Semantics of ±C(i.e. extended C−−)

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Programmation 1 – Compilateur COCass

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Notation

$\mathbb{Z}_{64}$ is the set of 64-bit signed integers, in which all calculations are done when not specified otherwise.

We write $(\rho : \mathcal{S} \to \mathbb{Z}_{64}) \in \mathcal{P}$ the environment, where $\mathcal{S}$ is the set of names of variables and functions, $(\mu : \mathbb{Z}_{64} \to \mathbb{Z}_8) \in \mathcal{M}$ the memory.
μ is read by blocks of 8 bytes : $\mu^{64}(i) \triangleq \sum_{k=0}^{7} 2^{8k}\mu(i+k)$.
$\rho_g \in \mathcal{P}$ is the global environment.

A flag is defined as an element of $\mathcal{E} \triangleq S \sqcup \{\texttt{brk}, \texttt{ret}, \texttt{cnt}, \texttt{nil}\}$: either an exception string or a special control flow keyword.
Intuitively, $\rho, \mu, \chi, v \vdash_\pi c \Rightarrow \rho', \mu', \chi', v'$ means that when c is executed under the environment ρ with the memory μ, the flag χ, and the previous value v, it updates it to the new environment and memory ρ' and μ', raises χ', and changes the value to v'.

In addition, we write $\text{fun}_\pi^n : \mathbb{Z}_{64} \to \texttt{code}$, a wrapper around ±C functions : $\text{fun}_\pi^n(a)(p_1, \cdots, p_n) = c$ updates the environment with $p_1, \cdots, p_n$ and executes the body of the function whose definition was given by the code c and stored at a. This way of considering functions allows in particular for function pointers.

For $\mu \in \mathcal{M}, v \in \mathbb{Z}_8, x \in \mathbb{Z}_{64}$ we write $\mu[x \mapsto v] : \begin{cases} x \mapsto v \\ y \mapsto \mu(y) \quad y \in \operatorname{dom}\mu \setminus \{x\} \end{cases}$
However we will usually use $\mu^{64}[x \mapsto v] \triangleq \mu[x+k \mapsto v_k \mid 0 \leqslant k < 8,\ v = \sum_{k=0}^{8} 2^{8k} v_k]$, i.e. the memory is written 8 bytes at a time.

A similar notation is used for ρ, ρ_g and fun_π^n.

1 Expressions

1.1 Reading values

For local and global variables :

$$\frac{x \in \operatorname{dom}\rho \qquad \rho(x) \in \operatorname{dom}\mu}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{VAR}\ x \Rightarrow \rho, \mu, \texttt{nil}, \mu^{64}(\rho(x))}(\textsc{Var})$$

i.e. reading a variable returns its contents and changes nothing to the memory.

$$\frac{\chi \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{VAR}\ x \Rightarrow \rho, \mu, \chi, v}(\textsc{Var}^\chi)$$

For constant integers :

$$\frac{}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{CST}\ n \Rightarrow \rho, \mu, \texttt{nil}, n}(\textsc{Cst})$$

$$\frac{\chi \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{CST}\ n \Rightarrow \rho, \mu, \chi, v}(\textsc{Cst}^\chi)$$

For strings :

$$\frac{s \text{ stored at } a \in \mathit{Addr}}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{STRING}\ s \Rightarrow \rho, \mu, \texttt{nil}, a}(\textsc{Str})$$

$$\frac{\chi \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{STRING}\ s \Rightarrow \rho, \mu, \chi, v}(\textsc{Cst}^\chi)$$

For arrays :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, i \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a \\ v_a + v_i \times 8 \in \operatorname{dom} \mu_a\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP2}(\texttt{S\_INDEX}, a, i) \Rightarrow \rho, \mu_a, \texttt{nil}, \mu_a^{64}(v_a + v_i \times 8)}(\textsc{Idx})$$

None of these are different from the original C−− semantics.

1.2 Unary operators without side-effects

Unary minus (same as C−−) :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_MINUS}, e) \Rightarrow \rho, \mu_e, \texttt{nil}, -v_e}(\textsc{Neg})$$

Unary bitwise negation (same as C−−) :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_NOT}, e) \Rightarrow \rho, \mu_e, \texttt{nil}, -v_e - 1}(\textsc{Not})$$

Indirection (added in ±C) :

$$\frac{x \in \operatorname{dom} \rho}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{OP1}(\texttt{M\_ADDR}, \texttt{VAR}\ x) \Rightarrow \rho, \mu, \texttt{nil}, \rho(x)}(\textsc{Var}^\&)$$

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, v_i \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_ADDR}, \texttt{OP2}(\texttt{S\_INDEX}, a, i)) \Rightarrow \rho, \mu_a, \texttt{nil}, v_a + v_i \times 8}(\textsc{Idx}^\&)$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_ADDR}, \texttt{OP1}(\texttt{M\_DEREF}, a)) \Rightarrow \rho, \mu_a, \texttt{nil}, v_a}(\textsc{Ptr}^\&)$$

