Matrix:

→ A rectangular array (arrangement) of mn number (real or complex) in m rows and n columns is called a Matrix of order m by n written as m×n

→ A m×n matrix is usually written as ①

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$$

→ This matrix is denoted in a simple form as

$$A = [a_{ij}]_{m\times n}$$

where a_{ij} is the element in the ith row and jth column

Type of Matrices

1) **Row Matrices:**

A matrix which has only one row is called a row matrix or row vector i.e

$$[1 \; -1 \; 2 \; 3]_{1\times 4} \quad [2 \; 5]_{1\times 2}$$

2) **column matrix:**

A matrix which has only one column is called a column matrix column vector

ie $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times 1} \quad \begin{bmatrix} -1 \\ 1 \\ 0 \\ 4 \end{bmatrix}_{4\times 1}$

3.) Square Matrix:

A matrix in which the number of rows is equal to the number of columns is called a square matrix. i.e

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 5 & 0 & 6 \end{pmatrix}_{3\times3} \quad \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}_{2\times2}$$

4) Null or zero Matrix:

A matrix in which each element is equal to zero is called a null matrix and is denoted by O

i.e
$$\begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

5) Diagonal Matrix:

A square matrix is called a diagonal matrix if all its non diagonal elements are zero

i.e
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix} \quad \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$$

6) Scalar Matrix:

A diagonal matrix whose all diagonal elements are equal is called a scalar matrix

i.e
$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

7) Identity or unit matrix:

- A diagonal matrix whose all diagonal elements are unity(1) is called a unit or identity matrix and is denoted by. I

i.e. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

8) Upper Triangular matrix:

- A square matrix in which all the entries below the diagonal are zero is called an upper triangular matrix

i.e $\begin{bmatrix} 4 & -3 & 2 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$

9) Lower Triangular Matrix:

- A square matrix in which all the entries above the diagonal are zero is called an Lower triangular matrix

i.e. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & -6 & 0 \\ 2 & 3 & 3 \end{bmatrix}$

10) Trace of a square matrix:

The sum of all the diagonal elements of square matrix is called the trace of a matrix

If $A = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$ then

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Appling $C_3 \to 2C_3 - C_2$, $C_4 \to 4C_4 - 6C_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Appling $C_2 \to \frac{C_2}{4}$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This in the normal form of $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$

Hence the Rank$(A) = \rho(A) = 2$

Example. Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 2 \\ 2 & -2 & 0 & 6 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ into normal form and hence find the rank

sol: Let the given matrix $A = \begin{bmatrix} 1 & -1 & 0 & 3 \\ 4 & 2 & 0 & 2 \\ 2 & -2 & 0 & 6 \\ 1 & -2 & 1 & 2 \end{bmatrix}$

Appling $R_2 \to R_2 - 4R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - R_1$

$$A \sim \begin{bmatrix} 1 & -1 & 0 & 3 \\ 0 & 6 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

Appling $C_2 \to C_2 + C_1$, $C_4 \to C_4 - 3C_1$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & -1 \end{bmatrix}$$

Appling $R_3 \leftrightarrow R_4$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & -10 \\ 0 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to 6R_3 + R_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 6 & 0 & -10 \\ 0 & 0 & 6 & -16 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_2 \to \frac{C_2}{6}, \ C_3 \to \frac{C_3}{6}$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -10 \\ 0 & 0 & 1 & -16 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + 10C_2$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -16 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$C_4 \to C_4 + 16C_3$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This in the normal form of $\begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$

Hence the Rank$(A) = \rho(A) = 3$

Ex Reduce the matrix $A = \begin{bmatrix} 1 & -1 & 3 & -2 \\ 0 & 1 & 2 & 3 \\ 5 & 1 & -1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$ into Normal form and hence find the rank

Sol Let the given matrix $A = \begin{bmatrix} 1 & -1 & 3 & -2 \\ 0 & 1 & 2 & 3 \\ 5 & 1 & -1 & 2 \\ 4 & 3 & 2 & 1 \end{bmatrix}$

