2) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ where $\bar{F} = 3xy\bar{i} - y^2\bar{j}$ & c is the curve $y = 2x^2$ in the xy-plane from (0, 0) to (1, 2).

Sol: Let $\bar{r} = x\bar{i} + y\bar{j}$

$$d\bar{r} = dx\bar{i} + dy\bar{j}$$

$$\int_C \bar{F}\cdot d\bar{r} = \int_C (3xy\bar{i} - y^2\bar{j})(dx\bar{i} + dy\bar{j})$$

Gn, $y = 2x^2$

$dy = 4x\,dx$

$x = 0$ to 1

$$= \int (3xy\,dx - y^2\,dy)$$

$$= \int_0^1 3x(2x^2)\,dx - (2x^2)^2\,4x\,dx$$

$$= \int_0^1 (6x^3 - 16x^5)\,dx$$

$$= \left[\frac{6x^4}{4} - \frac{16x^6}{6}\right]_0^1 \Rightarrow \left[\left\{\frac{3}{2}(1)^4 - \frac{8}{3}(1)^6\right\} - 0\right]$$

$$= \left[\frac{3}{2} - \frac{8}{3}\right] \Rightarrow \left[\frac{9-16}{6}\right] = \frac{-7}{6}$$

V.V.Imp 10M

3) Find the work done in moving a particle in the force field $\bar{F} = 3x^2\bar{i} + (2xz - y)\bar{j} + z\bar{k}$ along the straight line from

Sol: Gn, $F = 3x^2 i + (2xz - y)j + zk$

Equivalent OA is $\Rightarrow \frac{x-0}{} = \frac{y-0}{} = \frac{z-0}{}$

$$\int_C \bar{F}\cdot d\bar{r} = \int (3x^2\bar{i} + (2xz-y)\bar{j} + z\bar{k}) \cdot (dx\bar{i} + dy\bar{j} + dz\bar{k})$$

$$= \int 3x^2 dx + (2xz-y)dy + z\,dz$$

$$= \int_{t=0}^{1} 3(2t)^2\, 2dt + [2(2t)(3t) - t]\,dt + (3t)\,3dt$$

$4t$

$3(4t^2)2dt$

$24t^2$

$$= \int_0^1 (24t^2 + 12t^2 - t + 9t)\,dt$$

$$= \left[\frac{\overset{8}{\cancel{24}}t^3}{\cancel{3}} + \frac{\overset{4}{\cancel{12}}t^3}{\cancel{3}} - \frac{t^2}{2} + \frac{9t^2}{2}\right]_0^1$$

$$= \left[\left\{8(1)^3 + 4(1)^3 - \frac{1}{2}(1)^2 + \frac{9}{2}(1)^2\right\} - 0\right]$$

$$= \left[8 + 4 - \frac{1}{2} + \frac{9}{2}\right] \Rightarrow [12 + 4]$$

$$= 16.$$

4) If $\bar{F} = xy\bar{i} - z\bar{j} + x^2\bar{k}$ & 'C' is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ to $t = 1$. Evaluate $\int_C \bar{F}\cdot d\bar{r}$

Sol:- Gn, $\bar{F} = xy\bar{i} - z\bar{j} + x^2\bar{k}$

$d\bar{r} = dx\bar{i} + dy\bar{j} + dz\bar{k}$

Gn $x = t^2$ $\quad y = 2t$ $\quad z = t^3$

$dx = 2t\,dt$ $\quad dy = 2dt$ $\quad dz = 3t^2 dt$

t - varies from 0 to 1

$$\int_C \bar{F}\cdot d\bar{r} = \int (xy\bar{i} - z\bar{j} + x^2\bar{k}) \cdot (dx\bar{i} + dy\bar{j} + dz\bar{k})$$

$$= \int xy\,dx - z\,dy + x^2 dz.$$

$$= \int_{t=0}^{1} (t^2)(2t)\,2t\,dt - (t^3)\,2dt + (t^2)^2\,3t^2 dt$$

$$= \int_0^1 (4t^4 - 2t^3 + 3t^6)\,dt$$

$$= \left[\frac{4t^5}{5} - \frac{\cancel{2}t^3}{\cancel{4}_2} + \frac{3t^7}{7}\right]_0^1$$

$$= \left[\left\{\frac{4}{5}(1)^5 - \frac{1}{2}(1)^3 + \frac{3}{7}(1)^7\right\} - 0\right]$$

$$= \left[\frac{4}{5} - \frac{1}{2} + \frac{3}{7}\right] = \left[\frac{56 - 35 + 30}{70}\right] = \frac{51}{70}.$$

