EINSTEIN

COLLEGE OF ENGINEERING

Sir.C.V.Raman Nagar, Tirunelveli-12

Department of Mechanical Engineering

Subject Code: ME-2302/Dynamics of Machinery

Staff-In charge: **S.Suresh**

Year & Semester: **IIIrd - Vth**

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Sub Name: Dynamics of Machinery **Year/Class: III -Mechanical A & B**

NOTES Of LESSON

OBJECTIVE:

- To understand the method of static force analysis and dynamic force analysis of Mechanisms
- To study the undesirable effects of unbalance in rotors and engines.
- To understand the concept of vibratory systems and their analysis
- To understand the principles of governors and gyroscopes.

REFERENCES:

- Thomas Bevan, "Theory of Machines", CBS Publishers and Distributors, 1984.
- Ghosh A. and Mallick A.K., "Theory of Mechanisms and Machines", Affiliated East-West Press Pvt. Ltd., New Delhi, 1988.
- Shigley J.E. and Uicker J.J., "Theory of Machines and Mechanisms", McGraw-Hill,Inc., 1995.
- Rao J.S. and Dukkipati R.V., "Mechanism and Machine Theory ", Wiley-Eastern Limited, New Delhi, 1992.
- John Hannah and Stephens R.C., "Mechanics of Machines", Viva low-Priced Student Edition, 1999.
- Sadhu Singh "Theory of Machines" Pearson Education, 2002.

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UNIT – I - Force Analysis

UNIT – I - Force Analysis

(1) Introduction:

If the acceleration of moving links in a mechanism is running with considerable amount of linear and/or angular accelerations, inertia forces are generated and these inertia forces also must be overcome by the driving motor as an addition to the forces exerted by the external load or work the mechanism does.

(2) Newton's Law:

First Law

Everybody will persist in its state of rest or of uniform motion (constant velocity) in a straight line unless it is compelled to change that state by forces impressed on it. This means that in the absence of a non-zero net force, the center of mass of a body either is at rest or moves at a constant velocity.

Second Law

A body of mass *m* subject to a force **F** undergoes an acceleration **a** that has the same direction as the force and a magnitude that is directly proportional to the force and inversely proportional to the mass, i.e., $\mathbf{F} = m\mathbf{a}$. Alternatively, the total force applied on a body is equal to the time derivative of linear momentum of the body.

Third Law

The mutual forces of action and reaction between two bodies are equal, opposite and collinear. This means that whenever a first body exerts a force **F** on a second body, the second body exerts a force −**F** on the first body. **F** and −**F** are equal in magnitude and opposite in direction. This law is sometimes referred to as the *action-reaction law*, with **F** called the "action" and −**F** the "reaction"

(3) Types of force Analysis:

- Equilibrium of members with two forces
- Equilibrium of members with three forces
- Equilibrium of members with two forces and torque
- Equilibrium of members with two couples.
- Equilibrium of members with four forces.

(4) Principle of Super Position:

Sometimes the number of external forces and inertial forces acting on a mechanism are too much for graphical solution. In this case we apply the method of superposition. Using superposition the entire system is broken up into (n) problems, where n is the number of forces, by considering the external and inertial forces of each link individually. Response of a linear system to several forces acting simultaneously is equal to the sum of responses of the system to the forces individually. This approach is useful because it can be performed by graphically.

(5) Free Body Diagram:

A free body diagram is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body. Creating a free body diagram can make it easier to understand the forces, and torques or moments, in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem. The diagrams are also used as a conceptual device to help identify the internal forces—for example, shear forces and bending moments in beams—which are developed within structures.

(6) D'Alemberts Principle:

D'Alembert's principle, also known as the **Lagrange–d'Alembert principle**, is a statement of the fundamental classical laws of motion. It is named after its discoverer, the French physicist and mathematician Jean le Rond d'Alembert. The principle states that the sum of the differences between the forces acting on a system and the time derivatives of the momenta of the system itself along any virtual displacement consistent with the constraints of the system is zero.

(7) Dynamic Analysis of Four bar Mechanism:

A **four-bar linkage** or simply a **4-bar** or **four-bar** is the simplest movable linkage. It consists of four rigid bodies (called bars or links), each attached to two others by single joints or pivots to form closed loop. Four-bars are simple mechanisms common in mechanical engineering machine design and fall under the study of kinematics.

- Dynamic Analysis of Reciprocating engines.
- Inertia force and torque analysis by neglecting weight of connecting rod.
- Velocity and acceleration of piston.
- Angular velocity and Angular acceleration of connecting rod.
- Force and Torque Analysis in reciprocating engine neglecting the weight of connecting rod.
- Equivalent Dynamical System
- Determination of two masses of equivalent dynamical system

(8) Turning Moment Diagram:

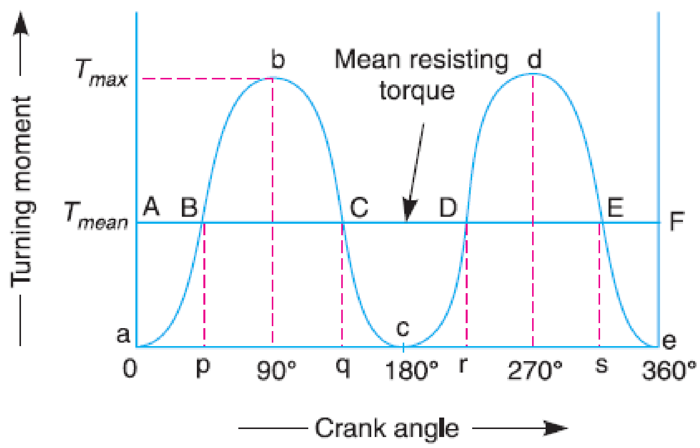
The turning moment diagram is graphical representation of the turning moment or crank effort for various positions of crank.

(9) Single cylinder double acting engine:

A turning moment diagram for a single cylinder double acting steam engine is shown in Fig. The vertical ordinate represents the turning moment and the horizontal ordinate represents the crank angle.

the turning moment on the crankshaft,

$$T = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$



Turning moment diagram for a single cylinder, double acting steam engine.

where F_P = Piston effort,

r = Radius of crank,

n = Ratio of the connecting rod length and radius of crank, and

θ = Angle turned by the crank from inner dead centre.

From the above expression, we see that the turning moment (T) is zero, when the crank angle (θ) is zero. It is maximum when the crank angle is 90° and it is again zero when crank angle is 180°.

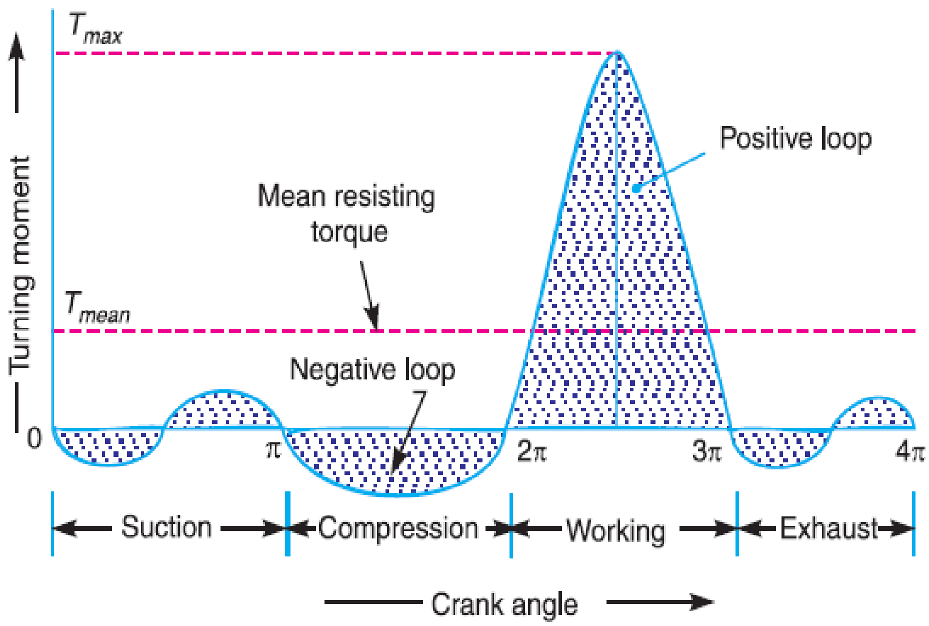
This is shown by the curve *abc* in Fig. and it represents the turning moment diagram for outstroke. The curve *cde* is the turning moment diagram for instroke and is somewhat similar to the curve *abc*.

Since the work done is the product of the turning moment and the angle turned, therefore the area of the turning moment diagram represents the work done per revolution. In actual practice, the engine is assumed to work against the mean resisting torque, as shown by a horizontal line AF. The height of the ordinate aA represents the mean height of the turning moment diagram. Since it is assumed that the work done by the turning moment per revolution is equal to the work done against the mean resisting torque, therefore the area of the rectangle $aAFe$ is proportional to the work done against the mean resisting torque.

For flywheel, have a look at your tailor's manual sewing machine.

(10) Turning moment diagram for 4-stroke I.C engine:

A turning moment diagram for a four stroke cycle internal combustion engine is shown in Fig. We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, *i.e.* 720° (or 4 π radians).

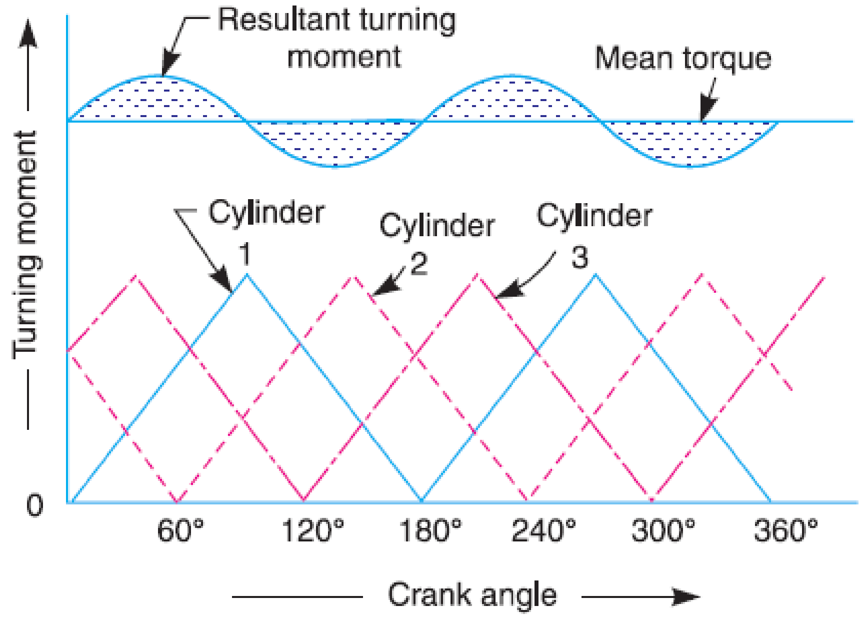


Turning moment diagram for a four stroke cycle internal combustion engine.

Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Fig. 16.2. During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained. During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases. During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Fig.

(11) Turning moment diagram for a multi cylinder engine:

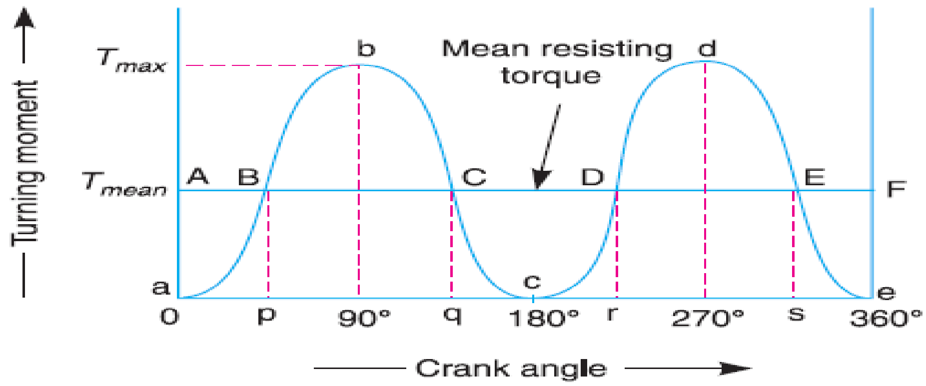
A separate turning moment diagram for a compound steam engine having three cylinders and the resultant turning moment diagram is shown in Fig. The resultant turning moment diagram is the sum of the turning moment diagrams for the three cylinders. It may be noted that the first cylinder is the high pressure cylinder, second cylinder is the intermediate cylinder and the third cylinder is the low pressure cylinder. The cranks, in case of three cylinders, are usually placed at 120° to each other.



Turning moment diagram for a multi-cylinder engine.

(12) Fluctuation of Energy:

The difference in the kinetic energies at the point is called the maximum fluctuation of energy.



The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Fig. We see that the mean resisting torque line *AF* cuts the turning moment diagram at points *B*, *C*, *D* and *E*. When the crank moves from *a* to *p*, the work done by the engine is equal to the area *aBp*, whereas the energy required is represented by the area *aABp*. In other words, the engine has done less work (equal to the area *a AB*) than the requirement. This amount of energy is taken from the flywheel and hence the speed of the flywheel decreases. Now the crank moves from *p* to *q*, the work done by the engine is equal to the area *pBbCq*, whereas the requirement of energy is represented by the area *pBCq*. Therefore, the engine has done more work than the requirement. This excess work (equal to the area *BbC*) is stored in the flywheel and hence the speed of the flywheel increases while the crank moves from *p* to *q*.

Similarly, when the crank moves from *q* to *r*, more work is taken from the engine than is developed. This loss of work is represented by the area *C c D*. To supply this loss, the flywheel gives up some of its energy and thus the speed decreases while the crank moves from *q* to *r*. As the crank moves from *r* to *s*, excess energy is again developed given by the area *D d E* and the speed again increases. As the piston moves from *s* to *e*, again there is a loss of work and the speed decreases. The variations of energy above and below the mean resisting torque line are called ***fluctuations of energy***. The areas *BbC*, *CcD*, *DdE*, etc. represent fluctuations of energy.

A little consideration will show that the engine has a maximum speed either at *q* or at *s*. This is due to the fact that the flywheel absorbs energy while the crank moves from *p* to *q* and from *r* to *s*. On the other hand, the engine has a minimum speed either at *p* or at *r*. The reason is that the flywheel gives out some of its energy when the crank moves from *a* to *p* and *q* to *r*. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy***.

(13) Fluctuation of Speed:

This is defined as the ratio of the difference between the maximum and minimum angular speeds during a cycle to the mean speed of rotation of the crank shaft.

(14) Maximum fluctuation of energy:

A turning moment diagram for a multi-cylinder engine is shown by a wavy curve in Fig. The horizontal line AG represents the mean torque line. Let a_1, a_3, a_5 be the areas above the mean torque line and a_2, a_4 and a_6 be the areas below the mean torque line. These areas represent some quantity of energy which is either added or subtracted from the energy of the moving parts of the engine.

A flywheel stores energy when the supply is in excess and releases energy when energy is in deficit.

Let the energy in the flywheel at $A = E$, then from Fig. we have

Energy at $B = E + a_1$

Energy at $C = E + a_1 - a_2$

Energy at $D = E + a_1 - a_2 + a_3$

Energy at $E = E + a_1 - a_2 + a_3 - a_4$

Energy at $F = E + a_1 - a_2 + a_3 - a_4 + a_5$

Energy at $G = E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$

$=$ Energy at A (*i.e.* cycle repeats after G)

Let us now suppose that the greatest of these energies is at B and least at E. Therefore,

Maximum energy in flywheel

$$= E + a_1$$

Minimum energy in the flywheel

$$= E + a_1 - a_2 + a_3 - a_4$$

$\therefore$ Maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$

$$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$$

(15) Coefficient of fluctuation of energy:

It may be defined as the **ratio of the maximum fluctuation of energy to the work done per cycle.** Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The work done per cycle (in N-m or joules) may be obtained by using the following two relations :

1. Work done per cycle $= T_{mean} \times \theta$

where T_{mean} = Mean torque, and

θ = Angle turned (in radians), in one revolution.

= 2π, in case of steam engine and two stroke internal combustion engines

= 4π, in case of four stroke internal combustion engines.

The mean torque (T_{mean}) in N-m may be obtained by using the following relation :

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s $= 2\pi N/60$

2. The work done per cycle may also be obtained by using the following relation :

$$\text{Work done per cycle} = \frac{P \times 60}{n}$$

where n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= $N/2$, in case of four stroke internal combustion engines.

(16) Coefficient of fluctuation of speed:

The difference between the maximum and minimum speeds during a cycle is called the *maximum fluctuation of speed*. The ratio of the maximum fluctuation of speed to the mean speed is called the *coefficient of fluctuation of speed*.

Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

∴ Coefficient of fluctuation of speed,

$$C_s = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \qquad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \qquad \text{...(In terms of linear speeds)}$$

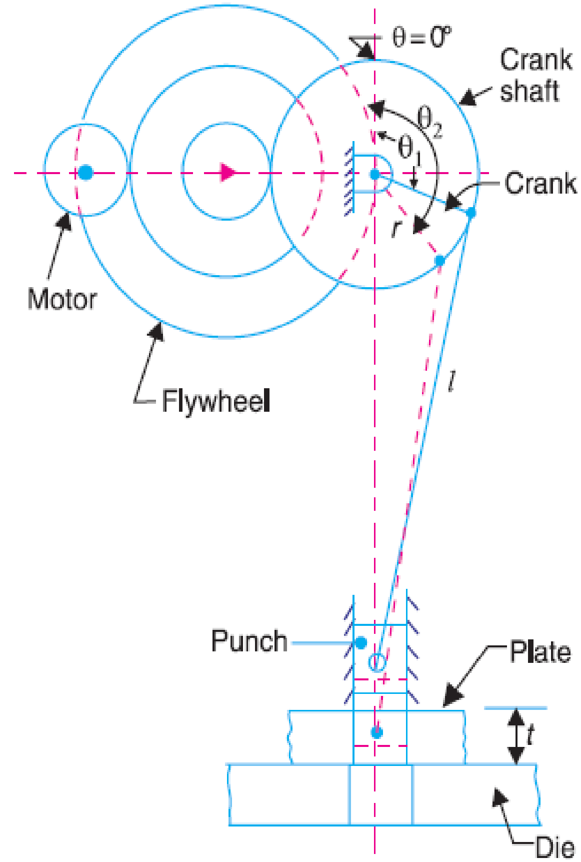
(17) Energy stored in flywheel:

A flywheel is a rotating mass that is used as an energy reservoir in a machine. It absorbs energy in the form of kinetic energy, during those periods of crank rotation when actual turning moment is greater than the resisting moment and release energy, by way of parting with some of its K.E, when the actual turning moment is less than the resisting moment.

(18) Flywheel in punching press:

The flywheels used for prime movers constitute a class of problems in which the resisting torque is assumed to be constant and the driving torque varies. flywheels used in punching, riveting and similar machines constitute another class of problems in which the actual(driving) turning moment provided by an electric motor is more or less constant but the resisting torque(load) varies.