Appling $R_3 \to R_3 - 5R_1$, $R_4 \to R_4 - 4R_1$

$$A \sim \begin{bmatrix} 1 & -1 & 3 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & -16 & 12 \\ 0 & 7 & -10 & 9 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & -3 & -8 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

Appling $R_1 \to R_1 - 9R_3$, $R_2 \to R_2 + 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

Hence $[A | I_3]$ reduce $[I_3 / B]$ then $B = A^{-1}$

$$B = A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$

→ Example: Find the inverse of $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 3 \\ 0 & 2 & 1 \end{bmatrix}$ by using Gauss-Jordon method

→ solution: let the given matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ -2 & 1 & 3 \\ 0 & 2 & -1 \end{bmatrix}$

By using Gauss-Jordon Method

let $[A / I_3] = \left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ -2 & 1 & 3 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$

Appling $R_2 \to R_2 + 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 7 & 11 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

Appling $R_2 \to R_2\left(\frac{1}{7}\right)$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{11}{7} & \frac{2}{7} & \frac{1}{7} & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

Appling $R_1 \to R_1 - 3R_2 \cdot R_3 - 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5/7 & 1/7 & -3/7 & 0 \\ 0 & 1 & \frac{11}{7} & 2/7 & 1/7 & 0 \\ 0 & 0 & -\frac{29}{7} & -\frac{1}{7} & -\frac{2}{7} & 1 \end{array}\right]$$

Appling $R_2 \to R_3 \left(-\frac{7}{29}\right)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{-5}{7} & 1/7 & -3/7 & 0 \\ 0 & 1 & \frac{11}{7} & 2/7 & 1/7 & 0 \\ 0 & 0 & 1 & 4/29 & 2/29 & -7/29 \end{array}\right]$$

Appling $R_1 \to R_1 + \frac{5}{7} \cdot R_3 \cdot R_2 \to R_2 - \frac{11}{7} R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 7/29 & \frac{-11}{29} & -5/29 \\ 0 & 1 & 0 & 2/29 & \frac{1}{29} & 11/29 \\ 0 & 0 & 1 & 4/29 & 2/29 & -7/29 \end{array}\right]$$

Hence $[A \mid I_3]$ reduce to $[I_3/B]$ then $B = A^{-1}$

$$B = A^{-1} \begin{bmatrix} 7/29 & \frac{-11}{29} & \frac{-5}{29} \\ 2/29 & 1/29 & 11/29 \\ \frac{4}{29} & 2/29 & -7/29 \end{bmatrix} \quad //$$

→ System of Non-Homogeneous linear Equations:

A system of m lines Equation in n unknowns can be written as

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Ex: Find whe ther the following Equation are consist or not If it is consistent solve them

$x+y+2z=2, 3x-2y-z=5, 2x-5y+3z=-4$

$x+4y+6z=0$

Sol: The given equations can be written in the matri form $AX=B$, we have

$$\begin{pmatrix} 1 & 1 & 2 \\ 3 & -2 & -1 \\ 2 & -5 & 3 \\ 1 & 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 2 \\ 5 \\ -4 \\ 0 \end{bmatrix} \quad \text{---- (1)}$$

consider Argument matrix $(A|B) = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 3 & -2 & -1 & 5 \\ 2 & -5 & 3 & -4 \\ 1 & 4 & 6 & 0 \end{pmatrix}$

Appling $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$, $R_4 \to R_4 - R_1$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & -9 & -1 & -8 \\ 0 & 2 & 4 & -2 \end{bmatrix}$$

Appling $R_3 \to R_3 - 9R_2$. $R_4 \to 4R_4 + R_2$

$$[A|B] \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 55 & -55 \\ 0 & 0 & 9 & -9 \end{bmatrix}$$

Appling $R_3 \to \frac{1}{55} R_3$, $R_4 \to \frac{1}{9} R_4$

$$(A|B) \sim \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Appling $R_4 \to R_4 - R_3$

$$(A|B) \sim \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & -8 & -7 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The number of Non zero rows in (A/B) is 3

