Semantics of $\pm C$ (i.e. extended C--)

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Notation

 \mathbb{Z}_{64} is the set of 64-bit signed integers, in which all calculations are done when not specified otherwise.

We write $(\rho: S \to \mathbb{Z}_{64}) \in \mathcal{P}$ the environment, where S is the set of names of variables and functions, $(\mu: \mathbb{Z}_{64} \to \mathbb{Z}_8) \in \mathcal{P}$ \mathcal{M} the memory.

 μ is read by blocks of 8 bytes : $\mu^{64}(i) \triangleq \sum_{k=0}^{7} 2^{8k} \mu(i+k)$. $\rho_q \in \mathcal{P}$ is the global environment.

A flag is defined as an element of $\mathcal{E} \triangleq S \sqcup \{brk, ret, cnt, nil\}$: either an exception string or a special control flow keyword.

Intuitively, $\rho, \mu, \chi, v \vdash c \Rightarrow \rho', \mu', \chi', v'$ means that when c is executed under the environment ρ with the memory μ , the flag χ , and the previous value v, it updates it to the new environment and memory ρ' and μ' , raises χ' , and changes the value to v'. Variants are used for toplevel declarations (no χ nor v but fun is added), and expressions (ρ is never modified and thus does not appear on the right side)

In addition, we write fun: $\mathbb{Z}_{64} \to \text{code}$, a wrapper around $\pm \mathbb{C}$ functions: $\text{fun}(a)(p_1, \dots, p_n) = c$ updates the environment with p_1, \dots, p_n and executes the body of the function whose definition was given by the code c and stored at a. This way of considering functions allows in particular for function pointers.

For
$$\mu \in \mathcal{M}, v \in \mathbb{Z}_8, x \in \mathbb{Z}_{64}$$
 we write $\mu[x \mapsto v] : \begin{cases} x \mapsto v \\ y \mapsto \mu(y) & y \in \text{dom } \mu \setminus \{x\} \end{cases}$

For $\mu \in \mathcal{M}, v \in \mathbb{Z}_8, x \in \mathbb{Z}_{64}$ we write $\mu[x \mapsto v] : \begin{cases} x \mapsto v \\ y \mapsto \mu(y) & y \in \text{dom } \mu \setminus \{x\} \end{cases}$ However we will usually use $\mu^{64}[x \mapsto v] \triangleq \mu[x + k \mapsto v_k \mid 0 \leqslant k < 8, \ v = \sum_{k=0}^8 2^{8k} v_k]$, i.e. the memory is written 8 bytes at a time.

A similar notation is used for ρ , ρ_g and fun.

 $alloc^i : \mathcal{M} \to \mathcal{P}(\mathbb{Z}_{64})$ is such that if $k \in alloc^i(\mu) \neq \bot$ then $\forall 0 \leqslant j < i, k+j \notin \text{dom } \mu$.

The domains as well are ommitted: " $\rho \in P$ ", " $\mu \in \mathcal{M}$ ", etc. are not explicit.

1 Expressions

Reading values

For local and global variables:

$$\frac{x\in\operatorname{dom}\rho\quad\rho(x)\in\operatorname{dom}\mu}{\rho,\mu,\operatorname{nil},v\vdash^{e}\operatorname{VAR}x\Rightarrow\mu,\operatorname{nil},\mu^{64}(\rho(x))}(\operatorname{Var})$$

i.e. reading a variable returns its contents and changes nothing to the memory.

$$\frac{\chi \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^e \mathtt{VAR} \; x \Rightarrow \mu, \chi, v}(\mathtt{VAR}^\chi)$$

For constant integers:

$$\begin{split} & \frac{}{\rho,\mu,\mathtt{nil},v \vdash^{e} \mathtt{CST} \ n \Rightarrow \mu,\mathtt{nil},n}(\mathtt{CST}) \\ & \frac{\chi \neq \mathtt{nil}}{\rho,\mu,\chi,v \vdash^{e} \mathtt{CST} \ n \Rightarrow \mu,\chi,v}(\mathtt{CST}^{\chi}) \end{split}$$

