UNIT - II

Shear force & Bending moment

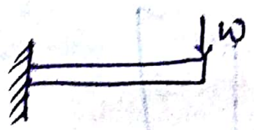
Shear force: The algebraic sum of vertical forces at any section of the beam to the right or left of the section is shear force.

Bending moment: The algebraic sum of moment of all forces acting to right (or) left of the section is known as bending moment.

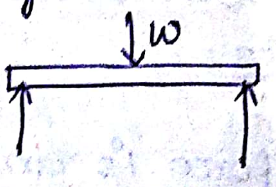
Beam: A structural member which is acted upon by a system of external loads at right angles to its axis is known as beam.

Types of Beam:

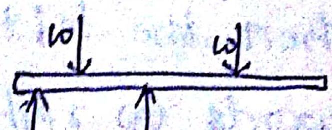
(1) Cantilever beam: The beam which is fixed at one end and free at other end.



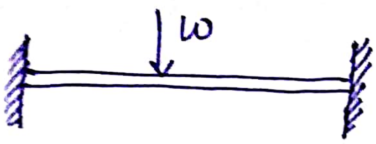
(2) simply supported beam: A beam supported or resting freely on the support at its both ends.



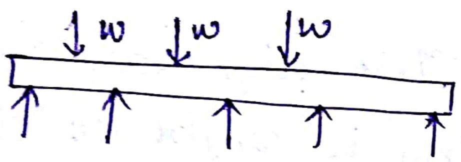
(3) Over hanging: If the end position of the beam is extended behind the support, is known as over hanging.



(4) **Rigidly fixed**: A beam which has both ends are fixed.

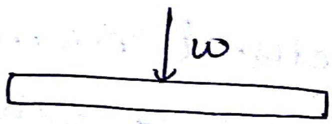


(5) **Continuous beam**: A beam which is provided with more than two supports is known as continuous beam.



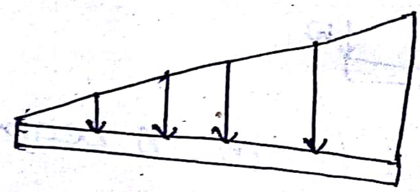
Types of loads:

(1) **Point load:**



(2) **Uniformly distributed load:**

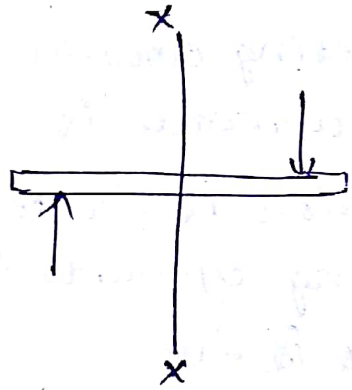
(3) **Uniformly varying load:**



* **Sign conventions for shear force and bending moments:**

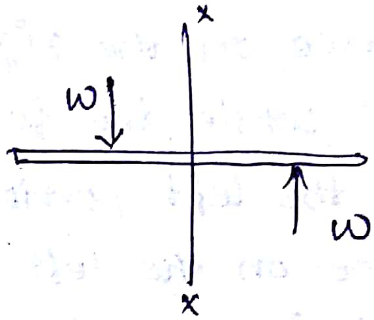
(1) **Shear force**: shear force is an unbalanced vertical force, it tends to slide one portion of the beam upwards or downwards with respect to others. we take shear force at a section as '+ve' when the left hand portion tends to slide upwards or right hand portion

tends to slide downwards. Similarly we take shear force at a section as '-ve' when the left hand portion tends to slide downwards or right hand portion tends to slide upwards.



'+ve' shear force

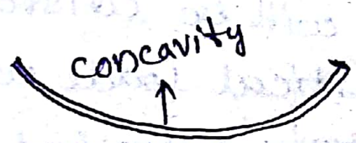
fig. (a)



'-ve' shear force

fig. (b)

(2) Bending moment :



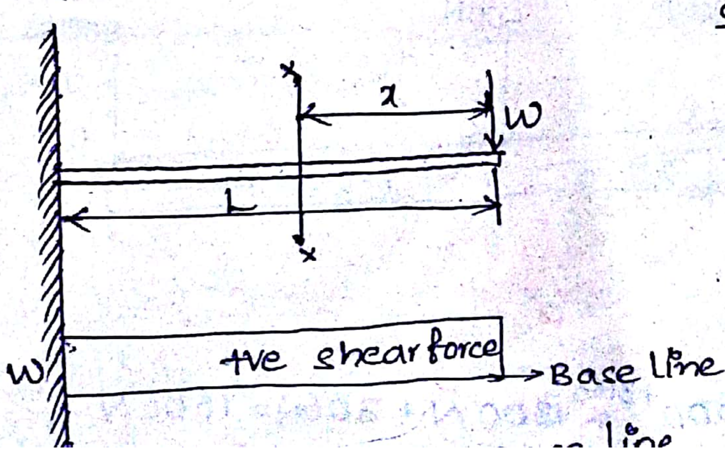
Sagging

'+ve' bending moment

Hogging

'-ve' bending moment

⊕ Shear force & Bending moment for cantilever beam at the free end:



Shear force,

$F_x = +W$

Bending moment,

$B.M = M_x = -Wx$

When, $x=0 \Rightarrow M_x = 0$

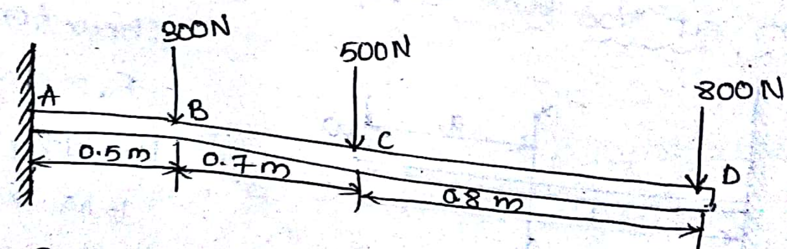
& $x = L \Rightarrow M_x = -WL$

⊛ Few important points to draw shear force & B.M.

1. Consider left (or) right portion of the section.
2. (i) Add the forces normal to the beam on one of the portion, if right portion of the section is chooseen a force on the right portion acting downwards is '+ve' while the force acting upwards is '-ve'.
   (ii) If the left portion of the section is chooeen a force on the left portion acting upwards is '+ve' while force acting downwards is '-ve'
3. The positive values of shear force & bending moment above the base line & -ve values of shear force & bending moment below the base line.
4. S.F will decrease (or) increase suddenly i.e, by vertical straight line at a section where there is a vertical point load.
5. S.F b/w any two vertical loads will be constant & hence the S.F diagram b/w vertical loads will be horizontal
6. B.M at two supports of the simply supported beam and at the free end of cantilever will be zero.

(Q) A cantilever beam of length 2m carries the point load as shown. Draw shear force & bending moment diagram for cantilever beam.

Sol:

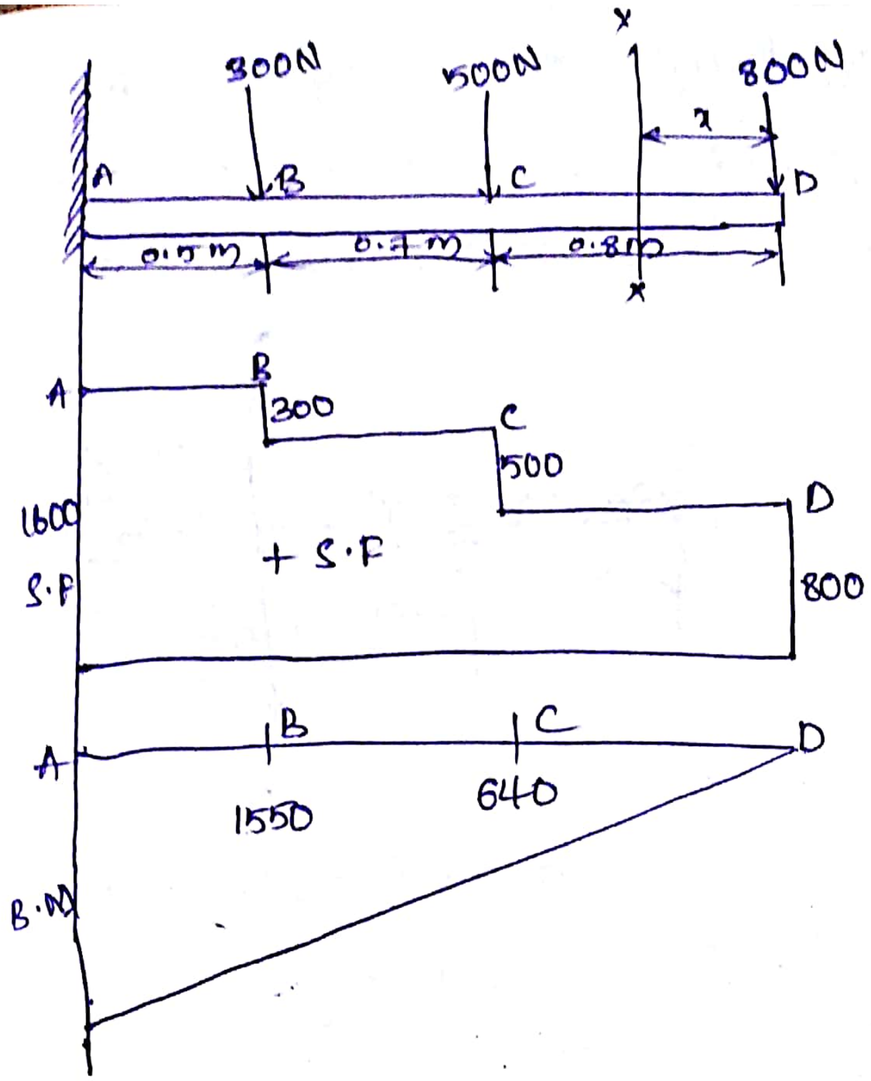


$F_D = 800\ N$

$F_B = 800 + 500 + 300 = 1300\ N + 300\ N = 1600\ N$

$F_C = 800 + 500 = 1300\ N$

$F_A = 1600\ N$



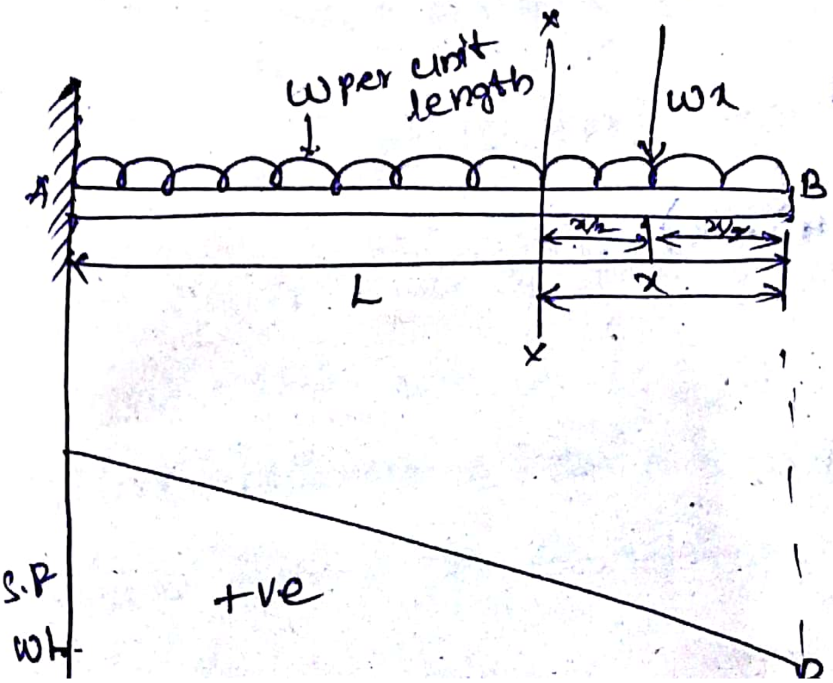
$M_x = Wx$

$M_D = -800 \times D$

$= 0$

$M_c =$

⊛ S.F & B.M for cantilever beam carrying UDL:



$S.F = F_x = +w \times x$

$F_B = w \times 0$

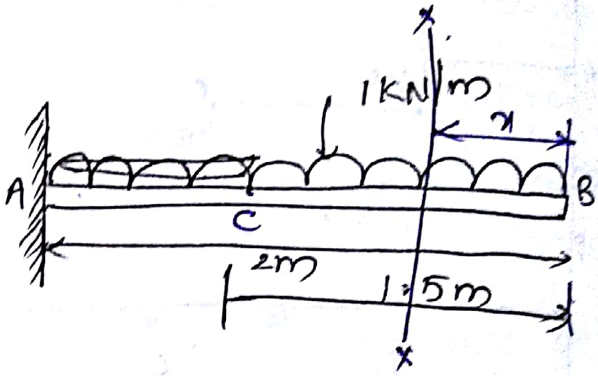
$= 0$

$F_A = wL$

$B.M = M_x = -wx \times (x/2)$

$= -\frac{wx^2}{2}$

Q A cantilever beam of length 2m carry's a UDL of 1 kN/m over the length of 1.5 m from free end draw the shear force & bending moment for cantilever beam.



sol:

$F_B = w \times 0$

$= 0$

$F_C = w \times 1.5$

$= 1.5 \times 10^3$ N/m

$F_A = 1.5 \times 10^3$ N/m

$M_B = -w \times 0 \left(\frac{0}{2}\right)$

$= 0$

$M_C = -1k \times (1.5) \times \left(\frac{1.5}{2}\right)$

$= -\frac{2.25}{2} = -1.125$ kN

$M_A = 1 \times (1.5) \times \left(\left(\left(\frac{1}{2}\right) + 0.25\right) + x\right)$

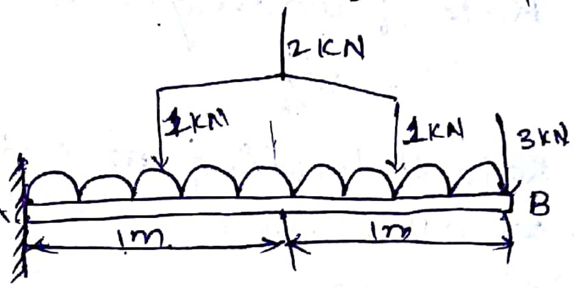
$= wx(x - 0.75)$

$= wx(x - 0.75)$

$= 1 \times 1.5\,(1.25)$

$= 1.875$

② A cantilever beam of length 2m carries a uniformly distributed load of 2 kN/m length over the whole length and a point load of 3 kN at the free end. Draw shear force & bending moment.