Dereferencing (added in ±C) :

$$\frac{\rho, \mu, \chi, v \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a \qquad v_a \in \operatorname{dom} \mu_a}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_DEREF}, a) \Rightarrow \rho, \mu_a, \texttt{nil}, \mu_a^{64}(v_a)}(\textsc{Ptr})$$

When the operand raises a non-`nil` flag :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \qquad \chi_e \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(op, e) \Rightarrow \rho, \mu_e, \chi_e, v_e}(\textsc{Op1}^\chi)$$

1.3 Binary operators

Multiplication (same as C−−) :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e_2 \Rightarrow \rho, \mu_2, \chi_2, v_2 \\ \rho, \mu_2, \chi_2, v_2 \vdash_\pi e_1 \Rightarrow \rho, \mu_1, \texttt{nil}, v_1\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP2}(\texttt{S\_MUL}, e_1, e_2) \Rightarrow \rho, \mu_1, \texttt{nil}, v_1 \times v_2}(\textsc{Mul})$$

Addition (same as C−−) :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e_2 \Rightarrow \rho, \mu_2, \chi_2, v_2 \\ \rho, \mu_2, \chi_2, v_2 \vdash_\pi e_1 \Rightarrow \rho, \mu_1, \texttt{nil}, v_1\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP2}(\texttt{S\_ADD}, e_1, e_2) \Rightarrow \rho, \mu_1, \texttt{nil}, v_1 + v_2}(\textsc{Add})$$

Subtraction (same as C−−) :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e_2 \Rightarrow \rho, \mu_2, \chi_2, v_2 \\ \rho, \mu_2, \chi_2, v_2 \vdash_\pi e_1 \Rightarrow \rho, \mu_1, \texttt{nil}, v_1\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP2}(\texttt{S\_SUB}, e_1, e_2) \Rightarrow \rho, \mu_1, \texttt{nil}, v_1 - v_2}(\textsc{Sub})$$

Division and remainder (same as C−−) :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e_2 \Rightarrow \rho, \mu_2, \chi_2, v_2 \qquad v_2 \neq 0 \\ \rho, \mu_2, \chi_2, v_2 \vdash_\pi e_1 \Rightarrow \rho, \mu_1, \texttt{nil}, v_1\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP2}(\texttt{S\_DIV}, e_1, e_2) \Rightarrow \rho, \mu_1, \texttt{nil}, v_1 \text{ div } v_2}(\textsc{Div})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad v_2 \neq 0 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2(S\_MOD},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \bmod v_2}(\textsc{Mod})$$

Shifts (added in ±C) :

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2(S\_SHL},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \times 2^{v_2}}(\textsc{Shl})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2(S\_SHR},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \operatorname{div} 2^{v_2}}(\textsc{Shr})$$

Let $\mathrm{dec}_{64} : \{\bot,\top\}^{64} \to \mathbb{Z}_{64}$ the function

$$(b_0,\cdots,b_{63}) \mapsto \sum_{i=0}^{63}(1 \texttt{ if } b_i \texttt{ else } 0) \times 2^i$$

and $\mathrm{bin}_{64} = {\mathrm{dec}_{64}}^{-1}$.
We can now define bitwise operators as follows (added in ±C).

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad (b_0^2,\cdots,b_{63}^2) = \mathrm{bin}_{64}(v_2) \qquad (b_0^1,\cdots,b_{63}^1) = \mathrm{bin}_{64}(v_1)}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2(S\_AND},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},\mathrm{dec}_{64}(b_0^1 \wedge b_0^2,\cdots,b_{63}^1 \wedge b_{63}^2),\mu_1}(\textsc{And})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad (b_0^2,\cdots,b_{63}^2) = \mathrm{bin}_{64}(v_2) \qquad (b_0^1,\cdots,b_{63}^1) = \mathrm{bin}_{64}(v_1)}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2(S\_OR},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},\mathrm{dec}_{64}(b_0^1 \vee b_0^2,\cdots,b_{63}^1 \vee b_{63}^2),\mu_1}(\textsc{Ior})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad (b_0^2,\cdots,b_{63}^2) = \mathrm{bin}_{64}(v_2) \qquad (b_0^1,\cdots,b_{63}^1) = \mathrm{bin}_{64}(v_1)}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2(S\_XOR},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},\mathrm{dec}_{64}(b_0^1 \oplus b_0^2,\cdots,b_{63}^1 \oplus b_{63}^2),\mu_1}(\textsc{Xor})$$

When one of the operands raises a non-`nil` flag :

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\chi_1,v_1 \qquad \chi_1 \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{OP2}(op,e_1,e_2) \Rightarrow \rho,\mu_1,\chi_1,v_1}(\textsc{Op2}^\chi)$$

1.4 Comparisons

All are the same as in C−−.