We have discussed that the function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crankshaft is constant and the input torque varies during the cycle. The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle. Such an application is found in punching press or in a rivetting machine. A punching press is shown diagrammatically in Fig. The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism. From Fig. we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle. Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied. On the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from



Operation of flywheel in a punching press.

$\theta = \theta_1$ to $\theta = \theta_2$ due to much more load than the energy supplied. Thus the flywheel has to absorb excess energy available at one stage and has to make up the deficient energy at the other stage to keep to fluctuations of speed within permissible limits. This is done by choosing the suitable moment of inertia of the flywheel.

Punching press and flywheel.

Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

Let d_1 = Diameter of the hole punched,

t_1 = Thickness of the plate, and

τ_u = Ultimate shear stress for the plate material.

$\therefore$ Maximum shear force required for punching,

$$F_S = \text{Area sheared} \times \text{Ultimate shear stress} = \pi d_1 . t_1 \tau_u$$

It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.

$\therefore$ Work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1. The energy supplied by the motor to the crankshaft during actual punching operation,

$$E_2 = E_1\left(\frac{\theta_2 - \theta_1}{2\pi}\right)$$

$\therefore$ Balance energy required for punching

$$= E_1 - E_2 = E_1 - E_1\left(\frac{\theta_2 - \theta_1}{2\pi}\right) = E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

This energy is to be supplied by the flywheel by the decrease in its kinetic energy when its speed falls from maximum to minimum. Thus maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = E_1\left(1 - \frac{\theta_2 - \theta_1}{2\pi}\right)$$

The values of θ_1 and θ_2 may be determined only if the crank radius *(r)*, length of connecting rod *(l)* and the relative position of the job with respect to the crankshaft axis are known. In the absence of relevant data, we assume that

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}$$

where t = Thickness of the material to be punched,

s = Stroke of the punch = $2 \times$ Crank radius $= 2r$.

By using the suitable relation for the maximum fluctuation of energy (ΔE) as discussed in the previous articles, we can find the mass and size of the flywheel.

<u>(19) Cam dynamics:</u>

Mechanism provides a non-linear I/O relationship. Different mechanism like single or multi-degree of freedom, intermittent motion mechanisms and linkages etc. have different I/O Relationship. When we can not obtain a certain functions from the well known mechanisms, we use a cam mechanism. It is a one degree of freedom mechanism of two moving links. One is cam and the other is follower.

- Rigid and elastic body cam system.
- Analysis of eccentric cam
- Problems on Cam –follower system.

(20) Example Problems:

1) The variation of crankshaft torque of a four cylinder petrol engine may be approximately represented by taking the torque as zero for crank angles 0° and 180° and as 260 Nm for crank angles 20° and 45°, the intermediate portions of the torque graph being straight lines. The cycle is being repeated in every half revolution. The average speed is 600 rpm. Supposing that the engine drives a machine requiring constant torque, determine the mass of the flywheel of radius of gyration 250 mm, which must be provided so that the total variation of speed shall be one percent.

2) A single cylinder vertical engine has a bore of 300 mm and a stroke of 400 mm. The connecting rod is 1 m long and the mass of the reciprocating parts is 140 kg. on the expansion stroke, with the crank at 30° from the top dead center, the gas pressure is 0.7 MPa. If the engine runs at 250 rpm, determine (i) net force acting on the piston (ii) resultant load on the gudgeon pin (iii) thrust on the cylinder walls, and (iv) the speed above which, other things remaining the same, the gudgeon pin load would be reversed in direction.

3) A vertical double acting steam engine has a cylinder 300 mm diameter and 450 mm stroke and runs at 200 rpm. The reciprocating parts has a mass of 225 kg and the piston rod is 50 mm diameter. The connecting rod is 1.2 m long. When the crank has turned through 125° from the top dead center the steam pressure above the piston is 30 kN/m^2 and below the piston is 1.5 kN/m^2. Calculate

(i) Crank-pin effort and

(ii) The effective turning moment on the crank shaft.

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UNIT – II BALANCING

UNIT – II BALANCING

(1) Introduction:

Balancing is the process of eliminating or at least reducing the ground forces and/or moments. It is achieved by changing the location of the mass centres of links. Balancing of rotating parts is a well known problem. A rotating body with fixed rotation axis can be fully balanced i.e. all the inertia forces and moments. For mechanism containing links rotating about axis which are not fixed, force balancing is possible, moment balancing by itself may be possible, but both not possible. We generally try to do force balancing. A fully force balance is possible, but any action in force balancing severe the moment balancing.

(2) Balancing of rotating masses:

The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

(3) Static balancing:

The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of the rotation. This is the condition for static balancing.

(4) Dynamic balancing:

The net couple due to dynamic forces acting on the shaft is equal to zero. The algebraic sum of the moments about any point in the plane must be zero.

(5) Various cases of balancing of rotating masses:

- Balancing of a single rotating mass by single mass rotating in the same plane.
- Balancing of a single rotating mass by two masses rotating in the different plane.
- Balancing of a several masses rotating in single plane.
- Balancing of a several masses rotating in different planes.

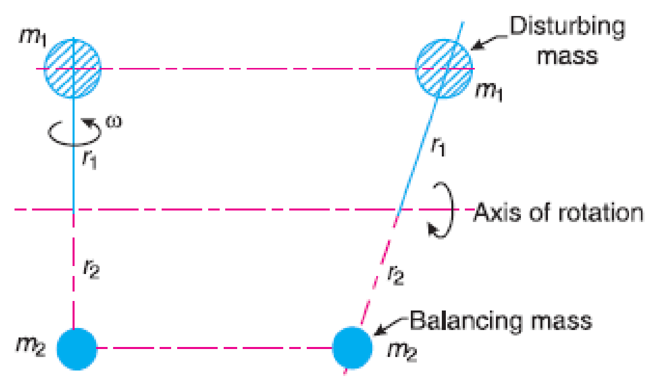
(6) Balancing of a single rotating mass by single mass rotating in the same plane:

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).

We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1 \quad \ldots (i)$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.



Balancing of a single rotating mass by a single mass rotating in the same plane.

Let r_2 = Radius of rotation of the balancing mass m_2 (*i.e.* distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2 \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$m_1 . \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Notes : 1. The product $m_2.r_2$ may be split up in any convenient way. But the radius of rotation of the balancing mass (m_2) is generally made large in order to reduce the balancing mass m_2.

2. The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

(7) Balancing of a single rotating mass by two masses rotating in the different plane:

We have discussed in the previous article that by introducing a single balancing mass in the same plane of rotation as that of disturbing mass, the centrifugal forces are balanced. In other words, the two forces are equal in magnitude and opposite in direction. But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for ***static balancing.***
2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero.

The conditions **(1)** and **(2)** together give ***dynamic balancing.*** The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

We shall now discuss both the above cases one by one.

Let l_1 = Distance between the planes A and L,
l_2 = Distance between the planes A and M, and
l = Distance between the planes L and M.

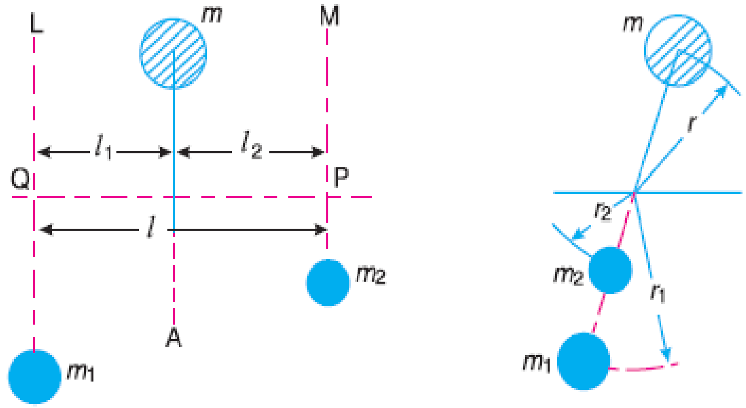


Fig. 21.2. Balancing of a single rotating mass by two rotating masses in different planes when the plane of single rotating mass lies in between the planes of two balancing masses.

We know that the centrifugal force exerted by the mass m in the plane A,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1 . \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore \quad m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \quad \ldots (i)$$

Now in order to find the magnitude of balancing force in the plane *L* (or the dynamic force at the bearing *Q* of a shaft), take moments about *P* which is the point of intersection of the plane *M* and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \quad \ldots (ii)$$

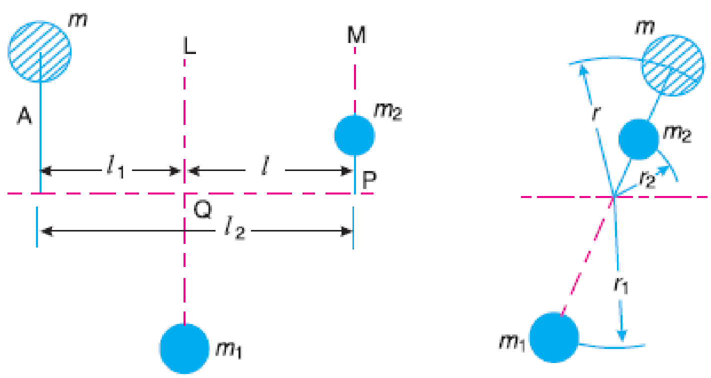
Similarly, in order to find the balancing force in plane *M* (or the dynamic force at the bearing *P* of a shaft), take moments about *Q* which is the point of intersection of the plane *L* and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\therefore \quad m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \quad \ldots (iii)$$

It may be noted that equation *(i)* represents the condition for static balance, but in order to achieve dynamic balance, equations *(ii)* or *(iii)* must also be satisfied.

2. *When the plane of the disturbing mass lies on one end of the planes of the balancing masses*



Balancing of a single rotating mass by two rotating masses in different planes, when the plane of single rotating mass lies at one end of the planes of balancing masses.

In this case, the mass *m* lies in the plane *A* and the balancing masses lie in the planes *L* and *M*, as shown in Fig. 21.3. As discussed above, the following conditions must be satisfied in order to balance the system, *i.e.*

$$F_C + F_{C2} = F_{C1} \quad \text{or} \quad m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

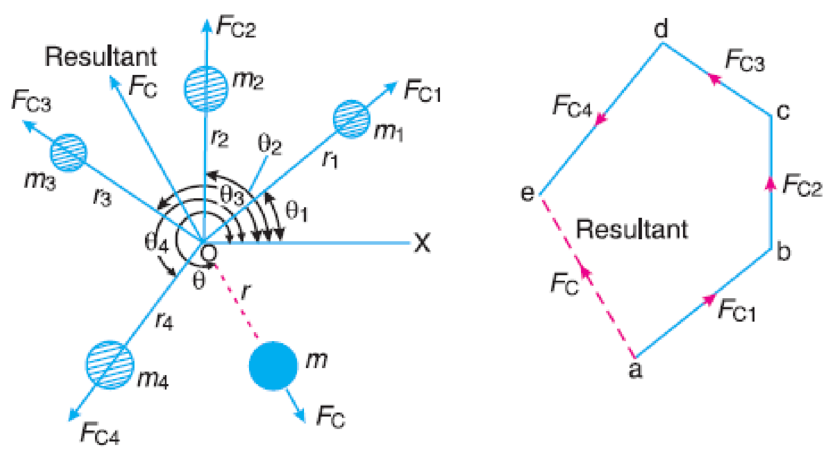
$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 . r_1 \quad \ldots (iv)$$

Now, to find the balancing force in the plane *L* (or the dynamic force at the bearing *Q* of a shaft), take moments about *P* which is the point of intersection of the plane *M* and the axis of rotation. Therefore

(8) Balancing of a several masses rotating in same plane:

Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let $\theta_1, \theta_2, \theta_3$ and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. . Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below :



(*a*) Space diagram. (*b*) Vector diagram.

Fig. 21.4. Balancing of several masses rotating in the same plane.

1. *Analytical method*

The magnitude and direction of the balancing mass may be obtained, analytically, as discussed below :

1. First of all, find out the centrifugal force* (or the product of the mass and its radius of rotation) exerted by each mass on the rotating shaft.

2. Resolve the centrifugal forces horizontally and vertically and find their sums, *i.e.* ΣH and ΣV . We know that

   Sum of horizontal components of the centrifugal forces,

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

and sum of vertical components of the centrifugal forces,

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

3. Magnitude of the resultant centrifugal force,

$$F_C = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

4. If θ is the angle, which the resultant force makes with the horizontal, then

$$\tan\theta = \Sigma V / \Sigma H$$

5. The balancing force is then equal to the resultant force, but in *opposite direction*.
6. Now find out the magnitude of the balancing mass, such that

$$F_C = m \cdot r$$

where m = Balancing mass, and
r = Its radius of rotation.

2. *Graphical method*

The magnitude and position of the balancing mass may also be obtained graphically as discussed below :

1. First of all, draw the space diagram with the positions of the several masses, as shown in Fig.
2. Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
3. Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc*, *cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).
4. Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig.
5. The balancing force is, then, equal to the resultant force, but in *opposite direction.*
6. Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m \cdot \omega^2 \cdot r = \text{Resultant centrifugal force}$$

or

$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

(9) Balancing of several masses rotating different plane:

When several masses revolve in different planes, they may be transferred to a *reference plane* (briefly written as *R.P.*), which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the reference plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and the reference plane. In order to have a complete balance of the several revolving masses in different planes, the following two conditions must be satisfied :

1. The forces in the reference plane must balance, *i.e.* the resultant force must be zero.
2. The couples about the reference plane must balance, *i.e.* the resultant couple must be zero.

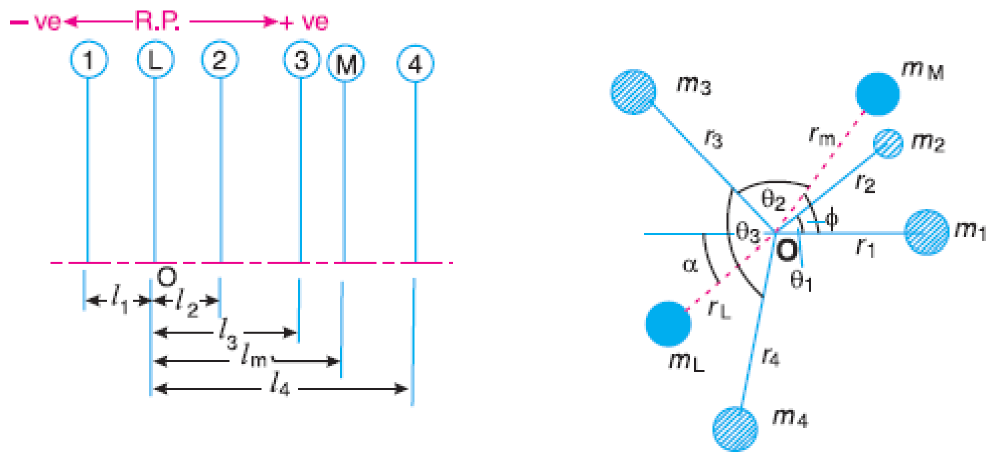
Diesel engine.

Let us now consider four masses m_1, m_2, m_3 and m_4 revolving in planes 1, 2, 3 and 4 respectively as shown in Fig. (*a*). The relative angular positions of these masses are shown in the end view [Fig. 21.7 (*b*)]. The magnitude of the balancing masses m_L and m_M in planes *L* and *M* may be obtained as discussed below :

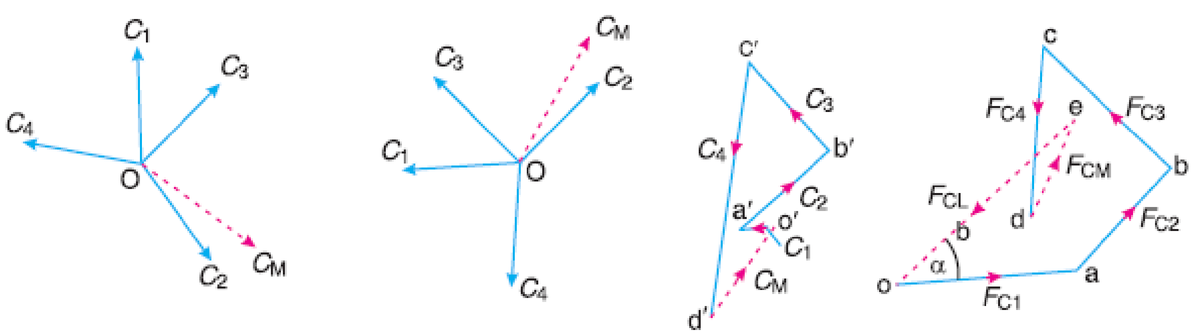
1. Take one of the planes, say *L* as the reference plane (*R.P.*). The distances of all the other planes to the left of the reference plane may be regarded as ***negative***, and those to the right as ***positive***.
2. Tabulate the data as shown in Table 21.1. The planes are tabulated in the same order in which they occur, reading from left to right.

Table 21.1

Plane (1)	*Mass (m)* (2)	*Radius(r)* (3)	*Cent.force* $\div \omega^2$ *(m.r)* (4)	*Distance from Plane L (l)* (5)	*Couple* $\div \omega^2$ *(m.r.l)* (6)
1	m_1	r_1	$m_1.r_1$	$-l_1$	$-m_1.r_1.l_1$
L(R.P.)	m_L	r_L	$m_L.r_L$	0	0
2	m_2	r_2	$m_2.r_2$	l_2	$m_2.r_2.l_2$
3	m_3	r_3	$m_3.r_3$	l_3	$m_3.r_3.l_3$
M	m_M	r_M	$m_M.r_M$	l_M	$m_M.r_M.l_M$
4	m_4	r_4	$m_4.r_4$	l_4	$m_4.r_4.l_4$



(*a*) Position of planes of the masses. (*b*) Angular position of the masses.



(*c*) Couple vector. (*d*) Couple vectors turned counter clockwise through a right angle. (*e*) Couple polygon. (*f*) Force polygon.

Fig. Balancing of several masses rotating in different planes.

3. A couple may be represented by a vector drawn perpendicular to the plane of the couple. The couple C_1 introduced by transferring m_1 to the reference plane through O is propor-

tional to $m_1.r_1.l_1$ and acts in a plane through Om_1 and perpendicular to the paper. The vector representing this couple is drawn in the plane of the paper and perpendicular to Om_1 as shown by OC_1 in Fig. (*c*). Similarly, the vectors OC_2, OC_3 and OC_4 are drawn perpendicular to Om_2, Om_3 and Om_4 respectively and in the plane of the paper.

4. The couple vectors as discussed above, are turned counter clockwise through a right angle for convenience of drawing as shown in Fig. 21.7 (*d*). We see that their relative positions remains unaffected. Now the vectors OC_2, OC_3 and OC_4 are parallel and in the same direction as Om_2, Om_3 and Om_4, while the vector OC_1 is parallel to Om_1 but in *opposite direction. Hence the ***couple vectors are drawn radially outwards for the masses on one side of the reference plane and radially inward for the masses on the other side of the reference plane.***

5. Now draw the couple polygon as shown in Fig. (*e*). The vector $d'o'$ represents the balanced couple. Since the balanced couple C_M is proportional to $m_M.r_M.l_M$, therefore

$$C_M = m_M \cdot r_M \cdot l_M = \text{vector } d'o' \quad \text{or} \quad m_M = \frac{\text{vector } d'o'}{r_M \cdot l_M}$$

From this expression, the value of the balancing mass m_M in the plane *M* may be obtained, and the angle of inclination ϕ of this mass may be measured from Fig. 21.7 (*b*).

