Unit - 1:

1. Explain Discrete random variables

Sol. A Discrete random variable has a countable number of possible values

2. If x_1, x_2 are two random variables and a, b are consonants then $E(ax_1 + bx_2)$?

3. The mean and variance of a binomial distribution are 6 & 3 respectively. Find the mode of the binomial distribution

Sol

mean $= np = 6$ mean $= np$

variance $= npq = 3$ variance $= npq$

$$\frac{np}{npq} = \frac{6^2}{3_1} = 2$$

mean $= np$

variance $= npq$

$$\frac{1}{q} = 2 \Rightarrow q = \frac{1}{2}$$

$$p + q = 1$$

$$p = 1 - q$$

$$p = 1 - \frac{1}{2} \quad p = \frac{1}{2}$$

$$np = 6$$

$$n\frac{1}{2} = 6$$

$$\boxed{n = 12}$$

mode of the binomial distribution is 12

4. If x is a poisson variant such that $P(x=0) = P(x=1) = k$. Determine k

Sol $P(x=0) = P(x=1) = k$

$$P(x=\bar{x}) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad P(x=\bar{x}) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P(x=0) = P(x=1)$$

$$\frac{e^{-\lambda}\lambda^0}{0!} = \frac{e^{-\lambda}\lambda^1}{1!} \qquad P(x=\bar{x}) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\frac{1}{1} = \frac{\lambda}{1}$$

$$\lambda = 1$$

$$P(x=1) = k$$

$$\frac{e^{-1} 1^{1}}{1!} = k$$

$$k = e^{-1} = \frac{1}{e}$$

$$\boxed{k = \frac{1}{e}}$$

5. If ~~x~~ $x = B(n,p)$ then write the condition under which r tends to a Poisson distribution

$P(x=r) = {}^nC_r\, p^r\, pq^{n-r}$

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

6. Define random variable

sol random variable :- A random variables is a variable whose value is unknown or a function that assigns values to each of an experiment's out comes

7. Define binomial distribution

sol:- Binomial distribution :- A frequency distribution of the possible number of successful out comes in a given number of trials in each of which there is the same probability of success.

8. If mean and variance of binomial variate are 12 and 4 then write binomial distribution.

sol mean $np = 12$

variance $npq = 4$

$\frac{np}{npq} = \frac{12}{4}$

$q = \frac{1}{3}$

$p+q = 1$

$p = 1-q$

$p = \cancel{0.066}\ \frac{2}{3}$

$np = 12 \Rightarrow n\frac{2}{3} = 12$

$n = 12 \times \frac{3}{2}$

$n = 18$

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

$= {}^{18}C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{18-r}$

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

$1-\frac{1}{3}$ $\frac{3-1}{3}$ $\frac{2}{3}$

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

9. List the property of probability distribution function.

sol. The sum of all probabilities for all possible values must equal 1.

10. Define poisson distribution

sol. Poisson distribution: A discrete probability distribution which gives the frequency of a number of independent events occurring in a fixed time

11. If k is a constant then what is the value of $E(ax+k)$?

sol By defin we have $E(x) = \sum_{i=1}^{n} a x_i$

consider $E(ax+k) = \sum_{i=1}^{n} a(x_i+k)$

$\sum_{i=1} = a x_i \qquad = \sum_{i=1}^{n} (a x_i + k x_i)$

$\sum_{i=1}^{n} a(x_i+k) = \sum_{i=1}^{n} (a x_i + k x_i) = \sum_{i=1}^{n} a x_i + k \sum_{i=1}^{n} x_i$

$= \sum_{i=1}^{n} a x_1 + \sum_{i=1}^{n} k x_i = E(x) + k$

$= Ex + k$

$= ax + k \quad E(x+k) = ax+k$

1. Random variable 'x' has the following probability function

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find the values of k

(ii) Evaluate $P(x<6)$, $P(x \geq 6)$

(iii) $P(0<x<5)$ (iv) mean v) variance.

sol.