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad v_1 = v_2}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CMP(C\_EQ},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},1}(\textsc{Eq}^\top)$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad v_1 < v_2}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CMP(C\_LT},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},1}(\textsc{Lt}^\top)$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad v_1 \leqslant v_2}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CMP(C\_LE},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},1}(\textsc{Le}^\top)$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad v_1 \neq v_2}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CMP(C\_EQ},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},0}(\textsc{Eq}^\bot)$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e_2 \Rightarrow \rho,\mu_2,\chi_2,v_2 \qquad \rho,\mu_2,\chi_2,v \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \qquad v_1 \not< v_2}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CMP(C\_LT},e_1,e_2) \Rightarrow \rho,\mu_1,\texttt{nil},0}(\textsc{Lt}^\bot)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1\not\leqslant v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_LE},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},0}(\textsc{Le}^\bot)$$

For optimisation purposes mostly, the comparison operators `C_NE`, `C_GT`, `C_GE` may be introduced by the compiler (not by the parser, however).
They are defined as

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1=v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_NE},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},0}(\textsc{Ne}^\bot)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1\leqslant v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_GT},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},0}(\textsc{Gt}^\bot)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1<v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_GE},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},0}(\textsc{Ge}^\bot)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1\neq v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_NE},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},1}(\textsc{Ne}^\top)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1\not\leqslant v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_GT},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},1}(\textsc{Gt}^\top)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\texttt{nil},v_1 \quad v_1\not< v_2}{\rho,\mu,\texttt{nil},v\vdash_\pi \texttt{CMP}(\texttt{C\_GE},e_1,e_2)\Rightarrow\rho,\mu_1,\texttt{nil},1}(\textsc{Ge}^\top)$$

When one of the operands raises a non-`nil` flag :

$$\frac{\rho,\mu,\chi,v\vdash_\pi e_2\Rightarrow\rho,\mu_2,\chi_2,v_2 \quad \rho,\mu_2,\chi_2,v_2\vdash_\pi e_1\Rightarrow\rho,\mu_1,\chi_1,v_1 \quad \chi_1\neq\texttt{nil}}{\rho,\mu,\chi,v\vdash_\pi \texttt{CMP}(op,e_1,e_2)\Rightarrow\rho,\mu_1,\chi_1,v_1}(\textsc{Cmp}^\chi)$$

1.5 Assignments

$$\frac{\rho,\mu,\chi,v\vdash_\pi e\Rightarrow\rho,\mu_e,\texttt{nil},v_e \quad x\in\operatorname{dom}\rho \qquad \rho(x)\in\operatorname{dom}\mu_e}{\rho,\mu,\chi,v\vdash_\pi \texttt{SET\_VAR}(x,e)\Rightarrow\rho,\mu_e^{64}[\rho(x)\mapsto v_e],\texttt{nil},v_e}(\textsc{Var}^\leftarrow)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e\Rightarrow\rho,\mu_e,\chi_e,v_e \qquad \chi_e\neq\texttt{nil}}{\rho,\mu,\chi,v\vdash_\pi \texttt{SET\_VAR}(x,e)\Rightarrow\rho,\mu_e,\chi_e,v_e}(\textsc{Var}^{\leftarrow\chi})$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e\Rightarrow\rho,\mu_e,\chi_e,v_e \quad \rho,\mu_e,\chi_e,v_e\vdash_\pi i\Rightarrow\rho,\mu_i,\texttt{nil},v_i \quad \rho(x)\in\operatorname{dom}\mu_i}{\rho,\mu,\chi,v\vdash_\pi \texttt{SET\_ARRAY}(x,i,e)\Rightarrow\rho,\mu_i^{64}[\rho(x)+v_i\times 8\mapsto v_e],\texttt{nil},v_e}(\textsc{Idx}^\leftarrow)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e\Rightarrow\rho,\mu_e,\chi_e,v_e \quad \rho,\mu_e,\chi_e,v_e\vdash_\pi i\Rightarrow\rho,\mu_i,\chi_i,v_i \qquad \chi_i\neq\texttt{nil}}{\rho,\mu,\chi,v\vdash_\pi \texttt{SET\_ARRAY}(x,i,e)\Rightarrow\rho,\mu_i,\chi_i,v_i}(\textsc{Idx}^{\leftarrow\chi})$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e\Rightarrow\rho,\mu_e,\chi_e,v_e \quad \rho,\mu_e,\chi_e,v_e\vdash_\pi a\Rightarrow\rho,\mu_a,\texttt{nil},v_a}{\rho,\mu,\chi,v\vdash_\pi \texttt{SET\_DEREF}(a,e)\Rightarrow\rho,\mu_a^{64}[v_a\mapsto v_e],\texttt{nil},v_e}(\textsc{Ptr}^\leftarrow)$$

$$\frac{\rho,\mu,\chi,v\vdash_\pi e\Rightarrow\rho,\mu_e,\chi_e,v_e \quad \rho,\mu_e,\chi_e,v_e\vdash_\pi a\Rightarrow\rho,\mu_a,\chi_a,v_a \qquad \chi_a\neq\texttt{nil}}{\rho,\mu,\chi,v\vdash_\pi \texttt{SET\_DEREF}(a,e)\Rightarrow\rho,\mu_a,\chi_a,v_a}(\textsc{Ptr}^{\leftarrow\chi})$$

