$2156.5 \times 25 = P' \times 700$

$\boxed{P' = 77N}$

* centroid & center of Gravity:- 3-UNIT

Any rigid body is made up of a large no. of particles & each particle is attracted towards the earth.

-> the force of attraction which is proportional to the mass of particle acts vertically downwards & is known as weight of the body.

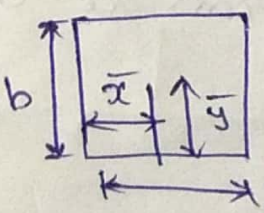
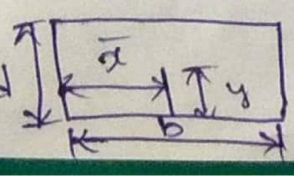
centroid:-

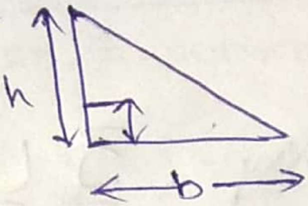
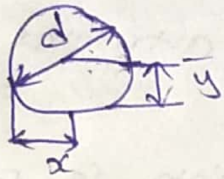
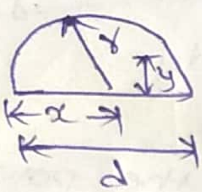
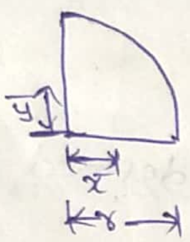
centroid (or) center of area is the point where the whole area of a plane fig. is Assumed to be concentrated.

center of gravity:-

center of gravity of a body is defined as the point through which resultant of the gravitational force weight acts for any orientation of body. Generally the term centroid is used for geometrical figures which have only areas but no mass & the term center of gravity is used when refering solid bodies having mass.

* centroids of plane geometrical shapes:-

		Area	$\bar{x}$	$\bar{y}$
1. square		b^2	$b/2$	$b/2$
2 Rectangle		bd	$b/2$	$d/2$

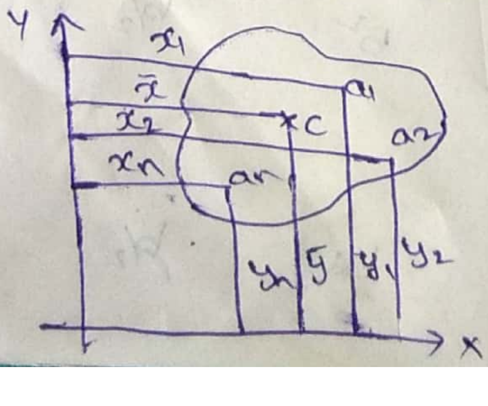
		Area	$\bar{x}$	$\bar{y}$
3. triangle		$\frac{1}{2}bh$	$b/3$	$h/3$
4. circle		πr^2	$d/2$	$d/2$
5. semicircle		$\frac{\pi r^2}{2}$	$d/2 = r$	$\frac{4r}{2\pi}$
6. Quadrant		πr^2	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$

* <u>centroid for complex shapes</u> :-

For simple areas such as circle, square, rectan-gle, triangle etc. the centroid can be easily located.

→ The complex shapes are considered as combi-nation of 2 or more simple shapes.

→ To determine the centroid of such shapes the method of composite areas can be used.



consider the lamina whose centroid is to be determine.

Divide the lamina into small areas. a_1, a_2, a_3 --- an whose centroids are at a distance x_1, x_2, x_3 --- x_n & y_1, y_2, y_3, --- y_n. from y-axis & x-axis respectively.

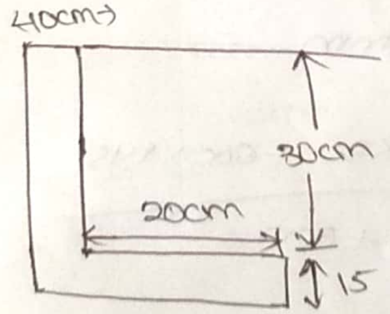
From the principle of moments.

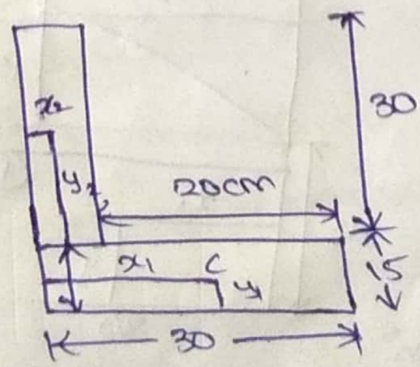
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n}{a_1 + a_2 + \cdots + a_n} = \frac{\Sigma ax}{\Sigma a} = \frac{\Sigma ax}{A}$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + \cdots + a_ny_n}{a_1 + a_2 + \cdots + a_n} = \frac{\Sigma ay}{A}$$

Σax = sum of product of areas of each component & its respective centroid distance from y-axis.

1) Find the centroid of plane lamina shown in figure.





$a_1 = 30 \times 15 = 450 cm^2$, $a_2 = 10 \times 30 = 300 cm^2$

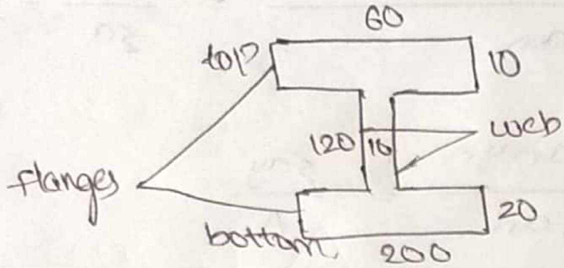
$x_1 = 30/2 = 15 cm$, $x_2 = 10/2 = 5 cm$

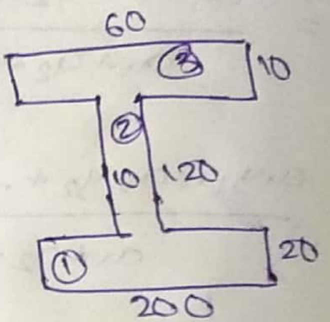
$y_1 = 15/2 = 7.5 cm$, $y_2 = 15 + 30/2 = 15 + 15 = 30 cm$

$$\bar{x} = \frac{450 \times 15 + 300 \times 5}{450 + 300} = 11\,cm$$

$$\bar{y} = \frac{450 \times 7.5 + 300 \times 30}{450 + 300} = 16.5\,cm.$$

2) Determine the centroid of I section of the dimensions in mm bottom flange = 200x20 & top flange = 60x10, web = 120x10.





