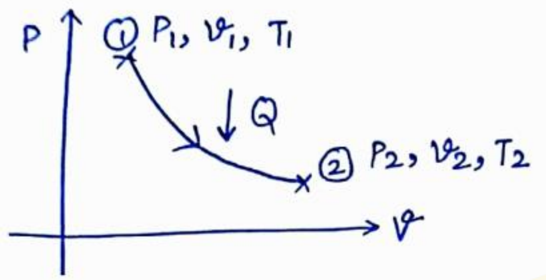
Heat and Mass Transfer

Heat → H.M.T

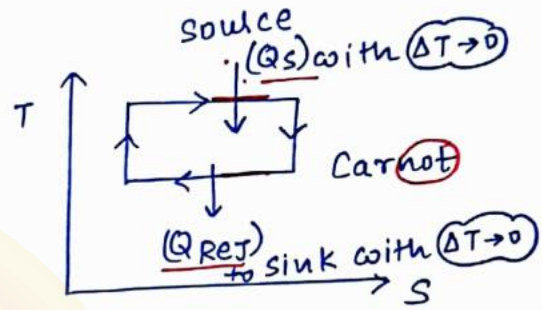
Heat → T.D. → Q in Joule

H.M.T → Q in $\frac{\text{Joule}}{\text{sec}}$ = watt → Rate



Rate → H.M.T.

Not T.D → only concern about final and initial tempr or amount of heat.



The main difference b/w Thermodynamic analysis and heat transfer analysis of a problem is that in thermodynamics, we deal with systems in equilibrium that is to bring a system from one equilibrium state to another equilibrium state, how much heat is required is the main criteria in thermodynamic analysis. But in heat transfer analysis we evaluate at what rate this change of state occurs by calculating rate of heat transfer in Joule/sec or watt.

MODES OF HEAT TRANSFER :-

① Conduction.

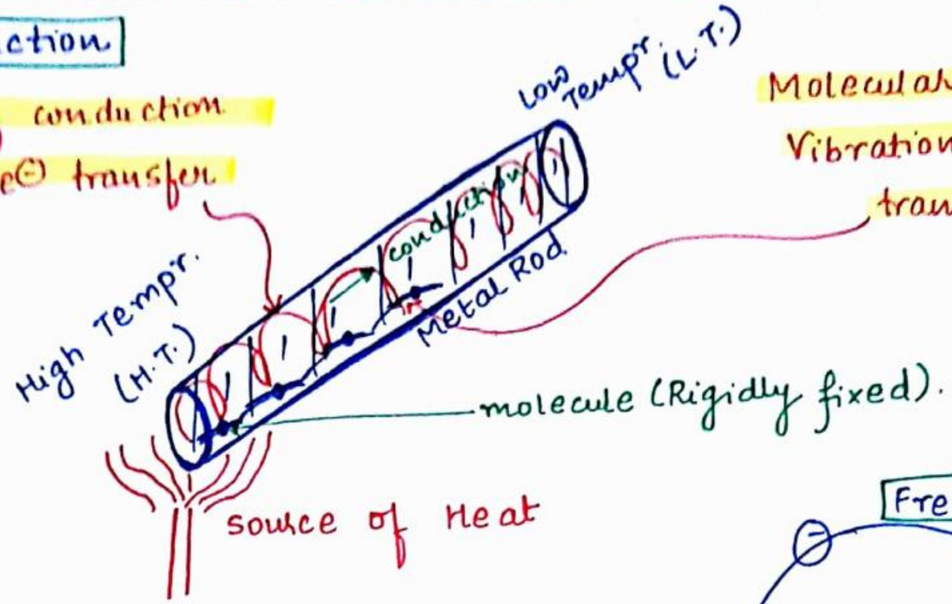
② Convection.

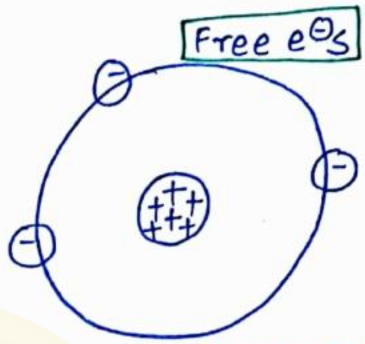
③ Radiation.

Conduction

(70%) of conduction
free $e^{\ominus}$ transfer

Molecular Lattice
Vibrational energy
transfer (30%)





30% and 70% only for metallic Bodies.

free $e^{\ominus}$s take energy & goes to excited state.

Best known metallic conductor :-

$\boxed{\text{Silver} \rightarrow K = 410 \ \frac{W}{mK}}$

then

$\boxed{\text{Copper} \rightarrow K = 385 \ W/mK}$

$\boxed{\text{Aluminium} \rightarrow K = 200 \ W/mK}$
(Low ρ)
density

$\boxed{\text{Steel} \rightarrow K = 17 \text{ to } 45 \ W/mK}$

Have plenty of free $e^{\ominus}$s.

All are metals &

also,

$$\boxed{K_{\text{pure metal}} > K_{\text{its alloy}}}$$

Ex :-

$$\boxed{K_{\text{Iron}} > K_{\text{steel}}}$$

$$\boxed{K_{\text{copper}} > K_{\text{Brass}} \text{ and Bronze also}}$$

Conduction :- is the mode of heat transfer which generally occurs in a solid body due to tempr. difference associated with Molecular Lattice vibrational energy transfer and also by free $e^{\ominus}$ transfer.

The reason behind all electrically good conductors are also in general good conductors of heat is that the presence of abundant (plenty / lot of) free electrons. $\boxed{\text{Ex}}$:- All Metals

$\boxed{\text{Notable}}$ Exception to the above statement is :- diamond whose thermal conductivity is 2300 W/m K

diamond → non-metal

DIAMOND → Due to molecular lattice arrangement
↳ order of arrangement perfect.

The highest thermal conductivity of diamond is due to its perfect crystalline molecular lattice arrangement.

$\boxed{\text{NON- METALS}}$

Crystalline

ordelly, Intimate arrangement of molecules.

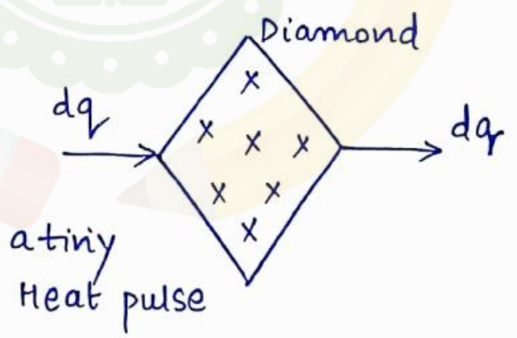
$\boxed{\text{Ex}}$:- ① Diamond
② quartz
③ Graphite

Amorphous

voids/gaps

$\boxed{\text{Ex}}$:- Glass

$K_{glass} = 1.5$ W/mK



$\boxed{\text{THERMAL CONDUCTIVITY DEF}^{N.} \text{ (K)}}$:- K is a thermophysical property of a material which tells about the ability of the material to allow the heat energy to get conducted through the material more rapidly or quickly.

(Thermophysical)
(property)

Insulators have very low thermal conductivity thereby prevent the heat conduction transfer rate.

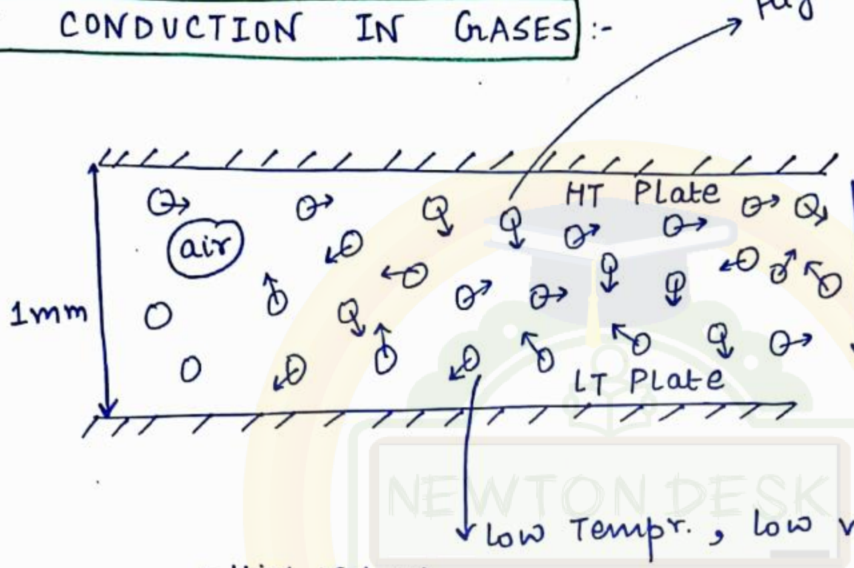
Ex - ① Asbestos → k = 0.2 W/mk

② Refractory Brick → k = 0.9 W/mk
(Furnaces)

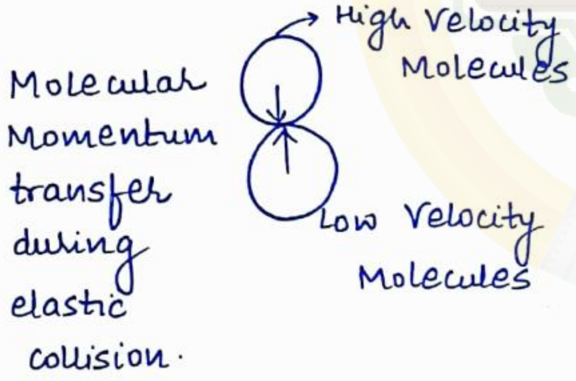
③ Glass wool → k = 0.075 W/mk

④ Polyurethene foam → k = 0.02 W/mk
(PUF)
(Refrigerator walls)

HEAT CONDUCTION IN GASES :-



From kinetic theory of gases
$\bar{c} \propto \sqrt{T}$
(RMS velocity) of gas molecule



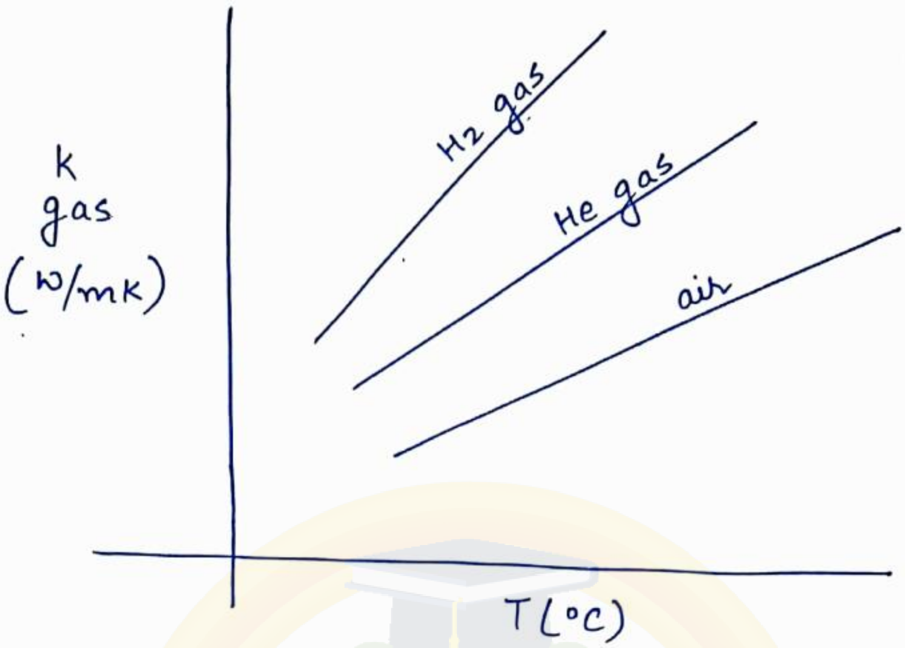
air is stagnant
gap is very narrow → rest

Heat conduction occurs in gases by molecular momentum transfer when high velocity, high tempr. molecules collide with the low velocity low tempr. molecules but in general gases are very bad conductors of heat.

$$\boxed{\begin{array}{l} K_{air} = 0.026\ W/mK \\ \text{(at Room conditions)} \end{array}}$$

As the tempr. of the gases increase, their thermal conductiv. also increased because at higher temperatures of gas, ~~rate of~~ greater molecular activity may result in more no. of collisions per unit time and hence more momentum transfer rate.

5



As T_{gas} ↑

⇒ K_{gas} ↑

(m²/sec) ν (K.V.) ↑ ⟶ kinematic viscosity

C_p ↑

ρ ↓

Liquids are better conductors of heat than gases.

$$K_{water} = 0.63\ W/mK$$

Among all the liquids,

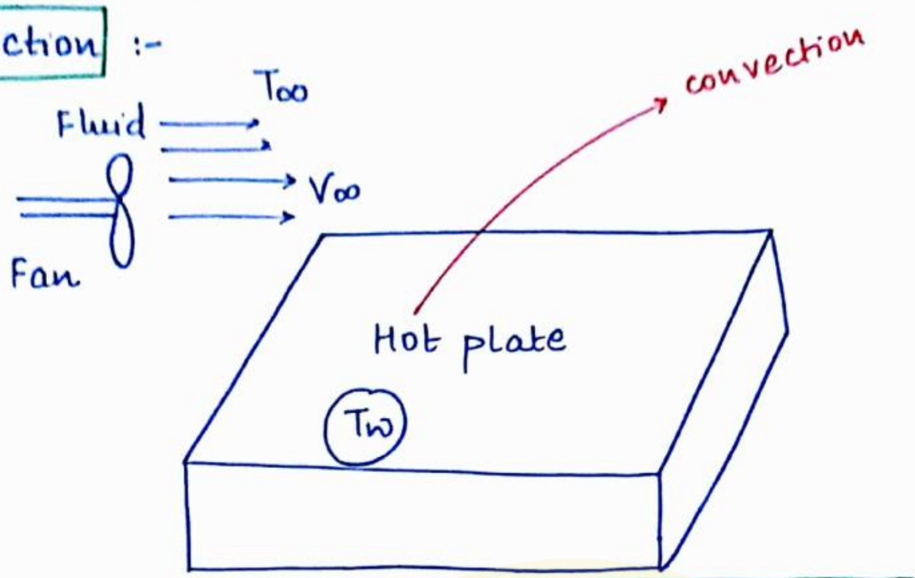
Mercury (Liquid metal) has highest thermal conductivity

$$K_{Hg} = 8.43\ W/mK$$

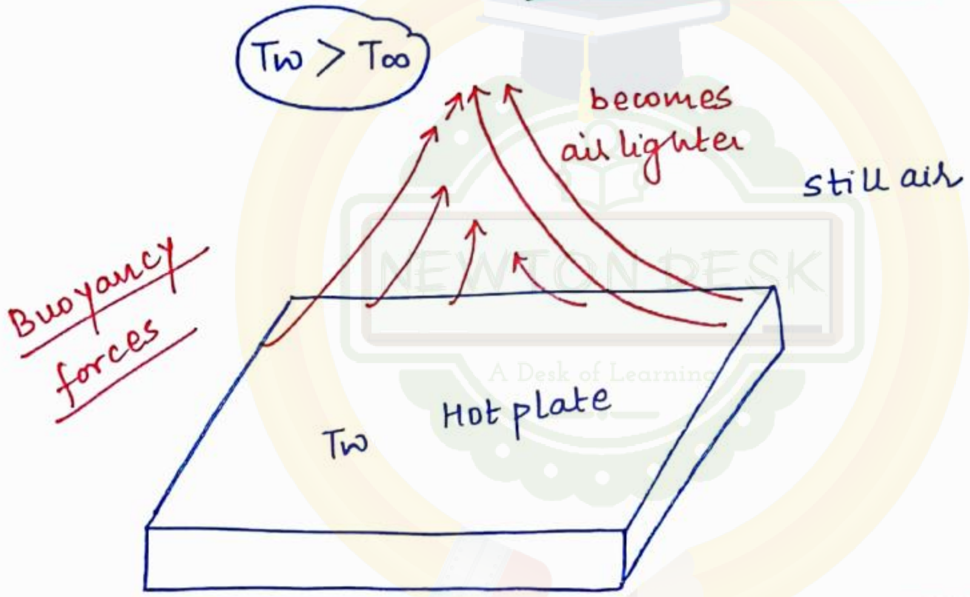
Hg (Thermometric fluid) ⟶ due to good conduction, good expansion, good low vapour pressure.

Hg → low V.P.
→ good volume expansion with heating
→ high 'k'.

Convection :-



Forced convection



Free / Natural convection Heat Transfer (H.T.)

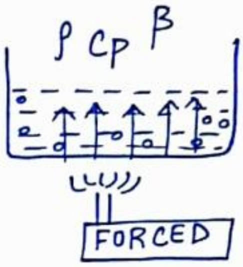
$$\frac{P}{\downarrow \rho} = RT\uparrow$$

Fluid can transfer /transport thermal energy in the form of enthalpy manifested by its Temp. ($\dot{m} C_p T$)

Convection is the mode of heat transfer which generally occurs between a solid surface and the surrounding fluid due to temperature difference associated with macroscopic Bulk motion of the fluid transporting thermal Energy. In case of forced convection heat transfer, this motion of the fluid is provided by an external agency like a fan or a blower or a pump whereas in free convection heat transfer, the motion of the fluid occurs naturally due to Buoyancy forces arising out of density changes of fluid (because of its tempr. change). 7

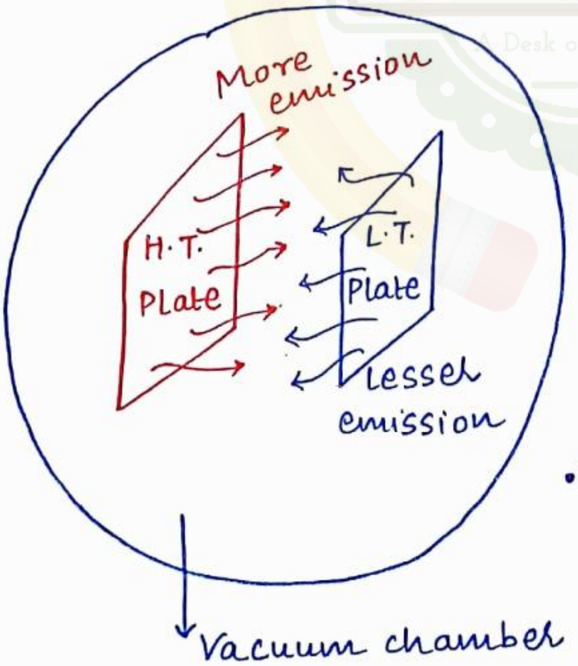
Conduction → internal phenomenon.

Convection → Boundary ——,,——.



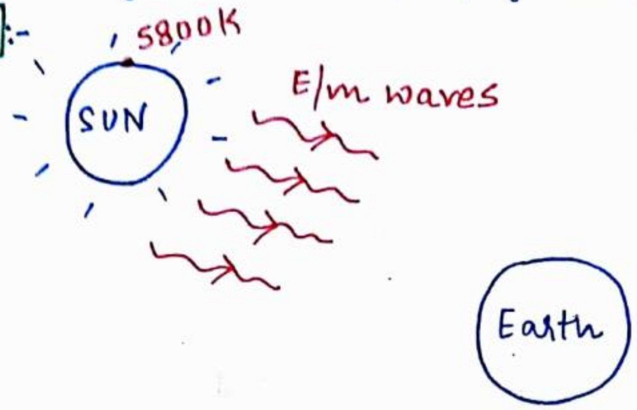
RADIATION :→

- All bodies at all Temperatures emit Thermal Radiation except the body at 0K (−273.15°C).
  (∵ $\eta \neq 100\%$ – carnot)
  (violates IInd Law)
- The rate of emission occurs in the form of electromagnetic waves which can propogate even through vacuum.
- The rate of energy emission $\propto T^4$ (T in K).

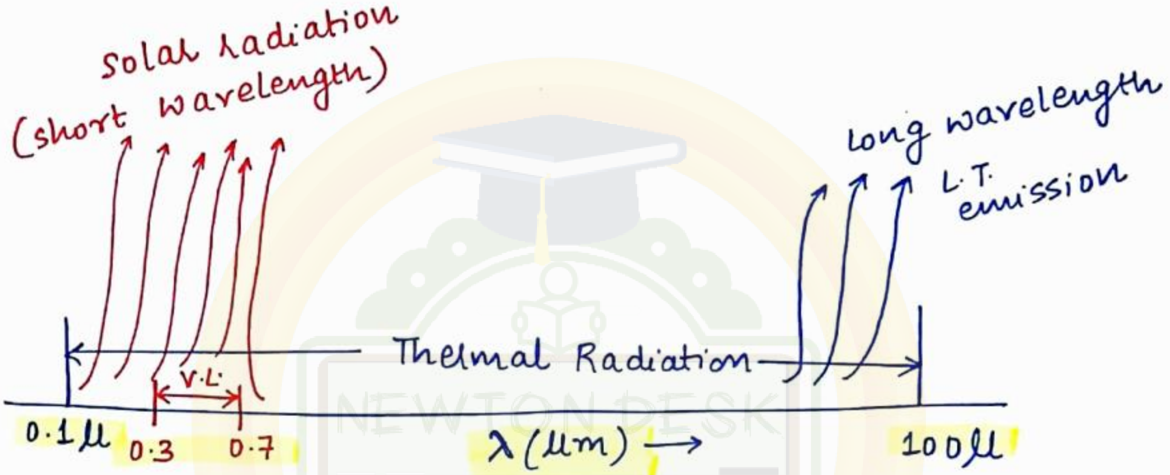


Radiation is the mode of heat transfer which does not require any material medium for its propogation and hence occurs by electromagnetic wave propogation travelling with a speed of light.

Ex :-



V.L. → visible light



E/m spectrum

$$T_K \longrightarrow E \xrightarrow{watt/m^2} \lambda\,(\mu m)$$

T_K	E	$\lambda\,(\mu m)$
↓ high	↓ very high	↓ small

NOTE :-

Radiation Mode of heat transfer completely predominates over conduction & convection particularly when the tempr. difference is sufficiently large.

Ex:- The Mode of H.T. between hot flue gases and Refractory brick walls in a large pulverised fuel fired Power Boiler is predominently by Thermal Radiation. (9)

Reason:- Large ΔT.

$$q_{cond} \propto (T_1 - T_2)\ ^\circ C \text{ (OR) } K$$

$$q_{conv} \propto (T_w - T_\infty)\ ^\circ C \text{ (OR) } K$$

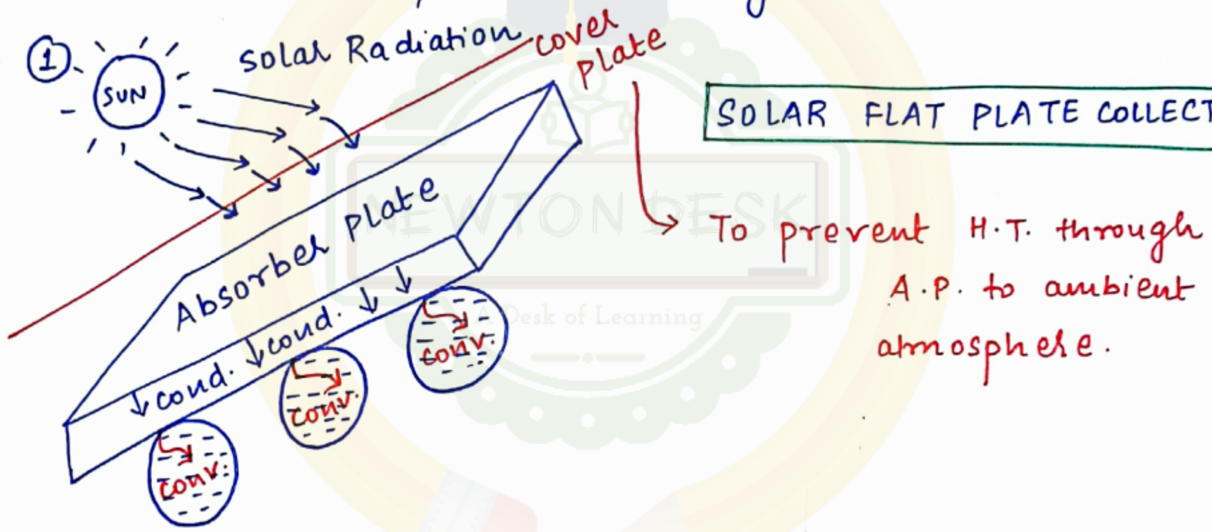
$$q_{Radiation} \propto (T_1^4 - T_2^4)\sigma$$

$\sigma \rightarrow$ steffan's Boltzman constant

$T_1, T_2 \rightarrow$ Kelvin

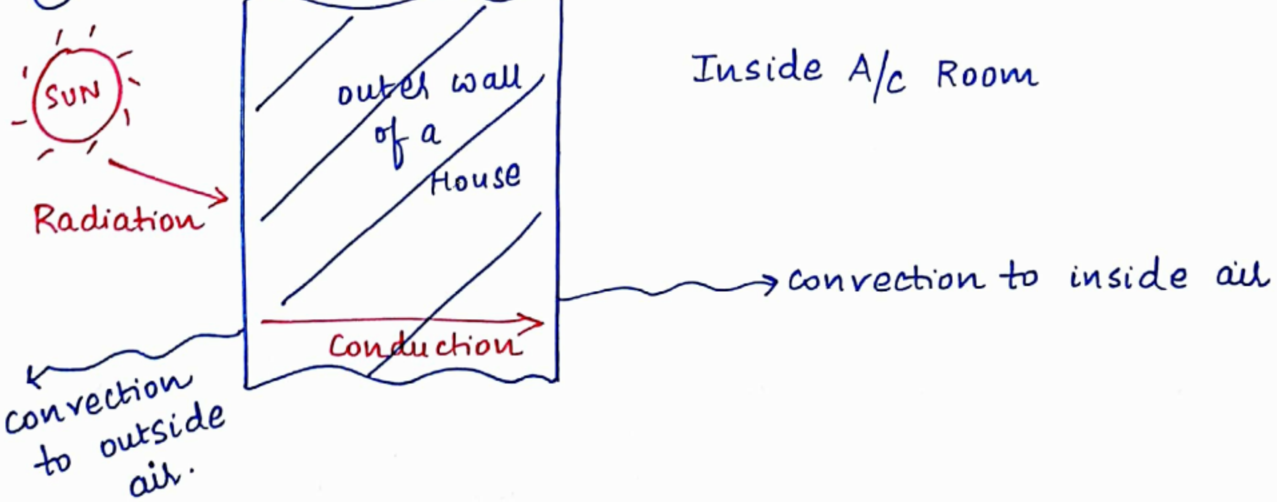
NOTE:- In any practical situation of heat transfer all the 3 modes of heat transfer may simultaneously exist.

Ex:- (1)

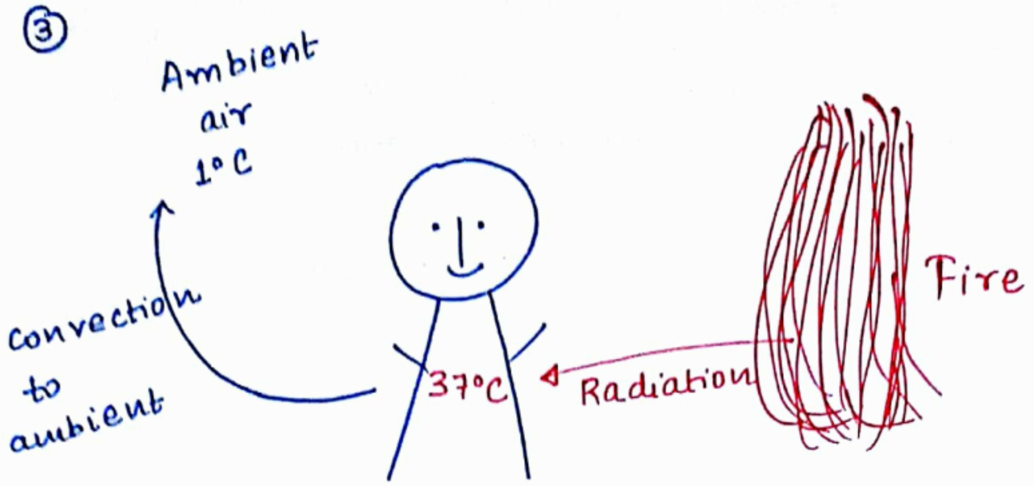


SOLAR FLAT PLATE COLLECTOR

(2)

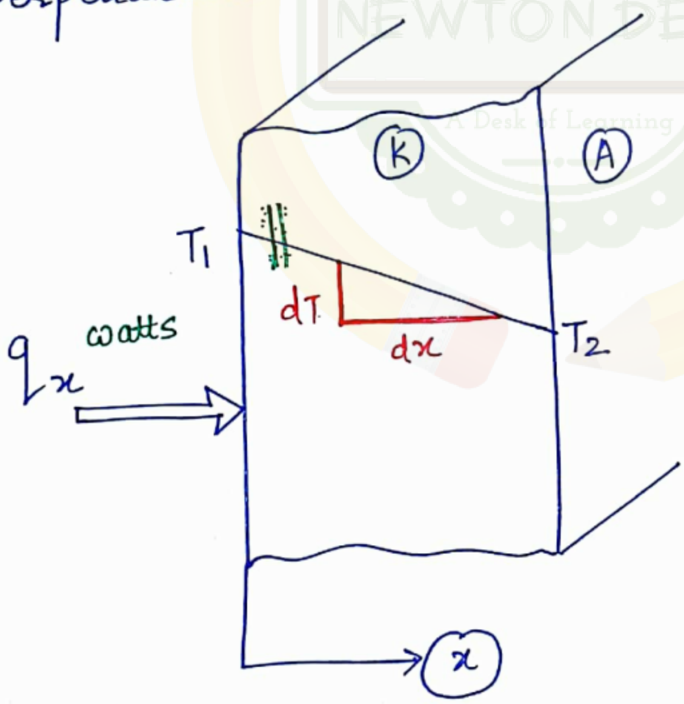


③



* $\boxed{\text{GOVERNING LAWS OF HEAT TRANSFER}}$:-

① **Fourier's law of Conduction** :- The law states that the rate of heat transfer by conduction along a given direction is directly proportional to the tempr. gradient along that direction and is also directly proportional to the area of heat transfer lying perpendicular to the direction of heat transfer.



$\frac{dT}{dx}$ → Tempr. gradient → very small

$$q_x \propto -\frac{dT}{dx}$$

(-ve sign shows that heat always flows in the direction of decreasing Temp. i.e. to satisfy clausius's statement of IInd law of T.D.)

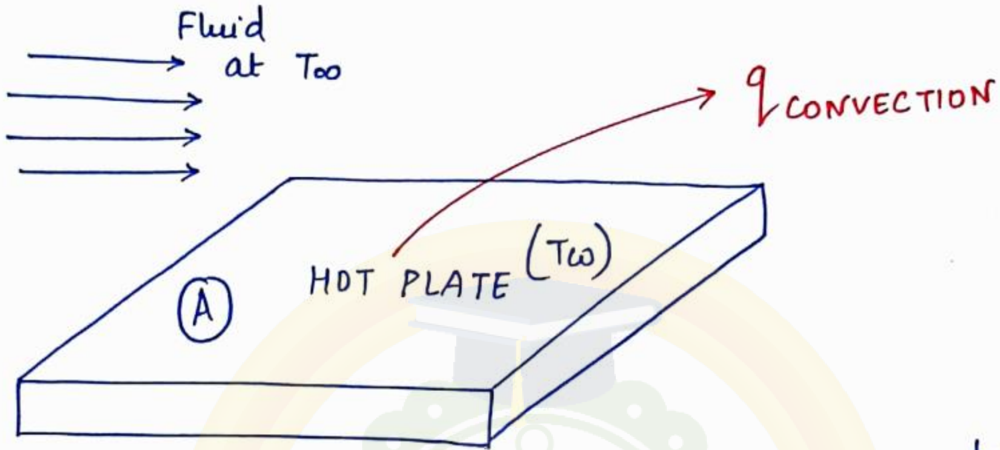
$\propto A$

Rate of heat transfer (H.T.)

$$\boxed{q_x = -KA\left(\frac{dT}{dx}\right) \text{ watt}}$$

$K \rightarrow$ Thermophysical property of Material of slab.

(2) NEWTONS LAW OF COOLING (for CONVECTION H.T.) -



The law states that the rate of heat transfer by convection between a solid body and a surrounding fluid is directly proportional to the tempr. difference between them and is also directly proportional to the area of contact or area of exposure between them.

$$q_{conv.} \propto (T_\omega - T_\infty)$$

$$\propto A$$

$$\boxed{q_{conv.} = hA(T_\omega - T_\infty) \text{ watt}}$$

h = convection heat transfer coefficient

(OR)

Film H.T. coefficient $\left(\text{watt}/m^2K\right)$. SI

also = $\text{watt}/m^2\,{}^\circ C$.

$\boxed{\text{NOTE}}$:- unlike thermal conductivity k, h is not a property of the material but it depends upon some of the thermophysical properties of the fluid like ρ, μ, C_p, K

In forced convection, H.T.

$$\boxed{h = f(\vec{V}, D, \underbrace{\rho, \mu, C_p, k}_{\text{Thermophysical properties of fluid}})}$$

which can influence our H.T. phenomenon.

$\vec{V}$ = Velocity of fluid.

D = characteristic Dimension of Body.

$\rho_w = 1000 \rho_a$

$C_{pw} = 4.186 \ KJ/Kg K$

$\mu_w > \mu_{air}$

$K_w > K_{air}$

water v/s air

In free convection H.T.,

$$\boxed{h = f(g, \beta, \Delta T, L, \underbrace{\mu, \rho, C_p, k}_{\text{properties of fluid}})}$$

g → Acc^n due to gravity.

β → Isobaric volume expansion coefficient of fluid.

$\Delta T = T_w - T_\infty$

L = characteristic Dimension of Body.

* **Ranges of 'h'** :-

1. Free Convection in Gases :- $h = 3$ to $25\ Watt/m^2K$.

2. Forced Convection in gases :- $h = 25$ to $400\ W/m^2K$.

3. ~~Forc~~ Free Convection in liquids :- $h = 250$ to $600\ W/m^2K$.

4. Forced convection in liquids :- $h = 600$ to $4000\ W/m^2K$

5. Condensation Heat transfer :- $h = 3000$ to $25{,}000\ W/m^2K$
   (vap. to liquid)

6. Boiling Heat Transfer :- $h = 5{,}000$ to $50{,}000\ W/m^2K$
   (Liq. to vapour)

* (3) **STEFAN - BOLTZMAN's Law of Radiation** :- The law states that the radiation energy emitted from the surface of a Black Body per unit time per unit area is directly proportional to the 4th power of the absolute tempr. of the Black Body.

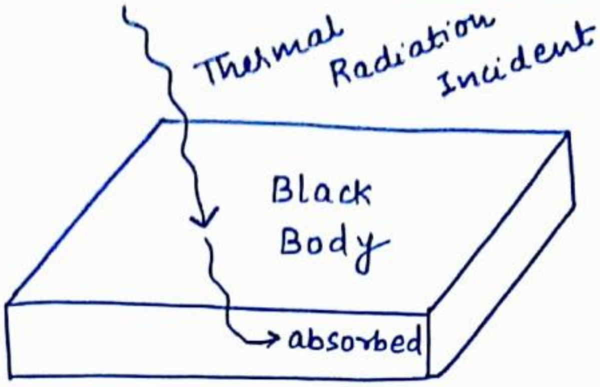
$$E_b \propto T^4 \text{ (T in kelvin only)}$$

$$\boxed{E_b = \sigma T^4 \ \frac{\text{Joule}}{\text{sec } m^2} = \frac{\text{watt}}{m^2}}$$

σ = stefan - Boltzman's constant

$\sigma = 5.67 \times 10^{-8}\ Watt/m^2K^4$

Black body is the Body which absorbs all the thermal Radiation incident or falling ~~above~~ or upon the body

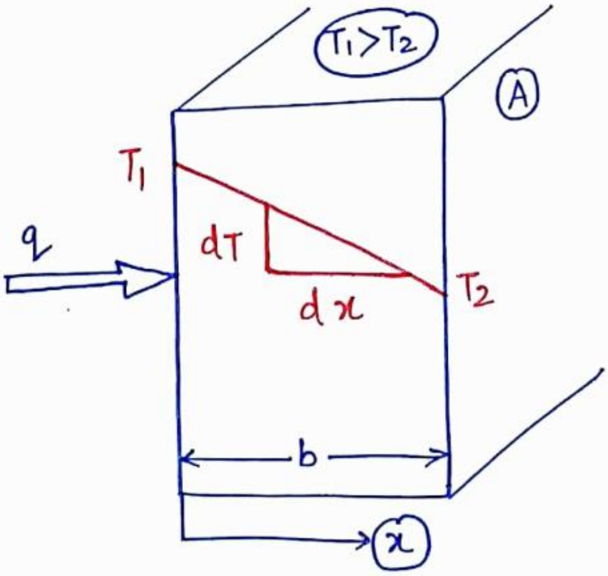


Black Body → Perfect absorber.
→ Ideal- emitter.
→ Diffusive in nature.

NOTE :- A Thermally black body absorbing all the incident thermal radiation falling upon it may not appear black in colour to the human eye.

Ex :- Ice and Snow.

CONDUCTION H.T. (Integration of Fourier's Law of Convection)



Assuming :- ① steady state H.T. conditions

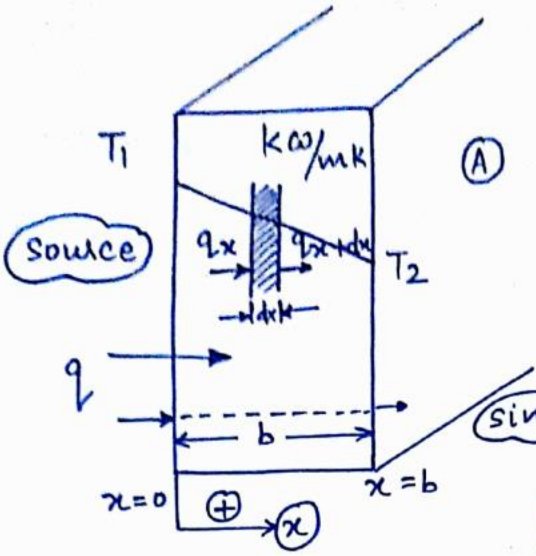
$T \neq f(\text{Time})$

② one - dimensional heat conduction

$T = f(x)$

at $x = 0 \Rightarrow T = T_1$
at $x = t \Rightarrow T = T_2$
Boundary cond^n's of a slab

③ uniform (or) constant 'K' of material.



$$\int_{x=0}^{x=b} q_x \, dx = \int_{T_1}^{T_2} -KA\, dT$$

$q_x \neq f(x)$ to satisfy steady state conditions.

i.e. $q_x = q_{x+dx}$

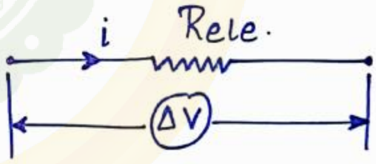
$\Rightarrow q_x \times b = KA(T_1 - T_2)$

$\Rightarrow$ Rate of conduction H.T. through slab $= q_x = \dfrac{KA(T_1-T_2)}{b}$ watt

$\Rightarrow$ q/A = H.T. Rate per unit area (or) Heat flux $= \dfrac{K(T_1-T_2)}{b}$ watt/m²

* Electrical Analogy of Heat Transfer :-

Electrical	Thermal
i (Amp) →	q (watt)
(ΔV) (or) Emf (volts) →	ΔT (°C) or K
R_{elec} (Ω) →	$R_{Thermal}$



ohm's law :-

$$R_{elec} = \frac{\Delta V}{i} \; \Omega$$

Thermal circuit

$$R_{Thermal} = \left(\frac{\Delta T}{q}\right) \; \text{K/watt}$$

$\therefore$ $\underset{(slab)}{(R_{Th})_{conduction}} = \dfrac{T_1 - T_2}{(q)} = \dfrac{(b)}{(KA)}$ K/watt

$\boxed{\text{NOTE}}$:- If the thickness of the slab is more and if its conductivity is low, then the conduction thermal resistance offered by the slab will be higher, hence heat current will be lesser.

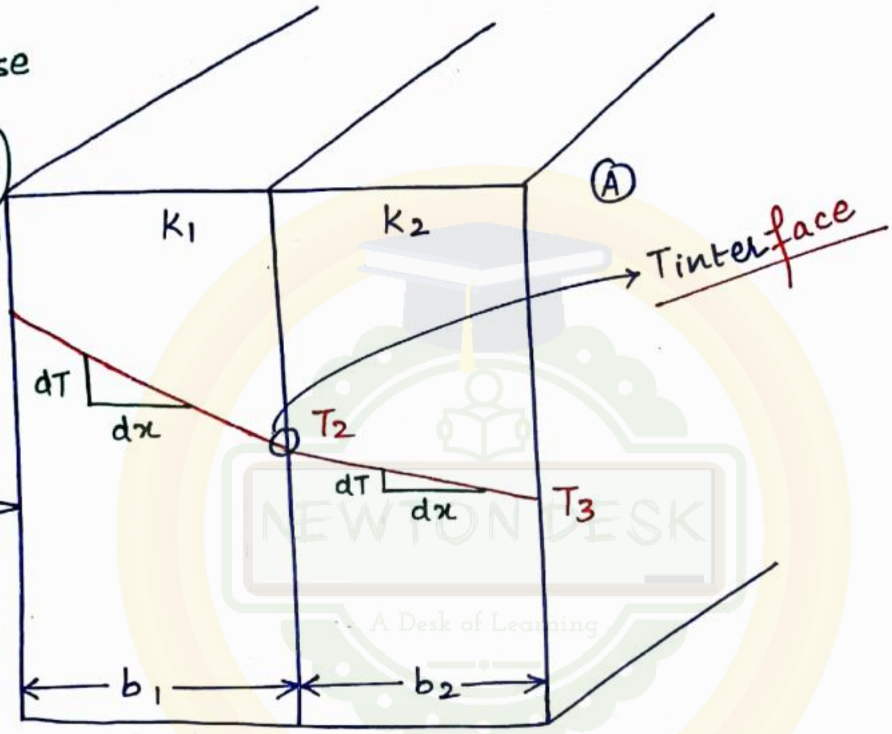
* **Conduction heat transfer through a composite slab** :-

($K_1 < K_2$) because

$\left(\dfrac{dT}{dx}\right)_{in ①} > \left(\dfrac{dT}{dx}\right)_{in ②}$

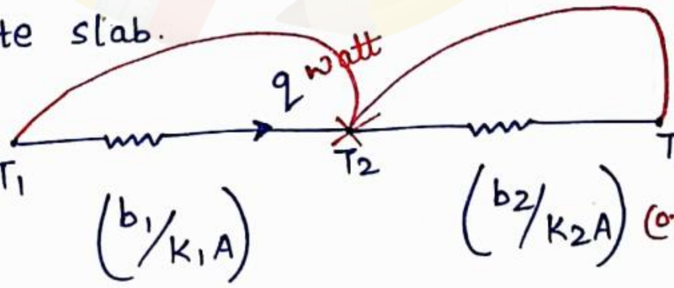
(q being same throughout)

$q = -KA\dfrac{dT}{dx}$



assume steady state one dimensional conduction H.T through the composite slab.

Thermal circuit :-



$q = \dfrac{T_1 - T_2}{(b_1/KA)}$ $\Rightarrow T_2 = \square$ °C

(or) $q = \dfrac{T_2 - T_3}{(b_2/K_2A)}$

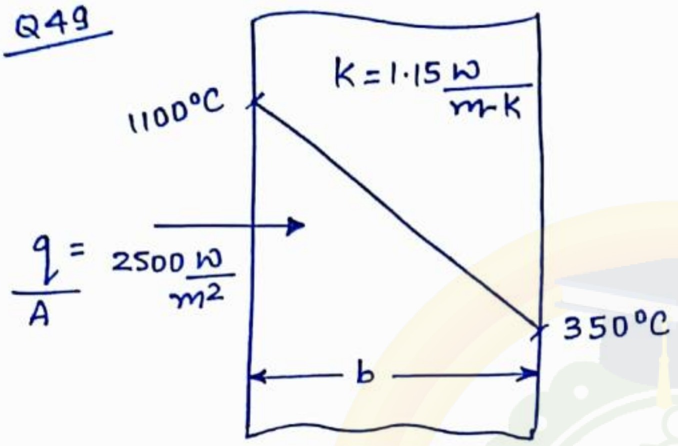
$\Rightarrow T_2 = \square$ °C

$\boxed{\text{Note}}$:- Area of H.T. will NOT change in the direction of heat flow in case of slabs.

$\therefore$ Rate of conduction H.T. through composite slab $= q = \dfrac{(T_1 - T_3)}{\dfrac{b_1}{K_1 A} + \dfrac{b_2}{K_2 A}}$ watt

$\therefore q/A = \text{Heat flux} = \dfrac{T_1 - T_3}{\dfrac{b_1}{K_1} + \dfrac{b_2}{K_2}}$ watt/m²

WB
Pg 70
Q49

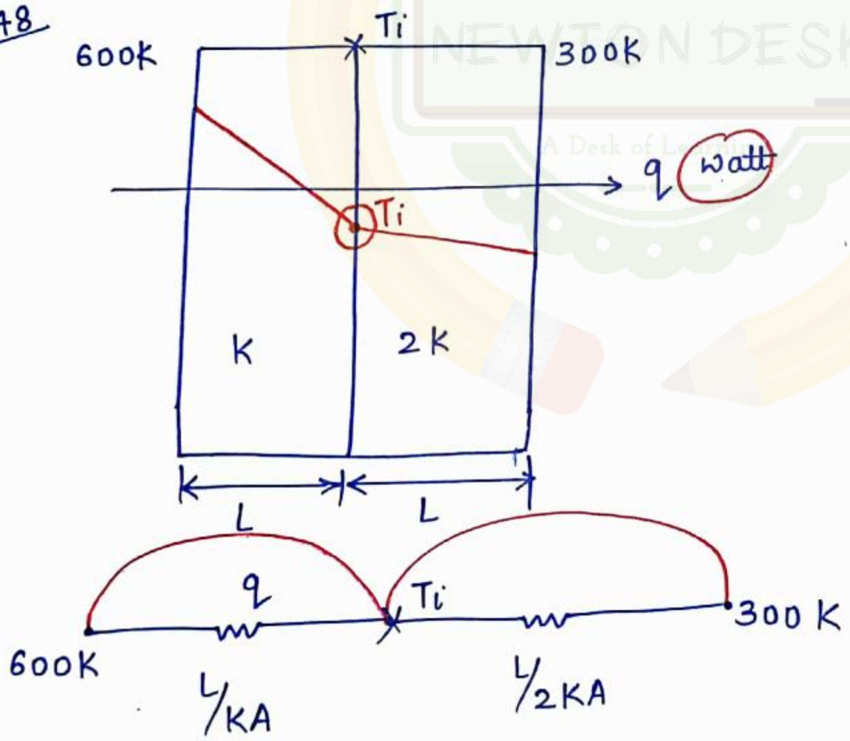


$q/A = \text{Heat flux} = \dfrac{K(T_1 - T_2)}{b}$

$\Rightarrow 2500 \dfrac{W}{m^2} = \dfrac{1.15\,(1100-350)}{b}$

$\Rightarrow b = 0.345 m$

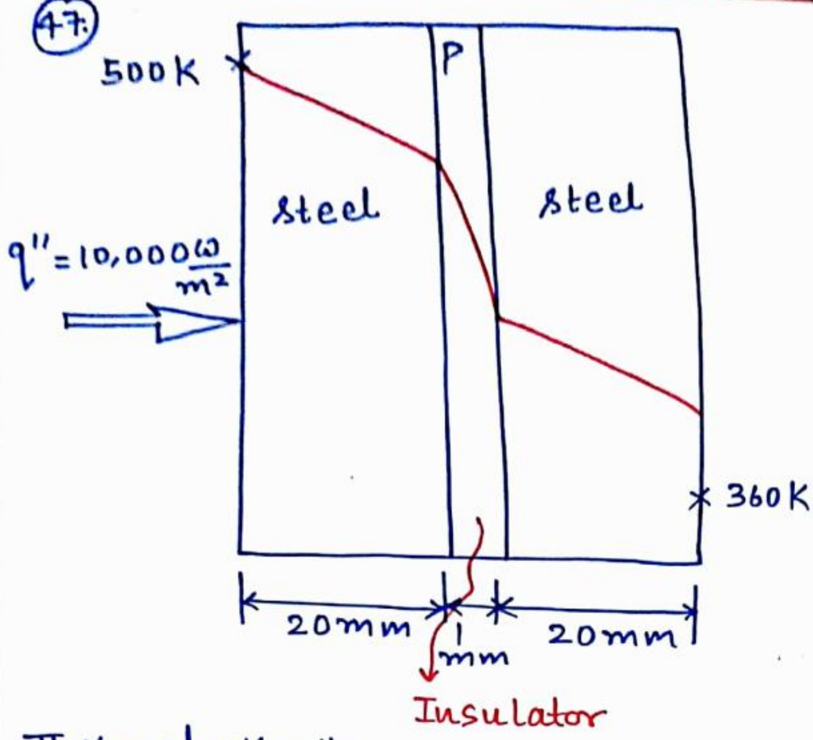
Q48



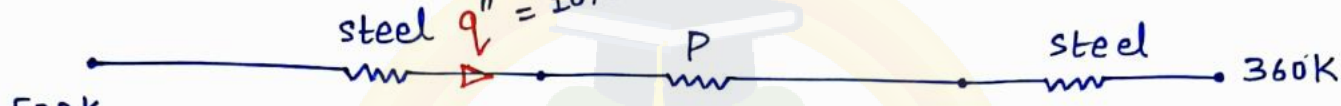
$q = \dfrac{600 - T_i}{\cancel{L}/\cancel{K}A} = \dfrac{T_i - 300}{\cancel{L}/2\cancel{KA}}$

$T_i = 400K$

47.



Thermal circuit



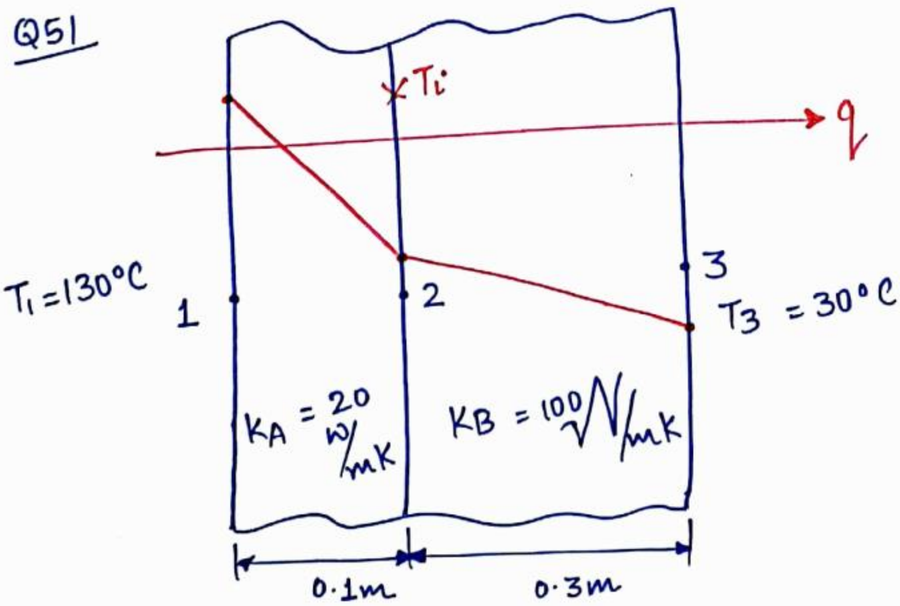
$$\frac{20}{1000 \times 10 \times A} \qquad \frac{1}{1000 \times K \times A} \qquad \frac{20}{1000 \times 10 \times A}$$

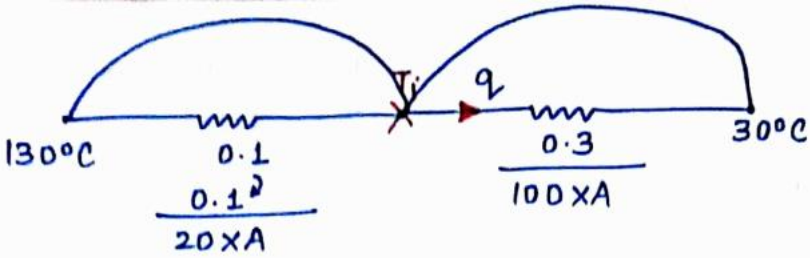
$$\therefore \text{Heat flux} = q/A = \overset{\text{flux}}{q''} = \frac{500-360}{\frac{200}{10{,}000} + \frac{1}{1000K} + \frac{20}{10{,}000}}\ W/m^2$$

$$= 10{,}000\ W/m^2$$

$$K = 0.1\ W/mK$$

Q51

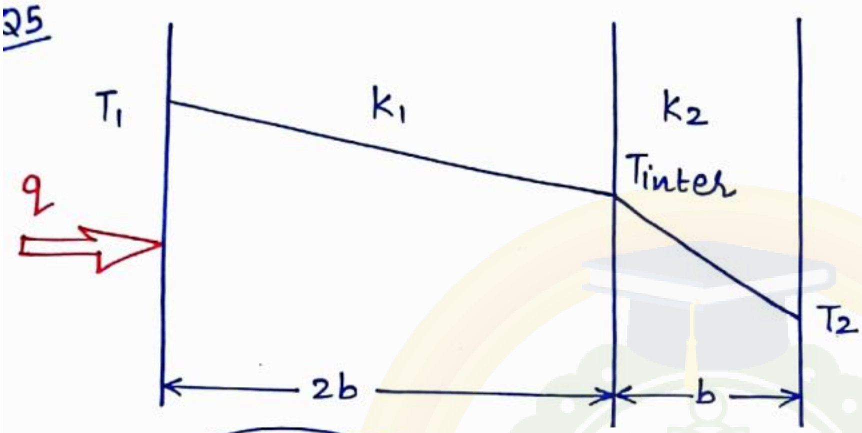




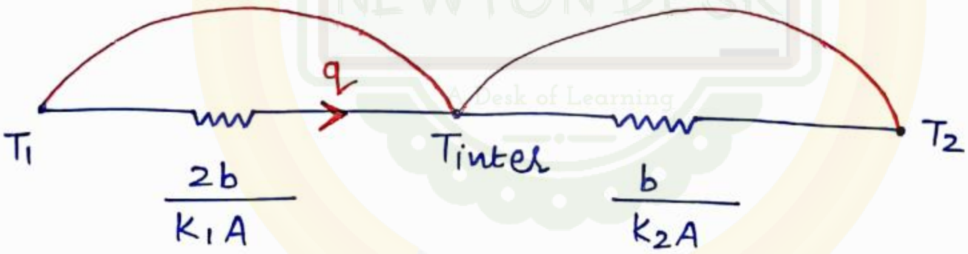
$$q = \frac{130 - T_i}{\frac{0.1}{20\times \cancel{A}}} = \frac{T_i - 30}{\frac{0.3}{1000\times \cancel{A}}}$$

$T_i = 67.5°C$

Q25



$k_1 > k_2$

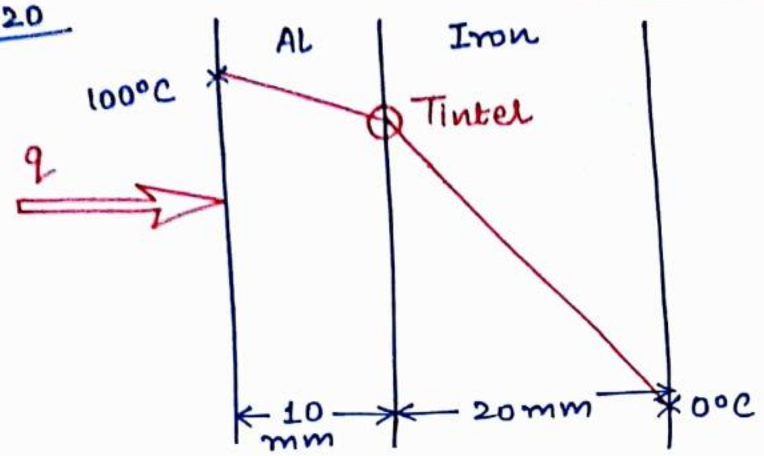


$$q = \frac{\cancel{T_1 - \text{Tinter}}}{\frac{2\cancel{b}}{k_1 \cancel{A}}} = \frac{\cancel{\text{Tinter} - T_2}}{\frac{\cancel{b}}{k_2 \cancel{A}}}$$

Put $\text{Tinter} = \left(\frac{T_1 + T_2}{2}\right)$ given

$\Rightarrow 2k_2 = k_1$

Q20

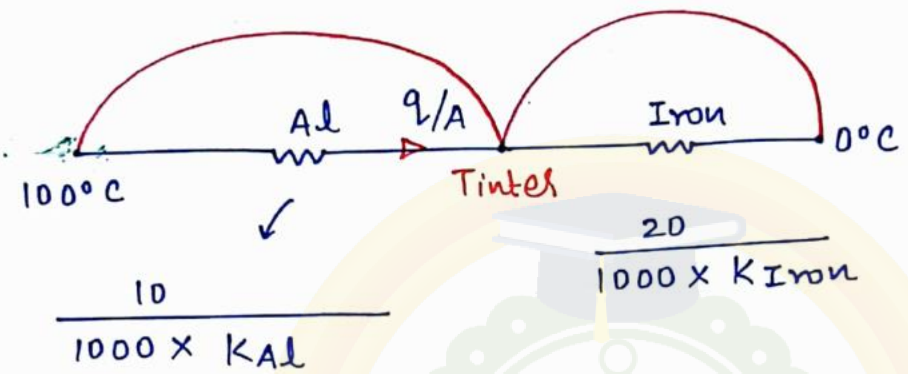


$$\frac{q}{A} = \frac{100 - T_{inter}}{\frac{10}{1000 \times K_{Al}}} = \frac{T_{inter} - 0}{\frac{20}{1000 \times K_{iron}}}$$

Put $\frac{K_{Al}}{K_{iron}} = 3$

$\therefore T_{inter} = 85.7°C$

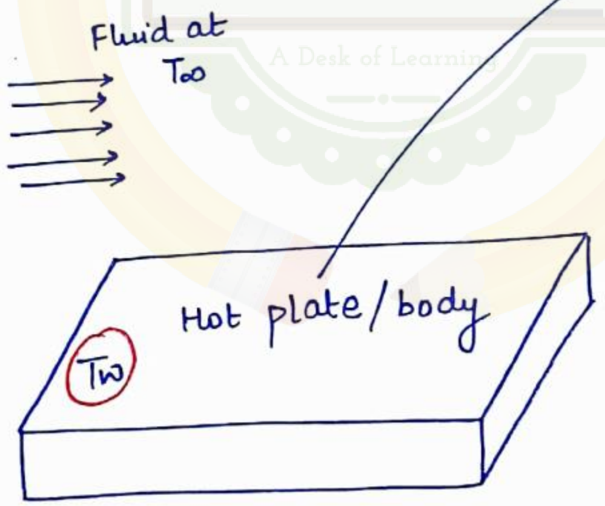
Thermal circuit



* $\boxed{\text{Convection Thermal Resistance}}$:-

$$q_{conv} = hA\,(T_w - T_\infty)$$

area of contact

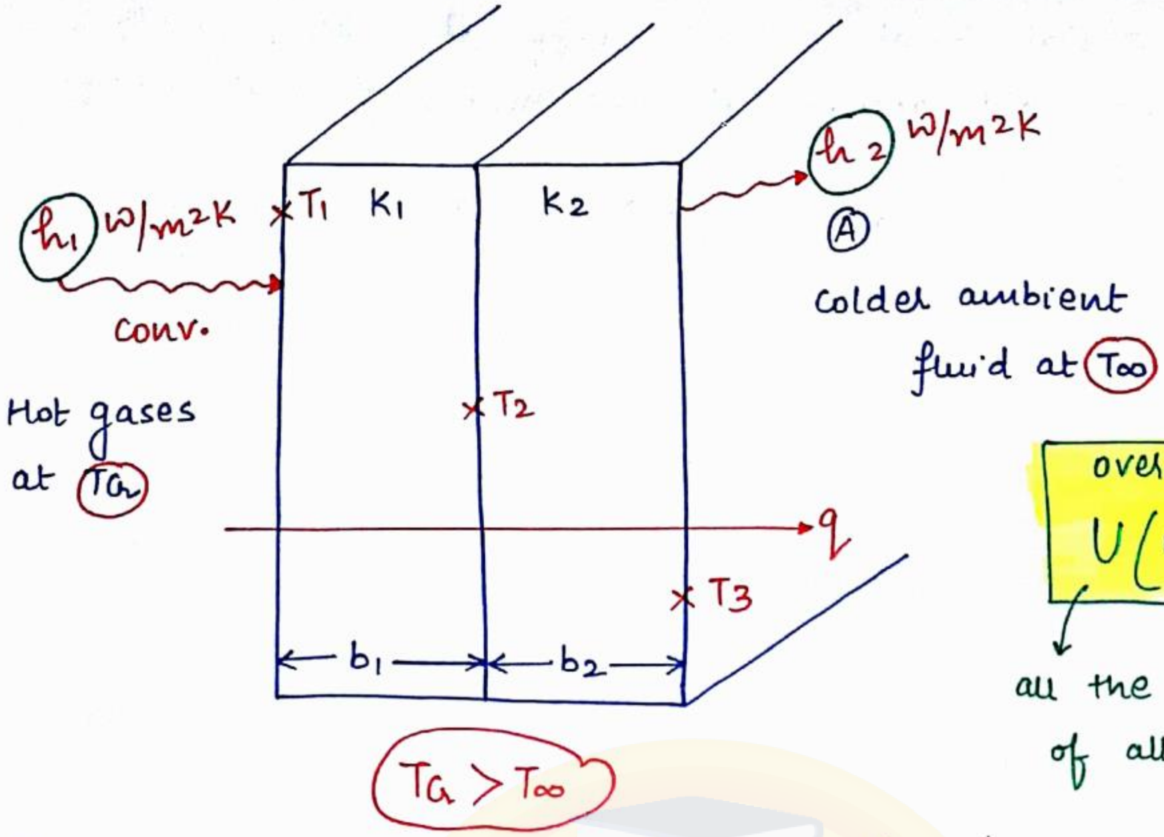


$$(R_{th}) = \frac{\Delta T}{q} \ K/watt$$

$$\therefore (R_{th})_{conv} = \left(\frac{T_w - T_\infty}{q}\right)$$

$$= \left(\frac{1}{hA}\right) K/watt$$

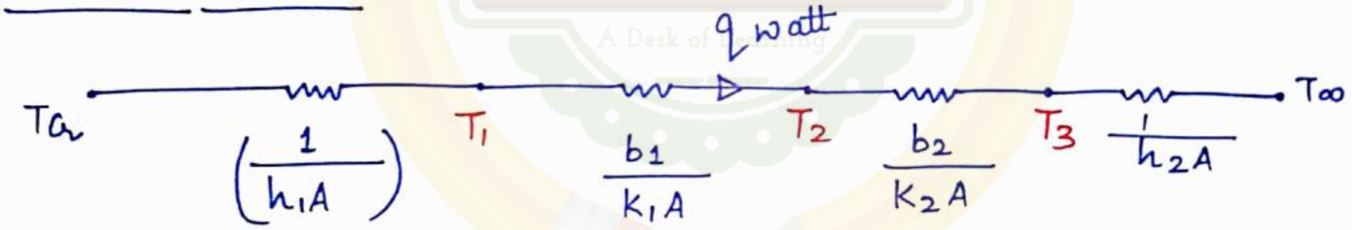
* $\boxed{\text{Conduction - Convection H.T. through a composite slab}}$:-



overall U (W/m²K) → all the Resistances of all the Modes

Assume steady state one dimensional conduction convection H.T. through composite slab b/w hot gases and the ambient colder fluid.

<u>Thermal circuit</u> :-



∴ Rate of H.T. between Hot gases and amb. fluid $= q = \dfrac{(T_a - T_\infty)\text{ watt}}{1/h_1A + \dfrac{b_1}{K_1A} + \dfrac{b_2}{K_2A} + 1/h_2A}$

⇒ Heat flux $= q/A = \dfrac{(T_a - T_\infty)\ \text{W/m}^2}{\left(1/h_1 + b_1/K_1 + b_2/K_2 + 1/h_2\right)}$ → (1)

Defining overall heat transfer coeff. U as the parameter which takes into account all the modes of heat transfer into a single entity that is

$$q = UA(\Delta T) \quad \text{i.e.} \quad q = UA(T_G - T_\infty) \longrightarrow (2)$$

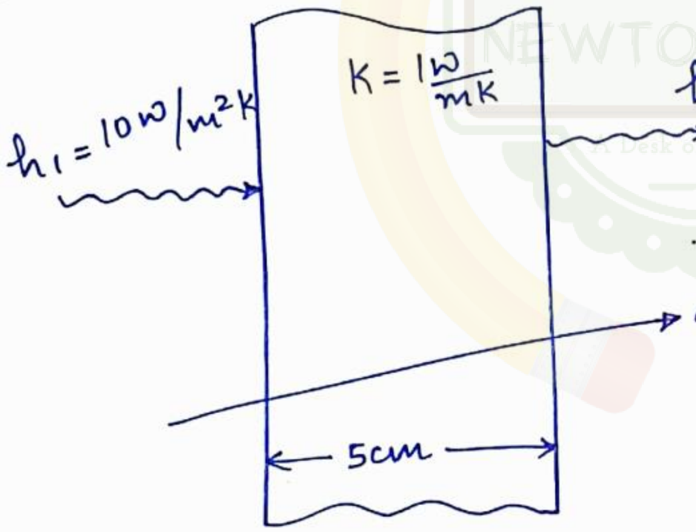
Comparing (1) & (2), we get,

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{K_2} + \frac{1}{h_2}}$$

U & h have same units (W/m^2K)

NOTE:- If U value is more, the Total thermal Resistance in the entire circuit will be lesser and hence H.T. transfer rate will be higher.

Q26



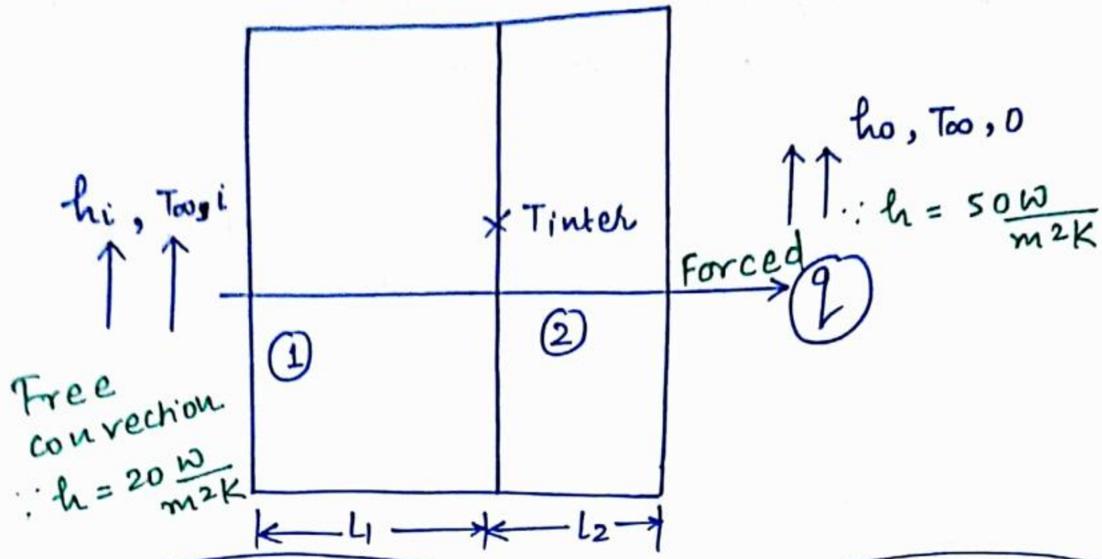
$$1/U = 1/h_1 + b/K + 1/h_2$$

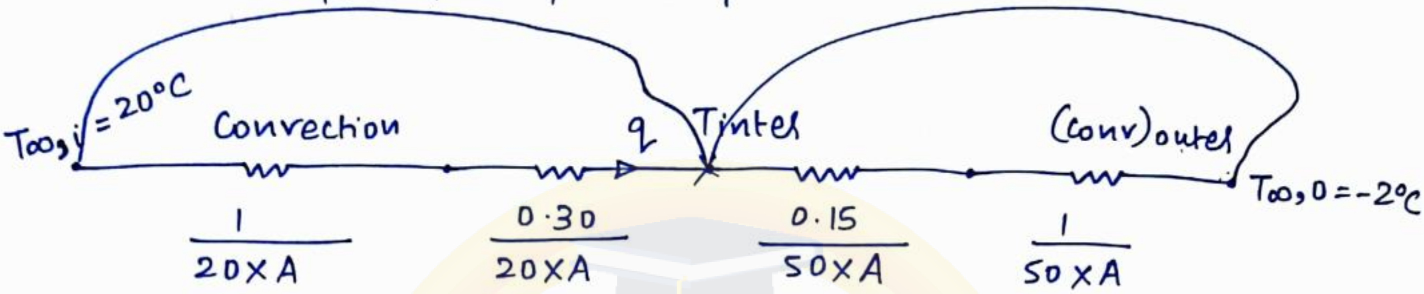
$$1/U = 1/10 + \frac{5}{100 \times 1} + \frac{1}{20}$$

$$\Rightarrow U = 5 W/m^2K$$

Q6

"NEXT PAGE"

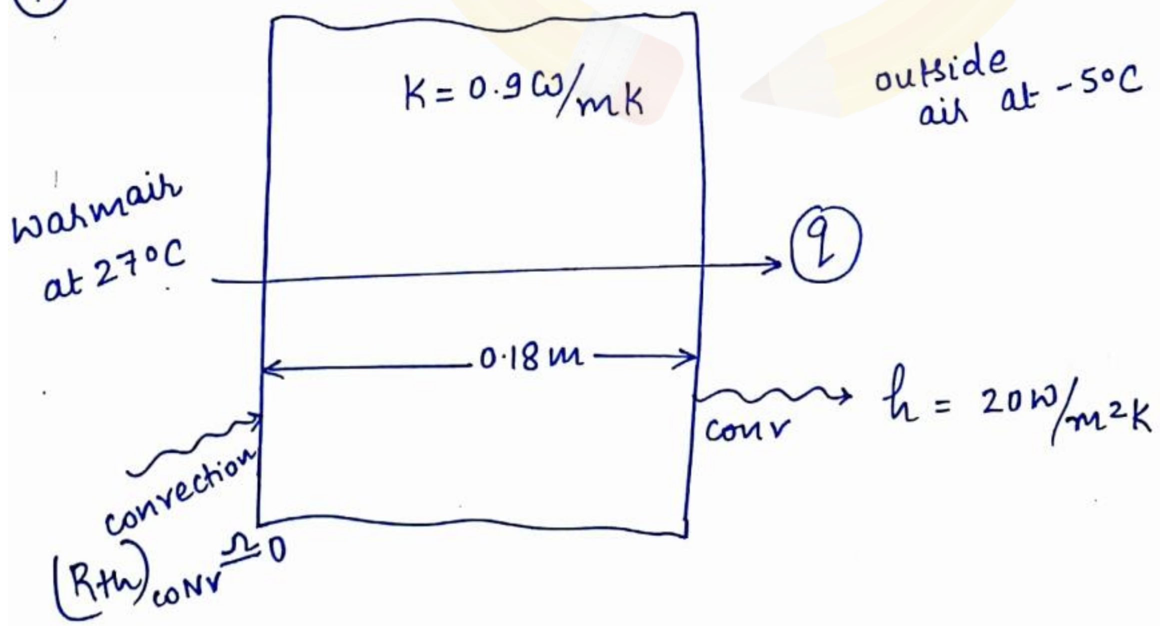




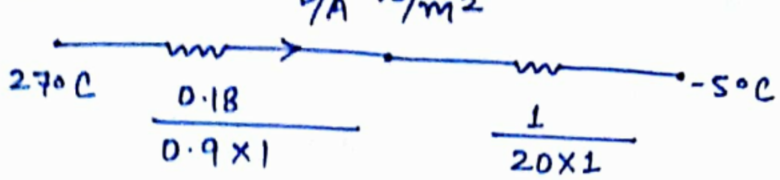
$$q = \frac{20 - T_{inter}}{\frac{1}{20\cancel{A}} + \frac{0.30}{20\cancel{A}}} = \frac{T_{inter} - (-2)}{\frac{0.15}{50\cancel{A}} + \frac{1}{50\cancel{A}}}$$

$$\Rightarrow \quad T_{inter} = 3.75°C$$

41

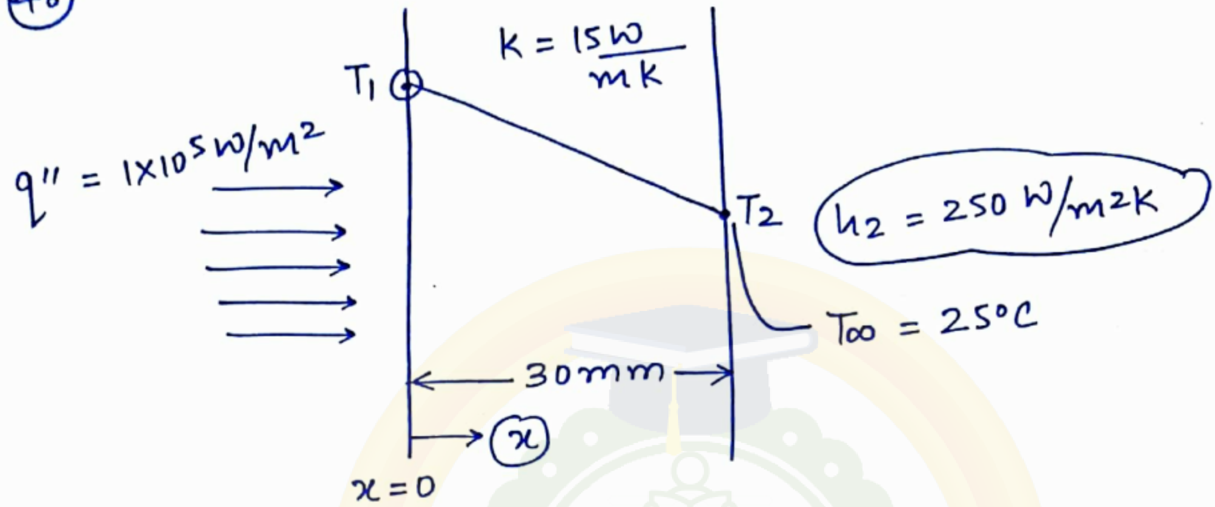


Thermal circuit

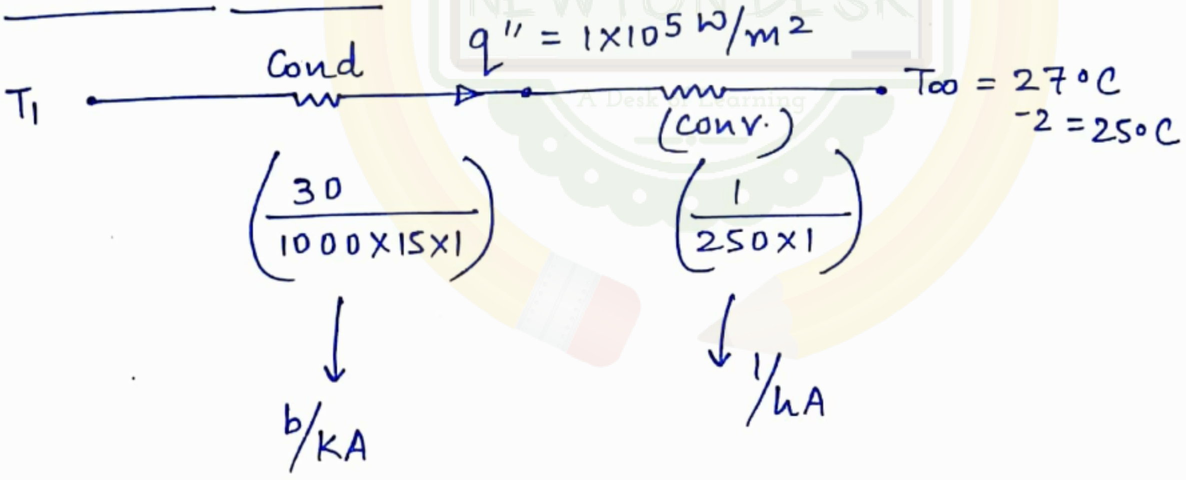


$$\therefore \; q/A = \frac{27-(-5)}{\frac{0.18}{0.9} + 1/20} = 128 \text{ W/m}^2$$

(4.6)



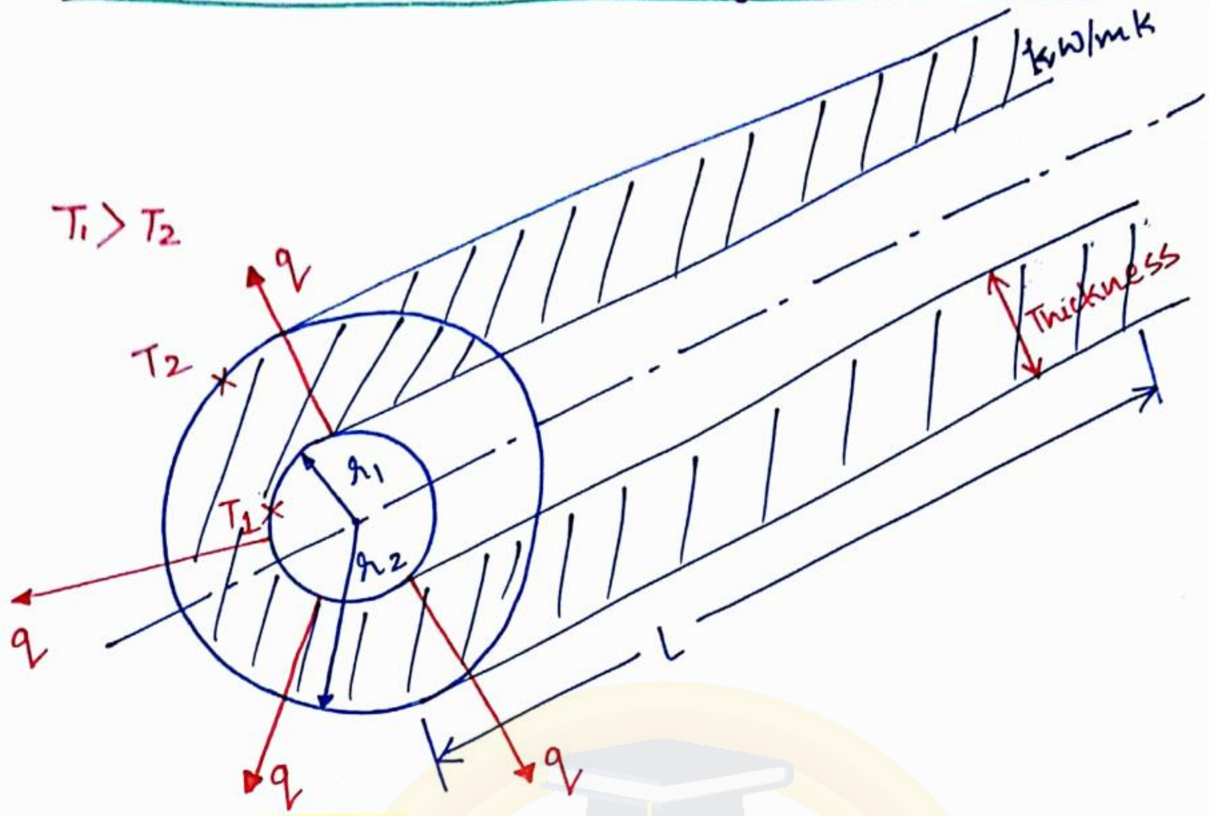
Thermal Circuit :-



$$q'' = \text{Heat flux} = \frac{T_1 - 25°C}{\frac{30}{15,000} + \frac{1}{250}} = 1\times10^5 \text{ W/m}^2$$

$$T_1 = 625°C$$

* Radial Conduction H.T. through a Hollow Cylinder :-

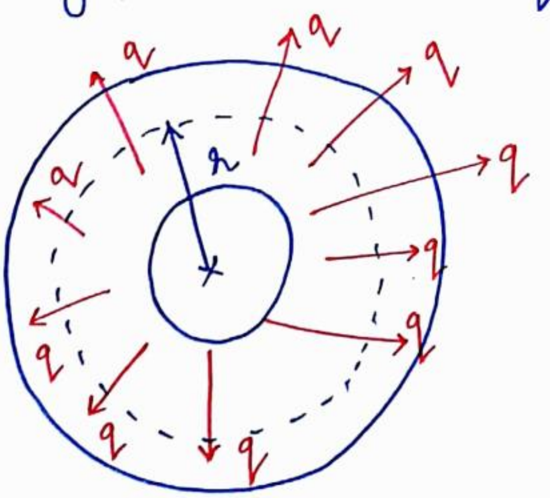


$T = f(r)$

Since temperature gradients are existing along the Radial dirn. It must conduct radially outwards from the inner cylindrical surface at T_1 to outer cylindrical surface at T_2.

