in the X-bar chart may occur due to intentional or unintentional change in process setting, deploying inexperienced operator, using different raw materials, and a minor failure of machine parts. Similarly, Change in level in R-chart may occur due to inexperienced operators, sudden increase in gear play, and greater variations in incoming raw materials.
- Trends or steady change in level:** Steady change in level in X-bar chart may occur due to continuous wear of tool and die, gradual deterioration of equipment, viscosity in a chemical process, build-up of chips in jigs and fixtures, etc. Similarly, steady change in level in an R-chart may occur due to an improvement in worker skill, a decrease in worker's skill due to fatigue, and gradual improvement in homogeneity of incoming materials.

- (iii) **Recurring cycles:** Recurring cycles of data in the X-bar chart may occur due to seasonal effect of incoming materials, recurring effect of temperature and humidity, periodic chemical/mechanical/psychological treatment, and periodic rotation of operators. Similarly, recurring cycles of data on R-chart may occur due to operator's fatigue and rejuvenation resulting from morning, noon and afternoon breaks and lubrication cycles.
- (iv) **Two populations:** Two populations in the X-bar chart may occur due to large differences in material quality, two or more machines working on the same chart, large differences in testing methods or equipments, etc. Similarly, two populations in an R-chart may occur due to different workers using the same chart and materials purchased from different suppliers.
- (v) **Mistakes:** There is a no. of sources of mistakes which leads in the process out of control. These sources include measuring equipment out of calibration, errors in calculation, error in using test equipment, and taking samples from different populations.

## 23.8 PROCESS CAPABILITY AND SPECIFICATION LIMITS

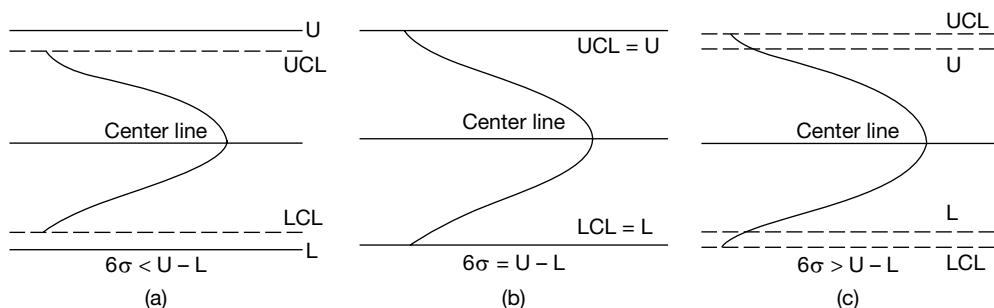
### **Central Limit Theorem**

If the population from which samples are taken is not normal, the distribution of sample average will tend towards normality provided that the sample size  $n$  is at least 4. This tendency increases as the sample size increases.

$$z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

### 23.8.1 Control Limits and Specification Limits

Control limits are established from the averages of subgroups of produced items, whereas the specification limits are the permissible variation of tolerance limits determined by the manufacturer. The difference between UCL and LCL is known as process spread. If we compare the process spread and the difference between specification limits, three situations may occur: (i) when the process spread ( $UCL - LCL = 6\sigma$ ) is less than the difference between the specification limits ( $U - L$ ), (ii) when the process spread ( $UCL - LCL = 6\sigma$ ) is equal to the difference between the specification limits ( $U - L$ ), and (iii) when the process spread ( $UCL - LCL = 6\sigma$ ) is greater than the difference between the specification limits ( $U - L$ ) (Figure 23.10).



**Figure 23-10:** Control limits and specification limits

**Case I:  $6\sigma < U - L$** 

Process spread is less than the difference between the specification limits is desirable since, in this situation defective items are not produced.

**Case II:  $6\sigma = U - L$** 

Process spread is equal to the difference between the specification limits is alarming situation however, defective items are not produced. In this case, the process may go out of control, therefore assignable causes of variation must be corrected as soon as they occur.

**Case III:  $6\sigma > U - L$** 

Process spread is greater than the difference between the specification limits is undesirable since, in this situation some defective items may be produced.

## 23.9 PROCESS CAPABILITY

To determine the process capability ( $6\sigma_0$ ) X-bar and R-chart should achieve the optimal quality improvement without investment in new equipment or modification. The quick method to determine the process capability is mentioned below as:

- (i) Take 20 subgroups of size 4 for a total 80 measurements. These subgroups should be selected randomly to avoid any biasing.
- (ii) Calculate the sample standard deviation  $S$  for each subgroup.
- (iii) Calculate the average sample standard deviation.

$$\bar{S} = \frac{\sum S}{g} = \frac{\sum S}{20}$$

- (iv) Estimate the population standard deviation,  $\sigma_0 = \frac{\bar{S}}{c_4}$
- (v) Process capability is equal to  $6\sigma_0$

$$(vi) \text{ Capability Index, } C_p = \frac{U - L}{6\sigma_0};$$

where  $C_p$  is capability index,  $U-L$  is tolerance,  
and  $6\sigma_0$  is process capability.

**Example 23.3:** Using the data provided in Example 23.1, find the process capability index. The upper and lower specification index are given as 50 mm and 30 mm, respectively.

**Solution:**

Given  $U = 50$  mm and  $L = 30$  mm. From Example 23.1, we find that the mean of standard deviation is 3.90 mm (as shown in Example 23.2) and for subgroup size of 4,  $c_4 = 0.9213$ .

$$\sigma_0 = \frac{\bar{S}}{c_4} = \frac{3.90}{0.9213} = 4.23$$

$$C_p = \frac{U - L}{6\sigma_0} = \frac{50 - 30}{6 \times 4.23} = 0.788$$

i.e.,

$$U - L < 6\sigma_0$$

Since process spread is greater than the difference between specification limits, it is undesirable since, in this situation, there is a chance to be produced some defective items.

## 23.10 LIMITATIONS OF CHART FOR VARIABLES

There are following limitations of control charts for variables:

- (a) It cannot be used for the attributes type quality characteristics.
- (b) If a product has a large number of variables, it will require individual X-bar and R-chart for each variable which is not possible. In this case, attributes chart is preferred.

## 23.11 CONTROL CHARTS FOR ATTRIBUTES

Attributes are the characteristics of a product or services which can be measured in terms of binary value such as pass-fail, accepted-rejected, good-bad, etc., but it cannot be measured in terms of numerical values such as weight, height, length, width, etc. There are two types of attributes:

- (a) Where measurement are not possible, for example, visually inspected items such as colour, missing parts, scratches and damage.
- (b) Where measurements are possible, but not made because of involvement of cost and time. For example, diameter of a wire can be measured using micrometer, but for easiness go-no go gauge is used to confirm the accuracy of the diameter.

Control charts for attributes can be used for both defects and defective analysis. Defects represent the individual quality characteristics of the product, but defective represents the entire product. A defective product may have a number of defects.

There are two different groups of control charts for attributes: one is for defective, i.e. p-chart which is based on binomial distribution and other one is for defects, i.e. c-chart and u-chart based on the Poisson distribution.

### 23.11.1 p-Chart

p-Chart is the most versatile control chart which can be used for single quality characteristics (like X-bar and R-chart) as well as for multiple quality characteristics at a time to control the entire product. It can be established for controlling the performance of operator, work centre, department, plant, etc.

#### **p-Chart Construction for Constant Subgroup Size**

Trial centre line and control limits

$$\bar{p} = \frac{\sum_{j=1}^g p_j}{g}$$

where  $p_j = \frac{np_j}{n}$ ,  $j$  indicates no. of subgroups, total no. of subgroups are  $g$ ,  
 $n$  is number inspected in a subgroup,  
 $np_j$  is number defectives of  $j^{th}$  subgroup,  
 $p_j$  is fraction defective in  $j^{th}$  subgroup, and  
 $\bar{p}$  is average fraction defective.

$$\text{UCL} = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}; \quad \text{LCL} = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

### Revised Control Limits and Centre Line

If some of the fraction defectives of subgroups go beyond the control limits, a revised control chart is required. To establish the revised control chart, the subgroups having the fraction defectives out of control should be discarded in the calculation of revised average fraction defective. A sample of p-chart is shown in Figure 23.11.

$$\bar{p}_0 = \frac{\sum_{j=1}^g p_j - p_d}{g - g_d}$$

where  $p_d$  is fraction defectives in discarded subgroups,  
 $g_d$  is subgroups.

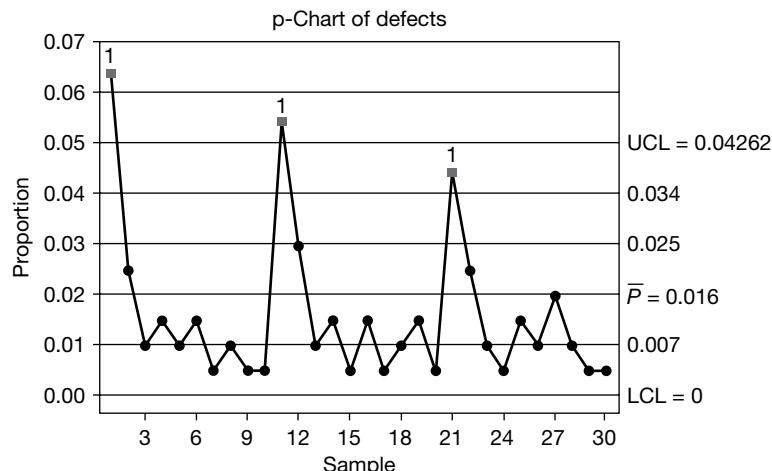


Figure 23-11: p-Chart

$$\text{UCL} = \bar{p}_0 + 3\sqrt{\frac{\bar{p}_0(1-\bar{p}_0)}{n}}; \quad \text{LCL} = \bar{p}_0 - 3\sqrt{\frac{\bar{p}_0(1-\bar{p}_0)}{n}}$$

**Example 23.4:** Inspection results of electric bulb produced by a company in a month of 25 working days are shown in Table 23.2. Daily production rate is 500 bulbs. Find the control limits using control charts for defectives.

Table 23-2: Daily production and defectives record of bulbs

Subgroup number	Number inspected (n)	Number defective (np)	Fraction defective (p)	Subgroup number	Number inspected (n)	Number defective (np)	Fraction defective (p)
1	500	18	0.036	14	500	5	0.01
2	500	5	0.01	15	500	2	0.004
3	500	14	0.028	16	500	8	0.016
4	500	6	0.012	17	500	10	0.02
5	500	2	0.004	18	500	12	0.024
6	500	9	0.018	19	500	25	0.05
7	500	9	0.018	20	500	3	0.006
8	500	2	0.004	21	500	8	0.016
9	500	13	0.026	22	500	9	0.018
10	500	16	0.032	23	500	2	0.004
11	500	3	0.006	24	500	5	0.01
12	500	15	0.03	25	500	3	0.006
13	500	14	0.028				

**Solution:**

$$\bar{p} = \frac{\sum_{j=1}^g p_j}{g} = \frac{0.436}{25} = 0.01744$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.01744 + 3\sqrt{\frac{0.01744(1-0.01744)}{500}} = 0.035$$

$$LCL = 0.01744 - 3\sqrt{\frac{0.01744(1-0.01744)}{500}} = 0$$

From p-chart shown in Figure 23.12, it is observed that first and nineteenth data are out of control. We have to draw the revised control limits.

$$\bar{p}_0 = \frac{\sum_{j=1}^{g-d} p_j - p_d}{g - g_d} = \frac{0.436 - 0.036 - 0.05}{23} = 0.0152$$

$$UCL_{Revised} = \bar{p}_0 + 3\sqrt{\frac{\bar{p}_0(1-\bar{p}_0)}{n}} = 0.0152 + 3\sqrt{\frac{0.0152(1-0.0152)}{500}} = 0.0316$$

$$LCL_{Revised} = \bar{p}_0 - 3\sqrt{\frac{\bar{p}_0(1-\bar{p}_0)}{n}} = 0.0152 - 3\sqrt{\frac{0.0152(1-0.0152)}{500}} = 0$$

**Note:** Similarly, per cent defective chart and number defective chart can be found using following formula for control limits.

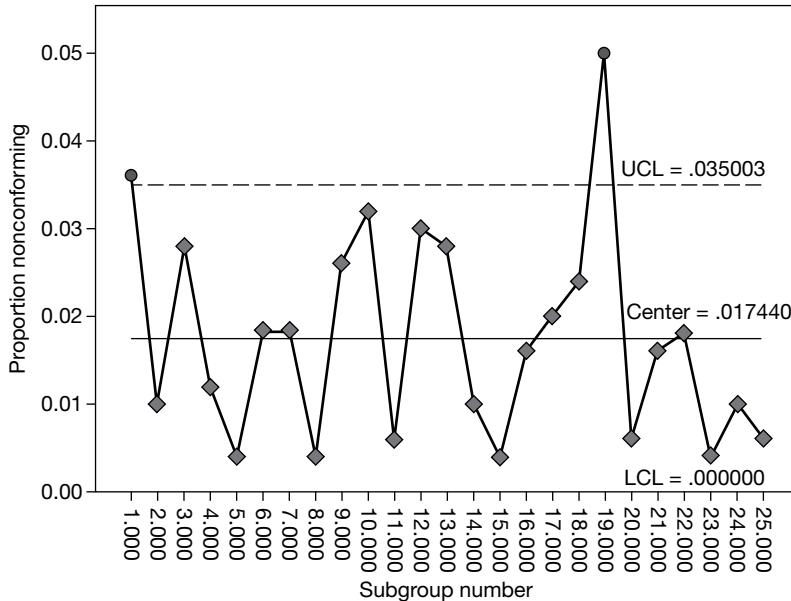


Figure 23-12: p-Chart

#### Per cent defective chart:

$$\text{Central line} = 100p_0$$

$$\text{UCL} = 100 \left[ p_0 + 3\sqrt{\frac{p_0(1-p_0)}{n}} \right]; \quad \text{LCL} = 100 \left[ p_0 - 3\sqrt{\frac{p_0(1-p_0)}{n}} \right]$$

#### Number defective chart:

$$\text{Central line} = np_0$$

$$\text{UCL} = n\bar{p}_0 + 3\sqrt{n\bar{p}_0(1-n\bar{p}_0)}; \quad \text{LCL} = n\bar{p}_0 - 3\sqrt{n\bar{p}_0(1-n\bar{p}_0)}$$

### 23.11.2 p-Chart Construction for Variable Subgroup Size

If the number inspected in subgroups varies from one subgroup to another, the control limits should be drawn individually for each subgroup as shown in Figure 23.13. Revised control limits can be drawn in similar way as in case of constant number inspected subgroups.

$$\bar{p} = \frac{\sum_{j=1}^g p_j}{g}$$

where  $p_j = \frac{np_j}{n_j}$ ,  $j$  indicates no. of subgroups, total no. of subgroups are  $g$ ,  
 $n$  is number inspected in a subgroup,  
 $np_j$  is number defectives of  $j^{th}$  subgroup,  
 $p_j$  is fraction defective in  $j^{th}$  subgroup, and  
 $\bar{p}$  is average fraction defective.

$$UCL_j = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_j}}; \quad LCL_j = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_j}}$$

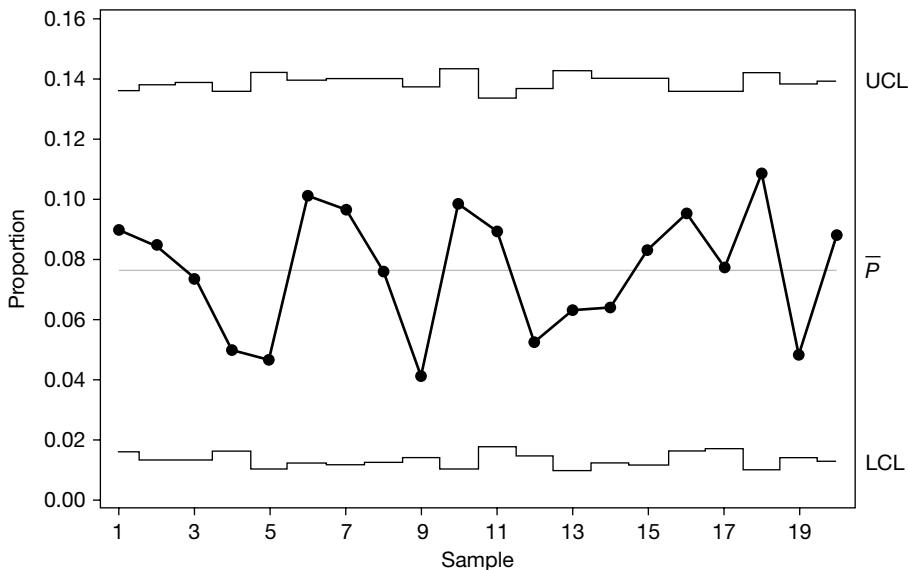


Figure 23-13: p-Chart for variable subgroup size

### Minimizing the Effect of Variable Subgroup Size

When the subgroup size varies from subgroup to subgroup, it presents a complex chart that is difficult to explain. By using average subgroup size, one limit can be calculated and placed on the control chart. A case of control chart is explained using an example as given below:

**Example 23.5:** There are one month data as given in Table 23.3. The production rate varies from day to day. Number of non-conforming components are also given with number inspected. Draw a p-chart and find the control limits for the data produced.

**Table 23-3:** Data for number inspected and number defectives

Subgroup no.	Number inspected ( $n_j$ )	Number defective ( $np_j$ )	Fraction defective ( $p_j$ )	Subgroup no.	Number inspected ( $n_j$ )	Number defective ( $np_j$ )	Fraction defective ( $p_j$ )
1	2385	47	0.02	14	1667	34	0.02
2	1451	18	0.012	15	2350	31	0.013
3	1935	74	0.038	16	2354	38	0.016
4	2450	42	0.017	17	1509	28	0.018
5	1997	39	0.02	18	2190	30	0.014
6	2168	52	0.024	19	2678	113	0.042
7	1941	47	0.024	20	2252	58	0.026
8	1962	34	0.017	21	1641	52	0.032
9	2244	29	0.013	22	1782	19	0.011
10	1238	39	0.032	23	1993	30	0.015
11	2289	45	0.02	24	2382	17	0.007
12	1464	26	0.018	25	2132	46	0.022
13	2061	47	0.023				

**Solution:**

$$\bar{p} = \frac{\sum_{j=1}^g p_j}{g} = \frac{\sum_{j=1}^g np_j}{\sum_{j=1}^g n_j} = \frac{1035}{50,515} = 0.020$$

$$\text{Control limits} = \bar{p} \pm 3 \sqrt{\frac{\bar{p}(1-\bar{p})}{n_j}} = 0.020 \pm 3 \sqrt{\frac{0.020(1-0.020)}{n_j}} = 0.02 \pm \frac{0.042}{\sqrt{n_j}}$$

Putting the values of  $n$  for different subgroup, we can get the individual control limits as shown in Figure 23.14.

We see that the data of 3<sup>rd</sup>, 19<sup>th</sup>, 21<sup>st</sup> and 24<sup>th</sup> are out of control. Therefore, we have to calculate the revised control limits.

$$\bar{p}_0 = \frac{\sum_{j=1}^g p_j - \sum_{d} p_d}{g - g_d} = \frac{0.514 - 0.038 - 0.042 - 0.032 - 0.007}{21} = 0.018$$

Using the new value of  $\bar{p}_0$ , we can calculate the new control limits for the individual subgroup.

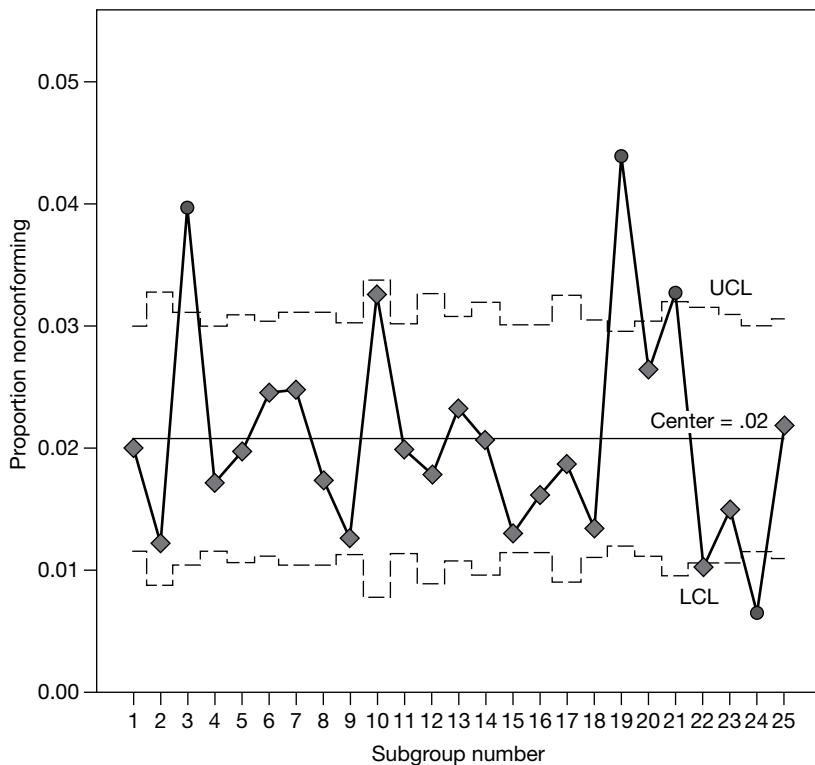


Figure 23-14: p-Chart for variable subgroup size

### Minimizing the Effect of Variable Subgroup Size

**Step 1:** Find the average subgroup size

$$n_{av} = \frac{\sum_{j=1}^g n_j}{g} = \frac{50515}{25} = 2020.5 \cong 2020$$

**Step 2:** Find the upper and LCLs using average subgroup size

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_{av}}} = 0.020 + 3\sqrt{\frac{0.020(1-0.020)}{2020}} = 0.0293$$

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n_{av}}} = 0.020 - 3\sqrt{\frac{0.020(1-0.020)}{2020}} = 0.0106$$

When an average subgroup size is used, there are four situations which occur between the control limits and the individual fraction defective values.

**Situation I:** When a point (subgroup fraction defective) falls inside the limits and its subgroup size is smaller than the average subgroup size, the individual control limits will be wider than that for average subgroup size; therefore, individual control limits are not needed. The data will be under control.

**Situation II:** When a point (subgroup fraction defective) falls inside the limits and its subgroup size is larger than the average subgroup size, and the difference in subgroup size is substantial, the individual control limits will be smaller than that for average subgroup size; therefore, individual control limits are needed to check the data that is either under control or out of control.

**Situation III:** When a point (subgroup fraction defective) falls outside the limits and its subgroup size is smaller than the average subgroup size and the difference in subgroup size is substantial, the individual control limits will be wider than that for average subgroup size, but individual control limits are needed to check the data that is either under control or out of control.

**Situation IV:** When a point (subgroup fraction defective) falls outside the limits and its subgroup size is larger than the average subgroup size, the individual control limits will be smaller than that for average subgroup size; therefore, individual control limits are not needed. The point will be out of control.

## 23.12 COUNT OF DEFECT CHART

There are two types of count of defect charts: (i) Count of defect chart (c-chart) (ii) and Count of defect per unit chart (u-chart)

### 23.12.1 c-Chart

In a c-chart, unit subgroup size is used and count of defects in each subgroup is inspected. A defective part can have a number of defects. A defect shows the quality characteristics, whereas defective represents the products having defects. There are following steps for constructing c-chart.

**Step 1:** Collect the data regarding the count of defects.

**Step 2:** Establish trial central line and control limits using following formula.

$$\bar{c} = \frac{\sum_{j=1}^g c}{g}$$

$$UCL = \bar{c} + 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

where  $\bar{c}$  is average count of defect.

If some data are out of control, establish a revised control limits by discarding the data out of control.

**Step 3:** Establish revised control limits

$$\bar{c}_0 = \frac{\sum_{j=1}^g c - c_d}{g - g_d}$$

$$\text{UCL} = \bar{c}_0 + 3\sqrt{\bar{c}_0}$$

$$\text{LCL} = \bar{c}_0 - 3\sqrt{\bar{c}_0}$$

where  $\bar{c}_0$  is average count of defect of revised data.

**Example 23.6:** There are 25 units of a product and the count of defects in each unit has been given in Table 23.4. Establish a control limits using c-chart.

**Table 23-4:** Data for count of defects

Unit no.	Count of defect	Unit no.	Count of defect
1	7	14	3
2	6	15	2
3	6	16	7
4	3	17	5
5	22	18	7
6	8	19	2
7	6	20	8
8	1	21	0
9	0	22	4
10	5	23	14
11	14	24	4
12	3	25	3
13	1		

**Solution:**

$$\bar{c} = \frac{\sum_{j=1}^g c}{g} = \frac{141}{25} = 5.64$$

$$\text{UCL} = \bar{c} + 3\sqrt{\bar{c}} = 5.64 + 3\sqrt{5.64} = 12.76$$

$$\text{LCL} = \bar{c} - 3\sqrt{\bar{c}} = 5.64 - 3\sqrt{5.64} = -1.484 \approx 0$$

5<sup>th</sup>, 11<sup>th</sup>, and 23<sup>rd</sup> data are out of control as shown in Figure 23.15; therefore, a revised control limit is required.

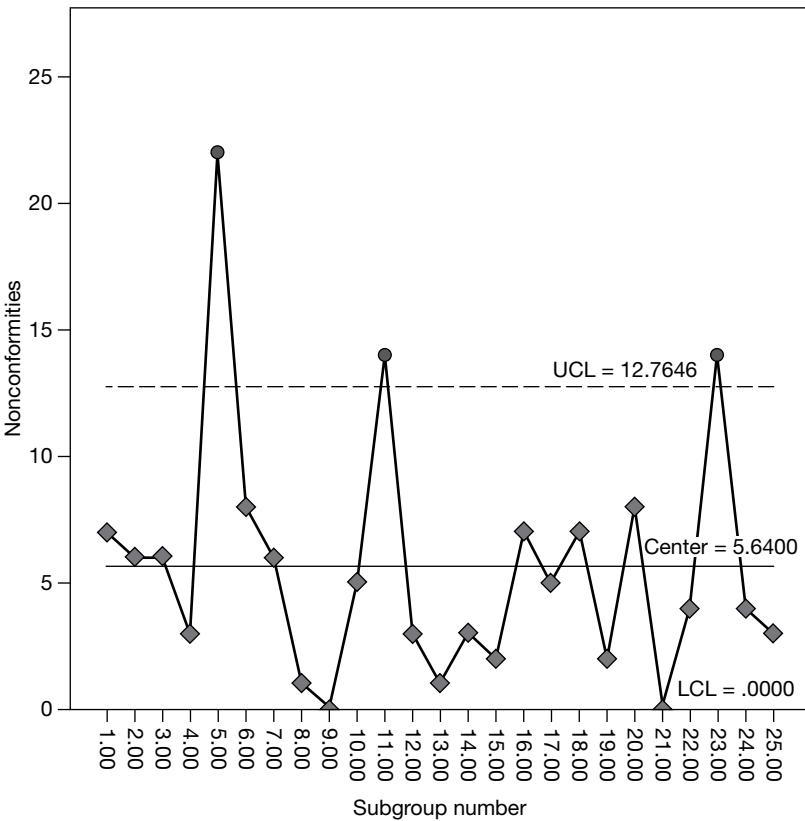


Figure 23-15: c-Chart

### Revised Control Limits

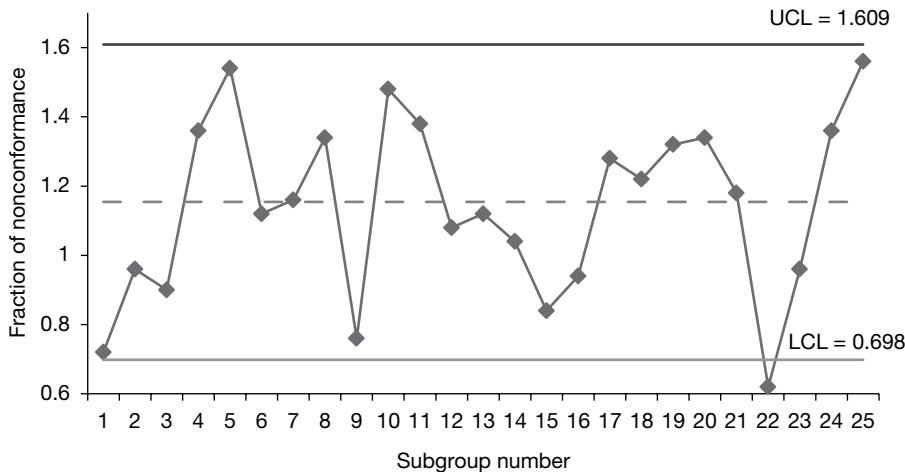
$$\bar{c}_0 = \frac{\sum_{j=1}^g c - c_d}{g - g_d} = \frac{141 - 22 - 14 - 14}{22} = 4.136$$

$$\text{UCL} = \bar{c}_0 + 3\sqrt{\bar{c}_0} = 4.136 + 3\sqrt{4.136} = 10.23$$

$$\text{LCL} = \bar{c}_0 - 3\sqrt{\bar{c}_0} = 4.136 - 3\sqrt{4.136} = -1.965 \approx 0$$

### 23.12.2 u-Chart

c-Chart can be used only when subgroup size is one but when subgroup size is more than one, u-chart is used. There are following steps to establish a u-chart (Figure 23.16);

**Figure 23-16:** u-Chart

**Step 1:** Collect the data regarding count of defect in each subgroup and find  $u$ .

$$u_j = \frac{c_j}{n}$$

where  $j$  is serial number of subgroup.