$a_1 = 20 \times 200 = 4000\,mm^2$, $a_2 = 120 \times 10 = 1200\,mm^2$

$a_3 = 60 \times 10 = 600\,mm^2$

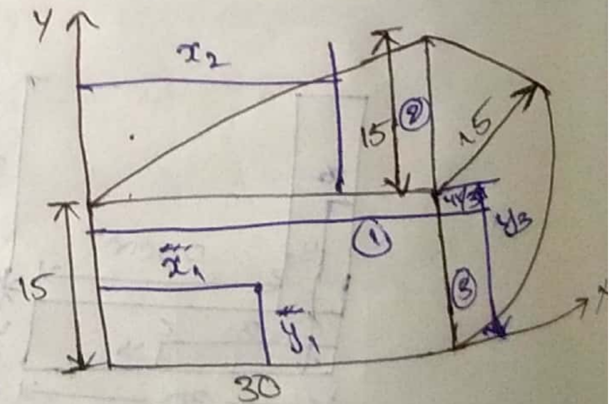
$y_1 = \frac{20}{2} = 10\,mm$, $y_2 = 20 + \frac{120}{2} = 80\,mm$

$y_3 = 20 + 120 + \frac{10}{2} = 145\,mm$

$$\bar{y} = \frac{4000 \times 10 + 1200 \times 80 + 600 \times 145}{4000 + 1200 + 600}$$

$\bar{y} = 38.44\,mm.$

3)



$a_1 = 15 \times 30 = 450\,mm^2$

$a_2 = \frac{1}{2} \times 15 \times \cancel{30}^{15} = 225\,mm^2$

$$a_3 = \frac{\pi \times 15 \times 15}{2} = 353.2\,mm^2$$

$x_1 = 30/2 = 15\,mm$

$x_2 = \frac{15}{3} = 5mm$, $x_3 = 30 + \frac{4\times 15}{3\times 3.14} = 36.36mm$

$h - \frac{h}{3} = \frac{2h}{3}$

$= \frac{2\times 30}{3} = 20mm.$

$y_1 = \frac{15}{2} = 7.5mm$, $y_2 = 15 + \frac{15}{3} = 20mm$

$y_3 = d/2 = r = 15mm.$

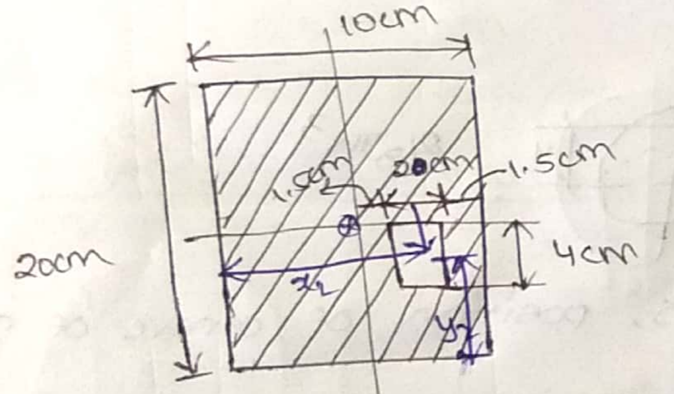
$$\bar{x} = \frac{450\times 15 + 225\times 20 + 353.2\times 36.36}{450 + 225 + 353.2}$$

$$\bar{x} = \frac{24092.352}{1028.2} = 23.43mm.$$

$$\bar{y} = \frac{\cancel{7.5\times 450 + 20\times 353.2 + 15\times 36}\; 450\times 7.5 + 225\times 20 + 353.2\times 15}{1028.2}$$

$\bar{y} = 12.81mm$

4)



from a rectangular lamina shown in fig of dimensions 10x20cm, a rectangular hole of 2x4cm is cut. Find the center of gravity of the lamina

Sol: $a_1 = 10\times 20 = 200cm^2$, $a_2 = 2\times 4 = 8cm^2$

$x_1 = 10/2 = 5cm$, $y_1 = \frac{20}{2} = 10cm$

$x_2 = 6.5 + \frac{2}{2} = 7.5cm$, $y_2 = 6 + 4/2 = 8cm$

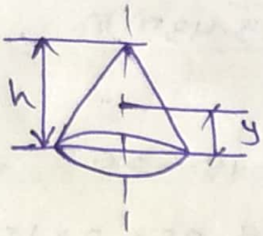
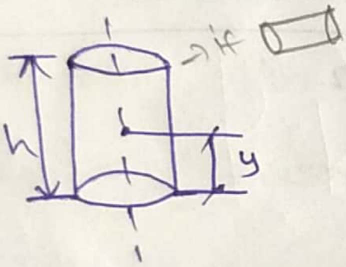
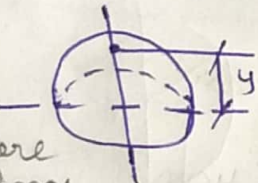
$$\bar{x} = \frac{a_1 x_1 - a_2 x_2}{a_1 - a_2}$$

$$\bar{x} = \frac{200 \times 5 - \cancel{80} \times 7.5}{200 - \cancel{80}} = \frac{\cancel{400}}{\cancel{128}} = \frac{940}{192}$$

$$\bar{x} = 4.89\,cm$$

$$\bar{y} = \frac{200 \times 10 - 8 \times 8}{200 - 8} = 10.08\,cm$$

<u>Centre of gravity for solids</u>:-

		volume	$\bar{x}$	$\bar{y}$
1) cone		$\frac{\pi r^2 h}{3}$		$h/4$
2) cylinder		$\pi r^2 h$		$h/2$
3) Hemisphere		$2/3\,\pi r^3$		$3/8\,r$

→ if (horizontal cylinder) $\bar{x} = h/2$, $\bar{y}$ = nothing.

4) segment of sphere next 3rd page

* For solid bodies, position of centre of gravity is given by,

$$\bar{x} = \frac{\Sigma mgx}{\Sigma mg} = \frac{\Sigma mx}{\Sigma m}.$$

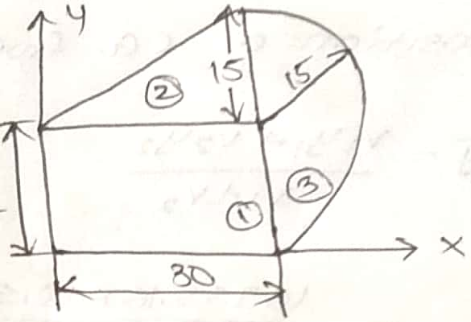
$$\text{lly}, \; \bar{y} = \frac{\Sigma my}{\Sigma m}$$

* If the densities of solid bodies are same, then ~~two~~ the mass or weights are replaced with

volumes.

$$\bar{x} = \frac{\Sigma Vx}{\Sigma V} ; \bar{y} = \frac{\Sigma Vy}{\Sigma V}.$$

Q) Locate the centroid of the composite shown in figure with respect to x and y axes.



Sol: $a_1 = 15 \times 30 = 450 mm^2$

$a_2 = \frac{1}{2} \times 30 \times 15 = 225 mm^2$

$a_3 = \frac{\pi \times 15^2}{2} = 353.43 mm^2$

$x_1 = \frac{30}{2} = 15$; $x_2 = \frac{2 \times 30}{3} = 20$; $x_3 = 30 + \frac{4 \times 15}{3 \times \pi}$

$= 36.37$

$y_1 = \frac{15}{2} = 7.5$; $y_2 = 15 + \frac{15}{3} = 20$; $y_3 = r = 15$

position of centroid from OY,

$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3} = \frac{450 \times 15 + 225 \times 20 + 353.43 \times 36.37}{450 + 225 + 353.43}$$

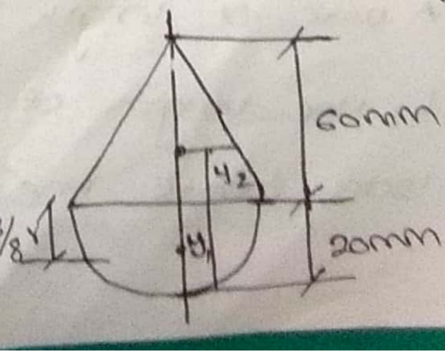
$= 23.43 mm.$

position of centroid from OX,

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3} = \frac{450 \times 7.5 + 225 \times 20 + 353.43 \times 15}{450 + 225 + 353.43}$$

$= 12.81 mm.$

Q) A solid hemisphere of 20mm radius supports a solid cone of same base and height 60mm as shown in fig. Locate the centre of gravity of the composite section.