Unlike in case of slabs, here the area of conduction heat transfer changes in the direction of heat flow.

At any Radius 'r', Area of conduction H.T. $= A = 2\pi r L$



$\therefore$ Fourier's law of conduction :-

Rate of Radial conduction

$$H.T. = q = -KA\left(\frac{dT}{dr}\right) \text{watt}$$

$$\Rightarrow q = -K\, 2\pi r L \left(\frac{dT}{dr}\right)$$

At $r = r_1 \Rightarrow T = T_1$

At $r = r_2 \Rightarrow T = T_2$

Assume: steady state,

one dimensional (Radial)

H.T.

$$\Rightarrow \int_{r_1}^{r_2} q \frac{dr}{r} = \int_{T_1}^{T_2} -2\pi K L \, dT$$

To satisfy the steady state H.T. conditions,

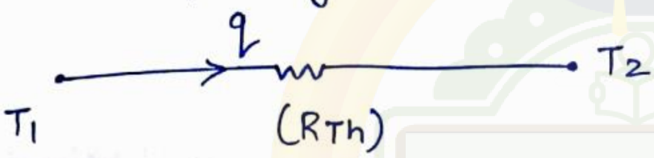
$q \neq f(r)$

i.e. $q_r = q_{r+dr}$

$$\Rightarrow q \ln\left(\frac{r_2}{r_1}\right) = 2\pi K L (T_1 - T_2)$$

$\Rightarrow$ Rate of Radial conduction H.T. $= \boxed{q = \dfrac{2\pi K L (T_1 - T_2)}{\ln(r_2/r_1)} \text{ (watt)}}$

The corresponding conduction thermal Resistance is

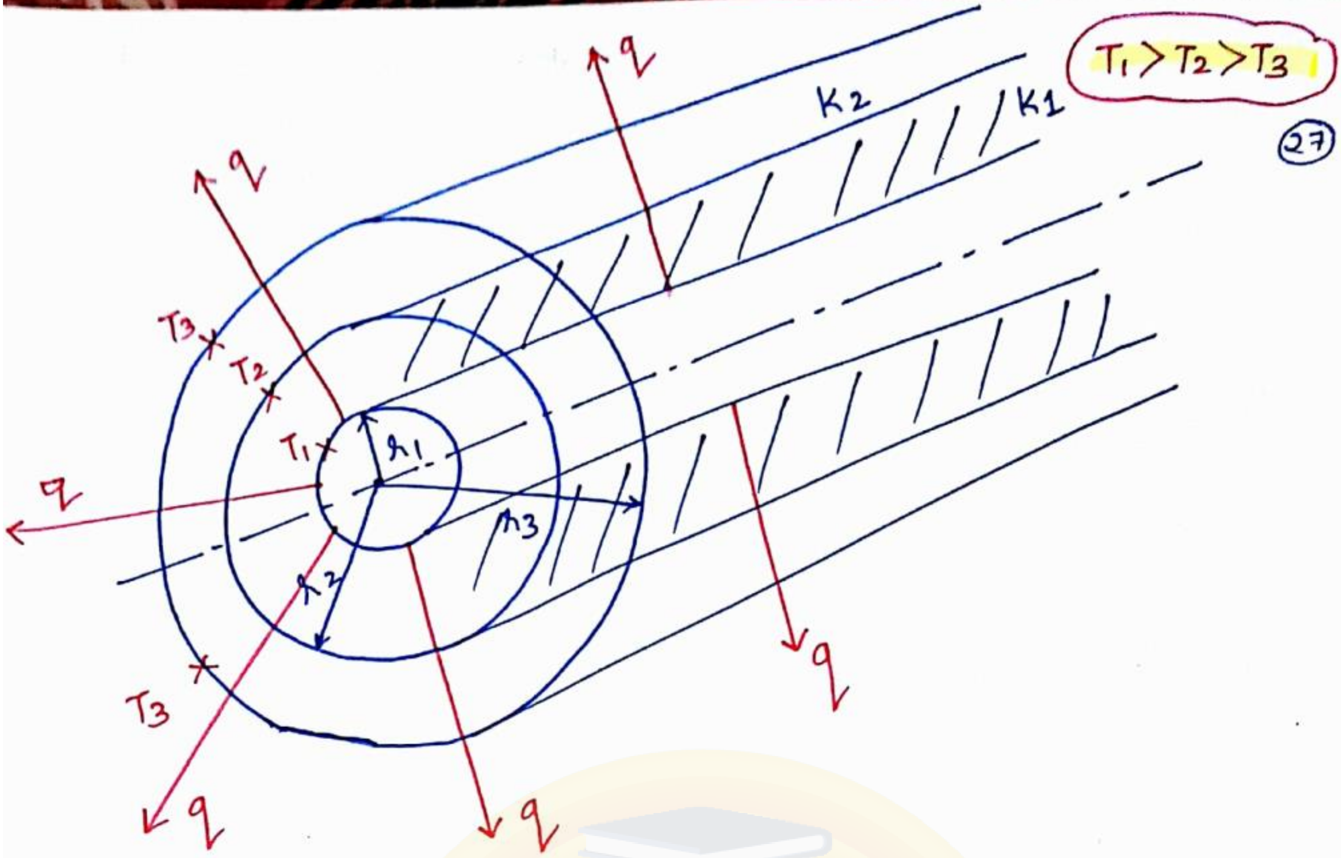


$$(R_{Th})_{(cond)\ \text{hollow cylinder}} = \left(\frac{T_1 - T_2}{q}\right) = \frac{\ln(r_2/r_1)}{2\pi K L} \ K/\text{watt}$$

NOTE:- If the thickness of the cylinder is very small and if the conductivity of material of cylinder is very high (a very thin copper cylinder) then the above conduction thermal Resistance almost becomes zero.

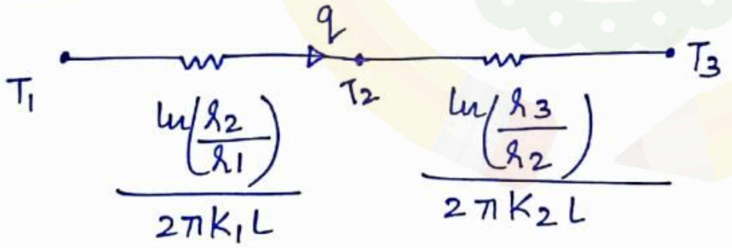
* Radial Conduction heat Transfer through a composite cylinder:-

" N. P. "



Since tempr. gradients are existing along the radial direction, heat must conduct radially outwards from the innermost cylindrical surface at T_1 which is at a radius of r_1 to outer-most cylindrical surface at T_3 which is at a radius of r_3.

Thermal circuit

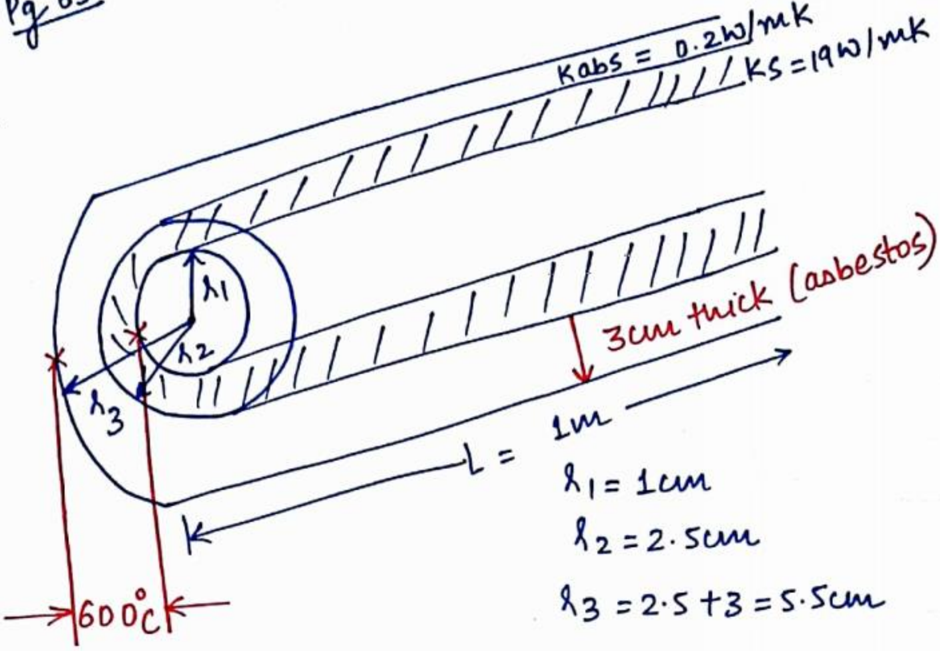


$\therefore$ Rate of Radial Conduction heat transfer (H.T.)

$$= q = \frac{(T_1 - T_3) \text{ watt}}{\frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi K_2 L}}$$

WB Q2 Pg 63

Sir



$r_1 = 1cm$

$r_2 = 2.5cm$

$r_3 = 2.5 + 3 = 5.5cm$

RW

$r_1 = 1cm$

$r_2 = 2.5cm$

$r_3 = 1.5 + 1$

$= 2.$

Thermal circuit

10°C   590°C

q watt

Steel   Asbestos

$$q = \frac{\Delta T}{R_{th}}$$

$$\frac{\ln\left(\frac{2.5}{1}\right)}{2\pi \times 19 \times 1} \qquad \frac{\ln\left(\frac{5.5}{2.5}\right)}{2\pi \times 0.2 \times 1}$$

600°C

$\therefore$ Rate of Radial conduction H.T.

$$\Rightarrow q = \frac{600}{\Sigma R_{th}}$$

$$= 944.7 \ W/m$$

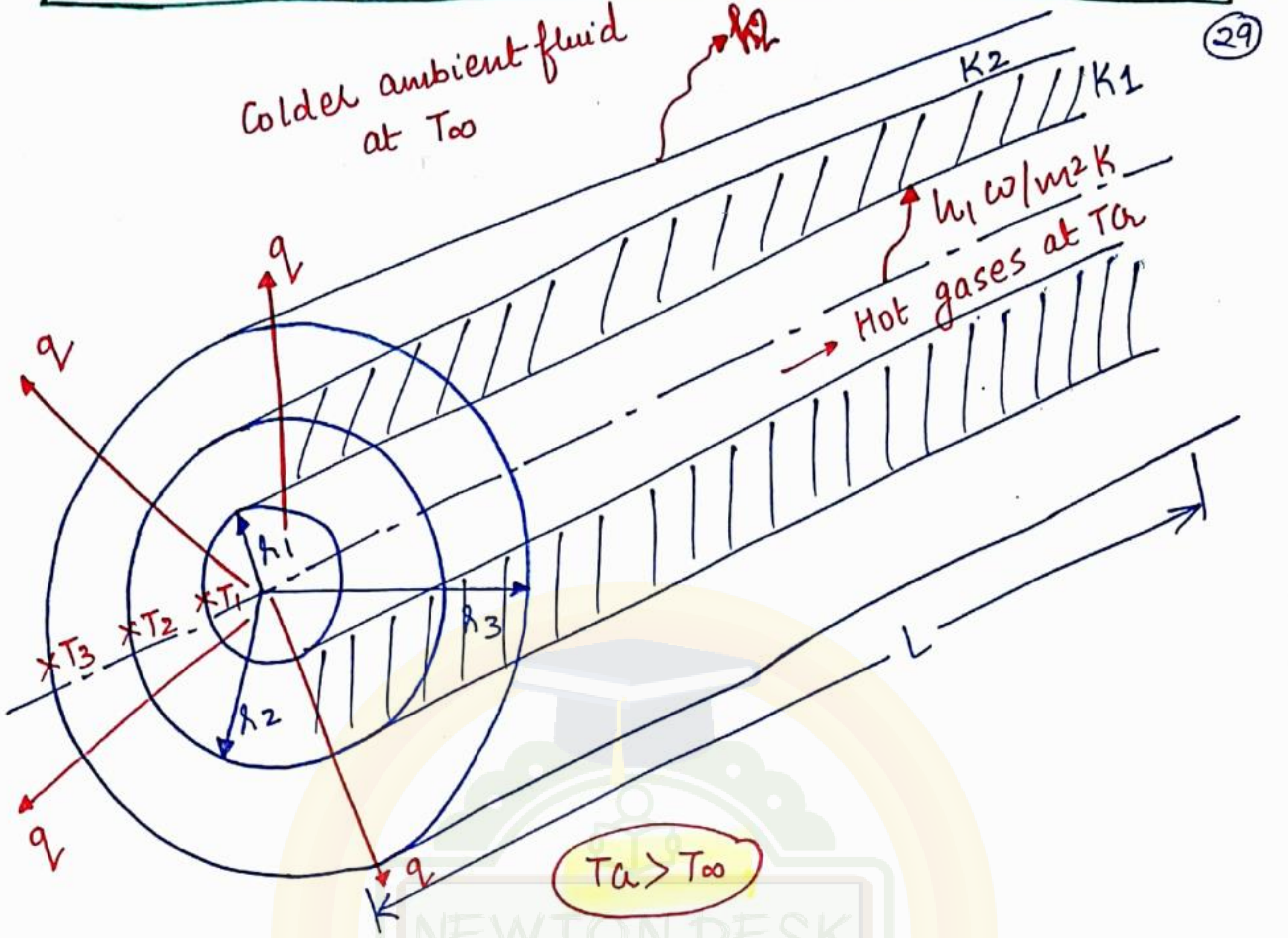
$K_{Steel} \simeq 100 \ K_{abs}$

$\downarrow$

$$(R_{Th})_{steel} <<< (R_{Th})_{Asb}$$

$$\Rightarrow \boxed{(\Delta T)_{across\ steel} <<< (\Delta T)_{across\ Asb.}}$$

* Radial Conduction- Convection H.T. through a composite cylinder :-



h_1 = Inside convective H.T. coefficient

h_2 = outside convective H.T. coefficient

assume steady state one dimensional Radial heat transfer b/w the hot gases and the ambient cold fluid through composite cylinder.

Thermal circuit :-

$$T_a \xrightarrow[\frac{1}{h_1(2\pi r_1 L)}]{(\text{conv})\text{inner}} T_1 \xrightarrow[\frac{\ln(r_2/r_1)}{2\pi K_1 L}]{\text{Cond}} T_2 \xrightarrow[\frac{\ln(r_3/r_2)}{2\pi K_2 L}]{\text{Cond}} T_3 \xrightarrow[\frac{1}{h_2(2\pi r_3 L)}]{(\text{conv})\text{outer}} T_\infty$$

q watt

$\therefore$ Rate of Radial H.T. $= q = \dfrac{2\pi r_1 L(T_a - T_\infty)\text{watt}}{2\pi r_1 L\left(\dfrac{1}{h_1 2\pi r_1 L} + \dfrac{\ln(r_2/r_1)}{2\pi K_1 L} + \dfrac{\ln(r_3/r_2)}{2\pi K_2 L} + \dfrac{1}{h_2(2\pi r_3 L)}\right)}$ ——→ (1)

Defining overall H.T. coeff. U_i that is based on the inside convection heat transfer area and overall H.T. coeff. U_o that is based on the outside convection H.T area. from the eqn :-

$$\Rightarrow q = U_i A_i \Delta T = U_o A_o \Delta T$$

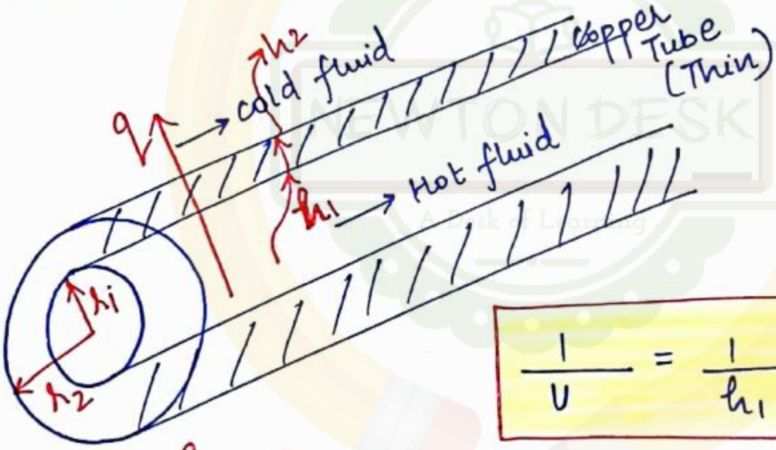
$$\Rightarrow q = U_i \underbrace{(2\pi r_1 L)}(T_a - T_\infty) = U_o \underbrace{(2\pi r_3 L)}(T_a - T_\infty) \text{ watt} \quad (2)$$

Comparing (1) and (2), we get

$$\boxed{\frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1}{K_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{K_2} \ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{r_3} \cdot {}^1/h_2}$$

$$\boxed{\frac{1}{U_o} = \frac{r_3}{r_1} \frac{1}{h_1} + \frac{r_3}{K_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{r_3}{K_2} \ln\left(\frac{r_3}{r_2}\right) + {}^1/h_2}$$

$\boxed{\text{NOTE}}$:- But in Heat Exchanger analysis, whether LMTD method or effectiveness NTU method, V can be calculated as



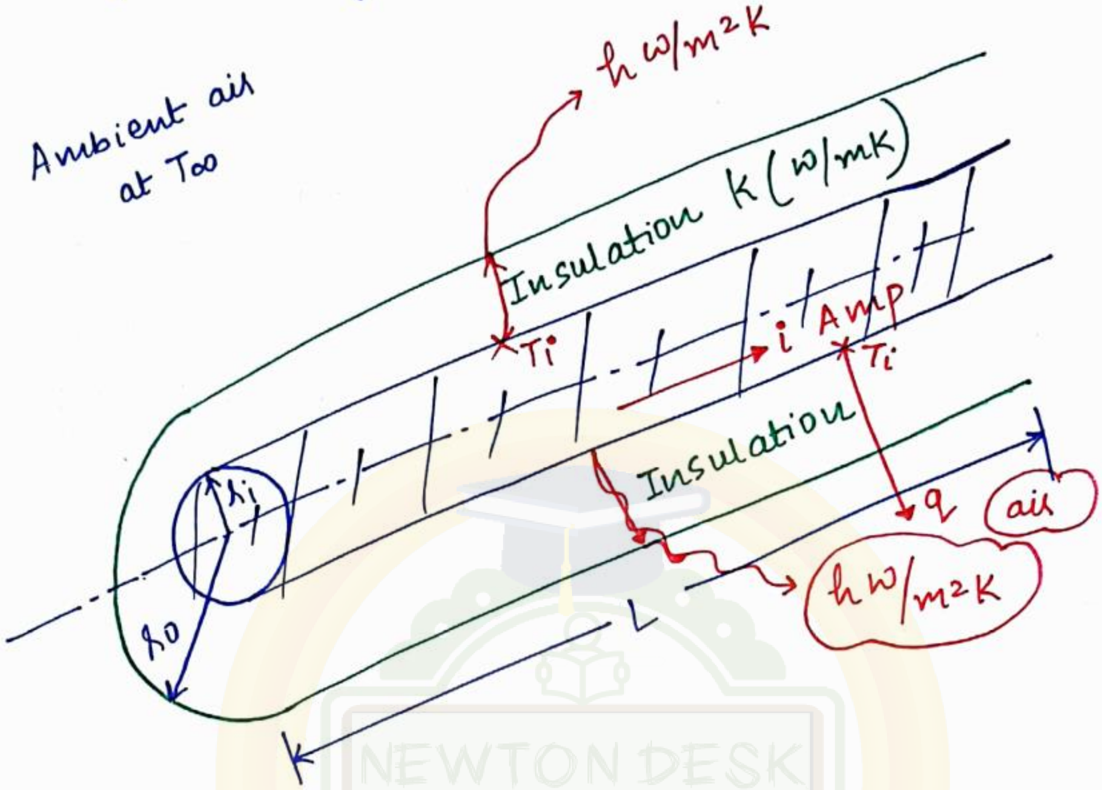
$r_i \simeq r_o$

$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}}$$

(Neglecting conduction Thermal Resistance of Tube wall)

* Critical Radius of Insulation

Note:- for sufficiently thin wires, putting the insulation around the wire may result in increase of heat transfer rate instead of decreasing it.



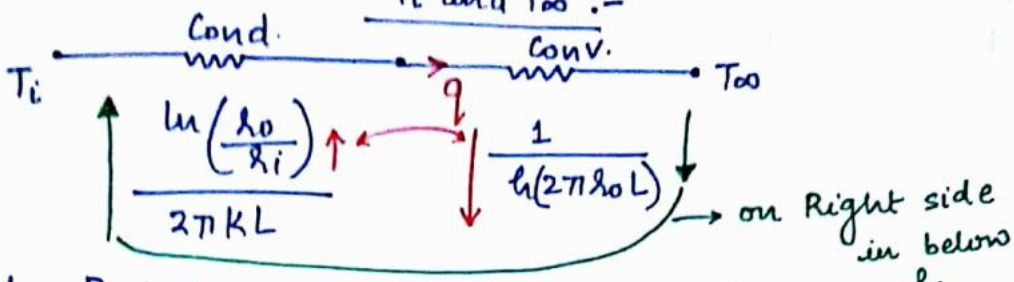
Consider a solid wire of radius r_i inside which heat is being generated by passing electric current. Let an insulation having a thermal conductivity 'K' is been wrapped around the wire upto the radius r_o.

The heat generated in the wire ~~is radially~~ due to passage of current is radially conducted through the insulation and then from the surface of the insulation heat is convected to the ambient fluid at $T\infty$. with a convective heat transfer coefficient of h W/m²K.

Assuming steady state H.T. conditions,

Let T_i be the surface tempr. of wire.

Drawing Thermal circuit for Radial H.T. b/w T_i and T_∞ :-



$\therefore$ Rate of Radial H.T. between wire and ambient $= q = \dfrac{(T_i - T_\infty)\text{ watt}}{\dfrac{\ln(r_o/r_i)}{2\pi KL} + \dfrac{1}{h 2\pi r_o L}}$

Treating all other parameters including h as constant in the above functional relationship. Now, q becomes a f^n of r_o only.

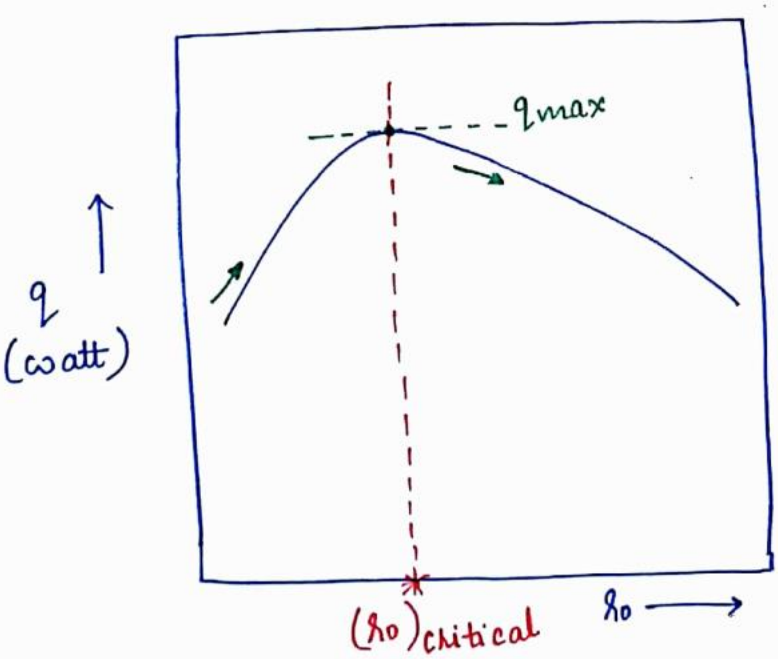
$$q = f(r_o) \text{ only}$$

For maximum H.T. rate,

$$\frac{dq}{dr} = 0 \Rightarrow \frac{d}{dr_o}\left[\frac{T_i - T_\infty}{\dfrac{\ln(r_o/r_i)}{2\pi KL} + \dfrac{1}{h 2\pi r_o L}}\right] = 0$$

$\Rightarrow r_o = \left(\dfrac{K_{Ins}}{h}\right)$ = This is called Critical Radius of Insulation.

* Physical Significance of Critical Radius of Insulation :-



for sufficiently thin wires whose radius is lesser than critical radius of insulation, putting the insulation around the wire will result in increase of H.T. rate instead of decreasing it. This happens so because initially when more and more insulation is being wrapped around the wire, there is a rapid decrease in convection, thermal resistance as compared to little increase of conduction thermal resistance, the overall effect being decrease in Total thermal resistance and increase of H.T. rate.

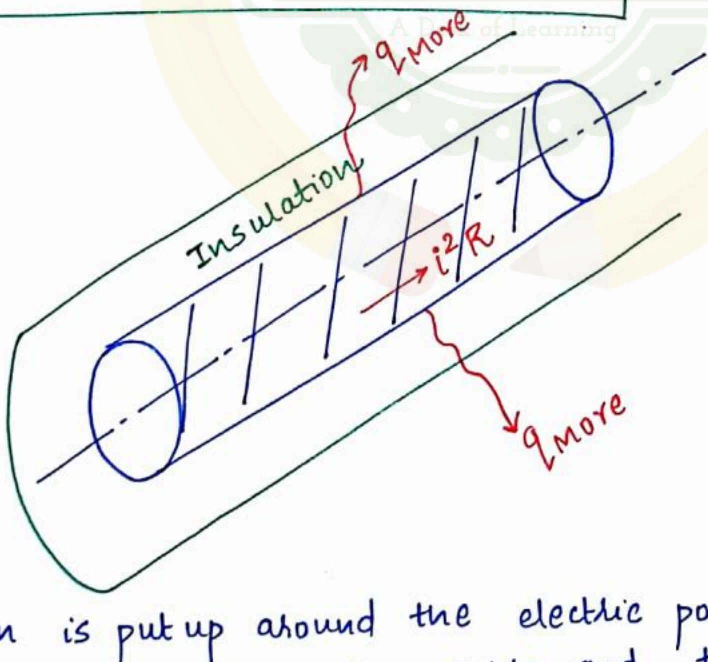
This continues upto critical radius of insulation beyond which any further insulation added shall decrease the heat transfer rate.

NOTE :- In case if the radius of the wire initially taken is already more than critical radius of insulation, any insulation wrapped around it shall directly decrease the H.T. rate.

Two Practical Applications of Critical Radius of Insulation :-

I) Electric Power Transmission Cables :-

$r_{critical} = k/h$

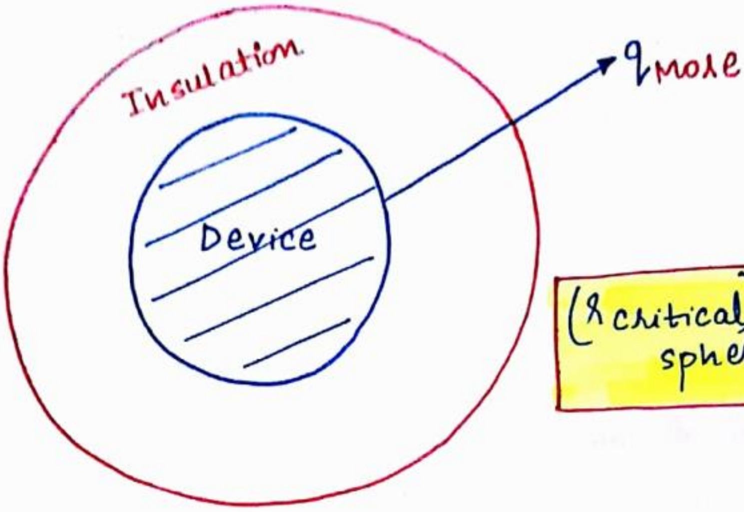


As $T_{cable} \uparrow \Rightarrow R_{electric} \uparrow$

Tempr. Resistance

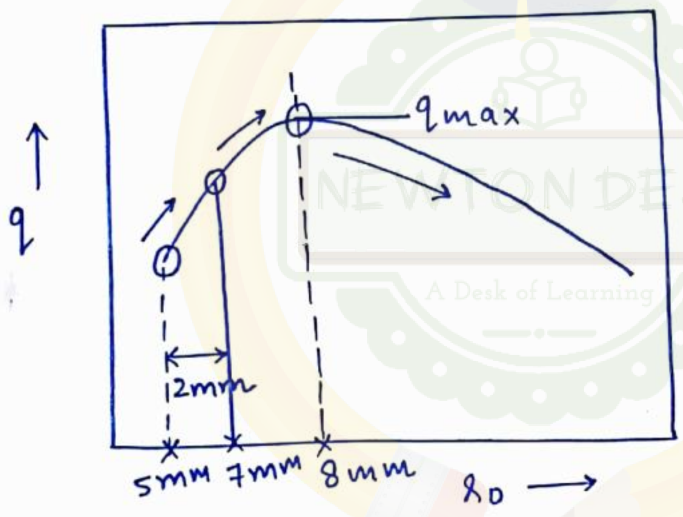
Insulation is put up around the electric power transmission cables to ↑ the H.T. Rate b/w the cable and the ambient so that the temp. of the cable can be maintained low thereby its electric resistance can be maintained low, thus transmitting more electric power.

② Spherical electronic (Semiconductor) Devices :-



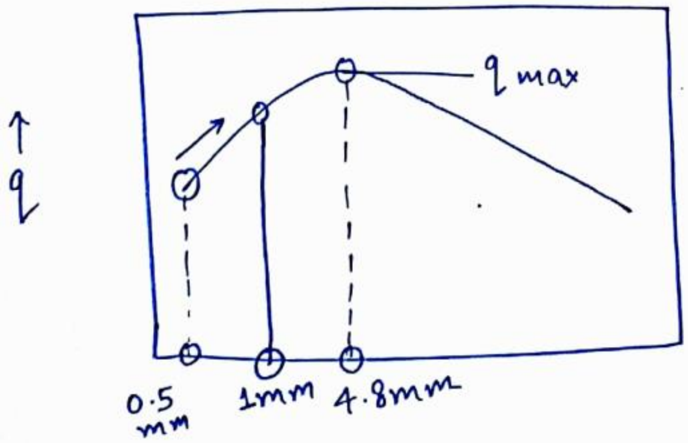
$$\boxed{(r_{critical})_{spherical} = \left(\frac{2K}{h}\right)}$$

WB
(40) $r_{critical} = \frac{K_{INS}}{h} = \left(\frac{0.08}{10}\right) m = 8\,mm$



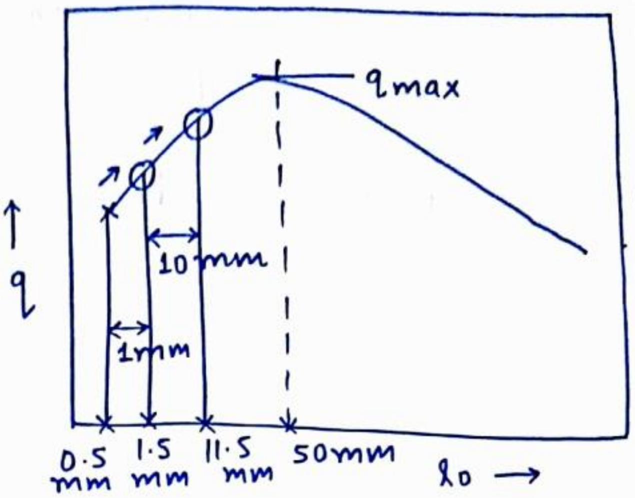
(c) ✓

(43) $r_{critical} = \frac{K_{Ins}}{h} = \frac{0.12}{(25)} = 4.8mm$



(32) $r_{critical} = \frac{K_{Ins}}{h}$

$= \left(\frac{0.5}{10}\right) = 50mm$



(a) ✓

As q from wire ↑

⇒ T_{wire} ↓ ⇒ $R_{electric}$ ↓

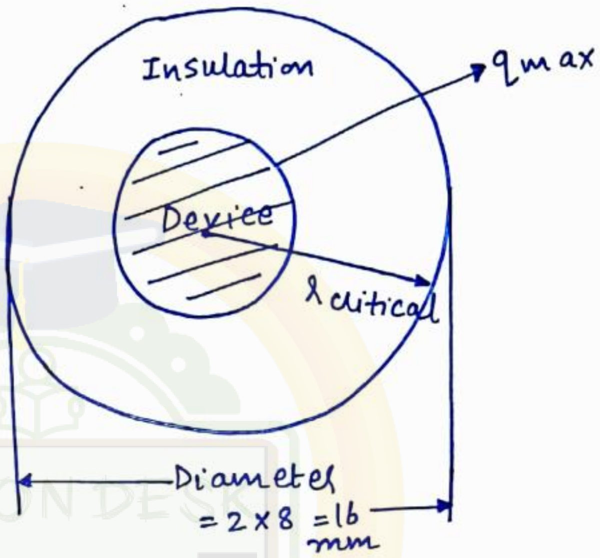
⇒ more electric Power can be transmitted

(30)

$(r_{critical})_{sph} = \frac{2K_{Ins}}{h}$

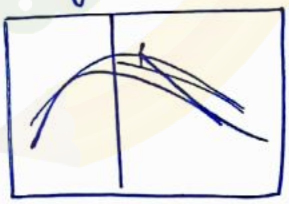
$= \frac{(0.04)}{10} \times 2$

$= 8\,mm$

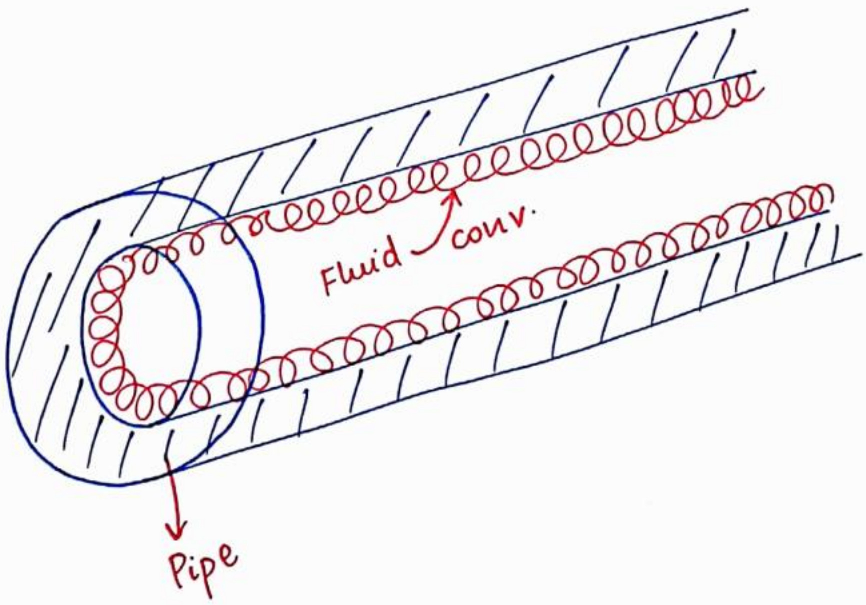


(29) c

provided the radius of cable initially taken is lesser than critical Radius of Insulation.



(42)



when Insulation is kept inside

⇒ convection area (or) contact area b/w fluid and pipe decreases

⇒ $R_{CONV} = \frac{1}{(hA)} \uparrow$

∴ Both $R_{cond} \uparrow$ and $R_{conv} \downarrow$ ⇒ H.T. rate always decrease

↓ $R_{Internal}$ ↓ $R_{surface}$

critical Radius NOT concept applicable

(a)

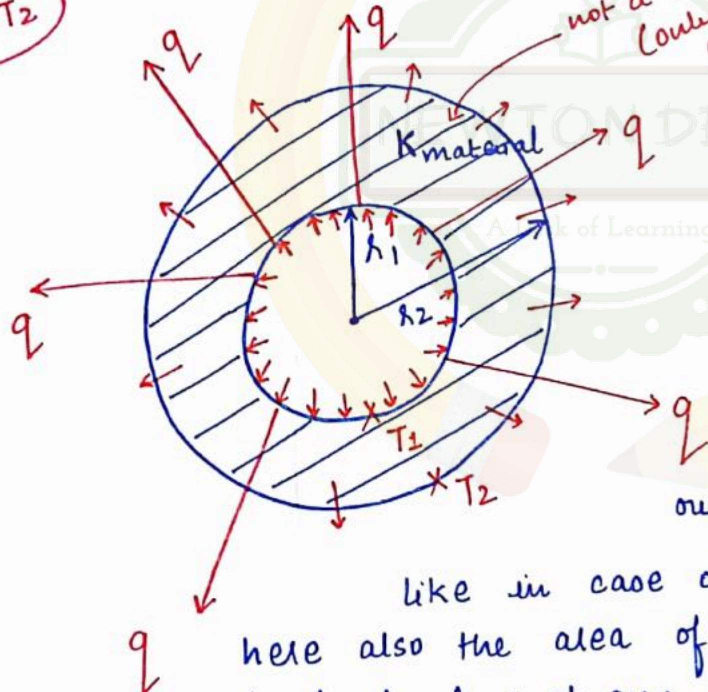
(Q 50)

$$r_{critical} = \frac{K_{INS}\,(\text{foam})}{h_o}$$

$$= \left(\frac{0.1}{2}\right) m = 5 cm.$$

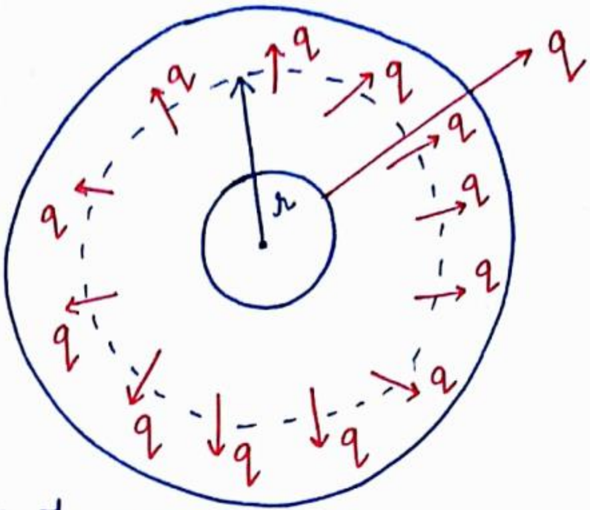
* $\boxed{\text{Radial Conduction H.T. through a hollow sphere}}$:-

$T_1 > T_2$



NOTE :- since tempr. gradients are existing along the radial direcn, assume steady state one dimensional conduction heat transfer between the inner spherical surface at T_1 to outer spherical surface at T_2.

like in case of cylindrical heat transfer, here also the area of conduction H.T. changes in the direction of heat flow. at any radius, 'r', area of conduction H.T. = $A = 4\pi r^2$.



Fourier's law of conduction :-

$\therefore$ Rate of Radial conduction $H.T. = q = -KA\frac{dT}{dr}$

$$\Rightarrow q = -K4\pi r^2\left(\frac{dT}{dr}\right)$$

$q \neq f(r)$ to satisfy steady state H.T. conditions $(q_r = q_{r+dr})$

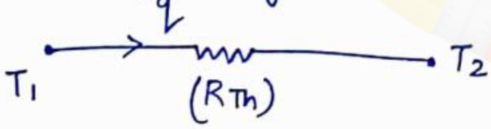
$$\Rightarrow \int_{r_1}^{r_2} q\frac{dr}{r^2} = \int_{T_1}^{T_2} -4\pi k\, dT$$

$$\Rightarrow q\left[-1/r\right]_{r_1}^{r_2} = 4\pi k(T_1 - T_2)$$

current (Thermal potential)

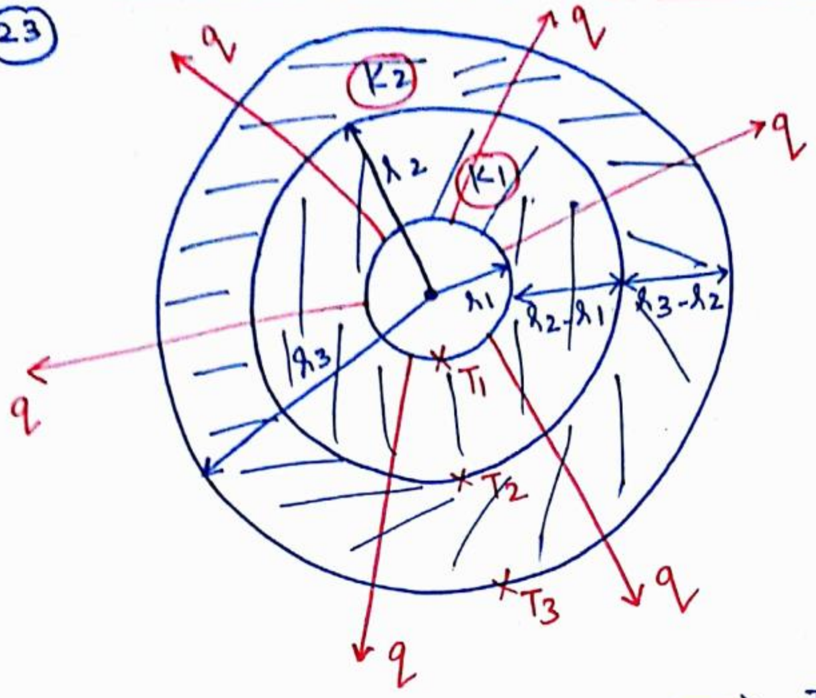
$$\Rightarrow q = \frac{4\pi k(T_1 - T_2)r_1 r_2}{r_2 - r_1}$$

The corresponding thermal Resistance for hollow sphere is :-



$$(R_{th})_{\substack{\text{hollow sphere}\\ \text{(conduction)}}} = \left(\frac{T_1 - T_2}{q}\right) = \frac{(r_2 - r_1)}{(4\pi k r_1 r_2)}\ K/watt$$

23



Thermal circuit

$$\frac{r_2 - r_1}{4\pi K_1 r_1 r_2} \qquad \frac{r_3 - r_2}{4\pi K_2 r_2 r_3}$$

$$q = \frac{T_1 - T_2}{\frac{\cancel{(r_2 - r_1)}}{4\pi K_1 r_1 \cancel{r_2}}} = \frac{T_2 - T_3}{\frac{\cancel{(r_3 - r_2)}}{4\pi K_2 \cancel{r_2} r_3}}$$

$$\Longrightarrow \frac{T_1 - T_2}{T_2 - T_3} = 2.5$$

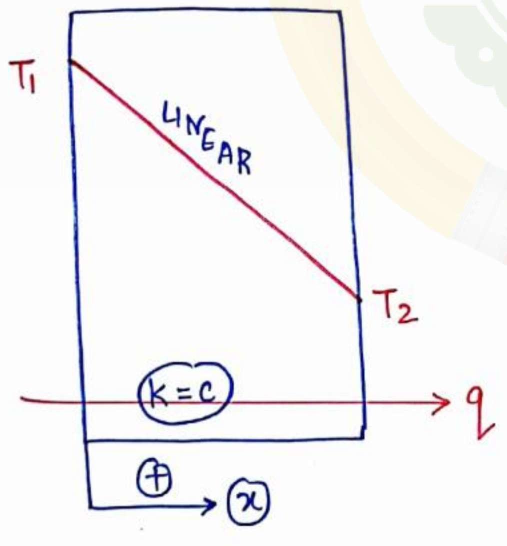
Thickness are equal

$$(r_2 - r_1) = (r_3 - r_2)$$

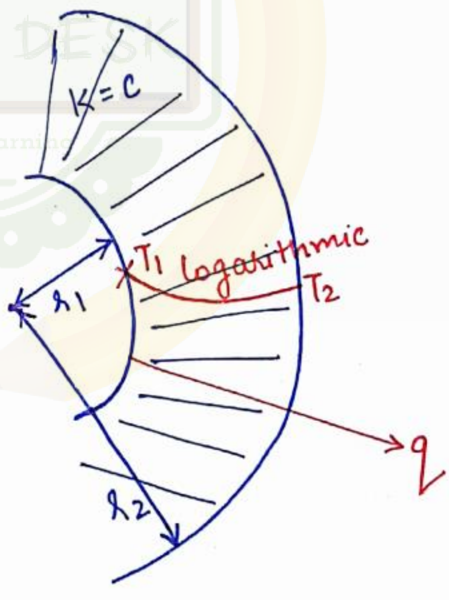
$$\frac{K_1}{K_2} = 1/2$$

$$\frac{r_1}{r_3} = 0.8$$

*

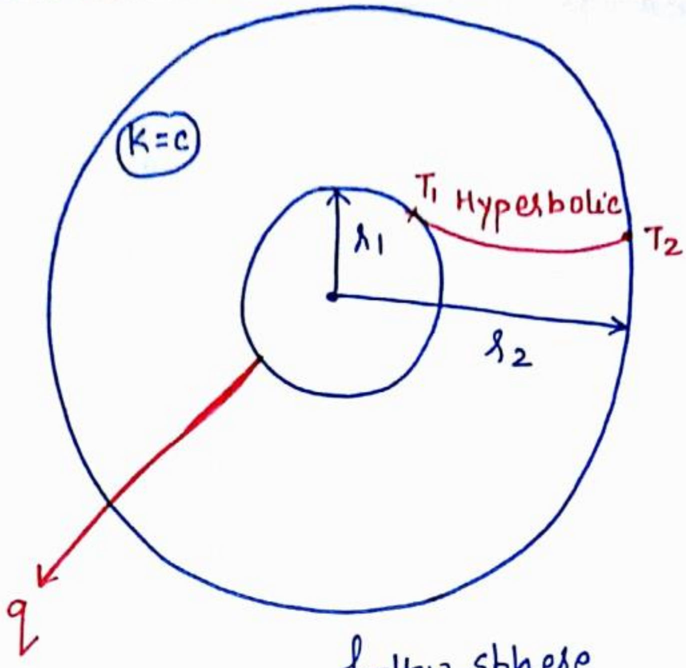


slab



hollow cylinder

*

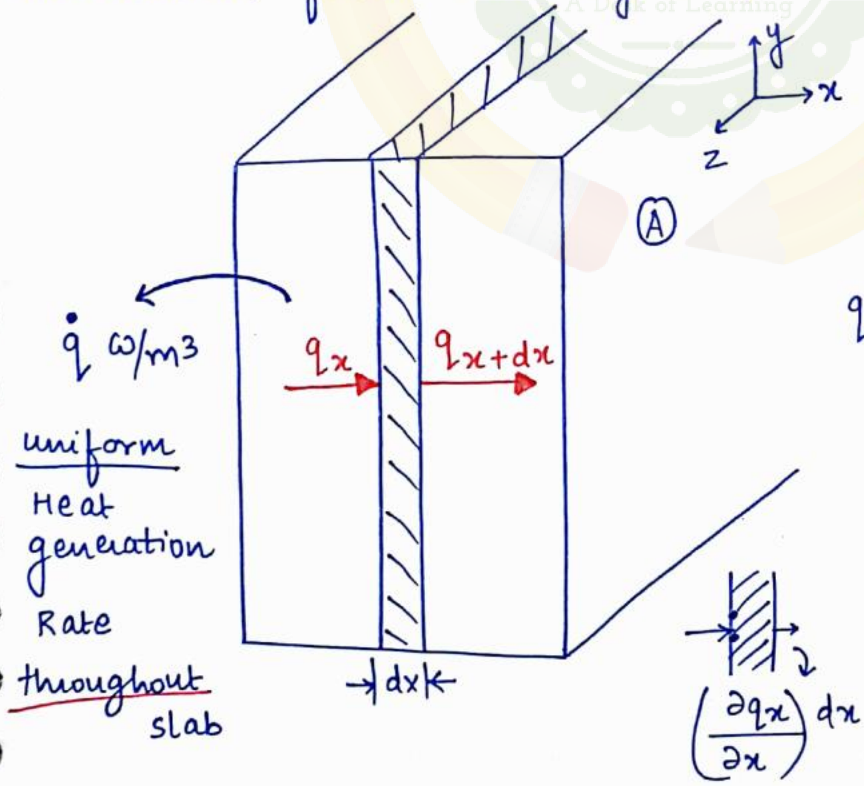


hollow sphere

$$R_{Th} = \frac{r_2 - r_1}{4\pi k r_1 r_2}$$

* Generalised (3D, steady or Unsteady, with or without heat generation) Conduction equation :-

Note :- heat can be generated inside a solid body either by supplying electric power or by exothermic chemical Rxn. or by thermonuclear fission. such diagram :-



Let q_x = Heat conducted into the element along x - direction

$$q_x = -KA\left(\frac{dT}{dx}\right) \text{ watt}$$

q_{x+dx} = Heat conducted out of the element along x- direction

$$= q_x + \frac{\partial}{\partial x}(q_x)\, dx \text{ watt}$$

Heat generated in the element = $\dot{q} A\, dx$ watt

Writing Energy balance for x-direction conduction through the element,

$q_x + q_{generated} = q_{x+dx}$ + Rate of change of I.E. of element wrt time.

internal energy

$$\cancel{q_x} + \dot{q}\,A\,dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + \frac{\partial}{\partial \tau}\underbrace{(mC_pT)}_{\text{I.E. of element}}$$

$\tau \rightarrow$ Time in sec

where m = Mass of element

$m = \rho \times (A\,dx)$

$$\dot{q}\cancel{A}\cancel{dx} = \frac{\partial}{\partial x}\left(-K\cancel{A}\frac{dT}{dx}\right)\cancel{dx} + \frac{\partial}{\partial x}\left(\rho\cancel{A}\cancel{dx}C_pT\right)$$

But all properties (K, ρ, C_p) being constant,

$$K\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p\left(\frac{\partial T}{\partial \tau}\right)$$

Writing the energy balance similarily for all the 3 Directional conductions that are occuring along x, y and z-directions simultaneously we get

$$K\frac{\partial^2 T}{\partial x^2} + K\frac{\partial^2 T}{\partial y^2} + K\frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p\left(\frac{\partial T}{\partial \tau}\right)$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \left(\frac{\rho C_p}{K}\right)\left(\frac{\partial T}{\partial \tau}\right)$$

Defining thermal diffusivity 'α', a thermophysical property of material as the ratio b/w the thermal conductivity of the material & its thermal capacity that is $\alpha = \left(\frac{K}{\rho C_p}\right)$ W/mK $\rightarrow \frac{m^2}{sec}$

Kg/m³ J/kgK

(ρC_p) $\rightarrow$ heat capacity (OR) heat storage ability.

$\boxed{\text{NOTE}}$:- α of a medium or material signifies the ability of the material to allow the heat energy to ~~kept~~ ~~di~~ get diffused or pass through the material more rapidly.

If the α of a material is more, if either thermal conductivity of material is more or if the heat capacity of the material is lesser. (41)

$$\alpha_{gases} > \alpha_{liquids}$$

Ex:- $\alpha_{air} > \alpha_{water}$

$K_{air} < K_{water}$

$(\rho C_p)_{air} <<< (\rho C_p)_{water}$ .... ——— Note over —.....

Now, derivation continues

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)}$$

If conditions are steady, then $\frac{\partial T}{\partial \tau} = 0$

$$T \neq f(\tau)$$

and If there is no heat generation, $\dot{q} = 0$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

$$\Rightarrow \boxed{\nabla^2 T = 0}$$

Laplace equ. in T.

JB Q14

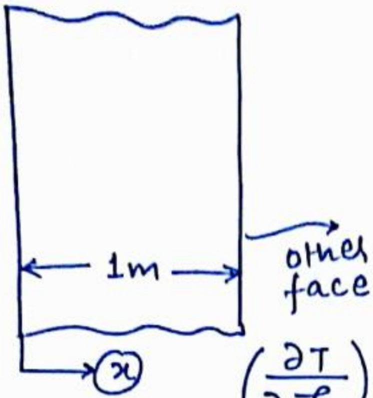
$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial t}\right)$$ ↳ Time

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2}\right)$$

But $\alpha = c$ (constant properties)

$$\frac{\partial T}{\partial t} \propto \frac{\partial^2 T}{\partial x^2}$$

Q27



$\left(\frac{\partial T}{\partial \tau}\right) = ?$

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{(\partial \tau)}$$

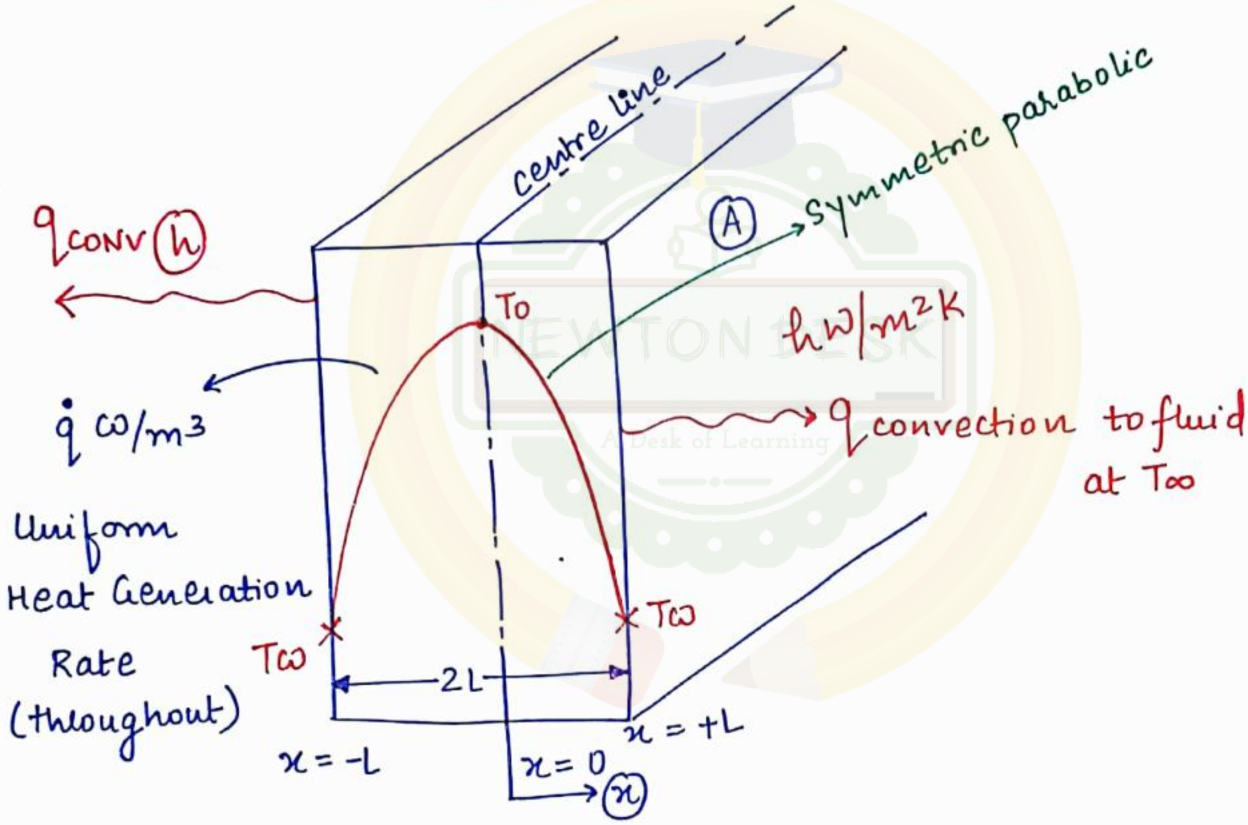
$$\frac{\partial T}{\partial \tau} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial T}{\partial \tau} = \alpha(40 + 60x)$$

Put $x = 1m$

$$\Rightarrow \left(\frac{\partial T}{\partial \tau}\right)_{at\ x=1m} = 2\times10^{-3}(100) = 0.2\,°C/Hr$$

* HEAT GENERATION IN A SLAB :-



To get Temp. distribution within the slab

Assume :- (i) steady state H.T. conditions ($T \neq f(\text{time})$)

To maintain this steady state conditions of the slab while generating heat, all the heat generated in the slab must be convected to a fluid either from one side of the slab or from both the sides.

② One - Dimensional heat conduction ($T = f(x)$) only.

③ Uniform heat generation rate and constant 'k' of material.

$$\frac{\partial^2 T}{\partial x^2} + \cancelto{0}{\frac{\partial^2 T}{\partial y^2}} + \cancelto{0}{\frac{\partial^2 T}{\partial z^2}} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\cancelto{0}{\frac{\partial T}{(\partial \tau)}}$$

1D 1D steady

$$\Rightarrow \frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \Rightarrow \frac{d^2T}{dx^2} = -\frac{\dot{q}}{k}$$

Integrating, $\frac{dT}{dx} = \frac{-\dot{q}}{k} x + C_1$

Integrating, $T = \frac{-\dot{q}}{k}\frac{x^2}{2} + C_1 x + C_2$

C_1 and C_2 are constants of $\int$ that are to be obtained from boundary conditions.

One Special boundary Condition is :-

At $x = +L$ and $x = -L \Rightarrow T = T_w$

i.e. both sides of slab are subjected to same Temp.

$\boxed{\text{NOTE}}$:- This can be possible only if both sides of the slab are subjected to the same fluid at the same tempr. with the same convective H.T. coefficient h. To satisfy this Boundary condn, C_1 must be zero.

Note over ....

The Tempr. of slab is max^m,

when $\frac{dT}{dx} = 0$

$$\Rightarrow \frac{-\dot{q}}{k} x + \cancelto{0}{C_1} = 0$$

$\Rightarrow x = 0$ i.e. we see max^m. Tempr. of slab at its centreline (or Mid-plane) of slab.

Note :- In case if both sides of the slab are at different tempr's. then C_1 would not be zero which means we would not see the maxm. tempr. at the mid-plane of the slab.

..over . . . .

Let the maxm. Tempr of slab be T_0.

$\therefore$ At $x = 0 \Rightarrow T = T_0$ $\left\{ \because \cancel{T}^{T_0} = -\frac{\dot{q}}{\cancel{k}} \cancel{\frac{x^2}{2}}^{0} + \cancel{C_1 x}^{0} + C_2 \right\}$

$\Rightarrow C_2 = T_0$

Therefore the tempr. distribution within the slab is :-

$$T = -\frac{\dot{q}}{K}\frac{x^2}{2} + T_0$$

$$\Rightarrow \boxed{T_0 - T = \frac{\dot{q}\,x^2}{2K}} \quad \text{--- (1)}$$

Parabolic Tempr. distribution.

At $x = +L$ (or) $x = -L \Rightarrow T = T_w$

$$T_0 - T_w = \frac{\dot{q}L^2}{2K} \quad \text{--- (2)}$$

$$\frac{(1)}{(2)} \Rightarrow \frac{T_0 - T}{T_0 - T_w} = \left(\frac{x}{L}\right)^2$$

Non-dimensional format of Tempr. distribution

The side wall tempr. T_w can be obtained from energy Balance eqn. for a steady state conditions of the slab that is heat generated in the slab = Heat convected from the slab to fluid

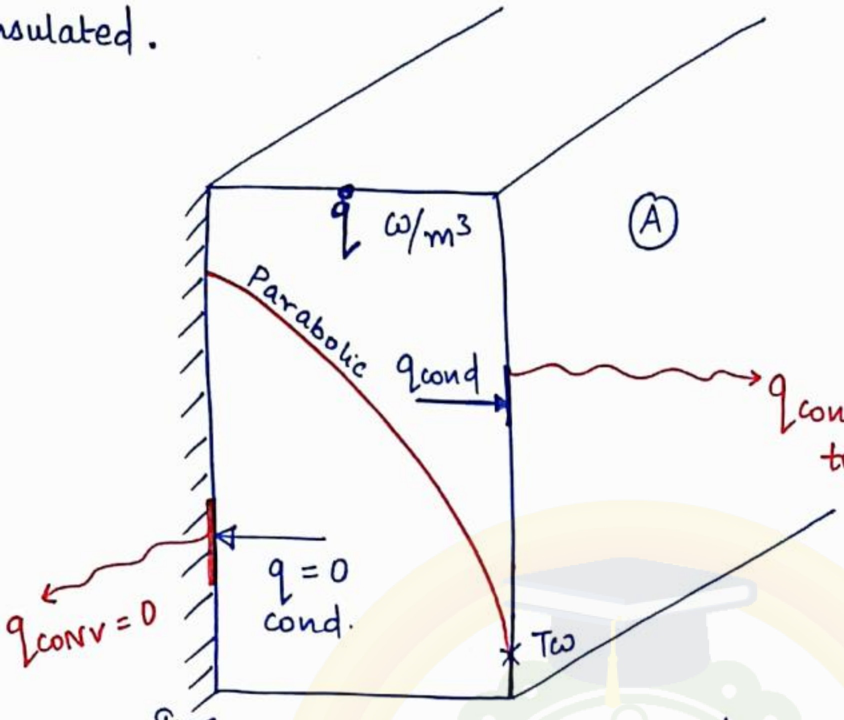
$$\Rightarrow \dot{q} \times (\cancel{2}L \times \cancel{A}) = \cancel{2}h\cancel{A}(T_w - T_\infty) \text{ watt}$$

$$\Rightarrow \boxed{T_w = \frac{\dot{q}L}{h} + T_\infty}$$

$$\Rightarrow T_0 \text{ (or) } T_{max} = \frac{\dot{q}L^2}{2K} + \frac{\dot{q}L}{h} + T_\infty$$

i.e. at the Mid Plane

The other extreme situation is one side of the slab perfectly insulated.

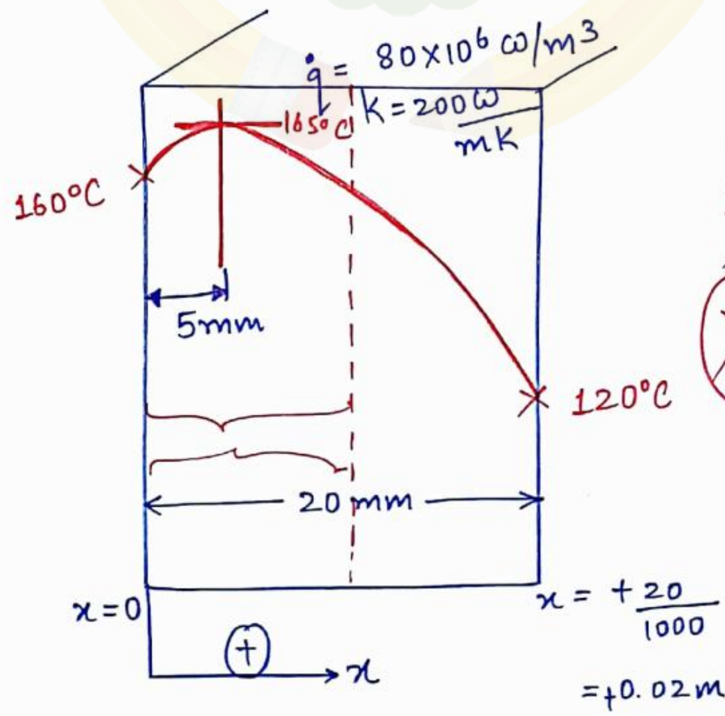


∴ Heat conducted at the Insulated surface $= 0$

$\Rightarrow -KA\left(\frac{dT}{dx}\right) = 0$ at the Insulated surface $\Rightarrow \frac{dT}{dx} = 0$ at the Insulated surface

$\Rightarrow$ T is maxm. at the Insulated surface.

WB Q17 and Q18



~~S~~ on the other side.

~~D~~ ← not symm.

S

~~I~~ ← no insulation

on the highest Tempr.

Since steady state, one dimensional heat conduction with uniform heat generation,

$$\frac{d^2T}{dx^2} + \frac{\dot q}{k} = 0$$

$$\frac{dT}{dx} = -\frac{\dot q}{k}x + C_1$$

$$\Rightarrow T = -\frac{\dot q}{k}\frac{x^2}{2} + C_1 x + C_2$$

At $x = 0 \Rightarrow T = 160°C$

$$160 = C_2$$

At $x = +0.02m \Rightarrow T = 120°C$

$$120 = \frac{-80\times10^6}{200}\frac{(0.02)^2}{2} + C_1(0.02) + 160$$

$$\Rightarrow C_1 = +2000$$

The tempr. of slab is Maximum, when $\frac{dT}{dx} = 0$

$$\Rightarrow 0 = -\frac{\dot q}{k}x + C_1$$

$$\Rightarrow 0 = -\frac{80\times10^6}{200}x + 2000$$

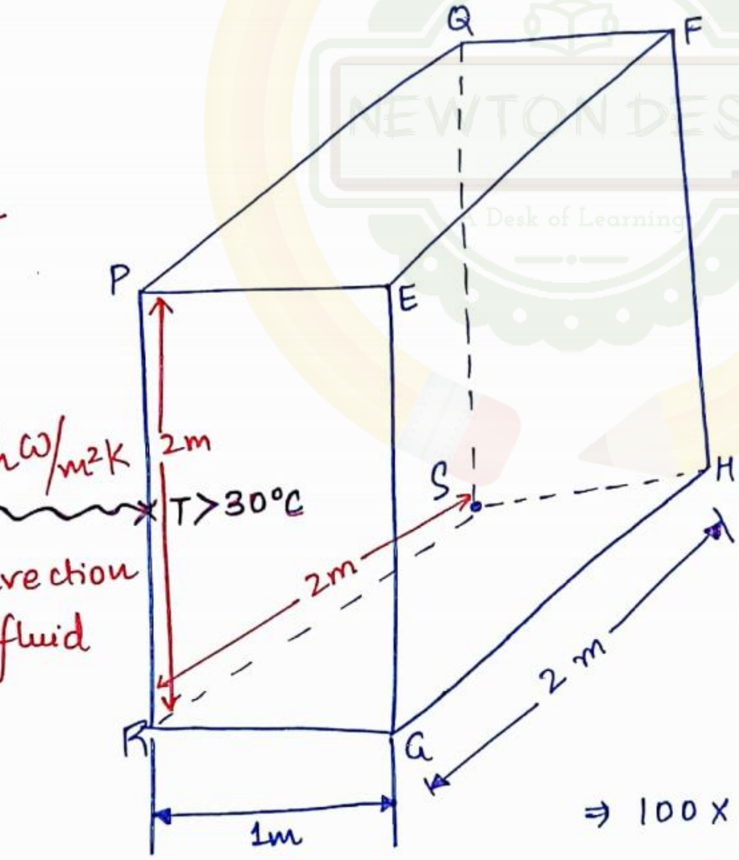
$$\Rightarrow x = 0.005\,m \text{ (location of } T_{max})$$

$$\therefore T_{max} = -\frac{80\times10^6}{200}\frac{(0.005)^2}{2} + 2000(0.005) + 160$$

$$= 165°C$$

Q19

10
20
30
40



$\therefore$ For steady state conditions of slab,

Heat generated in the slab = Heat convected from PQRS to fluid

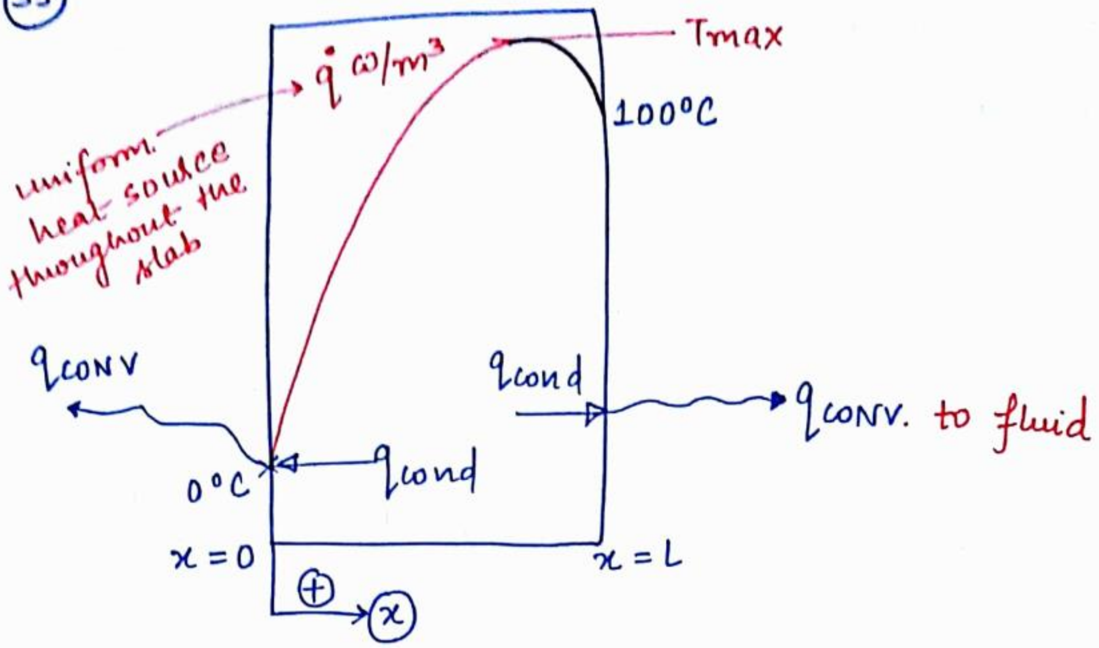
Newton's law of cooling

$$\Rightarrow \dot q \times \text{volume of slab} = h\,A_{PQRS}(T_{PQRS} - T_{fluid}) \text{ watt}$$

$$\Rightarrow 100\times(2\times1\times2)\text{watt} = 10\times2\times2(T_{PQRS} - 30)\text{watt}$$

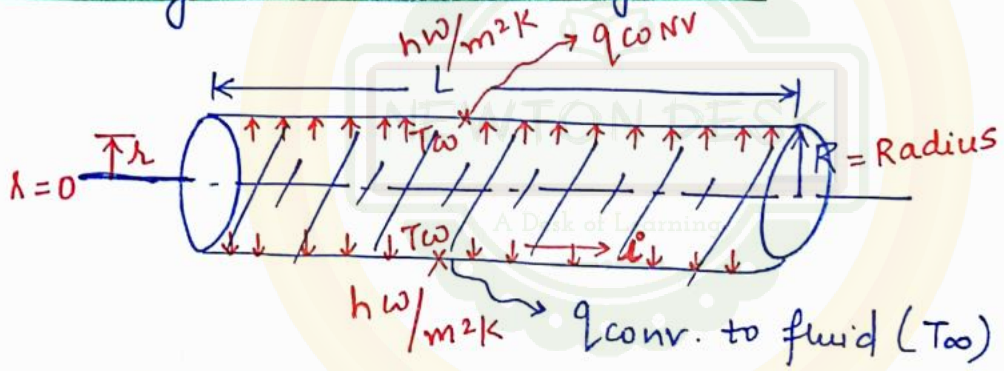
$$W/\cancel{m^3}\times\cancel{m^3}$$

$$T_{PQRS} = 40°C$$



convection at the Boundary is always preceeded by conduction at the boundary (for steady state H.T. conditions).

* **Heat generation in the cylinder** :-



$\dot{q}$ = uniform Heat generation rate

$$= \frac{i^2 \cdot R_{electric}}{\pi R^2 L} \quad watt/m^3$$

Objective :- To get Tempr. distribution within the cylinder i.e. $T = f(r)$

Assume :- ① steady state H.T. conditions $T \neq f(Time)$.

To maintain this steady state condition of the rod while generating heat, all the heat generated in the rod must be convective to a fluid surrounding the rod with a convective H.T. coefficient of h W/m²K

② One-dimensional (Radial) conduction H.T.

$$T = f(r) \text{ only}$$

③ Uniform Heat generation Rate & constant 'K' of Material.

The Generalized Heat conduction eqn. in cylindrical coordinate system is given by :-

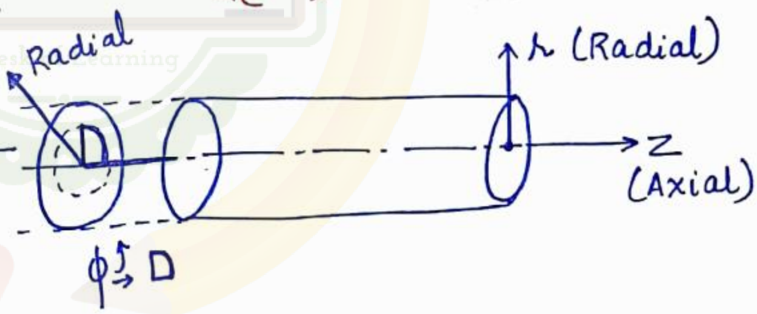
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}}}_{1D}^{\,0} + \underbrace{\cancel{\frac{\partial^2 T}{\partial z^2}}}_{1D}^{\,0} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\underbrace{\cancel{\left(\frac{\partial T}{\partial \tau}\right)}}_{\text{steady}}^{\,0}$$

$$\Rightarrow \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{-\dot{q}}{K}$$

$$\Rightarrow r\frac{d^2T}{dr^2} + \frac{dT}{dr} = \frac{-\dot{q}r}{K}$$

$d(vu) = u\,dv + v\,du$.

$$\Rightarrow \frac{d}{dr}\left(\overset{u}{r}\,\overset{v}{\frac{dT}{dr}}\right) = \frac{-\dot{q}r}{K}$$



Integrating w.r.t. 'r',

$$r\frac{dT}{dr} = \frac{-\dot{q}r^2}{2K} + C_1$$

$$\Rightarrow \frac{dT}{dr} = -\frac{\dot{q}r}{2K} + \frac{C_1}{r} \Rightarrow \left(\frac{dT}{dr}\right)_{at\, r=R} = \frac{-\dot{q}R}{2K} + \frac{C_1}{R} \longrightarrow ②$$

Integrating,

$$T = \frac{-\dot{q}r^2}{4K} + C_1 \log r + C_2$$

C_1 & C_2 are constants of $\int$ that are to be obtained from boundary condn's.

One boundary condition is :-

At $r = R$ (i.e. at the periphery of Rod) $\Rightarrow T = T_w$

The second boundary condition is:-

For steady state conditions of Rod, writting energy Balance:-

Heat generated in the Rod = Heat conducted Radially at the surface = Heat convected from surface to fluid.

$$\Rightarrow \dot{q} \times \pi R^2 L = -K(2\pi R L)\left(\frac{dT}{dr}\right)_{at\ r=R}$$

$$\Rightarrow \boxed{\left(\frac{dT}{dr}\right)_{at\ r=R} = \frac{-\dot{q}R}{2K}} \longrightarrow (1)$$

comparing eqⁿs (1) and (2),

C_1 must be zero

The tempr. of the rod is maximum, when

$$\left(\frac{dT}{dr}\right) = 0$$

$$\Rightarrow 0 = \frac{-\dot{q}r}{2K} + \cancel{\frac{C_1}{r}}^{\,0}$$

$\Rightarrow r = 0$ (i.e. at the axis of Rod)

i.e. we see Max. Tempr. at the axis of Rod

Let the maxᵐ. Tempʳ of Rod be T_o

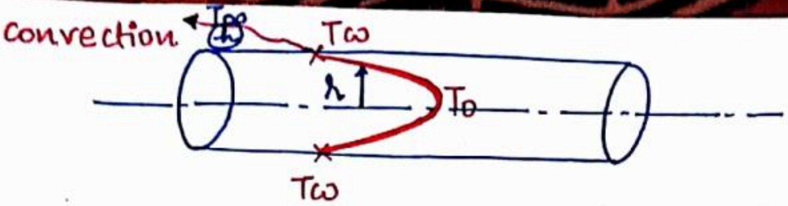
At $r = 0 \Rightarrow T = T_o$

$\Rightarrow T_o = C_2$

$\therefore$ The tempr. distribution within the rod is :-

$$T = \frac{-\dot{q}r^2}{4K} + T_o \Rightarrow \boxed{T_o - T = \frac{\dot{q}r^2}{4K}} \longrightarrow (3)$$

Parabolic Tempr Distribution.