For strings:

$$\begin{split} \frac{s \text{ stored at } a \in Addr}{\rho, \mu, \text{nil}, v \vdash^e \text{STRING } s \Rightarrow \mu, \text{nil}, a} (\text{Str}) \\ \frac{\chi \neq \text{nil}}{\rho, \mu, \chi, v \vdash^e \text{STRING } s \Rightarrow \mu, \chi, v} (\text{Cst}^\chi) \end{split}$$

For arrays:

$$\begin{split} \rho, \mu, \chi, v \vdash^e i \Rightarrow \mu_i, \chi_i, v_i \\ \rho, \mu_i, \chi_i, i \vdash^e a \Rightarrow \mu_a, \mathtt{nil}, v_a \\ v_a + v_i \times 8 \in \mathrm{dom}\,\mu_a \\ \hline \rho, \mu, \chi, v \vdash^e \mathtt{OP2}(\mathtt{S_INDEX}, a, i) \Rightarrow \mu_a, \mathtt{nil}, \mu_a^{64}(v_a + v_i \times 8) \end{split} (\mathrm{IDX}) \end{split}$$

None of these are different from the original $\mathrm{C}--$ semantics.

1.2 Unary operators without side-effects

Unary minus (same as C--):

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\mathtt{nil},v_e}{\rho,\mu,\chi,v\vdash^e\mathtt{OP1}(\mathtt{M_MINUS},e)\Rightarrow \mu_e,\mathtt{nil},-v_e}(\mathtt{NEG})$$

Unary bitwise negation (same as C--):

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\mathtt{nil},v_e}{\rho,\mu,\chi,v\vdash^e \mathtt{OP1}(\mathtt{M_NOT},e)\Rightarrow \mu_e,\mathtt{nil},-v_e-1}(\mathtt{NOT})$$

Indirection (added in $\pm C$):

$$\begin{split} \frac{x \in \text{dom}\,\rho}{\rho, \mu, \text{nil}, v \vdash^e \text{OP1}(\text{M_ADDR, VAR}\,x) \Rightarrow \mu, \text{nil}, \rho(x)} (\text{VAR}^\&) \\ \frac{\rho, \mu, \chi, v \vdash^e i \Rightarrow \mu_i, \chi_i, v_i}{\rho, \mu_i, \chi_i, v_i \vdash^e a \Rightarrow \mu_a, \text{nil}, v_a} \\ \frac{\rho, \mu, \chi, v \vdash^e \text{OP1}(\text{M_ADDR, OP2}(\text{S_INDEX}, a, i)) \Rightarrow \mu_a, \text{nil}, v_a + v_i \times 8} (\text{IDX}^\&) \\ \frac{\rho, \mu, \chi, v \vdash^e \text{OP1}(\text{M_ADDR, OP1}(\text{M_DEREF}, a)) \Rightarrow \mu_a, \text{nil}, v_a}{\rho, \mu, \chi, v \vdash^e \text{OP1}(\text{M_ADDR, OP1}(\text{M_DEREF}, a)) \Rightarrow \mu_a, \text{nil}, v_a} (\text{PTR}^\&) \end{split}$$

Dereferencing (added in $\pm C$):

$$\frac{\rho,\mu,\chi,v\vdash^e a\Rightarrow \mu_a,\mathtt{nil},v_a \qquad v_a\in \mathrm{dom}\,\mu_a}{\rho,\mu,\chi,v\vdash^e \mathtt{OP1}(\mathtt{M_DEREF},a)\Rightarrow \mu_a,\mathtt{nil},\mu_a^{64}(v_a)}(\mathtt{Ptr})$$

When the operand raises a non-nil flag:

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\chi_e,v_e \qquad \chi_e\neq \mathtt{nil}}{\rho,\mu,\chi,v\vdash^e \mathtt{OP1}(op,e)\Rightarrow \mu_e,\chi_e,v_e}(\mathtt{OP1}^\chi)$$

1.3 Binary operators

Multiplication (same as C--):

$$\begin{split} & \rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ & \rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1 \\ & \rho, \mu, \chi, v \vdash^e \mathtt{OP2}(\mathtt{S_MUL}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, v_1 \times v_2 \end{split} (\mathtt{MUL})$$

