V. TORSION

③

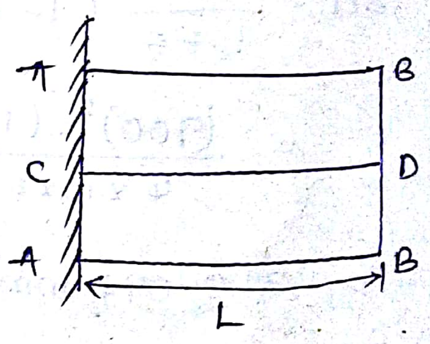
Torsion of shaft:

Shaft is said to be in torsion when equal end opposite torques are applied at the two end of the shaft.

→ Torque = product of force applied and radius the shaft.

→ Due to the applications of torques at two ends, shaft is subjected to twisting moments, this causes shear stresses and shear strains in the material of the shaft.

Deviation of shear stress produced in circular shaft subjected to torsion.

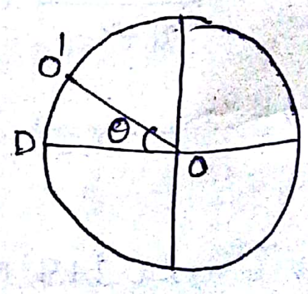


Distorsion = DO'

$$\text{stress} = \frac{DO'}{CD} = \frac{DO'}{L} = \tan\phi$$

$$\phi = \frac{DO'}{L}$$

from fig,



$$DO' = OD \times \theta = R\theta$$

$$\phi = \frac{R\theta}{L}$$

$$\text{Shear stress} = \frac{R\theta}{L}$$

$$C = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$C = \frac{\tau}{\frac{R\theta}{L}}$$

$$\frac{\tau}{R} = \frac{C\theta}{L}$$

C, θ, L are constants

$$\tau \propto R$$

$$\frac{\tau}{R} = \text{constant}$$

similarly,

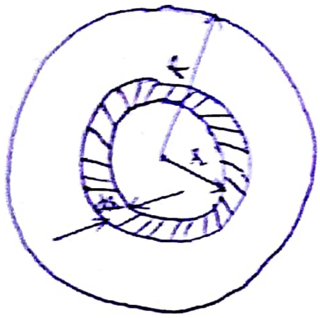
$$\frac{q}{r} = \text{constant}$$

$$\boxed{\frac{\tau}{R} = \frac{q}{r} = \frac{C\theta}{L}}$$ //

Assumptions made in a derivation of shear stress produced in the circular shaft subjected to torsion.

- → Material of the shaft is uniform throughout
- → Twist along the shaft is uniform.
- → The shaft is uniform circular cross section throughout.
- → cross section of shaft which are plain before twist remain plain after twist.
- → All radii which are straight before twist remains same straight after twist.

* ## Maximum torque transmitted by a circular Solid shaft :



Consider an elementary circular ring of thickness 'dr' at a distance 'r' from centre. Then area of the ring, dA = $2\pi r\,dr$

wkt,

$$\boxed{\frac{\tau}{R} = \frac{q}{r}}$$

where,

q = shear stress

Turning force on elementary circular ring is

= shear stress acting on the ring × area of ring

$$F = \sigma \times A$$

$$= \frac{\tau}{R} \cdot r \, dA$$

$$= \frac{\tau r^2}{R} (2\pi) \, dr$$

Twinning moment due to turning force on ring is, dT = turning force of ring × distance from a ring

$$dT = F \times r$$

$$= \frac{\tau r^2}{R} \cdot dA$$

Total twinning moment is obtained by integration of above equation from 0 to R

$$\int dT = \int_0^R \tau \frac{r^2}{R} \times dA$$

$$T = \int_0^R \tau \frac{r^2}{R} \times 2\pi \cdot r\, dr$$

$$T = \tau \cdot \frac{2\pi}{R} \int_0^R r^3\, dr$$

$$T = \tau \cdot \frac{2\pi}{R} \left[\frac{r^4}{4} \right]_0^R$$

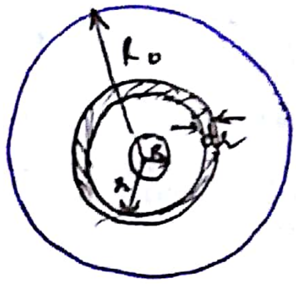
$$T = \tau \cdot \frac{2\pi}{R} \cdot \frac{R^4}{4}$$

