Chapter - 3

Sampling distribution and Testing of a Hypothesis

population:— population is the total of statistical data forming a subject of investigation the no. of units in the population is called population size. It is denoted by 'N' population may be two types finite population and infinite population.

If the no. of unit in the population is finite then is called finite population.

Eg:— The population of heights of the indians
the " " works in a factory.

If the population size 'N' is infinite then it is called infinite population.

SAMPLE :- A part of the population which is examine with a view to determine the population characteristics is called a sample. The no. of units in the sample is called sample size. It's denoted by 'n'

SAMPLING TECHNICS (or) SAMPLING METHODS

The technic of drawing samples from the population is called sampling methods (or) Technics. There are some important methods of sampling.

(1) Simple random sampling → It is the process of drawing a sample from a population in such a way each members of the population has an equal chance of being included in the sample. If each element of the population may be selected more than once then it is called sampling with replacement. Where as if the element cannot be selected more than once then it is called sampling without replacement. The no. of samples with replacement is N^n. The no. of samples without replacements is N_{C_n}.

(2) Stratified random sampling :- In this type of sampling the population is first subdivided into several groups these groups are called stratums. Then a sample is selected from each stratum at random. This type of process is called stratified random sampling.

(3) Classification of samples :- Samples are classified in two ways 1. Large samples

2. Small samples.

If the sample size 'n' is greater than = 30 ($n \geq 30$) then the sample is called large sample.

If the sample size ($n < 30$) then the sample is called ~~same~~ small sample

parameter :- parameter is a statistical method based on all units of population mean (μ), ~~population~~ standard deviation (σ), population variance (σ^2). population propotion (P)

Statistic :- Statistic is a statistical measure based on all elements of sample. Sample mean ($\bar{x}$), Sample standard deviation (s), Sample variance (s^2), Sample propotion (p).

Sampling distribution of statistic mean & variances :-

Sampling distribution of statistic is frequence distribution which is formed with varies values of statistic, calculated from different samples of the same size, drawn from the same population. Let us consider a finite population of size 'N' & drawn all posible random samples of size 'n'. Then will get $N_{C_n} = k$ then the sampling distribution of a statistic mean & variance becomes

Samples	Sample circuits	Sample mean	Sample variance
1.	$x_1, x_2 \dots x_n$	$\bar{x}_1 = \frac{\Sigma x}{n}$	$S_1^2 = \frac{\Sigma (x_i - \bar{x}_1)^2}{n}$
2.	$x_1, x_2 \dots x_n$	$\bar{x}_2 = \frac{\Sigma x}{n}$	S_2^2
3.	$x_1, x_2 \dots x_n$	$\bar{x}_3$	
K	$x_1, x_2 \dots x_n$	$\bar{x}_k$	S_k^2

NOTE:- In Sampling distributions the population mean = ~~Sampling distribution of mean~~

the mean of the Sampling distribution of means

$\mu = \mu_{\bar{x}}$

$\mu = E(\bar{x})$

Suppose the samples ~~of~~ from an Infinite that is with replacement.

then $\mu = \mu_{\bar{x}}$

$\mu = E(\bar{x})$

Suppose the Samples are drawn from a finite population that is without replacement then $\mu = E(\bar{x})$

$$V(\bar{x}) = \left(\frac{N-n}{N-1}\right) \cdot \frac{\sigma^2}{n}$$

Here, $\left(\frac{N-n}{N-1}\right)$ is called finite population correction factor.

Problem :-

→ The population 2, 3, 6, 8, 11 consider all possibles samples of size 2 which can be drawn with replacement from this population find

(a) population mean

(b) population standard deviation.

(c) ~~the~~ mean of the Sampling distribution of means

(d) Standard deviation of the Sampling distribution of mean.

Sol:-

2, 3, 6, 8, 11

$N = 5$

$n = 2$

No. of samples in case of with replacement

$= N^n = 5^2 = 25$

(i) population mean $(\mu) = \frac{2+3+6+8+11}{5} = 6$

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(ii) pop'n S.D $(\sigma) = \sqrt{\dfrac{\Sigma(x_i-\mu)^2}{n}}$

$$= \sqrt{\frac{(2-6)^2+(3-6)^2+(6-6)^2+(8-6)^2+(11-6)^2}{5}}$$

$$= \sqrt{\frac{16+9+0+4+25}{5}}$$

$$= 3.286$$

Samples	Sample units	Sample mean $(\bar{x})$	Sample variance
1	(2,2)	$2+2 = \frac{4}{2} = 2$	$(2-6)^2 = 16$
2	(2,3)	$2+3 = 5/2 = 2.5$	12.25
3	(2,6)	$2+6 = 8/2 = 4$	4
4	(2,8)	5	1
5	(2,11)	6.5	0.25
6	(3,2)	2.5	12.25
7	(3,3)	3	9
8	(3,6)	4.5	2.25
9	(3,8)	5.5	0.25
10	(3,11)	7	1
11	(6,2)	4	4
12	(6,3)	4.5	2.25
13	(6,6)	6	9
14	(6,8)	7	6.25
15	(6,11)	8.5	1
16	(8,2)	5	0.25
17	(8,3)	5.5	1
18	(8,6)	7	4
19	(8,8)	8	10.25
20	(8,11)	9.5	0.25
21	(11,2)	6.5	1
22	(11,3)	7	6.25
23	(11,6)	8.5	12.25
24	(11,8)	9.5	15
25	(11,11)	11	155

(iii) mean of sampling distribution means

$$\mu_{\bar{x}} = \frac{\Sigma\bar{x}}{15} = \frac{150}{25} = 6$$

$$\Rightarrow \mu_{\bar{x}} = \mu$$

(iv) S.D of sampling distribution means.

$$\sigma_{\bar{x}} = \sqrt{\frac{\Sigma(x-\mu)^2}{25}}$$

$$= \sqrt{\frac{135}{25}}$$

$$= 2.32$$

$$\sigma_{\bar{x}} \neq \sigma$$

A population consist of 5, 10, 14, 18, 13, 24 consider all possible samples of size 2 which can be drawn without replacement then find (i) pop' mean

(ii) pop'' std deviation.

(iii) the mean of the sampling distribution.

(iv) S.D of sampling distribution mean.

Sol:- $N = 6$ $n = 2$

In case of without replacement

Total samples $= {}^{N}C_{n}$

$= {}^{6}C_{2} = 15$

Population mean $= 14$

$\mu = 14$

(ii) population standard deviation

$$= \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

$$= \sqrt{\frac{(5-14)^2 + (10-14)^2 + (18-14)^2 + (13-14)^2 + (2\ldots}{6}}$$

$= 5.47$

Sample	Sample unit	Sample mean	$(\bar{x}-\mu)^2$
1	5,10	$15/2 = 7.5$	$(6.-5)^2 = 2.25$
2	5,14	9.5	$(-4.5)^2 = 2.25$
3	5,18	11.5	$(-2.5)^2 = 6.25$
4	5,13	9	$5^2 = 25$
5	5,24	14.5	$0.5^2 = 0.25$
6	10,14	12	$2^2 = 4$
7	10,18	14	$0 = 0$
8	10,13	11.5	$25^2 = 625$
9	10,24	17	$3^2 = 9$
10	14,18	16	$2^2 = 4$
11	14,13	$27 = 13.5$	$0.5^2 = 0.5$
12	14,24	$38 = 19$	$5^2 = 25$
13	18,13	$31 = 15.5$	$1.5^2 = 2.25$
14	18,24	$42 = 21$	$7^2 = 49$
15	13,24	$37 = 18.5$	$4.5^2 = 25$
		$\frac{210}{15}$	$\frac{214}{14}$

$\mu = 14$

$\sigma^2 = 15.28$

$\sigma = 3.9.$

→ If the population is 3, 6, 9, 15, 27. list all possible samples of size 3 that can be taken without replacement for the population & also calculate.

1. population mean
2. population S.D mean of the.
3. S.D of means
4. S.D of the S.D means.

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3, 6, 9, 15, 27

N = 5

n = 3

In case of without replacement

Total samples = $^{N}C_{n} = {}^{5}C_{3} = 10$

$$\text{pop}^{\text{ln}}\ \text{mean} = \frac{3+6+9+15+27}{5}$$

$= 12$

$$\text{pop}^{\text{ln}}\ S.D = \sqrt{\frac{(3-12)^2+(6-12)^2+(9-12)^2+(15-12)^2+(27-12)^2}{5}}$$

$= 8.485$

Sample	Sample limits	Sample mean	$(\bar{x}-\mu)^2$
1.	(3, 6, 9)	$\frac{3+6+9}{3} = 6$	$(6-12)^2 = 36$
2.	(3, 6, 15)	$\frac{3+6+15}{3} = 8$	$(8-12)^2 = 16$
3.	(3, 6, 27)	$\frac{3+6+27}{3} = 12$	$(12-12)^2 = 0$
4.	(3, 9, 15)	$\frac{3+9+15}{3} = 9$	$(9-12)^2 = 9$
5.	(3, 9, 27)	$\frac{3+9+27}{3} = 13$	$(13-12)^2 = 1$
6.	(3, 15, 27)	$\frac{3+15+27}{3} = 15$	$(15-12)^2 = 9$
7.	(6, 9, 15)	$\frac{6+9+15}{3} = 10$	$(10-12)^2 = 4$
8.	(6, 9, 27)	$\frac{6+9+27}{3} = 14$	$(14-12)^2 = 4$
9.	(6, 15, 27)	$\frac{6+15+27}{3} = 16$	$(16-12)^2 = 16$
10.	(9, 15, 27)	$\frac{9+15+27}{3}$ 17	$(17-12)^2 = 25$

$$\mu_{\bar{x}} = \frac{120}{10} = 12$$

$$\sigma_{\bar{x}} = \sqrt{\frac{120}{10}}$$

$= 3.46$

i.e, $\mu_{\bar{x}} = \mu$

$\sigma_{\bar{x}} \neq \sigma$

What is finite population correction factor n = 5 &

N = 200

$$\frac{N-n}{N-1} = \frac{200-5}{200-1} = \boxed{0.979}$$

Estimate:- An Estimate is a statement made to find an unknown pop'n parameter.

Estimator:- The procedure to determine an unknown pop'n parameter is called an Estimator.

Types of Estimation:- There are two types of Estimation to determine the statistic of the pop'n parameters namely the

point Estimation & Interval Estimation.

If an estimate of the population parameter is given by a single value then the Estimate is called point estimation of the parameter.

properties of estimator:-

A good Estimator satisfies the following properties

Efficiency.

→ Unbiasedness:-

A static is said to be unbiased estimate of parameter θ if $E(\hat{\theta}) = \theta$

i.e E[statistic] = parameter

→ Consistency:- A static $\hat{\theta}$ is said to be consistent of parameter θ if $E[\hat{\theta}] = \theta$

$V[\hat{\theta}] \to 0$ as n tends to ∞

→ Efficiency:- let $\hat{\theta}_1$ & $\hat{\theta}_2$ are unbiased estimate of parameter θ if $V[\hat{\theta}] < V(\hat{\theta})$ then $\hat{\theta}$ is the most efficient Estimator of parameter θ.

Interval Estimation:-

If an estimate the population parameter is given by an interval of $[\hat{\theta}_L, \hat{\theta}_U]$ then the Estimate is called interval Estimation of the parameter this interval estimation is also called confidence interval estimates of a parameter.

In an Interval Estimates of population parameter θ if we can find two quantities $\hat{\theta}_L$ & $\hat{\theta}_U$ based on sample observation drawn from the population such that the unknown parameter θ is included in the interval $[\hat{\theta}_L, \hat{\theta}_U]$ in a specified percentage of case then this interval is called confidence interval for parameter.

Formula's

→ specified percentage α% confidence limits for the population mean equal to $[\bar{x} - Z(\alpha/2)\,\sigma/\sqrt{n},\ \bar{x} + Z(\alpha/2)\,\sigma/\sqrt{n}]$

Here n = Large Sample size

i.e $n \geq 30$

$\bar{x}$ = Sample mean

σ = variance population std deviation

$Z_{\alpha/2}$ = the table value of Z at $\alpha/2$ % of size or level.