$\therefore$ The rank of argument matrix $(A|B] = 3$

and the number of Non zero rows in $(A]$ is 3

$$P(A) = 3$$

Hence we have $P(A) = P[A|B] = n = 3$

$\therefore$ The given system is consistent so it has unique solution

Then $\begin{vmatrix} 1 & 1 & 2 \\ 0 & -8 & -7 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{vmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 0 \end{pmatrix}$

write into equation form

$x + y + 2z = 2$ --(a), $-8y - 7z = -1$ -- (b)

$z = -1$ -- (c) from (c) $z = -1$

substituting $z = -1$ in (b) $\Rightarrow -8y + 7 = -1 \Rightarrow y = 1$

" " $z = -1$ $y = 1$ in (a) $\Rightarrow x + 1 - 2 = 2 \Rightarrow x = 3$

$\therefore x = 3$ $y = 1$, $z = -1$ is the solution

Ex: Find whether the following equations are consistent or not If it is consistent solve them $3x - y + 4z$

$3x + 2y - 3z = -2$, $6x + 5y - 5z = -3$

$\frac{x}{12+6} = \frac{y}{6-0} = \frac{z}{-6} = t$

$\frac{x}{18} = \frac{y}{6} = \frac{z}{-6} = t$

$x = 3t \cdot y = t \; z = -t$

$x_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3t \\ t \\ -t \end{pmatrix} \to t\begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = t x_3$

where x_3 is the Eijan vector corresspoinding to the Eijan value $\lambda = 4$

① Find the Eijan values and Eijan vector of the matrix

$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

characteristic eqn of matrix A is $|A - \lambda I| = 0$

$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$

characteristic eqn of matrix A of order 3

det of matix $(A - \lambda I) = 0$

$\to 8-\lambda[21 - 10\lambda + \lambda^2 - 16] + 6[-18 + 6\lambda + 8] + 2[24 - 14 + 2\lambda] = 0$

$\to 8-\lambda[\lambda^2 + 5 - 10\lambda] + 6[6\lambda - 10] + 2[10 + 2\lambda] = 0$

$\to 8\lambda^2 + 40 - 8\lambda - \lambda^3 - 5\lambda + 10\lambda^2 + 36\lambda - 60 + 20 + 4\lambda = 0$

$-\lambda^3 + 18\lambda^2 - 45\lambda = 0$

$\lambda = 3$ is one of the root

$$\begin{array}{r|rrr} 3 & -1 & 18 & -45 \\ & 0 & -3 & 45 \\ \hline & -1 & 15 & 0 \end{array}$$

$-\lambda^2 + 15 = 0$

$\lambda = 15, 0$

Eijan values of given matix are $\lambda = 0, 3, 15$ (11)

To Find the eijan vectors:

<u>case (i)</u> $\lambda = 0$ in $[A - \lambda I]x = 0$

$$\begin{pmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{pmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$8x - 6y + 2z = 0$ — (1) $-6x + 7y - 4z = 0$ — (2) $2x - 4y + 3y = 0$ — (3)

sole (1) & (2)

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\begin{vmatrix} -6 & 2 & 8 & -6 \\ 7 & -4 & -6 & 7 \end{vmatrix}$$

$$\frac{x}{24-14} = \frac{y}{-12+32} = \frac{z}{56-36} = t$$

$$\frac{x}{10} = \frac{y}{20} \quad \frac{z}{20} = t$$

$$x = t \quad y = 2t \quad z = 2t$$

$$x_1 = \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} t \\ 2t \\ 2t \end{vmatrix} \Rightarrow t \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = tX_1$$

X_1 is the Eijan vector corresponding to Eijan value $\lambda = 0$

<u>case (ii)</u> $\lambda = 3$

$$\begin{vmatrix} 8-3 & -6 & 2 \\ -6 & 7-3 & -4 \\ 2 & -4 & 3-3 \end{vmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$\begin{vmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{vmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$5x - 6y + 2z = 0$ — ① $-6x + 4y - 4z = 0$ — ② $2x - 4y = 0$ — ③

solve ② & ③

$$\begin{matrix} x & & y & & z & \\ 4 & & -4 & & -6 & 4 \\ -4 & \times & 0 & \times & 2 & \times\ -4 \end{matrix}$$

$$\frac{x}{-16} = \frac{y}{-8} = \frac{z}{24-8} = t$$

$$\frac{x}{-16} = \frac{y}{-8} = \frac{z}{16} = t$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{-2} = t$$