$\sum_{i=1}^{n} a(x_i+k)$

i) Find the values of k $\quad \sum_{i=1}^{n} (a x_1 + k x_1)$

we know that $\sum_{i=0}^{7} P_i = 1$ $\quad \sum_{i=1}^{n} a x_1 + \sum_{i=1}^{n} k x_1$

$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 1$

$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$E(x)+k$

$10k^2 + 9k = 1 \qquad ax+k$

$10k^2 + 9k - 1 = 0$

$10k^2 + 10k - 1k - 1 = 0 \qquad 10 \times 1$

$10k(k+1) - 1(k+1) = 0$

$10k - 1 = 0 \quad | \quad k+1 = 0$

$k = \frac{1}{10} \quad | \quad k \neq -1$

$\therefore$ k value is $\frac{1}{10} = 0.1$

(ii) $P(x<6)$

$= P_0 + P_1 + P_2 + P_3 + P_4 + P_5$

$= 0 + k + 2k + 2k + 3k + k^2$

$= k^2 + 8k = \frac{1}{100} + \frac{8}{10} = \frac{1+80}{100} = \frac{81}{100}$

(ii) Evaluate $P(X<6)$

$= P_0 + P_1 + P_2 + P_3 + P_4 + P_5$

$= 0 + k + 2k + 2k + 3k + k^2$

$= k^2 + 8k = (0.1)^2 + 8(0.1)$

$= \frac{1}{100} + \frac{8}{10} = \frac{81}{100}$ $\quad = 0.01 + 0.8 = 0.81$

$P(X \geq 6)$

$= P_6 + P_7$

$= 2k^2 + 7k^2 + k$

$= 9k^2 + k$

$= \frac{9}{100} + \frac{1}{10} = 9(0.1)^2 + 0.1 = 0.19$

$= \frac{19}{100}$

mean $\bar{x} = \sum_{i=0}^{7} p_i x_i$

variance $\sigma^2 = \sum_{i=0}^{7} p_i x_i^2 - (\bar{x})^2$

mean $= \sum_{i=0}^{7} p_i x_i$

49
12
15
66

(iii) $P(0<x<5)$

$= P_1 + P_2 + P_3 + P_4$

$= k + 2k + 2k + 3k$

$= 8k = \frac{8}{10} = 8(0.1) = 0.8$

(iv) Mean $\mu = \sum_{i=0}^{7} p_i x_i =$

$= P_0 x_0 + P_1 x_1 + P_2 x_2 + P_3 x_3 + P_4 x_4 + P_5 x_5 + P_6 x_6 + P_7 x_7$

$= 0(0) + 1(k) + 2(2k) + 3(2k) + 4(3k) + (k^2)5 + 6(2k^2) + 7(7k^2 + k)$

$= k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k$

$= 66k^2 + 30k = \frac{66}{100} + \frac{30}{100} = \frac{96}{100}$

$= 66(0.1)^2 + 30(0.1) = 0.96$

(iii) Let variance $V(x) = \sigma^2 = \sum_{i=0}^{7} p_i x_i^2 - \bar{x}^2$

$= 0(0) + k(1) + 2k(4) + 2k(9) + 3k(16) + k^2(25) + 2k^2(36) + (7k^2 + k)(49) - 0.9216$

$= k + 8k + 18k + 48k + 25k^2 + 72k^2 + 343k^2 + 49k - 0.9216$

$= 440k^2 + 124k - 0.9216$ $\quad \sigma^2 = \sum_{i=0}^{7} p_i x_i^2 - (\bar{x})^2$

$= 440(0.1)^2 + 124(0.1) - 0.9216$

$= 16.8 - 0.9216$

$= 15.8784$ $\quad \therefore$ standard deviation

$SD = \sqrt{\text{variance}} = \sqrt{15.8784}$
$= 3.98477101$

3. Let 'X' denote the sum of the two numbers that appear when a pair of fair dice is ~~thrown~~ tossed.

Determine (i) Distribution function
(ii) mean
(iii) variance.

Sol: Let X represent the sum of the numbers obtained when two unbiased dice are thrown.

(i) Distribution function

X can be 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

$P(x=2) = P[(1,1)] = \frac{1}{36}$

$P(x=3) = P[(1,2)(2,1)] = \frac{2}{36}$

$P(x=4) = P[(1,3)(2,2)(3,1)] = \frac{3}{36}$

$P(x=5) = P[(1,4)(2,3)(3,2)(4,1)] = \frac{4}{36}$

$P(x=6) = P[(1,5)(2,4)(3,3)(4,2)(5,1)] = \frac{5}{36}$

$P(x=7) = P[(1,6),(2,5),(3,4)(4,3)(5,2),(6,1)] = \frac{6}{36}$

$P(x=8) = P[(2,6),(3,5)(4,4)(5,3)(6,2)] = \frac{5}{36}$

~~$P(x=9) = P[(3,6)(4,5)(5,4)]$~~

$P(x=9) = P[(3,6)(4,5)(5,4)(6,3)] = \frac{4}{36}$

$P(x=10) = P[(4,6)(5,5)(6,4)] = \frac{3}{36}$

$P(x=11) = P[(5,6),(6,5)] = \frac{2}{36}$

$P(x=12) = P[(6,6)] = \frac{1}{36}$

x	2	3	4	5	6	7	8	9	10	11	12
P	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) mean $\bar{x} = \sum_{i=2}^{12} p_i x_i$

