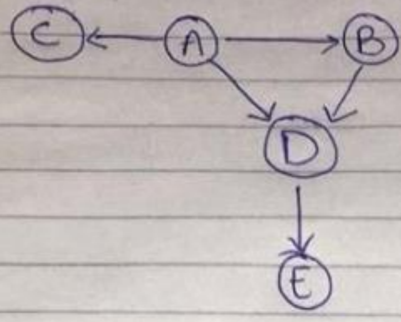
Q.1)

(a)



(b)(i) $P(E,\bar{M},S,\bar{B}) = ?$

$\Rightarrow P(E) \times P(\bar{M}) \times P(S/\bar{E},M) \times P(\bar{B}/\bar{M})$

$\Rightarrow (0.4) \times (0.9) \times (0.8) \times (0.9)$

$\Rightarrow \boxed{0.2592}$

(b)(ii) $P(+m \mid +s,+b,+e) = ?$

$\Rightarrow P(+m \mid +s,+b,+e) = \dfrac{P(+m,+s,+b,+e)}{P(+s,+b+e)}$

$\Rightarrow P(+m,+s,+b,+e) = P(+m) \times P(+s/+e,+m) \times P(+b/+m) \times P(+e)$

$\Rightarrow (0.1) \times (1) \times (1) \times (0.4)$

$\Rightarrow \boxed{0.04}$

$P(+s,+b,+e) = P(+s/+e,m) * P(m) \times P(+b/m) \times P(+e)$

$\Rightarrow (0.8) \times (0.9) \times (0.1) \times (0.4)$

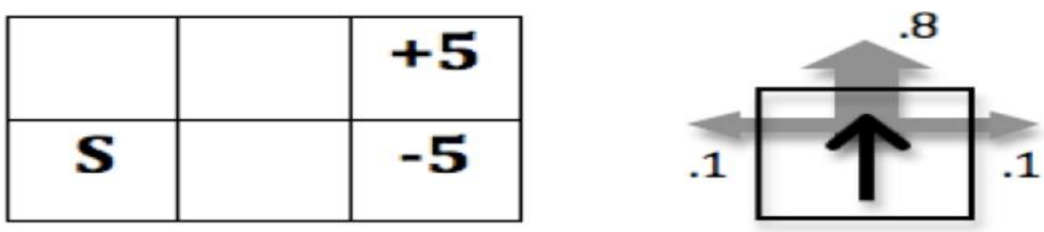
$\Rightarrow \boxed{0.0288}$

$P(+s,+b,+e) = 0.04 + 0.0288 = \boxed{0.0688}$

$$P(+m \mid +s,+b,+e) = \frac{P(+m,+s,+b,+e)}{P(+s,+b,+e)}$$

$$\Rightarrow \frac{0.04}{0.0688} = \boxed{0.5814} \text{ Ans.}$$

Q1: This gridworld MDP operates like to the one we saw in class. The states are grid squares, identified by their row and column number (row first). The agent always starts in state (1, 1), marked with the letter S. There are two terminal goal states, (2, 3) with reward +5 and (1, 3) with reward -5. Rewards are 0 in non-terminal states. (The reward for a state is received as the agent moves into the state.) The transition function is such that the intended agent movement (Up, Down, Left, or Right) happens with probability .8. With probability .1 each, the agent ends up in one of the states perpendicular to the intended direction. If a collision with a wall happens, the agent stays in the same state.



(a) (b)

(a) Gridworld MDP. (b) Transition function.

a) Write the optimal policy for each state? (5 points)

$S =$	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
$\pi^*(S) =$	Up	Left	NA	Right	Right	NA

b) Transition probabilities and reward is already given in question. Apply the first two rounds of value iteration updates for each state, with a discount of 0.9. (Assume V0 is 0 everywhere and compute Vi for times i = 1; 2). (10 points)

Apply the Bellman Equation:

$S =$	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
$V_0(S) =$	0	0	0	0	0	0
$V_1(S) =$	0	0	0	0	$0.8 \times 5.0 = 4.0$	0
$V_2(S) =$	0	$0.9 \times 0.8 \times 4$ $+0.1 \times -5 = 2.38$	0	$0.8 \times 0.9 \times 4.0 = 2.88$	$0.8 \times 5.0 = 4.0$	0

c) Suppose the agent does not know the transition probabilities. What does it need to be able do (or have available) in order to learn the optimal policy? (5 points)

The agent must be able to explore the world by taking actions and observing the effects.

Q6 Solution:

(a) What is $P(q_2 = Happy)$?

The question is to find the probability of Mr. X is ***Happy*** on day 2. It is given that the first day he was ***Happy***. For the second day, we need to find the transition probability $P(q_2 = Happy \mid q_1 = Happy)$.

$$P(q_2 = Happy \mid q_1 = Happy) = 0.8$$

(b) What is $P(o_2 = frown)$?