α% conference limit for the difference of two population means is $\cancel{[\bar{x}_1 - \bar{x}_2]}$

$$\left[(\bar{x}_1-\bar{x}_2)-Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\ ;\ (\bar{x}_1-\bar{x}_2)+Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}\right]$$

standard error of statistic

→ standard error of statistic equal to S.p(statistic)

$=\sqrt{v(\text{statistic})}$

eg:- $s.E(\bar{x})=\sqrt{V(\bar{x})}$

$=\sqrt{\sigma^2/n}$

$=\sigma/\sqrt{n}$

$(\bar{x}_1-\bar{x}_2)=\sqrt{V(\bar{x}_1-\bar{x}_2)}$

$=\sqrt{V(\bar{x}_1)+V(\bar{x}_2)}$

$=\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$

maximum error of an estimate for large sample
Since the sample mean estimate very rarely equals to the population mean μ. A point estimate is generally gives with statement of error which gives difference between estimate and quantity to be estimated i.e $|\bar{x}-\mu|$ maximum error and it is defined as

$M.E = Z_{\alpha/2}\cdot\sigma/\sqrt{n}$ $\quad n\geq 30$

→ what is the maximum error one can expect to make with probability 0.90 when using the mean of random sample 24 of size 64 to estimate the mean of the population with variance $(\sigma^2=2.56)$ and also construct 90% confidence limits for the true average.

→ $\sigma=\sqrt{2.56}$ $\quad n=64$

$=1.6$

$\alpha=0.90$

$\alpha/2=0.45$

$Z_{\alpha/2}=1.65$

$M.E = Z_{\alpha/2}(\sigma/\sqrt{n})$

$=1.65(1.6/\sqrt{64})$

$=0.33$

90% confidence limits

$[\bar{x}-Z_{\alpha/2}\,\sigma/\sqrt{n}\ ;\ \bar{x}+Z_{\alpha/2}\,\sigma/\sqrt{n}]$

$=\left[24-\frac{(1.65)(1.6)}{\sqrt{64}}\ ;\ 24+\frac{(1.65)(1.6)}{\sqrt{64}}\right]$

$=[24-0.33\ ;\ 24+0.33]$

$=[23.61\ ;\ 24.33]$

* A random sample of size 100 has a standard deviation with a mean of 26 what can you say about M.E with 95% confidence and also construct 95% confidence limit for the true average

sol:- $n=100$

$\bar{x}=26$

$\sigma = 5$

$\alpha = 0.95$

$\frac{\alpha}{2} = 0.475$

$t_{\alpha/2} = 1.96$

95% confidence limits

$[\bar{x} - \alpha/2 \cdot \sigma/\sqrt{n} ; \bar{x} + z_{\alpha/2}\, \sigma/\sqrt{n}]$

$= \left[26 - \frac{1.96 \cdot 5}{\sqrt{100}} ; 26 + \frac{1.96 \cdot 5}{\sqrt{100}}\right]$

$= [25.02 , 26.9]$

* Assuming that S.D is 20 how Large a random sample be taken to Essent with a probability 0.95 that the sample mean will not differ the true mean by max than 3 point.

Sol:-

$\sigma = 20$

$\alpha = 0.95$

$M.E = [\bar{x} - \mu] = 3$

$\alpha = 0.95$

$\alpha/2 = 0.475$

$z_{\alpha/2} = 1.96$

W.K.T

$M.E = z_{\alpha/2} \cdot \sigma/\sqrt{n}$

$\cancel{\Rightarrow \sqrt{n} = \frac{z_{\alpha/2} \cdot \sigma}{M.E}}$

$n =$

95% ~~confidence limit~~

$\left[\bar{x} - z_{\alpha/2} \cdot \sigma/\sqrt{n} ; \cancel{\bar{x} + z_{\alpha/2}\, \sigma/\sqrt{n}}\right]$

$\cancel{= 26 - \frac{1.96 \cdot 5}{\sqrt{100}} ; 26 +}$

$\Rightarrow \sqrt{n} = \frac{z_{\alpha/2} \cdot \sigma}{ME}$

$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{ME}\right]^2$

$= \left[\frac{(1.96)(20)}{3}\right]^2$ (∵ sample values Zeta not any information)

$= 170.73 \simeq 171$

It is desired to Estimate the mean no of hour continuous use until a certain computer will first requires repairs. If it can be now assumed that S.D is Equal to 48 hrs how large sample be needed so that one will be able to asset 90% confidence that the S.m may at most ~~10~~ hours.

$\sigma = 48$

$\alpha = 0.90$

$\alpha/2 = 0.45$ $\quad Z_{\alpha/2} = 1.65$

$M.E = (\bar{x} - \mu) = 10$

W.K.T

$M.E = Z_{\alpha/2} \times \sigma/\sqrt{n}$

$$\sqrt{n} = \frac{Z_{\alpha/2}\,\sigma}{ME} = \frac{1.65 \cdot 48}{10}$$

$n = (7.92)^2$

$n = 62.72 \approx 63$

In a study of an automobile insurance a random sample of 80 body repairs cost had a mean of 472.36 rupees and S.D of 62.35 rupees. If sample mean is used as a point estimate to the average repair cost with that confidence we can assert that M.E does not exceed 10.

$n = 80$

$\bar{x} = 472.36$

$\sigma = 62.35$

$M.E = 10$

$$M.E = \frac{Z_{\alpha/2} \cdot \sigma}{\sqrt{n}}$$

$(M.E/\sqrt{n}) = Z_{\alpha/2} \cdot \sigma$

$$Z_{\alpha/2} = \frac{(M.E)\sqrt{n}}{\sigma}$$

$$Z_{\alpha/2} = \frac{(10)\sqrt{80}}{62.35}$$

$= 1.434$

$\alpha/2 = 0.4236$

$\alpha = 0.8472$

$= 84.72\%$

→ In a certain factory there are two independent process for manufacturing the same item the avg. weight the sample of 200 items produced from process is found to be 250gm with a std deviation of 30gms while the corresponding another sample of 300 items with avg. of 300 and std deviation of 40 construct 1% confidence limits for the difference of 2 means

Note: 1% 99% are
Exp $\alpha = 99\%$

Note :- If 1% given then consider $\alpha = 99\%$.

If $5\% = 1-\alpha = 0.005$

$\alpha = 0.95$

If $10\% \to 1-\alpha = 0.10$

$\alpha = 90\%$.

Given

$n_1 = 700$

$\bar{x}_1 = 250$

$\sigma_1 = 30$

$n_2 = 300$

$\bar{x}_2 = 300$

$\sigma_2 = 40$

$1-\alpha = 0.01$

$\alpha = 0.99$

$\alpha/2 = 0.495$

$Z_{\alpha/2} = 2.58$

Sol: 17) 99% confidence limits of diff of mean

$$= \left((\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \; ; \; (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$= \left((250-300) - 2.58\sqrt{\frac{30^2}{700} + \frac{40^2}{300}} \; ; \; (250-300) + 2.58\sqrt{\frac{30^2}{700} + \frac{40^2}{300}}\right)$$

$$= \left[-50 - 6.637 \; ; \; -50 + 6.637\right]$$

$$= (56.637 \; ; \; -43.362)$$

→ two types of new cars produced in USA are tested for petrol mileage one sample is consisting of 42 cars average of 15 km per ltr. while other sample consisting of 80 cars average 11.5 km/m with population variants $\sigma_1^2 = 2$, $\sigma_2^2 = 1.5$. Construct 95% confidence limits for the difference of 2 avg.

Sol:- $\sigma_1 = \sqrt{2} = 1.41$

$\sigma_2 = \sqrt{1.5} = 1.22$

$\alpha = 0.95$

$\alpha/2 = 0.475$

$Z_{\alpha/2} = 1.96$

$n_1 = 42$, $n_2 = 80$

$\bar{x}_1 = 15$ km/m $\bar{x}_2 = 11.5$ km/m

$$\left((\bar{x}_1 - \bar{x}_2) - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \; ; \; (\bar{x}_1 - \bar{x}_2) + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right)$$

$$= \left((15 - 11.5) - 1.96\sqrt{\frac{2}{42} + \frac{1.5}{80}} \; ; \; (15 - 11.5) + 1.96\sqrt{\frac{2}{42} + \frac{1.5}{80}}\right)$$

$$(3.5 - 0.504 \; ; \; 3.5 + 0.504)$$

$$(2.996 \; ; \; 4.004)$$

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Testing of hypothesis

parameter → sample values $x \to \bar{x}$

In many cases we have to make decisions about population on the basis of only sample information, a parameter can be estimated from sample data. we can find a single no. for the parameter (or) an Interval of values. However there are many problems in which rather than the estimating the value of the parameter we need to decide where to accept (or) reject a statement about the parameter, this statement is called hypothesis & the decision making procedure about the hypothesis is called Testing of hypothesis.

STATISTICAL OF HYPOTHESIS:-

It is statement about the parameter ex:- the population mean is estimated from the sample mean.

(i) the teaching methods in the both schools are effective

(ii) Hypothesis mainly of two types

(1) Null Hypothesis & (2) Alternative Hypothesis

(1) A Null hypothesis is the hypothesis which asserts that there is no significant difference between the statistic & population parameters. It is denoted by H_0 for ex:- $H_0 = \mu \neq \mu_0$

(2) Alternative hypothesis:- Any hypothesis which contradicts the null is called Alternative hypothesis. It is denoted by H_a (or) H_1. for ex:- $H_1\ \mu \neq \mu_0$ (two tailed Alter)

$\mu > \mu_0$ (Right " ")

$\mu < \mu_0$ (Left " ")

Errors in testing of hypothesis:

The main objective in sampling theory is to draw valid conclusion about the pop'n parameters on the basis of sample results. we have two types of errors. In making decisions about the hypothesis.

(1) Decision Making

	H_0 true	H_0 false
Accept H_0	correct	wrong decision
Reject H_0	wrong decision	Correct

(1) If Type I Error If the null hypothesis is true but it is rejected by test procedure. then the error made is called Type 1 error. Type 2 error

(2) If null hypothesis H_0 false but it is accepted by test procedure then the error made is called type two error

Level of Significant:-

the Level of Significant is the confidence which is we reject (or) accept the null hypothesis. It is denoted by α. usually In practice we take either 5% (or) 1% level of significants.

Critical Region the critical region is formed based on the form of alternative hypothesis. If AH HA has not equal to sign. then the critical region is divided Equally. In left & Right tails if the form of HA is < sign. then the critical Region is taken in left tail. If AH HA is greater than sign. then the critical region is taken on the right tail.

one tailed & two tailed test!- If the alternative hypothesis In a test of statistical hypothesis be one tail then the test is called one tailed test. If AH h_1 has greater than sign then the corresponding test is called right tail test. If AH h_1 has Less than the sign then the corresponding test is called Left tailed test. If the AH h_1 in a test of a statistical hypothesis be two tailed then the test is called Two tailed test.

Procedure for Testing of hypothesis

various steps involved in testing of hypothesis are given below

1st step setup a null hypothesis h_0 by taking into consideration the nature of problem.

2nd step setup the AH h_1 so that we could decide whether we should use one tailed (or) two tailed test

3rd step:- choose an appropriate level of significant α.

4th step:- The test statistic by using the formula

$$z = \frac{t - E(t)}{\sqrt{V(t)}}$$

5th step: Now come back the calculated value of z with table value of z and we conclude as follows. If table value of z is greater than calculated value of z then we accept our null hypothesis otherwise we reject it. The critical ~~are~~ tables values of z at 1%, 5%, 10% are listed below:

table values of z

	1%	5%	10%
Two tailed Test	2.58	1.96	1.645
Right tailed test	2.33	1.645	1.28
Left Tailed test	−2.33	−1.645	−1.28

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Test of significant for single [illegible]

Large samples:-

Let $\bar{x}$ be the sample mean of size 'n' drawn from the population with mean μ & ~~significant~~ variance σ^2.