$= \frac{2\times1 + 3\times2 + 4\times3 + 5\times4 + 6\times5 + 7\times6 + 8\times5 + 9\times4 + 10\times3 + 11\times2 + 12\times1}{36}$

$= \frac{252}{36} = 7$

(iii) variance $= \sum_{i=2}^{12} p_i x_i^2 - (\bar{x})^2$

$= \frac{1\times2^2 + 2\times3^2 + 3\times4^2 + 4\times5^2 + 5\times6^2 + 6\times7^2 + 5\times8^2 + 4\times9^2 + 3\times10^2 + 2\times11^2 + 1\times12^2}{36} - 49$

$= \frac{1974}{36} - 49 = \frac{210}{36} = \frac{35}{6} = 5.833$

$\therefore$ standard deviation

$SD = \sqrt{\text{variance}} = \sqrt{5.833}$

$= 2.415$

Unit 2

1. Explain continous random variable.

Sol. Continous random variable is a random variable where the data can take infinitely many values.

2. $E(x) = 12x+1 \quad -1<x<10$

Find $E(x)$ =

$\int$ $12x$ $\quad x<10$

1 $\quad -1<x$

0 $\quad$ otherwise

5. write two application of normal distribution.

Sol. Application of normal distribution

a canned juice or a bag of cookies.

6. write the probability density function of normal distribution.

Sol. the normal density $f(x)$ is bell shaped curve that is symmetric about μ and the the normal density function with $\mu = 0, \sigma = 1$

7. If $\mu = 5$ and $\sigma = 2$, then write the pdf of the normal distribution

$\mu = 0 \quad \sigma = 1$ or μ and $\sigma = 2$

8. write the probability density function for the standard normal variate.

Sol. The probability density function is

$\phi(z) = P\{z \le x\} = 1\sqrt{2\pi} \int x - \infty \exp\{-z^2/2\}$

9. If $f(x) = k(2x^2)$ in $0 < x < 2$, then find k

Sol. $\int 2x^2 \quad x<2$

$=0$ otherwise

$\leq 2(1)^2 \quad \phi x = P(z=1)$

$= 2$

$= 1\sqrt{2\pi} \int x - \infty \exp\{-x^2/2\} dx$

$x = 1$

$\phi(x) = P(z=x) = \sqrt{2\pi} \int x - \infty \exp\{-x^2/2\}$

unit - 3

1. Define population and sample

~~A~~ Population:- A population is a Entire group that you want to draw conclusions ~~abt~~.

sample:- A sample is a specific group that you will collect data.

The size of sample is alway less than the total size of population.

~~2 param~~

2. Define parameter and statistic

sol: Parameter:- A parameter is a descriptive measure computed from an entire population of data

statistic:- A statistic is a descriptive measure computed from a ~~ep~~ ~~entire~~ sample of data

3. Define standard error of a statistic?

Sol:- The standard error of a statistic is the approximate standard deviation of a statistical sample population.

4. Define Estimator and estimate?

Sol:- Estimator :- An estimator is a function that maps a random sample to parameter estimate.

Estimate :- Estimate is a value that summarizes an observed sample.

$\hat{\theta} = t(x_1, x_2, \dots, x_n)$

5. Define point estimation and interval estimation

Sol: Point estimation is a single value estimate of a parameter

interval estimation:- An interval estimation gives you a range of values where the parameter is expected to lie.

Q6 Find population correction factor if n=5 and N=30

$$FPC = \sqrt{\frac{N-n}{N-1}}$$

Sol Given $n=5$, $N=30$

$$FPC = \sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{30-5}{30-1}} = \sqrt{\frac{25}{29}}$$

$= 0.9\cancel{3}/ = 0.928$

7. Define type I error

Sol. Type 1 error:- A kind of fault that occurs during the hypothesis testing process when a null hypothesis is rejected.

$$FPC = \sqrt{\frac{N-n}{N-1}}$$

8 write the test ~~stati~~ statistic of single mean in large samples $p.N^{*}P(1-P)$

Sol. Tests of single proportions are generally based on the binomal distribution with size parameter N and probability parameter P. For large sample size, this can be well approximated by a normal distribution with mean $N^{*}p$ and variance $N^{*}p(1-p)$.