1.6 Increments

On variables :

$$\frac{x \in \operatorname{dom}\rho \qquad \rho(x) = k \in \operatorname{dom}\mu \qquad \mu^{64}(k) = v_k}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{OP1}(\texttt{M\_POST\_INC}, \texttt{VAR}\ x) \Rightarrow \rho, \mu^{64}[k \mapsto v_k + 1], \texttt{nil}, v_k}(\textsc{Var}^{\bullet\uparrow})$$

$$\frac{x \in \operatorname{dom}\rho \qquad \rho(x) = k \in \operatorname{dom}\mu \qquad \mu^{64}(k) = v_k}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{OP1}(\texttt{M\_POST\_DEC}, \texttt{VAR}\ x) \Rightarrow \rho, \mu^{64}[k \mapsto v_k - 1], \texttt{nil}, v_k}(\textsc{Var}^{\bullet\downarrow})$$

$$\frac{x \in \operatorname{dom}\rho \qquad \rho(x) = k \in \operatorname{dom}\mu \qquad \mu^{64}(k) + 1 = v_k}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{OP1}(\texttt{M\_PRE\_INC}, \texttt{VAR}\ x) \Rightarrow \rho, \mu^{64}[k \mapsto v_k], \texttt{nil}, v_k}(\textsc{Var}^{\uparrow\bullet})$$

$$\frac{x \in \operatorname{dom}\rho \qquad \rho(x) = k \in \operatorname{dom}\mu \qquad \mu^{64}(k) - 1 = v_k}{\rho, \mu, \texttt{nil}, v \vdash_\pi \texttt{OP1}(\texttt{M\_PRE\_DEC}, \texttt{VAR}\ x) \Rightarrow \rho, \mu^{64}[k \mapsto v_k], \texttt{nil}, v_k}(\textsc{Var}^{\downarrow\bullet})$$

On arrays :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, v_i \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a \\ v_a + v_i \times 8 = k \qquad k \in \operatorname{dom}\mu_a\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_POST\_INC}, \texttt{OP2}(\texttt{S\_INDEX}, a, e)) \Rightarrow \rho, \mu_a^{64}[k \mapsto \mu_a(k) + 1], \texttt{nil}, \mu_a^{64}(k)}(\textsc{Idx}^{\bullet\uparrow})$$

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, v_i \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a \\ v_a + v_i \times 8 = k \qquad k \in \operatorname{dom}\mu_a\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_POST\_DEC}, \texttt{OP2}(\texttt{S\_INDEX}, a, e)) \Rightarrow \rho, \mu_a^{64}[k \mapsto \mu_a(k) - 1], \texttt{nil}, \mu_a^{64}(k)}(\textsc{Idx}^{\bullet\downarrow})$$

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, v_i \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a \\ v_a + v_i \times 8 = k \qquad k \in \operatorname{dom}\mu_a \qquad \mu_a^{64}(k) + 1 = v_k\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_PRE\_INC}, \texttt{OP2}(\texttt{S\_INDEX}, a, e)) \Rightarrow \rho, \mu_a[k \mapsto v_k], \texttt{nil}, v_k}(\textsc{Idx}^{\uparrow\bullet})$$

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, v_i \vdash_\pi a \Rightarrow \rho, \mu_a, \texttt{nil}, v_a \\ v_a + v_i \times 8 = k \qquad k \in \operatorname{dom}\mu_a \qquad \mu_a^{64}(k) - 1 = v_k\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_PRE\_DEC}, \texttt{OP2}(\texttt{S\_INDEX}, a, e)) \Rightarrow \rho, \mu_a[k \mapsto v_k], \texttt{nil}, v_k}(\textsc{Idx}^{\downarrow\bullet})$$