6. Now draw the force polygon as shown in Fig. (*f*). The vector *eo* (in the direction from *e* to *o*) represents the balanced force. Since the balanced force is proportional to $m_L.r_L$, therefore,

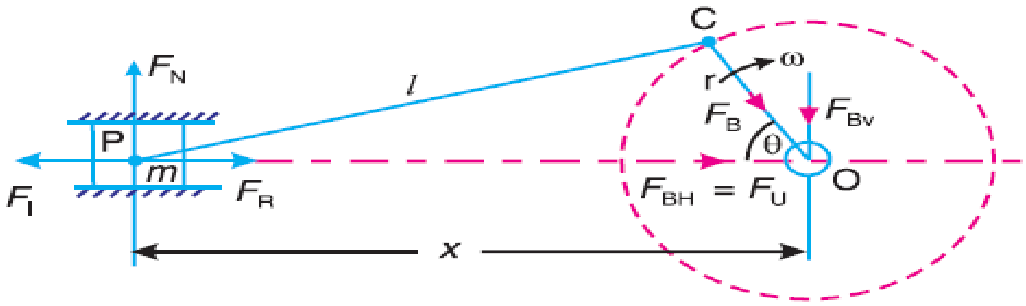
$$m_L \cdot r_L = \text{vector } eo \quad \text{or} \quad m_L = \frac{\text{vector } eo}{r_L}$$

From this expression, the value of the balancing mass m_L in the plane *L* may be obtained the angle of inclination α of this mass with the horizontal may be measured from Fig. (*b*).

(10) Balancing of Reciprocating masses:

Mass balancing encompasses a wide array of measures employed to obtain partial or complete compensation for the inertial forces and moments of inertia emanating from the crankshaft assembly. All masses are externally balanced when no free inertial forces or moments of inertia are transmitted through the block to the outside. However, the remaining internal forces and moments subject the engine mounts and block to various loads as well as deformities and vibratory stresses. The basic loads imposed by gas-based and inertial forces

(11) Primary and secondary unbalanced forces of reciprocating parts:



Reciprocating engine mechanism.

Let F_R = Force required to accelerate the reciprocating parts,

Let m = Mass of the reciprocating parts,

l = Length of the connecting rod PC,

r = Radius of the crank OC,

θ = Angle of inclination of the crank with the line of stroke PO,

ω = Angular speed of the crank,

n = Ratio of length of the connecting rod to the crank radius = l / r.

We have already discussed in Art. 15.8 that the acceleration of the reciprocating parts is approximately given by the expression,

$$a_R = \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

∴ Inertia force due to reciprocating parts or force required to accelerate the reciprocating parts,

$$F_I = F_R = \text{Mass} \times \text{acceleration} = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

We have discussed in the previous article that the horizontal component of the force exerted on the crank shaft bearing (*i.e.* F_{BH}) is equal and opposite to inertia force (F_I). This force is an unbalanced one and is denoted by F_U.

∴ Unbalanced force,

$$F_U = m \cdot \omega^2 \cdot r\left(\cos\theta + \frac{\cos 2\theta}{n}\right) = m.\omega^2 \cdot r\cos\theta + m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n} = F_P + F_S$$

The expression $(m \cdot \omega^2 \cdot r\cos\theta)$ is known as ***primary unbalanced force*** and $\left(m \cdot \omega^2 \cdot r \times \frac{\cos 2\theta}{n}\right)$ is called ***secondary unbalanced force***.

(12) Balancing of single cylinder engine:

A single cylinder engine produces three main vibrations. In describing them we will assume that the cylinder is vertical. Firstly, in an engine with no balancing counterweights, there would be an enormous vibration produced by the change in momentum of the piston, gudgeon pin, connecting rod and crankshaft once every revolution. Nearly all single-cylinder crankshafts incorporate balancing weights to reduce this. While these weights can balance the crankshaft completely, they cannot completely balance the motion of the piston, for two reasons. The first reason is that the balancing weights have horizontal motion as well as vertical motion, so balancing the purely vertical motion of the piston by a crankshaft weight adds a horizontal vibration. The second reason is that, considering now the vertical motion only, the smaller piston end of the connecting rod (little end) is closer to the larger crankshaft end (big end) of the connecting rod in mid-stroke than it is at the top or bottom of the stroke, because of the connecting rod's angle. So during the 180° rotation from mid-stroke through top-dead-center and back to mid-stroke the minor contribution to the piston's up/down movement from the connecting rod's change of angle has the same direction as the major contribution to the piston's up/down movement from the up/down movement of the crank pin. By contrast, during the 180° rotation from mid-stroke through bottom-dead-center and back to mid-stroke the minor contribution to the piston's up/down movement from the connecting rod's change of angle has the opposite direction of the major contribution to the piston's up/down movement from the up/down movement of the crank pin. The piston therefore travels faster in the top half of the cylinder than it does in the bottom half, while the motion of the crankshaft weights is sinusoidal. The vertical motion of the piston is therefore not quite the same as that of the balancing weight, so they can't be made to cancel out completely.

Secondly, there is a vibration produced by the change in speed and therefore kinetic energy of the piston. The crankshaft will tend to slow down as the piston speeds up and absorbs energy, and to speed up again as the piston gives up energy in slowing down at the top and bottom of the stroke. This vibration has twice the frequency of the first vibration, and absorbing it is one function of the flywheel.

Thirdly, there is a vibration produced by the fact that the engine is only producing power during the power stroke. In a four-stroke engine this vibration will have half the frequency of the first vibration, as the cylinder fires once every two revolutions. In a two-stroke engine, it will have the same frequency as the first vibration. This vibration is also absorbed by the flywheel.

(13) Balancing of inertial forces in the multi-cylinder engine:

In multi-cylinder engines the mutual counteractions of the various components in the Crank shaft assembly are one of the essential factors determining the selection of the Crank shafts configuration and with it the design of the engine itself. The inertial forces are Balanced if the common centre of gravity for all moving crankshaft-assembly components lies at the crankshaft's midpoint, i.e. if the crankshaft is symmetrical (as viewed from the front). The crankshaft's symmetry level can be defined using geometrical representations of 1st- and 2nd-order forces (star diagrams). The 2nd order star diagram for the four-cylinder in-line engine is asymmetrical, meaning that this order is characterized by substantial free inertial Forces. These forces can be balanced using two countershafts rotating in opposite directions at double the rate of the crankshaft (Lanchester system).

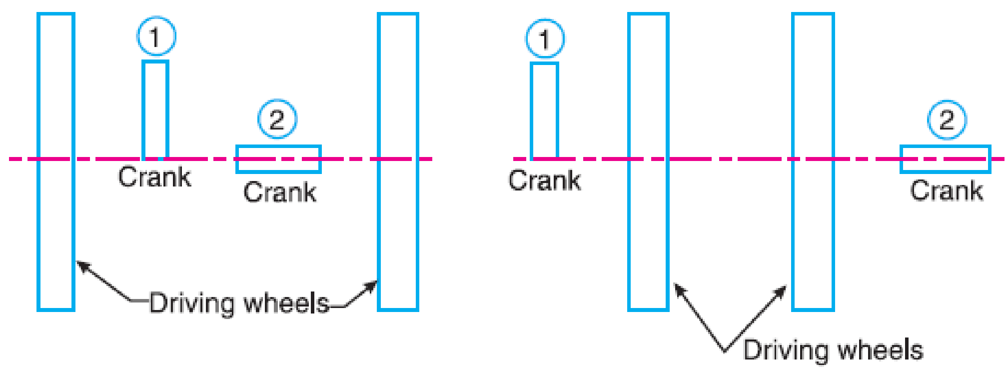
(14) Partial balancing of Locomotives:

The locomotives, usually, have two cylinders with cranks placed at right angles to each other in order to have uniformity in turning moment diagram. The two cylinder locomotives may be classified as :

1. Inside cylinder locomotives ; and 2. Outside cylinder locomotives.

In the ***inside cylinder locomotives***, the two cylinders are placed in between the planes of two driving wheels as shown in Fig. (*a*) ; whereas in the ***outside cylinder locomotives***, the two cylinders are placed outside the driving wheels, one on each side of the driving wheel, as shown in Fig. (*b*). The locomotives may be

(*a*) Single or uncoupled locomotives ; and (*b*) Coupled locomotives.



(*a*) Inside cylinder locomotives. (*b*) Outside cylinder locomotives.

(15) Variation of Tractive force:

The resultant unbalanced force due to the cylinders, along the line of stroke, is known as tractive force.

(16) Swaying Couple:

The couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clock wise and anticlockwise directions. Hence the couple is known as swaying couple.

The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line *YY* between the cylinders as shown in Fig. 22.5.

This couple has swaying effect about a vertical axis, and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as ***swaying couple.***

Let a = Distance between the centre lines of the two cylinders.

∴ Swaying couple

$$= (1-c)m.\omega^2.r\cos\theta\times\frac{a}{2}$$

$$-(1-c)m.\omega^2.r\cos(90°+\theta)\frac{a}{2}$$

$$= (1-c)m.\omega^2.r\times\frac{a}{2}(\cos\theta+\sin\theta)$$

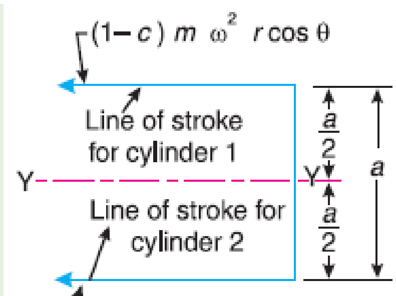


Fig. Swaying couple.

The swaying couple is maximum or minimum when $(\cos\theta+\sin\theta)$ is maximum or minimum. For $(\cos\theta+\sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta+\sin\theta)=0 \quad \text{or} \quad -\sin\theta+\cos\theta=0 \quad \text{or} \quad -\sin\theta=-\cos\theta$$

$$\therefore \quad \tan\theta=1 \quad \text{or} \quad \theta=45° \quad \text{or} \quad 225°$$

Thus, the swaying couple is maximum or minimum when θ = 45° or 225°.

∴ Maximum and minimum value of the swaying couple

$$= \pm(1-c)m.\omega^2.r\times\frac{a}{2}(\cos 45°+\sin 45°) = \pm\frac{a}{\sqrt{2}}(1-c)m.\omega^2.r$$

(17) Hammer blow:

The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as Hammer blow.

We have already discussed that the maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as ***hammer blow***.

We know that the unbalanced force along the perpendicular to the line of stroke due to the balancing mass *B*, at a radius *b*, in order to balance reciprocating parts only is $B.\omega^2.b \sin\theta$. This force will be maximum when $\sin\theta$ is unity, *i.e.* when $\theta = 90°$ or $270°$.

$\therefore$ Hammer blow $= B.\omega^2.b$ (Substituiting $\sin\theta = 1$)

The effect of hammer blow is to cause the variation in pressure between the wheel and the rail. This variation is shown in Fig. 22.6, for one revolution of the wheel.

Let *P* be the downward pressure on the rails (or static wheel load).

$\therefore$ Net pressure between the wheel and the rail

$$= P \pm B.\omega^2.b$$

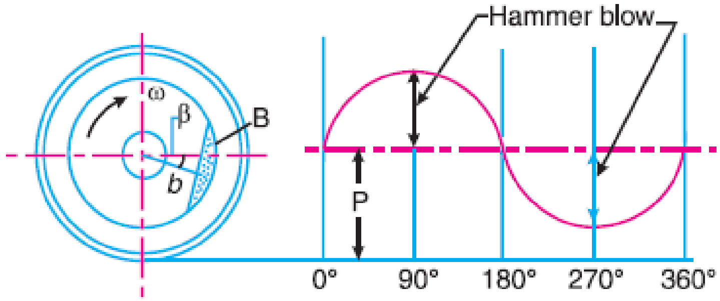


Fig. Hammer blow.

If $(P - B.\omega^2.b)$ is ***negative***, then the wheel will be lifted from the rails. Therefore the limiting condition in order that the wheel does not lift from the rails is given by

$$P = B.\omega^2.b$$

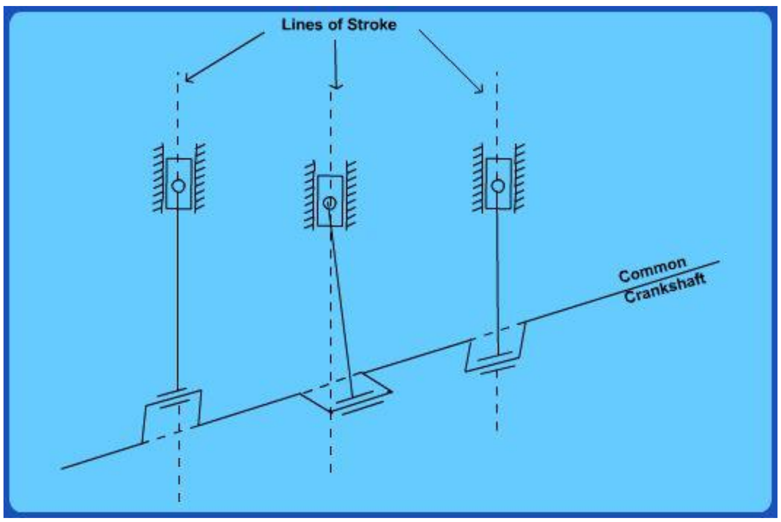
and the permissible value of the angular speed,

$$\omega = \sqrt{\frac{P}{B.b}}$$

(18) Balancing of Inline engines:

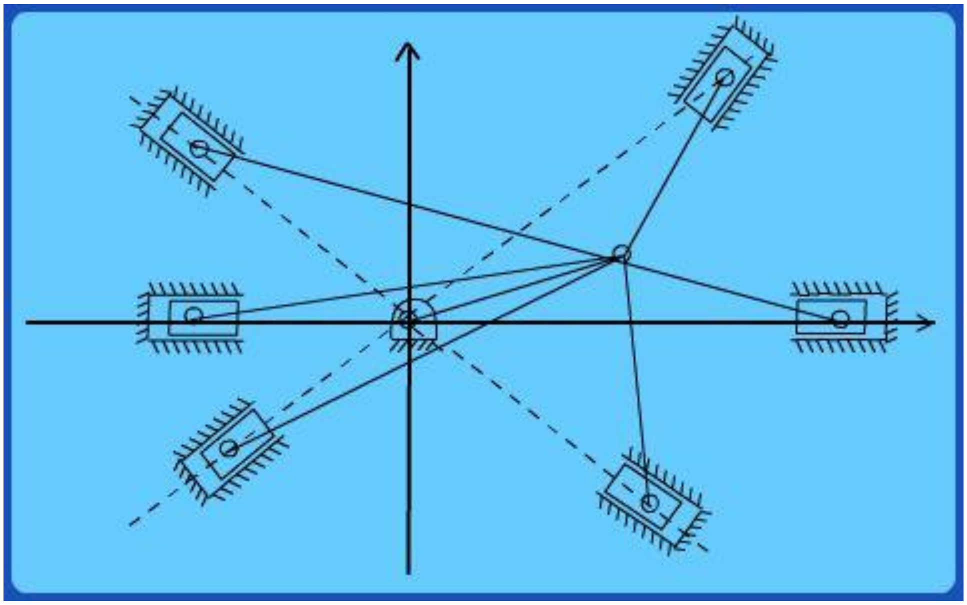
An in-line engine is one wherein all the cylinders are arranged in a single line, one behind the other as schematically indicated in Fig. Many of the passenger cars found on Indian roads such as Maruti 800, Zen, Santro, Honda City, Honda CR-V, and Toyota Corolla all have four cylinder in-line engines. Thus this is a commonly employed engine and it is of interest to us to understand the analysis of its state of balance.

For the sake of simplicity of analysis, we assume that all the cylinders are identical viz., r, ℓ, m_{rec} and m_{rot} are same. Further we assume that the rotating masses have been balanced out for all cylinders and we are left with only the forces due to the reciprocating masses.



(19) Balancing of radial engines:

A radial engine is one in which all the cylinders are arranged circumferentially as shown in Fig.These engines were quite popularly used in aircrafts during World War II. Subsequent developments in steam/gas turbines led to the near extinction of these engines. However it is still interesting to study their state of balance in view of some elegant results we shall discuss shortly. Our method of analysis remains identical to the previous case i.e., we proceed with the assumption that all cylinders are identical and the cylinders are spaced at uniform interval $\left(\frac{2\pi}{n}\right)$ around the circumference.



(20) Example Problems:

1) A shaft carries four rotating masses A, B, C and D which are completely balanced. The masses B, C and D are 50 kg, 80 kg and 70 kg respectively. The masses C and D make angles of 90° and 195° respectively with mass B in the same sense. The masses A, B, C and D are concentrated at radius 75 mm, 100 mm, 50 mm and 80 mm respectively. The plane of rotation of masses B and C are 250 mm apart. Determine (i) the magnitude of mass A and its angular position and (ii) the position planes A and D.

2) The cranks of a two cylinder, uncoupled inside cylinder locomotive are at right angles and are 325 mm long. The cylinders are 675 mm apart. The rotating mass per cylinders are 200 kg at crank pin and the mass of the reciprocating parts per cylinder is 240 kg. The wheel center lines are 1.5 m apart. The whole of the rotating and two thirds of the reciprocating masses are to be balanced and the balance masses are to be placed in the planes of the rotation of the driving wheels at a radius of 800 mm. Find (i) the magnitude and direction of the balancing masses. (ii) the magnitude of hammer blow (iii) variation in tractive force and (iv) maximum swaying couple at a crank speed of 240 rpm.

3) (i) Four masses m_1, m_2, m_3 and m_4 attached to a rotating shaft on the same plane are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if the radius of rotation is 0.2 m.

(ii) Explain with neat sketches, balancing of single revolving mass, by masses in two different planes in a rotating system

INDEX

UNIT –III FREE VIBRATIONS

UNIT –III FREE VIBRATIONS

(1) Introduction:

When a system is subjected to an initial disturbance and then left free to vibrate on its own, the resulting vibrations are referred to as free vibrations .**Free vibration** occurs when a mechanical system is set off with an initial input and then allowed to vibrate freely. Examples of this type of vibration are pulling a child back on a swing and then letting go or hitting a tuning fork and letting it ring. The mechanical system will then vibrate at one or more of its "natural frequencies" and damp down to zero.