At $r = R \Rightarrow T = T_w$

$$\therefore T_0 - T_w = \frac{\dot{q} R^2}{4K} \longrightarrow (4)$$

$$\frac{(3)}{(4)} \Rightarrow \frac{T_0 - T}{T_0 - T_w} = \left(\frac{r}{R}\right)^2$$ [Tempr. distribution in non-dimensional format]

The surface temperature T_w can be obtained from Energy Balance Eqn. for steady state conditions of the slab i.e..

Heat generated in the Rod = Heat convected to fluid from surface.

$$\Rightarrow \dot{q} \times (\pi R^2 \cancel{L}) = h \, 2\pi R \cancel{L} (T_w - T_\infty) \text{ watt}$$

$$\therefore T_w = \frac{\dot{q} R}{2h} + T_\infty$$

surface Tempr. of rod

$$\therefore T_0 \text{ (or) } T_{max} = \left(\frac{\dot{q} R^2}{4K} + \frac{\dot{q} R}{2h} + T_\infty\right)$$

Q.> A cylindrical uranium fuel rod of radius 5.0mm in a nuclear reactor is generating heat at the rate of 4×10^7 W/m³. The rod is cooled by a liquid (convective heat transfer coefficient is 1000 W/m²K) at 25°C. at steady state, the surface tempr. in K of the rod is

(a) 308

(b) 398 ✓

(c) 418

(d) 448

$$T_w = \frac{4 \times 10^7}{2 \times 1000} \times \frac{5}{1000} + 25^\circ C$$

$$T_w = 125^\circ C$$

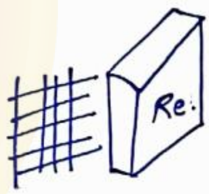
$$T_w = 398\,K$$

FINS

(EXTENDED SURFACES)

Fins are the projections protuding from a hot surface into ambient fluid and they are meant for increasing H.T. rate by increasing the surface area of Heat Transfer.

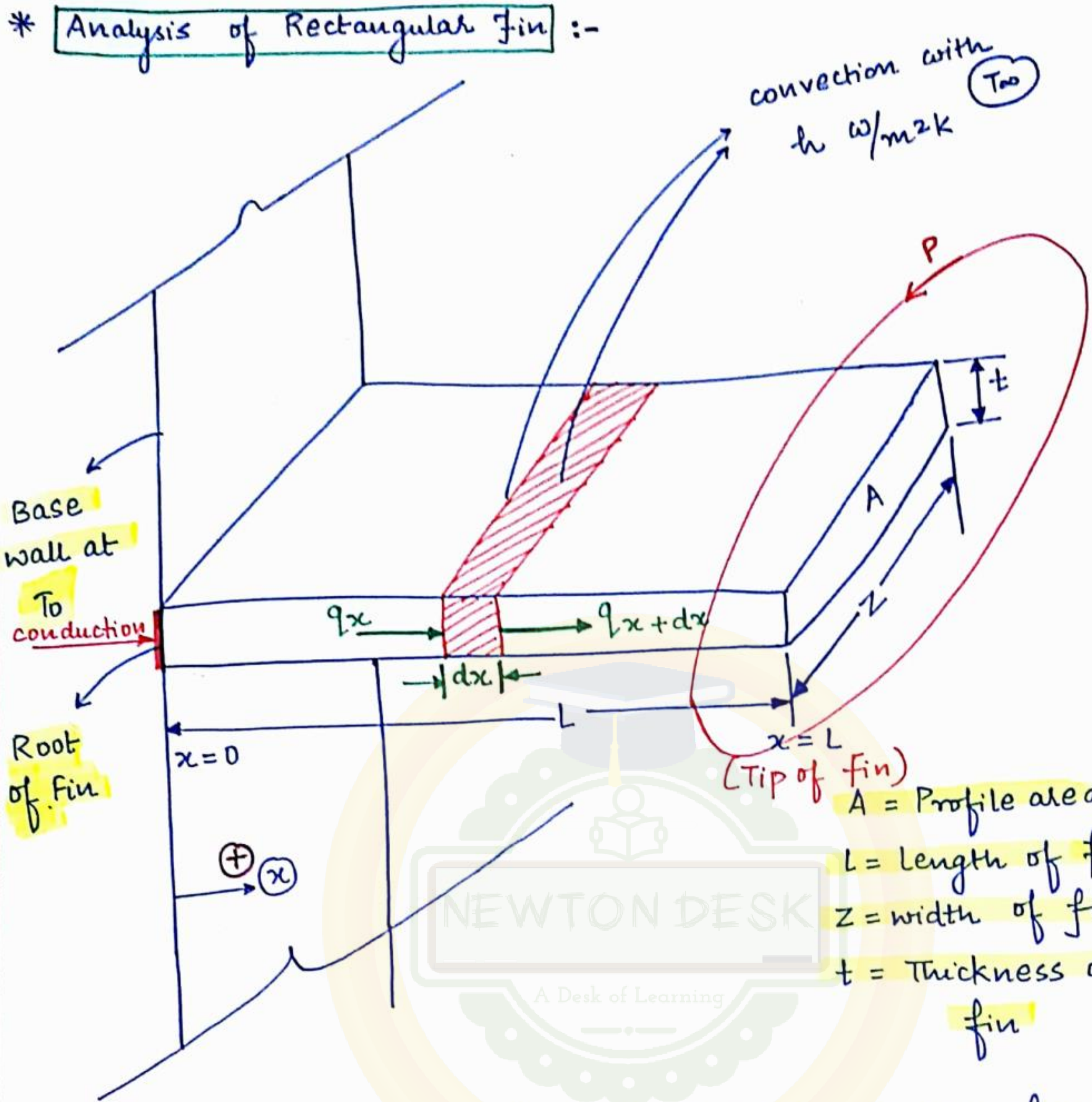
Ex :-
1. Air cooled I/c Engines.
2. Reciprocating Air compressors.
3. Refrigerator and A/c condenser units.
4. Electric motors and Transformers.
5. Electronic Devices.
6. Automobile Radiator.



Note :- fins are always used only when the convective H.T. coeffi's are relatively low that is with gases (air).

h_{low}.

* Analysis of Rectangular Fin :-



A = Profile area = Z × t
L = Length of fin
Z = width of fin
t = Thickness of fin

The actual mechanism of Heat Transfer in any fin is first heat gets conducted into the fin at its root and then while conducting along the length of the fin that is in the x- direction, it is also simultaneously convecting from the surface of the fin into the ambient fluid at T_∞ with a convective heat transfer coeffi. of h W/m²K.

Objective :- ① To get Temperature distribution within the fin that is $T = f(x)$.

② To get H.T. Rate through fin $q_{fin} = ?$

Assume :- Steady state Conditions of fins i.e. $T \neq f(\text{Time})$.

Consider a differential element of the fin of length dx at a distance of x from the root of fin.

Let q_x = Heat conducted into the element

$$= -KA\left(\frac{dT}{dx}\right)$$

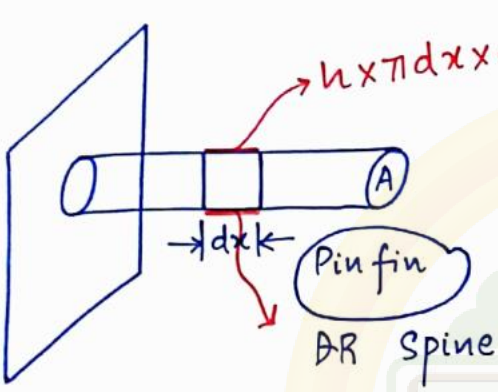
q_{x+dx} = Heat conducted out of the element

$$= q_x + \frac{\partial}{\partial x}(q_x)\,dx.$$

Heat convected from the surface of Fin = $hP\,dx\,(T-T_\infty)$

where P = perimeter of fin

$= (2z + 2t)$



writting Energy Balance for steady state conditions of element of fin,

$$q_x = q_{x+dx} + q_{\text{convected}}$$

$$\Rightarrow \cancel{q_x} = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + hP\,dx\,(T-T_\infty)$$

$$\Rightarrow 0 = \frac{\partial}{\partial x}\left(-KA\frac{dT}{dx}\right)\cancel{dx} + hP\cancel{dx}(T-T_\infty)$$

$$\Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{KA}(T-T_\infty) = 0$$

Let $(T-T_\infty) = \theta = f(x)$

$$\Rightarrow \frac{dT}{dx} = \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

and Put $m^2 = \left(\frac{hP}{KA}\right)$

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

This is a standard format of 2nd order Differential Equ. in θ whose solution can be given as :-

$$T - T_\infty = \theta = C_1 e^{-mx} + C_2 e^{mx}, \text{ where } m = \sqrt{\frac{hP}{KA}} \text{ /metre}$$

C_1 and C_2 are constant's of integration that are to be obtained from boundary conditions.

One common Boundary condition is :-

At $x = 0$ (i.e. at the Root of fin) $\Longrightarrow T = T_0$ and $\theta = \theta_0 = (T_0 - T_\infty)$

The second Boundary condition depends upon 3 different cases of fin :-

Case I :- Fin is infinitely long (OR) Very long Fin.

Then the tempr. at the tip of the fin will be essentially that of the ambient Fluid. i.e. at

at $x = \infty \Rightarrow T = T_\infty$ and hence $\theta = 0°$.

then the solution for Temperature distribution within the fin is :-

Tempr. distribution

$$\boxed{\frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}}$$

Heat Transfer

$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty) \text{ watt}}$$

for any Fin case,

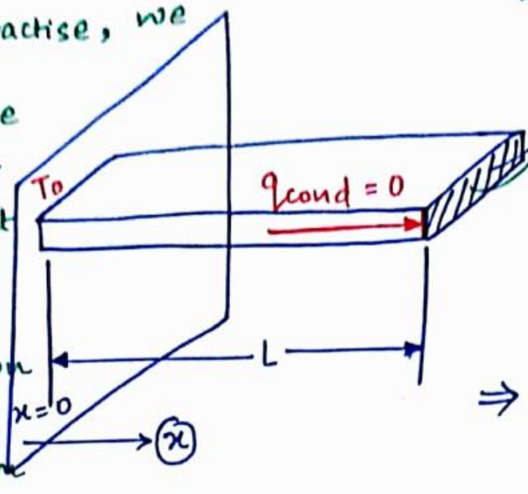
$q_{\text{through entire fin}}$ = Heat conducted into the Fin at its Root/base

$$= -KA\left(\frac{dT}{dx}\right)_{\text{at } x=0}$$

$\boxed{\text{Case 2}}$:- Fin is Finite in length but its Tip is insulated (adiabatic Tip)

small $\leftarrow (z \times t)$

In practise, we never insulate tip of a fin, but we neglect convection heat loss from Tip.



q_{conv} from Tip $= \underset{\text{small}}{h}\, A \underbrace{(T_{x=L} - T_\infty)}_{\text{small}}$ ($\because$ product is too small)

$q = 0$ convection from Tip

55

$\therefore$ Heat conducted into the Tip of Fin = 0

$$\Rightarrow -KA\left(\frac{dT}{dx}\right)_{at\ x=L} = 0 \quad \text{i.e. at the insulated Tip}$$

$$\Rightarrow \left(\frac{dT}{dx}\right)_{at\ x=L} = 0 \quad \Rightarrow \left(\frac{d\theta}{dx}\right)_{at\ x=L} = 0$$

$\downarrow$ 2nd boundary condition

Then the tempr. distribution within the Fin is :-

$$\boxed{\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = \frac{\cosh m(L-x)}{\cosh mL}}$$

Also

$$q_{fin} = \sqrt{hPKA}\ \_\_\_ = -KA\left(\frac{dT}{dx}\right)_{at\ x=0}$$

$$\downarrow$$

$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh mL \text{ watt}}$$

In any fin problem, if no case is mention, by default assume case II.

$\boxed{\text{Case 3}}$:- Fin is finite in length but its Tip is Uninsulated (i.e. convection heat loss from Tip also)

Then the solution for Tempr. distribution within the Fin is :-

$$\boxed{\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh mL_c}}$$

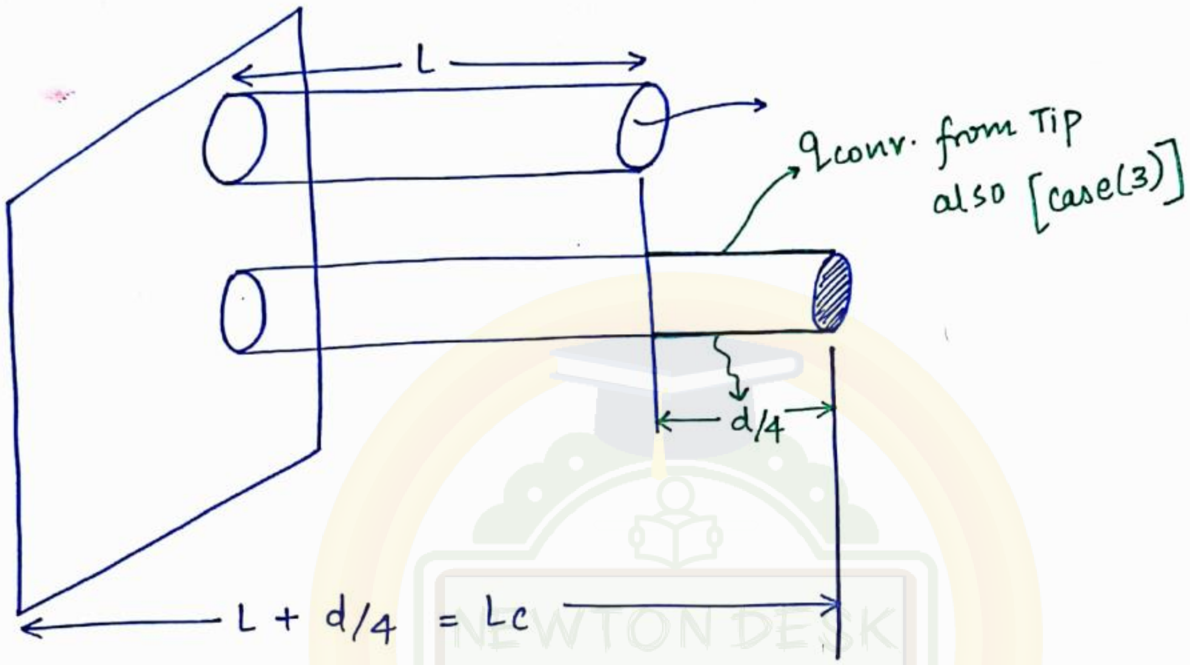
where L_c = corrected length of fin.

$= L + t/2$ (for Rectangular fin)

$= L + d/4$ (for Pin Fin)

Also, then

$$\boxed{q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\,\tanh mL_c}$$



WB Q34

$d = 5mm$

$L = 100\,mm$

$K = 400$

$T_0 = 130°C$

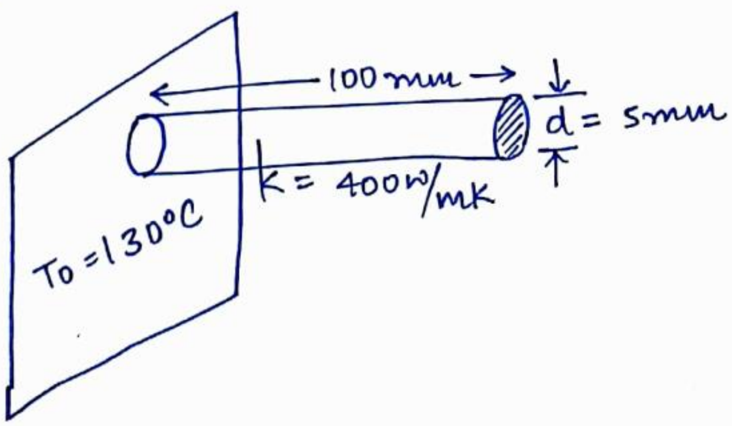
$T_\infty = 30°C$

$h = 40 W/m^2K$

$P =$

$m = \sqrt{\frac{hP}{KA}}$

SIR



Since No case is mentioned, Assume case (2) i.e.
insulated Tip case

$m = \sqrt{\dfrac{hP}{KA}}$ /metre

$P = \pi d = \boxed{}$

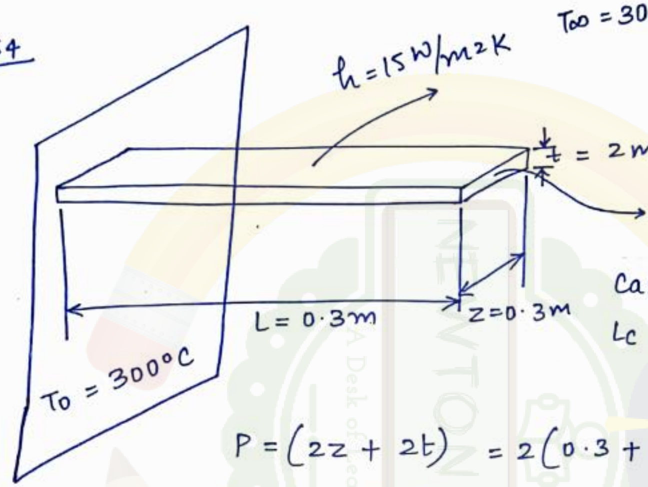
$A = \dfrac{\pi}{4} d^2 = \boxed{}$

$m = 8.944$/metre

$q_{fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\,\tanh mL$ watt

$= 5$ watt

Q54



Case (3)

$L_c = \left(L + t/2\right) = 0.301\,m$

$P = (2z + 2t) = 2(0.3 + 2/1000)$

$A = (z \times t)\,m^2$

$m = \sqrt{\dfrac{hP}{KA}} = 8.603$/metre

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L_c - x)}{\cosh mL_c}$$

Put $x = 0.3m$

$T_{(at\ x = 0.3m)} = \boxed{70.3}\ ^\circ C$

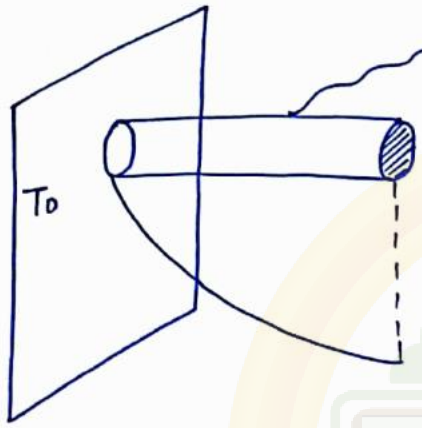
$q_{through\ fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\,\tanh mL_c$ watt

$= 281.1$ watt.

* FIN EFFICIENCY (η_{fin}) :-

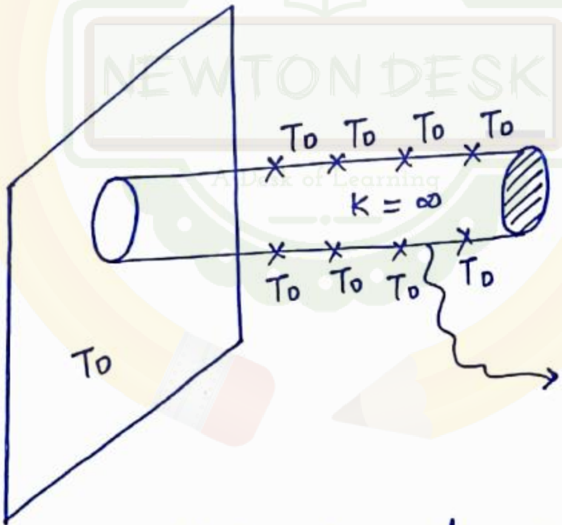
fin efficiency is defined as the ratio b/w actual heat transfer rate taking place through the fin and the maxm. possible heat transfer rate that can occur through the fin that is when the entire fin surface is at its root or base temperature.

$$\boxed{\eta_{fin} = \frac{q_{act}}{q_{max.possible}}}$$



$q_{act} = \sqrt{hPKA}\,(T_0 - T_\infty)\,\mathrm{Tanh}\,mL$

case (2)



$q_{max.possible} = h(PL)(T_0 - T_\infty)$

The entire fin will be at its Root Temp. (T_0) only when the Material of fin has infinite 'K'.

$$\eta_{fin} = \frac{\sqrt{hPKA}\,\cancel{(T_0 - T_\infty)}\,\mathrm{Tanh}\,mL}{hPL\,\cancel{(T_0 - T_\infty)}}$$

case (2)

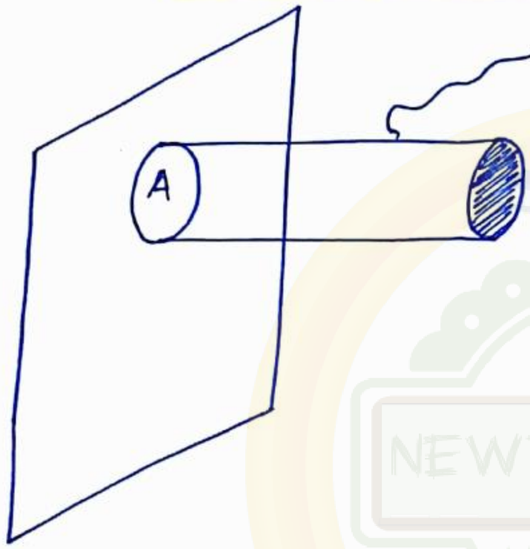
$$= \frac{\mathrm{Tanh}\,mL}{\sqrt{\frac{hP}{KA}} \times L} = \frac{\mathrm{Tanh}\,mL}{mL}.$$

$$\eta_{fin_{\text{case (3)}}} = \frac{\tanh mL_c}{mL_c} \qquad \eta_{fin} \propto \sqrt{K}.$$

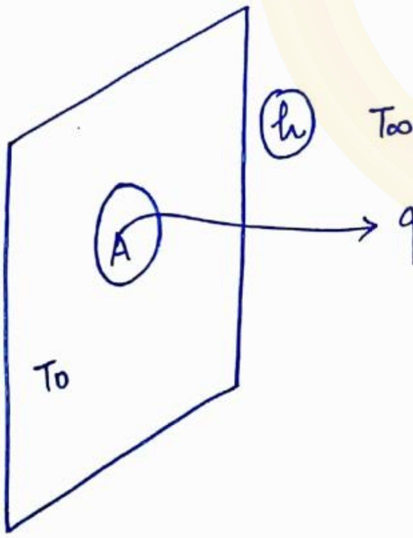
$$= 38.18\%$$

* $\boxed{\text{FIN effectiveness } (\epsilon_{fin})}$:- It is defined as the Ratio b/w heat transfer with fin and the heat transfer rate without fin.

$$\epsilon_{fin} = \frac{q_{with}}{q_{without}}$$



$$q_{with\ fin} = \sqrt{hPKA}\,(T_0 - T_\infty)\tanh mL$$



$$q_{without\ fin} = hA\,(T_0 - T_\infty)$$

$$\therefore \epsilon_{fin_{\text{Case (II) insulated Tip case}}} = \frac{\sqrt{hPKA}\,\cancel{(T_0 - T_\infty)}\tanh mL}{hA\,\cancel{(T_0 - T_\infty)}}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hA}{KP}}}$$

ϵ_{fin} tells about how much % increase in heat transfer rate, we are able to achieve (gain) by keeping the fins as compared to the case where there is no fin. If affectiveness of fin is low then fins are not worth keeping because they donot help us much in increasing the H.T. rate.

$$\epsilon_{fin} \propto 1/\sqrt{h}$$

This is the reason why fins are never used with water or condensation heat transfer where the convective H.T. coeff's are relative high which means effectiveness is low.

$$\epsilon_{fin} \propto \sqrt{\frac{P}{A}}$$

NOTE :- ∴ To have high effectiveness of fin,

Fins must be <u>thin</u> and closely spaced fins.

Ex :- Car Radiator.

$$\epsilon_{fin} \propto \sqrt{K}$$

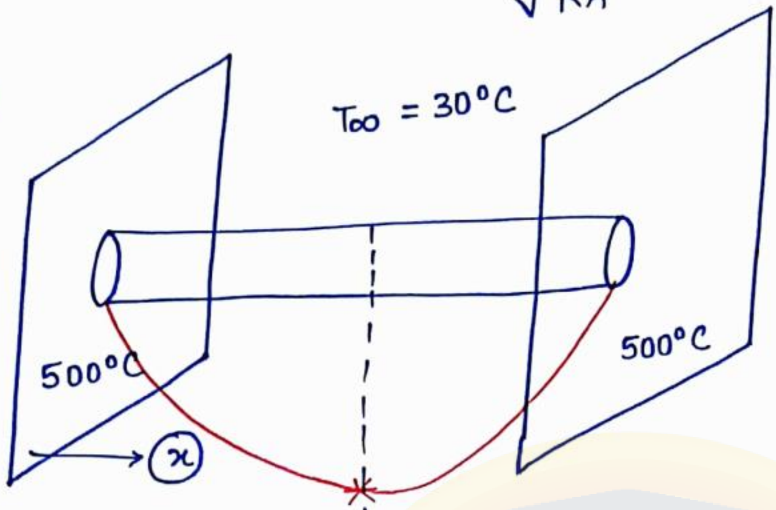
Fins →
- Moderately short
- No. of fins
- closely spaced fins
- Thin
- High Conductivity material (Al)

$$\boxed{\text{No. of fins Regulated} = n = \frac{\text{Total Heat to be dissipated}}{\text{H.T. Rate through each fin}}}$$

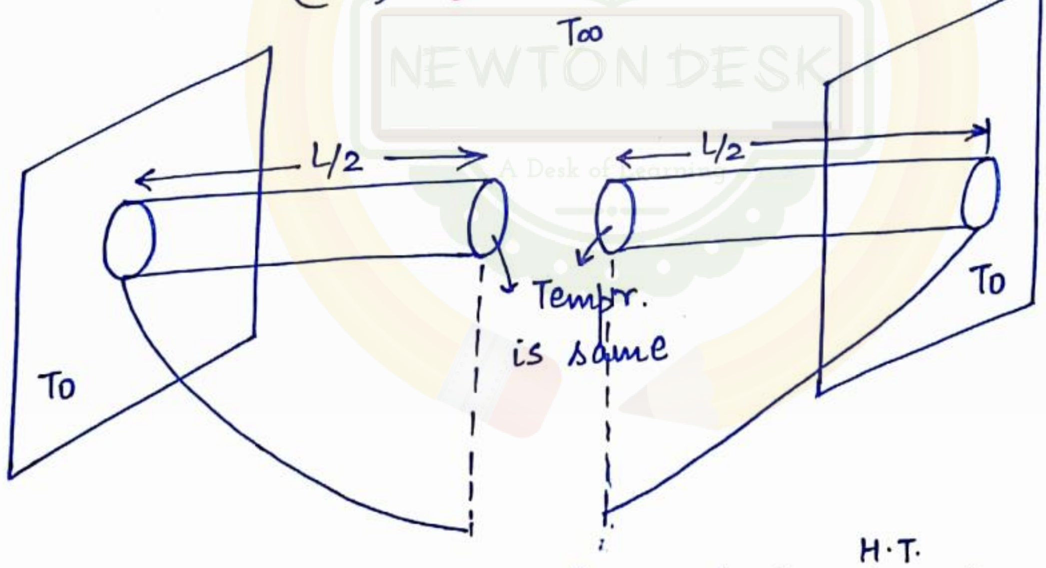
$$\epsilon_{fin}\ \text{infinitely long fin} = \frac{\sqrt{hPKA}\,\cancel{(T_0 - T_\infty)}}{hA\,\cancel{(T_0 - T_\infty)}}$$

$$= \sqrt{\frac{hP}{KA}}$$

21



$$\left(\frac{dT}{dx}\right) = ? = 0$$

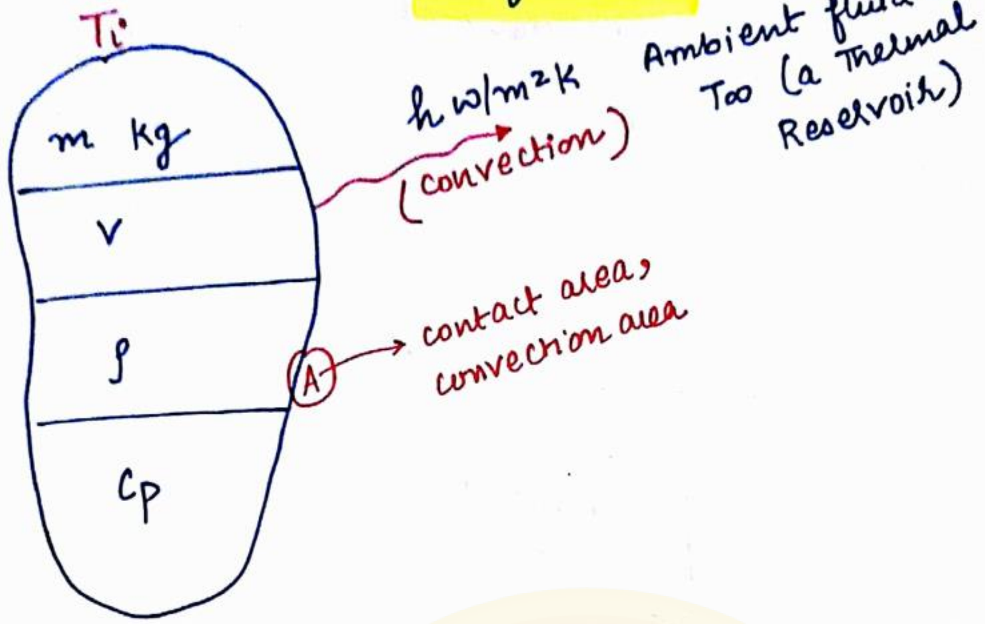


when Rods are Joined No conduction H.T. b/w the ends of Rod.

$$-KA\frac{dT}{dx} = 0 \Rightarrow \frac{dT}{dx} = 0.$$

* Unsteady State (OR) Transient Conduction Heat Transfer :-

$T = f(\text{Time})$



Consider a solid Body of mass 'm', volume 'V', density 'ρ', specific heat 'Cp' which is at an initial tempr. of 'Ti' (having preheated in a furnace) is suddenly exposed to an ambient fluid at T∞. Since the Body keeps on loosing heat by convection to the ambient fluid with a convective Heat transfer coefficient of h, The internal energy of the body keeps on decreasing as the time progresses which is manifested (showing) by decrease in tempr. of the body with respect to time.

$$T = f(\tau)$$

Let T_i = Initial tempr. of body at the instant of time τ = 0 sec, i.e. when the body is just exposed to fluid.

T = Tempr. of Body at any instant of Time 'τ sec', Later.

Writting the energy Balance for the body at any instant of Time 'τ' sec, (63)

The Rate of convection H.T. b/w body and Fluid = The Rate of Decrease of I.E. of body w.r.t. Time.

$$\Rightarrow hA(T-T_\infty) = -mC_p\left(\frac{dT}{d\tau}\right) \text{ Joule/sec}$$

$$= -\rho V C_p \left(\frac{dT}{d\tau}\right) \text{ J/sec}$$

J/KgK

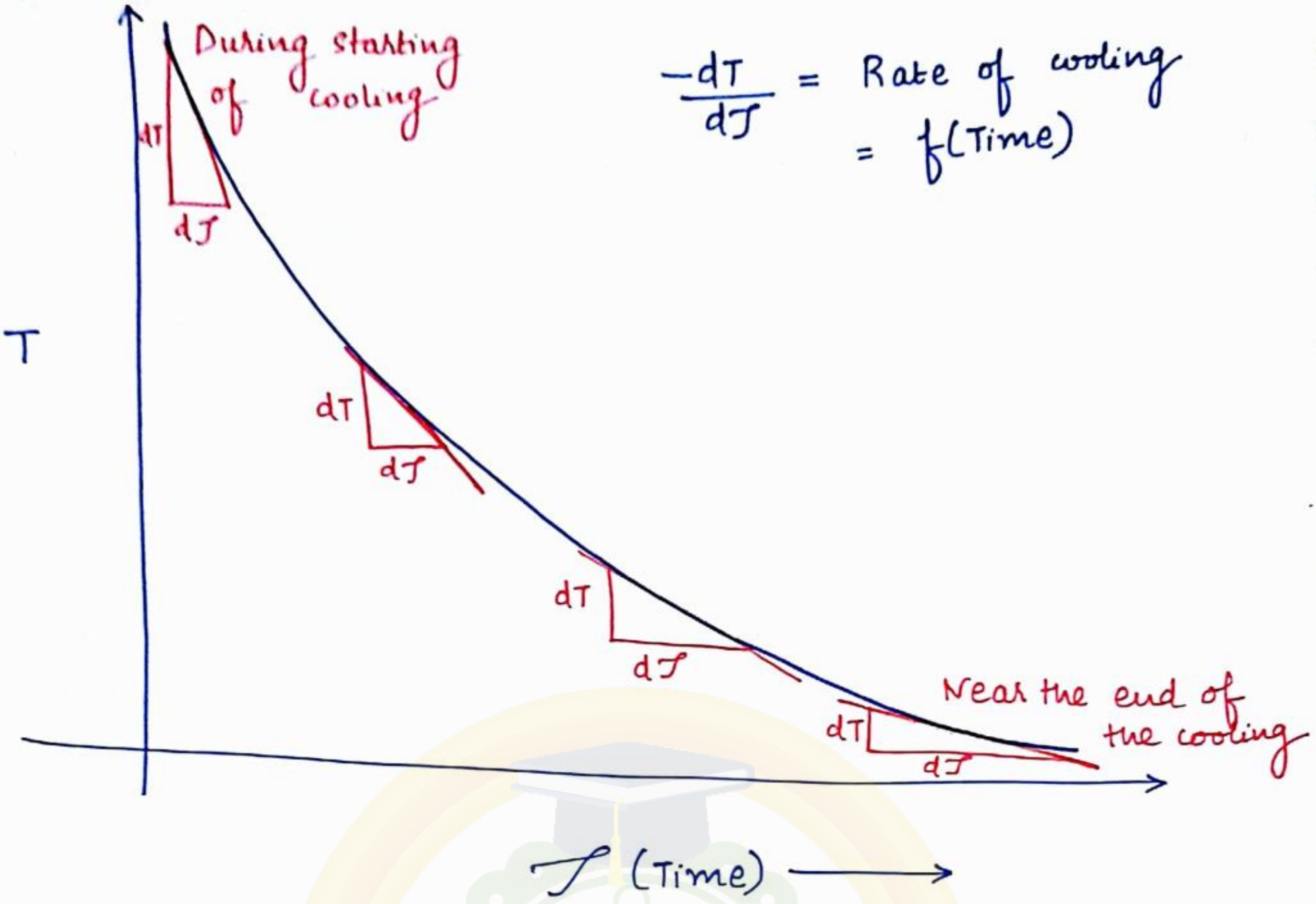
Treating all other parameters including h as constant and separating the variables time and temperature. we get

$$\int_{\tau=0\,sec}^{\tau}\left(\frac{hA}{\rho V C_p}\right)d\tau = \int_{T_i}^{T}\frac{-dT}{(T-T_\infty)}$$

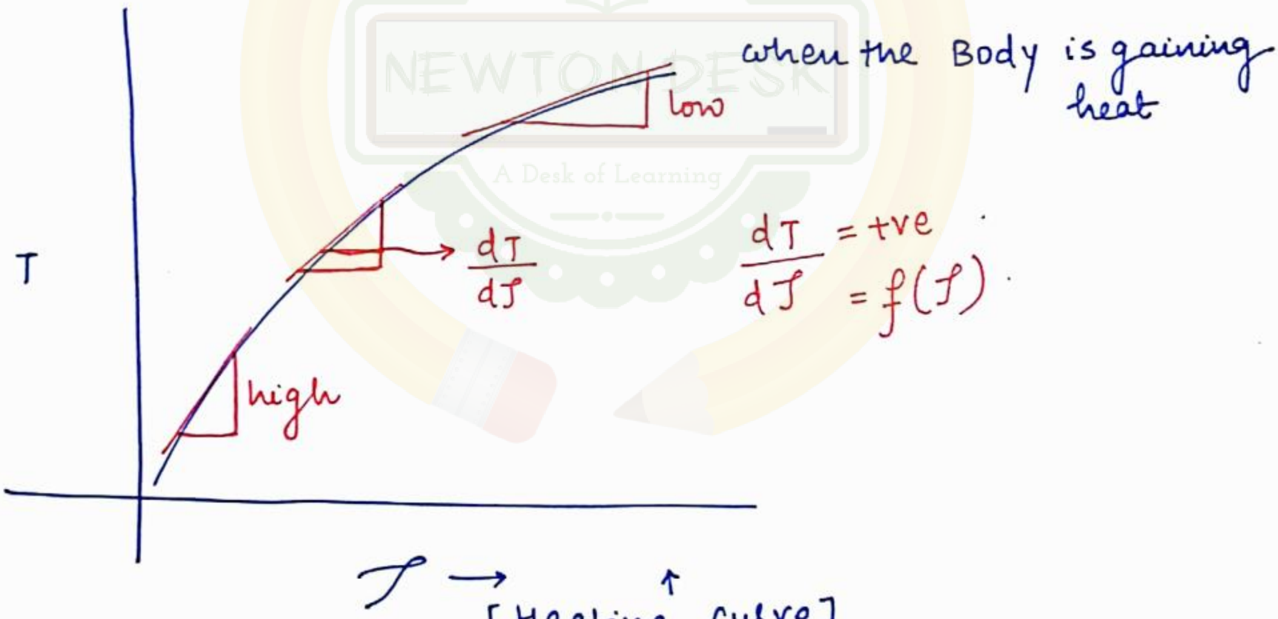
$$\Rightarrow \left(\frac{hA}{\rho V C_p}\right)\tau = \Big|\ln(T-T_\infty)\Big|_{T}^{T_i} = \ln_e\frac{(T_i-T_\infty)}{(T-T_\infty)}$$

$$\Rightarrow \boxed{e^{(hA/\rho V C_p)\tau} = \frac{(T_i-T_\infty)}{(T-T_\infty)}}$$

Note:- hence in any unsteady state H.T., the tempr. of the body changes exponentially wrt time as shown in the fig.



$$\frac{-dT}{dJ} = \text{Rate of cooling} = f(\text{Time})$$

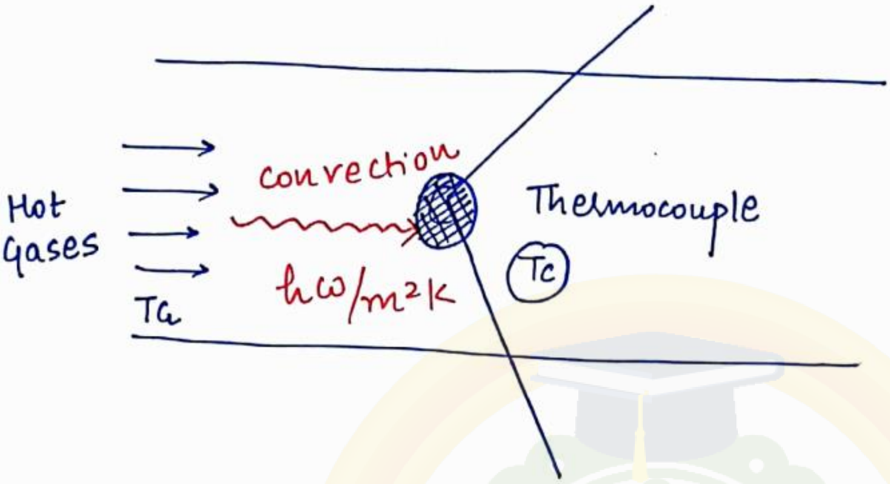


[Heating curve]

hence during unsteady state H.T., the rate of cooling/Heating of Body itself becomes a function of Time. Initially, the rate of cooling is very fast due to high convection heat transfer rate b/w body & fluid Because of large tempr. diff. prevailing b/w them. But as the time progresses this rate of cooling/Heating of body itself keeps on decreasing with time due to diminishing Heat transfer rate between them.

$$\frac{hA}{\rho V c_p} \rightarrow sec^{-1}$$

$\therefore \left(\frac{\rho V c_p}{hA}\right)$ has got the units of sec. This group is called Time constant of Body.

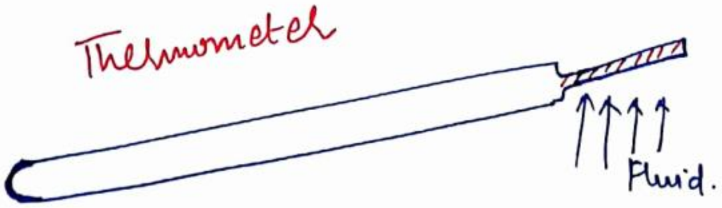


Time constant of Thermocouple bead because is very small (2 to 3 sec)

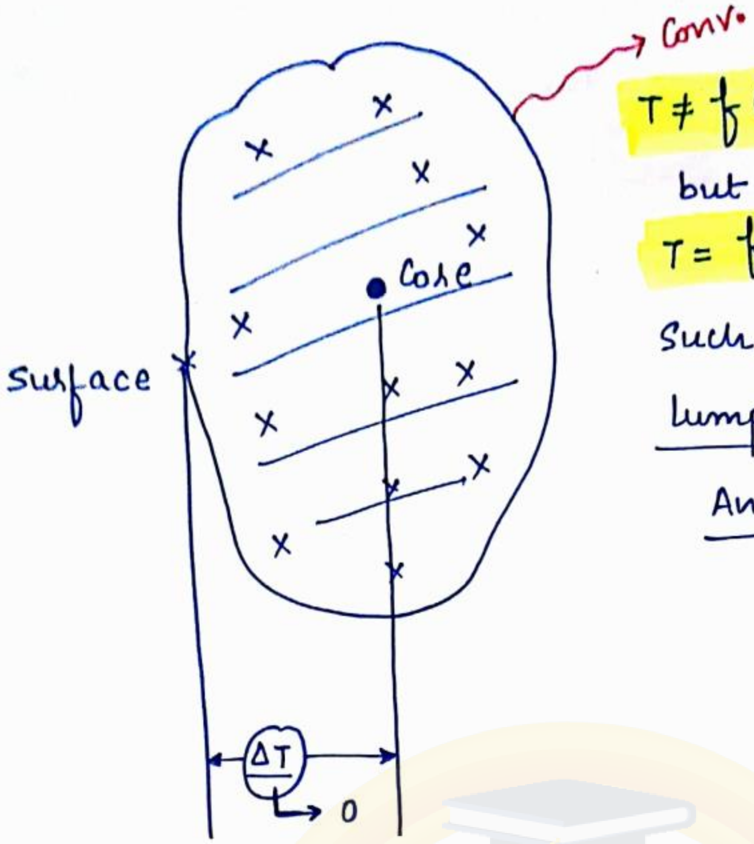
① $V/A = \frac{4/3\,\pi R^3}{4\pi R^2} = R/3$ (Size of Bead is very small)

② Heat Capacity of Material is small.

③ 'h' value is high.



In the above analysis done, it is assumed that the tempr. of the Body is uniform throughout its mass at any instant of time i.e. Internal tempr. gradients within the Body are neglected.



$T \neq f(\text{space})$

but

$T = f(\text{Time})$

Such analysis is called Lumped Heat Capacity Analysis.

* Criteria for lumped Heat analysis :-

$\boxed{\text{Biot No} < 0.1}$ where

$$\text{Biot No.} = \frac{hs}{K_{solid}}$$

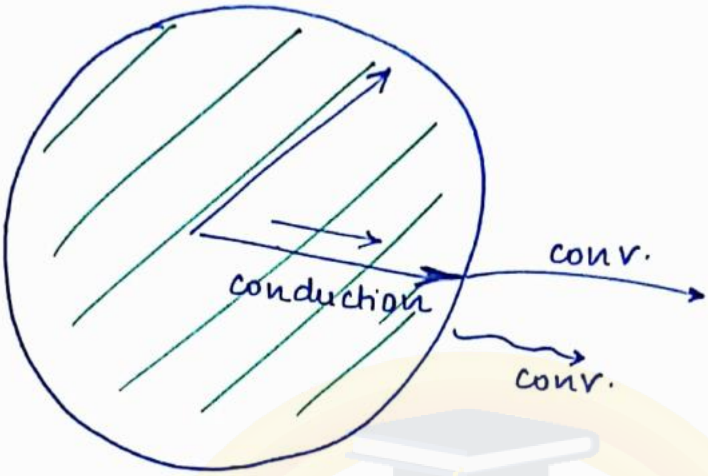
$$s = \frac{\text{Volume of Body}}{\text{surface area of Body}} = \frac{V}{A}$$

For spherical Body, $s = \dfrac{\frac{4}{3}\pi R^3}{4\pi R^2} = \left(\frac{R}{3}\right)$.

$$\boxed{\text{Biot No.} = \frac{s/KA}{1/hA}} = \frac{\text{Internal conductive Resistance offered By Body}}{\text{External convective No.}}$$

$$= \frac{ICR}{ECR}.$$

NOTE:- Low Biot No. values signifies that the body offers very little conductive resistance for any internal H.T. within the Body as compared to surface convective resistance thereby levelling the tempr. differences that may exist between any two locations within the body.



WB
Q36

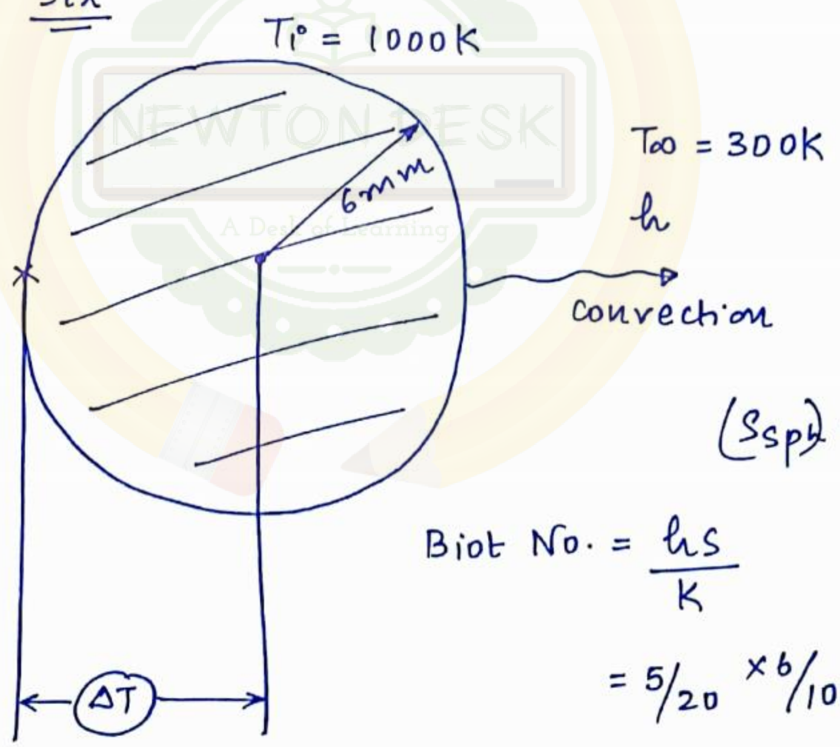
$\frac{9.04\times10^{-7}}{4.52\times10^{-4}}$

$\frac{2.15\times10^{-3}\ m^2}{20}$

$1/s$

0.00053

Sol



$(S_{sph}) = R/3$

$$\text{Biot No.} = \frac{hS}{K}$$

$$= 5/20 \times 6/1000 \times 1/3$$

$$= 0.0005$$

$$<< 0.1$$

(d) ✓

ICR <<< ECR

↓ cond ↓ conv.

(53) $d = 10\,mm$ $T_\infty = 300\,K$ $h = 1000\,W/m^2K$

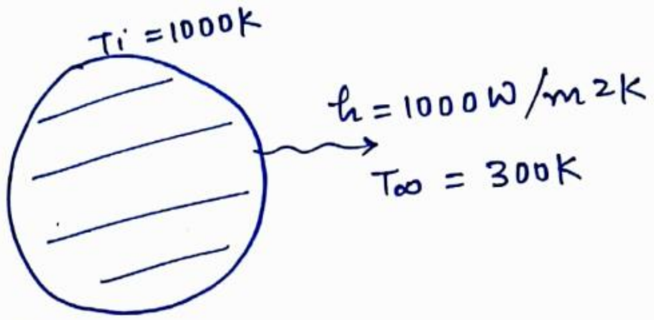
$T_i = 1000\,K$ $T_0 = 350\,K$ $k = 40\,W/mK$

$\tau = 16\,s$

$$e^{(hA/\rho V c_p)\tau} = \left(\frac{T_i - T_\infty}{T - T_\infty}\right)$$

SIR Since small steel ball,

Biot No. < 0.1



$$\underbrace{\left(\frac{hA}{\rho V c_p}\right)}_{\frac{1}{\text{Time constant}}} \times \tau = \ln\frac{(T_i - T_\infty)}{(T - T_\infty)} = \ln\left\{\frac{1000-300}{350-300}\right\}$$

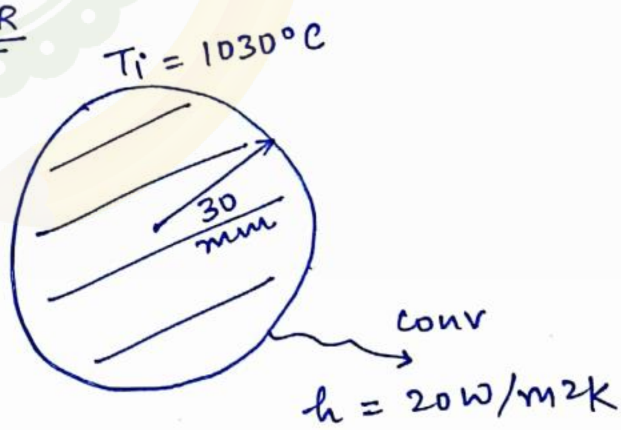
$$\Rightarrow \frac{1}{16} \times \tau = \ln\left(\frac{700}{50}\right) \Rightarrow \text{Time taken} = 42.2\,sec$$

(15)

$$\frac{4.83 \times 10^{-8}}{3.87 \times 10^{-4}} = \ln\frac{(1030 - 30)}{(T - 30)}$$

$$1.00T - 30T = 1030 - 30$$

SIR



Biot No. < 0.1 ($\because$ small steel Body)

$$\left(\frac{hA}{\rho V C_p}\right) \times \tau = \ln\frac{(T_i - T_\infty)}{T - T_\infty} = \ln\frac{(1030-30)}{(430-30)}$$

Put $A/V = (3/R) \Rightarrow \tau = 2144\,\text{sec}$ Time taken

(52) $r = \frac{0.01}{2 \times \cancel{1000}}\,m$

$L = 0.2\,m$ $T_\infty = 100°C$

$T_i = 750°C$ $T = 300°C$

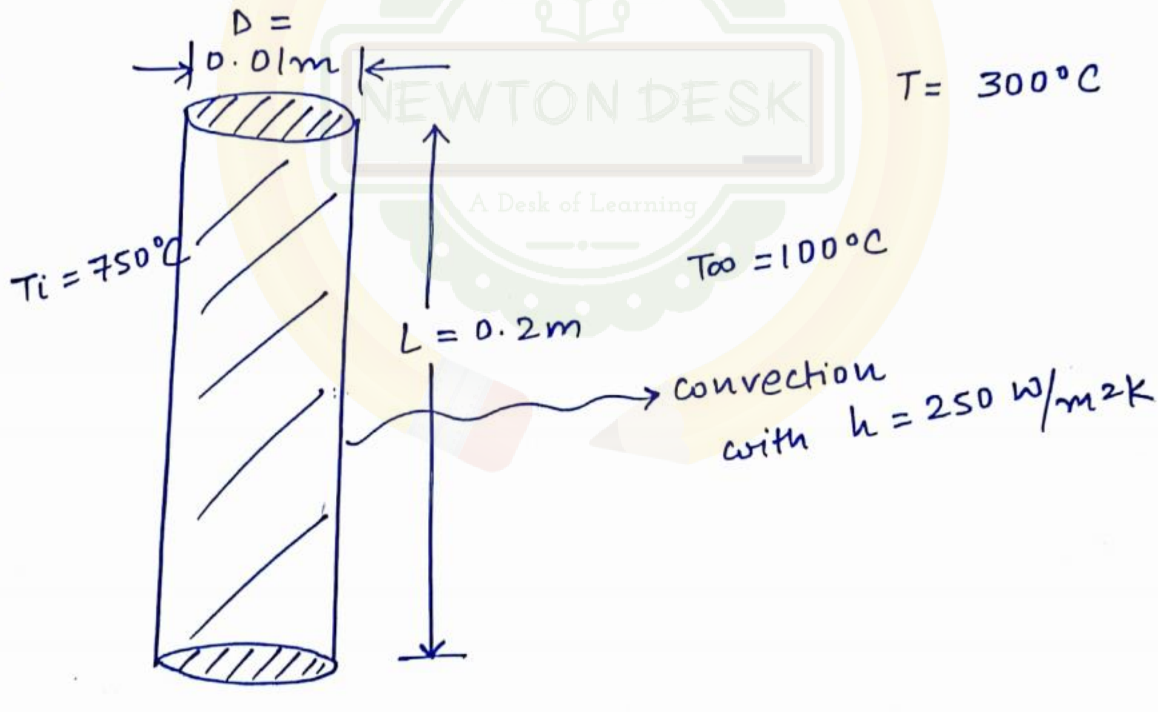
$h = 250\,W/m^2K$

$\rho = 7801\,kg/m^3$

$C = 473\,J/kg\,K$

$K = 43\,W/mK$

SIR

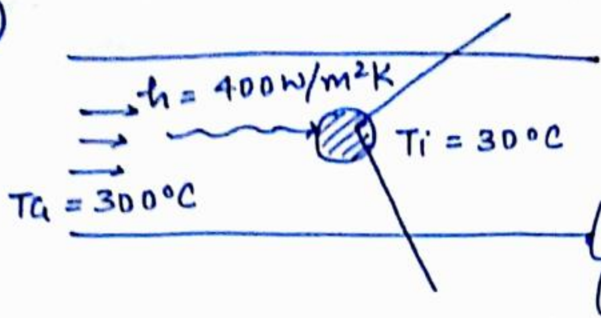


Since slender steel Rod, Biot No. < 0.1

$$\left(\frac{hA}{\rho V C_p}\right)\tau = \ln\left(\frac{T_i - T_\infty}{T - T_\infty}\right) = \ln\left(\frac{750-100}{300-100}\right)\tau = \text{time taken} = 42.4\,\text{sec}$$

$$A/V = \left[\frac{2(\pi/4)D^2 + \pi D L}{\pi/4 D^2 L}\right]$$

(39)



Thermocouple is a very small Body (i.e. Biot No<0.1)

$$\left(\frac{hA}{\rho V C_p}\right)\tau = \ln\frac{(T_i - T_\infty)}{(T - T_\infty)} = \ln\frac{(30-300)}{(298-300)}$$

Put $\frac{A}{V} = 3/R \Rightarrow \tau$ = Time taken = 4.9sec

(16) $d = 5mm$ $r = 2.5mm$

$T_i = 500K$

$T_\infty = 300K$

$K = 400$

$\rho = 9000$

$C = 385$

$h = 250$

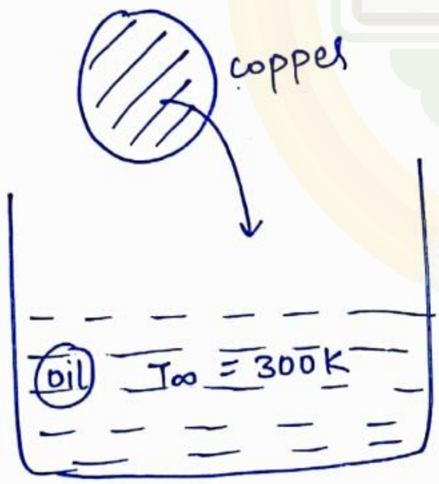
$\frac{\rho V C_p}{hA} = 7.55$

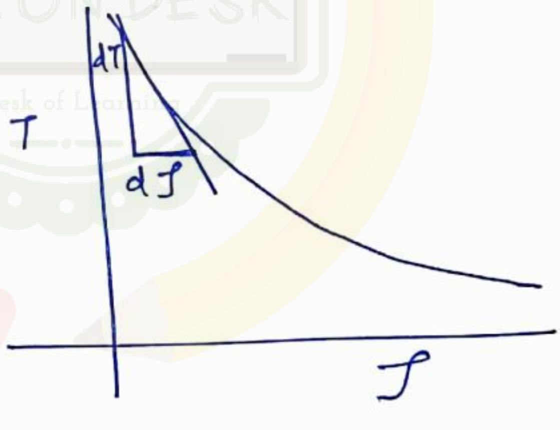
$\frac{hA}{\rho V C_p}$

$11.55 = \frac{500-300}{T-300}$

$0.086 \times$ $T - 300 = 500 - 300$

SIR





The Rate of fall of tempr. of ball at the begining of cooling $= \left(\frac{dT}{d\tau}\right)_{at\ \tau = 0sec}$

writting the energy balance for the body, at the very beginning of cooling.

The Rate of convection H.T. b/w body and oil bath = The Rate of decrease of IE of body wrt time

$$hA(T_i - T_\infty) = -\rho V C_p \left(\frac{dT}{d\tau}\right)_{at\ \tau=0\,sec} \text{ J/sec}$$

$$-\left(\frac{dT}{d\tau}\right)_{at\ \tau=0\,sec} = 17.3\ K/sec$$

Time	Temp
0 sec →	500 K
1 sec →	482.7 K
2 sec →	

$\}$ 17.3

$\}$ < 17.3

Put $\frac{A}{V} = 3/R$

Q57 $h = 400$
$K = 20$
$C = 200$
$\rho = 8500$

$\tau = 1sec$

SIR

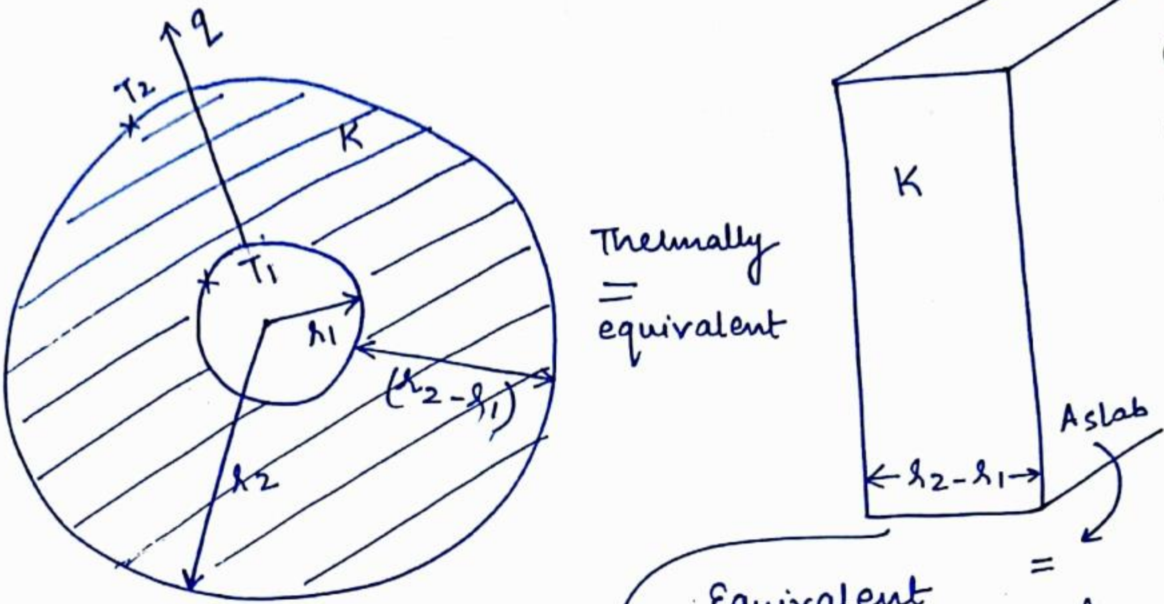
$$\frac{\rho V C_p}{hA} = 1\ sec$$

Put $\frac{V}{A} = R/3$

$R = 0.705\,mm$ $D = 1.4\,mm$

Q50B
1. c ✓ (clausiu's statement of IInd law)
3. d ✓
4. b ✓ $\alpha = \left(\frac{K}{\rho C_p}\right)$
7. a
8. d
9. c ($K_{solid\ ice} > K_{water} > K_{water\ vapour}$)
10. a $\left(\because \left(\frac{dT}{dx}\right)_{in\ 1} \text{ is higher}\right)$
11. c
12. d

(13)



Hollow sphere

$$(R_{th}) = \frac{r_2 - r_1}{4\pi K r_1 r_2}$$

(Slab)

$$R_{th} = \frac{r_2 - r_1}{K A_m}$$

The Hollow sphere & slab are said to be thermally equivalent only if both of them are having same total thickness and offer the same thermal Resistance.

Equating R_{th},

$$\frac{r_2 - r_1}{4\pi K r_1 r_2} = \frac{r_2 - r_1}{K A_m}$$

$$A_m = 4\pi r_1 r_2$$

$$A_m = 4 m^2$$

$4\pi r_1^2 = A_1 = 2 m^2$

$r_1 = \boxed{}$ m

$4\pi r_2^2 = A_2 = 8 m^2$

$r_2 = \boxed{}$ m

(22) b✓

24) K varies linearly w.r.t. Temp. i.e. we can take

$$T_{mean} = \frac{25+15}{2} = 20\ W/mK$$

$$\therefore q/A = \frac{K_{mean}\,\Delta T}{b}$$

$$= \left[\frac{20 \times (300-200)}{0.5}\right]$$

$$= 4000\ W/m^2$$

$$K = aT + c$$

$$25 = a(300) + c$$

$$15 = a(200) + c$$

$$q/A = K\frac{dT}{dx}$$

$$\int_{x=0}^{b} q/A\,dx = \int_{T_1}^{T_2} (aT+c)\,dT$$

$$q/A = \frac{1}{b}\left[\frac{a(T_1^2 - T_2^2)}{2} + c(T_1 - T_2)\right]$$

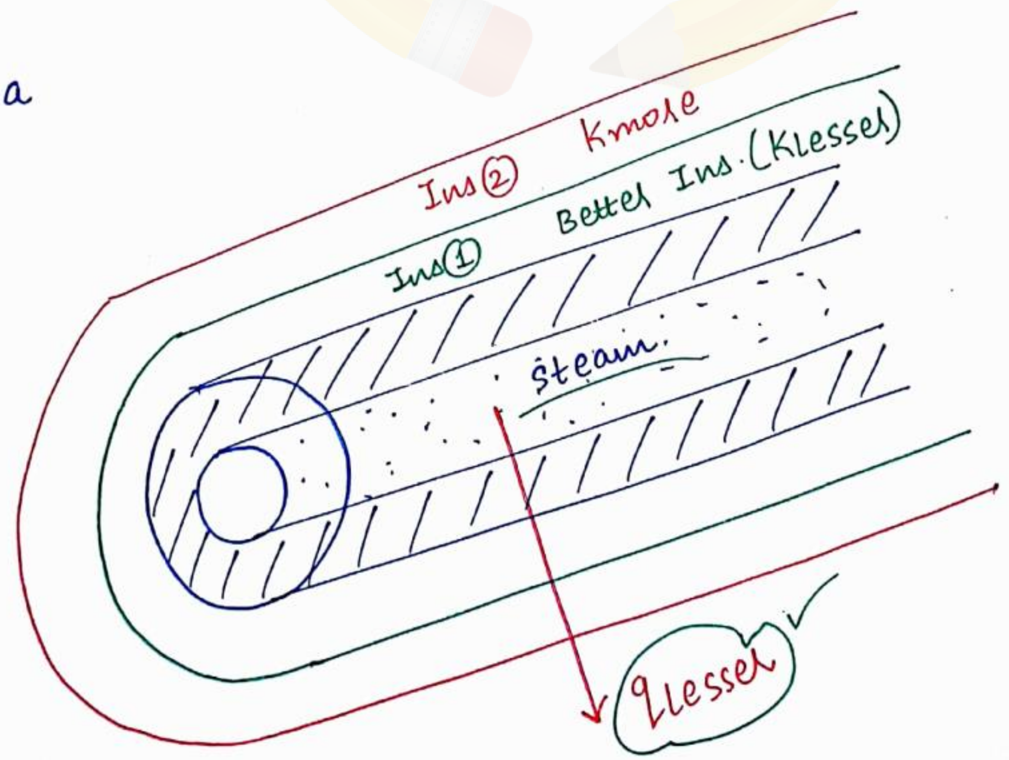
$\boxed{0.5m}$

$$= 4000\ W/m^2$$

25) $R_{th} = b/KA = \frac{0.1}{200 \times 5} = 10^{-4}\ K/watt$

28) a

31)



To have lesser H.T. Rate between steam pipe and ambient, Always keep better Insulator having lesser 'k' value immediately next to pipe.

Reason → In such arrangement $(R_{Th})_{Total}$ is higher.

(a) ✓

Q38 During unsteady state H.T.,

If Biot No. < 0.1

$T \neq f(\text{space})$ but $T = f(\text{Time})$

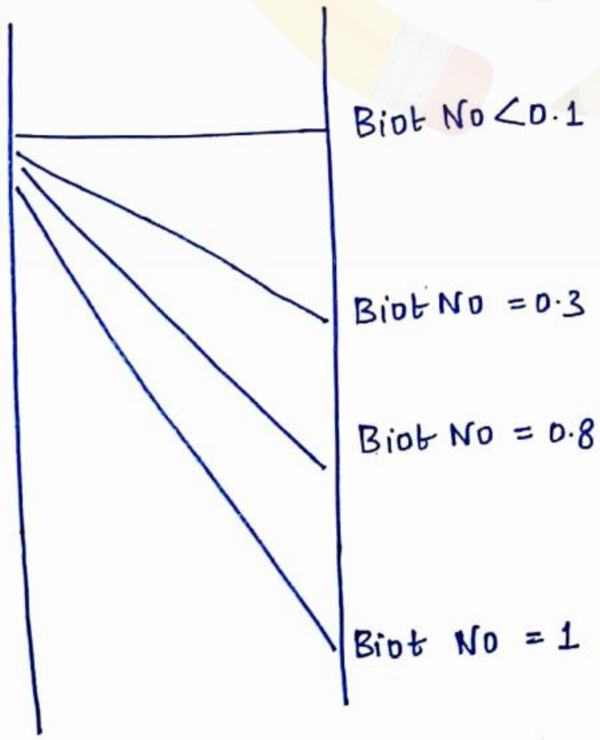
In Case if Biot No. > 0.1

Then $T = f(\text{Time, space (or) location})$

i.e. we observe Different temp's at different locations of body at a given instant of time

(i.e. Tempr. gradients appear)

∴ To get temperature of body at any particular location at a given instant of Time, Heisler charts must be used.



Put Biot No. = 1

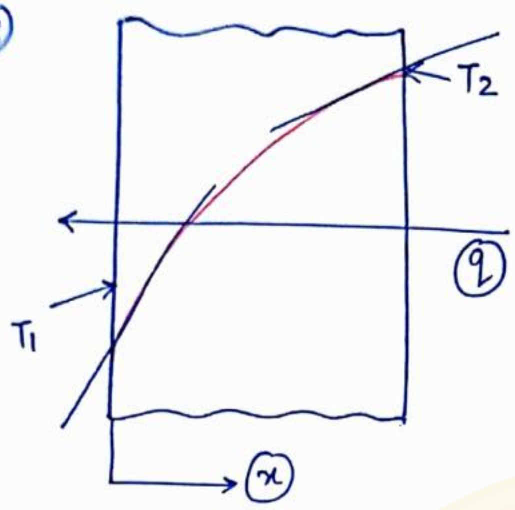
$\Rightarrow \underset{\text{cond.}}{\underset{\downarrow}{ICR}} = \underset{\text{conv.}}{\underset{\downarrow}{ECR}}$

i.e. Both Internal conductive Resistance and External convective Resistance become of equal significance.

44 $\dfrac{\ln(r_2/r_1)}{2\pi KL}$

45 b

58



$T_2 > T_1$

Fourier's law of cond :-

$$q = KA\frac{dT}{dx}$$

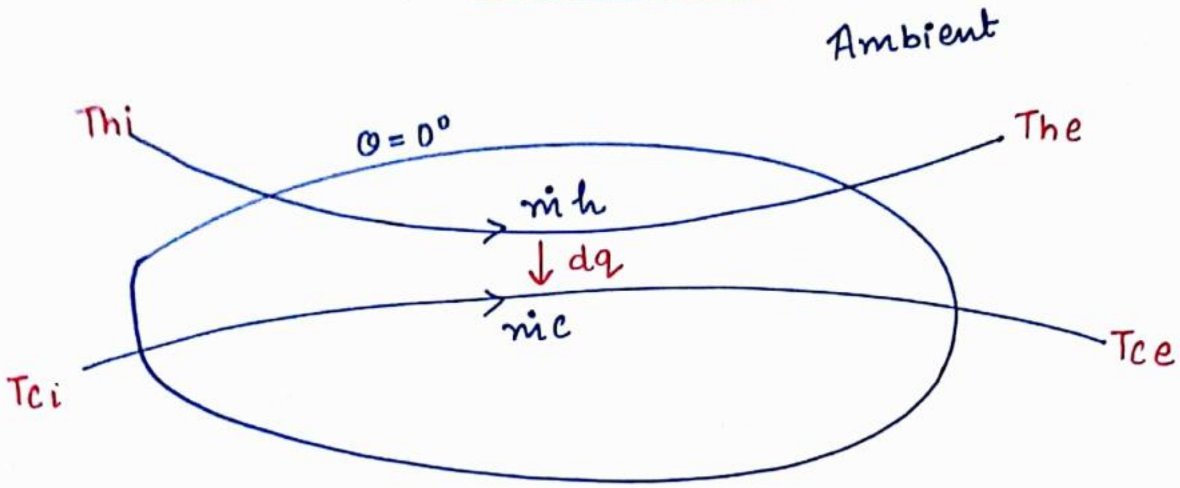
$$q/A = (K_0 + bT)\frac{dT}{dx}$$

But $q = C$ ($\because$ steady state H.T.)

$A = C$ ($\because$ slab) $\therefore$ As $x\uparrow \Rightarrow T\uparrow \Rightarrow (K_0+bT)\uparrow$

$\therefore \dfrac{q}{A} = \text{constant}$ $\Rightarrow \dfrac{dT}{dx}\downarrow$

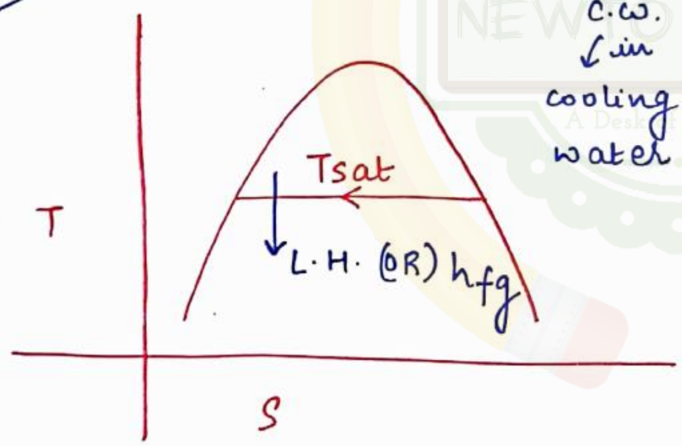
HEAT EXCHANGERS

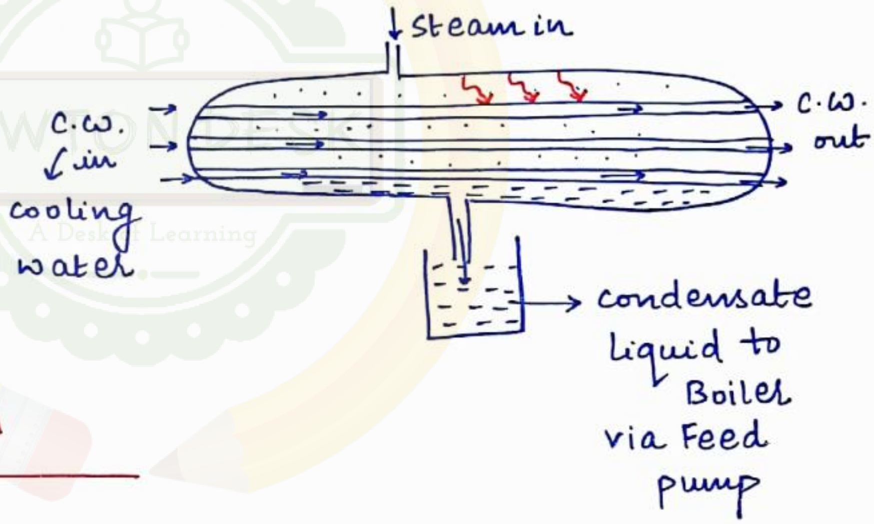


Heat Exchanger is a steady flow adiabatic open system in which two flowing fluids exchange or transfer heat b/w them without loosing or gaining any heat from the ambient.

Ex :- (1) Surface (steam) Condenser (steam to C.W.)

(2)



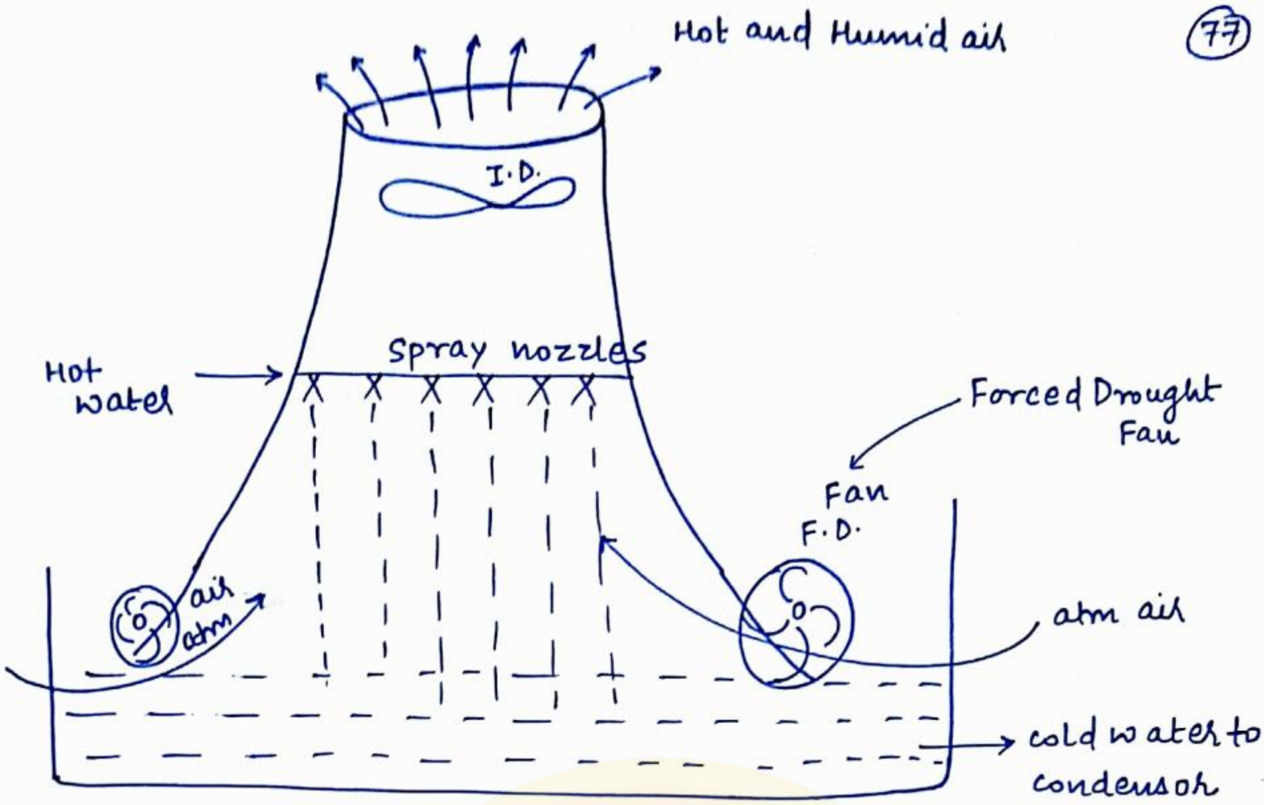


(2) Economiser (Hot flue gases to Feed water).

(3) Superheater (Hot flue gases to Dry saturated steam).

(4) Air Preheater (Hot flue gases to combustion air).

(5) Cooling Tower (Hot water to atm. air).



⑥ Jet condenser (steam to c.w.)
coolant water

⑦ Oil cooler (Hot oil to coolant water / air).

✓ Application of I^{st} law of T/D to any Heat Exchanger :-

Since H.E. being steady flow open system, writting SFEE

$$\cancel{Q}^{0} - \cancel{W}^{0} = \Delta H + \cancel{\Delta KE}^{0} + \cancel{\Delta PE}^{0}$$

$$\Rightarrow \therefore (\Delta H)_{HE} = 0$$

$$\Rightarrow (\Delta H)_{\text{hot fluid}} + (\Delta H)_{\text{cold fluid}} = 0$$

$$\Rightarrow -(\Delta H)_{\text{hot fluid}} = +(\Delta H)_{\text{cold fluid}}.$$

Hence, ∴ the rate of enthalpy decrease of hot fluid = The Rate of enthalpy increase of cold fluid.

$$\Rightarrow Q = \boxed{\dot{m}_h C_{Ph} (T_{hi} - T_{he}) = \dot{m}_c C_{Pc} (T_{ce} - T_{ci}) \text{ J/sec}}$$

Energy Balance Equ. (or) Heat Balance Equ.

$$\boxed{Q_{(P=C)} = \Delta H}$$

from T/D we know that H.T. in any constant pressure or isobaric process is equal to change in enthalpy of the fluid. also we assume that in any heat exchanger analysis, the pressures of both hot and cold fluid remains constant as they flow through heat Exchanger. By combining above 2 points, we may conclude that the rate of heat transfer b/w hot and cold fluids in any heat exchanger is equal to the rate of enthalpy change of either of the fluids.

Rate of H.T. in entire H.E.

$$\boxed{Q = \dot{m}_h c_{p_h} (T_{hi} - T_{he}) = \dot{m}_c c_{p_c} (T_{ce} - T_{ci}) \text{ watt}}$$

But the precaution is that do not use the above format of the equation for calculating enthalpy change of any fluid if it is undergoing phase change like steam condensation.

