Thermodynamics for Beginners – Chapter 9 THE FIRST LAW APPLIED TO OPEN SYSTEMS

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THE FIRST LAW
APPLIED TO
OPEN SYSTEMS

When the infinity of space and universe was first revealed he was terrified; he felt lost and humbled - he was no longer the central point of all but an insignificant, infinitely small grain of dust.
— Nicholas Berdyaev (*Man and Machine*)

So far, we have learned to apply the first law of thermodynamics to solve various kinds of thermodynamic problems associated with closed systems. In this chapter, we will learn to apply the first law of thermodynamics to open systems. An open system, like a closed system, allows heat and work to enter and/or leave the system. In addition, an open system allows mass to enter and/or leave the system. Study of open systems is important, since a large number of engineering applications involve open systems.

9.1 Example of an Open System

Consider a container to which water at room temperature is supplied from one end, and hot water is drawn from the other end, as shown in Figure 9.1. A heating coil is placed within the container to heat the water flowing through the container. A stirrer is used to stir the water, and thus does stirring work on the water.

Choose the open system to be the water present within the container. We may consider the dashed line of Figure 9.1 as the boundary of the system. It is however not correct. The boundary of the system should enclose only the water in the system, and should exclude the stirrer and the heating coil. The stirrer and the heating coil are part of the surroundings.
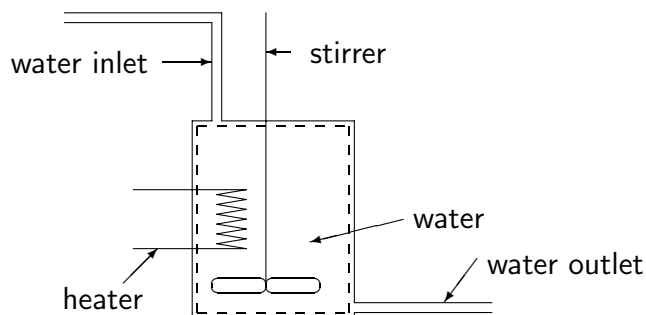

Figure 9.1 An example of an open system.

An open system is therefore best represented by a block diagram, as shown in Figure 9.2. The system comprises only the water in the container, marked by the boundary of the system, given by the dashed line of Figure 9.2, but excludes the heater and the stirrer.

Let the rate at which heat enters the system from the heater be $\dot{Q}_{in}$, and the rate at which work enters the system from the stirrer be $\dot{W}_{in}$. Let the rates at which mass enters and leaves the system be $\dot{m}_i$ and $\dot{m}_e$, respectively. Let the mass of the system at time t be m_s, and the rate of increase in mass within the system can be given by dm_s/dt. The energy of

the system at time t is represented by E_s, and the rate of energy accumulation within the system is represented by dE_s/dt.
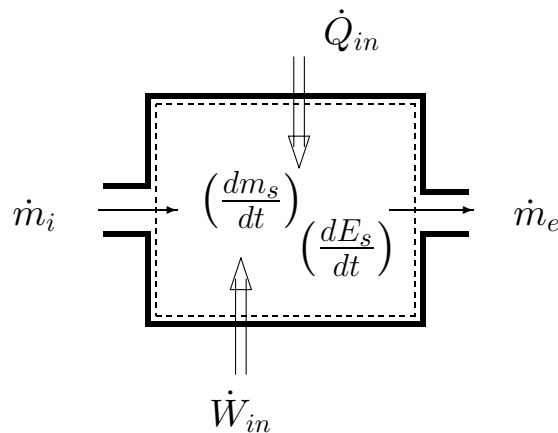
$$\dot{Q}_{in}$$

$$\dot{m}_i \longrightarrow \left(\frac{dm_s}{dt}\right) \left(\frac{dE_s}{dt}\right) \longrightarrow \dot{m}_e$$

$$\dot{W}_{in}$$

Figure 9.2 A block digram representing the open system of Figure 9.1.

Student : Teacher, permit me to ask you a question. You put a dot over Q_{in}, a dot over W_{in}, and dots over many other variables. You did not do that in the eight chapters that we have so far covered. Why are you doing that now? Why do you put dots over some of the variables?

Teacher It is good that you asked that question. Let me answer your question considering one variable, say, heat entering the system. When dealing with closed system, we considered the total amount of heat entering the system during the entire process, which we represented simply by the notation Q_{in}. In the open system presented above, we are considering the rate at which heat entering the system, that is, the heat entering the system per unit time. We choose to designate the rate at which an entity entering or leaving the system by putting a 'dot' over the entity, and therefore the rate at which heat entering the system is designated by the notation $\dot{Q}_{in}$. Note that Q_{in} takes the unit kJ, and $\dot{Q}_{in}$ takes the unit kJ/s, or kJ/hour, or kJ/day, or even kJ/year.

Student : I see now why you put dots over certain variables. Teacher, may I ask another question?

Teacher: Please, go ahead.

Student: What do you mean by the rate of increase in energy within the system?