To test whether the sample has been drawn from the population with mean μ_0 let us consider null hypothesis are $H_0 - \mu = \mu_0$

$H_1 - \mu \neq \mu_0$ (or) $\mu > \mu_0$ (or) $\mu < \mu_0$

Choose α % level of significants under the null hypothesis the test statistics is given by $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

If the ~~calculate~~ table value of Z is greater than calculated value of z at α% level of significants then we accept our null hypothesis. otherwise we reject the null hypothesis.

Test of significants for differents of two means in large samples.

Let $\bar{x}_1$ & $\bar{x}_2$ be the sample means of sizes n_1, n_2 drawn from the two population with means μ_1 & μ_2 & variances σ_1^2 & σ_2^2 two test whether there is any significant difference b/w two means let us consider null hypothesis h_0 as $\mu_1 = \mu_2$

h_1 as $\mu_1 \neq \mu_2$ (or) $\mu_1 > \mu_2$ (or) $\mu_1 < \mu_2$

choose as appropriate level of significants α now calculate the test statistics as $Z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

If $\sigma_1 = \sigma_2 = \sigma$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

If table value of z is greater than calculated value of z then we accept our null hypothesis otherwise we reject the null hypothesis.

problem's:-

→ A sample 400 items is taken from a population whose std. deviation is 10. the mean of the sample is 40. test whether the sample has come from a population with mean 38 at 5% level of significants.

Sol:— Given, $n = 400$

$\sigma = 10$

$\bar{x} = 40$

$\mu = 38$

~~Given~~

Now,

h_0 = the samples are taken from the population with $\mu = 38$.

h_1 = " " " " " " $\mu \neq 38$ (two tailed test)

$\alpha = 5\%$

Under H_0,

$$|Z| = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{40 - 38}{10/\sqrt{400}}$$

$$= 4$$

Now table value of z_α for two tailed test at 5% significant level is 1.96

The table value of z is less than the calculated value of z at 5% level so, we reject our null hypothesis.

→ It is claimed that a random sample of 49 tyre has a mean life of 15,200 KM. This sample was drawn from a population whose mean is 15,150 KM and Std. deviation of 1200 KM. Test the significance at 1% level.

Sol:— Given

$n = 49$

$\bar{x} = 15,200$

$\mu = 15,150$

$\sigma = 1200$

$H_0 = \mu = 15,150$

$H_1 = \mu \neq 15,150$

$\alpha = 1\%$

$$|Z| = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{15,200 - 15,150}{1200/\sqrt{49}}$$

$$= \frac{15,200 - 15,150}{171.42}$$

$$= 0.291$$

For two tailed test the table value of 1% significant is 2.58.

The table value of z is greater than the calculated value of z at 1% level so, we accept our null hypothesis.

→ A random sample of 60 workers the average time taken by them to get to work is 33.8 min with a std. deviation of 6.1 min. Can we reject the null hypothesis $\mu = 32.6$ min. In favour of alternative hypothesis $\mu > 32.6$. at 5% Level of significants

Sol:—

$n = 60$

$\bar{x} = 33.8$

$\sigma = 6.1$

$\mu = 32.6$

$H_0 = \mu = 32.6$

$H_1 = \mu > 32.6$ (Right tailed test)

$\alpha = 5\%$

under H_0,

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{33.8 - 32.6}{6.1/\sqrt{60}} = 1.523$$

For Right tailed test the table value of z at 5% level of significants 1.645. calculated value is 1.5 and table value is 1.645. ∴ the table value of t is greater than calculated value of z so, we accept our null hypothesis.

→ An ambulance service claims that it takes on average less than 10 minutes to be reaches its destination in emergency calls. A sample of 36 calls has a mean of 11 minutes and the variance of 16 min. Test the claim at 1% Level of significants.

Sol:—

Given,

$n = 36$

$\bar{x} = 11$

$\sigma^2 = 16$

$\sigma = \sqrt{16} = 4$

$\mu = 10$

$H_0 = \mu = 10$

$H_1 = \mu < 10$ (Left tailed test).

$\alpha = 1\%$

under H_0,

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{11 - 10}{4/\sqrt{36}}$$

$$= 1.5$$

For Left tailed test the table value of z at 1% Level of significants is -2.33. calculated value is greater than the table of z at 1% level of significants so, we reject the null hypothesis.

→ The Mean life time of a sample of 100 light tubes produced by a company is found to be ~~1000~~ 1560 hrs with population std deviation of 90 hr. test the hypothesis at 5% that the mean life time of tubes produced by the company is 1580 hrs.

Sol:— Given

$n = 100$

$\bar{x} = 1560$

$\mu = 1580$

$\sigma = 90$

$H_0 = \mu = 1580$

$H_1 = \mu \neq 1580$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{1560 - 1580}{90/\sqrt{100}}$$

$$= -2.22. \quad \boxed{\Rightarrow 2.22}$$

From two tailed test the table value of z at 5% level is 1.96 So table value is less than Calculated value of z. So we reject our null hypothesis.

→ In 64 randomly selected hours of production the mean and standard deviation of no of acceptions ance produced by an automatic stamping machine (or) 1.038 and 0.146 at 5% level of significants ~~than~~ this enables us to reject the null hypothesis $\mu = 1$ against the alternative hypothesis $\mu > 1$.

Sol:—

$n = 64$

$\bar{x} = 1.038$

$\sigma = 0.146$

$\mu = 1$

$H_0 = \mu = 1$

$H_1 = \mu > 1$ (Right tailed test)

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{1.038 - 1}{0.146/\sqrt{64}}$$

$$= 2.08.$$

5% $= 1.645$

for right tailed test the table value of z at 5% level is 1.645. So the calculated value is greater than Table value, so we reject our null hypothesis. $\therefore \mu > 1$

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→ The means of two Large Samples of Sizes 1000 and 2000 no. are 67.5 & 68 respectively. Can the Sample be regarded as drawn from the Same population of Std. deviation 2.5.

Sol:- Given,

$n_1 = 1000$

$n_2 = 2000$

$\bar{x}_1 = 67.5$

$\bar{x}_2 = 68$

$\sigma = 2.5$

Ho = The two Samples are taken from the Same population.

i.e $= \mu_1 = \mu_2$

H_1:- The two Samples are taken from the Same population $\mu_1 \neq \mu_2$

Here $\sigma_1 = \sigma_2 = \sigma = 2.5$

$$|z| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma^2}{n_1} + \frac{\sigma^2}{n_2}}}$$

$= -5.16 \quad = \boxed{5.6}$

Now for two tailed test at 1 Level of significance table value is less than calculated value of t

So, we reject our null hypothesis

→ A Researcher wants to know the intellegance of a student in a School. He Selected two groups of students. In the first group there are 150 students having the Mean IQ of 75 with a Std deviation 15 and in the second group there are 250 students having the Mean IQ of 70. with a Std deviation of 20. Test the hypothesis that the groups have to be taken from the Same population (or) not at 1% level.

Sol:- Given,

$n_1 = 150 \qquad n_2 = 250$

$\bar{x}_1 = 75 \qquad \bar{x}_2 = 70$

$\sigma_1 = 15 \qquad \sigma_2 = 20$

Ho = the two groups have been taken from the Same population.

i.e $\mu_1 = \mu_2$

H_1 = the two groups have not been taken from the same population

$\mu_1 \neq \mu_2$ (two tailed test)

$$|z| = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{75 - 70}{\sqrt{\frac{15^2}{150} + \frac{20^2}{250}}} = 2.83$$

from two tailed the table of z at 1% level of significant is 2.58. Calculated value is greater than the table value of z so, we reject our null hypothesis.

An average marks score by 32 boys is 72 with a std deviation of 8 while for that 36 girls is 70 with a std deviation of 6 does this indicates that the boys perform better than girls at 5% level of significants.

Sol: Given that

$n_1 = 32$

$\bar{x}_1 = 72$

$\sigma_1 = 8$

$n_2 = 36$

$\bar{x}_2 = 70$

$\sigma_2 = 6.$

H_0 = There is no difference b/w the performance of boys & girls.

$\mu_1 = \mu_2.$

H_1 = The boys perform better than the girls

$\mu_1 > \mu_2$ (Right tailed test).

$\alpha = 5\%.$

under H_0,

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{72 - 70}{\sqrt{\frac{8^2}{32} + \frac{6^2}{36}}}$$

$= 1.154$

For Right tailed test at 5% level of significants is 1.645 ∴ The calculated value of z is ~~greater~~ Less than the table value of z we accept our null hypothesis. Boys & girls performance same.

→ A company claims that its bulbs are superior to those of its main competitor if a study shows that a sample of 40 its bulbs have a mean life time of 647 hrs of continous use with a std devation of 27 hrs. while a sample of 40 bulbs made by its main competitor have a mean life of 638 hrs of continous use with a std devation 31 hrs. Test the significants b/w the difference of the two mean at 1% level.

Sol:- Given that,

$n_1 = 40$

$\bar{x}_1 = 647$

$\sigma_1 = 27$

$n_2 = 40$

$\bar{x}_2 = 638$

$\sigma_2 = 31$

H_0 is there no difference between the mean life of bulbs of 2 companies.

$\mu_1 = \mu_2$

H_1 = The companies bulbs are superior than main competitor.

$\mu_1 > \mu_2$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{647 - 638}{\sqrt{\frac{27^2}{40} + \frac{31^2}{40}}}$$

$= 1.384$

for right tailed test the table value of z at 1% level of significants is 2.33 ∴ The table value of Z is greater than calculated value of z So, we accept our null hypothesis.

→ A simple sample of the heigh of 6,400 English men has a mean of 67.85 inches & std devation of 2.56 inches. while a simple sample of heights of 1600 Austrlians has a mean of 68.55 inches and a std devation of 2.52 inches. do the data indicates that the Austrlians are on the average Taller than the English man use 1% level of significants

Sol: Given that,

$n_1 = 6400$ $\quad$ $n_2 = 1600$

$\bar{x}_1 = 67.85$ $\quad$ $\bar{x}_2 = 68.55$

$\sigma_1 = 2.56$ $\quad$ $\sigma_2 = 2.52$

H_0 is there is no difference b/w the average heights of English man and australians that is $\mu_1 = \mu_2$.

H_1 is on the average the heights of australians are taller than the English man $\mu_1 < \mu_2$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = -9.90 \quad \boxed{T = 9.90}$$

For left tailed test the table value of Z at 1% level of significant is -2.33. the calculated table value of Z is greater than calculated value of Z, so we accept our null hypothesis.

4. SMALL SAMPLES

Test the significance of single mean.

To test the sample mean of sample of size $(n<30)$ drawn from the population with mean 'μ' as a specified value (μ_0) when the population std deviation is not known

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

$\mu > \mu_0$

$\mu < \mu_0$

under H_0 $|t| = \frac{\bar{x} - \mu}{S/\sqrt{n-1}}$

where $S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

Now, compare the table value of with calculated value of $|t|$ at 'α' % level of significance with 'n-1' degrees of freedom. If table value of 't' is great then calculate value of 't' then we accept of Null hypothesis. otherwise we reject our null hypothesis.

Test of significance of difference of two mean

Let $\bar{x}, \bar{y}$ be the sample mean of the size N_1 and N_2 $(N_1 < 30)$ $(N_2 < 30)$ drawn from the two population to test significance diff between the two population mean is and why

Let us consider Null hypothesis

$H_0: \mu_1 = \mu_2$

$\Delta_1 = \mu_1 \neq \mu_2$ (or)

$\mu_1 > \mu_2$ (or)

$\mu_1 < \mu_2$

Under $H_0 \rightarrow |t| = \dfrac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

where $S = \sqrt{\dfrac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2}}$

(or)

$S = \sqrt{\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$

s - sample S.D

S = populf S.D

Compare the calculated value of $|t|$ with table value of 't' at "α"% level of significance with n_1+n_2-2

If table value of 't' is greater than calculated value of 't' we accept our null hypothesis.

otherwise, we reject our Null hypothesis

Note:- for two tailed test α ful of significance value of '$\alpha/2$' 'S.d' 'α'.

Q. The average breaking strength of the steel rod is specified to be 18.5 (1,000 pounds) to test this sample of 14 rods were tested the mean and S.D obtained were 17.5 and 1.95 respectively. Is the result of equipment significant.