$x = 2t \quad y = t \quad z = -2t$

$$x_2 = \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 2t \\ t \\ -2t \end{vmatrix} \Rightarrow t \begin{vmatrix} 2 \\ 1 \\ -2 \end{vmatrix} = tx_2$$

x_2 is the eigen vector corresponding to eigen value $\lambda = 3$

<u>case (iii)</u> $\lambda = 15$

$$\begin{vmatrix} 8-15 & 6 & 2 \\ -6 & 7-15 & -4 \\ 2 & -4 & 3-15 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$$\begin{vmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$-7x - 6y + 2z = 0$ — ① $-6x - 8y - 4z = 0$ — ② $2x - 4y - 12z = 0$ — ③

solve ② & ③

$$\begin{matrix} x & & y & & z & \\ -8 & & -4 & & -6 & -8 \\ -4 & \times & -12 & \times & 2 & \times\ -4 \end{matrix}$$

$$\frac{x}{96-16} = \frac{y}{-8-72} = \frac{z}{24+16} = t$$

$$\frac{x}{80} = \frac{y}{-80} = \frac{z}{40} = t$$

$$x/2 = y/-2 = z/1 = t$$

$$x = 2t \quad y = -2t \quad z = 1t$$

$$x_3 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ -t \\ 2t \end{pmatrix} \to t\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} = tx_3$$

where x_3 is the eigan vector corresponding to eigan value

$$\lambda = 15$$

(3) Find the eigan values and eigan vector of the matrix $A = \begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$

sol characteristis Eqn of matrix A

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

characteristic Eq of matrix order 3

$$\to -2-\lambda[-\lambda+\lambda^2-12] - 2[-2\lambda-6] - 3(4+1-\lambda) = 0$$

$$\to 2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 4\lambda + 12 + 9 + 3\lambda = 0$$

$$\to \lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\lambda_1 = 1$$

$$\to -\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$$

$\lambda = -3$ is one of the root

$$\begin{array}{r|rrrr} -3 & -1 & -1 & 21 & 45 \\ & 0 & 3 & -6 & -45 \\ \hline & -1 & 2 & 15 & 0 \end{array}$$

$$-\lambda^2 + 2\lambda + 15 = 0$$

$$\lambda^2 - 2\lambda - 15 = 0$$

$$\lambda^2 + 3\lambda - 5\lambda - 15 = 0$$

Cayley-Hamilton theorem:

statement: every square matrix satisfy its own characteristic equation

① verify cayley-Hamilton theorem for the following matrix and hence find A^{-1}, A^4

$$A = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$$

Given matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

characteristics eqn of matrix A $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

characteristic eqn of matrix of order 3

$\rightarrow 2-\lambda\,[4-2\lambda-2\lambda+\lambda^2-1] + 1[-2+\lambda+1] + 1[1-2+\lambda] = 0$

$\rightarrow 2-\lambda\,(\lambda^2-4\lambda+3) + 1(\lambda-1) + 1(\lambda-1) = 0$

$\rightarrow 2\lambda^2 - 8\lambda + 6 - \lambda^3 + 4\lambda^3 - 3\lambda + \lambda - 1 + \lambda - 1 = 0$

$\rightarrow -\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$

$\boxed{\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0}$ char. → A

The characteristics eqn to the matrix A is

$A^3 - 6A^2 + 9A - 4I = 0$ — ①

$A^2 = A \cdot A$

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$$

$$\begin{vmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{vmatrix} \rightarrow \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix}$$

form ①

$$A^3-6A^2+9A-4I=0$$

$$LHS = A^3-6A^2+9A-4I$$

$$\begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - 6\begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} + 9\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} - 4\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9+0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9+0 & -21+30-9+0 & 22-36+18-4 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ The matrix A satisfies its own characteristic eqn cayley-Hamilton theorm is verified.