(2) Basic elements of vibration system:

- Mass or Inertia
- Springiness or Restoring element
- Dissipative element (often called damper)
- External excitation

(3) Causes of vibration:

Unbalance: This is basically in reference to the rotating bodies. The uneven distribution of mass in a rotating body contributes to the unbalance. A good example of unbalance related vibration would be the "vibrating alert" in our mobile phones. Here a small amount of unbalanced weight is rotated by a motor causing the vibration which makes the mobile phone to vibrate. You would have experienced the same sort of vibration occurring in your front loaded washing machines that tend to vibrate during the "spinning" mode.

Misalignment: This is an other major cause of vibration particularly in machines that are driven by motors or any other prime movers.

Bent Shaft: A rotating shaft that is bent also produces the the vibrating effect since it losses it rotation capability about its center.

Gears in the machine: The gears in the machine always tend to produce vibration, mainly due to their meshing. Though this may be controlled to some extent, any problem in the gearbox tends to get enhanced with ease.

Bearings: Last but not the least, here is a major contributor for vibration. In majority of the cases every initial problem starts in the bearings and propagates to the rest of the members of the machine. A bearing devoid of lubrication tends to wear out fast and fails quickly, but before this is noticed it damages the remaining components in the machine and an initial look would seem as if something had gone wrong with the other components leading to the bearing failure.

(4) Effects of vibration:

(a)Bad Effects:

The presence of vibration in any mechanical system produces unwanted noise, high stresses, poor reliability, wear and premature failure of parts. Vibrations are a great source of human discomfort in the form of physical and mental strains.

(b)Good Effects:

A vibration does useful work in musical instruments, vibrating screens, shakers, relive pain in physiotherapy.

(5) Methods of reduction of vibration:

- -unbalance is its main cause, so balancing of parts is necessary.
- -using shock absorbers.
- -using dynamic vibration absorbers.
- -providing the screens (if noise is to be reduced)

(6) Types of vibratory motion:

- Free Vibration
- Forced Vibration

(7) Terms used vibratory motion:

(a)Time period (or)period of vibration:

It is the time taken by a vibrating body to repeat the motion itself.time period is usually expressed in seconds.

(b) Cycle:

It is the motion completed in one time period.

(c) Periodic motion:

A motion which repeats itself after equal interval of time.

(d)Amplitude (X)

The maximum displacement of a vibrating body from the mean position.it is usually expressed in millimeter.

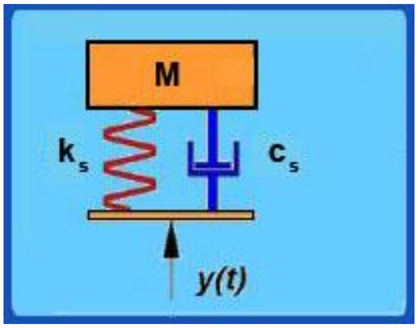
(e) Frequency (f)

The number of cycles completed in one second is called frequency

(8) Degrees of freedom:

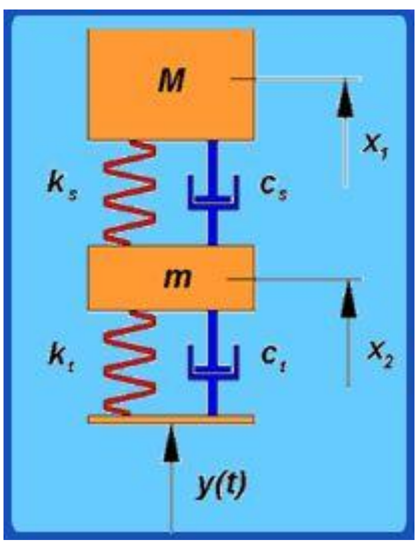
The minimum number of independent coordinates required to specify the motion of a system at any instant is known as D.O.F of the system.

(9) Single degree of freedom system:



The system shown in this figure is what is known as a Single Degree of Freedom system. We use the term degree of freedom to refer to the number of coordinates that are required to specify completely the configuration of the system. Here, if the position of the mass of the system is specified then accordingly the position of the spring and damper are also identified. Thus we need just one coordinate (that of the mass) to specify the system completely and hence it is known as a single degree of freedom system.

(10) Two degree of freedom system:



A two degree of freedom system With reference to automobile applications, this is referred as “quarter car” model. The bottom mass refers to mass of axle, wheel etc components which are below the suspension spring and the top mass refers to the mass of the portion of the car and passenger. Since we need to specify both the top and bottom mass positions to completely specify the system, this becomes a two degree of freedom system.

(11) Types of Vibratory motion:

The following types of vibratory motion are important from the subject point of view :

1. *Free or natural vibrations.* When no external force acts on the body, after giving it an initial displacement, then the body is said to be under ***free or natural vibrations.*** The frequency of the free vibrations is called ***free or natural frequency.***

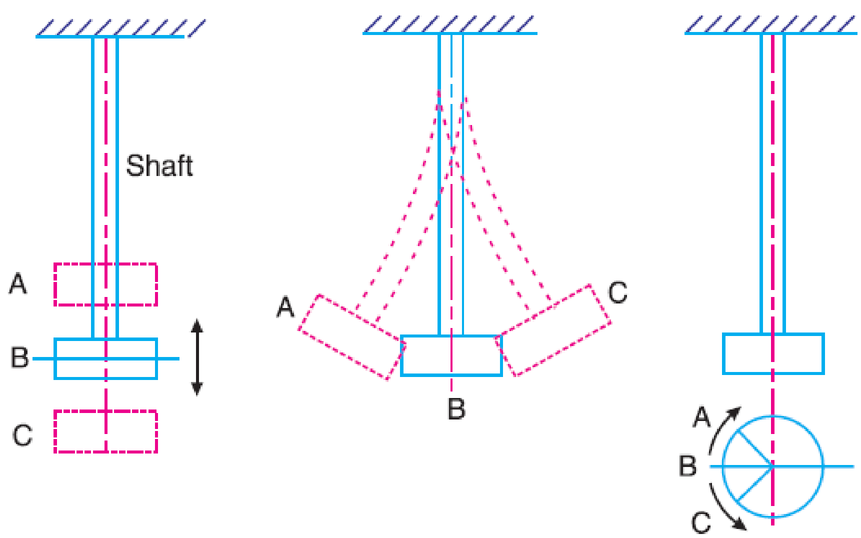
2. *Forced vibrations.* When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations.*** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

Note : When the frequency of the external force is same as that of the natural vibrations, resonance takes place.

3. *Damped vibrations.* When there is a reduction in amplitude over every cycle of vibration, the motion is said to be ***damped vibration.*** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

(12) Types of Vibration:

(a)Longitudinal vibration (b)Transverse Vibration (c)Torsional Vibration.



B = Mean position ; A and C = Extreme positions.

(a) Longitudinal vibrations. (b) Transverse vibrations. (c) Torsional vibrations.

(13) Longitudinal Vibration:

When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations known as longitudinal vibrations.

(14) Free undamped longitudinal vibrations;

When a body is allowed to vibrate on its own, after giving it an initial displacement, then the ensuring vibrations are known as free or natural vibrations. When the vibrations take place parallel to the axis of constraint and no damping is provided, then it is called free undamped longitudinal vibrations.

(15) Natural frequency of free undamped longitudinal vibration:

(15.a) Equilibrium method or Newton's method:

Consider a constraint (*i.e.* spring) of negligible mass in an unstrained position, .

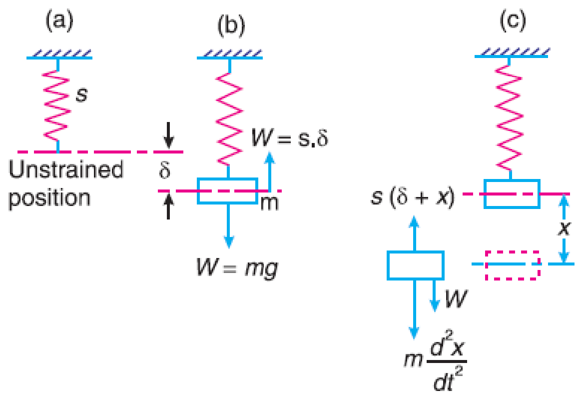
Let s = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/m.

m = Mass of the body suspended from the constraint in kg,

W = Weight of the body in newtons = $m.g$,

δ = Static deflection of the spring in metres due to weight W newtons, and

x = Displacement given to the body by the external force, in metres.



Natural frequency of free longitudinal vibrations.

In the equilibrium position, as shown in Fig. 23.2 (*b*), the gravitational pull $W = m.g$, is balanced by a force of spring, such that $W = s.\delta$.

Since the mass is now displaced from its equilibrium position by a distance x, as shown in Fig. (*c*), and is then released, therefore after time t,

Restoring force $= W - s(\delta + x) = W - s.\delta - s.x$

$= s.\delta - s.\delta - s.x = -s.x$ . . . ($\because W = s.\delta$) . . . (*i*)

and Accelerating force = Mass × Acceleration

$$= m \times \frac{d^2x}{dt^2}$$ . . . (Taking downward force as positive) . . . (*ii*)

Equating equations (*i*) and (*ii*), the equation of motion of the body of mass m after time t is

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \qquad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \qquad \ldots (iii)$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \qquad \ldots (iv)$$

Comparing equations (*iii*) and (*iv*), we have

$$\omega = \sqrt{\frac{s}{m}}$$

$\therefore$ Time period, $$t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$$

and natural frequency, $$f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \qquad \ldots (\because\ m.g = s.\delta)$$

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}}\text{Hz}$$

Note : The value of static deflection δ may be found out from the given conditions of the problem. For longitudinal vibrations, it may be obtained by the relation,

$$\frac{\text{Stress}}{\text{Strain}} = E \quad \text{or} \quad \frac{W}{A}\times\frac{l}{\delta} = E \quad \text{or} \quad \delta = \frac{W.l}{E.A}$$

where δ = Static deflection *i.e.* extension or compression of the constraint,

W = Load attached to the free end of constraint,

l = Length of the constraint,

E = Young's modulus for the constraint, and

A = Cross-sectional area of the constraint.

(15.b)Energy Method

In free vibrations, no energy is transferred into the system or from the system. Therefore, the total energy (sum of KE and PE)is constant and is same all the times.

We know that the kinetic energy is due to the motion of the body and the potential energy is with respect to a certain datum position which is equal to the amount of work required to move the body from the datum position. In the case of vibrations, the datum position is the mean or equilibrium position at which the potential energy of the body or the system is zero.

In the free vibrations, no energy is transferred to the system or from the system. Therefore the summation of kinetic energy and potential energy must be a constant quantity which is same at all the times. In other words,

$$\therefore \quad \frac{d}{dt}(K.E. + P.E.) = 0$$

We know that kinetic energy,

$$K.E. = \frac{1}{2} \times m \left(\frac{dx}{dt} \right)^2$$

This industrial compressor uses compressed air to power heavy-duty construction tools. Compressors are used for jobs, such as breaking up concrete or paving, drilling, pile driving, sand-blasting and tunnelling. A compressor works on the same principle as a pump. A piston moves backwards and forwards inside a hollow cylinder, which compresses the air and forces it into a hollow chamber. A pipe or hose connected to the chamber channels the compressed air to the tools.

Note : This picture is given as additional information and is not a direct example of the current chapter.

and potential energy, $$P.E. = \left(\frac{0 + s.x}{2} \right) x = \frac{1}{2} \times s.x^2$$

. . . ($\because$ *P.E.* = Mean force × Displacement)

$$\therefore \quad \frac{d}{dt} \left[\frac{1}{2} \times m \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} \times s.x^2 \right] = 0$$

$$\frac{1}{2} \times m \times 2 \times \frac{dx}{dt} \times \frac{d^2x}{dt^2} + \frac{1}{2} \times s \times 2x \times \frac{dx}{dt} = 0$$

or $$m \times \frac{d^2x}{dt^2} + s.x = 0 \quad \text{or} \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0$$ . . . (Same as before)

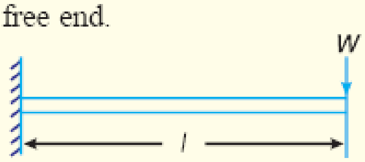
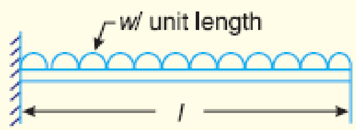
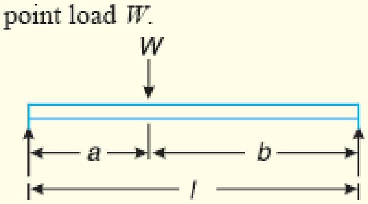
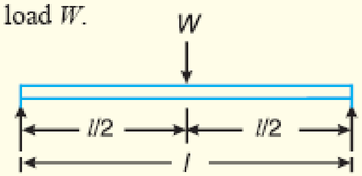
The time period and the natural frequency may be obtained as discussed in the previous method.

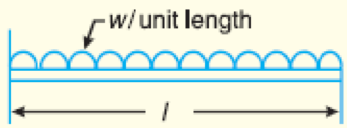
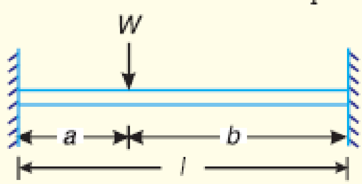
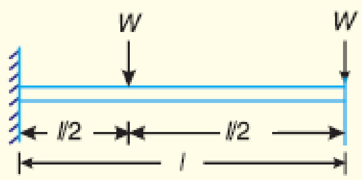
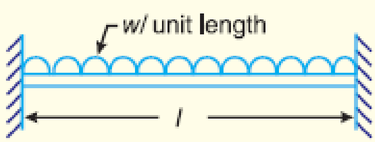
(c)Rayleigh's method

In this method, the maximum kinetic energy at mean position is made equal to the maximum potential energy at the extreme position.

(16) Equivalent stiffness of spring.

(1) Springs in series
(2) Springs in parallel
(3) Combined springs
(4) Inclined springs

S.No.	Type of beam	Deflection (δ)
1.	Cantilever beam with a point load W at the free end. 	$\delta = \frac{Wl^3}{3EI}$ (at the free end)
2.	Cantilever beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{wl^4}{8EI}$ (at the free end)
3.	Simply supported beam with an eccentric point load W. 	$\delta = \frac{Wa^2b^2}{3EIl}$ (at the point load)
4.	Simply supported beam with a central point load W. 	$\delta = \frac{Wl^3}{48EI}$ (at the centre)

S.No.	Type of beam	Deflection (δ)
5.	Simply supported beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{5}{384} \times \frac{wl^4}{EI}$ (at the centre)
6.	Fixed beam with an eccentric point load W. 	$\delta = \frac{Wa^3b^3}{3EIl}$ (at the point load)
7.	Fixed beam with a central point load W. 	$\delta = \frac{Wl^3}{192EI}$ (at the centre)
8.	Fixed beam with a uniformly distributed load of w per unit length. 	$\delta = \frac{wl^4}{384EI}$ (at the centre)

(17) Damping:

It is the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibrations.

(18) Types of damping:

(1) Viscous damping
(2) Dry friction or coulomb damping
(3) Solid damping or structural damping
(4) Slip or interfacial damping.

(19) Damping Coefficient:

The damping force per unit velocity is known as damping coefficient.

(20) Equivalent damping coefficient:

Dampers may be connected either in series or in parallel to provide required damping.

(21) Damped Vibration:

The vibrations associated with this resistance are known as damped vibrations.

(22) Damping factor:

Damping factor can be defined as the ratio of actual damping coefficient to critical damping coefficient.

The ratio of the actual damping coefficient (c) to the critical damping coefficient (c_c) is known as ***damping factor* or *damping ratio*.** Mathematically,

$$\text{Damping factor} = \frac{c}{c_c} = \frac{c}{2m.\omega_n} \qquad \ldots (\because c_c = 2\pi.\omega_n)$$

The damping factor is the measure of the relative amount of damping in the existing system with that necessary for the critical damped system.

Thus mainly three cases arise depending on the value of ξ

$\xi > 1 \Leftrightarrow$ *Overdamped System*

$\xi = 1 \Leftrightarrow$ *Critically damped System*

$\xi < 1 \Leftrightarrow$ *Underdamped System*

When $\xi \geq 1$ the system undergoes aperiodically decaying motion and hence such systems are said to be **Overdamped Systems**.

An example of such a system is a door damper – when we open a door and enter a room, we want the door to gradually close rather than exhibit oscillatory motion and bang into the person entering the room behind us! So the damper is designed such that $\xi \geq 1$

Critically damped motion ($\xi = 1$ a hypothetical borderline case separating oscillatory decay from a periodic decay) I the fastest decaying aperiodic motion.

When "ξ< 1", x(t) is a damped sinusoid and the system exhibits a vibratory motion whose amplitude keeps diminishing. This is the most common vibration case and we will spend most of our time studying such systems. These are referred to as **Underdamped systems**.

(23) Logarithmic decrement:

It is defined as the natural logarithm of ratio of any two successive amplitudes of an under damped system. It is a dimensionless quantity.

We define Damping factor ξ as

$$\xi = \frac{c}{2\sqrt{km}}$$

$$\text{such that } \frac{4km}{c^2} = \frac{1}{\xi^2}$$

$$\text{And} \quad \xi = \frac{c}{2m\omega_n}$$

$$\therefore \frac{c}{2m} = \xi\omega_n$$

(24) Transverse Vibration:

When the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft, then the vibrations known as transverse vibrations.

Consider a shaft of negligible mass, whose one end is fixed and the other end carries a body of weight W, as shown in Fig. 23.3.

Let s = Stiffness of shaft,

δ = Static deflection due to weight of the body,

x = Displacement of body from mean position after time t.

m = Mass of body = W/g

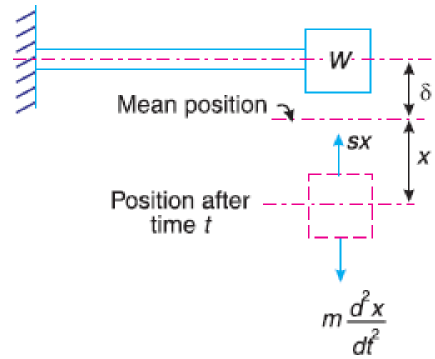


Fig. 23.3. Natural frequency of free transverse vibrations.

As discussed in the previous article,

$$\text{Restoring force} = -s.x \qquad \ldots (i)$$

$$\text{and accelerating force} = m \times \frac{d^2x}{dt^2} \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*), the equation of motion becomes

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \qquad \ldots (\text{Same as before})$$

Hence, the time period and the natural frequency of the transverse vibrations are same as that of longitudinal vibrations. Therefore

$$\text{Time period,} \quad t_p = 2\pi\sqrt{\frac{m}{s}}$$

$$\text{and natural frequency,} \quad f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

Note : The shape of the curve, into which the vibrating shaft deflects, is identical with the static deflection curve of a cantilever beam loaded at the end. It has been proved in the text book on Strength of Materials, that the static deflection of a cantilever beam loaded at the free end is

$$\delta = \frac{Wl^3}{3EI} \text{ (in metres)}$$

where W = Load at the free end, in newtons,

l = Length of the shaft or beam in metres,

E = Young's modulus for the material of the shaft or beam in N/m^2, and

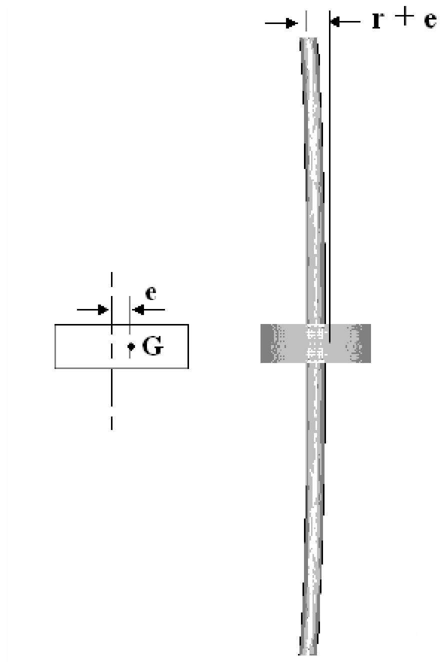
I = Moment of inertia of the shaft or beam in m^4.