$F_x = S.F = wx + 3$

$x = 0, \; F_B \Rightarrow w \times 0 + 3$

$F_B = 3 \text{ KN}$

$x = 2, \; F_A = (2 \times 2) + 3$

$= 7 \text{ KN}$

$M_x = \left[wx \cdot \frac{x}{2} + 3x \right]$

$M_B \Rightarrow x = 0 \Rightarrow M_B = 0$

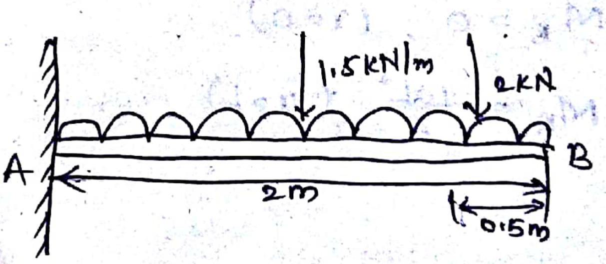
$x = 2, \; M_A = (2 \times 2) \frac{2}{2} + 3(2)$

$= 10 \text{ KN}$

③ A cantilever of length 2m carries a UDL of 1.5 kN/m run over the whole length and a point load of 2 kN at a distance of 0.5m from free end. Draw Shear force & Bending moment.

$\cancel{F_x = S.F = wx + 2}$

$\cancel{F_B = 2 \times (0.5) + 2}$



$x = 0, \; F_B = 2$

$F_A = (1.5) \times 2 + 2$

$= 3 + 2 = 5$

$M_B = (1.5)\, 2 \cdot \left(\frac{2}{2}\right) + 0$

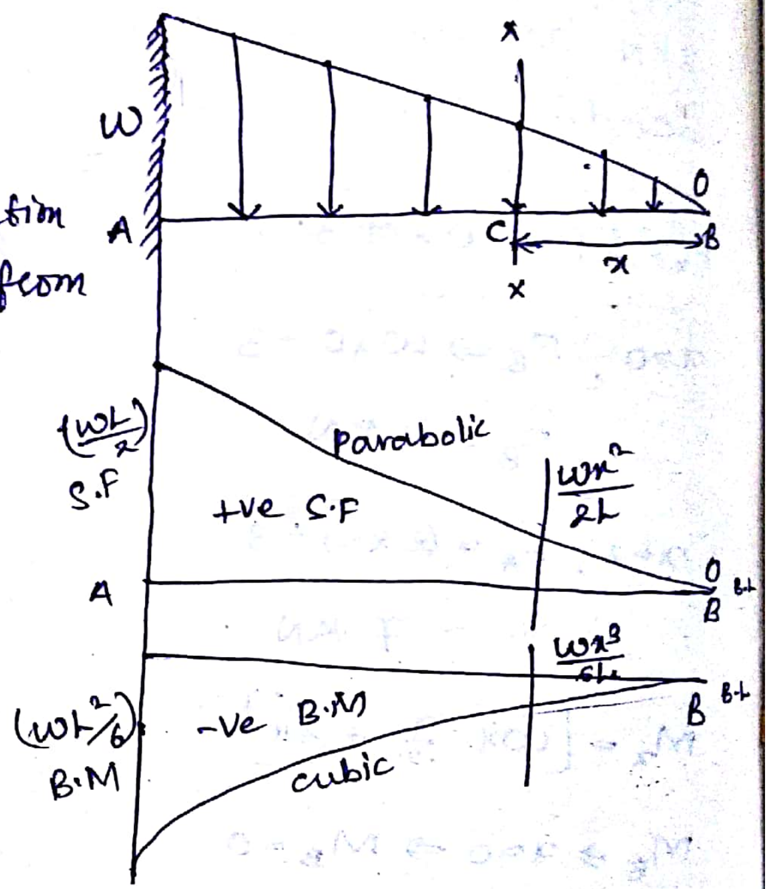
$= 3$

$M_A = (1.5)\, 2 \left(\frac{2}{2}\right) + 2(0.5)$

$= 3 + 1$

$= 4$

Shear force and Bending Moment for Cantilever beam carrying UVL:



Rate of loading at 'A'

$= w/L$

Rate of loading at section xx at a distance 'x' from 'B' $= \frac{wx}{L}$

$S.F = $ Area of Δ^{le} CxB

$= \frac{1}{2} \times (Cx) \times (xB)$

$= \frac{1}{2} \times \frac{wx}{L} \times L$

$= \frac{wx^2}{2L}$

$F_B = 0, (x=0)$

$F_A = \frac{wL}{2}, (x=L)$

$B.M = $ Total Load $\times (x/3)$
(Area of $\Delta^{le}_{(CxB)}$)

$= \frac{wx^2}{2L} \times \frac{x}{3}$

$= \frac{wx^3}{6L}$

$M_B = 0 ; (x=0)$

$M_A = \frac{wL^2}{6} ; (x=L)$

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⑥ A cantilever of length 4m carries a gradually weighing load zero at free end and 2kN/m at fixed end. Draw shear force and bending moment.

Rate of loading at 'A'

$= w/L = \frac{2K}{4}$

$= 0.5$ KN/m

rate of loading at x at a dist 'x' from 'B'

$= \frac{wx}{L}$

$F_B = 0$; $(x = 0)$

$F_A = \frac{2(4)}{2} = 4$ KN/m ; $(x = 4)$

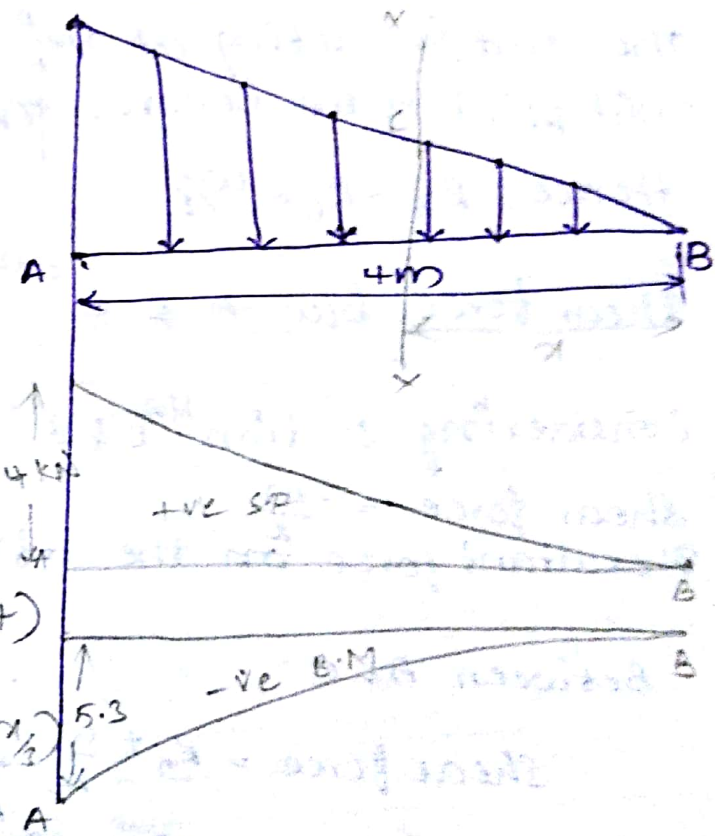
B.M = Area of $\triangle^{le}$ CXB $\times (x/3)$

$= \frac{wx^3}{6L}$

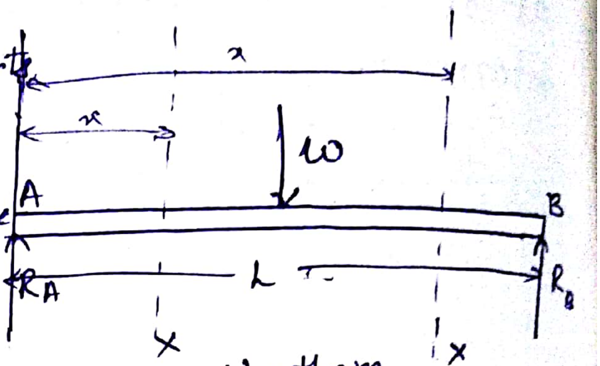
$M_B = 0$; $(x = 0)$

$M_A = \frac{2 \times (4)^3}{6(4)} = \frac{32}{6}$

$= -5.3$ KN/m



* Shear force and Bending moment for simply supported beam with Point load:



The reactions at supports will be equal to $W/2$ as the load is acting at the mid point of the beam.

Hence, $R_A = R_B = W/2$

Shear force b/w A & C considering 'x' b/w them is constant is $= \frac{W}{2}$

Considering section b/w C & B at a dist. 'x' from 'A' then

Shear force $= \frac{-W}{2}$

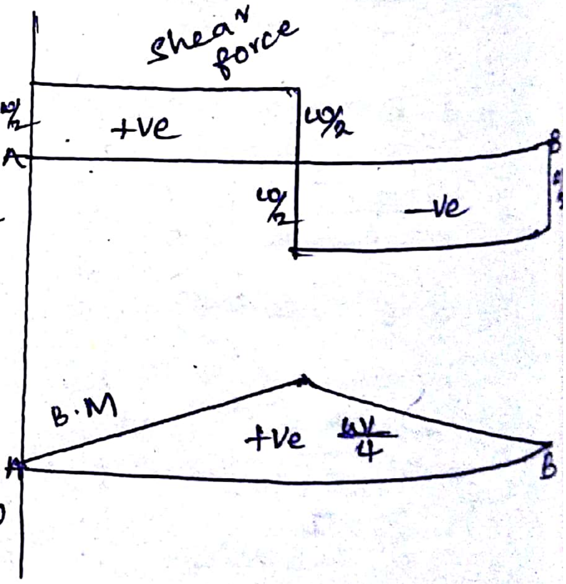
Resultant force on the left portion will be $= R_A - W$