On dereferences :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_POST\_INC}, \texttt{OP1}(\texttt{M\_DEREF}, e)) \Rightarrow \rho, \mu_e^{64}[v_e \mapsto \mu_e(v_e) + 1], \texttt{nil}, \mu_e(v_e)}(\textsc{Ptr}^{\bullet\uparrow})$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_POST\_DEC}, \texttt{OP1}(\texttt{M\_DEREF}, e)) \Rightarrow \rho, \mu_e^{64}[v_e \mapsto \mu_e(v_e) - 1], \texttt{nil}, \mu_e^{64}(v_e)}(\textsc{Ptr}^{\bullet\downarrow})$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e \qquad \mu_e^{64}(v_e) + 1 = k}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_PRE\_INC}, \texttt{OP1}(\texttt{M\_DEREF}, e)) \Rightarrow \rho, \mu_e[v_e \mapsto k], \texttt{nil}, k}(\textsc{Ptr}^{\uparrow\bullet})$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e \qquad \mu_e^{64}(v_e) - 1 = k}{\rho, \mu, \chi, v \vdash_\pi \texttt{OP1}(\texttt{M\_PRE\_DEC}, \texttt{OP1}(\texttt{M\_DEREF}, e)) \Rightarrow \rho, \mu_e[v_e \mapsto k], \texttt{nil}, k}(\textsc{Ptr}^{\downarrow\bullet})$$

1.7 Extended assignments

Let $op \in \texttt{bin\_op} \setminus \{\texttt{S\_INDEX}\}$.
On variables :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \\ x \in \operatorname{dom}\rho \qquad \rho(x) \in \operatorname{dom}(\mu_e) \qquad \rho(x) = k \qquad \mu_e^{64}(k) = u \\ \rho, \mu_e, \chi_e, v_e \vdash_\pi \texttt{OP2}(op, \texttt{CST}\ v, \texttt{CST}\ u) \Rightarrow \rho, \mu', \texttt{nil}, w\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OPSET\_VAR}(op, x, e) \Rightarrow \rho, \mu'[k \mapsto w], \texttt{nil}, w}(\textsc{Var}^{\leftarrow op})$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \qquad \chi_e \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OPSET\_VAR}(op, x, e) \Rightarrow \rho, \mu_e, \chi_e, v_e}(\textsc{Var}^{\leftarrow op\chi})$$

On arrays :

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \\ \rho, \mu_e, \chi_e, v_e \vdash_\pi i \Rightarrow \rho, \mu_i, \chi_i, v_i \\ t \in \operatorname{dom}\rho \qquad \rho(t) + v_i \times 8 = k \qquad k \in \operatorname{dom}\mu_i \\ \mu_i^{64}(k) = u \qquad \rho, \mu_i, \chi_i, v_i \vdash_\pi \texttt{OP2}(op, \texttt{CST}\ v, \texttt{CST}\ u) \Rightarrow \rho, \mu', \texttt{,nil}, w\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{OPSET\_ARRAY}(op, t, e_1, e_2) \Rightarrow \rho, \mu'[k \mapsto w], \texttt{nil}, w}(\textsc{Idx}^{\leftarrow op})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\chi_e,v_e \quad \rho,\mu_e,\chi_e,v_e \vdash_\pi i \Rightarrow \rho,\mu_i,\chi_i,v_i \qquad \chi_i \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{OPSET\_ARRAY}(op,t,e_1,e_2) \Rightarrow \rho,\mu_i,\chi_i,v_i}(\textsc{Idx}^{\leftarrow op\chi})$$

On dereferences :

$$\frac{\rho,\mu,\chi,v \vdash_\pi a \Rightarrow \rho,\mu_a,\chi_a,v_a \quad \rho,\mu_a,\chi_a,v_a \vdash_\pi e \Rightarrow \rho,\mu_e,\chi_e,v_e \quad v_e \in \operatorname{dom}\mu_e \qquad \mu_e^{64}(v_e) = u \quad \rho,\mu_e,\chi_e,v_e \vdash_\pi \texttt{OP2}(op,\texttt{CST}\ v,\texttt{CST}\ u) \Rightarrow \rho,\mu',\texttt{nil},w}{\rho,\mu,\chi,v \vdash_\pi \texttt{OPSET\_DEREF}(op,e_1,e_2) \Rightarrow \rho,\mu'[k \mapsto w],\texttt{nil},w}(\textsc{Ptr}^{\leftarrow op})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi a \Rightarrow \rho,\mu_a,\chi_a,v_a \quad \rho,\mu_a,\chi_a,v_a \vdash_\pi e \Rightarrow \rho,\mu_e,\chi_e,v_e \qquad \chi_e \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{OPSET\_DEREF}(op,e_1,e_2) \Rightarrow \rho,\mu_e,\chi_e,v_e}(\textsc{Ptr}^{\leftarrow op\chi})$$

1.8 Ternary operator

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\texttt{nil},v_e \qquad v_e = 0 \quad \rho,\mu_e,\texttt{nil},v_e \vdash_\pi e_\perp \Rightarrow \rho,\mu_\perp,\chi_\perp,v_\perp}{\rho,\mu,\chi,v \vdash_\pi \texttt{EIF}(e,e_\top,e_\perp) \Rightarrow \rho,\mu_\perp,\chi_\perp,v_\perp}(\textsc{Tern}^{\perp})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\texttt{nil},v_e \qquad v_e \neq 0 \quad \rho,\mu_e,\texttt{nil},v_e \vdash_\pi e_\top \Rightarrow \rho,\mu_\top,\chi_\top,v_\top}{\rho,\mu,\chi,v \vdash_\pi \texttt{EIF}(e,e_\top,e_\perp) \Rightarrow \rho,\mu_\top,\chi_\top,v_\top}(\textsc{Tern}^{\top})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\chi_e,v_e \qquad \chi_e \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{EIF}(e,e_\top,e_\perp) \Rightarrow \rho,\mu_e,\chi_e,v_e}(\textsc{Tern}^{\chi})$$