5) Find the work done by the force $\bar{F} = (2y+3)\bar{i} + xz\bar{j} + (yz-x)\bar{k}$ when it moves a particle from the pt $(0,0,0)$ to $(2,1,1)$ along the curve $x = 2t^2$, $y = t$ & $z = t^3$.

Sol:- Gn, $\bar{F} = (2y+3)\bar{i} + xz\bar{j} + (yz-x)\bar{k}$

$$d\bar{r} = dx\bar{i} + dy\bar{j} + dz\bar{k}$$

Also gn, $x = 2t^2$, $y = t$, $z = t^3$.

$$dx = 4t\,dt \quad dy = dt \quad dz = 3t^2\,dt.$$

when $x = 0, t = 0$. , when $y = 0, t = 0$, when $z = 0, t = 0$.

$x = 2, t = 1$ $\quad$ $y = 1, t = 1$ $\quad$ $z = 1, t = 1$

$\therefore$ t- varies from 0 to 1

$$\int_C \bar{F}\cdot d\bar{r} = \int \left((2y+3)\bar{i} + xz\bar{j} + (yz-x)\bar{k}\right)\cdot(dx\bar{i} + dy\bar{j} + dz\bar{k})$$

$$= \int (2y+3)\,dx + xz\,dy + (yz-x)\,dz$$

$$= \int_{t=0}^{1} [2t+3]\,4t\,dt + (2t^2)(t^3)\,dt + [t(t^3) - 2t^2]\,3t^2\,dt$$

$$= \int_{t=0}^{1} (8t^2 + 12t + 2t^5 + 3t^6 - 6t^4)\,dt.$$

$$= \left[\frac{8t^3}{3} + \frac{\cancel{12}^{6} t^2}{\cancel{2}} + \frac{\cancel{2}^{1} t^6}{\cancel{6}_3} + \frac{3t^7}{7} - \frac{6t^5}{5}\right]_0^1$$

$$= \left[\left\{\frac{8}{3}(1)^3 + 6(1)^2 + \frac{1}{3}(1)^6 + \frac{3}{7}(1)^7 - \frac{6}{5}(1)^5\right\} - 0\right]$$

$$= \left[\frac{8}{3} + 6 + \frac{1}{3} + \frac{3}{7} - \frac{6}{5}\right]$$

$$= \left[\frac{\cancel{560 + 1260 + 105 + 90 - 252}}{\cancel{210}}\right]$$

$$= \left[\frac{280 + 630 + 35 + 45 - 126}{105}\right] = \frac{864}{105} = \frac{288}{35}$$

08/05/20:-

6) Find the work done by $\bar{F} = (2x-y-z)\bar{i} + (x+y-z)\bar{j} + (3x-2y-5z)\bar{k}$ along a curve 'c' in the xy-plane given by.

i) $x^2+y^2=4$, $z=0$

ii) $x^2+y^2=9$, $z=0$.

Sol:- i) Gn, $\bar{F} = (2x-y-z)\bar{i} + (x+y-z)\bar{j} + (3x-2y-5z)\bar{k}$

Let $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

$\bar{dr} = dx\bar{i} + dy\bar{j} + dz\bar{k}$ in the xy plane, $z=0$, $\Rightarrow dz=0$.

$$\therefore \int_c \bar{F}\cdot d\bar{r} = \int_c (2x-y-z)\bar{i} + (x+y-z)\bar{j} + (3x-2y-5z)\bar{k} \,.\, (dx\bar{i} + dy\bar{j} + dz\bar{k})$$

$$= \int_C (2x-y)dx + (x+y)dy \quad \text{———} (1)$$

$[\because x = r\cos\theta$, $y = r\sin\theta]$

i) Gn curve is $x^2+y^2=4$

Let $x = 2\cos\theta$, $y = 2\sin\theta$

$dx = -2\sin\theta d\theta$ $\quad dy = 2\cos\theta d\theta$

Since the given curve represents circle.