Teacher: Consider the system shown in Figure 9.1 or in Figure 9.2. Let us suppose the temperature of the water in the tank increases with time, which means the internal energy of water increases with time. The rate at which the energy of the system increases is what we refer to as the rate of increase in energy within the system, and it is denoted by dE_s/dt. Note that the unit of dE_s/dt may be kJ/s, or kJ/hour, or kJ/day, or kJ/year.

Student: Thanks, Teacher.

9.2 Mass Balance for an Open System

The law of conservation of mass applied to the open system of Figure 9.2 yields the following:

rate of increase in mass within the system
$$= \quad \text{rate at which mass entering the system}$$
$$- \text{ rate at which mass leaving the system,}$$

which is equivalent to

$$\frac{dm_s}{dt} = \dot{m}_i - \dot{m}_e \tag{9.1}$$

where (dm_s/dt) represents the rate of increase in mass within the system, and $\dot{m}_i$ and $\dot{m}_e$ represent the respective rates at which mass entering and leaving the system.

Note that if $\dot{m}_i$ is larger than $\dot{m}_e$ then dm_s/dt will be positive denoting that mass accumulates within the system. If $\dot{m}_e$ is larger than $\dot{m}_i$ then dm_s/dt will be negative denoting that the mass within the system is decreased. If $\dot{m}_i$ is the same as $\dot{m}_e$ then dm_s/dt is zero so that the mass of the system remains a constant with time.

Integrating (9.1) over a time interval Δt $(= t_f - t_o)$, we get

$$\int_{m_{so}}^{m_{sf}} dm_s = \int_{t_o}^{t_f} \dot{m}_i \, dt - \int_{t_o}^{t_f} \dot{m}_e \, dt$$

which can be written as

$$m_{sf} - m_{so} = m_i - m_e \qquad (9.2)$$

where m_{so} is the mass of the system at the initial time t_o, m_{sf} is the mass of the system at the final time t_f, m_i is the total mass entering the system during the time interval Δt, and m_e is the total mass leaving the system during the time interval Δt.

Equations (9.1) and (9.2) are applicable for open systems with one inlet and one exit. If an open system has many inlets and exits for the mass to flow then (9.1) must be expanded to

$$\frac{dm_s}{dt} = \dot{m}_{i1} + \dot{m}_{i2} + \dot{m}_{i3} + \cdots - \dot{m}_{e1} - \dot{m}_{e2} - \dot{m}_{e3} - \cdots$$

and (9.2) must be expanded to

$$m_{sf} - m_{so} = m_{i1} + m_{i2} + m_{i3} + \cdots - m_{e1} - m_{e2} - m_{e3} - \cdots$$

where the subscripts $_{i1}$, $_{i2}$, $_{i3}$, ... stand for inlet 1, inlet 2, inlet 3, etc., and $_{e1}$, $_{e2}$, $_{e3}$, ... stand for exit 1, exit 2, exit 3, etc.

9.3 Energy Balance for an Open System

The energy of a closed system is changed only by heat and work interactions between the system and its surroundings. In an open system, however, an additional mechanism is seen to change the total energy of the system: that is, the flow of matter in and out of the system. When matter enters a system, it brings in some energy with it. Likewise, when matter leaves the system, it takes out some energy with it.

Therefore, the law conservation of energy, that is the first law, applied to the open system of Figure 9.2 yields the following:

rate of increase in energy within the system
= rate at which heat enters the system
+ rate at which work enters the system
+ rate at which energy enters the system with the mass flowing in
− rate at which energy leaves the system with the mass flowing out,

which is equivalent to

$$\frac{dE_s}{dt} = \dot{Q}_{in} + \dot{W}_{in} + \dot{E}_i - \dot{E}_e \tag{9.3}$$

where (dE_s/dt) is the rate of increase in energy within the system, $\dot{Q}_{in}$ is the rate at which heat enters the system, $\dot{W}_{in}$ is the rate at which work enters the system, $\dot{E}_i$ is the rate at which energy is brought in by the mass entering the system, and $\dot{E}_e$ is the rate at which energy is removed by the mass leaving the system.

Integrating (9.3) with respect to time over the time interval Δt yields

$$\Delta E_s = Q_{in} + W_{in} + \int_{t_o}^{t_f} \dot{E}_i \, dt - \int_{t_o}^{t_f} \dot{E}_e \, dt \tag{9.4}$$

where ΔE_s is the change in total energy content of the system during Δt, and Q_{in} and W_{in} are the respective amounts of net heat and net work entering the system during that period. The third and the fourth terms on the right hand side of (9.4) represent the energy entering and leaving the system with the mass entering and leaving the system, respectively. These terms are absent if the given system were a closed system, so that we get

$$\Delta E_s = Q_{in} + W_{in}$$

which is the familiar form of the first law of thermodynamics applied to closed systems, see equation (3.1).

Let us return to the first Law of thermodynamics applied to an open system, expressed by (9.3), and take a close look at each of the five terms given in (9.3).