* **TYPES OF HEAT EXCHANGER (CLASSIFICATIONS)**:-

① Direct transfer type H.E^s.

② Direct contact Type H.E^s

③ Regenerative (or) storage Type of H.E.S

① In Direct transfer type H.E's, both hot and cold fluids do not have any physical contact between them but the transfer of heat occurs between them through pipe wall of separation.

Ex :- ⓐ Surface Condenser.

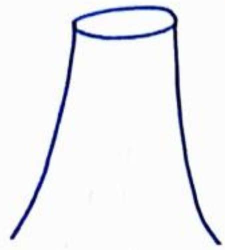
ⓑ oil cooler.

ⓒ Economiser.

ⓓ Airpreheater.

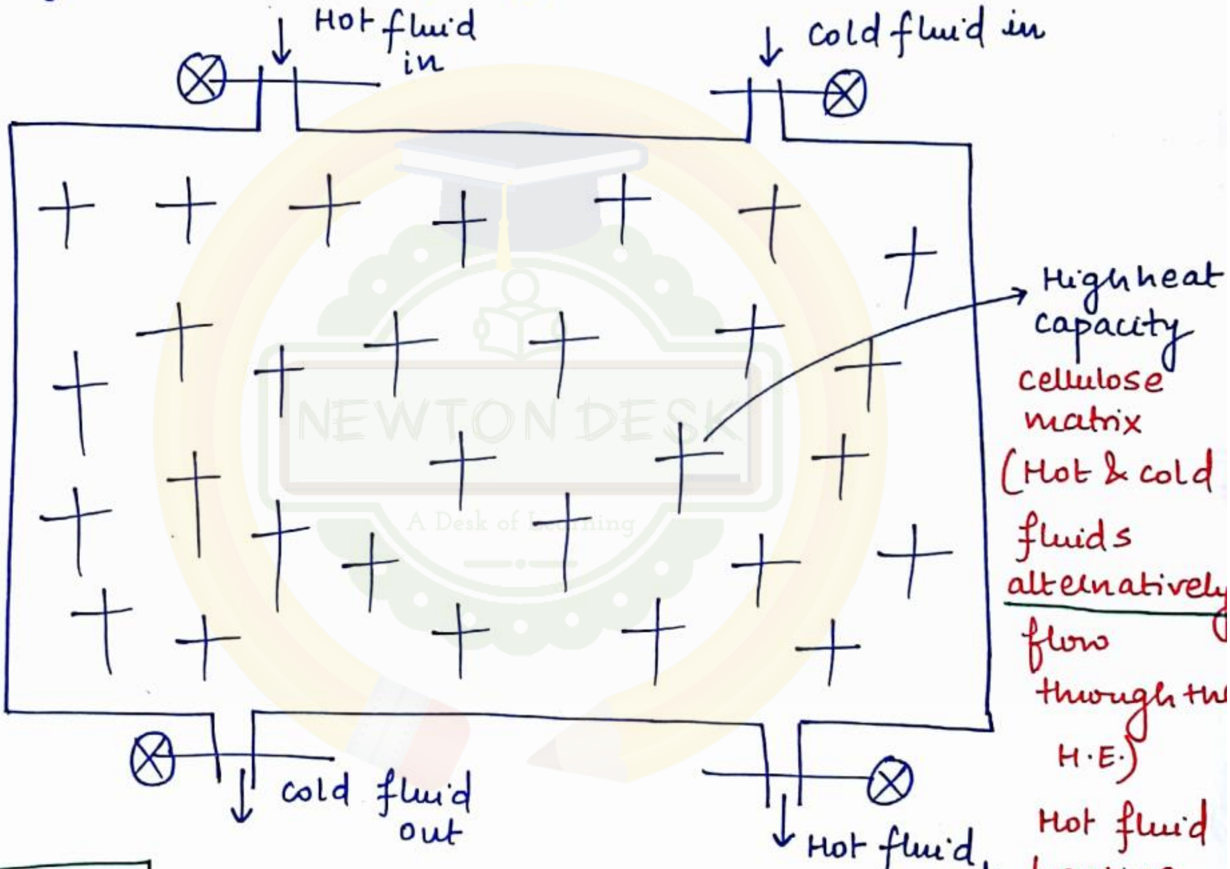
(2) In D.C.T.H.E., both hot and cold fluids physically mix-up with each other and exchange Q (heat) between them.

Example :- (a) Cooling Tower.



(b) Jet Condenser.

(3) In Regenerative or storage type H.E.,

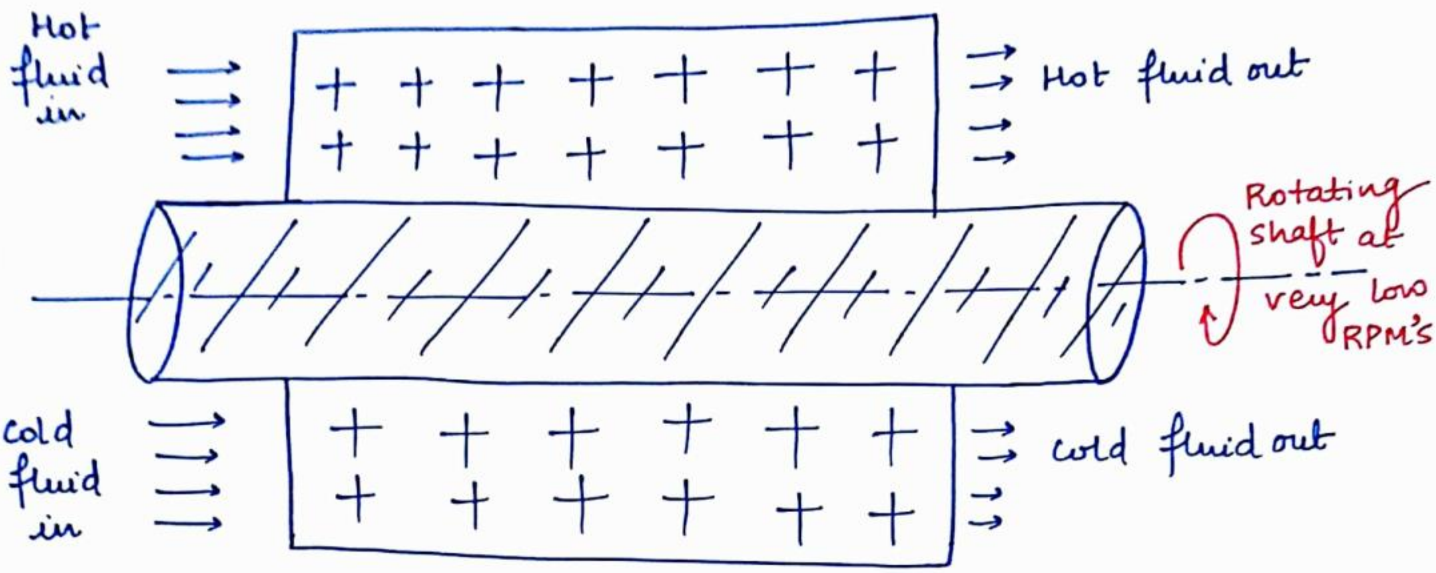


Hot fluid heating the Matrix and cold fluid picking up heat from it.

Disadvantage :- No continuity of flows

Practical application :- LJungstorm Air Preheater used in Gas Turbine Power Plants.

* Rotating Matrix Type Regenerative HE :-



Advantage :- ~~No need of stopping~~ continuity of fluid flows can be maintained
∴ No need of stopping and Restarting the fluid flows.

~~Disa~~

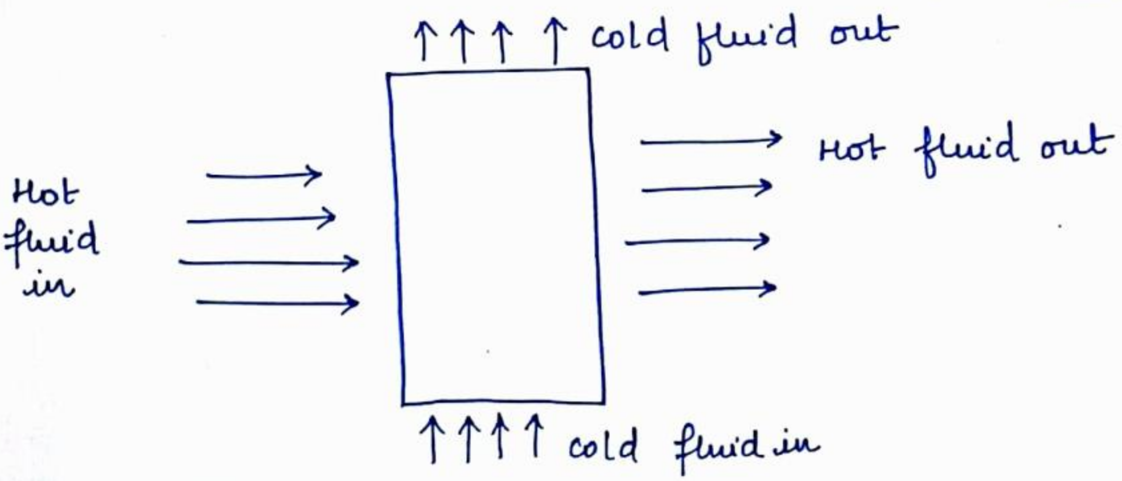
Disadvantage :- These may be some kind of fluid Mixing.

* Classification of Direct Transfer type HEs :-

① Parallel flow HE (Hot & cold fluids travel in ||el dirn)

② Counterflow HE (Hot & cold fluids travel in opposite dirn)

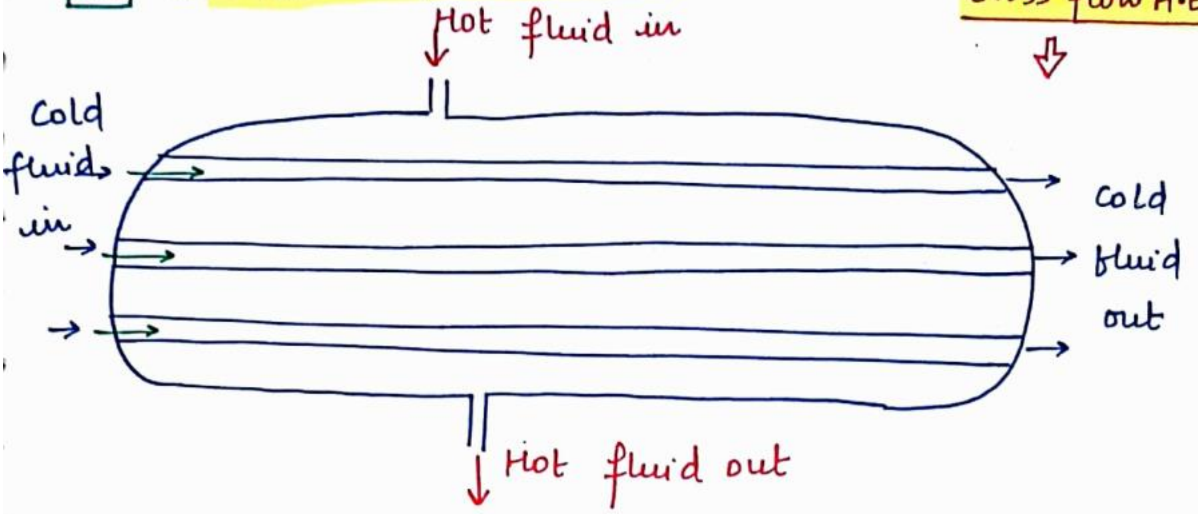
③ Cross flow HE (Hot and cold fluids travel in ⊥al dirn.) w.r.t. each other



Ex - (1) shell and Tube H.E. :-

Cross flow H.E.

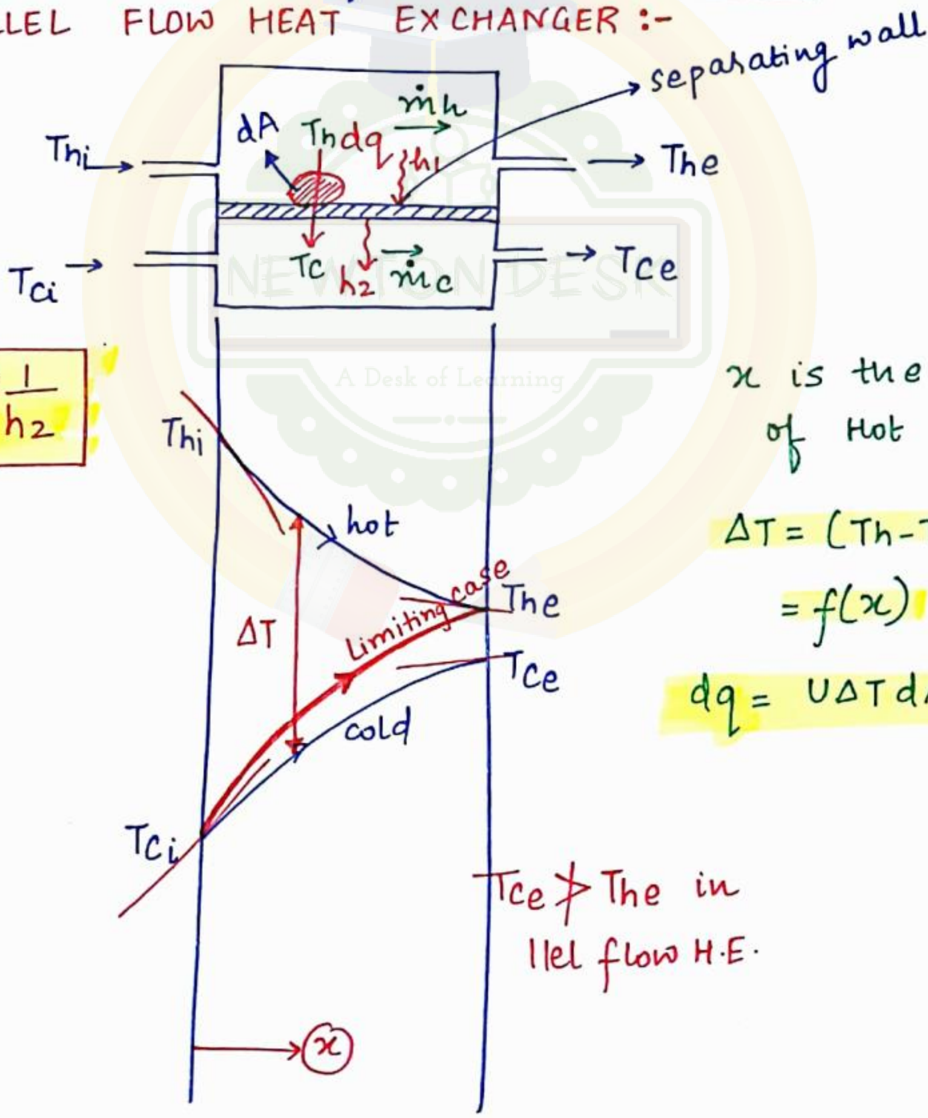
⇩



(2) Automobile Radiator.

* Temperature Profiles of Hot and Cold fluids :-

PARALLEL FLOW HEAT EXCHANGER :-



$$\boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}}$$

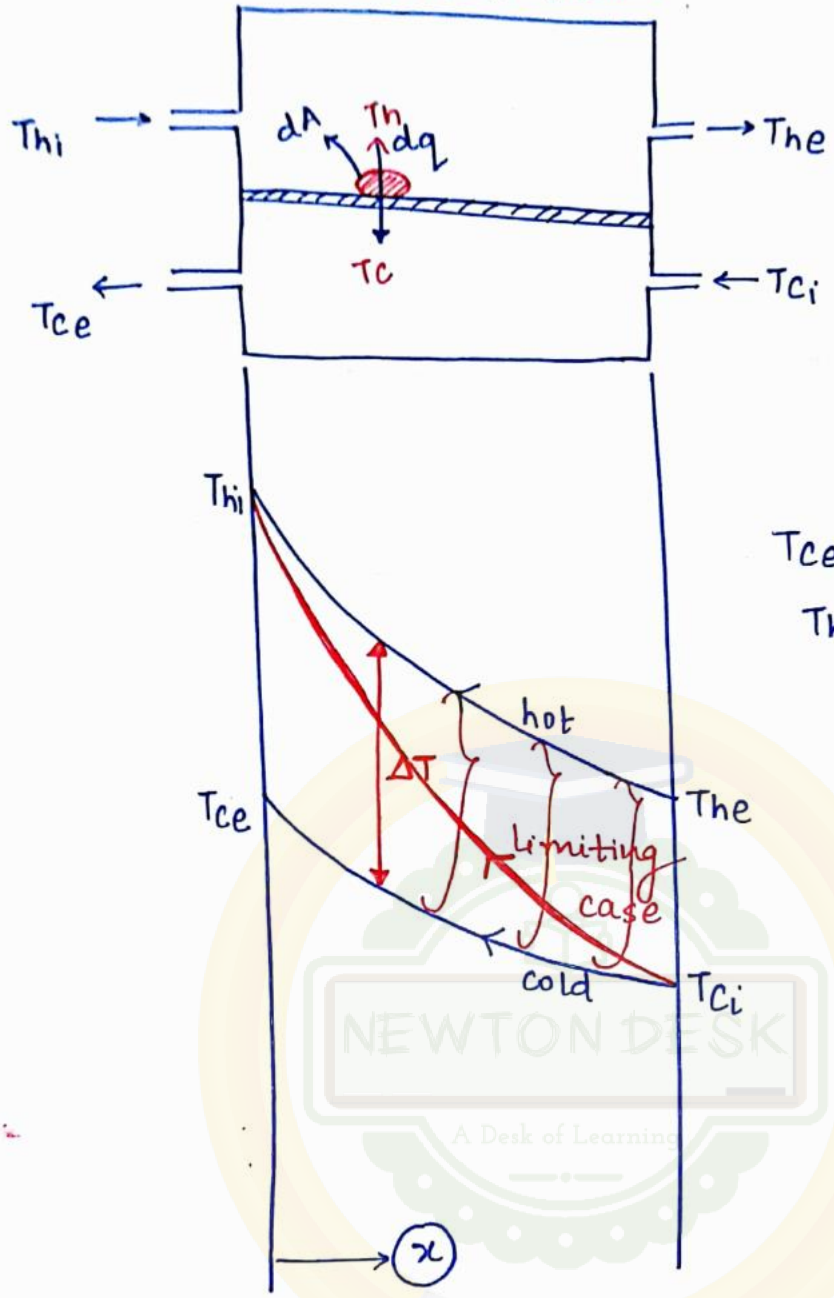
x is the direction of Hot fluid flow.

$$\Delta T = (T_h - T_c)$$
$$= f(x)$$

$$dq = U \Delta T \, dA$$

$T_{ce} \not> T_{he}$ in ||el flow H.E.

COUNTER FLOW :-



Tce can be greater than The only in counterflow H.E.

NOTE:-

The variation of ΔT with respect to 'x' is more pronounced in ‖ flow H.E. as compared to that in counter flow H.E.

Hence, the irreversibility associated with heat transfer is higher in parallel flow H.E. as compared to that in counterflow H.E..

$$\left[(\Delta S)_{\text{T.D. universe in parallel flow H.E.}} > (\Delta S)_{\text{uni. in counter flow}} \right]$$

Thus counter flow H.E. is thermodynamically more efficient than parallel flow H.E. (83)

Hence for the same H.T. rate required in both the cases, counterflow heat Exchanger occupies lesser heat transfer area or more compact in size than parallel flow Heat Exchanger.

* $\boxed{\text{M.T.D. (Mean Temperature Difference)}}$:-
$\boxed{(\Delta T_m)}$

It is the parameter which takes into account the variation of ΔT with respect to x (direction of hot fluid flow) by averaging it all along the length of the Heat Exchanger from inlet to exit. and hence is defined from the equation

$$\boxed{Q = UA\,\Delta T_m} \quad (1), \text{ where}$$

Q = Total H.T. rate between Hot and cold fluids in entire H.E.

U = overall H.T. coefficient

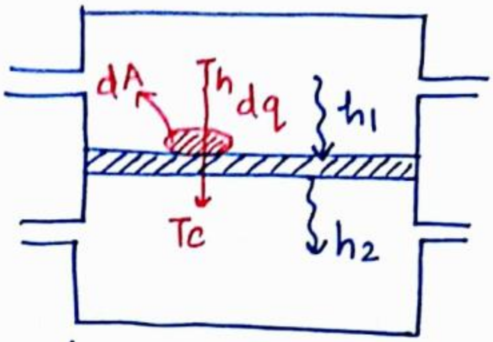
A = Total H.T. area of the H.E.

ΔT_m = MTD

$$\Delta T_m\ (\text{MTD}) = \frac{1}{A}\int_{\text{Inlet}}^{\text{Exit}} \Delta T \, dA$$

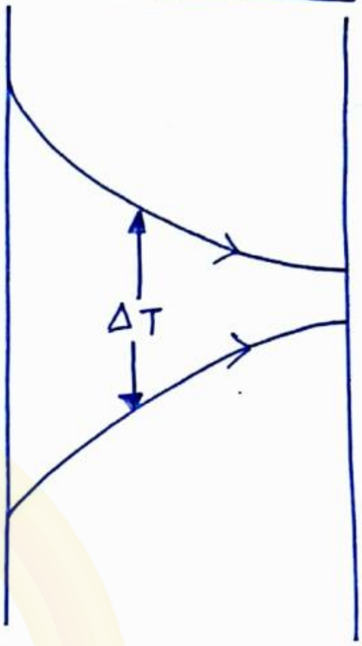
← on comparing (1) and (2).

$$dq = U \Delta T \, dA$$



$$\int_{Inlet}^{Exit} dq = \int_{Inlet}^{Exit} U \Delta T \, dA$$

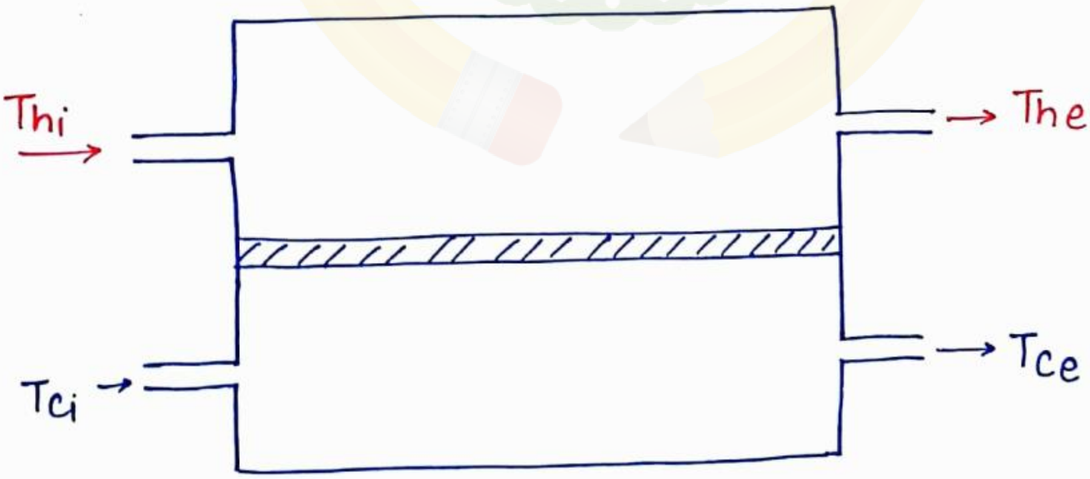
$$\Rightarrow \boxed{Q = U \int_{Inlet}^{Exit} \Delta T \, dA} \longrightarrow (2)$$



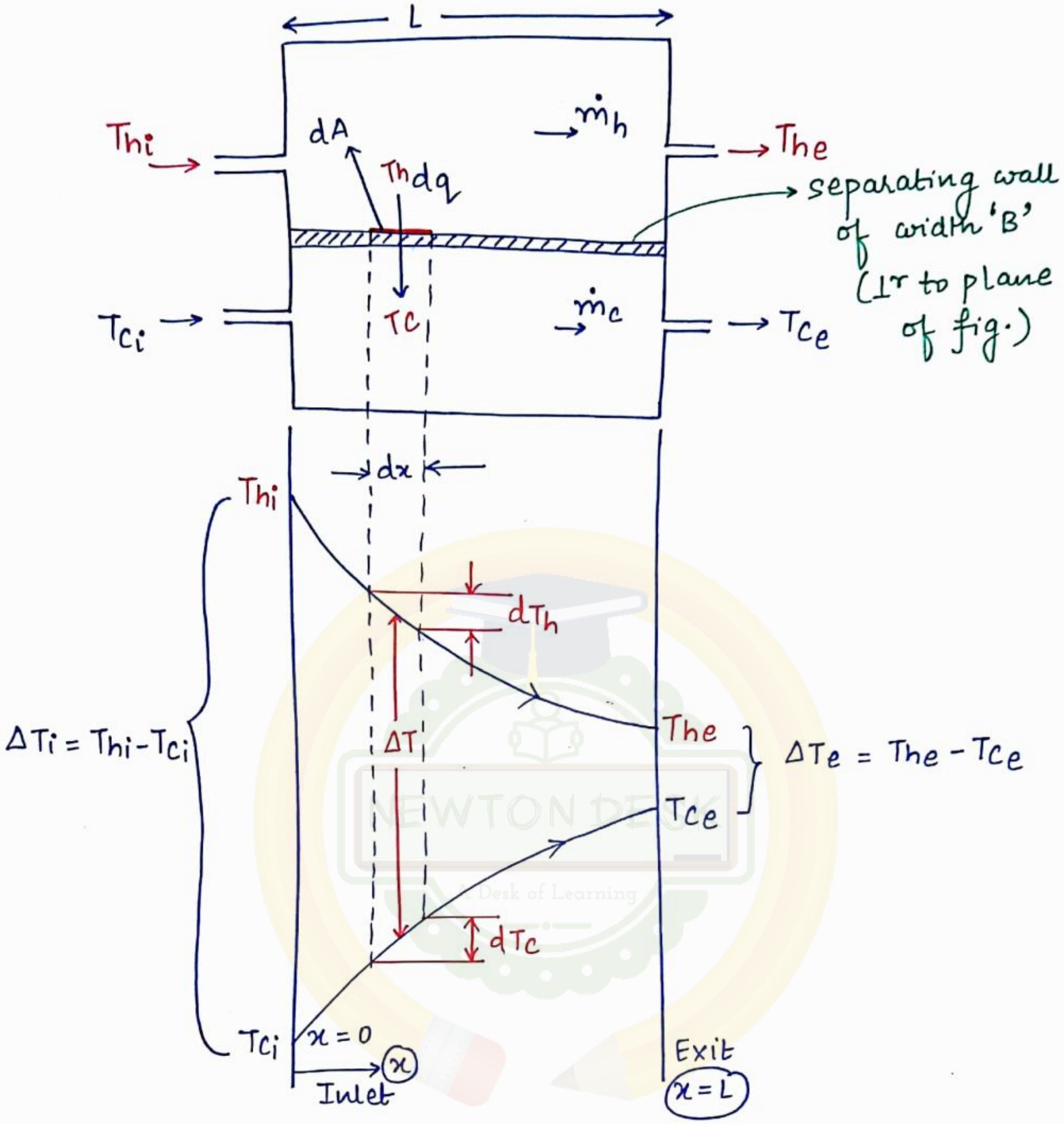
Comparing equ. (1) & equ (2)

$$\boxed{\Delta T_m \ (MTD) = \frac{1}{A} \int_{Inlet}^{Exit} \Delta T \, dA}$$

To Derive an equation for MTD of Parallel flow H.E :-



"N.P."



Consider differential H.T. Area dA of the H.E. of length 'dx' through which the differential H.T. rate b/w Hot and cold fluids is dq. Then $dq = U \Delta T \, dA$.

where $dA = B\,dx$.

and $\Delta T = T_h - T_c = f(x)$

$$\left[\begin{array}{l} \text{At } x = 0 \text{ (i.e. Inlet)} \Rightarrow \Delta T = \Delta T_i = T_{hi} - T_{ci} \\ \text{At } x = L \text{ (i.e. exit)} \Rightarrow \Delta T = \Delta T_e = T_{he} - T_{ce} \end{array}\right.$$

Also

$$dq = -\dot{m}_h c_{p_h} dT$$
$$= +\dot{m}_c c_{p_c} dT$$

$$\Delta T = T_h - T_c$$

$$d(\Delta T) = dT_h - dT_c$$

$$d(\Delta T) = -\frac{dq}{\dot{m}_h c_{p_h}} - \frac{dq}{\dot{m}_c c_{p_c}}$$

$$d(\Delta T) = -dq\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

$$\Rightarrow d(\Delta T) = -U\Delta T\, B\, dx\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

Now separating variables :-

$$\therefore \int_{\Delta T_i}^{\Delta T_e} -\frac{d(\Delta T)}{\Delta T} = \int_{x=0}^{L} UB\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right] dx$$

$$\Rightarrow \ln\frac{\Delta T_i}{(\Delta T_e)} = UBL\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

But $BL = A$ = Total H.T. area of H.E.

$$\therefore \ln\frac{\Delta T_i}{\Delta T_e} = UA\left[\frac{1}{\dot{m}_h c_{p_h}} + \frac{1}{\dot{m}_c c_{p_c}}\right]$$

Q = Total H.T. Rate for entire H.E. $= \dot{m}_h c_{p_h}(T_{hi} - T_{he})$

$= \dot{m}_c c_{p_c}(T_{ce} - T_{ci})$

$$\Rightarrow \ln \frac{\Delta T_i}{\Delta T_e} = UA\left[\frac{T_{hi}-T_{he}}{Q} + \frac{T_{ce}-T_{ci}}{Q}\right]$$

$$= \frac{UA}{Q}\left[\Delta T_i - \Delta T_e\right]$$

$$\Rightarrow \boxed{Q = UA\left[\frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}\right]}$$

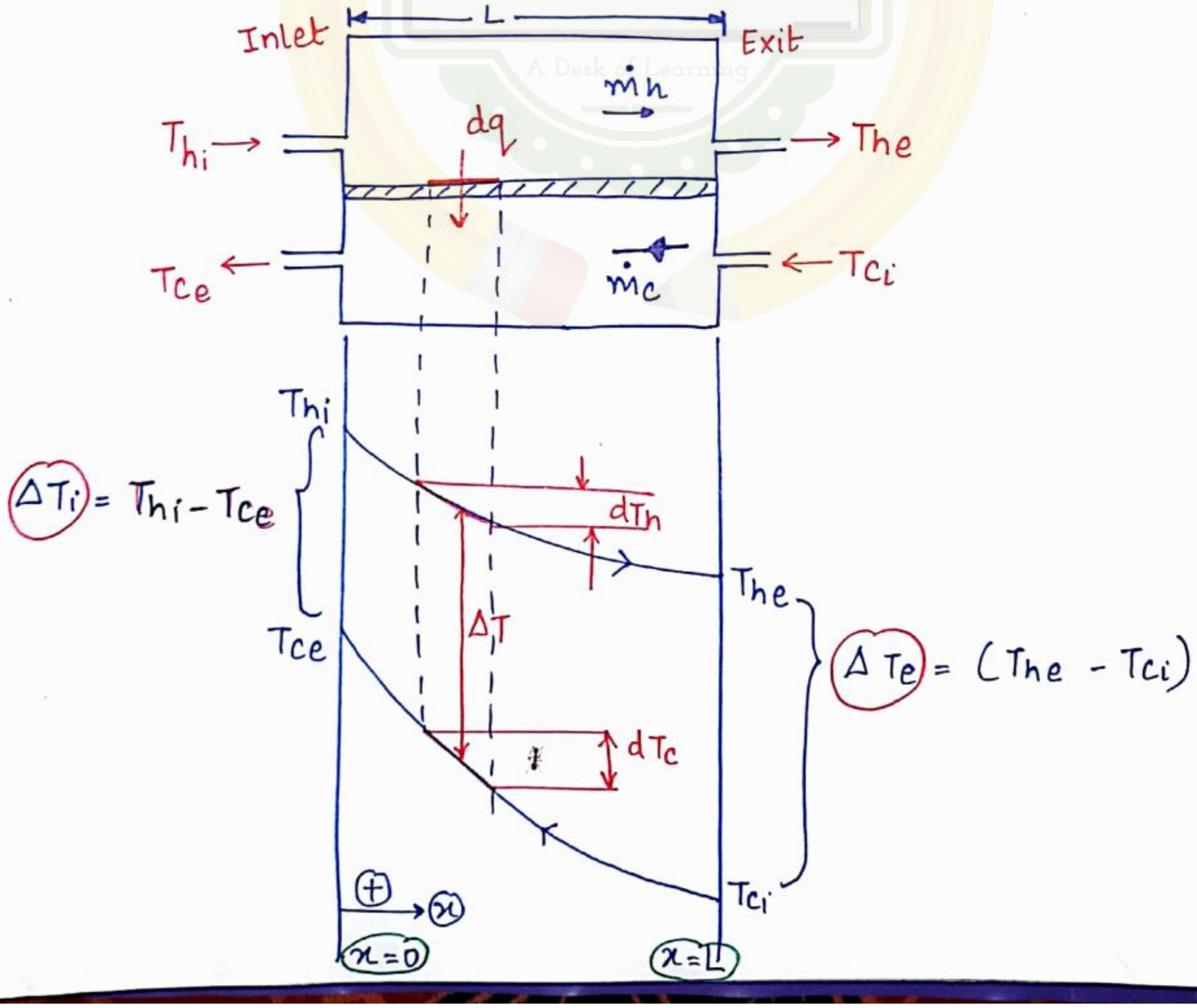
Comparing with

$$Q = UA\,\Delta T_m,$$

$$(\Delta T_m)_{||el} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

$$= (L.M.T.D.)_{||el\ flow}$$

* LMTD of counter flow Heat Exchanger :-



$$dq = U \Delta T \, B \, dx$$

$$dq = -\dot{m}_h c_{p_h} dT_h$$

$$= -\dot{m}_c c_{p_c} dT_c$$

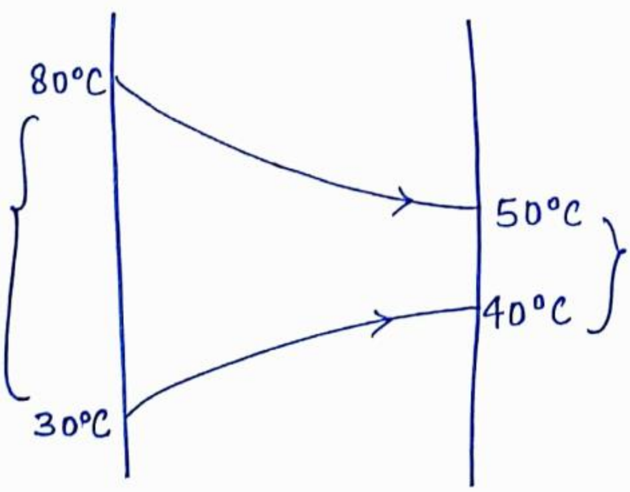
$$\boxed{(LMTD)_{counter} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}}$$

NOTE :- 1. Even though the formulae for LMTD is the same for both parallel flow and counterflow Heat Exchanger. The definitions of ΔT_i and ΔT_e are different between them.

***

2. For the same inlet and exit temperatures of both hot and cold fluids employed/used in parallel flow and counterflow Heat Exchangers, the L.M.T.D. of counterflow H.E. is greater than L.M.T.D. of parallel flow H.E. This is the reason why counterflow H.E. is more compact in size than parallel flow H.E. for the same Heat transfer rate required in both the cases.

WB Pg 74 5-1 = 4

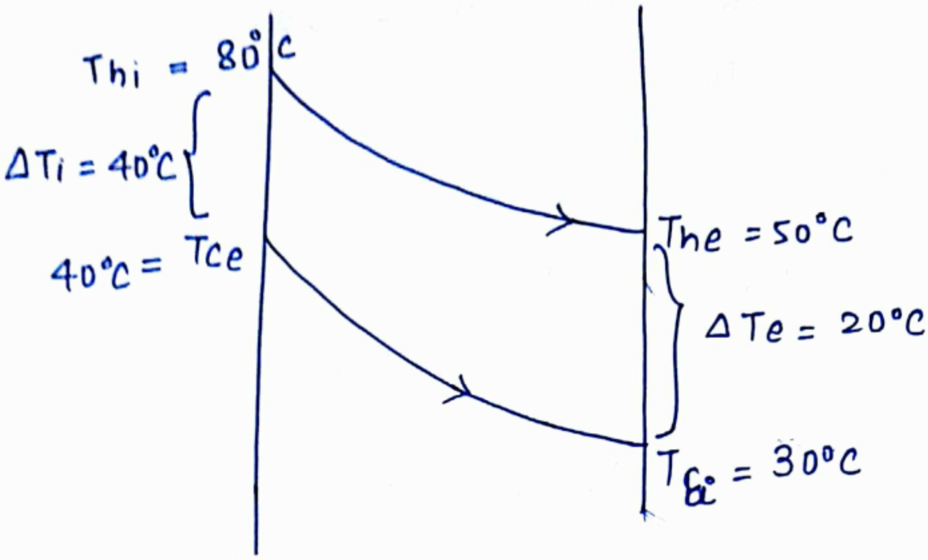


$\Delta T_i = 50°C$

$\Delta T_e = 10°C$

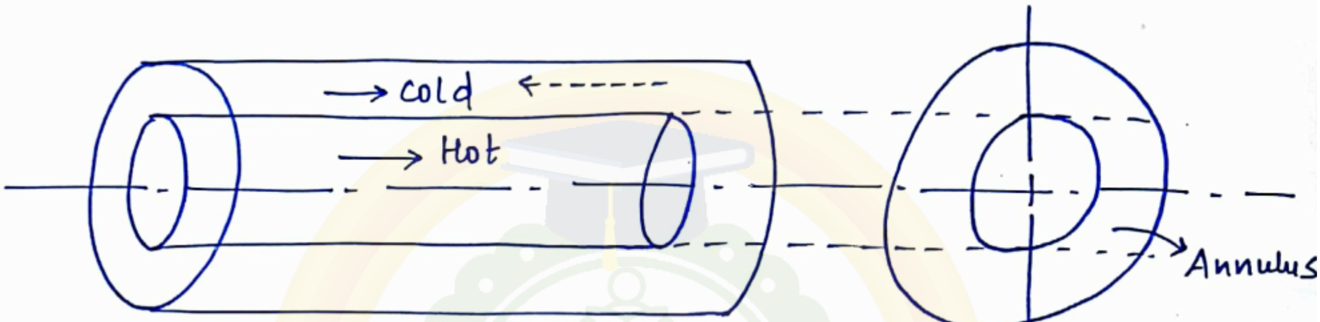
$$(LMTD)_{\parallel el} = \frac{50 - 10}{\ln \frac{50}{10}}$$

$$= 24.85°C$$



$$(LMTD)_{counter} = \frac{40-20}{\ln \frac{40}{20}}$$

$$= 28.85°C$$



Double Pipe

A Double Pipe HE can be either llel Mode (or) counterflow Mode.

Crossflow is never possible here.

$$(LMTD)_{\substack{crossflow \\ HE}} = (LMTD)_{\substack{counter \\ flow}} \times F$$

where F = correction Factor

$F < 1$

Here $F = 26/28.85$

$$\boxed{F = 0.9}$$

(15) $T_{ci} = 10°C$ $T_{hi} = 46°C$ $\dot{m}_h = 25\,L/s$

$T_{ce} = 38°C$ $T_{he} = ?$

$\dot{m}_c = 19\,lit/s$

$\rho = 1000\ kg/m^3$ $c_{pw} = 4186\ J/kg\,K$

$$\dot{m}_h c_{ph}(T_{hi} - T_{he}) = \dot{m}_c c_{pc}(T_{ce} - T_{ci})$$

$$25\ \cancel{25}(46 - T_{he}) = 19(38-10)$$

SIR

$C_{p\,hot\ water} = C_{p\,cold\ water}$

$\rho_{hot\ water} = \rho_{cold\ water}$

$\dot{m} = \rho \times$ volume flow Rate

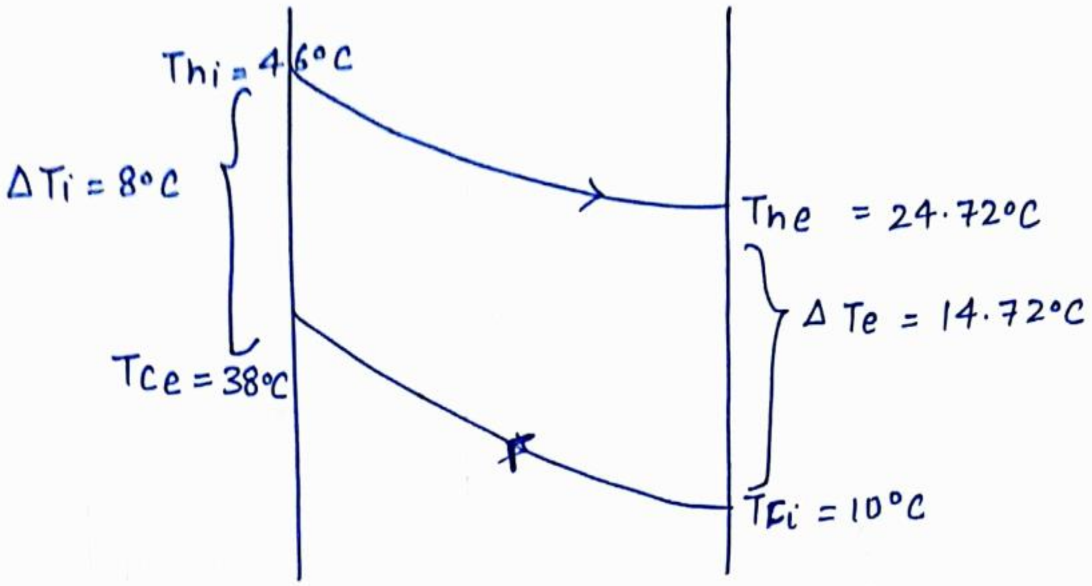
$\Rightarrow \dot{m} \propto$ volume flow Rate

Energy Balance Eqn :-

$$\dot{m}_h \cancel{c_{ph}}(T_{hi} - T_{he}) = \dot{m}_c \cancel{c_{pc}}(T_{ce} - T_{ci})$$

$$\Rightarrow 25(46 - T_{he}) = 19(38 - 10)$$

$$\boxed{T_{he} = 24.72°C}$$



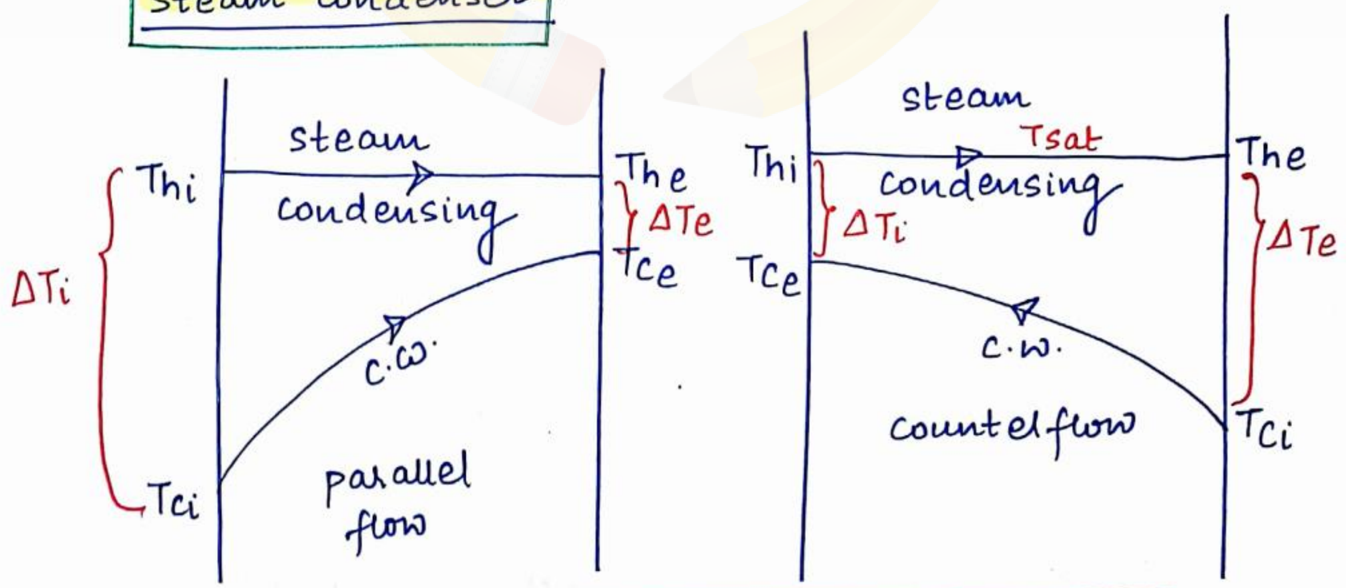
$$(LMTD)_{counterflow} = \frac{8 - 14.72}{\ln\left(\frac{8}{14.72}\right)}$$

$$= 11.02\,°C$$

* **Two special cases :-**

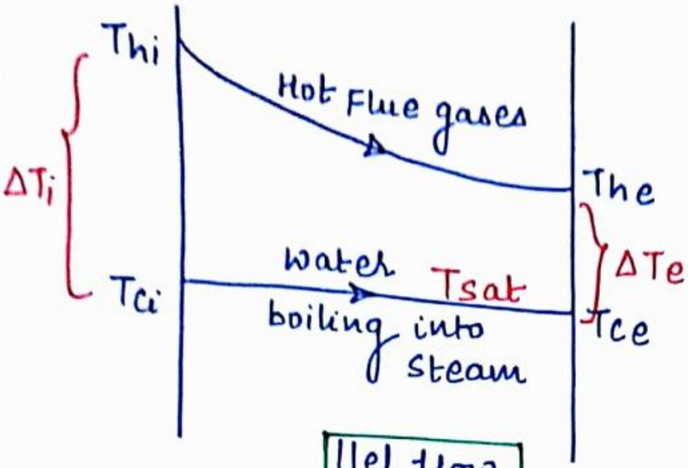
Case (I) :- when one of the fluids in the H.E. is undergoing phase change like a steam condenser (or) evaporator (or) steam generator.

Steam Condenser

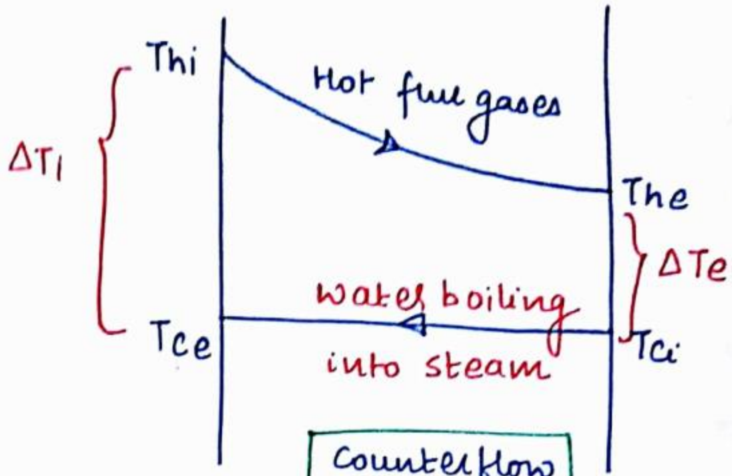


$$\boxed{(LMTD)_{\parallel el} = (LMTD)_{counterflow}}$$

Steam Generator



IIel flow

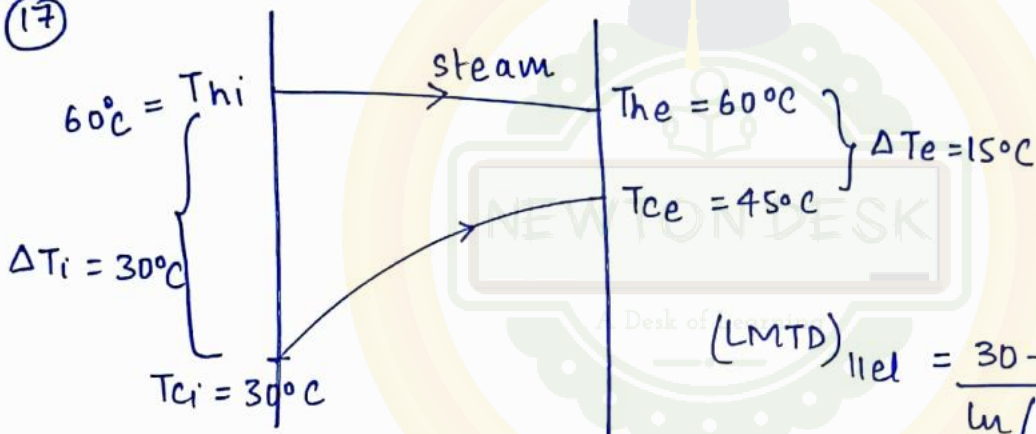


Counterflow

$$\boxed{(LMTD)_{\text{IIel}} = (LMTD)_{\text{counter}}}$$

WB

(17)



60°C

$T_{ci} = 30$

$T_{ce} = 45$

$100 \rightarrow 60$

$$(LMTD)_{\text{IIel}} = \frac{30-15}{\ln\left(\frac{30}{15}\right)} = (LMTD)_{\text{counter}}$$

$$= 21.6\,°C$$

~~Q.> When~~

CASE II :- When both Hot and cold fluids have equal capacity rates in a counter flow heat exchanger that is when

$\dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$ in a counterflow HE,

$(\dot{m} C_p)$ product is called heat capacity Rate.

Then from Energy-balance Eqn.,

$$\dot{m}_h \cancel{c_{p_h}} (T_{hi} - T_{he}) = \dot{m}_c \cancel{c_{p_c}} (T_{ce} - T_{ci})$$

$$\Rightarrow (T_{hi} - T_{he}) = (T_{ce} - T_{ci})$$

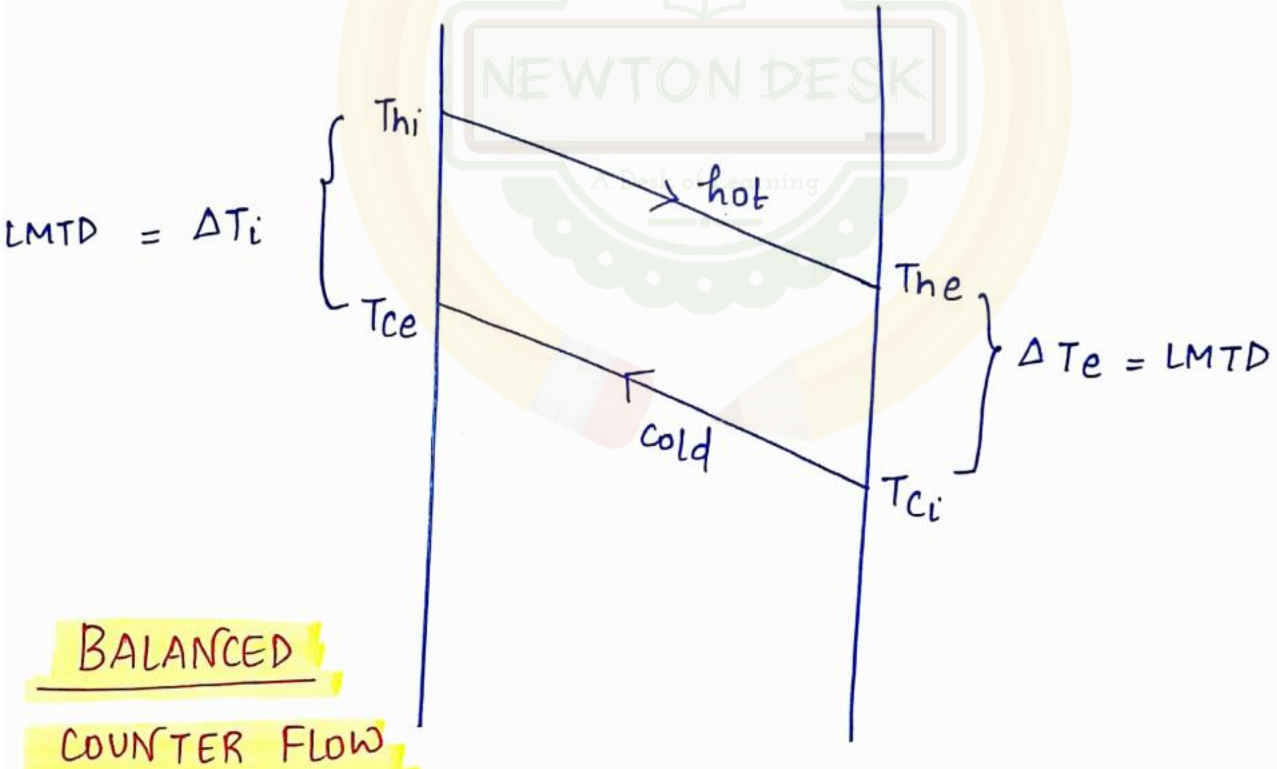
$$\Rightarrow (T_{hi} - T_{ce}) = (T_{he} - T_{ci})$$

$$\Delta T_i = \Delta T_e$$

Then $(LMTD)_{counterflow} = \dfrac{\Delta T_i - \Delta T_e}{\ln\left(\dfrac{\Delta T_i}{\Delta T_e}\right)} = \dfrac{0}{0}$ (undefined)

Then From L'hospital's Rule,

$(LMTD)_{counterflow}$ = Either ΔT_i (OR) ΔT_e



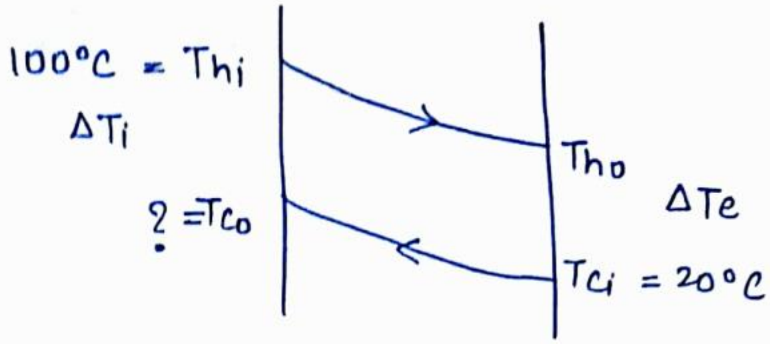
BALANCED COUNTER FLOW H.E.

(5)

LMTD $= 20^\circ C$

$\dot{m}_c = 2\dot{m}_h$

$c_{p_h} = 2\, c_{p_c}$



$$\cancel{e}^{20} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

$$\cancel{e}\ 20 = \frac{(100 - T_{co}) - (T_{ho} - 20^\circ)}{\ln\left(\frac{100 - T_{co}}{T_{ho} - 20}\right)}$$

$$\dot{m}_c\, c_{p_c}\, \Delta T_i = \dot{m}_h\, c_{p_c}\, \Delta T_e$$

$$\cancel{2}\cancel{\dot{m}_h}\, \cancel{c_{p_c}}\,(100 - T_{co}) = \cancel{\dot{m}_h}\, \cancel{2 c_{p_c}}\,(T_{ho} - 20^\circ)$$

$$T_{co} - T_{ho} = -20 - 100$$

$$T_{co} - 120 + T_{co} = -120$$

$$-T_{co} + T_{ho} = (120)$$

$$T_{ho} = 120 + T_{co}$$

$$T_{ho} - T_{co} = 120$$

$$\ln\left(\frac{100 - T_{co}}{T_{ho} - 20}\right) = -T_{ho} - T_{co} + 80 - 20$$

$$= -(T_{ho} + T_{co}) + 60$$

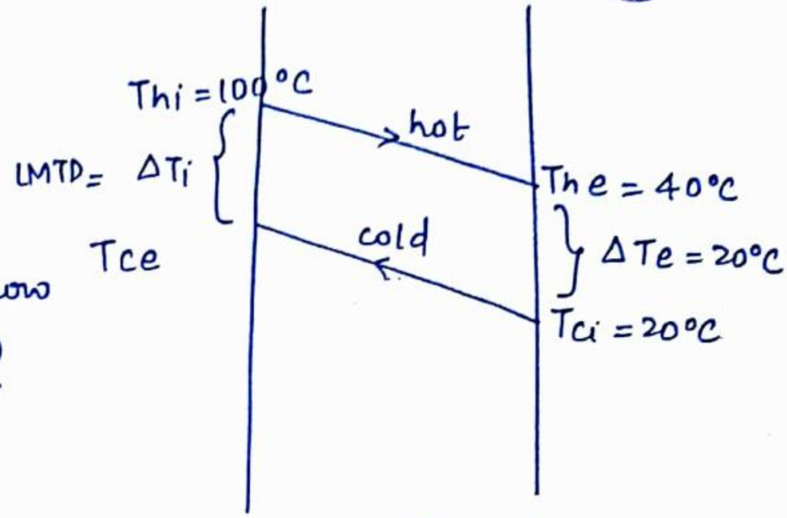
SIR

$\dot{m}_h = \frac{1}{2}\dot{m}_c$

$C_{p_h} = 2C_{p_c}$

$\Rightarrow \dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$

and HE is counterflow
(Balanced)



$LMTD = 20° = \Delta T_i = T_{hi} - T_{ce} =$

$100 - T_{ce}$

$T_{ce} = 100 - 20 = 80°C$

Q3 SIR $\dot{m}_h C_{p_h} = \dot{m}_c C_{p_c}$

and HE is counterflow (balanced)

$\therefore LMTD = T_{hi} - T_{ce}$

$= 30°C$

Q6 $C_{p_h} = 2$ $\quad C_{p_c} = 4$

$m_h = 5$ $\quad m_c = 10$

$T_{hi} = 150°C$ $\quad T_{ci} = 20°C$

$T_{ho} = 100°C$

$\dot{m}_h C_{p_h}(150° - 100°) = \dot{m}_c C_{p_c}(T_{ce} - 20°C)$

$T_{ce} = 32.5°C$

* DESIGN OF HE (LMTD METHOD) :-

In any design of Heat Exchanger, it is required to obtain the area of heat transfer in the Heat Exchanger needed for a given H.T. rate and hence to obtain the dimensions of Tubes that is d, L, n, P where d = Diameter of each Tube

L = length of HE.

n = No. of Tubes per pass

p = No. of passes

Given data :- ① Both the mass flow Rates of hot and cold fluids ($\dot{m}_h$ and $\dot{m}_c$)

② Both the sp. heats of fluids (C_{pc} and C_{ph}).

③ Overall H.T. coefficients (U) W/m^2K $\quad 1/U = 1/h_1 + 1/h_2$

④ Three Temperatures among 4 temps

for ex :- (T_{hi} , T_{ci} , T_{he})

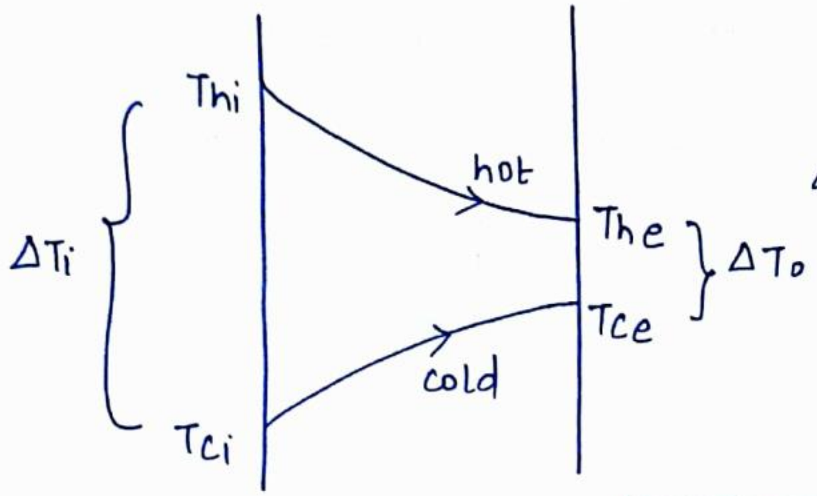
To find :- Area (A) of HE

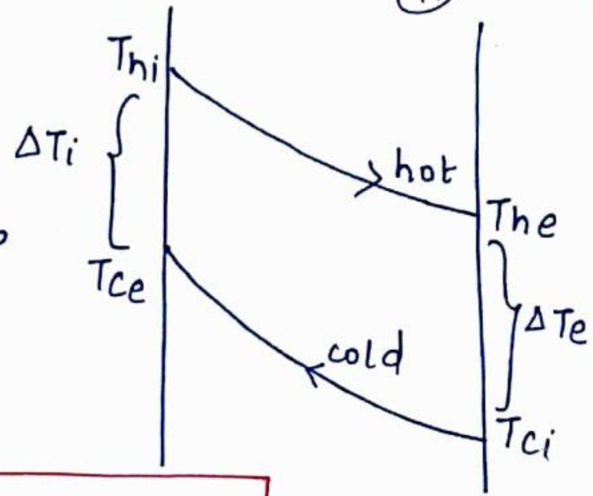
$d \quad L \quad n \quad p$

Sol :- Step 1 :- Calculate 4th unknown tempr. from Energy Balance eqn.

$$\boxed{\dot{m}_h C_{ph} (T_{hi} - T_{he}) = \dot{m}_c C_{pc} (T_{ce} - T_{ci})}$$

Step 2 :- Draw the Tempr. profiles of hot & cold fluids based on the type of HE to be designed and hence obtained LMTD of the H.E.





$$\boxed{LMTD = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}}$$

Step 3:- Calculate H.T. rate b/w hot and cold fluids from the rate of enthalpy change of either of the fluids.

$$\boxed{Q = \dot{m}_h \overset{J/KgK}{C_{Ph}} (T_{hi} - T_{he}) = \dot{m}_c C_{Pc} (T_{ce} - T_{ci})}$$

Step 4:- Calculate area of H.E. from

$$\boxed{Q = UA\Delta T_m}$$

$$\boxed{A = \frac{Q}{U\Delta T_m}}$$

Gate 2004 → Steam is condensing in a condenser at a tempr. of 120°C. The cooling water enters the condenser at a tempr. of 30°C and while flowing through the condenser at a mass flow rate of 1500°C kg/hr leaves the condenser at a tempr. of 80°C. If the overall H.T. co effi is 2000 W/m2K, the H.T. area req. for the condenser is

ⓐ 0.707 m²

ⓑ 7.07 m²

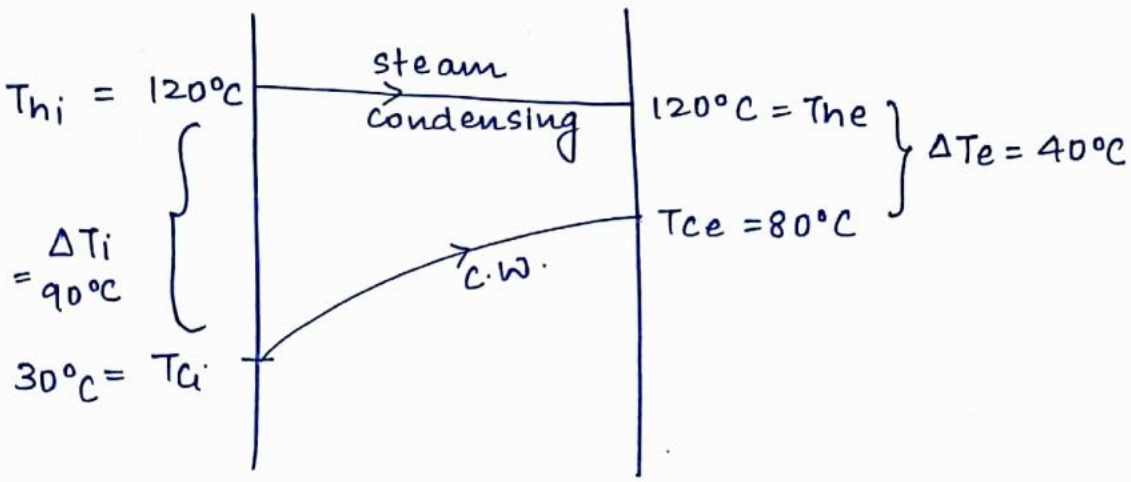
ⓒ 70.7 m²

ⓓ 141.4 m²

$$\dot{m}_h C_{ph}(T_{hi} - T_{ho}) = \dot{m}_c C_{pc}(T_{ce} - T_{co})$$

$$(120^\circ C - T_{ho}) = 1500 \times \quad (80 - 30)$$

SIR



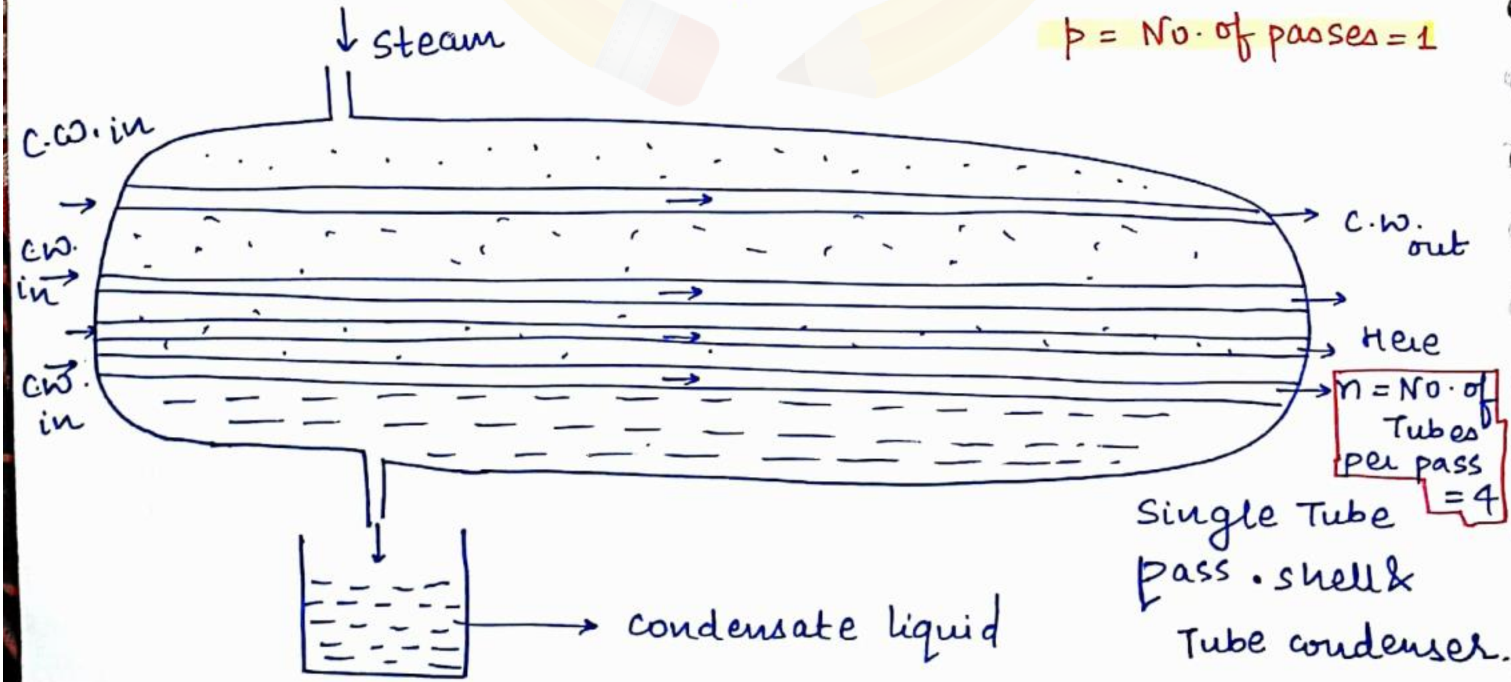
$$(LMTD)_{HE\ ||el} = \frac{90 - 40}{\ln\left(\frac{90}{40}\right)} = 61.65^\circ C$$

$$Q = UA\Delta T_m = \dot{m}_c \underbrace{C_{pc}}_{4.186\ KJ/KgK}(T_{ce} - T_{ci})\ \text{watt}$$

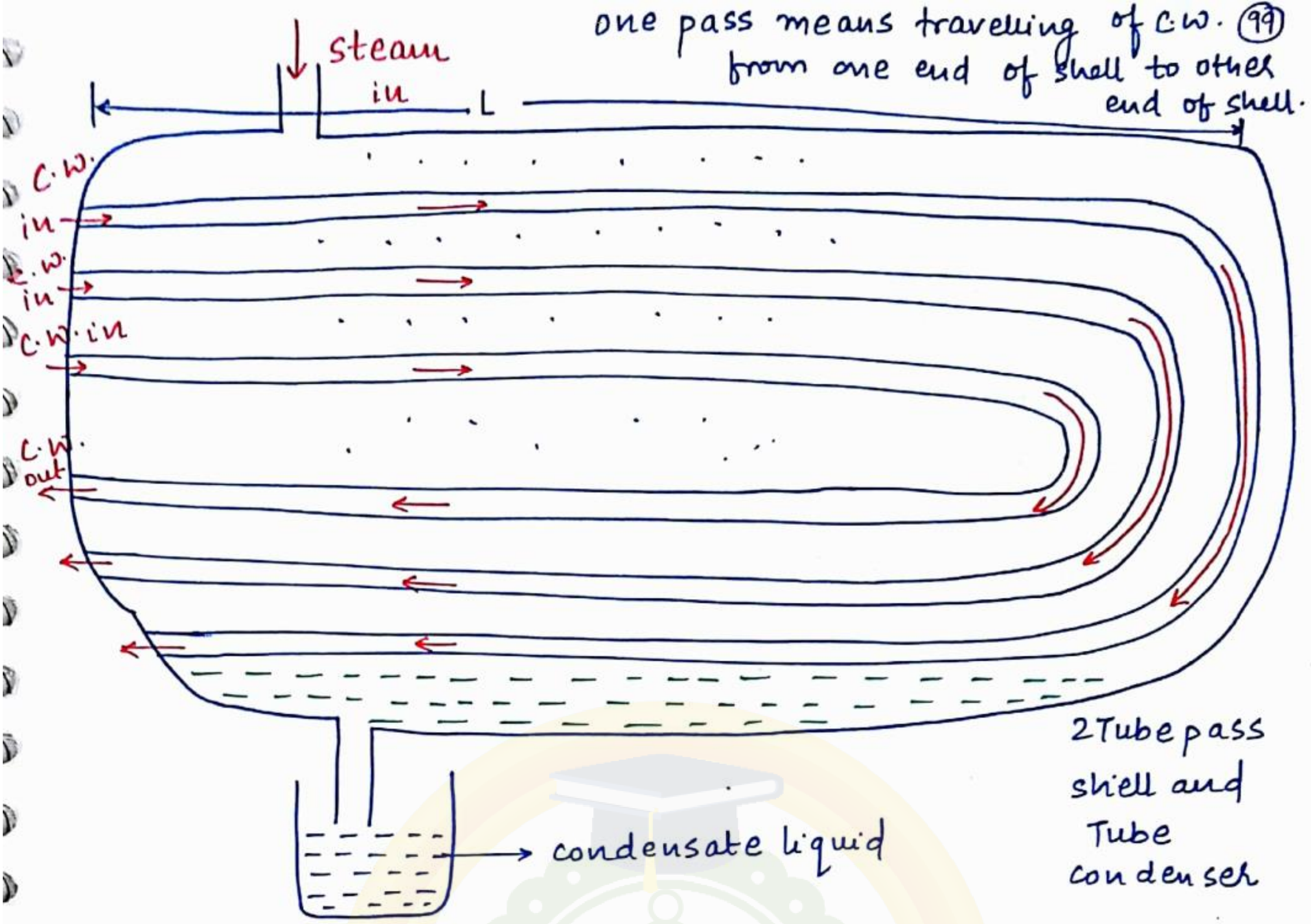
$$A = \frac{1500}{3600} \times \frac{4.86 \times 10^3 (80-30)}{2000 \times 61.65} = 0.707\ m^2$$

* $\boxed{\text{To get the dimensions of Tubes from the Area}}$:-

p = No. of passes = 1



Single Tube pass. shell & Tube condenser.



Here, No. of passes = $p = 2$

No. of Tubes per pass $= n = 3$

$$\boxed{A = \pi d L \times n \times P}$$

velocity

$$\dot{m}_{c.w.} = \rho \times \frac{\pi}{4} d^2 \vec{V} \times n \quad kg/sec$$

(Mass flow rate of c.w.)

Passes are provided only when the heat exchanger area req. is very large. For Ex:- steam condensers of a thermal power plant or chemical units. By providing passes, we can reduce the length of the H.E. req. for a given heat transfer rate needed so that the H.E. can be accomodated in the available floor space.

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(19) $\rho = 1100\ kg/m^3$

$C_p = 4.6\ kJ/kg\,K$

$65°C \longrightarrow 100°C$

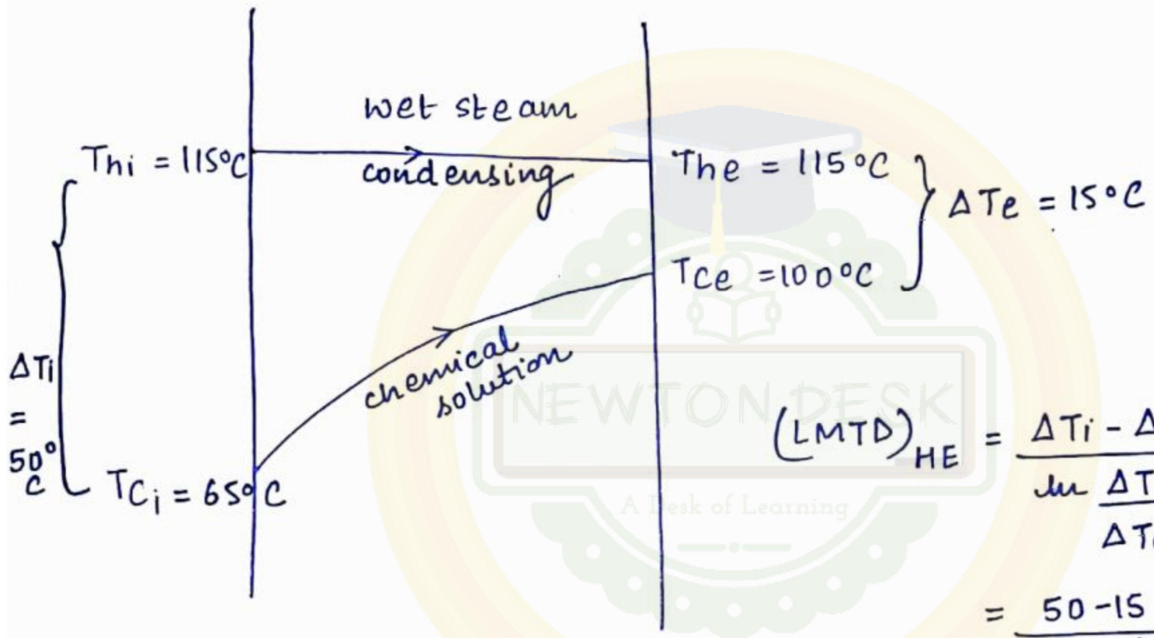
$m_{sol} = 11.8\ kg/s$

Note:- whenever any wet steam is cooled, it has to condense at constant tempr. liberating latent heat of condensation.