So, θ-varies from 0 to 2π.

From eqn (1)

$$\int_C \bar{F}\cdot d\bar{r} = \int_{\theta=0}^{2\pi} [2(2\cos\theta) - 2\sin\theta](-2\sin\theta d\theta) + (2\cos\theta + 2\sin\theta)(2\cos\theta d\theta)$$

$$= \int_{\theta=0}^{2\pi} [-8\sin\theta\cos\theta + 4\sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta] d\theta$$

$$= \int_{\theta=0}^{2\pi} [-4\sin\theta\cos\theta + 4(\sin^2\theta + \cos^2\theta)] d\theta$$

$$= \int_{\theta=0}^{\pi} [-2(2\sin\theta\cos\theta) + 4] d\theta$$

$$= \int_{\theta=0}^{\pi} [-2\sin 2\theta + 4] d\theta$$

$$= \left[\cancel{2}\left(\frac{-\cos 2\theta}{\cancel{2}}\right) + 4\theta\right]_0^{2\pi}$$

$$= [\cos 2\theta + 4\theta]_0^{2\pi} \Rightarrow [\{\cos 2(2\pi) + 4(2\pi)\} - \{\cos 2(0) + 0\}]$$

$$= [\cancel{1} + 8\pi - \cancel{1}] = 8\pi.$$

Adding L.H.S of eqⁿ (a), (b), (c), (d)

$1-e^{-\pi} - e^{-\pi} + 0 + 1 \quad = 2 - 2e^{-\pi}$

$= 2\left[1-e^{-\pi}\right] . = L.H.S$

$\therefore$ L.H.S = R.H.S.

$\therefore$ Green's theorem is verified.

7) Verify Green's theorem in plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where 'c' is the region bounded by $y = \sqrt{x}$ & $y = x^2$.

Sol: By Green's theorem in a plane

$$\oint_C M dx + N dy = \iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dx\, dy \longrightarrow (1)$$

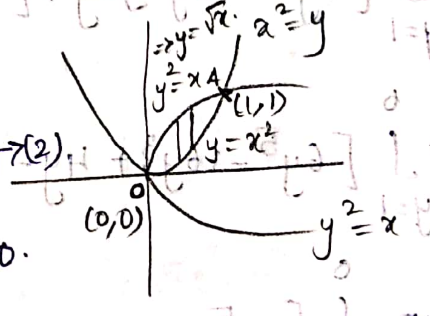
Gn, curves $y = \sqrt{x}$. & $y = x^2 \longrightarrow$ (ii)

$y^2 = x \longrightarrow$ (i)

Substitute eqⁿ(ii) in eqⁿ(i)

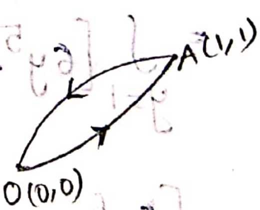
$(x^2)^2 = x \Rightarrow x^4 = x \Rightarrow x^3 = 1 . \Rightarrow x = 1$

Similarly $y = 1$.



L.H.S: $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy \longrightarrow (2)$

Consider two line segments i.e OA & AO.



Along OA: $y = x^2$, $dy = 2x\,dx$.

$x = (0, 1)$.

From eqⁿ(2)

$$\int_{x=0}^{1} \left[3x^2 - 8(x^2)^2\right]dx + \left[4x^2 - 6x(x^2)\right]2x\,dx$$

$$= \int_{x=0}^{1} \left[3x^2 - 8x^4\right]dx + \left[8x^3 - 12x^4\right]dx .$$

$$= \int_{x=0}^{1} [3x^2 - 8x^4 + 8x^3 - 12x^4]\,dx$$

$$= \int_{x=0}^{1} [3x^2 + 8x^3 - 20x^4]\,dx$$

$$= \left[\frac{\cancel{3}x^3}{\cancel{3}} + \frac{\overset{2}{\cancel{8}}x^4}{\cancel{4}} - \frac{\overset{4}{\cancel{20}}x^5}{\cancel{5}}\right]_0^1 \Rightarrow \left[x^3 + 2x^4 - 4x^5\right]_0^1$$

$$= \left[\{(1)^3 + 2(1)^4 - 4(1)^5\} - 0\right]$$

$$= [1+2-4] = 3-4 = -1 \longrightarrow (a)$$

Along AO:- $y^2 = x \Rightarrow 2y\,dy = dx$.