**Step 2:** Find the average value of  $u$ .

$$\bar{u} = \frac{\sum_{j=1}^g u_j}{g}$$

**Step 3:** Establish control limits.

$$\begin{aligned} \text{UCL} &= \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} \\ \text{LCL} &= \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} \end{aligned}$$

If some data be out of control, establish revise control limits.

$$\begin{aligned} \bar{u}_0 &= \frac{\sum_{j=1}^g u_j - \sum_{d} u_d}{g - g_d} \\ \text{UCL} &= \bar{u}_0 + 3\sqrt{\frac{\bar{u}_0}{n}} \\ \text{LCL} &= \bar{u}_0 - 3\sqrt{\frac{\bar{u}_0}{n}} \end{aligned}$$

**Example 23.7:** Establish a u-chart from the data given in Table 23.5.

**Table 23-5:** Data for count of defects per unit

Subgroup no.	Number inspected (n)	Count of defect (c)	Defect per unit ( $u = c/n$ )	Subgroup no.	Number inspected (n)	Count of defect (c)	Defect per unit ( $u = c/n$ )
1	50	36	0.72	14	50	52	1.04
2	50	48	0.96	15	50	42	0.84
3	50	45	0.9	16	50	47	0.94
4	50	68	1.36	17	50	64	1.28
5	50	77	1.54	18	50	61	1.22
6	50	56	1.12	19	50	66	1.32
7	50	58	1.16	20	50	37	1.34
8	50	67	1.34	21	50	59	1.18
9	50	38	0.76	22	50	31	0.62
10	50	74	1.48	23	50	48	0.96
11	50	69	1.38	24	50	68	1.36
12	50	54	1.08	25	50	78	1.56
13	50	56	1.12				

**Solution:**

$$\bar{u} = \frac{\sum_{j=1}^g u_j}{g} = \frac{28.85}{25} = 1.154$$

$$UCL = \bar{u} + 3\sqrt{\frac{\bar{u}}{n}} = 1.154 + 3\sqrt{\frac{1.154}{50}} = 1.609$$

$$LCL = \bar{u} - 3\sqrt{\frac{\bar{u}}{n}} = 1.154 - 3\sqrt{\frac{1.154}{50}} = 0.698$$

22<sup>nd</sup> point is out of control; therefore, revised control limits are required.

$$\bar{u}_0 = \frac{\sum_{j=1}^g u_j - u_d}{g - g_d} = 1.17$$

$$UCL = \bar{u}_0 + 3\sqrt{\frac{\bar{u}_0}{n}} = 1.17 + 3\sqrt{\frac{1.17}{50}} = 1.636$$

$$LCL = \bar{u}_0 - 3\sqrt{\frac{\bar{u}_0}{n}} = 1.17 - 3\sqrt{\frac{1.17}{50}} = 0.716$$

## 23.13 DEMERITS CLASSIFICATION AND CONTROL CHART

On the basis of severity of defects, they can be classified as critical defects, major defects and minor defects.

*Critical defects:* They are the defects in a product which result in hazardous or unsafe use. They also prevent the performance of the product.

*Major defects:* These are the defects in a product which result in failure of working or performance.

*Minor defects:* These are not severe and do not affect the working performance of a product, but affect the appearance or look of the product.

### Control Chart

Control chart is established and plotted for a count of demerits per unit. The demerit per unit is given by the formula:

$$D = w_c u_c + w_{ma} u_{ma} + w_{mi} u_{mi}$$

where  $D$  = demerits per unit;

$w_c, w_{ma}, w_{mi}$  = weights for critical, major, and minor defects, respectively;

$u_c, u_{ma}, u_{mi}$  = count of defects for critical, major, and minor defects, respectively.

If  $w_c, w_{ma}$ , and  $w_{mi}$  are 9, 3, and 1, respectively, the formula is

$$D = 9u_c + 3u_{ma} + 1u_{mi}$$

The value of  $D$  is calculated from the formula for each subgroup and central line and the  $3\sigma$  are calculated using following formulas:

$$D_0 = 9u_{0c} + 3u_{0ma} + 1u_{0mi}$$

$$\sigma_{0u} = \sqrt{\frac{9^2 u_{0c} + 3^2 u_{0ma} + 1^2 u_{0mi}}{n}}$$

$$UCL = D_0 + 3\sigma_{0u}; \quad LCL = D_0 - 3\sigma_{0u}$$

**Example 23.8:** Assuming that a 9:3:1 three-class weighing system is used, determine the central line and control limits when  $u_{0c} = 0.09$ ,  $u_{0ma} = 0.6$ , and  $u_{0mi} = 3.0$ , and  $n = 50$ . Also calculate the demerits per unit for 12<sup>th</sup> December when critical defects are 2, major defects are 24, and minor defects 150 for 50 units inspected on that day. Is the December 12 subgroup in control or out of control?

### Solution:

$$D_0 = 9u_{0c} + 3u_{0ma} + 1u_{0mi} = 9 \times 0.09 + 3 \times 0.6 + 1 \times 3 = 5.61$$

$$\sigma_{0u} = \sqrt{\frac{9^2 u_{0c} + 3^2 u_{0ma} + 1^2 u_{0mi}}{n}} = \sqrt{\frac{9^2 \times 0.09 + 3^2 \times 0.5 + 1^2 \times 3}{50}} = 0.54$$

$$UCL = D_0 + 3\sigma_{0u} = 5.61 + 3 \times 0.54 = 7.23; \quad LCL = D_0 - 3\sigma_{0u} = 5.61 - 3 \times 0.54 = 3.99$$

$$D_{\text{December 12}} = 9\left(\frac{2}{50}\right) + 3\left(\frac{24}{50}\right) + 1\left(\frac{150}{50}\right) = 4.8 \text{ (in control)}$$

## **23.14 LOT-BY-LOT ACCEPTANCE SAMPLING**

### **23.14.1 Introduction**

Lot-by-lot acceptance sampling is the basis to check the quality of products produced at different time intervals, on different machines, in different plants. Different lots of product are prepared based on the similarity in the production process. All the products in a lot cannot be inspected due to costly nature of inspection, destructive testing, repetitive nature of job, lack of automation, etc. Therefore, a sample(s) from each lot is selected for inspection; if number of defective is less or equal to prescribed minimum number, then the lot is accepted and otherwise lot is rejected. The acceptance or rejection of the lot is based on different sampling plans which have been discussed in the next section of this chapter.

### **23.14.2 Application of Sampling Plan**

There are following applications of acceptance sampling plan:

- (a) If the testing process used for the product is destructive, sampling plan is used in place of 100 per cent inspection.
- (b) If the testing process is costly, sampling plan is used in place of 100 per cent inspection.
- (c) Sampling plan may be used in a repetitive type of inspection to avoid fatigue or boredom.
- (d) When control charts are not available, sampling plan may be used.
- (e) When inspection process is not automated, sampling plan may be used.

### **23.14.3 Advantages of Sampling Plan**

There are following advantages of sampling plan over control chart or 100 per cent inspection process.

- (a) It is more economical due to fewer inspections.
- (b) There are less handling damages during inspection.
- (c) It is most suitable for the destructive type of testing.
- (d) There is upgradation of the inspection job from monotonous piece-by-piece to lot-by-lot decisions.
- (e) There is stronger motivation for quality improvement due to rejection of entire lot rather than the return of defectives.
- (f) Due to fewer inspectors, it is possible to simplify the recruiting, training and supervising activities.

### **23.14.4 Disadvantages/Limitations of Sampling Plan**

- (a) There are risks of accepting defective lots and rejecting good lots.
- (b) More time and efforts are devoted in the planning and documentation of the sampling plan.
- (c) Less information is provided about the product.

## 23.15 SAMPLING PLAN

The initial step of the sampling plan is lot formation. Lot formation can influence the effectiveness of the sampling plan. The guidelines for lot formation are given below as:

- (a) Lot must be homogeneous, i.e. the all the products in the lot is produced by the same machine, same operator, same input material, and so on.
- (b) Lots should be as large as possible. Since the sample sizes do not increase as rapidly as lot sizes, a lower inspection cost results with larger lot sizes.

The second step of the sampling plan is sample selection. The basic technique of sample selection is to assign a series of numbers to each product in the lot and select the products for sampling randomly. The random number can be generated manually or using software.

### Type of Sampling Plans

There are three types of sampling plans used in the lot-by-lot sampling. These plans are as given below:

- (a) Single sampling (SS) plan
- (b) Double sampling (DS) plan
- (c) Multiple sampling plan

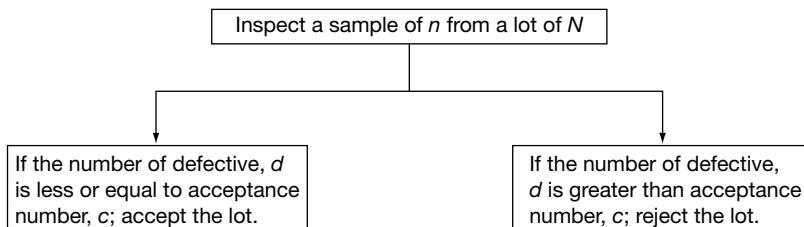
### 23.15.1 Single Sampling (SS) Plan

In SS plan, the decision about the lot is taken on the basis of quality of only one sample. The following are the procedures for SS plan:

- (a) A lot size ( $N$ ) of the product is taken to check the quality.
- (b) A sample size ( $n$ ) is selected randomly from the lot.
- (c) If the number of defectives in the sample exceeds the acceptance number ( $c$ ), the entire lot is rejected.
- (d) If the number of defectives in the sample is equal to or less than the acceptance number, the entire lot is accepted.

### Flow Chart for Single Sampling Plan

Flow chart for SS plan is given in shown in Figure 23.17.



**Figure 23-17:** Flow chart for single sampling plan

### **Operating Characteristics Curve for SS Plan**

There are following steps for plotting the operating characteristics (OCs) curve for single sampling Plan:

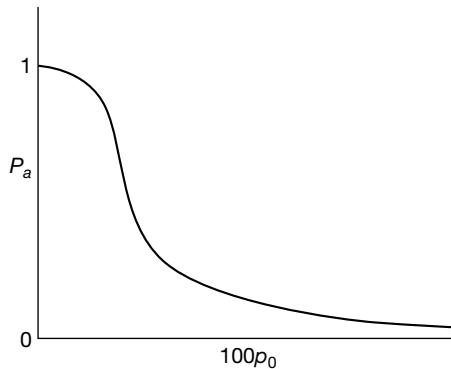
- Assume process average,  $P_0$  value between 0–8 per cent. Take at least 7–8,  $P_0$  values to plot smooth curve.
- Find the value of probability of acceptance,  $P_a$  using Poisson distribution table given in Appendix III of this book (take cumulative value).

$$P_a = P_{c \text{ or less}}$$

The Poisson distribution is used for sampling plans involving the number of defects or defects per unit rather than the number of defective parts. It is also used to approximate the binomial probabilities involving the number of defective parts when the sample ( $n$ ) is large and  $p_0$  is very small. When  $n$  is large and  $p_0$  is small, the Poisson distribution formula may be used to approximate the binomial. Using the Poisson distribution to calculate probabilities associated with various sampling plans is relatively simple because the Poisson tables can be used.

$$P_a = \frac{n p_0}{c} e^{-n p_0}$$

- Plot the curve between  $100P_0$  and  $P_a$ . The nature of the curve is shown in Figure 23.18.



**Figure 23.18:** OC curve for single sampling plan

**Example 23.9:** Draw the OC curve for single sampling plan; given  $N = 3000$ ;  $n = 100$ ;  $c = 2$ .

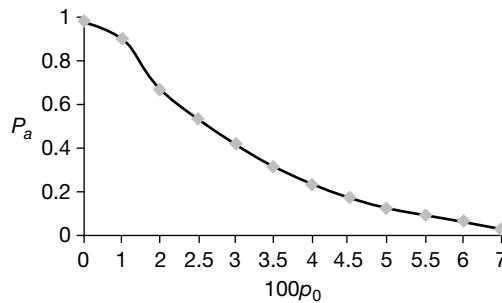
#### **Solution:**

To draw an OC curve, we have to take some arbitrary value of  $p_0$ . The value of  $p_0$  can be taken from 0.01 to 0.07 at close interval for smooth curve. The corresponding value of probability of acceptance  $P_a$  (cumulative) can be taken from Poisson distribution table given in the 'Appendix 4'. The graph is plotted between  $100p_0$  and  $P_a$ . The value of  $P_a = P_{2 \text{ or less}}$  corresponding to  $np_0$  is collected in Table 23.6.

**Table 23-6:**  $P_a$  corresponding to different value of  $np_0$  at  $c = 2$ 

$p_0$ (Process average)	$100p_0$ (Percentage process average)	$np_0$ (Number defective)	$P_a$ (Probability of acceptance)
0	0	0	1
0.01	1	1	0.92
0.02	2	2	0.677
0.025	2.5	2.5	0.543
0.03	3	3	0.423
0.035	3.5	3.5	0.321
0.04	4	4	0.238
0.045	4.5	4.5	0.174
0.05	5	5	0.125
0.055	5.5	5.5	0.093
0.06	6	6	0.062
0.07	7	7	0.029

The graph shown in Figure 23.19 is slight deviated from the ideal graph due to lack of exactness in values of  $P_a$  corresponding to different  $np_0$ .

**Figure 23-19:** Single sampling plan for  $N = 3000$ ,  $n = 90$ , and  $c = 1$ 

### 23.15.2 Double Sampling (DS) Plan

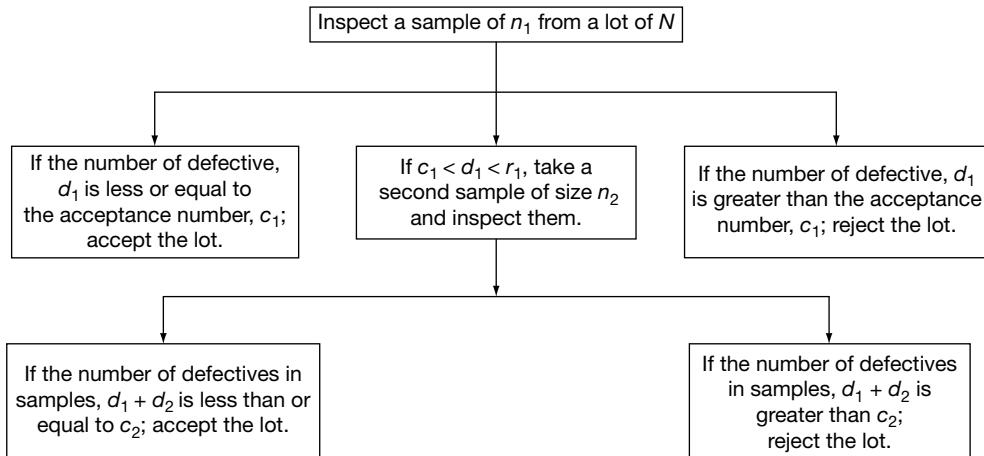
The decision about the sample is based on one or two samples depend on the acceptance and rejection number of first and second samples. The following are procedures for DS plan:

- A lot size ( $N$ ) of the product is taken to check the quality.
- Two sample sizes ( $n_1$  and  $n_2$ ); two acceptance numbers ( $c_1$  and  $c_2$ ); and two rejection numbers ( $r_1, r_2$ ) are specified.
- Take the first sample of size  $n_1$ .

- (d) If the number of defectives in the first sample does not exceed  $c_1$ , the lot is accepted and the second sample is not required.
- (e) If the number of defectives ( $d_1$ ) in the first sample exceeds  $r_1$ , the lot is rejected and the second sample is not required.
- (f) If the number defectives in the first sample are more than  $c_1$  but less than  $r_1$ , a second sample  $n_2$  is selected and inspected.
- (g) If the number of defectives in combined first and second sample does not exceed  $c_2$ , the lot is accepted.
- (h) If the number of defectives in combined first and second sample exceeds  $c_2$ , the lot is rejected.

### **Flow Chart for Double Sampling Plan**

Flow chart for the DS plan is shown in Figure 23.20



**Figure 23-20:** Flow chart for double sampling plan

### **OC Curve for Double Sampling Plan**

The following are the steps for plotting the OC curve for double sampling plan:

- (a) Assume process average,  $p_0$  value between 0 and 8 per cent. Take at least 7–8  $p_0$  values to plot smooth curve.
- (b) Find the value of probability of acceptance,  $P_a$  using Poisson distribution table given in ‘Appendix 4’ of this book (take cumulative value).

$$P_a = (P_a)_I + (P_a)_{II}$$

where  $(P_a)_I = (P_{c_1 \text{ or less}}) I$

and  $(P_a)_{II} = \sum_{c_1 < d_i}^{r_1 > d_i} (P_{d_i}) \left( P_{(c_2 - d_i) \text{ or less}} \right) I$

- (c) Plot the curve between  $100p_0$  and  $P_a$ . The nature of the curve is shown in Figure 23.21.

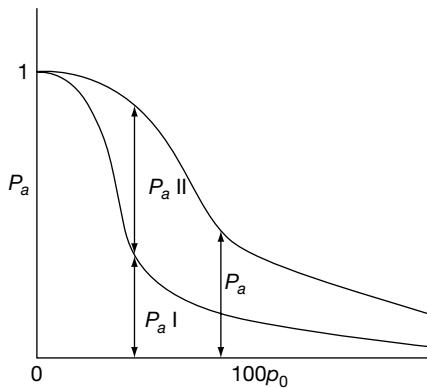


Figure 23-21: OC curve for double sampling plan

**Example 23.10:** Draw double sampling plan; given  $N = 3000$ ;  $n_1 = 200$ ;  $c_1 = 2$ ;  $r_1 = 5$ ;  $n_2 = 250$ ;  $c_2 = 5$ .

#### Solution:

For DS plan, we have to find the probability of acceptance of the first sample for the defective less than or equal to the acceptance number for the first sample, i.e., 2. If the number of defectives in the first sample is in between acceptance number (2) and rejection number (5) of the first sample, draw second sample of size 250. If the number of defective in the first sample is 3, the maximum defective allowed in the second sample will be 2 or less, so that total defective should not exceed the acceptance number of the second sample (i.e. 5). Similarly, if the number of defective in the first sample is 4, the maximum defective allowed in the second sample will be 1 or less. The probability of acceptance based on the second sample is found. Finally, the summation of the probabilities of acceptance based on the first sample and the second sample are combined as shown in Figure 23.22 using the following formula (Table 23.7).

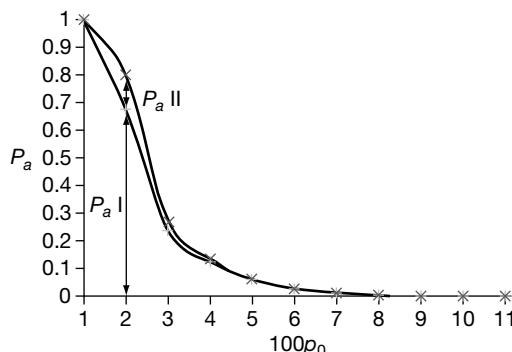


Figure 23-22: Double sampling plan for  $N = 3000$ ,  $n_1 = 200$ ;  $c_1 = 2$ ;  $r_1 = 5$ ;  $n_2 = 250$ ;  $c_2 = 5$ ;  $r_2 = 6$

$$\begin{aligned} (P_a)_I &= (P_{2 \text{ or less}})_I \\ (P_a)_{II} &= (P_3)_I (P_{2 \text{ or less}})_{II} + (P_4)_I (P_{1 \text{ or less}})_{II} \\ P_a &= (P_a)_I + (P_a)_{II} \end{aligned}$$

where

$(P_a)_I$  is probability of acceptance based on the basis of first sample;

$(P_{2 \text{ or less}})_I$  is probability of acceptance based on the first sample for the defective less or equal to 2;

$(P_a)_{II}$  is probability of acceptance based on second sample;

$(P_3)_I$  is probability of acceptance for the defective exactly equal to 3 in first sample;

$(P_{2 \text{ or less}})_{II}$  is probability of acceptance of for the defective less or equal to 2 in second sample;

$(P_4)_I$  is probability of acceptance for the defective exactly equal to 4 in first sample; and

$(P_{1 \text{ or less}})_{II}$  is probability of acceptance of for the defective less or equal to 1 in second sample.

**Table 23-7:** Calculation of probability of acceptance for double sampling plan (Values are taken from Poisson distribution table from the ‘Appendix 5’ of this book.)

$p_0$	$100p_0$	$n_1 p_0$	$(P_a)_I, (a)$	$(P_3)_I$	$(P_4)_I$	$n_2 p_0$	$(P_{2 \text{ or less}})_{II}$	$(P_{1 \text{ or less}})_{II}$	$(P_a)_{II}, (b)$	$P_a = (a) + (b)$
0	0	0	1			0			0	1
0.01	1	2	0.677	0.180	0.090	2.5	0.543	0.287	0.123	0.8
0.02	2	4	0.238	0.195	0.195	5	0.125	0.041	0.032	0.27
0.025	2.5	5	0.125	0.140	0.176	6.25	0.054	0.014	0.010	0.135
0.03	3	6	0.062	0.089	0.134	7.5	0.022	0.005	0.002	0.064
0.035	3.5	7	0.029	0.052	0.091	8.75	0.008	0.002	0.000	0.029
0.04	4	8	0.014	0.029	0.057	10	0.002	0.000	0.000	0.014
0.045	4.5	9	0.006	0.015	0.034	11.25	0.0007	0.000	0.000	0.006
0.05	5	10	0.002	0.007	0.019	12.5	0.000	0.000	0.000	0.002
0.055	5.5	11	0.001	0.004	0.010	13.75	0.000	0.000	0.000	0.001
0.06	6	12	0.000	0.002	0.005	15	0.000	0.000	0.000	0.000

### 23.15.3 Multiple Sampling Plan

Multiple sampling plan is very similar to the DS plane in which third sample, fourth sample and so on are used for the decision of acceptance and rejection of the lot. It depends on the number of defectives occurred in different samples.

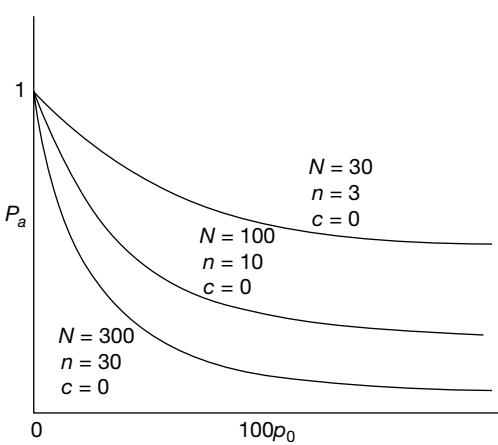
### 23.15.4 Treatment of Reject Lots

There are three important treatments of rejected lots discussed below as:

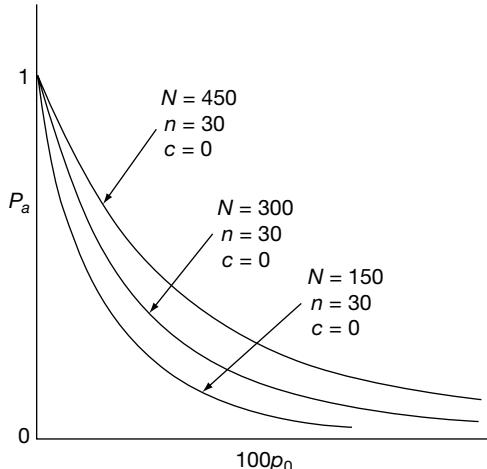
- The rejected lots can be sent to production facilities for rectification by production personnel. This treatment is not satisfactory, since it defeats the purpose of sampling inspection and slows production. However, if the products are badly needed, there may not be another choice.
- The rejected lots can be rectified at the consumer's site by the personnel of either producer side or consumer side. But in this case, there is a psychological disadvantage since all the consumer's personnel are aware that the producer 'A' had product rejected.
- The rejected lots can be returned to the producer for rectification. This is the only appropriate course of action, since it results in strong motivation to producer for quality improvement. Since shipping costs are paid in both directions by the producer for rejected lots.

### 23.16 PROPERTIES OF OC CURVES

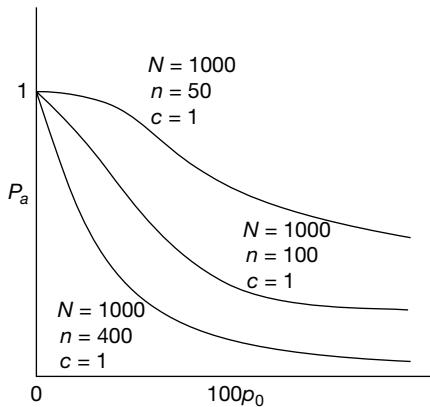
- For sample size as a fixed percentage of the lot size (10 per cent of lot size), curves become steeper for increasing sample size as shown in Figure 23.23.
- For fixed sample size, curves become similar to each other ( $n \geq 10$  per cent of  $N$ ) and identical for  $n < 10$  per cent of  $N$  as shown in Figure 23.24.
- For increasing sample size, curves become steeper as shown in Figure 23.25.
- For decreasing acceptance number, curves become steeper as shown in Figure 23.26.