We need to find the probability of observation ***frown*** on day 2. But we don't know the states whether he is happy or not on day 2 (we know he was happy on day 1). Hence, the probability of the observation is the sum of products of observation probabilities and all possible hidden state transitions.

$P(o_2 = frown) = P(o_2 = frown \mid q_2 = Happy) + P(o_2 = frown \mid q_2 = Angry)$

$= P(Happy \mid Happy) * P(frown \mid Happy) + P(Angry \mid Happy) * P(frown \mid Angry)$

$= (0.8 * 0.1) + (0.2 * 0.5) = 0.08 + 0.1 = 0.18$

(c) What is $P(q_2 = Happy \mid o_2 = frown)$?

Here, we need to find the probability of hidden state on day 2 as Happy given the observation on that day as frown. This conditional probability cannot be calculated directly. Hence, we apply Bayes' rule to solve as follows;

$P(q_2 = Happy \mid o_2 = frown) = (P(o_2 = f \mid q_2 = H) * P(q_2 = H)) / P(o_2 = f)$

$= (P(Happy \mid Happy) * P(frown \mid Happy)) / 0.18$

[Note: 0.18 is taken from the answer for question (b)]

$= (0.8 * 0.1) / 0.18 = 0.08 / 0.18 = 0.4444$

(d) What is $P(o_1 = frown\ o_2 = frown\ o_3 = frown\ o_4 = frown\ o_5 = frown,\ q_1 = Happy\ q_2 = Angry\ q_3 = Angry\ q_4 = Angry\ q_5 = Angry)$ if $\pi = [0.7, 0.3]$?

Here, we need to find the probability of the observation sequence "*frown frown frown frown frown*" given the state sequence "*Happy Angry Angry Angry Angry*". π is the initial probabilities.

$P(f\ f\ f\ f\ f,\ H\ A\ A\ A\ A)$

$= P(f \mid H) * P(f \mid A) * P(f \mid A) * P(f \mid A) * P(f \mid A) * P(H) * P(A \mid H) * P(A \mid H) * P(A \mid H) * P(A \mid H)$

$= 0.1 * 0.5 * 0.5 * 0.5 * 0.5 * 0.7 * 0.2 * 0.2 * 0.2 * 0.2$

$= 0.000007$

Q1. Given the following dataset: (2, 10), (2, 5), (8, 4), (5, 8), (7, 5), (6, 4), (1, 2), (4, 9)

Perform a k-Means clustering on this dataset. Initialize the centroids at (3, 7) and (9, 3). Use Manhattan distance as the distance metric. Solve for five iterations or until any other stopping condition is reached. List which points end up in which clusters.

Q2. How can you determine the optimal number of clusters (k) for a dataset in k-Means clustering? Explain the method briefly

Using the elbow method. Plot the error for different values of k. Look for points where the error drops sharply or stops dropping sharply.

Q3. Can k-Means handle categorical or qualitative data?

SOLUTION:

Iteration 1:
Centroids: (3, 7), (9, 3)

Data point Nearest Centroid
(2, 10)-> (3, 7)
(2, 5) -> (3, 7)
(8, 4) -> (9, 3)
(5, 8) -> (3, 7)
(7, 5) -> (9, 3)
(6, 4) -> (9, 3)
(1, 2) -> (9, 3)
(4, 9) -> (3, 7)

New Centroids:
(3, 7) - (2, 10), (2, 5), (5, 8), (4, 9) = (13/4, 32/4) = (3.25, 8)
(9, 3) - (8, 4), (7, 5), (6, 4), (1, 2) = (22/4, 15/4) = (5.5, 3.75)

Iteration 2:
Centroids: **(3.25, 8), (5.5, 3.75)**

Data point Nearest Centroid
(2, 10) -> (3.25, 8)
(2, 5) -> (3.25, 8)
(8, 4) -> (5.5, 3.75)
(5, 8) -> (3.25, 8)
(7, 5) -> (5.5, 3.75)
(6, 4) -> (5.5, 3.75)
(1, 2) -> (5.5, 3.75)
(4, 9) -> (3.25, 8)

New Centroids:
(3.25, 8) - (2, 10), (2, 5), (5, 8), (4, 9) = (13/4, 32/4) = **(3.25, 8)**
(5.5, 3.75) - (8, 4), (7, 5), (6, 4), (1, 2) = (22/4, 15/4) = **(5.5, 3.75)**

Since the centroids have not changed their positions, convergence is reached in iteration 3. The final clusters are as follows:

Cluster 1: (2, 10), (2, 5), (5, 8), (4, 9)
Cluster 2: (8, 4), (7, 5), (6, 4), (1, 2)