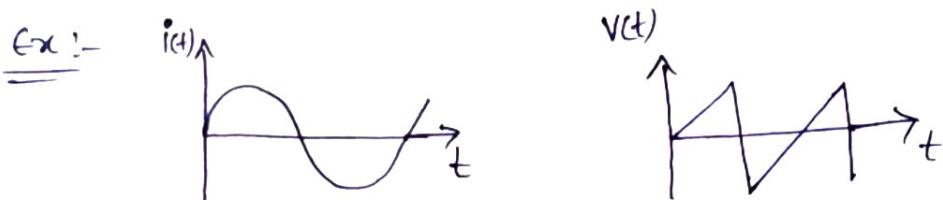


UNIT-IIA.C Circuits

Alternating Quantity : "A quantity which changes periodically its magnitude and direction with respect to time" is known as "Alternating quantity".

Alternating Current : (A.C) : "The current which changes periodically its magnitude and direction with respect to time, is called as " Alternating Current".



- it has magnitude & phase angle
- it has finite frequency
- it is dependent of the time
- it changes direction in +ve & -ve field.

Direct Current (D.C) : The current which maintains constant magnitude & only one direction



- It is independent of the time
- frequency is "zero(0)
- Magnitude is constant.

Cycle: A set of positive & negative instantaneous value of the alternating quantity.

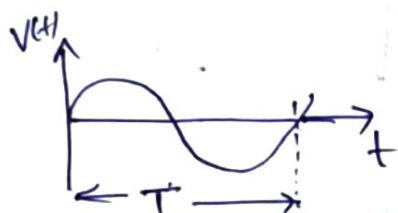
$$1 \text{ cycle} = 2\pi \text{ radians (or)} 360^\circ.$$

Frequency : (f) : Frequency is "no. of cycle per a second" for an alternating quantity.

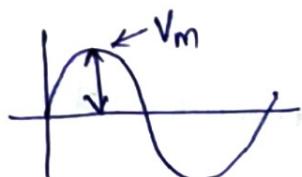
$$f = \frac{1}{T}$$

units "Hz" (Hertz)

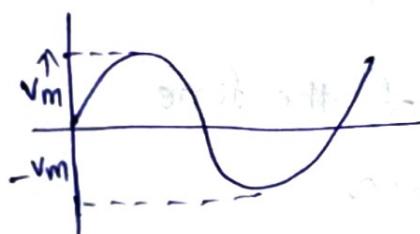
Time period (T) : It the time required by the alternating quantity to complete one cycle "



Maximum value :- The maximum value is the "Value reached by alternating quantity in total time 't' "

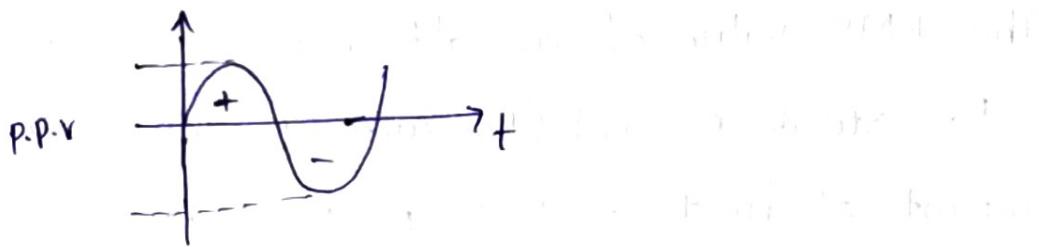


Peak value :- It is the maximum value of the wave during either positive or negative half cycle

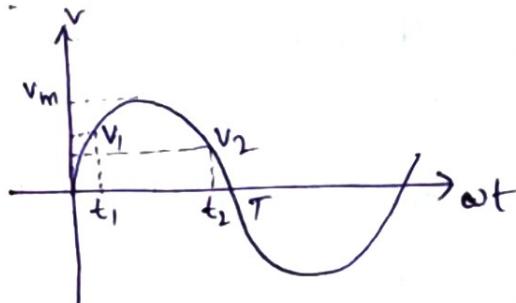


(2)

Peak to Peak Value:- It is maximum difference between peak to peak values of +ve and -ve cycles.



Instantaneous Value:- The value of alternating quantity at a particular instant time is known as instantaneous value.



where

$v_1$  is I.V of wave at  $t_1$ .

$v_2$  is I.V of wave at  $t_2$ .

Angular frequency ( $\omega$ ):-

It is the frequency expressed in electrical radians per second.

It is expressed as  $\Rightarrow 2\pi \times \text{Cycle/sec}$

$$\text{i.e } \boxed{\omega = 2\pi f} \text{ rad/sec} \quad \boxed{\omega = \frac{2\pi}{T}} \text{ rad/sec}$$

$$\theta = \omega t \quad \text{rad} \quad \text{or} \quad \boxed{\theta = \omega 2\pi f t} \quad \text{rad.}$$

Equation of Alternating quantity:-

For AC- Current  $i(t) = I_m \sin \omega t$  or  $i(t) = I_m \sin \theta$ .

where  $I_m \rightarrow$  maximum current

$\omega \rightarrow$  angular frequency.

## R.M.S Value (Root Mean Square Value)

### Effective Value

The RMS value of an alternating current is equivalent to steady current (D.C-current), which produces the same amount of heat as that produced by an alternating current when it passing through a same circuit for a some time (or) specified time.

(or)

The RMS -Value of an alternating quantity means that Square root of average of the squares of it's instantaneous values over a one - complete cycle.

→ For any alternating quantity  $f(t)$ , the time period  $T'$  then rms value is

$$f_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T f^2(t) dt} \Rightarrow \sqrt{\frac{\text{Area Covered by wave}}{\text{Length of base over a cycle}}}$$

$$\rightarrow \text{If } f(t) = (a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \dots) \\ + (b_1 \sin \omega t + b_2 \sin 2\omega t + \dots)$$

then

$$f_{\text{rms}} = \sqrt{a_0^2 + \frac{a_1^2 + a_2^2}{2} + \dots + \frac{b_1^2 + b_2^2}{2}}$$

$$\text{i.e } f_{\text{rms}} = \sqrt{\frac{f_1^2 + f_2^2 + f_3^2 + \dots + f_n^2}{n}}$$

Note:- For Alternating Current RMS value can be calculated for complete one cycle.

(3).

## Average Value:-

The Average Value can be defined as the direct steady current (D.C) which transfers charge across any circuit, the same amount of charge transferred by that alternating current during the same time for the same time.

(or)

The Average Value of an alternating quantity is defined as that value which is obtained averaging all the instantaneous values over a period of half cycle.

$$\text{i.e } f_{\text{avg}} = \frac{1}{T} \int_0^T f(t) dt, \quad \text{at } T = \frac{2\pi}{2} = \pi$$

$$\text{(ii) } f_{\text{avg}} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

$$\text{(iii) } f_{\text{avg}} = \frac{f_1 + f_2 + f_3 + \dots + f_n}{n}$$



Note:- Average value for AC-Signal is zero, for that we have to calculate average value for half cycle.

## Form Factor:- Form factor is the ratio of RMS & Average Values

$$\text{i.e. Form factor (F.F)} = K_f = \frac{\text{RMS Value}}{\text{Average value.}}$$

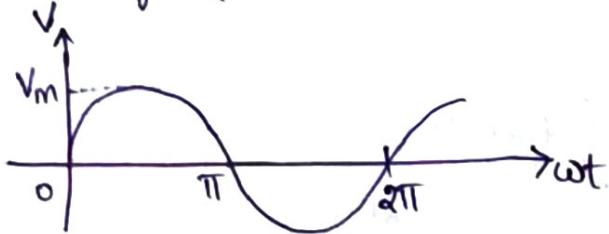
## Peak factor (or) Crest factor

Peak factor it is the ratio of Peak value (or) maximum value to the RMS Value

$$\text{i.e } P.F = \frac{\text{Maximum (or) Peak Value}}{\text{RMS Value.}}$$

Problem:- Determine RMS value for given Signal

(Q1) Derive the relation between RMS value & maximum value. for given signal.



Solution!-

For given signal is Sinusoidal Alternating voltage, its mathematical equation is

$$V = V_m \sin \omega t \quad \text{with time period } T = 2\pi$$

Then RMS - value is

$$\begin{aligned}
 V_{\text{rms}} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V^2 d(\omega t)} \\
 &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d(\omega t)} \\
 &= \sqrt{\frac{V_m^2}{2\pi} \int_0^{2\pi} \left[ \frac{1 - \cos 2\omega t}{2} \right] d(\omega t)} \\
 &= \sqrt{\frac{V_m^2}{2\pi} \cdot \frac{1}{2} \left[ \omega t - \frac{\sin 2\omega t}{2} \right]_0^{2\pi}}
 \end{aligned}$$

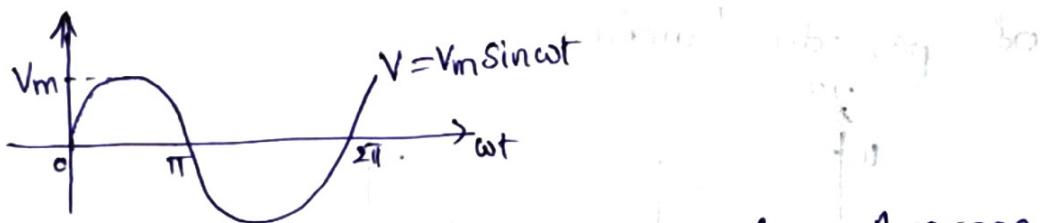
(4)

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}} \cdot \frac{1}{2} [2\pi - 0]$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \Rightarrow V_{rms} = 0.707 V_m$$