9. Write ~~about~~ about Null hypothesis?

Sol. The null hypothesis is a kind of hypothesis which explains the population parameter, whose purpose is to test the validity of the given experimental date

~~10. Define Left tailed test?~~

10. what is meant by level of significance

sol. The level of significance is the probability of rejecting the null hypothesis when it is true.

11. Define left tailed test.

Sol. A left tailed test is used when the alternative hypothesis states that the value of the parameter specified in the null hypothesis is less than the null hypothesis claims.

12. write about alternative hypothesis.

sol. The alternative hypothesis is one of the ~~two~~ mutually exclusive hypotheses in a hypothesis test. The alternative hypothesis states that a population parameter does not equal a ~~spe~~ specified value.

13. Define critical region

sol. A set of values for the test statistic for which null hypothesis is rejected.

14. Define type II error

sol. The probability of incorrectly failing to reject the null hypothesis.

15. write the test statistic for difference of two means in large sample

sol large sample confidence interval for the difference in two means. The difference between is 88 − 79 = 9 mmHg. For large samples we can calculate a 95% confidence interval for the difference in means as 9 − 1.96 × 0.81 to 9 + 1.96 × 0.81 which is 7.41 to 10.59 mm Hg.

9 − 1.96 × 0.81 to 9 + 1.96 × 0.81

7.41 to 10.59

1. Explain sampling and some important methods of sampling?

Sol. A sample is a subset of individuals from a large population. Sampling means selecting the group that you will actually collect data from in your research. Probability sampling methods include simple random sampling, systematic sampling, ~~state~~ stratified sampling, and cluster sampling.

2. A population consists of five numbers 2, 3, 6, 8, and 11. Consider all possible samples of size two which can be drawn with replacement from this propulation. Find i) the population mean ii) standard deviation of the population $\frac{\Sigma(x_i-\mu)^2}{N}$ iii) The mean of the sampling distribution of means iv) standard deviation of the sampling distribution of mean. $\frac{(x_i-\mu_c)^2}{}$

Sol Given population 2, 3, 6, 8, 11 $N=5$

i) population mean $\mu = \frac{2+3+6+8+11}{5}$

~~ii) variance of~~ $\mu = \frac{30}{5} = 6$

ii) variance of population $= \frac{\Sigma(x-\mu)^2}{N}$

$$= \frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5}$$

$$= \frac{16+9+0+4+25}{5} = \frac{54}{5} = 10.8$$

$S.D = \sqrt{\text{variance}} = \sqrt{10.8}$
$= 3.28$

$\sigma = \frac{\Sigma(x-\mu)^2}{N}$

iii) $n=2$, $N=5$ $\quad \sigma = \frac{\Sigma(x-\mu)^2}{N}$

no of samples $= {}^{N}C_{n} = {}^{5}C_{2} = 10$

(2,3) (2,6) (2,8) (2,11) (3,6) (3,8) (3,11) (6,8) (6,11) (8,11).

means

2.5, 4, 5, 6.5, 4.5, 5.5, 7, 7, 8.5, 9.5

$$\mu_c = \frac{2.5+4+5+6.5+4.5+5.5+7+7+8.5+9.5}{10}$$

$$\mu_c = \frac{60}{10} = 6$$

iv)

$$\sigma_x^2 = \frac{(2.5-6)^2+(4-6)^2+(5-6)^2+(6.5-6)^2+(4.5-6)^2+(5.5-6)^2+(7-6)^2+(7-6)^2+(8.5-6)^2+(9.5-6)^2}{10}$$

$$= \frac{(12.25)+4+1+0.25+2.25+0.25+1+1+6.25+12.25}{10}$$

$$\sigma_x^2 = \frac{40.43}{10} = 4.05$$

$$\sigma_x = \sqrt{4.05}$$
$$= 2.01$$

8. The means of two large sample of size 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches.

Sol Given $n_1 = 1000$ $\bar{x}_1 = 67.5$

$n_2 = 2000$ $\bar{x}_2 = 68.0$

$\sigma_1 = \sigma_2 = 2.5$ $n_1 \neq n_2$

Null hypothesis $H_0: \mu_1 = \mu_2$ (i.e two sample are drawn from the same population)

$H_1: \mu_1 \neq \mu_2$

Test statistics is

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1}-\frac{1}{n_2}}}$$

$$= \frac{67.5 - 68.0}{2.5\sqrt{\frac{1}{1000}-\frac{1}{2000}}}$$

$$= \frac{-0.5}{0.0ll} = -5.15$$

$|z_{cal}| = |-5.15| = 5.15$

At 5% los, $|z_{tab}| = 1.96$

$\therefore$ Here $|z_{cal}| > |z_{tab}|$

$\therefore H_0$ is rejected.

