Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$$p.r_2 = C \quad \text{or} \quad C = 0.1\ r_2 \text{ N/mm}$$

and the axial thrust transmitted to the frictional surface,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 2\,\pi \times 0.1\ r_2\,(1.25\ r_2 - r_2) = 0.157\,(r_2)^2$$

We know that mean radius of the frictional surface for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\ r_2 + r_2}{2} = 1.125\ r_2$$

We know that torque transmitted (T),

$$79.6 \times 10^3 = n.\mu.W.R = 2 \times 0.255 \times 0.157\,(r_2)^2 \times 1.125\ r_2 = 0.09\,(r_2)^3$$

$$\therefore \quad (r_2)^3 = 79.6 \times 10^3/0.09 = 884 \times 10^3 \quad \text{or} \quad r_2 = 96 \text{ mm } \textbf{Ans.}$$

and $\quad r_1 = 1.25\ r_2 = 1.25 \times 96 = 120$ mm **Ans.**

Axial thrust to be provided by springs

We know that axial thrust to be provided by springs,

$$W = 2\,\pi\, C\,(r_1 - r_2) = 0.157\,(r_2)^2 \quad \text{...[From}$$

$$= 0.157\,(96)^2 = 1447 \text{ N } \textbf{Ans.}$$

Example 10.25. *A single dry plate clutch transmits 7.5 kW at 900 r.p.m. The a… is limited to 0.07 N/mm². If the coefficient of friction is 0.25, find* **1.** *Mean radius and … the friction lining assuming the ratio of the mean radius to the face width as 4, and … inner radii of the clutch plate.*

Solution. Given : $P = 7.5$ kW $= 7.5 \times 10^3$ W ; $N = 900$ r.p.m or $\omega = 2\,\pi \times 900/60$; $p = 0.07$ N/mm² ; $\mu = 0.25$

1. *Mean radius and face width of the friction lining*

Let $\quad R$ = Mean radius of the friction lining in mm, and

$\quad w$ = Face width of the friction lining in mm,

Ratio of mean radius to the face width,

$$R/w = 4 \quad = w = \frac{R}{4}$$

We know that the area of friction faces,

$$A = 2\,\pi\, R.w$$

$\therefore$ Normal or the axial force acting on the friction faces,

$$W = A \times p = 2\,\pi\, R.w.p$$

We know that torque transmitted (considering uniform wear),

$$T = n.\mu.W.R = n.\mu\,(2\pi R.w.p)\,R$$

$$= n.\mu\left(2\pi R \times \frac{R}{4} \times p\right)R = \frac{\pi}{2} \times n.\mu.p.R^3$$

$$T = \frac{\pi}{2} \times 2 \times 0.25 \times 0.07\ R^3 = 0.055\ R^3 \text{ N-mm}$$

...($\because n = 2$, for single

We also know that power transmitted (P),

$$7.5 \times 10^3 = T.\omega = T \times 94.26$$

$$T = 7.5 \times 10^3/94.26 = 79.56 \text{ N-m} = 79.56 \times 10^3 \text{ N-mm} \quad ...(ii)$$

∴ From equations (i) and (ii),

$$R^3 = 79.56 \times 10^3/0.055 = 1446.5 \times 10^3 \text{ or } R = 113 \text{ mm } \textbf{Ans.}$$

$$w = R/4 = 113/4 = 28.25 \text{mm } \textbf{Ans.}$$

r and inner radii of the clutch plate

Let r_1 and r_2 = Outer and inner radii of the clutch plate respectively.

Since the width of the clutch plate is equal to the difference of the outer and inner radii,

$$w = r_1 - r_2 = 28.25 \text{ mm} \quad ...(iii)$$

Also for uniform wear, the mean radius of the clutch plate,

$$R = \frac{r_1 + r_2}{2} \text{ or } r_1 + r_2 = 2R = 2 \times 113 = 226 \text{ mm} \quad ...(iv)$$

From equations (iii) and (iv),

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$$r_1 = 127.125 \text{ mm ; and } r_2 = 98.875. \textbf{ Ans.}$$

Example 10.26. *A dry single plate clutch is to be designed for an automotive vehicle whose is rated to give 100 kW at 2400 r.p.m. and maximum torque 500 N-m. The outer radius of plate is 25% more than the inner radius. The intensity of pressure between the plate is not to 0.07 N/mm². The coefficient of friction may be assumed equal to 0.3. The helical springs d by this clutch to provide axial force necessary to engage the clutch are eight. If each spring ffness equal to 40 N /mm, determine the initial compression in the springs and dimensions of ction plate.*

Solution. Given : P = 100 kW; N = 2400 r.p.m. ; T = 500 N-m = 500 × 10³ N-mm ; 0.07 N/mm² ; μ = 0.3 ; Number of springs = 8 ; Stiffness = 40 N/mm

nsions of the friction plate

Let r_1 and r_2 = Outer and inner radii of the friction plate respectively.