Addition (same as C--):

$$\begin{split} & \rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ & \rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1 \\ & \rho, \mu, \chi, v \vdash^e \mathtt{OP2}(\mathtt{S_ADD}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, v_1 + v_2 \end{split} (\mathtt{Add})$$

Subtraction (same as C--):

$$\begin{split} & \rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ & \frac{\rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1}{\rho, \mu, \chi, v \vdash^e \mathtt{OP2}(\mathtt{S_SUB}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, v_1 - v_2} (\mathtt{SUB}) \end{split}$$

Division and remainder (same as C--):

$$\begin{split} & \rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \quad v_2 \neq 0 \\ & \frac{\rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1}{\rho, \mu, \chi, v \vdash^e \mathtt{OP2}(\mathtt{S_DIV}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, v_1 \ \mathrm{div} \ v_2} (\mathtt{DIV}) \end{split}$$

$$\begin{split} & \rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 & v_2 \neq 0 \\ & \frac{\rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1}{\rho, \mu, \chi, v \vdash^e \mathtt{OP2}(\mathtt{S_MOD}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, v_1 \bmod v_2} (\mathtt{Mod}) \end{split}$$

Shifts (added in $\pm C$):

$$\begin{split} &\rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ &\frac{\rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \mathtt{nil}, v_{1}}{\rho, \mu, \chi, v \vdash^{e} \mathtt{OP2}(\mathtt{S_SHL}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \mathtt{nil}, v_{1} \times 2^{v_{2}}} (\mathtt{SHL}) \\ &\frac{\rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2}}{\rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \mathtt{nil}, v_{1}} \\ &\frac{\rho, \mu, \chi, v \vdash^{e} \mathtt{OP2}(\mathtt{S_SHR}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \mathtt{nil}, v_{1}}{\rho, \mu, \chi, v \vdash^{e} \mathtt{OP2}(\mathtt{S_SHR}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \mathtt{nil}, v_{1}} (\mathtt{SHR}) \end{split}$$

Let $dec_{64}: \{\bot, \top\}^{64} \to \mathbb{Z}_{64}$ the function

$$(b_0,\cdots,b_{63})\mapsto \sum_{i=0}^{63}(1 \text{ if } b_i \text{ else } 0)\times 2^i$$

and $bin_{64} = dec_{64}^{-1}$.

We can now define bitwise operators as follows (added in $\pm C$).

$$\begin{split} & \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} & \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \mathtt{nil}, v_{1} \\ & (b_{0}^{2}, \cdots, b_{63}^{2}) = \mathrm{bin}_{64}(v_{2}) & (b_{0}^{1}, \cdots, b_{63}^{1}) = \mathrm{bin}_{64}(v_{1}) \\ \hline \rho, \mu, \chi, v \vdash^{e} \mathrm{OP2}(\mathbf{S_AND}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \mathtt{nil}, \mathrm{dec}_{64}(b_{0}^{1} \wedge b_{0}^{2}, \cdots, b_{63}^{1} \wedge b_{63}^{2}), \mu_{1} \\ \hline \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} & \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \mathtt{nil}, v_{1} \\ \hline (b_{0}^{2}, \cdots, b_{63}^{2}) = \mathrm{bin}_{64}(v_{2}) & (b_{0}^{1}, \cdots, b_{63}^{1}) = \mathrm{bin}_{64}(v_{1}) \\ \hline \rho, \mu, \chi, v \vdash^{e} \mathrm{OP2}(\mathbf{S_OR}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \mathtt{nil}, \mathrm{dec}_{64}(b_{0}^{1} \vee b_{0}^{2}, \cdots, b_{63}^{1} \vee b_{63}^{2}), \mu_{1} \\ \hline \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} & \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \mathtt{nil}, v_{1} \\ \hline (b_{0}^{2}, \cdots, b_{63}^{2}) = \mathrm{bin}_{64}(v_{2}) & (b_{0}^{1}, \cdots, b_{63}^{1}) = \mathrm{bin}_{64}(v_{1}) \\ \hline \rho, \mu, \chi, v \vdash^{e} \mathrm{OP2}(\mathbf{S_XOR}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \mathtt{nil}, \mathrm{dec}_{64}(b_{0}^{1} \oplus b_{0}^{2}, \cdots, b_{63}^{1} \oplus b_{63}^{2}), \mu_{1} \\ \hline \end{pmatrix} (\mathrm{XOR}) \end{split}$$