Problem:- Determine Average Value for given wave form

(a) Derive the relation between Average value & maximum value



Solution:- For given Alternating voltage wave form, Average value is zero for "total time period i.e  $2\pi$ ". So, let half time period  $\omega T = \pi$ , then

$$V_{avg} = \frac{1}{T} \int_0^T V \, d(\omega t)$$

$$= \frac{1}{\pi} \int_0^{\pi} V_m \sin(\omega t) \, d(\omega t)$$

$$= \frac{V_m}{\pi} \left[ -\cos \omega t \right]_0^{\pi}$$

$$= \frac{V_m}{\pi} \cdot [-[\cos \pi - \cos 0]]$$

$$= \frac{V_m}{\pi} \cdot [-[-1 - 1]]$$

$$V_{avg} = \frac{2V_m}{\pi}$$

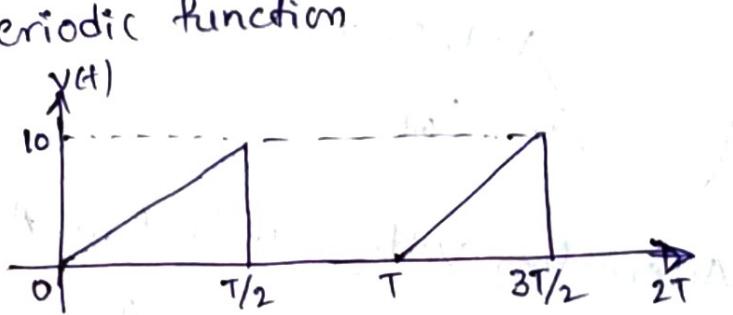
$$\Rightarrow V_{avg} = \frac{2V_m}{\pi} \Rightarrow V_{avg} = 0.637 V_m$$

For Sinusoidal Signal Form factor & Peak factor are

$$\text{Form-factor} = \frac{\text{RMS Value}}{\text{Avg Value}} = \frac{0.707 V_m}{0.637 V_m} = 1.11$$

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{rms value}} = \frac{V_{pp}}{0.707 V_m} = 1.414$$

Problem:- Determine RMS & Avg Values, form factor & peak factor of periodic function.



Solution:- From Given data, we have to calculate function equation with in the limits of  $0 < t < T$

i.e  $0 < t < T/2$  &  $T/2 < t < T$

so

$$v(t) \text{ points are } (0,0), (T/2, 10) \quad (T/2, 0) \quad (T, 0)$$

$$(v(t), t) \quad \cancel{v(t)} \Rightarrow v(t)-0 = \frac{10}{T/2} (t-0) \quad v(t)-0 = \frac{0-0}{T-T/2} (t-T/2)$$

$$v(t) = \frac{20}{T} t \quad v(t) = 0.$$

Now RMS Value is

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt}$$

$$= \sqrt{\frac{1}{T} \int_0^{T/2} \left(\frac{20}{T} t\right)^2 dt}$$

$$\int_{T/2}^T v(t) dt = 0.$$

(5)

$$\begin{aligned}
 V_{rms} &= \sqrt{\frac{1}{T_0} \int_{T_0}^{T_2} \frac{400}{T^2} t^2 dt} \\
 &= \sqrt{\frac{400}{T^3} \cdot \left( \frac{t^3}{3} \right) \Big|_0^{T_2}} \\
 &= \sqrt{\frac{400}{3 \cdot T^3} \left( \frac{T^3}{8} - 0 \right)} = \sqrt{\frac{400}{24}} \Rightarrow V_{rms} = 4.08
 \end{aligned}$$

Average Value:

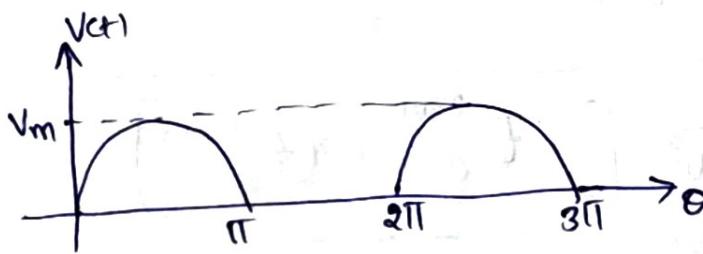
$$\begin{aligned}
 V_{Avg} &= \frac{1}{T} \int_0^T v(t) dt \\
 &= \frac{1}{T} \int_0^{T/2} \frac{20}{T} t dt + \int_{T/2}^T \frac{-20}{T} t dt \\
 &= \frac{20}{T^2} \int_0^{T/2} t dt \\
 &= \frac{20}{T^2} \left[ \frac{t^2}{2} \right]_0^{T/2} = \frac{20}{2T^2} \cdot \frac{T^2}{4} = \frac{20}{8} \Rightarrow V_{avg} = 2.5
 \end{aligned}$$

Then

$$\text{Form factor } F.F = \frac{\text{rms Value}}{\text{Avg Value}} = \frac{4.08}{2.5} = 1.632$$

$$\text{Peak value P.F} = \frac{\text{Maximum Value}}{\text{rms Value}} = \frac{10}{4.08} = 2.45$$

HW Determine the rms & Average Values of a half wave rectified sinusoidal voltage of a peak value  $V_m$



$$\text{Ans!-} \\ V_{rms} = \frac{V_m}{\sqrt{2}}$$

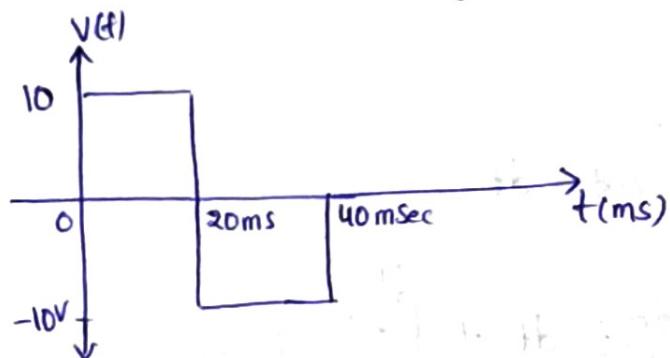
$$V_{avg} = \frac{V_m}{\pi}$$

problem! -

Obtain RMS - Value , Average value , form factor & peak value for a voltage of Symmetrical Square shape whose amplitude is 10V and time period is 40msec

Solution! -

From the given data Voltage wave form can be drawn as



For total time period of wave is 40msec , then Voltage equation for different intervals

$$\text{for } 0 < t < 20\text{ms} \rightarrow V(t) = 10\text{V}$$

$$20\text{ms} < t < 40\text{ms} \rightarrow V(t) = -10\text{V}$$

For RMS Value :-

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T (V(t))^2 dt}$$

then

$$V_{\text{rms}} = \sqrt{\frac{1}{40\text{ms}} \left[ \int_0^{20\text{ms}} (V(t))^2 dt + \int_{20\text{ms}}^{40\text{ms}} (V(t))^2 dt \right]}$$

$$= \sqrt{\frac{1}{40\text{ms}} \left[ \int_0^{20\text{ms}} (10)^2 dt + \int_{20\text{ms}}^{40\text{ms}} (-10)^2 dt \right]}$$

$$= \sqrt{\frac{100}{40 \times 10^{-3}}} \left[ (t) \Big|_0^{20\text{ms}} + (t) \Big|_{20\text{ms}}^{40\text{ms}} \right]$$

$$= \sqrt{\frac{100}{40\text{ms}}} \left[ 20\text{ms} - 0 + 40\text{ms} - 20\text{ms} \right]$$

$$V_{\text{rms}} = \sqrt{100} = \underline{10\text{V}}$$

(6)

For Average Value

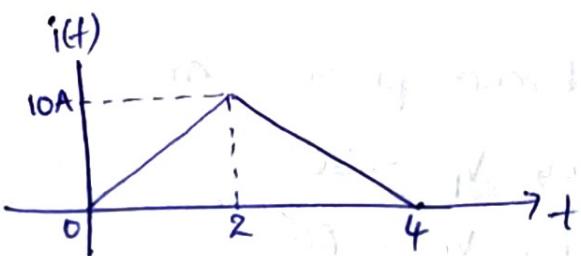
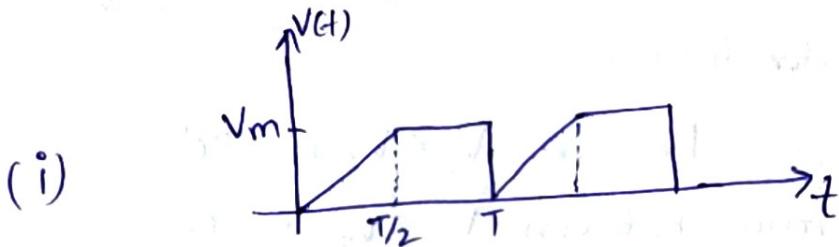
$$\begin{aligned}
 V_{avg} &= \frac{1}{T/2} \int_0^{T/2} v(t) dt \\
 &= \frac{1}{20ms} \int_0^{20ms} 10 dt \\
 &= \frac{10}{20ms} (t) \Big|_0^{20ms} \Rightarrow \frac{10}{20ms} (20ms - 0) \Rightarrow V_{avg} = 10
 \end{aligned}$$

$$\text{Form Factor} = \frac{V_{rms}}{V_{avg}} = \frac{10}{10} = 1$$

$$\text{Peak factor} = \frac{V_p}{V_{rms}} = \frac{10}{10} = 1$$

Q:- Find Average, RMS, Form factor & peak factor for given voltage wave form which is having 20V maximum value at  $0 < t < 0.1$  sec,  $0.3 < t < 0.4$  intervals and at the "OV" the interval of  $0.1 < t < 0.3$  sec.

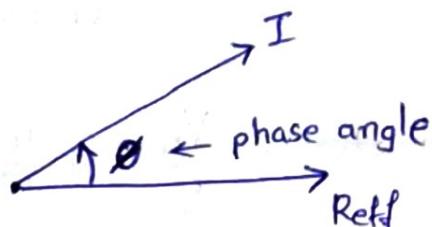
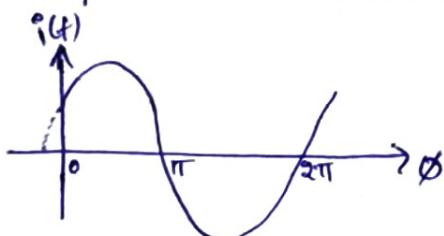
Q:- Determine Avg, RMS, F.F & P.F for given signals



Phase :- Phase is the relative position of the waveform with respect to zero position.