(25) Whirling speed of shaft:

The speed, at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.

No shaft can ever be perfectly straight or perfectly balanced. When an element of mass is a distance from the axis of rotation, centrifugal force, will tend to pull the mass outward. The elastic properties of the shaft will act to restore the “straightness”. If the frequency of rotation is equal to one of the resonant frequencies of the shaft, whirling will occur. In order to save the machine from failure, operation at such whirling speeds must be avoided.

When a shaft rotates, it may well go into transverse oscillations. If the shaft is out of balance, the resulting centrifugal force will induce the shaft to vibrate. When the shaft rotates at a speed equal to the natural frequency of transverse oscillations, this vibration becomes large and shows up as a whirling of the shaft. It also occurs at multiples of the resonant speed. This can be very damaging to heavy rotary machines such as turbine generator sets and the system must be carefully balanced to reduce this effect and designed to have a natural frequency different to the speed of rotation. When starting or stopping such machinery, the critical speeds must be avoided to prevent damage to the bearings and turbine blades. Consider a weightless shaft as shown with a mass M at the middle. Suppose the centre of the mass is not on the centre line.



The whirling frequency of a symmetric cross section of a given length between two points is given by:

$$N = 94.25\sqrt{\frac{E\ I}{m\ L^3}} \text{ RPM}$$

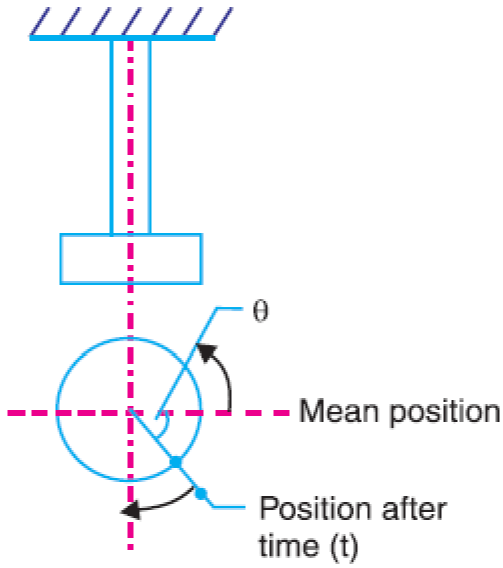
Where E = young's modulus, I = Second moment of area, m = mass of the shaft, L= length of the shaft between points

A shaft with weights added will have an angular velocity of N (rpm) equivalent as follows:

$$\frac{1}{N_N^2} = \frac{1}{N_A^2} + \frac{1}{N_B^2} + \cdots + \frac{1}{N_n^2}$$

(26) Torsional Vibration:

When the particles of the shaft or disc move in a circle about the axis of the shaft, then the vibrations known as tensional vibration



Natural frequency of free torsional vibrations.

Let θ = Angular displacement of the shaft from mean position after time t in radians,

m = Mass of disc in kg,

I = Mass moment of inertia of disc in kg-m^2 $= m.k^2$,

k = Radius of gyration in metres,

q = Torsional stiffness of the shaft in N-m.

$\therefore$ Restoring force $= q.\theta$ ... (*i*)

and accelerating force $= I \times \frac{d^2\theta}{dt^2}$ ... (*ii*)

Equating equations (*i*) and (*ii*), the equation of motion is

$$I \times \frac{d^2\theta}{dt^2} = -q.\theta$$

or
$$I \times \frac{d^2\theta}{dt^2} + q.\theta = 0$$

$$\therefore \quad \frac{d^2\theta}{dt^2} + \frac{q}{I} \times \theta = 0 \qquad \text{. . . } (iii)$$

The fundamental equation of the simple harmonic motion is

$$\frac{d^2\theta}{dt^2} + \omega^2.x = 0 \qquad \text{. . . } (iv)$$

Comparing equations (*iii*) and (*iv*),

$$\omega = \sqrt{\frac{q}{I}}$$

$\therefore$ Time period, $t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{q}}$

and natural frequency , $f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{q}{I}}$

The value of the torsional stiffness q may be obtained from the torsion equation,

$$\frac{T}{J}=\frac{C.\theta}{l} \quad \text{or} \quad \frac{T}{\theta}=\frac{C.J}{l}$$

$$q=\frac{C.J}{l} \qquad \ldots \left(\because \frac{T}{\theta}=q\right)$$

where C = Modulus of rigidity for the shaft material,

J = Polar moment of inertia of the shaft cross-section,

$= \frac{\pi}{32} d^4$; d is the diameter of the shaft, and

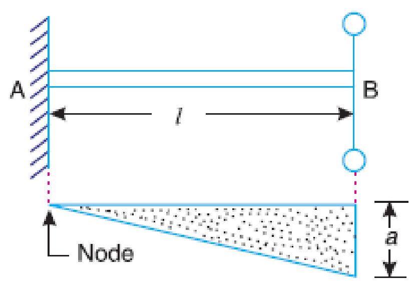
l = Length of the shaft.

(27) Torsional vibration of a single rotor system:

We have already discussed that for a shaft fixed at one end and carrying a rotor at the free end as shown in Fig. the natural frequency of torsional vibration,

$$f_n \quad \frac{1}{2}\sqrt{\frac{q}{I}} \quad \frac{1}{2}\sqrt{\frac{C.J}{l.I}}$$

$$\ldots \quad q \quad \frac{C.J}{l}$$



Free torsional vibrations of a single rotor system.

where C = Modulus of rigidity for shaft material,

J = Polar moment of inertia of shaft

$= \frac{}{32} d^4$

d = Diameter of shaft,

l = Length of shaft,

m = Mass of rotor,

k = Radius of gyration of rotor, and

I = Mass moment of inertia of rotor = $m.k^2$

A little consideration will show that the amplitude of vibration is zero at A and maximum at B, as shown in Fig. It may be noted that the point or the section of the shaft whose amplitude of torsional vibration is zero, is known as ***node***. In other words, at the node, the shaft remains unaffected by the vibration.

(28) Torsional vibration of a two rotor system:

Consider a two rotor system as shown in Fig. It consists of a shaft with two rotors at its ends. In this system, the torsional vibrations occur only when the two rotors *A* and *B* move in opposite directions *i.e.* if *A* moves in anticlockwise direction then *B* moves in clockwise direction at the same instant and *vice versa*. It may be noted that the two rotors must have the same frequency.

We see from Fig. that the node lies at point *N*. This point can be safely assumed as a fixed end and the shaft may be considered as two separate shafts *N P* and *N Q* each fixed to one of its ends and carrying rotors at the free ends.

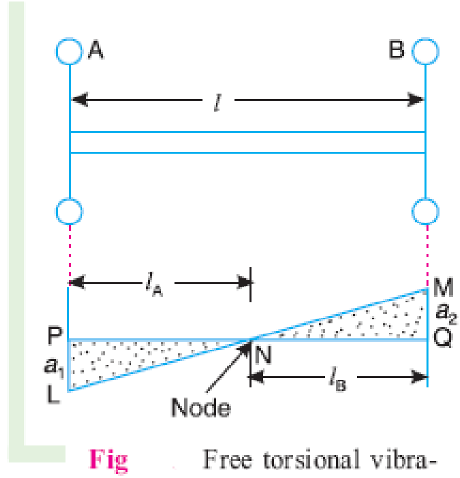


Fig Free torsional vibrations of a two rotor system.

Let l = Length of shaft,

l_A = Length of part *NP i.e.* distance of node from rotor *A*,

l_B = Length of part *NQ*, *i.e.* distance of node from rotor *B*,

I_A = Mass moment of inertia of rotor *A*,

I_B = Mass moment of inertia of rotor *B*,

d = Diameter of shaft,

J = Polar moment of inertia of shaft, and

C = Modulus of rigidity for shaft material.

Natural frequency of torsional vibration for rotor *A*,

$$f_{nA} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_A.I_A}} \qquad \ldots (i)$$

and natural frequency of torsional vibration for rotor *B*,

$$f_{nB} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_B.I_B}} \qquad \ldots (ii)$$

Since $f_{nA} = f_{nB}$, therefore

$$\frac{1}{2}\sqrt{\frac{C.J}{l_A.I_A}} \quad \frac{1}{2}\sqrt{\frac{C.J}{l_B.I_B}} \quad \text{or} \quad l_A \cdot I_A = l_B \cdot I_B \qquad \ldots (iii)$$

$$l_A \quad \frac{l_B.I_B}{I_A}$$

We also know that

$$l \quad l_A \quad I_B \qquad \ldots (iv)$$

(29) Torsionally equivalent shaft:

we have assumed that the shaft is of uniform diameter. But in actual practice, the shaft may have variable diameter for different lengths. Such a shaft may, theoretically, be replaced by an equivalent shaft of uniform diameter.

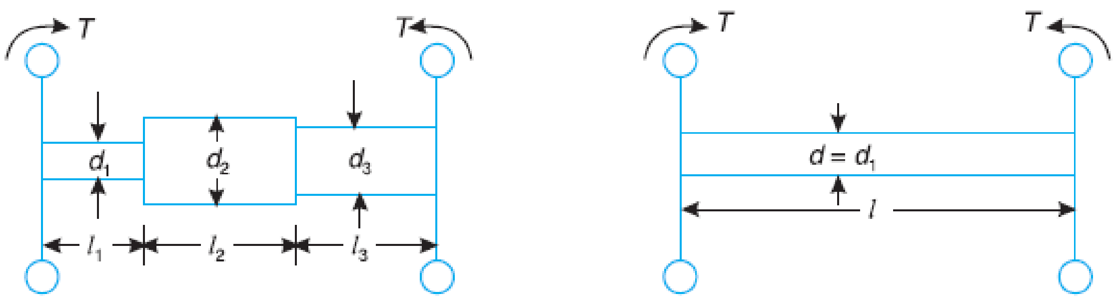
Consider a shaft of varying diameters as shown in Fig. (*a*). Let this shaft is replaced by an equivalent shaft of uniform diameter *d* and length *l* as shown in Fig. (*b*).These two shafts must have the same total angle of twist when equal opposing torques *T* are applied at their opposite ends.

Let d_1, d_2 and d_3 = Diameters for the lengths l_1, l_2 and l_3 respectively,

$_1$, $_2$ and $_3$ = Angle of twist for the lengths l_1, l_2 and l_3 respectively,

= Total angle of twist, and

J_1, J_2 and J_3 = Polar moment of inertia for the shafts of diameters d_1, d_2 and d_3 respectively.



(*a*) Shaft of varying diameters. (*b*) Torsionally equivalent shaft.

Fig 24.8

Since the total angle of twist of the shaft is equal to the sum of the angle of twists of different lengths, therefore

$$_1 \quad _2 \quad _3$$

or $$\frac{T.l}{C.J} \quad \frac{T.l_1}{C.J_1} \quad \frac{T.l_2}{C.J_2} \quad \frac{T.l_3}{C.J_3}$$

$$\frac{l}{J} \quad \frac{l_1}{J_1} \quad \frac{l_2}{J_2} \quad \frac{l_3}{J_3}$$

$$\frac{l}{\frac{}{32}\;d^4} \qquad \frac{l_1}{\frac{}{32}(d_1)^4} \qquad \frac{l_2}{\frac{}{32}(d_2)^4} \qquad \frac{l_3}{\frac{}{32}(d_3)^4}$$

$$\frac{l}{d^4} \quad \frac{l_1}{(d_1)^4} \quad \frac{l_2}{(d_2)^4} \quad \frac{l_3}{(d_3)^4}$$

In actual calculations, it is assumed that the diameter d of the equivalent shaft is equal to one of the diameter of the actual shaft. Let us assume that $d = d_1$.

$$\frac{l}{(d_1)^4} \quad \frac{l_1}{(d_1)^4} \quad \frac{l_2}{(d_2)^4} \quad \frac{l_3}{(d_3)^4}$$

or $$l \quad l_1 \quad l_2 \;\frac{d_1}{d_2}^{\,4} \quad l_3 \;\frac{d_1}{d_3}^{\,4}$$

This expression gives the length l of an equivalent shaft.

(30) Example Problems:

1) A spring mass system has spring stiffness of "k" N/m and a mass of "M" kg. It has the natural frequency of vibration as 12 Hz. An extra 2 kg mass is coupled to M and the natural frequency reduces by 2 Hz. Find the values of" k and "M".

2) A stepped shaft of 0.05 m in diameter for the first 0.6 m length, 0.08 m diameter for the next 1.8 m and 0.03 m diameter for the remaining 0.25 m length. While the 0.05 m diameter end is fixed, the 0.03 m diameter end of the shaft carries a rotor of mass moment of inertia 14.7 kg-m^2. If the modulus of elasticity of the shaft material is 0.83 x 10^{11} N/m^2, find the natural frequency of torsional oscillations, neglecting the inertia effect of the shaft.

3). Between a solid mass of 10 kg and the floor are kept two slabs of isolators, natural rubber and felt, in series. The natural rubber slab has a stiffness of 3000 N/m and an equivalent viscous damping coefficient of 100 N.sec/m. The felt slab has a stiffness of 12000 N/m and equivalent viscous damping coefficient of 330 N.sec/m. Determine the undamped and the damped natural frequencies of the system in vertical direction, neglecting the mass of isolators.

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UNIT –IV FORCED VIBRATIONS

UNIT- IV FORCED VIBRATION

(1) Introduction:

When a system is subjected continuously to time varying disturbances, the vibrations resulting under the presence of the external disturbance are referred to as forced vibrations.

Forced vibration is when an alternating force or motion is applied to a mechanical system. Examples of this type of vibration include a shaking washing machine due to an imbalance, transportation vibration (caused by truck engine, springs, road, etc), or the vibration of a building during an earthquake. In forced vibration the frequency of the vibration is the frequency of the force or motion applied, with order of magnitude being dependent on the actual mechanical system.

When a vehicle moves on a rough road, it is continuously subjected to road undulations causing the system to vibrate (pitch, bounce, roll etc). Thus the automobile is said to undergo forced vibrations. Similarly whenever the engine is turned on, there is a resultant residual unbalance force that is transmitted to the chassis of the vehicle through the engine mounts, causing again forced vibrations of the vehicle on its chassis. A building when subjected to time varying ground motion (earthquake) or wind loads, undergoes forced vibrations. Thus most of the practical examples of vibrations are indeed forced vibrations.

(2) Causes resonance:

Resonance is simple to understand if you view the spring and mass as energy storage elements – with the mass storing kinetic energy and the spring storing potential energy. As discussed earlier, when the mass and spring have no force acting on them they transfer energy back and forth at a rate equal to the natural frequency. In other words, if energy is to be efficiently pumped into both the mass and spring the energy source needs to feed the energy in at a rate equal to the natural frequency. Applying a force to the mass and spring is similar to pushing a child on swing, you need to push at the correct moment if you want the swing to get higher and higher. As in the case of the swing, the force applied does not necessarily have to be high to get large motions; the pushes just need to keep adding energy into the system.

The damper, instead of storing energy, dissipates energy. Since the damping force is proportional to the velocity, the more the motion, the more the damper dissipates the energy. Therefore a point will come when the energy dissipated by the damper will equal the energy being fed in by the force. At this point, the system has reached its maximum amplitude and will continue to vibrate at this level as long as the force applied stays the same. If no damping exists, there is nothing to dissipate the energy and therefore theoretically the motion will continue to grow on into infinity.

(3) Forced vibration of a single degree-of-freedom system:

We saw that when a system is given an initial input of energy, either in the form of an initial displacement or an initial velocity, and then released it will, under the right conditions, vibrate freely. If there is damping in the system, then the oscillations die away. If a system is given a continuous input of energy in the form of a continuously applied force or a continuously applied displacement, then the consequent vibration is called forced vibration. The energy input can overcome that dissipated by damping mechanisms and the oscillations are sustained.

We will consider two types of forced vibration. The first is where the ground to which the system is attached is itself undergoing a periodic displacement, such as the vibration of a building in an earthquake. The second is where a periodic force is applied to the mass, or object performing the motion; an example might be the forces exerted on the body of a car by the forces produced in the engine. The simplest form of periodic force or displacement is sinusoidal, so we will begin by considering forced vibration due to sinusoidal motion of the ground. In all real systems, energy will be dissipated, i.e. the system will be damped, but often the damping is very small. So let us first analyze systems in which there is no damping.

(4) Steady State Response due to Harmonic Oscillation:

Consider a spring-mass-damper system as shown in figure 4.1. The equation of motion of this system subjected to a harmonic force $F \sin \omega t$ can be given by

$$m\ddot{x} + kx + c\dot{x} = F \sin \omega t \tag{4.1}$$

where, m , k and c are the mass, spring stiffness and damping coefficient of the system, F is the amplitude of the force, w is the excitation frequency or driving frequency.

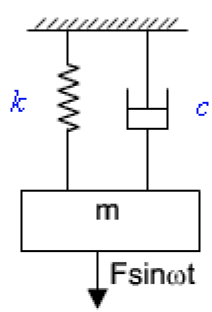


Figure 4.1 Harmonically excited system

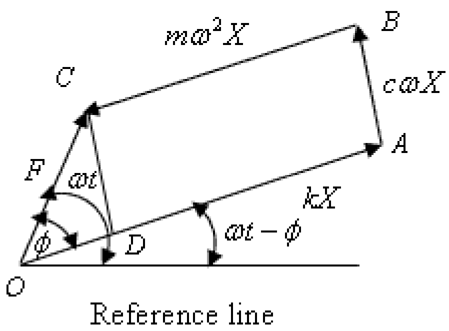


Figure 4.2: Force polygon

The steady state response of the system can be determined by solving equation(4.1) in many different ways. Here a simpler graphical method is used which will give physical understanding to this dynamic problem. From solution of differential equations it is known that the steady state solution (particular integral) will be of the form

$$x = X\sin(\omega t - \phi) \qquad (4.2)$$

As each term of equation (4.1) represents a forcing term viz., first, second and third terms, represent the inertia force, spring force, and the damping forces. The term in the right hand side of equation (4.1) is the applied force. One may draw a close polygon as shown in figure 4.2 considering the equilibrium of the system under the action of these forces. Considering a reference line these forces can be presented as follows.