When one of the operands raises a non-nil flag:

$$\begin{split} &\rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ &\rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \chi_1, v_1 \\ &\frac{\chi_1 \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^e \mathtt{OP2}(op, e_1, e_2) \Rightarrow \mu_1, \chi_1, v_1} (\mathtt{OP2}^\chi) \end{split}$$

1.4 Comparisons

All are the same as in C——.

$$\begin{array}{c} \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} = v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C}_\operatorname{EQ}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \end{array} (\operatorname{EQ}^{\top}) \\ \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} < v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C}_\operatorname{LT}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \end{array} (\operatorname{LT}^{\top}) \\ \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C}_\operatorname{LE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \end{array} (\operatorname{LE}^{\top}) \\ \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} \neq v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C}_\operatorname{EQ}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 0 \end{array} (\operatorname{EQ}^{\perp})$$

$$\begin{split} &\rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ &\rho, \mu_2, \chi_2, v \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1 \\ &\frac{v_1 \not< v_2}{\rho, \mu, \mathtt{nil}, v \vdash^e \mathtt{CMP}(\mathtt{C_LT}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, 0} (\mathtt{LT}^\perp) \\ &\rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ &\rho, \mu_2, \chi_2, v \vdash^e e_1 \Rightarrow \mu_1, \mathtt{nil}, v_1 \\ &\frac{v_1 \not\leqslant v_2}{\rho, \mu, \mathtt{nil}, v \vdash^e \mathtt{CMP}(\mathtt{C_LE}, e_1, e_2) \Rightarrow \mu_1, \mathtt{nil}, 0} (\mathtt{LE}^\perp) \end{split}$$

For optimisation purposes mostly, the comparison operators C_NE, C_GT, C_GE may be introduced by the compiler (not by the parser, however).

They are defined as

$$\begin{array}{c} \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} = v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_NE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 0 \\ \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GT}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 0 \\ \hline \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} < v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 0 \\ \hline \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} \neq v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_NE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GT}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline \rho, \mu, \chi, v \vdash^{e} e_{2} \Rightarrow \mu_{2}, \chi_{2}, v_{2} \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \operatorname{nil}, v_{1} \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GT}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline v_{1} \leqslant v_{2} \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e_{2}) \Rightarrow \mu_{1}, \operatorname{nil}, 1 \\ \hline \rho, \mu, \operatorname{nil}, v \vdash^{e} \operatorname{CMP}(\operatorname{C_GE}, e_{1}, e$$

When one of the operands raises a non-nil flag :

$$\begin{split} &\rho, \mu, \chi, v \vdash^e e_2 \Rightarrow \mu_2, \chi_2, v_2 \\ &\rho, \mu_2, \chi_2, v_2 \vdash^e e_1 \Rightarrow \mu_1, \chi_1, v_1 \\ &\frac{\chi_1 \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^e \mathtt{CMP}(op, e_1, e_2) \Rightarrow \mu_1, \chi_1, v_1} (\mathtt{CMP}^\chi) \end{split}$$

1.5 Assignments

$$\begin{split} &\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e} \\ &\frac{\rho, \mu_{e}, \chi_{e}, v_{e} \vdash^{e} \mathtt{OP1}(\mathtt{M_ADDR}, a) \Rightarrow \mu_{a}, \mathtt{nil}, v_{a}}{\rho, \mu, \chi, v \vdash^{e} \mathtt{SET}(a, e) \Rightarrow \mu_{a}^{64}[v_{a} \mapsto v_{e}], \mathtt{nil}, v_{e}} (\mathtt{PTR}^{\leftarrow}) \\ &\frac{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e}}{\rho, \mu_{e}, \chi_{e}, v_{e} \vdash^{e} \mathtt{OP1}(\mathtt{M_ADDR}, a) \Rightarrow \mu_{a}, \chi_{a}, v_{a}} \qquad \chi_{a} \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^{e} \mathtt{SET}(a, e) \Rightarrow \mu_{a}, \chi_{a}, v_{a}} \end{split}$$