The dE_s/dt term

For simple compressible systems, the total energy of the system, E_s, is only the internal energy of the system, U_s. If the system as a whole undergoes a change in its elevation then we will add in the gravitational potential energy term. If the velocity of the whole system changes then we will add in the translational kinetic energy term. As beginners in thermodynamics, we will not consider systems that could possess any forms of energy other than the internal energy, the gravitational potential energy and the translational kinetic energy.

The $\dot{Q}_{in}$ term

If there is a temperature difference between the system and the surroundings, then heat will be transferred across the system boundary. If the system is adiabatic, there will be no heat transfer across the boundary, and $\dot{Q}_{in} = 0$.

The $\dot{W}_{in}$ term

Three of the work forms that we often come across with open systems are discussed below.

Boundary Work ($\dot{W}_{\mathbf{boundary}}$):

> It is a work that results when the system boundary moves. We have learned about this work in detail in Chapter 7.

Shaft Work ($\dot{W}_{\mathbf{shaft}}$):

> It is a form of work that is realized when a shaft is given a motion by the fluid flowing through an open system, or when the motion of a shaft is used to do work on the fluid flowing through an open system.

Flow Work ($\dot{W}_{\mathbf{flow}}$):

> Work associated with a fluid mass flowing into or out of an open system is called flow work, and is relevant only to an open system. Let us consider an opening through which mass flows into a system as shown in Figure 9.3. The dashed box shows the volume dV of a very small mass dm that is being pushed into the system in time dt.
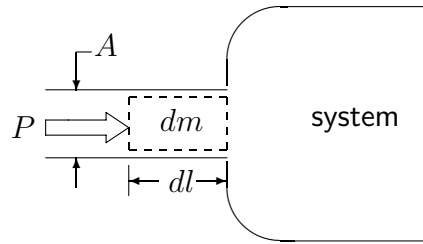
Figure 9.3   Schematic for flow work.

The fluid mass dm is taken to be sufficiently small for the properties within the fluid mass to be considered uniformly distributed. The pressure force acting on the fluid mass dm to push it into the system is given by the pressure P multiplied by the cross-sectional area A. The distance across which the fluid mass is pushed is given by dl. The work done in time dt to push the fluid mass dm into the system across the inlet is given by

$$dW_{flow} = (P\,A)\,(dl) = P\,dV = P\,v\,dm$$

since $A\,dl = dV$ and since $dV = v\,dm$, where v is the specific volume. Dividing the above expression by dt, we get the following:

$$\frac{dW_{flow}}{dt} = \frac{P\,v\,dm}{dt}$$

which can be written as

$$\dot{W}_{flow} = \dot{m}\,P\,v \tag{9.5}$$

The above equation gives the rate at which flow work is done to maintain a mass flow rate $\dot{m}$ at the inlet concerned. Flow work enters the system when matter flows into the system, and leaves the system when matter flows out of the system.

Any other form of Work ($\dot{W}_{\textbf{other}}$):

> There can be other forms of work such as work done to overcome fluid friction, electrical work, and so on. We shall not get into the details of all these types of work here. For our purposes, let us neglect any forms of work other than the three discussed above. Thus, we set $\dot{W}_{other} = 0$.

Expanding the work term in (9.3) in accordance with the discussion above, we get

$$\frac{dE_s}{dt} = \dot{Q}_{in} + (\dot{W}_{boundary})_{in} + (\dot{W}_{shaft})_{in} + (\dot{W}_{flow})_i - (\dot{W}_{flow})_e$$
$$+ \dot{E}_i - \dot{E}_e$$

When substituting in the above equation the flow work expressed by (9.5), we get

$$\frac{dE_s}{dt} = \dot{Q}_{in} + (\dot{W}_{boundary})_{in} + (\dot{W}_{shaft})_{in} + (\dot{m}\,P\,v)_i - (\dot{m}\,P\,v)_e$$
$$+ \dot{E}_i - \dot{E}_e \tag{9.6}$$

The term $\dot{E}$ at the inlet and at the exit

The total energy carried by a fluid in flow is made up of internal, kinetic and gravitational potential energies. Therefore, the energy flow rate, $\dot{E}$, can be expressed as

$$\dot{E} = \dot{m}\,u + \dot{m}\,\frac{c^2}{2} + \dot{m}\,g\,z \tag{9.7}$$

where $\dot{m}$ is the mass flow rate of the fluid, u the specific internal energy of the fluid, c the speed of the fluid, g the gravitational acceleration, and z the elevation at which the fluid flows. The fluid that enters/leaves the system may as well have of electrical, magnetic and other forms of energy, which we ignore since we deal mostly with simple compressible systems.