Two given samples are drawn from the same population

8. The means of two large samples of sizes 400 a

9. Random samples of 400 men and 600 women were asked wether they would like to have a flyover near the residence

200 men and ~~33~~ 325 women were favour of the proposal ~~test~~ test hypothesis that proportions of men and women in favour of the proposal are same at 5% level

Sol

Given

men n_1 400

women n_2 = 600

favour men $p_1 = \frac{200}{400} = 0.5$

proportion ~~favour~~ women $p_2 = \frac{325}{600} = 0.541$

1. Null hypothesis H_0: $H_0 = \mu_1 = \mu_2$

2. Alternative hypothesis: $H_1 = \mu_1 \neq \mu_2$

3. test statistic is $Z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

where $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$= \frac{400 \frac{200}{400} + 600 \frac{325}{600}}{400 + 600}$

$= \frac{200 + 325}{1000} = \frac{525}{1000} = 0.525$

$q = 1 - p = 1 - 0.525 = 0.475$.

$\therefore Z = \frac{0.5 - 0.541}{\sqrt{0.525 \times 0.475\left(\frac{1}{400} + \frac{1}{600}\right)}}$

$= \frac{-0.041}{0.032}$

$= -1.28$

$|z| = 1.28$

since $|z| < 1.96$. where level of significance is 5%

$\therefore H_0$ is accepted.

$$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

$$p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$$

Unit - 4

1. Find $t_{0.05}$ when $v = 16$

Sol. 1.746

2. Find $F_{0.01}(24, 19)$

Sol. 2.92

3. Write the one assumption of student's t-test.

Sol. The common assumptions made doing a t-test include those regarding the scale of measurement, random sampling, normality of data distribution, adequacy of sample size

4. Write the one use of t-test

Sol. The ~~[illegible]~~ t-test is one of many test used for the purpose of hypotesis testing ~~of~~ in statistics

5. write the one use of chi-square test

Sol A chi square test is used to help determine if observed results are in line with expected results

6 Find $F_{0.05}$ with $v_1 = 7$ and $v_2 = 15$

Sol. 2.71.

7. Write the formulae for significance of single mean in t-test

Sol. The formula for significance of single mean in t-test is $df = n-1$

8. Write the formula for significan of two means in t-test

Sol:- The formula for significance of two mean is test

$H_0: \mu_1 = \mu_2$

null hypothesis $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$

alternative hypothesis $A.H_1: \mu_1 > \mu_2$

9. Write the formula for chi - square test

Sol the formula for chi-square test is

$$x^2 = \Sigma (O_i - E_i)^2 / E_i$$

chi-sqr

$$x^2 = \frac{(O_i - E_i)^2}{E_i}$$

10. Write formula for f-test

Sol the formula for f-test is

$$f\text{ test} = \frac{\text{variance of the group mean}}{\text{mean of the within group variance}}$$

$$f\text{ test} = \frac{\text{variance of the group mean}}{\text{mean within group variance}}$$

11 Define small sample

Sol A sample of less than 30 units is considered as ~~a~~ small sample.

12. Find chi-squar value ~~of~~ for 1 degree of freedom at 5 % level of significance.

$df = n-1$

Sol 1° freedom

Level of significance $= 5\% = \frac{5}{100} = 0.05$

Chi-square value of 1° freedom at 5 % level of significance is. 3.841

Long answers

2. The number of automobiles accidents per week in a certain community are as follows: 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these frequencies in agreement with the ~~be~~ belief that accident conditions were the same ~~dur~~ during this 10 week period

Sol: Total accidents over the 10 weeks are
12+8+20+2+14+10+15+6+9+4 = 100
Null hypothesis, these accidents should be uniformly distributed over 10 weeks [illegible]
∴ the expected

The expected accidents the community per week = $\frac{100}{10} = 10$

(i) Null hypothesis:- Let H_0 is that the accidents on same in every community.