(or)

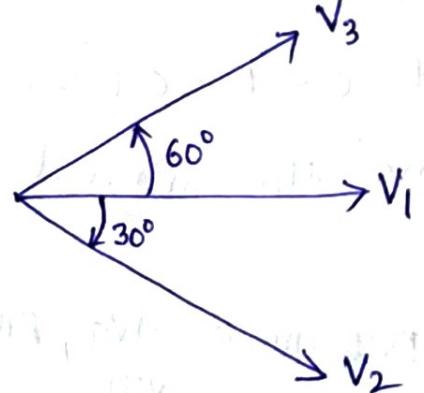
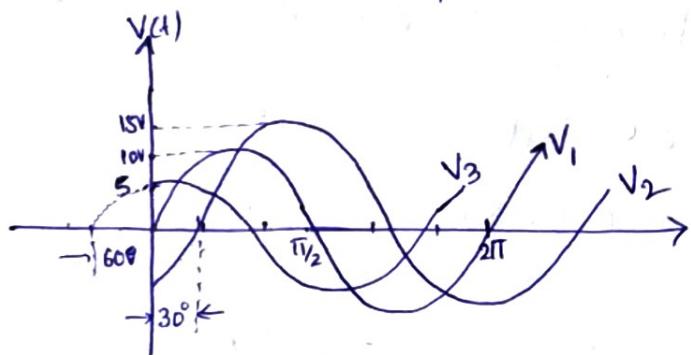
Phase is the relative position of the phase on phasors with respect to reference phase



Vector diagram.

Phase difference :-

It is the phase angle difference between two phasors on vectors



From the above vector diagram

We have phase difference between  $V_1$  &  $V_2$  is  $30^\circ$

We have phase difference between  $V_1$  &  $V_3$  is  $60^\circ$

And

if we take  $V_1$  is reference phasor then

$V_2$  is lags by  $V_1 \rightarrow 30^\circ$

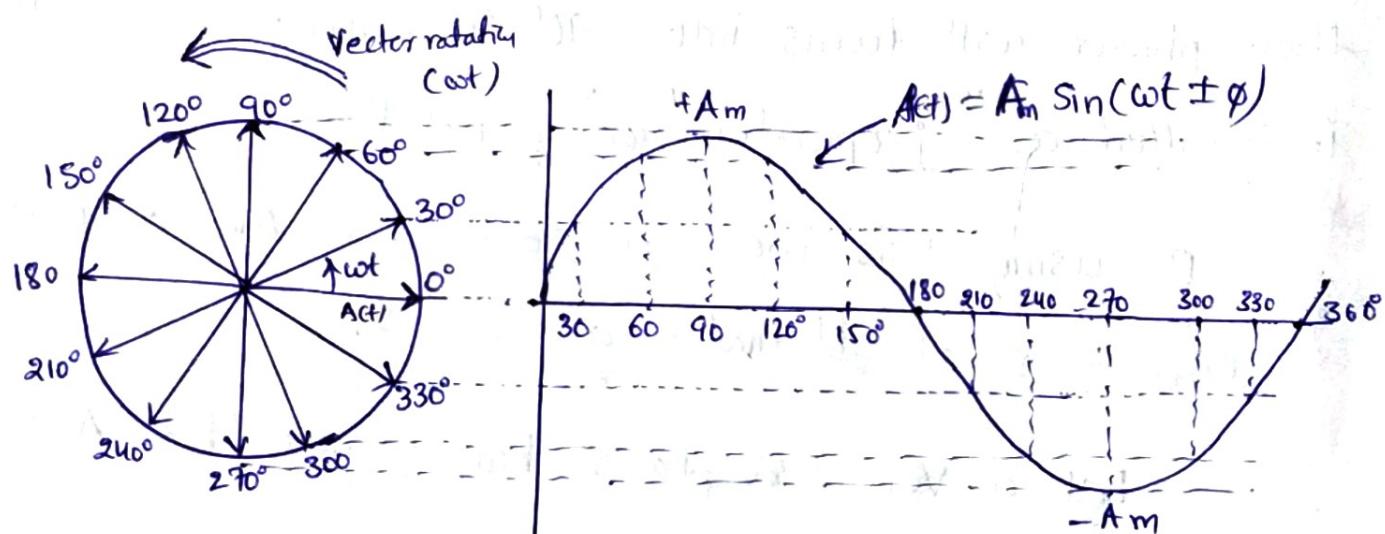
$V_3$  is lead by  $V_1 \rightarrow 60^\circ$ .

## Phasor representation of an Alternating Quantity :-

The graphical representation of sinusoidally varying alternating quantity by a straight line with an arrow, such line is called as "phasor" or "Vector"

The diagram which represents the different alternating quantities by individual phasors, which gives the exact phasor interrelationships is known as phasor diagram.

Let phasors are assumed to be rotated in anticlockwise direction with constant speed, as shown below



- Consider a phasor rotating in anticlockwise direction from the phasor angle  $0^\circ$  to  $30^\circ$ , then alternating quantity increases from 0 to upto  $30^\circ$  on y-axis
- And from  $30^\circ$  to  $60^\circ$  phasor rotated then again alternating quantity function will change, as i.e  $A(t) = A_m \sin \theta$

as like that

at  $90^\circ$  it is  $A(t) = A_m \underline{\hspace{2cm}}$

&  $180^\circ$  it is  $A(t) = 0$

&  $210^\circ$  &  $240^\circ$  it is  $A(t) = -A_m$  ... and so on,

the alternating quantity can be changed.

Like this this type of rotating vector is called phasor, and this diagram is called as "phasor diagram".

J-operator on j-notation:-

It is a quantity when it acts on any phasor then phasor will turns into  $90^\circ$  in anticlockwise direction it is called as "j-operator" or 'j-notation'.

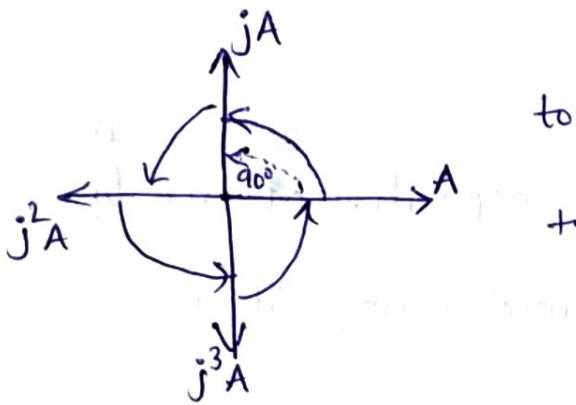
By using this we can solve the complex phasor calculations of alternating quantities.

Let a vector or phasor having magnitude A & phase angle  $0^\circ$ , then it shown as



if we apply  
j-operator then it rotates to  
 $90^\circ$  anticlockwise direction

i.e.  $A \angle 90^\circ$



$$\begin{aligned}
 & \text{to } \downarrow \quad A \angle 0^\circ \\
 & + \downarrow \quad A \angle 90^\circ \\
 & \quad \quad \quad \omega \\
 & + \downarrow \quad A \angle 180^\circ \\
 & \quad \quad \quad \downarrow \\
 & \quad \quad \quad A \angle 270^\circ
 \end{aligned}$$

$j = \sqrt{-1} = 1 \angle 90^\circ$   
 $j^2 = -1 = 1 \angle 180^\circ$   
 $j^3 = -j = 1 \angle 270^\circ$   
 $j^4 = 1 = 1 \angle 0^\circ$

Representation of Alternating Quantities :- It is by two forms,

### ① Rectangular form

It is the form is

Sum of real & imaginary parts

i.e  $z = a + jb$

→ It is useful for adding & Subtraction of A-Q

Add  $\rightarrow (R_1 + R_2) + j(X_1 + X_2)$

Sub  $\rightarrow (R_1 - R_2) + j(X_1 - X_2)$

⇒ Rectangular to polar conversion is

$$z = a + jb \Rightarrow r\angle\theta$$

then

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1}(b/a)$$

### ② Polar form

it is represents

as magnitude & phase angle

$$p = r\angle\theta$$

→ It is useful for multiplying & dividing

Multiply  $\rightarrow r_1 r_2 \angle(\theta_1 + \theta_2)$

Divide  $\rightarrow \frac{r_1}{r_2} \angle(\theta_1 - \theta_2)$

⇒ Polar to Rectangular form

$$r\angle\theta \Rightarrow z = a + jb$$

~~a = r \cos \theta~~

~~b = r \sin \theta~~

## Basic definitions :-

Impedance (Z) : It is the total opposition offered by an AC-circuit for the flow of current through it is called "Impedance"

The ratio of phasor voltage (V) to the phasor current (I) is called "Impedance"

$$\text{i.e } Z = R \pm jX \quad \text{or} \quad Z = \frac{V \angle \theta_1}{I \angle \theta_2}$$

→ Its magnitude value is  $Z = \sqrt{R^2 + X^2}$

→ Units are "Ω"

Reactance (X) : The opposition offered by reactive components i.e. inductor or capacitor in an A.C circuit for the flow of current through it is called "Reactance(X)"

i.e. imaginary part of impedance

$$Z = R \pm jX$$

for inductor it is " $X_L$ " & for capacitor it is " $X_C$ "

$X_L$  = inductive reactance

$$X_L = 2\pi f L$$

$X_C$  = capacitive reactance

$$X_C = \frac{1}{2\pi f C}$$

→ Units are "Ω"

(9).

Admittance :- (Y) : The reciprocal of impedance is called as Admittance (Y). (9) mark follow

→ Units are  $\text{S}$  (mho)

$$\text{i.e } Y = \frac{1}{Z} = \frac{1}{R \pm jX}$$

by Rationalizing the denominator we get

$$Y = \frac{R}{R^2 + X^2} \mp \frac{jX}{R^2 + X^2}$$

$$Y = G \mp jB$$

Where  $G \rightarrow$  conductance  $= \frac{R}{R^2 + X^2}$

$B \rightarrow$  Susceptance  $= \mp \frac{jX}{R^2 + X^2}$

Susceptance (B) : The imaginary part of admittance

is called as Susceptance (B)

→ Units are "Mho ( $\text{S}$ ) or Siemens"

$$B = \mp \frac{jX}{R^2 + X^2} \quad \text{where } \frac{-X}{R^2 + X^2} \rightarrow \text{inductive Susceptance}$$

$$\frac{+X}{R^2 + X^2} \rightarrow \text{capacitive Susceptance}$$

Conductance (G) : It is the real part of admittance

$$G = \frac{R}{R^2 + X^2} \quad \text{units are "S"}$$

## Real Power or Active Power or True Power or Average Power

### (or) Watt Power (P)

The product of r.m.s values of voltage and current with the cosine of angle between them is called "Active Power".

i.e

$$P = VI \cos \phi$$

→ Units are Watts.