Substituting from (9.7) for the energy entering and leaving the system with the flowing fluid in (9.6), we get

$$\frac{dE_s}{dt} = \dot{Q}_{in} + (\dot{W}_{boundary})_{in} + (\dot{W}_{shaft})_{in} + (\dot{m}\,P\,v)_i - (\dot{m}\,P\,v)_e$$

$$+ \left(\dot{m}\,u + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i - \left(\dot{m}u + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e$$

which can be regrouped as

$$\frac{dE_s}{dt} = \dot{Q}_{in} + (\dot{W}_{boundary})_{in} + (\dot{W}_{shaft})_{in}$$

$$+ \left(\dot{m}\,P\,v + \dot{m}\,u + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i$$

$$- \left(\dot{m}\,P\,v + \dot{m}u + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e$$

Using $h = u + Pv$ from (4.2) in the above equation, we get

$$\boxed{\begin{aligned}
\frac{dE_s}{dt} = {} & \dot{Q}_{in} + (\dot{W}_{boundary})_{in} + (\dot{W}_{shaft})_{in} \\[2mm]
& + \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i \\[2mm]
& - \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e
\end{aligned}} \qquad (9.8)$$

Equation (9.8) uses enthalpy which includes both the internal energy and the energy associated with pushing the fluid into or out of the open system. It is therefore the energy of a fluid stream flowing into or out of an open system is represented by enthalpy, kinetic energy and potential energy, and no reference will be made to flow work.

Equation (9.8) is the first law of thermodynamics applicable to open systems at any given time, and each term in this equation is a rate term

having the unit of energy per time. Equation (9.8) is, however, inadequate to completely describe the open systems. We need to use the mass balance given by (9.1), along with (9.8), to solve problems involving open systems.

Integrating (9.8) over the time interval Δt ($= t_f - t_o$), we get

$$\int_{E_{so}}^{E_{sf}} dE_s \;=\; \int_{t_o}^{t_f} \dot{Q}_{in} dt + \int_{t_o}^{t_f} (\dot{W}_{boundary})_{in} dt + \int_{t_o}^{t_f} (\dot{W}_{shaft})_{in} dt$$

$$+ \int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i dt$$

$$- \int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e dt$$

which can be rewritten as

$$
\boxed{
\begin{aligned}
E_{sf} - E_{so} \;=\;\; & Q_{in} + (W_{boundary})_{in} + (W_{shaft})_{in} \\[2mm]
& + \int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i dt \\[2mm]
& - \int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e dt
\end{aligned}
}
\qquad (9.9)
$$

where E_{so} is the energy of the system at the initial time t_o, E_{sf} is the energy of the system at the final time t_f, and Q_{in}, $(W_{boundary})_{in}$ and $(W_{shaft})_{in}$ are the respective amounts of net heat, net boundary work and net shaft work entering the system during the time interval Δt.

Equation (9.9) is the first law of thermodynamics applicable to open systems over a chosen time interval, such as $t_f - t_o$, and each term in this equation takes the unit of energy. Equation (9.9) is used together with the mass balance given by (9.2) to solve problems involving open systems.

Equations (9.8) and (9.9) are applicable for open systems with not more than one inlet and one exit. If an open system has many inlets and exits then we must include $\left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)$-term in (9.8), and $\int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right) dt$-term in (9.9), at the respective inlets and exits with appropriate signs; positive sign for inlet and negative sign for outlet.

9.4 Worked Examples

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Example 9.1

</div>

Nitrogen at 6 bar and 350 K is supplied to an insulated, rigid tank of volume 3 m^3. The tank initially contains nitrogen at 1 bar and 300 K. Determine the final temperature of nitrogen in the tank when its pressure reaches 6 bar. Calculate also the mass of nitrogen that has entered the tank. Assume that nitrogen behaves as an ideal gas, and that γ for nitrogen is a constant at 1.4. Molar mass of nitrogen is 28 kg/kmol.

Solution to Example 9.1

Figure 9.4 shows the rigid tank which is supplied with nitrogen at a constant inlet pressure of $P_i = 6$ bar and a constant inlet temperature of $T_i = 350$ K.
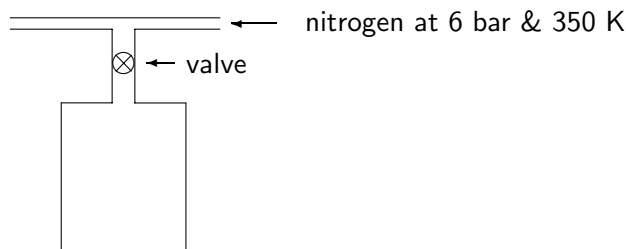

Figure 9.4 Charging of a rigid tank with nitrogen at 6 bar and 350 K.

The nitrogen in the tank is taken as the open system. Initially, it comprises nitrogen at $P_{so} = 1$ bar and $T_{so} = 300$ K, and is supplied with nitrogen until the final pressure in the tank becomes $P_{sf} = 6$ bar. The final temperature of nitrogen, that is T_{sf}, is to be found. Note that the subscripts $_{so}$ and $_{sf}$ denote the initial and the final states, respectively, of the nitrogen in the tank.

