Mechanical properties of common Eng. materials:

Elasticity: The property by virtue of a material which regain its original shape and size on removal of load.

Ex: Crystals, metallic substances, rubber etc.

Plasticity : Ability of material undergo under some degree of permanent deformation without rupture or failure and regain its own shape on removal of

Ex: Clay, lead, copper, Hard steel

Strength: Ability of material to resist failure under the action of stresses caused by load.

Hardness : Ability of material to resist cutting penetration, abrassion, indentation, scratching.

Hardenability: Degree of hardness that can be impaired to a metal particularly steel by the process of hardening.

Brittleness: Property of breaking a material without much permanent distortion.

Ex: Raw material

Ductility : Property of material by virtue of which it can be drawn into thin wires.

Ex: Mild steel

Scanned by CamScanner

Malleability: Ability of material to be flatten into thin sheets without cracking.

ex: copper, lead, steel, Aluminium etc.

Creep: The slow and progressive deformation of a material with time and constant stress.

Fatigue: Failure of material due to repeated stresses.

Toughness: It is a measure of amount of energy a material can absorb

Stiffness: Resistance of material to elastic deformation or deflection

Flexibility: Opposite to stiffness

Resistance: Capacity of material to absorb energy elastically.

Wear: Failure of material under combined action of abrasion and impact.

Endurance: Capacity of material to withstand repeated stresses. The maximum value is known as

Scanned by CamScanner

Stress : The force resistance per unit area offered by a body against deformation is known as stress.

$$\sigma = \frac{F}{A}$$ F – External force or Load
A – Cross sectional area

Units : N/m^2

Stress types (Types of stress):

Stress making normal stress (or) shear stress

(1) Normal stress : Normal stress is a stress which acts in a direction $\perp^{lar}$ to the area. It is further divided into tensil & compressive stress

(i) Tensile stress : The stress induced in a body when subjected to two equal and opposite pulls as a result of which there is an increase in length is known as tensile stress.

The ratio of increase in length to the original length is known as tensile strain.

(ii) Compressive stress : The stress induced in a body when subjected to two equal and opposite pushes as a result of which there is decrease in length of body is known as compressive stress.

The ratio of decrease in length to original length is called compressive strain.

(2) Shear stress : The stress induced in a body when subjected to equal and opposite forces which are acting tangentially across the

Scanned by CamScanner

resisting section as a result of which the body tends to shear of across the section is known as shear stress.

$$\tau = \frac{\text{shear resistance}}{\text{shear area}}$$

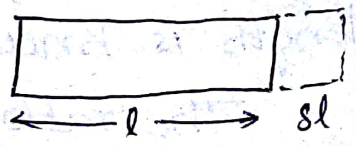
Strain : When the body is subjected to some external force, there is some change of dimensions of body. The ratio of change in dimension to the original dimension is known as strain.

$$\text{Strain} = \frac{\text{change in length}}{\text{Original length}} = \frac{\delta l}{l}$$

Types of strain :

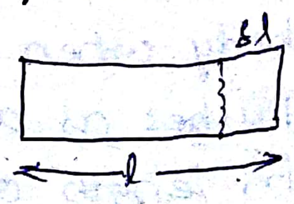
Tensile strain : The ratio of increase in length to original length is called tensile strain.

$$\text{Tensile strain} = \frac{l + \delta l}{l}$$



compressive strain : The ratio of decrease in length to original length is called compressive strain.

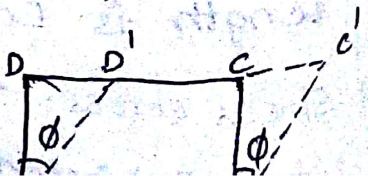
$$\text{Compressive strain} = \frac{l - \delta l}{l}$$



* Shearing strain :

Shear strain will be produced which will be measured by the angle through which the body distorts.

$$e_s = \frac{CC'}{BC} = \tan\phi$$



Hooke's law (or) Elastic modulli :

stress $\propto$ strain

$$\frac{\text{stress}}{\text{strain}} = \text{constant} = \text{Young's modulus.}$$

Modulus of Elasticity (or) Young's modulus :

$$E = \frac{\text{tensile stress } (\sigma)}{\text{tensile strain } (e)} \text{ (or) } \frac{\text{compressive stress}}{\text{compressive strain}}$$

Modulus of Rigidity (or) shear modulus

$$N = \frac{\text{shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

Factor of safety : The ratio of ultimate tensile stress to working stress (permissive stress)

Q) A square rod 20 mm x 20 mm is to carry an axial load of 100 KN. Calculate the shortening in a length of 50 mm. ($E = 2.14 \times 10^8$ K N/m^2)

sol:

$$\text{stress} = \frac{\text{Force}}{\text{area}} = \frac{100 \times 10^3}{20 \times 20 \times 10^{-3} \times 10^{-3}}$$

$$\text{strain} = \frac{\delta l}{l} = \frac{\delta l}{50}$$

$$E = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain} = \frac{\text{stress}}{E}$$

$$\frac{\delta l}{l} = \frac{\text{stress}}{E} \Rightarrow \delta l = \frac{\text{stress} \times l}{E}$$

$$= \frac{\dfrac{100 \times 10^3 \times 50 \times 10^{-3}}{20 \times 20 \times 10^{-3} \times 10^{-3}}}{2.14 \times 10^8 \times 10^3}$$

Scanned by CamScanner

$$\delta l = \frac{50}{4 \times 10^{-3}} \times \frac{1}{2.14 \times 10^{8}}$$

$$= \frac{50}{4 \times 2.14 \times 10^{5}}$$

Relation between stress and strain :

(1) One dimensional stress system

(2) Two dimensional stress system

(3) Three dimensional stress system

(1) One dimensional stress system : By hooke's law which states that the material is loaded with in the elastic limit, the normal stress developed is directly proportional to strain produced, which is the ratio of normal stress to corresponding strain is constant with in elastic limit.

$$E = \frac{\text{Normal stress}}{\text{Corresponding strain}}$$

(2) Two dimensional stress system :

i) longitudninal strain $= \frac{dl}{L}$

(ii) Lateral strain $= \frac{\delta d}{d}$ (or) $\frac{\delta b}{b}$

(iii) Poisson's ratio, $(\mu) = \frac{\text{Lateral strain}}{\text{Logitudinal strain}}$

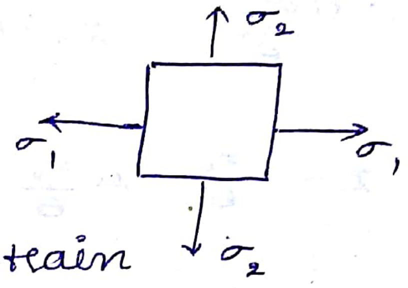
Scanned by CamScanner

lateral strain = $\mu \times$ longitudinal strain

* As, lateral strain is in opposite direction to the longitudinal strain, hence algebraically

lateral strain = $-\mu \times$ longitudinal strain

* Relation between stress and strain in 2D



let σ_1, σ_2 be the normal stresses in x & y direction.