(or)

The power in a.c circuit can be defined as the product of the r.m.s values of voltage and current with the sine of angle between them is ~~called~~ active component of the current is called "Active power".

## Reactive Power or Wattless Power or Quadrature Power (Q)

The product of r.m.s values of voltage & current with the sine of angle between them is called "Reactive power"

i.e

$$Q = VI \sin \phi$$

→ Units are VAR - Volts Amperes ~~per~~ ~~Resistance~~

(or)

The power in a.c circuit can be defined as the product of r.m.s value of voltage and reactive component of the current is called "Reactive Power Q".

## Apparent Power (S)

The product of r.m.s Values of voltage & current is called "Apparent Power" in a.c circuit

$$\text{i.e } S = V_{\text{rms}} \times I_{\text{rms}}$$

$$\text{or } S = \sqrt{P^2 + Q^2} \quad \text{or } S = P \pm jQ$$

## Power factor :-

Power factor is the "cosine of the angle between voltage phasor to current phasor in a.c circuit."

The term " $\cos\phi$ " is called power factor.

### Example :-

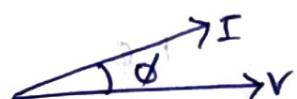
for Resistive load  $\rightarrow \cos\phi = 1$   
(Inphase)



Inductive load  $\rightarrow$  Current lags



Capacitive load  $\rightarrow$  Current leads



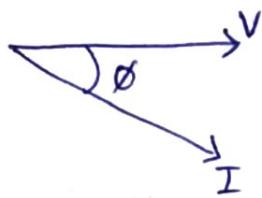
Power Triangle :- Power triangle is the geometrical representation of the apparent power, active power & reactive power.

We have power triangle for  
Inductive load  
Capacitive load.

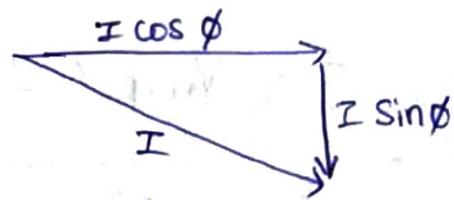
## Power Triangle for Inductive load :-

For inductive load current lags by voltage

i.e.



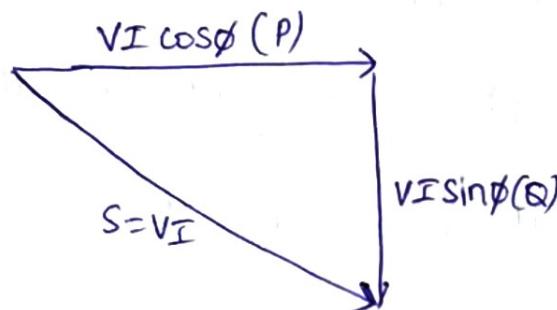
So



$I \cos \phi$  = active component

$I \sin \phi$  = reactive component

the power triangle is



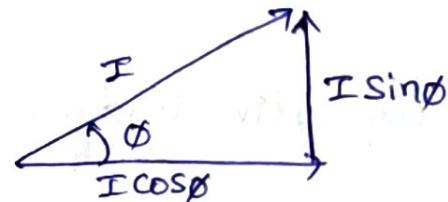
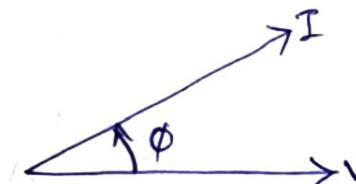
i.e.  $S = \sqrt{P^2 + Q^2}$

$$S = \sqrt{(VI \cos \phi)^2 + (VI \sin \phi)^2}$$

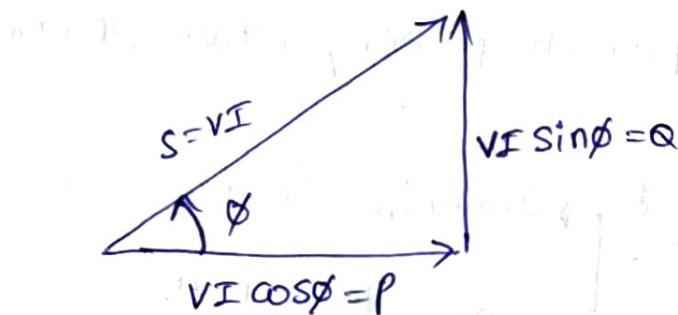
$$\underline{S = VI}$$

## Power Triangle for Capacitive load :-

For capacitive load current leads by voltage



Power Triangle is



$$S = \sqrt{P^2 + Q^2}$$

$$S = VI$$

Complex power :-

Complex power is the product of voltage to the conjugate current component (or product of conjugate voltage component to current).

$$C.P = V I^* \text{ or } V^* I$$

$$\begin{aligned} \text{Let } VI^* &= V e^{j\theta} \cdot I e^{-j(\theta+\phi)} \\ &= VI e^{-j\phi} \end{aligned}$$

$$VI^* = VI \cos\phi - j VI \sin\phi$$

$$S = \boxed{VI^* = P - jQ} \quad \text{is for Capacitive Circuit}$$

Similarly for inductive

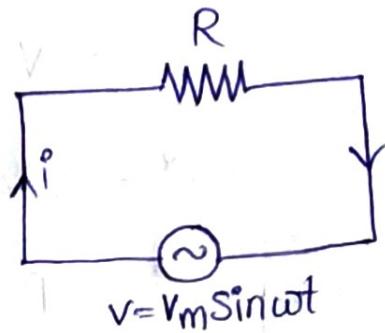
$$\boxed{S = V^* I = P + jQ} \quad \text{is for inductive circuit.}$$

## Analysis of Single-phase ac-circuits

### AC through Pure Resistance:-

#### (or) Sinusoidal response of Pure Resistive Circuit

Consider a simple circuit consisting of a pure resistance 'R' connected with sinusoidal voltage  $V = V_m \sin \omega t$ , as shown below



→ According to Ohm's law the current in circuit is

$$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$$

$$\text{i.e. } i = \left(\frac{V_m}{R}\right) \sin \omega t \quad \rightarrow ①$$

→ Compare eq ① with standard current equation

$$i = I_m \sin (\omega t \pm \phi)$$

$$\text{then } I_m = \frac{V_m}{R} \quad \& \quad \phi = 0^\circ$$

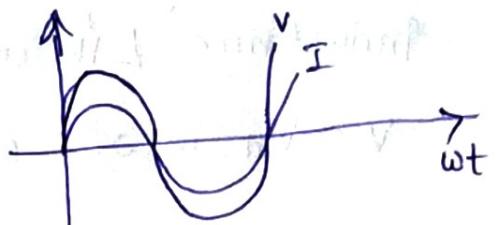
→ The maximum current value is  $I_m = \frac{V_m}{R}$

→  $\phi = 0$  represent voltage & currents are inphase so

$$i = I_m \sin \omega t \quad \text{at } \phi = 0$$

→ The instantaneous power is product of instantaneous values of voltage and current

$$\begin{aligned} P_i &= V \times i \\ &= V_m \sin \omega t \times I_m \sin \omega t \\ &= \frac{V_m I_m}{2} \sin^2 \omega t \end{aligned}$$



$$P_{inst} = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos(2\omega t) \quad \text{②}$$

In eq ② the term  $\frac{V_m I_m}{2} \cos(2\omega t)$  is having double frequency

So this value becomes zero,

→ The average power value is equal to instantaneous power of constant power component  $\frac{V_m I_m}{2}$

$$P_{avg} = \frac{V_m I_m}{2}$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

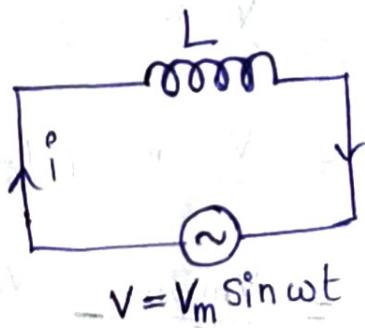
$$P_{avg} = V_{rms} \cdot I_{rms}$$

For pure resistive circuit power is

$$P = VI \quad \text{Watts.}$$

## AC - through Pure Inductance

consider a simple circuit consisting of a pure inductance 'L' connected with sinusoidal voltage  $V = V_m \sin \omega t$  as shown below.



For pure inductive circuit current lags by voltage  $90^\circ$ , it can be proved as

Voltage across inductor is  $V = L \frac{di}{dt}$

$$\text{So } V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t$$

$$i = \int di = \int \frac{V_m}{L} \sin \omega t$$

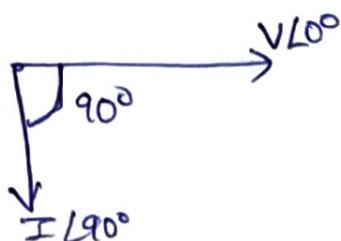
$$= \frac{V_m}{L} \cdot \frac{-\cos \omega t}{\omega}$$

$$i = \frac{V_m}{\omega L} \sin(\omega t - \frac{\pi}{2})$$

$$i = I_m \sin(\omega t - \frac{\pi}{2}) \quad \text{where } I_m = \frac{V_m}{X_L} \quad \& \quad \phi = -\frac{\pi}{2}$$

Above equation gives that current lags by voltage  $90^\circ$

vector diagram is



→ The instantaneous power

$$P_{\text{inst}} = V \times I^{\circ} = V_m \sin \omega t \times I_m \sin(\omega t - \frac{\pi}{2})$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$\because \sin 2A = 2 \sin A \cos A$$

$$P_{\text{inst}} = \frac{-V_m I_m}{2} \sin(2\omega t)$$

→ The Average <sup>power</sup> value for pure inductive is zero, why because we have double frequency term ( $\sin 2\omega t$ )

so

$$P_{\text{avg}} = 0$$

Pure inductance never consumes power

### AC - through Pure Capacitance

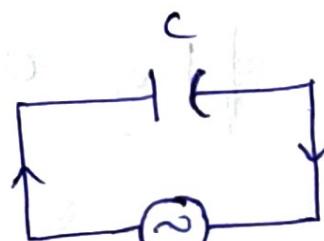
consider a simple circuit consisting of a pure capacitor  $C$  Farads connected with sinusoidal voltage  $V = V_m \sin \omega t$ .