to Find A^{-1}

we multiply eqn by A^{-1}

$$A^{-1}(A^3-6A^2+9A-4I)=0$$

$$A^2-6A+9A^0-4A^{-1}=0$$

$$A^2-4A^{-1}-6A+9I=0$$

$$4A^{-1}=A^2-6A+9I$$

$$A^{-1}=\frac{1}{4}(A^2-6A+9I)$$

$$A^{-1}=\frac{1}{4}\left(\begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} - 6\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} + \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}\right)$$

$$A^{-1} = \boxed{}$$

(17)

$$A^{-1} = \frac{1}{4}\left(\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}\right) \rightarrow A^{-1} = \begin{bmatrix} 3/4 & 1/4 & -1/4 \\ 1/4 & 3/4 & 1/4 \\ -1/4 & 1/4 & 3/4 \end{bmatrix}$$

$$A^4 = A^3 \cdot A$$

$$A^4 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 44+21+21 & -22-42-21 & 22+21+42 \\ -22-42-21 & 44+21+21 & -22-42-21 \\ 22+21+42 & -22-42-21 & 44+21+21 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 86 & -85 & 85 \\ -85 & 86 & -85 \\ 85 & -85 & 86 \end{bmatrix}$$

(2) verify cayley-Hamition theorem and also find A^{-1}, A^4

(1) $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

sol " "

characteristis eqn of matrix $A\,[A-\lambda I]=0$

$$\begin{vmatrix} 1-\lambda & 2 & -2 \\ -1 & 3-\lambda & 0 \\ 0 & -2 & 1-\lambda \end{vmatrix} = 0$$

characteristis eqn of matrix of order 3

$$1-\lambda\,[3-3\lambda-\lambda+\lambda^2]-2\,[-1+\lambda]-2(2)=0$$

$$\rightarrow 1-\lambda\,[\lambda^2-4\lambda+3]-2(\lambda-1)-4=0$$

$$\rightarrow \lambda^2-4\lambda+3-\lambda^3+4\lambda^2-3\lambda-2\lambda+2-4=0$$

$$\rightarrow -\lambda^3+5\lambda^2-9\lambda+1=0$$

The characteristic eqn of matrix A is

$$A^3-5A^2+9A-I=0 \quad \text{— (1)}$$

$$A^2 = A \cdot A$$

$[\because D = P^T A P]$

$$D = \begin{bmatrix} 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(2) Determine diagonal matrix orthogonally simiar to the real symmetrix matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ & also find modal matrix

characteristics eqn of matrix A is

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

characterisitic eqn of matrix A of order 3 is

$$\Rightarrow 8-\lambda(21-10\lambda+\lambda^2-16)+6[-18+6\lambda+8]+2(24-14+2\lambda)=0$$

$$\Rightarrow -\lambda^3+18\lambda^2-45\lambda=0$$

$\lambda = 3$ is one of the root

$$\begin{array}{c|ccc} 3 & -1 & 18 & -45 \\ & 0 & -3 & 45 \\ \hline & -1 & 15 & \boxed{0} \end{array}$$

$$\lambda = 15, 0$$

$\therefore$ Eigen values are $\lambda = 0, 3, 15$

To find the Eigan vectors

<u>case (i)</u> $\lambda = 0$ in $(A - \lambda I)X = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$$\begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$8x - 6y + 2z = 0$ — ① $-6x + 7y - 4z = 0$ — ② $2x - 4y + 3y = 0$ — ③

solve ① & ②

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\begin{vmatrix} -6 & 2 & 8 & -6 \\ 7 & -4 & -6 & 7 \end{vmatrix}$$

$$\frac{x}{24-14} = \frac{y}{-12+32} = \frac{z}{56-36} = t$$

$x = t, \; y = 2t, \; z = 2t$

$$X_1 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} t \\ 2t \\ 2t \end{bmatrix} \to t \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \Rightarrow X_1 t$$

X_1 is the eigan vector corresponding to $\lambda = 0$

case(ii) $\lambda = 3$

$(A - \lambda I) X = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = 0$$

$5x - 6y + 2z = 0$ — ① $-6x + 4y - 4z = 0$ — ② $2x - 4y + 0z = 0$ — ③

solve ① & ③

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\begin{vmatrix} -6 & 2 & 5 & -6 \\ -4 & 0 & 2 & -4 \end{vmatrix}$$

$$\frac{x}{0+8} = \frac{y}{4-0} = \frac{z}{-20+12} = t$$

$x = 2t, \; y = t, \; z = -2t$

$$X_2 = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow t \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \to t X_2$$