Sol: $v_1 = \frac{2\pi r^3}{3} = \frac{2\pi \times 20^3}{3}$

$= 16755.16 mm^3$

$V_2 = \frac{\pi r^2 h}{3} = \frac{\pi \times 20^2 \times 60}{3} = 25132.74 mm^3$

$y_1 = 3/8\, r = 20 - 7.5 = 12.5$

$y_2 = h/4 = 20 + \frac{60}{4} = 35 mm.$

∴ position of C.G. from OX,

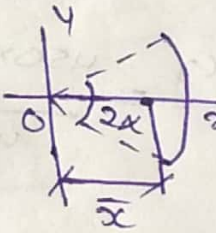
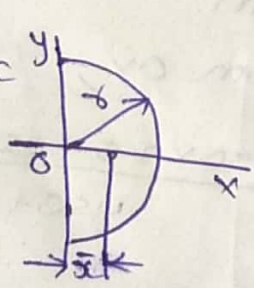
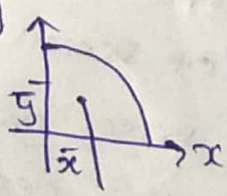
$$\bar{y} = \frac{V_1 y_1 + V_2 y_2}{V_1 + V_2}$$

$$= \frac{16755.16 \times 12.5 + 25132.74 \times 35}{16755.16 + 25132.74}$$

$= 26 mm.$

<u>Centroid of lines</u> :-

$$\bar{x} = \frac{l_1 x_1 + l_2 x_2}{l_1 + l_2}, \quad \bar{y} = \frac{l_1 y_1 + l_2 y_2}{l_1 + l_2}$$

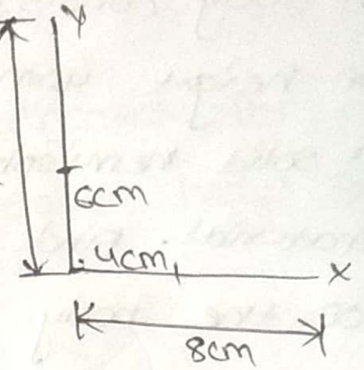
		volume	$\bar{x}$	$\bar{y}$
1. segment of arc		$2\alpha \cdot r$	$\frac{r \sin\alpha}{\alpha}$	0
2. semicircular arc		πr	$\frac{2r}{\pi}$	0
3. Quater circular arc		$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$

Q) A wire of length 20cm is bent in the form of L, the length of short leg 8cm & long leg is 12cm. locate the centroid.

$l_1 = 8cm$; $l_2 = 12cm$

$x_1 = 4cm$; $x_2 = 0$

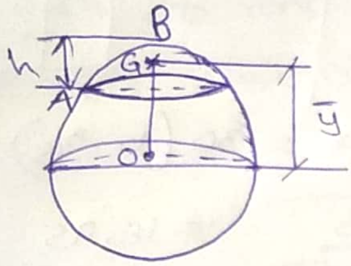
$y_1 = 0$; $y_2 = 6cm$



$$\bar{x} = \frac{8 \times 4 + 12 \times 0}{8+12} = 1.6cm$$

$$\bar{Y} = \frac{8 \times 0 + 12 \times 6}{8+12} = 3.6cm$$

* <u>segment of sphere</u>

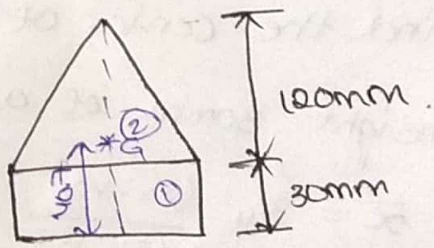


$$\bar{y} = \frac{3}{4}\frac{(2r-h)^2}{(3r-h)}$$

It is measured from centre of sphere.

For 8

Q) A solid body formed by ~~joting~~ joining the base of a right circular cone of height 'H' to the equal base of right circular cylinder of height 'h'. Calculate the distance of the center of mass of the solid from its plane face, when H = 120mm & h = 30mm



$$V_1 = \pi r^2 h$$

$$= 3.14 \times r^2 \times 30 = 94.2r^2$$

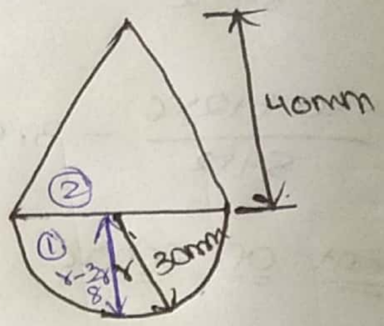
$$V_2 = \frac{\pi r^2 \times 120}{3} = 125.6r^2$$

$$y_1 = \frac{30}{2} = 15\,mm$$

$$y_2 = 30 + \frac{120}{4} = \cancel{90}\ 60mm$$

$$\bar{y} = \frac{\cancel{r^2}(94.2\cancel{r^2} \times 15 + 125.6 \times 60)}{\cancel{r^2}(94.2 + 125.6)} = 40.7\,mm.$$

A body consists of a right circular body cone of height 40mm and radius 30mm placed on a solid hemisphere of radius 30mm of the same material. Find the position of centre of gravity of the body.



Sol: $V_1 = \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 30 \times 30 \times 30$

$V_1 = 56548.6\ mm^3$

$V_2 = \frac{\pi r^2 h}{3} = 37680\ mm^3$

$y_1 = \cancel{3/8\, r} = \cancel{11.25mm}$ $\quad r - 3/8\, r = 30(1 - 3/8)$

$= 18.75$

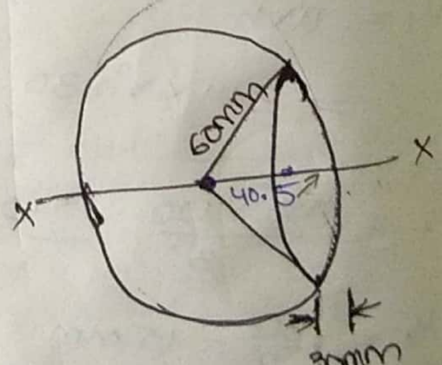
$y_2 = 30 + \frac{40}{4} = \cancel{10\ mm} = 30 + 10 = 40$

$$\bar{y} = \frac{56548.6 \times \overset{18.75}{\cancel{11.25}} + 37680 \times 40}{56548.6 + 37680}$$

$\bar{y} = 27.24mm.$

Find the center of gravity of a segment of height 30mm of a sphere of radius 60mm.