1.6 Increments

$$\begin{split} &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_ADDR},a) \Rightarrow \mu_{a}, \mathrm{nil}, v_{a} \\ &k = \mu_{a}^{64}(v_{a}) \\ \hline &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_POST_INC},a) \Rightarrow \mu_{a}^{64}[v_{a} \mapsto k+1], \mathrm{nil}, k} (\mathrm{POST}^{\uparrow}) \\ &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_ADDR},a) \Rightarrow \mu_{a}, \mathrm{nil}, v_{a} \\ &k = \mu_{a}^{64}(v_{a}) \\ \hline &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_POST_DEC},a) \Rightarrow \mu_{a}^{64}[v_{a} \mapsto k-1], \mathrm{nil}, k} (\mathrm{POST}^{\downarrow}) \\ &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_ADDR},a) \Rightarrow \mu_{a}, \mathrm{nil}, v_{a} \\ &k = \mu_{a}^{64}(v_{a}) \\ \hline &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_PRE_INC},a) \Rightarrow \mu_{a}^{64}[v_{a} \mapsto k+1], \mathrm{nil}, k+1} (\mathrm{PRE}^{\uparrow}) \\ &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_ADDR},a) \Rightarrow \mu_{a}, \mathrm{nil}, v_{a} \\ &k = \mu_{a}^{64}(v_{a}) \\ \hline &\rho,\mu,\chi,v \vdash^{e} \mathrm{OP1}(\mathrm{M_PRE_DEC},a) \Rightarrow \mu_{a}^{64}[v_{a} \mapsto k-1], \mathrm{nil}, k-1} (\mathrm{PRE}^{\downarrow}) \end{split}$$

1.7 Extended assignments

Let $op \in bin_op \setminus \{S_INDEX\}.$

$$\begin{array}{c} \rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e} \\ \rho, \mu_{e}, \chi_{e}, v_{e} \vdash^{e} \mathrm{OP1}(\mathrm{M_ADDR}, a) \Rightarrow \mu_{a}, \chi_{a}, v_{a} \\ v_{a} \in \mathrm{dom}\,\mu_{a} \qquad \mu_{a}^{64}(v_{a}) = u \\ \hline \rho, \mu_{a}, \chi_{a}, v_{a} \vdash^{e} \mathrm{OP2}(op, \mathrm{CST}\,v_{a}, \mathrm{CST}\,v_{e}) \Rightarrow \mu', \mathrm{nil}, w \\ \hline \rho, \mu, \chi, v \vdash^{e} \mathrm{OPSET}(op, a, e) \Rightarrow \mu'[v_{a} \mapsto w], \mathrm{nil}, w \\ \hline \rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e} \\ \rho, \mu_{e}, \chi_{e}, v_{e} \vdash^{e} \mathrm{OP1}(\mathrm{M_ADDR}, a) \Rightarrow \mu_{a}, \chi_{a}, v_{a} \\ \hline \chi_{a} \neq \mathrm{nil} \\ \hline \rho, \mu, \chi, v \vdash^{e} \mathrm{OPSET}(op, a, e) \Rightarrow \mu_{a}, \chi_{a}, v_{a} \end{array} \tag{OPSET}^{\chi})$$

1.8 Ternary operator

$$\begin{split} &\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \mathtt{nil}, v_{e} \qquad v_{e} = 0 \\ &\frac{\rho, \mu_{e}, \mathtt{nil}, v_{e} \vdash^{e} e_{\perp} \Rightarrow \mu_{\perp}, \chi_{\perp}, v_{\perp}}{\rho, \mu, \chi, v \vdash^{e} \mathtt{EIF}(e, e_{\top}, e_{\perp}) \Rightarrow \mu_{\perp}, \chi_{\perp}, v_{\perp}} (\mathtt{TERN}^{\perp}) \\ &\frac{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \mathtt{nil}, v_{e} \qquad v_{e} \neq 0}{\rho, \mu_{e}, \mathtt{nil}, v_{e} \vdash^{e} e_{\top} \Rightarrow \mu_{\top}, \chi_{\top}, v_{\top}} (\mathtt{TERN}^{\top}) \\ &\frac{\rho, \mu, \chi, v \vdash^{e} \mathtt{EIF}(e, e_{\top}, e_{\perp}) \Rightarrow \mu_{\top}, \chi_{\top}, v_{\top}}{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e} \qquad \chi_{e} \neq \mathtt{nil}} (\mathtt{TERN}^{\chi}) \\ &\frac{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e} \qquad \chi_{e} \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^{e} \mathtt{EIF}(e, e_{\top}, e_{\perp}) \Rightarrow \mu_{e}, \chi_{e}, v_{e}} (\mathtt{TERN}^{\chi}) \end{split}$$