Since the outer radius of the friction plate is 25% more than the inner radius, therefore

$$r_1 = 1.25\, r_2$$

We know that, for uniform wear,

$$p.r_2 = C \text{ or } C = 0.07\, r_2 \text{ N/mm}$$

load transmitted to the friction plate,

$$W = 2\pi C (r_1 - r_2) = 2\pi \times 0.07\, r_2 (1.125\, r_2 - r_2) = 0.11\, (r_2)^2 \text{ N} \quad ...(i)$$

We know that mean radius of the plate for uniform wear,

$$R = \frac{r_1 + r_2}{2} = \frac{1.25\, r_2 + r_2}{2} = 1.125\, r_2$$

∴ Torque transmitted (T),

$$500 \times 10^3 = n.\mu.W.R = 2 \times 0.3 \times 0.11\, (r_2)^2 \times 1.125\, r_2 = 0.074\, (r_2)^3 \quad ...(\because n = 2)$$

∴

$$(r_2)^3 = 500 \times 10^3/0.074 = 6757 \times 10^3 \text{ or } r_2 = 190 \text{ mm } \textbf{Ans.}$$

and $r_1 = 1.25\, r_2 = 1.25 \times 190 = 273.5$ mm **Ans.**

Initial compression of the springs

We know that total stiffness of the springs,

s = Stiffness per spring × No. of springs = $40 \times 8 = 320$ N/mm

Axial force required to engage the clutch,

$W = 0.11\, (r_2)^2 = 0.11\, (190)^2 = 3970$ N ...[From equa

∴ Initial compression in the springs

$= W/s = 3970/320 = 12.5$ mm **Ans.**

Example 10.31. *A conical friction clutch is used to transmit 90 kW at 1500 r.p.m cone angle is 20° and the coefficient of friction is 0.2. If the mean diameter of the bearin 375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm², find the dimen conical bearing surface and the axial load required.*

Solution. Given : $P = 90$ kW $= 90 \times 10^3$ W ; $N = 1500$ r.p.m. or $\omega = 2\,\pi \times 15$ rad/s ; $\alpha = 20°$; $\mu = 0.2$; $D = 375$ mm or $R = 187.5$ mm ; $p_n = 0.25$ N/mm²

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respect

b = Width of the bearing surface in mm, and

T = Torque transmitted.

We know that power transmitted (P),

$$90 \times 10^3 = T.\omega = T \times 156$$

$$\therefore \quad T = 90 \times 10^3/156 = 577 \text{ N-m} = 577 \times 10^3 \text{ N-mm}$$

nd the torque transmitted (T),

$$577 \times 10^3 = 2\,\pi\,\mu\,p_n.R^2.b = 2\pi \times 0.2 \times 0.25\,(187.5)^2\,b = 11\,046\,b$$

$$\therefore \quad b = 577 \times 10^3/11\,046 = 52.2 \text{ mm} \quad \textbf{Ans.}$$

We know that $r_1 + r_2 = 2R = 2 \times 187.5 = 375$ mm ...

nd $\quad r_1 - r_2 = b \sin \alpha = 52.2 \sin 20° = 18$ mm ...

From equations (*i*) and (*ii*),

$$r_1 = 196.5 \text{ mm, and } r_2 = 178.5 \text{ mm} \quad \textbf{Ans.}$$

xial load required

Since in case of friction clutch, uniform wear is considered and the intensity of pressu aximum at the minimum contact surface radius (r_2), therefore

$$p_n.r_2 = C \text{ (a constant) or } C = 0.25 \times 178.5 = 44.6 \text{ N/mm}$$

We know that the axial load required,

$$W = 2\pi C\,(r_1 - r_2) = 2\pi \times 44.6\,(196.5 - 178.5) = 5045 \text{ N} \quad \textbf{Ans.}$$