→ The stress σ_1 will produce strain in the direction of x and also in the direction of y. whereas longitudinal strain in x direction and lateral strain in y direction

then, $x = \frac{\sigma_1}{E}$

$y = -\mu \frac{\sigma_1}{E}$

→ The stress σ_2 will produce strain in the x & y directions, whereas lateral strain in x direction and longitudinal strain y direction.

then, $x = -\mu \frac{\sigma_2}{E}$

$y = \frac{\sigma_2}{E}$

Now, total strain in the direction of x due to stresses σ_1 & σ_2 is $= \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$ //

Similarly, total strain in the direction of y due to stresses σ_1 & σ_2 is $= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$ //

Scanned by CamScanner

* Relation between stress and strain in 3D:

σ_1 | σ_2 | σ_3

$x = \sigma_1/E$ | $x = -\mu \frac{\sigma_2}{E}$ | $x = -\mu \frac{\sigma_3}{E}$

$y = -\mu \frac{\sigma_1}{E}$ | $y = \frac{\sigma_2}{E}$ | $y = -\mu \frac{\sigma_3}{E}$

$z = -\mu \frac{\sigma_1}{E}$ | $z = -\mu \frac{\sigma_2}{E}$ | $z = \frac{\sigma_3}{E}$

$$\therefore x = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

$$\therefore y = -\mu \frac{\sigma_1}{E} + \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E}$$

$$z = -\mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} + \frac{\sigma_3}{E}$$

② A rod of length 150 cm long and of diameter 2cm and force is 20 kN. If the modulus of elasticity of the rod is 2×10^5 N/m². Determine (i) stress, (ii) strain & (iii) Elongation of rod.

Sol: $L = 150$ cm

$D = 2$ cm

$P = 20$ kN

$E = 2 \times 10^5$ N/m²

$$\text{Stress} = \frac{F}{A} = \frac{(20) \times 10^3}{\pi r^2} = \frac{20 \times 10^3}{\pi (1)^2} = 63.6 \text{ N/mm}^2$$

$$E = \frac{\text{stress}}{\text{strain}} \Rightarrow \delta l = \frac{\text{stress}}{E} \times l$$

$$\delta l = \frac{63.6}{2 \times 10^5} \times 150$$

$$\text{strain} = \frac{\delta l}{l}$$

$$\text{strain} = \frac{63.6}{2 \times 10^5} \times \frac{\cancel{150}}{\cancel{150}}$$

$$\text{strain} = \frac{63.6}{2} \times 10^{-5}$$

② A tensile test was conducted on a mild steel bar and the following data obtain from the test.

(i) Diameter of steel bar 3cm

(ii) Gauge length of bar 20 cm

(iii) load of elastic limit is 250 kN

(iv) Extension at load of 150 kN is 0.21 mm. Max. load is 380 kN. Total extension is 60 mm, diameter of rod at failure is 2.25 cm. determine young's modulus(E) stress at elastic limit, percentage elongation, percentage dec in area.

Sol: WKT,

(i) Young's modulus, $e = \frac{\text{stress}}{\text{strain}}$

$$\text{Stress} = \frac{F}{A} = \frac{150}{\pi r^2} = \frac{150 \times 10^3}{\pi \times (1.5)^2 \times 10^{-4}}$$

$$(\because A = 7.06 \times 10^{-4}\, m^2)$$

$$\Rightarrow \text{stress}(\sigma) = 21220.9 \times 10^4 \; N/m^2$$

$$\text{strain} = \frac{0.21 \times 10^{-3}}{20 \times 10^{-2}}$$

$$e = 21220.9 \times 10^4 \times \frac{20 \times 10}{0.21} = 202.095 \; GN/m^2$$

(ii) stress at elastic limit $= \frac{\text{load at E.L}}{\text{Area}} = \frac{250 \times 10^3}{7.06 \times 10^{-4}}$

$= 353.6 \; MN/m^2$

Scanned by CamScanner

(iii) % Elongation $= \frac{\text{Total ↑se in length}}{\text{Original length}} \times 100$

$$= \frac{\cancel{60}^{3} \times 10^{-3}}{\cancel{20} \times 10^{-2}} \times 100$$

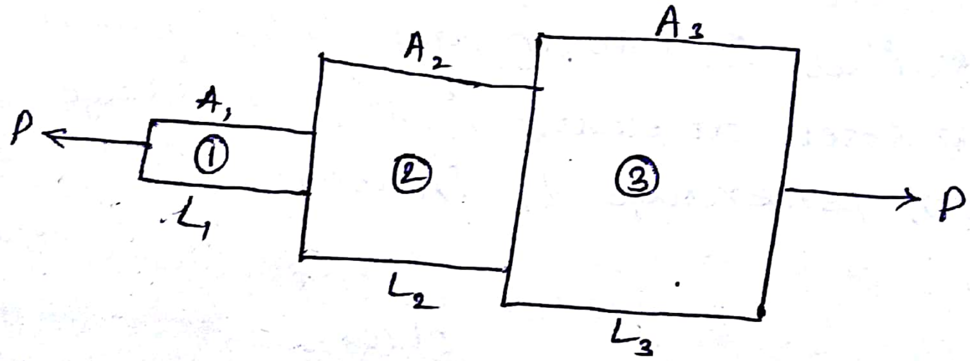
$$= 30\,\%$$

(iv) % decrease in area $= \frac{\text{Original area} - \text{Area of failure}}{\text{Original area}}$

$$= \frac{7.06 \times 10^{-4} - \pi (1.12 \times 10^{-2})^2}{7.06 \times 10^{-4}}$$

$$= \cancel{353.6\ MN/m^2}\ \ 43.7\,\%$$

* **Analysis of Bars on Varying sections :**



Each section is subjected to same axial load P at the stresses and strains and change in length will be difference.

→ The total change in length will be obtaining by adding changes in length of individual section.

Stresses for section ①, ② & ③ :

Stress, $\sigma_1 = \frac{P}{A_1}$

$$\sigma_2 = \frac{P}{A_2}$$

$$\sigma_3 = \frac{P}{A_3}$$

Scanned by CamScanner

strains for section ①, ② & ③ :

$$\text{strain}, e_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E} \quad \text{—} ①$$

$$e_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E} \quad \text{—} ②$$

$$e_3 = \frac{\sigma_3}{E} = \frac{P}{A_3 E} \quad \text{—} ③$$

$$e_1 = \frac{dL_1}{L_1} \quad \text{—} ④$$

$$e_2 = \frac{dL_2}{L_2} \quad \text{—} ⑤$$

$$e_3 = \frac{dL_3}{L_3} \quad \text{—} ⑥$$

comparing ①, ②, ③ with ④, ⑤, ⑥

we get,

$$\frac{P}{A_1 E} = \frac{dL_1}{L_1} \; ; \; \frac{P}{A_2 E} = \frac{dL_2}{L_2} \; ; \; \frac{P}{A_3 E} = \frac{dL_3}{L_3}$$