Between A & C

Shear force $= R_A + \frac{W}{2}$

$= W/2 + W/2$

$= W$



Between C & B

Shear force $= R_A - W$

$= W/2 - W$

$= -W/2$

Shear force from $W/2$ to $-W/2$

For bending moment between A & C consider 'x' in between them

B/w A & C

$M_x = R_A \times x$

$= \frac{W}{2} \times x$

$M_A \Rightarrow x=0 \Rightarrow M_A=0$

$M_c \Rightarrow x=\frac{L}{2} \Rightarrow M_c=\frac{WL}{4}$

Between C & B

$M_x = R_A \times x - W(x-L/2)$

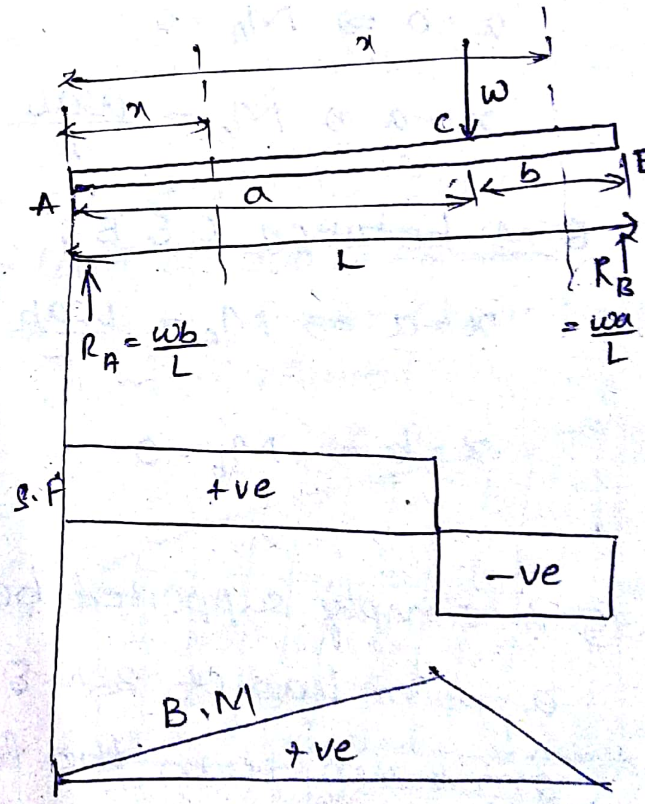
$M_c \Rightarrow x=\frac{L}{2} \Rightarrow M_c = \frac{WL}{4}$

$M_B \Rightarrow x=L \Rightarrow M_B=0$

⊛ Shear force and Bending moment for simply supported beam with eccentric load:

$R_A \times 0 + R_B \times L = W \times a$

$R_B = \frac{Wa}{L}$



$\because R_A + R_B = W$

$R_A = W - \frac{Wa}{L}$

$R_A = \frac{W(L-a)}{L}$

$R_A = \frac{Wb}{L}$

A & C:

$S.F = M_x = R_A \times x$

$F_A = R_A \times 0 = 0 \quad ;(x=0)$

$F_c = R_A \times a = aR_A \quad ;(x=a)$

C & B:

$S.F = R_A - W$

$$= \frac{wb}{L} - w$$

$$= \frac{w(b-L)}{L}$$

$$= -\frac{wa}{L}$$

so, S.F is varying from $\frac{wb}{L}$ to $-\frac{wa}{L}$

$S.F_{(B)} = 0$

B.M between A & C :

$$M_x = R_A \times x$$

$$= \frac{wb}{L} \times x$$

$x = 0 \Rightarrow M_A = 0$

$x = a \Rightarrow M_c = \frac{wab}{L}$

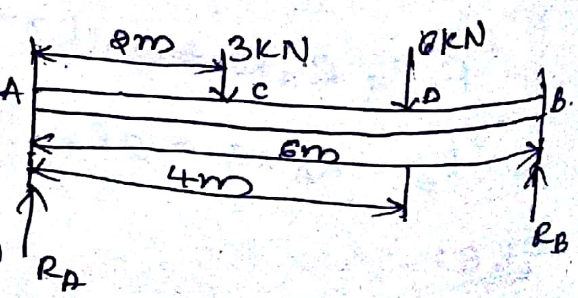
B.M between C & B :

$x = a \Rightarrow M_c = \frac{wab}{L}$

$x = b \Rightarrow M_b = 0$

Q A simply supported beam of length 6m carries a point load of 3kN & 6kN at a distance of 2m & 4m from the left end. Draw shear force and Bending moment.

sol:



$3+6 = R_A + R_B$

$R_A + R_B = 9$

about 'A'

$3(2) + 6(4) = R_B(6) + R_A(0)$

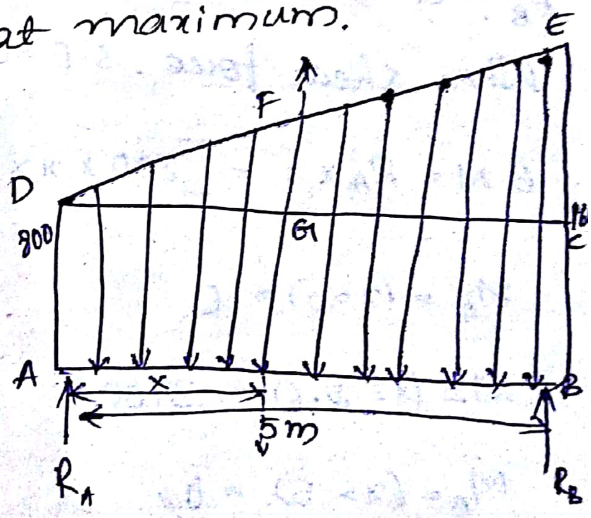
$R_B = 5$

$R_A = 9-5$

$= 4$

S.F. Between A & C :

② A simply supported beam of l = 5cm carries load of uniformly increasing load of 800 N/m at one end to 1600 N/m at the other. Draw shear force and bending moment at maximum.



Sol:

□^{le} load = 800×5

$= 4000$

△^{le} load = $\frac{1}{2} \times 800 \times 5$

$= 2000$

$\therefore$ T.L = $4000 + 2000$

$= 6000$

$R_A \times 0 + R_B \times 5 = 4000 \times \frac{5}{2} + 2000 \times \frac{2}{3}(5)$

$R_B = 3333.3$

$R_A + R_B = 6000$

$R_A = 6000 - 3333.3$

$= 2666.7$

Rate of loading $= w + \frac{wx}{L}$

$= 800 + \frac{800x}{5}$

$= 800 + 160x$

Total load on AX = Area of AXGD + Area of Δ^{le} DFG

$= 800 \times x + \frac{1}{2} \times x \times 160x$

$= 800x + 80x^2$

$S.F = R_A - (800x + 80x^2)$

$= 2666.7 - (800x + 80x^2)$

$F_A = (x=0) \Rightarrow 2666.7$

$F_B = (x=5) \Rightarrow -3333.3$

When shear force. $S.F = 0$, then $x = 2.63$

$B.M = R_A \times x - \left[800 \times x \times \frac{x}{2} + 80x^2 \times \frac{x}{3}\right]$

$M_A = (x=0) = 0$

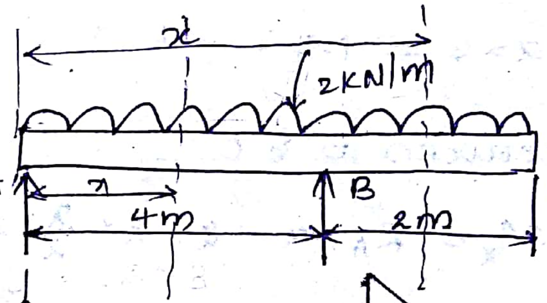
$M = (x = 2.6) = 3760.7$

$M_B = (x = 5) = 0$

Overhanging beam :

* Point of contraflexure : It is a point where its bending moment is zero, after changing its sign from +ve to −ve (or) viceversa.

Q) Draw Shearforce & Bending moment for overhanging beam for UDL of 2 KN/m over the entire length as shown in the figure. Also locate the point of contraflexure.



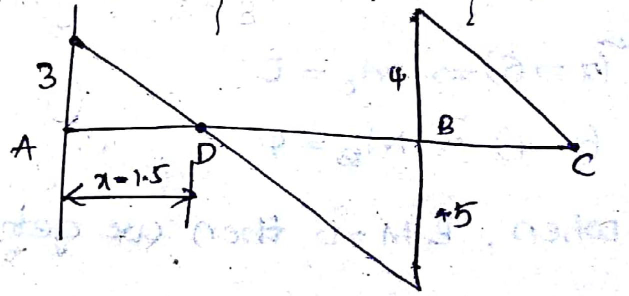
Sol: $R_A \times 0 + R_B \times 4 = (2\times 6) \times \frac{6}{2}$

$R_B = \frac{36}{4} = 9$

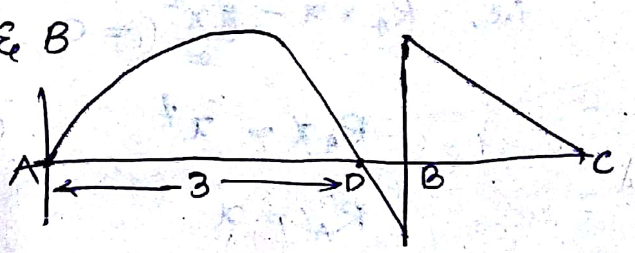
$R_A + R_B = 12$

$12 - R_B = R_A$

$\therefore R_A = 3$



Shear force between A & B



$S.F \Rightarrow F_x = R_A - 2x$

$F_A \Rightarrow x=0 \Rightarrow F_A = 3-0 = 3$

$F_B \Rightarrow x=4 \Rightarrow F_B = -5$

When, $S.F=0 \Rightarrow x = 1.5$

i.e, $R_A - 2x = 0$

$3 - 2x = 0$

$x = 3/2 = 1.5$

B & C

$S.F \Rightarrow F_x = R_A + R_B - (2\times 4) - 2(x-4)$

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$F_B \Rightarrow x=4 \Rightarrow F_B = 4$

$F_C \Rightarrow x=6 \Rightarrow F_C = 0$

Bending moment between A & B

$M_x = R_A \times x - 2 \times x \times \frac{x}{2}$ ——— ⊛

$= R_A x - \frac{\cancel{2} x^2}{\cancel{2}}$

$= R_A x - x^2$

$M_A \Rightarrow x=0 \Rightarrow M_A = 0$

$x=4 \Rightarrow M_B = -4$

between B & C

$M_x = R_A \times x - 2 \times x \times \frac{x}{2} + R_B \times (x-4)$

when $(x \Rightarrow 6) \Rightarrow M_c = 0$

$(x=4) \Rightarrow M_B = 4$

when, B.M = 0 then we get point of contraflexure.

i.e, $R_A \times x - \frac{\cancel{2} x^2}{\cancel{2}} = 0$ ($\because$ from ⊛)

$R_A x = x^2$

$R_A = x$

$\therefore x = 3$ //

Relation b/w S.F, B.M and Loading:

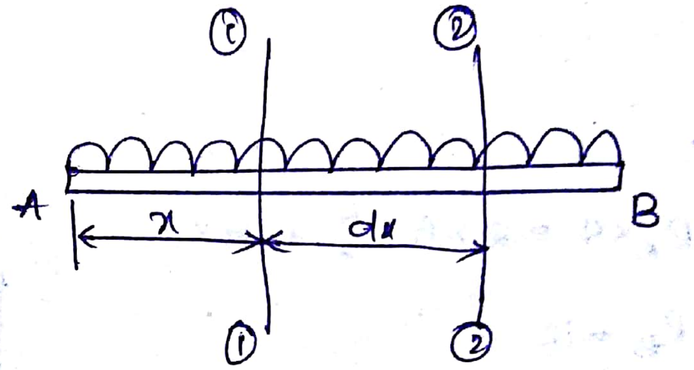
Forces

$F = ①$

$F+dF = ①$

$M = ①$

$M+dM = ①$

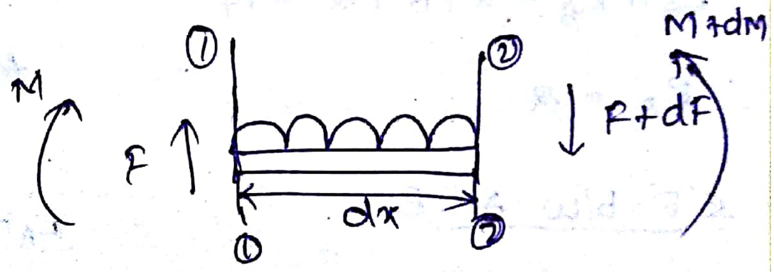


$F\uparrow - ①$

M & $M+dM \longrightarrow ①$ & $②$

$F+dF \longrightarrow ② \downarrow$

$wdx \downarrow$



Forces

$$F - wdx - (F+dF) = 0$$

$$-wdx = dF$$

$$\boxed{\frac{dF}{dx} = -w}$$

Moments

$$\cancel{M} - wdx \cdot \frac{dx}{2} + Fdx = \cancel{M} + dM$$

$$w\frac{dx^2}{2} + Fdx = dM$$

$$\boxed{\frac{dM}{dx} = F}$$

previous
(Q) question.