... fitted with a cone clutch

Range of speed

We know that range of speed

$$= N_2 - N_1 = 154.4 - 133.8 = 20.7 \text{ r.p.m. } \textbf{Ans.}$$

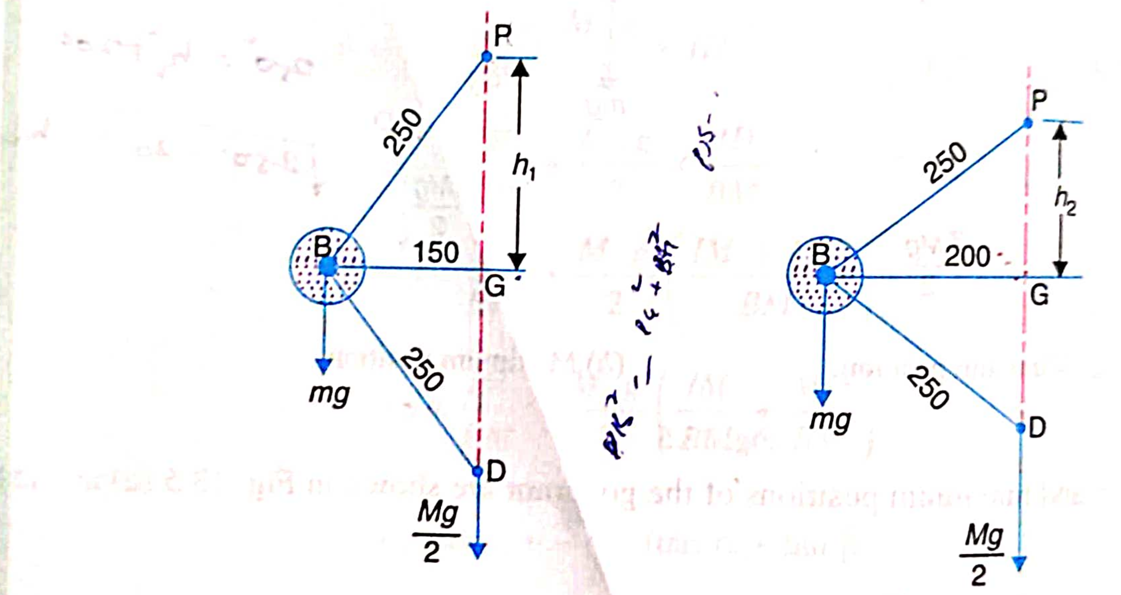
Example 18.3. *The arms of a Porter governor are each 250 mm long and piv... governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. T... of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 2... maximum speed. Determine the speed range of the governor. If the friction at the sleeve is... of 20 N of load at the sleeve, determine how the speed range is modified.*

Solution. Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 30$ kg ; $r_1 = 150$ mm ; $r_2 =$

First of all, let us find the minimum and maximum speed of the governor. The min... maximum position of the governor is shown in Fig. 18.6 (*a*) and (*b*) respectively.

Let N_1 = Minimum speed when $r_1 = BG = 150$ mm, and

N_2 = Maximum speed when $r_2 = BG = 200$ mm.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.6

Speed range of the governor

From Fig. 18.6 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm}$$

We know that

$$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{5 + 30}{5} \times \frac{895}{0.2} = 31\,325$$

$\therefore$ $N_1 = 177$ r.p.m.

From Fig. 18.6 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm}$$

We know that

$$(N_2)^2 = \frac{m + M}{m} \times \frac{895}{h_2} = \frac{5 + 30}{5} \times \frac{895}{0.15} = 41\,767$$

$\therefore$ $N_2 = 204.4$ r.p.m.

We know that speed range of the governor

$$= N_2 - N_1 = 204.4 - 177 = 27.4$$

Speed range when friction at the sleeve is equivalent of 20 N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29\,500$$

$$\therefore \quad N_1 = 172 \text{ r.p.m.}$$

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44\,200$$

$$\therefore \quad N_2 = 210 \text{ r.p.m.}$$

We know that speed range of the governor

$$= N_2 - N_1 = 210 - 172 = 38 \text{ r.p.m}$$

Example 18.4. *In an engine governor of the Porter type, t... and 250 mm respectively and pivoted on the axis of rotation. Th... the mass of each ball is 2 kg and friction of the sleeve together... gear is equal to a load of 24 N at the sleeve. If the limiting in... vertical are 30° and 40°, find, taking friction into account, rang...*

Solution. Given : $BP = 200$ mm $= 0.2$ m ; $BD = 250$ mm ... $F = 24$ N ; $\alpha_1 = 30°$; $\alpha_2 = 40°$

First ... minimum and maximum speed ... of the gover...