SIR

LMTD Method

To find the area of H.T., (A) = ?



$$(LMTD)_{HE} = \frac{\Delta T_i - \Delta T_e}{\ln \frac{\Delta T_i}{\Delta T_e}}$$

$$= \frac{50-15}{\ln\left(\frac{50}{15}\right)} = 29.07°C$$

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}$$ (Neglecting Conduction Thermal Resistance)

$$\frac{1}{U} = \frac{1}{5} + \frac{1}{10}$$

$$\Rightarrow U = \boxed{3.33}\ kW/m^2K$$

$Q = UA\Delta T_m$

$A = \dfrac{Q}{U\Delta T_m} = \dfrac{\dot m_c C_{p_c}(T_{ce} - T_{ci})}{U\Delta T_m}$

$= \dfrac{11.8 \times 4.6\,(100-65)}{3.33 \times 29.07} = \boxed{19.62}\, m^2$

$A = \pi d L \times n \times p = 19.62\, m^2 \quad \Rightarrow p = 3.75$

$= 4$ passes ✓

$\dot m_c = \rho \times \dfrac{\pi}{4} d^2 \vec{V} \times n$ kg/sec $= 11.8$ kg/sec

n = no. of Tubes per pass

$= 18.21 \simeq 19$ Tubes ✓

(21)

$A_p \leftarrow$ ||el, $A_o \leftarrow$ counter $\Big\}$ $\dot m_h = 1$ kg/s, $T_{hi} = 80°C$, $T_{ho} = 50°C$

$\dot m_c = 2$ kg/s

$T_{ci} = 10°C$

$U_{||el} = U_{counter}$

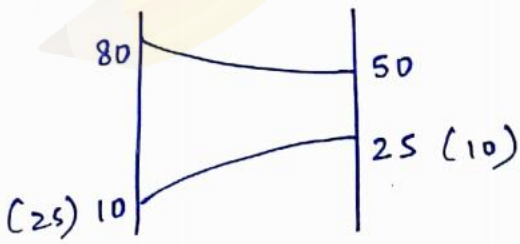
$\dfrac{A_c}{A_p} = ?$

$\dot m_h \cancel{C_{p_h}} (T_{hi} - T_{ho}) = \dot m_c \cancel{C_{p_c}} (T_{co} - T_{ci})$

$1\,(80-50) = 2\,(T_{co} - 10)$

$\boxed{T_{co} = 25°C}$

$Q = UA\Delta T_m$



$\Delta T_m = \dfrac{\Delta T_i - \Delta T_o}{\ln \dfrac{\Delta T_i}{\Delta T_o}}$

$\Delta T_m = \dfrac{45}{1.029}$

$\dfrac{15}{0.31}$

$\boxed{\dfrac{30}{43.73\,A} = U}$

$\dfrac{30}{47.10\,A} = U$

$\dfrac{47.10}{43.73} =$ 1.07

SIR Parallel Counter

$\dot{m}_h = \dot{m}_h = 1\,kg/sec$

$T_{hi} = T_{hi} = 80°C$

$T_{he} = T_{he} = 50°C$

$\dot{m}_c = \dot{m}_c = 2\,kg/sec$

$T_{ci} = T_{ci} = 10°C$

$c_{p_h} = c_{p_c}$ (same fluid in both the HEs)

$U_p = U_c$

$$Q = \dot{m}_h \cancel{c_{p_h}} (T_{hi} - T_{he}) = \dot{m}_c \cancel{c_{p_c}} (T_{ce} - T_{ci})$$

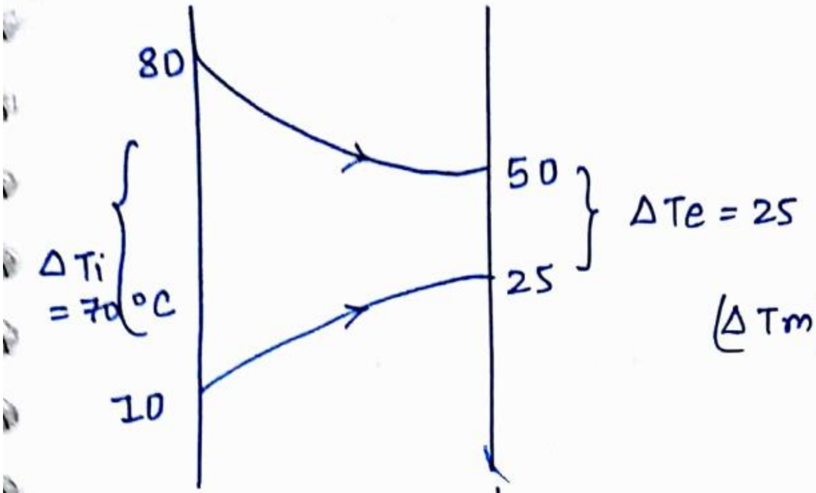
$$1 \times (80 - 50) = 2 (T_{ce} - 10)$$

$T_{ce} = 25$ (same in both the HEs)

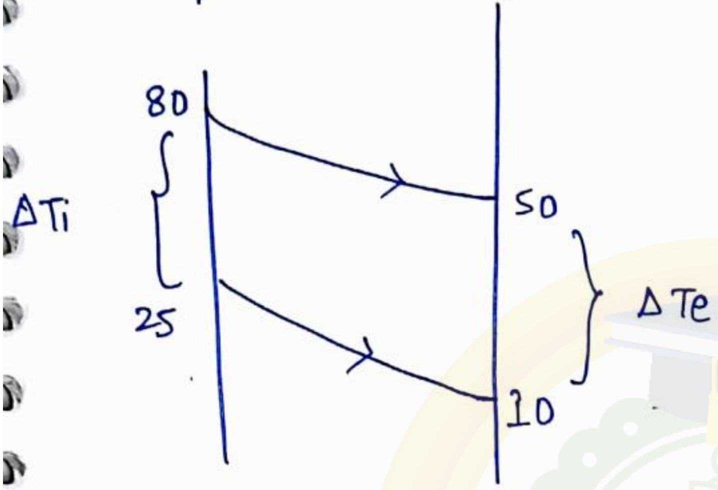
(Rate of HT) Q also same in both the HES

$$Q = UA\Delta T_m$$

$$\frac{A_c}{A_p} = \frac{(\Delta T_m)_p}{(\Delta T_m)_c} = \left(\frac{43.7}{47.1}\right) = 0.927$$



$(\Delta T_m)_{prl} = 43.7\ °C$



$(\Delta T_m)_{counter} = 47.1°C$

* **PERFORMANCE of HE's :-**

① **Effectiveness (ϵ) of HE :-**

It is defined as the ratio between actual heat transfer rate taking place b/w hot and cold fluids and the maxm. possible H.T. rate b/w them.

$$\boxed{\epsilon_{HE} = \frac{q_{act.}}{q_{max.\ possible}}}$$

$\boxed{q_{act.}}$ = Rate of enthalpy change of either of the fluids

$= \boxed{\dot{m}_h C_{p_h} (T_{hi} - T_{he}) = \dot{m}_c C_{p_c} (T_{ce} - T_{ci})}$

$\boxed{q_{max.\ possible} = (\dot{m} C_p)_{small} (T_{hi} - T_{ci})}$, where $(\dot{m} C_p)_{small}$ being the smaller capacity rate b/w the hot & cold fluids i.e. smaller value of $\dot{m}_h C_{p_h}$ & $\dot{m}_c C_{p_c}$.

$(\dot{m}C_p)$ product is called as heat capacity Rate.

Why $(\dot{m}C_p)_{small}$?

when steam is condensing,

$$(\dot{m}C_p)_{steam} \to \infty$$

because when steam is undergoing phase change,

$$(\Delta T)_{steam} = 0$$

$$\therefore Q = mC_p \Delta T$$

$$\Rightarrow \boxed{(mC_p)_{steam} = \frac{Q}{\Delta T} = \frac{Q}{0} \to \infty}$$

***

continues previous page.......

$$\boxed{\text{If } \dot{m}_h C_{p_h} < \dot{m}_c C_{p_c},}$$

Then

$$\boxed{\epsilon_{HE} = \frac{\cancel{\dot{m}_h C_{p_h}}(T_{hi} - T_{he})}{\cancel{\dot{m}_h C_{p_h}}(T_{hi} - T_{ci})} = \frac{(T_{hi} - T_{he})}{(T_{hi} - T_{ci})}}$$

on the other hand,

$$\boxed{\text{If } \dot{m}_c C_{p_c} < \dot{m}_h C_{p_h},}$$

Then

$$\boxed{\epsilon_{HE} = \frac{\cancel{\dot{m}_c C_{p_c}}(T_{ce} - T_{ci})}{\cancel{\dot{m}_c C_{p_c}}(T_{hi} - T_{ci})} = \frac{T_{ce} - T_{ci}}{(T_{hi} - T_{ci})}}$$

only one of the 2 formulae given above can be used at a time. only under the special case of both the capacity rates being equal on both Hot ~~fluid~~ side & cold side then only any formulae of effectiveness given above can be used.

(20) b ✓

(14) $T_{ci} = 25°C$ $T_{hi} = 100°C$

$T_{co} = 42°C$ $T_{ho} = 50°C$

6267 $m_w = 1.5 kg/s$ $C_{pw} = 4178 J/kgK$

2130 $m_o = 1 kg/s$ $C_{po} = 2130 J/KgK$

$$\frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}} = 0.66$$

SIR

$\dot{m}_h C_{ph} < \dot{m}_c C_{pc}$

$(1 \times 2130) < (1.5 \times 4178)$

Since hot side capacity Rate is smaller,

$$\epsilon_{HE} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}} = \frac{100 - 50}{100 - 25} = 0.667$$

(2.09)

(12) $C_p = 4.18$

water → $T_{hi} = 80°C$ $m_w = 0.5$

air → $T_{ci} = 30°C$ $C_p = 1$ $m_a = 2.09$

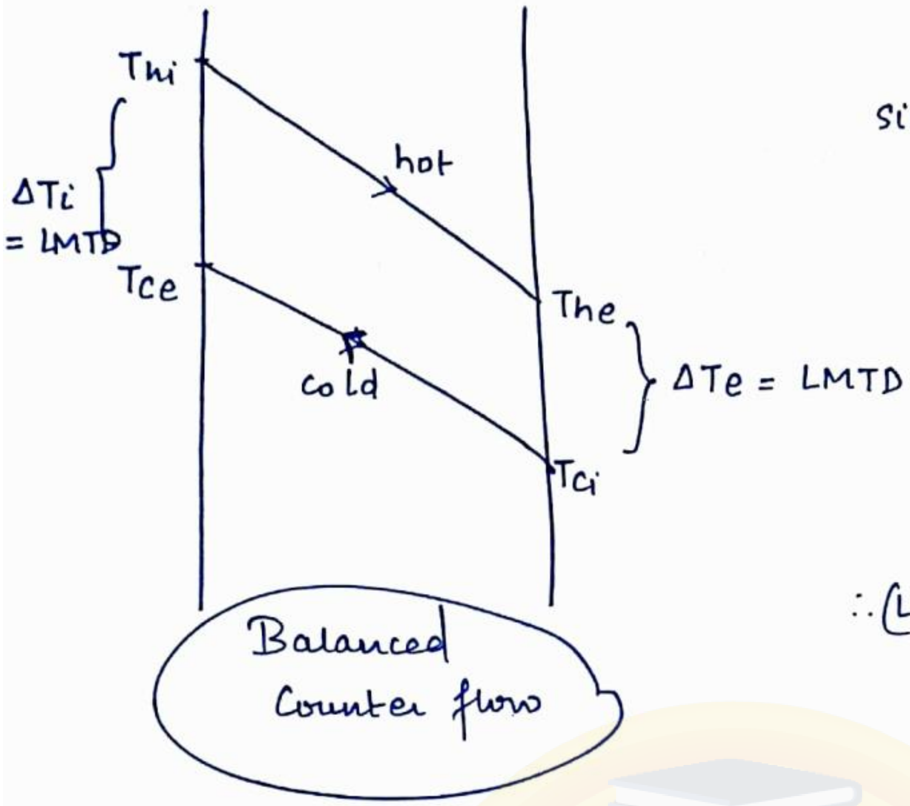
$\Delta T_i = \Delta T_e$

$T_{hi} - T_{ce} = T_{he} - T_{ci}$

SIR

Here $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$ & HE is counterflow

$\boxed{\text{Balanced counter flow HE}}$



$LMTD = T_{hi} - T_{ce}$

Since capacity Rates being equal any formula of 'ε' can be used

$$\varepsilon = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} \Rightarrow 0.8 = \frac{T_{ce} - 30}{80 - 30}$$

$$\Rightarrow T_{ce} = 70°C$$

$$\therefore (LMTD)_{counter} = 80 - 70 = 10°C$$

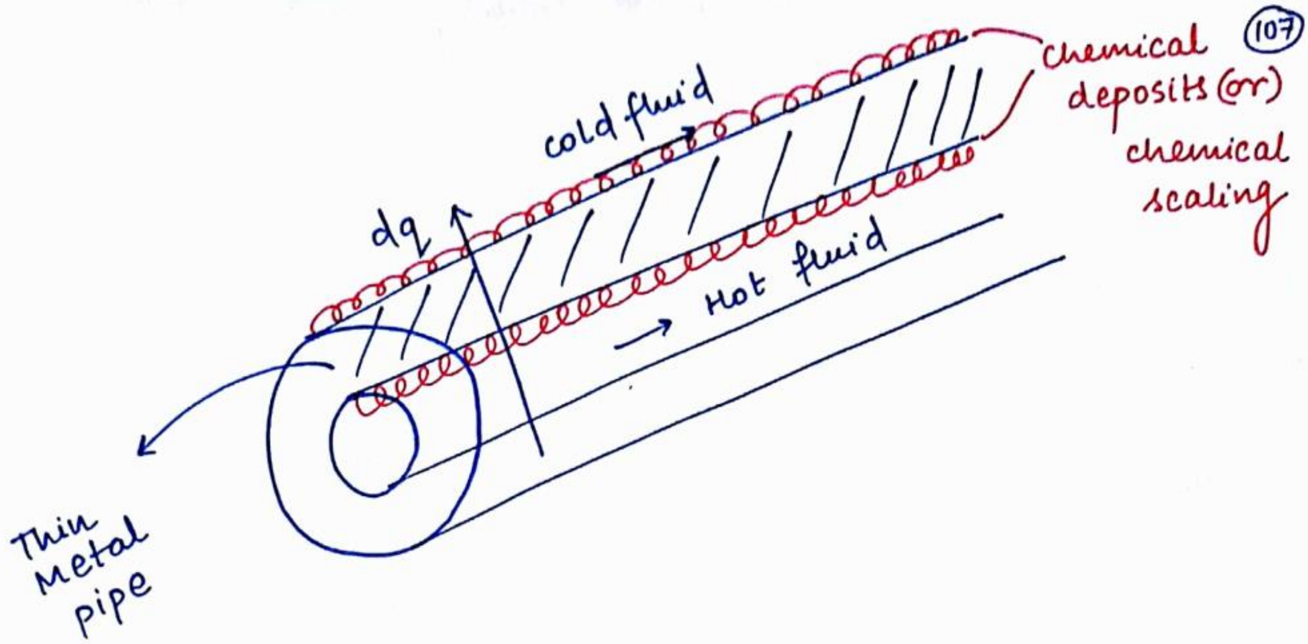
$\boxed{\text{NOTE}}$:- Practically, $\boxed{\varepsilon_{any\ H.E.} = 0.5 \text{ to } 0.8}$

② **Number of Transfer units (NTU)** :-

$$\boxed{NTU = \frac{UA}{(\dot{m}\,C_P)_{small}}}$$

Since NTU is being directly proportional to area of H.E., it indicates overall size of the H.E.

"N.P"



Fouling Factor (F) :- F.F. is the factor which takes into account the thermal resistance offered by any chemical scaling or deposits that are formed on the H.T. surfaces on both hot sides and cold side.

Usually, $F \rightarrow 0.0005\ m^2 K/watt$ (Kelvin)

(or)

$\rightarrow 0.0004$ —,,—

(or)

$\rightarrow 0.0006$ —,,—

with fouling on both hot and cold sides, 'U' can be calculated as :-

$$\boxed{\frac{1}{U_{Dirty}} = \frac{1}{h_1} + F_1 + \frac{1}{h_2} + F_2}$$

F_1 and F_2 are Fouling factors on hot side and cold side resp.

$\therefore$ As a result of Fouling :-

1. U decrease.
2. NTU $\downarrow$
3. $\epsilon_{HE} \downarrow$
4. H.T. Rate in HE $(Q) \downarrow$

Practically, NTU $\longrightarrow$ 1.15, 1.5, 2, 2.5 NTU $\not>$ 18 NTU = 1, > 1, < 1

Now,

$$NTU = \frac{UA}{(\dot{m} C_p)_{small}}$$

$U \rightarrow W/m^2K$

$C_p \rightarrow \frac{\text{Joule}}{\text{kg K}}$

③ * Capacity Rate Ratio (C) :- $0 \le C \le 1$

$$\boxed{C = \frac{(\dot{m} C_p)_{small}}{(\dot{m} C_p)_{big}}}$$

$C = 0$ if $(\dot{m} C_p)_{big} \longrightarrow \infty$

when one of the fluids is undergoing phase change (vap. to liq.)

like in steam condenser.

(or) Evaporator.

(or) Boiler.

For any HE, $\boxed{\epsilon = f(NTU, C)}$

For parallel flow H.E.,

$$\boxed{\epsilon_{\substack{H.E.\\ \parallel el}} = \frac{1 - e^{-(1+c)NTU}}{1+c}}$$

For counter flow H.E.,

$$\boxed{\epsilon_{\substack{H.E.\\ counter}} = \frac{1 - e^{-(1-c)NTU}}{1 - c\,e^{-(1-c)NTU}}}$$

* TWO SPECIAL CASES Regarding Effectiveness :-

Case I → when one of the fluids is undergoing phase change like in steam condenser.

Then $C = 0$

$$\Rightarrow \boxed{\epsilon_{\parallel el} = 1 - e^{-NTU}}$$

and

$$\Rightarrow \boxed{\epsilon_{counter} = 1 - e^{-NTU}}$$

Case II → when both hot and cold fluids have equal capacity rates i.e. when $\dot{m}_h c_{ph} = \dot{m}_c c_{pc}$

$$\Rightarrow C = 1$$

Then
$$\boxed{\epsilon_{\parallel el} = \frac{1 - e^{-2NTU}}{2}}$$

and
$$\boxed{\epsilon_{counter} = 0/0}$$ undefined

Then from L' hospital's Rule

$$\boxed{\epsilon_{counter} = \frac{NTU}{1+NTU}}$$

Balanced counter flow H.E.

* Effectiveness (ϵ) - NTU Method :-

The main objective of this NTU method is to obtain both the exit temperatures of hot and cold fluids that is the and tce for a given heat exchanger area A.

<u>Given data</u> :- ① Both the mass flow rates of hot and cold fluids i.e. $\dot{m}_h$ and $\dot{m}_c$ (kg/sec).

② Both the sp. heats of hot and cold fluids C_{ph} & C_{pc}.

③ Overall H.T. coefficient 'U' W/m^2K.

④ Only two Inlet temperatures of fluids (T_{hi} and T_{ci})

⑤ Area of HE (A).

<u>To find</u> :- $T_{he} = ?$

$T_{ce} = ?$

<u>Solution</u> :- step ① → Calculate both the capacity rates of hot & cold fluids that is $\dot{m}_h C_{ph}$ and $\dot{m}_c C_{pc}$ and hence obtain capacity rate ratio C.

$$\boxed{C = \frac{(\dot{m} C_p)_{small}}{(\dot{m} C_p)_{big}}}$$

step ② → Calculate $\boxed{NTU = \frac{UA}{(\dot{m} C_p)_{small}}}$

step ③ → Calculate ϵ of H.E.

since $\epsilon = f(NTU, C)$.

step ④ → ϵ

if $\dot{m}_c C_{pc}$ is smaller: $\left(\frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}\right)$

if $\dot{m}_h C_{ph}$ is smaller: $\left(\frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}\right)$

select only one formula of 'ε' based on which fluid has smaller capacity rate and hence obtain only one exit tempr. of the fluids.

Step 5 :- Calculate the other exit tempr. of the fluids from the Energy balance eqn. (or) Heat Balance eqn.

$$\dot{m}_h C_{ph} (T_{hi} - T_{he}) = \dot{m}_c C_{pc} (T_{ce} - T_{ci})$$

(22) $m_w = 7500$ kg/hr counterflow

$T_{ci} = 15°C$

$T_{ho} = ?$

air $T_{hi} = 185°C$ $m_a = 8000$ kg/hr

$U = 145$ W/m²K $A = 20$ m²

SIR

$(C_p)_a = 1.005$ KJ/kg K

$C_{p\,water} = 4.186$ KJ/kg K

$C = 0.2400$

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}} = \frac{145 \times A}{\left(\frac{8000}{3600} \times 1005\right)}$$

$$NTU = 0.0649\,A$$

$$\epsilon = \frac{1 - e^{-(1-c)NTU}}{1 - c\, e^{-(1-c)NTU}}$$

$$\epsilon = \frac{1 - e^{-0.76 \times 0.0649 A}}{1 - 0.24\, e^{-0.76 \times 0.0649 A}}$$

$$\epsilon = \frac{1 - e^{-0.049 A}}{1 - 0.24 e^{-0.049 A}}$$

SIR $\dot{m}_h C_{ph} < \dot{m}_c C_{pc}$

$8000 \times 1.005 < 7500 \times 4.186$

$$C = \frac{\dot{m}_h C_{ph}}{\dot{m}_c C_{pc}} = 0.256$$

$$NTU = \frac{UA}{\dot{m}_h C_{ph}} = 1.298$$

$\dot{m}_h \to$ kg/sec, $C_{ph} \to$ J/kg K

$$\epsilon_{counter} = \frac{1 - e^{-(1-C)NTU}}{1 - C\, e^{-(1-C)NTU}}$$

$$= \boxed{0.686}$$

Since $\dot{m}_h C_{ph} < \dot{m}_c C_{pc}$ i.e. hot side capacity Rate being small.

$$\epsilon = \frac{(T_{hi} - T_{he})}{(T_{hi} - T_{ci})} = 0.686$$

$$\Rightarrow \frac{105 - T_{he}}{105 - 15} = 0.686$$

$$\Rightarrow T_{he} = \boxed{43.26}\ °C$$

Exit tempr. of air.

(18) ||el $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$

$\dot{m}_h = 1$ kg/s

$C_p = 4$ KJ/Kg K

$T_{hi} = 102°C$

$T_{ho} = ?$

$T_{ci} = 15°C$

$T_{co} = ?$

$U = 1 \times 10^3$ W/m²K

$A = 5 m^2$

$$2\epsilon = 1 - e^{-2NTU}$$

Sol $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$ and HE is parallel flow type

$$\epsilon_{HE} = \frac{1 - e^{-NTU}}{2}$$

$$NTU = \left(\frac{UA}{\dot{m}C_p}\right)_{small} = \frac{(1000 \times 5)}{(1 \times 4000)} = 1.25$$

W/m²K

J/kg K

$$\Rightarrow \epsilon_{HE} = 0.459$$

since capacity rates being equal,
any formula of ϵ can be used

$$\epsilon = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}} \Rightarrow 0.459 = \frac{T_{ce} - 15}{102 - 15} \Rightarrow T_{ce} = 54.9°C.$$

Q9

$\dot{m}_c = 1 \text{ kg/sec}$

$A = 60 \text{ m}^2$

$U = 25 \text{ W/m}^2\text{K}$

$C_p = 1 \text{ kJ/kg K}$

Sol $\dot{m}_h = \dot{m}_c$

$C_{ph} = C_{pc}$

$\Rightarrow \dot{m} C_{ph} = \dot{m} C_{pc}$

$C = 1$

& HE is counterflow (Balanced)

$$\epsilon = \frac{NTU}{1 + NTU}$$

$$NTU = \frac{UA}{(\dot{m}C_p)_{small}} = \left(\frac{25 \times 60}{1 \times 1000}\right) = 1.5$$

$$(\epsilon_{HE}) = \frac{1.5}{1 + 1.5} = 0.6$$

(8) $\epsilon = 0.8$ counter

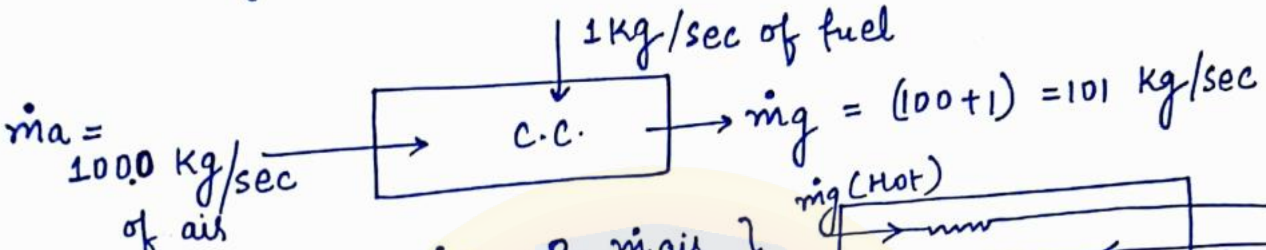
$NTU = ?$

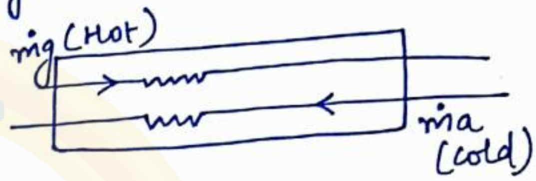
$$\epsilon = \frac{NTU}{1+NTU}$$

$$0.8 + 0.8NTU = NTU$$

$$\boxed{NTU = 4}$$

SIR In Gas Turbine Power plants, $A/F \simeq 100:1$ Ratios





$\dot{m}_g \simeq \dot{m}_{air}$

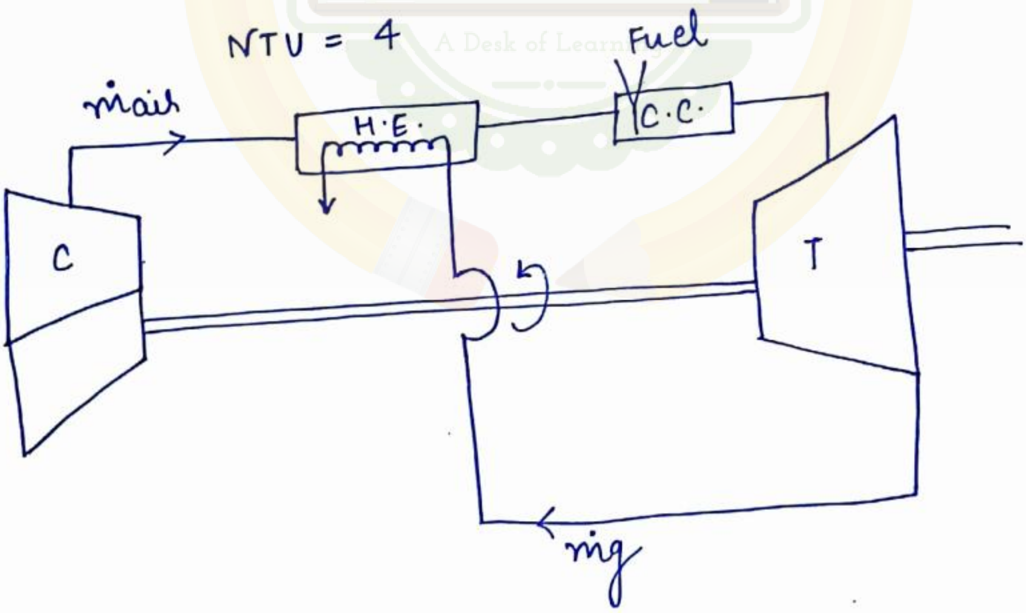
$C_{pg} \simeq C_{p\,air}$

$\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$

$\Rightarrow C = 1$

$$\epsilon_{counter} = \frac{NTU}{1+NTU} = 0.8$$

$NTU = 4$



(1) a

(2) a

(10) $$\frac{1}{U_{Dirty}} = \frac{1}{U_{clean}} + F.$$

$$F = \frac{1}{2000} \ m^2K/watt$$

$$\frac{1}{U_{Dirty}} = \frac{1}{400} + \frac{1}{2000}$$

$$U_{Dirty} = 333 \ W/m^2K$$

(13) d ($\because$ $C_{phase\ change} = 0$)

(11) (c) ✓

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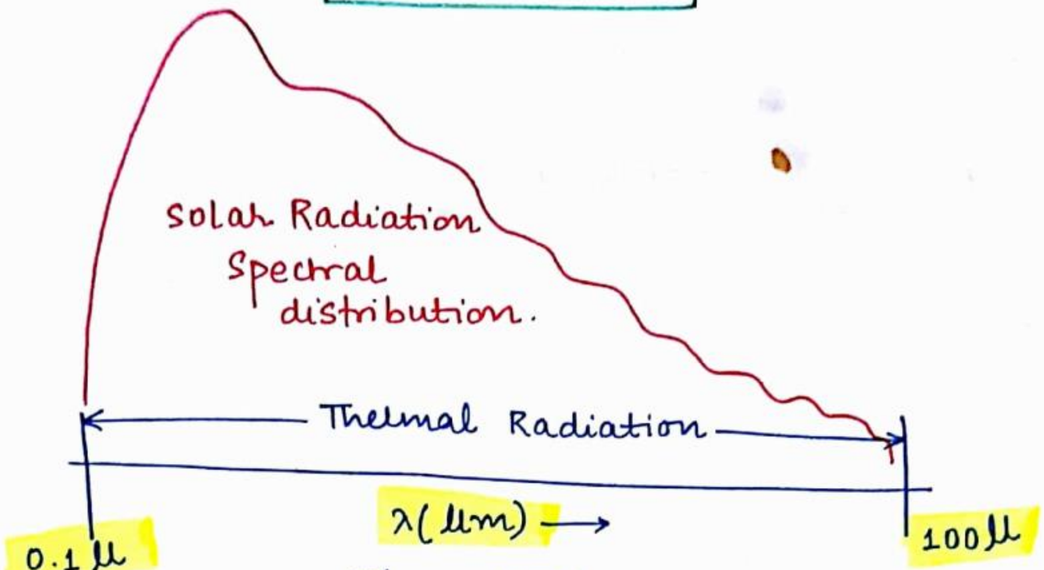
RADIATION

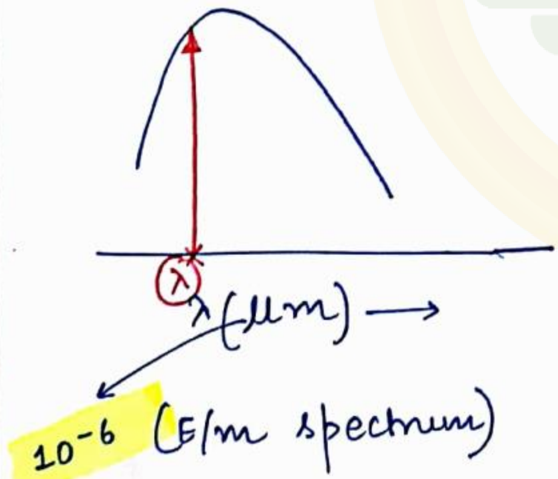


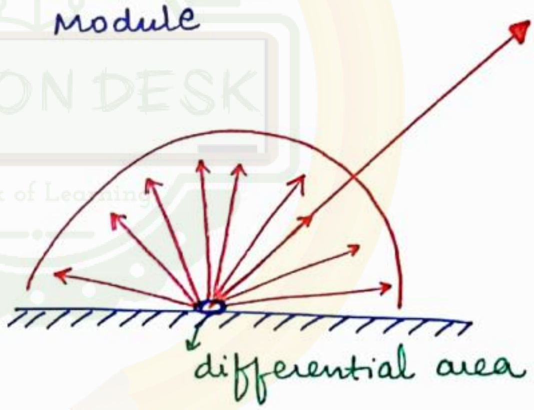
E/m spectrum

Thermal Radiation

↙ Wavelength module

↘ Directional Module



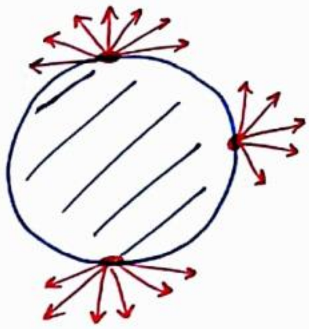


Any Body at a given Tempr. emit Thermal Radiation at all probable wavelength on E/m spectrum and in all possible hemispherical directions.

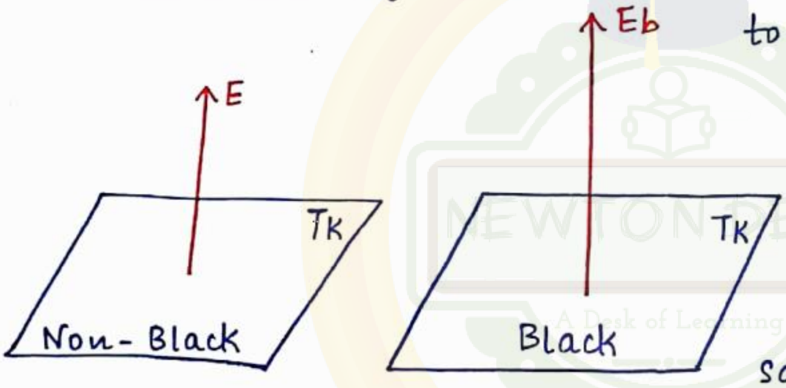
* Basic Definitions in Radiation Heat Transfer :-

② **Total hemispherical emissive power (E)** :- It is defined as the radiation energy emitted from the surface of a Body per unit time per unit area in all possible hemispherical directions integrated over all the wavelengths.

$$E \rightarrow \frac{\text{Joule}}{\text{sec } m^2} = \frac{\text{watt}}{m^2}$$



② **Total Emissivity (ϵ)** :- It is defined as the ratio between total hemispherical emissive power of a non-black Body and total hemispherical emissive power of a Black Body both being at the same tempr.



$$\boxed{\epsilon = \frac{E}{E_b}}$$

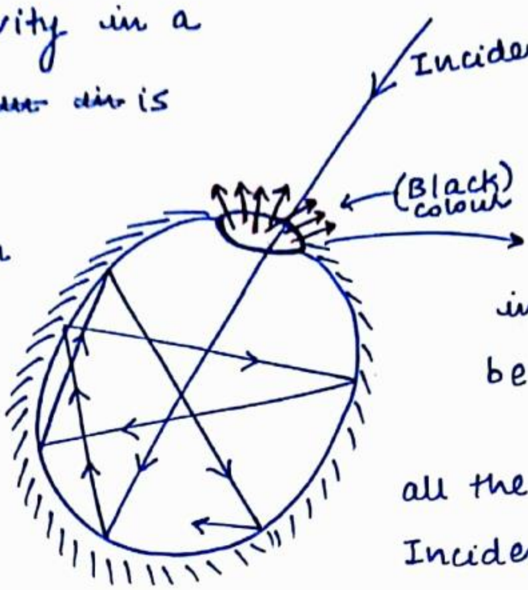
For any body, $\epsilon \leq 1$.

For Black Body, $\epsilon_b = 1$.

Black Body is the Body which absorbs all the thermal Radiation incident or falling upon the Body.

Examples :- ① A small hole (Or) a tiny cavity in a furnace wall is blackbody.

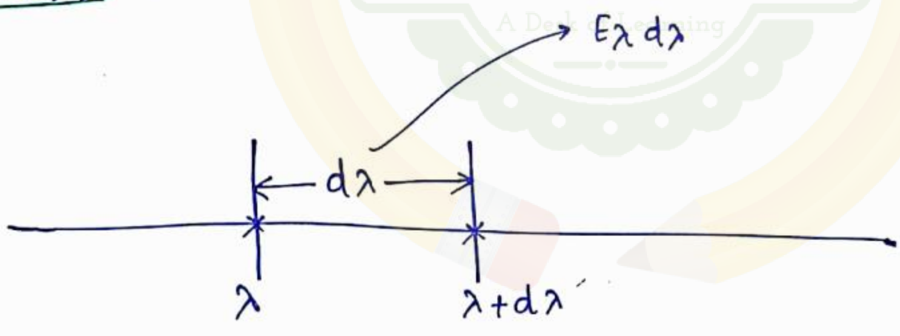
② In Radiation analysis, SUN is also treated as Black Body



→ A Narrow (Or) Tiny cavity in a hollow container can be treated as Black Body. because all the thermal Radiation Incident upon the cavity will get absorbed after multiple ~~the~~ Internal Reflections.

* Black Body → Perfect absorber
→ Ideal emitter
→ Diffusive

③ $\boxed{\text{Monochromatic (OR) Spectral hemispherical emissive Power}}$ $\boxed{(E_\lambda)}$:-



E/M spectrum → λ(μm)

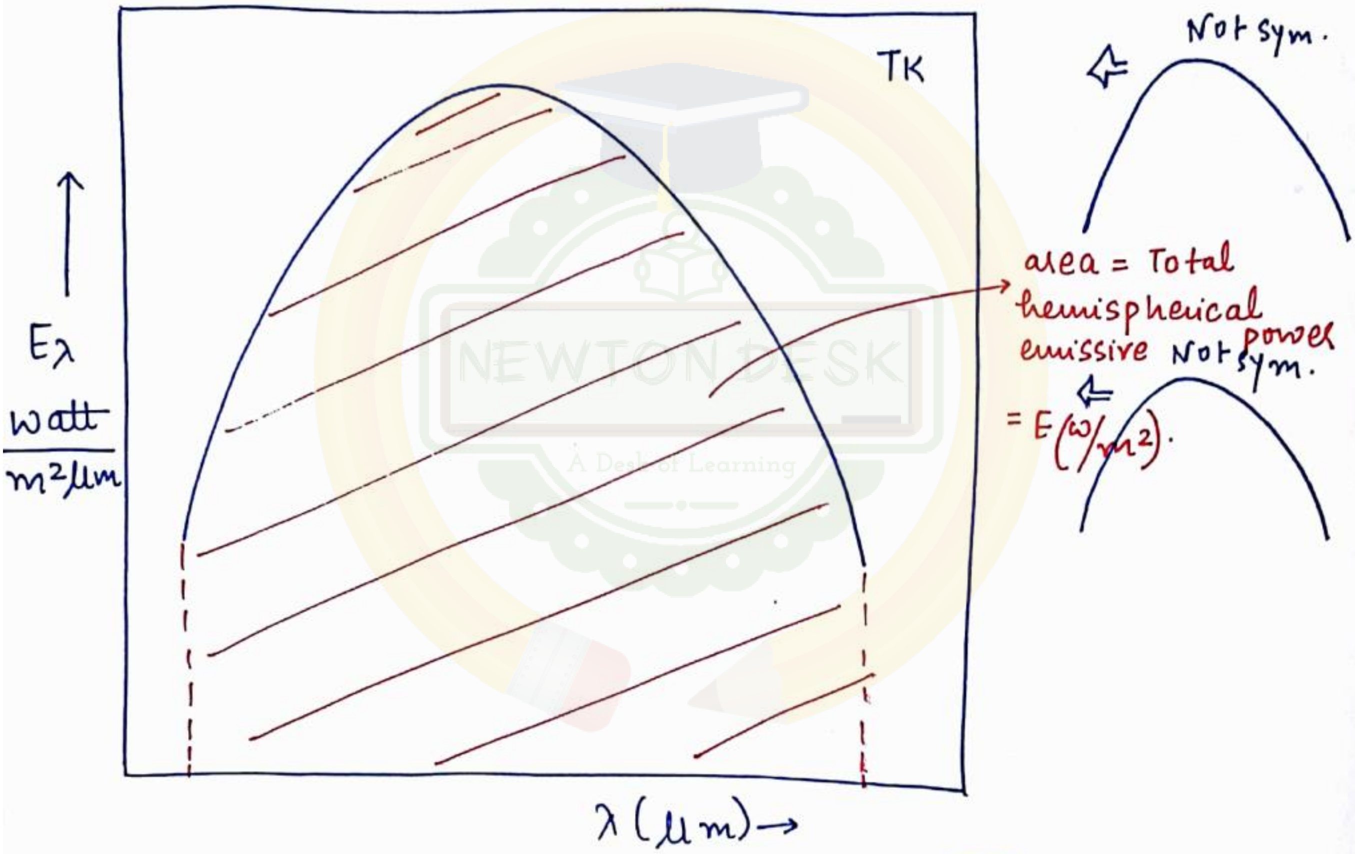
$d\lambda$ = Differentially small Increment in wavelength λ.

'E_λ' at a particular wavelength 'λ' is defined as the quantity which when multiplied by '$d\lambda$' shall give the Radiation energy emitted from the surface of a body per unit time per unit area in the wavelength region 'λ' to '$\lambda + d\lambda$' 119

$$E_\lambda \rightarrow \frac{\text{Joule}}{\text{sec } m^2 \mu m} = \frac{\text{watt}}{m^2 \mu m}.$$

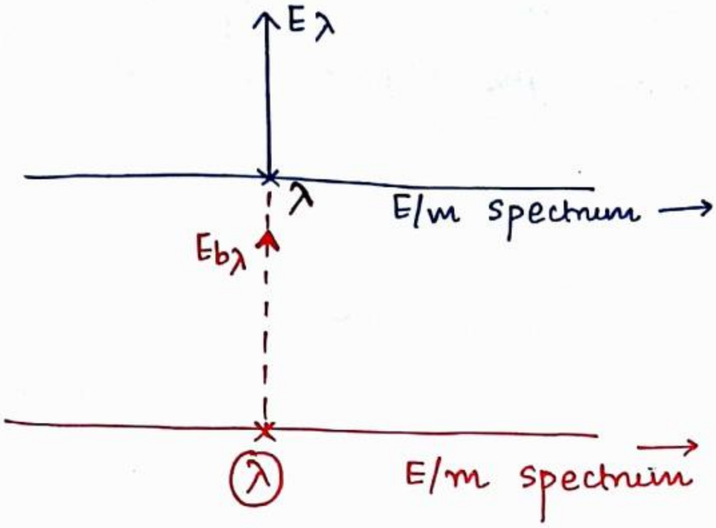
For any given body at a given tempr.,

$$\boxed{E_\lambda = f(\lambda)}$$



$\therefore$ E = Total emissive power $= \int_0^\infty E_\lambda \, d\lambda \ \frac{\omega}{m^2}$.

④ Monochromatic or spectral Emissivity :→ (ϵ_ν)



It is defined as the ratio b/w Monochromatic hemispherical emissive power of a non-black body and the —"—"—"— black body. Both Being at the same temperature and wavelength.

$$\boxed{\epsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}}} \Rightarrow \boxed{E_\lambda = \epsilon_\lambda E_{b\lambda}}$$

$$\epsilon = \frac{E}{E_b} = \frac{\int_0^\infty E_\lambda \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda} = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda}$$

If $\epsilon_\lambda \neq f(\lambda)$ (or) Rather constant,

Then

$$\epsilon = \frac{\epsilon_\lambda \cancel{\int_0^\infty E_{b\lambda} \, d\lambda}}{\cancel{\int_0^\infty E_{b\lambda} \, d\lambda}}$$

$$\boxed{\epsilon = \epsilon_\lambda}$$

Such Body whose monochromatic emissivity 'ϵ_λ' is independent of wavelength 'λ' ~~are~~ (or) rather remaining constant is known as grey Body (or) grey surface. 121

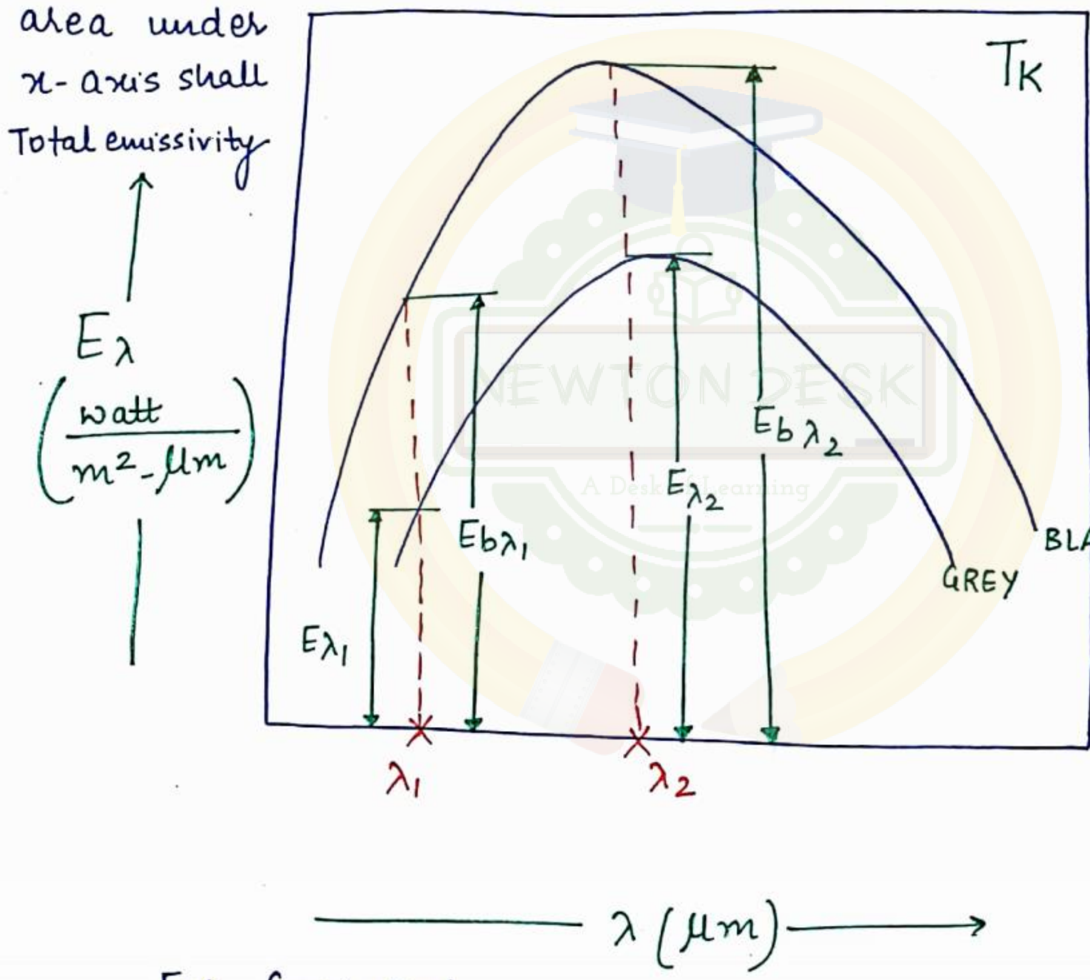
Grey body

∴ for Grey body

$\epsilon_\lambda = C$

* Significance of GRey Body (Physical) :-

The ratio b/w the area under bottom curve on x-axis & the area under the top curve on x-axis shall be equal to Total emissivity of grey body which happens to be equal to its monochromatic emissivity.

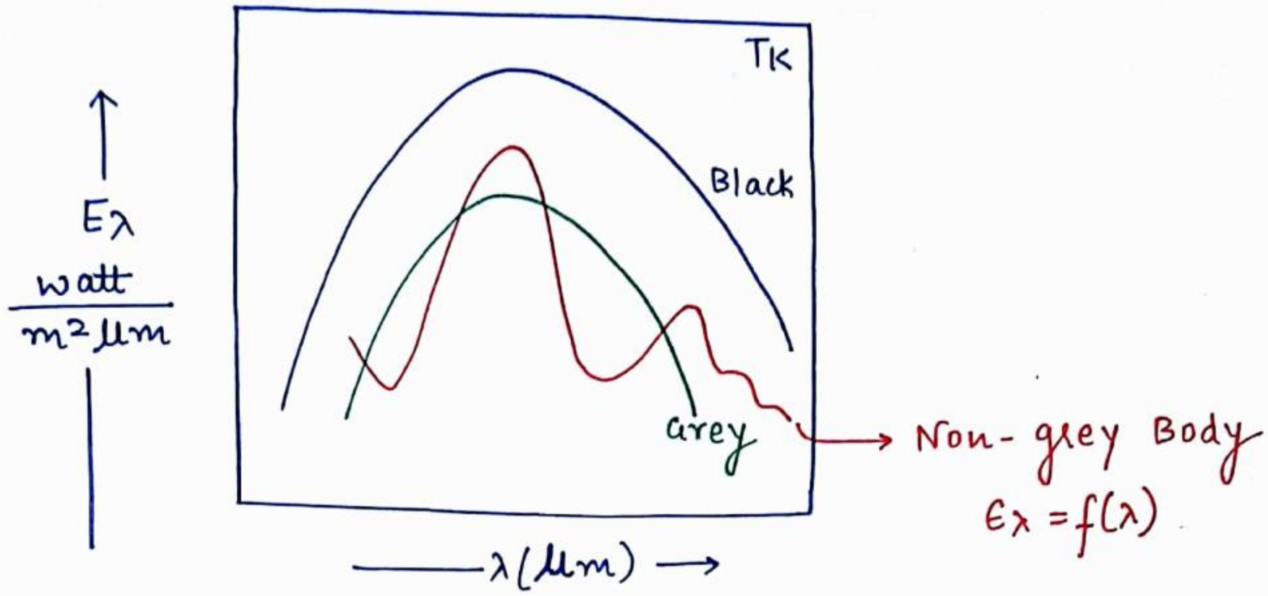


For Grey Body,

$$\epsilon_\lambda \neq f(\lambda)$$

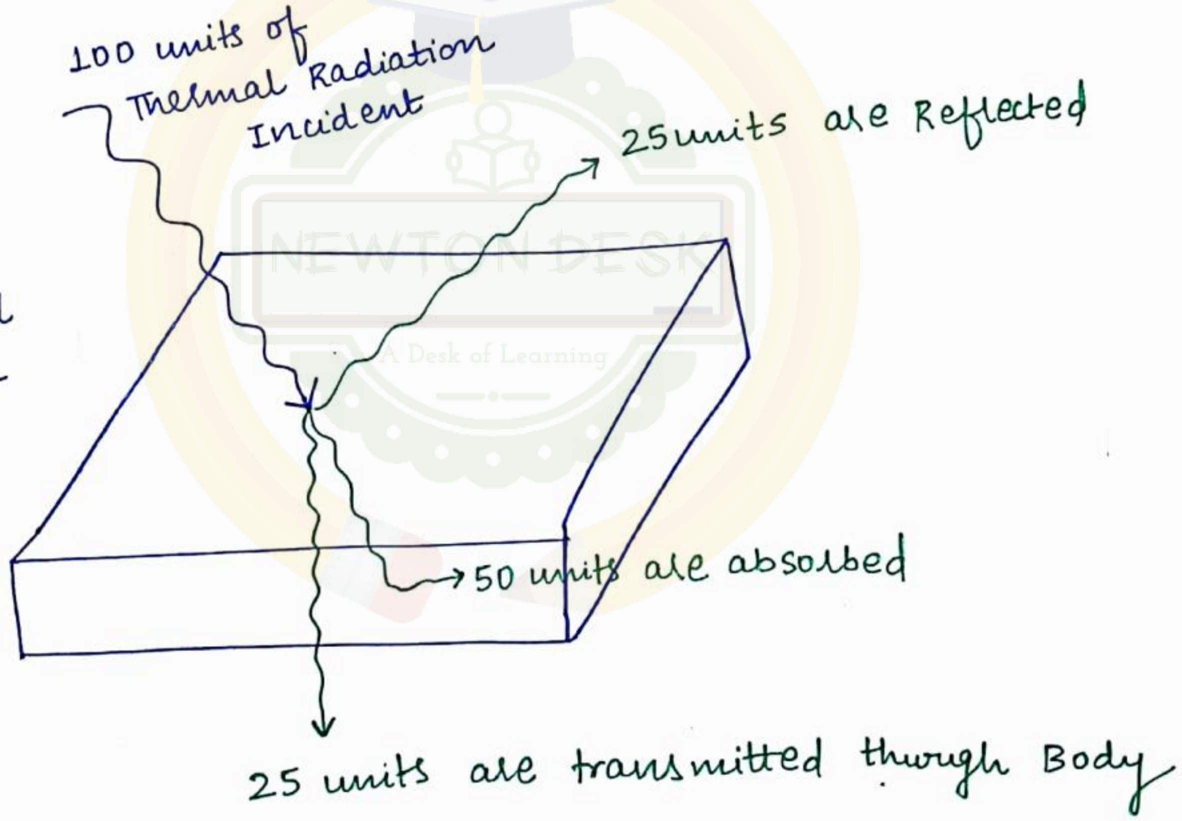
$$\Rightarrow \epsilon_{\lambda_1} = \epsilon_{\lambda_2}$$

$$\Rightarrow \frac{E_{\lambda_1}}{E_{b\lambda_1}} = \frac{E_{\lambda_2}}{E_{b\lambda_2}}$$



* $\boxed{\text{Absorptivity } (\alpha), \text{ Reflectivity } (\rho) \text{ and Transmissivity } (\tau)}$:-

For Black body, which absorbs all thermal Radiation incident, $\alpha_b = 1$



All the above Radiation properties defined ~~are~~ change with wavelength of incident ~~instant~~ thermal Radiation, surface Roughness of the body and the fixed Tempr. ⟶

Absorptivity $(\alpha) = \frac{50}{100} = 0.5$ = fraction of Radiation energy incident upon a surface which is absorbed by it.

Reflectivity $(\rho) = \frac{25}{100} = 0.25$ = fraction of Radiation energy incident upon a surface which is Reflected by it.

Transmissivity $(\tau) = \frac{25}{100} = 0.25$ = fraction of Radiation energy incident upon a surface which is transmitted through it.

$\therefore$ For any surface, $\alpha + \rho + \tau = 1$

✓ for opaque surface, which does not Transmit any energy,

$\tau = 0$

✓ $\therefore$ for opaque surface, $\alpha + \rho = 1$.

✓ for black body, which absorbs all Thermal Radiation Incident, $\alpha_b = 1$

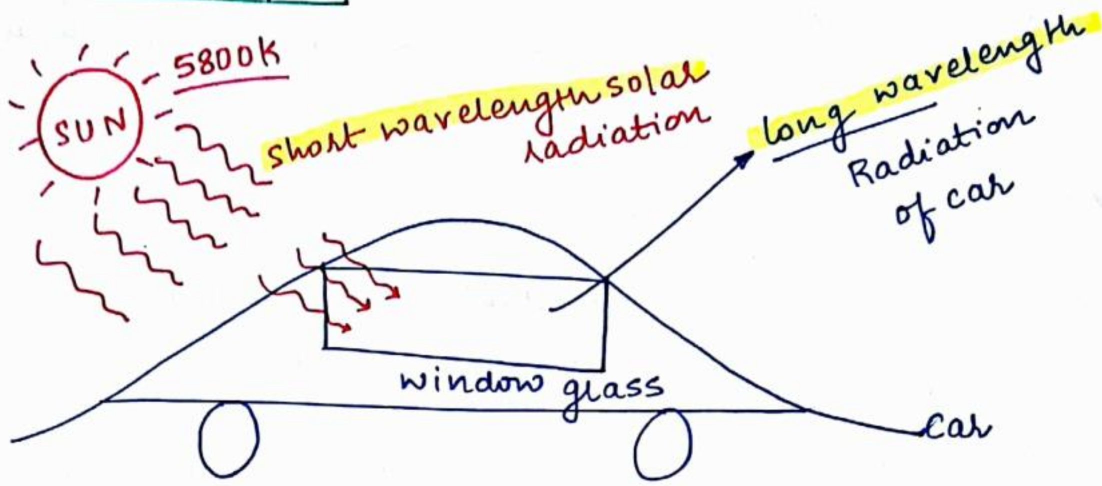
✓ Metals have high Reflectivity (ρ) as compared to Non-metals.

$\boxed{\text{NOTE}}$:- Metal surfaces are more reflecting to the thermal Radiations. This is the reason why metallic surfaces are made of Cu (or) Al are generally used as radiation shields in the furnaces to reduce radiation heat Exchange.

✓ Gases like O_2, N_2 etc have high Transmissivity (τ) i.e. Transparent to thermal Radiation.

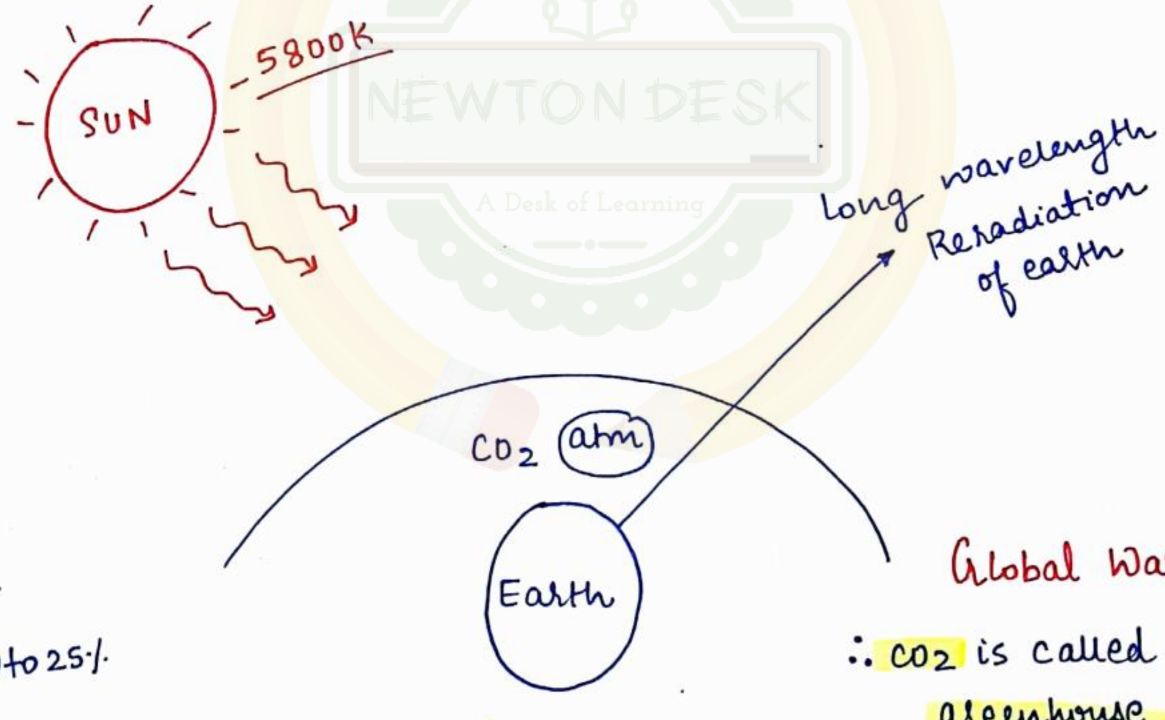
* Practical Example :-

(1)



The window glass of a car is very much transparent to the short wavelength solar radiation falling upon it. But the same window glass almost becomes opaque to the long wavelength re-radiation given by inside of the car. thus trapping energy inside the car hence increasing its temperature.

(2)



Desert
↓
p = 20 to 25%
(R.H.)

Global Warming

∴ CO_2 is called greenhouse gas.

H_2O (w.v.) is also greenhouse gas.

as the surface roughness of a Body decreases by polishing it, the reflectivity of the surface will increase. (125)

Hence, Eg:- highly polished 'Al' or 'Cu' shields having very good reflectivity are generally used in the furnaces to reduce radiation heat exchange.

* LAWS OF THERMAL RADIATION :-

① **Kirchoff's Law of Thermal Radiation** :- The law states that whenever a Body is in thermal Eqm. with its surroundings, its emissivity is equal to its absorptivity.

valid for Black nor Black. for eqm. Then or non-eqm. Thermal

$$\boxed{\epsilon = \alpha}$$

A good absorber is always a good emitter.

Ex:- for a black body, $\alpha_b = 1$
$\epsilon_b = 1$.

② **Planck's Law of Thermal Radiation** :-

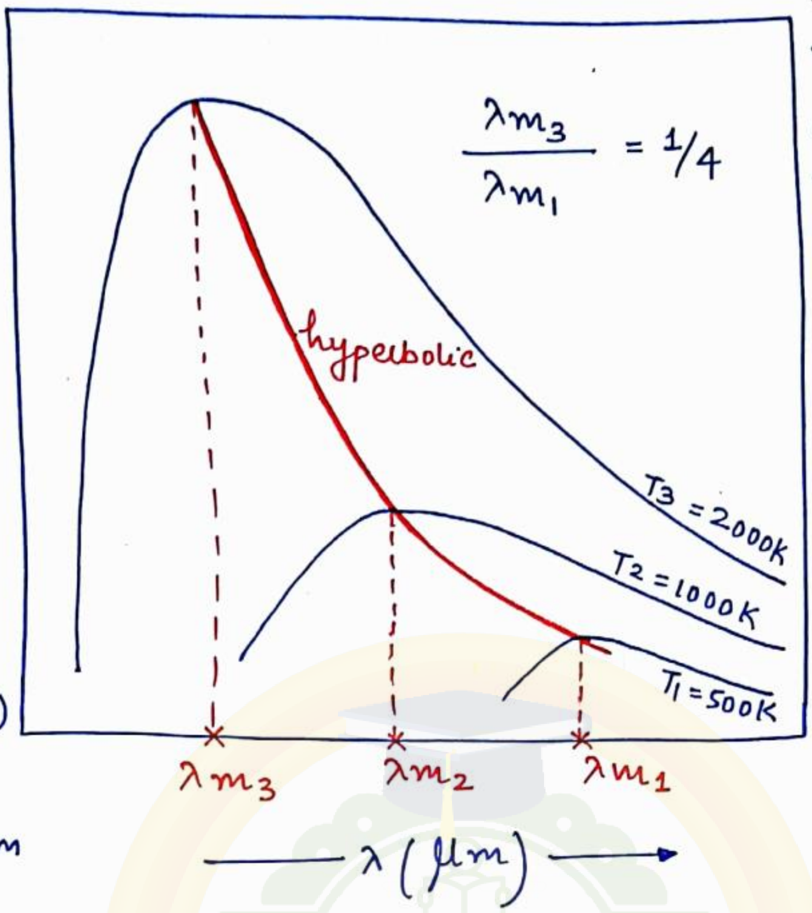
$$E_{b\lambda} = f(\lambda, T) \quad T \text{ in Kelvin}$$

The law states that the monochromatic emissive power of a Black body is dependent on both absolute tempr. of Black Body and also on wavelength of ~~ene~~ Radiation energy emitted 'λ'.

$$E_{b\lambda} = \frac{2\pi C_1}{\lambda^5 \left(e^{C_2/\lambda T} - 1\right)} \quad \text{watt}/m^2\text{-}\mu m$$

C_1 and C_2 are experimental constants.

The above functional relationship among the 3 variables can be graphically represented as :-



$E_{b\lambda}$ $\left(\frac{watt}{m^2 \mu m}\right)$

FOR SUN (BLACK BODY)

$\lambda m = \frac{2898}{5800}$

$\lambda m \simeq 1/2\ \mu m$

The ratio between the area under Top-most curve on x-axis and the area under Bottom most curve on x-axis will be equal to $4^4 = \boxed{256}$.

λm = wavelength at which $E_{b\lambda}$ is maximum at a given absolute Temperature of Black Body.

At a given absolute temperature of a Black Body as wavelength 'λ' increases, $E_{b\lambda}$ also increase reaches a maximum and then decreases.

Also as absolute tempr. of Black Body increases (each time getting doubled). $E_{b\lambda}$ value enormously increases but now most of thermal radiation at higher ~~the~~ tempr.'s will be shifted to shorter wavelengths.

As T increases $\Rightarrow$ λm decreases.

i.e. $\lambda m \propto \frac{1}{T}$

i.e. $\boxed{\lambda m\, T = \text{a constant} = 2898\ \mu m\text{-}K}$ → ③

This eqn. is called Wein's displacement law.

wein's displacement law

Optical pyrometer to measure very high Temps. uses

$$\boxed{\lambda_m T = c}$$

4. $\boxed{\text{STEFAN - BOLTZMAN' Law}}$:- The law states that the Total hemispherical emissive power of a Black body is directly proportional to the fourth power of the absolute temperature of the black Body.

$$E_b \propto T^4 \text{ (T in K only)}$$

$$\Rightarrow E_b = \sigma T^4 \text{ watt/m}^2$$

σ = Stefan - Boltzman' constant

$= 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

$$\Rightarrow E_b = \int_0^\infty E_{b\lambda}\, d\lambda = \int_0^\infty \frac{2\pi c_1}{\lambda^5\left(e^{c_2/\lambda T} - 1\right)} d\lambda$$

$$= \sigma T^4 \text{ W/m}^2$$

Planck's law

Stefan - Boltzman' Law

Wein's displacement Law

For a Non-Black Body, whose emissivity is ϵ, The Total hemispherical emissive power of Non-Black Body $= E = \epsilon E_b \ \frac{\text{watt}}{\text{m}^2}$

$$E = \epsilon \sigma T^4 \text{ W/m}^2$$

If 'A' is the Total surface area of Non-Black body,

The Total Radiation energy emitted from entire Non-Black Body $= EA$ watt

$= \epsilon \sigma T^4 A$ watt

Pg. 80

29 +1

= 30 $E_1 = 500\,W/m^2$ —— T_1 $\epsilon = C$

$E_2 = 1200\,W/m^2$ —— T_2 $\frac{T_1}{T_2}$

SIR $E_1 = \epsilon_1 \sigma T_1^4 = 500\,W/m^2$, $E_2 = \epsilon_2 \sigma T_2^4 = 1200\,W/m^2$

But $\epsilon_1 = \epsilon_2$

$$\Rightarrow \left(\frac{T_1}{T_2}\right)^4 = \frac{500}{1200}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(5/12\right)^4 = 0.803$$

32 Sun is a black body

From wein's displacement law,

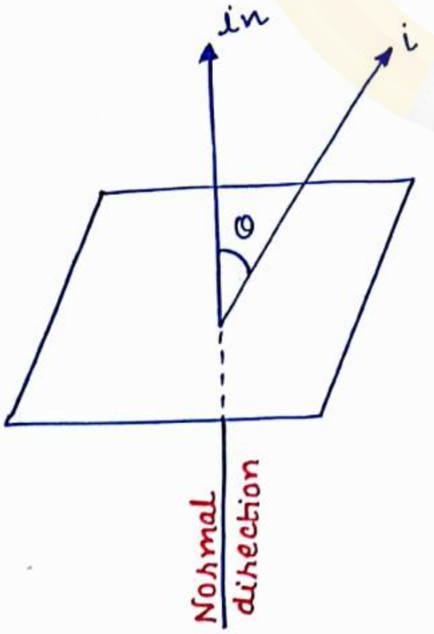
$$\lambda m_1 T_1 = \lambda m_2 T_2$$

$$5800 \times 0.50 = \lambda m_2 \times 1000$$

$$\lambda m_2 = 2.90\,\mu m$$

22/09/2016

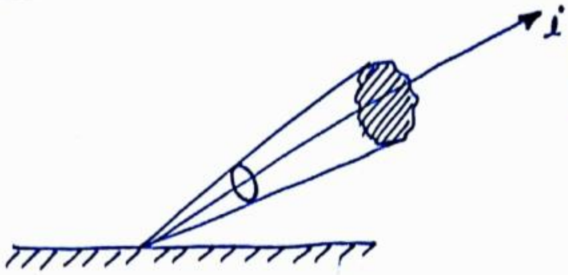
5 $\boxed{\text{LAMBERT'S COSINE LAW}}$:-



$$\boxed{i = i_n \cos\theta}$$

i_n = Normal Intensity of Radiation.

i = intensity of Radiation along any direction making an angle 'θ' wrt Normal direction



$\frac{\text{Joule}}{\text{sec } m^2\text{-Steradian}}$

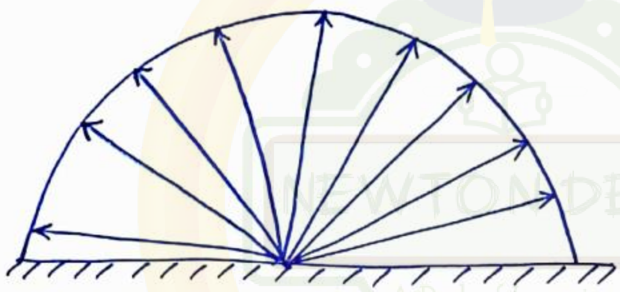
$= \frac{\text{watt}}{m^2\text{-steradian}}$

Steradian is the unit of solid angle.

i → Intensity of Radiation 'i' along a given specified dirn. is defined as the radiation energy emitted from the surface of the body per unit time per unit area normal to that direction and per unit solid angle about that dirn.

$$i = \frac{dE}{d\omega} \ \text{watt}/m^2\text{- steradian}$$

Total hemispherical emissive power $= E = \int i \, d\omega$ watt/m^2.



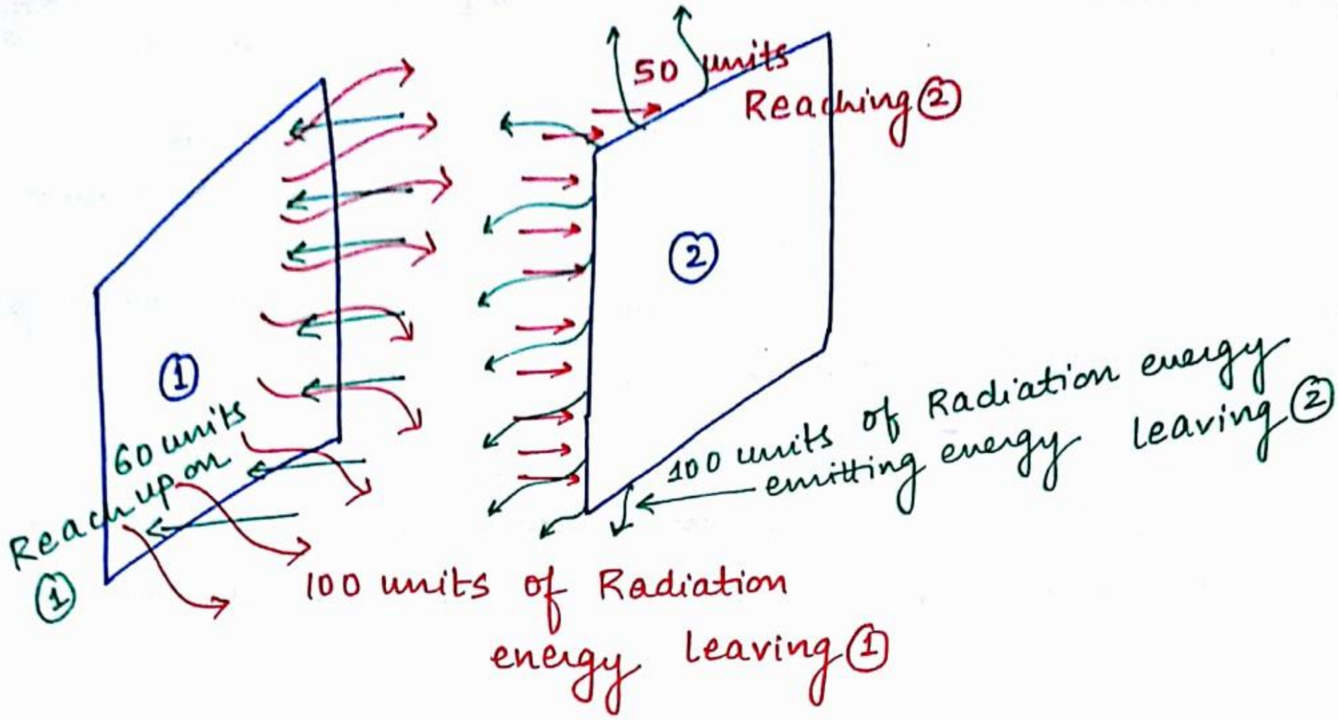
A Diffuse surface has the same intensity of Radiation along all the directions.

i.e. for Diffuse surface, i is independent of Direction.

Ex:- Blackbody is a diffuse surface.

∴ for Black body, $E_b = \pi i$ in ω/m^2.

* SHAPE FACTOR (OR) VIEW FACTOR (OR) CONFIGURATION FACTOR :-



$F_{12} = \frac{50}{100} = 0.5$ = Fraction of Radiation energy leaving surface ① that reaches surface ②

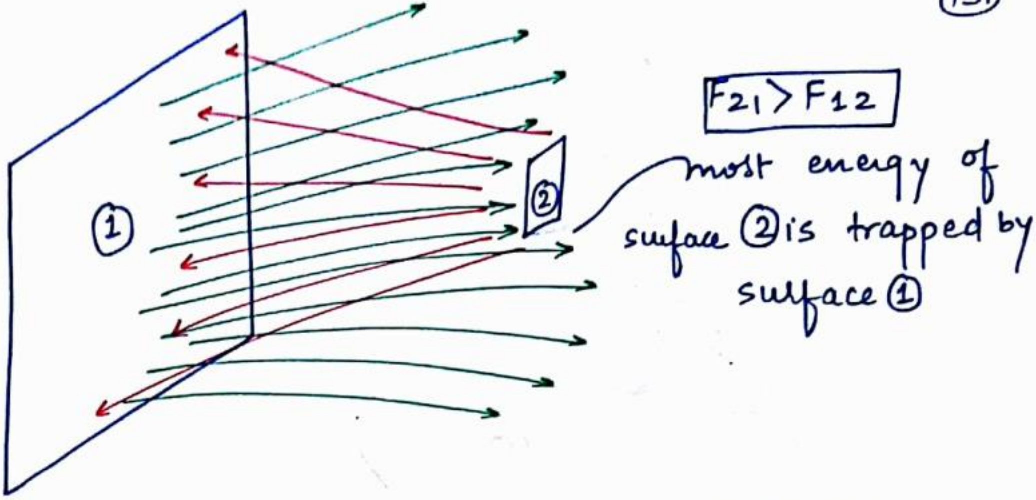
$F_{21} = \frac{60}{100} = 0.6$ = Fraction of Radiation energy leaving surface ② that reaches surface ①.

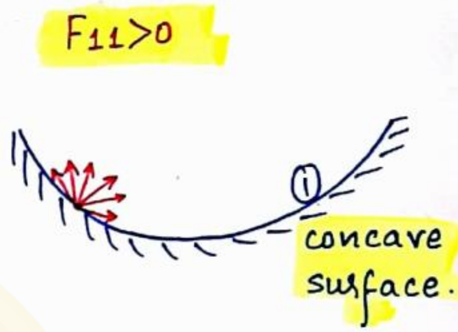
In General,

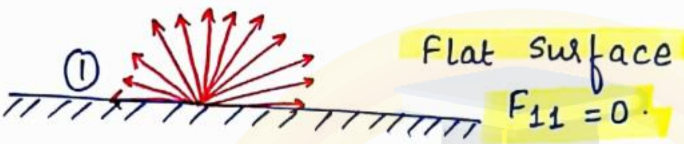
F_{mn} = Fraction of Radiation energy leaving surface 'm' that reaches surface 'n'.

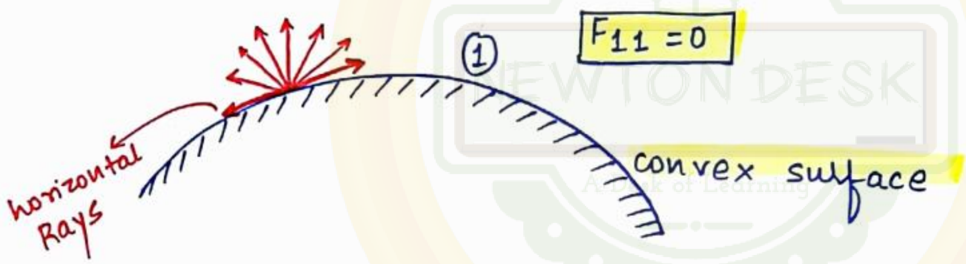
$$\boxed{0 \le F_{mn} \le 1}$$

NOTE - The shape Factor b/w 2 surfaces / bodies is independent of their temperatures, their emissivities, but depends only on how the 2 surfaces are geometrically oriented with respect to each other. (Also their sizes & shapes can influence the shape factor).







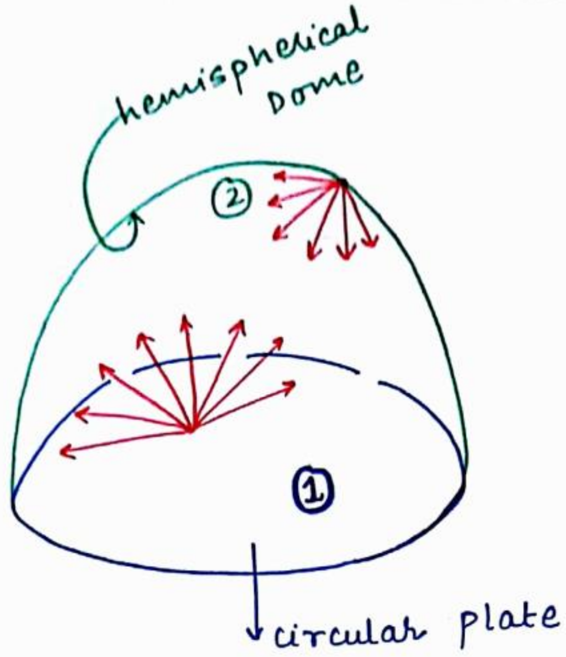


✓ when one Body (or) a surface is completed surrounded by another Body, the shape factor of the inner Body wrt the outer Body / surface is equal to 1.

Examples :- 1

" N.P. "

(1)



$F_{12} = 1$

$F_{21} < 1$

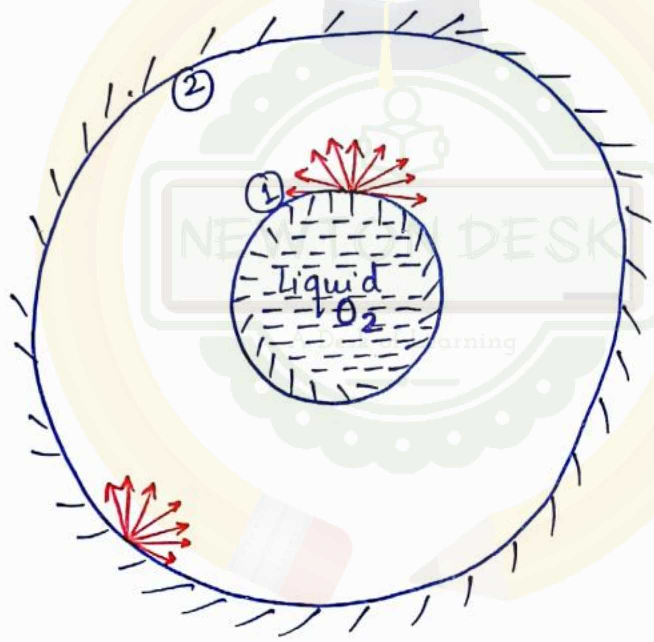
$F_{22} < 1$

$\boxed{F_{21} + F_{22} = 1}$

(2) Two concentric spherical surfaces.

Cryogenics

0K → 113K



$F_{12} = 1$

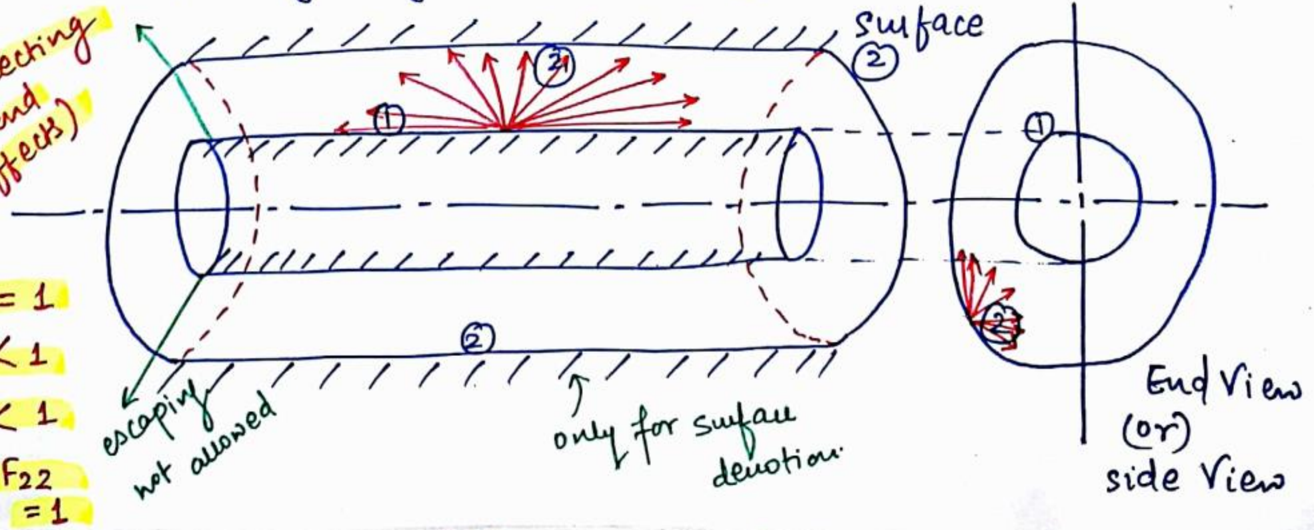
$F_{21} < 1$

$F_{22} < 1$

$\boxed{F_{21} + F_{22} = 1}$

(3) Two Infinitely long Concentric cylindrical surfaces :-

(Neglecting end effects)



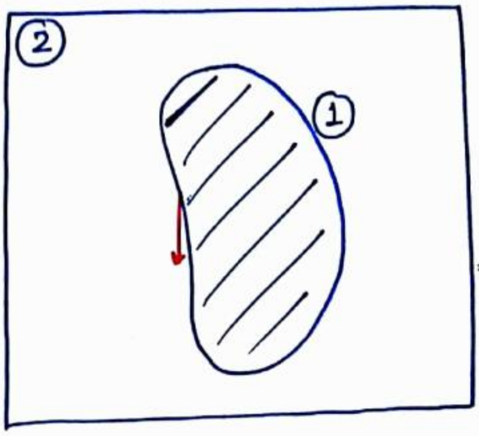
$F_{12} = 1$

$F_{21} < 1$

$F_{22} < 1$

$F_{21} + F_{22} = 1$

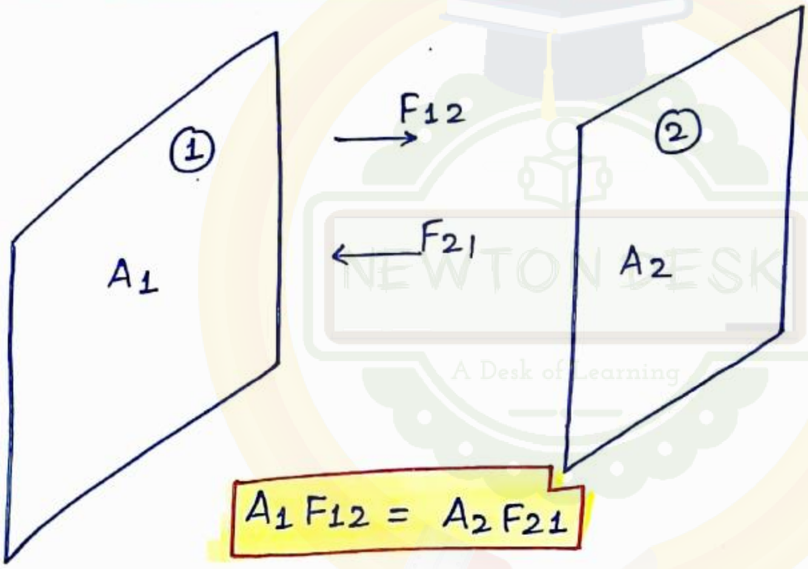
(4) A Body kept in enclosure / Room.