- Spring force = $kx = kX\sin(\omega t - \phi)$ (This force will make an angle $\omega t - \phi$ with the reference line, represented by line OA).
- Damping force = $c\dot{x} = c\omega X\cos(\omega t - \phi)$ (This force will be perpendicular to the spring force, represented by line AB).
- Inertia force = $m\ddot{x} = -m\omega^2 X\sin(\omega t - \phi)$ (this force is perpendicular to the damping force and is in opposite direction with the spring force and is represented by line BC) .
- Applied force = $F\sin\omega t$ which can be drawn at an angle ωt with respect to the reference line and is represented by line OC.

From equation (1), the resultant of the spring force, damping force and the inertia force will be the applied force, which is clearly shown in figure 4.2.

It may be noted that till now, we don't know about the magnitude of X and ϕ which can be easily computed from Figure 2. Drawing a line CD parallel to AB, from the triangle OCD of Figure 2,

$$F^2 = (c\omega X)^2 + (kX - m\omega^2 X)^2$$

$$X = \frac{F}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}}$$

$$\phi = \tan^{-1}\frac{c\omega}{k - m\omega^2}$$

From the previous module of free-vibration it may be recalled that

- Natural frequency $\omega_n = \sqrt{\frac{k}{m}}$
- Critical damping $c_c = 2m\omega_n$
- Damping factor or damping ratio $\zeta = \frac{c}{c_c}$
- Hence, $\frac{c\omega}{k} = \frac{c}{c_c}\frac{c_c\omega}{k} = \zeta\frac{2m\omega_n\omega}{k} = 2\zeta\frac{\omega}{\omega_n}$

$$X = \frac{F/k}{\sqrt{\left(1 - \frac{m}{k}\omega^2\right)^2 + \left(\frac{c\omega}{k}\right)^2}}$$

$$\Rightarrow \quad \frac{Xk}{F} = \frac{1}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

$$\tan\phi = \frac{2\zeta(\omega/\omega_n)}{1 - (\omega/\omega_n)^2}$$

or

$$\phi = \tan^{-1}\left(\frac{2\zeta(\omega/\omega_n)}{1 - (\omega/\omega_n)^2}\right)$$

As the ratio $\frac{F}{k}$ is the static deflection (X_0) of the spring, $\frac{Xk}{F} = \frac{X}{X_0}$ is known as the magnification factor or amplitude ratio of the system

(5) Forced vibration with damping:

In this section we will see the behaviour of the spring mass damper model when we add a harmonic force in the form below. A force of this type could, for example, be generated by a rotating imbalance.

$$F = F_0 \cos(2\pi f t).$$

If we again sum the forces on the mass we get the following ordinary differential equation:

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos(2\pi f t).$$

The steady state solution of this problem can be written as:

$$x(t) = X \cos(2\pi f t - \phi).$$

The result states that the mass will oscillate at the same frequency, f, of the applied force, but with a phase shift φ.

The amplitude of the vibration "X" is defined by the following formula.

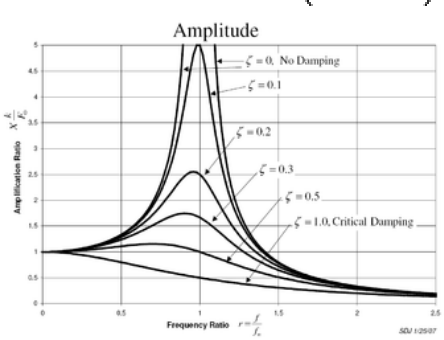
$$X = \frac{F_0}{k} \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}.$$

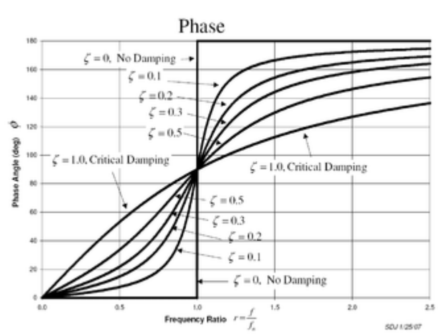
Where "r" is defined as the ratio of the harmonic force frequency over the undamped natural frequency of the mass–spring–damper model.

$$r = \frac{f}{f_n}.$$

The phase shift , φ, is defined by the following formula.

$$\phi = \arctan\left(\frac{2\zeta r}{1-r^2}\right).$$





The plot of these functions, called "the frequency response of the system", presents one of the most important features in forced vibration. In a lightly damped system when the forcing frequency nears the natural frequency ($r \approx 1$) the amplitude of the vibration can get extremely high. This phenomenon is called **resonance** (subsequently the natural frequency of a system is often referred to as the resonant frequency). In rotor bearing systems any rotational speed that excites a resonant frequency is referred to as a critical speed.

If resonance occurs in a mechanical system it can be very harmful – leading to eventual failure of the system. Consequently, one of the major reasons for vibration analysis is to predict when this type of resonance may occur and then to determine what steps to take to prevent it from occurring. As the amplitude plot shows, adding damping can significantly reduce the magnitude of the vibration. Also, the magnitude can be reduced if the natural frequency can be shifted away from the forcing frequency by changing the stiffness or mass of the system. If the system cannot be changed, perhaps the forcing frequency can be shifted (for example, changing the speed of the machine generating the force).

The following are some other points in regards to the forced vibration shown in the frequency response plots.

At a given frequency ratio, the amplitude of the vibration, X, is directly proportional to the amplitude of the force F_0 (e.g. if you double the force, the vibration doubles)

With little or no damping, the vibration is in phase with the forcing frequency when the frequency ratio $r < 1$ and 180 degrees out of phase when the frequency ratio $r > 1$

When $r \ll 1$ the amplitude is just the deflection of the spring under the static force F_0. This deflection is called the static deflection δ_{st}. Hence, when $r \ll 1$ the effects of the damper and the mass are minimal.

When $r \gg 1$ the amplitude of the vibration is actually less than the static deflection δ_{st}. In this region the force generated by the mass ($F = ma$) is dominating because the acceleration seen by the mass increases with the frequency. Since the deflection seen in the spring, X, is reduced in this region, the force transmitted by the spring ($F = kx$) to the base is reduced. Therefore the mass–spring–damper system is isolating the harmonic force from the mounting base – referred to as vibration isolation. Interestingly, more damping actually reduces the effects of vibration isolation when $r \gg 1$ because the damping force ($F = cv$) is also transmitted to the base.

(6) Rotating unbalance forced vibration:

One may find many rotating systems in industrial applications. The unbalanced force in such a system can be represented by a mass *m* with eccentricity *e* , which is rotating with angular velocity as shown in Figure 4.1.

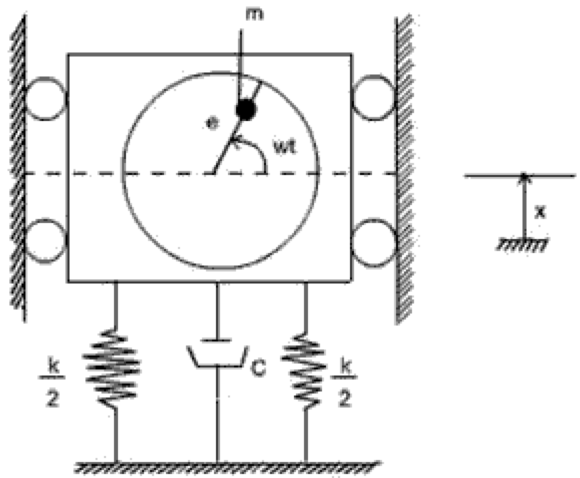


Figure 4.1 : Vibrating system with rotating unbalance

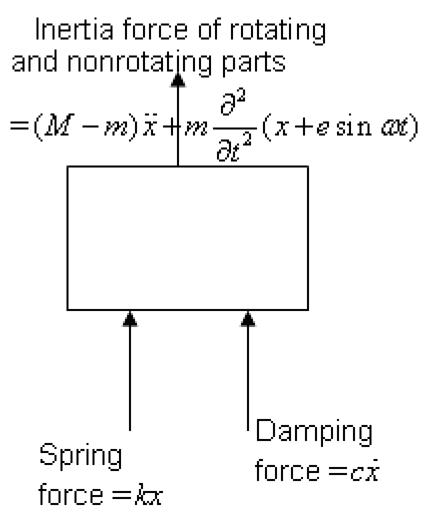


Figure 4.2. Freebody diagram of the system

Let x be the displacement of the nonrotating mass (M-m) from the static equilibrium position, then the displacement of the rotating mass m is $x+e\sin\omega t$

From the freebody diagram of the system shown in figure 4.2, the equation of motion is

$$(M-m)\ddot{x}+m\frac{\partial^2}{\partial t^2}(x+e\sin\omega t)+kx+c\dot{x}=0 \tag{4.1}$$

or $$M\ddot{x}+k\dot{x}+cx=me\omega^2\sin\omega t \tag{4.2}$$

This equation is same as equation (1) where *F* is replaced by $me\omega^2$. So from the force polygon as shown in figure 4.3

$$me\omega^2=\sqrt{\left\{(-M\omega^2+k)^2+c\omega^2\right\}X^2} \tag{4.3}$$

or $$X=\frac{me\omega^2}{\sqrt{(k-M\omega^2)^2+(c\omega)^2}} \tag{4.4}$$

or $$\frac{X}{e}=\frac{\frac{m\omega}{M}}{\sqrt{\left(\frac{k}{M}-\omega^2\right)^2+\left(\frac{c}{M}\omega\right)^2}} \tag{4.5}$$

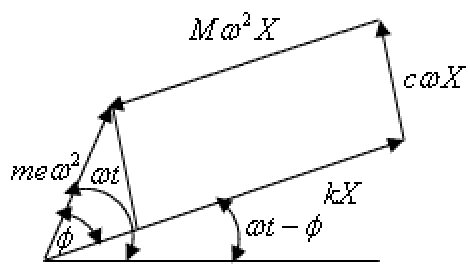


Figure 4.3: Force polygon

or $$\frac{X}{e}=\frac{\frac{m\omega}{M}}{\sqrt{\left(\frac{k}{M}-\omega^2\right)^2+\left(\frac{c}{M}\omega\right)^2}} \tag{4.6}$$

and
$$\tan\phi = \frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2} \tag{4.7}$$

So the complete solution becomes

$$x(t) = x_1 e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \phi_1\right) + \frac{me\omega^2}{\sqrt{(k - M\omega^2)^2 + (c\omega)^2}}\sin\left(\omega t - \phi\right) \tag{4.8}$$

(7) Vibration Isolation and Transmissibility:

When a machine is operating, it is subjected to several time varying forces because of which it tends to exhibit vibrations. In the process, some of these forces are transmitted to the foundation – which could undermine the life of the foundation and also affect the operation of any other machine on the same foundation. Hence it is of interest to minimize this force transmission. Similarly when a system is subjected to ground motion, part of the ground motion is transmitted to the system as we just discussed e.g., an automobile going on an uneven road; an instrument mounted on the vibrating surface of an aircraft etc. In these cases, we wish to minimize the motion transmitted from the ground to the system. Such considerations are used in the design of machine foundations and in order to understand some of the basic issues involved, we will study this problem based on the single d.o.f model discussed so far.

we get the expression for force transmitted to the base as follows:

$$F_T = \sqrt{(kX_0)^2 + (c\Omega X_0)^2}$$

$$X_0 = X_g\sqrt{\frac{k^2 + (c\Omega)^2}{(k - (m\Omega)^2)^2 + (c\Omega)^2}}$$

(8) Vibration Isolators:

Consider a vibrating machine; bolted to a rigid floor (Figure 2a).The force transmitted to the floor is equal to the force generated in the machine. The transmitted force can be decreased by adding a suspension and damping elements (often called vibration isolators) Figure 2b , or by adding what is called an inertia block, a large mass (usually a block of cast concrete), directly attached to the machine (Figure 2c).Another option is to add an additional level of mass (sometimes called a seismic mass, again a block of cast concrete) and suspension (Figure 2d).

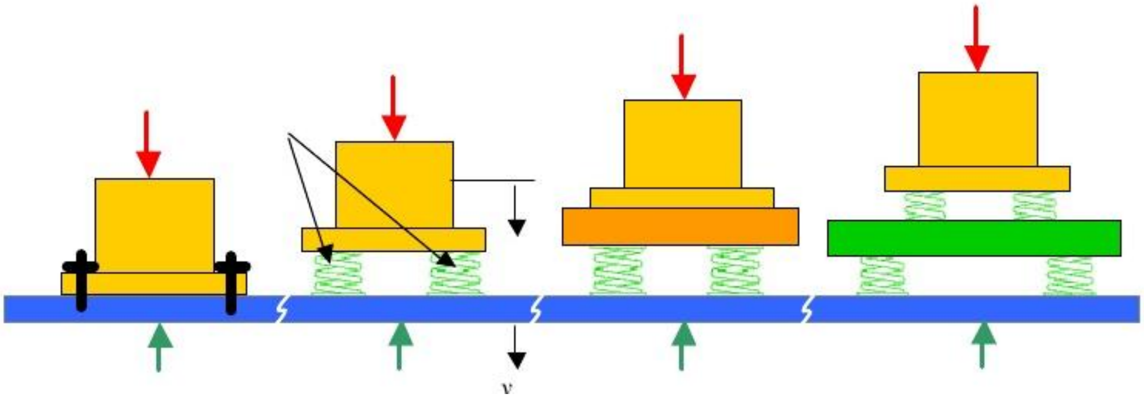


Figure 2.Vibration isolation systems: a) Machine bolted to a rigid foundation b) Supported on isolation springs, rigid foundation c) machine attached to an inertial block. d) Supported on isolation springs, non-rigid foundation (such as a floor); or machine on isolation springs, seismic mass and second level of isolator springs

When oscillatory forces arise unavoidably in machines it is usually desired to prevent these forces from being transmitted to the surroundings. For example, some unbalanced forces are inevitable in a car engine, and it is uncomfortable if these are wholly transmitted to the car body. The usual solution is to mount the source of vibration on sprung supports. Vibration isolation is measured in terms of the motion or force transmitted to the foundation. The lesser the force or motion transmitted the greater the vibration isolation

Suppose that the foundation is effectively rigid and that only one direction of movement is effectively excited so that the system can be treated as having only one degree of freedom.

(9) Response without damping:

The amplitude of the force transmitted to the foundations is Where ***k*** is the Stiffness of the support and ***x(t)*** is the displacement of the mass ***m***.
The governing equation can be determined by considering that the total forcing on the machine is equal to its mass multiplied by its acceleration (Newton’s second law)

The ratio (transmitted force amplitude) / (applied force amplitude) is called the **transmissibility.**

$$\text{Transmissibility} = \left|\frac{F_T}{F}\right| = \frac{1}{\left|1 - \frac{\omega^2}{\omega_n{}^2}\right|} = \frac{1}{\left|1 - \frac{f^2}{f_n{}^2}\right|}$$

The transmissibility can never be zero but will be less than 1 providing $\frac{\omega}{\omega_n} > \sqrt{2}$ or $\frac{f}{f_n} > \sqrt{2}$ otherwise it will be greater than 1.

(10) Example Problems:

1) A mass of 10 kg is suspended from one end of a helical spring, the other end being fixed. The stiffness of the spring is 10 N/mm. The viscous damping causes the amplitude to decrease to one tenth of the initial value in four complete oscillations. If a periodic force of 150 cos 50 t N is applied at the mass in the vertical direction, find the amplitude of the forced vibrations. What is its value at resonance?

2) A machine supported symmetrically on four springs has a mass of 80 kg. The mass of the reciprocating parts is 2.2 kg which move through a vertical stroke of 100 mm with simple harmonic motion. Neglecting damping, determine the combined stiffness of the spring so that the force transmitted to foundation is 1/20th of the impresses force. The machine crank shaft rotates at 800 rpm.

If under working conditions, the damping reduces the amplitudes of successive vibrations by 30%, find (i) the force transmitted to the foundation at resonance and (ii) the amplitude of vibration at resonance.

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UNIT- V MECHANISM FOR CONTROL

(25) Effect of Gyroscopic couple on a Naval Ship during Rolling

(26) Effect of Gyroscopic couple on a 4-wheel drive

(27) Example Problems

UNIT- V MECHANISM FOR CONTROL

Governor

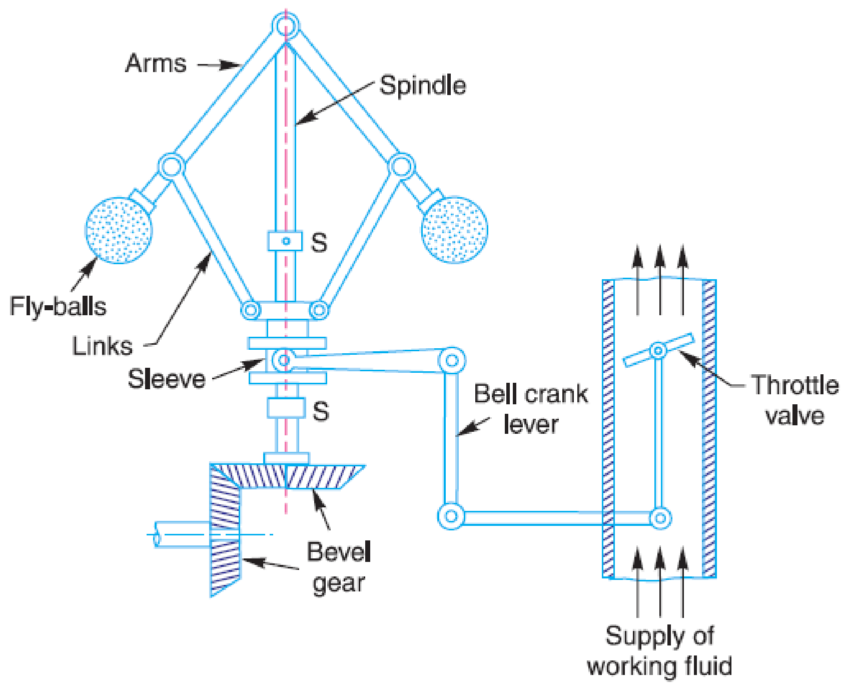
(1)Introduction:

A **centrifugal governor** is a specific type of governor that controls the speed of an engine by regulating the amount of fuel (or working_fluid) admitted, so as to maintain a near constant speed whatever the load or fuel supply conditions. It uses the principle of proportional control.

It is most obviously seen on steam engines where it regulates the admission of steam into the cylinder(s). It is also found on internal combustion engines and variously fuelled turbines, and in some modern striking_clocks.

(2)Principle of Working:

Note : When the balls rotate at uniform speed, controlling force is equal to the centrifugal force and they balance each other.



Power is supplied to the governor from the engine's output shaft by (in this instance) a belt or chain (not shown) connected to the lower belt wheel. The governor is connected to a throttle valve that regulates the flow of working_fluid (steam) supplying the prime mover (prime mover not shown). As the speed of the prime mover increases, the central spindle of the governor rotates at a faster rate and the kinetic energy of the balls increases. This allows the two masses on lever arms to move outwards and upwards against gravity. If the motion goes far enough, this motion causes the lever arms to pull down on a thrust_bearing, which moves a beam linkage, which reduces the aperture of a throttle valve. The rate of working-fluid entering the cylinder is thus reduced and the speed of the prime mover is controlled, preventing over speeding.