The tank is insulated and therefore no heat interaction occurs between the system and the surroundings. There is no shaft work. Since the tank is rigid, there is no boundary work either. The system has only one inlet which is the nitrogen supply line and no exit. Therefore, (9.9) applied to the system for the

entire charging period becomes

$$E_{sf} - E_{so} = \int_{t_o}^{t_f} \left(\dot{m}\, h + \dot{m}\, \frac{c^2}{2} + \dot{m}\, g\, z \right)_i dt$$

Since the system is stationary, the kinetic energy of the system does not change. The potential energy of the system changes with nitrogen entering the system. Let us however ignore the change in potential energy, and take the change in total energy E_s of the system be the change in internal energy U_s alone. Also, by neglecting the kinetic and potential energies at the inlet, the above equation is reduced to

$$U_{sf} - U_{so} = \int_{t_o}^{t_f} (\dot{m}\, h)_i\, dt \tag{9.10}$$

where U_{sf} and U_{so} are the internal energies of the nitrogen in the tank at time t_f and t_o, respectively.

Pressure and temperature in the nitrogen supply line remain constant throughout the charging process, and therefore enthalpy at the inlet of the system also remains constant at h_i. Using this information, (9.10) can be simplified to

$$U_{sf} - U_{so} = h_i \int_{t_o}^{t_f} \dot{m}_i\, dt = h_i\, m_i$$

where m_i is the total mass of nitrogen entering the tank during the entire charging process.

Replacing the internal energies of the above equation by specific internal energies, we get

$$m_{sf}\, u_{sf} - m_{so}\, u_{so} = h_i\, m_i \tag{9.11}$$

where m_{sf} and m_{so} are the mass of the nitrogen in the tank at time t_f and t_o, respectively, and u_{sf} and u_{so} are the specific internal energies of the nitrogen in the tank at time t_f and t_o, respectively, which are assumed to be uniform throughout the system.

The mass balance given by (9.2), when applied to the given system, reduces to

$$m_{sf} - m_{so} = m_i \tag{9.12}$$

Combining (9.11) and (9.12) to eliminate m_i, we get

$$m_{sf} u_{sf} - m_{so} u_{so} = h_i\, (m_{sf} - m_{so}) \tag{9.13}$$

In classical thermodynamics, the absolute values of u and h are not known. Only the changes in u and h are known. Therefore, (9.13) is transformed to a

convenient form using the following standard procedure. Replacing the h-term in (9.13) using $h = u + Pv$, we get

$$m_{sf} u_{sf} - m_{so} u_{so} = (u + Pv)_i \ (m_{sf} - m_{so})$$

which can be rearranged to give

$$m_{sf} (u_{sf} - u_i) - m_{so} (u_{so} - u_i) = (Pv)_i \ (m_{sf} - m_{so}) \tag{9.14}$$

Since nitrogen is assumed to behave as an ideal gas, the differences in specific internal energies of (9.14) can be expressed in terms of the differences in temperatures using $\Delta u = \int C_v \, dT$. Taking C_v as a constant and using the ideal gas equation of state $Pv = RT$, (9.14) can be rewritten as

$$m_{sf} \, C_v \, (T_{sf} - T_i) - m_{so} \, C_v \, (T_{so} - T_i) = R \, T_i \ (m_{sf} - m_{so})$$

which can be rearranged, using the ideal gas relationship $C_p = C_v + R$ and the definition of $\gamma = C_p/C_v$, to give the following:

$$
\begin{aligned}
m_{sf} \left(C_v \, T_{sf} - C_v \, T_i - R \, T_i \right) &= m_{so} \left(C_v \, T_{so} - C_v \, T_i - R \, T_i \right) \\
m_{sf} \left(C_v \, T_{sf} - C_p \, T_i \right) &= m_{so} \left(C_v \, T_{so} - C_p \, T_i \right) \\
m_{sf} \left(T_{sf} - (C_p/C_v) \, T_i \right) &= m_{so} \left(T_{so} - (C_p/C_v) \, T_i \right) \\
m_{sf} \left(T_{sf} - \gamma \, T_i \right) &= m_{so} \left(T_{so} - \gamma \, T_i \right) \tag{9.15}
\end{aligned}
$$

Since the masses m_{sf} and m_{so} are unknown, they can be expressed using the ideal gas equation of state by

$$m_{sf} = \frac{P_{sf} \, V_{sf}}{R \, T_{sf}} \quad \text{and} \quad m_{so} = \frac{P_{so} \, V_{so}}{R \, T_{so}} \tag{9.16}$$

Since the tank is rigid, its volume remains a constant throughout. Therefore

$$V_{sf} = V_{so} \tag{9.17}$$

Using (9.16) and (9.17), we can rewrite (9.15) as follows:

$$\frac{P_{sf}}{T_{sf}} \left(T_{sf} - \gamma \, T_i \right) = \frac{P_{so}}{T_{so}} \left(T_{so} - \gamma \, T_i \right) \tag{9.18}$$

Substituting the numerical values known from the problem statement in (9.18), we determine the value of T_{sf} as follows:

$$\frac{6 \text{ bar}}{T_{sf}} \times (T_{sf} - 1.4 \times 350 \text{ K}) = \frac{1 \text{ bar}}{300 \text{ K}} \times (300 \text{ K} - 1.4 \times 350 \text{ K})$$

$$6 \times \left(1 - 1.4 \times \frac{350 \text{ K}}{T_{sf}} \right) = \left(1 - 1.4 \times \frac{350}{300} \right)$$

$$T_{sf} = 443 \text{ K}$$

The mass of nitrogen that entered the tank can be found using (9.16) as follows:

$$
\begin{aligned}
m_{sf} - m_{so} &= \frac{P_{sf} V_{sf}}{R T_{sf}} - \frac{P_{so} V_{so}}{R T_{so}} \\
&= \left[\frac{600 \times 3}{(8.314/28) \times 443} - \frac{100 \times 3}{(8.314/28) \times 300} \right] \text{ kg} \\
&= (13.7 - 3.4) \text{ kg} = 10.3 \text{ kg}
\end{aligned}
$$

Example 9.2

Rework **Example 9.1** assuming that the tank was empty at the initial state.