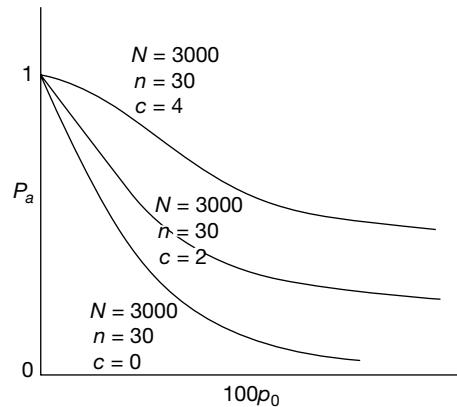
**Figure 23-23:** OC curves for increasing sample size and fixed percentage of lot size



**Figure 23-24:** OC curves for fixed sample size



**Figure 23-25:** OC curves for increasing sample size

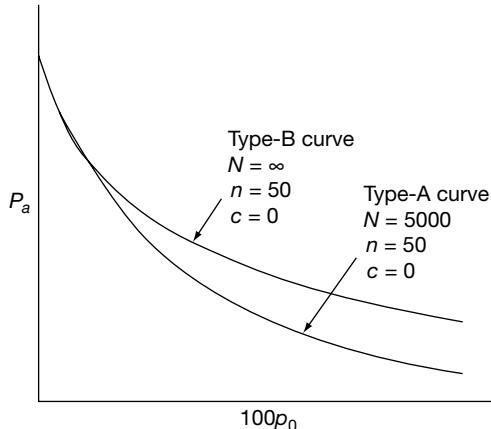


**Figure 23-26:** OC curves for decreasing acceptance number

### 23.16.1 Type-A and Type-B Curves

Type-A OC curves give the probability of acceptance of an isolated finite lot. With a finite situation, the hypergeometric distribution is used to calculate the acceptance. As the lot size of type-A curve increases; it approaches the type-B curve and will become almost identical when the lot size is at least 10 times the sample size ( $n/N \leq 0.10$ ).

In type-B curve, it is assumed that the lots come from a continuous stream of product, and therefore the calculations are based on an infinite lot size. The binomial distribution is the exact distribution for calculating the probability of acceptance; however, Poisson distribution is used in place of Binomial distribution since it is a good approximation. Type-A and Type-B curves are shown in Figure 23.27.



**Figure 23-27:** Type-A and Type-B OC curves

## 23.17 CONSUMER–PRODUCER RELATIONSHIP

The producer always wants ‘all good lots should be accepted’ and the consumer always wants ‘all defective lots should be rejected’. Only one ideal situation for both is 100 per cent inspection and the sampling plan for 100 per cent inspection is vertical as shown in Figure 23.28. Thus, a sampling plan always carries some risks for both producer as well as consumer.

The producer’s risk is represented by a symbol  $\alpha$  as the probability of rejection of good lot. The values of  $\alpha$  range from 0.01 to 0.10 but generally, its value is taken as 0.05.  $\alpha$  is the probability of rejection and thus its value cannot be shown on OC curve. The probability of acceptance, i.e.  $P_a = 1 - \alpha$  corresponding to AQL is shown on OC curve, i.e. 95 per cent (Figure 23.28). AQL is acceptable quality level corresponding to producer’s risk.

The consumer’s risk is the probability of acceptance of defective lots. It is represented by the symbol  $\beta$ . The value of  $\beta$  is used as 0.10. The probability of acceptance of defective lot corresponding to limiting quality level (LQL) is shown on an OC curve as  $P_a = 0.10$ , i.e. 10 per cent as shown in Figure 23.29.

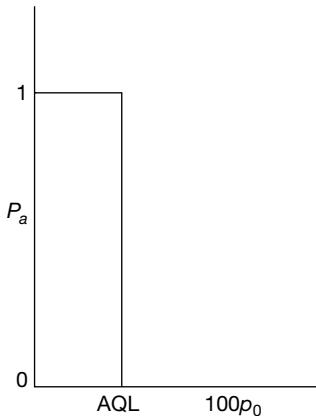


Figure 23-28: OC curve for 100 per cent inspection

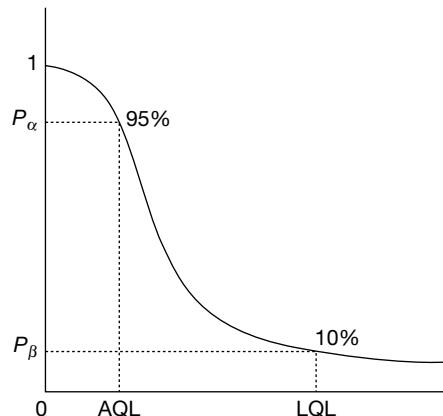
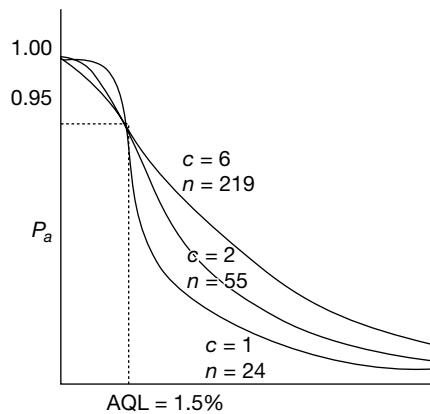


Figure 23-29: Producer's and consumer's risks

## 23.18 SAMPLING PLAN DESIGN

### 23.18.1 Sampling Plan for Stipulated Producer's Risk

If the producer’s risk  $\alpha$  and its corresponding AQL are given, a family of sampling plan can be produced as shown in Figure 23.30. For example, in Figure 23.30, producer’s risk,  $\alpha$ , is taken as 0.05 and corresponding AQL is taken as 1.5 per cent. Each plan passes through  $P_a = 0.95$  and  $P_{0.95} = 0.015$ . Therefore, each plan will ensure that sample of 1.5 per cent defective will be rejected 5 per cent of the time or accepted 95 per cent of the time. The sampling plans can be produced considering different value of  $c$  (acceptance number) and corresponding value of  $np_0$  is taken at  $P_a = 0.95$ . To simplify the calculation, the values of  $np_0$  for different values of  $c$  at  $P_a = 0.95$  and  $P_a = 0.10$  are given in Table 23.8. The values of  $np_0$  in Table 23.8 are calculated by interpolation of the value taken from Poisson distribution table given in the ‘Appendix 4’ of the book.



**Figure 23-30:** Sampling plan for stipulated producer's risk

**Table 23-8:**  $np$  Values for corresponding  $c$  values

<b>c</b>	$P_a = 0.99$ ( $\alpha = 0.01$ )	$P_a = 0.95$ ( $\alpha = 0.05$ )	$P_a = 0.90$ ( $\alpha = 0.10$ )	$P_a = 0.10$ ( $\beta = 0.10$ )	$P_a = 0.05$ ( $\beta = 0.05$ )	$P_a = 0.01$ ( $\beta = 0.01$ )	Ratio of $p_{0.10}/p_{0.95}$
0	0.010	0.051	0.105	2.303	2.996	4.605	44.890
1	0.149	0.355	0.532	3.890	4.744	6.638	10.946
2	0.436	0.818	1.102	5.322	6.296	8.406	6.509
3	0.823	1.366	1.745	6.681	7.754	10.045	4.890
4	1.279	1.970	2.433	7.994	9.154	11.605	4.057
5	1.785	2.613	3.152	9.275	10.513	13.108	3.549
6	2.330	3.286	3.895	10.532	11.842	14.571	3.206
7	2.906	3.981	4.656	11.771	13.148	16.000	2.957
8	3.507	4.695	5.432	12.995	14.434	17.403	2.768
9	4.130	5.426	6.221	14.206	15.705	18.783	2.618
10	4.771	6.169	7.021	15.407	16.962	20.145	2.497
11	5.428	6.924	7.829	16.598	18.208	21.490	2.397
12	6.099	7.690	8.646	17.782	19.442	22.821	2.312
13	6.782	8.464	9.470	18.958	20.668	24.139	2.240
14	7.477	9.246	10.300	20.128	21.886	25.446	2.177
15	8.181	10.035	11.135	21.292	23.098	26.743	2.122

$$P_a = 0.95; p_{0.95} = 0.015$$

$$\text{For } c = 1, np_{0.95} = 0.355$$

$$n = \frac{np_{0.95}}{p_{0.95}} = \frac{0.355}{0.015} = 23.66 \geq 24$$

For  $c = 2$ ,  $np_{0.95} = 0.818$

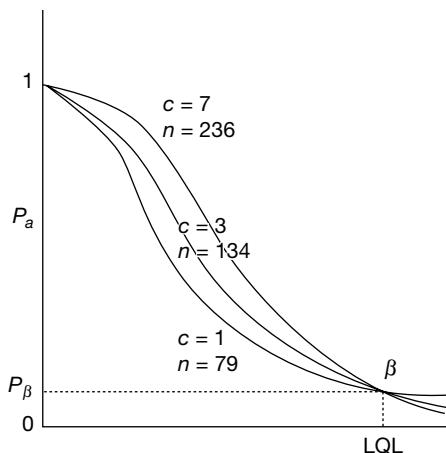
$$n = \frac{np_{0.95}}{p_{0.95}} = \frac{0.818}{0.015} = 54.53 \cong 55$$

For  $c = 6$ ,  $np_{0.95} = 3.286$

$$n = \frac{np_{0.95}}{p_{0.95}} = \frac{3.286}{0.015} = 219.06 \cong 219$$

### 23.18.2 Sampling Plan for Stipulated Consumer's Risk

If the consumer's risk  $\beta$  and its corresponding LQL are given, a family of sampling plan can be produced as shown in Figure 23.31. For example, in Figure 23.31, consumer's risk,  $\beta$ , is taken as 0.1 and corresponding LQL is taken as 5 per cent. Each plan passes through  $P_a = 0.1$  and  $p_{0.1} = 0.05$ . Therefore, each plan will ensure that sample of 5 per cent defective will be accepted 10 per cent of the time or rejected 90 per cent of the time. The sampling plans can be produced considering different value of  $c$  (acceptance number) and corresponding value of  $np_0$  is taken at  $P_a = 0.1$ . To simplify the calculation, the values of  $np_0$ , for different value of  $c$  at  $P_a = 0.1$ ,  $np_0$  can be taken from Table 23.8.



**Figure 23-31:** Sampling plan for stipulated consumer's risk

$$P_\beta = 0.1; p_{0.1} = 0.05$$

For  $c = 1$ ,  $np_{0.1} = 3.89$

$$n = \frac{np_{0.10}}{p_{0.10}} = \frac{3.89}{0.05} = 77.8 \cong 79$$

For  $c = 3$ ,  $np_{0.1} = 6.681$

$$n = \frac{np_{0.10}}{p_{0.10}} = \frac{6.681}{0.05} = 133.62 \cong 134$$

For  $c = 7$ ,  $np_{0.1} = 11.771$

$$n = \frac{np_{0.10}}{p_{0.10}} = \frac{11.771}{0.05} = 235.42 \cong 236$$

### 23.18.3 Sampling Plan for Stipulated Producer's and Consumer's Risk

$\alpha = 0.05$ , AQL = 1.5%,  $\beta = 0.10$ , LQL = 5%

Ratio of  $\frac{p_{0.10}}{p_{0.95}} = \frac{0.05}{0.015} = 3.333$  lies between  $c = 5$  and 6 from Table 23.8.

Thus, the plan exactly meets the consumer's stipulation of LQL = 5% for  $\beta = 0.10$

$$P_\beta = 0.1; p_{0.1} = 0.05$$

For  $c = 5$ ,  $np_{0.1} = 9.275$

$$n = \frac{np_{0.10}}{p_{0.10}} = \frac{9.275}{0.05} = 185.5 \cong 186$$

For  $c = 6$ ,  $np_{0.1} = 10.532$

$$n = \frac{np_{0.10}}{p_{0.10}} = \frac{10.532}{0.05} = 210.64 \cong 211$$

The plan exactly meets the producer's stipulation of AQL = 1.5% for  $\alpha = 0.05$

$$P_a = 0.95; p_{0.95} = 0.015$$

For  $c = 5$ ,  $np_{0.95} = 2.613$

$$n = \frac{np_{0.95}}{p_{0.95}} = \frac{2.613}{0.015} = 174.2 \cong 174$$

For  $c = 6$ ,  $np_{0.95} = 3.286$

$$n = \frac{np_{0.95}}{p_{0.95}} = \frac{3.286}{0.015} = 219.06 \cong 219$$

Exactly meet the consumer's risk and comes as close as possible to producer's stipulation.

$c = 5, n = 186$  and

$c = 6, n = 211$

For  $c = 5, n = 186$ ,  $p_{0.95} = \frac{np_{0.95}}{n} = \frac{2.613}{186} = 0.0141 = 1.41\%$

For  $c = 6, n = 211$ ,  $p_{0.95} = \frac{np_{0.95}}{n} = \frac{3.286}{211} = 0.01557 = 1.56\%$  more closer to 1.5%

Exactly meet the producer's risk and comes as close as possible to consumers stipulation.

$$c = 5, n = 174 \text{ and}$$

$$c = 6, n = 219$$

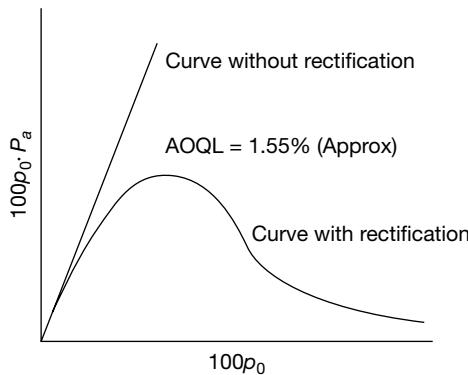
$$\text{For } c = 5, n = 174, p_{0.10} = \frac{np_{0.10}}{p_{0.10}} = \frac{9.275}{174} = 0.053 = 5.3\%$$

$$\text{For } c = 6, n = 219, p_{0.10} = \frac{np_{0.10}}{p_{0.10}} = \frac{10.532}{219} = 0.048 = 4.8\% \text{ more closer to } 5\%$$

For lowest sample size  $c = 5, n = 174$  is suitable sampling plan and for largest sample size  $c = 6, n = 219$  is suitable sampling plan.

### 23.19 AVERAGE OUTGOING QUALITY

The average outgoing quality (AOQ) is the quality that leaves the inspection operation. It is assumed that any rejected lots have been rectified with 100 per cent good products. When rectification is not done, the quality of incoming and outgoing lots will be same and can be represented by a straight line as shown in Figure 23.32.  $\text{AOQ} = 100p_0 \cdot P_a$ . This formula does not account for described defectives, however it is close enough for practical purposes and is much simpler to use.



**Figure 23-32:** AOQ curve

Since the rejected lots are rectified, the AOQ is always better than the incoming quality. In fact, there is a limit which is given the name average outgoing quality limit (AOQL).

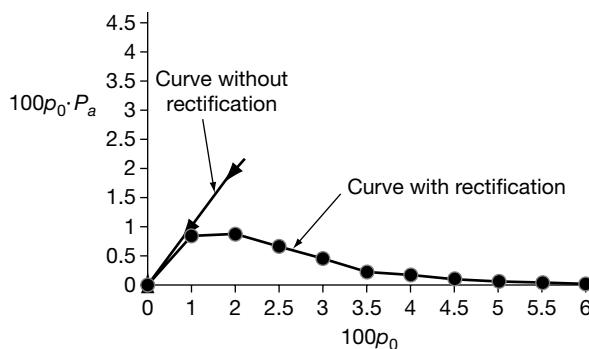
**Example 23.11:** Draw the AOQ curve for  $N = 5000; n = 200, c = 3$ .

**Solution:**

To draw AOQ curve, consider the different values of  $p_0$ , and find the corresponding value of  $P_a$  using Poisson distribution table. The values of  $np_0$ ,  $P_a$ , and  $100p_0P_a$  are given in Table 23.9. The graph between  $100p_0 \cdot P_a$  and  $100p_0$  is plotted as shown in Figure 23.33.

**Table 23-9:** Calculation of Average Outgoing Quality

$p_0$ (Process average)	$100p_0$ (Percentage process average)	$np_0$ (Number defective)	$P_a$ (Probability of acceptance)	$100p_0 \cdot P_a$
0	0	0	1	0
0.01	1	2	0.857	0.857
0.02	2	4	0.433	0.866
0.025	2.5	5	0.265	0.662
0.03	3	6	0.151	0.453
0.035	3.5	7	0.081	0.283
0.04	4	8	0.043	0.172
0.045	4.5	9	0.021	0.094
0.05	5	10	0.009	0.045
0.055	5.5	11	0.005	0.027
0.06	6	12	0.002	0.012
0.07	7	14	0.001	0.007

**Figure 23-33:** AOQ curve

## 23.20 AVERAGE SAMPLE NUMBER

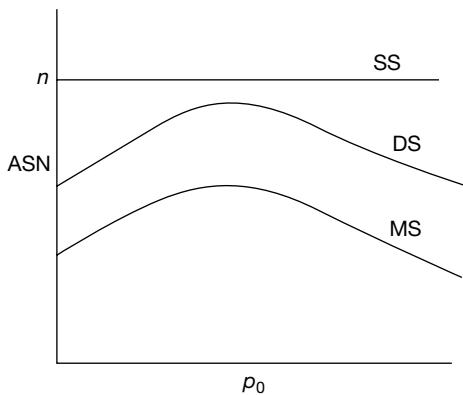
Average sample number (ASN) is the number of items inspected by the consumer. It is used to compare the average number of items inspected in single, double and multiple sampling plans. In SS plan, the ASN is fixed equal to sample size but in DS plan, to find the ASN is more complex. Since the ASN in DS plan depends on whether the second sample is taken or not.

For SS; ASN =  $n$

For DS; ASN =  $n_1 + (1 - P_1)n_2$  and  $P_1 = P_{c_1 \text{ or less}} + P_{r_1 \text{ or more}}$

For MS; ASN =  $n_1 P_1 + (n_1 + n_2)P_2 + \dots + (n_1 + n_2 + \dots + n_k)P_k$

The nature of ASN curves is shown in Figure 23.34.



**Figure 23-34:** ASN curves for single, double and multiple sampling plans

**Example 23.12:** Draw the ASN curves for SS plan  $N = 5000$ ;  $n = 350$ ,  $c = 2$ ; and for DS  $N = 5000$ ;  $n_1 = 250$ ;  $c_1 = 2$ ;  $r_1 = 5$ ;  $n_2 = 200$ ;  $c_2 = 5$ ;  $r_2 = 6$ .

**Solution:**

For SS plan, the ASN is fixed as  $n = 350$

For DS plan,

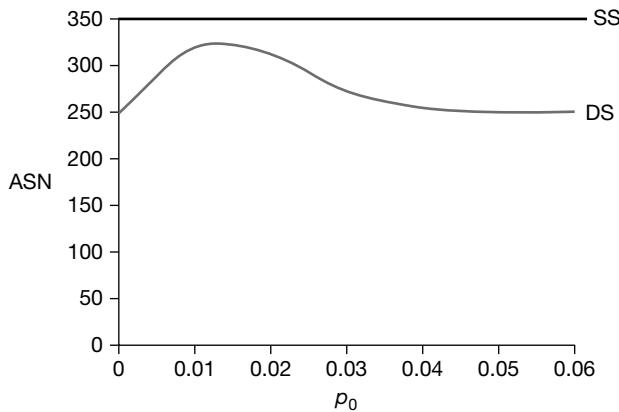
$$\text{ASN} = n_1 + (1 - P_1)n_2 \text{ and } P_1 = P_{c_1 \text{ or less}} + P_{r_1 \text{ or more}}$$

In this problem, we can write  $\text{ASN} = n_1 + (P_3 + P_4) \cdot n_2$ , where  $P_3$  and  $P_4$  are the probabilities of exactly 3 and 4 defectives, respectively, in the first sample.

Taking the value of  $P_3$  and  $P_4$  from Poisson distribution table for different values of process average  $p_0$ , we get the value of ASN as shown in Table 23.10. The nature of ASNs for the given value of sample size and acceptance number for SS and DS are shown in Figure 23.35.

**Table 23-10:** ASN corresponding to different  $p_0$

$p_0$ (Process average)	$n_1 p_0$	$P_3$	$P_4$	$\text{ASN} = n_1 + (P_3 + P_4) \cdot n_2$
0.01	2.5	0.214	0.134	319.6
0.02	5	0.140	0.176	313.2
0.03	7.5	0.04	0.074	272.8
0.04	10	0.007	0.019	255.2
0.05	12.5	0.0015	0.004	251.1
0.06	15	0	0.001	250.2

**Figure 23-35:** ASN curve

### 23.21 AVERAGE TOTAL INSPECTION

Average total inspection (ATI) is the number of items inspected by both consumer as well as producer. Similar to ASN, this curve also provides the information about the number of items inspected in different sampling plans.

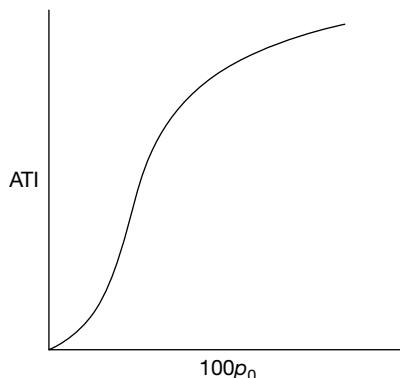
For SS;

$$\text{ATI} = n + (1 - P_a)(N - n)$$

For DS:

$$\text{ATI} = n_1 P_{a_1} + (n_1 + n_2) P_{a_2} + (1 - P_a)(N - n), \text{ where } P_a = P_{a_1} + P_{a_2}$$

ATI curve for SS plan is shown in Figure 23.36.

**Figure 23-36:** ATI curve for single sampling plan

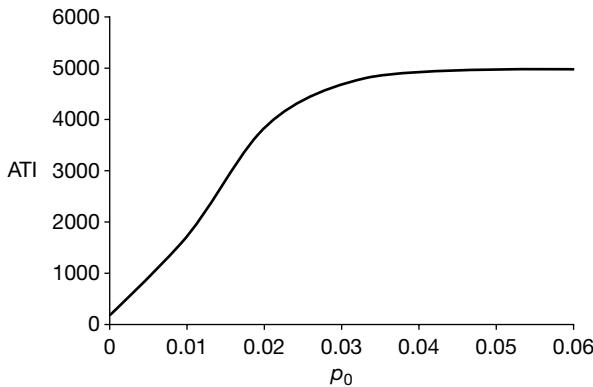
**Example 23.13:** Draw the ATI curve for  $N = 5000$ ,  $n = 200$ ,  $c = 2$

**Solution:**

To draw the ATI curve, at first we find the probability of acceptance corresponding to different values of  $p_0$ . These values of  $P_a$  are used to find the ATI as discussed in this section. The different values of ATI corresponding to different values of  $p_0$  are given in Table 23.11, and the curve is drawn in Figure 23.37.

**Table 23-11:** ATI values for  $N = 5000$ ,  $n = 200$ ,  $c = 2$

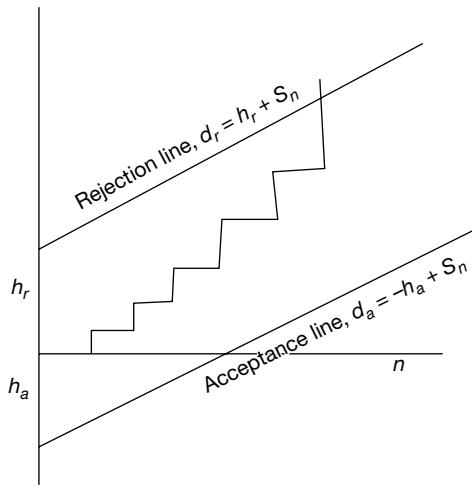
$p_0$ (Process average)	$np_0$	$P_a$	$ATI = n + (1 - P_a)(N - n)$
0	0	1	200
0.01	2	0.677	1750.4
0.02	4	0.238	38.57.6
0.03	6	0.062	4702.4
0.04	8	0.014	4932.8
0.05	10	0.002	4990.4
0.06	12	0.000	5000



**Figure 23-37:** ATI curve for  $N = 5000$ ,  $n = 200$ ,  $c = 2$

## 23.22 SEQUENTIAL SAMPLING PLAN

It is similar to multiple sampling plans except that the sequential sampling can continue indefinitely. In practice, the plan is truncated after the number inspected equals to three times of the number inspected by a corresponding SS plan. This plan is employed for costly inspection or destructive testing, usually has a subgroup size of 1. This plan is also known as item-by-item plan. This plan is shown in Figure 23.38.



**Figure 23-38:** Sequential sampling plan

The sequential sampling plan is defined by the producer's risk,  $\alpha$  and its process quality  $p_\alpha$  and consumer's risk,  $\beta$  and its process quality  $p_\beta$

$$d_a = -h_a + S \times n;$$

$$d_r = h_r + S \times n;$$

$$h_a = \log\left(\frac{1-\alpha}{\beta}\right) / \left[ \log\left(\frac{p_\beta}{p_\alpha}\right) + \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \right];$$

$$h_r = \log\left(\frac{1-\beta}{\alpha}\right) / \left[ \log\left(\frac{p_\beta}{p_\alpha}\right) + \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \right];$$

$$S = \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) / \left[ \log\left(\frac{p_\beta}{p_\alpha}\right) + \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \right]$$

where

$S$  = slope of the line;

$h_r$  = intercept for the rejection line;

$h_a$  = intercept for the acceptance line;

$p_\beta$  = fraction defective for consumer's risk;

$p_\alpha$  = fraction defective for producer's risk;

$\beta$  = consumer's risk;

$\alpha$  = producer's risk;

$d_a$  = number of defectives for acceptance; and

$d_r$  = number of defectives for rejection.

**Example 23.14:** Find the equations for acceptance and rejection line for sequential sampling plan, if  $\alpha = 0.05$ ;  $p_\alpha = 0.08$ ;  $\beta = 0.10$ ; and  $p_\beta = 0.30$ .

**Solution:**

$$h_a = \log\left(\frac{1-\alpha}{\beta}\right) \left/ \left[ \log\left(\frac{p_\beta}{p_\alpha}\right) + \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \right] \right.$$

$$= \log\left(\frac{1-0.05}{0.10}\right) \left/ \left[ \log\left(\frac{0.3}{0.08}\right) + \log\left(\frac{1-0.08}{1-0.30}\right) \right] \right. = 1.41$$

$$h_r = \log\left(\frac{1-\beta}{\alpha}\right) \left/ \left[ \log\left(\frac{p_\beta}{p_\alpha}\right) + \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \right] \right.$$

$$= \log\left(\frac{1-0.10}{0.05}\right) \left/ \left[ \log\left(\frac{0.3}{0.08}\right) + \log\left(\frac{1-0.08}{1-0.30}\right) \right] \right. = 1.812$$

$$S = \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \left/ \left[ \log\left(\frac{p_\beta}{p_\alpha}\right) + \log\left(\frac{1-p_\alpha}{1-p_\beta}\right) \right] \right.$$

$$= \log\left(\frac{1-0.08}{1-0.8}\right) \left/ \left[ \log\left(\frac{0.3}{0.08}\right) + \log\left(\frac{1-0.08}{1-0.30}\right) \right] \right. = 0.459$$

$$d_a = -h_a + S \times n = -1.41 + 0.459n$$

$$d_r = h_r + S \times n = 1.812 + 0.459n$$



## SUMMARY

This is one of the important chapters of statistical quality control. In this chapter, we have demonstrated the control charts for variables and attributes. Purposes of control charts have been discussed. X-bar and R-chart and X- bar and S have been demonstrated as control charts for variables. p-chart, c-chart and u-chart have been demonstrated as control charts for attributes. p-chart has been used for defectives and c-chart and u-chart have been used for defects. Process capability and capability index have been explained. Finally, control chart for demerits has been explained. Quality control technique using lot-by-lot sampling plan is also included in this chapter. This is normally used where use of control charts are not beneficial or possible. The applications, advantages and disadvantages have been discussed. Single, double, multiple, sequential and sampling plans have been explained with some examples. In addition, design of sampling plans has been analysed; and ASN and ATI AOQL have been introduced.