(h being in metres) ...(*iv*)

xample 18.9. *A Proell governor has equal arms of length 300 mm. The upper and lower e arms are pivoted on the axis of the governor. The extension arms of the lower links are m long and parallel to the axis when the radii of rotation of the balls are 150 mm and he mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the peed of the governor.*

olution. Given : $PF = DF = 300$ mm ; $BF = 80$ mm ; $r_1 = 150$ mm; $r_2 = 200$ mm ; g ; $M = 100$ kg ;

rst of all, let us find the minimum and maximum speed of the governor. The minimum and position of the governor is shown in Fig. 18.13.

et

N_1 = Minimum speed when radius of rotation, $r_1 = FG = 150$ mm ; and

N_2 = Maximum speed when radius of rotation , $r_2 = FG = 200$ mm.

om Fig. 18.13 (*a*), we find that height of the governor,

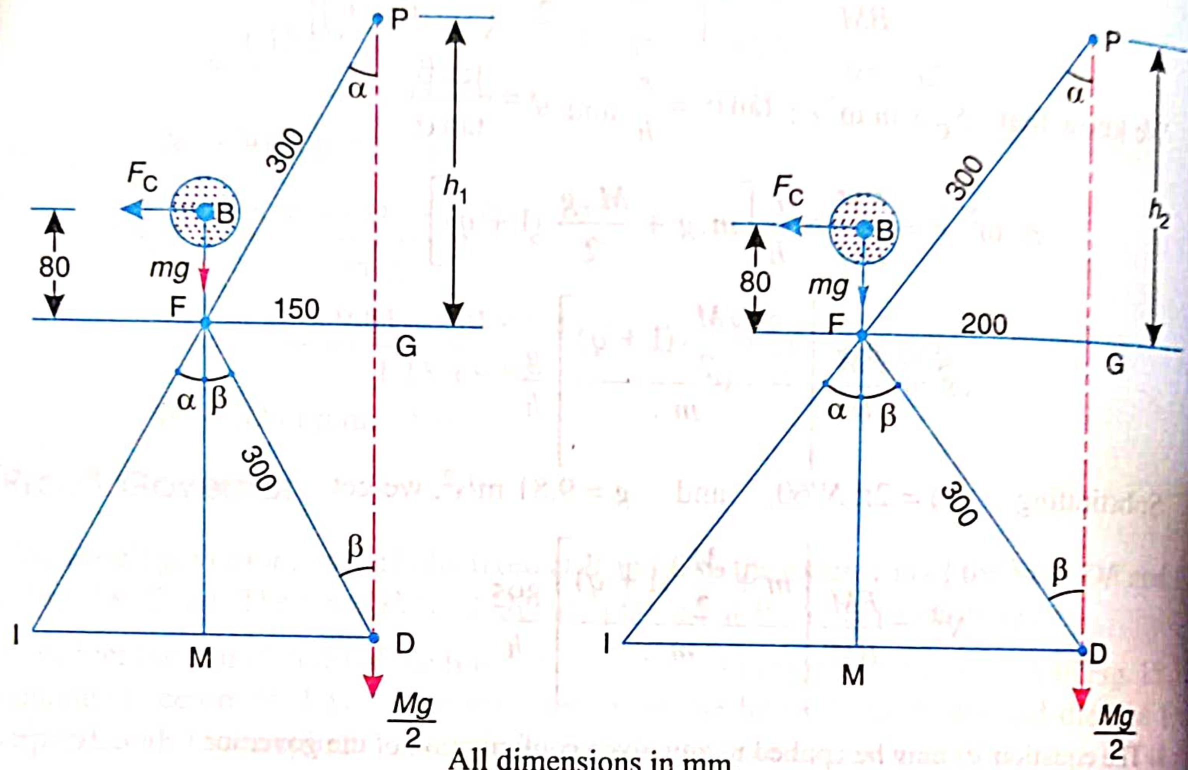
$$h_1 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm} = 0.26 \text{ m}$$

and $FM = GD = PG = 260 \text{ mm} = 0.26 \text{ m}$

$\therefore$ $BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$

We know that $$(N_1)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_1} \qquad \ldots(\because \alpha = \beta \text{ or } q$$

$$= \frac{0.26}{0.34}\left(\frac{10+100}{10}\right)\frac{895}{0.26} = 28\,956 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$



All dimensions in mm.

(*a*) Minimum position. (*a*) Maximum position.