$F_{12} = 1$ $\quad F_{22} < 1$

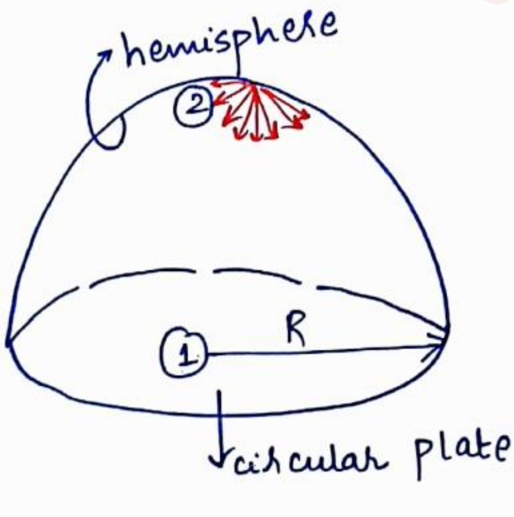
$F_{21} < 1$ $\quad F_{21} + F_{22} = 1$

* RECIPROCITY ~~THEOREM~~ Relation between shape Factors :-



$$\boxed{A_1 F_{12} = A_2 F_{21}}$$

Q.>



$F_{22} = ?$

Sol>

$F_{12} = 1$

$F_{21} = \frac{A_1}{A_2} \cancel{F_{12}}^{\,1}$

$= \frac{\pi R^2}{2\pi R^2}$

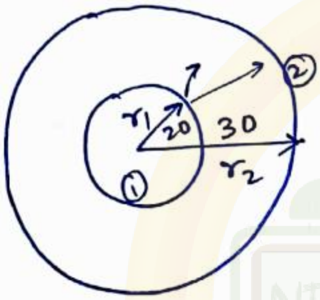
$= 1/2$

$F_{21} + F_{22} = 1$

$\Rightarrow F_{22} = 1 - F_{21}$

$\Rightarrow F_{22} = 1/2$

(35)



$F_{12} = 1$

$F_{21} < 1$

$F_{22} < 1$

$F_{21} + F_{22} = 1$

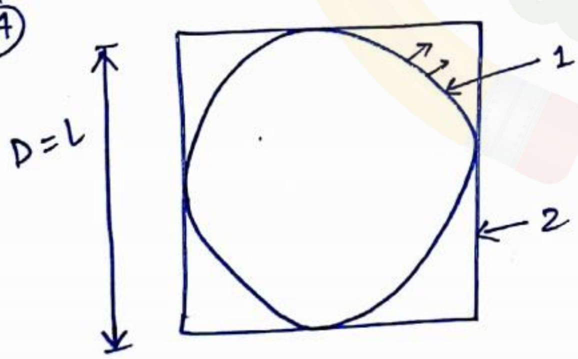
$A_1 F_{12} = A_2 F_{21}$

$A_1 \; 1 = A_2 F_{21}$

$\frac{\cancel{4\pi}(20)^2}{\cancel{4\pi}(30)^2} = F_{21}$

$4/9$

(24)



F_{21}

$A_1 \underline{F_{12}} = A_2 F_{21}$

1

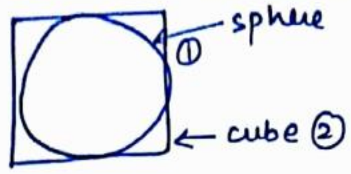
$\frac{A_1}{A_2} = F_{21}$

$\frac{\frac{\pi}{4} \cancel{D^2}}{\cancel{D^2}} = F_{21}$

x^2

SIR

$D = L$

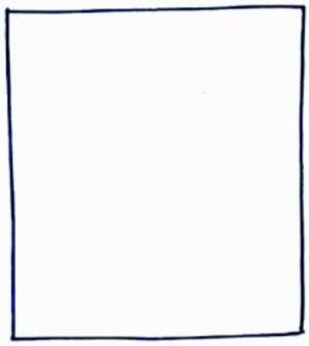


$F_{12} = 1$

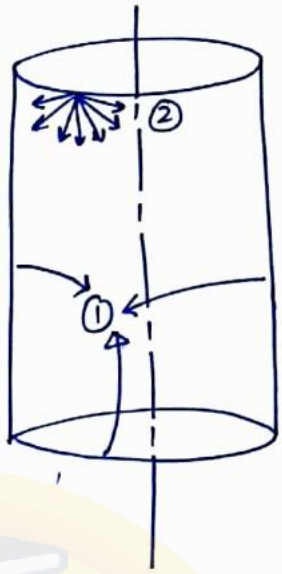
$A_1 F_{12} = A_2 F_{21}$

$$F_{21} = \frac{A_1}{A_2} = \frac{4\pi\left(\frac{D}{2}\right)^2}{6 \times D^2} = \frac{\pi}{6}.$$

(19)



SIR



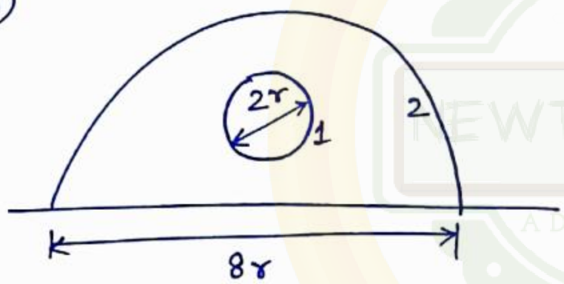
① → circular Base + → cylindrical surface

$F_{21} = 1$

$A_1 F_{12} = A_2 F_{21}$

$$F_{12} = \frac{A_2}{A_1} = \frac{\pi/4\, d^2}{\frac{\pi}{4} d^2 + \pi d h} = \frac{d}{(d+4h)}$$

(31)



$F_{21} = ?$

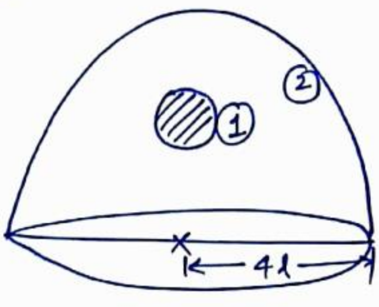
$F_{1-2} = 1$

$A_1 F_{12} = A_2 F_{21}$

$$4\pi (r)^2 \cancel{F_{12}}^{1} = 2\pi (4l)^2 F_{21}$$

$$\frac{\cancel{4\pi r^2}}{2 \times \cancel{4} \times 4 \times 4\, \cancel{l^2}} = F_{21}$$

SIR



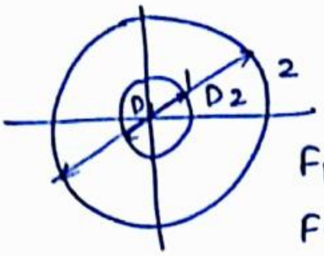
② → circular Base + → hemispherical dome

$F_{12} = 1$

$$F_{21} = A_1/A_2 = \frac{4\pi l^2}{\underbrace{(4l)^2\pi}_{\text{circular}} + \underbrace{2\pi(4l)^2}_{\text{hemisphere}}}$$

$= 1/12$

(28)



$A_1 F_{12} = A_2 F_{21}$

$\frac{\pi}{4}\cancel{(D_1)^2}^{\pi D_1} = \frac{\pi}{4}\cancel{(D_2)^2}^{\pi D_2} F_{21}$

$F_{12} = 1$

$F_{21} + F_{22} = 1$

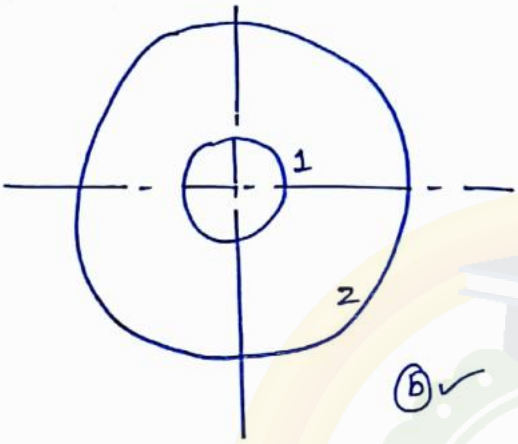
$F_{21} = \frac{D_1^2}{D_2^2}$

$F_{22} = 1 - \left(\frac{D_1}{D_2}\right)$ ✓

πr^2

$\pi (D/2)^2$

SIR



$F_{12} = 1 (\because \infty \text{ long})$

$A_1 F_{12} = A_2 F_{21}$

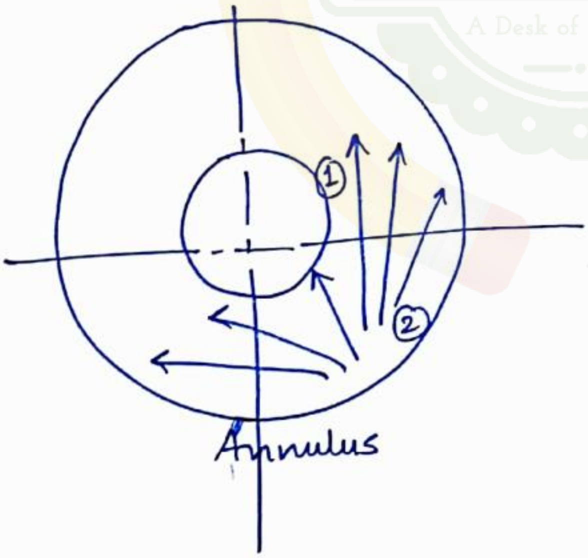
$F_{21} = \frac{A_1}{A_2} = \frac{\pi D_1 L}{\pi D_2 L} = D_1/D_2$

(b) ✓

$F_{21} + F_{22} = 1$

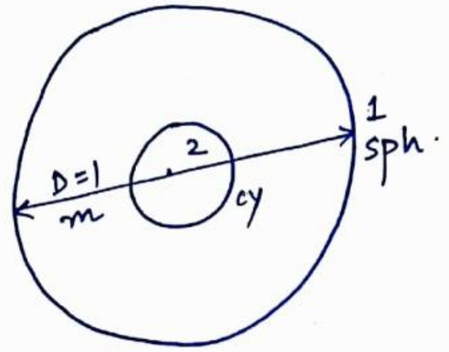
$\Rightarrow F_{22} = 1 - D_1/D_2$

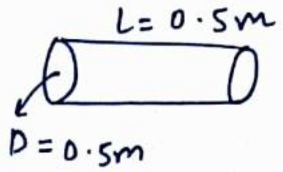
(42)



(d) ✓

3

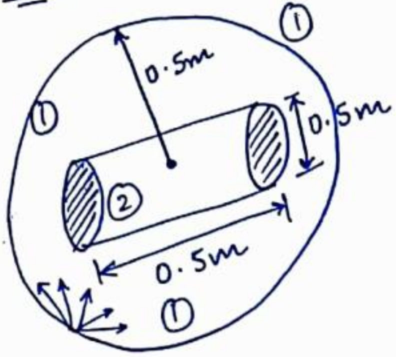




F_{11}

$A_1 F_{12} = A_2 F_{21}$

SIR



$F_{21} = 1$

$A_1 F_{12} = A_2 F_{21}$

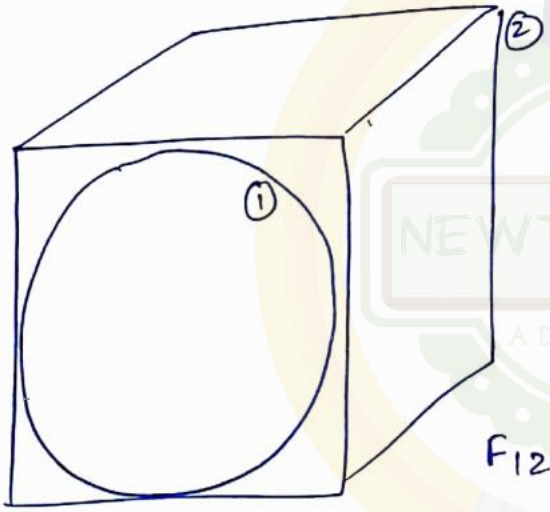
$$F_{12} = \frac{A_2}{A_1} = \frac{2\left(\frac{\pi}{4}\right)(0.5)^2 + \pi(0.5)(0.5)}{4\pi(0.5)^2}$$

$= 0.375$

$F_{11} + F_{12} = 1 \Rightarrow F_{11} = 1 - F_{12}$

$= 0.625$

23



$A_1 = 0.6$

$A_2 = ?$

$A_1 F_{12} = A_2 F_{21}$

$F_{21} = 0.004$

$F_{12} = 1$

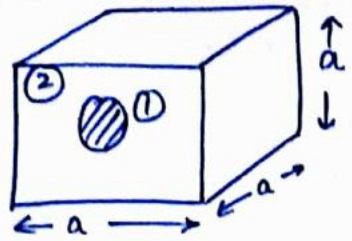
$0.6 \cdot 1 = A_2 \, 0.004$

$\frac{0.6}{0.004} = 6L^2$

$\frac{0.\cancel{6}}{0.004 \times \cancel{6}} = L^2$

$0.1 \qquad = 5$

(23) SIR



$F_{12}=1$

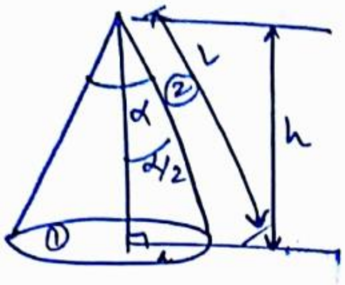
$A_1F_{12}= A_2F_{21}$

$0.6\times 1 = 6a^2\times 0.004$

$\Rightarrow a = 5m$

(20)

F_{21} ✓
F_{12} ✓



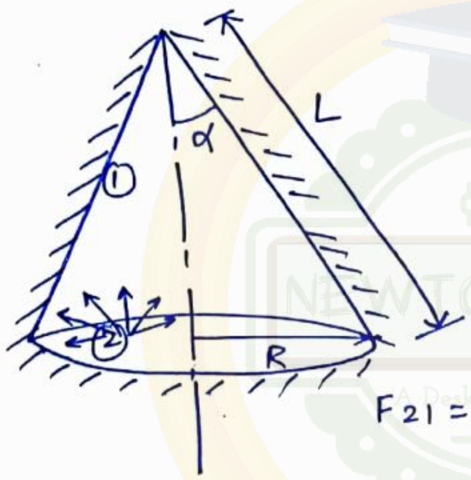
$\sin(\alpha/2) = \cancel{\dfrac{h}{\sqrt{h^2+r^2}}} = \dfrac{r}{L}$ ✓

$\underline{F_{12}=1}$

$h^2 = p^2+b^2$

$h = \sqrt{h^2+r^2}$

SIR



$A_1F_{12} = A_2F_{21}$

$F_{12} = \dfrac{A_2}{A_1}(1) = \dfrac{\pi R^2}{\pi R L}$

$= \dfrac{R}{L}$

$= \sin\alpha$

$F_{21}=1$

* **Summation Rules among shape Factors** :-

If there are 'n' no. of surfaces involved in any Radiation heat Exchange Problem,

Then $F_{11} + F_{12} + F_{13} + \text{-----------} + F_{1n} = 1$

$F_{21} + F_{22} + F_{23} + \text{-----------} + F_{2n} = 1$

$\vdots$

$F_{n1} + F_{n2} + F_{n3} + \text{-----------} + F_{nn} = 1$

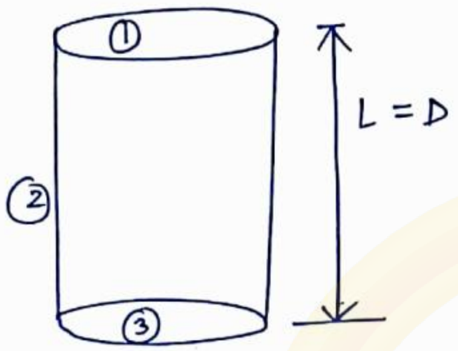
If any particular surface is either flat surface or convex surface then its self shape factor becomes zero.

Even now, the Reciprocity relation is valid between any two surfaces.

Ex :- $A_1 F_{13} = A_3 F_{31}$

$A_2 F_{2n} = A_n F_{n2}$

WB 2



$F_{13} = 0.17$

$F_{12} = ?$

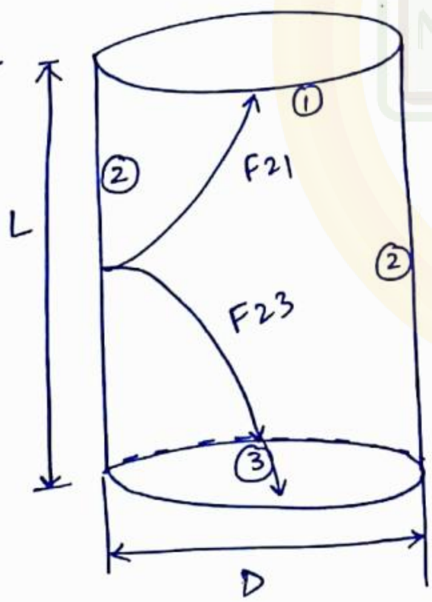
$A_1 F_{13} = A_3 F_{31}$

$\cancel{\pi r^2}\, 0.17 = \cancel{\pi r^2}\, F_{31}$

$F_{31} = \cancel{F}F_{12} + F_{23}$

$0.17 = F_{12} + 0.83$

SIR



$F_{13} = 0.17$

Summation Rule :-

$\cancel{F_{11}}^{\,0} + F_{12} + F_{13} = 1$

Flat

$\therefore F_{12} = 1 - F_{13}$

$= 1 - 0.17$

$= 0.83$

$F_{33} = 0 \qquad F_{22} = ?$

$$\boxed{\begin{array}{c} A_2 F_{23} = A_3 F_{32} \\ \boxed{\underline{F_{21}} + F_{22} + \underline{F_{23}} = 1} \\ F_{22} \end{array}}$$

from symmetry of figure,

$$\boxed{F_{21} = F_{23}}$$

Summation Rule : $F_{21} + F_{22} + F_{23} = 1$

$$\therefore F_{22} = 1 - 2F_{21}$$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1 \times F_{12}}{A_2} = \frac{\pi/4 \, D^2}{\pi D \times D} \times 0.83 = \frac{0.83}{4}$$

$$\therefore F_{22} = 1 - 2\left(\frac{0.83}{4}\right) = 0.585$$

* **Equilateral Angular Duct** :-

From symmetry of Figure, $\boxed{F_{12} = F_{13}}$

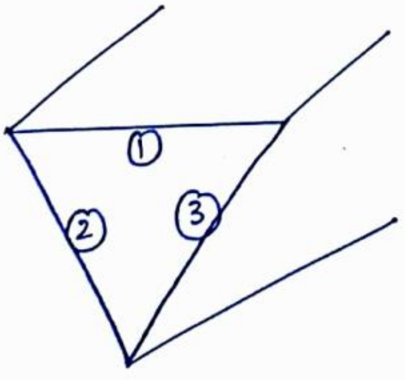
Since Duct is very long, Neglecting end effects,

$$\cancel{F_{11}} + F_{12} + F_{13} = 1$$

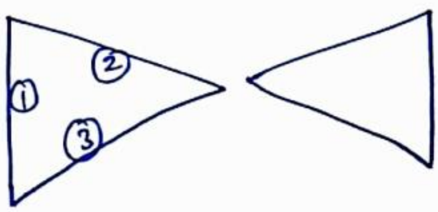
0

$$\therefore \boxed{F_{12} = F_{13} = 1/2}$$

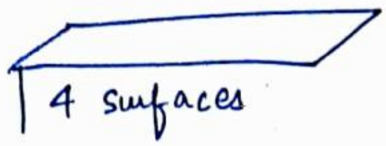
Now,



$F_{12} = 1/2$



21



$F_{11} = 0.1$ $\quad A_1 = 4$

$F_{12} = 0.4$ $\quad A_4 = 2$

$F_{13} = 0.25$ $\quad A_1 F_{14} = A_4 F_{41}$

$F_{41} = ?$ $\quad \frac{4 \times 0.25}{2}$ 0.5 ✓

$0.1 + 0.4 + 0.25 + F_{14} = 1$

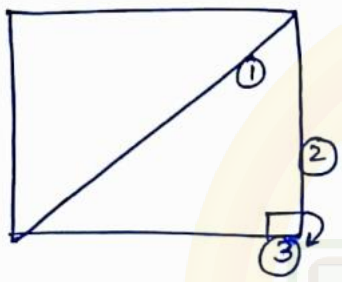
SIR $\quad F_{11} + F_{12} + F_{13} + F_{14} = 1$

$$F_{14} = 1 - (F_{11} + F_{12} + F_{13}) = 0.25$$

$$F_{14} A_1 = A_4 F_{41}$$

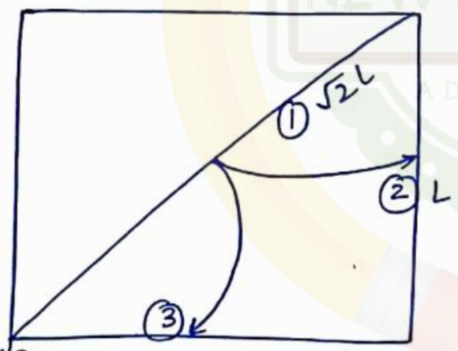
$$F_{41} = \frac{A_1}{A_4} \times 0.25 = 0.5 ✓$$

33



$F_{12} + F_{11} + F_{13} = 1$

SIR



Very long sq. Duct

From symmetry of fig,

$F_{12} = F_{13}$

Since very long Duct, Neglecting end effects,

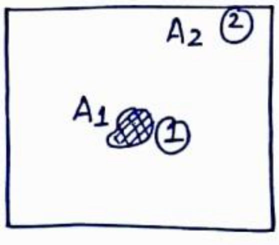
$$\underset{0}{\cancel{F_{11}}} + F_{12} + F_{13} = 1$$

$$\Rightarrow F_{12} + F_{13} = 1/1 \Rightarrow F_{12} = F_{13} = 1/2$$

$$A_1 F_{12} = A_2 F_{21}$$

$$F_{21} = \frac{A_1}{A_2} \times F_{12} = \frac{\sqrt{2} L \times 1}{L \times 1} \times 1/2 = 0.707$$

when a very small Body is kept in very large enclosure/Room,



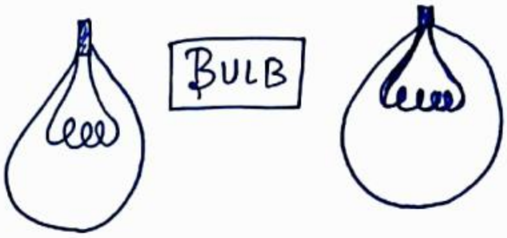
$F_{12} = 1$

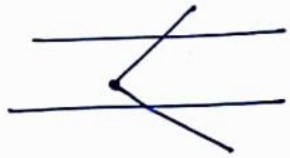
$F_{21} = \frac{A_1}{A_2} \longrightarrow 0$

$A_1 <<< A_2$

$A_1/A_2 \longrightarrow 0$

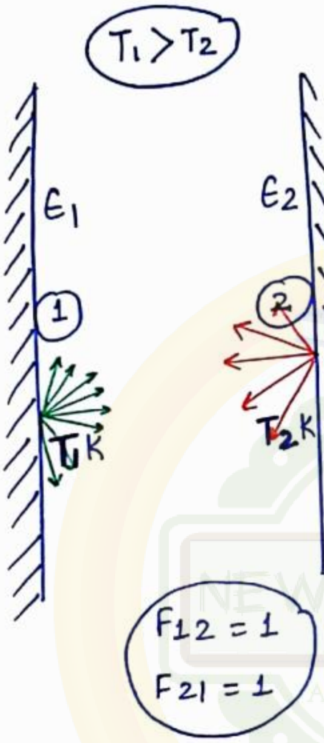
Examples :- ① Filament in a bulb ② Thermocouple in a duct.





Case I :-

* Radiation Heat Exchange b/w two Infinitely Parallel Plates :-



Assume :- ① steady state H.T. conditions i.e. $T \neq f(\text{Time})$

② Surfaces are Diffuse and Grey.

Diffuse ↓ $i \neq f(\text{direction})$.

Grey ↓ $\epsilon_\lambda \neq f(\lambda)$

Then,
Net Radiation heat Exchange between ① & ② per unit area

$$= \left(\frac{q}{A}\right)_{\substack{net\\1-2}} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

$\left(\frac{watt}{m^2}\right)$ ↓ Radiation flux

Incase If both the surfaces are black, Then $\epsilon_1 = \epsilon_2 = 1$

$$\Rightarrow \left(\frac{q}{A}\right)_{\substack{net\\1-2}} = \sigma(T_1^4 - T_2^4)\ \omega/m^2$$

✓ Not violates 2nd law

✓ Radiation is a 2 way Phenomenon.

(2.9) $1.75\times10^{10}\times$

(143)

992.25

$\frac{661.5}{10^3}$ kW

$$\left(\frac{q}{A}\right)_{net\ 1-2} = \frac{5.67\times10^{-8}(400^4 - 300^4)}{\frac{1}{0.8}+\frac{1}{0.8}-1}$$

$$= 0.66\times10^3\ W/m^2$$

(40) Gap → dont matter

since large plates,

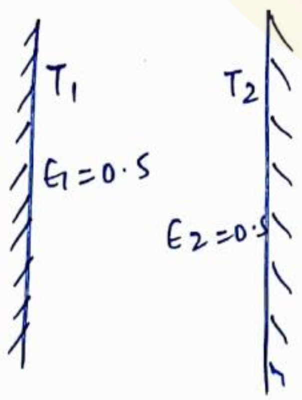
$$\left(q/A\right)_{net\ 1-2} = \frac{5.67\times10^{-8}(100^4 - 400^4)}{1/0.5 + 1/0.25 - 1} = 11.05\times10^3\ W/m^2$$

$$1000 = \frac{5.67\times10^{-8}(T_1^4 - T_2^4)}{1/0.5 + 1/0.5 - 1}$$

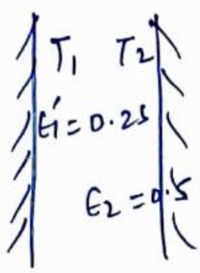
$$Q = \frac{\_\_\_\_}{1/0.25 + 1/0.25 - 1}$$

$$\frac{1000}{Q} = \frac{1/0.25 + 1/0.25 - 1}{1/0.5 + 1/0.5 - 1}$$

SIR (11)



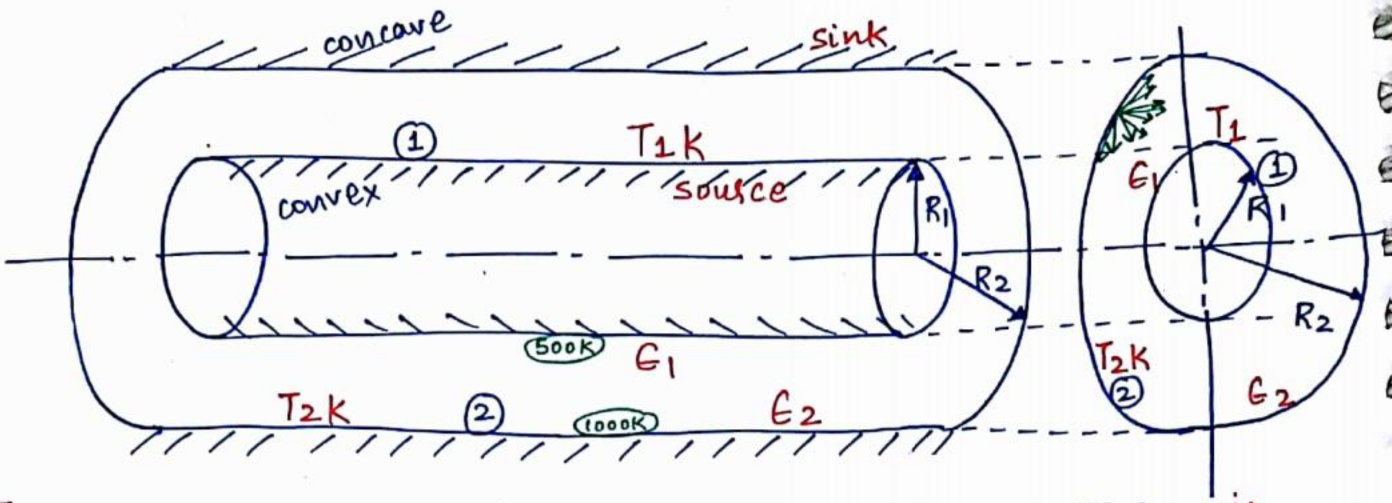
$$(q/A)_{net\ 1-2} = \frac{\sigma(T_1^4 - T_2^4)}{1/0.5 + 1/0.5 - 1} = 1000\ W/m^2 \rightarrow (1)$$



$$\left(\frac{q'}{A}\right)_{net\ 1-2} = \frac{\sigma(T_1^4 - T_2^4)}{1/0.25 + 1/0.5 - 1} = ? \rightarrow (2)$$

$$(2)/(1) \Rightarrow \frac{q'}{A} = 600\ W/m^2$$

Case 2 :- Radiation Heat Exchange between two infinitely Long concentric cylindrical surfaces :-



The outer surface of the inner cylinder at $T_1 K$ is exchanging heat by radiation with the inner surface of the outer cylinder at $T_2 K$.

let $T_1 > T_2$

Assume :- ① Steady state H.T. conditions $T \neq f(\text{Time})$.

② surfaces are diffuse and grey.

$F_{12} = 1$ ($\because$ Infinitely Long cylinders)

$F_{21} < 1$

The Net Radiation Heat Exchange between ① & ②

$$\Rightarrow q_{1-2} = \frac{\sigma (T_1^4 - T_2^4) A_1}{1/\epsilon_1 + \frac{A_1}{A_2}\left(1/\epsilon_2 - 1\right)} \text{ watt}$$

it is not a flux

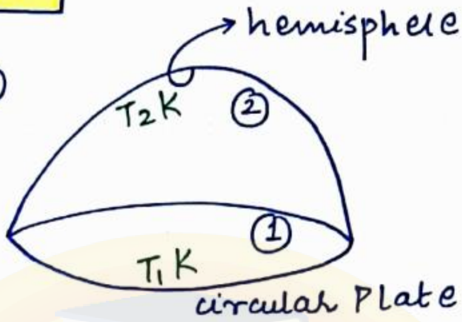
This case 2 formulae given is applicable for all the Radiation Heat Exchange cases b/w 2 surfaces whenever one shape factor between them is equal to 1 and the other shape factor is less than 1.

The same formulae is also valid even when $T_2 > T_1$ But the precaution is that always number the inner surface as 1 and the outer surface as 2

Here formulae key below,

$$\boxed{\frac{A_1}{A_2} = \frac{R_1}{R_2}}$$

Few Examples of Case 2 formulae :- Ex :- 1



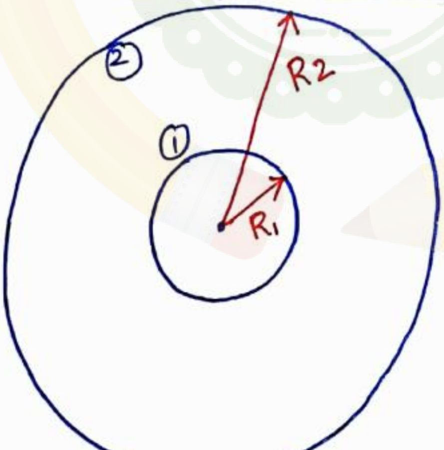
$F_{12} = 1$
$F_{21} < 1$

Here,

$$\frac{A_1}{A_2} = \frac{\pi R^2}{2\pi R^2} = \frac{1}{2}$$

$$\frac{A_1}{A_2} = \frac{1}{2}$$

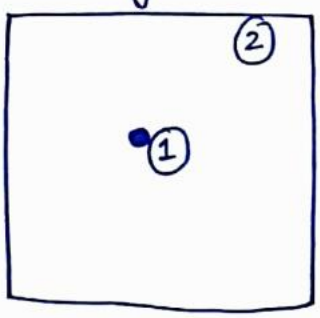
Ex :- 2 Two concentric spherical surfaces



$F_{12} = 1$
$F_{21} < 1$

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^2$$

Ex :- 3 When a very small Body kept in very large enclosure.



$$\frac{A_1}{A_2} \to 0$$

$F_{12} = 1$
$F_{21} < 0$

1 Filament
2 Thermocouple

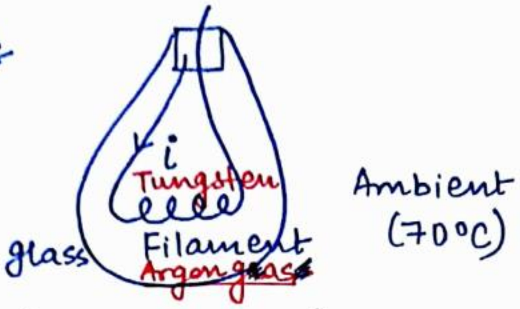
$$q_{(1-2)net} = \frac{\sigma(T_1^4 - T_2^4)A_1}{1/\epsilon_1 + \frac{A_1}{A_2}\left(1/\epsilon_2 - 1\right)}$$

$$q_{(1-2)net} = \sigma(T_1^4 - T_2^4)A_1\epsilon_1 \text{ watt}$$

WB
8

$$q_{1-2} = \sigma(T_1^4 - T_2^4) A_1 \epsilon_1$$

SIR



writting the energy Balance for steady state conditions of filament,

Electric power of Bulb = P = Rate of Heat generation in the Filament = Net Radiation heat Exchange b/w filament and Ambient

(Filament is a very small Body in large ambient).

$$P = \sigma(T_{fil.}^4 - T_{ambient}^4) A_{filament} \times \epsilon_{filament}$$

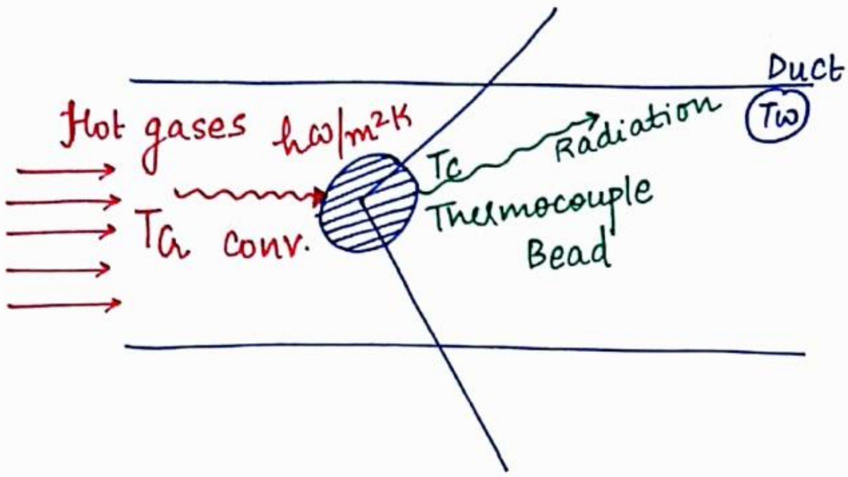
($A_{filament} = \pi d L$)

$$75 = 5.67 \times 10^{-8} (T_{fil}^4 - T_{amb}^4) \text{ — " — " —}$$

$$75 = 5.67 \times 10^{-8} (T_{fil}^4 - (70+273)^4) \times \pi \times \frac{0.1}{1000} \times \frac{5}{100} \times 1 \text{ watt}$$

$$T_{filament} = 3029 \text{ K}$$

* To get the Error in Temp. measurement of gases by using Thermocouple :-



Ta = ~~Thermocouple~~ True gas Temp.

Tc = Temp. Recorded by Thermocouple

Tw = Duct wall temp.

NOTE :- Thermocouple recieves heat By convection from the hot flowing gases with a convective coefficient of h which inturn exchanges heat by thermal Radiation with the duct walls.

over......... for steady state conditions of thermocouple bead,

writting energy Balance,

The Rate of convection H.T. b/w Hot gases and thermocouple bead = Net Radiation Heat Exchange between T/c and Duct walls (Thermocouple is a very small body in large Duct)

Newton's Law of cooling :-

$$h \cancel{A} (\overbrace{T_a - T_c}^{\text{error}}) = \sigma (T_c^4 - T_w^4) \cancel{A}\ \epsilon_{T/c} \quad \text{watt}$$

$(T_a - T_c) \rightarrow$ °C (or) K

$T_c^4 - T_w^4$: Kelvin

(39) $V = 4\,m/s$, $T = 400°C$,

SIR To get convective H.T. coefficient 'h' :-

$$Nu = \frac{hd}{k} = 0.5(Re)^{1/2} = (0.5)\left(\frac{\vec{V}d\rho}{\mu}\right)^{1/2}$$

$$\Rightarrow h = 162.3\ W/m^2K$$

$$162.3(T_a - 580) = 5.67\times10^{-8}\left[(580+273)^4 - (400+273)^4\right]$$

$\times 0.3$ watt

$$\boxed{T_a = 613.9°C}$$

Error in measurement $= 613.9 - 580.0 = 33.9°C$

Comment → (1) $\vec{V}$ should be ↑

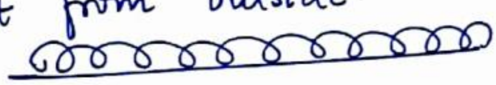
(2) . - - - -

$\boxed{\text{NOTE(SIR)}}$ → The error in the measurement can be reduced by:

(a) ↑ing the convective H.T. coefficient 'h' that is by providing more velocity to the gases.

(b) By ↓ing the emissivity of thermocouple Bead that is by polishing it.

(c) By ↑ing the duct wall tempr. T_w that is by insulating the duct from outside.

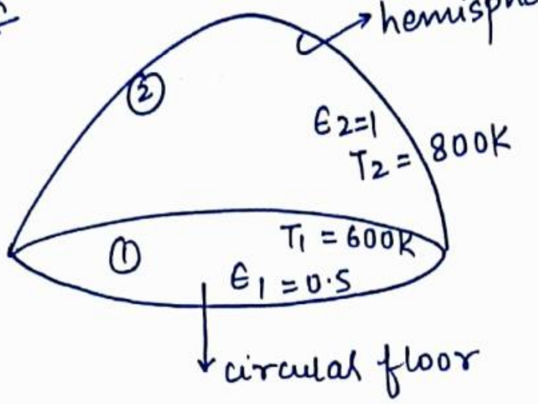


→ asbestos rope

34 $r = 1\,m$ $\quad T_w = 800K \quad K = 5.68 \times 10^{-8}$

$\epsilon = 1 \quad T_a = 600K$

SIR



$F_{12} = 1$

$F_{21} < 1$ → $\boxed{\text{case 2 formula}}$

$$q_{(1-2)net} = \frac{\sigma (T_1^4 - T_2^4) \overset{\pi R^2}{A}}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\cancel{\left(\frac{1}{\epsilon_2} - 1\right)} \to 0} \text{ watt} = \frac{5.67 \times 10^{-8} (600^4 - 800^4) \times \pi \times 1^2}{(1/0.5)} \text{ watt}$$

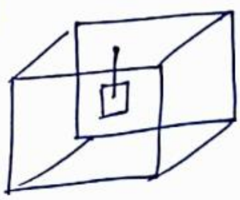
$= \ominus 24.9\, KW$

dont enter in integer type

∵ Qn. says Roof to floor

−ve sign shows that Net Radiation heat exchange is from 2 to 1 i.e. from Roof to floor.

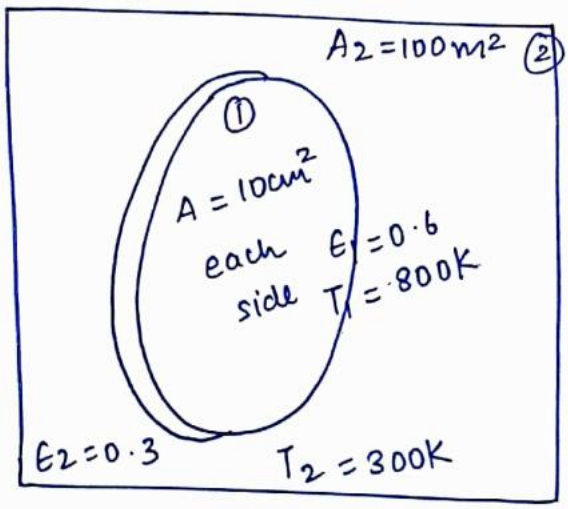
10



45.6

$$\frac{\sigma (T_1^4 - T_2^4) A}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

SIR



$F_{12} = 1$

$F_{21} < 1$

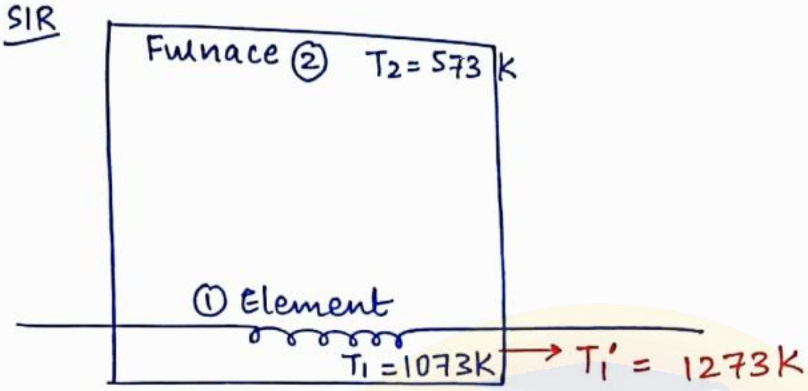
Case 2 formula

Total area of plate = 10 + 10 = 20 cm^2 = 20×10^{-4} m^2 = A_1

$$q_{(1-2)} = \frac{\sigma(T_1^4 - T_2^4)\times A_1}{1/\epsilon_1 + A_1/A_2(1/\epsilon_2 - 1)} = \frac{5.67\times10^{-8}(800^4 - 600^4)\times 20\times10^{-4}}{1/0.6 + \frac{20\times10^{-4}}{100}(1/0.3 - 1)}$$

= 27.32 watt.

Q9 SIR



$$\frac{4.015\times10}{1000^4 - 3} = \frac{8\times1000}{Q}$$

Element is a very small body in large furnace.

$$(q_{(1-2)})_{net} = \sigma(1073^4 - 573^4)\,\epsilon_{element}\times 1 \overset{\text{flux}}{=} 8\ KW/m^2$$

$$(q'_{(1-2)})_{net} = \sigma(1273^4 - 573^4)\,\epsilon_{element}\times 1 = ? \longrightarrow ②$$

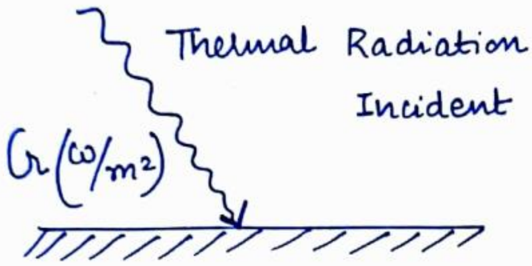
Special Case of Case 2

$$\frac{②}{①} \Rightarrow (q'_{1-2}) = 16.5\ KW/m^2.$$

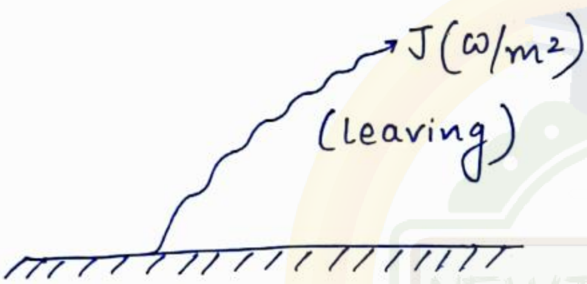
* RADIATION NETWORKS :-

from this radiation network technique we can calculate Net Radiation heat Exchange b/w any two finite (or) infinite diffuse grey (or) Black surfaces.

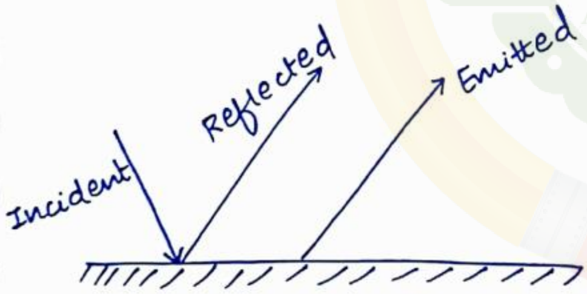
Irradiation (G) and Radiosity (J) :-



The Total thermal Radiation incid-ent upon a surface per unit time and per unit area is known as irradiation G.



The Total thermal Radiation Leaving a surface per unit time per unit area is known as Radiosity J.



J = Emitted Energy + Reflected part of Incident Energy.

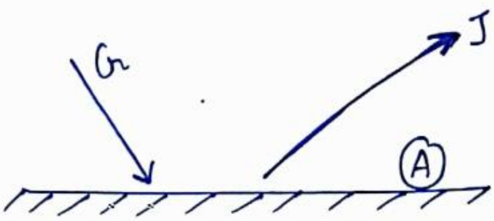
$$J = E + \rho G \quad \omega/m^2$$

$$J = \epsilon E_b + \rho G \quad W/m^2$$

$$\therefore \; J = \epsilon E_b + (1-\epsilon) G \quad W/m^2 \Rightarrow$$

$$G = \frac{J - \epsilon E_b}{(1-\epsilon)}$$

put in next eqn.



For any surface,

$\alpha + \rho + \tau = 1$

For opaque surface

$\tau = 0$

$\alpha + \rho = 1$

$\rho = 1 - \alpha$

$\rho = (1 - \epsilon)$

B at kirchoff's law,

$\alpha = \epsilon$

Hence, the net Radiation heat exchange b/w a surface of area A and all of its surroundings is given by

$$(q_{net})_{\text{surface - surroundings}} = (J - G) \, A \text{ watt}$$

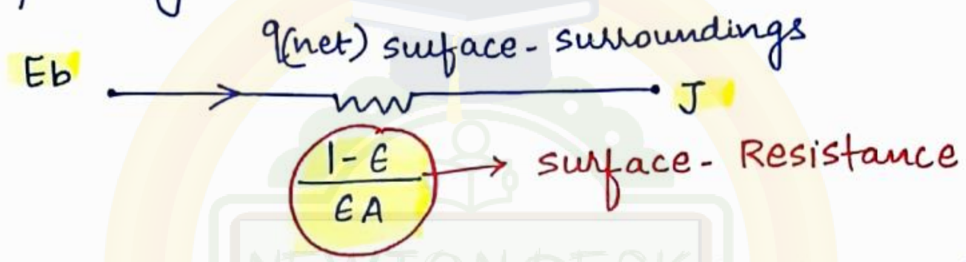
$$\Rightarrow G = \frac{J - \epsilon E_b}{(1-\epsilon)}$$

$$= \left[\epsilon E_b + (1-\epsilon) G - G\right] A \text{ watt}$$

Eliminating G, we get

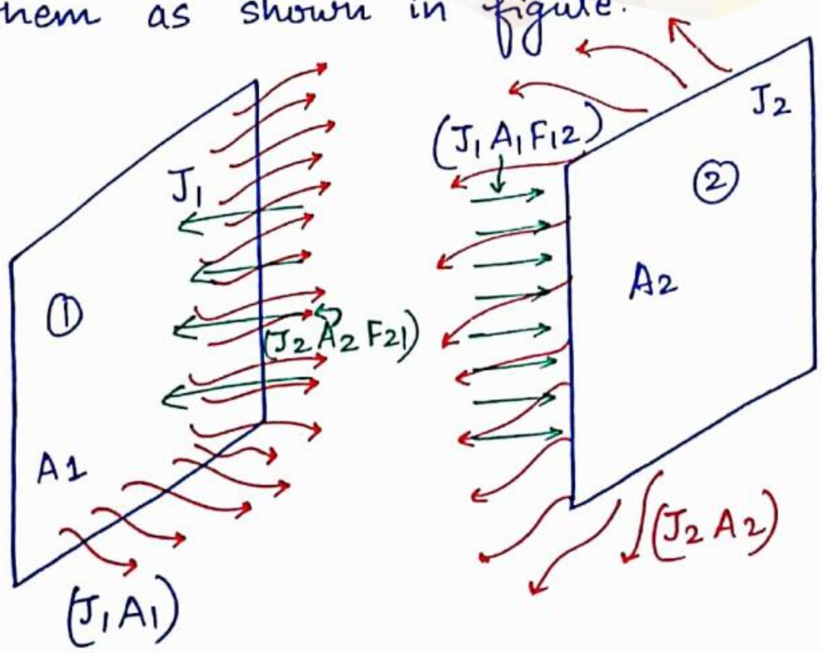
$$(q_{net})_{\text{surface - surroundings}} = \frac{E_b - J}{\left(\frac{1-\epsilon}{\epsilon A}\right)} \text{ watt.}$$

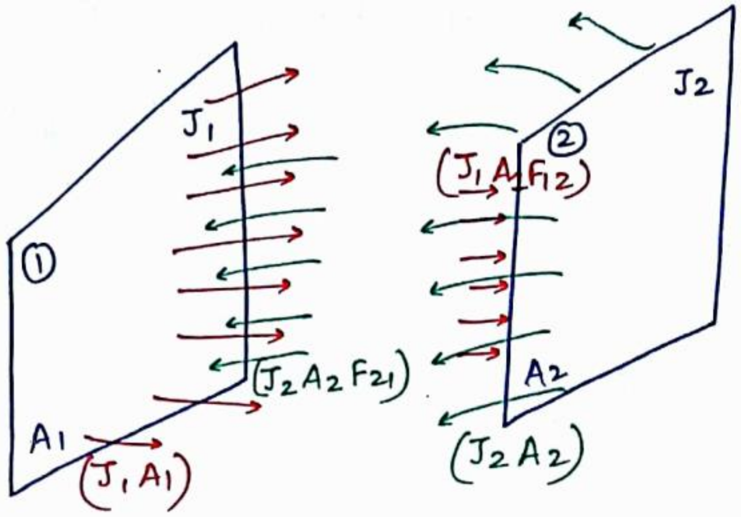
The corresponding thermal Radiation circuit is



This surface resistance will exist at the surface which is exchanging heat by Radiation with all of its surroundings.

Consider two finite surfaces of areas A_1 and A_2 having radiosities J_1 and J_2 exchanging heat by thermal Radiation b/w them as shown in figure.





Out of the Total Radiation Energy leaving surface 1, the energy that reaches surface 2 is $J_1 A_1 F_{12}$. Similarily, out of the Total Radiation energy leaving surface 2, the energy that reaches surface 1 is $J_2 A_2 F_{21}$. Therefore,

$\therefore$ Net Radiation Heat Exchange between 1 & 2

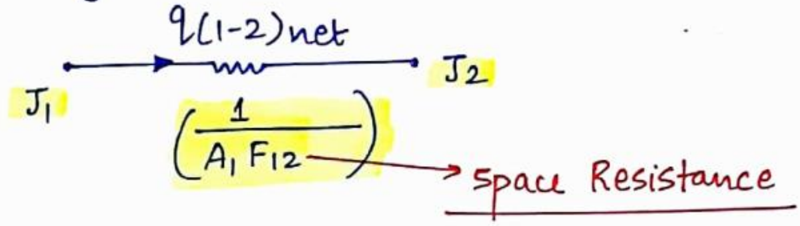
$$= q_{(1-2)net} = J_1 A_1 F_{12} - J_2 A_2 F_{21} \text{ watt}$$

But $A_1 F_{12} = A_2 F_{21}$ (Reciprocity Relation)

$$\therefore q_{(1-2)net} = (J_1 - J_2) A_1 F_{12}$$

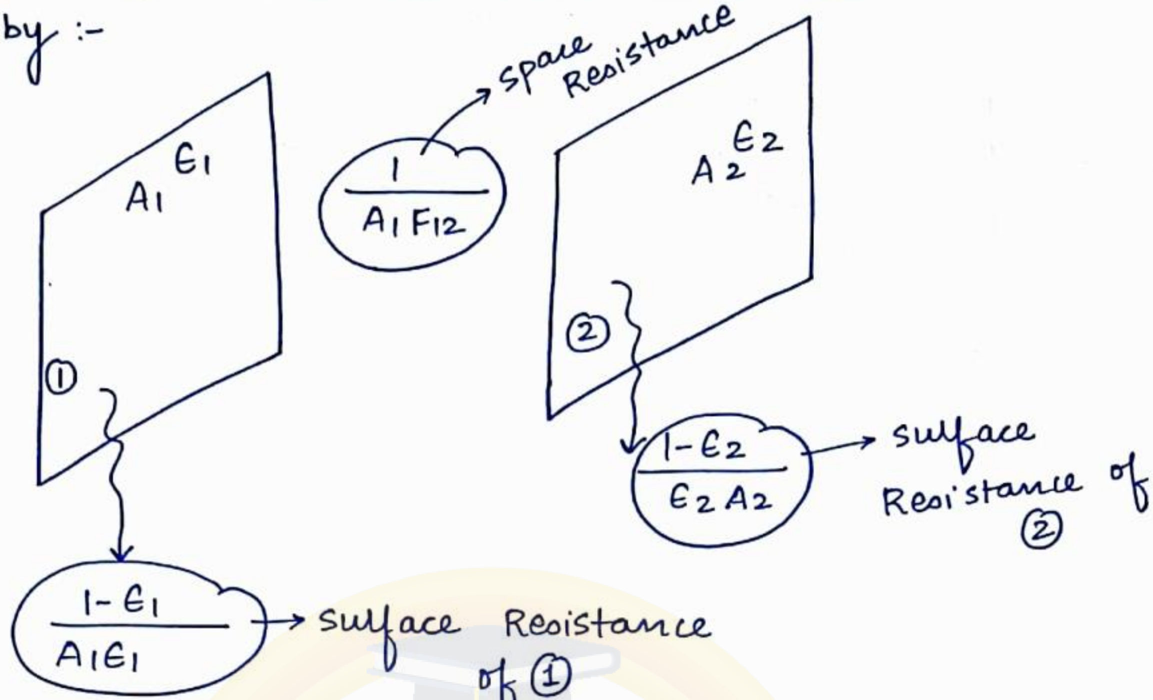
$$\boxed{q_{(1-2)net} = \frac{(J_1 - J_2)}{\left(\frac{1}{A_1 F_{12}}\right)} \text{ watt}}$$

The corresponding Radiation circuit is



This space Resistance shall exist in the space or the vacuum prevailing b/w the 2 surfaces which are exchanging heat by thermal Radiation b/w them.

Hence, the complete Radiation network for Radiation heat exchange b/w 2 finite diffused grey surfaces is given by :-



Total Radiation Network :-

$$E_{b1} \xrightarrow[\frac{1-\epsilon_1}{\epsilon_1 A_1}]{} J_1 \xrightarrow[\frac{1}{A_1 F_{12}}]{q_{(1-2)net}} J_2 \xrightarrow[\frac{1-\epsilon_2}{\epsilon_2 A_2}]{} E_{b2}$$

$E_{b1} = \sigma T_1^4$, $E_{b2} = \sigma T_2^4$

$$\boxed{\therefore \left(q_{(1-2)net}\right) = \frac{\sigma\left(T_1^4 - T_2^4\right) \text{ watt}}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}}$$

In case, if both the surfaces are black,

Then $\epsilon_1 = \epsilon_2 = 1$

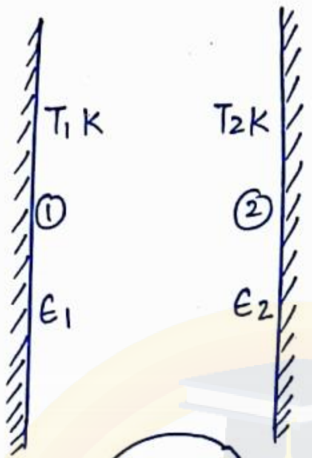
Then Radiation Network is :-

$$E_{b1} \xrightarrow[\left(\frac{1}{A_1 F_{12}}\right)]{q_{(1-2)net}} E_{b2}$$

$$\therefore \; q_{(1-2)net} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{A_1 F_{12}}\right)} \text{ watt.}$$

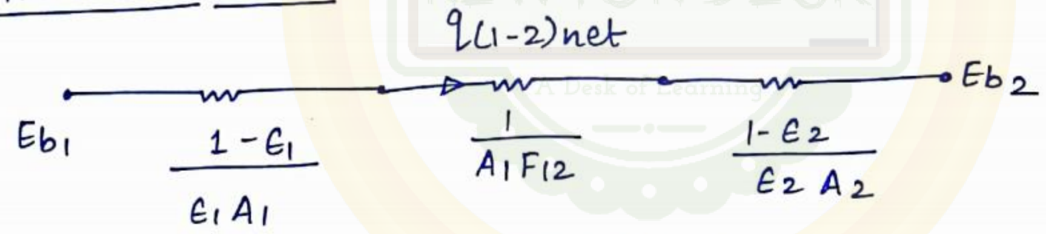
* APPLICATIONS OF RADIATION NETWORKS METHODS:-

Case I :- Two infinitely large parallel planes



$F_{12} = 1$
$F_{21} = 1$

Radiation Network :-

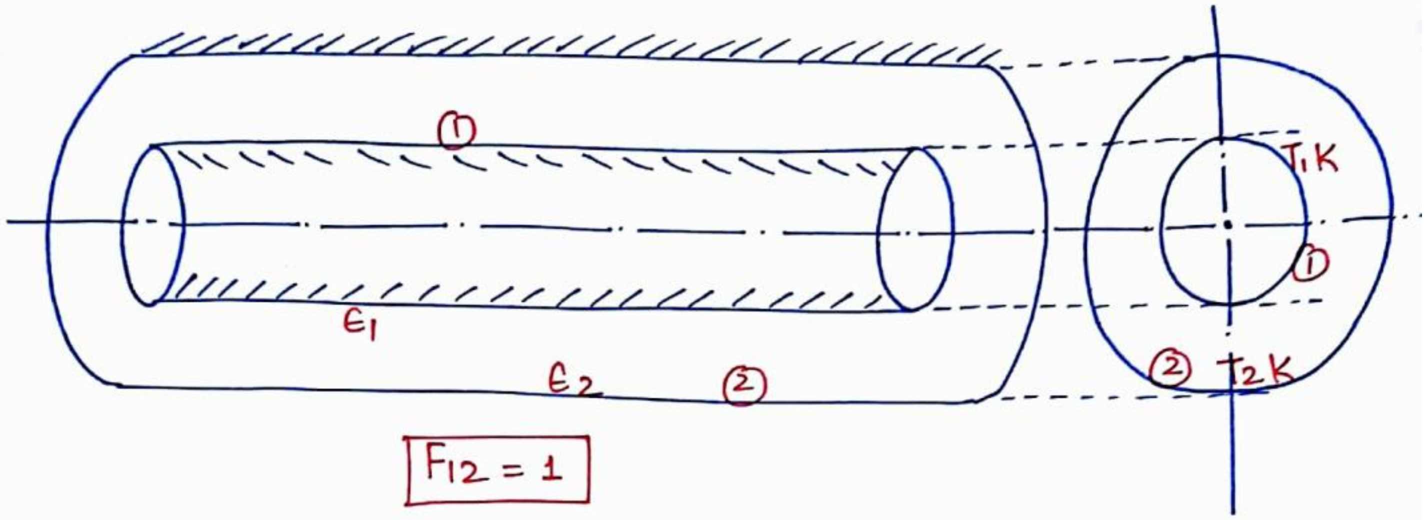


Put $A_1 = A_2 = 1$ (∵ Flux being calculated)

$$\therefore \left(q/A\right)_{\substack{net \\ 1-2}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1 \times 1} \cancel{- \frac{1}{1}} + \cancel{\frac{1}{1 \times 1}} + \frac{1}{\epsilon_2 \times 1} - \frac{1}{1}} \; W/m^2$$

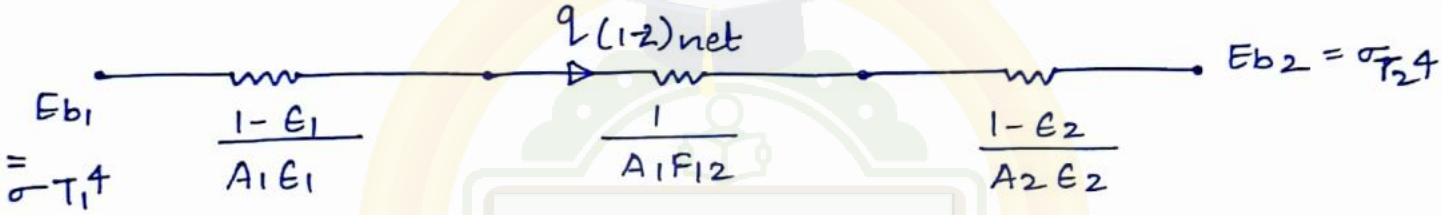
$$= \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

* Case 2 :- Infinitely long concentric cylindrical surfaces :-



$$\boxed{F_{12} = 1}$$

Radiation Network



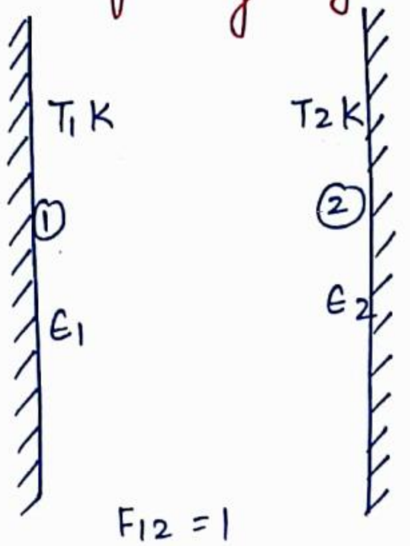
$$\therefore \quad q_{(1-2)net} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1 A_1} - \cancel{\frac{1}{A_1}} + \cancel{\frac{1}{A_1 \times 1}} + \frac{1}{\epsilon_2 A_2} - \frac{1}{A_2}}$$

Multiply N & D with A_1

$$\boxed{q_{(1-2)net} = \frac{\sigma (T_1^4 - T_2^4) A_1}{1/\epsilon_1 + A_1/A_2 (1/\epsilon_2 - 1)} \text{ watt}}$$

Case 3 :- RADIATION SHIELD :-

Infinitely large Planes

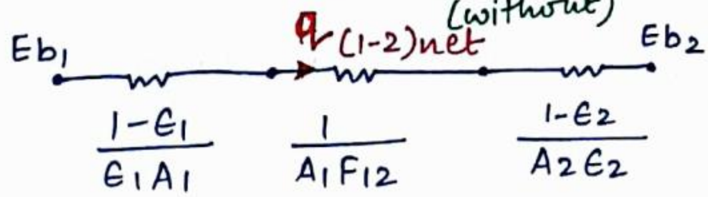


$F_{12} = 1$

$F_{21} = 1$

$A_1 = A_2 = 1$ ($\because$ flux)

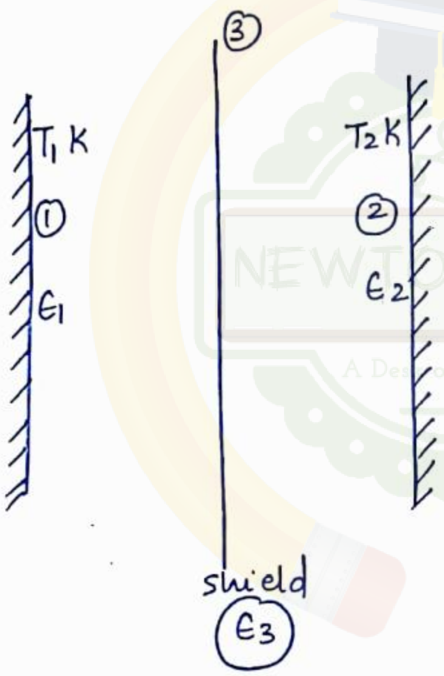
Radiation Network without shield :-

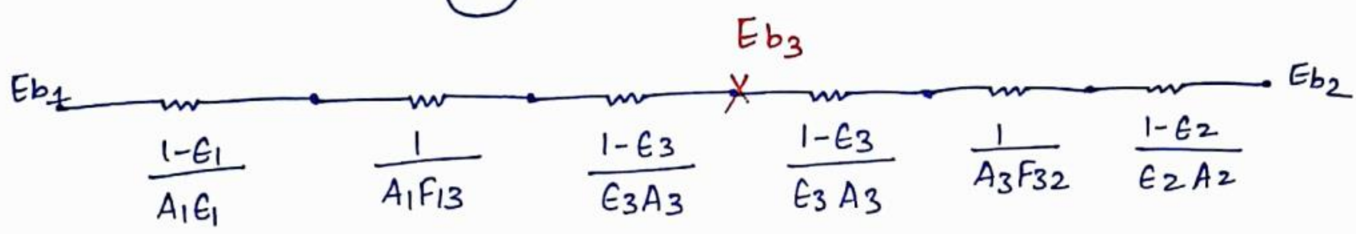


$$\left(\frac{q}{A}\right)_{(1-2)\,net} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\epsilon_1 + 1/\epsilon_2 - 1} \ \frac{W}{m^2}$$

without any shield

Radiation Network with shield :-





Put $F_{13} = F_{32} = 1$ Put $A_1 = A_2 = A_3 = 1$

($\because$ flux being calculated)

$$\Rightarrow \left(q/A\right)_{net\ 1-2} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\epsilon_1 + 2/\epsilon_3 + 1/\epsilon_2 - 2} \ W/m^2$$

(with shield)

Hence to Reduce the Radiation heat Exchange by a very good percentage. The shield kept b/w the planes must have very low emissivity or very high reflectivity for which highly polished Al 'or' Cu shields are generally used.

$\epsilon = 1 - \rho$

(low)

$\epsilon_3 \rightarrow$ has to be low.
To ↑ $(2/\epsilon_3)$ To ↓ $(q/A)_{net}$
hence more diffi. for H.T. Radiation q

Each shield kept between the planes shall bring in 3 additional resistances extra into the network out of which 2 are surface resistances and the remaining one is space resistance. hence, if there are 'n' no. of shields being kept b/w the plates, there would be total '2n + 2' no. of surface resistances and 'n+1' no. of space resistances in the entire radiation network drawn with 'n' shields.

* $\boxed{\text{SPECIAL CASE}}$:- In case if all the surfaces (planes as well as shields) have the same emissivity 'ε', then without any shield i.e.

Then $$\left(\frac{q}{A}\right)_{net\ 1-2 \text{(without any)}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(2/\epsilon - 1\right)}\ \omega/m^2$$

and $$\left(\frac{q}{A}\right)_{net\ 1-2 \text{(with one shield)}} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(4/\epsilon - 2\right)}$$

$$= \frac{1}{2}\,\frac{\sigma\left(T_1^4 - T_2^4\right)}{\left(2/\epsilon - 1\right)}$$

$$= \frac{1}{2} \times \left(q/A\right)_{net\ 1-2 \text{(without any shield)}} \quad \boxed{\therefore 50\%\ \text{drop}}$$

Thus If there are 'n' no. of shields being used, Then (provided all have same ε)

$$\boxed{\left(\frac{q}{A}\right)_{\substack{net\\1-2}} = \left(\frac{1}{n+1}\right)\left(\frac{q}{A}\right)_{\substack{net\\1-2}} \; W/m^2}$$

with 'n' shields (without any shield)

No. of shields	% Reduction in Heat Exchange
1 $\rightarrow$	50%
2 $\rightarrow$	66.6%
3 $\rightarrow$	75%
4 $\rightarrow$	80%

(37) $\cancel{\frac{3.58}{52.58}}$

SIR

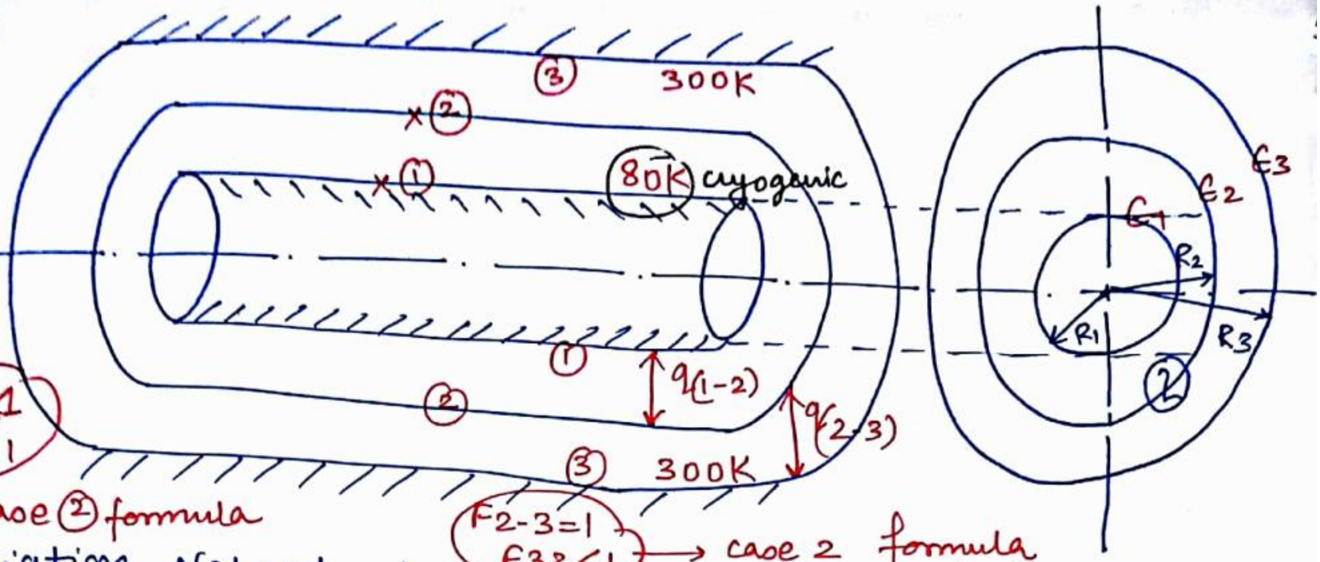
$$\left(q/A\right)_{1-2 \text{ without any shield}} = \frac{\sigma(T_1^4 - T_2^4)}{1/0.3 + 1/0.8 - 1} \; W/m^2$$

$$\left(q/A\right)_{\text{with (AL) one shield}} = \frac{\sigma(T_1^4 - T_2^4)}{1/0.3 + 2/0.04 + 1/0.8 - 2} \; W/m^2$$

$\therefore$ % Reduction in Radiation Heat Exchange $= \left[\frac{q_{without} - q_{with}}{q_{without}}\right] \times 100\%$

$= 93.18\%$

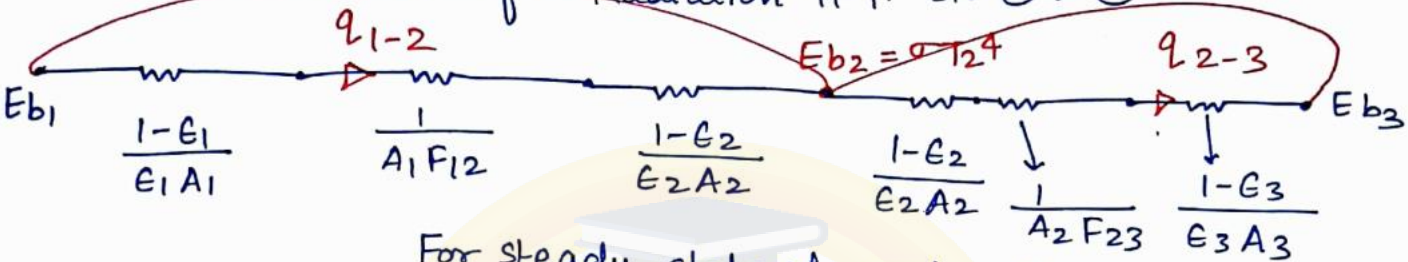
38



$F_{12} = 1$
$F_{21} < 1$
↳ case ② formula

$F_{2-3} = 1$
$F_{32} < 1$ → case 2 formula

Radiation Network for Radiation H.T. b/w ① & ③ :-



For steady state of surface ②,

$$(q_{1-2})_{net} = (q_{2-3})_{net}$$

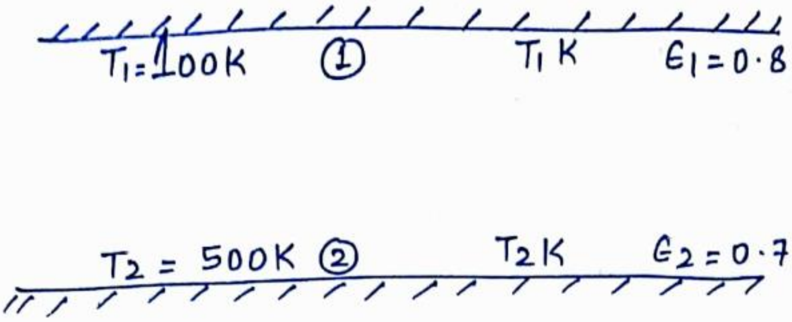
$$\Rightarrow \frac{\cancel{\sigma}(T_1^4 - T_2^4)A_1}{1/\epsilon_1 + A_1/A_2\left(1/\epsilon_2 - 1\right)} = \frac{\cancel{\sigma}(T_2^4 - T_3^4)A_2}{1/\epsilon_2 + A_2/A_3\left(1/\epsilon_3 - 1\right)}$$

Put $A_1/A_2 = R_1/R_2$ $\qquad T_2 = \boxed{280.8}\ K$

$A_2/A_3 = R_2/R_3$

23/9/2016

(26)

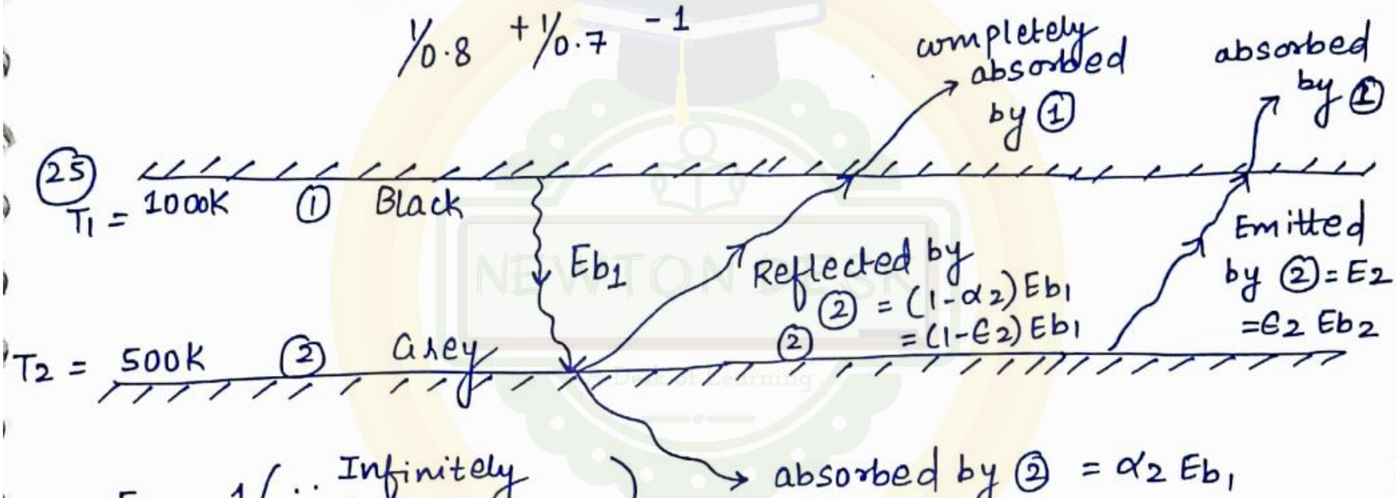


Case ① formula.

$$q_{(A)net_{1-2}} = \frac{\sigma(T_1^4 - T_2^4)}{1/\epsilon_1 + 1/\epsilon_2 - 1}$$

$$= \frac{5.67 \times 10^{-8}(1000^4 - 500^4)}{1/0.8 + 1/0.7 - 1} = 31.7 \times 10^3 \, W/m^2.$$

(25)



$F_{12} = 1$ ($\because$ Infinitely large planes)

$F_{21} = 1$

Irradiation of ① = Total thermal Radiation Incident upon ① $= (1-\epsilon_2)Eb_1 + \epsilon_2 Eb_2$

$$G_1 = (1-\epsilon_2)\sigma T_1^4 + \epsilon_2 \sigma T_2^4 \; \frac{W}{m^2}$$

$G_2 =$

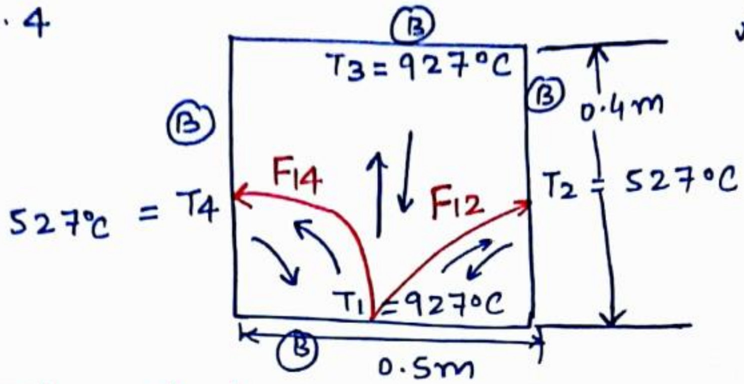
$$G_1 = 19.5 \, kW/m^2$$

(41)

"N.P"

(41) 0.5×0.4

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

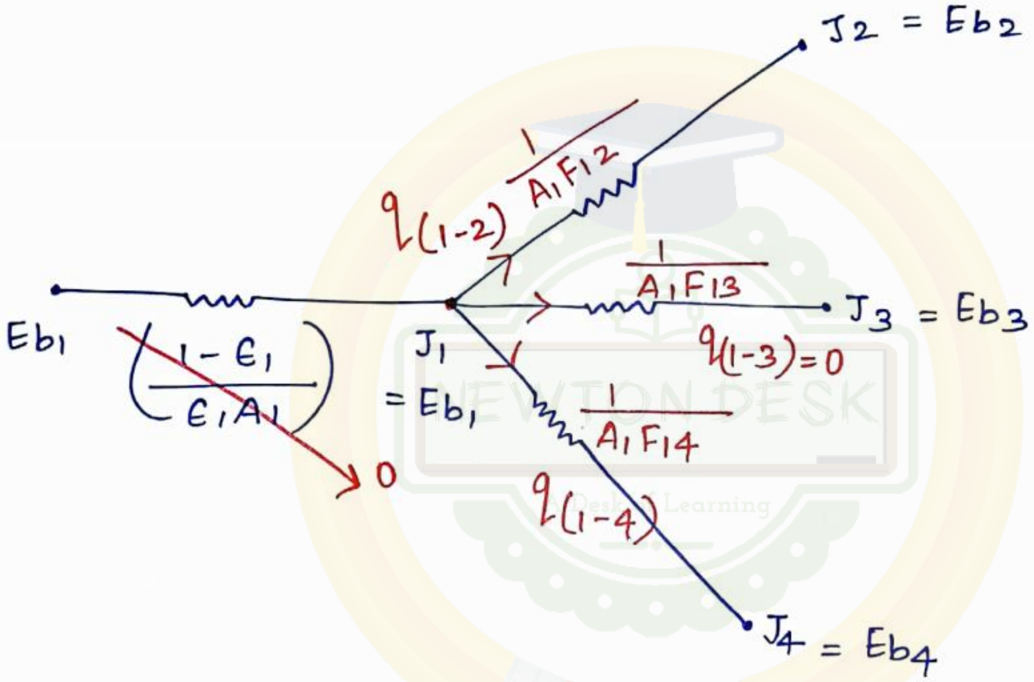


$F_{12} = 0.26$

From symmetry of figure,

$$F_{12} = F_{14} = 0.26$$

$$F_{13} = 1 - (2 \times F_{12}) = \boxed{}$$



∴ Total net Radiation heat loss from (1) $= 2 \times q_{(1-2)net}$

$T_1 = T_3$

$$= 2 \times \frac{E_{b1} - E_{b2}}{\left(\frac{1}{A_1 F_{12}}\right)}$$

$$= \frac{2 \times \sigma (T_1^4 - T_2^4)}{\frac{1}{0.5 \times 1 \times 0.26}}$$ watt/metre length of Duct

$= \boxed{}$ watt.

