

# MATHS

## SHORT ANSWERS:-

1. Explain Discrete random variables.

1. If the Random variable "x" takes only a finite (or) countable number of values then it is called Discrete Random variable  
for ex—
- 1) No. of Head in tossing three coins.
  - 2) No. of rooms in buildings.

2. If  $x_1, x_2$  are two random variables and  $a, b$  are constants then  $E(ax_1 + bx_2)$ ?

Sol:-

$$\underline{E(ax + b)} = aE[x] + b.$$

$$\begin{aligned} E(ax_1 + bx_2) &= E(ax_1) + E(bx_2) \\ &= aE[x_1] + bE[x_2] \end{aligned}$$

3. The mean and variance of a binomial distribution are 6 & 3 respectively find the mode of the binomial distribution?

Sol:-

$$np = 6$$

$$npq = 3$$

$$q = \frac{npq}{np} = \frac{3}{6}$$

$$p = 1 - q = 1 - \frac{3}{6} = \frac{1}{2}$$

$$n \times \frac{1}{2} = 6$$

$$n = 12$$

$$(n+1)p - 1 \leq r \leq (n+1)p$$

$$\frac{13}{2} - 1 \leq r \leq \frac{13}{2} \quad \left[ 5.5 \leq r \leq 6.5 \right] \\ \text{(mode = 6)}$$

Ex 10  $X$  is a poisson variate such that  $p(X=0) = p(X=1) = k$ . Determine  $k$ .

Sol:-

$$e^{-1}$$

$$p(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$p(X=0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-\lambda}$$

$$p(X=1) = \frac{e^{-\lambda} \lambda^1}{1!} = \lambda e^{-\lambda}$$

Given  $p(X=0) = p(X=1) = k$

$$\Rightarrow e^{-\lambda} = \lambda e^{-\lambda} = k$$

$$\Rightarrow e^{-1} = \lambda e^{-\lambda}$$

$$\therefore \lambda = 1$$

$$k = e^{-1} = e^{-1}$$

! If  $X = B(n, p)$  then write the conditions under which  $X$  tends to a poisson distribution

$$(i) - n \rightarrow \infty$$

$$(ii) - p \rightarrow 0$$

$$\frac{1}{n} \rightarrow 0$$

$$\lambda = np$$

6. Define a random variable.

Sol:- The process of assigning a numerical 'x' called random variable for other words. Let 'x' be the sample space associated with random experiment a real value function.

7. Define binomial distribution.

Sol:- A random variable  $x$  is said to follow binomial distribution if it assumes only non-negative values &  $x$  probability mass function is given by

$$p(x) = n C x \cdot p^x \cdot q^{n-x}, x; 0, 1, 2, \dots, n$$

8. If the mean and variance of the binomial variate are 12 and 4 then write binomial distribution.

Sol:-  $np = 12$  — (1)

$$npq = 4$$
 — (2)

$$(2) \div (1) \quad q = \frac{4}{12}$$

$$p = 1 - q = 1 - \frac{4}{12} = \frac{1}{3}$$

$$n = \frac{12}{p} = 36.$$

9. List the properties of probability distribution function.

- A. 1.  $0 \leq p(x_i) \leq 1$   
2.  $p(x_1) + p(x_2) + \dots + p(x_n) = 1$

The first

10. Define poisson distribution?

Sol- A random variable 'x' is said to follow a poisson distribution if it assumes only non-negative values & its probability mass function is given

by

$$p(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} ; x=0, 1, \dots, \infty$$

11. If K is a constant, then what is the value of  $E(ax + K)$ ?

Sol-

1. Explain continuous random variable.

Sol:- A Random variable  $X$  which can take values continuously i.e., which takes all possible values in a given interval is called continuous random variable.

Ex:- Height, age, time.

2. If  $f(x) = \begin{cases} (x+1) & -1 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$

Find  $E(X)$ .

Sol:-  $E(X) = \int_{-1}^1 (x+1) dx$

$$= \left( \frac{x^2}{2} + x \right) \Big|_{-1}^1 = \left( \frac{1}{2} + 1 \right) - \left( \frac{1}{2} - 1 \right) = 2$$

3. If  $f(x) = ke^{-x/5}$ ,  $x > 0$  is a probability density function then find  $k$ .

Sol:-



4. If  $f(x) = k(2x+3)$  in  $0 < x < 2$ , then find  $k$ .

sol

5. write to application of normal distribution

sol:- The normal distribution can be used to approximate binomial & poisson distribution. It has a wide use in testing statistical hypothesis & test of significance. It helps us to estimate parameters from the statistic and to find confidence limits of parameters.