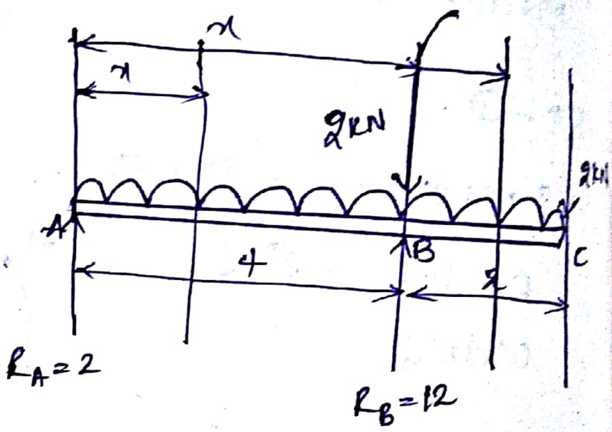
sol:

$R_B \times 4 = 2 \times 6 \times \frac{6}{2} + 2 \times 6$

$R_B = 12$

$R_A + R_B = 2 \times 6 + 2 = 14$

$\therefore R_A = 2$



S.F b/w A & B :

$F_x = R_A - 2x$

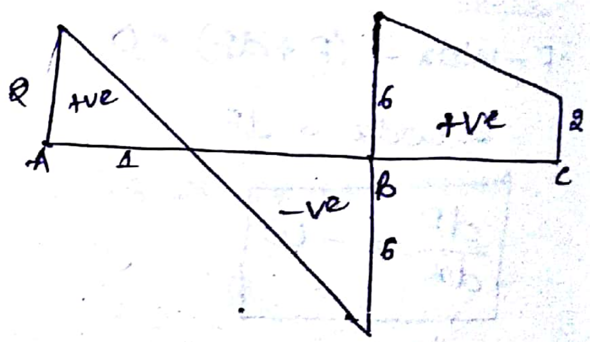
$F_A = 2$

$F_B = -6$

when, S.F = 0

i.e, $R_A - 2x = 0$

$x = 1$

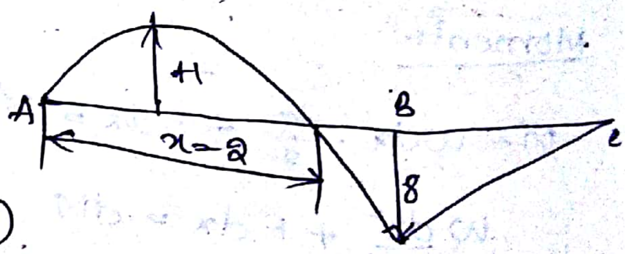


B/w B & C :

$F_x = R_A + R_B - 2 \times 4 - 2(x-4)$

$F_B = +6$

$F_C = +2$



B.M b/w A & B :

$M_x = R_A \times x - 2 \times \frac{x^2}{2}$

$x = 0 \Rightarrow M_A = 0$

$x = 4 \Rightarrow M_B = -8$

$x = 1 \Rightarrow M_D = +1$

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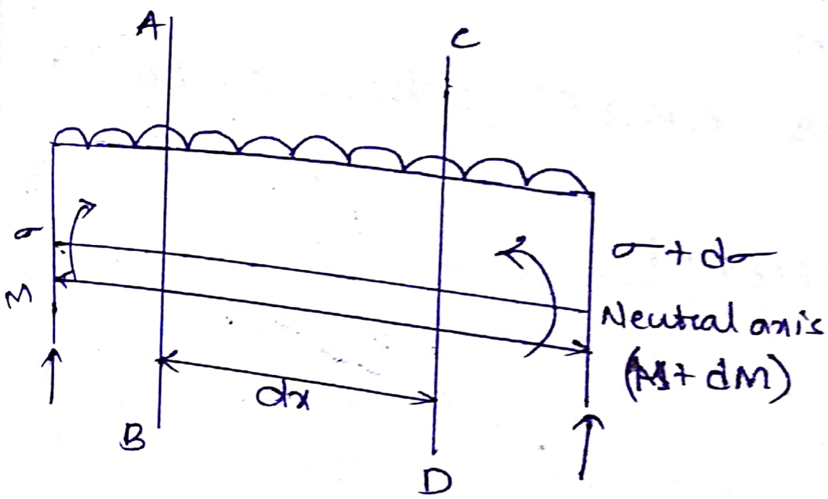
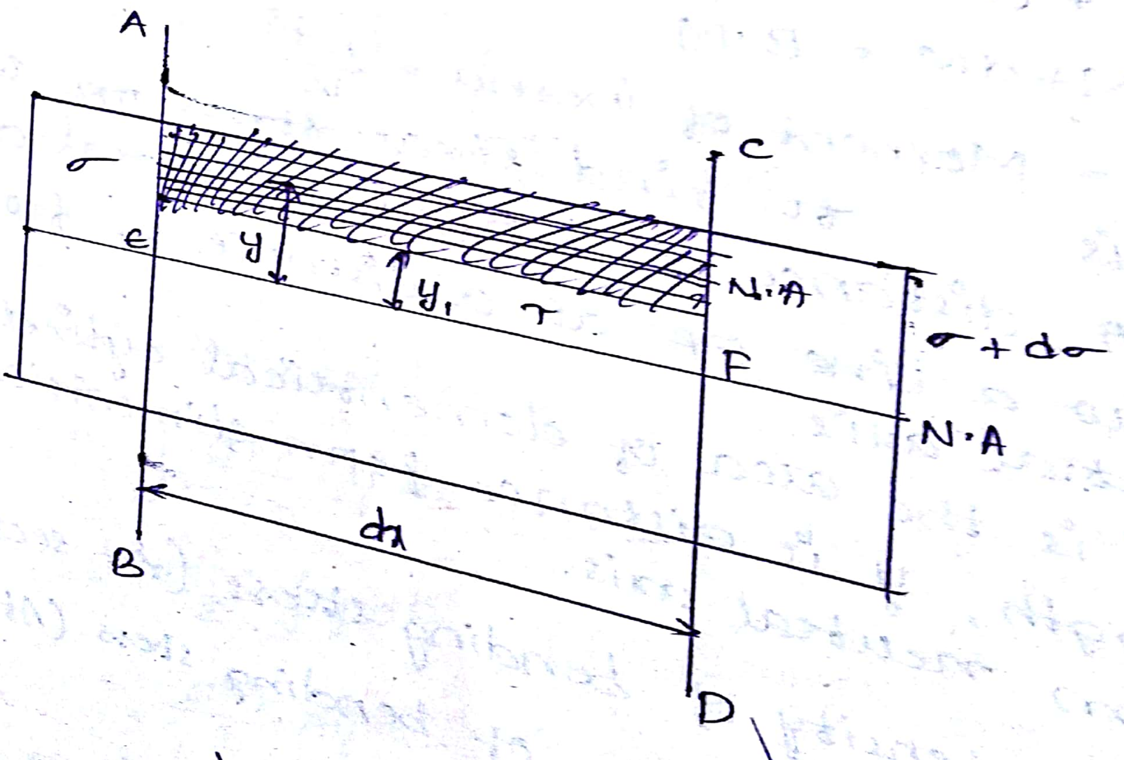
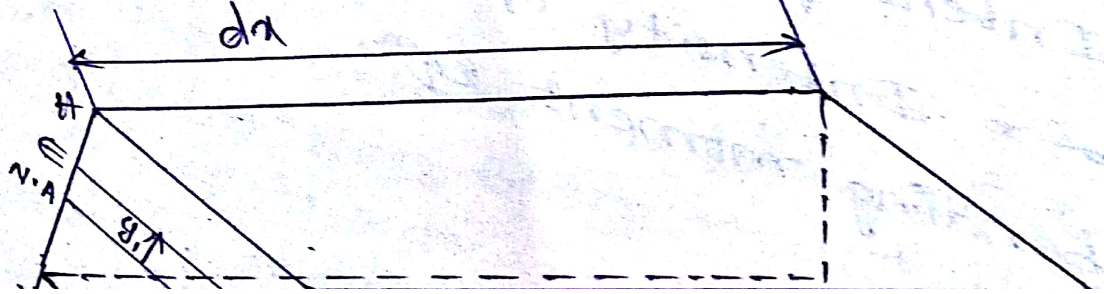
* Shear stresses in beams:







Consider the simply supported beam carrying U.D.L. S.F & B.M will be varying along the length of the beam

→ consider x-sections AB & CD where at AB

$F = S.F$,

M = Bending moment

At section CD;

$F + dF = S.F$

$M + dM = B.M$

I = Moment of inertia $= \frac{bd^3}{12}$

→ It is to find shear stress on section AB at 'A' distance 'y₁' from the neutral axis.

→ Draw a line EF at a distance 'y_1' from the neutral axis.

→ 'dA' is the area of elementical cylinder 'dx' is length, y is distance from elimentrical cylinder from neutral axis.

σ = Intensity of bending stress (At section AB)

$\sigma + d\sigma$ = Intensity of bending stress (At section CD)

from bending moment Eqn

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\boxed{\sigma = \frac{M}{I} \times y}$$

At section 'AB' Bending stress,

$$\boxed{\sigma = \frac{M}{I} \times y}$$

At section 'CD' Bending stress

$$\sigma + d\sigma = \left(\frac{M + dM}{I}\right) \times y$$

force at section 'AB' of the elementrical cylinder

$\sigma = F/A$

$\sigma = F/dA$

$F = \sigma \times dA$

force at section 'CD'

forces differs at section AB & CD which are acting along the same line but are in opposite direction on net unbalanced force on elementrical cylinder is

Net unbalanced force

$$= \left(\frac{M+dM}{I} \times y\right) dA - \frac{M}{I} \times y \, dA$$

$$= \frac{dM}{I} \times y \, dA$$

Total unbalanced force

$$= \int \frac{dM}{I} \times y \, dA$$

$$= \frac{dM}{I} \int y \, dA$$

$$= \frac{dM}{I} \times A\bar{y}$$

due to total unbalanced force acting on a part of a beam, then beam may fail due to shear. Hence in order to the above part may not fail by shear the horizontal section of the beam at the level 'EF' must offer shear resistance.

$\therefore$ Shear resistance (S.F) at level EF = T.U.F

$$= \frac{dM}{I} A\bar{y}$$

→ Area on which τ is acting is equal to shear force = $\tau \times b \times dx$

Equating ② - values of S.F

$$\frac{dM}{I} \times A\bar{y} = \tau \times b \times dx$$

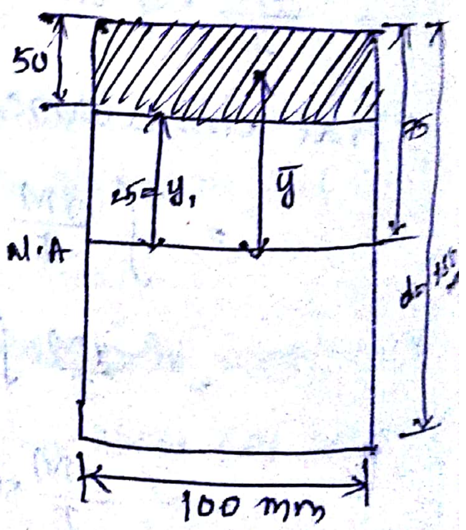
$$\tau = \frac{dM}{dx} \times \frac{A\bar{y}}{Ib} \qquad \left(\because F = \frac{dM}{dx}\right)$$

$$\boxed{\tau = F \times \frac{A\bar{y}}{Ib}}$$

② A wooden beam 100 mm wire & 150 mm beam is simply supported beam over a span of 4m at S.F at a section of beam is 4500N. Find the shear stresses at a distance of 25mm from the neutral axis.