$$\boxed{T = \tau \frac{\pi R^3}{2}}$$ (or)

$$\boxed{T = \frac{\tau \pi D^3}{16}}$$

③ A solid shaft of 150 mm diameter is used to transmit torque. Find the maximum torque transmitted by the shaft, if the max. shear stress induced to the shaft is $45\ N/mm^2$.

* Torque transmitted by Hollow Circular Sha



$a = dA = 2\pi r\, dr$

$$\frac{T}{R_0} = \frac{q}{r}$$

$$q = \frac{r}{R_0} \cdot T$$

$$\sigma = \frac{F}{A} \Rightarrow F = \sigma A$$

Turning force $= \sigma \times A = q \times dA$

$$= \frac{r}{R_0} \times 2\pi r\, dr$$

$$= \frac{\tau}{R_0} \times 2\pi r^2\, dr$$

Turning moment $= F \times \perp^{lar}$ dist.

$$= \frac{\tau}{R_0} \cdot 2\pi r^2 dr \cdot r$$

$$= \frac{\tau}{R_0} \cdot 2\pi r^3\, dr$$

$$\int dT = \int_{R_i}^{R_o} \frac{\tau}{R_0} \cdot 2\pi r^3\, dr$$

$$T = \frac{\tau}{R_0} \cdot 2\pi \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{\tau}{R_0} \cdot \frac{\pi}{2} \left[R_o^4 - R_i^4\right]$$

$$= \frac{\pi}{2} \frac{\tau}{R_0} \left[\frac{D_o^4}{16} - \frac{D_i^4}{16}\right]$$

$$= \frac{\pi}{\cancel{2}} \cdot \frac{\tau}{D_o} \times \cancel{2} \left[\frac{D_o^4 - D_i^4}{16}\right]$$

$$T = \frac{\pi \tau}{16 D_o} \left[D_o^4 - D_i^4\right]$$

Power transmitted by shaft:

$$P = \frac{2\pi NT}{60}$$

$$= \omega \times T \text{ watts} \quad \left[\because \omega = \frac{2\pi N}{60}\right]$$

polar modulus:

$$Z = \frac{J}{R}$$

solid shaft, $J = \frac{\pi}{32} D^4$

Hollow shaft, $J = \frac{\pi}{32}(D_o^4 - D_i^4)$

solid, $Z = \frac{\pi}{16} D^3$

Hollow, $Z = \frac{\pi}{16}\left[\frac{D_o^4 - D_i^4}{D}\right]$

Q In a hollow circular shaft of outer and inner dia. of 20 cm & 10 cm respectively. If Shear stress is not to exceed 40 N/mm². Find the maximum torque which the shaft can safely transmit.

Q Determine the diameter of solid steel shaft which will transmit 90 kwatt at 160 rpm also determine the length of the shaft, if the twist must not exceed 1° over the entire length. The max. shear stress is limited to 60 N/mm². Take value of modulus of rigidity as 8×10^4 N/mm².