(ii) Alternative hypothesis: the no. of accidents decrease in the same community

(iii) Level of significance consider $\alpha = 5\% = 0.05$
⇒ Tabular value χ^2_α at 5% level with
D.o.f $= \nu = n-1 = 10-1 = 9$ is 16.919

(iv) Test statistic: $\chi^2 = \frac{\sum (O_i - E_i)^2}{E_i}$

$= \frac{266}{100} = 2.66$

(Oi) Observed	(Ei) Expected	$O_i - E_i$	$(O_i - E_i)^2$
12	10	2	4
8	10	-2	4
20	10	10	100
2	10	-8	64
14	10	4	16
10	10	0	0
15	10	5	25
6	10	-4	16
9	10	-1	1
4	10	-6	36
			$\sum (O_i - E_i)^2 = 266$

5. Conclusion: If the calculated value of χ^2 > tabulated value of χ^2
∴ H_0 is rejected.

3. A random sample of 10 boys had the following I.Q's:
70, 120, 110, 101, 88, 83, 95, 98, 107, 100
Do these data support the assumption of a population mean I.Q of 100.

Sol: Total I.Q of 100 boys is

$\frac{70+120+110+101+88+83+95+98+107+100}{10}$

$= \frac{9720}{100} = 97.2$

The expected I.Q's of 10 boys is 97.2

(i) Null hypothesis:- The level of I.Q's same in the 10 boys.

(ii) Alternative hypothesis:- The level of I.Q's Increase in the 10 boys.

(iii) Level of significance:- consider 5% = 0.05

Tabulated value χ^2 at level of dof $= v = n-1 = 10-1 = 9$ is 16.919

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$
70	97.2	-27.2	739.84
120	97.2	22.8	519.84
110	97.2	12.8	163.84
101	97.2	3.8	14.44
88	97.2	-9.2	84.64
83	97.2	-14.2	201.64
95	97.2	-2.2	4.84
98	97.2	0.8	0.64
107	97.2	9.8	96.04
100	97.2	2.8	7.84
			1833.6

(iv) Test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

$= \frac{1833.6}{97.2}$

$= 18.86$

(v) Conclusion: If the calculated value of χ^2_c ~~or~~ Tabulated value the χ^2

$\therefore$ It is ~~rejected~~ accepted

4. In one sample of 8 observations from a normal population, the sum of the squares of devitions of the sample values from the sample means is 84.4 and in another sample of 10 observations it was 102.6. Test at 5% level whether the population have the same variance.

sol:- $n_1 = 8$ $n_2 = 10$

$(x_1 - \bar{x}) = 84.4$, $(y_1 - \bar{y}) = 102.6$

(i) Null hypothesis:- If $H_0: \sigma_1^2 = \sigma_2^2$

(ii) Alternative hypothesis:- If $H_1: \sigma_1^2 \neq \sigma_2^2$

(iii) Level of significance:- Consider 5% = 0.05

Here f_α is with DoF $(n_1 - 1), (n_2 - 1)$
$= (8-1), (10-1) = (7, 9)$

$f_{0.05}(7, 9) = \cancel{3.68}\ 3.29$

$(n_1 - 1)\ (n_2 - 1)$

(iv) Test statistics:-

Consider $f = \frac{S_1^2}{S_2^2}$ or $\frac{S_2^2}{S_1^2}$

$= \frac{\text{Greater value}}{\text{lower value}}$ $\qquad \frac{(x_1 - \bar{x})^2}{n_1 - 1}$

Here $S_1 = \frac{(x_1 - \bar{x})^2}{n_1 - 1} = \frac{84.4}{8-1} = \frac{84.4}{7} = 12.0$

$S_2 = \frac{(y_1 - \bar{y})^2}{n_2 - 1} = \frac{102.6}{10-1} = \frac{102.6}{9} = 11.4$

$\therefore \frac{S_1^2}{S_2^2} = \frac{(12.05)^2}{(11.4)^2} = \frac{145.2025}{\cancel{129.96}}$

$|F| = 1.057$ $\qquad \frac{(y_1 - \bar{y})^2}{n_2 - 1}$

(v) Conclusion:-

$\therefore |F| < F_\alpha$ the the null hypothesis is accepted.

i.e calculated value of 'f' is < tabulated value of f_α

$\therefore$ Null hypothesis is accepted.