For pure capacitive

circuit current leads by

voltage  $90^\circ = \frac{\pi}{2}$ , it can be

proved as



$$V = V_m \sin \omega t$$

→ The instantaneous charge in capacitor is

$$q = C \cdot V = C \cdot V_m \sin \omega t$$

→ Current  $i$  is rate of charge so  $i = \frac{dq}{dt}$

$$i = \frac{d(C V_m \sin \omega t)}{dt}$$

by standart current  
Bustia  $i = I_m \sin(\omega t \pm \phi)$

$$= C \cdot V_m \cdot (\omega \cos \omega t)$$

$$i = \left( \frac{V_m}{\sqrt{C}} \right) \sin \left( \omega t + \frac{\pi}{2} \right) \rightarrow I_m \sin \left( \omega t + \frac{\pi}{2} \right)$$

The equation give that current leads by voltage  $90^\circ = \frac{\pi}{2}$

i.e.



→ The instantaneous power

$$P_{inst} = V \times i = V_m \sin \omega t \times I_m \sin \left( \omega t + \frac{\pi}{2} \right)$$

$$P_{inst} = \frac{V_m I_m}{2} \sin(2\omega t)$$

→ The average power value is zero for pure capacitive circuit, because it has double frequency term ( $\sin 2\omega t$ )

$P_{avg} = 0$

Pure capacitor never consumes the power.

Note:- For pure inductive & capacitive circuits never consumes the power, because there are energy stored component circuits.

## Analysis of Series RL-Circuit :-

Consider a series RL-circuit connected with alternating voltage  $V = V_m \sin \omega t$  as shown below

In a given circuit, the current 'I' drawn by RL, then it causes two voltage drops :-

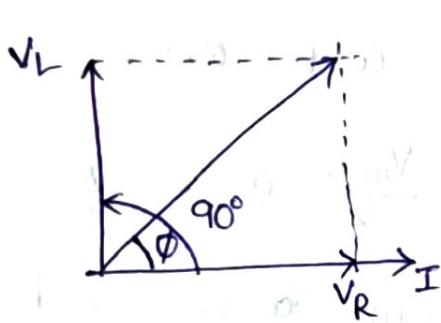
→ Voltage drop across pure resistance  $V_R = IR$

→ Voltage drop across pure inductance  $V_L = IX_L$

If we will take voltages should be a phasors, then according to KVL, the addition of phasor is

$$\bar{V} = \bar{V}_R + \bar{V}_L = \bar{IR} + \bar{IX}_L$$

by above equation their phasor diagram & voltage triangle is

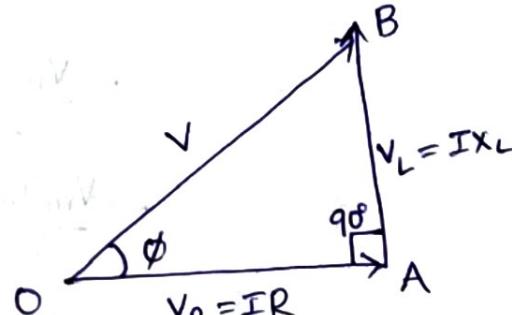


phasor diagram

From phasor diagram

$V_R \rightarrow$  is inphase with I

I → is lags  $V_L$  by  $90^\circ$



voltage Triangle

From  $\triangle OAB$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{(IR)^2 + (IX_L)^2}$$

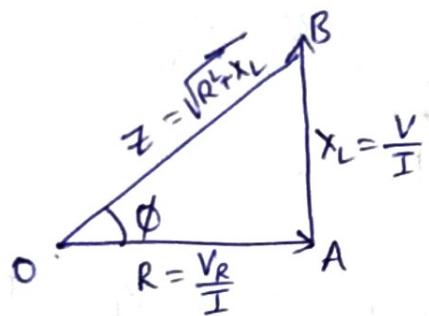
$$V = I \cdot \sqrt{R^2 + X_L^2} = I \cdot Z$$

## Impedance & Impedance Triangle:-

The impedance for RL-Circuit is  $Z = R + jX_L$  (Rectangular)

where  $X_L = 2\pi f L$

$Z = |Z| \angle \phi$  (Polar form)



from  $\triangle OAB$

$$\tan \phi = \frac{X_L}{R}$$

$$\sin \phi = \frac{R}{Z}$$

$$\cos \phi = \frac{X_L}{Z}$$

→ It can be seen that current lags by voltage by ' $\phi$ '

$$\text{so } V(t) = V_m \sin \omega t$$

$$I(t) = I_m \sin(\omega t - \phi)$$

→ The instantaneous Power  $P_{\text{inst}} = V \times i$

$$= V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

$$= V_m I_m \sin \omega t \cdot \sin \omega t - \phi$$

$$= \frac{V_m I_m}{2} (\cos \phi - \cos(2\omega t - \phi))$$

$$= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos(2\omega t - \phi)$$

→ In above equation the second term  $\cos(2\omega t - \phi)$  has double frequency & average value for cycle is zero

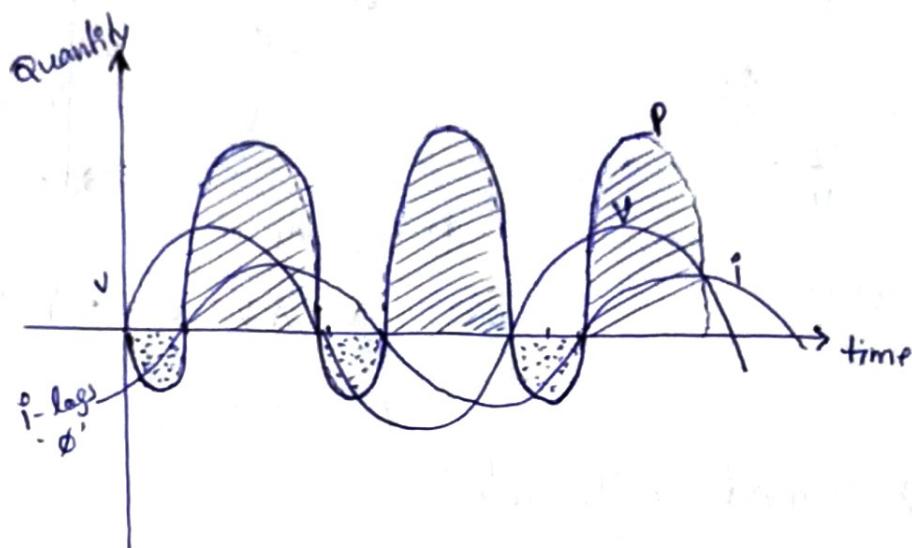
$$P_{\text{avg}} = \frac{V_m I_m}{2} \cos \phi$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$$P = VI \cos\phi$$

w.r.t.

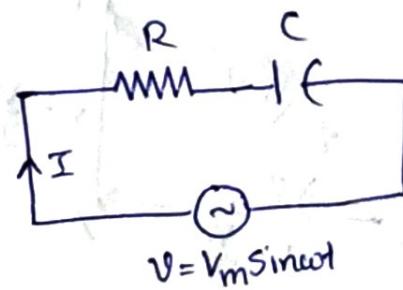
→ The ~~phasor~~ waveforms for  $V$ ,  $I$  & power is



### Analysis of Series RC-Circuit :

Consider a series RC-circuit connected with a source  $V = V_m \sin\omega t$ , as shown below

The given circuit draws the current ' $I$ ', causes the two voltage drops



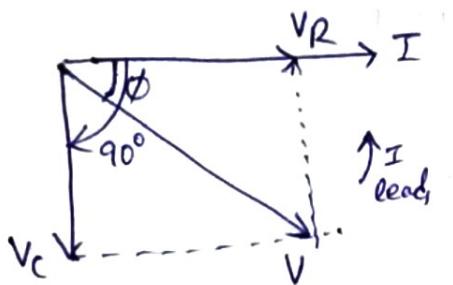
→ Voltage drop across pure resistance  $V_R = IR$

→ Voltage drop across pure capacitance  $V_C = IX_C$

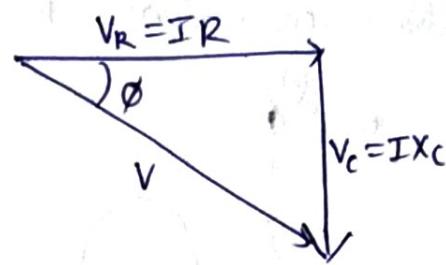
For a given circuit, apply KVL then by addition of two phasors

$$\bar{V} = \bar{V}_R + \bar{V}_C = \bar{IR} + \bar{IX}_C$$

From the above equation their phasor diagram & voltage triangle is



Phasor diagrams



From LOAB

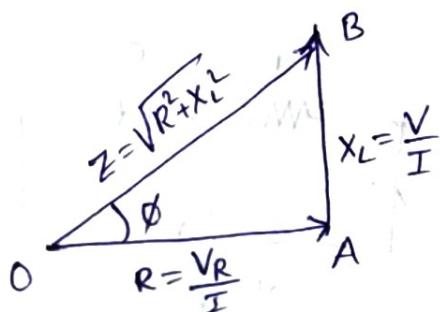
$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$\boxed{V = IZ'}$$

Impedance & Impedance Triangle :-

The impedance for RC-circuit is  $Z = R - jX_L$  (rectangular form)  
 $Z = |Z| L - \phi$  (polar form)

$$X_C = \frac{1}{2\pi f C}$$



From LOAB

$$\tan \phi = \frac{X_L}{R}$$

$$\sin \phi = \frac{R}{Z}$$

$$\cos \phi = \frac{X_L}{Z}$$

→ It can be seen that current leads by voltage  $\phi$

$$\text{So } V_{CA} = V_m \sin \omega t$$

$$i(t) = I_m (\sin \omega t + \phi)$$

→ The instantaneous Power  $P_{inst} = V \times i$

$$= V_m \sin \omega t \times I_m \sin(\omega t + \phi)$$

$$= V_m I_m (\sin \omega t \cdot \sin \omega t + \phi)$$

$$P_{\text{inst}} = \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos(2\omega t + \phi)$$