Sol: $\bar{x} = 3/4 \frac{(2r-h)^2}{(3r-h)}$



$= 3/4 \frac{(2 \times 60 - 30)^2}{(3 \times 60 - 30)}$

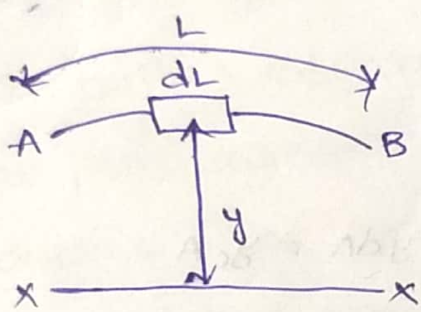
$= 3/4 \times \frac{8100}{150} = 40.5mm$

Theorem of Pappus and Guldinus

pappus and Guldinus developed a theorem for finding the surface areas & volumes of body's of revolution.

Theorem: 1 (To find surface Area)

The surface area generated by rotating any plane curve about a non intersecting axis in its plane is equal to the product of the length of the curve and distance travelled by its centroid.



Let AB is a plane curve of length 'L'. consider an element length dL and its centroid is at a distance y from xx axis. as shown in figure.

The centroid of entire area is y_c.

The surface area generated by revolving dL

$$dA = 2\pi y \times dL$$

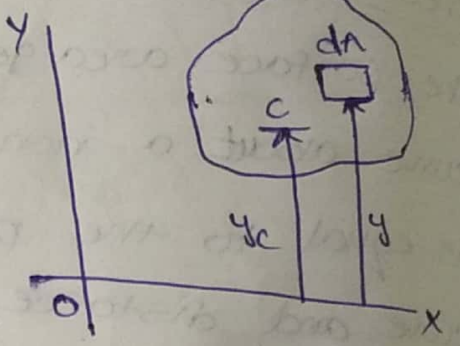
$$A = 2\pi \int_0^L y \, dL$$

But $y_c = \frac{\int y \, dL}{L}$ $\Rightarrow$ $\int y \, dL = y_c \cdot L$

* $\boxed{A = 2\pi y_c L}$

Theorem: 2 (To find volume)

The volume of the solid generated by rotating any plane figure about non intersecting axis in its plane is equal to the product of area of the figure and distance travelled by its centroid.



consider element area dA at a distance 'y' from ox axis.

volume generated by revolving dA

$$dv = 2\pi y \, dA$$

$$V = 2\pi \int y \, dA$$

$$y_c = \frac{\int y \, dA}{A} \Rightarrow \int y \, dA = y_c A$$

$$* \boxed{V = 2\pi y_c A}$$

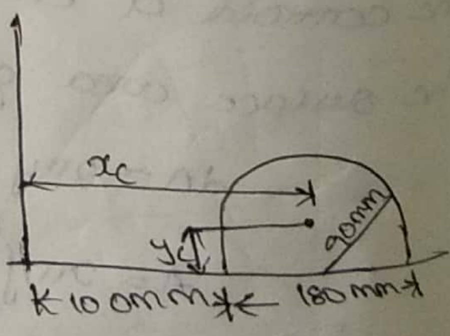
1) Find the volume generated by revolving a semi circular area about x axis & y-axis.

Sol: w.r.t x-axis

$$V = 2\pi y_c A$$

w.r.t y-axis

$$V = 2\pi x_c A$$



$$A = \frac{\pi r^2}{2} = \frac{3.14 \times 90 \times 90}{2}$$

$A = \cancel{12723.5}$ 12722.45 mm^2

$$y_c = \frac{4r}{3\pi} = \frac{4 \times 90 \times 7}{3 \times 22} = 38.18 \text{ mm}$$

π value in the calci shift $\times 10^x = \pi$

$V = 2 \times 3.14 \times 38.18 \times 12728.5$

$V = 3054694.5\ mm^3$

$x_c = r = \frac{180}{2} + 100 = 190mm$

$A = 12728.5$

$V = 2 \times 3.14 \times 190 \times 12728.5$

$V = (7197797.176)$

$V = 15189321 mm^3$

Moment of Inertia:-

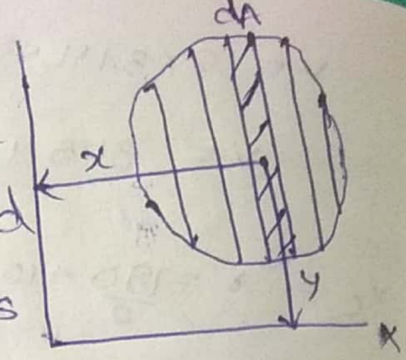
The moment of Inertia of an object is defined by the distribution of mass around an axis.
We know that, the moment of a force about a point is the product of the force and perpendicular dist. b/w the point & the line of action of the force. This moment is also called as first moment of force. (P.x) (Force x distance).
→ If this moment is again multiplied by the ⊥^lr distance b/w the point & line of action of force then this quantity is called as moment of the moment of a force. or second moment of force. or moment of Inertia. $(Px)x = Px^2$

Units:- mm^4 or m^4

Moment of Inertia can be use find out by the following Methods

1) By Routh's Rule
2) By Integration.

By Integration :-



Consider a plane figure whose moment of Inertia is required to be found out about XX axis & YY axis as shown in figure.

Let us divide the whole area into no of strips.

consider one of the strips.

Let dA = Area of the strip.

x = distance of the centre of the gravity of the strip about YY axis.

y = distance of the centre of the gravity of the strip about XX axis.

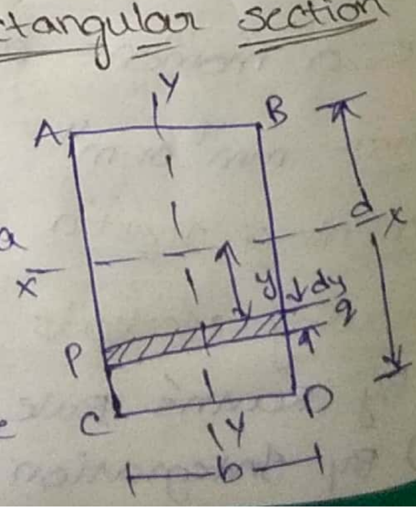
WKT, the moment of Inertia of the strip about YY axis

$$\boxed{I_{y\cancel{x}} = dA\,x^2}$$

Now, the moment of inertia of the strip ø whole area about YY & XX axis respectively may be found out by integrating above eqns.

$$\boxed{\begin{aligned} I_{YY} &= \Sigma\, dA\, x^2 \\ I_{XX} &= \Sigma\, dA\, y^2 \end{aligned}}$$

*Moment of Inertia of a Rectangular section



Consider a rectangular section ABCD whose moment of Inertia (M.I) is required to find out

b = width of the rectangle

d = depth of the rectangle.

Area of the strip, $A = b \times dy$

Moment of inertia about xx axis $= (b \times dy) \times y^2$

Moment of Inertia of the whole section is found out by Integrating the above eqn.