1.9 Sequence

$$\frac{\rho,\mu_0,\chi_0,v_0 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\chi_1,v_1 \quad \cdots \quad \rho,\mu_{n-1},\chi_{n-1},v_{n-1} \vdash_\pi e_n \Rightarrow \rho,\mu_n,\chi_n,v_n}{\rho,\mu_0,\chi_0,v_0 \vdash_\pi \texttt{ESEQ}\ [e_1;\cdots;e_n] \Rightarrow \rho,\mu_n,\chi_n,v_n}(\textsc{Seq}^n)$$

1.10 Function call

Works for both a toplevel function and a function pointer :

$$\frac{\rho,\mu_{n+1},\chi_{n+1},v_{n+1} \vdash_\pi e_n \Rightarrow \rho,\mu_n,\chi_n,v_n \quad \cdots \quad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\texttt{nil},v_1 \quad f \in \operatorname{dom}\operatorname{fun}_\pi^n \quad \rho_g,\mu_1,\texttt{nil},0 \vdash_\pi \operatorname{fun}_\pi^n(f)(v_1,\cdots,v_n) \Rightarrow \rho_f,\mu_f,\chi_f,v_f}{\rho,\mu_n,\chi_n,v_n \vdash_\pi \texttt{CALL}(f,[e_1;\cdots;e_n]) \Rightarrow \rho,\mu_f,\chi_f,v_f}(\textsc{Call}^n)$$

$$\frac{\rho,\mu_{n+1},\chi_{n+1},v_{n+1} \vdash_\pi e_n \Rightarrow \rho,\mu_n,\chi_n,v_n \quad \cdots \quad \rho,\mu_2,\chi_2,v_2 \vdash_\pi e_1 \Rightarrow \rho,\mu_1,\chi_1,v_1 \quad \chi_1 \neq \texttt{nil}}{\rho,\mu_{n+1},\chi_{n+1},v_{n+1} \vdash_\pi \texttt{CALL}(f,[e_1;\cdots;e_n]) \Rightarrow \rho,\mu_1,\chi_1,v_1}(\textsc{Call}^{\chi})$$

2 Code

2.1 Expressions

An expression as statement is simply executed. If a non-`nil` flag is raised, it will be skipped anyway.

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\chi_e,v_e}{\rho,\mu,\chi,v \Vdash_\pi \texttt{CEXPR}\ e \Rightarrow \rho,\mu_e,\chi_e,v_e}(\textsc{Expr})$$

2.2 Conditional branching

If only `nil` is raised after the evaluation of the condition, one of the two branches is executed.

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e \qquad v_e = 0 \qquad \rho, \mu_e, \texttt{nil}, v_e \vdash_\pi c_\bot \Rightarrow \rho_\bot, \mu_\bot, \chi_\bot, v_\bot}{\rho, \mu, \chi, v \vdash_\pi \texttt{CIF}(e, c_\top, c_\bot) \Rightarrow \rho, \mu_\bot, \chi_\bot, v_\bot} (\textsc{If}^\bot)$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e \qquad v_e \neq 0 \qquad \rho, \mu_e, \texttt{nil}, v_e \vdash_\pi c_\top \Rightarrow \rho_\top, \mu_\top, \chi_\top, v_\top}{\rho, \mu, \chi, v \vdash_\pi \texttt{CIF}(e, c_\top, c_\bot) \Rightarrow \rho, \mu_\top, \chi_\top, v_\top} (\textsc{If}^\top)$$

Note that the branch is allowed to modify the memory and raise flags, but not change the environment : ρ is preserved.

For all other flags, neither of the branches is executed.

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \qquad \chi_e \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{CIF}(e, c_\top, c_\bot) \Rightarrow \rho, \mu_e, \chi_e, v_e} (\textsc{If}^\chi)$$

2.3 Blocks

$$\frac{\rho, \mu, \chi, v \vdash_\pi c \Rightarrow \rho', \mu', \chi', v' \qquad \rho', \mu', \chi', v' \vdash_\pi \texttt{CBLOCK}\ S \Rightarrow \rho'', \mu'', \chi'', v''}{\rho, \mu, \chi, v \vdash_\pi \texttt{CBLOCK}(c :: S) \Rightarrow \rho, \mu'', \chi'', v''} (\textsc{Block}^1)$$

$$\frac{}{\rho, \mu, \chi, v \vdash_\pi \texttt{CBLOCK}\ [] \Rightarrow \rho, \mu, \chi, v} (\textsc{Block}^0)$$

Again for blocks, the memory may be changed and flags may be raised, but the environment is preserved.