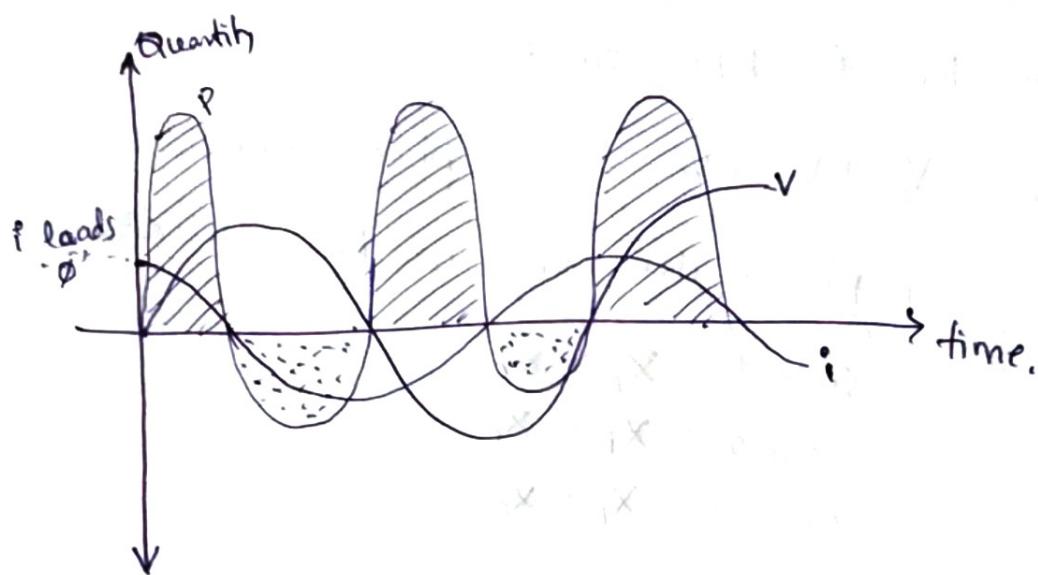
In above equation the cosine term "cos2wt" has double frequency, hence its average power value is zero, so

$$P_{\text{avg}} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \phi$$

$$P_{\text{avg}} = VI \cos \phi$$

watts

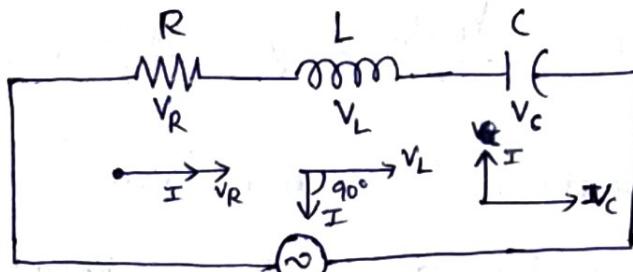
→ Waveforms for V, I & Power is



### Analysis of Series RLC Circuit :-

Consider a circuit consisting of resistance R ohms, pure inductance L Henries and capacitance C farads connected in series with supply  $V = V_m \sin \omega t$ , as shown below.

When Supply is connected, then current 'I' draws and it causes 3-voltage drops



$$V = V_m \sin \omega t$$

→ Due to current 'I', there are different voltage drops across R, L & C with are given by

→ Drop across resistance R is  $V_R = IR$

→ Drop across inductance L is  $V_L = IX_L$

→ Drop across Capacitance C is  $V_C = IX_C$

By According KVL for RLC-circuit

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad \text{--- is phasor addition.}$$

For ~~the~~ drawing phasor diagram, we have to ~~not~~ consider

3-cases as

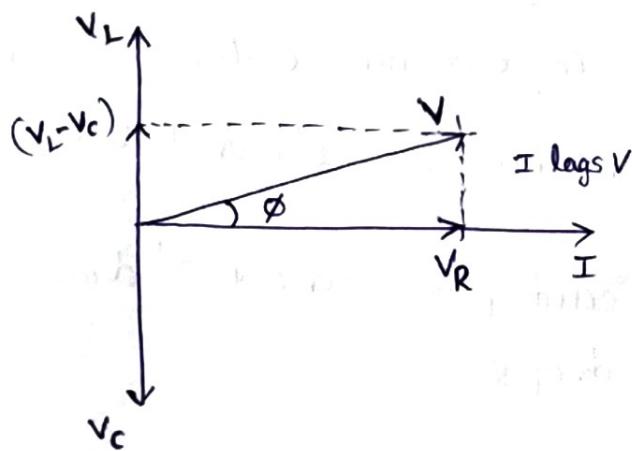
case 1 :  $X_L > X_C$

case 2 :  $X_L < X_C$

case 3 :  $X_L = X_C$

Case 1 :-  $X_L > X_C$

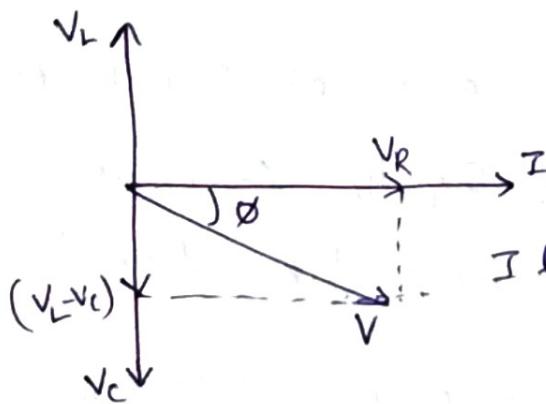
When  $X_L > X_C$  then  $V_L > V_C$ , so resultant of  $(V_L - V_C)$  is towards ' $V_L$ ' i.e. leading I,



→ the circuit behaves as  
inductive nature

case 2 :-  $X_L < X_C$

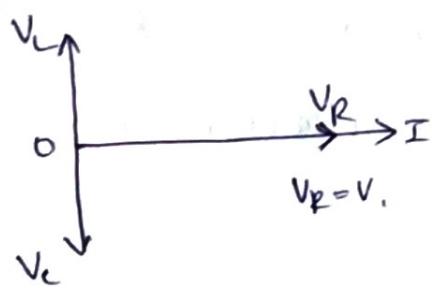
When  $X_L < X_C$  then  $V_L < V_C$ , so resultant of  $(V_L - V_C)$  is "-ve" is towards ' $V_C$ ' i.e. lagging 'I' with angle  $\phi$ .



i.e. the circuit behaves as capacitive nature.

case 3 :-  $X_L = X_C$

when  $X_L = X_C$  then  $V_L = V_C$ , so resultant of this is zero, means cancel each other,  $\phi = 0$



i.e. the circuit behaves pure resistive nature.

→ This condition  $X_L - X_C = 0$  (or)  $X_L = X_C$  is called "Resonance condition."

## Problems:-

- ① A coil has resistance of  $4\Omega$  and an inductance of  $9.55 \text{ mH}$ . Calculate (i) the reactance (ii) the impedance (iii) the current taken from a  $240V, 50\text{Hz}$  supply.
- ② Draw the phasor diagram for a coil of  $80\text{mH}$  &  $60\Omega$  resistor connected to  $200V, 100\text{Hz}$  supply. Calculate the circuit impedance & the current taken from the supply.
- ③ A coil takes a current of  $2.5A$  at  $0.8$  lagging power factor from a  $220V, 60\text{Hz}$ , single phase source. If the coil is modeled by a series RL-circuit find  
(i) the complex power in the coil  
(ii) the values of R & L.
- ④ Develop the phasor diagram for a series RC-Circuit with  $R = 10\Omega$  &  $C = 10\mu\text{F}$  & excited with  $1-\phi, 230V$  supply.
- ⑤ A coil of resistance  $5\Omega$  and inductance  $120\text{mH}$  in series with a  $100\mu\text{F}$  capacitor, is connected to  $300V, 50\text{Hz}$  supply. Find  
(a) The current flowing  
(b) The phase difference between Supply voltage & current  
(c) The voltage across the coil  
(d) The voltage across the capacitor  
(e) Draw the phasor diagram.

## Resonance in Series RLC-circuit

In a series RLC-circuit, when  $X_L = X_C$  or  $(X_L - X_C) = 0$  i.e. when reactance of circuit becomes to zero, then it is called as "the circuit is at ~~at~~"series resonance".

- At this time circuit behaves a "purely resistive"
- And the power factor  $\cos \phi = 1$
- At this time it draws maximum current.

### Applications:-

- ① It is used in Tuning circuits
- ② In inverters
- ③ Wave trapping circuits.

### Resonance frequency ( $f_r$ ) for Series RLC-circuit:-

In a ~~series~~ RLC-circuit at which ~~the~~ frequency the resonance occurs, that frequency called as "Resonance frequency ( $f_r$ )."

i.e. 
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Hz.

Let ' $f_r$ ' be the frequency at which resonance occurs in series RLC-circuit

→ At resonant frequency  $f_r$  the reactance is zero

$$\text{i.e } X_L = X_C \quad \text{so } X_L - X_C = 0.$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ rad.}$$

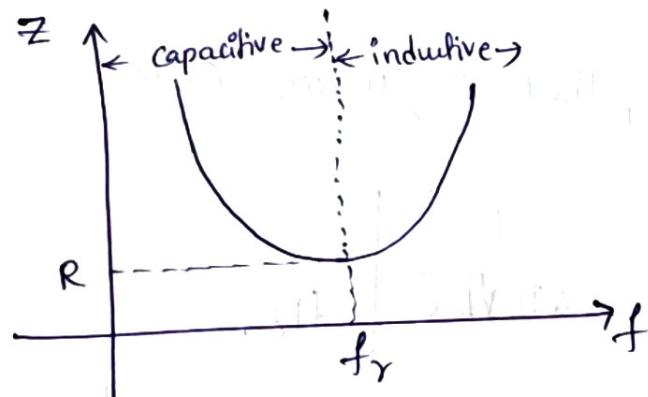
Angular resonance frequency

→ If  $f_r > f$  then circuit is Capacitive nature

$f_r < f$  then circuit is inductive nature

$f_r = f$  is pure resistive nature

→ It can be shown as



→ At resonance current is maximum  $I_m = \frac{V}{R}$

→ The power at resonance  $P_m = I_m^2 R$ .