Fig. 18.13

Now from Fig. 18.13 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm} = 0.224$$

and $FM = GD = PG = 224 \text{ mm} = 0.224 \text{ m}$

$\therefore$ $BM = BF + FM = 80 + 224 = 304 \text{ mm} = 0.304 \text{ m}$

We know that $$(N_2)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_2} \qquad \ldots(\because \alpha = \beta \text{ or}$$

$$= \frac{0.224}{0.304}\left(\frac{10+100}{10}\right)\frac{895}{0.224} = 32\,385 \quad \text{or} \quad N_2 = 180 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 180 - 170 = 10 \text{ r.p.m.} \textbf{ Ans.}$$

Note : The example may also be solved as discussed below :

From Fig. 18.13 (*a*), we find that

$\sin\alpha = \sin\beta = 150 / 300 = 0.5$ or $\alpha = \beta = 30°$

and $MD = FG = 150 \text{ mm} = 0.15 \text{ m}$

$FM = FD\cos\beta = 300\cos 30° = 260 \text{ mm} = 0.26 \text{ m}$

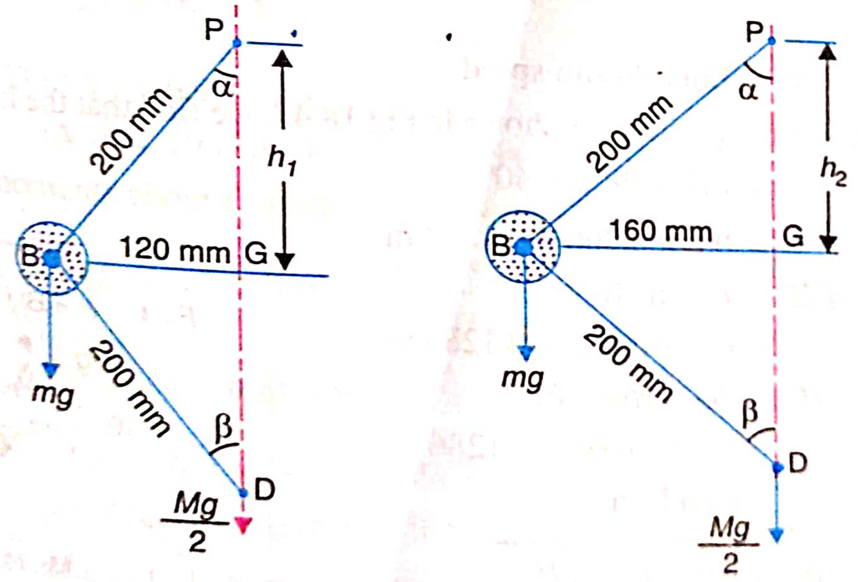
$\therefore$ Coefficient of insensitiveness $= \dfrac{N'' - N'}{N} = \dfrac{F_B}{F_C} = \dfrac{F_S}{F_C} \times \dfrac{y}{2x}$ ...(*xi*)

Example 18.30. *A Porter governor has equal arms 200 mm long pivoted on the axis of ion. The mass of each ball is 3 kg and the mass on the sleeve is 15 kg. The ball path is 120 mm the governor begins to lift and 160 mm at the maximum speed. Determine the range of speed. If the friction at the sleeve is equivalent to a force of 10 N, find the coefficient of insensitiveness.*

Solution. Given : $BP = BD = 200$ mm $= 0.2$ m ; $m = 3$ kg ; $M = 15$ kg ; $r_1 = 120$ mm $= 0.12$ m ; 60 mm $= 0.16$ m ; $F = 10$ N

e of speed

First of all, let us find the minimum and maximum speed of rotation.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.46

The minimum and maximum position of the balls is shown in Fig 18.46 (*a*) and (*b*) respec

Let

N_1 = Minimum speed, and

N_2 = Maximum speed.

From Fig. 18.46 (*a*), $h_1 = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(0.2)^2 - (0.12)^2} = 0.16$ m

and from Fig. 18.46 (*b*), $h_2 = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(0.2)^2 - (0.16)^2} = 0.12$ m

We know that $$(N_1)^2 = \frac{m + M}{m} \times \frac{895}{h_1} = \frac{3 + 15}{3} \times \frac{895}{0.16} = 33\,563$$

$\therefore$ $N_1 = 183.2$ r.p.m.