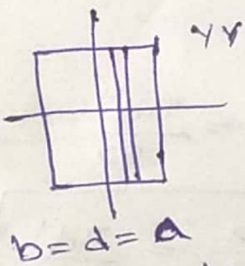
$$I_{xx} = \int_{-d/2}^{+d/2} (b\,dy)\, y^2 = b\left(\frac{y^3}{3}\right]_{-d/2}^{d/2}$$

$$= b/3 \left[(d/2)^3 - (-d/2)^3\right]$$

$$= b/3 \left[\frac{d^3}{8} + \frac{d^3}{8}\right] = \frac{\cancel{2}bd^3}{3 \times \cancel{8}_4} = \frac{bd^3}{12}.$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

$$\boxed{I_{yy} = \frac{db^3}{12}}$$



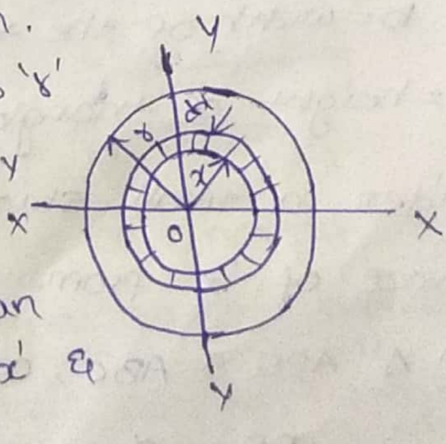
* For square section $b = d = a$
where a is side of a square

M.I ; $\boxed{I = \frac{a^4}{12}}$ $\because I_{xx} = I_{yy}$

Moment of Inertia of a circular section:-

M.I of a circular section.

consider a circle of radius 'r' with center 'o' and xx & yy be two axis of reference through 'o'. Now, consider an elemental ring of radius 'x' & thickness dx.



The area of the ring = $2\pi x \times dx$

M.I of the ring about XX or YY axis = $(2\pi x dx) x^2$
$= 2\pi x^3 dx$.

*Moment of Inertia of the whole section about the central axis:- (polar moment of Inertia)

$I_{xx} + I_{yy} = I_{zz} \rightarrow \perp^{r}$ axis theorem.

Total M.I about ZZ axis,

$$I_{zz} = \int_0^r 2\pi x^3 dx$$

$$= 2\pi \int_0^r x^3 dx = \frac{2\pi}{4}\left[x^4\right]_0^r = \frac{2\pi}{4}(r^4 - 0)$$

$$= \frac{2\pi r^4}{4} = \frac{\pi r^4}{2}$$

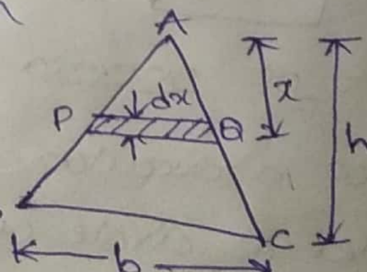
Sub, $r = d/2$,

$$I_{zz} = \frac{\pi (d/2)^4}{2} = \frac{\pi d^4}{32}$$

$$\boxed{I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi d^4}{64}}$$

* Moment of Inertia of a Triangular section

Consider a triangular section ABC.



Let b = width of the triangle
h = height of triangle

Consider a small strip PQ of thickness dx at a distance of 'x' from the vertex 'A'.

From Δ^{le} APQ & ABC, are similar Δ^{le}'s

$$\frac{PQ}{BC} = \frac{x}{h}, \quad PQ = \frac{x \times b}{h}$$

Scanned with CamScanner

Area of the strip $= \frac{bx}{h} \times dx$

M.I of the strip about base BC.

$$I_{BC} = \left(\frac{bx}{h} dx\right) (h-x)^2$$

M.I for whole section:-

$$I_{BC} = \int_0^h \left(\frac{bx}{h}\right) dx \, (h-x)^2 = b/h \int_0^h x (h-x)^2 \, dx$$

$$\frac{b}{h} \int_0^h x[h^2 + x^2 - 2xh] \, dx = \frac{b}{h} \int_0^h (xh^2 + x^3 - 2x^2 h) \, dx$$

$$= b/h \left\{ h^2 \frac{x^2}{2} + \frac{x^4}{4} - 2h \frac{x^3}{3} \right]_0^h$$

$$= b/h \left[\frac{h^2}{2} [h^2 - 0] + 1/4 [h^4 - 0] - \frac{2h}{3} [h^3 - 0] \right]$$

$$= b/h \left[\frac{h^4}{2} + \frac{h^4}{4} - \frac{2h^4}{3} \right]$$

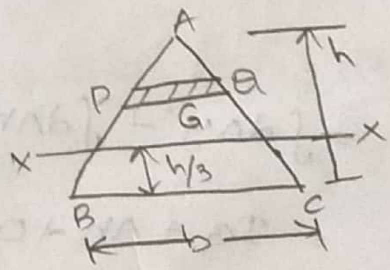
$$= b/h \left[\frac{12h^4 + 6h^4 - 16h^4}{24} \right] = b/h \left[\frac{\cancel{2}h^4}{\cancel{24}_{12}} \right]$$

$$= \frac{bh^4}{12h} = \frac{bh^3}{12} \Rightarrow \boxed{I_{BC} = \frac{bh^3}{12}}$$ MI about the base of triangular section.

2m

Note:-

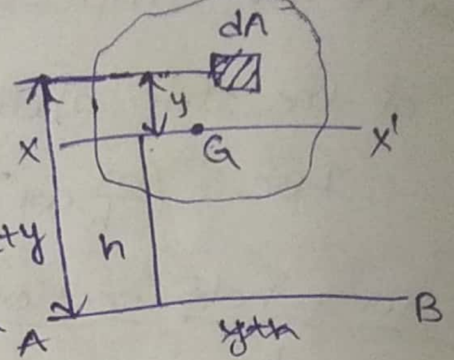
M.I of a Triangular section about the centroidal axis XX $\boxed{I_{XX} = \frac{bh^3}{36}}$



Parallel Axis theorem:- (2m statement)

It states that the moment of Inertia of an area about any axis is equal to the moment of Inertia about a parallel Axis passing through its centroid plus the area multiplied by the square of the dist. b/w the axis.

$$\boxed{I_{AB} = I_G + Ah^2}$$



$I_G = \Sigma dA\,y^2$

where I_G = moment of Inertia about centroidal Axis.

'A' is the area of the given figure. 'h' dist. b/w centroidal axis & any given axis. I_{AB} = moment of inertia about axis AB.

Proof :-

Consider an elemental Area dA of lamina which is at a dist. of y from centroidal axis XX'.

$I_G = \Sigma dA\,y^2$

From diagram dist. of dA from given axis AB is $(h+y)$

$$I_{AB} = \int dA\,(y+h)^2$$

$$= \int dA\,(y^2+h^2+2hy) = \int dA\,y^2 + \int dA\,h^2 + \int dA\,2hy$$

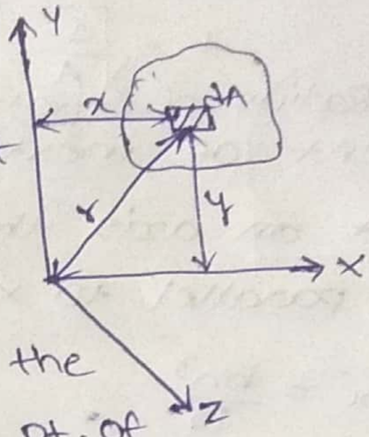
$$= \int dA\,y^2 + \int dA\,h^2 + 2h\underbrace{\int dA\,y}_{0}$$

$$= I_G + Ah^2 + 0$$

$$\boxed{\therefore I_{AB} = I_G + Ah^2}$$

→ $\int dA\,y$ represents moment of Area w.r.t centroidal Axis XX. This is equal to the product of area $(\int dA = A)$ and dist. of centroid from XX' axis the dist. b/w them is zero.