(1) b

(4) c

(5) b

(6) $E = \int_0^{\infty} E_\lambda \, d\lambda = 150(12-3) + 300(25-12) = 5250 \ W/m^2$

(7) c

(12) c

(13) c

(15) $J = E + \rho G$

$J = 32 + 0.6 \times 93 = 87.8 \ W/m^2$

(16) $J = E + (1-\epsilon)G$

$12 = 10 + (1-\epsilon)20$

$\epsilon = 0.9$

(17) c

(18) $E_b = \pi i_n = \sigma T^4 \ \omega/m^2$

$i_n = \left(\frac{\pi}{\sigma T^4}\right)^{-1}$

$= \frac{5.67 \times 10^{-8} \times 800^4}{\pi}$ watt/m²- steradian

$i = i_n \cos 50$

$= 4750$ watt/m²- steradian

(27) c

CONVECTION

Note :- In any convection Heat transfer problem it is req. to obtain the convective heat transfer coefficient 'h' for a given boundary conditions prevailing b/w Body and fluid and hence to obtain the convection heat transfer rate b/w them from newton's law of cooling i.e.

$$\boxed{q = h_{conv.} A \Delta T}$$

Convection

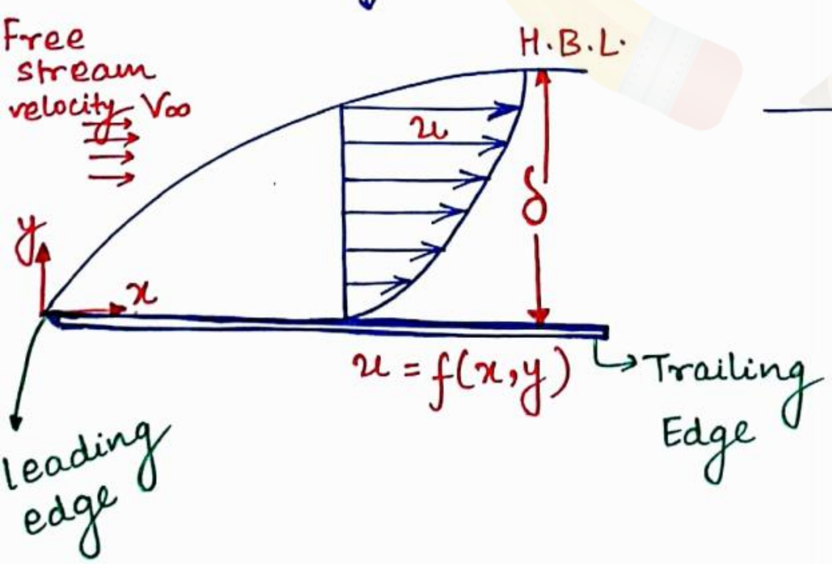
Forced Convection H.T.

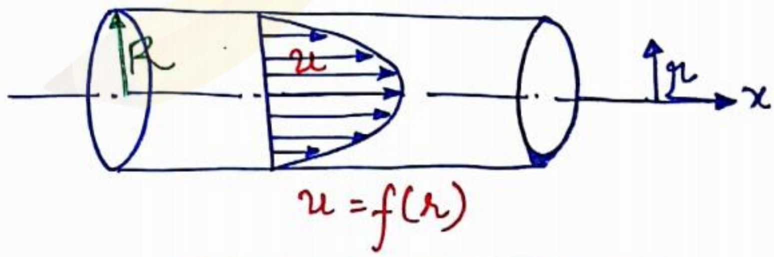
velocity is evident.

- Flow over Flat plates
- Flow through pipes/Ducts (Internal flows)

Free (OR) Natural Convection

No velocity evident but the flow occurs naturally due to Buoyancy forces arising out of density changes of fluid.

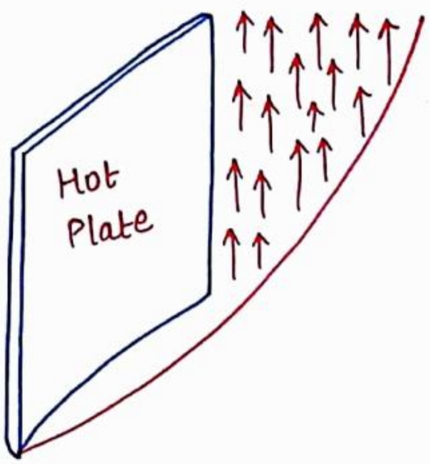




Mass flow Rate

$$= \dot{m} = \rho \times \pi R^2 \, \vec{V}_{mean} \quad \text{kg/sec}$$

Free convection



$$\frac{P}{\rho\downarrow} = RT\uparrow$$

* FORCED CONVECTION HEAT TRANSFER :-

In any forced convection heat transfer,

$$h = f(\vec{V}, D, \underbrace{\rho, \mu, C_p, K}_{\text{Thermophysical Properties of fluid.}})$$

$\vec{V}$ = velocity of fluid (m/sec)

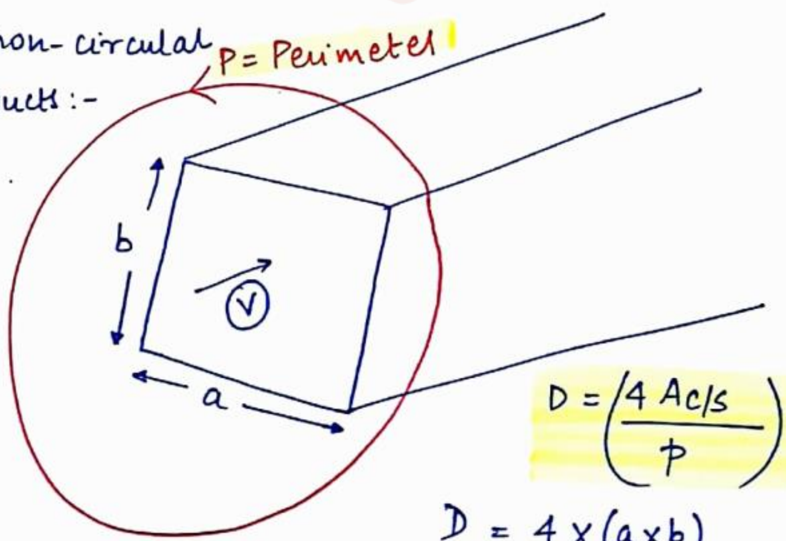
D = characteristic Dimension of Body

(Dimension of Body i.e. used in the calculation of dimensionless No.s).

$$D = \frac{4 \times \frac{\pi}{4} D^2}{\pi D}$$

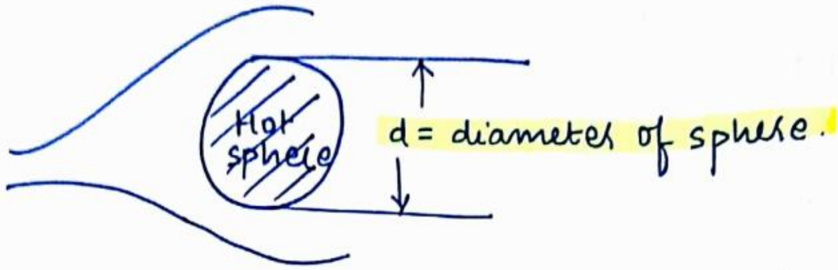
$= D$

For non-circular Ducts :-



$$D = \left(\frac{4\, A_{c/s}}{P}\right)$$

$$D = \frac{4 \times (a \times b)}{2(a+b)}$$



* from Buckingham- π Theorem of dimensional analysis which states that if there are Total 'n' no. of variables in any functional relationship both dependent and Independent and if all the variables put together contain 'm' no. of fundamental dimentions, then the functional relationship among the variables can be expressed in terms of ('n -m') no. of dimensionless π-Terms.

Here $n = 7$

$m = 4(M, L, T, \theta)$ → Temp^r

$\therefore$ from Theorem,

no. of dimensionless π-terms $= 7-4 = 3$.

Let the π- terms be π_1, π_2 and π_3.

To get $\pi_1 = ?$ $\pi_2 = ?$ and $\pi_3 = ?$

- $D \rightarrow L$
- $V \rightarrow LT^{-1}$
- $\rho \rightarrow ML^{-3}$
- $Pa\text{-}sec = \mu \Rightarrow ML^{-1}T^{-1}$

- $\dfrac{\cancel{M}LT^{-2}}{\cancel{M}\theta} = \dfrac{Nm}{kgK} = \dfrac{J}{kgK} = C_p \Rightarrow L^2T^{-2}\theta^{-1}$

- $\dfrac{MLT^{-2}}{T\theta} = \dfrac{N\cancel{m}}{sec\,\cancel{m}K} = \dfrac{J}{sec\,mK} = \dfrac{W}{mK} = K \rightarrow MLT^{-3}\theta^{-1}$

- $\dfrac{W}{m^2K} = h \rightarrow MT^{-3}\theta^{-1}$.

choose 'm' no. of Repeating variables in such a way that

① All of them put together contain all the fundamental dimensions.

② They themselves should NOT from a dimensionless group.

∴ Select h, V, D, ρ as Repeating variables.

Then

$$\pi_1 = (h^{a_1} \cdot V^{b_1} \cdot D^{c_1} \cdot \rho^{d_1})\mu.$$

$$\pi_2 = (h^{a_2} \cdot V^{b_2} \cdot D^{c_2} \cdot \rho^{d_2})C_p.$$

$$\pi_3 = (h^{a_3} \cdot V^{b_3} \cdot D^{c_3} \cdot \rho^{d_3})K.$$

Now, To get π_1 :-

$$M^0L^0T^0\theta^0 = (MT^{-3}\theta^{-1})^{a_1}(LT^{-1})^{b_1}(L)^{c_1}(ML^{-3})^{d_1} \times ML^{-1}T^{-1}.$$

For Mass 'M' :- $0 = a_1 + d_1 + 1$

For ~~Mass~~ length 'L' :- $0 = b_1 + c_1 - 3d_1 - 1$

For Time 'T' :- $0 = -3a_1 - b_1 - 1$

For Tempr. 'θ' :- $0 = -a_1$

$a_1 = 0$
$b_1 = -1$
$c_1 = -1$
$d_1 = -1$

$$\Rightarrow \boxed{\pi_1 = \frac{\mu}{VD\rho}}$$

To get π_2 :- $M^0L^0T^0\theta^0 = (MT^{-3}\theta^{-1})^{a_2}(LT^{-1})^{b_2}(L)^{c_2}(ML^{-3})^{d_2} \times L^2T^{-2}\theta^{-1}$

For mass 'M' :- $0 = a_2 + d_2$

For length 'L' :- $0 = b_2 + c_2 - 3d_2 + 2$

For time 'T' :- $0 = -3a_2 - b_2 - 2$

For ~~time~~ Temp. 'θ' :- $0 = -a_2 - 1$

$\Rightarrow$ $a_2 = -1$, $b_2 = 1$, $c_2 = 0$, $d_2 = 1$

$$\Rightarrow \boxed{\pi_2 = \left(\frac{\rho V C_p}{h}\right)}$$

To get π_3 :-

$$M^0L^0T^0\theta^0 = (MT^{-3}\theta^{-1})^{a_3}.(LT^{-1})^{b_3}.(L)^{c_3}.(ML^{-3})^{d_3}(MLT^{-3}\theta^{-1})$$

$a_3 = -1$
$b_3 = 0$
$c_3 = -1$
$d_3 = 0$

$\therefore \pi_3 = \frac{K}{hD}$

hence,

$$\pi_1 = \frac{\mu}{VD\rho}.$$

$$\pi_2 = \frac{\rho V C_p}{h}.$$

$$\pi_3 = \frac{K}{hD}.$$

Let

$\pi_{[4]} = \frac{1}{\pi_1} = \frac{VD\rho}{\mu}$ = Reynold's No. = (Re)

$\pi_5 = \frac{1}{\pi_3} = \frac{hD}{K}$ = Nusselt No. = (Nu)

also → ((dimensionless heat transfer coefficient)

$\pi_6 = \frac{\pi_1 \pi_2}{\pi_3} = \frac{\mu C_p}{K}$ = Prandtl No. = (Pr)

→ all μ, C_p, K → Thermophy. properties

$\pi_7 = 1/\pi_2 = \frac{h}{\rho V C_p}$ = Stanton No. = (St)

hence, important in GATE

$\therefore$ In any forced Convection H.T.,

From Theorem,

$$Nu = f(Re, Pr)$$

$$\frac{hD}{K} = f\left(\frac{VD\rho}{\mu}, \frac{\mu C_p}{K}\right)$$

our aim to get this here outside.

* PHYSICAL SIGNIFICANCE OF DIMENSIONLESS NO.'S IN forced Convection Heat Transfer :-

① **Reynold's No (Re)** :- Reynolds No. is defined as the Ratio b/w inertia forces and viscous forces to which a flowing fluid is subjected to

$$Re. = \frac{I.F.}{V.F.}$$

$$Re = \frac{VD\rho}{\mu}$$

$$Re = \frac{VD}{\left(\frac{\mu}{\rho}\right)}$$

$$Re = \frac{VD}{\nu}$$ where ν = Kinematic viscosity of fluid in $\frac{m^2}{sec}$

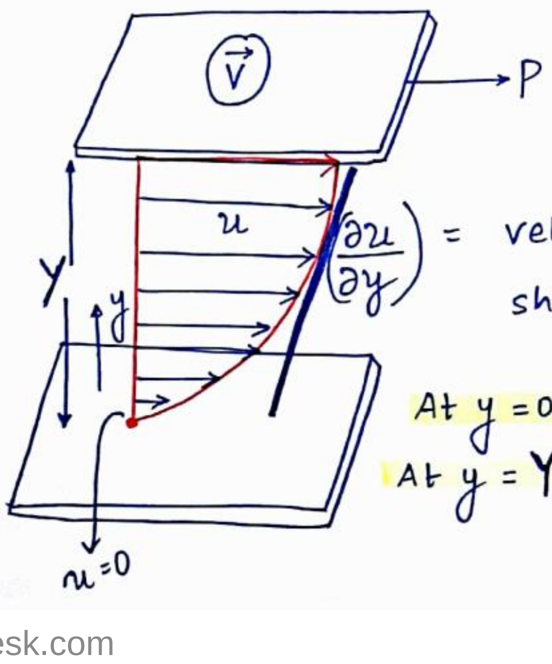
$$\nu = \left(\frac{\mu}{\rho}\right)$$

ν (K.V.) signifies Momentum Diffusion Rate through fluid layers.

✓ viscosity is a property of fluid by virtue of which there is a resistance offered by one layer of the fluid over its adjacent layer against the relative motion between them.

✓ Newton's law of Viscosity :-

$F_{net} = m \cdot a$, $a \to 0$



$\left(\frac{\partial u}{\partial y}\right)$ = velocity gradient (or) Rate of shear strain (or) Rate of angular deformation.

At $y = 0 \Rightarrow u = 0$

At $y = Y \Rightarrow u = V$

The law states :-

$$\text{Shear stress} \propto \left(\frac{\partial u}{\partial y}\right)$$

$$\tau \propto \left(\frac{\partial u}{\partial y}\right)$$

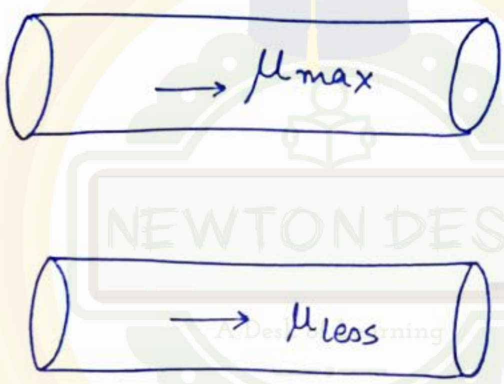
$$\boxed{\tau = \mu\left(\frac{\partial u}{\partial y}\right) \text{ Pascal}}$$

property of a fluid. ← μ = Dynamic viscosity of fluid in Pa-sec

$$\mu_{air} = 18 \times 10^{-6} \text{ Pa-sec}$$

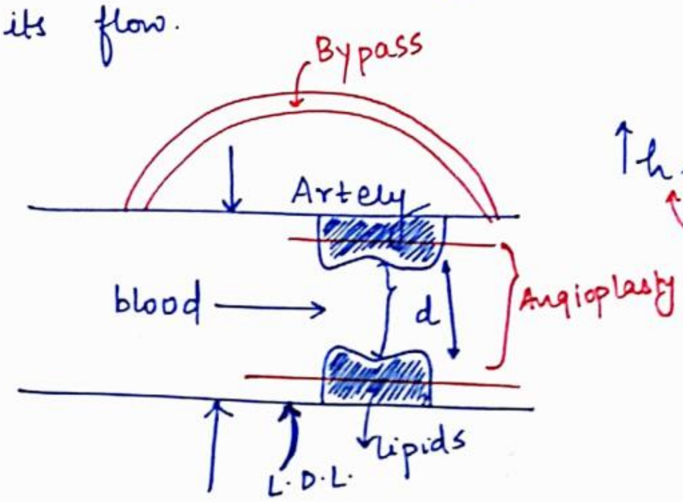
(At Room conditions)

*



Note :-

A fluid having more viscosity need to be supplied with more amount of energy for a given volume flow rate in a given pipe diameter because it suffers more frictional resistance against its flow.

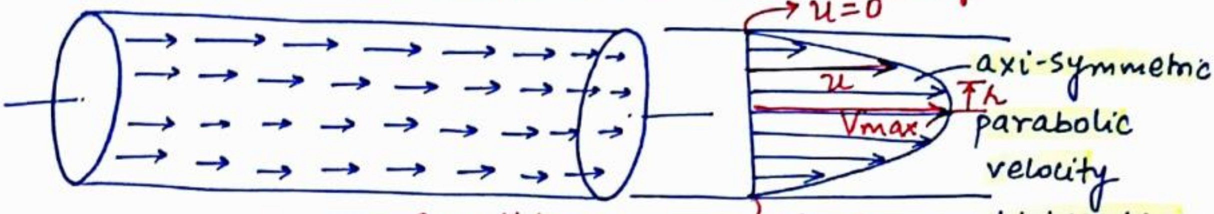


$$\uparrow h_f \downarrow = \frac{fLv^2}{2gd\downarrow} \quad \mu\downarrow$$

Re is the criteria To tell whether the fluid flow is laminar or turbulent. for incompressible flow through pipes as well as ducts, (171)

- If Re < 2000 (lower critical Re. No.) ⇒ Flow is laminal flow/Viscous flow.

English $\boxed{\text{LAMINA} \to \text{moving Planes}}$ → No intermixing



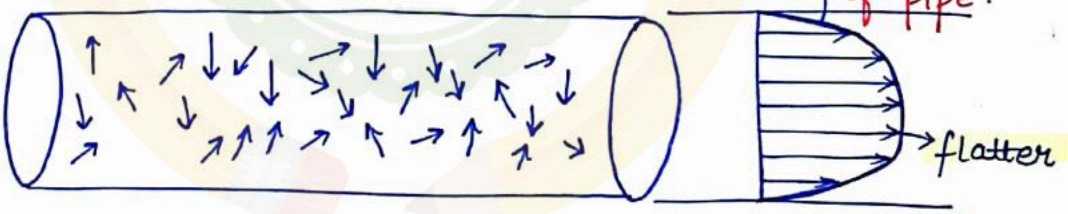
one layer of fluid Just smoothly slides over its adjacent layer without any intermingling (or) Intermixing of fluid particles among fluid layers.

$\boxed{u = f(r)}$

$\boxed{\text{Ex}}$:- ① The flow of highly viscous lubricant oil at relative low velocities in a small diameter pipe is generally laminar.

- If Re > 4000 (upper critical Re. No.) ⇒ Flow is turbulent

logrithmic velocity distribution at a given cross-section of pipe.

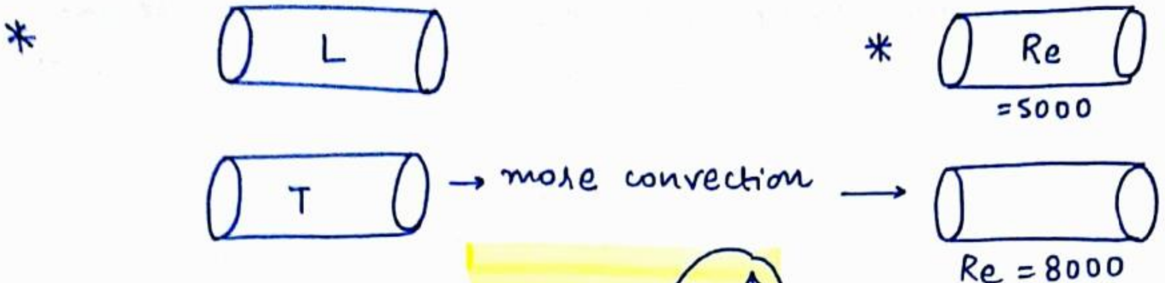


$u = f(r)$

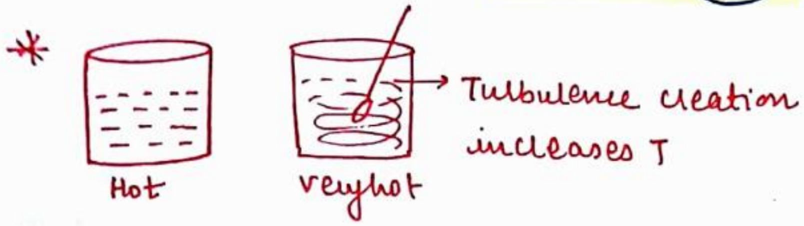
Continuous intermingling (OR) Intermixing of fluid particles among fluid layers with chunks of fluid Mass jumping from one layer to another layer carrying more Mass, Momentum and energy with them.

$\boxed{\text{Ex}}$- The flow of any gas even at low velocities inside a pipe or duct are generally turbulent flows.

Generally, more often we come across turbulent flows during flow through pipes.



To ↑ Re, $\vec{V}$ ↑

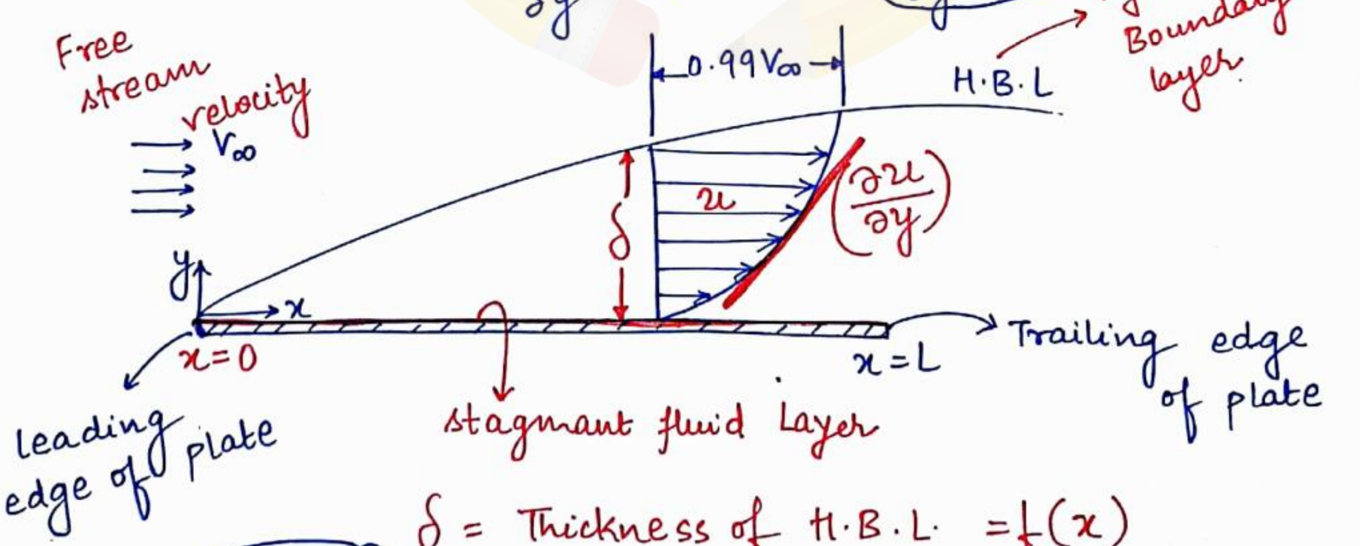


✓ For any given fluid in a given pipe, the convective h.T. coefficients in turbulence flow's are always greater than those in laminar flows. also, for a given turbulent flow of a fluid in a given pipe, as the Reynold's no. increases (that is by ↑ing the velocity of fluid), the convective H.T. coefficient 'h' also increased.

* $\boxed{\text{FOR Incompressible Flow over Flat plates}}$:-

At any given x, $\frac{\partial u}{\partial y} = f(y)$

$\frac{\partial u}{\partial y}$ at $y=0$ with maxm.



δ = Thickness of H.B.L. $= f(x)$

$u = f(x,y)$

Hydrodynamic Boundary layer is defined as a thin region formed over the flat plate inside which velocity gradients are seen in the normal direction to the plate (flat). These velocity gradients are formed due to the viscous nature or rather "due to" momentum diffusion through the fluid layers in the normal direction to the plate (y - direction). Outside this HBL, everywhere free stream velocities prevail (existing) that is no viscous influence felt. 173

The Boundary conditions of HBL are :-

at any x, measured from leading edge of plate,

At any given x,

At $y = 0 \Rightarrow u = 0$

At $y = \delta \Rightarrow u = 0.99\, V_\infty$

At $y = \delta \Rightarrow \dfrac{\partial u}{\partial y} = 0$

At $y = 0 \Rightarrow \dfrac{\partial^2 u}{\partial y^2} = 0$

Also, at any given x,

$\left(\dfrac{\partial u}{\partial y}\right) = f(y)$

with maxm. value of $\dfrac{\partial u}{\partial y}$ at $y = 0$

$\delta = f(x)$

26/9/2016

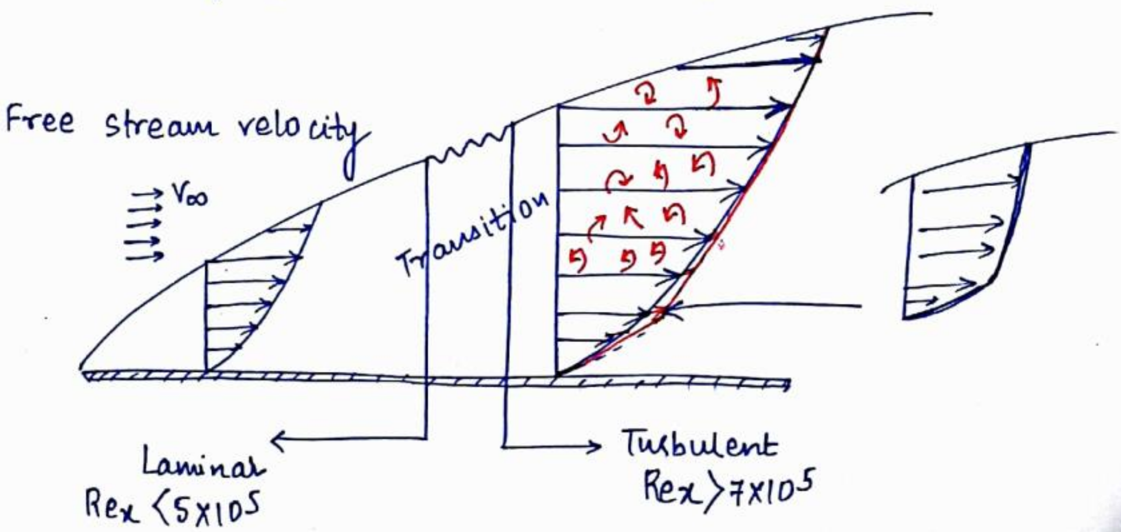
Let local Reynold's No. $= \boxed{Re_x = \left(\dfrac{V_\infty x \rho}{\mu}\right) = \dfrac{V_\infty x}{\nu}}$.

The flow over flat plate is laminar,

If $\boxed{Re_x < 5 \times 10^5}$

The flow over flat plate is Turbulent,

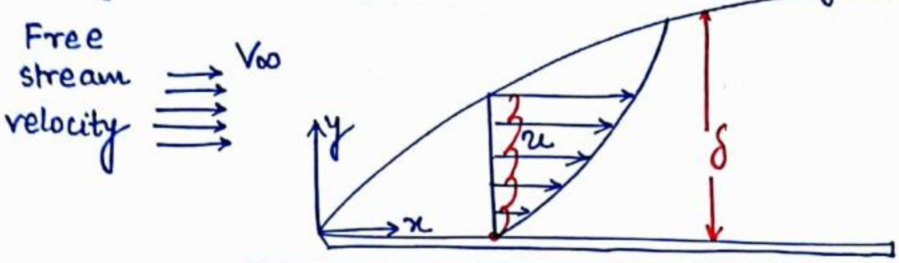
if $\boxed{Re_x > (6.5 \text{ to } 7 \times 10^5)}$



NOTE :- Most commonly we come across laminar flows during flow over flat plates and turbulent flows during flow through pipes or Ducts.

for laminar flow over flat plate, the velocity distribution within the hydrodynamic Boundary layer is given as :-

at any given x, measured from leading edge of plate,



$$\boxed{\frac{u}{V_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3}$$

where δ = HBL Thickness

δ_h // $= f(x)$

$$\boxed{\delta = \frac{4.64x}{\sqrt{Re_x}}} \quad \text{(OR)} \quad \boxed{\frac{5.0x}{\sqrt{Re_x}}}$$

(Blausiu's solution)

$$\boxed{Re_x = \left(\frac{V_\infty x}{\nu}\right)}$$

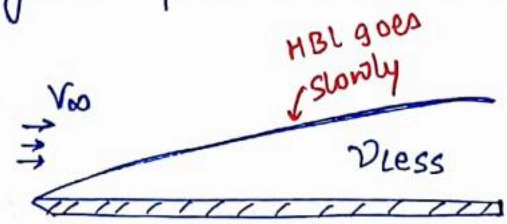
Now, $\delta = \dfrac{5.0x}{\sqrt{\dfrac{V_\infty x}{\nu}}}$ ← kinematic viscosity (property of fluid)

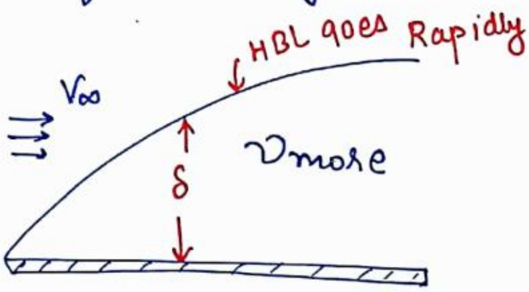
$\Rightarrow \boxed{\delta \propto x^{1/2}}$

we may also conclude that,

$\Rightarrow \boxed{\delta \propto \sqrt{\nu}}$

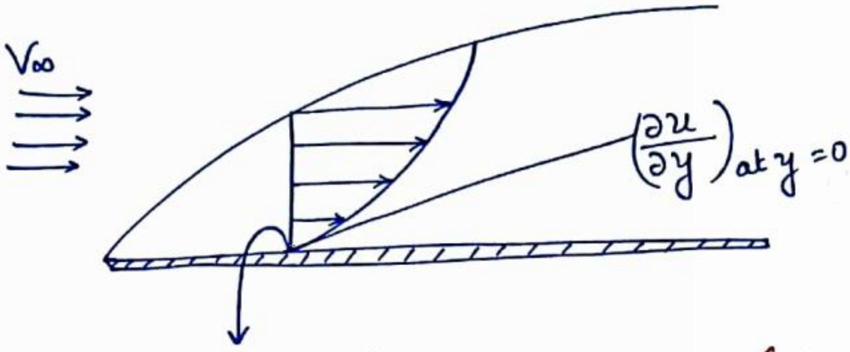
Note :- Therefore a fluid having more kinematic viscosity shall form thicker HBL as compared to the fluid having lesser K.V. for a given free stream velocity.





K.V. (ν) ⟶ Momentum diffusivity

Newton's law of viscosity for flow over flat plate :-



τ_ω = local wall shear stress (at any given x)

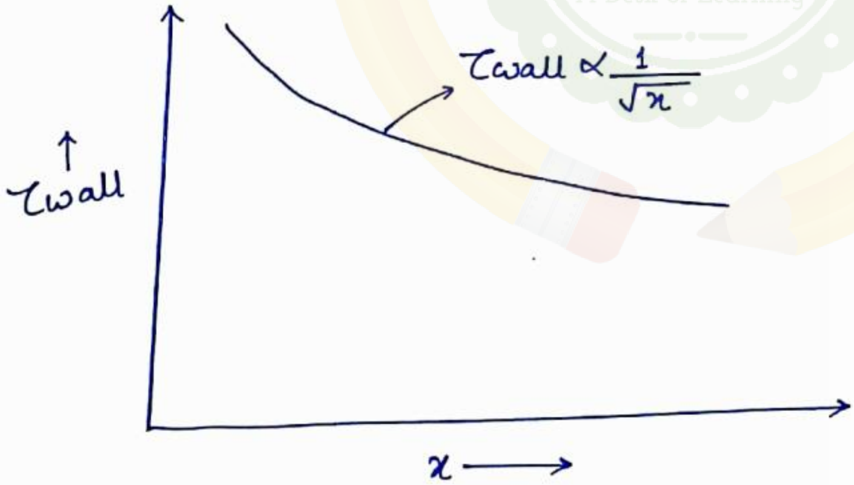
$$= \mu\left(\frac{\partial u}{\partial y}\right)_{at\ y=0} = \left(\mu V_\infty \frac{3}{2\delta}\right) \text{Pascal}$$

{after differentiating
$$\frac{u}{V_\infty} = \frac{3}{2}(y/\delta) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$$
and put $y=0$ }

$$\boxed{\tau_\omega = \left(\mu V_\infty \frac{3}{2\delta}\right) \text{Pascal}}$$

since $\delta \propto x^{1/2}$

$$\Rightarrow \boxed{\tau_{wall} \propto x^{-1/2}}$$



Note :- This local wall shear stress physically indicates the frictional drag (or) the drag resistance exerted by the fluid on the flat plate.

$$\boxed{\underset{\text{(average shear stress)}}{\overline{\tau}} = 2 \times \left(\tau_{wall}\right)_{at\ x=L}}$$

$$\boxed{\text{Drag force on Plate} = \tau \times \text{Area of Plate}}$$

② **Nusselt No. (Nu)** :-

$$Nu = \frac{hD}{K_{fluid}} = \frac{(D/KA)}{\left(\frac{1}{hA}\right)}$$

Nu resembles Biot No.

$$\frac{hD}{K_{fluid}} \qquad \frac{hs}{K_{solid}}$$

$$Nu = \frac{hD}{K_{fluid}} = \frac{(D/KA)}{(1/hA)} = \frac{\text{Conduction Resistance offered by fluid if it were stationary}}{\text{surface convective Resistance}}$$

since fluids being bad conductors of heat,

Numerator > Denominator

$\therefore$ $\boxed{\text{Nu is always} > 1.}$

Nu is also called as dimensionless H.T. coefficient.

③ **Prandtl No. (Pr)** :- Prandtl is the only dimensionless no. which is the property of a fluid defined as the ratio between kinematic viscosity of the fluid and its thermal diffusivity.

$$Pr = \frac{K.V.}{T.D.} = \left(\frac{\nu}{\alpha}\right) = \frac{\frac{\mu}{\rho}}{\frac{K}{\rho C_p}} = \left(\frac{\mu C_p}{K}\right)$$

$\mu \rightarrow$ Pa-sec

$C_p \rightarrow$ J/kgK

$K \rightarrow$ W/mK

Pr of air → 0.65 to 0.73

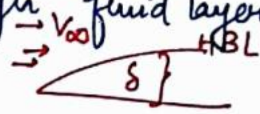
Pr of water → 2 to 6

Pr of liquid Metals like Hg is very low since their 'K' value is very high.

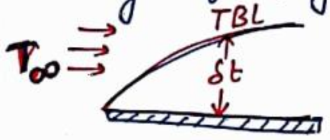
For lubricating oils (engine oils),

Pr may go upto 100.

(ν) → signifies Momentum Diffusivity Rate through fluid layers.
(K.V.)

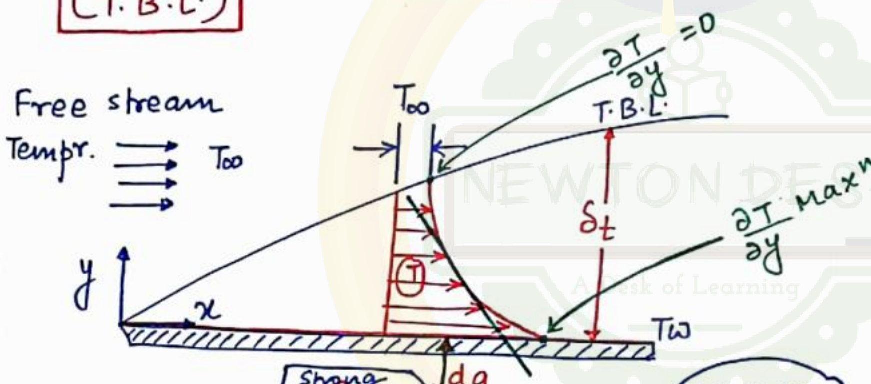


α (T.D.) → signifies Heat Energy Diffusion Rate through fluid layers.



Prandtl No. signifies the relative magnitudes of Momentum diffusion Rate and heat Energy diffusion Rate that are occuring through the fluid layers in the normal direction to the plate simultaneously in the Respected Boundary layers i.e. H.B.L. and T.B.L.
Thermal Boundary layer.

* THERMAL BOUNDARY LAYER (T.B.L.) -



$T = f(x, y)$

$T_w > T_\infty$

δ_t = thickness of T.B.L. = $f(x)$.

The thermal Energy transported by Moving fluid = $\dot{m} C_p T$.

The actual mechanism of Heat transfer in any convection is first heat gets conducted through the stagnant fluid layer at $y = 0$ and then while conducting through the fluid layers in the normal direction to the plate (y-direction). Heat is also simultaneously transported / convected away by the ~~[illegible]~~ moving fluid layers entering & leaving from the Boundary layer in the form of $\dot{m} c_p T$ (J/sec)

Defn. of T.B.L. :- Just similar to H.B.L. inside which velocity gradients are seen in the normal dirn. to the plate, Thermal Boundary level is also a thin Region inside which tempr. gradients are present in the normal dirn. to the plate. These Tempr. gradients are formed due to the heat transfer b/w the plate & flowing fluid.

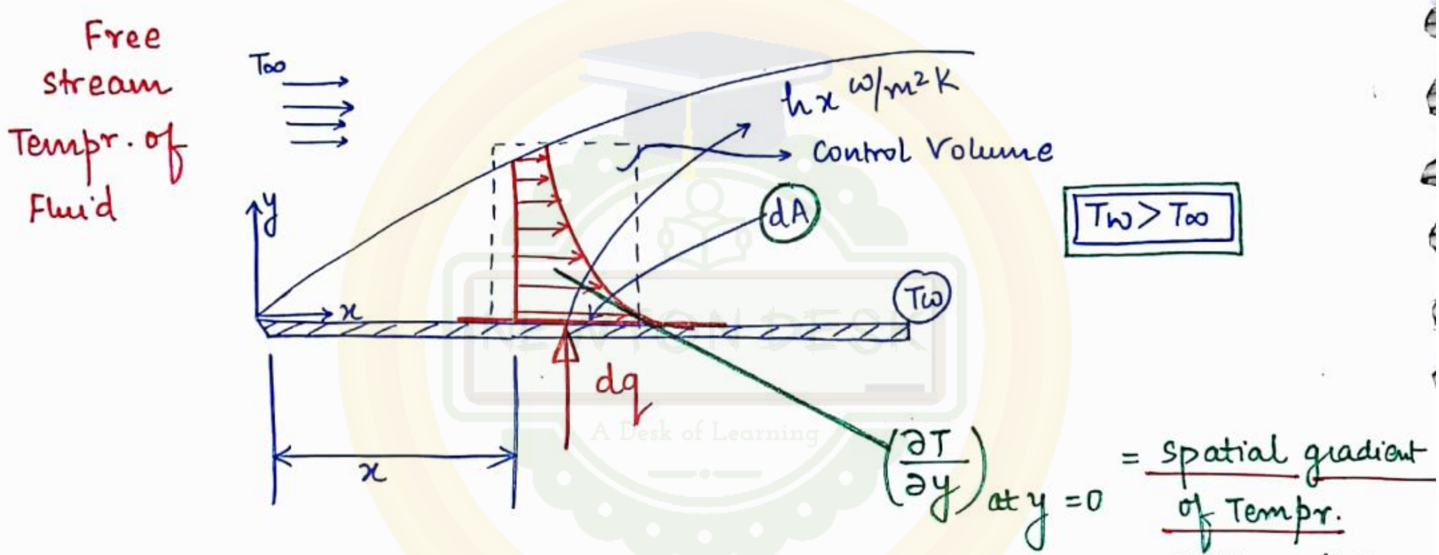
The Boundary conditions of T.B.L. are :-

At any x, measured from leading edge of plate,

$$\begin{bmatrix} \text{At } y=0 \Rightarrow T = T_w \\ \text{At } y=\delta_t \Rightarrow T = T_\infty \\ \text{At } y=\delta_t \Rightarrow \frac{\partial T}{\partial y} = 0 \\ \text{At } y=0 \Rightarrow \frac{\partial^2 T}{\partial y^2} = 0 \end{bmatrix}$$

at any given x, $\frac{\partial T}{\partial y} = f(y)$ with maximum value of $\frac{\partial T}{\partial y}$ at $y=0$.

$\boxed{\text{Energy Balance for a differential Control volume of T.B.L. are}}$:-



Consider a differential control volume of Area 'dA' at a distance of 'x' measured from the leading edge of the plate. The C.V. is extending from the plate upto the edge of the Boundary layer.

Writting the energy Balance for steady state conditions of C.V.,

Heat conducted into the C.V. through stagnant fluid layer at $y=0$ = Heat convected from the hot plate at T_w to free stream fluid at T_∞.

Fourier's law of Conduction | Newton's law of cooling

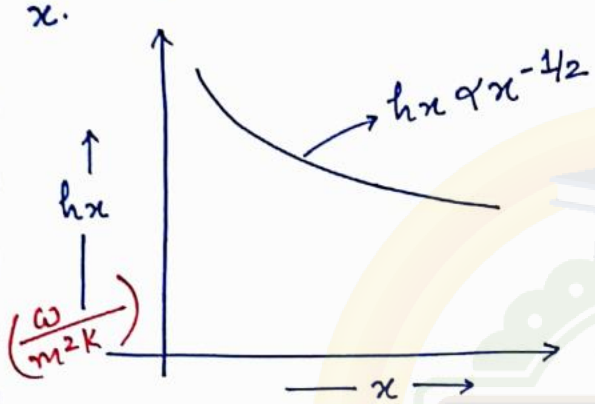
$$-k_f \cancel{dA} \left(\frac{\partial T}{\partial y}\right)_{\text{at } y=0} = h_x \cancel{dA} (T_w - T_\infty)$$

$$\Rightarrow \boxed{h_x = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{at\ y=0}}{(T_w - T_\infty)} \quad \frac{watt}{m^2\text{-}K}}$$

$\left(\frac{T}{y}\right) f^n \rightarrow$ find करो

local convective heat transfer coefficient. (H.T.)

Since the spatial gradient of Tempr. within the fluid at $y=0$ keeps on decreasing with increase of x, the local convective H.T. coefficient 'h_x' also must be decreasing with increase of x.



Gate 2009

Q:> A coolant fluid at 30°C flows over an isothermal flat plate which is maintained at a Tempr. of 100°C. The Boundary layer Tempr. distribution within the fluid at some location of x measured from the leading edge is $T = 30 + 70\, e^{(-y)}$ where y is the distance measured in the normal dirn. to plate. If Thermal conductivity of fluid is 1W/mk. The local convective h.T. coefficient h_x at that location will be ______?

(a) 10 W/m²K

(b) 5 W/m²K

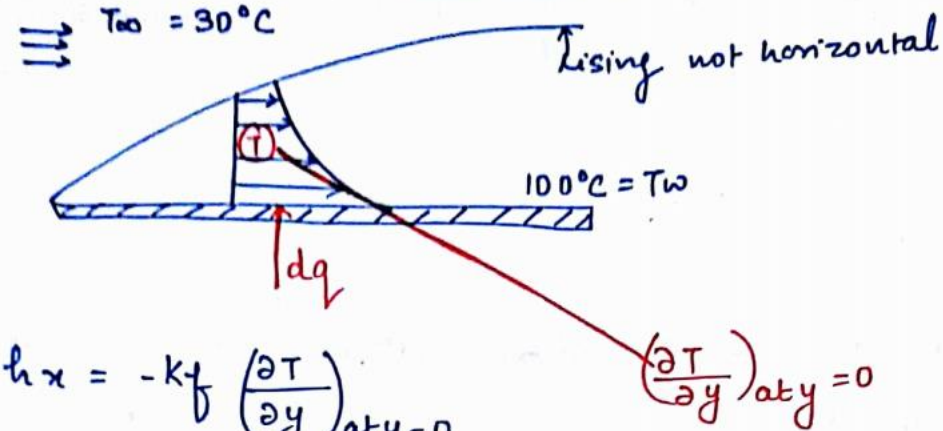
(c) 1 W/m²K

(d) 2 W/m²K

Sol:> $\frac{\partial T}{\partial y} = 70 + 70 e^{-y}(-1)$

(1) ✓

SIR



$$\therefore h_x = \frac{-k_f \left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{T_w - T_\infty}$$

$$h_x = \frac{-1 \times \frac{\partial}{\partial y}\left(30 + 70e^{-y}\right)_{at\, y=0}}{(100 - 30)}$$

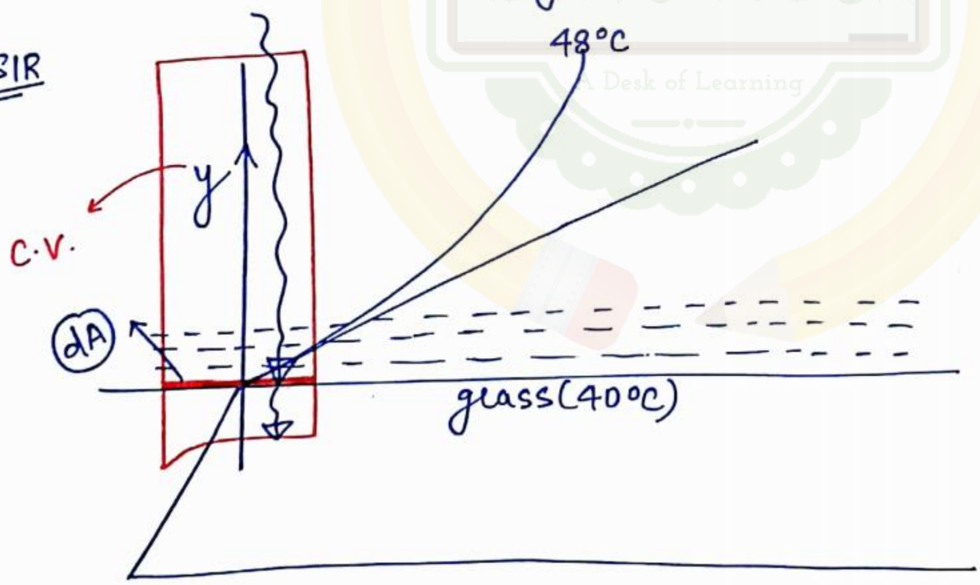
$$h_x = 1 W/m^2K$$

Pg 84
(4) (5) (4) $T_\infty = 48°C$

$T_w = 40°C$ $K_w = 0.6\ W/mK$ $\frac{dT}{dy} = 1 \times 10^4 \frac{K}{m}$

$K_g = 1.2 W/mK$

SIR



Consider a differential Control Volume of Area 'dA' which is extending from the free stream water at 48°C and well into the glass plate.

Writing the energy Balance for steady state conditions of differential C.V.,

Heat ~~conducted~~ convected from the free stream water at 48°C to stagnant water layer at 40°C } = Heat Conducted through stagnant water layer at Interface } = Heat Conducted through glass plate

Newton's   Fourier's   Fourier's

$$\Rightarrow h\cancel{dA}(48-40) = K_w\cancel{dA}\left(\frac{dT}{dy}\right)_{\text{within water at interface}}$$

$$= K_{glass}\cancel{dA}\left(\frac{dT}{dy}\right)_{\text{within glass at Interface}}$$

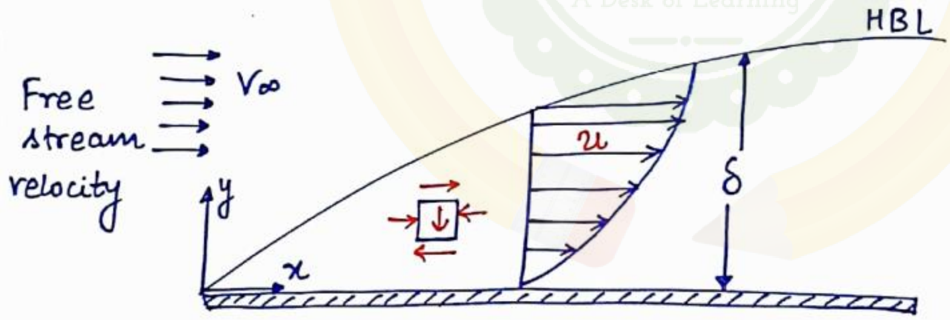
$$\Rightarrow h = \frac{0.6\times1\times10^4}{(48-40)}$$

$$= 750\ W/m^2K$$

$$\left(\frac{dT}{dy}\right)_{\text{within glass at interface}} = \frac{0.6\times(1\times10^4)}{1.2}$$

$$= 0.5\times10^4\ K/\text{metre}$$

* $\boxed{\text{Momentum Equation of H.B.L.}}$:- $\boxed{\text{(F.M.)}}$



Assume :- ①

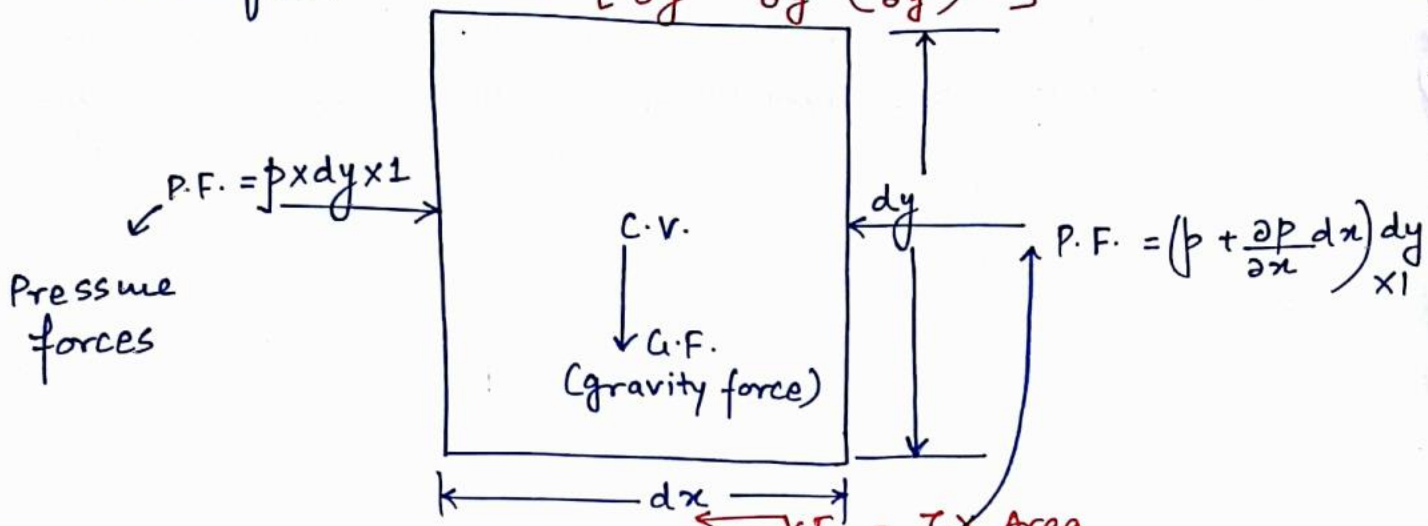
Steady, 2D, Incompressible flow ($\rho = C$)

$u = f(x,y)$ $v = f(x,y)$

② Unit depth of element (⊥r to Plane of fig.)

③ Real fluid flow.

viscous force $\leftarrow \vec{V.F.} = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)dy\right](dx \times 1)$



$V.F. = \tau \times Area$

$P.F. = \left\{p + \left(\frac{\partial P}{\partial x}\right)dx\right) dy \times 1 \qquad = \mu \times \left(\frac{\partial u}{\partial y}\right) \times (dx \times 1).$

Newton's IInd Law of Motion :-

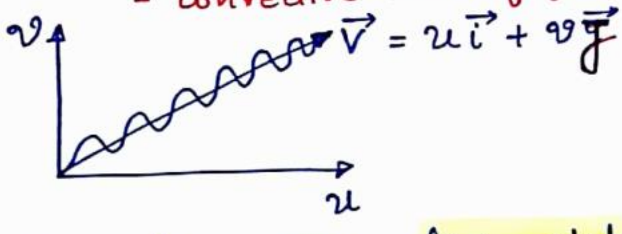
$\overset{\oplus}{\overrightarrow{\Sigma F_x}} = m \overset{\oplus}{\overrightarrow{a_x}}$ (Conservation of linear Momentum)

Net algebraic sum of all the forces acting on fluid element along x- direction = Rate of change of linear momentum of element along x- direction.

The Resulting Momentum Eqn. of H.B.L. is :-

$$\boxed{\underbrace{\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right)}_{\text{convective accln. of fluid along } x} = \underbrace{\left(\frac{\mu}{\rho}\right)}_{\text{K.V. (Kinematic viscosity)}}\frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho}\left(\frac{dP}{dx}\right)}$$

Navier- stokes eqn. of Motion along x.



For flow over flat plates,

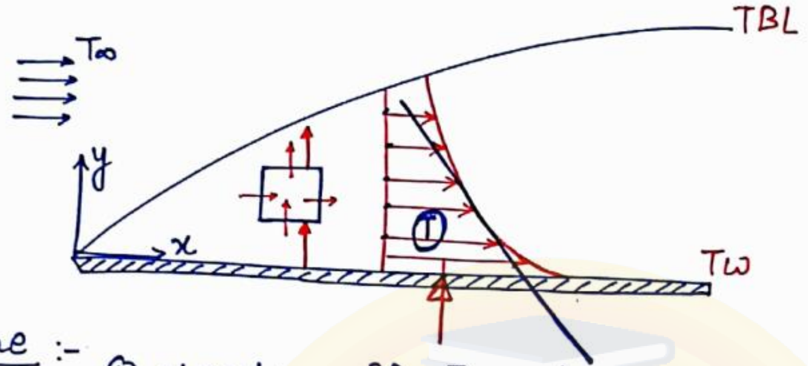
$$\boxed{\frac{dP}{dx} = 0}$$

and Put $\frac{\mu}{\rho} = \nu$ = K.V. of fluid m^2/sec.

$$\Rightarrow \boxed{u\left(\frac{\partial u}{\partial x}\right) + v\frac{\partial u}{\partial y} = \underset{K.V.}{\nu} \cdot \frac{\partial^2 u}{\partial y^2}}$$

↳ Momentum equation of H.B.L.

* **ENERGY EQUATION OF T.B.L. :-**



Assume :-

1. steady, 2D, Incompressible flow ($\rho = c$)
   $u = f(x,y)$ $v = f(x,y)$ $T = f(x,y)$ (strong, weak)
2. Constant fluid properties (ρ, μ, c_p, k)
3. Unit depth of element ($\perp$ ar to plane of fig.)
4. Negligible conduction along x- direction.

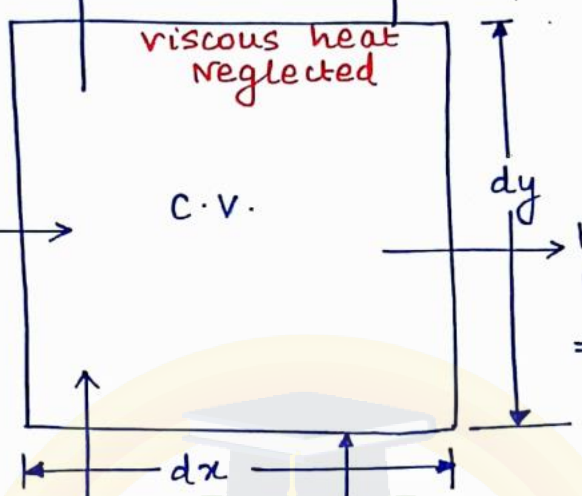
The Thermal energy transported / convected by flowing fluid = $\dot{m} c_p T$ J/sec

Heat convected through Top face = $\boxed{\rho(dx\times 1)\left(v+\frac{\partial v}{\partial y}\cdot dy\right)C_p\left(T+\frac{\partial T}{\partial y}\cdot dy\right)}$

Heat conducted through Top face

$= \boxed{-k_f(dx\times 1)\left[\frac{\partial T}{\partial y}+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)dy\right]}$

$\boxed{\text{Six Energies}}$



Heat convected through Left face = $\boxed{\rho(dy\times 1)uC_pT}$

Heat convected through Right face

$= \boxed{\rho(dy\times 1)\left[u+\frac{\partial u}{\partial x}\cdot dx\right]C_p\left[T+\frac{\partial T}{\partial x}\cdot dx\right]}$

Heat convected through bottom face = $\boxed{\rho(dx\times 1)vC_pT}$

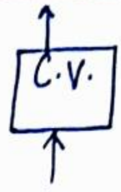
Heat conducted through Bottom face = $\boxed{-k_f(dx\times 1)\frac{\partial T}{\partial y}}$

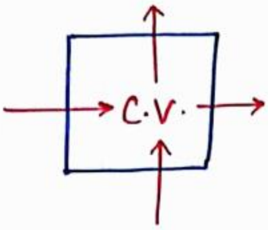
Writting the Energy Balance for steady state conditions of C.V. we get,

$$\left.\begin{array}{l}\text{Heat Conducted through Bottom face}\\ \quad + \\ \text{Heat Convected through Left face}\\ \quad + \\ \text{Heat Convected through Bottom face}\end{array}\right\} = \left\{\begin{array}{l}\text{Heat conducted thro. Top face}\\ \quad + \\ \text{Heat convected thro. Right face}\\ + \text{ Heat convected through Top face}\end{array}\right.$$

The Resulting energy eqn. of T.B.L. is :-

$$\underbrace{\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)}_{\text{Net heat convected to C.V.}} = \underbrace{\left(\frac{K}{\rho C_p}\right)\frac{\partial^2 T}{\partial y^2}}_{\text{Net heat conducted out of C.V.}}$$





But $\frac{K}{\rho C_p} = \alpha =$ T.D. of fluid (m^2/sec)

$$\underbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}}_{\text{(Tempr. field)}}$$ Energy eqn. of T.B.L.

Momentum eqn. of HBL :- $$\underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}}_{\text{(flow field)}}$$

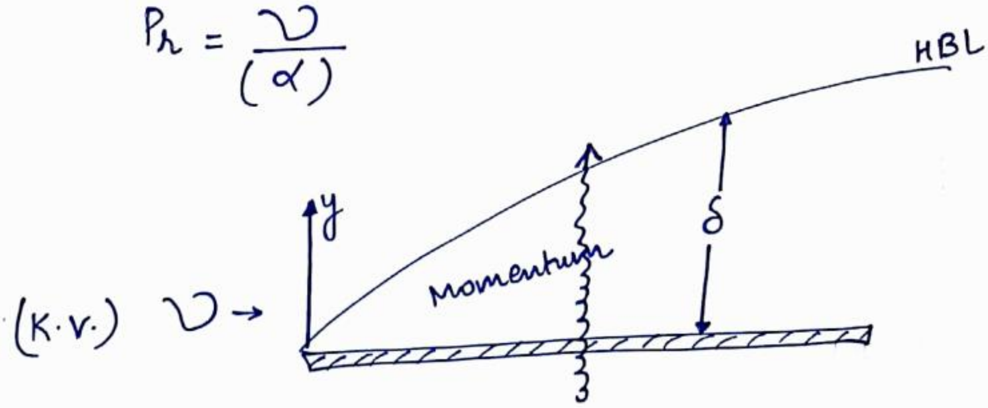
There are 2 different Equations of H.B.L :-

There is a striking similarity b/w the momentum eqn. of H.B.L. and the Energy eqn. of T.B.L. The solution to both these second order Differential Equations would exactly be the same if K.V. (Kinematic Viscosity) ν of the fluid is equal to its thermal diffusivity 'α'

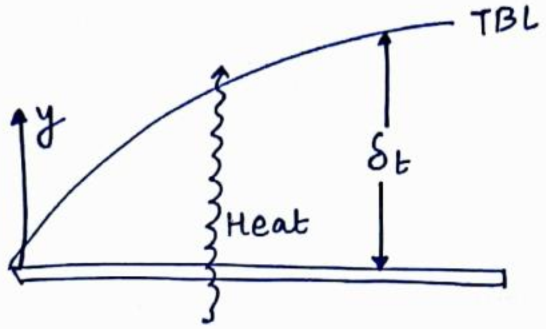
* Significance of Prandtl No :- (Physical)

$$P_r = \frac{\nu}{(\alpha)}$$

$\delta \propto \sqrt{\nu}$



(T.D.) α →



If kinematic viscosity of the fluid is more, then viscous influence of the fluid is felt farther away into free stream thereby making the value of δ relatively more.

If the thermal diffusivity 'α' is more then the heating affects of the hot plate is felt farther away into the free stream thereby making the value of δ_t relatively more.

Thus prandtl No., a ratio b/w ν and α can tell about the relative magnitudes of HBL thickness δ and TBL thickness δ_t at a given location of 'x' measured from the leading edge of plate. → (or) same.

If $Pr > 1$	If $Pr < 1$	If $Pr \simeq 1$
$\Rightarrow \nu > \alpha$	$\Rightarrow \nu < \alpha$	$\Rightarrow \nu \simeq \alpha$
$\Rightarrow \delta > \delta_t$	$\Rightarrow \delta < \delta_t$	$\Rightarrow \delta \simeq \delta_t$
$\Rightarrow \frac{\delta_t}{\delta} < 1$	$\Rightarrow \frac{\delta_t}{\delta} > 1$	$\Rightarrow \frac{\delta_t}{\delta} \simeq 1$

for a given fluid,

$$\boxed{\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}}$$

Pr is a property of fluid.

(2) d ✓

(3) b ✓

(6) $Pr = \frac{\mu C_p}{K} = \frac{0.001 \times 1000}{1}$

$\Rightarrow \delta = \delta_t = 1mm$

(22) a

(20) d

(24) for liquid Metals (Hg)

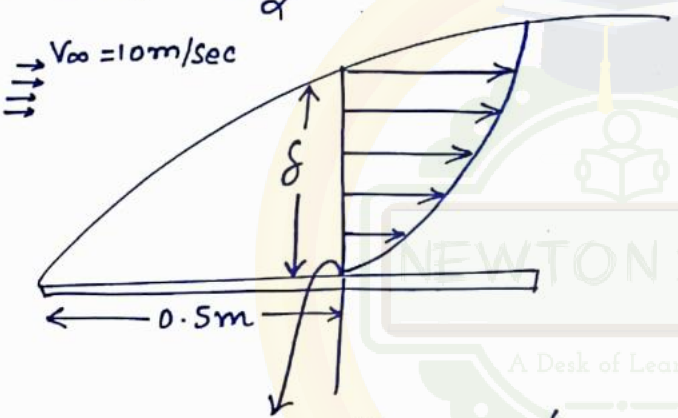
Pr is very low

$\nu <<< \alpha$

(b) ✓ $\Rightarrow \delta <<< \delta_t$

(30) $Pr = 1 = \frac{\nu}{\alpha}$

31R



$$Re_x = \frac{V_\infty x}{\nu} = \left(\frac{10 \times 0.5}{30 \times 10^{-6}}\right) = 1.6 \times 10^5$$

$$\delta_{at\ x = 0.5m} = \frac{5.0x}{\sqrt{Re_x}} = \frac{5 \times 0.5}{\sqrt{1.6 \times 10^5}} = 6.123mm$$

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} P_r^{-1/3}$$

$$(\delta_t)_{at\ x = 0.5m} = 5.96\ mm$$ Ans

continue

By $\int$ing the Energy Eqn. of T.B.L. that is

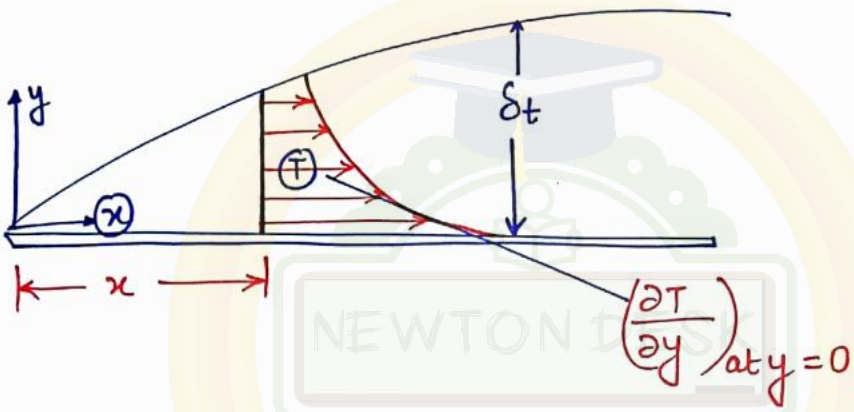
i.e. $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

with the help of its boundary conditions, we get the Tempr. distribution within the T.B.L. as :-

At any given x, measured from leading Edge of plate,

$$\boxed{\left(\frac{T - T_w}{T_\infty - T_w}\right) = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3}$$

where $\delta_t = f(x)$



hence, $$\left(\frac{\partial T}{\partial y}\right)_{at\, y=0} = (T_\infty - T_w)\frac{3}{(2 \cdot \delta_t)}$$

$$h_x = \text{local conv. H.T. coefficient} = \frac{-k_f\left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{T_w - T_\infty}$$

$$h_x = \frac{-k_f\,(T_\infty - T_w)}{(T_w - T_\infty)}\frac{3}{2\delta_t}$$

$$\boxed{h_x = \left(\frac{3k}{2\delta_t}\right) \; W/m^2K}$$

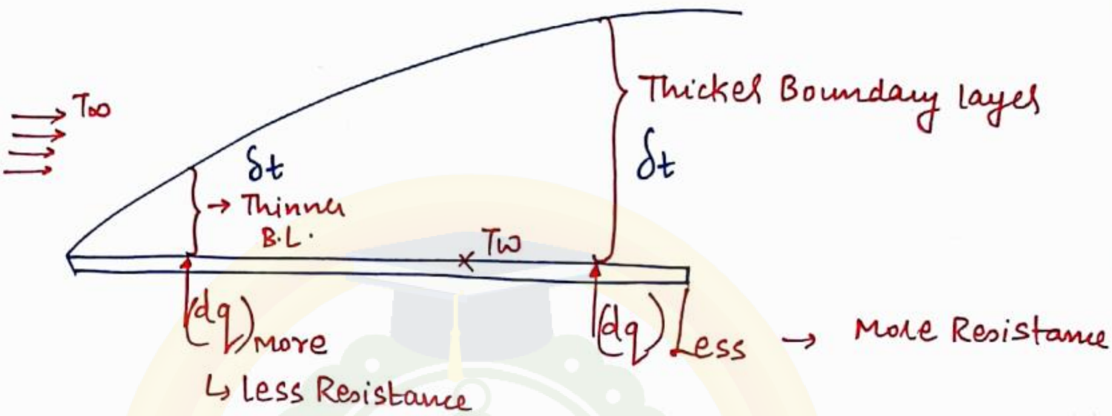
$$\delta \propto x^{1/2}$$

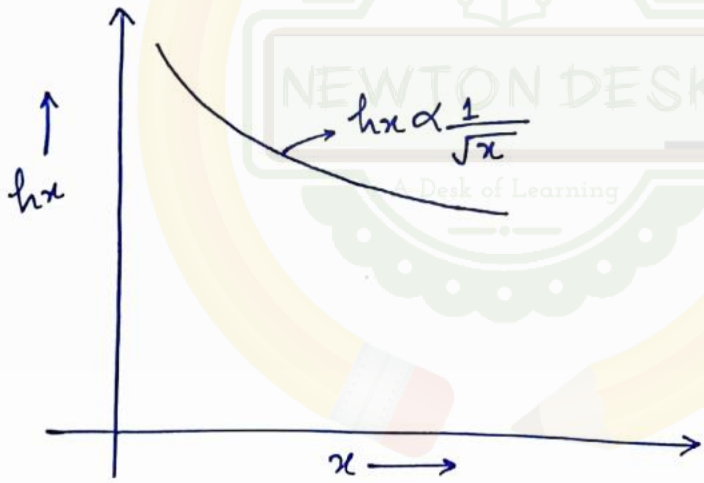
But $\dfrac{\delta_t}{\delta} \neq f(x)$ → a property

since $\dfrac{\delta_t}{\delta} = f(Pr)$

$Pr \neq f(x)$

$$\Rightarrow \boxed{\delta_t \propto x^{1/2}} \Rightarrow \boxed{h_x \propto x^{-1/2}}$$





h_x decreases with ↑ of x because the thicker Boundary layer at a greater value of x shall offer more thermal Resistance against the heat flow between the hot plate and the free stream fluid at T_∞.

$h_x \rightarrow K, \delta_t \rightarrow K, \delta, Pr \longrightarrow K, x, Re_x, Pr$

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$

$$\delta = \frac{4.64x}{\sqrt{Re_x}}$$

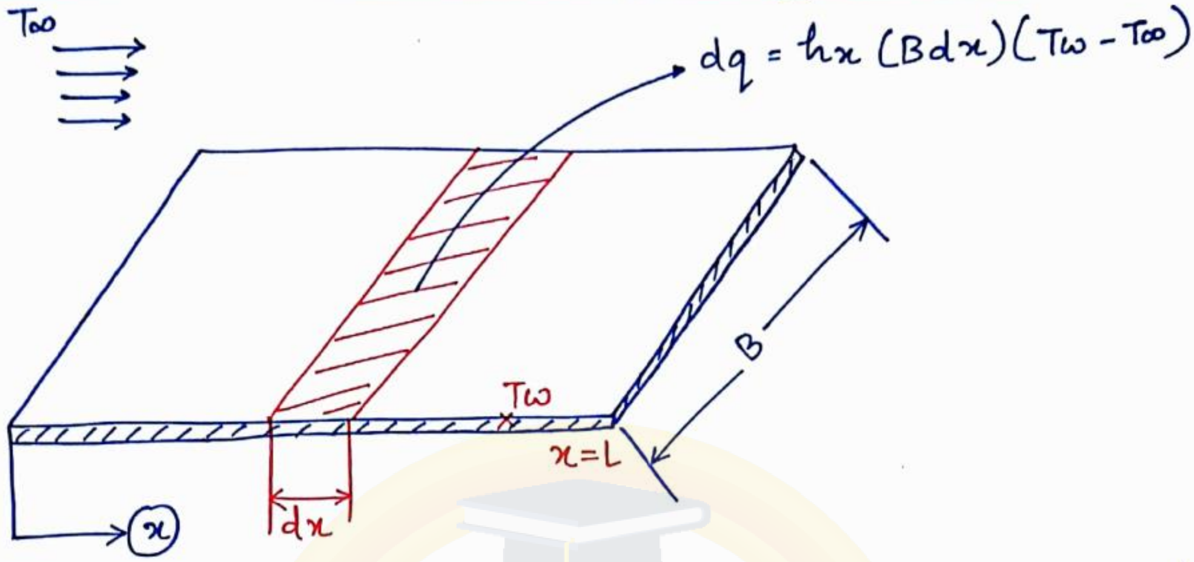
Let local Nusselt No. $= Nu_x = \left(\dfrac{h_x x}{K}\right)$

∴ The local convective H.T. coeff. h_x for Laminar Boundary layer over flat plate can be obtained from:-

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2} \cdot Pr^{1/3}$$

Local Nusselt's No.