Sol:- Given

$\mu = 18.5$

$x = 17.85$

$n = 14$

$s = 1.955$

$H_0 = \mu = 18.5$

$H_1 = \mu \neq 18.5$ (two tailed test)

$|t| = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}} = \dfrac{17.85 - 18.5}{1.955/\sqrt{13}}$

$\Rightarrow 1.19$

For the two tailed test the table value of 't' at $\alpha/2$

$\alpha/2 = \dfrac{0.05}{2} = 0.025$

with $n-1 = 13$ degree of freedom is

$t = (0.025) = 2.160$

therefore the table value of $t >$ calculated value

So we accept our Null hypotesis.

Q. A Machine is design to produced insulating washers for electrical devices of average thickness of 0.025 cm a Random sample of 10 washers found to have a thickness of 0.024 cm with S.D of 0.002 cm. Test the significant of given claim at 1% of significant.

Sol:-

$H_0 = \mu = 0.025$

$H_1 = \mu \neq 0.025$

Given

$n = 10$

$\bar{x} = 0.024$

$S = 0.002$

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$$|t| = \frac{0.025 - 0.024}{0.002/\sqrt{10-1}} = \frac{\bar{x}-\mu}{s/\sqrt{n-1}}$$

$\Rightarrow 1.5$

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$ with

$\Rightarrow n-1 = 9$ degrees of freedom.

$t(9) \cdot 0.005 = 3.250$

Q. The height of 10 students are found to be 70, 67, 68, 67, 61, 68, 70, 64, 64, 66 inches. It is reversable to believe that the average height is greater than 64 inches. test at 1% Level of Significance. $\mu = 64$

Sol:- $H_0 = \mu = 64$

$H_1 = \mu > 64$ (Right tailed test)

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{70+67+67+68+61+68+70+64+64+66}{10}$$

$\Rightarrow \boxed{66 = \bar{x}}$

$$s = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(70-66)^2+(67-66)^2+(67-66)^2+(68-66)^2+(61-66)^2+(68-66)^2+(70-66)^2+(64-66)^2+(64-66)^2+(66-66)^2}{9}}$$

$= 3.16$

$$|t| = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{66-64}{3.16/\sqrt{9}} = 1.89$$

$\therefore$ for one tailed test the table of value 't' at $\alpha = 1\%$.

$\alpha = 0.01$ with $((n-1) = 9)$ degree of freedom is 2.82

$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

value of 't'. So we accept our null hypothesis.

Q. The life time of electric bulbs for a random sample 10 from a large consignment given as 1.2, 4.6, 3.9, 4.1, 5.2, 3.8, 3.9, 4.3, 4.4 & 5.6 hours. Can we accept the hypothesis that the average life time of the bulb is 4 hours

$\mu = 4$.

Sol:- $\mu = 4$

$H_0 := \mu = 4$

$H_1 := \mu \neq 4$ (two tailed test)

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{12+4.6+3.9+4.8+5.2+3.8+3.9+4.3+4.4+5.6}{10}$$

$\bar{x} = 4.1$

$$s = \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n-1}}$$

$$s = \sqrt{\frac{(1.2-4.1)^2+(4.6-4.1)^2+(3.9-4.1)^2+(4.8-4.1)^2+(5.2-4.1)^2+(3.8-4.1)^2+(3.9-4.1)^2+(4.3-4.1)^2+(4.4-4.1)^2+(5.6-4.1)^2}{9}}$$

$s = 1.174$

$$|t| = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{4.1-4}{1.174/\sqrt{9}} = 0.255$$

For two tailed test the table value of t-test at

$\alpha/2 = \frac{0.05}{2} = 0.025$ with $(n-1 = 9)$ degree of freedom

$\rightarrow$

Sol^n— $n_1 = 16$, $s_1 = 10$, $s_2 = 8$

$\bar{x} = 107$, $n_2 = 14$, $\bar{y} = 112$

$H_0 = \mu_1 = \mu_2$

$H_1 = \mu_1 \neq \mu_2$

$$S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{16 \times 10^2 + 14 \times 8^2}{16 + 14 - 2}}$$

$$= 9.44$$

$$|t| = \frac{\bar{x} - \bar{y}}{S \Big/ \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{107 - 112}{9.44 \Big/ \sqrt{\frac{1}{16} + \frac{1}{14}}}$$

$$= 1.447.$$

For two tailed test $\alpha/2$ is $0.05/2$ is 0.025 with $n_1 + n_2 - 2 = 28$ as degree of freedom is 2.048

$\therefore$ The table value of 't' is > then calculated value So we accept of Null hypothesis.

→ following Random samples are measurements of heat producing capacity of specimen of coal from two mines. Use 1% level of significants to test whether it is reasonable to assumes that the variances of two populations samples are equal

Mine A	Mine B
8,260	7,950
8,130	7,890
8,350	7,900
8,070	8,140
8,340	7,920
	7,840

Sol:-

$n_1 = 5$

$n_2 = 6$

H_0 = The variance of population sample are Equal

H_1 = " " " " are not Equal.

$$\bar{x} = \frac{\Sigma x_i}{n_1} =$$

$$\bar{y} = \frac{\Sigma y_i}{n_2} =$$

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$$\bar{x} = \frac{\Sigma x_i}{n_1} = 8230$$

$$\bar{y} = \frac{\Sigma y_i}{n_2} = 7940$$

x Mine A	y Mine B	$(x-\bar{x})^2$	$(y-\bar{y})^2$
8260	7950	900	~~1600~~ 100
8130	7891	10,000	~~10,000~~ 2500
8350	7,900	14400	~~8100~~ 1600
8070	8,140	25600	4000 ~~22500~~
8340	7,920	25600	1400
	7,840		
		63000	54600

$$S_1^2 = \frac{\Sigma(x_i-\bar{x})^2}{n_1-1} = \frac{63000}{4}$$

$= 15750$

$$S_2^2 = \frac{\Sigma(y_i-\bar{y})^2}{n_2-1}$$

$$\frac{54600}{5} = 10920$$

$$F = \frac{S_1^2}{S_2^2} = \frac{15750}{10920} = 1.44$$

1%.

table value of $F(n_1-1, n_2-1) = F_{(4,5)}^{1\%} = 11.39$

→ In two Independents samples of sizes 8 & 10 The sum squares of deviations of sample values from mean is 84.4 and in another sample, it is 102.6. Test whether the diff of variances of population is significant or not

Sol:- $n_1 = 8$, $n_2 = 10$

$(x-\bar{x})^2 = 84.4$

$(y-\bar{y})^2 = 102.6$

H_0 = there is no significant diff. variances of population sample.

H_1 = there is a significant diff of variances of population sample.

$$S_1^2 = \frac{\Sigma(x_i-\bar{x})^2}{n_1-1} = \frac{84.4}{8-1} = \frac{84.4}{7} = 12.05$$

$$S_2^2 = \frac{\Sigma(y_i-\bar{y})^2}{n_2-1}$$

$$= \frac{102.6}{10-1} = 11.4$$

$$F = \frac{S_1^2}{S_2^2} = \frac{12.05}{11.4} = 1.05$$

Table value of $F(n_1-1, n_2-1) = 3.29$

Table value greater than calculated value, so we accept our null hypothesis.

Chi - square test (χ^2 test)

Chi-square test is used to test whether diff b/w observed & expected frequency are significant.

Chi Square test is mainly used to test an Independent (Quality variables), to test the goodness of fit. we use this test to decide whether the difference b/w observed & expected frequency is significant (or) not. The chi square test is given by

$$\chi^2 = \Sigma\left[\frac{(O_i - E_i)^2}{E_i}\right]$$

If the table value of chi square at alpha level of significance with $(n-1)$ degrees of freedom Is greater than the calculated value of chi square then we accept our null hypothesis otherwise we reject our null hypothesis. This is carry chi square test for goodness of fit.

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Chi Square Test for Independence of 'Attribute'

An Attributes means a quality (or) characteristics in this case we set the null hypothesis as H_0 = there is no association between the attributes. that is the two attributes are Independent

To test this hypothesis the chi square test as given by $\chi^2 = \sum_{i=1, j=1}^{n, m} \left[\frac{(O_{ij} - E_{ij})^2}{E_{ij}}\right]$

where $E_{ij} = \dfrac{i^{th} \text{ row total} \times j^{th} \text{ column total}}{\text{Grand total}(N)}$

Now compare with this calculated value of chi square with table value of chisquare at α level of significants with $(R-1)(C-1)$. degrees of freedom.

where R = No. of Rows
C = No. of columns.

If table value of chisquare is greater than the calculated value of chisquare then we accept our null hypothesis.

The no. of automobiles accidents per ~~week~~ week in a certain community area are as follows. are these frequency in agreement with the belief that accident conditions for the same during this Ten week period?

O_i
12
8
20
2
14
10
15
6
9
4
100

H_0 = The accident conditions were the same during the ten weeks

H_1 = The accident conditions are diff during the 10 weeks

Sol:-

O_i	E_i	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
12	10	4	4/10=0.4
8	10	4	4/10=0.4
20	10	100	100/10=10
2	10	64	64/10=6.4
14	10	16	16/10=1.6
10	10	0	0/10=0
15	10	25	25/10=2.5
6	10	16	16/10=1.6
9	10	1	1/10=0.1
4	10	36	36/10=3.6

$P(x)$ = accoring of accident in 10 weeks. $= \frac{1}{10}$.

E = xpected frequency

$E_i = N \times P(x)$
$= 100 \times \frac{1}{10} = 10$

for Each week.

the table value at 5% Level of Siginificant with $(n-1) = 10-1 = 9$ degrees of freedom . 16.919 so table value of chisquare is Less than calculated value chisquare . So, we reject our null hypothesis

→ A die is thrown 264 times with the following results test the dies biased (or) not.

No. appeard on die	frequency
1	40
2	32
3	28
4	58
5	54
6	52

Sol:- Given,

N = 264

n = 6

probability of getting a no. in throwing a die $p(x) = 1/6$

Now the Expected frequency is $= N \times p(x)$

$= 264 \times \frac{1}{6} = \frac{264}{6} = 44.$

H_0 = The die is unbaised

H_1 = The die is baised.

Oi frequency	Ei	$(Oi-Ei)^2$	$(Oi-Ei)^2/Ei$
40	44	16	16/44 = 0.36
32	44	144	144/44 = 3.27
28	44	256	256/44 = 5.81
58	44	196	196/44 = 4.45
54	44	100	100/44 = 2.27
52	44	64	64/44 = 1.45
			17.61

Table of chi square at 5% Level of significant 11.07 . Calculated value is greater than table value So we reject our null hypothesis.

Q. pair of dies thrown 360 and freq of each sum is indicated below.

sum	frequency	$E_i = N \times p(x)$ $360 \times 1/36$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
2	8	10	4	0.4
3	24	20	16	0.8
4	35	30	25	0.83
5	37	40	9	0.23
6	44	50	36	0.72
7	65	60	25	0.41
8	51	50	1	0.02
9	42	40	4	0.1
10	26	30	16	0.533
11	14	20	36	1.8
12	14	10	16	1.6
				7.439

Would you say that the dies are faired on the basis of x^2 test at 5% level of significant

Sol:- p(sum of 2) = p(1,1)

= 1/36

p(sum of 3) = p(1,2),(2,1) = 2/36 = 1/18

p(sum 4) [(1,3),(2,2),(3,1)]

=> 3/36 = 1/12

p(sum 5) = [(2,3),(1,4),(3,2),(4,1)]

=> 4/36 = 1/9

p(sum 6) = [(1,5)(5,1)(3,3)(4,2)(2,4)]

= 5/36 = 0.138

p(Sum 7) = (1,6)(6,1)(5,2)(2,5)(4,3)(3,4)

6/36 = 1/6 = P(7)

p(sum 8) = P[(4,4)(5,3)(3,5)(6,2)(2,6)] 5/36

p(sum 9) = [(5,4)(4,5)(6,3)(3,6)]

= 4/36

P(sum 10) = (5,5)(6,4)(4,6)

= 3/36

p(sum 11) = (5,6)(6,5)

= 2/36 = 1/18

p(sum 12) = (6,6)

= 1/36

$x^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 7.439$

table value of x of 5% level of significant (n-1) = (11-1)

= 15.307 degree of freedom

Table vale x^2 > calculated value

Accepted our null hypothesis

=> A sample analysis of examination result of 500 were made it was found that 220 students had failed 170 had secured 3rd class 90 where placed in second class and 20 got first class. Do this data reads that with general examination result which is the ratio 4:3:2:1 for various category respectly.