Perpendicular Axis Theorem :- (2m)



If I_{xx} & I_{yy} be the moment of inertia of a lamina about mutually $\perp^{lr}$ axis x & y, then the moment of inertia of a lamina about z axis normal to the lamina & passing through the pt. of intersection of x & y axis is given by

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

Proof:-

Consider elemental Area dA at a dist x, y, r from y, x & z axis respectively.

Moment of Inertia of lamina about z-axis

$$I_{zz} = \int dA\, r^2 \qquad \because x^2 + y^2 = r^2$$

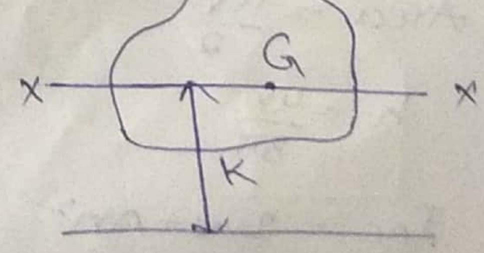
$$= \int dA(x^2+y^2) = \int dA\, x^2 + \int dA\, y^2$$

$$= I_{yy} + I_{xx}$$

$$\boxed{\therefore I_{zz} = I_{yy} + I_{xx}}$$

The moment of inertia about z-axis (I_{zz}) is called polar moment of inertia & is denoted by J or I_p

Radius of Gyration :-



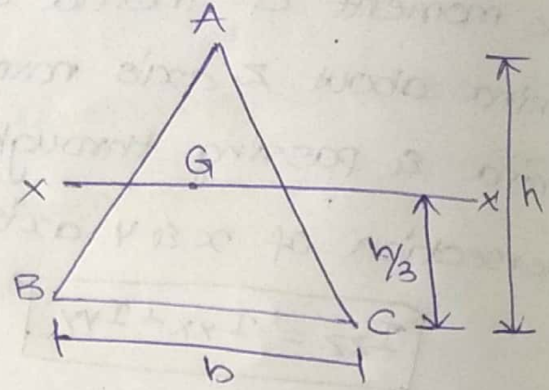
Consider the entire area is concentrated at a point on the lamina the distance of this point from the given reference axis is called radius of gyration.

$I = AK^2$ with resp to x axis then

$K = \sqrt{\frac{I}{A}}$ ~~k~~ is. $K = \sqrt{\frac{I_{xx}}{A}}$

K is Radius of Gyration. wrt. y-axis, $K = \sqrt{\frac{I_{yy}}{A}}$

*Moment of inertia of a triangular section about an axis through its center of gravity and parallel to xx axis.



$I_{BC} = \frac{bh^3}{12}$

$I_{xx} = \frac{bh^3}{36}$

$I_{BC} = I_{xx} + Ah^2$

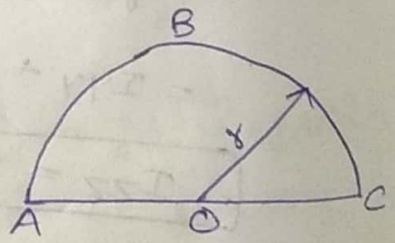
$I_{xx} = I_{BC} - Ah^2$

$= \frac{bh^3}{12} - (\frac{1}{2} bh)(h/3)^2$

$= \frac{bh^3}{12} - \frac{bh^3}{18} = bh^3 [\frac{1}{12} - \frac{1}{18}]$

$\boxed{I_{xx} = \frac{bh^3}{36}}$

*Moment of Inertia of a semi circular section



$I_{AC} = \frac{1}{2}\left(\frac{\pi d^4}{64}\right) \quad (\because \frac{d}{2} = r)$

$= \frac{1}{2} \times \frac{\pi r^4 \times 2}{64}$

$= \frac{8\pi r^4}{64} = \frac{\pi r^4}{8}$

$\text{Area} = \frac{\pi r^2}{2}$

$h = \frac{4r}{3\pi}$

$I_{AC} = I_{xx} + Ah^2$

$I_{xx} = I_{AC} - Ah^2$

$= \frac{\pi r^4}{8} - \left(\frac{\pi r^2}{2}\right)\left(\frac{4r}{3\pi}\right)^2$

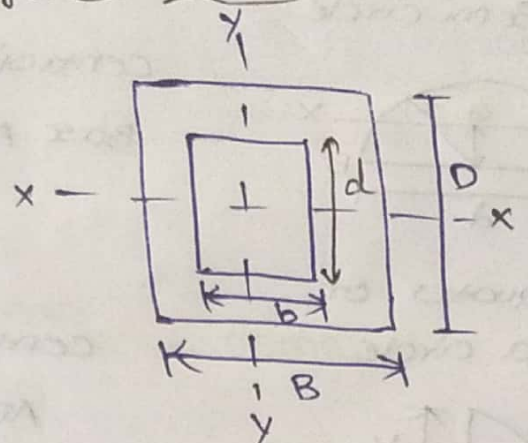
$$= \frac{\pi r^4}{8} - \frac{\pi r^2}{2} \times \frac{16 r^2}{9\pi^2} = \frac{\pi r^4}{8} - \frac{8 r^4}{9\pi}$$

$$= r^4 \left[\pi/8 - \frac{8}{9\pi} \right]$$

$$\boxed{I_{xx} = 0.109 r^4}$$

M.I of a hollow rectangular section

M.I about x-x axis

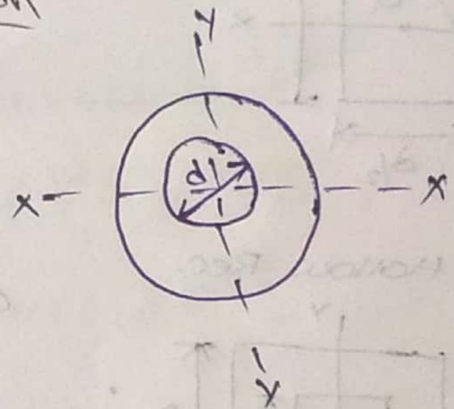


$$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$I_{xx} = \frac{BD^3 - bd^3}{12}$$

$$I_{yy} = \frac{DB^3 - dB^3}{12}$$

M.I for hollow circular section

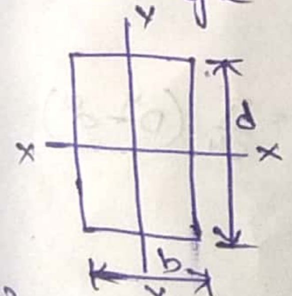
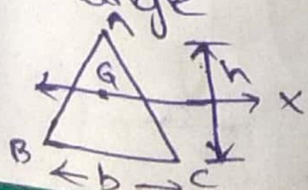


$$I_{xx} = I_{yy} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

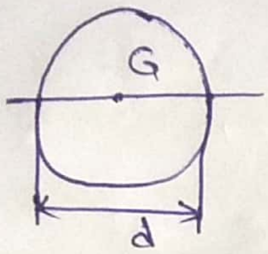
$$= \frac{\pi (D^4 - d^4)}{64}$$

$$I_{zz} = \frac{\pi D^4 - \pi d^4}{32} = \frac{\pi (D^4 - d^4)}{32}$$

Moment of inertia of standard sections

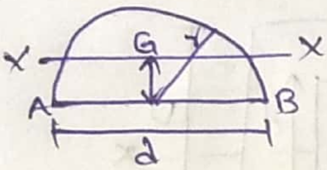
Shape	Axis	M.I
1) Rectangle 	centroidal axis xx	$I_{xx} = \frac{bd^3}{12}$
	centroidal axis y-y	$I_{yy} = \frac{db^3}{12}$
	Base A-B, $I_{AB} = \frac{bd^3}{3}$	
2) Triangle 	centroidal axis x-x	$I_{xx} = \frac{bh^3}{36}$
	Base B-C	$I_{BC} = \frac{bh^3}{12}$