Similarly $$(N_2)^2 = \frac{m + M}{m} \times \frac{895}{h_2} = \frac{3 + 15}{3} \times \frac{895}{0.12} = 44750$$

$\therefore$ $N_2 = 211.5$ r.p.m.

We know that range of speed

$$= N_2 - N_1 = 211.5 - 183.2 = 28.3 \text{ r.p.m.}$$ **Ans.**

Coefficient of insensitiveness

We know that coefficient of insensitiveness,

$$\frac{N'' - N'}{N} = \frac{F}{(m + M)\,g} = \frac{10}{(3 + 15)\,9.81} = 0.0566 = 5.66\%$$ **Ans.**

Example 18.31. *The following particulars refer to a Proell governor with open arms :*

Fig. 21.10

Example 21.5. *A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine :*

1. The magnitude of the masses at A and D ; 2. the distance between planes A and D ; and 3. the angular position of the mass at D.

Solution. Given : $m_B = 18$ kg ; $m_C = 12.5$ kg ; $r_B = r_C = 60$ mm $= 0.06$ m ; $r_A = r_D = 80$ mm $= 0.08$ m ; $\angle BOC = 100°$; $\angle BOA = 190°$

1. ***Magnitude of the masses at A and D***

Let M_A = Mass at A,

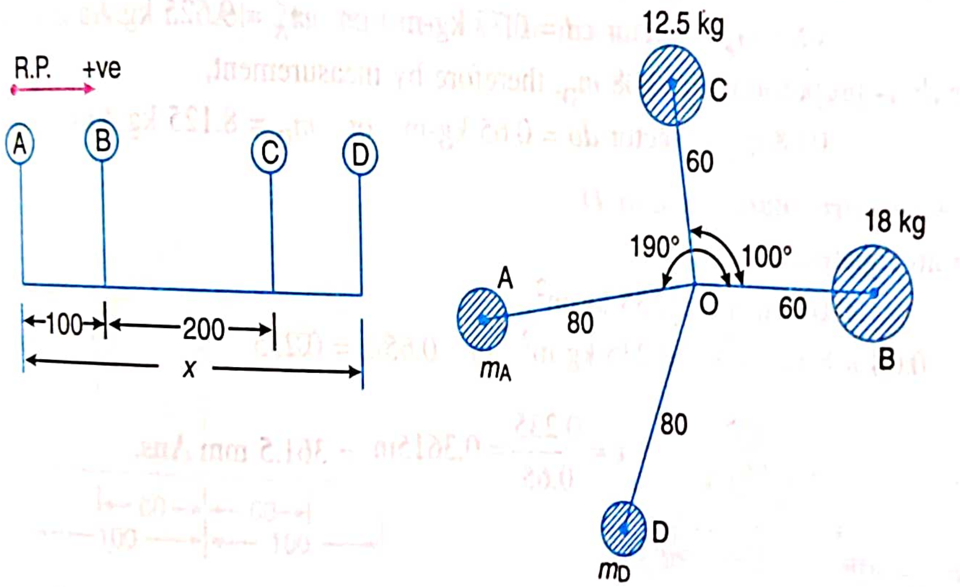
M_D = Mass at D, and

x = Distance between planes A and D.

The position of the planes and angular position of the masses is shown in Fig. 21.11 (*a*) ... respectively. The position of mass *B* is assumed in the horizontal direction, *i.e.* along *OB*. ... the plane of mass *A* as the reference plane, the data may be tabulated as below :