① Ans: $D_i = 10\ cm = 10 \times 10\ mm$

$D_o = 20\ cm = 200\ mm$

$$T = \frac{\pi}{16}\tau\left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$$= \frac{\pi}{16} \times 40\left[\frac{(200)^4 - (100)^4}{200}\right]$$

$$= 58904862.25\ N\text{-}mm$$

② Ans: $\omega = \frac{2\pi N}{60}$

$N = 160\ rpm$

$P = 90\ kW$

Modulus of rigidity, $C = 8 \times 10^4\ N/mm^2$

twist, $\theta = 1° = \frac{\pi}{180}$ rad ; $\tau = 60\ N/mm^2$

$$\frac{90 \times 10^3}{\frac{2\pi(160)}{60}} = T$$

$$T = \frac{60 \times 90 \times 10^3}{320 \times 3.14}$$

$$= 5374.20\ kN\text{-}m$$

$$T = \frac{\pi}{16}\tau D^3$$

$$D^3 = \frac{16T}{\pi . \tau} = \frac{16 \times 5374.2}{3.14 \times 60} = 456.4 \times 10^{?}mm$$

$\therefore D = 7.699 \times 10\ mm$

$$\frac{\tau}{R} = \frac{C\theta}{L}$$

$$\therefore L = C\theta . \frac{R}{\tau} = 8 \times 10^4 \times \frac{\pi}{180} \times (38.49) = 895.24$$

③ A ...
trans...
max...
shaft...

Sol: ...

④ Find ...
circul...
trans...

Sol:

③ A hollow shaft of external dia 120 mm transmits 300 kW power at 200 rpm. Determine max. internal dia., if the max. stress in the shaft is not exceeds 60 N/mm²

Sol: External dia, $d_o = 120$ mm

$N = 200$ rpm

$P = 300$ kW

shear stress, $\tau = 60$ N/mm²

$$P = \omega \times T$$

$$T = \frac{P}{\omega} = \frac{P}{\frac{2\pi N}{60}} = \frac{300 \times 10^3 \times 60}{2\pi \times (200)}$$

$$= 14.33 \text{ kN-mm}$$

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o}\right]$$

$$14.33 \times 10^3 = \frac{\pi}{16}(60)\left(\frac{(120)^4}{(120)} - \frac{D_i^4}{120}\right)$$

$$-\frac{14.33 \times 10^3}{60 \times \pi} \times 16 + (120)^3 = \frac{D_i^4}{120}$$

$$D_i^4 = 207213961.8$$

$$D_i = 119.$$

④ Find the max. shear stress induced in solid circular shaft of dia. 15cm when the shaft transmits 150 kW power at 180 rpm.

Sol:

$D = 15$ cm

$P = 150$ kW

$N = 180$ rpm

$$T = \frac{\pi}{16} \tau D^3 \Rightarrow \tau = \frac{16T}{\pi} \cdot \frac{1}{D^3}$$

$$P = \omega \times T$$

$$T = \frac{P}{\omega} = \frac{150 \times 10^3 \times 60}{2\pi \times (180)}$$

$$T = 7961.78$$

$$\tau = \frac{16 \times (7961.78)}{3.14} \cdot \frac{1}{(15 \times 10)^3}$$

$$= 0.012 \ N/mm^2$$

(5) A Hollow shaft is to transmit 300kW power at 80rpm if the shear stress if not exceed 60 N/mm² and internal diameter 0.6 of external dia. Find the external & internal dia. assuming max. torque as 1.4 mean torque.

Sol: $P = 300 \times 10^3 \ W$

$N = 80 \ rpm$

$\tau = 60 \ N/mm^2$

$T = 1.4$

$D_i = 0.6 \ D_o$

$$T = \frac{\pi}{16} \tau \left(\frac{D_o^4 - (0.6 D_o)^4}{D_o} \right)$$

$$T = \frac{\pi}{16} \tau \left(\frac{D_o^4 (0.4)}{D_o} \right)$$

$$D_o^3 = \frac{T \times 16}{\pi \times \tau} \times \frac{1}{0.4}$$

$$= \frac{1.4 \times 16}{3.14 \times 60} \times \frac{1}{0.4}$$

$$=$$

(6) A cylindrical shell 90 cm long and 20 cm internal dia. having thickness of metal has 8mm is filled with fluid and atmospheric pressure. If an additional 20 cm^3 of fluid is pumped into cylinder. (i) Find pressure exerted by the fluid on the cylinder?