3) circle



Diametral axis $I = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$

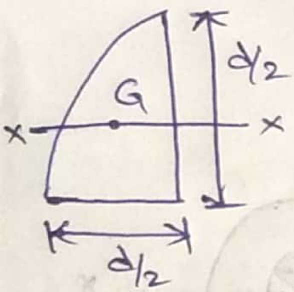
4) semi circle



centroidal axis $I_{xx} = 0.11 r^4$

Base AB $I_{AB} = \frac{\pi d^4}{128} = \frac{\pi r^4}{8}$

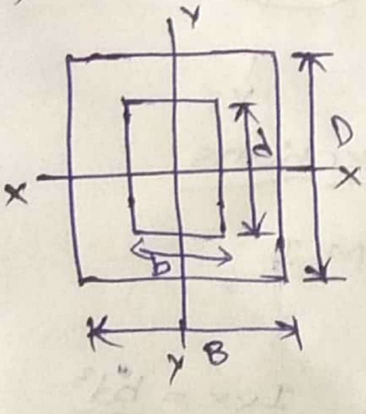
5) quater of a circle



centroidal axis, $I_{xx} = 0.055 r^4$

AB $I_{AB} = \frac{\pi d^4}{256}$

6) Hollow Rec.



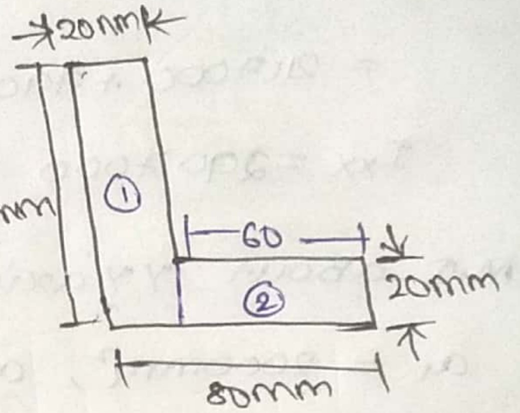
centroidal axis $I_{xx} = \frac{BD^3 - bd^3}{12}$

$I_{yy} = \frac{DB^3 - db^3}{12}$

7) Hallow circle

diameter axis $I = \frac{\pi}{64}(D^4 - d^4)$

1) Find the M.I about the centroidal xx axis & y-y axis of the angle section as shown in figure.



$a_1 = 20 \times 100 = 2000\,mm^2$

$a_2 = 60 \times 20 = 1200\,mm^2$

M.I about xx axis.

$y_1 = \frac{100}{2} = 50mm$

$y_2 = \frac{20}{2} = 10mm$

$\bar{y} = \frac{2000 \times 50 + 1200 \times 10}{2000 + 1200} = 35mm$

M.I for ① Rectangle,

$I_{G_1} = \frac{bd^3}{12} = \frac{20 \times 100^3}{12} = 1666666.6\,mm^4$

$= 1.667 \times 10^6\,mm^4$

$a_1 = 2000\,mm^2$

$h_1 = 50 - 35 = 15mm$

$I_{xx_1} = I_{G_1} + A_1 h_1^2$

$= 1.667 \times 10^6 + 2000 \times 15 \times 15$

$= 2117000\,mm^4 = 2.117 \times 10^6\,mm^4$

M.I for ② Rectangle,

$I_{G_2} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 40{,}000$

$a_2 = 1200\,mm^2$

$h_2 = 35 - 10 = 25mm$

$I_{YY_1} = I_{G_2} + A_2 h_2^2 = 40000 + 1200 \times 25 \times 25$

$= 7{,}90{,}000\,mm^4$

M.I about XX axis for the whole section

$I_{XX} = I_{XX_1} + I_{YX_1}$

$= 2117000 + 790000$

$I_{XX} = 2907000 = 2.907 \times 10^6 \, mm^4$

M.I about YY axis,

$a_1 = 2000 mm^2, \; a_2 = 1200 mm^2$

$x_1 = \frac{20}{2} = 10mm, \; x_2 = \frac{\cancel{20+}60}{2} = \cancel{40mm} \; 50mm$

$\bar{x} = \frac{2000 \times 10 + 1200 \times 40}{2000 + 1200} = 25mm$

$I_{G_1} = \frac{db^3}{12} = \frac{100 \times 20 \times 20 \times 20}{12} = 66666.6$

$= 6.6 \times 10^4 mm^4$

$h_1 = 25 - 10 = 15mm$

$I_{YY_1} = I_{G_1} + Ah_1^2$

$I_{YY_1} = 6.6 \times 10^4 + 2000 \times 15 \times 15 = 516000 mm^4$

M.I for ② rectangle,

$I_{G_2} = \frac{db^3}{12} = \frac{20 \times 60 \times 60 \times 60}{12} = 360000 mm^4$

$a_2 = 1200$

$h_2 = \cancel{50} \; 25 = 25mm$

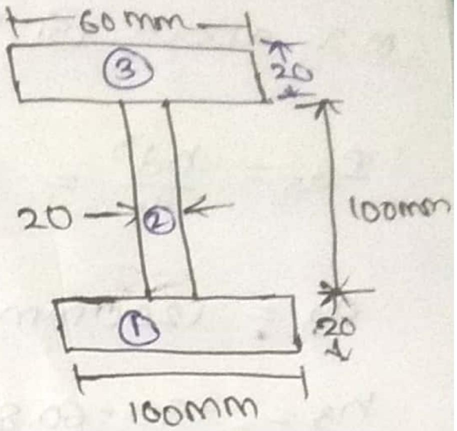
$I_{YY_2} = I_{G_2} + A_2 h^2$

$= 1110000 mm^4$

$I_{YY} = I_{YY_1} + I_{YY_2} = 1110000 + 516000$

$= 1626000 mm^4$

2)



$a_1 = 20 \times 100 = 2000mm^2$

$a_2 = 20 \times 100 = 2000mm^2$

$a_3 = 60 \times 20 = 1200mm^2$

$y_1 = \frac{20}{2} = 10mm$

$y_2 = 20 + \frac{100}{2} = 70mm$

$y_3 = 20 + 100 + \frac{20}{2} = 130mm$

$$\bar{y} = \frac{2000 \times 10 + 2000 \times 70 + 1200 \times 130}{2000 + 2000 + 1200}$$

$\bar{y} = 60.76\,mm \simeq 60.8mm$

M.I about ① Rectangle,

$I_{G_1} = \frac{bd^3}{12} = \frac{20^3 \times 100}{12} = (1.667 \times 10^6 mm^4)$
$= 6.6 \times 10^4 mm^4$