1.9 Sequence

$$\rho, \mu_0, \chi_0, v_0 \vdash^e e_1 \Rightarrow \mu_1, \chi_1, v_1 \\ \dots \\ \frac{\rho, \mu_{n-1}, \chi_{n-1}, v_{n-1} \vdash^e e_n \Rightarrow \mu_n, \chi_n, v_n}{\rho, \mu_0, \chi_0, v_0 \vdash^e \mathtt{ESEQ} [e_1; \dots; e_n] \Rightarrow \mu_n, \chi_n, v_n} (\mathtt{SEQ}^n)$$

1.10 Function call

Works for both a toplevel function and a function pointer :

$$\rho, \mu_{n+1}, \chi_{n+1}, v_{n+1} \vdash^{e} e_{n} \Rightarrow \mu_{n}, \chi_{n}, v_{n} \dots$$

$$\rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \text{nil}, v_{1}$$

$$f \in \text{dom } \rho \qquad \rho(f) \in \text{dom fun}$$

$$\rho_{g}, \mu_{1}, \text{nil}, 0 \vdash^{e} \text{fun}(\rho(f))(v_{1}, \dots, v_{n}) \Rightarrow \rho_{f}, \mu_{f}, \chi_{f}, v_{f}$$

$$\rho, \mu_{n}, \chi_{n}, v_{n} \vdash^{e} \text{CALL}(f, [e_{1}; \dots; e_{n}]) \Rightarrow \mu_{f}, \chi_{f}, v_{f}$$
(Callⁿ)

$$\begin{split} \rho, \mu_{n+1}, \chi_{n+1}, v_{n+1} \vdash^{e} e_{n} \Rightarrow \mu_{n}, \chi_{n}, v_{n} \\ & \cdots \\ \rho, \mu_{2}, \chi_{2}, v_{2} \vdash^{e} e_{1} \Rightarrow \mu_{1}, \chi_{1}, v_{1} \\ & \frac{\chi_{1} \neq \mathtt{nil}}{\rho, \mu_{n+1}, \chi_{n+1} \vdash^{e} \mathtt{CALL}(f, [e_{1}; \cdots; e_{n}]) \Rightarrow \mu_{1}, \chi_{1}, v_{1}} (\mathtt{CALL}^{\chi}) \end{split}$$

2 Code

2.1 Expressions

An expression as statement is simply executed. If a non-nil flag is raised, it will be skipped anyway.

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\chi_e,v_e}{\rho,\mu,\chi,v\vdash^c \mathtt{CEXPR}\ e\Rightarrow \rho,\mu_e,\chi_e,v_e}(\mathtt{EXPR})$$

2.2 Conditional branching

If only nil is raised after the evaluation of the condition, one of the two branches is executed.

$$\begin{split} & \rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \mathtt{nil}, v_{e} \quad v_{e} = 0 \\ & \frac{\rho, \mu_{e}, \mathtt{nil}, v_{e} \vdash^{c} c_{\perp} \Rightarrow \rho_{\perp}, \mu_{\perp}, \chi_{\perp}, v_{\perp}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CIF}(e, c_{\top}, c_{\perp}) \Rightarrow \rho, \mu_{\perp}, \chi_{\perp}, v_{\perp}} (\mathtt{IF}^{\perp}) \\ & \frac{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \mathtt{nil}, v_{e} \quad v_{e} \neq 0}{\rho, \mu_{e}, \mathtt{nil}, v_{e} \vdash^{c} c_{\top} \Rightarrow \rho_{\top}, \mu_{\top}, \chi_{\top}, v_{\top}} (\mathtt{IF}^{\top}) \\ & \frac{\rho, \mu_{e}, \mathtt{nil}, v_{e} \vdash^{c} c_{\top} \Rightarrow \rho_{\top}, \mu_{\top}, \chi_{\top}, v_{\top}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CIF}(e, c_{\top}, c_{\perp}) \Rightarrow \rho, \mu_{\top}, \chi_{\top}, v_{\top}} (\mathtt{IF}^{\top}) \end{split}$$

Note that the branch is allowed to modify the memory and raise flags, but not change the environment : ρ is preserved.