Total change in length of bar

$$dL = dL_1 + dL_2 + dL_3$$

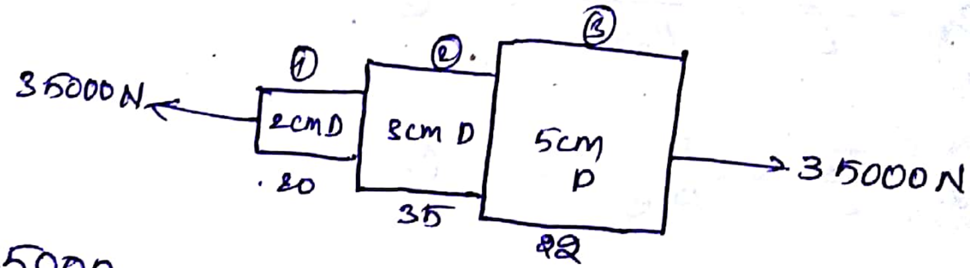
$$= \frac{P}{A_1 E} \cdot L_1 + \frac{P}{A_2 E} \cdot L_2 + \frac{P}{A_3 E} \cdot L_3$$

$$= \frac{P}{E} \left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right)$$

Scanned by CamScanner

ⓐ An axial pull of 35000 N is acting on a bar consisting of three lengths. if the youngs modulus is 2.1×10^5 N/mm². Determine stresses in each section & total extension of bar.

sol:



$$\sigma_1 = \frac{35000}{\pi (1 \times 10^{-2})^2} = 111.4 \text{ N/mm}^2$$

$$\sigma_2 = \frac{35000}{\pi (1.5 \times 10^{-2})^2} = 49.51 \text{ N/mm}^2$$

$$\sigma_3 = \frac{35000}{\pi (2.5 \times 10^{-2})^2} = 11.8 \text{ N/mm}^2$$

strains:

$$e_1 = \frac{\sigma_1}{E} = \frac{111.4}{2.1 \times 10^5}$$

$$dL_1 = \frac{111.4 \times 20 \times 10^{-2}}{2.1 \times 10^5} =$$

$$dL_2 = \frac{49.51}{2.1 \times 10^5} \times 35 \times 10^{-2} =$$

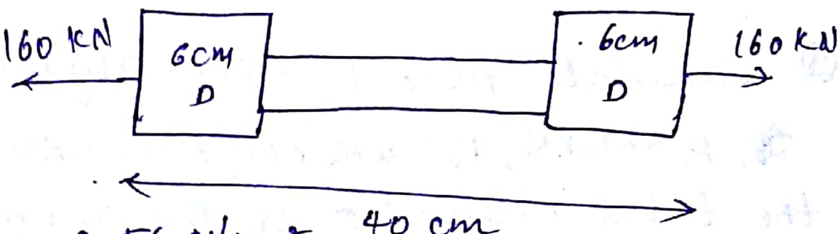
$$dL_3 = \frac{11.8}{2.1 \times 10^5} \times 22 \times 10^{-2} =$$

$$dL = dL_1 + dL_2 + dL_3$$

$$=$$

② A bar is subjected to tensile load of 160 KN. If the strength in the middle portion is limited to 150 N/mm^2. Determine the diameter of middle portion. Find the length of middle portion, if the total elongation of bar is 0.2mm ($E = 2.1 \times 10^5\ N/mm^2$)

Sol:



$$\sigma_1 = \frac{160 \times 10^3}{\pi (3 \times 10)^2} = 0.56\ N/mm^2$$

$$\sigma_3 = \frac{160 \times 10^3}{\pi (3 \times 10)^2} = 0.56\ N/mm^2$$

$$\sigma_2 = 150\ N/mm^2$$

$$\Rightarrow \frac{160 \times 10^3}{\pi (r^2)} = 150$$

$$r^2 = \frac{160 \times 10^3}{\pi \times 150} = 339.70 \Rightarrow r = 18.43$$

$$\therefore D = 36.86\ mm$$

$L_1 + L_2 + L_3 = 40$

$(\because L_1 = L_3)$

$2L_1 + L_2 = 40$

$L_2 = 40 - 2L_1$

$$dL_3 + dL_1 = \frac{0.56}{2.1 \times 10^5} \times 2L_1$$

$$dL_2 = \frac{\cancel{0.56}(150)}{2.1 \times 10^5} \times (40 - 2L_1)$$

$$dL = dL_1 + dL_2 + dL_3$$

$$0.2 = \frac{0.56}{2.1 \times 10^5} \times 2L_1 + \frac{150}{2.1 \times 10^5}(40 - 2L_1)$$

Scanned by CamScanner

$$0.2 = \frac{1}{2.1\times10^5}\left[1.12L + 150(40-2L_1)\right]$$

$$2.1\times10^5 \times 0.2 = 1.12L + 6000 - 300L_1$$

$$0.44\times10^5 = 6000 - 298.88L_1$$

$$298.88L_1 = 6000 - 44.$$

③ Calculate force P_2 necessary for equilibrium. If $P_1 = 45\,kN$, $P_3 = 450\,kN$, $P_4 = 130\,kN$. Determine the the total elongation of the member assuming young's modulous as $2.1\times10^5\,N/m^2$.

Sol:

$P_1 = 45\,kN$

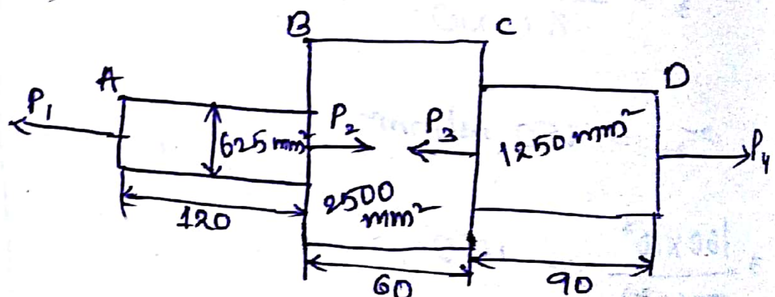
$L_1 = 120\,cm$

$L_2 = 60\,cm$

$P_3 = 450\,kN$

$L_3 = 90\,cm$

$P_4 = 130\,kN$



$$P_1 + P_3 = P_2 + P_4$$

$$45 + 430 = P_2 + 130$$

$$P_2 = 365\,kN \;//$$

Increase in length AB.

$$\frac{P_1L_1}{A_1E} = 0.41\,mm$$

decrease in length BC,

$$\frac{P_2 L_2}{A_2 E} = 0.36 \text{ mm}$$

increase in length CD

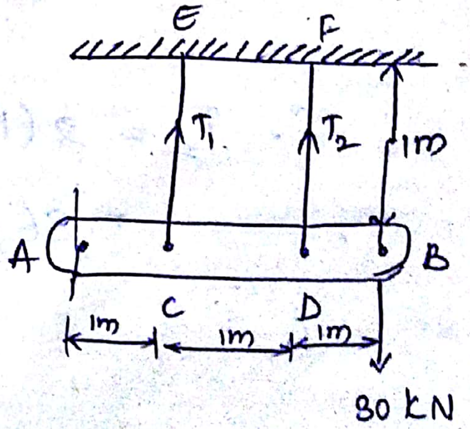
$$\frac{P_3 L_3}{A_3 E} = 0.44 \text{ mm}$$

Now total change in length is,

$= 0.41 - 0.36 + 0.44$

$= 0.49$ mm //

④ A rigid body (bar) ACBD is pinjed at A and supported in a horizontal position by 2 identical steel wires. A vertical load of 30 kN is applied at B. Find the tensile forces T_1 & T_2 induced in these wires by vertical loads.