## MULTIPLE-CHOICE QUESTIONS

- Which of the following is the source of variation in quality parameter?  
(a) process (b) materials  
(c) environment (d) all the above
  - Which of the following is the type of variation in the production parts?  
(a) With-in-piece variation  
(b) Piece-to-piece variation  
(c) Time-to-time variation  
(d) All the above
  - The central limit theorem is based on  
(a) Binomial distribution  
(b) Poisson distribution  
(c) Normal distribution  
(d) Exponential distribution
  - X-bar and R-chart are used for  
(a) Variables (b) Attributes  
(c) Both variable and attributes (d) None of these
  - R-Chart is best suitable for the subgroup size  
(a) less than 10  
(b) more than 10 and less than 20  
(c) more than 20 and less than 50  
(d) more than 50 and less than 100
  - For larger subgroup size, which of the following is used in place of R-chart?  
(a) p-Chart (b) c-Chart  
(c) u-Chart (d) s-Chart
  - Process is called out of control if  
(a) About 2/3 (67 per cent) of the points lie near the centre line.  
(b) A few points (5 per cent) lie closer to control limits.  
(c) Points are balanced on both sides of the centre line.  
(d) Six points in the row continuously increasing or decreasing.
  - For a good quality process,  
(a) Control limits should lie beyond the specification limits  
(b) Control limits should lie under the specification limits  
(c) Control limits should coincide the specification limits  
(d) None of these

9. The ratio of the difference of specification limits and the process spread is known as
  - (a) Quality index
  - (b) Process index
  - (c) Process capability
  - (d) Capability index
10. The desired capability index should be
  - (a) Less than 1
  - (b) Greater than 1
  - (c) Equal to 1
  - (d) None of these
11. Which of the following is used as a control chart for defectives?
  - (a) X-bar and R-chart
  - (b) p-Chart
  - (c) c-Chart
  - (d) u-Chart
12. For smaller subgroup size, the
  - (a) control limits will be wider and will move away from the centre line
  - (b) control limits will be closer to the centre line
  - (c) there will be no effect on the control limits
  - (d) none of these
13. Which of the following is known as count of defect chart?
  - (a) X-bar and R-chart
  - (b) p-Chart
  - (c) c-Chart
  - (d) u-Chart
14. The critical defects in the product
  - (a) result in hazardous or unsafe use.
  - (b) result in failure of working or performance.
  - (c) do not affect the working performance of a product, but affect the appearance or look of the product.
  - (d) none of these
15. The operating characteristic (OC) curve shows the probability of
  - (a) rejection for every possible true percentage of defectives
  - (b) acceptance for every possible true percentage of defectives
  - (c) making type-I errors for various percentages of defectives
  - (d) none of the above

## REVIEW QUESTIONS

1. What are the sources of variations in the quality of a product? Discuss the different types of the variables.
2. What is the difference between the variables and attributes? Discuss the method to produce the control chart for variables.

3. What are the objectives of control charts for the variables?
4. How do you decide that whether the process is under control or out of control?
5. What are the reasons for the process out of control?
6. What is process capability? How do you determine the process capability index?
7. Discuss the control charts for attributes.
8. What do you mean by sampling plan? What are the advantages of sampling plan over the control chart?
9. How do you plot the operating characteristic curve? Take the arbitrary data and explain in detail.
10. How do you differentiate between single sampling plan and double sampling plan? Explain with the flow charts.
11. Discuss the properties of OC curve.
12. What is the difference between Type-A and Type-B curves?
13. Explain the procedure of sampling plan design considering the producer risk and consumer risk.
14. Write short notes on the following:
  - (a) Average outgoing quality
  - (b) Average sample number
  - (c) Average total inspection
  - (d) Sequential sampling plan

## **Answers**

1. (d)      2. (d)      3. (c)      4. (a)      5. (a)      6. (d)      7. (d)      8. (b)      9. (d)  
 10. (b)     11. (b)     12. (a)     13. (c)     14. (a)     15. (b)

## **EXERCISES**

1. Following data (in millimetre) regarding a product manufactured are shown in Table 23.12. Find the control limits for X-bar and R-chart.

**Table 23-12:** Dimensions in mm

Subgroup no.	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$
1	40	36	46	41	37
2	36	34	34	36	40
3	65	68	69	59	64
4	39	44	38	40	34
5	40	34	41	43	42
6	43	46	41	41	44
7	37	36	41	38	33
8	50	51	52	49	48

*(Continued)*

**Table 23-12:** (Continued)

<b>Subgroup no.</b>	<b>X<sub>1</sub></b>	<b>X<sub>2</sub></b>	<b>X<sub>3</sub></b>	<b>X<sub>4</sub></b>	<b>X<sub>5</sub></b>
<b>9</b>	42	42	43	36	47
<b>10</b>	39	39	38	38	41
<b>11</b>	38	41	37	37	37
<b>12</b>	40	47	40	35	38
<b>13</b>	41	45	38	42	39
<b>14</b>	45	43	50	45	42
<b>15</b>	34	39	35	29	33
<b>16</b>	36	34	40	29	41
<b>17</b>	42	58	44	28	38
<b>18</b>	35	38	32	37	33
<b>19</b>	51	48	55	45	56
<b>20</b>	40	45	40	37	38
<b>21</b>	39	35	42	40	39
<b>22</b>	39	39	39	36	42
<b>23</b>	38	35	36	38	43
<b>24</b>	41	43	38	44	39

2. Draw the control chart for the data given in Table 23.13.

**Table 23-13:** The mean dimension and standard deviation of the subgroups of 12 elements

<b>Subgroup number</b>	<b>Mean dimension, <math>\bar{X}</math></b>	<b>Standard deviation, s</b>	<b>Subgroup number</b>	<b>Mean dimension, <math>\bar{X}</math></b>	<b>Standard deviation, s</b>
<b>1</b>	36	5.54	<b>14</b>	38	2.16
<b>2</b>	32	1.82	<b>15</b>	40	2.55
<b>3</b>	62	5.54	<b>16</b>	29	3.16
<b>4</b>	35	5.16	<b>17</b>	32	6.59
<b>5</b>	36	5.08	<b>18</b>	38	12.54
<b>6</b>	40	3.46	<b>19</b>	30	1.94
<b>7</b>	32	4.36	<b>20</b>	50	4.35
<b>8</b>	46	0.82	<b>21</b>	36	2.55
<b>9</b>	38	5.54	<b>22</b>	35	3.94
<b>10</b>	34	2.41	<b>23</b>	35	3.44
<b>11</b>	33	3.10	<b>24</b>	36	4.55
<b>12</b>	36	7.09	<b>25</b>	40	1.94
<b>13</b>	36	7.09			

3. Inspection results of CFL produced by a company in a month of 25 working days are shown in Table 23.14. The daily production rate is 200 CFLs. Find the control limits using control charts for defectives.

**Table 23-14:** Daily production and defectives record of bulbs

<b>Subgroup number</b>	<b>Number inspected</b>	<b>Number defective</b>	<b>Subgroup number</b>	<b>Number inspected</b>	<b>Number defective</b>
<b>1</b>	200	9	<b>14</b>	200	2
<b>2</b>	200	2	<b>15</b>	200	1
<b>3</b>	200	7	<b>16</b>	200	4
<b>4</b>	200	3	<b>17</b>	200	5
<b>5</b>	200	0	<b>18</b>	200	6
<b>6</b>	200	5	<b>19</b>	200	12
<b>7</b>	200	5	<b>20</b>	200	1
<b>8</b>	200	1	<b>21</b>	200	4
<b>9</b>	200	6	<b>22</b>	200	5
<b>10</b>	200	8	<b>23</b>	200	3
<b>11</b>	200	2	<b>24</b>	200	2
<b>12</b>	200	8	<b>25</b>	200	2
<b>13</b>	200	7			

4. There are one month data as given in Table 23.15. The production rate varies from day to day. Number of defective components is also given with number inspected. Draw a p-chart and find the control limits for the data produced.

**Table 23-15:** The number of defectives found the various subgroups

<b>Subgroup no.</b>	<b>Number inspected</b>	<b>Number defective</b>	<b>Subgroup no.</b>	<b>Number inspected</b>	<b>Number defective</b>
<b>1</b>	2435	37	<b>14</b>	1546	24
<b>2</b>	1245	28	<b>15</b>	2249	21
<b>3</b>	1848	84	<b>16</b>	2263	48
<b>4</b>	2389	32	<b>17</b>	1438	28
<b>5</b>	1876	29	<b>18</b>	2042	40
<b>6</b>	2056	62	<b>19</b>	2578	113
<b>7</b>	1832	37	<b>20</b>	2174	68
<b>8</b>	1847	24	<b>21</b>	1541	42
<b>9</b>	2139	29	<b>22</b>	1642	29
<b>10</b>	1185	29	<b>23</b>	1843	10
<b>11</b>	2166	35	<b>24</b>	2292	27
<b>12</b>	1376	36	<b>25</b>	2042	56
<b>13</b>	1933	57			

5. There are 25 units of a product and the count of defects in each unit has been given in Table 23.16. Establish a control limits using c-chart.

**Table 23-16:** Count of defects in the different units of a product

Unit no.	Count of defects	Unit no.	Count of defects
1	12	14	8
2	11	15	7
3	11	16	12
4	8	17	10
5	28	18	12
6	14	19	8
7	12	20	8
8	6	21	0
9	5	22	9
10	10	23	20
11	19	24	9
12	8	25	8
13	0		

6. Establish a u-chart from the data given in Table 23.17.

**Table 23-17:** Count of defects for the subgroups of 20 units

Subgroup no.	Number inspected	Count of defects	Subgroup no.	Number inspected	Count of defects
1	20	12	14	20	17
2	20	16	15	20	14
3	20	15	16	20	16
4	20	23	17	20	22
5	20	25	18	20	21
6	20	17	19	20	22
7	20	19	20	20	13
8	20	22	21	20	20
9	20	12	22	20	11
10	20	24	23	20	16
11	20	26	24	20	13
12	20	18	25	20	26
13	20	19			

7. Draw the OC curve for a single sampling plan; given  $N = 5000$ ;  $n = 200$ ;  $c = 2$ .
8. Draw a double sampling plan; given  $N = 5000$ ;  $n_1 = 100$ ;  $c_1 = 2$ ;  $r_1 = 5$ ;  $n_2 = 300$ ;  $c_2 = 5$ ;  $r_2 = 6$ .
9. Draw the AOQ curve for  $N = 3000$ ;  $n = 100$ ,  $c = 2$ .
10. Draw the ASN curves for  $N = 3000$ ;  $n = 350$ ,  $c = 2$ ; and  $N = 5000$ ;  $n_1 = 200$ ;  $c_1 = 2$ ;  $r_1 = 6$ ;  $n_2 = 300$ ;  $c_2 = 5$ ;  $r_2 = 6$ .
11. Draw the ATI curve for  $N = 3000$ ,  $n = 100$ ,  $c = 2$ .
12. Find the equations for acceptance and rejection line for sequential sampling plan, if  $\alpha = 0.05$ ;  $p_\alpha = 0.06$ ;  $\beta = 0.10$ ; and  $p_\beta = 0.20$ .



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# CHAPTER ▶ 24

## Six-Sigma, ISO 9000 and 14000

### 24.1 INTRODUCTION

Six-sigma ( $6\sigma$ ) is a process improvement methodology developed at Motorola in the year 1989 to reduce defects in its processes. Its goal was to minimize the defect rate to 3.4 defects per million opportunities (DPMOs); this is a virtually defect free environment. The roots of Six Sigma can be traced back to Carl Frederick Gauss, who introduced the concept of the normal curve.  $6\sigma$  as a measurement standard in product variation can be traced back to the 1920s when Walter Shewhart showed that three sigma from the mean is the point where a process requires correction. Many measurement standards (process capability, zero defects, etc.) later came on the scene, but the credit of ' $6\sigma$ ' goes to a Motorola engineer named Bill Smith.

### 24.2 $6\sigma$ MEASUREMENT

Six-sigma simply means a measure of quality that strives for near perfection. Six-sigma is a disciplined, data-driven approach and methodology for eliminating defects (driving toward six standard deviations between the mean and the nearest specification limit) in any process from manufacturing to transactional and from product to service. The areas under  $\sigma$ ,  $2\sigma$  and  $3\sigma$  are shown in Figure 24.1. Under  $3\sigma$  limits (known as control limits), the percentage of conformance is 99.73 per cent and the percentage of non-conformance is 0.27 per cent, i.e. 2700 ppm (part per million) defects.

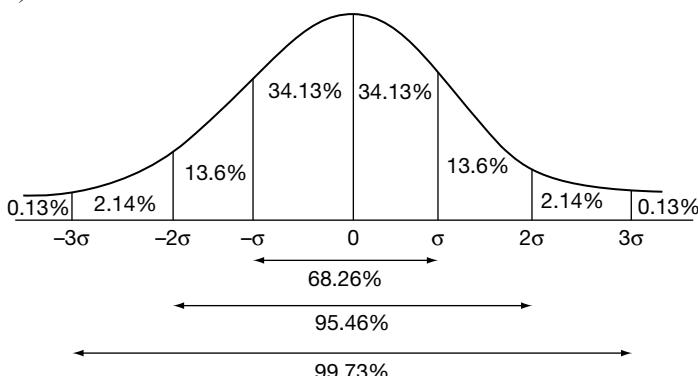
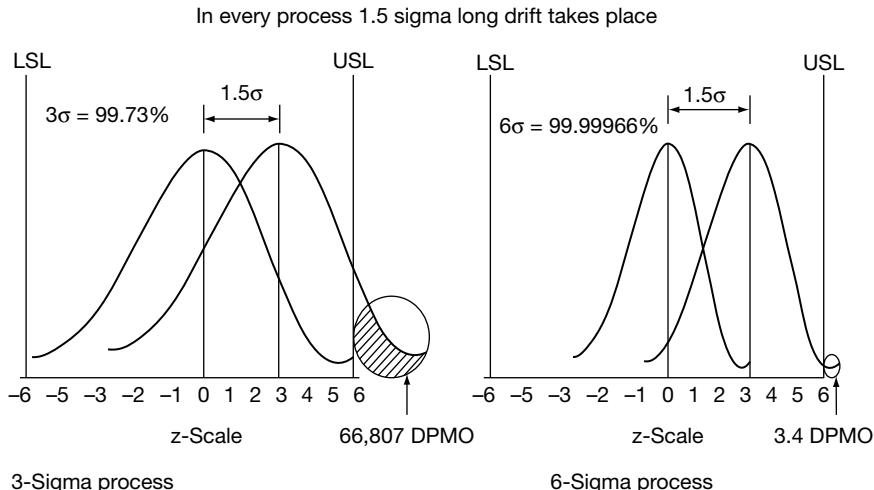


Figure 24-1: Areas under  $\sigma$ ,  $2\sigma$  and  $3\sigma$

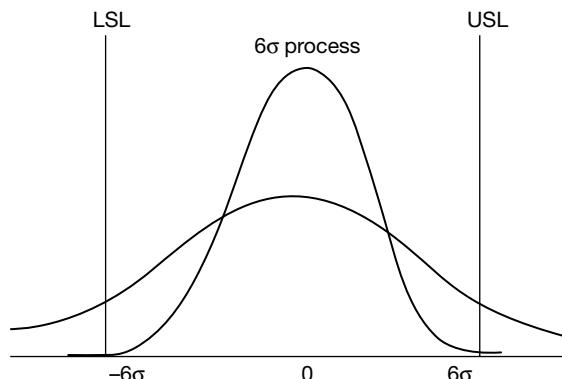
To achieve  $6\sigma$ , a process must not produce more than 3.4 defects per million opportunities. A  $6\sigma$  defect is defined as anything outside of customer specifications. A  $6\sigma$  opportunity is then the total quantity of chances for a defect. The difference between  $3\sigma$  and  $6\sigma$  processes is shown in Figure 24.2. There is drift of  $1.5\sigma$  between  $3\sigma$  and  $6\sigma$ .



**Figure 24-2:** Difference between  $3\sigma$  and  $6\sigma$  processes

The value of standard deviation in  $6\sigma$  is lesser than the  $3\sigma$ . If we improve the system quality-wise, the value of  $\sigma$  can be minimized so that the defect per million opportunities (DPMO) can be minimized to 3.4. The relationship between  $6\sigma$  and specification limits can be understood by Figure 24.3. The  $6\sigma$  limits lies under the specification limits.

The main objective of the  $6\sigma$  methodology is to implement the measurement-based strategy that focuses on process improvement and variation reduction through the application of  $6\sigma$ .



**Figure 24-3:** Relationship between  $6\sigma$  limits and specification limits

This can be accomplished through the use of two methodologies: DMAIC (define, measure, analyse, improve and control) and DMADV (define, measure, analyse, design and verify). The 6σ DMAIC process is an improvement system for existing processes falling below specification and looking for incremental improvement.

The six-sigma DMADV process is an improvement system used to develop new processes or products at six-sigma quality levels. It can also be employed if a current process requires more than just incremental improvement. The DMAIC methodology, instead of the DMADV methodology, should be used when a product or process is in existence, but is not meeting customer specification or is not performing adequately. The DMADV methodology, instead of the DMAIC methodology, should be used when a product or process is not in existence and one needs to be developed, and the existing product or process has been optimized (using either DMAIC or not) and still does not meet the level of customer specification or six-sigma level.

## **24.3 DMAIC METHODOLOGY**

The DMAIC methodology uses a stepwise structure. Steps generally are sequential; however, some activities from various steps may occur concurrently or may be iterative. The five steps of DMAIC are discussed below as:

### **24.3.1 Define**

Define the problem, process and project goals. In Six Sigma, it is imperative that the problem is specifically defined. The problem should be clearly established in quantitative terms, i.e. it should not be defined in vague terms. The description of the problem should include the issues related to feeling of customers and/or business as well as how long the issue has existed. Hence, identify the customer, the project goals, and timeframe for completion. The appropriate types of problems have unlimited scope and scale, from employee problems to issues with the production process or marketing. Regardless of the type of problem, it should be systemic part of an existing, steady-state process wherein the problem is not a one-time event.

### **24.3.2 Measure**

Measure the current process or performance. Identify what data is available and from what source. Develop a plan to gather it. Measure and collect data that will determine the factors that have influence over the outcome of the process or procedure. The feedback of people who manufacture products, feedback from customers who use the products and the way the product is processed, are all measured. The team also takes a look at business growth strategies. This can be known as the data collection step too. All relevant data, important to the product, and the processes followed to manufacture the product is collected at this stage.

### **24.3.3 Analyse**

Analyse the current performance to isolate the problem. Through analysis (both statistical and qualitative), begin to formulate and test hypotheses about the root cause of the problem. The data is analysed using statistical tools to assess whether the problem is real (and solvable) or

random, which makes it unsolvable within the six-sigma framework. It is important to analyse the feedback given by customers, as they are the end users of the product and the product needs to match their needs. In this stage, the root cause of the problem is identified. A process chart, here, helps the team in understanding where the process of manufacturing the product has gone wrong.

#### **24.3.4 Improve**

Improve the problem by selecting a solution. Based on the identified root cause in the prior step, directly address the cause with an improvement. Brainstorm potential solutions, prioritize them based on customer requirements, make a selection, and test to see if the solution resolves the problem. A complete new process chart is then made, which highlights the changes and improvements to be incorporated, in order to do away with defects. The concepts of total quality management and lean manufacturing are used in this stage. Documentation accompanies the new process chart, which provides the changes made in the process, in detail. Work at this stage becomes easy, if the team has collected enough data.

#### **24.3.5 Control**

This is the last step in DMAIC model. After the new process is designed, the organization replaces the old process with the new one. The team closely monitors the working of the new process and ensures that there are no problems in the new process. They monitor the performance of the new process and ensure that the products manufactured are defect free. If there are any further changes to be made, the team makes changes and again measures the performance of the process. Under proper guidance and observance of the team, the new process is adopted by the organization. Once the solution has resolved the problem, the improvements must be standardized and sustained over time. The standard-operating-procedures may require revision, and a control plan should be put in place to monitor ongoing performance. The project team transitions the standardized improvements and sustaining control plan to the process players and closes out the project.

### **24.4 SIX-SIGMA BELTS**

There are four levels in six-sigma training; and each level is defined by a belt. These levels in the respective order are yellow belt, green belt, black belt and master black belt. These belts are discussed in detail as:

**Yellow belt:** This is the lowest level in six-sigma. The person trained at this level holds the responsibility of smaller process improvement, but is not in-charge of a team under him. He has the basic knowledge about six-sigma processes.

**Green belt:** Green belt is the next level of expertise in six-sigma processes. Usually, the person trained or certified with this level gets an opportunity to head and lead the project. Usually the projects allotted to him/her are quite challenging and require him/her to be much more responsible.

**Black belt:** At this level, the person in-charge needs to be responsible in six-sigma implementation at all levels. He/she has many teams under him/her and he/she guides them and trains them to achieve the goals successfully. He/she also has to play the role of a mentor or teacher and teach his subordinates about six-sigma programmes. He/she leads the team and provides training to green belts and yellow belts.

**Master black belt:** This is the highest level and the expert is the mentor of black belts. He/she is the trainer, the teacher and the person who is responsible for implementation of six-sigma processes at all levels. He/she is the head and leads many projects at a time. A master black belt is responsible for company's strategies and training the black belts in six-sigma methods.

### **Limitations/Drawbacks of 6σ**

The main limitations or drawback of six-sigma are summarized below as:

- (a) System interaction is not considered; this is an uncoordinated projects.
- (b) Processes are improved independently.
- (c) There is a lack of consideration for human factors.
- (d) Significant infrastructure investment is required.
- (e) This is complicated for some tasks.
- (f) The goal of six-sigma (3.4 defects per million opportunities) is absolute, but this is not always an appropriate goal and does not need to be adhered to rigorously.
- (g) It is only about quality.

## **24.5 ISO 9000**

ISO 9000 (International Organization for Standardization) is a list of standards published by an international standards writing body. The ISO 9000 standards are a collection of formal international standards, technical specifications, technical reports, handbooks and web-based documents on quality management. The standards define the practices that are universally recognized and accepted for assuring that organizations consistently understand and meet the needs of their customers. The ISO 9000 series consists of four primary standards supported by several other documents.

- (a) **ISO 9000:2000 – Fundamentals and vocabulary:** This standard defines the fundamental terms and vocabulary used in the ISO 9000 family. The standard also includes the eight quality management principles which were used to develop ISO 9001 and ISO 9004. These eight management principles are customer focused, leadership, involvement of people, process approach, system approach to management, continual improvement, factual approach to decision-making, and mutually beneficial supplier relationship.
- (b) **ISO 9001:2000 – Requirements:** This standard describes the requirements for establishing and maintaining a quality management system (QMS) in an organization. The organization needs to assess and demonstrate its ability to provide products that meet customers' and regulatory requirements, and thereby enhance customer satisfaction.

- (c) **ISO 9004:2000 – Guidelines for performance improvements:** This standard provides guidance for continual improvement and is used for performance improvement of an organization. While ISO 9001 aims on quality assurance to enhance customer satisfaction, ISO 9004 takes in a broader perspective of quality management and provides guidance for future improvement.

The ISO 9000 standards were first published in 1987, revised for the first time in 1994, revised for the second time in 2000, and revised for the third time in 2008. Standards are reviewed every five years to ensure that they are relevant to the current practice and satisfy the needs of customers.

## 24.6 EIGHT MANAGEMENT PRINCIPLES

ISO 9000 is based on eight management principles.

- (a) Organizations depend on their customers and therefore should understand current and future customer needs, meet customer requirements and strive to exceed customer expectations.
- (b) Leaders should establish unity of purpose, i.e., goal and provide direction and resources to the organization to achieve organization's objectives. Leadership, aiming to create an internal environment in which people are fully involved.
- (c) People who are essence of an organization should be involved at all levels. Their full involvement enables their abilities to be used in favour of the organization.
- (d) Organizations need to do more than simply monitor process outputs. They must also control all the process inputs, such as people, facilities/equipment, material and methods. Also, they must establish appropriate controls over the transformation activities.
- (e) System approach to management, leading to improved effectiveness and efficiency through identification, understanding and management of interrelated processes.
- (f) Continual improvement should be a permanent objective of the organization. PDCA cycle has been discussed in Chapter 22 as a part of continual improvement.
- (g) Effective decisions should be based on the analysis of data and information.
- (h) An organization and its suppliers are interdependent on each other. Therefore, a mutually beneficial relationship should be established between them to enhance the ability of both to create value.

## 24.7 MAJOR CHANGES BETWEEN THE 1994 AND 2000 VERSIONS OF THE ISO 9001 STANDARD

The standards in 1994 are less generic and more focused on manufacturing industries. But the new standards are more generic and less biased to manufacturing industries. It can be used by all types of organizations, regardless of size and product nature. All the requirements of this new standard may not be mandatory for all organizations. As the distinction between ISO 9001, ISO 9002 and ISO 9003 has been removed, a clause 1.2 in the new standard allows companies to exclude certain requirements of (Section 7) product realization that are not relevant to them. The statement of Clause 1.2 is given below as:

All requirements of this International Standard are generic and are intended to be applicable to all organizations, regardless of type, size and product provided. Where any requirement(s) of this International Standard cannot be applied due to the nature of an organization and its product, this can be considered for exclusion. Where exclusions are made, claims of conformity to this International Standard are not acceptable unless these exclusions are limited to requirements within clause 7, and such exclusions do not affect the organization's ability, or responsibility, to provide a product that fulfils customer and applicable regulatory requirements.

The standard in the version ISO 9001:2000 retains a large part of ISO 9001:1994, but the 20 requirements (management responsibility; quality systems; contract review; design control; documentation and data control; purchasing; control of customer supplied product; product identification and traceability; process control; control of inspection; measuring and test equipments; inspection and testing; inspection and test status; control of non-conforming product; corrective and preventive action; handling, storage, packaging, preservation and delivery; control of quality records; internal quality audits; training; servicing; statistical techniques) have been grouped in five sections such as QMS; management responsibility; resource management; product realization; and measurement, analysis and improvement.

The new standard has also reduced the amount of documentation up to some extent. Documented procedures have been reduced from eighteen to six, although the organization, if required, may document other procedures, instructions, etc.

The new requirements in ISO 9001:2000 increase the emphasis on:

- (a) On the role of top management;
- (b) 'Customer focus' to ensure 'involvement of top management for determining customer requirements';
- (c) Consideration of statutory and regulatory requirements;
- (d) Establishment of measurable quality objectives at relevant functions and levels;
- (e) Establishment of internal communication processes to ensure effective communication of QMS objectives within the organization;
- (f) On resource availability, by adding separate requirements for 'infrastructure' and 'work environment';
- (g) Determination of training effectiveness;
- (h) Monitoring of information on customer satisfaction as a measure of system effectiveness;
- (i) Analysis of collected data to demonstrate the suitability and effectiveness of QMS;
- (j) 'Continual improvement' of the effectiveness of the QMS.

## 24.8 IMPLEMENTING ISO 9000 QMS

An ISO 9000:2000 QMS can be implemented using the following the steps:

1. **Determination of the organization's goals for implementing a QMS:** Identify the goals which you would like to achieve through a QMS. These goals may be customer

satisfaction, increased market share, improved communications and morale in the organization, greater efficiency and profitability, etc. Another objective of implementing a QMS may be a demonstration of compliance through third-party certification, which may be requested by an important client or required for enlisting as a supplier to large companies, e.g., original equipment manufacturers (OEMs).