$y = 1$ to 0.

From eq^n(2)

$$\int_{y=1}^{0} [3(y^2)^2 - 8y^2]\,2y\,dy + [4y - 6(y^2)y]\,dy$$

$$= \int_{y=1}^{0} [6y^5 - 16y^3]\,dy + [4y - 6y^3]\,dy$$

$$= \int_{y=1}^{0} [6y^5 - 16y^3 + 4y - 6y^3]\,dy$$

$$= \int_{y=1}^{0} [6y^5 - 22y^3 + 4y]\,dy$$

$$= \left[\frac{\cancel{6}y^6}{\cancel{6}} - \frac{\overset{11}{\cancel{22}}y^4}{\cancel{4}_2} + \frac{\overset{2}{\cancel{4}}y^2}{\cancel{2}}\right]_1^0 \Rightarrow \left[y^6 - \frac{11y^4}{2} + 2y^2\right]_1^0$$

$$= \left[0 - \left\{(1)^6 - \frac{11(1)^4}{2} + 2(1)^2\right\}\right] = \left[-1 + \frac{11}{2} - 2\right].$$

$= \left[\frac{2+11\cancel{+}4}{2}\right] = -\frac{6+11}{2} = +\frac{5}{2} \longrightarrow (b)$

Adding eq^n (a) & (b)

$$-1+\frac{5}{2} = \frac{-2+5}{2} = \frac{3}{2}.$$

R.H.S:- $\iint_R \left[\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right] dx\,dy.$

Let $M = 3x^2 - 8y^2$, $N = 4y - 6xy$ $x = 0$ to 1

$y = x^2$ to $\sqrt{x}$

$\frac{\partial M}{\partial y} = -16y$ $\frac{\partial N}{\partial x} = -6y$

$$\iint_R \left[\frac{\partial N}{\partial y} - \frac{\partial M}{\partial x}\right] dx\,dy = \int_{x=0}^{1}\int_{y=x^2}^{\sqrt{x}} \left[-6y + 16y\right] dx\,dy$$

$$= \int_{x=0}^{1}\left[\int_{y=x^2}^{\sqrt{x}} 10y\,dy\right] dx$$

$$= \int_{x=0}^{1} \cancel{10}^{5}\left[\frac{y^2}{\cancel{2}}\right]_{x^2}^{\sqrt{x}} dx$$

$$= \int_{x=0}^{1} 5\left[(\sqrt{x})^2 - (x^2)^2\right] dx$$

$$= \int_{x=0}^{1} 5\left[x - x^4\right] dx \Rightarrow \int_{x=0}^{1} (5x - 5x^4)\,dx.$$

$$= \left[\frac{5x^2}{2} - \frac{\cancel{5}x^5}{\cancel{5}}\right]_0^1 \Rightarrow \left[\left\{\frac{5}{2}(1)^2 - (1)^5\right\} - 0\right]$$

$$= \left[\frac{5}{2} - 1\right] \Rightarrow \left[\frac{5-2}{2}\right] = \frac{3}{2}.$$

$\therefore$ L.H.S = R.H.S

Hence Green's theorem is Verified.

2015 Stokes Theorem (3m)

[Transformation b/w Line Integral & Surface Integral].

