

**Example 14.10.** A four-wheeled trolley car of mass 2500 kg runs on rails, which are 1.5 m apart and travels around a curve of 30 m radius at 24 km/hr. The rails are at the same level. Each wheel of the trolley is 0.75 m in diameter (i.e. each of the two axles is driven by a motor running in a direction opposite to that of the wheels at a speed of five times the speed of rotation of the wheels. The moment of inertia of each axle with gear and wheels is 18 kg-m<sup>2</sup>. Each motor with shaft and gear pinion has a moment of inertia of 12 kg-m<sup>2</sup>. The centre of gravity of the car is 0.9 m above the rail level. Determine the vertical force exerted by each wheel on the rails taking into consideration the centrifugal and gyroscopic effects. State the centrifugal and gyroscopic effects on the trolley.



**Solution.** Given :  $m = 2500 \text{ kg}$  ;  $x = 1.5 \text{ m}$  ;  $r = 30 \text{ m}$

$$v = 24 \text{ km/h} = 6.67 \text{ m/s} ; r_w = 0.375 \text{ m} ; G = 18 \text{ kg-m}^2 ; I_E = 12 \text{ kg-m}^2 ; h = 0.9 \text{ m}$$

The weight of the trolley ( $W = m.g$ ) will be equally distributed over the four wheels, which will act downwards. The reaction between the wheels and the road surface of the same magnitude will act upwards.

$$\therefore \text{Road reaction over each wheel} = W/4 = m.g/4 = 2500 \times 9.81/4 = 6131.25 \text{ N}$$

We know that angular velocity of the wheels.

$$\omega_w = v/r_w = 6.67/0.375 = 17.8 \text{ rad/s} \quad \text{www.EngineeringBooksPDF.com}$$

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$$\text{and angular velocity of precession, } \omega_p = v/R = 6.67/30 = 0.22 \text{ rad/s}$$

$\therefore$  Gyroscopic couple due to one pair of wheels and axle,  $C_w = 2 I_w \omega_w \omega_p = 2 \times 18 \times 17.8 \times 0.22 = 141 \text{ N-m}$  and gyroscopic couple due to the rotating parts of the motor and gears,

$$C_E = 2 I_E \omega_E \omega_p = 2 \dots \dots (0 \omega = G \omega_w)$$

$$= 2 \times 12 \times 17.8 \times 0.22 = 470 \text{ N-m}$$

$$\therefore \text{Net gyroscopic couple, } C = C_w - C_E = 141 - 470 = -329 \text{ N-m}$$

... (-ve sign is used due to opposite direction of motor) Due to this net gyroscopic couple, the vertical reaction on the rails will be produced. Since  $C_E$  is greater than  $C_w$ , therefore the reaction will be vertically downwards on the outer wheels and vertically upwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheel be  $P$  newton.

$$P/2 = C/r = 329/2 \times 1.5 = 109.7 \text{ N} \quad \text{We know that centrifugal force,}$$

$$F_c = m.v^2/R = 2500 (6.67)^2/30 = 3707 \text{ N}$$

$$\therefore \text{Overturning couple, } C_o = F_c \times h = 3707 \times 0.9 = 3336.3 \text{ N-m}$$

This overturning couple is balanced by the vertical reactions which are vertically upwards on the outer wheels and vertically downwards on the inner wheels. Let the magnitude of this reaction at each of the outer or inner wheels be  $Q$  newton.

$$\therefore Q/2 = C_o/2r = 3336.3/2 \times 1.5 = 1112.1 \text{ N}$$

We know that vertical force exerted on each outer wheel.

$$P_o = W/4 + Q/2 = 6131.25 + 1112.1 = 7243.35 \text{ N}$$

and vertical force exerted on each inner wheel.

$$P_i = W/4 - Q/2 = 6131.25 - 1112.1 = 5019.15 \text{ N}$$

## 14.8. Effect of Gyroscopic Couple on a Naval Ship during Rolling

We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

In case of rolling of a ship, the axis of precession (i.e. longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

**Example 14.4.** *The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/h and steers to the left in a curve of 75 m radius.*

**Solution.** Given:  $m = 8 \text{ t} = 8000 \text{ kg}$ ;  $k = 0.6 \text{ m}$ ;  $N = 1800 \text{ r.p.m.}$  or  $\omega = 2\pi \times 1800/60 = 188.5 \text{ rad/s}$ ;  $v = 100 \text{ km/h} = 27.8 \text{ m/s}$ ;  $R = 75 \text{ m}$

We know that mass moment of inertia of the rotor,  $I = m \cdot k^2 = 8(0.6)^2$

$$= 2880 \text{ kg-m}^2$$

and angular velocity of precession,

$$\omega_p = v/R = 27.8 / 75 = 0.37 \text{ rad/s}$$

We know that

gyroscopic couple,

$$C = I \omega_p = 2880 \times 188.5 \times 0.37 = 200866 \text{ N-m} = 200.866 \text{ kN-m}$$

Ans.

We have discussed in Art. 14.6, that when the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.