2. **Find information about the ISO 9000 family:** The persons leading the implementation of an ISO 9000 QMS need to understand the requirements of ISO 9001:2000 as read with ISO 9000:2000 and ISO 9004:2000. The concerned information such as quality management principles, frequently asked questions (FAQs), guidance on clause 1.2 of ISO 9001:2000, guidance on the documentation requirements of ISO 9001:2000 and other brochures are available on website of ISO.
3. **Appoint a consultant, if necessary:** If the organization does not have adequate competence to develop a QMS, he/she may appoint a consultant. Before doing so, it is good to check his/her background, knowledge about the product realization processes of the organization, and experience in helping other organizations to achieve their stated goals, including certification. It is possible to appoint a consultant only for the training of key staff and then the staff can carry out further training and development of the system.
4. **ISO awareness and training:** Raise awareness about QMS requirements among the entire employee that affect quality. Plan and provide specific training on how to develop quality manuals, on procedures, on QMS planning, on how to identify and implement improvement processes, and on how to audit compliance with the QMS, etc.
5. **Gap analysis:** Find the gaps between the existing QMS and the QMS requirements of ISO 9001. Plan how to bridge these gaps. The gap analysis may be carried out through self-assessment or by the external consultant.
6. **Product realization processes:** Review the clause 7 of ISO 9001:2000 relating to product realization to determine the requirements that are applicable or not applicable for the company's QMS. The processes covered by this clause are: customer-related processes, design and development, purchasing, production and service provision, and control of measuring and monitoring devices. Suppose a company is not responsible for preparing the design of a product, he/she can exclude the requirement for design and development from QMS but the reasons for doing so should be mentioned in the quality manual.
7. **Staffing:** Decide the responsibilities of the persons involved in developing and documenting the QMS, appoint the management representative overseeing the implementation of the QMS. The project steering committee may also be established to oversee progress and provide resources wherever required.
8. **Planning a time frame:** Prepare a plan to fill the gaps identified in Step 5 to develop the QMS processes. Incorporate the activities to be performed, resources required, responsibilities and an estimated completion time for each activity should be in the plan. Clauses 4.1 and 7.1 of ISO 9001:2000 provide information that should be used when developing the plan. The total time required for each phase (planning,

documentation, implementation and evaluation) depends on the extent of the gaps in the existing QMS.

9. **Draft a quality manual:** Draft a quality manual that should include the following information:
  - (a) Include how the QMS applies to the products, processes, locations and departments of the organization;
  - (b) Justification for excluding any requirement as decided in step 6 above;
  - (c) Documentation procedures for QMS;
  - (d) Description of interaction between product realization processes and other management, measurement and improvement processes; and
  - (e) Draft the quality policy and quality objectives for the organization;
  - (f) Take the feedback of staff concerned in the organization about Quality Manual and the documented procedures before the Quality Manual and procedures are approved for issue and use;
  - (g) The effective date of implementation should also be decided.
10. **Carry out internal audits:** During three to six months of implementation of the QMS, the trained auditors should carry out one or two internal audits covering all activities for the QMS, and concerned management should take corrective action on the audit findings. If required, revise the manuals, procedures and objectives. After each internal audit, the top management should review the effectiveness of the system and provide necessary resources for corrective actions and improvements.
11. **Apply for certification:** On satisfactory completion of Step 10, and if the company decides to obtain third-party certification, he/she can make an application for certification to an accredited certification body.
12. **Conduct periodic evaluations:** After certification, the organization should periodically conduct internal audits to review the effectiveness of the QMS and see how it can be improved continually.

#### **24.8.1 Advantages of Implementing ISO 9000 Quality Systems**

Following advantages may be observed from implementing ISO 9000 quality systems:

- (a) Improvements in customer focus and process orientation within organization.
- (b) Continual improvement.
- (c) Consistency throughout the organization.
- (d) Strengthened relationships between the organization and its suppliers and customers, and among suppliers/customers within that organization.
- (e) Quality assurance to customers.
- (f) Improved management decision-making.
- (g) Institutionalized training in methods and procedures essential to quality.
- (h) Reduced dependence upon individuals.

## 24.8.2 Major Differences between ISO 9001:2000 and ISO 9001:2008

ISO 9001:2000 and ISO 9001:2008 use the same numbering system to organize the standard. However, some important clarifications and modifications were made. These changes are summarized below.

**Outsourcing processes:** The new (2008) standard makes it clear that an outsourced process is still part of your QMS even though it is performed by a party that is external to your organization. The new standard emphasizes the need to ensure that outsourced processes comply with all customer and legal requirements.

**Documentation:** ISO 9001:2000 Part 4.2.3 gave the impression that all external documents needed to be identified and controlled. This has now been clarified. The new standard says that you need to identify and control only those external documents that you need in order to be able to plan, operate, and control your QMS.

**Management representative:** The old standard did not explicitly say that the management representative must be a member of the organization's own management; outsiders were sometimes appointed, instead. This loophole has now been closed. ISO 9001:2008 now makes it clear that the management representative must be a member of the organization's own management. Outsiders may no longer perform this important function.

**Infrastructure:** For ISO 9001:2000 (Part 6.3), the term 'infrastructure' includes buildings, workspaces, equipment, software, utilities and support services like transportation and communications. ISO 9001:2008 has now added information systems to the previous list of support services.

**Work environment:** ISO 9001:2008 says that the term 'work environment' refers to working conditions. These working conditions include physical and environmental conditions, as well as things like noise, temperature, humidity, lighting, and weather. According to the new standard, all of these conditions need to be managed in order to help ensure that product requirements are being met. While ISO 9001:2000, Part 6.4, says that you are expected to manage the work environment that your organization needs in order to be able to ensure that all product requirements are being met.

**Monitoring and measuring equipments:** ISO 9001:2008, Part 7.6, refers to the need to control monitoring and measuring equipment, the old standard talked about controlling devices. Since the term device can refer to almost anything from a literary contrivance to a machine, its meaning wasn't exactly clear. The new ISO 9001 standard has removed this ambiguity by using the term equipment.

**Internal audit records:** Both old and new standards refer to the need to establish a procedure to define how internal audits should be planned, performed, reported and recorded (Part 8.2.2). However, the old standard did not explicitly state that audit records must actually be maintained. While ISO 9001:2008 now explicitly says that you must maintain a record of your internal audit activities and results.

### 24.8.3 Sections of ISO 9001:2008 Standard

**Section 1:** Scope

**Section 2:** Normative reference

**Section 3:** Terms and definitions

**Section 4:** Quality management system

*Section 4.1:* Introduction to QMS; and document hierarchy

*Section 4.2:* Documentation requirements; customer, statutory, and regulatory requirements; document control procedure; and record control procedure

**Section 5:** Management responsibility

*Section 5.1:* Top management commitment

*Section 5.2:* Customer focus

*Section 5.3:* Quality policy

*Section 5.4:* QMS planning and quality objectives

*Section 5.5:* Responsibility, authority and communication; management representatives; and internal communication

*Section 5.6:* Management review

**Section 6:** Resource management

*Section 6.1:* Provision of resources

*Section 6.2:* Human resources

*Section 6.3:* Infrastructure

*Section 6.4:* Work environment

**Section 7:** Product realization

*Section 7.1:* Planning of product realization

*Section 7.2:* Customer-related processes; contract review; production planning; communication; customer communication; authority communication

*Section 7.3:* Design and development

*Section 7.4:* Purchasing; purchasing documents; supplier assurance and approval; verification of purchased materials; supplier performance review; and supplier and contractor communication

*Section 7.5:* Control of production and service provision; identification and traceability; customer property; systems of preservation of products; housekeeping policy; cleaning policy; pest control policy; control of visitors and contractors; dispatch and distribution; maintenance; waste management; and crisis management

*Section 7.6:* control of monitoring and measurement equipment

**Section 8:** Measurement, analysis and improvement

*Section 8.1:* Management, analysis, and improvement of QMS

*Section 8.2:* Monitoring and measurement; customer satisfaction; internal audits; monitoring and measurement of processes; and monitoring and measurement of product

*Section 8.3:* Control of non-conforming product; and product recall

*Section 8.4:* Analysis of data

*Section 8.5:* Corrective action; preventive action; and improvement

## 24.9 EMS: INTRODUCTION

All business, regardless of size, operation and activity, have some impact on the environment. An environmental management system (EMS) is a tool that allows a company to identify and address his particular environmental impacts. Environmental issues should not be considered in isolation to other management issues. By adopting an EMS, a company is making a commitment to incorporate environmental issues into existing management systems.

An EMS is a framework that helps a company in achieving its environmental goals through consistent efforts and control of its operations. It is the part of the company's management system used to develop and implement its environmental policy and manage environmental aspects. The better control improves the environmental performance of the company. The EMS itself does not dictate a level of environmental performance that must be achieved. The proactive approach of EMS can help a company reduce the risk of non-compliance and improve the health and safety for employees and the society. An EMS can also address non-regulated issues, such as energy conservation, and can promote stronger operational control and employee stewardship.

### 24.9.1 Basic Elements of EMS

Some basic elements of EMS are given below as:

- (a) Review of the environmental goals of a company.
- (b) Analysis of environmental impacts and legal requirements.
- (c) Determination of environmental objectives and targets to reduce environmental impacts and comply with legal requirements.
- (d) Programmes implementation to meet these objectives and targets.
- (e) Monitoring and measuring the progress in achieving the objectives.
- (f) Employees' environmental awareness and competence.
- (g) Review the progress of the EMS and making improvements.

### **Advantages of EMS**

The following are advantages of EMS:

- (a) It helps the company to improve its environmental performance.
- (b) It enhances the compliance with environmental laws and regulations.
- (c) It prevents the environmental pollution.
- (d) It helps in conservation of natural resource.
- (e) It also attracts the new customers/markets for the company.
- (f) It increases the efficiency at reduced costs.
- (g) It also enhances the morale of employees.
- (h) It enhances the image with public, regulators, lenders, investors, etc.
- (i) It reduces the waste collection, treatment and disposal costs.
- (j) It reduced risk and liability associated with poor environmental performance.
- (k) It improves relationships with key stakeholders, e.g. customers, suppliers, staff, regulatory authorities and local communities.

### ***Limitations of EMS***

The following are limitations of EMS:

- (a) Internal labour costs represent the bulk of the EMS resources expended by most organizations.
- (b) It requires consulting assistance.
- (c) It requires outside training of personnel.

### **24.9.2 EMS: Step-by-Step Plan**

There are following step-by-step plan for implementation of EMS in an organization:

- (a) Top management commitment to meet the environmental goals.
- (b) Select a leader for leading the environmental management team.
- (c) Prepare the budget and schedule devoted to EMS.
- (d) Form a cross-functional team to implement, control and monitor the programme.
- (e) Involve employees in achieving the environmental goal.
- (f) Review of the plan.
- (g) Establish the reviewed plan.
- (h) Identify the environmental impact of the organization.
- (i) Identify objectives and targets in managing these environmental impacts.
- (j) Establish the environmental policy and objectives.
- (k) Set the procedures and prepare the documents.
- (l) Plan changes, if required.
- (m) Train the employees for EMS implementation.
- (n) Implement EMS.
- (o) Audit the performance of the EMP (environmental management programme) on an ongoing basis. Measure what targets are being achieved. Identify if objectives need to be changed.
- (p) Regularly review the EMP performance; identify areas of the program that can be improved.
- (q) Reset the objectives and targets in managing these environmental impacts accordingly.
- (r) Certification by third party, i.e. ISO 14000.

### **24.10 ISO 14000**

The ISO 14000 family is primarily concerned with environmental management. This means what the organization does to minimize harmful effects on the environment caused by its activities, and achieve continual improvement of its environmental performance. The world's first standard for EMS was BS 7750 developed by the British Standard Institution (BSI) in 1992. ISO 14000 series developed by the International Organization for Standardization is based on BS 7750. ISO 14001 was finalized in 1996.

## 24.10.1 Coverage of ISO 14000 Series

ISO14000 series covers the following area:

- (a) Environmental management systems (ISO 14001, 14004)
- (b) Environmental performance evaluation (ISO 14014, 14015, 14031)
- (c) Environmental auditing (ISO 14010, 14011, 14012, 14013, 14014)
- (d) Life cycle assessment (ISO 14040, 14041, 14042, 14043)
- (e) Environmental labelling (ISO 14020, 14021, 14022, 14023, ISO 14024)
- (f) Environmental aspects in product standards (ISO 14060)

## 24.10.2 Five Major Sections of ISO 14001

- (a) *Environmental policy*: The first section establishes an organization's policy and commitment as it relates to the energy and environment.
- (b) *Planning*: The second section identifies energy and environmental issues, and defines the initiatives and resources needed to achieve the environmental policy and economic goals.
- (c) *Implementation and operations*: The third section describes the procedures, programmes and responsibilities necessary to implement the key initiatives to achieve goals.
- (d) *Checking and corrective action*: The fourth section monitors and assesses the effectiveness of energy and environmental management activities on regular basis.
- (e) *Management review*: The fifth section, a high-level evaluation of the management system as a whole to determine its overall effectiveness in terms of driving continual improvement and achieving business goals.

## 24.10.3 The ISO 14001 Seventeen Elements

The ISO 14001 standard breaks the five major sections down into seventeen elements described below:

- (a) *Environmental policy*: The policy drives the commitment of the organization to maintain and improve its environmental performance. By documenting and publicizing the policy, the organization demonstrates a commitment to the management of environmental issues from the top management levels.
- (b) *Environmental aspects*: Environmental aspects are the organizational activities that have the potential to interact with the environment in some way, potentially posing a risk if they are not managed properly.
- (c) *Legal and other requirements*: The legal and other requirements are those requirements that the organization is expected to comply with on a continual basis. Besides legal requirements, these could be the expectations of the local community/society or other local organizations.
- (d) *Objectives and targets*: Objectives and targets fix the goals of an organization's EMS. Drawing on the information gained in this aspect, an organization develops goals for improving its performance in regard to specific activities.

- (e) *Environmental management programmes (EMPs)*: These programmes define the methods that will be used by the organization to achieve its objectives and targets.
- (f) *Structure and responsibility*: Just like an organizational chart, the structure and responsibility of the organization defines the authority structure for (EMP). The responsibility defines who is responsible for what within the organization.
- (g) *Training, awareness and competence*: This element of an EMS defines what training and minimum competence levels are required to ensure that environmental risks are managed appropriately, who receives the training, and how often.
- (h) *Communications*: The communications element of an EMS defines the ways to handle the environmental issues.
- (i) *EMS documentation*: The documentation element defines the structure of the EMS.
- (j) *Document control*: This element focuses on the maintenance and control of EMS documents required to run the EMP.
- (k) *Operational control*: The operational control element focuses on the level of operation control that is applied to minimize the environmental risks within the organization.
- (l) *Emergency preparedness and response*: This EMS element outlines the procedures by which the organization responds to environmental emergencies, and the maintenance of a minimum level of preparedness.
- (m) *Monitoring and measurement*: This element describes how an organization monitors its environmental performance, what procedures are used to measure the appropriate data sources, and how often they are measured.
- (n) *Non-conformance, corrective and preventative action*: This EMS element outlines how an organization investigates, prevents and corrects non-conformances.
- (o) *Records*: The EMS records describe how the organization handles and controls the larger scope of documents related to the EMS, such as training records, compliance reports and letters to regulators.
- (p) *EMS internal audit*: The auditing element of the EMS outlines how an organization audits its environmental performance.
- (q) *Management review*: This element of the EMS defines how the top management coordinates performance reviews and drives the process of continual improvement.



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## SUMMARY

Different versions of ISO 9000 and their sections and clauses have been introduced. ISO 9000 is a continual quality improvement programme which sets some standards. These standards are to be followed by the organizations to achieve the quality related goals. ISO 14000 series is concerned with environmental aspects of an organization. EMS and its implementation have been discussed in detail. The various aspects of ISO 9000 and 14000 have been outlined properly.

## MULTIPLE-CHOICE QUESTIONS

1. The concept of ‘six-sigma’ was developed by
  - (a) Toyota
  - (b) Motorola
  - (c) Bell
  - (d) IBM
2. The defects per million opportunity in six-sigma is equal to
  - (a) 2700
  - (b) 3.4
  - (c) 1.2
  - (d) None of the above
3. DMAIC stands for
  - (a) Define, Measure, Analyse, Improve, and Control
  - (b) Design, Measure, Analyse, Improve, and Control
  - (c) Define, Maintain, Access, Inform, and Control
  - (d) Design, Maintain, Access, Inform, and Control
4. Which of the following DOES NOT belong to six-sigma belts?
  - (a) Yellow belt
  - (b) Green belt
  - (c) Black belt
  - (d) Red belt
5. Which of the following is the advantage of six-sigma?
  - (a) There is a lack of consideration for human factors
  - (b) Significant infrastructure investment is required
  - (c) This is complicated for some tasks
  - (d) This improves the reputation in market
6. ISO stands for
  - (a) International Organization for Standardization
  - (b) Indian Standard Organization
  - (c) International Standardization for Organizations
  - (d) Indian Organization for Standardization
7. ISO 9000 is concerned with
  - (a) Quality management systems
  - (b) Environmental management systems
  - (c) Inventory management systems
  - (d) None of these
8. ISO 14000 is concerned with
  - (a) Quality management systems
  - (b) Environmental management systems
  - (c) Inventory management systems
  - (d) None of these

9. How does the ISO help the organization?
  - (a) Provides organizations with information about quality management systems
  - (b) Provide the training for the documentation
  - (c) Helps organizations become certified under the international standard
  - (d) All the above
10. The ISO applies to software engineering is
  - (a) ISO 9000:2004
  - (b) ISO 9001:2000
  - (c) ISO 9002:2001
  - (d) ISO 9003:2004
11. The ISO 9000 series consists of three primary standards of which ISO 9000 pertains to
  - (a) Vocabulary
  - (b) Requirements
  - (c) Guidelines for performance improvements
  - (d) All the above
12. The ISO 9000 series consists of three primary standards of which ISO 9001 pertains to
  - (a) Vocabulary
  - (b) Requirements
  - (c) Guidelines for performance improvements
  - (d) All the above
13. The ISO 9000 series consists of three primary standards of which ISO 9004 pertains to
  - (a) Vocabulary
  - (b) Requirements
  - (c) Guidelines for performance improvements
  - (d) All the above
14. ISO 14060 pertains to
  - (a) Environmental management systems
  - (b) Environmental performance evaluation
  - (c) Environmental auditing
  - (d) Environmental aspects in product standards
15. ISO 14040 pertains to
  - (a) Environmental management systems
  - (b) Life cycle assessment
  - (c) Environmental auditing
  - (d) Environmental aspects in product standards

## Answers

1. (b)      2. (b)      3. (a)      4. (d)      5. (d)      6. (a)      7. (a)      8. (b)      9. (d)  
10. (b)     11. (a)     12. (b)     13. (c)     14. (d)     15. (b)

## REVIEW QUESTIONS

1. What do you mean by six-sigma concept? Explain it.
2. Explain the basic procedure of implementation of six-sigma, i.e., DMAIC.

3. What are the various types of six-sigma belts that are given for the six-sigma trainers? Explain the meaning of the belts.
4. What are the limitations of six-sigma?
5. Explain the objectives of various standards of ISO 9000.
6. Discuss the eight management principles of ISO 9000.
7. Explain the various steps used for the implementation of ISO 9000:2000.
8. What are the advantages of using ISO 9000:2000?
9. What are the differences between ISO 9001:2000 and ISO 9001:2008 versions?
10. What do you mean by the environmental management system (EMS)? Write its advantages and disadvantages.
11. Discuss the step-by-step plan for implementation of EMS in an organization.
12. Discuss the 17 elements of ISO 14001.



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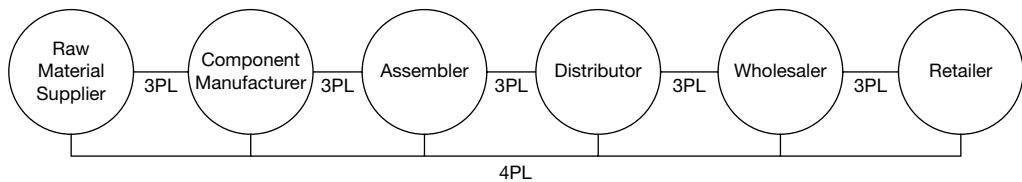
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## Supply Chain Management

### 25.1 INTRODUCTION

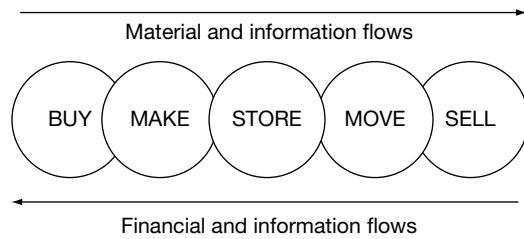
Supply chain management (SCM) is a management technique to coordinate all the activities, processes and parties to fulfil the demand of the target customer. It is a management technique to coordinate all the activities involved in product design and development to fulfil the demand of the customer, i.e., from the supplier end to consumer end. There are a number of entities in a supply chain that are supplier, assembler or manufacturer, distributor, warehousing, retailer, customer, etc. A standard supply chain is shown in Figure 25.1. All the entities of a supply chain are interlinked with a third party logistics service provider (3PL) and/or a coordinator of all the activities in a supply chain, i.e., fourth party (4PL) service provider. Generally, 3PL is considered as a part of 4PL.



**Figure 25-1:** Entities of a supply chain

SCM may be defined in a different ways. Simchi Levi (2000) defined SCM as ‘Supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses, and stores, so that merchandise is produced and distributed at the right quantities, to the right locations, and at the right time, in order to minimize systemwide costs while satisfying service level requirements.’ The Institute for supply chain defined SCM as ‘The design and management of seamless, value-added process across organizational boundaries to meet the real needs of the end customer’. The Supply Chain Council defined SCM as ‘Managing supply and demand, sourcing raw materials and parts, manufacturing and assembly, warehousing and inventory tracking, order entry and order management, distribution across all channels, and delivery to the customer’. The activities of an internal supply chain can be explained as shown in Figure 25.2.

In an internal supply chain or supply chain at the micro level, the main activities are to buy the raw materials or sub components, make and assemble them, store the finished products, distribute and sell them.



**Figure 25-2:** Activities of an internal supply chain

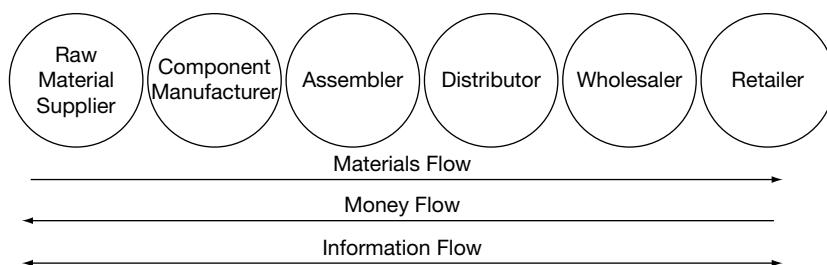
## 25.2 FOUR FUNDAMENTALS OF SUPPLY CHAIN (4FS OF SCM)

There are four fundamentals (Sweeney 2002) as discussed below:

*First fundamental (Objective):* The first fundamental of the SCM is to achieve high levels of customer service in target markets/segments and to optimize total supply chain investment and cost. This service/cost approach has long been regarded as central or the main objective of a SCM. This approach requires companies to have a clear understanding of both issues. Customer service requirements, dictated by the marketplace, set the specification for the supply chain.

*Second fundamental (Philosophy):* The second fundamental of the SCM is to create and manage the strong link of the supply chain to supply the right product of the right quality to the right customer at the right time. Since, every product or service is delivered to the final consumer through a series of often complex movements between companies which comprise the complete chain. Inefficiency anywhere in the chain will result in the chain as a whole failing to achieve its true competitive potential. Supply chains are increasingly competing with other supply chains rather than the companies simply competing with other companies.

*Third fundamental (Manage the Flows):* For a supply chain to achieve its maximum level of effectiveness and efficiency, material flows, money flows and information flow throughout, the chain must be managed in an integrated and holistic manner, driven by the overall service and cost objectives. The directions of these flows are shown in Figure 25.3.



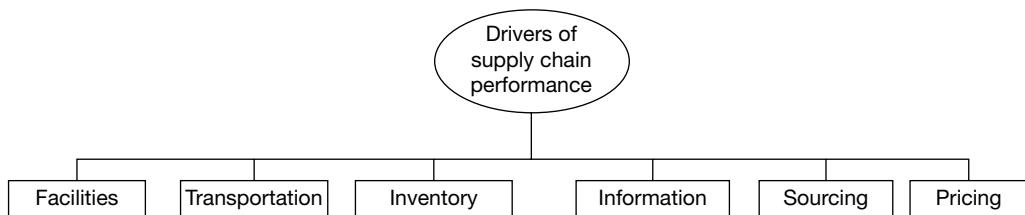
**Figure 25-3:** Three flows in supply chain

Materials or the product flow in the downstream supply chain, i.e. from supplier and to the consumer or retailer end; and, money, i.e. finance flows from the consumer end to supplier end (upstream supply chain). The demand related information flows from retailer end to the supplier end, but, there are some other information like product design and development flows from both ends in both directions.

*Fourth fundamental (Relationships):* SCM is not a ‘zero-sum’ game based on adversarial relationships (arm’s length relationship), rather, it needs to be a ‘win-win’ game based on partnership approaches. This point is relevant to the interactions between the key ‘internal’ supply chain functions of buy, make, store, move and sell, as well as to relationships between an organization and its external customers and suppliers (strategic relationship). Arm’s length relationship is concerned with single interaction or one time purchase of goods. But, strategic relationship is based on long term relationship and contract is made for a continuous supply of goods in place of one time supply. The main aim of the strategic relationship among the supply chain partners is to achieve a common objective combined.

## 25.3 DRIVERS OF SUPPLY CHAIN PERFORMANCE

The performance of supply chain depends on a number of factors. Some of the important drivers of supply chain performance have been discussed in this text. These drivers are facilities, transportation, inventory, information, sourcing and pricing, as shown in Figure 25.4.



**Figure 25-4:** Drivers of supply chain performance

**Facilities:** There are a number of facilities that are to be located as per requirement of the markets. Mainly, two facilities, manufacturing and storage, are very important. We have already discussed the factors influencing the facility location in Chapter 2 in this text. The main objective with the facility location is to minimize the transportation cost, inventory-carrying cost, and manufacturing cost of the product with availability of suppliers, markets, powers, raw materials, etc.

**Transportation:** Transportation covers the entire logistics services required for a supply chain and it is an important link of a supply chain to connect the various entities. It includes all modes of transportation, i.e. sea, air, and land (rail and road) transport. In the scenario of global supply chain, all the multinational products are available in the market because of the availability of low cost transportation services. This may help the supply chain manager to retain the existing customer and to expand the target markets.

**Inventory:** Inventory of raw materials, work-in-process, and finished product, improve the responsiveness and efficiency of a supply chain. But, at the same time, it increases the cost of the supply chain. Inventory is required at various points in the supply chain to meet the uncertainty in demand and supply, delivery lead time and incentives for larger supplies. A number of models have been proposed by the researchers to minimize the inventory and maximize the flexibility and agility of a supply chain. Some of the major inventory related strategies are vendor-managed inventory (VMI), risk pooling, lean manufacturing, and just-in-time manufacturing. The concepts of lean manufacturing and just-in-time have already been discussed in Chapter 8 (Manufacturing Systems).

**Vendor-managed inventory (VMI):** It is an important tool of the manufacturers to minimize the inventory at the shop floor. The inventory is completely managed by the vendor and supplied to the manufacturer as per requirement. But, in this situation all the vendors must be located very near to the manufacturing plant.