Mechanical stops may be used to limit the range of throttle motion, as seen near the masses in the image at right.

The direction of the lever arm holding the mass will be along the vector_sum of the reactive_centrifugal_force vector and the gravitational force.

(3) Classification of governors:

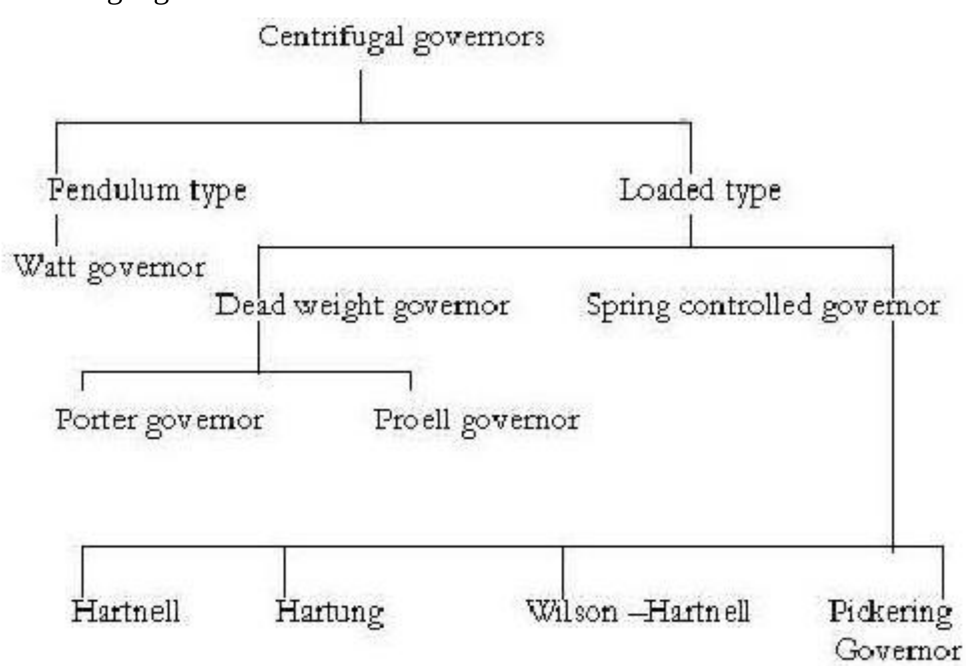
Governors are classified based upon two different principles.

These are:

1. Centrifugal governors

2. Inertia governors

Centrifugal governors are further classified as –



(4) Height of governor

It is the vertical distance between the centre of the governor halls and the point of intersection between the upper arms on the axis of spindle is known as governor height. It is generally denoted by h.

(5) Sleeve lift

The vertical distance the sleeve travels due to change in the equilibrium Speed is called the sleeve lift. The vertical downward travel may be termed as Negative lift

(6) Isochronism

This is an extreme case of sensitiveness. When the equilibrium speed is constant for all radii of rotation of the balls within the working range, the governor is said to be in isochronism. This means that the difference between the maximum and minimum equilibrium speeds is zero and the sensitiveness shall be infinite.

(7) Stability

Stability is the ability to maintain a desired engine speed without Fluctuating. Instability results in hunting or oscillating due to over correction. Excessive stability results in a dead-beat governor or one that does not correct sufficiently for load changes

(8) Hunting

The phenomenon of continuous fluctuation of the engine speed above and below the mean speed is termed as hunting. This occurs in over- sensitive or isochronous governors. Suppose an isochronous governor is fitted to an engine running at a steady load. With a slight increase of load, the speed will fall and the sleeve will immediately fall to its lowest position. This shall open the control valve wide and excess supply of energy will be given, with the result that the speed will rapidly increase and the sleeve will rise to its higher position. As a result of this movement of the sleeve, the control valve will be cut off; the supply to the engine and the speed will again fall, the cycle being repeated indefinitely. Such a governor would admit either more or less amount of fuel and so effect would be that the engine would hunt.

(9) Sensitiveness

A governor is said to be sensitive, if its change of speed s from no Load to full load may be as small a fraction of the mean equilibrium speed as possible and the corresponding sleeve lift may be as large as possible.

Suppose

$\omega 1$ = max. Equilibrium speed

$\omega 2$ = min. equilibrium speed

ω = mean equilibrium speed = $(\omega 1+ \omega 2)/2$

Therefore sensitiveness = $(\omega 1- \omega 2)/2$

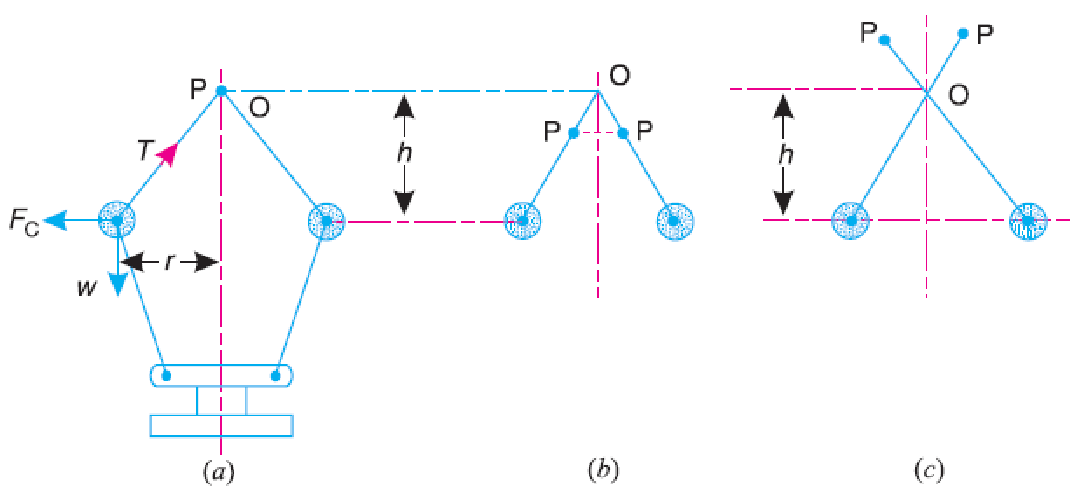
(10) Characteristics and qualities of centrifugal governor:

For satisfactory performance and working a centrifugal governor should possess The following qualities.
a. On the sudden removal of load its sleeve should reach at the top most position at Once.
b. Its response to the change of speed should be fast.
c. Its sleeve should float at some intermediate position under normal operating Conditions.
d. At the lowest position of sleeve the engine should develop maximum power.
e. It should have sufficient power, so that it may be able to exert the required force At the sleeve to operate the control & mechanism

(11)Watt governor:

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways :

1. The pivot *P*, may be on the spindle axis as shown in Fig. (*a*).
2. The pivot *P*, may be offset from the spindle axis and the arms when produced intersect at *O*, as shown in Fig (*b*).
3. The pivot *P*, may be offset, but the arms cross the axis at *O*, as shown in Fig (*c*).



Watt governor.

Let m = Mass of the ball in kg,

w = Weight of the ball in newtons = $m.g$,

T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (F_C) acting on the ball, **2.** the tension (T) in the arm, and **3.** the weight (w) of the ball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

or $$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2 \qquad \ldots (i)$$

When g is expressed in m/s^2 and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

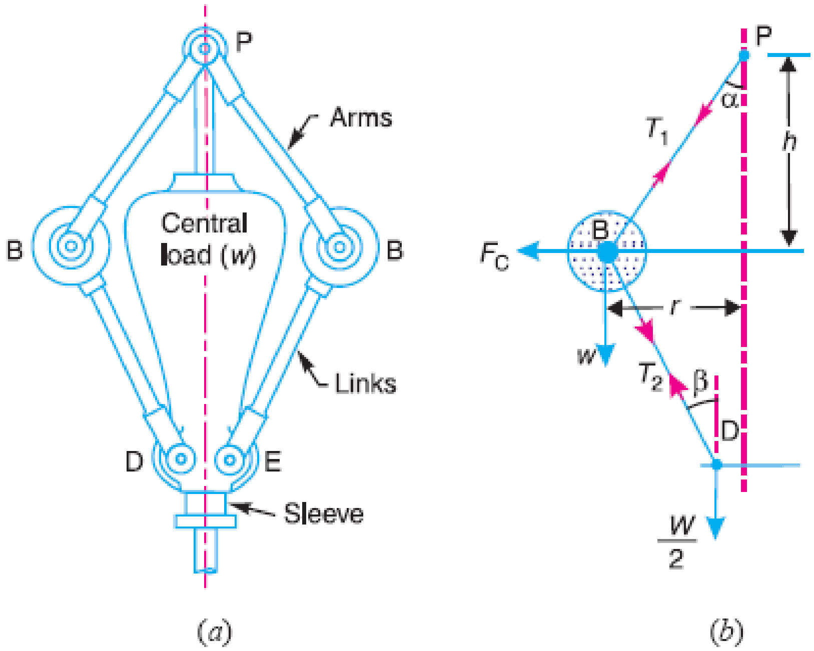
$$\therefore \quad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots (\because g = 9.81 \text{ m/s}^2) \ldots (ii)$$

Note : We see from the above expression that the height of a governor h, is inversely proportional to N^2. Therefore at high speeds, the value of h is small. At such speeds, the change in the value of h corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds *i.e.* from 60 to 80 r.p.m.

(12) Porter governor:

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. (*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Consider the forces acting on one-half of the governor as shown in Fig. (*b*).



Porter governor.

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T_1 = Force in the arm in newtons,

T_2 = Force in the link in newtons,

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

(13)Proell governor:

The Proell governor has the balls fixed at B and C to the extension of the links DF and EG, as shown in Fig. (a). The arms FP and GQ are pivoted at P and Q respectively.

Consider the equilibrium of the forces on one-half of the governor as shown in Fig (b). The instantaneous centre (I) lies on the intersection of the line PF produced and the line from D drawn perpendicualr to the spindle axis. The prependicular BM is drawn on ID.

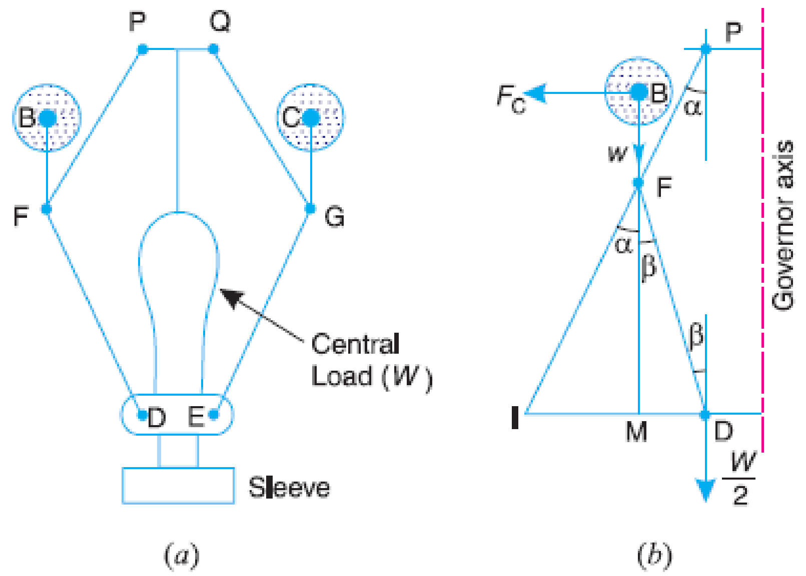
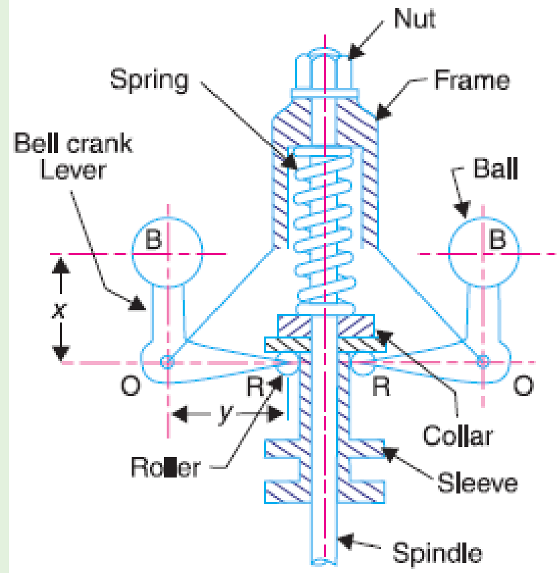


Fig. Proell governor.

(14) Hartnell governor:

A Hartnell governor is a spring loaded governor as shown in Fig. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.



Hartnell governor.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

r_2 = Maximum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve at ω_2 in newtons,

F_{C1} = Centrifugal force at ω_1 in newtons = $m\,(\omega_1)^2\,r_1$,

F_{C2} = Centrifugal force at ω_2 in newtons = $m\,(\omega_2)^2\,r_2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

(15) Hartung governor:

A spring controlled governor of the Hartung type is shown in Fig. (*a*). In this type of governor, the vertical arms of the bell crank levers are fitted with spring balls which compress against the frame of the governor when the rollers at the horizontal arm press against the sleeve.

Let S = Spring force,

F_C = Centrifugal force,

M = Mass on the sleeve, and

x and y = Lengths of the vertical and horizontal arm of the bell crank lever respectively.

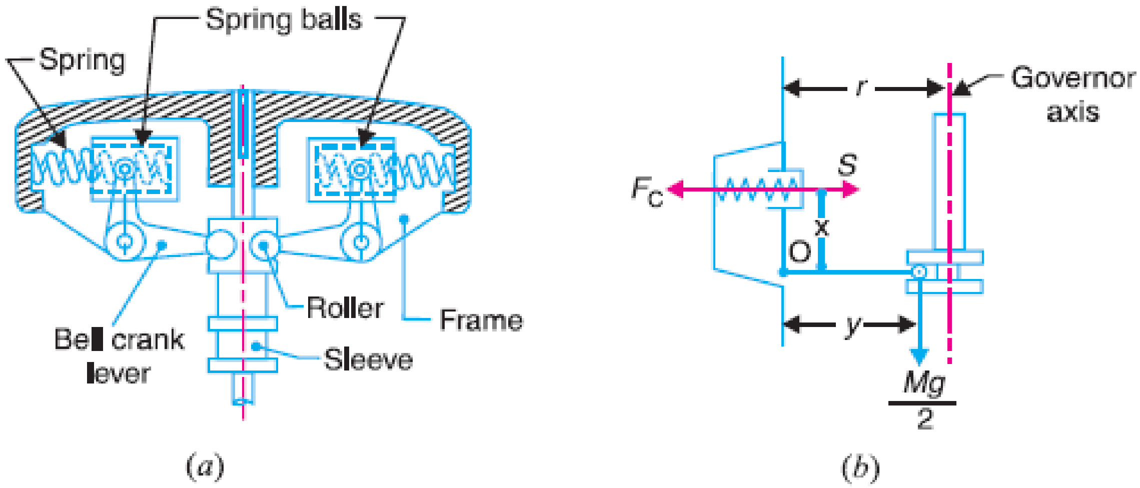


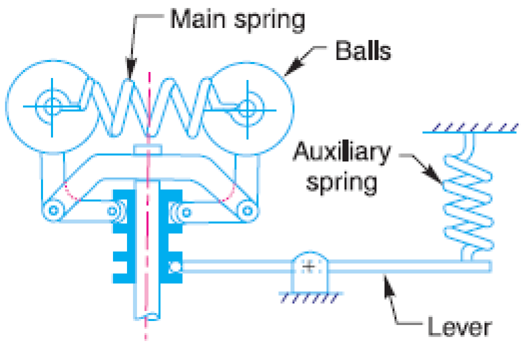
Fig. Hartung governor.

Fig. (*a*) and (*b*) show the governor in mid-position. Neglecting the effect of obliquity of the arms, taking moments about the fulcrum O,

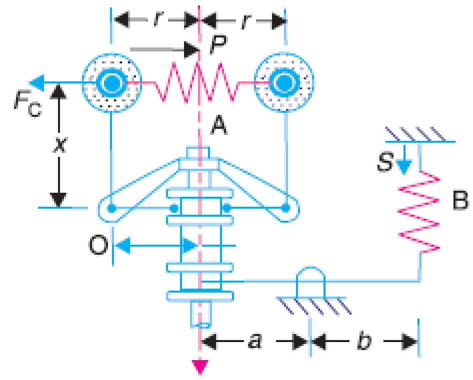
$$F_C \times x = S \times x + \frac{M.g}{2} \times y$$

(16) Wilson Hartnell governor:

A Wilson-Hartnell governor is a governor in which the balls are connected by a spring in tension as shown in Fig. An auxiliary spring is attached to the sleeve mechanism through a lever by means of which the equilibrium speed for a given radius may be adjusted. The main spring may be considered of two equal parts each belonging to both the balls. The line diagram of a Wilson-Hartnell governor is shown in Fig.



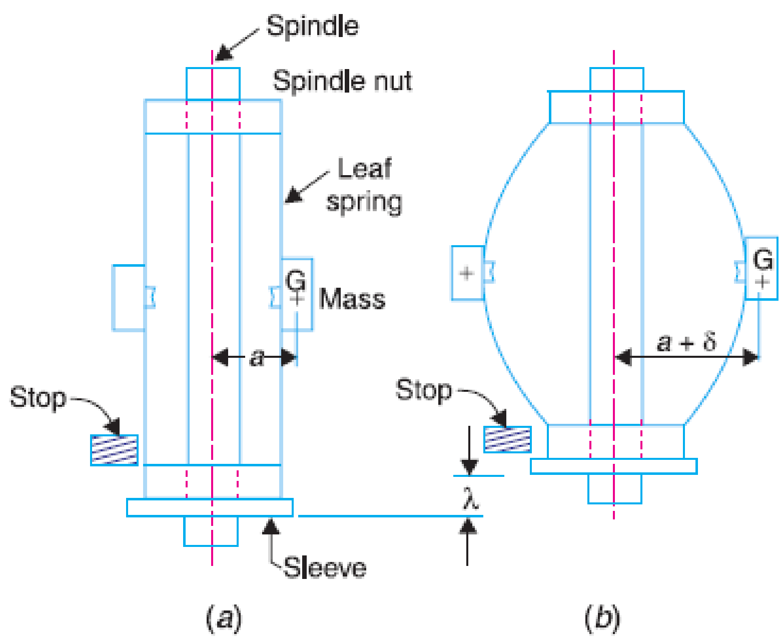
Wilson-Hartnell governor.



Line diagram of Wilson-Hartnell governor.

(17)Pickering governor:

A Pickering governor is mostly used for driving gramophone. It consists of *three straight leaf springs arranged at equal angular intervals round the spindle. Each spring carries a weight at the centre. The weights move outwards and the springs bend as they rotate about the spindle axis with increasing speed.