$\bar{y} = 25+25$

$= 50$



$$\tau = F \times \frac{A\bar{y}}{Ib}$$

Area of section = 100×50

$= 5000$

$$I = \frac{bd^3}{12} = \frac{100 \times (150)^3}{12}$$

$I = 281 \times 10^5$

$$\tau = 4500 \times \frac{5000 \times 50}{281 \times 10^5 \times 100}$$

$\tau = 0.4\ N/mm^2$

* Shear stress for Rectangle :

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

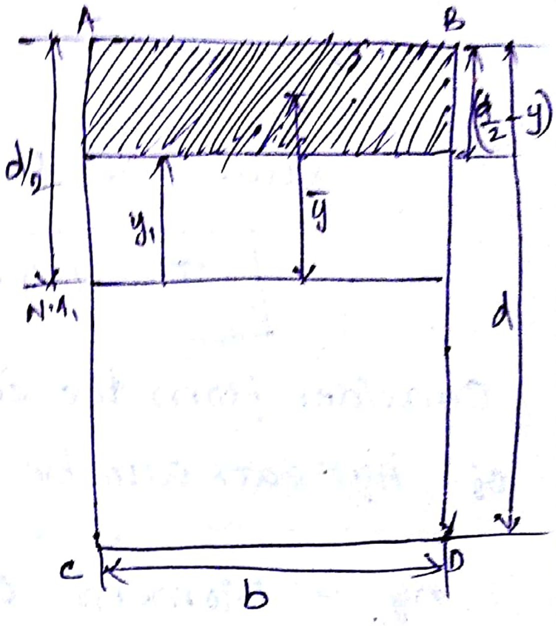
$$A = b \times \left(\frac{d}{2} - y\right)$$

$$\bar{y} = y + \frac{1}{2}\left(\frac{d}{2} - y\right)$$

$$= y + \frac{d}{4} - \frac{y}{2}$$

$$= \frac{y}{2} + \frac{d}{4}$$

$$= \frac{1}{2}\left(y + \frac{d}{2}\right)$$



$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

At Top, $y = d/2$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right)$$

$$= 0$$

At Neutral axis, $y = 0$

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - 0\right)$$

$$\tau = \frac{Fd^2}{8I}$$

$$\tau = \frac{Fd^2}{8 \times \frac{bd^3}{12}}$$

$$\tau = \frac{F\cancel{d^2}}{\cancel{8}_2} \times \frac{\cancel{12}^3}{b\cancel{d^3}}$$

$$\boxed{\tau = 1.5\frac{F}{bd}} \quad \text{—— (1)}$$

sl Average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area of section}}$

$$\tau_{avg} = \frac{F}{b \times d} \quad \text{—— (2)}$$

Substitute (2) in eq (1)

$$\boxed{\tau = 1.5 \times \tau_{avg}}$$

Consider from the equation $\tau = \frac{A\bar{y}}{Ib}$, the v of $A\bar{y}$ can also be calculated

(2)

$A\bar{y}$ = Moment of shaded area about neutral ax

Consider a strip of thickness dy at a dist y' from neutral axis.

dA = area of the strip = $b \times dy$

Moment of area 'dA' about the neutral axis

$$= dA \times y$$

$$\text{Moment} = y \times b \times dy$$

Moment of shaded area about N·A is obt by integrating above eqn b/w limits y to $d/2$ we get

$$= \int_{y}^{d/2} y b \, dy$$

$$= b \int_{y}^{d/2} y \, dy$$

$$= b\left(\frac{y^2}{2}\right)_{y}^{d/2}$$

$$= \frac{b}{2}\left(-y^2 + \frac{d^2}{4}\right)$$

$$A\bar{y} = \frac{bd^2}{8} - \frac{by^2}{2}$$

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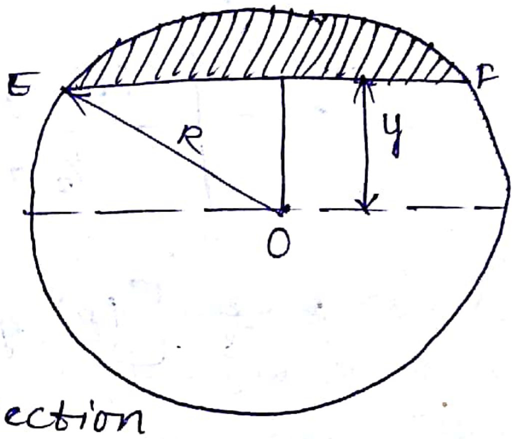
Equate moment of shaded area about the neutral axis $= A\bar{y}$

$$A\bar{y} = \frac{b}{2}\left[\frac{d^2}{4} - y^2\right]$$

$$\therefore \boxed{\tau = \frac{F}{2I}\left[\frac{d^2}{4} - y^2\right]}$$

* **Shear stress for circular section:**

W.K.T,



$$\tau = F \times \frac{A\bar{y}}{Ib}$$

dA = Area of strip

dy = thickness

R is radius of circular section

F is shear force

$A\bar{y}$ = moment of shaded area above neutral axis

I = Moment of I of whole section

b = Width of beam at level 'EF'

Consider a strip of thickness dy at a distance 'y' from neutral axis & area dA then

$$dA = b \times dy$$

$$= 2\sqrt{R^2 - y^2}\, dy$$

$$\left(\because b = EF = EB + BF = 2EB = 2\sqrt{R^2 - y^2}\right)$$

Moment of this area 'dA' about neutral axis $= dA \times y$

$$= 2\sqrt{R^2 - y^2}\, dy \times y$$

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→ Moment of whole shaded area about N.A. obtained by integrating above eqn' b/w the limits 'y' to 'R'

$$A\bar{y} = \int_y^R 2y\sqrt{R^2-y^2}\,dy$$

$$= -\int_y^R (-2y)\sqrt{R^2-y^2}\,dy$$

$$= -\left[\frac{(R^2-y^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_y^R$$

$$= -\frac{2}{3}\left[(R^2-y^2)^{3/2}\right]_y^R$$

$$= \frac{2}{3}(R^2-y^2)^{3/2}$$

Substitute the values in of $A\bar{y}$ in τ eqn

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

$$= F \times \frac{\frac{2}{3}(R^2-y^2)^{3/2}}{Ib}$$

$$= \frac{2F}{3}\,\frac{(R^2-y^2)^{3/2}}{I\sqrt{R^2-y^2}}$$

$$\therefore \boxed{\tau = \frac{F}{3I}(R^2-y^2)}$$

At $y=R$, $\tau = 0$

At $y=0$ i.e, at neutral axis

$$\tau_{max} = \frac{F}{3I}R^2$$

$$\tau_{max} = \frac{F}{3 \times \frac{\pi d^4}{64}} \times R^2$$

$$= \frac{FR^2}{3} \times \frac{64}{\pi d^4}$$

$$= \frac{FR^2}{3} \times \frac{64}{\pi (2R)^4} = \frac{FR^2}{3} \times \frac{\cancel{64}\,4}{\pi \times \cancel{16} R^4}$$

$$= \frac{4}{3} \times \frac{F}{\pi R^2}$$

$$\tau_{avg} = F/A$$

$$= F/\pi R^2$$

$$\boxed{\tau_{max} = \frac{4}{3} \tau_{avg}}$$

(Q) A rectangular beam of 100 mm wide & 250 mm beam is subjected to max. shear force of 50 kN. Determine avg. shear stress & max. shear stress, shear stress at a distance of 25 mm above the neutral axis.

$$\tau_{avg} = \frac{F}{A} = \frac{F}{bd} = \frac{50 \times 10^3}{100 \times 250} = 2\ N/mm^2$$

$$\tau_{max} = 1.5\ \tau_{avg}$$

$$= 1.5\ (2)$$

$$= 3\ N/mm^2$$

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4} - y^2\right]$$

Here, $y = 25$

$$= \frac{50 \times 10^3}{2 \times \frac{bd^3}{12}} \left(\frac{(250)^2}{4} - (25)^2\right)$$

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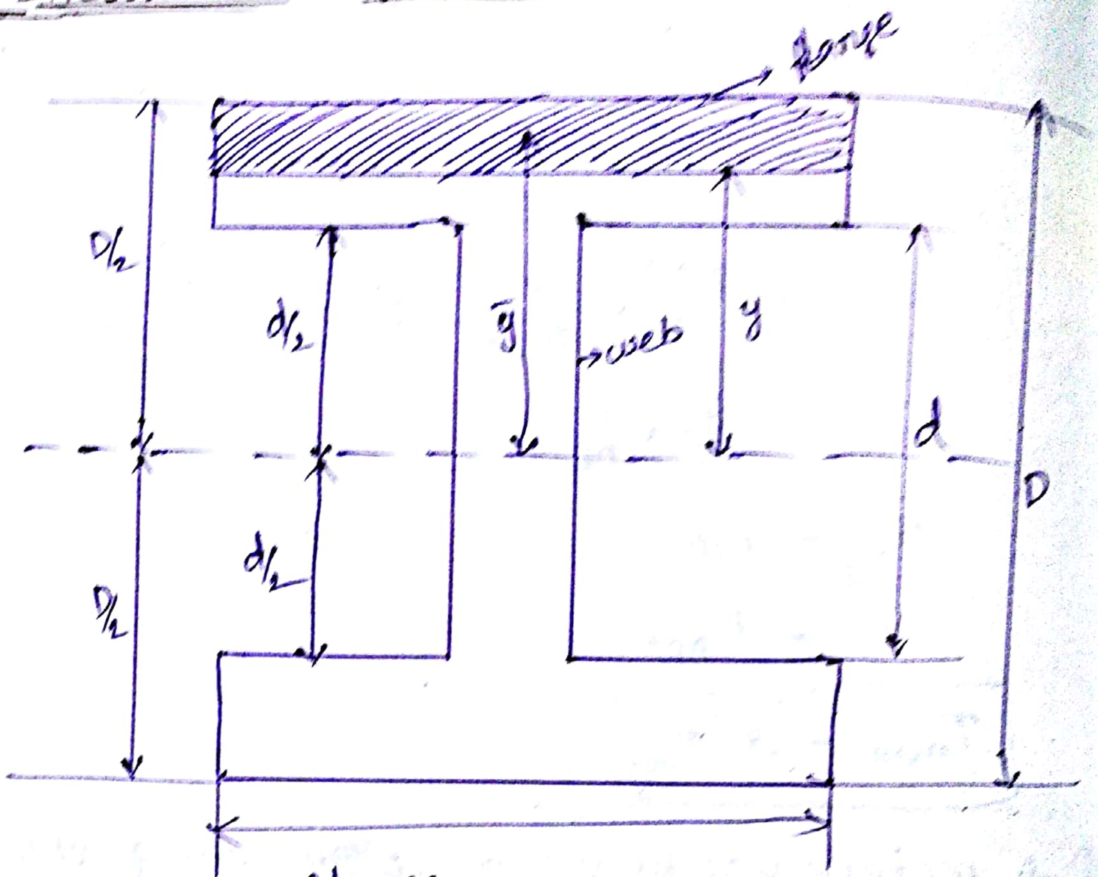
* Shear stress of I - section :



B = width of flange

D = Depth

b = width of web

d = depth.

Flange :

$$\text{Area} = l \times b = B \times \left(\frac{D}{2} - y \right)$$

$$\bar{y} = y + \frac{1}{2}\left(\frac{D}{2} - y \right)$$

$$= y + \frac{D}{4} - \frac{y}{2}$$

$$= \frac{D}{4} + \frac{y}{2}$$

$$= \frac{1}{2}\left(\frac{D}{2} + y \right)$$

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

$$= F \times \frac{\cancel{B} \times \left(\frac{D}{2} - y \right) \times \frac{1}{2}\left(\frac{D}{2} + y \right)}{I \cancel{B}} \qquad (b = B)$$

$$= \frac{F \times \left(\left(\frac{D}{2}\right)^2 - y^2 \right) \times \frac{1}{2}}{I}$$

$$= \frac{F}{2I} \left[\left(\frac{D}{2}\right)^2 - (y)^2 \right]$$

$$= \frac{F}{2I} \left(\frac{D^2}{4} - y^2 \right)$$

At top of the flange,

$$y = D/2 \Rightarrow \tau_{min} = 0$$

At bottom of the flange,

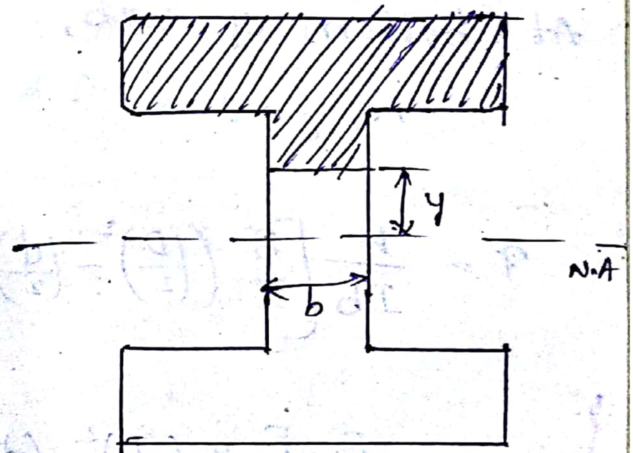
$$y = d/2$$

$$\tau_{max} = \frac{F}{2I} \left(\frac{D^2}{4} - \frac{d^2}{4} \right) //$$

<u>Web:</u>

Area = $l \times b$

$$= B \times \left(\frac{D}{2} - d/2 \right)$$



$$\bar{y} = \frac{d}{2} + \frac{1}{2} \left(\frac{D}{2} - \frac{d}{2} \right)$$

$$= \frac{d}{2} + \frac{D}{4} - \frac{d}{4}$$

$$= \frac{D}{4} + \frac{d}{4}$$

$$= \frac{1}{2} \left(D/2 + d/2 \right)$$

web = $l \times b$ = $B \times \left(\frac{d}{2} - y \right)$

$$\bar{y} = y + \frac{1}{2} \left(\frac{d}{2} - y \right)$$

$$= y + \frac{d}{4} - \frac{y}{2}$$

$$= \frac{1}{2} \left(\frac{d}{2} + y \right)$$

$$\tau = F \times \frac{A\bar{y}}{Ib}$$

$$= \frac{F \times \left[B \times \left(\frac{D}{2} - \frac{d}{2}\right) \times \frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right)\right] + b \times \left(\frac{d}{2} - y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)}{Ib}$$

$$= \frac{F\left(\frac{B}{2}\left[\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right] + \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y^2\right]\right)}{Ib}$$

At the top of web,

$y = d/2$

$$\Rightarrow \tau = \frac{F}{Ib}\left(\frac{B}{2}\left(\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right)\right)$$

At bottom of web,

i.e, $y = 0$

$$\tau = \frac{F}{Ib}\left[\frac{B}{2}\left(\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right) + \frac{b}{2}\left(\frac{d}{2}\right)^2\right]$$

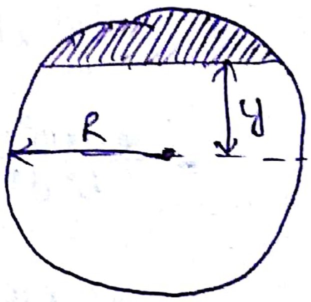
$$= \frac{F}{Ib}\left[\frac{B}{2}\left(\left(\frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right) + \frac{bd^2}{8}\right]$$

(8) A circular beam of 100mm diameter is subjected to shear force of 5KN. calculate average shear stress, max. shear stress and shear stress at a distance of 40 mm from neutral axis.