## Band width and Quality factor (Q) for Resonance Circuit

In a series RLC-circuit, at resonance condition we have the two half power frequencies which are due to reactive components, as

$$\left(\omega_L - \frac{1}{\omega_C}\right) = \pm R$$

Consider two half power frequencies as

$$\left(\omega_1 L - \frac{1}{\omega_1 C}\right) = +R \quad \text{--- (1)}$$

$$\left(\omega_2 L - \frac{1}{\omega_2 C}\right) = -R \quad \text{--- (2)}$$

→ Add eq (1) + eq (2), then

$$(\omega_1 + \omega_2) L - \left(\frac{1}{\omega_1} + \frac{1}{\omega_2}\right) \frac{1}{C} = 0$$

$$(\omega_1 + \omega_2) L = \left(\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right) \frac{1}{C}$$

$$\omega_1 \omega_2 = \frac{1}{LC} \quad \text{--- (3)} \quad \text{but } \omega_r = \frac{1}{LC}$$

so

$$\boxed{\omega_1 \omega_2 = \omega_r^2}$$

$$\boxed{f_1 f_2 = f_r^2}$$

→ Subtract eq (1) - eq (2), then

$$(\omega_2 - \omega_1) + \frac{(\omega_2 - \omega_1)}{\omega_1 \omega_2} \cdot \frac{1}{LC} = \frac{2R}{L} \quad \begin{matrix} \text{multiply } \frac{1}{L} \\ \text{on both sides} \end{matrix}$$

$$(\omega_2 - \omega_1) + (\omega_2 - \omega_1) = \frac{2R}{L}$$

$$\therefore \omega_1 \omega_2 = \frac{1}{LC}$$

$$(\omega_2 - \omega_1) = \frac{R}{L}$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

$$\boxed{\text{Band width} = B.W = f_2 - f_1 = \frac{R}{2\pi L}}$$

$$\Delta f = \frac{R}{4\pi L}$$

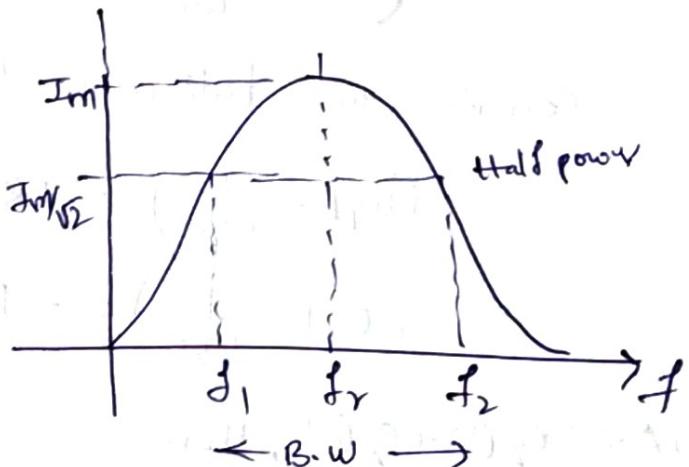
→ And we have

Upper cut-off frequency

$$f_2 = f_r + \Delta f$$

lower cut-off frequency

$$f_1 = f_r - \Delta f$$



Quality factor:-

It ratio of voltage across L (or) C with supply voltage.

i.e. voltage magnification in circuit.

$$Q = \frac{V_L}{V} \cos \frac{V_C}{V} \quad (\text{or}) \quad \frac{X_L}{R} \cos \frac{X_C}{R}$$

for RL → ~~Q = V\_L / V~~

$$Q = \frac{\omega_r L}{R} \rightarrow$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

for RC →

$$Q = \frac{1}{R} \sqrt{\frac{C}{L}}$$

i.e. Same.

## Parallel RLC Circuits

(pro)- In series-parallel circuit A & B are in parallel and in series with 'c'. The impedances are

$$Z_A = 4+j3 \Omega, Z_B = 4-j5 \Omega \text{ & } Z_C = 2+j8 \Omega.$$

If the current  $I_C = (25+j0)$ , calculate

- (i) Branch current (ii) total power
- (iii) Branch voltage (iv) phasor diagram.

## Resonance in Parallel Circuit :-

In a parallel circuit, when susceptance  $\phi$  is becomes zero, then circuit is said to be in resonance. i.e.  $(B_L - B_C) = 0$ .

→ Power factor is unity. & current is minimum.

→ It is used in oscillators

→ complex communication circuit & filter.

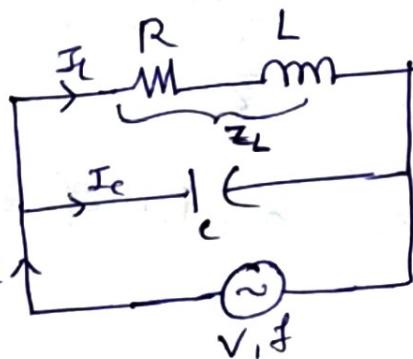
Consider a parallel circuit which is having series of R & L in parallel with 'C' as shown below

This circuit is called

Tuned circuit. If impedance

of circuit is  $Z_L$  and reactance I

are  $X_L$  &  $X_C$  then



at resonance we have relation

$$Z_L^2 = X_L X_C$$

$$R^2 + (2\pi f_r L)^2 = (2\pi f_r L) \times \frac{1}{2\pi f_r C}$$

$$R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$2\pi f_r = \frac{1}{L} \sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

i.f  $\frac{R^2}{L^2} \ll \frac{1}{LC}$  then

$$f_r = \frac{1}{2\pi \sqrt{LC}}$$

it is same as ~~resonance~~ series resonance

→ Quality factor is current magnification

i.e  $\frac{\frac{V}{Z_L}}{\frac{V}{Z_D}} = \frac{\frac{1}{RC}}{\sqrt{\frac{L}{C}}}$

then Quality factor

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

## Three-phase balanced Circuits :-

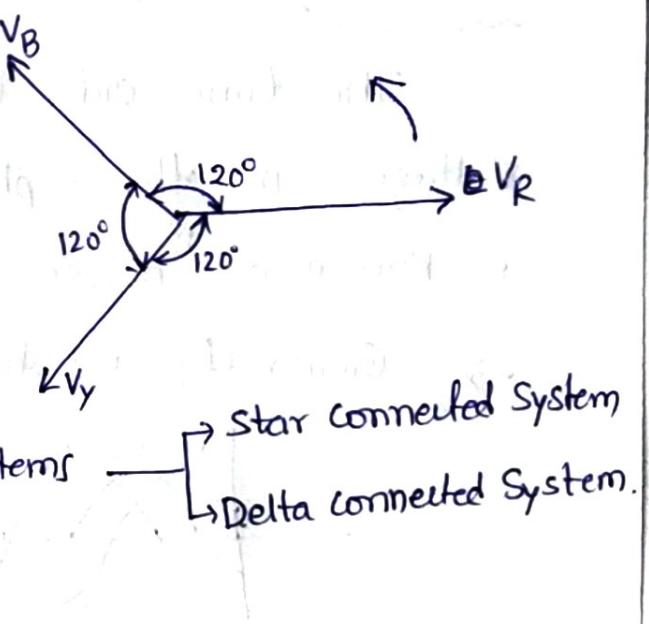
Three-phase Circuit system :- The system which is having three voltages with same magnitude & same frequency ~~but~~ and having each one with a phase difference  $120^\circ$ . As follows

$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin(\omega t - 120^\circ)$$

$$V_B = V_m \sin(\omega t - 240^\circ)$$

$$\text{or } V_m \sin(\omega t + 120^\circ)$$



→ we have 2-types of 3-φ Systems

## Advantages of 3-φ System :-

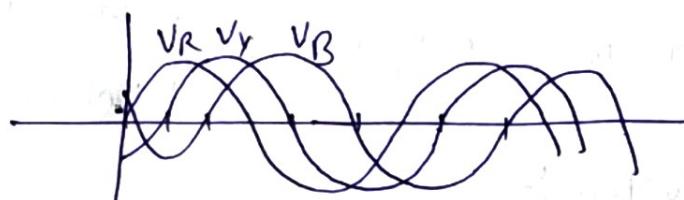
- By using 3-φ system output power increases, it is 1.5 times of 1-φ system power.
- In 3-φ system ~~copper~~ designing of transmission and distribution it require less copper as compared with 1-φ system.
- By using 3-φ system we can produce rotating magnetic field.
- In 3-φ system instantaneous power is constant
- Three phase system give steady output.

- The 3-φ system having better regulation, with 1-φ System.
- 3-φ can be used to supply the 1-φ load system.
- 3-φ can be rectified into DC-Supply with low ripple factor.
- Parallel operation is easy in 3-φ system.

### Phase sequence:-

The time order in which the voltages in three-phases reach their maximum values is known as "phase sequence".

- Generally we have R-Y-B is phase sequence.



Phase voltage: ( $V_{ph}$ ) The voltage which is measured between a line and neutral, called as phase voltage.

$$\text{Ex: } \overline{V_{RN}} = \overline{V_{YN}} \text{ or } \overline{V_{BN}}$$

Line voltage: ( $V_L$ ): The voltage measured between any two lines in a 3-phase system.