X_2 is the eigan vector corresponding to $\lambda = 3$

case (iii) $\lambda = 15$

$$|A - \lambda I| X = 0$$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$-7x - 6y + 2z = 0$ — ① $\quad -6x - 8y - 4z = 0$ — ② $\quad 2x - 4y - 12z = 0$ — ③

solve ① & ②

$$\begin{matrix} x & y & z \\ -6 & 2 & -7 & -6 \\ -4 & -12 & 2 & -4 \end{matrix}$$

$$\frac{x}{72+8} = \frac{y}{4-84} = \frac{z}{28+2} = t$$

$$x = 2t \quad y = -2t \quad z = t$$

$$X_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2t \\ -2t \\ t \end{bmatrix} \Rightarrow t \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \Rightarrow tX_3$$

X_3 is the Eigan vector corresponding to Eigan value $\lambda = 15$

To find modal matrix P.

The modal matrix P having the normalized Eigan vectors as its collumn vectors

length of Eigan vectors $X_1 = \sqrt{1+4+4} = 3$

length of Eigan vectors $X_2 = \sqrt{4+1+4} = 3$

length of Eigan vectors $X_3 = \sqrt{4+4+1} = 3$

The normaized Eigan vectors are

$$X_1 = \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \end{pmatrix} \quad X_2 = \begin{pmatrix} 2/3 \\ 1/3 \\ -2/3 \end{pmatrix} \quad X_3 = \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix}$$

$$P = \left[\frac{X_1}{\|X_1\|} \quad \frac{X_2}{\|X_2\|} \quad \frac{X_3}{\|X_3\|} \right]$$

$$P = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$P^{-1} = P^{T}$$

$$P^{T} = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$D = P^{T}AP$$

$$D = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

(3) Determine a diagonal matrix orthogonally similar to real symmtric matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ also find nodal matrix

characteristicy eqn of matrix A is

$$(A - \lambda I) = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

characteristicy eqn of matrix A of order 3 is

$$\rightarrow 6-\lambda[9-3\lambda-3\lambda+\lambda^2-1]+2[-6+2\lambda+2]+2[2-6+2\lambda]=0$$

$$\rightarrow 6\lambda^2-36\lambda+48-\lambda^3+6\lambda^2-8\lambda+4\lambda-8+4\lambda-8=0$$

$$\rightarrow -\lambda^3+12\lambda^2-36\lambda+32=0$$

$\lambda = 2$ is one of the root

$$\begin{array}{r|rrrr} 2 & -1 & 12 & -36 & 32 \\ & 0 & -2 & 20 & -32 \\ \hline & -1 & 10 & -16 & 0 \end{array}$$

$$P = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$P^{-1} = P^T$$

$$P^T = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$D = P^T A P$$

$$D = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

(3) Determine a diagonal matrix orthogonally similar to real symmtric matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ also find nodal matrix

characteristics eqn of matrix A is

$$(A - \lambda I) = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

characteristics eqn of matrix A of order 3 is

$$\rightarrow 6-\lambda\{9-3\lambda-3\lambda+\lambda^2-1\}+2\{-6+2\lambda+2\}+2\{2-6+2\lambda\}=0$$

$$\rightarrow 6\lambda^2-36\lambda+48-\lambda^3+6\lambda^2-8\lambda+4\lambda-8+4\lambda-8=0$$

$$\rightarrow -\lambda^3+12\lambda^2-36\lambda+32=0$$

$\lambda = 2$ is one of the root

$$\begin{array}{r|rrrr} 2 & -1 & 12 & -36 & 32 \\ & 0 & -2 & 20 & -32 \\ \hline & -1 & 10 & -16 & 0 \end{array}$$

i) Given Quadratic form

$Q = x_1^2 + 5x_2^2 + x_3^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$

$Q = X^T A X$

$$Q = [x_1 \; x_2 \; x_3] \begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 1 & 1 & 3 \\ x_2 & 1 & 5 & 1 \\ x_3 & 3 & 1 & 1 \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