Sol:- H_0 = the obtained result conformity with general examination result

N = 500

n = 4

Expected freq with ratio of 4:3:2:1

P(failed student) = 4/10 = 0.4

p(third class) = 3/10 = 0.3

p(second ") = 2/10 = 0.2

p(first ") = 1/10 = 0.1

O_i	E_i	$(O_i-E_i)^2$	$\Sigma(O_i-E_i)^2/E_i$
220	200	400	2
170	150	400	2.667
90	100	100	1
20	50	900	18
			23.667

$$\chi^2 = \frac{\Sigma(O_i-E_i)^2}{E_i} = 23.667$$

Table value of χ^2 of 5% significant $(n-1)=(4-1)=3$ degree of freedom

Table of χ^2 < calculated value

So reject the Null hypothesis

8) On the bases of information given below about the treatment of 200 patients suffering from disease state the New treatment is comparatively superior to the Convention treatment

	Favourable	Not favourable	Total	
New	O_1 60	30	90	R_1
Convention	O_2 40	70	110	R_2
	100	100	200	

Sol:– H_0 = There is no difference b/w New & convention treatment

$N = 200$

Now Expected frequency can be calculated as follows:–

$$E_{11} = \frac{R_1 C_1}{N} = \frac{90 \times 100}{200} = 45$$

$$E_{12} = \frac{R_1 C_2}{N} = \frac{90 \times 100}{200} = 45$$

$$E_{21} = \frac{R_2 \times C_1}{N} = \frac{110 \times 100}{200} = 55$$

$$E_{22} = \frac{R_2 \times C_2}{N} = \frac{110 \times 100}{200} = 55$$

O_{ij}	E_{ij}	$(O_{ij}-E_{ij})^2$	$\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$
60	45	225	5
30	45	225	5
40	55	225	~~4.09~~ 4.09
70	55	225	4.09
			18.18

table = χ 25% $(R-1)(C-1)$ (χ 5%)

$(2-1)(2-1) = \chi$ 5% = 3.84

degrees of freedom

→ From multicylinder development of making disc of a super conducting material 50 discs are made by each method and they are checked for super conductivity when cooled with liquid. Test the significant difference between the proportions of super conductors at 5% level

	Ist Method	IInd "	IIIrd "	IVth "	Tot
Super conductors	31	42	22	25	12
failures	19	8	28	25	80
total	50	50	50	50	20

Sol:- H_0:- There is no significant difference b/w proportions of super conductors

H_1:- There is a significant difference b/w proportions of super conductors.

$N = 200.$

Now the expected frequencies can be calculated as follows

$$E_{11} = \frac{R_1 \times C_1}{N} = \frac{120 \times 50}{200} = 30$$

$$E_{12} = \frac{R_1 \times C_2}{N} = \frac{120 \times 50}{200} = 30$$

$$E_{13} = \frac{R_1 \times C_3}{N} = \frac{120 \times 50}{200} = 30$$

$$E_{14} = \frac{R_1 \times C_4}{200} = \frac{120 \times 50}{200} = 30$$

$$E_{21} = \frac{R_2 \times C_1}{N} = \frac{80 \times 50}{200} = 20$$

$$E_{22} = \frac{R_2 \times C_2}{N} = \frac{80 \times 50}{200} = 20$$

$$E_{23} = \frac{R_2 \times C_3}{N} = \frac{80 \times 50}{200} = 20$$

$$E_{24} = \frac{R_2 \times C_4}{N} = \frac{80 \times 50}{200} = 20$$

O_{ij}	E_{ij}	$(O_{ij}-E_{ij})^2$	$\frac{(O_{ij}-E_{ij})^2}{E_{ij}}$
31	30	1	1/30
42	30	144	144/30
22	30	64	64/30
25	30	25	25/30
19	20	1	1/20
8	20	144	144/20
28	20	64	64/20
25	20	25	25/20
			19.49

Table value of chi square at 5% level of significants with $(R-1)(C-1)$ $(2-1)(4-1) = 1\times3 = 3$ degrees of freedom 7.815 calculated value of chi square is greater than table value of chi square So we reject our null hypothesis

i.e, there is significant difference b/w the proportions of super conductors

→ From the following data find whether there is any significant difference of liking in a habit of taking soft drinks among the categories of employees

	Clerk	Teachers	Others	Total
Pepsi	10	25	65	100 R_1
thumsup	15	30	65	110 R_2
fanta	50	60	30	140 R_3
Total	75 C_1	115 C_2	160 C_3	350

Sol:

H_0:- There is no significant difference in liking of the habits of soft drinks among the categories of employees

H_1:- there is a significant difference in liking of the habits of soft drinks among the categories of employees

$N = 350$

$E_{11} = \frac{100 \times 75}{350}$

$= 21.42$

$E_{12} = \frac{100 \times 115}{350}$

$= 32.85$

$E_{13} = \frac{100 \times 160}{350}$

$= 45.71$

$E_{21} = \frac{110 \times 75}{350}$

$= 23.57$

$E_{22} = \frac{110 \times 115}{350}$

$= 36.14$

$E_{23} = \frac{110 \times 160}{350}$

$= 50.28$

$E_{31} = \frac{140 \times 75}{350}$

$= 30$

$E_{32} = \frac{140 \times 115}{350}$

$= 46$

$E_{33} = \frac{140 \times 160}{350}$

$= 64$

O_i	E_{ij}	$(O_{ij} - E_{ij})^2$	$\frac{(O_{ei} - E_{ij})^2}{E_{ij}}$
10	21.~~42~~	$(10 - 21.\cancel{42})^2 = 121$	$121/21 = 5.76$
25	33.~~85~~	$(\cancel{33} - 25)^2 = 64$	$64/33 = 1.93$
65	46.~~71~~	361	$361/46 = 7.84$
15	24.~~7~~	81	$81/24 = 3.37$
30	36.~~14~~	36	$36/36 = 1$
65	50.~~28~~	225	$225/50 = 4.5$
50	30	400	$400/30 = 13.33$
60	46	196	$196/46 = 4.26$
30	64	1156.	$1156/64 = 18.06$
			60.05

Table value of chisquare at 5% level of significats with $(R-1)(C-1)$ $(3-1)(3-1)$ $= 4$ degrees of freedom 9.488

Calculated value is greater then the table value so we reject our null hypothesis.

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→ Four coins are tossed 160 times and following results for obtained using binominal distribution test for the goodness of fit.

No. of heads	Observed frequency
0	17
1	52
2	54
3	31
4	6

Sol:- $N = 160$, $n = 4$

H_0: Binomial distribution can be fitted to the given data.

H_1 = Binominal " cannot be fitted to the given data.

Here,

p: probability of getting a head. $\Rightarrow 1/2$.

$q = 1 - p \quad 1 - 1/2 = 1/2$

Now, Binomial distribution $p(x) = {}^{n}C_x\, p^x q^{n-1} \quad {}^4C_x \left(\frac{1}{2}\right)^x$

Now, Expected frequency

$E = N \cdot p(x)$

$x = 0 \Rightarrow N\,p(0) = 160\left[{}^4C_0 (1/2)^0 (1/2)^{4-0}\right]$

$= 10$

$x = 1 \Rightarrow N\,p(1) = 160 \cdot {}^4C_1 (1/2)^1 (1/2)^{4-1}$

$= 40$

$x = 2 \Rightarrow N\,p(2) = 160 \; {}^4C_2 (1/2)^2 (1/2)^{4-2}$

$= 60$

$x = 3 \Rightarrow N\,p(3) = 160 \; {}^4C_3 (1/2)^3 (1/2)^{4-3}$

$= 40$

$x = 4 \Rightarrow N\,p(4) = 160 \; {}^4C_4 (1/2)^4 (1/2)^{4-4}$

$= 10$

No. of heads	Observation frequency O_i	(E_i)	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
0	12	10	49	49
1	52	40	144	3.6
2	54	60	30	0.6
3	31	40	81	2.045
4	6	10	16	1.6
				12.745

V. Functions of Complex Variables

→ **Complex Number:** A No. of the form $x+iy$ where x & y are real numbers & $i^2=-1$ is called an imaginary unit is known as complex number. Here; x is the real part of $x+iy$ & y is the imaginary part of $x+iy$.

→ **Complex conjugate:** complex conjugate if z is a complex no. then the no. obtained by changing the sign of its imaginary part is called complex conjugate i.e $x-iy$

→ **Modulus (or) Absolute value:** The no. $r=\sqrt{x^2+y^2}$ is called the modulus of $x+iy$ and it is written as $|z|$ (or) $|x+iy|$

→ **Function of a complex variable:** If for every value of z a unique value is associated then the unique value is said to be function of a complex variable i.e, $f(z)=u(x,y)+iv(x,y)$

Here; $u(x,y)$ & $v(x,y)$ are real value functions of x & y and are known as real & imaginary parts of function of z.

→ **Limit:** If l is said to be the limit of $f(z)$ as z approaches z_0 & is defined by $\lim_{z\to z_0} f(z)=l$

→ **Continuity:** A function of $f(z)$ is said to be continuous at z_0 if $f(z_0)$ is exist and $\lim_{z\to z_0} f(z)=f(z_0)$ i.e, the limiting value of $f(z)$ as z approaches to z_0 coincides with the value of $f(z_0)$

→ **Differentiability (or) Derivatives of complex functions**

A function $f(z)$ is said to be derivable at point z_0 if the limit $f'(z_0)=\lim_{\Delta z\to 0}\frac{f(z_0+\Delta z)-f(z_0)}{\Delta z}$

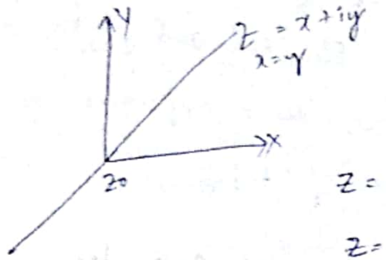
Where $\Delta=z-z_0$

$$\lim_{z-z_0\to 0}\frac{f(z_0+z-z_0)-f(z_0)}{z-z_0}$$

$$\frac{f(z)-f(z_0)}{z-z_0}$$

Then the function of $f(z)$ is derivable at z_0.

Note:- The above limit should be same along ~~the~~ any path from z to z_0



$z=0,\ y=0$

$z=0,\ x=0$

$z=0,\ x=y$

Note:- If a function is differentiable at a point then it is continuous but the converse need not be true.

Analytical Function: A function $f(z)$ is said to be Analytical at the point z_0 iff $f(z)$ differentiable ~~at~~ each of every point of the neighbourhood of z_0

Cauchi's Riemann Equations:- A necessary and sufficient conditions of $f(z)$ is to be analytical

(or) (i) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$ are continuous functions.

(ii) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

→ Show that $f(z) = xy + iy$ is every where continous but is not analytic.

Sol:- $f(z) = xy + iy$

W.K.T → $f(z) = u(x,y) + iv(x,y)$

$= u + iv$

then $u = xy$; $v = y$

$\frac{\partial u}{\partial x} = 1 \cdot y$

$\frac{\partial u}{\partial x} = y$

$\frac{\partial u}{\partial y} = x$

$\frac{\partial v}{\partial x} = 0$

$\frac{\partial v}{\partial y} = 1$

Now, C-R eq^n

$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

$\frac{\partial u}{\partial y} \neq \frac{\partial v}{\partial x}$

C.R are not satisfied.