7. Use F test to test significance difference variance of two diets, at 5% level of significance.

$\frac{336}{12}$

Diet A	25	32	30	34	24	14	32	24	30	31	35	25	-	-	-
Diet B	44	34	22	10	47	31	40	30	32	35	18	21	35	29	24

Sol $n_1 = 12$ $\qquad n_2 = 15$

(i) Null hypothesis:- If $\sigma_1^2 = \sigma_2^2$

(ii) Alternative hypothesis:- If $\sigma_1^2 \neq \sigma_2^2$

(iii) Level of significance:- Consider 5% = 0.05

$f_\alpha = 0.05$ consider $(n_1, n_2) = (12, 15)$

$f_{\alpha\ 0.05}(12, 15) = 2.62$

(iv) Test statistics:-

$$f = \frac{S_{x_1}^2}{S_{y_1}^2}$$

where $S_{x_1}^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_1 - \bar{x})^2$

$S_y^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_1 - \bar{y})^2$

Diet A

x_1	$x_1 - \bar{x}$	$(x_1 - \bar{x})^2$
25	-3	9
32	4	16
30	2	4
34	6	36
24	-4	16
14	-14	196
32	4	16
24	-4	16
30	2	4
31	3	9
35	7	49
25	-3	9
		$\Sigma(x_1-\bar{x})^2 = 380$

$\bar{x} = \frac{336}{12} = 28$

Diet B:

Y_1	$Y_1 - \bar{Y}$	$(Y_1 - \bar{Y})^2$
44	14	196
34	4	16
22	-8	64
10	-20	400
47	17	289
31	1	1
40	10	100
30	0	0
32	2	4
35	5	25
18	-12	144
21	-9	81
35	5	25
24	-6	36
22	-8	64

$\bar{x} = \frac{450}{15} = 30$

$F = \frac{S_x^2}{S_y^2}$

$S_x^2 = \frac{1}{n_1 - 1} \sum_{i=1}^{n_1} (x_1 - \bar{x})^2$

$= \frac{1}{12-1} 380$

$= \frac{380}{11} = 34.54$

$S_y^2 = \frac{1}{n_2 - 1} \sum_{i=1}^{n_2} (y_2 - \bar{y})^2$

$= \frac{1}{15-1} 1410$

$= \frac{1410}{14} = 100.71$

$\cancel{\frac{S_x^2}{S_y^2} = \frac{34.54}{100.71} = 0.342}$

$F = \frac{S_y^2}{S_x^2} = \frac{100.71}{34.54} = 2.91$

Conclusion:- calculated value is > $F_{0.05}$ Tabulated value of F ∴ Null hypothesis is ~~accepted~~ rejected.

8. Given the following contingence table for hair colour and eye colour. find the value of chi-square. is there good association between the two.

Hair colour

		Fair	Brown	Black	Total
Eye colour	Blue	15	5	20	40
	Gray	20	10	20	50
	Brown	25	15	20	60
	Total	60	30	60	150

Sol:-

(i) Null hypothesis:- Let H_0 there is no difference between hair colour and eye colours

(ii) Alternative hypothesis:- Let H_1 there is difference between hair colour and eye colour

→ Expected frequency is as follows table

$$\text{Expected frequency} = \frac{\text{row total} \times \text{column total}}{\text{grand total}}$$

$$I = \frac{40 \times 60}{150} = 16 \qquad II = \frac{50 \times 30}{150} = 10$$

$$III = \frac{60 \times 60}{150} = 24.$$

O_i	E_i	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
15	16	-1	1	0.0625
5	16	-11	121	7.5625
20	16	4	16	1
20	10	10	100	10
10	10	0	0	0
20	10	10	100	10
25	24	-1	1	0.0416
15	24	-9	81	3.375
20	24	-4	16	0.6666
				=32.7082

(iii) Level of significance:- Consider 5% $= 0.05$

$(n_1 - 1, n_2 - 1) = (3-1, 3-1) = 2,2$

$f_{0.05}(2,2) = 19.00$

(iv) Test statistics:- consider $\chi^2 = \frac{(O_i - E_i)^2}{E_i}$

$= 32.7082$

(v) Conclusion:- The calculated value of χ^2 is ~~greater~~ ge ≤ Tabulated value of χ^2

∴ Null hypothesis is rejected.

Unit-5

1. Write C-R equations in cartesian form

Sol. If $u(x,y)$ and $v(x,y)$ are the real and imaginary parts of the same analytic function of $z = x+iy$. show that in plot using cartesian coordinates the lines of constant intersect the lines of constant at right angles

2. write the defination Harmonic function.