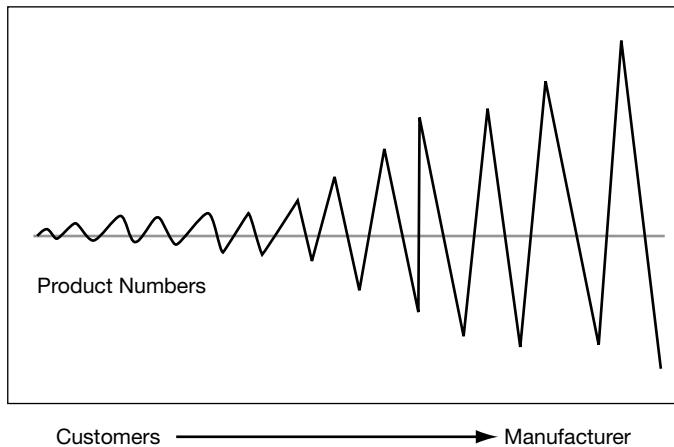
## 25.4 RISK POOLING

Risk pooling is a statistical concept that suggests that demand variability can be reduced if one can aggregate demand, for example, across locations, across products or even across time. This is really a statistical concept that suggests that aggregation reduces variability and uncertainty. For example, if demand is aggregated across different locations, it becomes more likely that high demand from one location will be offset by low demand from other location. This reduction in variability allows a decrease in safety stock and therefore reduces average inventory. But, risk pooling is useful only when the variation in demand in two locations are opposite in nature. Risk pooling concept is very useful in inventory management, warehouse location and product flow, transportation, push–pull strategy, postponement, and product design.

## 25.5 BULLWHIP EFFECT

The goal of any supply chain is to deliver the right quality of goods and services to customers in the most efficient way possible. To meet this goal, each link along the supply chain must not only function as efficiently as possible; it must also coordinate and integrate with links both upstream and downstream in the chain. The biggest problem in a lean supply chain is accuracy in demand planning. Unforeseen in demand overestimates the volume of supply at the supplier end with changes in production. Production and supply issues, then impact the consumer end of the supply chain and the effects ripple up and down the chain. This is often referred to as the bullwhip effect. Variation in demand from customer end to supplier end is shown in Figure 25.5. In this figure, average demand remains same but the variability increases from customer end to supplier end. This is known as bullwhip effect.

The bullwhip effect can be explained as a phenomena detected by the supply chain where orders sent to the manufacturer/supplier creates a larger variability than the sales to the end customer. This variance can interrupt the smoothness of the supply chain process as each link in the supply chain will over or underestimate the product demand resulting in exaggerated



**Figure 25-5:** Bullwhip effect

fluctuations. Bullwhip effect results in increased safety stock, reduced service level, inefficient allocation of resources, and increased transportation cost.

There are some **factors that contribute to the bullwhip effect** in supply chains are given below:

*Artificial demand creation:* Some time demand of the product increases due to sales promotion or offer given to the product. This unexpected increase in demand results in variation in demand estimation of the retailer and other links of the supply chain and manages the increased safety stock towards the upstream supply chain. But, in reality, the mean demand remains same throughout the supply chain.

*Lack of communication:* Due to lack of communication or information, managers can perceive a product demand quite differently within different links of the supply chain and therefore order different quantities.

*Free return policies:* Customers may intentionally overstate demands due to shortages and then cancel when the supply becomes adequate again, without return forfeit retailers will continue to exaggerate their needs and cancel orders; resulting in excess material.

*Batch ordering:* Companies may not immediately place an order with their supplier; often accumulating the demand first, due to the discount provided by the supplier for large order size or due to economies of scale. Companies may order weekly or even monthly. This creates variability in the demand as there may be a surge in demand at some stage followed by no demand after.

*Anticipation of increase in price:* If there is an anticipation that the price of a specific product is to be increased in the near future, the customer starts to purchase more without increasing the consumption, which results in bullwhip effect.

*Demand information:* Relying on past demand information to estimate current demand information of a product does not take into account any fluctuations that may occur in demand over a period of time.

There are following ways to *minimize the bullwhip effect*:

*Reduce uncertainty:* Uncertainty can be reduced by sharing proper information. It is possible if point of sale data is shared by all the supply chain partners centrally. The forecasting policies and forecasts should be shared by the all supply chain partners.

*Reduce variability:* To reduce the variability in demand, promotion should be eliminated and everyday low pricing strategies should be followed.

*Reduce lead times:* Lead time of ordering and supply can be reduced using electronic data interchange (EDI) and cross docking respectively. In the case of cross docking, the warehouse acts just like a coordination point. Without storing in the warehouse, the product is directly supplied to the customer from the manufacturer. Warehouse can be used for repackaging and coordination purpose only.

*Strategic partnership:* The inventory can be minimized by the VMI and sharing the data centrally to all the supply chain partners.

## 25.6 SUPPLY CHAIN INFORMATION SYSTEMS

E-business is concerned with the use of the Internet to link companies with their suppliers, customers and other supply chain partners. As a business concept, it has evolved significantly since its introduction in the 1990s in parallel with the rapid development of information technology (IT) during that period. SCM is concerned with integration of activities both with and between organizations. IT plays a crucial role in SCM as a key enabler of supply chain integration (SCI). The main role of IT in supply chain is given below:

- To collect information on each product from production to delivery and provide complete visibility for all parties,
- To access any data in the system from a single-point-of-contact,
- To analyse, plan activities, and make trade-offs based on information from the entire supply chain.

There are many advantages of e-business in the supply chain. These advantages are: Streamlining the procurement process, connecting buyers and sellers, coordinating SCM, better post-sales service, better sales and marketing efficiencies, and improved inter-organizational efficiencies in the selling company.

### 25.6.1 IT Based Tools for SCM

A number of IT-based SCM tools have been in use to provide intelligent decision support to SCM. They can be applied in day-to-day operations, operational planning or strategic planning to redesign the supply chain resources. Some of the major IT tools used in the supply chain are discussed in the following paragraphs:

**Electronic data interchange (EDI):** EDI is the inter-organizational exchange of business documentation/information in a structured machine-processable form. It consists of standardized electronic message formats for common business documents such as a request for quotations, purchase orders, invoices and other standard business correspondence documents. These electronic transaction sets enable the computer in one company to communicate with the computer in another organization without actually producing paper documents. All human efforts required to sort and transport the document are eliminated.

A fully integrated EDI solution adds speed and efficiency to business processes, enabling the organization to maximize resources, minimize waste and increase customer satisfaction. The key benefits of EDI can be explained as:

- (a) Reduced transaction costs across the supply chain of an organization through a reduction in labour and material costs, communication costs and administrative costs,
- (b) Greater accuracy of information flow while reducing the likelihood of costly errors,
- (c) Reduced inventory and its associated costs,
- (d) Enabling quicker response to address changes in the marketplace, and
- (e) Easier for customers to do business with the company.

**Enterprise resource planning (ERP):** ERP is a system that connects different functions within an organization, as well as an organization's supply chain partners (i.e. suppliers, distributors, third party logistics providers), enabling the various business partners and organizational entities to share information, such as order status, product schedules, sales records, as well as to plan production, logistics and marketing promotions. (Gunasekaran and Ngai 2004). ERP is a comprehensive planning and control framework that has evolved over a long horizon of time. It finds its applications in materials requirement planning (MRP), manufacturing requirement planning (MRP II), relational database management systems (RDBMS) and 4<sup>th</sup> generation computer languages (4GL). It also influences just-in-time (JIT) and computer-integrated manufacturing (CIM) and takes advantage of latest IT developments such as client-server computing and the Internet.

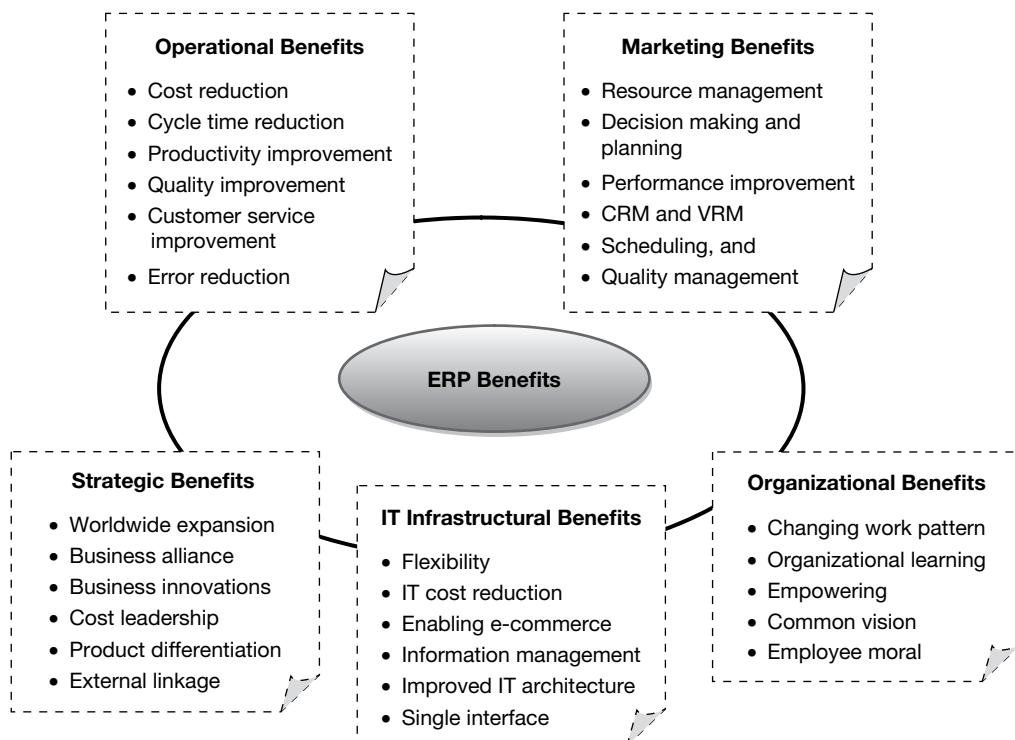
Su and Yang (2010) found impact of ERP on SCM competencies. They demonstrated the relationships between ERP and supply chain competitiveness as shown in Figure 25.6. The key benefits of ERP can be explained as:

- (a) ERP links all the activities in the organization with customer orders and thus the customer becomes the key focus of all departments.
- (b) An ERP system regulates the flow of goods in the supply chain. It captures and consolidates related data from the retailer that can be used to change the production schedule quickly.
- (c) ERP as a tool can enhance overall performance by reduction of costs, increased per capita productivity and improved quality of goods and services. However, Akkermans et al. (2003) identify some shortcomings in ERP as lack of extended enterprise functionality, limited flexibility in adapting to changes in the environment,

limited advanced decision support functionality; and lack of (web-enabled) modularity. They identified a number of key SCM trends for which ERP provides support. These trends are given as:

- (a) Mass customization of products;
- (b) Standardization of processes and data; and
- (c) Integration of globalized businesses.

**Electronic/Internet commerce:** Internet is transforming the entire nature of supply chains by eliminating middlemen, making commerce more democratic and creating a frictionless economy. Internet commerce is changing the manner in which the entire supply chain is managed today. It is reducing entry barriers for the new entrants and the costs of operations for the existing ones at the same time and also offering the customers a wider choice for selection. Internet also helps in e-tendering. E-tendering is the process of sending RFx to suppliers and receiving responses electronically. The three types of RFx commonly used for sourcing are: RFI (request for information), RFP (request for proposal), and RFQ (request for quotation). RFIs typically involve a potential buyer asking a seller to provide additional information on a product or process.



**Figure 25-6:** ERP benefits and supply chain competencies

RFQs involve a potential buyer requesting a specific price for given items, while RFPs tend to include both a quote and a qualitative description of the work to be done.

**Bar coding:** A bar code is a group of parallel bars (usually blocks) of varying widths separated by light spaces (usually white) of varying width. A scanner is used to read the bars and spaces and it uses software to interpret their meaning. In the supply chain, the accurate, rapid identification of products and use of this information in controlling the entire process have been key factors. The following are the benefits of bar code technology in the supply chain:

- (a) Rapid data entry,
- (b) Enhanced data accuracy,
- (c) Reduced the materials handling activities,
- (d) Easy verification of orders at receiving and shipping, and
- (e) Improved customer service.

**Information and communication technology (ICT):** ICT also significantly enhances the supply chain performance through faster and widespread communication. Applications of radio frequency identification, satellite communications and image processing technologies have overcome the problems caused by product movement and geographic decentralization. Improved customer service is provided in the form of the more timely definition of tasks, quicker transport tracing, and faster transfer of sales and inventory information. The availability of shared information has led to increase efficiency and effectiveness across the supply chain.

**Radio frequency identification (RFID):** The unique number, called EPC (electronic product code), is encoded in a RFID tag. There are three types of RFID tags; these are: passive tags – simply store data and draw power from a reader whose electromagnetic wave induces a current in the tag's antenna for short-range communication (up to 10 m); semi-passive tags – use an integral battery to run the chip's circuitry, but draw power from the reader to communicate; and active tags – are capable of communicating over greater distances (up to 100 m) but are currently far more expensive. RFID tags enable us with an increased visibility and accuracy of the asset pool. This visibility and accuracy impacts the operating costs, lead times, inventory visibility and accuracy, and customer service.

The automatic identification of products in the store would increase the inventory visibility and its accuracy. Stock out levels can be decreased as consequences of the increased inventory visibility. Decreased stock out increases sales; and ultimately increases profits and customer service. Finally, inventory levels can be reduced, increasing the ROI.

The benefit of an EPC code is primarily derived from the ability to automatically pinpoint the exact location of goods and documents anywhere within an extended enterprise. Such ability leads to the benefits: enhance supply-chain control, security and authentication, and enhanced customer service. The RFID technology can promote customer service by allowing faster checkouts, returns, and personalization of service. It improves supply chain performance by reducing costs, inventory levels, lead times, and stock outs; increasing quality, manufacturing flexibility, inventory visibility, inventory record accuracy, order accuracy, customer service, and the collaboration among supply chain members.

**Sourcing:** One of the main drivers of supply chain is sourcing. The main aims of the sourcing are cost saving, increased capacity and ability to take advantage of offshore advantage. The most difficult phase of outsourcing is transition of business processes from the company to its suppliers to increase the innovations in technology of the supplier and to increase the production capacity at low cost. The main barriers in sourcing are loss of institutional knowledge, poor communication with suppliers, mismatch of firm cultures, high transportation cost and larger lead time owing to the coverage of wider geographical area, investment in larger inventory size owing to chances of logistics failure and unstable business and taxation policy of supplier government owing to instability. Supplier selection, evaluation, and quota allocation are the part of sourcing (Kumar et. al. 2008, 2011; Singh et. al. 2010). There are many activities, which are not part of the core competencies of an organization, are outsourced to the third party. A number of activities such as, logistics, warehousing, distribution, packaging, etc. are outsourced to the third-party logistics service providers (Kumar and Singh 2012; Bansal and Kumar 2013; Bansal et al. 2014).

**Pricing:** Pricing fixes the prices of goods and services in the supply chain. It affects the behaviour of consumers. The behaviour of consumer or satisfaction level of the consumer affects the supply chain performance. Pricing of a product depends on the market structure, demand and supply, competitor's strategy etc.



## SUMMARY

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In this chapter, SCM is introduced; four fundamentals of the SCM have been discussed. Drivers of supply chain outlined and explained in detail. Finally, various issues in SCM, such as, risk pooling, bullwhip effect have been highlighted. Various tools of information technology like ERP, e-commerce, bar coding, ICT and RFID have been introduced.

## MULTIPLE-CHOICE QUESTIONS

1. A supply chain is a network of firms that perform activities required
  - (a) to find the similar products
  - (b) to facilitate inventory
  - (c) to create synergy
  - (d) to produce and deliver goods to consumers
2. Bar coding is concerned with
  - (a) enhanced data accuracy
  - (b) reduced the materials handling activities
  - (c) easy verification of orders at receiving and shipping
  - (d) all the above
3. Activities which are undertaken before the final operation (Manufacturing/assembly) are termed as
  - (a) Downstream supply chain
  - (b) Upstream supply chain
  - (c) Pre-supply chain
  - (d) Post-supply chain

4. The activity related to supplier evaluation, selection, and purchasing the materials is known as
  - (a) Sourcing
  - (b) Outsourcing
  - (c) Contracting
  - (d) All the above
5. The functions that are not the core competencies of an organization are performed by the outsiders of the organization. This activity is known as
  - (a) sourcing
  - (b) outsourcing
  - (c) procurement
  - (d) all the above
6. The major benefit of single sourcing is
  - (a) lower administration cost
  - (b) consistency in quality
  - (c) speed in delivery
  - (d) all the above
7. The major benefit of multiple sourcing is
  - (a) low material cost
  - (b) volume flexibility
  - (c) continuity of supply
  - (d) all the above
8. Electronic data interchange helps in
  - (a) risk pooling
  - (b) reducing the bullwhip effect
  - (c) forecasting
  - (d) none of these
9. A supply chain is essentially a sequential link between
  - (a) customer and retailer
  - (b) supplier and manufacturer
  - (c) suppliers and customers
  - (d) supplier and retailer
10. Lead time and order cycle time are similar to
  - (a) customer service time
  - (b) order placement time
  - (c) replenishment time
  - (d) real-time service time
11. Risk pooling is useful only when
  - (a) the variation in demand in two locations is opposite in nature.
  - (b) the variation in demand in two locations is similar in nature.
  - (c) the variation in demand in two locations has no relationship.
  - (d) none of these.
12. Which of the following is a third-party logistics providers?
  - (a) Tata motors
  - (b) MUL
  - (c) Honda motors
  - (d) TCIL
13. Five basic transportation modes are
  - (a) air carrier, motor carrier, pipelines, railways and water carriers
  - (b) air carrier, motor carrier, trucks, canals and robotics
  - (c) air carrier, motor carrier, water routes, railways and trucks
  - (d) air carrier, motor carrier, pipeline, water routes and ocean vessels
14. Which of the following is the supply chain flow?
  - (a) materials flow
  - (b) information flow
  - (c) money flow
  - (d) all the above

15. Bullwhip effect is concerned with
- the price of share in the capital markets
  - overestimates the volume of supply at the supplier end with changes in production due to unforeseen in demand
  - underestimate the product demand resulting in exaggerated fluctuations
  - both (b) and (c)

## Answers

1. (d)      2. (d)      3. (b)      4. (a)      5. (b)      6. (d)      7. (d)      8. (b)      9. (c)  
10. (c)     11. (a)     12. (d)     13. (d)     14. (d)     15. (d)

## REVIEW QUESTIONS

- What do you mean by supply chain management? Explain upstream supply chain and downstream supply chain.
- Discuss the four fundamentals of supply chain in brief.
- What are the drivers of supply chain? Discuss the influences of these drivers.
- Write short notes on: (a) risk pooling and (b) bullwhip effect.
- Explain the factors influencing the bullwhip effect and the ways to minimize the bullwhip effect.
- Write the short notes on the basic IT tools used in the supply chain management.



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# CHAPTER ▶ 26

## Decision-Making

### 26.1 INTRODUCTION

Decision-making is a process of taking action in a particular environment. In the process of decision-making, a decision-maker makes a choice of action among the number of alternative actions available in a particular environment. This theory helps the decision-maker in identifying the alternative with the highest expected value (probability of obtaining a possible value). The decision-making process relies on the information available about the alternatives. The reliability of information in any decision situation can run the whole gamut from scientifically derived hard data to subjective interpretations and from certainty about decision outcomes (deterministic information) to uncertain outcomes represented by probabilities and fuzzy numbers. This diversity in the type and quality of information about a decision problem calls for methods and techniques that can assist in information processing.

A decision-making system consists of a set of goals and objectives, a system of priorities, an enumeration of the alternatives of feasible and viable actions, a projection of the consequences associated with different alternatives and a system of choice criteria by which the most preferred course of action is taken. It provides an analytical and systematic approach to depict the expected result of a situation when alternative managerial actions and outcomes are compared.

**Course of action and state of nature:** Course of action is the decision taken by the decision-makers. A state of nature is a future condition, which is not under the control of the decision-maker; and it may be linked to various courses of action. It can be probabilistic or conditional, such as a state of the economy, a weather condition, a political development, etc. The states of nature are usually not determined by the action of an individual or an organization. These are the result of an ‘act of God’ or the result of many situations pushing in various directions.

**Pay-off:** An outcome (numerical value) resulting from each possible combination of alternatives and states of nature is called *pay-off*. The pay-off values are always conditional values because of unknown states of nature. Pay-off is measured within a specified period, which is called the *decision horizon*. Pay-off can also be measured in terms of money market share, or other measures. Pay-offs considered in most decisions are monetary.

## 26.2 DECISION-MAKING ENVIRONMENTS

There are four types of decision-making environments: certainty, uncertainty, risk and conflict.

### 26.2.1 Decision-Making under Certainty

In this case, the decision-maker has the perfect information of the consequences of every decision choice with certainty. Obviously, an alternative should be selected that yields the largest return (pay-off) for the known future (state of nature). For example, the decision to either invest in public provident fund or invest in a Kisan Vikas Patra/life insurance policy/National Saving Certificate (NSC) is the one in which investors have complete information about the future (matured amount) because payment would be made when it is due.

### 26.2.2 Decision-Making under Uncertainty

In this case, the decision-maker cannot specify probabilities with which the various states of nature (futures) will occur. However, this is not the case of decision-making under ignorance because possible states of nature are known. For example, the probability that the share of a company will grow 10 times after 5 years is not known. Thus, if the decision-maker does not know with certainty which state of nature will occur, then he/she is said to be making a decision under uncertainty. The five commonly used criteria for decision-making under uncertainty are as follows:

1. Optimistic approach (Maximax).
2. Conservative/pessimistic approach (Maximin).
3. Minimax regret approach (Minimax regret).
4. Equally likely (Laplace criterion).
5. Criterion of realism with  $\alpha$  (Hurwicz criterion).

**Optimistic approach:** In this approach, a decision-maker has an optimistic view about the output. The decision with the largest possible pay-off is chosen. If the pay-off table was in terms of costs, the decision with the lowest cost or maximum profit would be chosen.

**Conservative/Pessimistic approach:** In this approach, a decision-maker has a pessimistic view about the outcome. For each decision, the minimum pay-off is listed and then the decision corresponding to the maximum of these minimum pay-offs is selected. Therefore, it is also known as maximin approach. The minimum possible pay-off is maximized. If the pay-off was in terms of costs, the maximum costs would be determined for each decision and then the decision corresponding to the minimum of these maximum costs is selected. Hence, the maximum possible cost is minimized.

**Minimax regret approach:** In this approach, an opportunity loss table is prepared. This is done by calculating the difference between each pay-off and the largest pay-off for each state of nature. Then, using this regret table, the maximum regret for each possible decision is listed. The decision chosen is the one corresponding to the minimum of the maximum regrets.

**Equally likely (Laplace) criterion:** Equally likely criterion, also called Laplace criterion, finds the decision alternative with the highest average pay-off. In this approach, first the average pay-off for every alternative is calculated. Then the alternative with maximum average pay-off is chosen.

**Criterion of realism with  $\alpha$  (Hurwicz criterion):** Often called weighted average, the criterion of realism (or Hurwicz) is a compromise between an optimistic and a pessimistic decision. In this method, first select the coefficient of realism  $\alpha$  with a value between 0 and 1. When  $\alpha$  is close to 1, the decision-maker is optimistic about future, and when  $\alpha$  is close to 0, the decision-maker is pessimistic about the future. For a specific course of action, there may be a number of natures of state and corresponding pay-offs. Under this approach, the resultant pay-off for that course of action is calculated as sum of product of the coefficient of realism and pay-off using the following equation:

$$\text{Pay-off} = \alpha \times (\text{Maximum pay-off}) + (1 - \alpha) \times (\text{Minimum pay-off})$$

Table 26.1 shows the format of the pay-off table.

**Table 26-1:** Pay-off table

	State of nature, $N_1$	State of nature, $N_2$
Decision, $S_1$	Pay-off, $P_{11}$	Pay-off, $P_{12}$
Decision, $S_2$	Pay-off, $P_{21}$	Pay-off, $P_{22}$

Table 26.1 shows the format of pay-off table. For the course of action  $S_1$ , there are two states of nature:  $N_1$  and  $N_2$  and the corresponding pay-offs are  $P_{11}$  and  $P_{12}$ . Similarly, for the course of action  $S_2$ , there are two states of nature:  $N_1$  and  $N_2$  and the corresponding pay-offs are  $P_{21}$  and  $P_{22}$ .

**Example 26.1:** A company XYZ is thinking about the three decision alternatives: introduction of a new product by replacing the existing product at a much higher price ( $S_1$ ) or effecting a moderate change in the composition of the existing product at a small increase in price ( $S_2$ ) or bringing a minor change in the composition of the existing product with a negligible increase in price ( $S_3$ ). The three possible states of nature or events are: high increase in sales ( $N_1$ ), no change in sales ( $N_2$ ) and decrease in sales ( $N_3$ ). The marketing department of the company has worked out the pay-offs in terms of yearly net profits for the strategies of each of the three events (expected sales). The profits (pay-offs) for different courses of action under various states of nature are shown in Table 26.2.

**Table 26-2:** Pay-off table

Strategy	State of nature		
	$N_1$	$N_2$	$N_3$
$S_1$	750,000	350,000	200,000
$S_2$	550,000	500,000	50,000
$S_3$	350,000	350,000	350,000

Which strategy should the company choose on the basis of (a) optimistic approach, (b) conservative approach, (c) minimax regret approach, (d) equally likely (Laplace) criterion and (e) Hurwicz criterion (use  $\alpha = 0.3$ )?

**Solution:**

- (a) Optimistic approach (Table 26.3)

**Table 26-3:** Pay-off table

<b>Strategy</b>	<b>State of nature</b>			<b>Maximum</b>
	<b>N<sub>1</sub></b>	<b>N<sub>2</sub></b>	<b>N<sub>3</sub></b>	
$S_1$	750,000	350,000	200,000	<b>750,000</b>
$S_2$	550,000	500,000	50,000	550,000
$S_3$	350,000	350,000	350,000	350,000

The maximum among maximum pay-offs is Rs 750,000. Thus strategy  $S_1$  should be chosen.

- (b) Conservative or pessimistic approach (Table 26.4)

**Table 26-4:** Pay-off table

<b>Strategy</b>	<b>State of nature</b>			<b>Minimum</b>
	<b>N<sub>1</sub></b>	<b>N<sub>2</sub></b>	<b>N<sub>3</sub></b>	
$S_1$	750,000	350,000	200,000	200,000
$S_2$	550,000	500,000	50,000	50,000
$S_3$	350,000	350,000	350,000	<b>350,000</b>

The maximum among minimum pay-offs is Rs 350,000. Thus, strategy  $S_3$  should be chosen.

- (c) Minimax regret approach (Table 26.5)

**Table 26-5:** Opportunity cost table

<b>Strategy</b>	<b>State of nature</b>			<b>Maximum</b>
	<b>N<sub>1</sub></b>	<b>N<sub>2</sub></b>	<b>N<sub>3</sub></b>	
$S_1$	750,000 – 750,000 = 0	500,000 – 350,000 = 150,000	350,000 – 200,000 = 150,000	<b>150,000</b>
	550,000 – 200,000 = 350,000	500,000 – 0 = 50,000	350,000 – 50,000 = 300,000	
$S_3$	750,000 – 350,000 = 400,000	500,000 – 350,000 = 150,000	350,000 – 350,000 = 0	<b>400,000</b>

The minimum among maximum opportunity costs is Rs 150,000. Thus, strategy  $S_1$  should be chosen.