2.4 Loops

$$\frac{}{\rho, \mu, \chi, v \vdash_\pi \texttt{None} \Rightarrow \rho, \mu, \chi, v} (\textsc{None})$$

$$\frac{\rho, \mu, \chi, v \vdash_\pi f \Rightarrow \rho_f, \mu_f, \chi_f, v_f}{\rho, \mu, \chi, v \vdash_\pi \texttt{Some}\ f \Rightarrow \rho_f, \mu_f, \chi_f, v_f} (\textsc{Some})$$

A loop with a false condition stops :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e \qquad v_e = 0}{\rho, \mu, \chi, v \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{true}) \Rightarrow \rho, \mu', \texttt{nil}, v} (\textsc{While}^{\bot, \texttt{true}})$$

Except in the case of a `do-while` :

$$\frac{\rho, \mu, \chi, v \vdash_\pi c \Rightarrow \rho_c, \mu_c, \chi_c, v_c \qquad \rho, \mu_c, \chi_c, v_c \vdash_\pi f \Rightarrow \rho, \mu_f, \texttt{nil}, v_f \qquad \rho, \mu_f, \texttt{nil}, v_f \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{true}) \Rightarrow \rho, \mu_w, \chi_w, v_w}{\rho, \mu, \chi, v \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{false}) \Rightarrow \rho, \mu_w, \chi_w, v_w} (\textsc{While}^{\texttt{false}})$$

A loop continues normally if its condition is nonzero :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e \qquad v_e \neq 0 \qquad \rho, \mu_e, \texttt{nil}, v_e \vdash_\pi c \Rightarrow \rho_c, \mu_c, \chi_c, v_c \qquad \chi_c \notin \{\texttt{brk}, \texttt{cnt}\} \qquad \rho, \mu_c, \chi_c, v_c \vdash_\pi f \Rightarrow \rho, \mu_f, \chi_f, v_f \qquad \rho, \mu_f, \chi_f, v_f \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{true}) \Rightarrow \rho, \mu_w, \chi_w, v_w}{\rho, \mu, \chi, v \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{true}) \Rightarrow \rho, \mu_w, \chi_w, v_w} (\textsc{While}^{\top, \texttt{true}})$$

A flag skips the loop :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \qquad \chi_e \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{true}) \Rightarrow \rho, \mu_e, \chi_e, v_e} (\textsc{While}^{\chi, \texttt{true}})$$

$$\frac{\chi \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{CWHILE}(e, c, f, \texttt{false}) \Rightarrow \rho, \mu, \chi, v} (\textsc{While}^{\chi, \texttt{false}})$$

`cnt` executes the finally clause before continuing as normal :

$$\frac{\begin{array}{c}\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\texttt{nil},v_e \qquad v_e \neq 0\\ \rho,\mu_e,\texttt{nil},v_e \vdash_\pi c \Rightarrow \rho_c,\mu_c,\texttt{cnt},v_c\\ \rho,\mu_c,\texttt{nil},v_c \vdash_\pi f \Rightarrow \rho,\mu_f,\chi_f,v_f\\ \rho,\mu_f,\chi_f,v_f \vdash_\pi \texttt{CWHILE}(e,c,f,\texttt{true}) \Rightarrow \rho,\mu_w,\chi_w,v_w\end{array}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CWHILE}(e,c,f,\texttt{true}) \Rightarrow \rho,\mu_w,\chi_w,v_w}(\textsc{While}^{\texttt{cnt,true}})$$

$$\frac{\begin{array}{c}\rho,\mu,\chi,v \vdash_\pi c \Rightarrow \rho_c,\mu_c,\texttt{cnt},v_c\\ \rho,\mu_c,\texttt{nil},v_c \vdash_\pi f \Rightarrow \rho,\mu_f,\chi_f,v_f\\ \rho,\mu_f,\chi_f,v_f \vdash_\pi \texttt{CWHILE}(e,c,f,\texttt{true}) \Rightarrow \rho,\mu_w,\chi_w,v_w\end{array}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CWHILE}(e,c,f,\texttt{false}) \Rightarrow \rho,\mu_w,\chi_w,v_w}(\textsc{While}^{\texttt{cnt,false}})$$