Sol: Radius = 50 mm

shear force, F = 5KN

$y = 40$ mm



$$\tau = \frac{F}{3I}(R^2 - y^2)$$

$$\because I = \frac{\pi d^4}{64} = \frac{\pi (100)^4}{64}$$

$$= 4906250$$

$$\tau = \frac{5\times10^3}{3\times 4906250}((50)^2 - (40)^2)$$

$$\tau = 0.3057 \text{ N/mm}^2$$

$$\tau_{avg} = \frac{F}{A} = \frac{5\times10^3}{\pi(50)^2} = 0.63 \text{ N/mm}^2$$

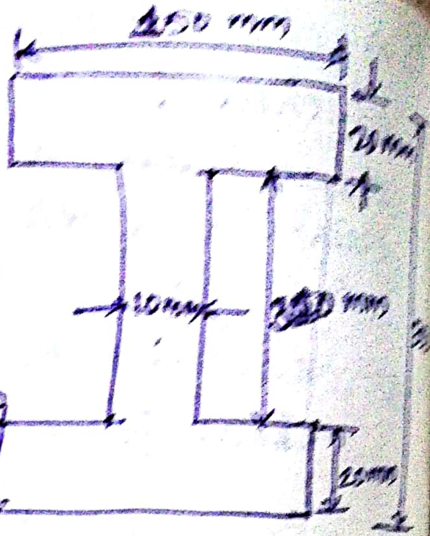
$$\tau_{max} = \frac{4}{3}\tau_{avg}$$

$$= \frac{4}{3}(0.63)$$

$$= 0.848 \text{ N/mm}^2$$

(9) An I-section beam 350 mm × 150 mm has web thickness of 10mm & flange thickness of 20mm. If the shear force acting on the section is 40 kN. Find the max. shear stress deloped in I-section.

Sol: Shear force, $F = 40\,kN$



$$\tau_{max} = \frac{F}{Ib}\left[\frac{B}{2}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) + \frac{bd^2}{8}\right]$$

$$= \frac{40\times10^3}{\frac{bd^3}{12}\times b}\left[\frac{B}{2}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) + \frac{bd^2}{8}\right]$$

$$= \frac{40\times10^3\times12}{b^2d^3}\left[\frac{150}{2}\left(\frac{(350)^2}{4} - \frac{(310)^2}{4}\right) + \frac{10\times310^2}{8}\right]$$

$$= \frac{40\times10^3\times12}{(10)^2\times(310)^3}\left[75\left(\frac{26400}{4}\right) + \frac{96100\times10}{8}\right]$$

$$= \frac{480\times10^4}{10^2\times(310)^3}\left[75(6600) + \frac{961}{8}\times10^3\right]$$

$$\tau_{max} = \frac{F}{Ib}\left[\frac{B}{2}\left(\frac{D^2}{4} - \frac{d^2}{4}\right) + \frac{bd^2}{8}\right]$$

Here, $I = \frac{BD^3}{12} - \frac{bd^3}{12}$

$$= \frac{150\times350^3}{12} - \frac{140\times310^3}{12}$$

$$= 188375833.4\ mm$$

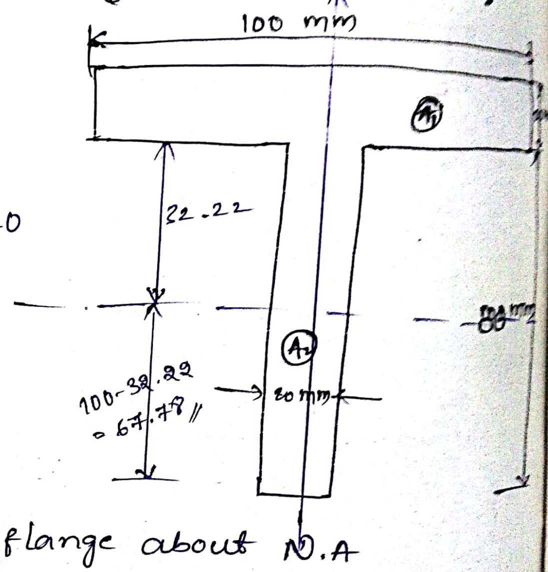
$$\tau_{max} = \frac{45\times10^3}{I\times10}\left[\frac{150}{2}\left(\frac{350^2}{4} - \frac{310^2}{4}\right) + \frac{10\times310^2}{8}\right]$$

$$= \frac{45\times10^2}{I}\left[75(6600) + 120\times10^3\right]$$

$$= 2.38\times10^{-5}\,(615\times10^3)$$

$$\tau_{max} = 14.637\ N/mm^2$$

(2) shear force acting on a section of a beam is 50 kN. The section of the beam of T-shape of dimensions 100 mm × 100 mm × 20 mm as shown in fig. The moment of inertia about the horizontal neutral axis is 314.221×10^4 mm⁴. Calculate shear stress at neutral axis & at the section of flange.



$$y = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

A_1 = flange = 100×20

A_2 = web = 80×20

$y_1 = \frac{20}{2} = 10$

$y_2 = 20 + \frac{80}{2} = 60$

$y = 32.22$

→ shear stress for flange about N.A

$$\tau = \frac{F \cdot A\bar{y}}{Ib}$$

$$\bar{y} = \left(32.2 - \frac{20}{2}\right)$$

$$= 22.2$$

$b = 100$

$A = 2000$

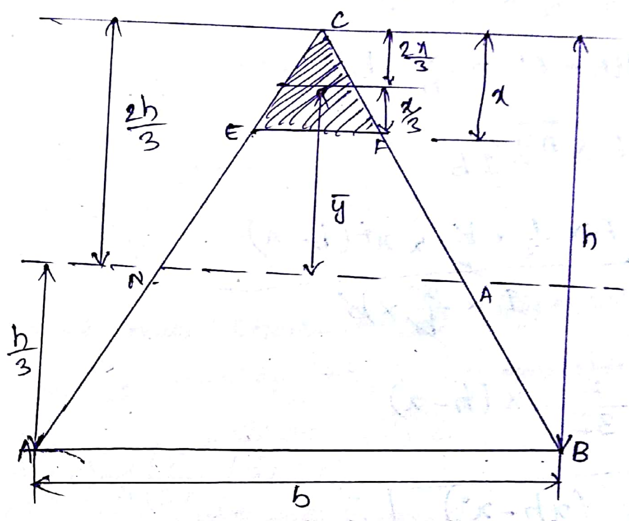
$\bar{y} = 22.2$

$I = \frac{bd^3}{12} = \frac{100 \times (20)^3}{12} = 66666.66$ (not about neutral axis)

$$\tau = 50 \times 10^3 \times \frac{2000 \times 22.2}{314.221 \times 100}$$ //

(*) The shear force acting on a beam at a section. shear force of the section of the beam is Δ^{lar}, base of 'b' and and an altitude of 'h'. Beam is placed with its base horizontal. Find the max. shear stress at neutral axis.

sol:



$$\tau = F \times \frac{A\bar{y}}{Ib}$$

$A\bar{y}$ is moment of shaded area about above neutral axis = Area of Δ^{le} CEF x dist. of centre of gravity of Δ^{le} CEF from neutral axis.

from similar Δ^{les} CEF & ABC

$$\frac{AB}{EF} = \frac{h}{x}$$

$$EF = \frac{x}{h} \times AB$$

$$= \frac{x}{h} \times b$$

$$A = \frac{1}{2} \times EF \times x = \frac{1}{2} \times \frac{x}{h} \times b \times x$$

$$= \frac{1}{2}\left(x^2 \, b/h\right)$$

$$= \frac{F}{3I}\left(\frac{h^2}{4}\right)$$

$$= \frac{F}{(3)\frac{bh^3}{36}} \times \frac{h^2}{4}$$

$$= \frac{Fh^{\cancel{2}}}{\cancel{12}} \times \frac{\cancel{36}^{3}}{bh^{\cancel{3}}}$$

$$\therefore \boxed{\tau_{max} = \frac{3F}{bh}}$$

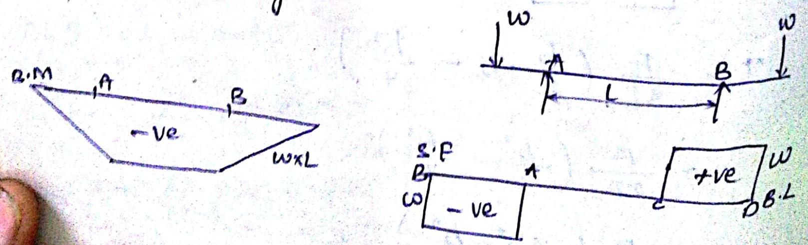
& $\tau_{apex} = 0$, $\tau_{base} = 0$

$$\therefore \boxed{\tau_{N.A} = \frac{8}{3}\left(\frac{F}{bh}\right)}$$

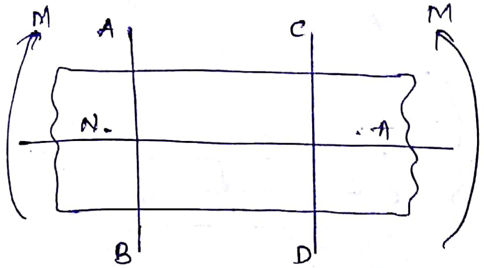
* FLEXURAL STRESSES *

PURE BENDING (or) SIMPLE BENDING :

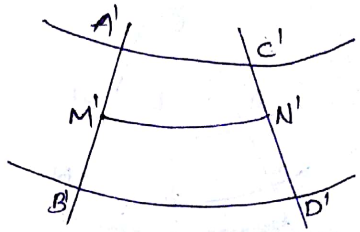
→ If the length of the beam is subjected to constant bending moment and no shear force i.e, S.F = 0. When stresses will be setup in the length of beam due to bending moment only and that length of the beam is said to be pure bending or simple bending.



THEORY OF SIMPLE BENDING :



Before bending



After bending

ASSUMPTION :

→ A material of the beam is homogeneous and isotropic.

→ The value of young's modulus is same in tension and compression

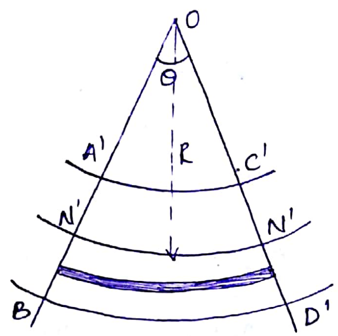
→ The transfer sections which were plane before bending will remain same after bending.

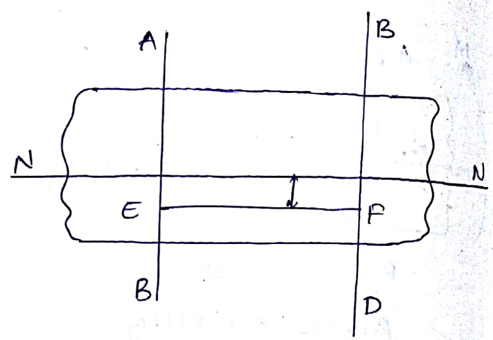
→ The beam is initially strained and longitudinal filaments are bent into circular arcs with a common centre of curvature.

→ The radius of curvature is large compared with dimensions of cross section.

→ Each layer of the beam is feel to expand or contract independently of layer above or below it.