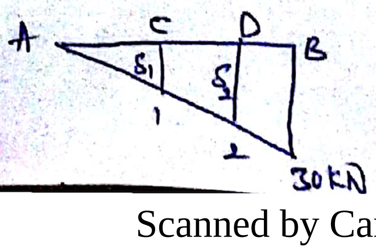
Sol:

$A_1 = A_2$

$L_1 = L_2$

$E_1 = E_2$

$$\frac{\delta_1}{\delta_2} = \frac{AC}{AD}$$



Scanned by CamScanner

$$\frac{\delta_1}{\delta_2} = \frac{1}{2}$$

$$\boxed{\delta_2 = 2\delta_1}$$

$$\delta_1 = EC = \frac{P_1 L_1}{A_1 E_1} = \frac{T_1 L_1}{A_1 E_1}$$

$$\delta_2 = FD = \frac{P_2 L_2}{A_2 E_2} = \frac{T_2 L_2}{A_2 E_2}$$

$$2\left(\frac{T_1 \cancel{L_1}}{\cancel{A_1}\cancel{E_1}}\right) = \frac{T_2 \cancel{L_2}}{\cancel{A_2}\cancel{E_2}} \qquad (\because \delta_2 = 2\delta_1)$$

$$\boxed{2T_1 = T_2} \qquad (\because A_1 = A_2,\ E_1 = E_2,\ L_1 = L_2)$$

$$T_1 \times 1 + T_2 \times 2 = 30 \times 3 \qquad \text{(upward forces = downward forces)}$$

$$T_1 + 2T_2 = 90$$

$$T_1 + 2(2T_1) = 90 \qquad (\because T_2 = 2T_1)$$

$$5T_1 = 90$$

$$T_1 = 18$$

$$\therefore T_2 = 2(18) = 36$$

* **Analysis of uniformly tapered circular rod:**

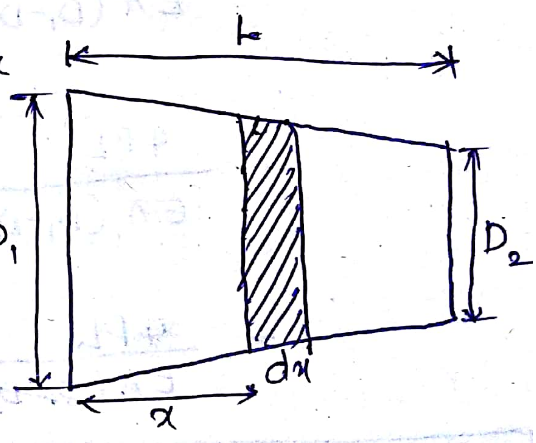
A bar uniformly tapered from a diameter D_1 at one end to the diameter D_2 at other end as shown.

→ 'P' is axial tensile load on the bar. 'L' is total length of the bar and 'E' is young's modulus.

→ consider a small element of length 'dx' of the bar at a distance 'x' from left end.

→ Let the diameter of bar be D_x at a distance x from left end, then

$$D_x = D_1 - \frac{(D_1 - D_2)}{L}x$$



$$D_x = D_1 - kx$$

$$A_x = \frac{\pi}{4} D_x^2$$

$$= \frac{\pi}{4}(D_1 - kx)^2$$

$$\text{Stress} = \frac{P}{A_x} = \frac{P}{\frac{\pi}{4}(D_1 - kx)^2} = \frac{4P}{\pi(D_1 - kx)^2}$$

$$\text{strain} = \frac{\text{stress}}{E} = \frac{4P/\pi(D_1 - kx)^2}{E}$$

$$= \frac{4P}{E\pi(D_1 - kx)^2}$$

$$\text{Total extension} = \int_0^L \frac{4P}{E\pi} \cdot \frac{1}{(D_1 - kx)^2}$$

$$= \frac{4P}{E\pi} \int_0^L \frac{1}{(D_1 - kx)^2}$$

multiply & divide with '$-k$'

Scanned by CamScanner

$$= \frac{4P}{E\pi} \cdot \frac{1}{-K} \int_0^L (D_1 - Kx)^{-2} \cdot (-K)$$

$$= \frac{-4P}{EK\pi} \left[\frac{(D_1 - Kx)^{-2+1}}{-2+1} \right]_0^L \qquad \left(\because f^n(x) \cdot f'(x) = \frac{f^{n+1}(x)}{n+1} \right)$$

$$= \frac{4P}{EK\pi} \left[(D_1 - Kx)^{-1} \right]_0^L$$

$$= \frac{4P}{EK\pi} \left[\frac{1}{(D_1 - KL)} - \frac{1}{(D_1)} \right]$$

$$= \frac{4PL}{E\pi(D_1 - D_2)} \left[\frac{1}{D_1 - \left(\frac{D_1 - D_2}{\cancel{L}}\right)\cancel{L}} - \frac{1}{D_1} \right]$$

$$\left(\because K = \frac{D_1 - D_2}{L} \right)$$

$$= \frac{4PL}{E\pi(D_1 - D_2)} \left[\frac{1}{D_2} - \frac{1}{D_1} \right]$$

$$= \frac{4PL}{E\pi \cancel{(D_1 - D_2)}} \left(\frac{\cancel{(D_1 - D_2)}}{D_1 D_2} \right)$$

$$\boxed{\text{Total extension} = \frac{4PL}{E\pi} \left(\frac{1}{D_1 D_2} \right)}$$

//

Q) Find the modulus of elasticity for a rod which tappered uniformly from 30 mm to 15 mm diameter in a length of 350 mm a rod is subjected to an axial load of 5.5 kN and extension of rod is 0.25 mm

Sol:

$D_1 = 30$ mm

$D_2 = 15$ mm

$P = 5.5$ kN

$L = 350$ mm

Scanned by CamScanner

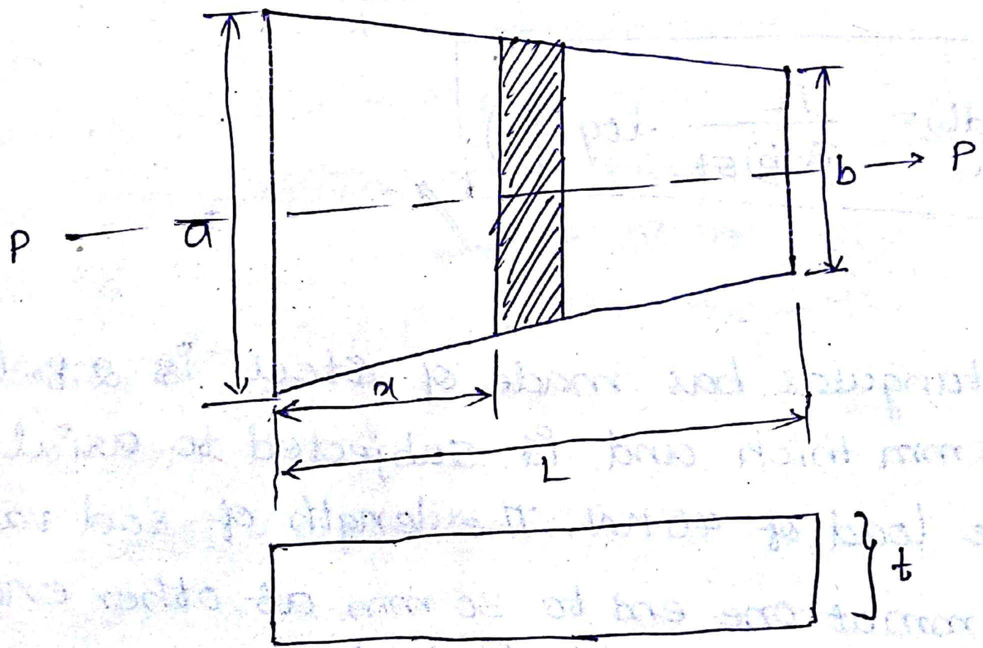
WKT,

$$\text{Total extension} = \frac{4PL}{E\pi}\left(\frac{1}{D_1 D_2}\right)$$

$$0.25 = \frac{4(5.5)\times10^3\times350}{E\pi}\left(\frac{1}{30\times15}\right)$$

$$\therefore E = \frac{4\times5.5\times10^3\times350}{0.25(\pi)}\left(\frac{1}{30\times15}\right)$$

* Analysis of uniformly tapered Rectangular Bar:



$$w = a - \frac{(a-b)}{L}x$$

thickness $= t$

Area = width x thickness

$$= \left[a - \frac{(a-b)}{L}x\right]t$$

$$\text{Stress} = \frac{P}{A} = P\Big/\left(a - \frac{(a-b)}{L}x\right)\times t$$

$$\text{strain} = \frac{\sigma_1}{E}$$

$$= \frac{P/\left[a-\frac{(a-b)}{L}x\right]t}{E}$$

$$\text{Total extension} = \int_0^L \frac{P}{\left[a-\frac{(a-b)}{L}x\right]Et}\,dx$$

$$= \frac{P}{Et}\int_0^L \frac{1}{(a-kx)}\,dx \qquad \left(\because k = \left(\frac{a-b}{L}\right)\right)$$

$$= \frac{P}{Et}\left[\log(a-kx)\cdot\left(-\frac{1}{k}\right)\right]_0^L$$

$$= \frac{-P}{kEt}\log\left(\frac{a-kL}{a}\right) = \frac{PL}{(a-b)Et}\left[\log a - \log\left(a-\frac{(a-b)}{L}L\right)\right]$$

$$\boxed{\text{total extension},\ (dL) = \frac{PL}{(a-b)Et}\cdot\log\left(\frac{a}{b}\right)}$$

Q) A rectangular bar made of steel is 2.8m long and 15 mm thick and is subjected to axial tensile load of 40 kN. The length of rod varies of 75 mm at one end to 30 mm at other end. Find extension if $E = 2\times10^5\ N/mm^2$.

Sol: WKT,

$$\text{Total extension}, dL = \frac{PL}{Et(a-b)}\log\left(\frac{a}{b}\right)$$

Given,

$P = 40\ kN$

$a = 75\ mm$

$b = 30\ mm$

$L = 2.8\ m$ & $E = 2\times10^5\ N/mm^2$

from,

total extension, $\delta L = \frac{PL}{Et(a-b)} \cdot \log\left(\frac{a}{b}\right)$

$$= \frac{40\times10^3 \times 2.8\times10^3}{2\times10^5 \times 15(75-30)} \log\left(\frac{75}{30}\right)$$

$$\delta l \simeq 0.76 \text{ mm}$$

② The extension in a rectangular steel bar of length 400mm and thickness 10mm is found to be 0.21 mm. The bar tapers uniformly in width from 100 mm to 50 mm, If $E = 2\times10^5$ N/mm², determine axial load on the bar.

Sol: Given,

$dL = 0.21$ mm

$L = 400$ mm

$t = 10$ mm

$a = 100$ mm , $b = 50$ mm

$E = 2\times10^5$ N/mm²

WKT,

$$dL = \frac{PL}{Et(a-b)} \log\left(\frac{a}{b}\right)$$

$$P = \frac{(dL).Et(a-b)}{L} \times \frac{1}{\log(a/b)}$$

Scanned by CamScanner

$$P = \frac{0.21 \times 2 \times 10^5 \times 10\ (100-50)}{400} \times \frac{1}{\log\left(\frac{100}{50}\right)}$$

$$P = \frac{0.42 \times 10^6 (50)}{400} \times \frac{1}{\log 2}$$

$$P = 174401.22 \ N/mm$$

$$P = 174.4 \ kN/mm$$

* Analysis of bars of composite section :

A bar made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to axial tensile or compressive load is called as composite bar.

→ The extension (or) compression in each bars is equal. Hence, deformation per unit length i.e, strain in each bar is equal.

→ The total external load on composite bar is equal to sum of loads carried by each different material.

→ P = total load

L = length of composite bar & lengths of bars of different material.

A_1 & A_2 = Area of cross section of bar 1 & bar 2

E_1 & E_2 are Young's modulus of bar 1 & bar 2

P_1 & P_2 are loads sheared by bar 1 & bar 2

σ_1 & σ_2 are stresses inducing in bar 1 & bar 2

Scanned by CamScanner

Now,

Total load on composite bar = sum of load carried by the 2 bars

$$P = P_1 + P_2 \quad \text{—(1)}$$

stress in bar 1 = $\dfrac{\text{load carried by bar 1}}{\text{Area of cross section of bar 1}}$

$$\sigma_1 = P_1/A_1$$

$$P_1 = \sigma_1 A_1 \quad \text{—(2)}$$

lly, $\sigma_2 = P_2/A_2$

$$P_2 = \sigma_2 A_2 \quad \text{—(3)}$$

$$\therefore \boxed{P = \sigma_1 A_1 + \sigma_2 A_2} \quad \text{—(4)}$$

strain in bar 1 = $\dfrac{\text{stress in bar 1}}{\text{Young's modulus}}$

$$e_1 = \frac{\sigma_1}{E_1}$$

Similarly, $e_2 = \dfrac{\sigma_2}{E_2}$

but, strain in bar 1 = strain in bar 2

$$\therefore \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad \text{—(5)}$$

→ The stresses σ_1 & σ_2 can be determined from eqn (4) & (5)