- (d) Equally likely (Laplace) criterion (Table 26.6)

**Table 26-6:** Pay-off table

Strategy	State of nature			Average
	$N_1$	$N_2$	$N_3$	
$S_1$	750,000	350,000	200,000	$(750,000 + 350,000 + 200,000)/3 = 433,333$
$S_2$	550,000	500,000	50,000	$(550,000 + 500,000 + 50,000)/3 = 366,666$
$S_3$	350,000	350,000	350,000	$(350,000 + 350,000 + 350,000)/3 = 350,000$

Equally likely probability is  $1/3$  because there are three states of nature and probability of each is same. The maximum among average values is Rs 433,333. Thus strategy  $S_1$  should be chosen.

- (e) Hurwicz criterion (use  $\alpha = 0.3$ ) (Table 26.7)

**Table 26-7:** Pay-off table

Strategy	State of nature			Resultant pay-off
	$N_1$	$N_2$	$N_3$	
$S_1$	750,000	350,000	200,000	$0.3(750,000) + (1 - 0.3)200,000 = 365,000$
$S_2$	550,000	500,000	50,000	$0.3(550,000) + (1 - 0.3)50,000 = 200,000$
$S_3$	350,000	350,000	350,000	$0.3(350,000) + (1 - 0.3)350,000 = 350,000$

The maximum pay-off is Rs 365,000. Thus, strategy  $S_1$  should be selected.

**Example 26.2:** A fruit merchant, who supplies fruits to a retailer, desires to determine the optimal daily order size for the fruit. The retailer buys the fruit at the rate of Rs 50 per kg and sells at the rate of Rs 60 per kg. If the order size is more than the demand, the excess quantity can be sold at Rs 45 per kg using door-to-door selling; otherwise, the opportunity cost is Rs 8 per kg for unsatisfied portion of the demand. Based on the past experience, it is found that the demand varies from 40 kg to 120 kg in steps of 20 kg. The possible values of the order size are from 30 kg to 150 kg in steps of 30 kg. Determine the optimal order size using (a) optimistic approach, (b) conservative approach, (c) minimax regret approach, (d) equally likely (Laplace) criterion and (e) Hurwicz criterion (use  $\alpha = 0.4$ ).

#### **Solution:**

Purchase price of the fruit = Rs 50 per kg

Selling price to the retailer = Rs 60 per kg

Selling price for door-to-door selling = Rs 45 per kg

Profit in retail shop = Rs 60 – 50 = Rs 10 per kg

Profit in door-to-door selling = Rs 45 – 50 = –Rs 5 per kg

Opportunity cost not meeting the demand = Rs 8 per kg

Suppose  $D_j$  represents the demand for  $j = 1, 2, \dots, 5$  and  $Q_i$  represents the supply for  $i = 1, 2, \dots, 5$ ; the profits (pay-offs) can be calculate using the following formula for supply higher than the demand and demand higher than the supply, respectively:

Net profit

$$P = \begin{cases} D_j \times 10 - (Q_i - D_j) \times 5 & \text{if } Q_i > D_j \\ Q_i \times 10 - (D_j - Q_i) \times 8 & \text{if } D_j > Q_i \end{cases}$$

The calculation of opportunity cost is shown in Table 26.8 and the final pay-off table is shown in Table 26.9.

**Table 26-8:** Pay-off table

	State of nature (Demand, $D_j$ )					
	40 kg	60 kg	80 kg	100 kg	120 kg	
Strategy (Supply, $Q_i$ )	30 kg	$30 \times 10 - 10 \times 8 = 220$	$30 \times 10 - 30 \times 8 = 60$	$30 \times 10 - 50 \times 8 = -100$	$30 \times 10 - 70 \times 8 = -260$	$30 \times 10 - 90 \times 8 = -420$
	60 kg	$40 \times 10 - 20 \times 5 = 300$	$60 \times 10 - 0 \times 5 = 600$	$60 \times 10 - 20 \times 8 = 440$	$60 \times 10 - 40 \times 8 = 280$	$60 \times 10 - 60 \times 8 = 120$
	90 kg	$40 \times 10 - 50 \times 5 = 150$	$60 \times 10 - 30 \times 5 = 450$	$80 \times 10 - 10 \times 5 = 750$	$90 \times 10 - 10 \times 8 = 820$	$90 \times 10 - 30 \times 8 = 660$
	120 kg	$40 \times 10 - 80 \times 5 = 0$	$60 \times 10 - 60 \times 5 = 300$	$80 \times 10 - 40 \times 5 = 600$	$100 \times 10 - 20 \times 5 = 900$	$120 \times 10 - 0 \times 5 = 1200$
	150 kg	$40 \times 10 - 110 \times 5 = -150$	$60 \times 10 - 90 \times 5 = 150$	$80 \times 10 - 70 \times 5 = 450$	$100 \times 10 - 50 \times 5 = 750$	$120 \times 10 - 30 \times 5 = 1050$

**Table 26-9:** Final pay-off table

	State of nature (Demand, $D_j$ )					
	40 kg	60 kg	80 kg	100 kg	120 kg	
Strategy (Supply, $Q_i$ )	30 kg	220	60	-100	-260	-420
	60 kg	300	600	440	280	120
	90 kg	150	450	750	820	660
	120 kg	0	300	600	900	1200
	150 kg	-150	150	450	750	1050

- (a) Optimistic approach (Table 26.10)

**Table 26-10:** Pay-off table

Strategy (Supply, $Q_j$ )	State of nature (Demand, $D_j$ )					Maximum
	40 kg	60 kg	80 kg	100 kg	120 kg	
30 kg	220	60	-100	-260	-420	220
60 kg	300	600	440	280	120	600
90 kg	150	450	750	820	660	820
120 kg	0	300	600	900	1200	<b>1200</b>
150 kg	-150	150	450	750	1050	1050

The maximum among maximum pay-offs is Rs 1200. Thus, the order size should be 120 kg.

- (b) Conservative approach (Table 26.11)

**Table 26-11:** Pay-off table

Strategy (Supply, $Q_j$ )	State of nature (Demand, $D_j$ )					Minimum
	40 kg	60 kg	80 kg	100 kg	120 kg	
30 kg	220	60	-100	-260	-420	-420
60 kg	300	600	440	280	120	120
90 kg	150	450	750	820	660	<b>150</b>
120 kg	0	300	600	900	1200	0
150 kg	-150	150	450	750	1050	-150

The maximum among minimum pay-offs is Rs 150. Thus, the order size should be 90 kg.

- (c) Minimax regret approach (Table 26.12)

**Table 26-12:** Opportunity cost table

Strategy (Supply, $Q_j$ )	State of nature (Demand, $D_j$ )					Maximum
	40 kg	60 kg	80 kg	100 kg	120 kg	
30 kg	80	540	850	1160	1620	1620
60 kg	0	0	310	620	1080	1080
90 kg	150	150	0	80	540	540
120 kg	300	300	150	0	0	<b>300</b>
150 kg	450	450	300	150	150	450

The minimum among maximum opportunity costs is Rs 300. Thus, the order size should be 120 kg.

- (d) Equally likely (Laplace) criterion (Table 26.13)

**Table 26-13:** Pay-off table

Strategy (Supply, $Q_j$ )	State of nature (Demand, $D_j$ )					Average
	40 kg	60 kg	80 kg	100 kg	120 kg	
30 kg	220	60	-100	-260	-420	-100
60 kg	300	600	440	280	120	348
90 kg	150	450	750	820	660	566
120 kg	0	300	600	900	1200	<b>600</b>
150 kg	-150	150	450	750	1050	450

The maximum among average values is Rs 600. Thus, the order size should be 120 kg.

- (e) Hurwicz criterion (Use  $\alpha = 0.4$ ) (Table 26.14)

**Table 26-14:** Pay-off table

Strategy (Supply, $Q_j$ )	State of nature (Demand, $D_j$ )					$\text{Max. value} \times \alpha + \text{Min. value} \times (1-\alpha)$
	40 kg	60 kg	80 kg	100 kg	120 kg	
30 kg	220	60	-100	-260	-420	-164
60 kg	300	600	440	280	120	312
90 kg	150	450	750	820	660	418
120 kg	0	300	600	900	1200	<b>480</b>
150 kg	-150	150	450	750	1050	330

The maximum value is Rs 480. Thus, the order size should be 120 kg.

### 26.2.3 Decision-making under Risk

In this case, the decision-maker has less than complete knowledge of the consequence of every decision choice because it is not definitely known which outcome will occur. This means there is more than one state of nature and he makes an assumption of the probability with which each state of nature will occur.

Decision-making under risk is a probabilistic decision situation in which more than one state of nature exists and the decision-maker has sufficient information to assign probability values to the likely occurrence of each of these states. Knowing the probability distribution of the states of nature, the best decision is to select that course of action which has the largest expected pay-off value. The expected (average) pay-off of course of action is the sum of all possible pay-offs of the states of nature weighted by the probabilities of those pay-offs occurring.

**1. Expected monetary value (EMV):** The EMV for a given course of action is the weighted sum of possible pay-offs for states of nature. It is obtained by summing the pay-offs for each course of action multiplied by the probabilities associated with each state of nature. The expected (or mean) value is the long-run average value that would result if the decision were repeated a large number of times. Mathematically, EMV is expressed as follows:

$$\text{EMV}(d_i) = \sum_{j=1}^N (P_{ij} \times p_j)$$

where  $N$  is the number of possible states of nature;  $p_j$  is a probability of occurrence of the state of nature,  $N_j$ ;  $P_{ij}$  is the pay-off associated with the state of nature  $N_j$  and course of action  $S_i$ .

**Example 26.3:** An FMCG retailer purchases a perishable product at the rate of Rs 150 per kg daily and sells at the rate of Rs 200 per kg. The unsold product at the end of the day can be disposed off the next day at a salvage value of Rs 120 per kg. Past sales have ranged from 22 to 25 kg per day. Table 26.15 shows the record of sales for the past 100 days:

**Table 26-15:** Sales record

<b>Amount sold (kg)</b>	22	23	24	25
<b>Number of days</b>	30	40	20	10

Find the amount of the product (kg) the retailer should purchase to optimize the profit.

**Solution:**

Let  $S_i$  ( $i = 1, 2, \dots, 4$ ) be the possible course of action (the amount of the product purchased in kg) and  $N_j$  ( $j = 1, 2, \dots, 4$ ) be the possible state of nature (daily demand). Now

$$\text{Profit} = \text{Selling price} - \text{Cost} = \text{Rs } (200 - 150) = \text{Rs } 50$$

$$\text{Loss} = \text{Rs } (150 - 120) = \text{Rs } 30$$

$$\text{Conditional profit} = \text{Profit} \times \text{amount sold (kg)} - \text{Loss} \times \text{amount unsold (kg)}$$

$$= \begin{cases} 50S & \text{if } N \geq S \\ (200 - 150)N - 30(S - N) = 80N - 30S & \text{if } S > N \end{cases}$$

The pay-off table is shown in Table 26.16.

**Table 26-16:** Pay-off table (Conditional profit values)

	State of nature (Demand per day, $N_i$ )				Expected pay-off due to course of action = $\sum_{j=1}^4 \text{Prob } p_j \times \text{Payoff of}$ <b>course of action <math>i</math> (<math>p_{ij}</math>)</b>
Probability	0.3	0.4	0.2	0.1	
Course of action (Purchase per day, $S_j$ )	22	23	24	25	
22	1100	1100	1100	1100	1100
23	1070	1150	1150	1150	1126
24	1040	1120	1200	1200	1120
25	1010	1090	1170	1250	1098

EMV is 1126 because this is maximum profit among the profits from various course of action. Thus, the daily purchase should be 23 kg of the product.

**2. Expected opportunity loss (EOL):** The EOL approach is an alternative approach to minimize the EOL. It is also called expected value of regret. EOL is the difference between the highest profit (pay-off) for a state of nature and the actual profit obtained for the particular course of action taken. In other words, EOL is the amount of pay-off that is lost by not selecting the course of action that has the greatest pay-off for the state of nature that actually occurs. The course of action with minimum EOL is recommended.

The result of EOL will always be the same as that obtained by EMV criterion. Mathematically, EOL is expressed as follows:

$$\text{EOL} (\text{State of nature, } N_j) = \sum_{j=1}^N l_{ij} p_j$$

where  $l_{ij}$  is the opportunity loss due to state of nature ( $N_j$ ) and course of action ( $S_i$ ),  $p_j$  is the probability of occurrence of state of nature,  $N_j$ .

**Example 26.4:** Using the information given in Example 26.3, find the expected opportunity loss for each course of action and show the optimum course of action.

#### **Solution:**

The pay-off table 26.16 is reproduced here again as shown below:

Pay-off table: conditional profit values

State of nature (Demand per day, $N_j$ )					Expected pay-off due to course of action = $\sum_{j=1}^4 \text{Prob } p_j \times \text{Payoff of}$ course of action $i$ ( $p_{ij}$ )
<b>Probability</b>	0.3	0.4	0.2	0.1	
<b>Course of action (Purchase per day, <math>S_j</math>)</b>	22	23	24	25	
22	1100	1100	1100	1100	1100
23	1070	1150	1150	1150	1126
24	1040	1120	1200	1200	1120
25	1010	1090	1170	1250	1098

The opportunity loss can be calculated as the difference between maximum pay-off and individual pay-off for particular state of nature as shown in Table 26.17.

**Table 26-17:** Opportunity loss table

State of nature (Demand per day)					Expected pay-off due to course of action = $\sum_{j=1}^4 \text{Prob } p_j \times \text{Opportunity loss due}$ to course of action $i$ ( $I_{ij}$ )
<b>Probability</b>	0.3	0.4	0.2	0.1	
<b>Course of action (Purchase per day)</b>	22	23	24	25	
22	1100 – 1100 = 0	1150 – 1100 = 50	1200 – 1100 = 100	1250 – 1100 = 150	55
23	1100 – 1070 = 30	1150 – 1150 = 0	1200 – 1150 = 50	1250 – 1150 = 100	<b>29 (Optimal decision)</b>
24	1100 – 1040 = 60	1150 – 1120 = 30	1200 – 1200 = 0	1250 – 1200 = 50	35
25	1100 – 1010 = 90	1150 – 1090 = 60	1200 – 1170 = 30	1250 – 1250 = 0	57

The minimum opportunity loss is 29. Thus the daily purchase 22 kg of the product is optimum size.

3. **Expected value of perfect information:** In the case of decision-making under risk, each state of nature is associated with the probability of its occurrence. But, if the decision-maker can get *perfect information* about the occurrence of various states of nature, then he/she will be able to select a course of action that yields the desired pay-off for whatever state of nature that actually occurs.

EMV or EOL criterion helps the decision-maker select a particular course of action that optimizes the expected pay-off without any additional information. The *expected value of perfect information* (EVPI) is the maximum amount of money the decision-maker has to pay to get additional information about the occurrence of various states of nature before a decision has to be made. Mathematically, it is expressed as

$$\begin{aligned} EVPI &= (\text{Expected profit with perfect information, i.e EPPI}) \\ &\quad - (\text{Expected profit without perfect information, i.e. EMV}) \\ &= \sum_{j=1}^N p_j \max_j(P_{ij}) - EMV \end{aligned}$$

where  $P_{ij}$  is the best pay-off when course of action  $S_i$  is taken in the presence of state of nature  $N_j$ ;  $p_j$  is the probability of state of nature  $N_j$ ;  $EMV$  is the maximum EMV.

**Example 26.5:** Using the information given in Example 26.3, find the *EVPI*.

**Solution:**

$EMV$  from the solution of Example 26.3 is Rs 1126.

State of nature	Probability	Course of action	Profit from optimal course of action	Weighted profit
$N_1 = 22 \text{ kg}$	0.3	$S_1 = 22 \text{ kg}$	1100	330
$N_2 = 23 \text{ kg}$	0.4	$S_2 = 23 \text{ kg}$	1150	460
$N_3 = 24 \text{ kg}$	0.2	$S_3 = 24 \text{ kg}$	1200	240
$N_4 = 25 \text{ kg}$	0.1	$S_4 = 25 \text{ kg}$	1250	125
$\sum_{j=1}^N p_j \max_j(P_{ij})$				1155

Therefore

$$\begin{aligned} EVPI &= (\text{Expected profit with perfect information}) \\ &\quad - (\text{Expected profit without perfect information}) \\ &= \sum_{j=1}^N p_j \max_j(P_{ij}) - EMV \\ &= 1155 - 1126 \\ &= \text{Rs } 29 \end{aligned}$$

## 26.3 DECISION TREE ANALYSIS

Decision tree analysis requires construction of a diagram, which is known as a *decision tree*. A decision tree consists of *nodes*, *branches*, *probability estimates* and *pay-offs*.

There are two types of nodes: *decision nodes* (or *act nodes*) denoted by square ( $\square$ ) and *chance nodes* (*event nodes*) represented by circle ( $\circ$ ). Decision nodes indicate the places at which the decision-maker must select one action from the available *alternative courses of action*. The alternative courses of action are shown as *branches* or *arcs* emerging out of a decision node. Each course of action results in a *chance node*. The chance node indicates a point at which the decision-maker will discover the response to his/her decision.

*Branches* start from and connect various nodes. There are two types of branches: *decision branches* and *chance branches*. Each branch leading away from a decision node represents a course of action to be chosen at a decision point. A branch leading away from a chance node represents the state of nature of a set of chance events. Probabilities of the states of nature are written alongside respective chance branches. These probabilities are the likelihood that the chance outcome will assume the value assigned to a particular branch.

Any branch not followed by either a decision or chance node is called a *terminal branch*. A terminal branch can represent either a course of action or a chance outcome. The terminal points are mutually exclusive points, that is, exactly one course of action will be chosen for each terminal point. The *pay-off* can be positive (i.e. revenue or sales) or negative (i.e. expenditure or cost) and can be associated either with decision branches or chance branches.

If a node is a chance node, then the expected return is calculated as the sum of the products of the probabilities of the branches emanating from the chance node and their respective expected returns. If a node is a decision node, then the expected return is calculated for each of its branches and the highest return is selected. The procedure continues until the initial node is reached. The expected return for this node corresponds to the maximum expected return obtainable from the decision sequence.

**Example 26.6:** Considering the data given in Example 26.3, draw a decision tree and find the optimum course of action.

#### Solution:

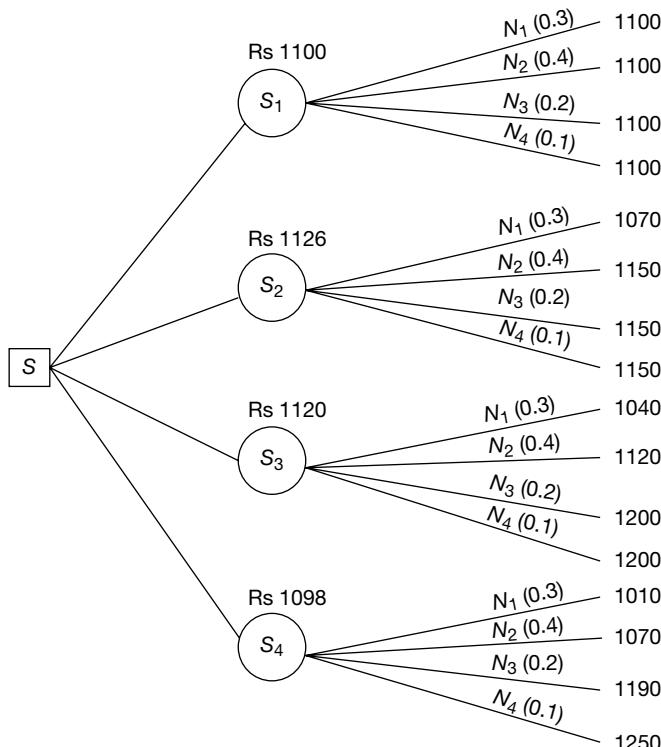
From Example 26.3 the pay-off Table 26.16 is reproduced as follows.

**Pay-off table:** conditional profit values

		States of nature (Demand per day)			
Probability		0.3	0.4	0.2	0.1
Course of action (Purchase per day)		$N_1 = 22 \text{ kg}$	$N_2 = 23 \text{ kg}$	$N_3 = 24 \text{ kg}$	$N_4 = 25 \text{ kg}$
$S_1 = 22 \text{ kg}$		1100	1100	1100	1100
$S_2 = 23 \text{ kg}$		1070	1150	1150	1150
$S_3 = 24 \text{ kg}$		1040	1120	1200	1200
$S_4 = 25 \text{ kg}$		1010	1090	1170	1250

In Figure 26.1, square S shows the decision node and circles  $S_1, S_2, S_3$  and  $S_4$  show the chance nodes.  $N_1, N_2, N_3$ , and  $N_4$  are four probability estimates their corresponding values are 0.3, 0.4, 0.2, and 0.1. The values shown right-hand side of Figure 26.1 is pay-offs.

$S_2$  (23 kg per day) is an optimum strategy to purchase the apple because sum of products of probability estimates and pay-offs for  $S_2$  is maximum, i.e. Rs 1126. The expected profits from  $S_1$ ,  $S_3$ , and  $S_4$  are Rs 1100, Rs 1120, and Rs 1098, respectively.



**Figure 26-1:** Decision tree



## SUMMARY

In this chapter, we have discussed about the decision-making process under risk and uncertainty. Various methods for decision-making under risk and under uncertainty have been discussed with the help of some numerical examples. In addition to this, a decision tree method to take decision is also discussed.

## MULTIPLE-CHOICE QUESTIONS

1. A decision-maker
  - (a) creates the alternative action
  - (b) makes a choice of action among the number of alternative actions
  - (c) is an authority to decide about a job as per his/her perception
  - (d) all the above
2. Course of action is
  - (a) the decision taken by the decision-makers
  - (b) the alternatives available
  - (c) not under the control of the decision-maker
  - (d) none of these
3. A state of nature is
  - (a) the nature of the decision-maker
  - (b) the nature of decision taken
  - (c) the future condition, which is not under control of decision-making
  - (d) none of these.
4. Pay-off is
  - (a) the output of the decision taken
  - (b) the optimal decision
  - (c) the results from the possible combination of alternatives and states of nature
  - (d) none of these
5. Decision regarding the investment in share market is an example of
  - (a) decision under certainty
  - (b) decision under risk
  - (c) decision under uncertainty
  - (d) none of these
6. Maximum among the minimum pay-off is selected under
  - (a) Optimistic approach
  - (b) Pessimistic approach
  - (c) Laplace criterion
  - (d) Hurwicz criterion
7. Minimum among the maximum pay-off is selected under
  - (a) Optimistic approach
  - (b) Minimax regret approach
  - (c) Laplace criterion
  - (d) Hurwicz criterion
8. Compromise between an optimistic and a pessimistic approach is used in
  - (a) Optimistic approach
  - (b) Minimax regret approach
  - (c) Laplace criterion
  - (d) Hurwicz criterion

9. Opportunity cost table is prepared by finding
  - (a) the difference between the largest pay-off and each pay-off
  - (b) the difference between the largest pay-off and each pay-off
  - (c) the difference between minimum and second minimum pay-off
  - (d) none of these
10. If the decision-maker has sufficient information to assign probability values to the likely occurrence of each of the states of nature, this is known as
  - (a) Decision-making under risk
  - (b) Decision-making under certainty
  - (c) Decision-making under uncertainty
  - (d) None of these
11. The expected value of perfect information (EVPI) is
  - (a) the maximum amount of money the decision-maker gains due to the additional information about the occurrence of various states of nature
  - (b) the maximum amount of money the decision-maker has to pay to get additional information about the occurrence of various states of nature
  - (c) the value of additional information in terms of its importance, about the occurrence of various states of nature
  - (d) none of these
12. Expected opportunity loss (EOL) is
  - (a) the difference between the highest profit for a state of nature and the actual profit obtained for the particular course of action taken
  - (b) the amount of pay-off that is lost by not selecting the course of action that has the greatest pay-off for the state of nature that actually occurs
  - (c) both (a) and (b)
  - (d) none of these
13. In a decision tree analysis, decision node is represented by
  - (a)  $\Delta$
  - (b)  $\circ$
  - (c)  $\square$
  - (d) D
14. Analytic hierarchy process (AHP) is used for
  - (a) Single criteria decision-making
  - (b) Multiple criteria decision-making
  - (c) Both (a) and (b)
  - (d) None of these
15. In a decision tree analysis, chance node is represented by
  - (a)  $\Delta$
  - (b)  $\circ$
  - (c)  $\square$
  - (d) D

## Answers

1. (b)    2. (a)    3. (c)    4. (c)    5. (c)    6. (b)    7. (b)    8. (d)    9. (a)  
 10. (a)   11. (b)   12. (c)   13. (c)   14. (b)   15. (b)

## REVIEW QUESTIONS

- Define the term ‘decision-making’. What are the different classifications of decision-making?
- What do you mean by course of action and states of nature?
- Explain the process of decision-making under uncertainty.
- Discuss the process of decision-making under risk.
- What do you mean by decision tree analysis? Discuss its use in the decision-making process.

## EXERCISES

- A company ABC is thinking about the three decision alternatives: introduction of a new product by replacing the existing product at a much higher price ( $S_1$ ) or effecting a moderate change in the composition of the existing product at a small increase in price ( $S_2$ ) or bringing a minor change in the composition of the existing product with a negligible increase in price ( $S_3$ ). The three possible states of nature or events are high increase in sales ( $N_1$ ), no change in sales ( $N_2$ ) and decrease in sales ( $N_3$ ). The marketing department of the company has worked out the pay-offs in terms of yearly net profits for the strategies of each of the three events (expected sales). The profits (pay-offs) for different courses of action under various states of nature are shown in Table 26.18.

**Table 26-18:** Pay-off table

Strategy	State of nature		
	$N_1$	$N_2$	$N_3$
$S_1$	900,000	500,000	350,000
$S_2$	700,000	650,000	200,000
$S_3$	500,000	500,000	500,000

- Which strategy should the company choose on the basis of (a) optimistic approach, (b) conservative approach, (c) minimax regret approach, (d) equally likely (Laplace) criterion and (e) Hurwicz criterion (use  $\alpha = 0.4$ )?
- A fruit merchant, who supplies bananas to a hawker, desires to determine the optimal daily order size for the bananas. The hawker buys the bananas at the rate of Rs 50 per dozen and sells at the rate of Rs 80 per dozen. If the order size is more than the demand, the excess quantity can be sold at Rs 30 per dozen using door-to-door selling; otherwise, the opportunity cost is Rs 10 per dozen for unsatisfied portion of the demand. Based on the past experience, it is found that the demand varies from 20 dozen to 100 dozen in steps of 20 dozen. The possible values of the order size are from 30 dozen to 120 dozen in steps of 30 dozen. Determine the optimal order size using (a) optimistic approach, (b) conservative approach, (c) minimax regret approach, (d) equally likely (Laplace) criterion and (e) Hurwicz criterion (use  $\alpha = 0.4$ ).

- 
3. In Exercise Problem no. 3, the demand is probabilistic. The probability of the demand 20, 40, 60, 80, and 100 dozen are 0.2, 0.3, 0.1, 0.2, and 0.2, respectively. Find the amount of bananas the hawker should purchase to optimize the profit using (a) expected monetary value, (b) expected opportunity loss, and (c) expected value of perfect information.