For all other flags, neither of the branches is executed.

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\chi_e,v_e \qquad \chi_e\neq \mathtt{nil}}{\rho,\mu,\chi,v\vdash^c \mathtt{CIF}(e,c_\top,c_\bot)\Rightarrow \rho,\mu_e,\chi_e,v_e}(\mathtt{IF}^\chi)$$

2.3 Blocks

$$\begin{split} & \rho, \mu, \chi, v \vdash^{c} c \Rightarrow \rho', \mu', \chi', v' \\ & \frac{\rho', \mu', \chi', v' \vdash^{c} \mathtt{CBLOCK} \ S \Rightarrow \rho'', \mu'', \chi'', v''}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CBLOCK} (c :: S) \Rightarrow \rho, \mu'', \chi'', v''} (\mathtt{BLOCK}^{1}) \\ & \overline{\rho, \mu, \chi, v \vdash^{c} \mathtt{CBLOCK} \ [] \Rightarrow \rho, \mu, \chi, v} (\mathtt{BLOCK}^{0}) \end{split}$$

Again for blocks, the memory may be changed and flags may be raised, but the environment is preserved.

2.4 Loops

A loop with a false condition stops:

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\mathtt{nil},v_e \qquad v_e=0}{\rho,\mu,\chi,v\vdash^c \mathtt{CWHILE}(e,c,f,\mathtt{true})\Rightarrow \rho,\mu',\mathtt{nil},v}(\mathtt{WHILE}^{\perp,\mathtt{true}})$$

Except in the case of a do-while:

$$\begin{split} \rho, \mu, \chi, v \vdash^{c} c &\Rightarrow \rho_{c}, \mu_{c}, \chi_{c}, v_{c} \\ \rho, \mu_{c}, \chi_{c}, v_{c} \vdash^{e} f &\Rightarrow \mu_{f}, \mathtt{nil}, v_{f} \\ \frac{\rho, \mu_{f}, \mathtt{nil}, v_{f} \vdash^{c} \mathtt{CWHILE}(e, c, f, \mathtt{true}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CWHILE}(e, c, f, \mathtt{false}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}} (\mathtt{While}^{\mathtt{false}}) \end{split}$$

A loop continues normally if its condition is nonzero:

$$\begin{split} & \rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \mathtt{nil}, v_{e} & v_{e} \neq 0 \\ & \rho, \mu_{e}, \mathtt{nil}, v_{e} \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, \chi_{c}, v_{c} & \chi_{c} \not\in \{\mathtt{brk}, \mathtt{cnt}\} \\ & \rho, \mu_{c}, \chi_{c}, v_{c} \vdash^{e} f \Rightarrow \mu_{f}, \chi_{f}, v_{f} \\ & \frac{\rho, \mu_{f}, \chi_{f}, v_{f} \vdash^{c} \mathtt{CWHILE}(e, c, f, \mathtt{true}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CWHILE}(e, c, f, \mathtt{true}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}} \end{split}$$