`brk` interrupts the loop but is not retransmitted :

$$\frac{\begin{array}{c}\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\texttt{nil},v_e \qquad v_e \neq 0\\ \rho,\mu_e,\texttt{nil},v_e \vdash_\pi c \Rightarrow \rho_c,\mu_c,\texttt{brk},v_c\end{array}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CWHILE}(e,c,f,\texttt{true}) \Rightarrow \rho,\mu_c,\texttt{nil},v_c}(\textsc{While}^{\texttt{brk,true}})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi c \Rightarrow \rho_c,\mu_c,\texttt{brk},v_c}{\rho,\mu,\chi,v \vdash_\pi \texttt{CWHILE}(e,c,f,\texttt{false}) \Rightarrow \rho,\mu_c,\texttt{nil},v_c}(\textsc{While}^{\texttt{brk,false}})$$

2.5 Control flow

$$\frac{}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CBREAK} \Rightarrow \rho,\mu,\texttt{brk},0}(\textsc{Break})$$

$$\frac{\chi \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CBREAK} \Rightarrow \rho,\mu,\chi,v}(\textsc{Break}^{\chi})$$

$$\frac{}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CCONTINUE} \Rightarrow \rho,\mu,\texttt{cnt},0}(\textsc{Continue})$$

$$\frac{\chi \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CCONTINUE} \Rightarrow \rho,\mu,\chi,v}(\textsc{Continue}^{\chi})$$

$$\frac{}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CRETURN None} \Rightarrow \rho,\mu,\texttt{ret},0}(\textsc{Return}^{\texttt{None}})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\texttt{nil},v_e}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CRETURN}(\texttt{Some } e) \Rightarrow \rho,\mu_e,\texttt{ret},v_e}(\textsc{Return}^{\texttt{Some}})$$

$$\frac{\chi \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CRETURN None} \Rightarrow \rho,\mu,\chi,v}(\textsc{Return}^{\texttt{None}\chi})$$

$$\frac{\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\chi_e,v_e \qquad \chi_e \neq \texttt{nil}}{\rho,\mu,\texttt{nil},v \vdash_\pi \texttt{CRETURN}(\texttt{Some } e) \Rightarrow \rho,\mu_e,\chi_e,v_e}(\textsc{Return}^{\texttt{Some}\chi})$$

2.6 Local variable declarations

First, the obvious :

$$\frac{}{\rho,\mu,\chi,v \vdash_\pi \texttt{CLOCAL } [] \Rightarrow \rho,\mu,\chi,v}(\textsc{Local}^{0})$$

$$\frac{\chi \neq \texttt{nil}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CLOCAL } d \Rightarrow \rho,\mu,\chi,v}(\textsc{Local}^{\chi})$$

There are never functions defined in a `CLOCAL`, only `CDECL` :

$$\frac{\begin{array}{c}\rho,\mu,\chi,v \vdash_\pi e \Rightarrow \rho,\mu_e,\texttt{nil},v_e\\ k \in alloc^8(\mu_e) \qquad \rho' = \rho[w \mapsto k] \qquad \mu' = \mu_e[k \mapsto v_e]\\ \rho',\mu',\texttt{nil},v_e \vdash_\pi \texttt{CLOCAL } S \Rightarrow \rho_s,\mu_s,\chi_s,v_s\end{array}}{\rho,\mu,\chi,v \vdash_\pi \texttt{CLOCAL}(\texttt{CDECL}(w,e) :: S) \Rightarrow \rho_s,\mu_s,\chi_s,v_s}(\textsc{Local}^{1})$$

2.7 Throw

If a flag is already raised, skip the `CTHROW` :

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \qquad \chi_e \neq \texttt{nil}}{\rho, \mu, \chi, v \vdash_\pi \texttt{CTHROW}(s, e) \Rightarrow \rho, \mu_e, \chi_e, v_e}(\textsc{Throw}^\chi)$$

Otherwise raise the new exception $s \in S$:

$$\frac{\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \texttt{nil}, v_e}{\rho, \mu, \chi, v \vdash_\pi \texttt{CTHROW}(s, e) \Rightarrow \rho, \mu_e, s, v_e}(\textsc{Throw})$$

2.8 Switch

$$\frac{\begin{array}{c}\rho, \mu, \chi, v \vdash_\pi e \Rightarrow \rho, \mu_e, \chi_e, v_e \\ \rho, \mu_e, \chi_e, v_e \vdash_\pi \texttt{CBLOCK}(L(v_e)) \Rightarrow \rho, \mu_l, \chi_l, v_l\end{array}}{\rho, \mu, \chi, v \vdash_\pi \texttt{CSWITCH}(e, L, c) \Rightarrow \rho, \mu_l, \chi_l, v_l}(\textsc{Switch})$$

Where for $L = [(j_1, l_1); \cdots; (j_n, l_n)]$, $L(v_e)$ is defined as follows :
Let $I_i = \{j_1, \cdots, j_i\}$ for $1 \leqslant i \leqslant n$, $I_{n+1} = \mathbb{Z}_{64}$.
$\tilde{j} \triangleq \min_{1 \leqslant i \leqslant n+1}\{i \mid v_e \in I_i\}$, finally $L(v_e) \triangleq [l_{\tilde{j}}; \cdots; l_n; c]$.