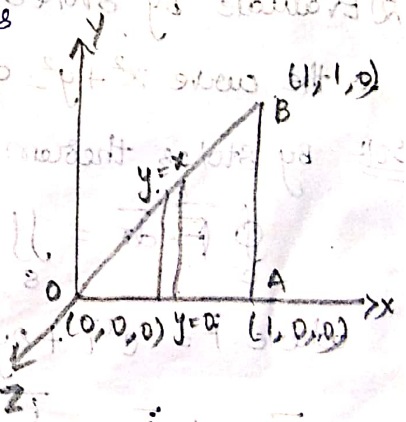
Let 's' be a open surface bounded by a closed non intersecting curve 'c'. If $\bar{F}$ is any differentiable vector pt function Hence $\oint_C \bar{F}\cdot\bar{dr} = \int_S \text{curl } \bar{F}\cdot\bar{n}\, ds$ where 'c' is transversed in the +ve direction & $\bar{n}$ is unit outward drawn normal at any pt of the surface.

Cartesian form:- Let $\bar{F} = F_1\bar{i} + F_2\bar{j} + F_3\bar{k}$.

Let the unit normal vector $\bar{n}$ drawn outward make angles α, β, γ with the +ve direction of x, y, z axis. Then Stokes theorem can also be written as

$$\oint_C F_1 dx + F_2 dy + F_3 dz = \iint_S \left[\left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\cos\alpha + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\cos\beta + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{y}\right)\cos\gamma\right] ds.$$

1) Evaluate by stokes theorem $\oint_C (x+y)dx + (2x-z)dy + (y+z)dz$ where c is the boundary of the Δle with vertices (i) (0,0,0), (1,0,0), & (1,1,0).



Sol: By stokes theorem

$$\oint_C \bar{F}\cdot\bar{dr} = \iint_S \text{curl } \bar{F}\cdot\bar{n}\, ds \longrightarrow (1)$$

Then $\bar{F} = (x+y)\bar{i} + (2x-z)\bar{j} + (y+z)\bar{k}$

where 's' is the surface of the Δle OAB which lies in the xy plane. Since 'z' coordinates in O(0,0,0) A(1,0,0) B(1,1,0) is '0'

$\therefore \bar{n} = \bar{k}$, $ds = dx\,dy$.

$$\therefore \text{curl}\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x+y) & (2x-z) & (y+z) \end{vmatrix}.$$

$$= \bar{i}\left[\frac{\partial}{\partial y}(y+z) - \frac{\partial}{\partial z}(2x-z)\right] - \bar{j}\left[\frac{\partial}{\partial x}(y+z) - \frac{\partial}{\partial z}(x+y)\right] +$$

$$\bar{k}\left[\frac{\partial}{\partial x}(2x-z) - \frac{\partial}{\partial y}(x+y)\right]$$

$$= \bar{i}[1+1] - \bar{j}[0] + \bar{k}[2-1]$$

$$\text{curl}\,\bar{F} = 2\bar{i} + \bar{k}$$

$$\therefore \iint_S \text{curl}\,\bar{F}\cdot\bar{n}\,ds = \int_{x=0}^{1}\int_{y=0}^{x} dy\,dx.$$

$$= \int_{x=0}^{1} [y]_0^x\,dx$$

$$= \int_{x=0}^{1} x\,dx \Rightarrow \left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}.$$

21/5/20

2) Evaluate by stoke's theorem $\oint_C (e^x dx + 2y\,dy - dz)$ where 'c' is the curve $x^2+y^2=9$ & $z=2$:

Sol:- By stokes theorem

$$\oint_C \bar{F}\cdot\bar{dr} = \iint_S (\nabla\times\bar{F})\cdot\bar{n}\,ds \longrightarrow (1)$$

where $\bar{F} = e^x\bar{i} + 2y\bar{j} - \bar{k}$

$$\nabla\times\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^x & 2y & -1 \end{vmatrix} = \bar{i}\left[\frac{\partial}{\partial y}(-1) - \frac{\partial}{\partial z}(2y)\right] - \bar{j}\left[\frac{\partial}{\partial x}(-1) - \frac{\partial}{\partial z}(e^x)\right] + \bar{k}\left[\frac{\partial}{\partial x}(2y) - \frac{\partial}{\partial y}(e^x)\right].$$

$$= \bar{i}[0-0] - \bar{j}[0-0] + \bar{k}[0-0]$$

$$\nabla\times\bar{F} = 0$$

$$\therefore \iint_S (\nabla\times\bar{F})\cdot\bar{n}\,ds = 0.$$

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