* **AVERAGE Convective Heat Transfer coefficient ($\bar{h}$)** :→



Consider a small differential strip of the plate of length 'dx' where the differential heat transfer rate b/w the strip and fluid is dq then

$$dq = h_x B (T_w - T_\infty) dx$$

$$\int_{x=0}^{L} dq = \int_{x=0}^{L} h_x B (T_w - T_\infty) dx$$

⇒ Total H.T. Rate from entire plate $= Q = \int_{x=0}^{L} h_x B (T_w - T_\infty)\, dx$ ⟶ 1

But Interms of $\bar{h}$, Total H.T. Rate between entire plate and fluid $= Q = \bar{h} \times (B \times L)\, T_w - T_\infty$ ⟶ 2

equating 1 & 2, we get

$$\boxed{\bar{h} = \frac{1}{L}\int_{x=0}^{L} h_x\, dx}$$

we know $h_x \propto x^{-1/2}$

$$\Rightarrow h_x = c x^{-1/2}$$

put $x = L$ on both sides

$$h_{x=L} = c L^{-1/2} \Rightarrow c = \frac{h_{x=L}}{L^{-1/2}}$$

(local convective H.T. coefficient at $x=L$) {i.e. trailing edge}

$$\bar{h} = \frac{1}{L}\int_{x=0}^{L} c x^{-1/2}\,dx = \frac{1}{L} c \left[\frac{x^{-1/2+1}}{-1/2+1}\right]_{x=0}^{L}$$

$$\Rightarrow \bar{h} = \frac{1}{L} \times \frac{h_{x=L}}{L^{-1/2}} \left[\frac{L^{1/2}}{1/2}\right]$$

$$\Rightarrow \boxed{\bar{h} = 2(h_{x=L})\ W/m^2 K}$$

27/9/16

Hence the average convective H.T. coefficient for the entire plate will be equal to twice the local convective H.T. coefficient at the trailing edge (i.e. $x = L$)

we know that

$$\frac{h_x x}{k} = \text{local Nu. No.} = Nu_x = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$

Put $x = L$ on both sides

$$\frac{h_{x=L} \times L}{k} = 0.332\, Re_L^{1/2}\, Pr^{1/3}$$

where $Re_L = \dfrac{V_\infty L \rho}{\mu}$

local Reynolds No. at Trailing edge.

$$\Rightarrow \frac{2h_{x=L} \times L}{k} = 0.664\, Re_L^{1/2}\, Pr^{1/3}.$$

$$\Rightarrow \boxed{\frac{\bar{h} L}{K} = \overline{Nu} = \text{Average Nu. No.} = 0.664\, Re_L^{1/2}\, Pr^{1/3}}$$

Also $\boxed{\overline{Nu} = 2(Nu_x)_{at\ x=L}}$

Pg 85
Q(11)

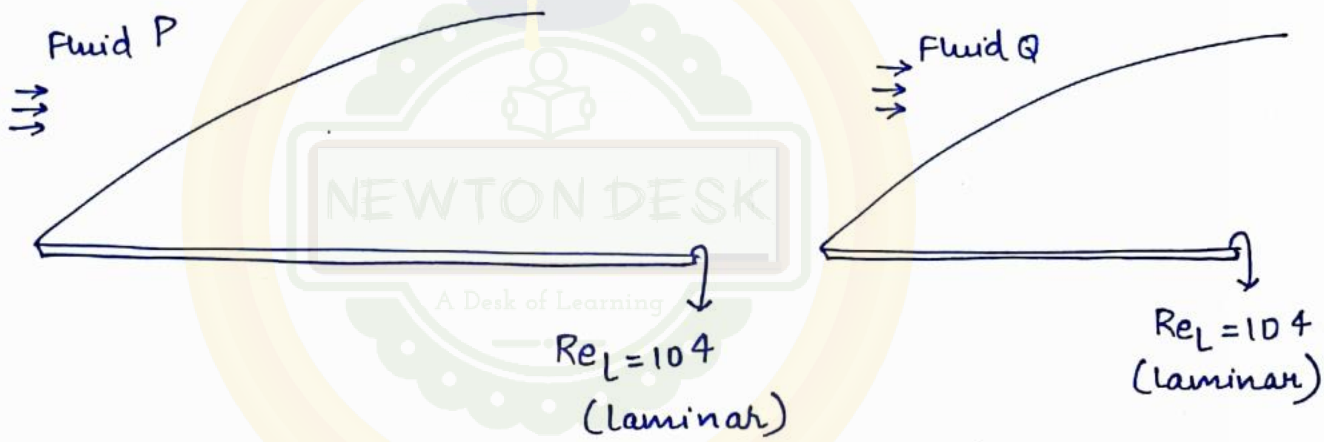
$\left(\frac{\delta}{\delta_t}\right)_P = 1/2 \quad \left(\frac{\delta}{\delta_t}\right)_Q = 2 \quad \underset{Both}{Re} = 10^4 \quad Pr_P = 1/8 \quad Nu_P = 35$

$(Pr)_Q = ? \quad (Nu)_Q = ?$

$$\left(\frac{\delta_t}{\delta}\right) = \frac{1}{1.026} Pr^{1/3}$$

$$\frac{1}{2} = \frac{1}{1.026} Pr^{1/3}$$

SIR



$$\left(\frac{\delta_t}{\delta}\right)_P = 2 \qquad \left(\frac{\delta_t}{\delta}\right)_Q = 1/2$$

$$Pr_P = 1/8 \qquad Pr_Q = ?$$

we know that,

$$\frac{\delta_t}{\delta} = \frac{1}{1.026} Pr^{-1/3}$$

For fluid P,

$$2 = \left(\frac{\delta_t}{\delta}\right)_P = \frac{1}{1.026}(Pr_P)^{-1/3} \longrightarrow (1)$$

For Q, $\frac{1}{2} = \left(\frac{\delta t}{\delta}\right)_Q = \frac{1}{1.026}(Pr_Q)^{-1/3} \longrightarrow$ ②

$$\frac{②}{①} \Rightarrow \frac{1}{2\times2} = \frac{(Pr_Q)^{-1/3}}{(Pr_P)}$$

$$\Rightarrow \boxed{Pr_Q = 8}$$

$\overline{Nu}_P = 0.664\ Re_L^{1/2}\ Pr_P^{1/3} = 35 \longrightarrow$ ③

$\overline{Nu}_Q = 0.664\ Re_L^{1/2}\ Pr_Q^{1/3} = ?$ — ④

$$\frac{④}{③} \Rightarrow \overline{Nu}_Q = 140.$$

(26) $h_x = a x^{-0.1}$

$$\left(\frac{\bar{h}}{h_x}\right)_x = ?$$

SIR $\frac{\bar{h}}{h_x} = \dfrac{1/x \int_0^x h_x\,dx}{h_x} = \dfrac{1/x \int_0^x \cancel{a} x^{-0.1}\,dx}{\cancel{a} x^{-0.1}} = 1.11$

(29) $\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta t}\right) - \frac{1}{2}\left(\frac{y}{\delta t}\right)^3$ $Nu_x = ?$

$\delta t \leftarrow$ TBL

SIR At any given x,

$$\left(\frac{\partial T}{\partial y}\right)_{at\ y=0} = (T_\infty - T_w)\frac{3}{2\delta t}$$

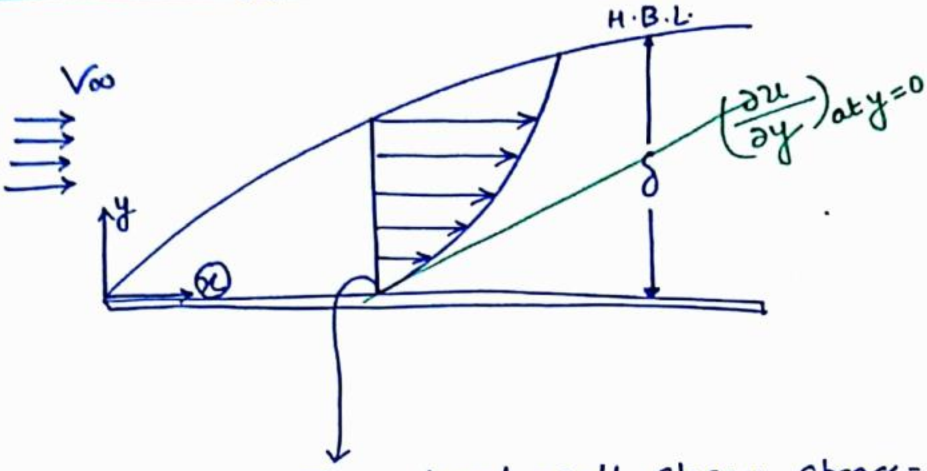
$$\therefore h_x = \frac{-k_f\left(\frac{\partial T}{\partial y}\right)_{at\ y=0}}{T_w - T_\infty} = \left(\frac{3K}{2\delta t}\right)$$

$\therefore$ local Nu. No. = $Nu_x = \left(\frac{h_x x}{K}\right) = \frac{3\cancel{K}}{2\delta t} \times \frac{x}{\cancel{K}} = \left(\frac{3x}{2\delta t}\right)$

Local Nu. No. based on T.B.L. thickness

(Put) $x = \delta t \Rightarrow \underset{\text{based on } \delta t}{(Nu_x)} = \frac{3\times\delta t}{2\times\delta t} = 1.5.$

* Analogy between Fluid friction and Heat Transfer (Reynold's-Colburn analogy) :-



τ_w = local wall shear stress $= \mu\left(\frac{\partial u}{\partial y}\right)_{at\, y=0}$

$$= \mu V_\infty \frac{3}{2\delta} = \mu V_\infty \frac{3}{2\times \frac{4.64x}{\sqrt{Re_x}}}$$

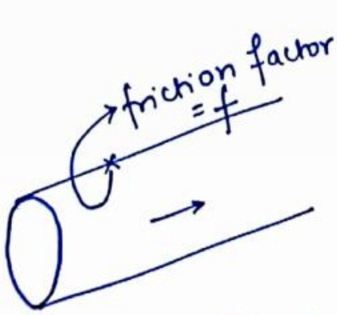
$$= \mu V_\infty \times \frac{3}{2\times \frac{4.64x}{\sqrt{\frac{V_\infty x \rho}{\mu}}}} \text{ Pascal} \quad \text{—— (1)}$$

In F.M., local wall shear stress =

$$\tau_{w_{all}} = C_{fx} \times \frac{\rho V_\infty^2}{2} \text{ Pa} \quad \text{—— (2)}$$

C_{fx} = local skin friction coefficient (OR) local drag coefficient.

$= 0.004$ (OR) 0.005 {a dimensionless parameter.}



Equating (1) & (2), we get

$$\boxed{0.332\, Re_x^{-1/2} = \frac{C_{fx}}{2}}$$

$$Nu_x = 0.332\, Re_x^{1/2} \cdot Pr^{1/3}$$

$$\Rightarrow \left(\frac{Nu_x}{Re_x Pr}\right) = \frac{0.332\, Re_x^{1/2} \cdot Pr^{1/3}}{Re_x Pr}$$

Stanton No. = $\boxed{St = \frac{Nu}{Re\, Pr}} = \frac{Nu}{Pe}$ [Pe → Peclet No]

$\therefore$ local stanton No. $= St_x = \frac{Nu_x}{Re_x Pr}$

$$\Rightarrow St_x = \frac{\frac{h_x \cancel{x}}{\cancel{k}}}{\frac{V_\infty \cancel{x} \rho}{\cancel{\mu}} \times \frac{\cancel{\mu} C_p}{\cancel{k}}}$$

$$\Rightarrow \boxed{St_x = \frac{h_x}{(V_\infty \rho C_p)} = \frac{1}{\eta_2}}$$

$$\Rightarrow \boxed{St_x = \frac{1}{\eta_2}}$$

The product of (Re Pr) is called as Peclet No. (Pe). significant in liquid Metal cooling of nuclear Reactors.

$$\therefore \boxed{St_x = \left(\frac{Nu}{Pe}\right)}$$

also, $St_x = \left(0.332\, Re_x^{-1/2}\right) Pr^{-2/3}$

$$\boxed{St_x\, Pr^{2/3} = \frac{C_{fx}}{2}}$$ → Reynold's Analogy

* PHYSICAL Significance of Reynold's Analogy :-

from this Reynolds analogy, we can predict the value of local convective H.T. coefficient h_x at a given location of x measured from the leading edge just by knowing the local skin friction coefficient C_{fx} at the same location of x even when there is no Heat Transfer b/w the plate and the flowing fluid.

Pg 87

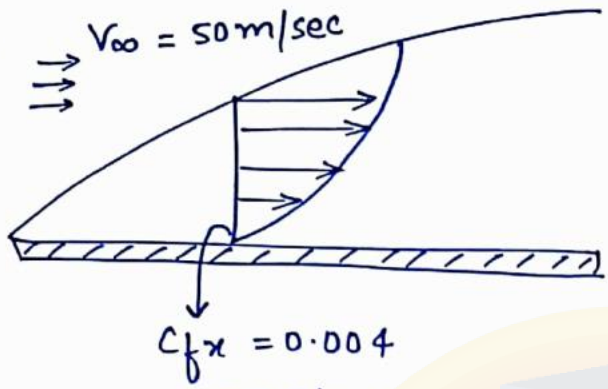
31 $V_\infty = 50 m/s$

$C_{fx} = 0.004$

$h_x = ?$

$$St_x = \frac{h_x}{\rho V_\infty C_p}$$

SIR



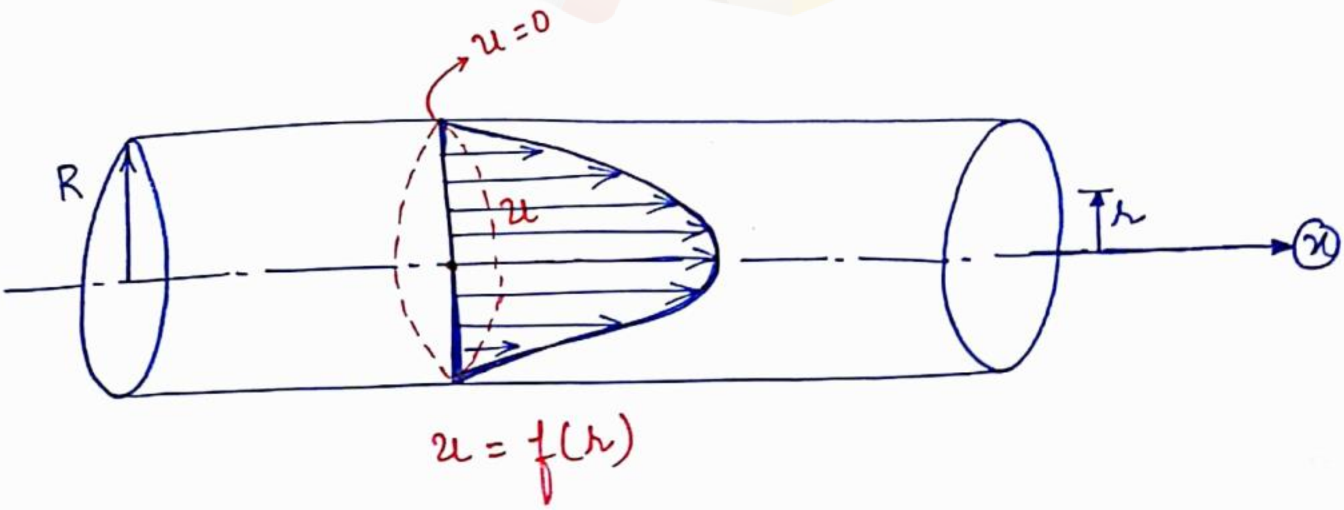
$$P_r = \frac{\mu C_p}{k} = 0.653$$ ($C_p \to$ J/kg K)

$$St_x P_r^{2/3} = \frac{C_{fx}}{2} = \frac{0.004}{2}$$

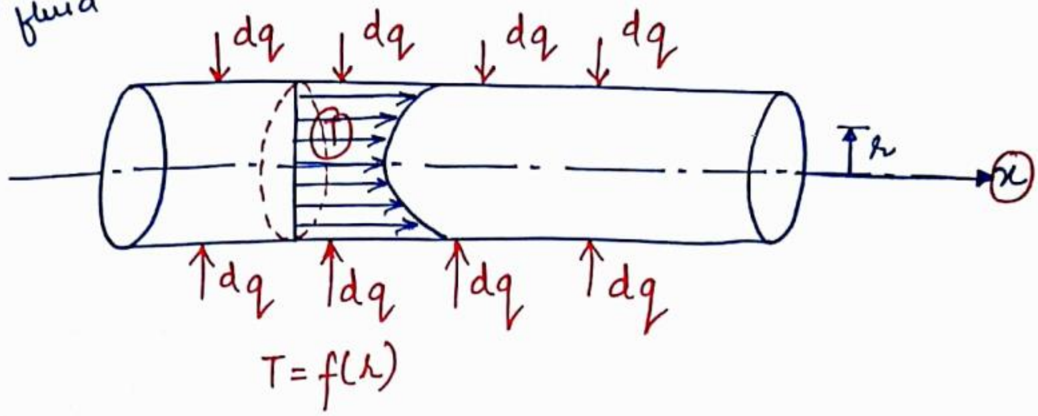
$$\Rightarrow St_x = \text{local stanton No.} = 2.65 \times 10^{-3} = \frac{h_x}{\rho V_\infty C_p} \Rightarrow h_x = 116.9 \text{ W/m}^2$$

($C_p \to$ J/kg K)

$\boxed{\text{FORCED CONVECTION in Flow through Pipes/Ducts}}$:-

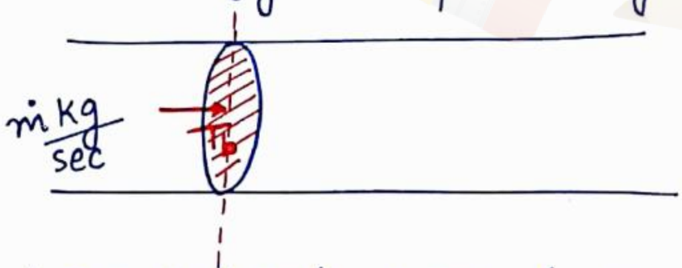


Mass flow rate of flowing fluid $\boxed{\dot{m} = \rho \times \pi R^2 V_{mean} \ kg/sec}$



Just like velocity of fluid layers being a function of 'r' (measured from the axis) at a given cross-section of the pipe whenever there is fluid flow happening in the pipe due to viscous influence of the fluid, whenever there is H.T. b/w the pipe boundary and the flowing fluid, the tempr. of fluid layers also become a function of r at a given cross-section.

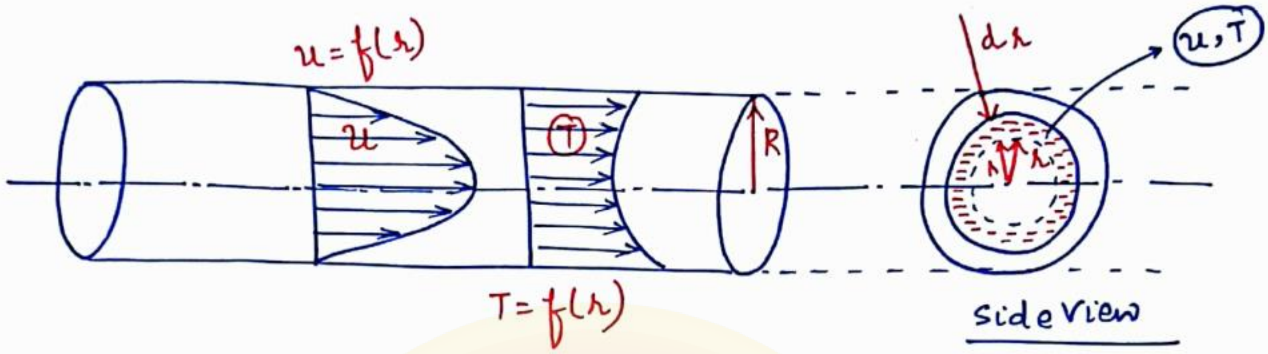
Tb (Bulk Mean Tempr. of Fluid) :- Tb of fluid at a given c/s of pipe is defined as the temperature which takes into account the variation of tempr. of fluid layers with respect to 'r' at that c/s of the pipe and thus indicates the total thermal Energy transported by the fluid through the c/s.



∴ The thermal energy/enthalpy transported by fluid through the c/s of the pipe $= \dot{m} C_p T_b = \rho \pi R^2 V_{mean} C_p T_b \ J/sec$

If there is any kind of H.T. b/w the pipe boundary and the flowing fluid. This T_b of fluid must change in the direction of fluid flow.

* $\boxed{\text{TO DERIVE an expression for } T_b \text{ of fluid}}$:-



Consider a given c/s of the pipe at which the velocity distribution and the tempr. distribution are as shown in the figure. Consider a differential elemental ring of fluid flow in the c/s of the pipe at a radius of 'r' measured from the axis. Let 'dr' be the differential radius of the elemental ring then

$d\dot{m}$ = Differential mass flow Rate of fluid through the elemental Ring = $\rho \times 2\pi r\, dr\, u$.

∴ Differential Thermal energy (or) enthalpy transported by fluid through elemental Ring = $d\dot{m}\, C_p T$ J/sec

$= \rho\, 2\pi r\, dr\, u\, C_p T$ J/sec

∴ Total Thermal Energy transported by fluid through entire cross-section of pipe $= \int_0^R \rho\, 2\pi r\, u\, C_p T\, dr$. ⟶ (1)

But Interms of T_b,

Total Thermal Energy transported by fluid through entire cross-section $= \rho \pi R^2 V_{mean} C_p T_b$. J/sec ⟶ (2)

Equating (1) & (2),

we get $\cancel{\rho}\cancel{\pi}R^2 V_{mean} \cdot \cancel{c_p} \cdot T_b = \int_0^R \cancel{\rho}\, 2\cancel{\pi} r u \cancel{c_p} T\, dr$

$$R^2 V_{mean} T_b = \int_0^R 2 u r T\, dr$$

$$\Rightarrow \boxed{T_b = \frac{2\int_0^R r u T\, dr}{R^2 V_{mean}}}$$

(21) (Pg 86) $u(r,x) = C_1$

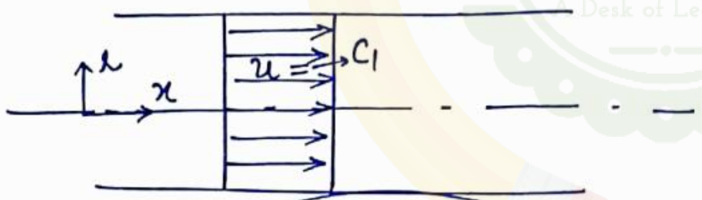
$$T(r,x) = C_2\left[1 - \left(\frac{r}{R}\right)^3\right]$$

$$T_m = \frac{2}{U_m R^2}\int_0^R u(r,x)\, T(r,x)\, r\, dr$$

$$T_m = \frac{2}{U_m R^2}\int_0^R C_1 C_2 \left(1 - \left(\frac{r}{R}\right)^3\right) r\, dr$$

$T_m =$

SIR $u(r,x) = C_1$

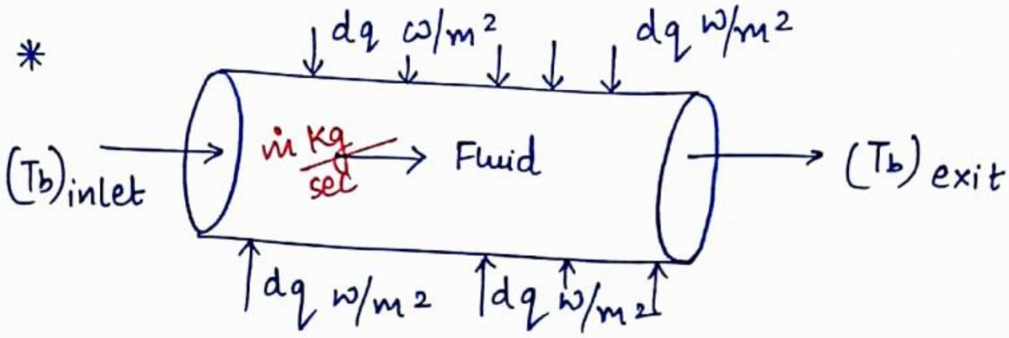


$V_{mean} = C_1 = V_m$

$$\therefore T_b = \frac{2}{\cancel{U_m} R^2}\int_0^R \cancel{C_1} C_2 \left[r - \frac{r^4}{R^3}\right] dr$$

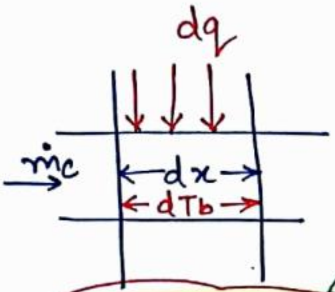
$$T_b = 0.6 C_2$$

*



Total H.T. Rate b/w entire Pipe and flowing fluid

$$= \dot{m} c_p (\Delta T)_b = \dot{m} c_p (T_{b\,exit} - T_{b\,inlet}) \text{ watt}$$



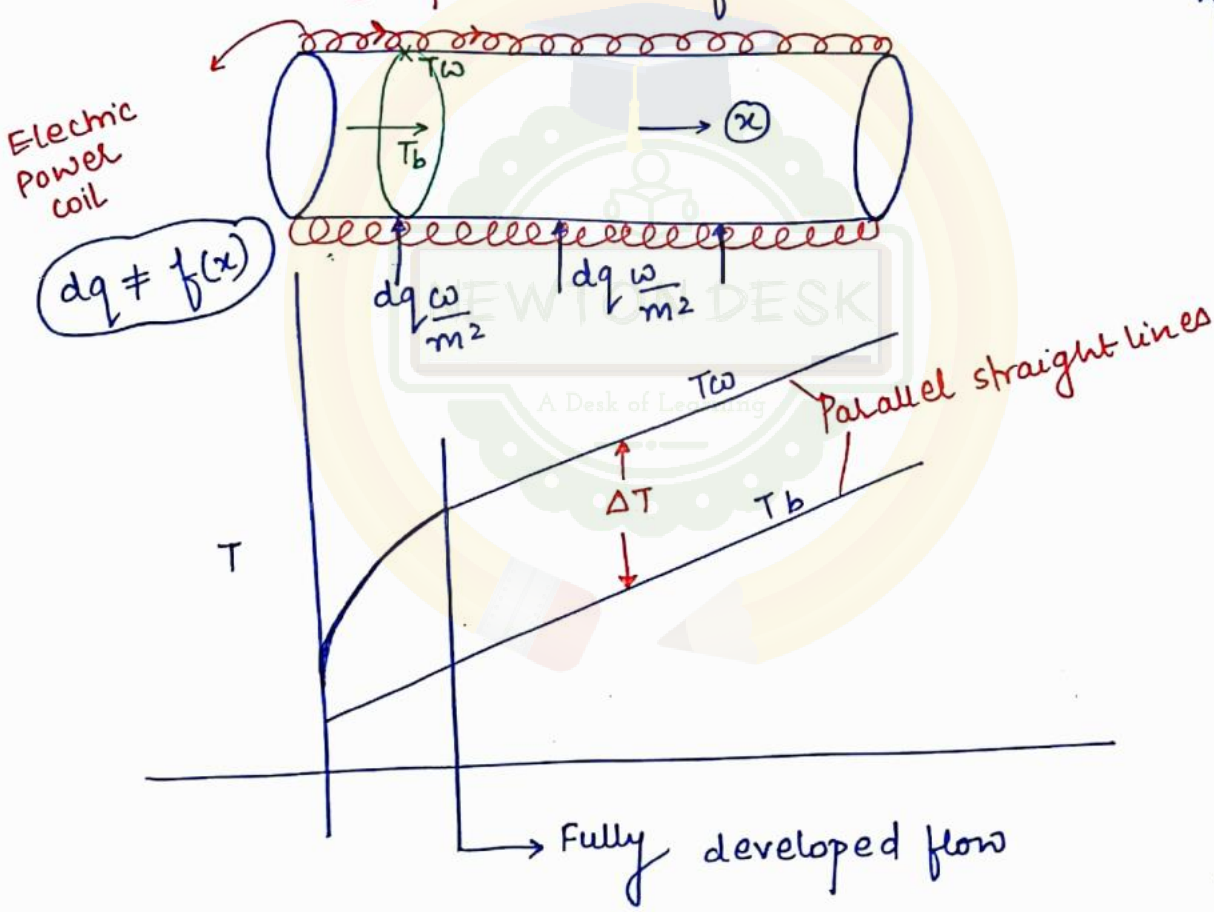
$$dq = \dot{m}_c \, c_p \, dT_b$$

enthalpy changes indicated by this Term.

differential form

* VARIATION OF T_b :-

i Amp constant Heat flux conditions.



case (A)

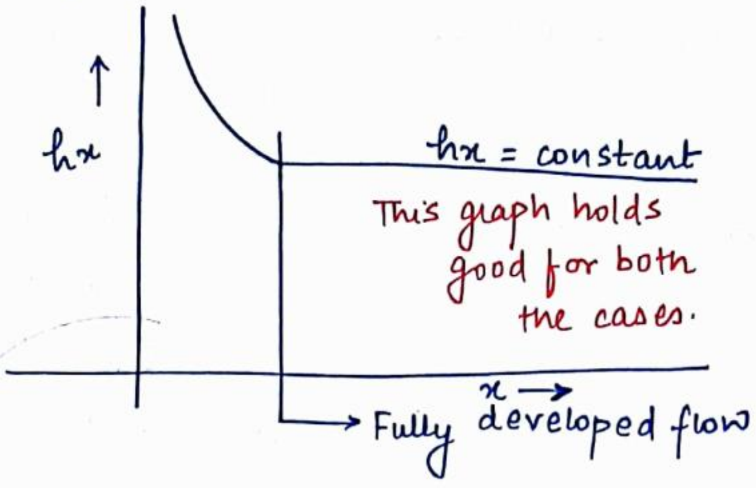
$dq \neq f(x)$

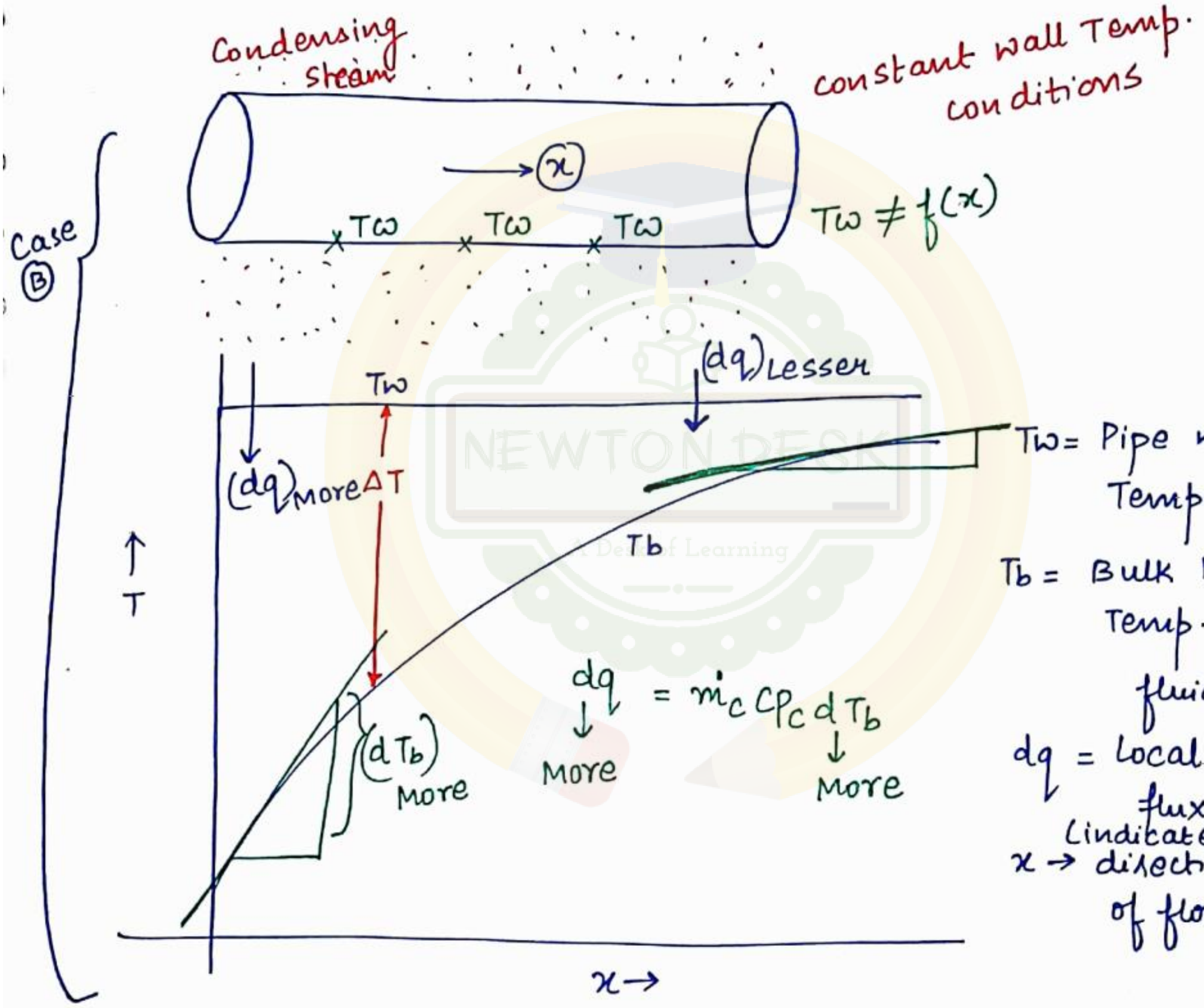
Newton's law of cooling at any given x :-

$$dq = \text{local heat flux} = h_x \times 1 \times (T_w - T_b) \,(W/m^2)$$

$$dq = h_x \times 1 \times (\Delta T)$$

* Variation continues....





T_w = Pipe wall Temperature

T_b = Bulk Mean Temp. of fluid

dq = Local Heat flux

$x \rightarrow$ (indicates the) direction of flow

Unlike in case of flow over flat plates, the local convective H.T. coefficient h_x remains constant in the direction of fluid flow during both constant heat flux conditions as well as constant wall tempr. conditions. ⟶ After case Ⓐ ⟶ T_b should ↑ in the dirn. of fluid i.e. during constant heat flux conditions, since both dq and h_x are remaining constant in the dirn. of fluid flow, the ~~local~~ ΔT value also must remain constant in the dirn. of fluid flow. But since T_b has to ↑ in the dirn. of fluid flow (Because the fluid is getting heat), T_w value also must increase in the direction of fluid flow in such a way that $T_w - T_b$ shall remain the same at any x.

After case Ⓑ ⟶ During constant wall tempr. conditions, since T_w is remaining constant and T_b has to increase in the dirn. of fluid flow, the ΔT value must be decreasing in the dirn. of fluid flow.

Since h_x is remaining constant & ΔT is ↓ing in the dirn. of fluid flow, the local heat flux dq must decrease in the dirn. of fluid flow. This is evident from the decreasing slope of (dT_b) with respect to x.

✓ When dq = constant $\neq f(x)$

⇒ T_w is increasing with x

and

✓ when T_w = constant

⇒ dq is decreasing with x.

Hence, it is just not possible to maintain both constant heat flux conditions and constant wall tempr. conditions simultaneously at the same time.

h_x = constant

Local Nusselt No. → Diameter

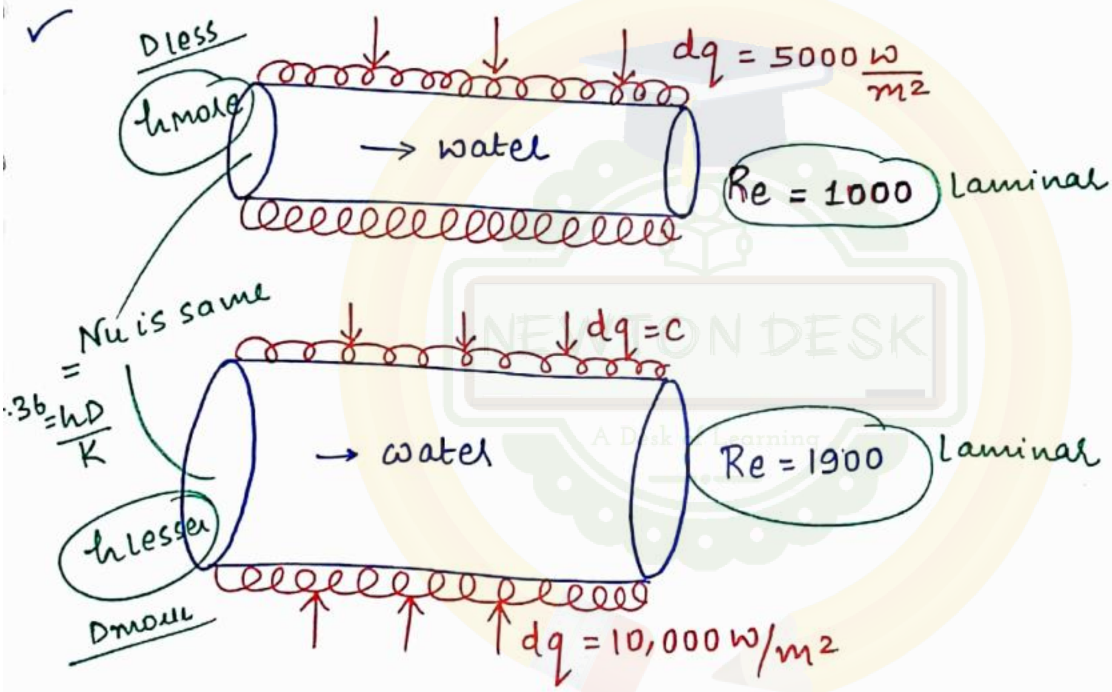
$$Nu_x = \frac{h_x D}{K} = \text{constant}$$

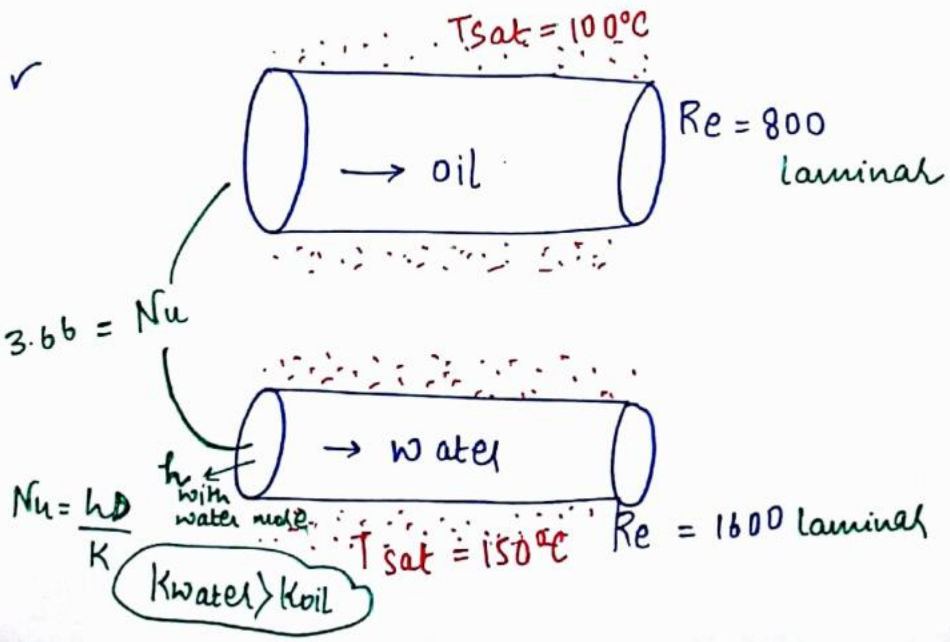
(During both constant heat flux and constant wall tempr. conditions).

∴ For fully developed laminar flow through Pipes/Ducts, (Re < 2000)

$$\boxed{Nu = \frac{hD}{K} = 4.36}$$ (During constant Heat flux conditions)

$$\boxed{Nu = \frac{hD}{K} = 3.66}$$ (During constant wall Tempr. conditions)





→ if Velocity doubles, then

$$Re = \frac{\rho v d}{K}$$

Re = 1600

No change in Nu No.

For fully developed Turbulent flow through pipes or Ducts :-

'h' can be obtained from :-

$$\boxed{Nu = \frac{hD}{k} = 0.023\, Re^{0.8}\, Pr^{n}}$$

Mcadam's equation (OR) Dittus- Boelter Equation

$n = 0.4$ for heating of fluid.

$n = 0.3$ for cooling of fluid.

Pg 87

(28) $D = 25mm$

$V_{mean} = 1.0\ m/s$

$\rho = 1000\ kg/m^3$

$\mu = 7.25 \times 10^{-4}\ NS/m^2$

$k = 0.625\ W/m.K$

$Pr = 4.85$

$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$

$\cancel{\frac{\rho VD}{K} = 40 = Re}$

SIB

$$Nu = \frac{hD}{k} = 0.023 \left(\frac{VD\rho}{\mu}\right)^{0.8} \left(\frac{\mu c_p}{k}\right)^{0.4}$$

$D = 0.025m$

$v = 1m/sec$

$h = 4,613.6\ W/m^2K$ ↑high.

Pg 28

(7) $1m \times 0.5m$ $Nu = 0.023\, Re^{0.8}\, Pr^{0.33}$

$T^{w}_{\infty} = 20°C$

$V_\infty = 10m/s$

$T_\infty = 30°C$

$K = 0.025$

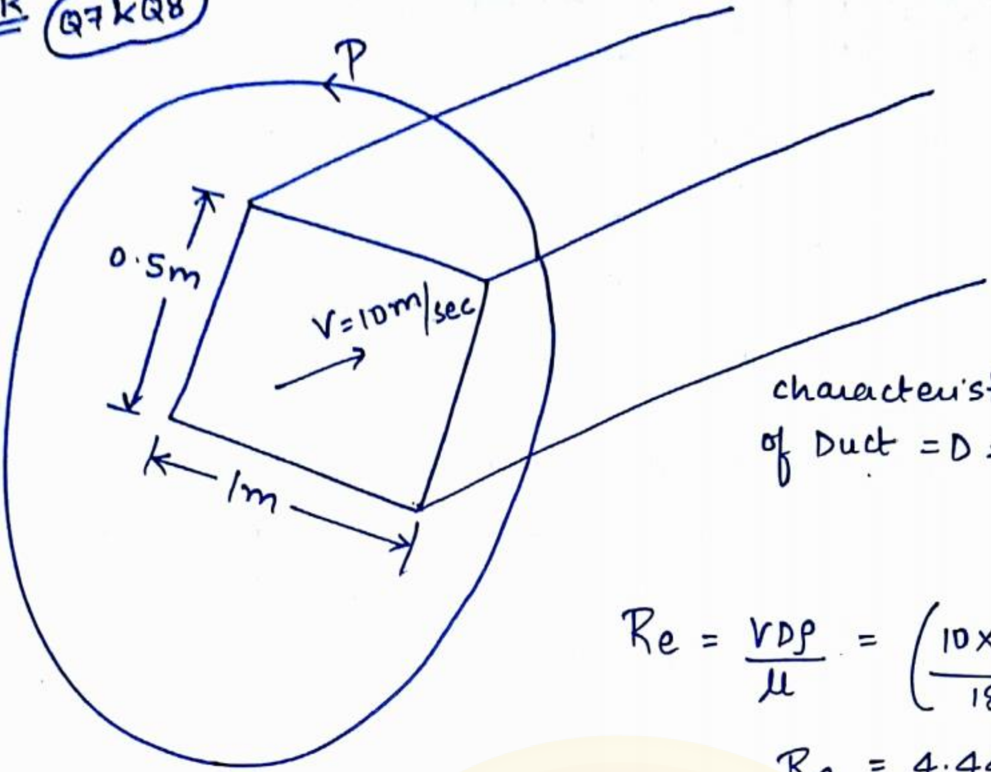
$\mu = 18\ \mu Pas$

$Pr = 0.73$

$\rho = 1.2$

$Nu = 3.4$

SIR (Q7 & Q8)



characteristic dimension of Duct $= D = \frac{4A_{c/s}}{P} = \frac{4\times(1\times0.5)}{2(1+0.5)}$

$= 0.667\,m$

$$Re = \frac{V D \rho}{\mu} = \left(\frac{10\times0.667\times1.2}{18\times10^{-6}}\right)$$

$$Re = 4.44\times10^{5}$$

Since $Re > 4000 \Rightarrow$ Flow is turbulent
(Internal flow)

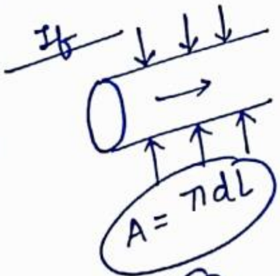
$$Nu = \frac{hD}{K} = 0.023\, Re^{0.8}\, Pr^{0.33} \Rightarrow h = 25.6\ W/m^2K$$

$\therefore$ H.T. Rate per unit length of Duct $= h\times(P\times L)(T_\infty - T_\infty)$

$= 25.6\times2(1+0.5)\times1(30-20)$

$= 769$ watt.

If



(18) and (19)

$C_p = 4.18\times10^3\ J/Kg\,K$

$\dot m = 0.01\ Kg/s$

$T_w = 20^\circ C$

$D = 50\,mm$

$L = 3\,m$

$q_w \rightarrow W/m^2$

$q_w = 2500x$

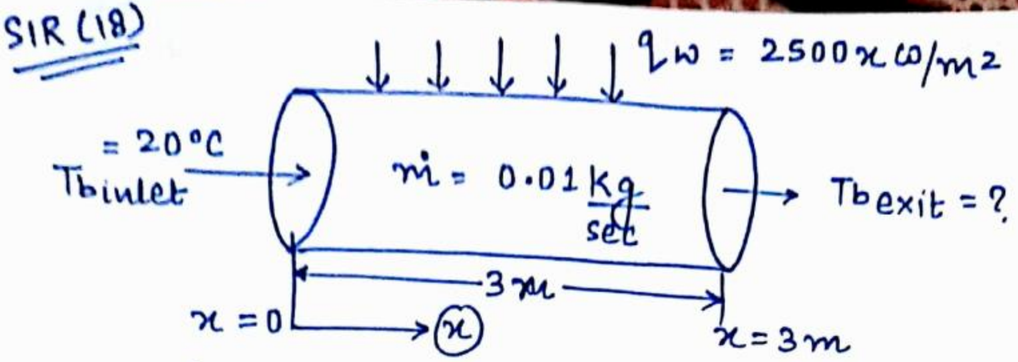
$T_{mean} = ?\ T_b = ?$

$dq = \dot m c_p\, dT_b$

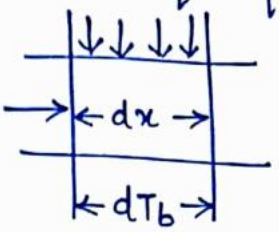
$2500 =$

$179.42 + 20 =$

SIR (18)



$dq = q_w \times \text{Area} = q_w \times \pi D \times dx$



Differential H.T. Rate through differential H.T. area of length $dx = dq = \dot{m} C_p dT_b$

$$q_w \times \pi D dx = \dot{m} C_p dT_b$$

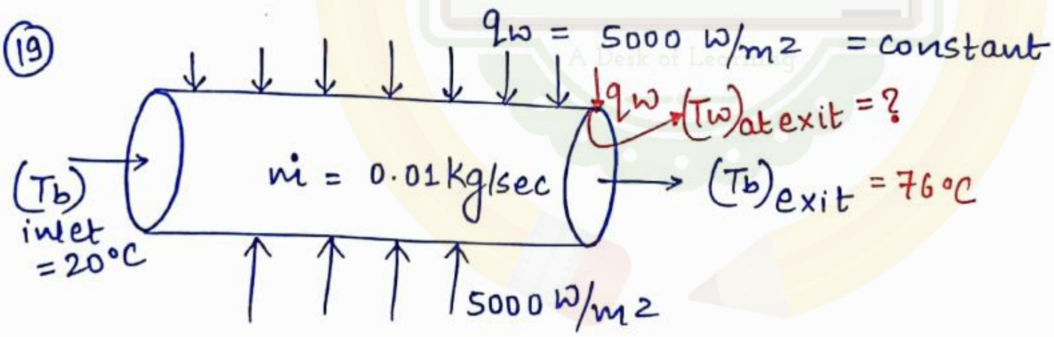
$$\int_{x=0}^{3m} 2500x \times \pi D dx = \int_{(T_b)_{inlet}}^{(T_b)_{exit}} \dot{m} C_p dT_b$$

$$\Rightarrow \underbrace{2500 \times \frac{3^2}{2} \times \pi \times \frac{50}{1000}}_{} = 0.01 \times 4180 (T_{b\,exit} - 20)$$

Total H.T. Rate for entire pipe

$$\Rightarrow \boxed{(T_b)_{exit} = 62°C}$$

(19)



Total H.T. Rate between entire pipe and water = Heat flux × Total H.T. area

$$= 5000 \times (\pi D \times L) = \dot{m} C_p (T_{b\,exit} - T_{b\,inlet})$$

$$= 0.01 \times 4180.0 (T_{b\,exit} - 20)$$

$$\Rightarrow (T_{b\,exit}) = 76°C$$

Newton's law of cooling at exit :-

q_w = Heat flux at exit = $h_x \times 1 (T_{w\,at\,exit} - T_{b\,at\,exit})$

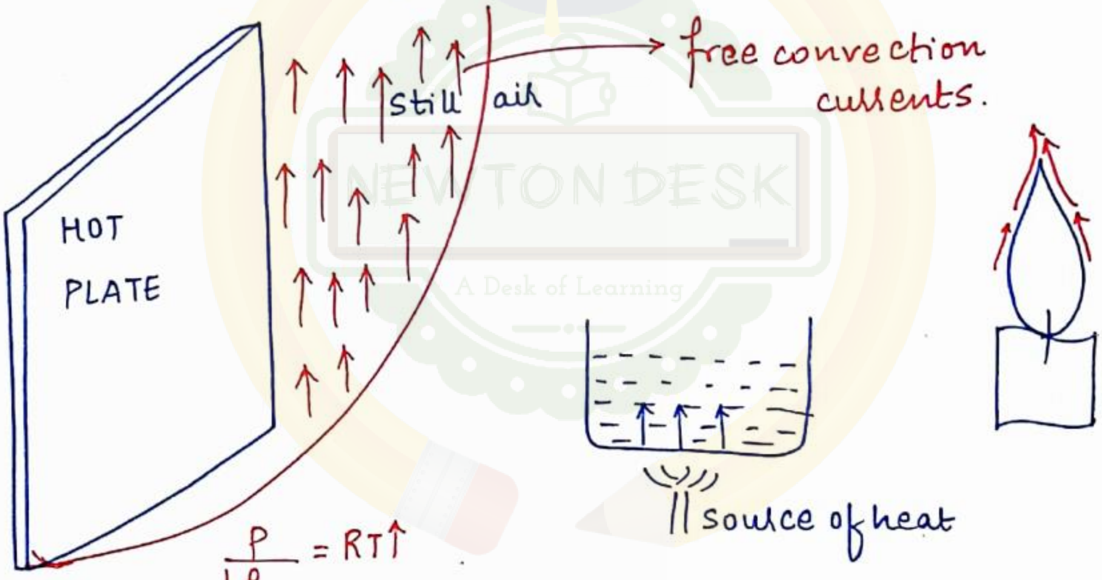
$5000 = 1000 \times 1 (T_{w\,at\,exit} - 76)\ W/m^2$

$$\boxed{T_{w\,at\,exit} = 81^\circ C}$$

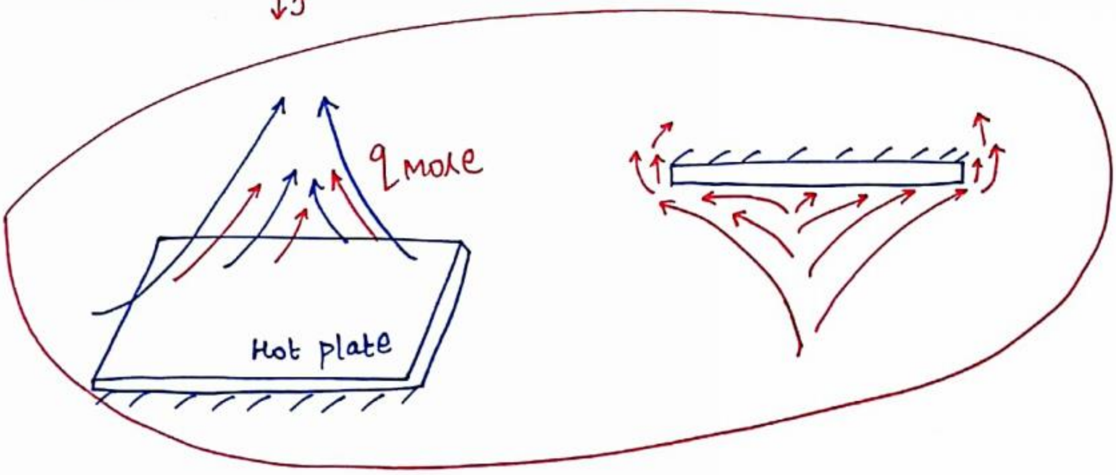
FREE / CONVECTION
NATURAL

Throughout the subject, isobaric Heat transfer only.

No velocity evident but the flow occurs naturally due to Buoyancy forces arising out of density changes of fluid.



$\dfrac{P}{\downarrow \rho} = RT\uparrow$



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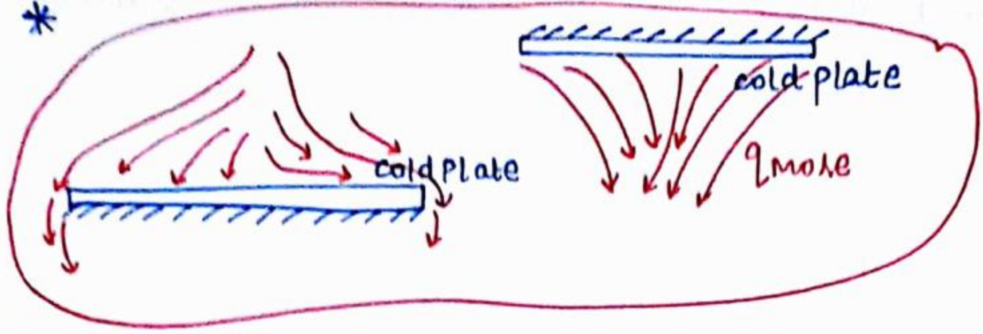

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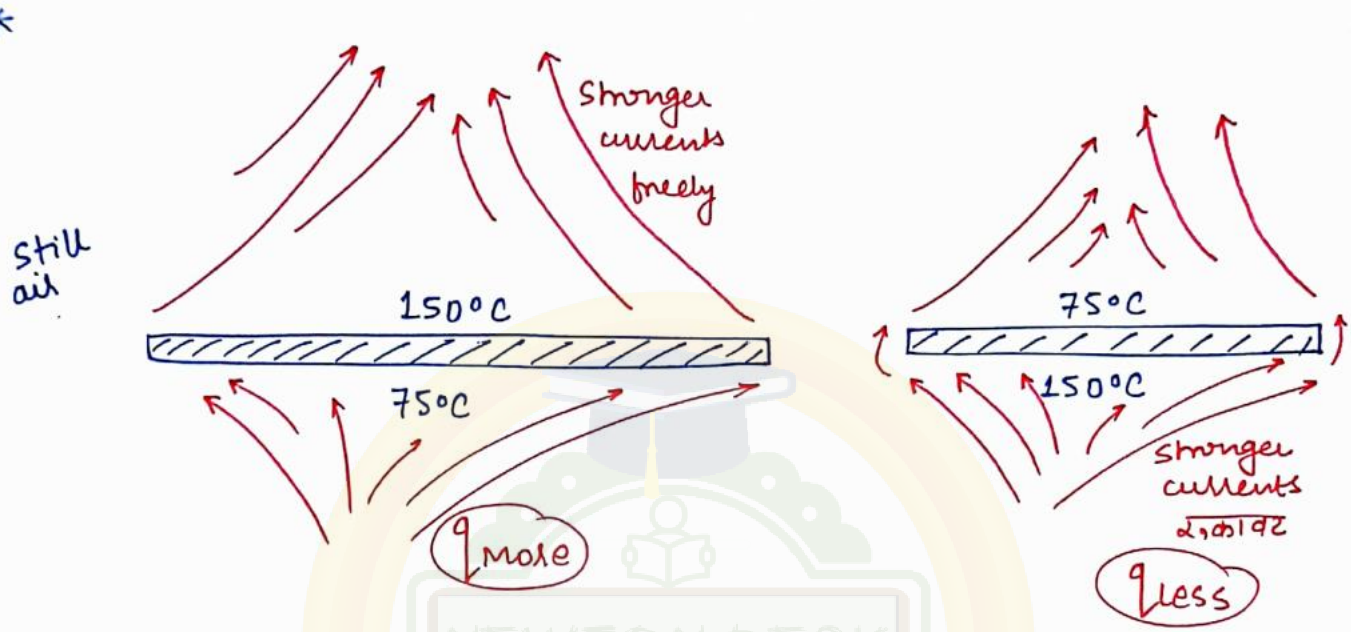


In any free convection heat transfer,

$$h = f(g, \beta, \Delta T, L, \underbrace{\mu, \rho, c_p, k}_{\text{Thermophysical Properties of fluid}})$$

g = Acceleration due to gravity.

β = isobaric Volume expansion coefficient of fluid.

$$= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p \Big/ \text{Kelvin}$$

For ideal gas like air,

$$\beta = \frac{1}{T_{mean}} \Big/ K,$$

where T_{mean} = Mean film Temp. of fluid in K

$$= \frac{1}{\left(\frac{T_w + T_\infty}{2}\right) \text{in } K}$$

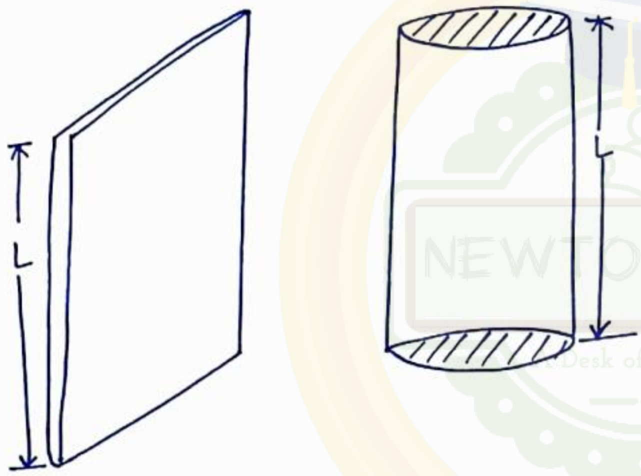
High-β $\Rightarrow$ More ΔV
Ger
$\Rightarrow$ Higher $\Delta \rho$
$\Rightarrow$ Stronger Buoyancy forces

$\beta_{air} > \beta_{water}$

- $\Delta T = (T_w - T_\infty)$ (T_w: Body; T_∞: fluid.)

- L = characteristic Dimension of Body (Dimension of Body i.e. used in the calculation of Dimensionless No.s)

For vertical plates and cylinder,

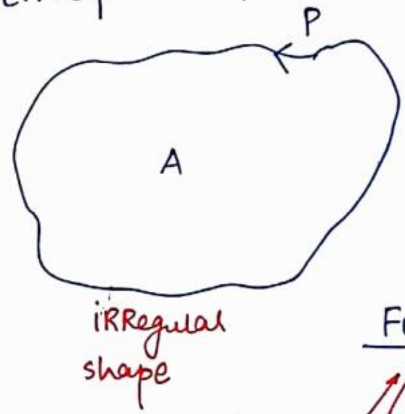


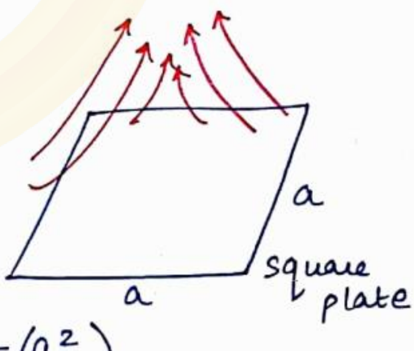
For Horizontal cylinder,

For Horizontal plate,
(irregular shape)

$\boxed{L = \frac{A}{P}}$

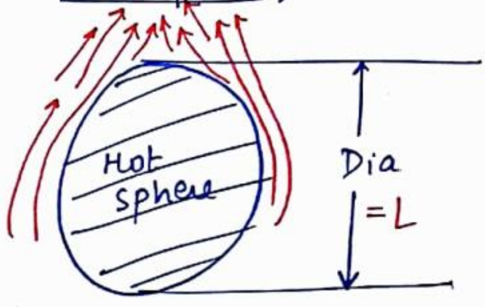
For ex:-





$L = \left(\frac{a^2}{4a}\right)$

For sphere,



All the variables in free convection H.T. are grouped into 3 Dimens-ionless numbers from dimensional analysis which are given as

① Grashoff No = $\boxed{Gr = \frac{g\beta \Delta T L^3}{\nu^2}}$ → only for gas

$= \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{viscous Force})^2}$

where ν = K.V. of fluid.

Gr signifies the Magnitude of Buoyancy forces since it contains β.

Gr replaces Reynold's No (∵ No velocity) in Free convection heat transfer.

② Nussel's No. $\boxed{Nu = \frac{hL}{K}}$

③ Prandtl No. $\boxed{Pr = \left(\frac{\mu C_p}{K}\right)}$

∴ In any free convection H.T.,

$$\boxed{Nu = f(\underbrace{Gr, Pr}_{\text{common powers}})}$$

In forced convection H.T.,

$Nu = f(Re, Pr)$ ↓ different exponents.

The product of Gr Pr is called

Rayleigh No. (Ra)

Usually the functional Relationship appears as :-

$$\boxed{\frac{hL}{K} = Nu = C(Gr\,Pr)^m}$$

C and m are constants which vary from case to case.

$m = 1/4$ for Laminar flow.

$m = 1/3$ for Turbulent flow.

The flow in free convection H.T. is decided as laminar or Turbulent based on the value of $(Gr\,Pr)$ product that is Rayleigh No. (Ra)

If $Gr\,Pr < 10^9 \Rightarrow$ Flow is laminar.
i.e. (Ra)

If $Gr\,Pr > 10^9 \Rightarrow$ Flow is Turbulent.

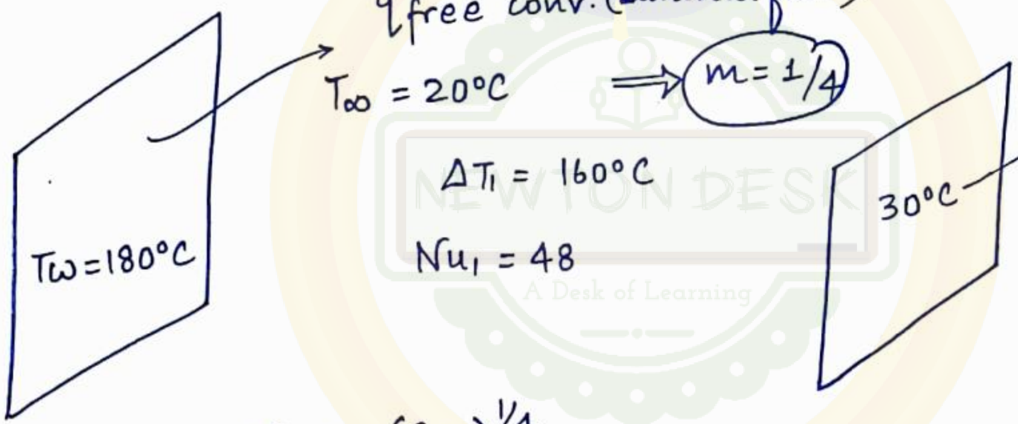
WB
(15) Pg 85

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2} = \frac{180-20}{\nu^2} g\beta L^3$$

$$Gr = 160$$

$$Gr =$$

SIR



q free conv. (laminar flow)

$T_\infty = 20°C \Rightarrow m = 1/4$

$\Delta T_1 = 160°C$

$Nu_1 = 48$

$T_\infty = 20°C$

q free conv

$\Delta T_2 = 10°C$

$Nu_2 = ?$

$$Nu \propto (Gr)^{1/4}$$

$$Nu \propto (\Delta T)^{1/4}$$

$$\frac{Nu_2}{Nu_1} = \left(\frac{\Delta T_2}{\Delta T_1}\right)^{1/4} \Rightarrow Nu_2 = 48 \times \left(\frac{10}{160}\right)^{1/4} = 24$$

(17) $T_\infty = 25°C$ $\quad C_p = 2.5 \times 10^3$ J/kg K

$A = 0.01 m^2$ $\quad T_w = 225°C$ $\quad \frac{dT}{dt} = -0.02$ K/s

$m = 4$ kg

(17) SIR

Free convection

$T_\infty = 25°C$

writing the Energy Balance for the plate,

The Rate of convection H.T. b/w plate and fluid = The Rate of decrease of I.E. of plate wrt time

$$hA(T_p - T_\infty) = -mC_p \left(\frac{dT}{dt}\right) \text{ J/sec} \quad (C_p \rightarrow \text{J/kg-K})$$

$$h \times 0.1 \times (225 - 25) = 4 \times 2500 \times 0.02 \text{ J/sec}$$

$$h = 10 \text{ W/m}^2\text{K}$$

(16) $Nu = \frac{hD}{K} = 25$ ($D \rightarrow 0.1$ m)

$h = 7.5 \text{ W/m}^2\text{K}$

(25) a $m = 1/4$

(12) $Re = 1500$

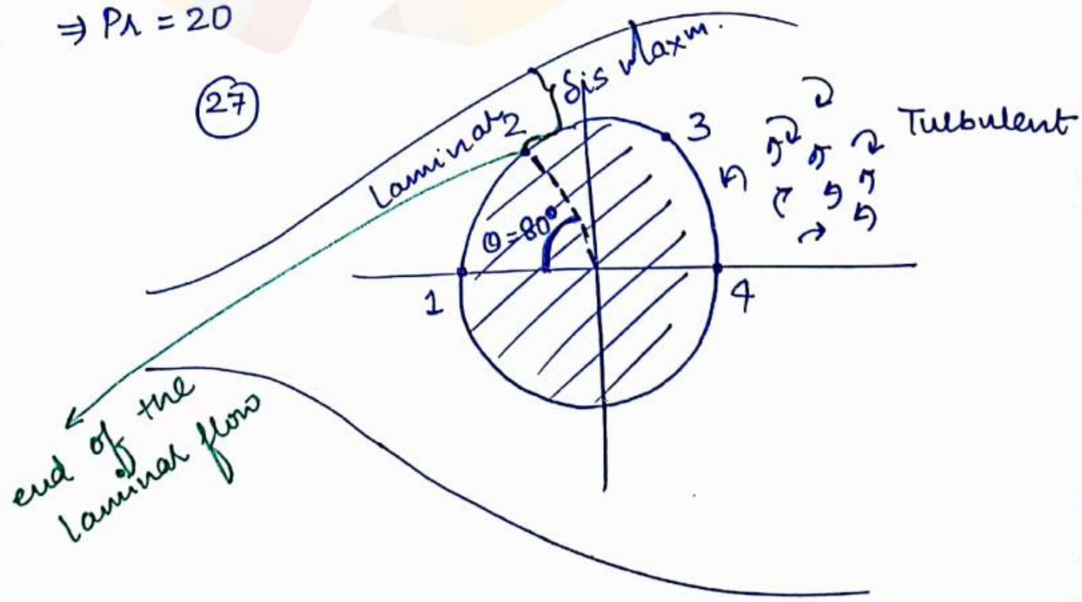
flow is Laminar

$\frac{hD}{K} = Nu = 4.36 \Rightarrow h = 43.6 \text{ W/m}^2\text{K}$ (during constant heat flux conditions.)

$\frac{hD}{K} = 3.66 \Rightarrow h = 36.6 \text{ W/m}^2\text{K}$ (During constant wall Temp. conditions.)

(10) $St = \frac{Nu}{Re\,Pr} \Rightarrow Pr = 20$

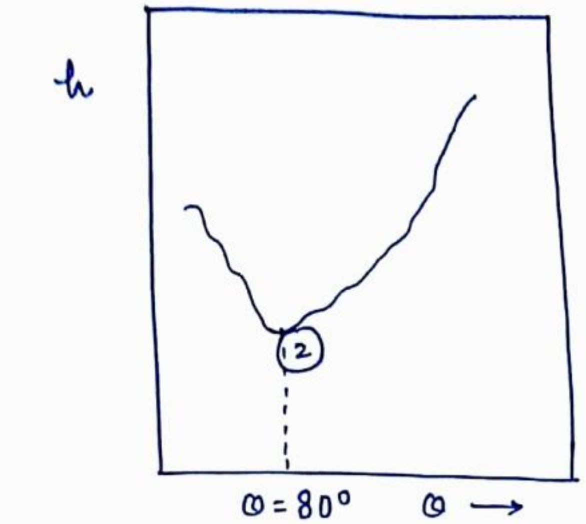
(27)



(13) a

(14) d

(23) c



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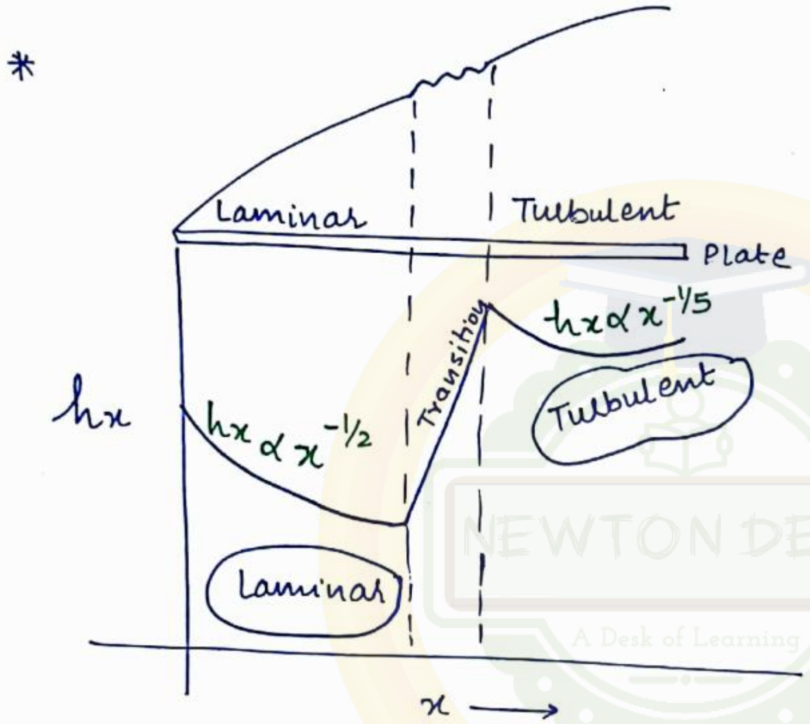
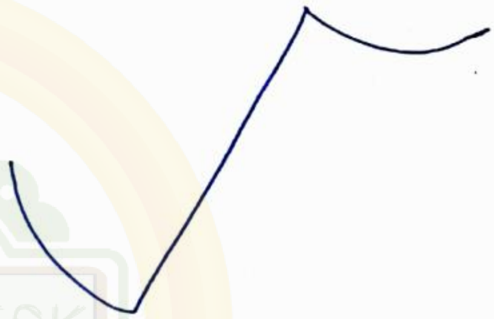


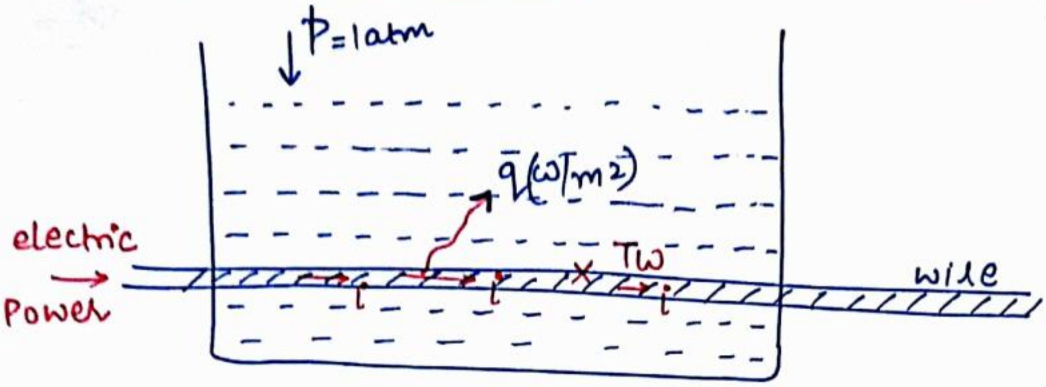


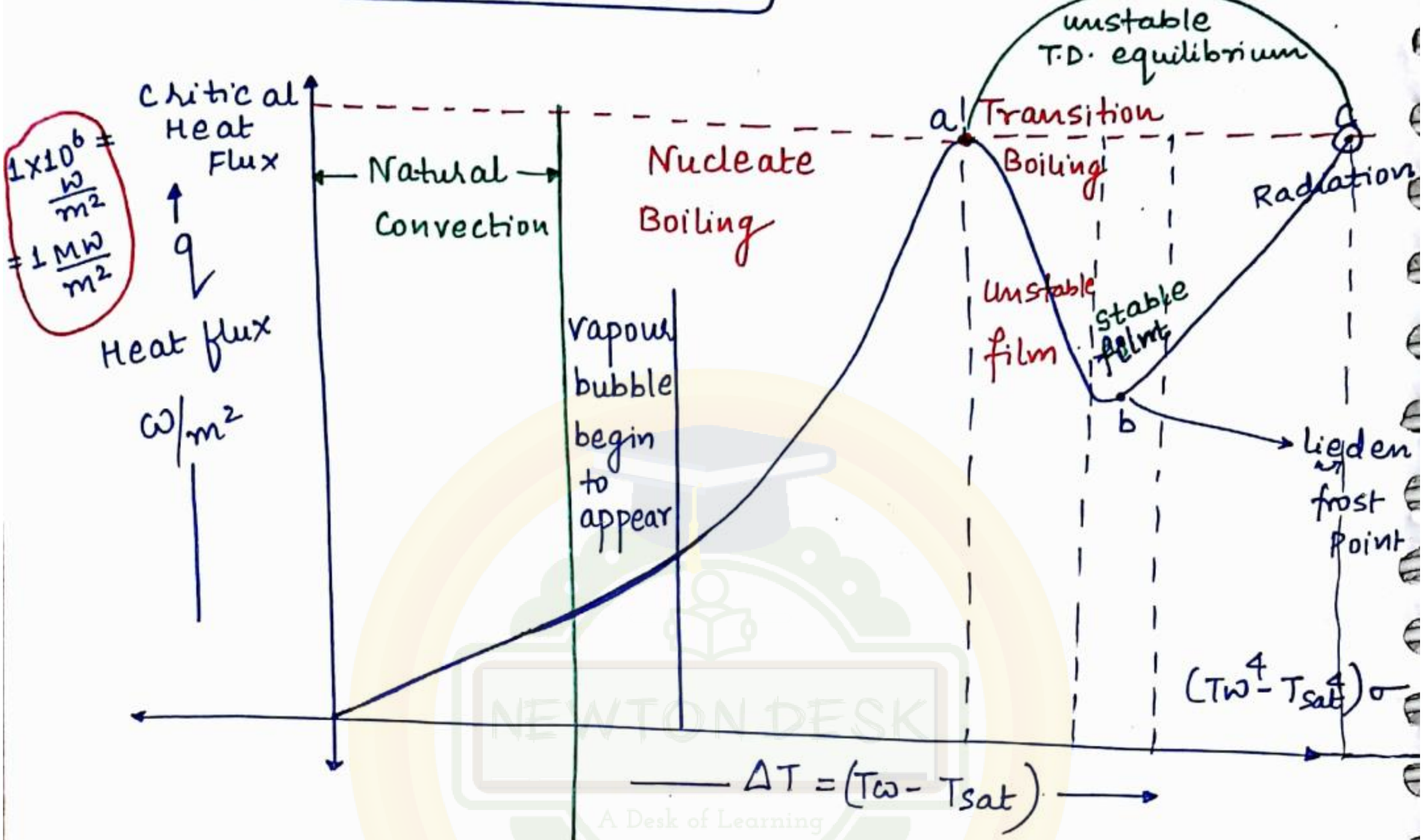
* POOL BOILING CURVE :-

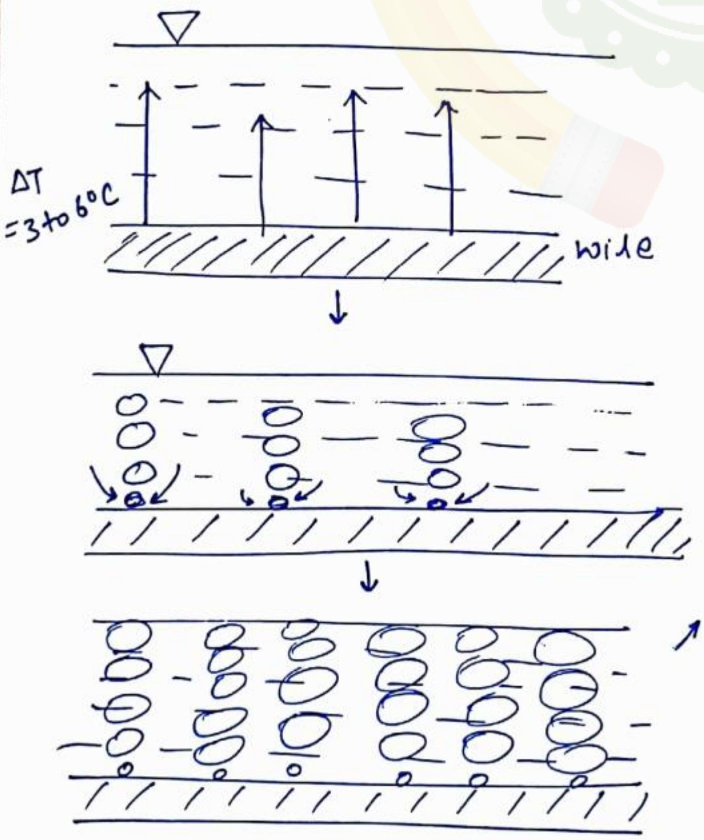
The Boiling of any liquid can begin only when the liquid comes into contact with a solid surface whose tempr. is greater than the saturation temperature ~~corresponding~~ of the liquid corresponding to its saturation pressure.

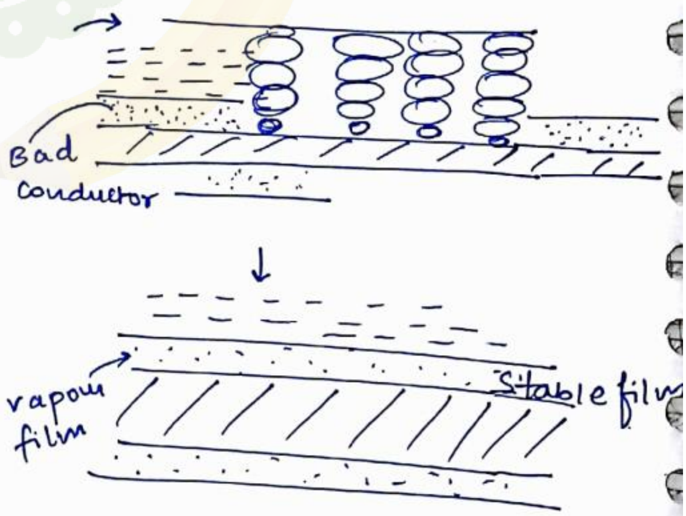
Ex :- At a p_{sat} = 101.3 KPa (abs)

$\Rightarrow$ (T_{sat}) of water = 100°C.


P = 1atm
q (W/m2)
electric Power
Tw
i
wire


Critical Heat Flux
1x10^6 W/m2 = 1 MW/m2
q
Heat flux W/m2
Natural Convection
Nucleate Boiling
vapour bubble begin to appear
a
Transition Boiling
unstable T.D. equilibrium
unstable film
stable film
b
Radiation
Liedenfrost Point
(Tw^4 - Tsat^4)σ
ΔT = (Tw - Tsat)


ΔT = 3 to 6°C
wire


Bad conductor
vapour film
Stable film

Beyond point (a)