<u>Modular ratio</u>: The ratio of E_1 & E_2

i.e, Modular ratio = $\dfrac{E_1}{E_2}$

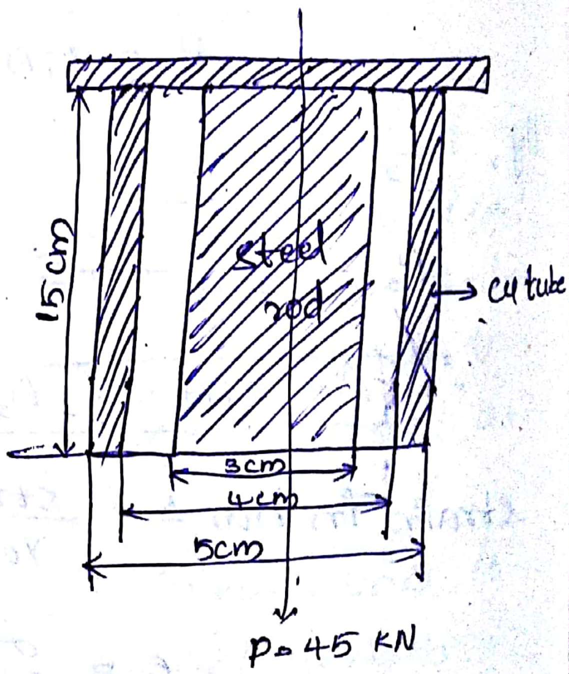
Scanned by CamScanner

(a) A steel rod of 50 diameter is enclosed centrally in a hollow copper tube of external diameter 5cm & internal diameter 4cm. Composite bar is then subjected to an axial pull of 45 kN. If the length of each bar is 15 cm. Determine the stresses in the rod & tube, load carried by each bar. Take E for steel is 2.1×10^5 N/mm² & E for copper is 1.1×10^5 N/mm².

Sol:

$$P = P_s + P_c$$

$$45kN = \sigma_s A_s + \sigma_c A_c \quad \text{—} ①$$



$$A_s = \frac{\pi}{4}(d_s^2) = \frac{\pi}{4}(3)^2$$

$$= 7.065 \text{ cm}^2$$

$$A_c = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(5^2 - 4^2)$$

$$= \frac{\pi}{4}(9)$$

$$= 7.065 \text{ cm}^2$$

$$① \Rightarrow 45 \times 10^3 = 7.065(\sigma_s + \sigma_c)$$

$$\sigma_s + \sigma_c = \frac{45 \times 10^3}{7.065} = 6369.42 \text{ N/cm}^2$$

from,

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{\sigma_c}{E_c} \times E_s$$

$$\sigma_s = \frac{\sigma_c}{1.1 \times \cancel{10^5}} \times 2.1 \times \cancel{10^5}$$

$$\sigma_s = 1.90\, \sigma_c$$

Now,

$$1.90\,\sigma_c + \sigma_c = 6369.42 \text{ N/cm}^2$$

$$2.9\,\sigma_c = \frac{6369.42}{(10)^2} \text{ N/mm}^2$$

$$2.9\,\sigma_c = 63.69$$

$$\sigma_c = 21.96 \text{ N/mm}^2$$

$$\sigma_s = 63.69 - 21.96$$

$$\sigma_s = 41.73 \text{ N/mm}^2$$

$$P_s = \sigma_s A_s = (41.73)(7.065 \times 10^{+2}) \text{mm}^2 \cdot \text{N/mm}^2$$

$$\therefore P_s = 294.82 \times 10^2 \text{ N}$$

$$P_c = \sigma_c A_c = (21.96)(7.065 \times 10^2)$$

$$\therefore P_c = 155.14 \times 10^2 \text{ N}$$

Thermal stresses:

Thermal stresses are stressed induced in the body due to change in temperature.

→ Thermal stresses are set up in a body when the temperature of the body is raised (or) lowered on a body is not allowed to expand (or) contact freely.

L → Original length

T → Raise in temp.

E → Young's modulus

$\alpha \rightarrow$ coefficient of linear expansion

$\delta l \rightarrow$ extension of rod due to raise in temp.

If the rod is free to expand then the expression is given by.

$$\boxed{\delta L = \alpha T L} \text{ —— } (1)$$

Suppose an external compressive load 'P' is applied at B'. so that the rod is decreased in its length from $(L+\delta l)$ to L

The compressive strain $= \dfrac{\text{decrease in length}}{\text{original length}}$

$$e = \frac{\delta l}{L+\delta l}$$

$$e = \frac{\alpha T L}{L+\alpha T L} \quad (\because \text{ from } (1))$$

$$\therefore e = \alpha T$$

$$E = \frac{\sigma}{e}$$

$$\sigma = E \times e$$

$$= \alpha T E$$

and load on the rod = stress x Area

$$P = \alpha T E \times A$$

→ If the ends of the body are fixed to rigid support so that its expansion is prevented, then compressive stress & strain will be set up in the rod, these stresses and strains are known as thermal stresses & thermal strains.

Scanned by CamScanner

Thermal strain, $e = \frac{\text{extension prevented}}{\text{original length}} = \frac{dL}{L}$

Thermal strain $(e) = \frac{\alpha T L}{L} = \alpha T$

Thermal stress, $(\sigma) = \text{strain} \times E$

$= \alpha T \times E$

→ Thermal stress is also known as temp. stress & Thermal strain is known as temp. strain.

→ stress & strain when its supports yield by an amount equal to 'δ' then the actual expansion is equal to = expansion due to raise in temp. - Heat (E

$= \alpha T L - \delta$

actual strain $= \frac{\text{actual expansion}}{\text{Original Length}} = \frac{\alpha T L - \delta}{L}$

actual stress = actual strain $\times E = \frac{\alpha T L - \delta}{L} \times E$

(Q) A rod is 2m long at a temp. of 10°c. find expansion of the rod when the temp. is raised to 80°c. If these expansion is prevented find stress induced in the material of the rod. Take young's modulus as 1.0×10^5 MN/m² & $\alpha = 0.000012$ per °c.

sol: length of rod = 2m

$T_1 = 10°c$, $T_2 = 80°c$, $E = 1.0 \times 10^5$ MN/m²

$\alpha = 0.000012/°c$

expansion $= \alpha T E$

$= 0.000012 \times 70 \times 1.0 \times 10^5$

$= 0.168$ cm

Scanned by CamScanner

Thermal stress $= \alpha T E$

$= 84\ N/mm^2$

② A steel rod of 3 cm dia & 5cm long is connected to 2-grips and rod is maintained at a temp. of 95°C. Determine the stress and pull exerted, when the temp. falls to 30°C. if the ends do not yield & the ends yield by 0.12 cm. $E = 2 \times 10^5\ MN/m^2$, $\alpha = 12 \times 10^{-6}/°C$.

sol: Given,

$L = 5cm$

$T_1 = 95°;\ T_2 = 30°C$

Thermal stress $= \alpha T E$

$= 12 \times 10^{-6} \times 65 \times 2 \times 10^5$

$= 156 \times 10^6\ N/m^2$

$\sigma = P/A \Rightarrow P = \sigma \times A$

$= 156 \times 10^6 \times \frac{\pi}{4}(3)^2$

stress after yield $= \left(\frac{\alpha TL - \delta}{L}\right) \times E$

$= ?$

* Definitions :

1) strain energy:- The energy which is stored in a body due to straining effect is known as strain energy.
2) Resiliance: The total S.E stored in a body.
3) Proof resiliance: The max. S.E stored in a body.
4) Modulus of resiliance: It is ratio of proof resiliance to volume of body.

Scanned by CamScanner

* Expression for S.E stored in a body when load is applied gradually :

S.E stored in a body is equal to workdone by the applied load in stretching the body. The tensile load 'P' increases gradually from '0' to value of 'P' and the extension of the body increases from '0' to the value of 'x'.

