

1. 8

$$P(A=1)$$

$$P(B=1|A=1) \dots$$

2. $\theta_{ML} = \text{argmax}$

$$\theta_{A_1} = 5/13$$

$$\theta_{B_1|A_1} = 2/5$$

$$\theta_{B_1|A_0} = 3/8$$

$$\theta_{C_1} = 5/13$$

$$\theta_{D_1|\theta_{A_1}, C_1} = 1/2$$

$$\theta_{D_1|B_1, C_0} = 2/3$$

$$\theta_{D_1|B_0, C_1} = 1/3$$

$$\theta_{D_1|B_0, C_0} = 4/5$$

3.

$$p(\theta_A) = \text{Beta}(\theta_A | \alpha, \beta) = \text{Beta}(\theta_A | 2, 1)$$

$$\begin{aligned} \theta_A &= p(A=1, D) = \int_{\theta_A} p(\theta_A | D) \theta_A = \frac{\alpha_A + \#(A=1)}{\alpha_A + \#(A=1) + \beta + \#(A=0)} \\ &= \frac{2 + 5}{2 + 5 + 1 + 8} = \frac{7}{16} \end{aligned}$$

$$\theta_{B_1|A_1} = \frac{\alpha + \#(B=1|A=1)}{\alpha + \#(B=1|A=1) + \beta + \#(B=1|A=0)}$$

$$= \frac{3 + 2}{3 + 2 + 2 + 3} = \frac{5}{10} = \frac{1}{2}$$

$$\theta_{B_1|A_0} = \frac{2 + \#(B=1|A=0)}{\alpha + \#(B=1|A=0) + \beta + \#(B=0|A=0)}$$

$$= \frac{3 + 3}{3 + 3 + 1 + 5} = \frac{6}{12} = \frac{1}{2}$$

$$\theta_{C_1} = \frac{\alpha + \#(C=1)}{\alpha + \#(C=1) + \beta + \#(C=0)} = \frac{2 + 5}{2 + 5 + 1 + 8} = \frac{7}{16}$$

$$\theta_{D_1|B_1,C_1} = \frac{2 + \#(D=1, B=1, C=1)}{\alpha + \#(D=1, B=1, C=1) + \beta + \#(D=0, B=1, C=1)}$$

$$= \frac{1 + 1}{1 + 1 + 1 + 1} = \frac{1}{2}$$

$$\theta_{D_1|B_1,C_0} = \frac{\alpha + \#(D=1, B=1, C=0)}{\alpha + \#(D=1, B=1, C=0) + \beta + \#(D=0, B=1, C=0)}$$

$$= \frac{1 + 2}{1 + 2 + 1 + 1} = \frac{3}{5}$$

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$$\theta_{\alpha, \beta, \gamma} = \frac{\alpha + \#(D=1, B=0, C=1)}{\alpha + \#(D=1, B=0, C=1) + \beta + \#(D=0, B=0, C=1)}$$

$$= \frac{1 + 1}{1 + 1 + 1 + 2} = \frac{2}{5}$$

$$\theta_{\alpha, \beta, \gamma} = \frac{\alpha + \#(D=1, B=0, C=0)}{\alpha + \#(D=1, B=0, C=0) + \beta + \#(D=0, B=0, C=0)}$$

$$= \frac{1 + 4}{1 + 4 + 1 + 1} = \frac{5}{7}$$