Expression for Bending moment :





stress : 'R' is radius of neutral layer N'N'

'θ' is angle subtended at 'O' & A'B' and C'D' are produced.

strain variation along the depth of beam :

In a original length of layer EF = dx and also length of neutral layer NN = dx .

→ After bending the length of neutral layer NN' will remain unchanged, but length of layer E'F' will increase.

Hence;

$$N'N' = NN = dx$$

$$N'N' = R \times \theta$$

$$E'F' = (R+y)\theta$$

Increase in length of layer $EF = E'F' - N'N'$

$$= (R+y)\theta - R \times \theta$$

$$= y\theta$$

Strain in the layer = $\frac{\text{change in length}}{\text{original length}}$

$$e = \frac{y\theta}{R\theta} = y/R$$

Stress variation :

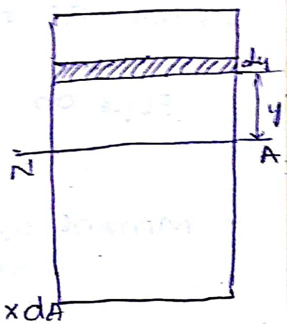
$$E = \frac{\sigma}{e}$$

$$E = \frac{\sigma}{y/R}$$

$$\boxed{\frac{E}{R} = \frac{\sigma}{y}}$$

Neutral axis : Neutral axis of any transverse section of a beam is defined as line of intersection of neutral layer with transverse section.

$$\sigma = \frac{E}{R} \times y$$



Now, consider a small layer at a distance 'y' from neutral axis. Let 'dA' is area of layer.

Now, force on the layer, $F = \sigma \times dA$

$$F = \frac{E}{R} \times y \times dA$$

Total force on beam is obtained by integrating above eqn, $\int \frac{E}{R} \times y \times dA$

→ But for pure bending, there is no force on section of the beam

$$\therefore \int \frac{E}{R} \times y \times dA = 0$$

$$y \times dA = 0$$

Where $y \times dA$ represents moment of area dA about neutral axis.

$\int y \, dA$ represents 'moment of entire area of the section about the neutral axis.

Moment of Resistance:

Due to pure bending the layers above neutral axis are subjected to compressive stress, whereas the layer below neutral axis are subjected to tensile stresses.

→ Due to these stresses forces will be acting on the layers. This forces will have moment about neutral axis. The total moment of these forces about neutral axis for a section is known as moment of resistance.

Force on a layer, $F = \sigma \times dA$

$$F = \frac{E}{R} \times y \times dA$$

→ Moment of this force about neutral axis,

$$= \frac{E}{R} \times y \times dA \times y$$

$$= \frac{E}{R} \times y^2 dA$$

Total moment of force of section of beam.

$$M = \int \frac{E}{R} \times y^2 \times dA$$

$$= \frac{E}{R} \int y^2 dA$$

where $\int y^2 dA$ = Moment of inertia

$$M = \frac{E}{R} \times I$$

$$\frac{M}{I} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}} \quad \left(\because \frac{E}{R} = \frac{\sigma}{y}\right)$$

→ bending moment equation

8 A steel plate of width 120 mm and of thickness 20 mm is bent into a circular arc of radius 10 m. Determine max. stress induced and bending moment which will produce max. stress. $E = 2 \times 10^5 \, N/mm^2$

Sol: $w = 120 \, mm$

$t = 20 \, mm$

$R = 10 \times 10^3 \, mm$

$E = 2 \times 10^5 \, N/mm^2$

$\sigma = \frac{E}{R} \times y$

$y = \frac{t}{2} = \frac{20}{2} = 10$

$\sigma_{max} = \frac{E}{R} \times y$

$= \frac{2 \times 10^5}{10 \times 10^3} \times 10 = 200 \, N/mm^2$

$M = \frac{E}{R} \times I$

$I = \frac{bt^3}{12} = \frac{120(20)^3}{12} = 8 \times 10^4 \, mm^4$

$M = \frac{2 \times 10^5 \times (20)^3}{10 \times 10^3} \times 8 \times 10^4$

$M = 1.6 \, kNm$

SECTION MODULUS:

Section modulus is defined as ratio of moment of inertia of section about neutral axis to the distance of outermost layer from the neutral axis.

$$\boxed{Z = \frac{I}{y_{max}}}$$

'σ' will be max. when y is maximum

from, $\frac{M}{I} = \frac{\sigma}{y}$

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

$$\boxed{M = \sigma_{max} \cdot Z}$$

Rectangular section:

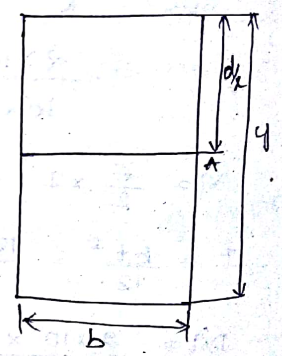
$$Z = \frac{I}{y_{max}}$$

$$I = \frac{bd^3}{12}$$

$$y_{max} = d/2$$

$$Z = \frac{bd^3}{12} \cdot \frac{2}{d}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

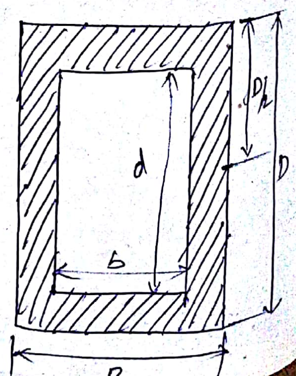


Hollow rectangular section:

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

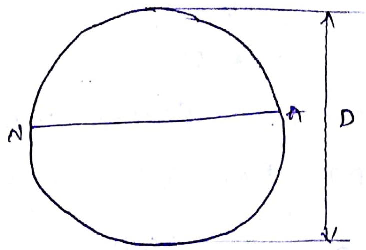
$$y_{max} = \frac{D}{2}$$

$$\boxed{Z = \frac{BD^3 - bd^3}{6D}}$$



Circular section :

$$Z = \frac{I}{y_{max}}$$



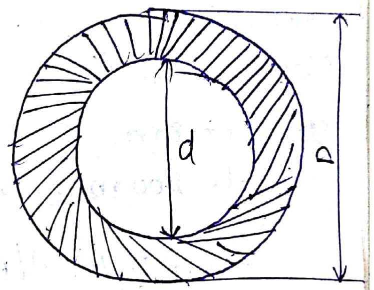
$$I = \frac{\pi D^4}{64} \quad , \quad y_{max} = \frac{D}{2}$$

$$Z = \frac{\pi}{64} D^4 \times \frac{2}{D}$$

$$\boxed{Z = \frac{\pi D^3}{32}}$$

Hallow circular section :

$$Z = \frac{I}{y_{max}}$$

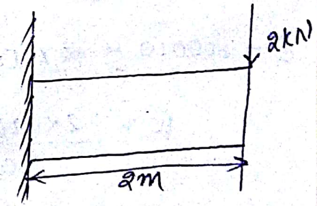


$$I = \frac{\pi}{64}(D^4 - d^4)$$

$$\boxed{Z = \frac{\pi}{32D}(D^4 - d^4)}$$

(2) A cantilever of length 2m fails when a load of 2kN is applied at free end. If the section of the beam be 40 mm × 60 mm. Find the stress at the failure.

Sol:



$W = 2kN \ , \ L = 2m$

$D \times d = 40 \times 60$ mm

$$y_{max} = \frac{60}{2} = 30$$

$$I = \frac{bd^3}{12}$$

$$M = \sigma_{max} \times Z$$

$$\sigma_{max} = M/Z$$

$$Z = \frac{bd^2}{6} = \frac{40 \times (60)^2}{6}$$

$$= 24000$$

$M = W \times L$ (when cantilever beam is with pt-load)

$M = 2 \times 2 \times 10^6 = 4000000$ Nmm

$$\sigma_{max} = \frac{4 \times 10^6}{24 \times 10^3} = 166.66 \text{ N/mm}^2$$

② A rectangular beam of 200 mm deep and 300 mm wide is simply supported over a span of 8 M which is uniformly distributed load per meter. The beam may carry if the bending stress is not to exceed 120 N/mm².

Sol: $l = 8m$

$d = 200$ mm, $b = 300$ mm

$\sigma_{max} = 120 \text{ N/mm}^2$

$M = Z \cdot \sigma_{max}$ —— ①

$$Z = \frac{bd^2}{6} = \frac{300 \times (200)^2}{6} = 2 \times 10^6 \text{ mm}^2$$

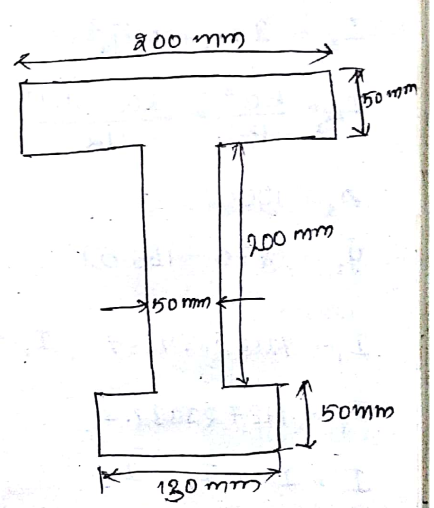
$$M = \frac{W \times 8^2}{8} \Rightarrow M = 8000 W$$

$$① \Rightarrow 8000W = 2 \times 10^6 \times 120$$

$$W = \frac{2 \times 120 \times 10^6}{8000}$$

$$= 30 \text{ kN/m}$$

(9) A cast iron, brass subjected to bending has a cross section of I-form with unequal flanges. The demension of the section are as shown in figure. Find the position of neutral axis and moment of inertia of the section about the neutral axis. If the max. bending moment on the section is 40MNmm. Determine max. bending stress.



Given,

$M = 40\ MNmm$

$\sigma_{max} = ?$

$\frac{M}{I} = \frac{\sigma}{y}$

$\sigma_{max} = \frac{M}{I} \times y$

$$y = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$A_1 = 130 \times 50\ mm = 6500$ | $y_1 = \frac{50}{2} = 25\ mm$

$A_2 = 200 \times 50\ mm = 10000$ | $y_2 = 50 + \frac{200}{2} = 150$

$A_3 = 200 \times 50\ mm = 10000$ | $y_3 = 50 + 200 + \frac{50}{2} = 275$

$$y = \frac{6500(25) + 10000(150) + 10000(275)}{6500 + 10000 + 10000}$$

$= 166.5$

$I_{N.A} = I_1 + I_2 + I_3$

$I_1 = I_{xx_1} + A_1\bar{y}_1^2$

$I_{xx_1} = \frac{bd^3}{12} = \frac{130(50)^3}{12}$

$A_1 = 6500$

$\bar{y}_1 = (166.5 - 25)$

$I_2 = I_{xx_2} + A_2\bar{y}_2^2$

$I_{xx_2} = \frac{bd^3}{12} = \frac{50(200)^3}{12}$

$A_2 = 10000$

$\bar{y}_2 = (166.5 - 150)$

$I_3 = I_{xx_3} + A_3\bar{y}_3^2$

$I_{xx_3} = \frac{bd^3}{12} = \frac{200(50)^3}{12}$

$A_3 = 10000$

$\bar{y}_3 = (275 - 166.5)$

$I_1 = 131498791.7$, $I_2 = 36055833.33$

$I_3 = 118723333.3$

$I = I_1 + I_2 + I_3$

$= 286277958.4$ //

$\frac{M}{I} = \frac{\sigma}{y} \Rightarrow \sigma_{max} = \frac{M}{I} \times y$

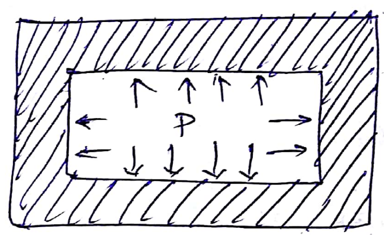
$= \frac{40 \times 10^6}{286277958.4} \times 166.5$

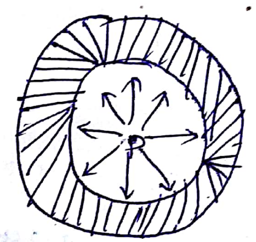
$\sigma_{max} = 23.26 \ N/mm^2$ //

Thin cylinder

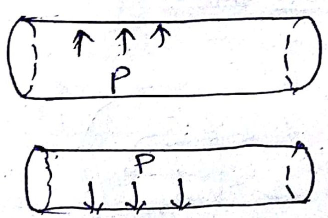
Thin cylinder: If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}^{th}$ of internal diameter, the cylindrical vessel is known as thin cylinder.