6. write the probability density function of normal distribution?

sol:- The general formula for the probability density function of the normal distribution is

$$f(x) = \frac{e^{-(x-\mu)^2 / (2\sigma^2)}}{\sigma\sqrt{2\pi}} \quad (\text{or})$$

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$$

7. If  $\mu=5$ , and  $\sigma=2$ , then write the p.d.f. of the normal distribution

Sol:-

$$\mu=5$$

$$\sigma=2$$

8. Write the probability density function for the standard normal variate.

Sol:- The Eq<sup>n</sup> for the standard normal distribution is

$$f(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

9. If  $f(x) = k(2x^2)$  in  $0 < x < 2$ , then find  $k$

Sol:- The probability density function is continuous function

$$\int_0^2 k(2x^2) dx = 1$$

$$k(2) \left[ \frac{x^3}{3} \right]_0^2 = 1$$

1. Define population and sample?

sol:- population is the total of statistical data forming a subject of investigation. The no. of units in the population is called population. It is denoted by 'N'.

A part of the population which is examined with a view to determine the population characteristics is called a sample. It is denoted by 'n'.

2. Define parameter and statistic?

A parameter is a statistical method based on all units of pop. mean ( $\mu$ ), std deviation ( $\sigma$ ), pop. variance ( $\sigma^2$ ), population proportion ( $P$ ).

Statistic is a statistical measure based on all elements of sample. Sample mean ( $\bar{x}$ ), sample std deviation ( $s$ ), sample variance ( $s^2$ ), sample proportion ( $P$ ).

3. Define std error of a statistic?

sol:- The standard of statistic 't' is the std deviation of the sampling distribution of the statistic. i.e., S.E of sample mean is the std deviation of the sampling distribution of sample mean.

$$S.E = \frac{\sigma}{\sqrt{n}}$$



4. Define Estimator & estimate?

Sol:- An estimate is a statement made to find an unknown pop'n parameter. The procedure to determine an unknown pop'n parameter is called an estimator.

5. Define point estimation & Interval Estimation.

A. If an estimate of the population parameter is given by a single value, then the estimate is called a point estimation.

If an estimate of a population parameter is given by two different values b/w

which the parameter may be considered to lie, then the estimate is called an Interval Estimation.

6. Find the pop'n correction factor if  $n = 5$  &  $N = 30$

Sol:- 
$$\frac{N-n}{N-1} = \frac{30-5}{30-1} = \frac{15}{29}$$

7. Define type I error?

A. If type I error the null hypothesis is true but it is rejected by test procedure. Then the error made is called type I error.

8. Write the test statistic of single mean in large samples?

A. If the size of the sample  $n \geq 30$ , then it is said to be large sample.

9. Write about Null Hypothesis?

A. A Null hypothesis is the hypothesis which asserts that there is no significant difference between the statistic & pop'n parameters. It is denoted by  $H_0$  for

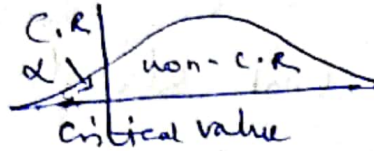
Ex:-  $H_0 = \mu \neq \mu_0$

10. What is mean by level of significance.

A. The level of significance is the confidence which is we reject (or) accept the null hypothesis. It is denoted by  $\alpha$  usually. In practice we take either 5% (or) 1% level of significance.

11. Define Left tailed test

A. If  $H_1$  has  $<$  sign, the critical region is taken in the left side of the distribution



12. Write about alternative hypothesis

A. Any hypothesis which contradicts the null is called Alternative hypothesis

It is denoted by  $H_a$  (or)  $H_1$ . for

ex:  $H_1: \mu \neq \mu_0$ .

13. Define critical region

A. The critical region is formed based on the form of alternative hypothesis

If  $H_1$  has not equal to sign.

then the critical region is divided equally.

In left & right tails is the form of  $H_1$  is  $<$  sign. then the critical region is taken in left tail. If  $H_1$  is greater sign then the critical region is taken on the right tail.



14. Define type II error?  
 A. If null hypothesis  $H_0$  falls but it is accepted by test procedure then the error made is called type two error.

15. Write the statistic of test for difference of two mean in large samples.

A. The test statistic as

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

If  $\sigma_1 = \sigma_2 = \sigma$

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$



1. Find  $t_{0.05}$  when  $v=16$ .

Sol:- 1.746.