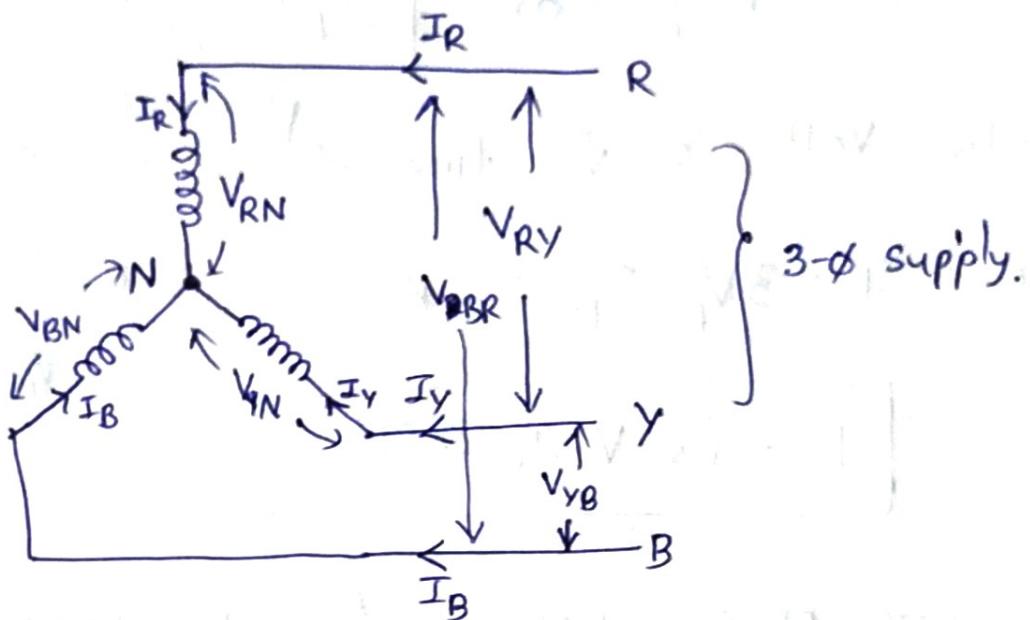
$$\text{Ex: } \overline{V_{RY}}, \overline{V_{YB}}, \overline{V_{BR}}$$

$$\overline{V_{RY}} = \overline{V_R} - \overline{V_Y}$$

$$\overline{V_{YB}} = \overline{V_Y} - \overline{V_B}$$

## Voltage & Current relations in 3-φ Star connected system

The circuit diagram for a 3-φ balanced star connected circuit with phase sequence R-Y-B as shown below



For a given 3-φ star connected system has

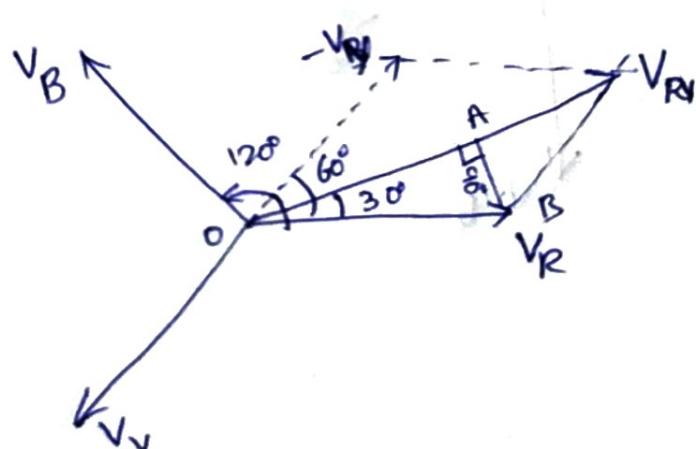
$$\rightarrow \text{Phase voltages } V_{ph} = V_{RN} = V_N = V_{BN}$$

$$\& \text{phase currents } I_{ph} = I_R = I_Y = I_B$$

$$\rightarrow \text{Line voltages } V_L = V_{RY} = V_{YB} = V_{BR}$$

$$\& \text{line current } I_L = I_R = I_Y = I_B.$$

$\rightarrow$  The phasor diagram for line & phase voltages is



$$V_{RY} = V_R - V_Y$$

$$V_{YB} = V_Y - V_B$$

$$V_{BR} = V_B - V_R$$

From  $\Delta OAB$   $\cos 30 = \frac{OC}{OA} \Rightarrow \frac{V_L/2}{V_{ph}} = \frac{\sqrt{3}}{2}$   $\therefore \frac{\frac{V_{RY}}{2}}{V_R} = \frac{OC}{OA}$

So, In Star connected system

$$V_L = \sqrt{3} V_{ph}$$

and

$$I_L = I_{ph}$$

Thus line voltage is  $\sqrt{3}$  times of phase voltage

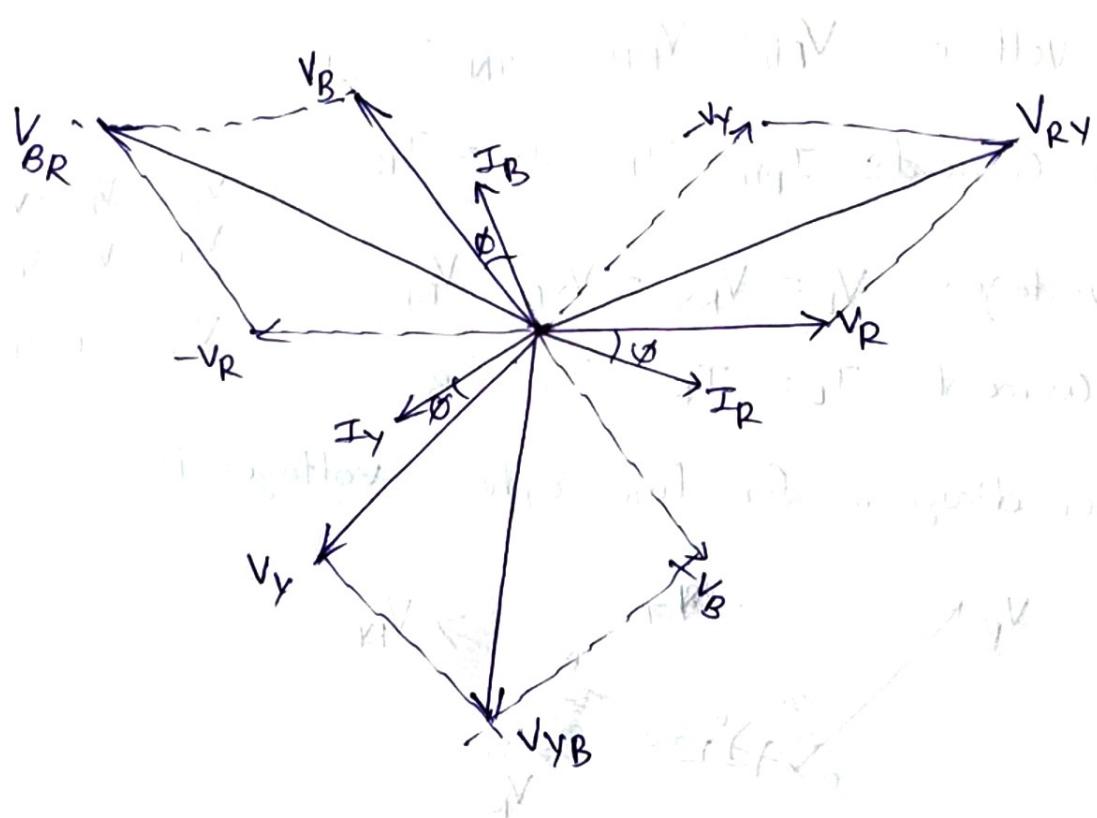
→ Power  $P = 3V_{ph} I_{ph} \cos \phi$  for 3-φ system

So

$$P = \sqrt{3} V_L I_L \cos \phi$$

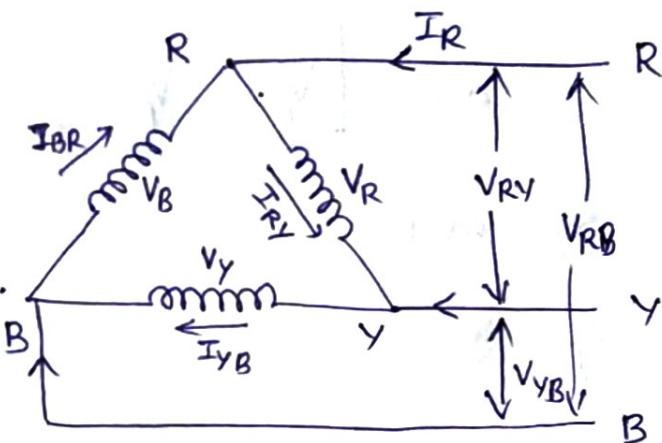
$$\therefore V_{ph} = \frac{V_L}{\sqrt{3}}$$

→ The 3-φ phasor diagram for star connected inductive load is



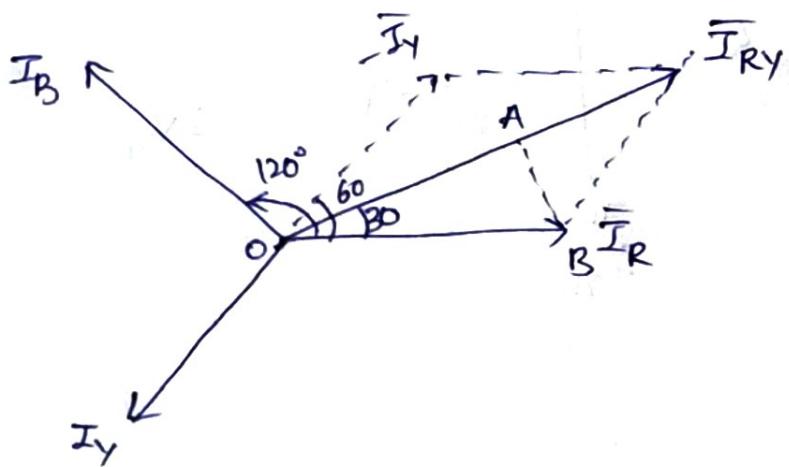
## Voltage & current relations in 3-φ Delta connected System!-

The circuit diagram for a 3-φ Delta connected system with phase sequence R-Y-B as shown below.



For a given 3-φ delta connected system has

- phase voltages  $V_R = V_Y = V_B = V_{ph}$
- phase currents  $I_{ph} = I_{RY} = I_{YB} = I_{BR}$
- line voltages  $V_L = V_{RY} = V_{YB} = V_{BR}$
- line currents  $I_L = I_R = I_Y = I_B$
- The phasor diagram for line current & phase currents



From Triangle OAB  $\cos \phi = \frac{OC}{OA} = \frac{I_R/2}{I_{RY}}$

$$\frac{\sqrt{3}}{2} = \frac{I_L/2}{I_{ph}}$$

So In delta connected System

$$I_L = \sqrt{3} I_{ph}$$

and

$$V_{ph} = V_{PL}$$

→ The Power in 3-φ System is

$$P_{ph} = 3 V_{ph} I_{ph} \cos \phi$$

$$P = \sqrt{3} V_L I_L \cos \phi \text{ watt}$$

→ The 3-φ phasor diagram for delta connected inductive load is