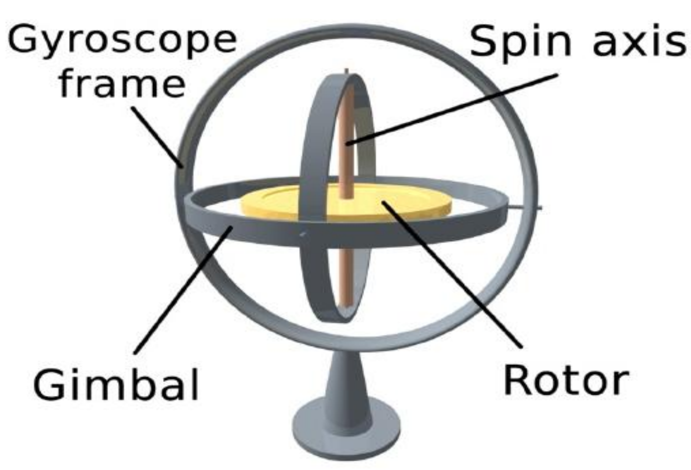
(18) Difference between a flywheel and a governor:

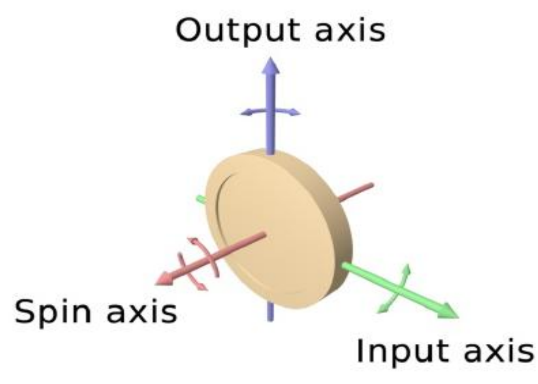
S.no	Flywheel	Governor
1	It is provided on the engine and fabricating machines viz., rolling mills; punching machines; shear machines, presses etc.	It is provided on prime movers such as engines and turbines
2	Its function is to store available mechanical energy when it is in excess of the load requirements and to part with the same when the available energy is less than that required by the load.	Its function is to regulate the supply of driving fluid producing energy, according to the load requirements so that at different loads almost a constant speed is maintained.
3.	In engines it takes care of fluctuations of speed during thermodynamic cycle.	It take care of fluctuation of speed due to variation of load over range of working of engines and turbines.
4.	It works continuously from cycle to cycle.	It works intermittently, i.e. only when there is change in the load.
5.	In fabrication machines it is very economical to use it as its use reduces capital investment on prime movers and their running expenses.	But for governor, there would have been unnecessarily more consumption of driving fluid thus it economizes its consumption

Gyroscope and its applications

(19) Gyroscope

A gyroscope is a device for measuring or maintaining orientation, based on the principles of conservation of angular momentum. A mechanical gyroscope is essentially a spinning wheel or disk whose axle is free to take any orientation. This orientation changes much less in response to a given external torque than it would without the large angular momentum associated with the gyroscope's high rate of spin. Since external torque is minimized by mounting the device in gimbals, its orientation remains nearly fixed, regardless of any motion of the platform on which it is mounted. Gyroscopes based on other operating principles also Exit, such as the electronic, microchip-packaged MEMS gyroscope devices found in consumer electronic devices, solid state ring lasers, fiber optic gyroscopes and the extremely sensitive quantum gyroscope. Applications of gyroscopes include navigation (INS) when magnetic compasses do not work (as in the Hubble telescope) or are not precise enough (as in ICBMs) or for the stabilization of flying vehicles like radio-controlled helicopters or UAVs. Due to higher precision, gyroscopes are also used to maintain direction in tunnel mining.



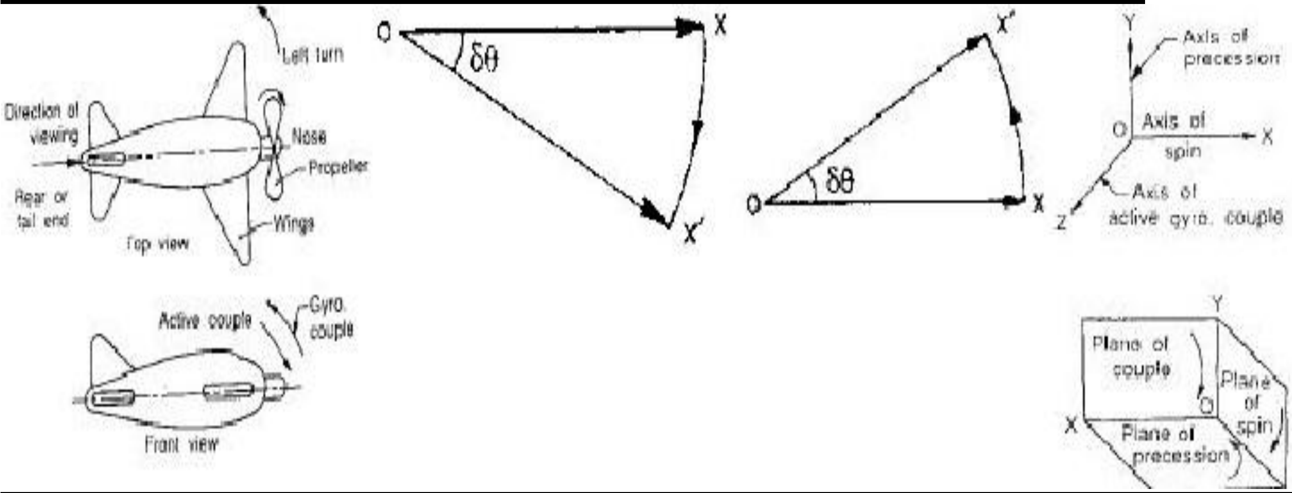


(20) Description and diagram:

Diagram of a gyro wheel. Reaction arrows about the output axis (blue) correspond to forces applied about the input axis (green), and vice versa. Within mechanical systems or devices, a conventional gyroscope is a mechanism comprising a rotor journal led to spin about one axis, the journals of the rotor being mounted in an inner gimbal or ring, the inner gimbal is journal led for oscillation in an outer gimbal which is journal led in another gimbal. So basically there are three gimbals. The **outer gimbal** or ring which is the gyroscope frame is mounted so as to pivot about an axis in its own plane determined by the support. This outer gimbal possesses one degree of rotational freedom and its axis possesses none. The next **inner gimbal** is mounted in the gyroscope frame (outer gimbal) so as to pivot about an axis in its own plane that is always perpendicular to the pivotal axis of the gyroscope frame (outer gimbal). This inner gimbal has two degrees of rotational freedom. Similarly, next **innermost gimbal** is attached to the inner gimbal which has three degree of rotational freedom and its axis posses two. The axle of the spinning wheel defines the spin axis. The rotor is journaled to spin about an axis which is always perpendicular to the axis of the innermost gimbal. So, the rotor possesses four degrees of rotational freedom and its axis possesses three. The wheel responds to a force applied about the input axis by a reaction force about the output axis.

The behavior of a gyroscope can be most easily appreciated by consideration of the front wheel of a bicycle. If the wheel is leaned away from the vertical so that the top of the wheel moves to the left, the forward rim of the wheel also turns to the left. In other words, rotation on one axis of the turning wheel produces rotation of the third axis.

(21) Effect of the Gyroscopic Couple on an Aero plane



(22) EFFECT OF GYROSCOPIC COUPLE

This couple is, therefore, to raise the nose and dip the tail of the aero plane.

Notes

1. When the aero plane takes a right turn under similar Conditions as discussed above, the effect of the reactive Couple will be to dip the nose and raise the tail of the aero plane.
2. When the engine or propeller rotates in anticlockwise direction when viewed from the rear or tail end and the aero plane takes a left turn, then the effect of reactive gyroscopic couple will be to dip the nose and raise the tail of the aero plane.

3. When the aero plane takes a right turn under similar Conditions as mentioned in note 2 above, the effect of Reactive gyroscopic couple will be to raise the nose and dip the of the aero plane.

4. When the engine or propeller rotates in clockwise direction when viewed from the front and the aero plane takes a left turn, then the effect of reactive gyroscopic couple will be to raise the tail and dip the nose of the aero plane.

5. When the aero plane takes a right turn under similar conditions as mentioned in note4 above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aero plane.

(23) Effect of gyroscopic couple on ship

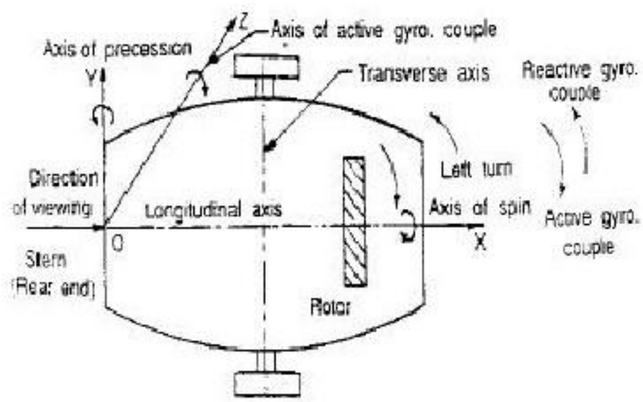
The top and front views of a naval ship are shown in fig. The fore end of the ship is called bow and the rear end is known as stern or aft. The left hand and the right hand sides of the ship, when viewed from the stern are called port and star board respectively. We shall now discuss the effect of gyroscopic couple in the naval ship in the following three cases:

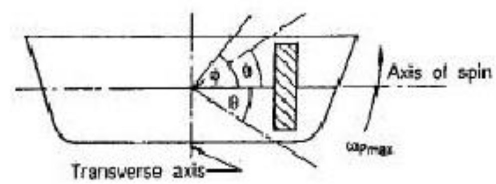
1. Steering

2. Pitching, and

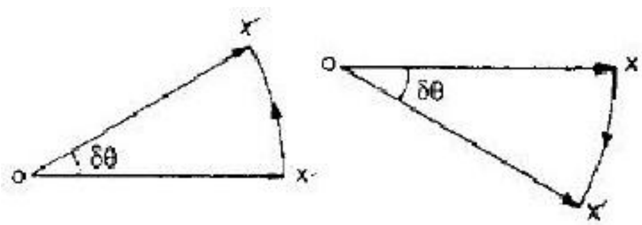
3. Rolling

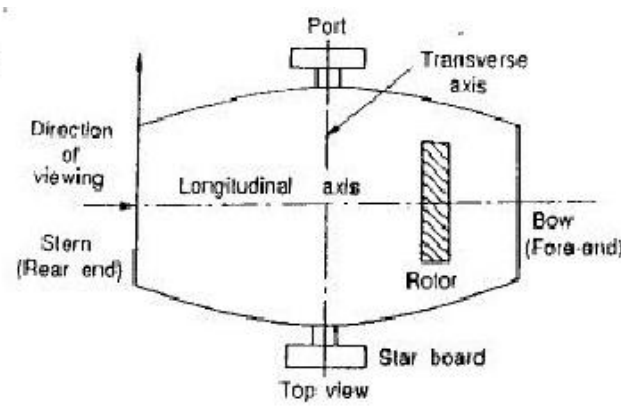
(24) Effect of Gyroscopic Couple on a Naval Ship during pitching& Steering

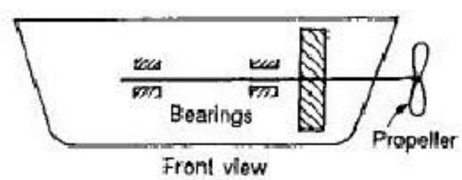




(a) Pitching of a naval ship.







Steering is the turning of a complete ship in a curve towards left or right, while it moves forward, considers the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. below. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aero plane as discussed in Art.

When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction ox as shown in Fig. A1. As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from ox to ox'. The vector xx' now represents the active gyroscopic couple and is perpendicular to ox. Thus the plane of active gyroscopic couple is perpendicular to xx' and its direction in the axis OZ for left hand turn is clockwise as shown in Fig below. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (i.e in anticlockwise direction). The effect of this reactive gyroscopic couple is to raise the bow and lower the stern.

Notes

1. When the ship steers to the right under similar condition as discussed above, the effect of the reactive gyroscopic couple, as shown in Fig. B1, will be to raise the stern and lower the bow.
2. When the rotor rotates in the anticlockwise direction, when viewed from the stern and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to lower the bow and raise the stern.
3. When the ship is steering to the right under similar conditions as discussed in note 2 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.
4. When the rotor rotates in the clockwise direction when viewed from the bow or fore end and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to raise the stern and lower the bow.
5. When the ship is steering to the right under similar conditions as discussed in note 4 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.
6. The effect of the reactive gyroscopic couple on a boat propelled by a turbine taking left or right turn.

(25) Effect of Gyroscopic couple on a Naval Ship during Rolling:

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship. In case of rolling of a ship, the axis of precession (i.e. longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

(26) Effect of Gyroscopic couple on a 4-wheel drive:

Consider the four wheels A, B, C and D of an automobile locomotive taking a turn towards left as shown in Fig. 14.11. The wheels A and C are inner wheels, whereas B and D are outer wheels. The centre of gravity ($C.G.$) of the vehicle lies vertically above the road surface.

Let m = Mass of the vehicle in kg,

W = Weight of the vehicle in newtons = $m.g$,

r_W = Radius of the wheels in metres,

R = Radius of curvature in metres ($R > r_W$),

h = Distance of centre of gravity, vertically above the road surface in metres,

x = Width of track in metres,

I_W = Mass moment of inertia of one of the wheels in kg-m^2,

ω_W = Angular velocity of the wheels or velocity of spin in rad/s,

I_E = Mass moment of inertia of the rotating parts of the engine in kg-m^2,

ω_E = Angular velocity of the rotating parts of the engine in rad/s,

G = Gear ratio = ω_E / ω_W,

v = Linear velocity of the vehicle in m/s = $\omega_W . r_W$

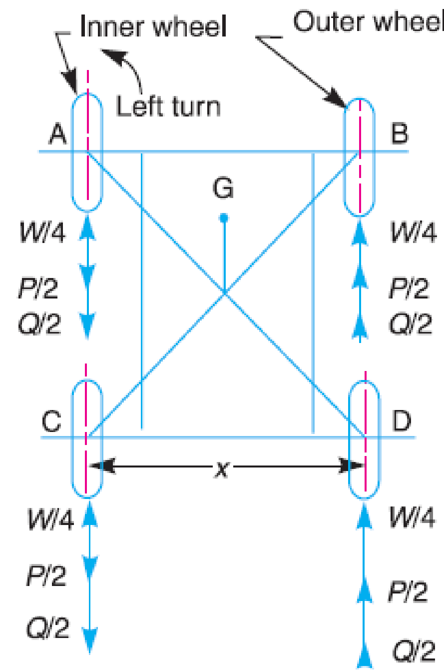


Fig. 14.11. Four wheel drive moving in a curved path.

A little considereation will show, that the weight of the vehicle (W) will be equally distributed over the four wheels which will act downwards. The reaction between each wheel and the road surface of the same magnitude will act upwards. Therefore

Road reaction over each wheel $= W/4 = m.g/4$ newtons

Let us now consider the effect of the gyroscopic couple and centrifugal couple on the vehicle.

1. ***Effect of the gyroscopic couple***

Since the vehicle takes a turn towards left due to the precession and other rotating parts, therefore a gyroscopic couple will act.

We know that velocity of precession,

$$\omega_P = v/R$$

$\therefore$ Gyroscopic couple due to 4 wheels,

$$C_W = 4\, I_W.\omega_W.\omega_P$$

and gyroscopic couple due to the rotating parts of the engine,

$$C_E = I_E.\omega_E.\omega_P = I_E.G.\omega_W.\omega_P \qquad \ldots (\because\ G = \omega_E/\omega_W)$$

$\therefore$ Net gyroscopic couple,

$$C = C_W \pm C_E = 4\, I_W.\omega_W.\omega_P \pm I_E.G.\omega_W.\omega_P$$
$$= \omega_W.\omega_P\,(4\, I_W \pm G.I_E)$$

The ***positive*** sign is used when the wheels and rotating parts of the engine rotate in the same direction. If the rotating parts of the engine revolves in opposite direction, then ***negative*** sign is used.

Due to the gyroscopic couple, vertical reaction on the road surface will be produced. The reaction will be vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be P newtons. Then

$$P \times x = C \quad \text{or} \quad P = C/x$$

$\therefore$ Vertical reaction at each of the outer or inner wheels,

$$P/2 = C/2x$$

Note: We have discussed above that when rotating parts of the engine rotate in opposite directions, then –ve sign is used, *i.e.* net gyroscopic couple,

$$C = C_W - C_E$$

When $C_E > C_W$, then C will be –ve. Thus the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels.

2. *Effect of the centrifugal couple*

Since the vehicle moves along a curved path, therefore centrifugal force will act outwardly at the centre of gravity of the vehicle. The effect of this centrifugal force is also to overturn the vehicle. We know that centrifugal force,

$$F_C = \frac{m \times v^2}{R}$$

$\therefore$ The couple tending to overturn the vehicle or overturning couple,

$$C_O = F_C \times h = \frac{m.v^2}{R} \times h$$

This overturning couple is balanced by vertical reactions, which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at the two outer or inner wheels be Q. Then

$$Q \times x = C_O \quad \text{or} \quad Q = \frac{C_O}{x} = \frac{m.v^2.h}{R.x}$$

$\therefore$ Vertical reaction at each of the outer or inner wheels,

$$\frac{Q}{2} = \frac{m.v^2.h}{2R.x}$$

$\therefore$ Total vertical reaction at each of the outer wheel,

$$P_O = \frac{W}{4} + \frac{P}{2} + \frac{Q}{2}$$

and total vertical reaction at each of the inner wheel,

$$P_I = \frac{W}{4} - \frac{P}{2} - \frac{Q}{2}$$

A little consideration will show that when the vehicle is running at high speeds, P_I may be zero or even negative. This will cause the inner wheels to leave the ground thus tending to overturn the automobile. In order to have the contact between the inner wheels and the ground, the sum of $P/2$ and $Q/2$ must be less than $W/4$.

(27) Example Problems:

1) A ship is propelled by a turbine rotor which has a mass of 5 tonnes and a speed of 2100 rpm. The rotor has a radius of gyration of 0.5 m and rotates in a clockwise direction when viewed from the stern. Find the gyroscopic effect in the following conditions. (i) the ship sails at a speed of 30 km/hr and steers to the left in curve having 60 m radius. (ii) the ship pitches 6° above and 6° below the horizontal position. The bow is descending with its maximum velocity. The motion due to pitching is simple harmonic and the periodic time is 20 seconds. (iii) the ship rolls at a certain instant it has an angular velocity of 0.03 rad/sec clockwise when viewed from stern.

2) The length of the upper and lower arms of a porter governor are 200 mm and 250 mm respectively. Both the arms are pivoted on the axis of rotation. The central load is 150 N, the weight of each ball is 20 N and the friction on the sleeve together with the resistance of the operating gear is equivalent to a force of 30 N at the sleeve. If the limiting inclinations of the upper arms to the vertical are 30° and 40°, determine the range of speed of the governor.