$a_1 = 2000mm^2$

$h_1 = 60.8 - 10 = 50.8mm.$

$I_{xx_1} = I_{G_1} + A_1 h_1^2$

$= \overset{6.6\times10^4}{1.667 \times 10^4} + 2000 \times 50.8 \times 50.8$

$= (6828280mm^4)\ 5227280mm^4$

M.I about ② Rectangle,

$I_{G_2} = \frac{bd^3}{12} = \frac{100^3 \times 20}{12} = 1.667 \times 10^6 mm^4$

$a_2 = 2000mm^2$

$h_2 = 70 - 60.8 = 9.2mm$

$I_{xx_2} = 1836280\ mm^4$

Scanned with CamScanner

M.I about ③ Rectangle,

$$I_{G_3} = \frac{bd^3}{12} = \frac{60 \times 20^3}{12} = 40000\,mm^4$$

$$a_3 = 1200\,mm^2$$

$$h_3 = 130 - 60.8 = 69.2$$

$$I_{xx_3} = I_{G_3} + a_3 h_3^2$$

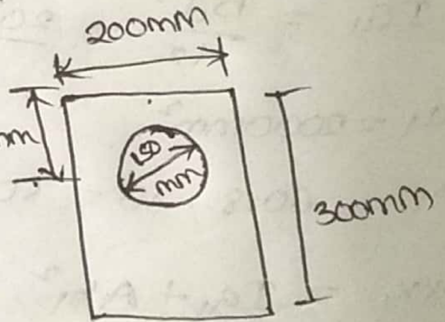
$$= 5786368\,mm^4$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$= 5227280 + 1836280 + 5786368$$

$$I_{xx} = 12849928\,mm^4. = 12.8 \times 10^6\,mm^4$$

3) Find M.I of the hollow section shown in figure about an axis passing through its centre of gravity or parallel to XX axis



Soll $a_2 = \pi r^2 = \pi \times \left(\frac{150}{2}\right)^2$, $a_1 = 200 \times 300 = 60000\,mm^2$

$a_2 = 17671.4$

$y_1 = \frac{300}{2} = 150\,mm$, $y_2 = 300 - 100 = 200\,mm$

$\bar{y} = 129.12\,mm$

$$I_{G_1} = \frac{bd^3}{12} = \frac{200 \times 300^3}{12} = 450 \times 10^6\,mm^4$$

$h_1 = \cancel{130} \; 150 - 129.12 = 20.9\,mm$

$$I_{G_2} = \frac{\pi d^4}{64} = \frac{3.14 \times (150)^4}{64} = \begin{cases} 24837890.63\,mm^4 \\ 24.8 \end{cases}$$

$$= 24850488.76 = 24.85 \times 10^6\,mm^4$$

$$h_2 = 200 - 129.1 = 70.9\,mm$$

$$I_{xx_1} = I_{G_1} + A h_1^2$$

$$= 450 \times 10^6 + 60000 \times (20.9)^2$$

$$= 476208600 = 476.2 \times 10^6$$

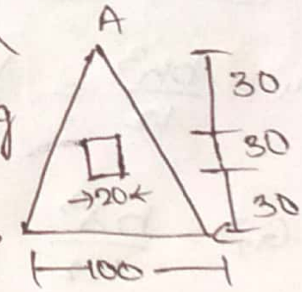
$$I_{xx_2} = I_{G_2} + A h_2^2$$

$$= 113680770.2 = 113.6 \times 10^6$$

$$I_{xx} = \cancel{589889370.2}\ 476.2 \times 10^6 - 113.6 \times 10^6$$

$$I_{xx} = 362.6 \times 10^6\,mm^4.$$

4) A rectangular hole is made in a triangular section as show in fig. Determine the M.I of the section about xx axis passing through the centre of gravity & the base BC.



Sym about

Sol: $a_1 = \frac{1}{2} \times 90 \times 100 = 4500\,mm^2$

$a_2 = 20 \times 30 = 600\,mm^2$

$y_1 = h/3 = \frac{90}{3} = 30\,mm$, $y_2 = 30 + \frac{30}{2} = 45\,mm$

$\bar{y} = 27.69\,mm$

M.I about the axis passing through C.G:-

$$I_{G_1} = \frac{bh^3}{36} = \frac{100 \times (90)^3}{36} = 2025 \times 10^3$$

$h_1 = 30 - 27.69 = 2.31 mm$

$IG_2 = \frac{bd^3}{12} = \frac{20 \times 30^3}{12} = 45000 mm^4$

$h_2 = 45 - 27.7 = 17.3 mm$

$Ixx_1 = IG_1 + A_1 h_1^2$

$= \cancel{2025023805 mm^4} \quad 2048805 mm^4$

$Ixx_2 = IG_2 + A_2 h_2^2$

$= 224574 mm^4$

$Ixx = Ixx_1 + Ixx_2$

$= 1824231 mm^4$

M.I of the whole section about the base BC:-

$IG_1 = \frac{bh^3}{12} = \frac{100 \times 90^3}{12} = 6075000 mm^4 = Ixx_1 \quad \frac{bh^3}{12}$

$IG_2 = \frac{bd^3}{12} = \frac{20 \times 30^3}{12} = 45000 mm^4$

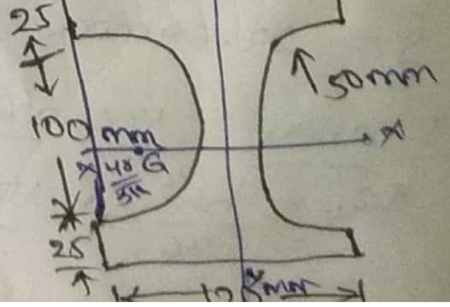
$a_2 = 600 mm$

$h_2 = 30 + \frac{30}{2} = 45 mm$

$Ixx_2 = 1260000$

$Ixx = 4815 \times 10^3 mm^4.$

5) Fig. shows the cross section of cast iron beam. Determine the moments of inertia of the section about horizontal & vertical axis.



passing through the centroid of the section.

Sol M.I (of rectangle) about horizontal axis

$$I_{G_1} = \frac{bd^3}{12} = \frac{120 \times 150^3}{12} = 3375 \times 10^4 \, mm^4$$

$$I_{G_2} = \frac{\pi r^4}{4} = \frac{\pi \times 50^4}{4} = 4908738.52$$

$$490.87 \times 10^4 mm^4$$

$$I_{xx} = I_{G_1} - I_{G_2} = 28841.2 \times 10^3$$

$$= 2884.1 \times 10^4 mm^4.$$

M.I about vertical axis :-

$$I_{G_1} = \frac{db^3}{12} = \frac{150 \times 120^3}{12} = 216 \times 10^5 mm^4 = I_{yy_1}$$

$$I_{G_2} = \frac{\pi r^4}{4} = 0.11 \times r^4 = 0.11 \times 50^4$$

$$= 687500 mm^4$$

I_{yy} = Area of circle $a = \frac{\pi r^2}{2} = 3926.99 mm^2$

$$h = 60 - \frac{4 \times r}{3\pi} = 38.77 mm$$

$$I_{yy_2} = I_{G_2} + ah^2$$

$= \cancel{6593254.7}$ 6590224.95mm^4

(2 semi circles) for only one semi circle.

for both $I_{yy_2} = 6590224.95 \times 2$

$$= 13180449.9 mm^4.$$

$$I_{yy} = I_{yy_1} - I_{yy_2}$$

$$= 216 \times 10^5 - 131.8 \times 10^5$$

$$= 842 \times 10^4 mm^4$$