→ The load 'P' performs work in stretching the body this work will be stored in the body as S.E which is recoverable after load 'P' is removed.

P → gradually applied load

x → extension of the body.

A → cross sectional area.

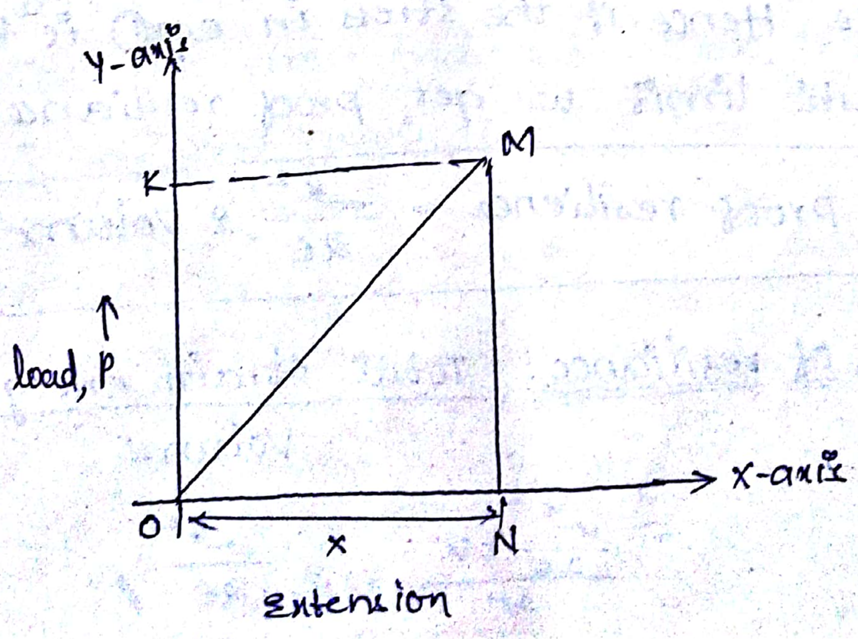
L → length of the body

V → Volume of body

E → Young's modulus

U → S.E stored in a body

σ → stress induced in a body.



Scanned by CamScanner

Work done by load = area of load extension curve

$= \text{area of } \triangle^{le}\ OMN$

$= \frac{1}{2} \times (ON) \times (MN)$

$= \frac{1}{2} \times (x) \times (P)$

$$\text{strain}(e) = \frac{\text{extension}(x)}{L}$$

$$x = e \times L$$

$$\boxed{x = \frac{\sigma}{E} \times L} \text{ --- (a)}$$

$$\sigma = \frac{P}{A}$$

$$\boxed{P = \sigma A} \text{ --- (b)}$$

$$\text{Work done} = \frac{1}{2} \times \frac{\sigma}{E} \times L \times \sigma \times A$$

$$= \frac{1}{2} \times \frac{\sigma^2}{E} \times V$$

→ But workdone by load is stretching the body is equal to S.E stored in the body.

∴ Energy stored in the body, $U = \frac{1}{2} \times \frac{\sigma^2}{E} \times V$

<u>Proof resiliance</u>: The maximum energy stored in a body without deformation is known as proof resilience. Hence if the stress in eqn(1) is taken at the elastic limit. we get proof resiliance

$$\boxed{\therefore \text{ proof resilience} = \frac{\sigma^{*2}}{2E} \times \text{volume}}$$

<u>Modulus of resiliance</u>: $= \frac{\text{Total strain energy}}{\text{Volume}}$

$$= \frac{\frac{\sigma^{*2}}{2E} \times V}{V} = \frac{\sigma^{*2}}{2E}$$

Scanned by CamScanner

* Expression for SE stored in a body when the load is applied suddenly :

When the load is applied suddenly to a body, the load is constant throughout the process of deformation of the body.

Work done by the load = Load × extension

$= P \times (x)$

The max. S.E stored in a body is given by

$$\boxed{U = \frac{\sigma^2}{2E} \times A \times L}$$

Equating S.E stored in a body to work done

$$\frac{\sigma^2}{2E} \times A \times L = P \times x$$

$$\frac{\sigma^{\cancel{2}}}{2\cancel{E}} \times A \times \cancel{L} = P \times \frac{\cancel{\sigma}}{\cancel{E}} \times \cancel{L}$$

$$\frac{\sigma}{2} A = P$$

$$\sigma = \frac{2P}{A}$$ //

from the above equation, it is clear that max. stress induced due to suddenly applied load is twice the stress induced when the same load is applied gradually.

Scanned by CamScanner

① A tensile load of 60 kN is gradually applied to a circular bar of 4 cm dia & 5 m long. If value of $E = 2\times10^5\ N/mm^2$. Determine the stretch in the rod, stress in the rod & strain in the rod.

Sol: $P = 60\ kN$

$L = 5\ m$

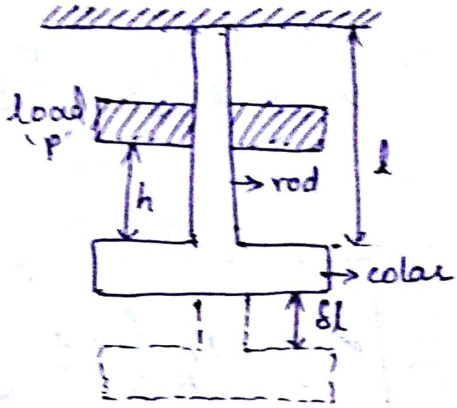
$E = 2\times10^5\ N/mm^2$

$d = 4\ cm$

$A = \frac{\pi}{4}(4)^2$

② Calculate instantaneous stress produced in bar $10\ cm^2$ area & 3 cm long by sudden application of tensile load of unknown magnitude. If the extension of bar due to suddenly applied load is 1.5 mm determine the suddenly applied load. $E = 2\times10^5\ N/mm^2$

* Expression when load applied with impact: for S.E stored in body



$E = \frac{\sigma}{e}$

$e = \frac{\sigma}{E}$

$\frac{dl}{l} = \frac{\sigma}{E}$

$dl = \frac{\sigma}{E} \times l$

work done = load × distance

$= P \times (h + dl)$ —— ①

strain energy $= \frac{1}{2} \frac{\sigma^2}{E} \times A \times l$ —— ②

Equating ① & ②

$P(h + dl) = \frac{1}{2} \frac{\sigma^2}{E} \times A \times l$

$P\left(h + \frac{\sigma}{E} \times l\right) = \frac{1}{2} \frac{\sigma^2}{E} \times A \times l$

$Ph + \frac{P\sigma}{E} \times l = \frac{1}{2} \frac{\sigma^2}{E} \times A \times l$

$\frac{1}{2} \frac{\sigma^2}{E} \times A \times l - \frac{P\sigma}{E} l - Ph = 0$

$\sigma^2 - \frac{2\cancel{E}}{A\cancel{l}}\left(\frac{P\sigma}{\cancel{E}} \times \cancel{l}\right) - Ph\left(\frac{2E}{Al}\right) = 0$