(ii) Hoop stress induced, take $E = 2 \times 10^5 \, N/m^2$ & $\mu = 0.3$

(7) A water main of 80 cm dia contain water pressure head of 100m. If the weight density of water is 9810 N/m^3. Find the thickness of metal requires for water main. given the permissible stress as 20 N/mm^2

(8) A boiler shell is to be made of 15mm thick plate having a limiting tensile stress of 120 N/mm^2. If the efficiencies of longitudinal and circumferential joints are 70% and 30% respectively. Determine

(i) Max. permissible dia. of shell of an internal pressure of 20 N/mm^2

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5 m

Sol:- thickness, $t = 15$ mm

Tensile stress, $\sigma = 120 \, N/mm^2$

(i) $\sigma_t = \dfrac{Pd}{2\eta_l t} \Rightarrow 120 = \dfrac{20(d)}{2\left(\frac{70}{100}\right)(15)}$

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$$d = \frac{\overset{6}{\cancel{120}} \times 2 \times \frac{70}{100} \times 15}{\cancel{20}} = 126$$

$$\sigma_l = \frac{Pd}{2\eta_c t} \Rightarrow d = \frac{120 \times 2 \times \frac{30}{100} \times 15}{20} =$$

(ii) $d = 1.5\ m$

$$\sigma_l = \frac{Pd}{2\eta_c t}$$

$$120 = \frac{P(1.5 \times 1000)}{2\left(\frac{30}{100}\right)(15)}$$

$$P = \frac{120 \times 2 \times 30 \times 15}{1.5 \times 1000 \times 100} = 1.44\ N/mm^2$$

$$\sigma_c = \frac{Pd}{2\eta_l t} \Rightarrow Pd = \sigma_c (2)\eta_l t$$

$$P = \frac{120 \times 2 \times 70 \times 15}{100 \times 1.5 \times 10^3} = 1.68\ N/m$$

⑦ **Sol:** diameter, $d = 80\ cm$.

$h = 100\ m$

$\rho = 9810\ N/m^3$

$$P = \rho g h = 9810 \times 9.81 \times 100$$

$$= 9623610\ N/m^2$$

$$\sigma = \frac{Pd}{2t} \Rightarrow t = \frac{1}{\frac{2\sigma}{Pd}} = \frac{9623610 \times 80 \times 10^{-2}}{2 \times 20 \times 10^6 \times 10}$$

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Sol: (6) Given,

$L = 90\ cm$

$d = 20\ cm$

$\delta V = 20\ cm^3$

$t = 8\ mm$

$P = ?$

$$\frac{\delta V}{V} = 2e_1 + e_2 = 2\left(\frac{Pd^2}{2tE}\left(1-\frac{\mu}{2}\right)\right) + \left(\frac{PdL}{2tE}\left(\frac{1}{2} -\right.\right.$$

Principal stresses and strains

→ The planes which do not have shear stress are known as principal plane.

→ These carry only normal stresses

→ The normal stresses acting on principal plane are known as principal stress.

→ There are two methods for determining stresses on oblique section.

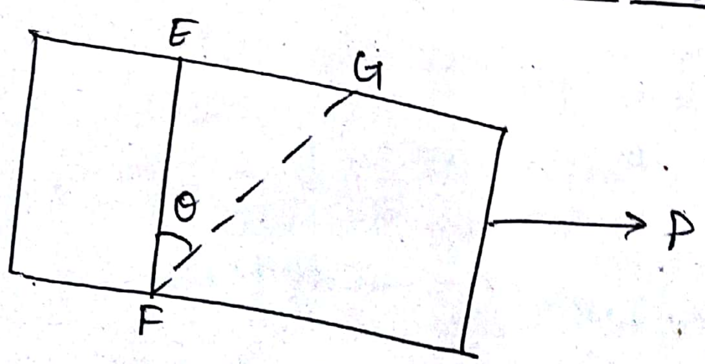
1) Analytical method
2) Graphical method

(1) Analytic method for determining stress on oblique section are :

(i) member subjected to direct stress in one plane.