Charact Eqn is $|A - \lambda I| = 0$

$$\rightarrow \begin{vmatrix} 1-\lambda & 1 & 3 \\ 1 & 5-\lambda & 1 \\ 3 & 1 & 1-\lambda \end{vmatrix} = 0$$

$\rightarrow (1-\lambda)\left[(5-\lambda)(1-\lambda)-1\right] - 1\left[1(1-\lambda)-3\right] + 3\left[1-3(5-\lambda)\right] = 0$

$\lambda^3 - 7\lambda^2 + 36 = 0$ — (1)

$\lambda = -2$	1	-7	0	36
	0	-2	18	-36
	1	-9	18	0

$\lambda^2 - 9\lambda + 18 = 0$

$\lambda(\lambda - 6) - 3(\lambda - 6) = 0$

$\lambda = -2, 3, 6.$

Rank of = 3 (no. of non zero values)

Index (p) = 2 (+ve Eigan values)

signature (s) = 1 (diff b/w +ve & -ve Eigan values)

Nature = Indefinite (have +ve as well as -ve Eigan valluy)

(ii) Given QF

$Q = 6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$

$Q = X^T A X$

$$= (x_1 \; x_2 \; x_3) \begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline x_1 & 6 & -2 & 2 \\ x_2 & -2 & 3 & -1 \\ x_3 & 2 & -1 & 3 \end{array} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

charc. Ean $(A - \lambda I) = 0$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$\rightarrow 6-\lambda(9-3\lambda-3\lambda+\lambda^2-1)+2(-6+2\lambda+2)+2(2-6+2\lambda)=0$

$\rightarrow 6\lambda^2-36\lambda+48-\lambda^3+6\lambda^2-8\lambda+4\lambda-8+4\lambda-8=0$

$-\lambda^3+12\lambda^2-36\lambda+32=0$

$\lambda=2$ is one of the root

$$\begin{array}{r|rrrr} 2 & -1 & 12 & -36 & 32 \\ & 0 & -2 & 20 & 32 \\ \hline & -1 & 10 & -16 & \boxed{0} \end{array}$$

$\lambda^2-10\lambda+16=0$

$\lambda = 2, 8$

Rank $= 3$

Index $(P) = 3$

Signature $(S) = 3$

Nature $=$ +ve definite

iii) Given QF

$Q = x_1^2+4x_2^2+x_3^2-4x_1x_2+2x_3x_1-4x_2x_3$

$Q = X^T A X$

$$= (x_1\ x_2\ x_3) \begin{array}{c|c} & \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix} \end{array} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{pmatrix}$$

Charact eqn $(A - \lambda I) = 0$

$$\begin{bmatrix} 1-\lambda & -2 & 1 \\ -2 & 4-\lambda & -2 \\ 1 & -2 & 1-\lambda \end{bmatrix} = 0$$

$\rightarrow 1-\lambda\,[4-4\lambda-\lambda+\lambda^2-4]+2\,(-2+2\lambda+2)+1\,[4-4+\lambda]=0$

$\lambda^2-5\lambda-\lambda^3+5\lambda^2-4\lambda-\lambda=0$

$-\lambda^3+6\lambda^2=0$

$\lambda=0,0,6$ are the eigan values

Rank: No of non zero values $=1$

Index $=1$

signature $=1$

nature $=$ +ve semi definite

II Reduce the Quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canical form by orthogonal transformation & also find Index, rank, signature nature of the Quadratic form

Given QF is

$Q=3x^2+5y^2+3z^2-2yz+2zx-2xy$

$Q=X^T A X$

$$Q=(x\ y\ z)\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

charateristic of A is $|A-\lambda I|=0$

$$\begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix} = 0$$

$\rightarrow 3-\lambda\,[15-5\lambda-3\lambda+\lambda^2-1]+1\,[-3+\lambda+1]+1\,[1-5+\lambda]=0$

$-\lambda^3+11\lambda^2-36\lambda+36=0$

$\lambda=2$ is one of the root