$\sigma^2 - \left(\frac{2P}{A}\right)\sigma - Ph\frac{2E}{Al} = 0$

Here, $a = 1$, $b = -\frac{2P}{A}$, $c = -\frac{Ph2E}{Al}$

(∵ by comparing above eqn. with $ax^2 + bx + c = 0$ to find the roots)

$$\sigma = \frac{\frac{2P}{A} \pm \sqrt{\frac{4P^2}{A^2} + 4\left(\frac{2PhE}{Al}\right)}}{2}$$

$$\sigma = \cancel{2}\left(\frac{\frac{P}{A} \pm \sqrt{\frac{P^2}{A^2} + \frac{2PhE}{Al}}}{\cancel{2}}\right)$$

$$\therefore \boxed{\sigma = \frac{P}{A}\left(1 \pm \sqrt{1 + \frac{2AhE}{Pl}}\right)}$$

Case (1): If δl is small

work done = energy stored (strain energy)

$$Ph = \frac{\sigma^2}{2E} \times Al$$

$$\sigma = \sqrt{\frac{2EPh}{Al}}$$ //

Case (2): when, $h = 0$

$$\sigma = \frac{2P}{A}$$ //

Ⓠ A weight of 10 kN falls by 30 mm on a collar rigidly attached to a vertical bar 4m long and 1000 mm² in section. Find the instantaneous extension of the bar. Take E = 210 G Pascals. Derive the formula

Given,

$P = 10\,kN = 10 \times 10^3\,N$

$h = 30\,mm = 30 \times 10^{-3}\,m$

$l = 4m$

$A = 1000 \times (10^{-3})^2\,m^2$; $E = 210 \times 10^9$ pascals

Scanned by CamScanner

WKT,

$$\sigma = \frac{P}{A}\left(1 \pm \sqrt{1 + \frac{2AhE}{Pl}}\right)$$

$$\sigma = \frac{10\times10^3}{1000\times10^{-6}}\left(1 \pm \sqrt{1 + \frac{2\times10^{-3}\times30\times10^{-3}\times210\times10^9}{10\times10^3\times4}}\right)$$

$$\sigma = \frac{10^4}{10^{-3}}\left(1 \pm \sqrt{1 + \frac{\cancel{2}\times\overset{15}{\cancel{30}}\times\overset{21}{\cancel{210}}\times\cancel{10^2}}{\cancel{10}\times\underset{2}{\cancel{4}}\times\cancel{10^2}}}\right)$$

$$\sigma = 10^7\left(1 \pm \sqrt{1 + 15\times21}\right)$$

$$\sigma = 10^7\left(1 \pm \sqrt{316}\right)$$

$$\sigma = 18.77\times10^7\ N/m^2$$

WKT,

$$\delta L = \frac{\sigma}{E}\times l$$

$$\delta l = \frac{18.77\times10^7}{210\times10^9}\times4$$

$$= 0.35\times10^{-2}\ m$$

② An unknown weight fall on through height 10mm on a collar rigidly attached to lower end of vertical bar 500 cm long and 600 mm² in section. If the max extension of rod is to be 2mm. What is corresponding stress and magnitude of unknown weight. ($E = 2\times10^5 N/mm^2$)

Sol: $h = 10mm$

$l = 500\ cm = 5000\ mm$

$A = 600\ mm^2$

$dl = 2\times10^{\cancel{3}}\cancel{m}mm = 2\ mm$

$E = 2\times10^5\ N/mm^2$

Scanned by CamScanner

$$\sigma = \frac{P}{A}\left(1 \pm \sqrt{1+\frac{2AhE}{Pl}}\right) \quad \text{—} ①$$

WKT, $\sigma = (e) \times E$

$$\sigma = \frac{\delta l}{l} \times E$$

$$\sigma = \frac{2}{5000} \times 2 \times 10^5$$

$$\sigma = \frac{4}{5} \times 10^2 = 0.8 \times 10^{+2} \ N/mm^2$$

from ①

$$0.8 \times 10^{+2} = \frac{P}{600}\left(1 \pm \sqrt{1+\frac{2(600)(10)(2\times10^5)}{P(5000)}}\right)$$

$$\frac{0.8}{1/600} \times 10^2 = P\left(1 \pm \sqrt{1+\frac{24\times10^5}{5P}}\right)$$

$$80 \times 600 = \frac{P}{\sqrt{P}}\left(\sqrt{P} \pm \sqrt{P+4.8\times10^5}\right)$$

$$80 \times 600 = \sqrt{P}\left(\sqrt{P} \pm \sqrt{P+4.8\times10^5}\right)$$

$$80 \times 600 = P \pm \sqrt{P^2+(4.8\times10^5)P}$$

$$48\times10^3 - P = \pm\sqrt{P^2+(4.8\times10^5)P}$$

$$(48\times10^3-P)^2 = P^2+(4.8\times10^5)P$$

$$2304\times10^6 + \cancel{P^2} - 96\times10^3 P = \cancel{P^2} + 4.8\times10^5 P$$

$$2304\times10^6 = 96\times10^3 P + 4.8\times10^5 P$$

$$2304\times10^6 = (576\times10^3)P$$

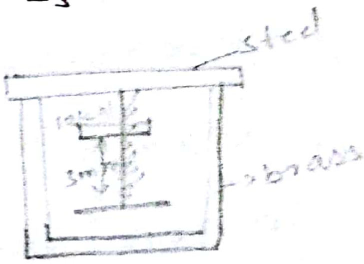
$$\frac{2304}{576} \times \frac{10^6}{10^3} = P$$

$$\therefore P = 4\times10^3 \ N$$

$$P = 4 \ KN$$

Scanned by CamScanner

③ A vertical compound fine tie member fixed rigidly at its upper end consists of steel rod 2.5 m long and 20 mm dia placed with in an equally long brass tube 21 mm internal dia and 30 mm in external dia. The rod and the tube is fixed together at the ends. The compound member is then suddenly loaded in tension by a weight of 10 KN falling through a height of 3 mm onto a flanch fixed to its lower end. calculate max. stresses in steel and brass, if $E_s = 2\times10^5\ N/mm^2$ & $E_b = 1.0\times10^5\ N/mm^2$.



sol: Given,

$l_s = 2.5\ m$; $l_b = 2.5\ m$

$d_s = 20\ mm$; $d_{b_1} = 21\ mm$, $d_{b_2} = 30\ mm$

$w = 10\ kN$; $h = 3\ mm$

$E_b = 1.0\times10^5\ N/mm^2$, $E_s = 2\times10^5\ N/mm^2$

W.D = S.E

$$P(h+\delta l) = \frac{1}{2}\frac{\sigma^2}{E}\cdot Al$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\sigma_s = \frac{\sigma_b}{E_b}\times E_s$$

$$U = U_s + U_b$$

$$= \frac{1}{2}\frac{\sigma_b^2}{E_b}A_bL_b + \frac{1}{2}\frac{\sigma_s^2}{E_s}A_sL_s$$

$$= \frac{1}{2}\left(\frac{\sigma_b^2}{E_b}A_bl_b + \frac{\sigma_b^2}{E_s^2 E_b}\ldots A_sl_s\right.$$