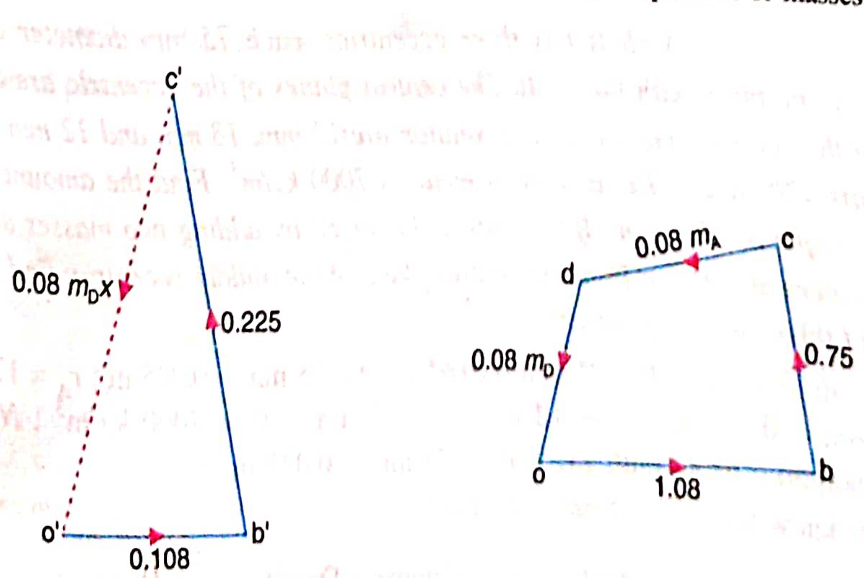
Table 21.5

Plane (1)	Mass (m) kg (2)	Eccentricity (r) m (3)	Cent. force ÷ ω^2 (m.r) kg-m (4)	Distance from plane A(l)m (5)	Couple ÷ ω^2 (m.r.l) kg-m^2 (6)
(R.P.)	m_A	0.08	0.08 m_A	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	0.08 m_D	x	0.08 m_D . x



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.



(c) Couple polygon. (*d*) Force polygon.

Fig. 21.11

side of the couple polygon (vector $c'\,o'$) is proportional to 0.08 $m_D.x$. By measurement, we find t

$$0.08\ m_D.x = \text{vector } c'\,o' = 0.235 \text{ kg-m}^2$$

In Fig. 21.11 (*b*), draw *OD* parallel to vector $c'\,o'$ to fix the direction of mass *D*.

Now draw the force polygon, to some suitable scale, as shown in Fig. 21.11 (*d*), from data given in Table 21.5 (column 4), as discussed below :

1. Draw vector *ob* parallel to *OB* and equal to 1.08 kg-m.
2. From point *b*, draw vector *bc* parallel to *OC* and equal to 0.75 kg-m.
3. For the shaft to be in complete dynamic balance, the force polygon must be a cl figure. Therefore from point *c*, draw vector *cd* parallel to *OA* and from point *o* c vector *od* parallel to *OD*. The vectors *cd* and *od* intersect at *d*. Since the vector c proportional to 0.08 m_A, therefore by measurement

$$0.08\ m_A = \text{vector } cd = 0.77 \text{ kg-m} \quad \text{or} \quad m_A = 9.625 \text{ kg} \textbf{ Ans.}$$

and vector *do* is proportional to 0.08 m_D, therefore by measurement,

$$0.08\ m_D = \text{vector } do = 0.65 \text{ kg-m} \quad \text{or} \quad m_D = 8.125 \text{ kg} \textbf{ Ans.}$$

2. Distance between planes A and D

From equation (*i*),

$$0.08\ m_D.x = 0.235 \text{ kg-m}^2$$

$$0.08 \times 8.125 \times x = 0.235 \text{ kg-m}^2 \quad \text{or} \quad 0.65\ x = 0.235$$

$$\therefore \qquad x = \frac{0.235}{0.65} = 0.3615\text{m} = 361.5 \text{ mm } \textbf{Ans.}$$

3. Angular position of mass at D

By measurement from Fig. 21.11 (*b*), we find that the angular position of mass at D mass *B* in the anticlockwise direction, *i.e.* $\angle BOD = 251°$ **Ans.**

Example 23.2. *A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibration. Assume $E = 200\ GN/m^2$ and shaft diameter = 50 mm.*

Solution. Given : $l = 0.75$ m ; $m = 90$ kg ; $a = AC = 0.25$ m ; $E = 200\ GN/m^2 = 200 \times 10^9$ N/m^2; $d = 50$ mm $= 0.05$ m

The shaft is shown in Fig. 23.7.

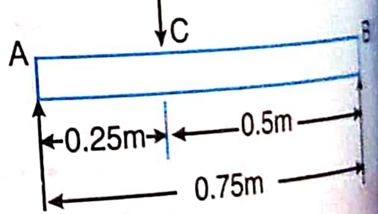


Fig. 23.7

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4\ m^4$$

$$= 0.307 \times 10^{-6}\ m^4$$

and static deflection at the load point (*i.e.* at point *C*),

$$\delta = \frac{Wa^2b^2}{3EIl} = \frac{90 \times 9.81 (0.25)^2 (0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 0.75} = 0.1 \times 10^{-3}\ m$$

$\ldots (\because b = BC = 0.5$

We know that natural frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.1 \times 10^{-3}}} = 49.85 \text{ Hz} \quad \textbf{Ans.}$$

Example 23.4. *A shaft 50 mm diameter and 3 metres long is simply supported at the ends*
rries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from the left support.
ung's modulus for shaft material is 200 GN/m². Find the frequency of transverse vibration.