Solution to Example 9.2

Since the tank was empty at the initial state, $m_{so} = 0$, and therefore (9.13) reduces to

$$ u_{sf} = h_i $$

which can be rewritten, using $h = u + Pv$, as

$$ u_{sf} = (u + Pv)_i $$

Assuming ideal gas properties for nitrogen with constant specific heats, the above equation can be worked out as follows:

$$
\begin{aligned}
u_{sf} &= u_i + R T_i \\
u_{sf} - u_i &= R T_i \\
C_v (T_{sf} - T_i) &= R T_i \\
C_v T_{sf} &= (C_v + R) T_i \\
T_{sf} &= \frac{C_p}{C_v} T_i = \gamma T_i = 1.4 \times 350 \text{ K} = 490 \text{ K}
\end{aligned}
$$

Substituting the numerical value of T_{sf} in the ideal gas equation of state at the final state, the mass of nitrogen that entered the tank can be found as

$$ m_{sf} = \frac{600 \times 3}{(8.314/28) \times 490} \text{ kg} = 12.4 \text{ kg} $$

Example 9.3

Air at 15 bar and 500 K is supplied to a rigid tank of volume 1 m³ containing air initially at 1 bar and 300 K, until the air in the tank reaches 10 bar and 450 K. Determine the mass of air fed to the tank and the heat interaction between the tank and the surroundings. Assume that air behaves as an ideal gas, and that R and γ for air are 287 J/kg·K and 1.4, respectively.

Solution to Example 9.3

Initially the air in the tank is at 1 bar and 300 K, and therefore the initial mass of air in the tank is

$$m_{so} = \frac{(10^5 \text{ Pa}) \times (1 \text{ m}^3)}{(287 \text{ J/kg} \cdot \text{K} \times (300 \text{ K})} = 1.16 \text{ kg} \tag{9.19}$$

Finally the air in the tank is at 10 bar and 450 K, and therefore the final mass of air in the tank is

$$m_{sf} = \frac{(10^6 \text{ Pa}) \times (1 \text{ m}^3)}{(287 \text{ J/kg} \cdot \text{K} \times (450 \text{ K})} = 7.74 \text{ kg} \tag{9.20}$$

Therefore, the mass of air added to the tank is 6.58 kg.

To determine the heat interaction between the system and the surroundings, let us use the following procedure. The problem to be solved here is similar to the problem of **Example 9.1**, except for the heat interaction between the system and the surroundings. Therefore, (9.9) applied to the given system becomes

$$E_{sf} - E_{so} = Q_{in} + \int_{t_o}^{t_f} \left(\dot{m}\, h + \dot{m}\, \frac{c^2}{2} + \dot{m}\, gz \right)_i dt$$

Neglecting the potential and kinetic energy changes of the system and of the inlet stream, the above equation is reduced to

$$U_{sf} - U_{so} = Q_{in} + \int_{t_o}^{t_f} (\dot{m}\, h)_i\, dt$$

which can be rewritten, in terms of the specific internal energies, as

$$m_{sf}\, u_{sf} - m_{so}\, u_{so} = Q_{in} + h_i\, m_i \tag{9.21}$$

where h_i is the specific enthalpy of the incoming nitrogen taken as a constant, and m_i is the mass of nitrogen entering the tank during the entire charging process, given by $\int_{t_o}^{t_f} \dot{m}_i \, dt$.

Mass balance (9.2) applied to the system yields

$$m_{sf} - m_{so} = m_i \tag{9.22}$$

Combining (9.21) and (9.22) to eliminate m_i and rearranging the resulting equation, we get

$$Q_{in} = m_{sf} \, u_{sf} - m_{so} \, u_{so} - h_i \, (m_{sf} - m_{so}) \tag{9.23}$$

Replacing the specific enthalpy h_i in (9.23) by $(u_i + RT_i)$ applicable for an ideal gas, we get

$$
\begin{aligned}
Q_{in} &= m_{sf}(u_{sf} - u_i - RT_i) - m_{so}(u_{so} - u_i - RT_i) \\
&= m_{sf}[C_v(T_{sf} - T_i) - RT_i] - m_{so}[C_v(T_{so} - T_i) - RT_i] \\
&= m_{sf}(C_v T_{sf} - C_p T_i) - m_{so}(C_v T_{so} - C_p T_i) \\
&= m_{sf} C_v(T_{sf} - \gamma T_i) - m_{so} C_v(T_{so} - \gamma T_i)
\end{aligned}
\tag{9.24}
$$

in which m_{so} and m_{sf} are known from (9.19) and (9.20), $T_{so} = 300$ K, $T_{sf} = 450$ K and $T_i = 500$ K. We can calculate C_v using $R/(\gamma - 1)$ as 717.5 J/kg·K. Substituting these numerical values in (9.24), we get $Q_{in} = -1055$ kJ, which is to say that about 1055 kJ of heat is lost to the surroundings during the entire charging process.