## REFERENCES AND FURTHER READINGS

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# Appendix 1

## STANDARD NORMAL PROBABILITIES

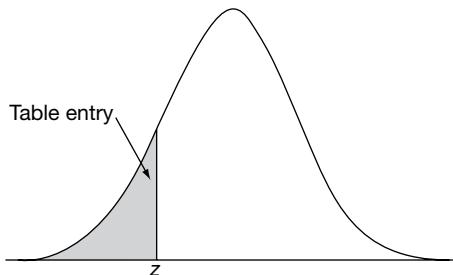


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

<b><math>z</math></b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985

<b><i>z</i></b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

## STANDARD NORMAL PROBABILITIES

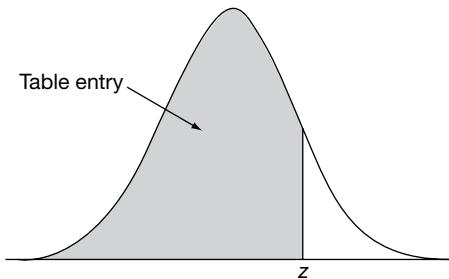


Table entry for  $z$  is the area under the standard normal curve to the left of  $z$ .

<b><i>z</i></b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.03</b>	<b>0.04</b>	<b>0.05</b>	<b>0.06</b>	<b>0.07</b>	<b>0.08</b>	<b>0.09</b>
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015



# Appendix 2

**Gamma Function Table for ( $1 \leq p \leq 2$ )**

$p$	$\Gamma(p)$	$p$	$\Gamma(p)$	$p$	$\Gamma(p)$	$p$	$\Gamma(p)$
1.00000	1.00000	1.25000	0.90640	1.50000	0.88623	1.75000	0.91906
1.01000	0.99433	1.26000	0.90440	1.51000	0.88659	1.76000	0.92137
1.02000	0.98884	1.27000	0.90250	1.52000	0.88704	1.77000	0.92376
1.03000	0.98355	1.28000	0.90072	1.53000	0.88757	1.78000	0.92623
1.04000	0.97844	1.29000	0.89904	1.54000	0.88818	1.79000	0.92877
1.05000	0.97350	1.30000	0.89747	1.55000	0.88887	1.80000	0.93138
1.06000	0.96874	1.31000	0.89600	1.56000	0.88964	1.81000	0.93408
1.07000	0.96415	1.32000	0.89464	1.57000	0.89049	1.82000	0.93685
1.08000	0.95973	1.33000	0.89338	1.58000	0.89142	1.83000	0.93969
1.09000	0.95546	1.34000	0.89222	1.59000	0.89243	1.84000	0.94260
1.10000	0.95135	1.35000	0.89115	1.60000	0.89352	1.85000	0.94561
1.11000	0.94740	1.36000	0.89018	1.61000	0.89468	1.86000	0.94869
1.12000	0.94359	1.37000	0.88931	1.62000	0.89592	1.87000	0.95184
1.13000	0.93993	1.38000	0.88854	1.63000	0.89724	1.88000	0.95507
1.14000	0.93642	1.39000	0.88785	1.64000	0.89864	1.89000	0.95838
1.15000	0.93304	1.40000	0.88726	1.65000	0.90012	1.90000	0.96177
1.16000	0.92980	1.41000	0.88676	1.66000	0.90167	1.91000	0.96520
1.17000	0.92670	1.42000	0.88636	1.67000	0.90330	1.92000	0.96877
1.18000	0.92373	1.43000	0.88604	1.68000	0.90500	1.93000	0.97240
1.19000	0.92089	1.44000	0.88581	1.69000	0.90678	1.94000	0.97610
1.20000	0.91817	1.45000	0.88566	1.70000	0.90864	1.95000	0.97988
1.21000	0.91558	1.46000	0.88560	1.71000	0.91057	1.96000	0.98374
1.22000	0.91311	1.47000	0.88563	1.72000	0.91258	1.97000	0.98768
1.23000	0.91075	1.48000	0.88575	1.73000	0.91467	1.98000	0.99171
1.24000	0.90852	1.49000	0.88595	1.74000	0.91683	1.99000	0.99581
1.25000	0.90640	1.50000	0.88623	1.75000	0.91906	2.00000	1.00000

Properties of gamma function to calculate the higher values:

$$\Gamma(1 + n) = n\Gamma(n); (1 + n) > p$$

# Appendix 3

Control chart factors										
<i>n</i>	<i>A</i>	<i>A2</i>	<i>A3</i>	<i>c4</i>	<i>B3</i>	<i>B4</i>	<i>B5</i>	<i>B6</i>	<i>d2</i>	<i>1/d2</i>
2	2.121	1.880	2.659	0.7979	0.000	3.267	0.000	2.606	1.128	0.8862
3	1.732	1.023	1.954	0.8862	0.000	2.568	0.000	2.276	1.693	0.5908
4	1.500	0.729	1.628	0.9213	0.000	2.266	0.000	2.088	2.059	0.4857
5	1.342	0.577	1.427	0.9400	0.000	2.089	0.000	1.964	2.326	0.4299
6	1.225	0.483	1.287	0.9515	0.030	1.970	0.029	1.874	2.534	0.3946
7	1.134	0.419	1.182	0.9594	0.118	1.882	0.113	1.806	2.704	0.3698
8	1.061	0.373	1.099	0.9650	0.185	1.815	0.179	1.751	2.847	0.3512
9	1.000	0.337	1.032	0.9693	0.239	1.761	0.232	1.707	2.970	0.3367
10	0.949	0.308	0.975	0.9727	0.284	1.716	0.276	1.669	3.078	0.3249
11	0.905	0.285	0.927	0.9754	0.321	1.679	0.313	1.637	3.173	0.3152
12	0.866	0.266	0.886	0.9776	0.354	1.646	0.346	1.610	3.258	0.3069
13	0.832	0.249	0.850	0.9794	0.382	1.618	0.374	1.585	3.336	0.2998
14	0.802	0.235	0.817	0.9810	0.406	1.594	0.399	1.563	3.407	0.2935
15	0.775	0.223	0.789	0.9823	0.428	1.572	0.421	1.544	3.472	0.2880
16	0.750	0.212	0.763	0.9835	0.448	1.552	0.440	1.526	3.532	0.2831
17	0.728	0.203	0.739	0.9845	0.466	1.534	0.458	1.511	3.588	0.2787
18	0.707	0.194	0.718	0.9854	0.482	1.518	0.475	1.496	3.640	0.2747
19	0.688	0.187	0.698	0.9862	0.497	1.503	0.490	1.483	3.689	0.2711
20	0.671	0.180	0.680	0.9869	0.510	1.490	0.504	1.470	3.735	0.2677
21	0.655	0.173	0.663	0.9876	0.523	1.477	0.516	1.459	3.778	0.2647
22	0.640	0.167	0.647	0.9882	0.534	1.466	0.528	1.448	3.819	0.2618
23	0.626	0.162	0.633	0.9887	0.545	1.455	0.539	1.438	3.858	0.2592
24	0.612	0.157	0.619	0.9892	0.555	1.445	0.549	1.429	3.895	0.2567
25	0.600	0.153	0.606	0.9896	0.565	1.435	0.559	1.420	3.931	0.2544

# Appendix 4

**THE POISSON DISTRIBUTION  $P(c) = (np_0/c!)e^{-np_0}$**

(Cumulative Values are in Parentheses)

c	$np_0$				
	0.1	0.2	0.3	0.4	0.5
0	0.905 (0.905)	0.819 (0.819)	0.741 (0.741)	0.670 (0.670)	0.607 (0.607)
1	0.091 (0.996)	0.164 (0.983)	0.222 (0.963)	0.268 (0.938)	0.303 (0.910)
2	0.004 (1.000)	0.016 (0.999)	0.033 (0.996)	0.054 (0.992)	0.076 (0.986)
3		0.010 (1.000)	0.004 (1.000)	0.007 (0.999)	0.013 (0.999)
4				0.001 (1.000)	0.001 (1.000)

c	$np_0$				
	0.6	0.7	0.8	0.9	1.0
0	0.549 (0.549)	0.497 (0.497)	0.449 (0.449)	0.406 (0.406)	0.368 (0.368)
1	0.329 (0.878)	0.349 (0.845)	0.359 (0.808)	0.366 (0.772)	0.368 (0.736)
2	0.099 (0.977)	0.122 (0.967)	0.144 (0.952)	0.166 (0.938)	0.184 (0.920)
3	0.020 (0.997)	0.028 (0.995)	0.039 (0.991)	0.049 (0.987)	0.061 (0.981)
4	0.003 (1.000)	0.005 (1.000)	0.008 (0.999)	0.011 (0.998)	0.016 (0.997)
5			0.001 (1.000)	0.002 (1.000)	0.003 (1.000)

c	$np_0$				
	1.1	1.2	1.3	1.4	1.5
0	0.333 (0.333)	0.301 (0.301)	0.273 (0.273)	0.247 (0.247)	0.223 (0.223)
1	0.366 (0.693)	0.361 (0.662)	0.354 (0.627)	0.345 (0.592)	0.335 (0.558)
2	0.201 (0.900)	0.217 (0.879)	0.230 (0.857)	0.242 (0.834)	0.251 (0.809)
3	0.074 (0.974)	0.087 (0.966)	0.100 (0.957)	0.113 (0.947)	0.126 (0.935)
4	0.021 (0.995)	0.026 (0.992)	0.032 (0.989)	0.039 (0.986)	0.047 (0.982)
5	0.004 (0.999)	0.007 (0.999)	0.009 (0.998)	0.011 (0.997)	0.014 (0.996)
6	0.001 (1.000)	0.001 (1.000)	0.002 (1.000)	0.003 (1.000)	0.004 (1.000)

c	$np_0$				
	1.6	1.7	1.8	1.9	2.0
0	0.202 (0.202)	0.183 (0.183)	0.165 (0.165)	0.150 (0.150)	0.135 (0.135)
1	0.323 (0.525)	0.311 (0.494)	0.298 (0.463)	0.284 (0.434)	0.271 (0.406)
2	0.258 (1.000)	0.264 (0.758)	0.268 (0.731)	0.270 (0.704)	0.271 (0.677)
3	0.138 (0.999)	0.149 (0.907)	0.161 (0.892)	0.171 (0.875)	0.180 (0.857)
4	0.055 (0.994)	0.064 (0.971)	0.072 (0.964)	0.081 (0.956)	0.090 (0.947)
5	0.018 (0.976)	0.022 (0.993)	0.026 (0.990)	0.031 (0.987)	0.036 (0.983)
6	0.005 (0.921)	0.006 (0.999)	0.008 (0.998)	0.010 (0.997)	0.012 (0.995)
7	0.001 (0.783)	0.001 (1.000)	0.002 (1.000)	0.003 (1.000)	0.004 (0.999)
8					0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>2.1</b>	<b>2.2</b>	<b>2.3</b>	<b>2.4</b>	<b>2.5</b>
0	0.123 (0.123)	0.111 (0.111)	0.100 (0.100)	0.091 (0.091)	0.082 (0.082)
1	0.257 (0.380)	0.244 (0.355)	0.231 (0.331)	0.218 (0.309)	0.205 (0.287)
2	0.270 (0.650)	0.268 (0.623)	0.265 (0.596)	0.261 (0.870)	0.256 (0.543)
3	0.189 (0.839)	0.197 (0.820)	0.203 (0.799)	0.203 (0.779)	0.214 (0.757)
4	0.099 (0.938)	0.108 (0.928)	0.117 (0.916)	0.125 (0.904)	0.134 (0.891)
5	0.042 (0.980)	0.048 (0.976)	0.054 (0.970)	0.060 (0.064)	0.067 (0.953)
6	0.015 (0.995)	0.017 (0.993)	0.021 (0.991)	0.024 (0.985)	0.028 (0.986)
7	0.004 (0.999)	0.005 (0.998)	0.007 (0.998)	0.008 (0.995)	0.010 (0.396)
8	0.001 (1.000)	0.002 (1.000)	0.002 (1.000)	0.003 (0.995)	0.003 (0.999)
9				0.001 (1.000)	0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>2.6</b>	<b>2.7</b>	<b>2.8</b>	<b>2.9</b>	<b>3.0</b>
0	0.074 (0.074)	0.067 (0.067)	0.061 (0.061)	0.055 (0.055)	0.050 (0.050)
1	0.193 (0.267)	0.182 (0.249)	0.170 (0.231)	0.160 (0.215)	0.149 (0.199)
2	0.251 (0.518)	0.245 (0.494)	0.238 (0.469)	0.231 (0.446)	0.224 (0.423)
3	0.213 (0.736)	0.221 (0.715)	0.223 (0.692)	0.224 (0.670)	0.224 (0.647)
4	0.141 (0.877)	0.149 (0.864)	0.156 (0.848)	0.162 (0.832)	0.168 (0.815)
5	0.074 (0.951)	0.080 (0.944)	0.087 (0.935)	0.094 (0.926)	0.101 (0.916)
6	0.032 (0.983)	0.036 (0.980)	0.041 (0.976)	0.045 (0.972)	0.050 (0.966)
7	0.012 (0.995)	0.014 (0.994)	0.016 (0.992)	0.019 (0.990)	0.022 (0.988)
8	0.004 (0.999)	0.005 (0.959)	0.006 (0.598)	0.007 (0.997)	0.008 (0.996)
9	0.001 (1.000)	0.001 (1.000)	0.002 (1.000)	0.002 (0.999)	0.003 (0.939)
10				0.001 (1.000)	0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>3.1</b>	<b>3.2</b>	<b>3.3</b>	<b>3.4</b>	<b>3.5</b>
0	0.045 (0.045)	0.041 (0.041)	0.037 (0.037)	0.033 (0.033)	0.030 (0.030)
1	0.140 (0.185)	0.130 (0.171)	0.122 (0.159)	0.113 (0.146)	0.106 (0.136)
2	0.216 (0.401)	0.209 (0.380)	0.201 (0.360)	0.193 (0.339)	0.185 (0.321)
3	0.224 (0.625)	0.223 (0.603)	0.222 (0.582)	0.219 (0.558)	0.216 (0.537)
4	0.173 (0.798)	0.178 (0.781)	0.182 (0.764)	0.186 (0.744)	0.189 (0.726)
5	0.107 (0.905)	0.114 (0.895)	0.120 (0.884)	0.126 (0.870)	0.132 (0.858)
6	0.056 (0.961)	0.061 (0.956)	0.066 (0.950)	0.071 (0.941)	0.077 (0.935)
7	0.025 (0.986)	0.028 (0.984)	0.031 (0.981)	0.035 (0.976)	0.038 (0.973)
8	0.010 (0.996)	0.011 (0.995)	0.012 (0.993)	0.015 (0.991)	0.017 (0.990)
9	0.003 (0.999)	0.004 (0.999)	0.005 (0.998)	0.006 (0.997)	0.007 (0.597)
10	0.001 (1.000)	0.001 (1.000)	0.002 (1.000)	0.002 (0.999)	0.002 (0.999)
11				0.001 (1.000)	0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>3.6</b>	<b>3.7</b>	<b>3.8</b>	<b>3.9</b>	<b>4.0</b>
0	0.027 (0.027)	0.025 (0.025)	0.022 (0.022)	0.020 (0.020)	0.018 (0.018)
1	0.098 (0.125)	0.091 (0.116)	0.085 (0.107)	0.079 (0.099)	0.073 (0.091)
2	0.177 (0.302)	0.169 (0.285)	0.161 (0.268)	0.154 (0.253)	0.147 (0.238)
3	0.213 (0.515)	0.209 (0.494)	0.205 (0.473)	0.200 (0.453)	0.195 (0.433)
4	0.191 (0.706)	0.193 (0.687)	0.194 (0.667)	0.195 (0.648)	0.195 (0.628)
5	0.138 (0.844)	0.143 (0.830)	0.148 (0.815)	0.152 (0.800)	0.157 (0.785)
6	0.083 (0.927)	0.088 (0.918)	0.094 (0.909)	0.099 (0.899)	0.104 (0.889)
7	0.042 (0.969)	0.047 (0.965)	0.051 (0.960)	0.055 (0.954)	0.060 (0.949)
8	0.019 (0.988)	0.022 (0.987)	0.024 (0.984)	0.027 (0.981)	0.030 (0.979)
9	0.008 (0.996)	0.009 (0.996)	0.010 (0.994)	0.012 (0.993)	0.013 (0.992)
10	0.003 (0.999)	0.003 (0.999)	0.004 (0.998)	0.004 (0.997)	0.005 (0.997)
11	0.001 (1.000)	0.001 (1.000)	0.001 (0.999)	0.002 (0.999)	0.002 (0.999)
12			0.001 (1.000)	0.001 (1.000)	0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>4.1</b>	<b>4.2</b>	<b>4.3</b>	<b>4.4</b>	<b>4.5</b>
0	0.017 (0.017)	0.015 (0.015)	0.014 (0.014)	0.012 (0.012)	0.011 (0.011)
1	0.068 (0.085)	0.063 (0.078)	0.058 (0.072)	0.054 (0.066)	0.050 (0.061)
2	0.139 (0.224)	0.132 (0.210)	0.126 (0.198)	0.119 (0.185)	0.113 (0.174)
3	0.190 (0.414)	0.185 (0.395)	0.180 (0.378)	0.174 (0.359)	0.169 (0.343)
4	0.195 (0.609)	0.195 (0.590)	0.193 (0.571)	0.192 (0.551)	0.190 (0.533)
5	0.160 (0.769)	0.163 (0.753)	0.166 (0.737)	0.169 (0.720)	0.171 (0.704)
6	0.110 (0.879)	0.114 (0.867)	0.119 (0.856)	0.124 (0.844)	0.128 (0.832)
7	0.064 (0.943)	0.069 (0.936)	0.073 (0.929)	0.078 (0.922)	0.082 (0.914)
8	0.033 (0.976)	0.036 (0.972)	0.040 (0.969)	0.043 (0.965)	0.046 (0.960)
9	0.015 (0.991)	0.017 (0.989)	0.019 (0.988)	0.021 (0.986)	0.023 (0.983)
10	0.006 (0.997)	0.007 (0.996)	0.008 (0.996)	0.009 (0.995)	0.011 (0.994)
11	0.002 (0.999)	0.003 (0.999)	0.003 (0.999)	0.004 (0.999)	0.004 (0.998)
12	0.001 (1.000)	0.001 (1.000)	0.001 (1.000)	0.001 (1.000)	0.001 (0.999)
13					0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>4.6</b>	<b>4.7</b>	<b>4.8</b>	<b>4.9</b>	<b>5.0</b>
0	0.010 (0.010)	0.009 (0.009)	0.008 (0.008)	0.008 (0.008)	0.007 (0.007)
1	0.046 (0.056)	0.043 (0.052)	0.039 (0.047)	0.037 (0.045)	0.034 (0.041)
2	0.106 (0.162)	0.101 (0.153)	0.095 (0.142)	0.090 (0.135)	0.084 (0.125)
3	0.163 (0.325)	0.157 (0.310)	0.152 (0.294)	0.146 (0.281)	0.140 (0.265)
4	0.188 (0.513)	0.185 (0.495)	0.182 (0.476)	0.179 (0.460)	0.176 (0.441)
5	0.172 (0.685)	0.174 (0.669)	0.175 (0.651)	0.175 (0.635)	0.176 (0.617)

<b>c</b>	<b><math>np_0</math></b>				
	<b>4.6</b>	<b>4.7</b>	<b>4.8</b>	<b>4.9</b>	<b>5.0</b>
6	0.132 (0.817)	0.136 (0.805)	0.140 (0.791)	0.143 (0.778)	0.146 (0.763)
7	0.087 (0.904)	0.091 (0.896)	0.096 (0.887)	0.100 (0.878)	0.105 (0.868)
8	0.050 (0.954)	0.054 (0.950)	0.058 (0.945)	0.061 (0.939)	0.065 (0.933)
9	0.026 (0.980)	0.028 (0.978)	0.031 (0.976)	0.034 (0.973)	0.036 (0.969)
10	0.012 (0.992)	0.013 (0.991)	0.015 (0.991)	0.016 (0.989)	0.018 (0.987)
11	0.005 (0.997)	0.006 (0.927)	0.006 (0.997)	0.007 (0.996)	0.008 (0.995)
12	0.002 (0.999)	0.002 (0.929)	0.002 (0.999)	0.003 (0.999)	0.003 (0.928)
13	0.001 (1.000)	0.001 (1.000)	0.001 (1.000)	0.001 (1.000)	0.001 (0.999)
14					0.001 (1.000)
<b>c</b>	<b><math>np_0</math></b>				
	<b>6.0</b>	<b>7.0</b>	<b>8.0</b>	<b>9.0</b>	<b>10.0</b>
0	0.002 (0.002)	0.001 (0.001)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
1	0.015 (0.017)	0.006 (0.007)	0.003 (0.003)	0.001 (0.001)	0.000 (0.000)
2	0.045 (0.062)	0.022 (0.022)	0.011 (0.014)	0.005 (0.006)	0.002 (0.002)
3	0.089 (0.151)	0.052 (0.081)	0.029 (0.043)	0.015 (0.021)	0.007 (0.009)
4	0.134 (0.285)	0.091 (0.172)	0.057 (0.100)	0.034 (0.055)	0.019 (0.028)
5	0.161 (0.446)	0.128 (0.300)	0.092 (0.192)	0.051 (0.116)	0.038 (0.066)
6	0.161 (0.607)	0.149 (0.449)	0.122 (0.314)	0.091 (0.207)	0.063 (0.129)
7	0.138 (0.745)	0.149 (0.598)	0.140 (0.454)	0.117 (0.324)	0.090 (0.219)
8	0.103 (0.848)	0.131 (0.729)	0.140 (0.594)	0.132 (0.456)	0.113 (0.332)
9	0.069 (0.917)	0.102 (0.831)	0.124 (0.718)	0.132 (0.588)	0.125 (0.457)
10	0.041 (0.958)	0.071 (0.922)	0.099 (0.817)	0.119 (0.707)	0.125 (0.582)
11	0.023 (0.981)	0.045 (0.947)	0.072 (0.889)	0.097 (0.804)	0.114 (0.696)
12	0.011 (0.992)	0.026 (0.973)	0.048 (0.937)	0.073 (0.877)	0.095 (0.791)
13	0.005 (0.997)	0.014 (0.987)	0.030 (0.967)	0.050 (0.927)	0.073 (0.864)
14	0.002 (0.999)	0.007 (0.994)	0.017 (0.984)	0.032 (0.959)	0.052 (0.918)
15	0.001 (1.000)	0.002 (0.997)	0.009 (0.993)	0.019 (0.978)	0.035 (0.951)
16		0.002 (0.999)	0.004 (0.997)	0.011 (0.989)	0.022 (0.973)
17		0.001 (1.000)	0.002 (0.999)	0.006 (0.995)	0.013 (0.986)
18			0.001 (1.000)	0.003 (0.998)	0.007 (0.993)
19				0.001 (0.999)	0.004 (0.997)
20				0.001 (1.000)	0.002 (0.999)
21					0.001 (1.000)

<b>c</b>	<b><math>np_0</math></b>				
	<b>11.0</b>	<b>12.0</b>	<b>13.0</b>	<b>14.0</b>	<b>15.0</b>
0	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
1	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
2	0.001 (0.001)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
3	0.004 (0.005)	0.002 (0.002)	0.001 (0.001)	0.000 (0.000)	0.000 (0.000)
4	0.010 (0.015)	0.005 (0.007)	0.003 (0.004)	0.001 (0.001)	0.001 (0.001)
5	0.022 (0.037)	0.013 (0.020)	0.007 (0.011)	0.004 (0.005)	0.002 (0.003)
6	0.041 (0.078)	0.025 (0.045)	0.015 (0.026)	0.009 (0.014)	0.005 (0.008)
7	0.065 (0.143)	0.044 (0.089)	0.028 (0.054)	0.017 (0.031)	0.010 (0.018)
8	0.089 (0.232)	0.066 (0.155)	0.046 (0.100)	0.031 (0.062)	0.019 (0.037)
9	0.109 (0.341)	0.087 (0.242)	0.066 (0.166)	0.047 (0.109)	0.032 (0.069)
10	0.119 (0.460)	0.105 (0.347)	0.086 (0.252)	0.066 (0.175)	0.049 (0.118)
11	0.119 (0.579)	0.114 (0.461)	0.101 (0.353)	0.084 (0.259)	0.066 (0.184)
12	0.109 (0.688)	0.114 (0.575)	0.110 (0.463)	0.099 (0.358)	0.083 (0.267)
13	0.093 (0.781)	0.106 (0.681)	0.110 (0.573)	0.106 (0.464)	0.096 (0.363)
14	0.073 (0.854)	0.091 (0.772)	0.102 (0.675)	0.106 (0.570)	0.102 (0.465)
15	0.053 (0.907)	0.072 (0.844)	0.088 (0.763)	0.099 (0.669)	0.102 (0.567)
16	0.037 (0.944)	0.054 (0.898)	0.072 (0.835)	0.087 (0.756)	0.096 (0.663)
17	0.024 (0.968)	0.038 (0.936)	0.055 (0.890)	0.071 (0.827)	0.085 (0.748)
18	0.015 (0.983)	0.026 (0.962)	0.040 (0.930)	0.056 (0.883)	0.071 (0.819)
19	0.008 (0.991)	0.016 (0.978)	0.027 (0.957)	0.041 (0.924)	0.056 (0.875)
20	0.005 (0.996)	0.010 (0.988)	0.018 (0.975)	0.029 (0.953)	0.042 (0.917)
21	0.002 (0.998)	0.006 (0.994)	0.011 (0.986)	0.019 (0.972)	0.030 (0.947)
22	0.001 (0.999)	0.003 (0.997)	0.006 (0.992)	0.012 (0.984)	0.020 (0.967)
23	0.001 (1.000)	0.002 (0.999)	0.004 (0.996)	0.007 (0.991)	0.013 (0.980)
24		0.001 (1.000)	0.002 (0.998)	0.004 (0.995)	0.008 (0.988)
25			0.001 (0.999)	0.003 (0.998)	0.005 (0.993)
26			0.001 (1.000)	0.001 (0.999)	0.003 (0.996)
27				0.001 (1.000)	0.002 (0.998)
28					0.001 (0.999)
29					0.001 (1.000)

# Appendix 5

## 'np' VALUE FOR CORRESPONDING 'C' VALUE

<b>c</b>	$P_a = 0.99$ ( $\alpha = 0.01$ )	$P_a = 0.95$ ( $\alpha = 0.05$ )	$P_a = 0.90$ ( $\alpha = 0.10$ )	$P_a = 0.10$ ( $\beta = 0.10$ )	$P_a = 0.05$ ( $\beta = 0.05$ )	$P_a = 0.01$ ( $\beta = 0.01$ )	Ratio of $p_{0.10}/p_{0.05}$
0	0.010	0.051	0.105	2.303	2.996	4.605	44.890
1	0.149	0.355	0.532	3.890	4.744	6.638	10.946
2	0.436	0.818	1.102	5.322	6.296	8.406	6.509
3	0.823	1.366	1.745	6.681	7.754	10.045	4.890
4	1.279	1.970	2.433	7.994	9.154	11.605	4.057
5	1.785	2.613	3.152	9.275	10.513	13.108	3.549
6	2.330	3.286	3.895	10.532	11.842	14.571	3.206
7	2.906	3.981	4.656	11.771	13.148	16.000	2.957
8	3.507	4.695	5.432	12.995	14.434	17.403	2.768
9	4.130	5.426	6.221	14.206	15.705	18.783	2.618
10	4.771	6.169	7.021	15.407	16.962	20.145	2.497
11	5.428	6.924	7.829	16.598	18.208	21.490	2.397
12	6.099	7.690	8.646	17.782	19.442	22.821	2.312
13	6.782	8.464	9.470	18.958	20.668	24.139	2.240
14	7.477	9.246	10.300	20.128	21.886	25.446	2.177
15	8.181	10.035	11.135	21.292	23.098	26.743	2.122

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