Solution. Given : d = 50 mm = 0.05 m ; l = 3 m, W_1 = 1000 N ; W_2 = 1500 N ;
750 N; E = 200 GN/m² = 200 × 10⁹ N/m²

The shaft carrying the loads is shown in Fig. 23.13

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\,\text{m}^4$$

e static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3EIl}$$

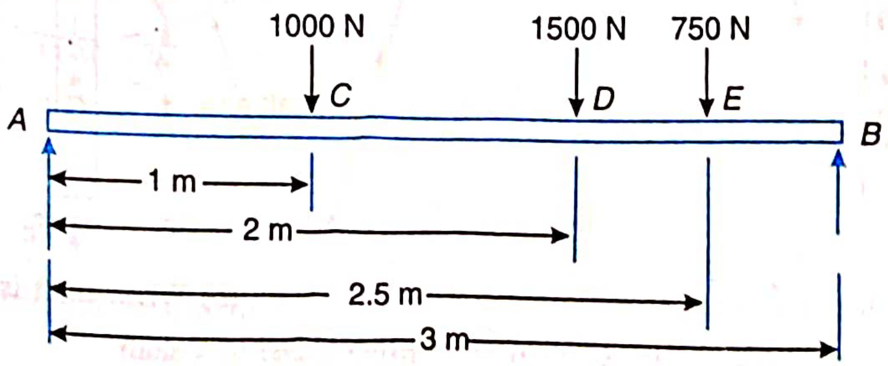


Fig. 23.13

∴ Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3}\ \text{m}$$

...(Here

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3} \text{ m}$$

. . . (Here a = 2 m, and b = 1 m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750\,(2.5)^2\,(0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 2.12 \times 10^{-3} \text{ m}$$

. . . (Here a = 2.5 m, and b = 0.5 m)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz}$$ **Ans.**

Example 23.5. *Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m^3, and Young's modulus is 200 GN/m^2. Assume the shaft to be freely supported.*

to convert the stored energy in the fuel into mechanical energy, or work.

Note : This picture is given as additional information

Solution. Given : $d = 20$ mm $= 0.02$ m ; $l = 0.6$ m ; $m_1 = 1$ kg ; $\rho = 40$ Mg/m^3 $= 40 \times 10^6$ g/m^3 $= 40 \times 10^3$ kg/m^3 ; $E = 200$ GN/m^2 $= 200 \times 10^9$ N/m^2

The shaft is shown in Fig. 23.15.

We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.02)^4 \text{ m}^4$$

$$= 7.855 \times 10^{-9} \text{ m}^4$$

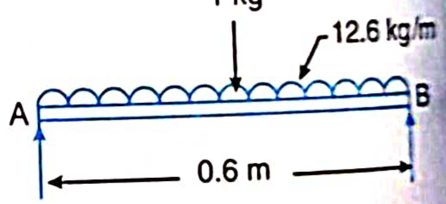


Fig. 23.15

Since the density of shaft material is 40×10^3 kg/m^3, therefore mass of the shaft per metre length,

$$m_S = \text{Area} \times \text{length} \times \text{density} = \frac{\pi}{4}(0.02)^2 \times 1 \times 40 \times 10^3 = 12.6 \text{ kg/m}$$

We know that static deflection due to 1 kg of mass at the centre,

$$\delta = \frac{Wl^3}{48\,EI} = \frac{1 \times 9.81(0.6)^3}{48 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 28 \times 10^{-6} \text{ m}$$

and static deflection due to mass of the shaft,

$$\delta_S = \frac{5\,wl^4}{384\,EI} = \frac{5 \times 12.6 \times 9.81(0.6)^4}{384 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 0.133 \times 10^{-3} \text{ m}$$

∴ Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta + \frac{\delta_S}{1.27}}} = \frac{0.4985}{\sqrt{28 \times 10^{-6} + \frac{0.133 \times 10^{-3}}{1.27}}}$$

$$= \frac{0.4985}{11.52 \times 10^{-3}} = 43.3 \text{ Hz}$$

Let N_c = Whirling speed of a shaft.

We know that whirling speed of a shaft in r.p.s. is equal to the frequency of transverse vibration in Hz , therefore

$$N_c = 43.3 \text{ r.p.s.} = 43.3 \times 60 = 2598 \text{ r.p.m.}$$ **Ans.**