$f(z) = xy + iy$

$f(z_0) = f(x_0, y_0)$

$= x_0 y_0 + i y_0$

$\lim_{z \to z_0} f(z) = \lim_{z \to z_0} [xy + iy]$

$= \lim_{\substack{x \to x_0 \\ y \to y_0}} (xy + iy)$

$= x_0 y_0 + i y_0$

$= f(z_0)$

f(z) is continuous every where but CR Eqⁿ are not satisfied.

$\therefore$ f(z) is not analytic.

→ Show that the function $f(z) = \sqrt{|xy|}$ is not Analytic at the horizon although C.R Eqⁿ are satisfied at that point

Sol:- $f(z) = \sqrt{|xy|} + i0$

W.K.T $\rightarrow f(z) = u(x,y) + iv(x,y)$

$u(x,y) = \sqrt{|xy|}$; $v(x,y) = 0$

At $(0,0) \Rightarrow u(0,0) = \sqrt{|0 \times 0|} = 0$

$v(0,0) = 0$

$\Delta x = x - x_0$; $\Delta y = y - y_0$

$\Delta x = x$; $\Delta y = y$

At $(0,0) \Rightarrow \Delta x = 0$; $\Delta y = 0$

$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{u(x + \Delta x, y) - u(x, y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(0 + \Delta x, 0) - u(0,0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(\Delta x, 0) - u(0,0)}{\Delta x}$$

$$\Rightarrow \lim_{\Delta x \to 0} \frac{u(0,0) - u(0,0)}{\Delta x}$$

$= 0$

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{u(x, y + \Delta y) - u(x, y)}{\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(0, \Delta y) - u(0,0)}{\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(0,0) - u(0,0)}{\Delta y} = 0$$

$$\frac{\partial v}{\partial x} = \lim_{\Delta x \to 0} \frac{v(x+\Delta x, y) - v(x,y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{0-0}{\Delta x} = 0$$

$$\frac{\partial v}{\partial y} = \lim_{\Delta y \to 0} \frac{v(x, y+\Delta y) - v(x,y)}{\Delta y}$$

$$= \frac{0-0}{\Delta y} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = 0$$

$$u_y = -v_x$$

C-R eq^n are not satisfied.

$$f(z) = \sqrt{|xy|} + i0$$

$$f(z_0) = f(0) = 0$$

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z - 0}$$

$$= \lim_{z \to 0} \frac{\sqrt{|xy|} - 0}{x+iy}$$

$$\Rightarrow \lim_{z \to 0} \frac{\sqrt{|xy|}}{x+iy}$$

$$= \lim_{z \to 0} \frac{\sqrt{|xy|}\,(x-iy)}{(x+iy)(x-iy)}$$

$$\Rightarrow \lim_{z \to 0} \frac{\sqrt{|xy|}\,(x-iy)}{x^2 - (i^2)(y^2)}$$

$$\Rightarrow \lim_{z \to 0} \frac{\sqrt{|xy|}\,(x-iy)}{x^2+y^2}$$

$$= \lim_{\substack{x \to 0 \\ y \to 0}} \frac{\sqrt{|xy|}\,(x-iy)}{x^2+y^2}$$

Along x- axis as $z \to 0$ put $y=0$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{0}\,(x-i0)}{x^2+0} = 0$$

Along y-axis as $z \to 0$ put $x=0$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{0}\,(0-iy)}{0+y^2} = 0$$

Along $x=y$ as $z \to 0$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{yy}\cdot(y-iy)}{y^2+y^2}$$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{\cancel{y}\,(1-i)\cancel{y}}{2\cancel{y^2}}$$

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{(1-i)}{2}$$

$$= \frac{1-i}{2}$$

As $f'(0)$ takes diff values $f(z)$ is not continuous at origin even satisfied C.R Eqn.

prove that the function $f(z)$ defined by $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$ $(z \neq 0)$. $f(z)=0$ is continuous the C.R Eqn are satisfied at the origin but $f'(0)$ does not exist.

Sol:- $$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$$

$$= \frac{x^3 + x^3 i - y^3 + y^3 i}{x^2+y^2}$$

$$= \frac{(x^3-y^3) + i(x^3+y^3)}{x^2+y^2}$$

$$= \frac{x^3-y^3}{x^2+y^2} + i\left[\frac{x^3+y^3}{x^2+y^2}\right]$$

$$= u(x,y) + i\,v(x,y)$$

where $u(x,y) = \frac{x^3-y^3}{x^2+y^2}$; $v(x,y) = \frac{x^3+y^3}{x^2+y^2}$

$u(0,0) = 0$; $v(0,0) = 0$

$f(z) = u(x,y) + i\,v(x,y)$

$f(0) = u(0,0) + i\,v(0,0)$

$= 0 + i0$

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$$\frac{\partial u}{\partial x} = \lim_{\Delta x \to 0} \frac{u(x+\Delta x, y) - u(x,y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{u(0+\Delta x, 0) - u(0,0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x - 0}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}} = 1$$

$$\frac{\partial u}{\partial y} = \lim_{\Delta y \to 0} \frac{u(x, y+\Delta y) - u(x,y)}{\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{u(0, 0+\Delta y) - u(0,0)}{\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{-\cancel{\Delta y}}{\cancel{\Delta y}} = -1$$

$$\frac{\partial v}{\partial x} = \lim_{\Delta x \to 0} \frac{v(x+\Delta x, y) - v(x,y)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{v(0+\Delta x, 0) - v(0,0)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}} = 1$$

$$\frac{\partial v}{\partial y} = \lim_{\Delta y \to 0} \frac{v(x, y+\Delta y) - v(x,y)}{\Delta y}$$

$$= \lim_{\Delta y \to 0} \frac{v(0, 0+\Delta y) - v(0,0)}{\Delta y}$$

$$\lim_{\Delta y \to 0} \frac{\cancel{\Delta y}}{\cancel{\Delta y}} = 1$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \; ; \; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} = -1 = 1$$

$$\Rightarrow 1 = -1$$

CR Eqn are satisfied

$$f'(0) = \lim_{z \to 0} \frac{f(z) - f(0)}{z - 0}$$

$$= \lim_{z \to 0} \frac{\frac{(x^3 - y^3) + i(x^3 + y^3)}{x^2 + y^2} - 0}{x + iy}$$

$$= \lim_{z \to 0} \frac{(x^3 - y^3) + i(x^3 + y^3)}{(x^2 + y^2)(x + iy)}$$

$$= \lim_{z \to 0} \frac{(x^3 - y^3) + i(x^3 + y^3)}{(x^2 + y^2)(x + iy)} \cdot \frac{x - iy}{(x - iy)}$$

$$\lim_{z \to 0} \frac{(x^3 - y^3) + i(x^3 + y^3)}{(x^2 + y^2)(x + iy)} \times \frac{x - iy}{x - iy}$$

$$\lim_{z \to 0} \frac{[(x^3 - y^3) + i(x^3 + y^3)](x - iy)}{(x^2 + y^2)(x^2 - i^2 y^2)}$$

$$\lim_{z \to 0} \frac{[(x^3 - y^3) + i(x^3 + y^3)](x - iy)}{(x^2 + y^2)(x^2 + y^2)}$$

case :- $z=0$, along x-axis put $y=0$

$$f'(0) = \lim_{z\to 0} \frac{((x^3-0)+i(x^3+0))[x-i0]}{(x^2+0)(x^2+0)}$$

$$= \frac{(x^3+ix^3)x}{x^4}$$

$$= \frac{x^3(1+i)x}{x^4}$$

$$= \frac{\cancel{x^4}(1+i)}{\cancel{x^4}}$$

$$= 1+i$$

case :- $z=0$, along y axis - put $x=0$

$$f'(0) = \lim_{z\to 0} \frac{[(0-y^3)+i(0+y^3)[0-iy]}{(0+y^2)(0+y^2)}$$

$$= \frac{(-y^3+iy^3)\cdot(-iy)}{y^4}$$

$$= \frac{y^3(-1+i)\cdot(-iy)}{y^4}$$

$$= \frac{\cancel{y^4}(i-1)(-i)}{\cancel{y^4}}$$

$$= -i(i-1).$$

case :- $z=0$ along $x=y$,

$$f'(0) = \lim_{z\to 0} \frac{[(y^3-y^3)+i(y^3+y^3)](y-iy)}{(y^2+y^2)(y^2+y^2)}$$

$$= \frac{0+i(2y^3(y-iy)}{2y^2\cdot 2y^2}$$

$$= \frac{\cancel{2}iy^{\cancel{3}}\cdot\cancel{y}(1-i)}{\cancel{4}y^{\cancel{4}}} = \frac{i(1-i)}{2}$$

$f'(0)$ takes different values in different path. $f(0)$ is not differentiable $0+i0$.

$f(0) = 0+i0$ at $f'(0)$ does not exist

① Show that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic.

Sol:- $u = e^{-x}(x\sin y - y\cos y)$

$= e^{-x} x\sin y - e^{-x} y\cos y$

$\frac{\partial u}{\partial x} = \sin y\left[(1\cdot e^{-x}) + x(-e^{-x})\right] - y\cos y(-e^{-x})$

$= \sin y\left[e^{-x} - x e^{-x}\right] + e^{-x} y\cos y.$

$= e^{-x}\sin y - x\sin y e^{-x} + e^{-x} y\cos y$

$\frac{\partial^2 u}{\partial x^2} = \sin y(-e^{-x}) - \sin y\left[e^{-x} - x e^{-x}\right] + y\cos y\,[-e^{-x}]$

$= -e^{-x}\sin y - e^{-x}\sin y + x\sin y e^{-x} - y\cos y e^{-x}$

$= -2e^{-x}\sin y + x\cdot e^{-x}\sin y - y e^{-x}\cos y$

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(x e^{-x}\sin y - y\cdot e^{-x}\cos y\right)$

1) $\sin x = \cos x$
2) $(\cos x) = -\sin x$

$= x e^{-x}\cos y - e^{-x}\left[\cos y + y(-\sin y)\right]$

$= x e^{-x}\cos y - e^{-x}\cos y + e^{-x} y\sin y$

$\frac{\partial^2 u}{\partial y^2} = x e^{-x}(-\sin y) - e^{-x}(\sin y) + e^{-x}\left[1\cdot\sin y + y\cos y\right]$

$= -x e^{-x}\sin y + \cancel{e^{-x}\sin y} + e^{-x}\sin y + e^{-x} y\cos y$

$= -x e^{-x}\sin y + e^{-x} y\cos y + 2e^{-x}\sin y$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

$= -2e^{-x}\sin y + x e^{-x}\sin y - y e^{-x}\cos y$
$- x e^{-x}\sin y + y e^{-x}\cos y + 2e^{-x}\sin y$

$= 0$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$

$\therefore u$ is a harmonic function.

② Verify that $u = x^2 - y^2 - y$ is harmonic.

It enough to show that

Sol:- $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Given $u = x^2 - y^2 - y$

$\frac{\partial u}{\partial x} = 2x$

$\frac{\partial^2 u}{\partial x^2} = 2$

$\frac{\partial u}{\partial y} = 0 - 2y - 1$

$$\frac{\partial^2 u}{\partial y^2} = -2$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2-2=0$$

$\therefore u$ is a harmonic function

(I) show that $u = e^x \cos y$ is harmonic?

Sol:- Given $u = e^x \cos y$

$$\frac{\partial u}{\partial x} = e^x \cos y$$

$$\frac{\partial^2 u}{\partial x^2} = e^x \cos y$$

$$\frac{\partial u}{\partial y} = e^x(-\sin y) = -e^x \sin y$$

$$\frac{\partial^2 u}{\partial y^2} = -e^x(\cos y) = -e^x \cos y$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{d^2 u}{\partial y^2} = e^x \cancel{\cos y} - e^x \cancel{\cos y}$$

$$= 0$$

$\therefore u$ is a harmonic function.

(II) show that $u(x,y) = x^3 - 3xy^2$ is harmonic

Sol:- Given $u = x^3 - 3xy^2$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 ; \frac{\partial^2 u}{\partial x^2} = 6x$$

$$\frac{\partial u}{\partial y} = 0 - 3x(2y) ; \frac{\partial^2 u}{\partial y^2} = -6x$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6x - 6x = 0$$

$\therefore u$ is harmonic function.