2. Find  $t_{0.01}(24, 29)$

Sol:- 2.49

3. write the one assumption of student's t-test.

Sol:- Let  $\bar{x}$  be the mean of a random sample of size  $n$ , taken from a normal population having the mean  $\mu$  and the variance  $\sigma^2$ , and sample variance  $s^2 = \sum \frac{(x_i - \bar{x})^2}{n-1}$ , then

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$  is a random variable having the  $t$ -distribution with  $\nu = n - 1$  degree of freedom.

4. write one use of t-test

Sol:- 1. To test the significance of the sample mean, when population variance is not given.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

$\bar{x}$  = mean of the sample  
 $n$  = size of the sample  
 $\sigma$  = std. deviation.  
 $\mu$  = pop. mean.

6. Write the one use of chi-square test?  
 sol- chi-square test is used to test whether  
 diff b/w observed & expected frequency  
 Chi square test is mainly used to  
 test an Independent of fit. we use  
 this test to decide whether the diff  
 b/w observed & expected frequency  
 is significant (or) not.

$$\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$$

6. find  $F_{0.05}$  with  $v_1 = 7$  &  $v_2 = 15$   
 sol- 2.914

7. Write the formulae for significance of  
 single mean in t-test.

sol-

$$|t| = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

8. Write the formulae for significance of two means in t-test.

Sol:  $t = \frac{\bar{x} - \bar{y}}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

9. Write the formula for chi-square test.

Sol:  $\chi^2 = \sum \left[ \frac{(O_i - E_i)^2}{E_i} \right]$

10. Write the formula for F-test.

Sol:  $F = \frac{S_1^2}{S_2^2} \quad \text{(or)} \quad \frac{S_2^2}{S_1^2}$

$F_{cal} = \frac{\text{Greater variance}}{\text{Smaller variance}}$

11. Define Small sample?

Sol: If the size of the sample  $n < 30$ , then it is said to be small sample (or) Exact sample.

12. Find chi-square value for 1 degree of freedom at 5% level of significance.

Ans:  $\chi^2_{(1)} = 3.841$

(or)  $\chi^2_{(1)} = 3.841$

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1. Write C-R Equations in Cartesian form?

Sol: A necessary and sufficient condition of  $f(z)$  is to be analytical (or)

$\frac{\partial u}{\partial x}$ ,  $\frac{\partial u}{\partial y}$ ,  $\frac{\partial v}{\partial x}$ ,  $\frac{\partial v}{\partial y}$  are continuous functions.

2. Write the definition Harmonic function

3. Define Analytic function?

Sol: A function  $f(z)$  is said to be analytical at the point  $z_0$  if  $f(z)$  is differentiable at each of every point of the neighbourhood of  $z_0$ .

11. Write the C-R Eqn in polar form.

Ans:- Let  $z = re^{i\theta}$  and  $f(z) = u + iv$ .  
Then the C-R Eqn in polar form is  
$$\frac{1}{r} \frac{\partial v}{\partial \theta} = \frac{\partial u}{\partial r}$$
$$-\frac{\partial u}{\partial \theta} = r \frac{\partial v}{\partial r}$$

5. Write the Statement of Cauchy's Theorem?

Ans:- If a function  $f(z)$  is analytic on  
and with in a simple closed curve

where  $\oint_C f(z) dz = 0$ .

6. Evaluate  $\int_0^{1+i} (x^2 + iy) dz$  along the  
path  $y = x^2$

7. Write Generalized Cauchy's Integral  
~~the~~ formula

Sol:- If  $f(z)$  is analytic on and inside  
in a simple closed curve and  
 $z=a$  is any point  
with in 'C' then,

$$\int_C \frac{f(z)}{(z-a)^{n+1}} dz = \frac{2\pi i}{n!} f^{(n)}(a)$$

8. Find the value  $\oint_C \frac{e^z}{z-4} dz$ , where  
C is  $|z|=2$ .

1. Write the definition conjugate Harmonic function

soln-

2. Define Laplace Eqn

soln-

3. ~~Def~~ Show that  $u(x, y) = x^3 - 3xy^2$  is harmonic



12. Find 'r' such that  $f(x, y) = x^3 + 3rxy^2$  be harmonic.

13. Using Milne-Thomson <sup>method</sup> find  $f(z)$  given that  $u(x, y) = 3x^2 - 3y^2 + 6ixy$ .