Sol: harmonic function. mathematical function of two variables having the property that its values at any point is equal to the average of its values along any circle around that point

3. Define Analytic function

Sol. Analytic function is a function that is locally given by a convergent power series.

4. write the C-R equations in polar form

Sol C-R equation in polar form

$\partial r = \partial u \partial x \cos\theta + \partial u \partial y \sin\theta, du$

$\partial\theta = -\partial u \partial x r \sin\theta + \partial u \partial x r \cos\theta$

. Write the statement of cauchy's theorem

Sol. Cauchy's theorem, states that if G is a finite group and p is a prime number dividing the order of G (the number of element in G), then G contains an element of order p.

Q. 7. write Generalized cauchy's integral formula

Sol. f(z) is entire, since C is a simple closed curve and z=2 is inside C. Cauchy's integral formula says that the integral is $2\pi i f(2) = 2\pi i e4$

9. write the defination Conjugate Harmonic function

Sol. A real valued function defined on a connected open set is said to have a conjugate function if and only if they are respectively the real and imaginary parts of a holomorphic function.

10. Define laplace equation

Sol. The ~~lan~~ laplace equation, $u_{xx} + u_{yy} = 0$ is the simplest such equation describing this condition ~~to~~ in two dimension. If the coefficients a, b and c are not constant by ~~be~~ depending on x and y, then the equation is cald ~~laptrore equation~~ elliptic

11. Show that $u(x,y) = x^3 - 3xy^2$ is harmonic

Sol: ~~do~~ $u(x,y) = 3x^3 - 3xy^2$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 \qquad \frac{\partial u}{\partial y} = 0 - 6xy$$

$$\frac{\partial^2 u}{\partial x^2} = 6x + 6 \qquad \frac{\partial^2 u}{\partial y^2} = -6x$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \neq 0$$

$\therefore$ This not harmonic

$2\pi i f(2) = 2\pi i e4$

5. Find the analytic function f(z) if $u - v = (x-y)(x^2+4xy+y^2)$

Sol

$$u - v = (x-y)(x^2+4xy+y^2)$$

$$u - v = x^3 - x^2y + 4x^2y - 4xy^2 + xy^2 - y^3$$

$$\boxed{u - v = x^3 - y^3 + 3x^2y - 3xy^2} \rightarrow (1)$$

Diff with respect to x eqn (1)

$$\boxed{\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = 3x^2 + 6xy - 3y^2} \rightarrow (2)$$

Diff with respect to y eqn (2)

$$\boxed{\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} = -3y^2 + 3x^2 - 6xy} \rightarrow (3)$$

Find $f(z) = u + iv$

$$\boxed{f'(z) = \frac{du}{dx} + i\frac{dv}{dx}}$$

For C-R eqn we know that

$$\frac{du}{dx} = \frac{dv}{dy}$$

$$\frac{du}{dy} = -\frac{dv}{dx}$$

eqn (iii) become

$$\boxed{-\frac{du}{dx} - \frac{dv}{dx} = -3y^2 + 3x^2 - 6xy} \quad \text{(iv)}$$

by adding eq (iii) & (iv)

$$\frac{du}{dx} - \frac{dv}{dx} = 3x^2 \cancel{+6xy} - 3y^2$$

$$-\frac{du}{dx} - \frac{dv}{dx} = 3x^2 - 6xy + 3y^2$$

$$-2\frac{dv}{dx} = 6x^2 - 6y^2$$

$$\frac{dv}{dx} = \frac{6x^2 - 6y^2}{-2}$$

$$\frac{dv}{dx} = 3y^2 - 3x^2$$

by substraction eq (iii) & (iv)

$$\frac{du}{dx} - \frac{dv}{dx} = 3x^2 + 6xy - 3y^2$$

$$-\frac{du}{dx} - \frac{dv}{dx} = 3x^2 - 6xy - 3y^2$$

$$2\frac{du}{dx} = 12xy$$

$$\frac{du}{dx} = 6xy$$

sub $\frac{du}{dx}$, $\frac{dv}{dx}$ in $f'(z)$

$$f'(z) = 6xy + i(3y^2 - 3x^2)$$

$$f'(z) = 6xy + 3iy^2 - 3ix^2$$

by milne theorem method we express $f'(z)$ in term of z by putting $x = z$ & $y = 0$

$$f'(z) = i(-3z^2)$$

$$f'(z) = -3iz^2$$

by integrating

$$f(z) = -3i\frac{z^3}{3} \qquad \boxed{f(z) = -iz^3 + c}$$