Milne - Thomson Method

(1) $f(z) = u(x,y) + iv(x,y) = u + iv$

(2) $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

(3) In $f'(z)$, use CR Eq'n

(4) In $f'(z)$, Substitute x by z and y by $= 0$

(5) we get $f'(z)$ in terms of x.

(6) Integrate $f'(z)$, we get $f(z)$

problems

① find most general analytic function whose real part $u = x^2 - y^2 - x$

Sol:- Given, $u = x^2 - y^2 - x$

$$\frac{\partial u}{\partial x} = 2x - 1 \; ; \frac{\partial u}{\partial y} = -2y$$

$$f(z) = u + iv$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

using CR Eq'n

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \; ; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \Rightarrow \frac{\partial v}{\partial x} = \frac{-\partial u}{\partial y}$$

$$f'(z) = \frac{\partial u}{\partial x} + i\left(\frac{\partial v}{\partial x}\right)$$

$$= \frac{\partial u}{\partial x} + i\left(-\frac{\partial u}{\partial y}\right)$$

$$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$= 2x - 1 - i(-2y)$$

$$f'(z) = 2x - 1 + i(2y)$$

By Milne - Theorem method, put $x = z$ and $y = 0$

$$f'(z) = 2z - 1 + i(0)$$

$$= 2z - 1$$

integrate $f'(z)$, we get,

$$\int f'(z)dz = \int (2z - 1)dz$$

$$\Rightarrow f(z) = \int 2z\,dz - \int 1dz$$

$$= 2\frac{z^2}{2} - z + c$$

$$= z^2 - z + c$$

② Find regular function whose Imaginary part is $e^{-x}(x\cos y + y\sin y)$ and also find real part.

Sol:- Let $v = e^{-x}(x\cos y + y\sin y)$

$$= xe^{-x}\cos y + e^{-x}y\sin y$$

$$\frac{\partial v}{\partial x} = \cos y(1 \cdot e^{-x} - xe^{-x}) + y\sin y(-e^{-x})$$

$$= e^{-x}\cos y - \cos y\,xe^{-x} - e^{-x} \cdot y\sin y$$

$$\frac{\partial v}{\partial y} = xe^{-x}(-\sin y) + e^{-x}[1 \cdot \sin y + y(\cos y)]$$

$= -xe^{-x}\sin y + e^{-x}\sin y + e^{-x} y\cos y$

consider

$f(z) = u + iv$

$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

from C.R conditions,

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$

$= -xe^{-x}\sin y + e^{-x}\sin y + e^{-x}y\cos y$

$+ i[e^{-x}\cos y - xe^{-x}\cos y - e^{-x}y\sin y]$

By Milne Thomson Method, put $x=z, y=0$

$f'(z) = -ze^{-z}\sin 0 + e^{-z}\sin 0 + e^{-z}\, 0\cos 0$

$+ i[e^{-z}\cos 0 - z\, e^{-z}\cos 0 + e^{-z}\, 0\sin 0]$

$= i[e^{-z} \cdot z\; e^{-z} + 0]$

$= i[e^{-z} \cdot z\; e^{-z}]$

$f'(z) = ie^{-z}(1-z)$

Taking integration, on both sides,

$\int f'(z)\,dz = \int ie^{-z}(1-z)\,dz$

$f(z) = i\int(1-z)e^{-z}\,dz$

$1-z \qquad -1 \qquad -0$

$e^{-z} \qquad -e^{-z} \qquad e^{-z}$

$f(z) = i[-e^{-z}(1-z) + e^{-z}] + c$

$= i[-\cancel{e^{-z}} + ze^{-z} + \cancel{e^{-z}}] + c$

$= i\, ze^{-z} + c$

which is required regular function.

To find Real part:-

$f(z) = u + iv$ —① $z = x+iy$

$e^{ix} = \cos x + i\sin x$

$e^{-ix} = \cos x - i\sin x$

$f(z) = ize^{-z} + c$ —②

$\Rightarrow u + iv = ize^{-z} + c$

$\Rightarrow u + iv = i(x+iy)e^{-(x+iy)} + c$

$= i(x+iy) \cdot e^{-x} \cdot e^{-iy} + c$

$= i(x+iy) \cdot e^{-x}(\cos y - i\sin y] + c$

$= ie^{-x}[x\cos y - ix\sin y + iy\cos y - i^2 y\sin y]$

$+c$

$= ie^{-x}[x\cos y - ix\sin y + iy\cos y + y\sin y] + c$

$= e^{-x}[ix\cos y - i^2 x\sin y + i^2 y\cos y + iy\sin y] + c$

$= e^{-x}[ix\cos y + x\sin y - y\cos y + iy\sin y] + c$

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$= e^{-x}[(x\sin y - y\cos y) + i(x\cos y + y\sin y)] + c$

$= e^{-x}(x\sin y - y\cos y) + i e^{-x}(x\cos y + y\sin y) + c$

$= u(x,y) + i v(x,y)$

$u = e^{-x}(x\sin y - y\cos y)$

→ find the Analytic function, $f(z) = u + iv$, interms of z where, $u - v = (x-y)(x^2+4xy+y^2)$.

Sol'n:— Let $f(z) = u + iv$ — ①

$i f(z) = i(u + iv)$

$= iu + i^2 v$

$i f(z) = iu - v$ —②

①+② $f(z) + i f(z) = u + iv + iu - v$

$= f(z)(1+i) = (u - v) + i(u+v)$

$= (1+i) f(z) = (u - v) + i(u+v)$

let $U = u - v$; $V = u + v$

$\frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}$

$(1+i) f(z) = U + iV$; $\frac{\partial U}{\partial y} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}$

differentiating on both sides with respect to x

$(1+i) f'(z) = \frac{\partial U}{\partial x} + i\frac{\partial V}{\partial x}$

By CR Eq'n, $\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$, $\frac{\partial V}{\partial x} = -\frac{\partial U}{\partial y}$

$(1+i) f'(z) = \frac{\partial U}{\partial x} - i\frac{\partial U}{\partial y}$

$= \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}\right) - i\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}\right]$ — ③

$u - v = (x-y)(x^2+4xy+y^2)$

Differentiate w.r.to 'x'

$\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}\right) = (x^2+4xy+y^2) + (x-y)(2x+4y)$

$u - v = (x-y)(x^2+4xy+y^2)$

Differentiate w.r.to y

$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} = (-1)(x^2+4xy+y^2) + (x-y)(4x+2y)$

Substitute these values eqn ③

③ ⇒ $(1+i) f'(z) = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}\right) - i\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}\right]$

$= [x^2+4xy+y^2] + (x-y)(2x+4y)$

$- i[(-1)(x^2+4xy+y^2) + (x-y)(4x+2y)]$

$= (x^2+4xy+y^2) + (x-y)(2x+4y) + i(x^2+4xy+y^2)$

$- i(x-y)(4x+2y)]$

using Milne Thomson method

put $x = z$, & $y = 0$

$$(1+i) f'(z) = z^2 + z(2z) + i(z^2) - i(z)(4z)$$

$$= z^2 + 2z^2 + i(z^2) - i4z^2$$

$$= 3z^2 + i(-3z^2)$$

$$= 3z^2 - i3z^2$$

$$(1+i) f'(z) = 3z^2 (1-i)$$

$$f'(z) = \left(\frac{1-i}{1+i}\right)(3z^2)$$

integrate on both sides.

$$\int f'(z)\,dz = \frac{1-i}{1+i}\int 3z^2\,dz$$

$$\Rightarrow f(z) = \frac{1-i}{1+i}\cdot \cancel{3}\frac{z^3}{\cancel{3}} + c$$

$$\Rightarrow f(z) = \frac{1-i}{1+i}\cdot x^3 + c = \frac{(1-i)^2}{(1+i)(1-i)} z^3 + c$$

$$f(z) = \frac{(1-i)^2}{(1-i)(1+i)} z^3 + c$$

$$= \frac{1+i^2-2i}{1-i^2} z^3 + c$$

$$= \frac{\cancel{1}-\cancel{1}-2i}{1+1}\cdot z^3 + c$$

$$= \frac{-\cancel{2}i}{\cancel{2}}\cdot z^3 + c$$

$$f(z) = -i z^3 + c.$$

find the analytic function $f(z) = u + iv$, of $u - v = e^x(\cos y - \sin y)$.

Sol: Let $f(z) = u + iv$ —— ①

$$i f(z) = i[u + iv]$$

$$= iu - v \text{ —— ②}$$

① + ②

$$f(z) + i f(z) = u + iv + iu - v$$

$$(1+i) f(z) = (u-v) + i(u+v)$$

Let $U = u - v$; $V = u + v$

$$\frac{\partial U}{\partial x} = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}$$

$$\frac{\partial U}{\partial y} = \frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}$$

$$(1+i) f(z) = U + iV$$

$$(1+i) f'(z) = \frac{\partial U}{\partial x} + i\frac{\partial V}{\partial x}$$

C-R eq^n, $\frac{\partial U}{\partial x} = \frac{\partial V}{\partial y}$

$$\frac{\partial U}{\partial y} = -\frac{\partial V}{\partial x}$$

$$(1+i) f'(z) = \frac{\partial U}{\partial x} + i\frac{\partial V}{\partial x}$$

$$= \frac{\partial U}{\partial x} - i\frac{\partial U}{\partial y}$$

$= \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}\right) - i\left[\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y}\right]$

Given, $u - v = e^x(\cos y - \sin y)$

Diff w.r.to x

$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x} = e^x(\cos y - \sin y)$

Diff w.r.to y

$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial y} = e^x(-\sin y - \cos y)$

$(1+i)f'(z) = e^x(\cos y - \sin y) - i[e^x(-\sin y - \cos y)]$

$= e^x(\cos y - \sin y) + ie^x(\sin y + \cos y)$

By using Milne Thomson method

put $x = z$; $y = 0$

$(1+i)f'(z) = e^z(\cos 0 - \sin 0) + ie^z(\sin 0 + \cos 0)$

$(1+i)f'(z) = e^z(1-0) + ie^z(0+1)$

$(1+i)f'(z) = e^z + ie^z$

$\cancel{(1+i)}f'(z) = e^z\cancel{(1+i)}$

$\Rightarrow f'(z) = e^z$

integrate on both sides,

$= \int f'(z)\,dz = \int e^z\,dz$

$= f(z) = e^z + c$

→ Determine the analytic function $f(z)$ given that $3u + 2v = y^2 - x^2 + 16x$.

Sol— Given, $3u + 2v = y^2 - x^2 + 16x$

Diff w.r.to x

$\Rightarrow 3\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial x} = -2x + 16$ — ①

$3u + 2v = y^2 - x^2 + 16x$

Diff w.r.to y

$\Rightarrow \frac{3\partial u}{\partial y} + \frac{2\partial v}{\partial y} = 2y$ — ②

① $\Rightarrow 3\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial x} = -2x + 16$

② $\Rightarrow \frac{3\partial u}{\partial y} + 2\frac{\partial v}{\partial y} = 2y$.