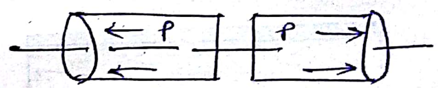
Thin cylindrical vessels subjected to internal pressure:





(a) Across the axis (b) $\perp^{lar}$ to the axis





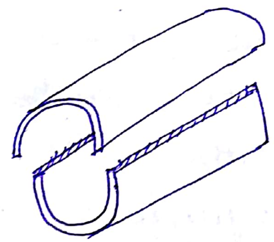
Stresses in thin cylindrical vessel subjected to internal pressure:

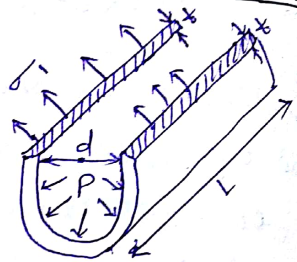
→ circumference stress (hoop stresses)

The stress acting along the circumference of cylinder is known as circumference stress.

→ The stress acting along the length of the cylinder is known as longitudinal stress.

* Expression for circumferential stress:





→ Force due to internal pressure $= P \times A$

$= P \times d \times L$ — ①

→ Force due to circumferential stress $= \sigma_1 \times A$

$= \sigma_1 \times (tL + tL)$

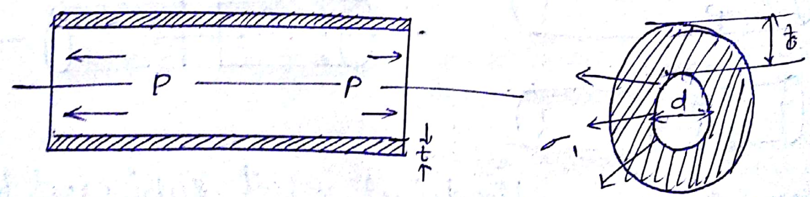
$= 2\sigma_1 (tL)$ — ②

from ① & ②

$P \times d \times \cancel{L} = 2\sigma_1 t \cancel{L}$

$$\boxed{\sigma_1 = \frac{Pd}{2t}}$$

* Expression for longitudinal stress:



→ Force due to internal pressure $= P \times A$

$= P \times \frac{\pi}{4} d^2$ — ①

→ Force due to longitudinal stress $= \sigma_2 \times A$

$= \sigma_2 \times \pi d \times t$ — ②

from Eqn ① & ②

$P \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$

$$\boxed{\sigma_2 = \frac{Pd}{4t}}$$

Max. shear stress :

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2} = \frac{Pd}{8t}$$

(2) A cylindrical pipe of diameter 1.5m & thickness 1.5cm is subjected to an internal fluid pressure of 1.2 N/m². Determine longitudinal stress & circumferential stress.

sol: Given,

$d = 1.5m$

$t = 1.5\,cm = 0.015\,m$

$$\sigma_1 = \frac{Pd}{2t} = \frac{1.2 \times 10^3 \times 1.5}{2 \times 0.015} = 60\ N/mm^2$$

$$\sigma_2 = \frac{Pd}{4t} = \frac{1.2 \times 10^3 \times 1.5}{4 \times 0.015} = 30\ N/mm^2$$

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*** Effect of internal pressures on the dimensions of cylindrical shell:**

circumferential strain	longitudinal strain
$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$	$e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$
$e_1 = \frac{Pd}{2tE} - \mu\frac{Pd}{4tE}$	$e_2 = \frac{Pd}{4tE} - \mu\frac{Pd}{2tE}$
$e_1 = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$ —— (1)	$e_2 = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$ —— (2)

$$e_1 = \frac{\text{change in circumference}}{\text{original circumference}}$$

change in circumference = final − initial circum.

$= \pi(d+\delta d) - \pi d$

$= \pi\delta d$

Original circumference = πd

$$e_1 = \frac{\cancel{\pi}\,\delta d}{\cancel{\pi} d} = \frac{\delta d}{d} \quad \text{—— (3)}$$

from (1) & (3)

$$\frac{Pd}{2tE}\left(1 - \frac{\mu}{2}\right) = \frac{\delta d}{d}$$

$$\Rightarrow \boxed{\delta d = \frac{Pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)}$$

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$$e_2 = \frac{\text{change in length}}{\text{original length}}$$

$$e_2 = \frac{\delta L}{L} \quad \text{--- (4)}$$

from (2) & (4)

$$\frac{Pd}{2tE}\left(\frac{1}{2} - \mu\right) = \frac{\delta L}{L}$$

$$\Rightarrow \boxed{\delta L = \frac{PdL}{2tE}\left(\frac{1}{2} - \mu\right)}$$

volume strain $= \frac{\delta V}{V}$

$$V = A \times L$$
$$= \frac{\pi}{4} d^2 \times L$$

change in volume = final − initial

$$= \frac{\pi}{4}(d+\delta d)^2 \times (\cancel{\delta L} + L) - \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4}(d^2 + \delta d^2 + 2d\delta d)(\delta L + L) - \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4}(d^2 \delta L + d^2 L + \delta d^2 \delta L + \delta d^2 L + 2d\delta d \cdot \delta L + 2d\,\delta d \cdot L) - \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4}(d^2 \delta L + (\delta d)^2 \delta L + (\delta d)^2 \cdot L + 2d\delta d(\delta L + L))$$

by neglecting small values

$$= \frac{\pi}{4}(2dL \cdot \delta d + \delta L \cdot d^2)$$

Original volume $= \frac{\pi}{4} d^2 \times L$

$\therefore$ volumetric strain $= \dfrac{\frac{\pi}{4}(2dL\,\delta d + \delta l \cdot d^2)}{\left(\frac{\pi}{4}\right) d^2 \times L}$

$= \dfrac{2dL\,\delta d + \delta l \cdot d^2}{d^2 L}$

$= \dfrac{2L\,\delta d + \delta l \cdot d}{dL}$

$= 2\dfrac{\delta d}{d} + \dfrac{\delta l}{L}$

$\dfrac{\delta V}{V} = 2e_1 + e_2$

$\boxed{\delta V = V(2e_1 + e_2)}$ //

(*) Calculate change in circumference, change in length & change in volume of cylindrical shell of 100 cm dia, 1 cm thick & 5 m long when subjected to internal pressure of $3\,N/mm^2$. Take values of ($E = 2\times10^5\,mm^2$ & $\mu = 0.3$)

Sol: Given, $d = 100\,cm$

$t = 1\,cm$

$L = 5\times10^{-2}\,cm$; $P = 3\,N/mm^2$

$E = 2\times10^5\,N/mm^2$; $\mu = 0.3$

$\delta d = \dfrac{Pd^2}{2tE}\left(1 - \dfrac{\mu}{2}\right) = ?$

$\delta L = \dfrac{PdL}{2tE}\left(\dfrac{1}{2} - \mu\right) = ?$

$\delta v = V \cdot \left(2\left(\dfrac{Pd}{2tE}\left(1 - \dfrac{\mu}{2}\right)\right) + \dfrac{Pd}{2tE}\left(\dfrac{1}{2} - \mu\right)\right) = ?$

(2) A closed cylindrical vessel made of steel plate $t = 4mm$ with place ends carry's fluid under pressure of $2N/mm^2$ the dia of cylinder is 25cm & length is 75cm. Calculate the longitudinal & hoop stresses in the cylindrical well. Determine change in length, diameter & volume of cylinder. Take, $E = 2 \times 10^5 \ N/mm^2$ & poissons ratio $(\mu) = 0.286$

sol: Given,

$t = 4mm$

$P = 3 \ N/mm^2$

$L = 75cm = 750 \ mm$

dia. $= 25cm = 250mm$

circumferential stress $= \frac{Pd}{2t} = \frac{3 \times 250}{2 \times 4}$

longitudinal stress $= \frac{Pd}{4t} = \frac{3 \times 250}{4 \times 4}$

change in dia., $\delta d = \frac{Pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$

change in length, $\delta L = \frac{PdL}{2tE}\left(\frac{1}{2} - \mu\right)$

$= \frac{3 \times 250 \times 750}{2 \times 4 \times 2 \times 10^5}\left(\frac{1}{2} - 0.286\right)$

change in volume, $\delta V = \left(2\frac{\delta d}{d} + \frac{\delta L}{L}\right) \cdot V$

③ A cylindrical vessel whose ends are closed by means of rigid flange plates is made of steel plate 3mm thick. The length & internal diameter of vessels are 50cm & 25 cm respectively. Determine longitudinal & circumferential stresses in the cylinder shell due to an internal fluid pressure of 3 N/mm². Calculate increase in length, diameter & volume of vessel. Take, $E = 2\times10^5 N/mm^2$ & $\mu = 0.3$.

Given, $t = 3mm$; $E = 2\times10^5 N/mm^2$

$P = 3N/mm^2$; $L = 50cm$

$\mu = 0.3$; $d = 25$ cm

circumferential stress $= \frac{Pd}{2t} = \frac{3\times250}{2\times3}$

longitudinal stress $= \frac{Pd}{4t} = \frac{3\times250}{4\times3}$

change in diameter, $\delta d = \frac{Pd^2}{2tE}(1-\mu/2)$

change in length, $\delta L = \frac{PdL}{2tE}(\frac{1}{2}-\mu)$

change in volume, $\delta V = V.\left(2\frac{\delta d}{d}+\frac{\delta L}{L}\right)$

$= \frac{\pi}{4}d^2\times L\left(2\frac{\delta d}{d}+\frac{\delta l}{L}\right)$

* Thin

∴ H

* Change shell du

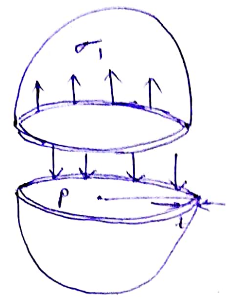
e_1

e_1

fr

cd

* **Thin Spherical shell :**



→ The force 'p' which has the tendency to split the Shell is $= p \times \frac{\pi}{4} d^2$

The area resisting the force is $= \pi d \times t$

∴ Hoops stress or circumference stress,

$$\sigma_1 = \frac{p \times \frac{\pi}{4} d^2}{\pi d t}$$

along x-axis $\boxed{\sigma_1 = \frac{Pd}{4t}}$

along y-axis $\boxed{\sigma_2 = \frac{Pd}{4t}}$

* Change in dimensions of a thin spherical shell due to an internal pressure:

$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$= \frac{\sigma_1}{E} - \mu \frac{\sigma_1}{E} \quad (\because \sigma_1 = \sigma_2)$$

$$= \frac{Pd}{4tE}(1-\mu) \text{ ——— } 1$$

$$e_1 = \frac{\delta d}{d} \text{ ——— } 2$$

from 1 & 2

$$\frac{\delta d}{d} = \frac{Pd}{4tE}(1-\mu) \,//$$

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change in diameter, $\delta d = \frac{Pd^2}{4tE}[1-\mu]$

$\frac{\delta V}{V}$ (volumetric strain)

$V = \frac{4}{3}\pi r^3$ (or) $\frac{\pi d^3}{6}$

on differentiating

$\delta V = \frac{\pi}{6} 3d^2 \, d(d)$

$\frac{\delta V}{V} = \frac{\cancel{\frac{\pi}{6}} \cdot 3\cancel{d^2} \, d(d)}{\cancel{\frac{\pi}{6}} \cancel{d^3}}$

$\frac{\delta V}{V} = \frac{3}{d} d(d)$

$\frac{\delta V}{V} = \frac{3Pd}{4tE}[1-\mu]$

$\boxed{\delta V = \frac{3Pd}{4tE}(1-\mu) \cdot V}$ Change in volume

(Q) A vessel in the shape of spherical shell of 1.2 m internal dia and 12 mm shell thickness is subjected to pressure of 1.6 N/mm². Determine stress induced in material of vessel.

(2) A spherical shell of internal dia 0.9 m & thickness of 10 mm is subjected to an internal pressure of 1.4 N/mm². Determine increase in diameter & increase in ~~dia~~ vol. Take, $E = 2\times10^5$ N/mm², $\mu = 1/3$

(8) Sol: Given,

Internal dia. = 1.2 m = 1200 mm

thickness = 12 mm

pressure = 1.6 N/mm^2

$$\therefore \sigma = \frac{Pd}{4t} = \frac{1.6 \times 1200}{4 \times 12}$$

$= 40\ N/mm^2$

(9) Sol: Given,

Internal dia = 0.9 m = 900 mm

Thickness = 10 mm

pressure = 1.4 N/mm^2

$$\delta d = \frac{Pd^2}{4tE}(1-\mu)$$

$$= \frac{(900)^2 . (1.4)}{4 \times 10 \times 2 \times 10^5}(1-\tfrac{1}{3})$$

$= 0.094$ mm

$$\delta V = \frac{3Pd}{4tE}(1-\mu) . V$$

$$V = \frac{4}{3}\pi r^3 = \frac{\pi}{6}d^3 = 381510\ m^3$$

$\delta V = 12017.8\ m^3$

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