Example 9.4

A tank contains 0.25 kg of nitrogen at 300 kPa and 300 K. It is discharged until its pressure becomes 100 kPa. If the tank is well insulated, what will be the final temperature in the tank? Determine the amount of nitrogen discharged from the tank. Assume ideal gas behaviour, and γ for nitrogen to be 1.4.

Solution to Example 9.4

Figure 9.5 shows discharging of nitrogen from a tank. Initially the tank is filled with $m_{so} = 0.25$ kg of nitrogen at $P_{so} = 300$ kPa and $T_{so} = 300$ K. It is

discharged until its pressure becomes $P_{sf} = 100$ kPa. We need to determine the final temperature T_{sf} of the nitrogen remaining in the tank, and the amount of nitrogen discharged from the tank.
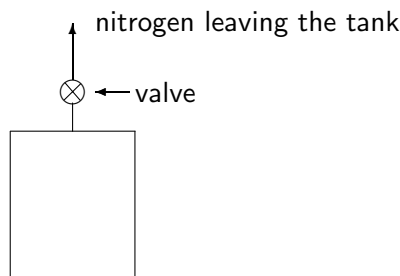

Figure 9.5 Discharging of nitrogen from a tank.

Arguments very similar to those presented in the **Solution to Example 9.1** reduces (9.9) applied to the discharging process to

$$E_{sf} - E_{so} = -\int_{t_o}^{t_f} \left(\dot{m}\, h + \dot{m}\, \frac{c^2}{2} + \dot{m}\, gz \right)_e dt$$

which can be simplified, by neglecting the potential and kinetic energy changes, to

$$U_{sf} - U_{so} = -\int_{t_o}^{t_f} \dot{m}_e\, h_e dt \qquad (9.25)$$

Unlike in the charging process considered in the previous examples, the specific enthalpy h_e at the discharging point is a variable, and is equal to the specific enthalpy of the nitrogen in the tank, that is h_s. Thus, h_e varies from h_{so} to h_{sf} which are the respective initial and final specific enthalpies of the nitrogen in the tank. Integrating (9.25) would therefore not be possible unless the exact variation of h_e, or h_s, as a function of time is known. Since we do not know that, we need to approximate h_e to a constant. The most suitable approach would be to consider h_e as the average value between its initial and final values. That would be

$$h_e = \frac{h_{so} + h_{sf}}{2} = \text{constant} \qquad (9.26)$$

Substituting h_e of (9.26) in (9.25), we get

$$U_{sf} - U_{so} = -\left(\frac{h_{so} + h_{sf}}{2} \right) m_e \qquad (9.27)$$

where m_e is the mass of nitrogen leaving the tank during the entire discharging process, given by $\int_{t_o}^{t_f} \dot{m}_e \, dt$.

Rewriting (9.27) in terms of specific internal energies, we get

$$m_{sf} u_{sf} - m_{so} u_{so} = - \left(\frac{h_{so} + h_{sf}}{2} \right) m_e \tag{9.28}$$

Applying (9.2) to the discharging process, we get

$$m_{sf} - m_{so} = -m_e \tag{9.29}$$

Combining (9.28) and (9.29) to eliminate m_e, we get

$$m_{sf} \, u_{sf} - m_{so} \, u_{so} = \left(\frac{h_{so} + h_{sf}}{2} \right) (m_{sf} - m_{so}) \tag{9.30}$$

which can be rearranged to give

$$m_{sf} \, (2 \, u_{sf} - h_{so} - h_{sf}) = m_{so} \, (2 \, u_{so} - h_{so} - h_{sf})$$

Using $u = h - Pv$, the above equation can be written as

$$m_{sf} \, (h_{sf} - h_{so} - 2 \, P_{sf} v_{sf}) = m_{so} \, (h_{so} - h_{sf} - 2 \, P_{so} v_{so})$$

Now, transform the differences in specific enthalpies into differences in temperatures using $\Delta h = \int C_p \, dT$, with the assumption that nitrogen behaves as an ideal gas. Taking C_p as a constant, and using the ideal gas equation of state $Pv = RT$ in the above equation, we get

$$m_{sf} \, [C_p \, (T_{sf} - T_{so}) - 2 \, R \, T_{sf}] = m_{so} \, [C_p \, (T_{so} - T_{sf}) - 2 \, R \, T_{so}]$$

Substituting $m_{so} = 0.25$ kg, $T_{so} = 300$ K, $\gamma = 1.4$, $R = (8.314/28)$ kJ/kg·K, and therefore $C_p = \gamma \, R/(\gamma - 1) = 1.039$ kJ/kg·K, we get

$$m_{sf} \, (0.445 \, T_{sf} - 311.7) = 33.4 - 0.260 \, T_{sf} \tag{9.31}$$

We need to determine T_{sf} from (9.31), but we do not know m_{sf}. Therefore, we need to look for another independent equation containing T_{sf} and m_{sf}. Applying the ideal gas equation of state at the final state, we get

$$m_{sf} = \frac{P_{sf} \, V_{sf}}{R \, T_{sf}} \tag{9.32}$$

of which we know $P_{sf} = 100$ kPa, but V_{sf} is unknown.