CR Eq^n, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

① $\Rightarrow \frac{3\partial u}{\partial x} - \frac{2\partial v}{\partial y} = -2x + 16$ — ③

② $\Rightarrow \frac{3\partial u}{\partial y} + \frac{2\partial u}{\partial x} = 2y$ — ④

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$$③ \times 2 \Rightarrow \cancel{6\frac{\partial u}{\partial x}} - \frac{4\partial u}{\partial y} = -4x+32$$

$$④ \times 3 \Rightarrow \cancel{6\frac{\partial u}{\partial x}} + 9\left(\frac{\partial u}{\partial y}\right) = 6y$$

$$-13\frac{\partial u}{\partial y} = -4x+32-6y$$

$$\frac{\partial u}{\partial y} = \frac{1}{13}(4x - 6y + 32)$$

$$③ \times 3 \Rightarrow 9\frac{\partial u}{\partial x} - \cancel{6\frac{\partial u}{\partial y}} = -6x+48$$

$$④ \times 2 \Rightarrow 4\frac{\partial u}{\partial x} + \cancel{6\frac{\partial u}{\partial y}} = 4y$$

$$13\frac{\partial u}{\partial x} = -6x+48+4y$$

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{1}{13}[48 - 6x + 4y]$$

$$f(z) = u + iv$$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$= \frac{1}{13}[48 - 6x + 4y] + i\left(\frac{1}{13}(-4x - 6y + 32)\right]$$

from Milne Thomson Method,

put $x = z$; $y = 0$

$$i(z) = \frac{1}{13}[48 - 6z] + i\frac{1}{13}(-4z+32]$$

$$= \frac{1}{13}[48 - 6z + i(32 - 4z)]$$

$$f'(z) - \frac{1}{13}[48 - 6z + i(32 - 4z)]$$

$$\int f'(z)\,dz = \frac{1}{13}\int (48 - 6z) + i(32 - 4z)\,dz$$

$$= \frac{1}{13}\left[48 - \cancel{6}^{3}\frac{z^2}{\cancel{2}} + i\left(32z - 4\frac{z^2}{2}\right]\right.$$

$$f(z) = \frac{1}{13}\left[48z - 3z^2 + i(32z - 2z + c)\right] + c$$

complex integration

→ Evaluate $\int_0^{1+i} (x^2 - iy)\,dz$ along the path

(i) $y = x$ (ii) $y = x^2$.

Sol: Let $z = x + iy$

$$dz = dx + i\,dy$$

Given limits $z = 0 + i0$

$$\Rightarrow x = 0;\ y = 0$$

$$z = 1 + i$$

$$\Rightarrow x + iy = 1 + i$$

$$x = 1;\ y = 1$$

$$(1, 1)$$

$$\int_{(0,0)}^{(1,1)} (x^2 - iy)\,dz$$

(i) $y = x$

$dy = dx$

$$\int_{(0,0)}^{(1,1)} x^2 - i$$

$$\int_{(0,0)}^{(1,1)} (x^2 - iy)\,dz \qquad \begin{matrix} y = x \\ dy = dx \end{matrix}$$

$$= \int_{x=0}^{1} (x^2 - iy)(dx + i\,dy)$$

$$= \int_{x=0}^{1} (x^2 - ix)(dx + i\,dx)$$

$$= \int_{x=0}^{1} (x^2 - ix)\,dx\,(1+i)$$

$$= (1+i)\int_{x=0}^{1} (x^2 - ix)\,dx$$

$$= (1+i)\left[\int_{x=0}^{1} x^2\,dx - i\int_{x=0}^{1} x\,dx\right]$$

$$= (1+i)\left[\left(\frac{x^3}{3}\right)_0^1 - i\left(\frac{x^2}{2}\right)_0^1\right]$$

$$; (1+i)\left[\left(\frac{1}{3}\right) - i\left(\frac{1}{2}\right)\right] = (1+i)\left(\frac{1}{3} - \frac{i}{2}\right)$$

(ii) Given $y = x^2$

$dy = 2x\,dx$

$$\int_{(0,0)}^{(1,1)} (x^2 - iy)\,dz$$

$$= \int_{x=0}^{1} (x^2 - iy)(dx + i\,dy)$$

$$= \int_{x=0}^{1} (x^2 - ix^2)(dx + i2x\,dx)$$

$$= \int_{x=0}^{1} x^2(1-i)\,dx\,(1+2xi)$$

$$= (1-i)\int_{x=0}^{1} x^2(1+2xi)\,dx$$

$$= (1-i)\left[\int_{x=0}^{1} x^2\,dx + i\int_{x=0}^{1} 2x^3\,dx\right]$$

$$= (1-i)\left[\left(\frac{x^3}{3}\right)_0^1 + i2\left[\frac{x^4}{4}\right]_0^1\right]$$

$$= (1-i)\left[\frac{1}{3} + 2i\frac{1}{4_2}\right] = (1-i)\left[\frac{1}{3} + \frac{i}{2}\right]$$

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→ Evaluate $\int_c (x+y)dx + x^2 y\,dy$ along $y = 3x$ between $(0,0)$ and $(3,9)$

Sol:— Let $z = x + iy$

$dz = dx + i\,dy$

Given $y = 3x$

$dy = 3dx$

x limits 0 to 3

$$\int_c (x+y)\,dx + x^2 y\,dy = \int_{x=0}^{3} \left[(x+3x)dx + x^2(3x)(3dx)\right]$$

$$= \int_{x=0}^{3} dx\left[x + 3x + 9x^3\right]$$

$$= \frac{x^2}{2} + \frac{3x^2}{2} + 9\cdot\frac{x^4}{4}\Big]_0^3$$

$$= 2x^2 + \frac{9x^4}{4}\Big]_0^3$$

$$= 2(3^2) + \frac{9\cdot 3^4}{4} = 18 + \frac{9(81)}{4}$$

$$= \frac{801}{4}$$

Cauchy's Integral theorem

If a function $f(z)$ is analytic on and with in a simple closed curve c, then $\int_c f(z)\,dz = 0$

Cauchy's Integral formula:—

Let $f(z)$ be an analytic function every where on and within a closed curve c, if $z = a$ is any point with in c, then

$$\int_c \frac{f(z)}{(z-a)}\,dz = 2\pi i f(a)$$

Cauchy's generalization of integral formula:—

If $f(z)$ is analytic on and within a simple closed curve and if $z = a$ is any point with in 'c' then,

$$\int \frac{f(z)}{(z-a)^{n+1}}\,dz = \frac{2\pi i}{n!} f^n(a)$$

→ Evaluate $\int_c \frac{e^z}{z-4} dz$ where c is $|z|=2$

Sol:- $z = x+iy$

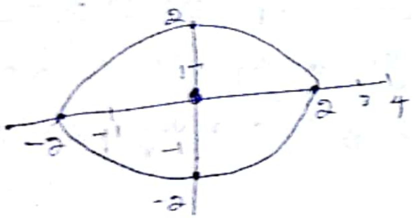
$|z| = \sqrt{x^2+y^2}$ $x^2+y^2 = r^2$

Given $|z| = 2$

$\sqrt{x^2+y^2} = 2$

$\Rightarrow x^2+y^2 = 2^2$

This is the circle with radius 2; origin 0



Given, $\int_c \frac{e^z}{z-4} dz$

which is in the form of

$\int_c \frac{f(z)}{z-a} dz$

where $f(z) = e^z$

and $a = 4$

$z = a$ i.e, $z = 4$ is out side the region, By Cauchy integral theorem,

$\int f(z)\, dz = 0$

i.e $\int \frac{f(z)}{z-4} = \int_c \frac{e^z}{z-4} dz = 0.$

→ Evaluate $\int \frac{dz}{z^8(z+4)}$ where c is the circle $|z|=2$

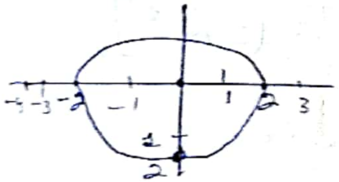
Sol Let $f(z) = 1/z+4$

$\int_c \frac{1/z+4}{(z-0)8} dz$

$|z| = 2$

$\sqrt{x^2+y^2} = 2$

$\Rightarrow x^2+y^2 = 2^2$ is a circle with radius 2, as center is $(0,0)$



$z = 0$

$z = -4$

f (z) is analytic at all the points inside C.

it is analytic except at $z=-4$ But this point is outside. the point $z=0$ is inside the circle

Hence by Cauchy integral formula.

$$\int_C \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} f^n(a)$$

from the problem,

$f(z) = 1/z+4$, $\quad z-a = z-0$ i.e, $a=0$

$$\int \frac{1/z+4}{(z-0)^8} dz$$

$$\Rightarrow \int \frac{\frac{1}{z+4}}{(z-0)^{7+1}} dz = \frac{2\pi i}{7!} \cdot f^7(0)$$

$$= \frac{2\pi i}{7!} \frac{d^7}{dz^7}(1/z+4)$$

$$\frac{d^n}{dx^n} f^n(x) = (-1)^n . n! [f(x)]^{n+1}$$

$$= \frac{2\pi i}{7!}(-1)^7\, 7! \left(\frac{1}{(z+4)}\right)^{7+1}$$

$$\int \frac{1/z+4}{z^8} dz = \frac{2\pi i}{7!} \cdot (-1)^7 . 7! \left[(z+4)^{-8}\right]_{z=0}$$

$$= -2\pi i (z+4)^{8} \Big]_{z=0}$$

put $z=0$

$$= -2\pi i \cdot \frac{1}{(0+4)^8}$$

$$= \frac{-2\pi i}{48}$$

$$= \frac{-\pi i}{32768}$$

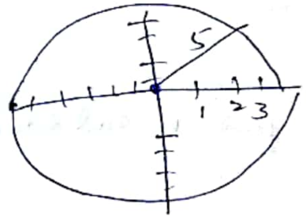
Evaluate $\int \frac{z^2+4}{z-3} dz$ where 'c' is (a) $|z|=5$. (b) $|z|=2$

Given $|z|=5$

Sol:

$\sqrt{x^2+y^2}=5$

$x^2+y^2=5^2$ is a circle with radius is 5, centre 0



Given $\int \frac{z^2+4}{z-3} dz$

let $f(z)=z^2+4$; $a=3$ is inside the circle

By cauchy's integral formula.

$$\int \frac{f(z)}{z-a} dz = 2\pi i f(a)$$

$$\int \frac{z^2+4}{z-3} dz = 2\pi i [z^2+4]_{z=a=3}$$

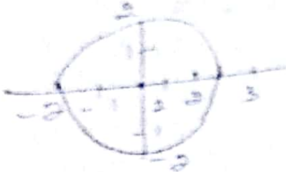
$$= 2\pi i [9+4]$$

$$= 26\pi i$$

(b) $|z| = 2$

$\sqrt{x^2+y^2} = 2$

$x^2+y^2 = 2^2$ is a circle with radius 2 & its centre 0



$$\int_c \frac{z^2+4}{z-3}\, dz$$

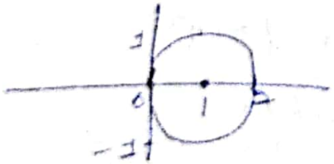
i.e., $f(z) = z^2+4$, $z = a = 3$ point is outside the circle

Hence 3 lie by Cauchy's theorem

$$\int_c \frac{z^2+4}{z-3}\, dz = 0$$

→ calculate $\int_c z^{-2}\, dz$ where c is $|z-1| = 1$

Sol:- given $|z-1| = 1$ is a circle with radius 1 as centre 1



$$\int z^{-2}\, dz = \int_c \frac{1}{z^2}\, dz = \int \frac{1}{(z-0)^{1+1}}\, dz$$

$f(z) = 1$; $a = 0$; $n = 1$

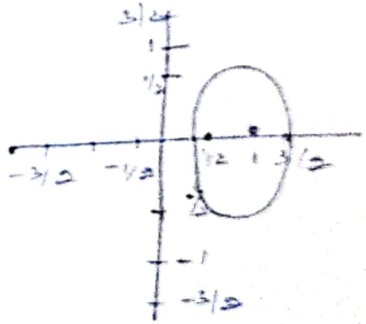
$z = a = 0$ is outside the circle $|z-1| = 1$

By Cauchy's theorem

$$\int \frac{1}{z^2}\, dz = 0$$

(ii) Evaluate $\int \frac{\log z}{(z-1)^2}\, dz$ where c is $|z-1| = 1/2$

Sol:- Given $|z-1| = 1/2$ is a circle with radius $1/2$ as centre 1



$f(z) = \log z$; $a = 1$ inside the circle

By Cauchy's integral formula

$$\int \frac{\log z}{(z-1)^{2+1}}\, dz = \frac{2\pi i}{2!} f^{2}(1)$$

$$= \pi i \frac{d^2}{dz^2}(\log z)_{z=1}$$

$$= \pi i \frac{d}{dz}\left(\frac{1}{z}\right)\Big]_{z=1}$$

$$= \pi i \left[-\frac{1}{z^2}\right]_{z=1}$$

$$= -\pi i$$