(ii) member subjected to direct stress in two mutually ⊥lar directions.

(i) Member subjected to direct stress in one plane:



Area of EF = EF x 1

, A = EF

$$\therefore \sigma = \frac{P}{EF} = \frac{P}{A}$$

Area of FG = FG × 1

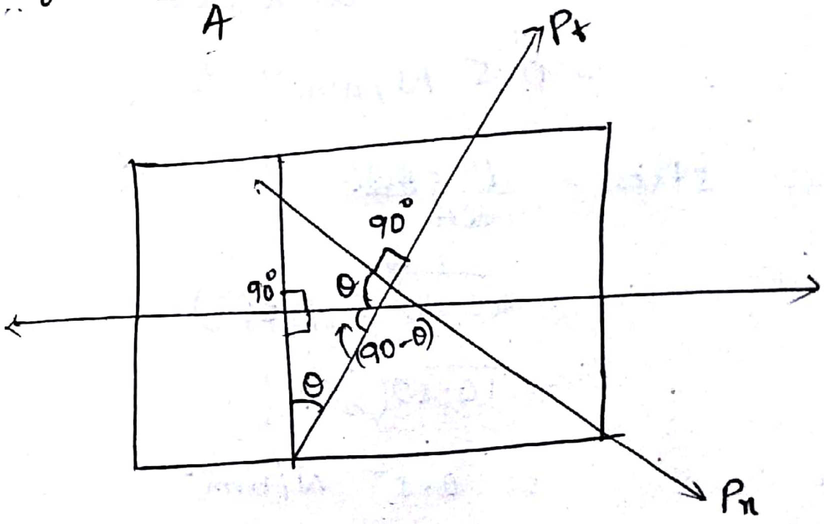
from fig, $\cos\theta = \frac{EF}{FG}$

$$FG = \frac{EF}{\cos\theta}$$

$$\text{Area of } FG = \frac{EF}{\cos\theta} \times 1$$

$$= \frac{A}{\cos\theta}$$

$$\therefore \sigma = \frac{P\cos\theta}{A}$$



$P_n = P\cos\theta$

$P_t = P\sin\theta$

$$\sigma_n = \frac{F}{A} = \frac{P\cos\theta}{A/\cos\theta}$$

$$= \frac{P\cos^2\theta}{A}$$

if, $\theta = 0$

$\boxed{\sigma_n = \frac{P}{A}}$ → maximum

$$\sigma_t = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{2A}\sin 2\theta$$

If, $\theta = 45°$ & $135°$

$\boxed{\sigma \quad P}$ → max.

(6) A rectangular bar of cross sectional area 10000 mm^2 is subjected to an axial load 20 kN. Determine normal & shear stresses on a section which is inclined at an angle of 30° with normal cross section of a bar.

sol: $A = 10,000\ mm^2$

$P = 20\ kN$

Normal stress, $\sigma_n = \frac{P\cos^2\theta}{A} = \frac{\cancel{10000}}{\cancel{20} \times 1000} \cos^2\theta$

$= \cancel{0.5}\ N/mm^2 \cos^2\theta = 2\ N/mm^2$

Shear stress $= \frac{P\sin 2\theta}{2A}$ $= 1.5\ N/mm^2$

$= 2(0.5)(\cos 60°)$

$= (\cancel{0.5})(\cancel{\tfrac{1}{2}})(\tfrac{1}{2})$

$= 0.5\ N/mm^2$

(2) Find the dia. of circular bar which is subjected to an axial pull of 160 kN. If the max. allowable shear stress on any section is 65 N/mm^2.

sol: $P = 160\ kN$

$A = \pi r^2$

$\sigma_t = 65\ N/mm^2$

$\sigma_t = \frac{P}{2A}$

$65 = \frac{160 \times 10^3}{2 \times \pi \times \frac{D^2}{4}}$

$D^2 = 1567.85 \Rightarrow D = 39.59\ mm$

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