Since the tank is rigid, its volume remains a constant throughout. Therefore

$$V_{sf} = V_{so} \tag{9.33}$$

Applying the ideal gas equation of state at the initial state, we get

$$m_{so} = \frac{P_{so} V_{so}}{R T_{so}} \tag{9.34}$$

Combining (9.32), (9.33) and (9.34), we get

$$m_{sf} = \left(\frac{P_{sf}}{R T_{sf}}\right) \times \left(\frac{m_{so} R T_{so}}{P_{so}}\right) = \frac{P_{sf} T_{so} m_{so}}{P_{so} T_{sf}}$$

which gives

$$m_{sf} = 25/T_{sf} \tag{9.35}$$

Eliminating m_{sf} from (9.31) using (9.35), we get

$$0.260 \, T_{sf}^2 - 22.3 \, T_{sf} - 7792.5 = 0$$

which gives

$$T_{sf} = 221.2 \text{ K}$$

Substituting the value of T_{sf} in (9.35), we get

$$m_{sf} = 0.113 \text{ kg}$$

The amount of nitrogen discharged from the tank is therefore given by

$$m_{so} - m_{sf} = 0.25 \text{ kg} - 0.113 \text{ kg} = 0.137 \text{ kg}$$

9.5 Summary

- Mass balance for an open system is given by

$$\frac{dm_s}{dt} = \dot{m}_i - \dot{m}_e \tag{9.1}$$

where the subscript s stands for system, i for inlet, and e for exit.

- Mass balance for an open system applied over a time interval Δt ($= t_f - t_o$) is given by

$$m_{sf} - m_{so} = m_i - m_e \tag{9.2}$$

where m_{sf} is the mass of the system at the final time t_f, m_{so} is the mass of the system at the initial time t_o, m_i is the total mass entering the system during the time interval Δt, and m_e is the total mass leaving the system during the time interval Δt.

- Equations (9.1) and (9.2) are applicable for open systems with not more than one inlet and one exit. If an open system has many inlets and exits for the mass to flow then (9.1) must be expanded to

$$\frac{dm_s}{dt} = \dot{m}_{i1} + \dot{m}_{i2} + \dot{m}_{i3} + \cdots - \dot{m}_{e1} - \dot{m}_{e2} - \dot{m}_{e3} - \cdots$$

and (9.2) must be expanded to

$$m_{sf} - m_{so} = m_{i1} + m_{i2} + m_{i3} + \cdots - m_{e1} - m_{e2} - m_{e3} - \cdots$$

where the subscripts $i1$, $i2$, $i3$, ... stand for inlet 1, inlet 2, inlet 3, etc., and $e1$, $e2$, $e3$, ... stand for exit 1, exit 2, exit 3, etc.

- Energy balance (that is, the first law of thermodynamics) for an open system is given by

$$\frac{dE_s}{dt} = \dot{Q}_{in} + (\dot{W}_{boundary})_{in} + (\dot{W}_{shaft})_{in}$$

$$+ \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i$$

$$- \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e \tag{9.8}$$

- Energy balance for an open system applied over a time interval Δt ($= t_f - t_o$) is given by

$$E_{sf} - E_{so} = Q_{in} + (W_{boundary})_{in} + (W_{shaft})_{in}$$

$$+ \int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_i dt$$

$$- \int_{t_o}^{t_f} \left(\dot{m}\,h + \dot{m}\,\frac{c^2}{2} + \dot{m}\,gz \right)_e dt \tag{9.9}$$

where E_{sf} is the energy of the system at the final time t_f, E_{so} is the energy of the system at the initial time t_o, and Q_{in}, $(W_{boundary})_{in}$ and $(W_{shaft})_{in}$ are the respective amounts of net heat, net boundary work and net shaft work entering the system during the time interval Δt.

- Equations (9.8) and (9.9) are applicable for open systems with not more than one inlet and one exit. If an open system has many inlets and many exits then we must include $\left(\dot{m} \, h + \dot{m} \, \frac{c^2}{2} + \dot{m} \, gz \right)$-term in (9.8), and $\int_{t_o}^{t_f} \left(\dot{m} \, h + \dot{m} \, \frac{c^2}{2} + \dot{m} \, gz \right) dt$-term in (9.9), at the respective inlets and exits with appropriate signs; positive sign for inlet and negative sign for outlet.