A flag skips the loop:

$$\begin{split} &\frac{\rho,\mu,\chi,v \vdash^{e} e \Rightarrow \mu_{e},\chi_{e},v_{e}}{\rho,\mu,\chi,v \vdash^{c} \mathtt{CWHILE}(e,c,f,\mathtt{true}) \Rightarrow \rho,\mu_{e},\chi_{e},v_{e}} (\mathtt{WHILE}^{\chi,\mathtt{true}}) \\ &\frac{\chi \neq \mathtt{nil}}{\rho,\mu,\chi,v \vdash^{c} \mathtt{CWHILE}(e,c,f,\mathtt{false}) \Rightarrow \rho,\mu,\chi,v} (\mathtt{WHILE}^{\chi,\mathtt{false}}) \end{split}$$

cnt executes the finally clause before continuing as normal :

$$\begin{split} \rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \text{nil}, v_{e} & v_{e} \neq 0 \\ \rho, \mu_{e}, \text{nil}, v_{e} \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, \text{cnt}, v_{c} \\ \rho, \mu_{c}, \text{nil}, v_{c} \vdash^{e} f \Rightarrow \mu_{f}, \chi_{f}, v_{f} \\ \hline \frac{\rho, \mu_{f}, \chi_{f}, v_{f} \vdash^{c} \text{CWHILE}(e, c, f, \text{true}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}}{\rho, \mu, \chi, v \vdash^{c} \text{CWHILE}(e, c, f, \text{true}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}} \end{split} \\ (\text{WHILE}^{\text{cnt,true}})$$

$$\frac{\rho, \mu, \chi, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, \text{cnt}, v_{c}}{\rho, \mu_{c}, \text{nil}, v_{c} \vdash^{e} f \Rightarrow \mu_{f}, \chi_{f}, v_{f}} \\ \frac{\rho, \mu_{f}, \chi_{f}, v_{f} \vdash^{c} \text{CWHILE}(e, c, f, \text{true}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}}{\rho, \mu, \chi, v \vdash^{c} \text{CWHILE}(e, c, f, \text{false}) \Rightarrow \rho, \mu_{w}, \chi_{w}, v_{w}} \end{split}$$

brk interrupts the loop but is not retransmitted:

$$\begin{split} & \rho, \mu, \chi, v \vdash^e e \Rightarrow \mu_e, \mathtt{nil}, v_e & v_e \neq 0 \\ & \frac{\rho, \mu_e, \mathtt{nil}, v_e \vdash^c c \Rightarrow \rho_c, \mu_c, \mathtt{brk}, v_c}{\rho, \mu, \chi, v \vdash^c \mathtt{CWHILE}(e, c, f, \mathtt{true}) \Rightarrow \rho, \mu_c, \mathtt{nil}, v_c} (\mathtt{WHILE}^{\mathtt{brk}, \mathtt{true}}) \\ & \frac{\rho, \mu, \chi, v \vdash^c c \Rightarrow \rho_c, \mu_c, \mathtt{brk}, v_c}{\rho, \mu, \chi, v \vdash^c \mathtt{CWHILE}(e, c, f, \mathtt{false}) \Rightarrow \rho, \mu_c, \mathtt{nil}, v_c} (\mathtt{WHILE}^{\mathtt{brk}, \mathtt{false}}) \end{split}$$

2.5 Control flow

$$\frac{\rho,\mu,\operatorname{nil},v\vdash^c\operatorname{CBREAK}\Rightarrow\rho,\mu,\operatorname{brk},0}{\chi\neq\operatorname{nil}}(\operatorname{Break}^\chi)$$

$$\frac{\chi\neq\operatorname{nil}}{\rho,\mu,\chi,v\vdash^c\operatorname{CBREAK}\Rightarrow\rho,\mu,\chi,v}(\operatorname{Break}^\chi)$$

$$\frac{\rho,\mu,\operatorname{nil},v\vdash^c\operatorname{CCONTINUE}\Rightarrow\rho,\mu,\operatorname{cnt},0}{\rho,\mu,\chi,v\vdash^c\operatorname{CCONTINUE}\Rightarrow\rho,\mu,\chi,v}(\operatorname{CONTINUE}^\chi)$$

$$\frac{\chi\neq\operatorname{nil}}{\rho,\mu,\chi,v\vdash^c\operatorname{CCONTINUE}\Rightarrow\rho,\mu,\chi,v}(\operatorname{CONTINUE}^\chi)$$

$$\frac{\rho,\mu,\chi,v\vdash^c\operatorname{CRETURN}\operatorname{None}\Rightarrow\rho,\mu,\operatorname{ret},0}{\rho,\mu,\operatorname{nil},v\vdash^c\operatorname{CRETURN}(\operatorname{Some}e)\Rightarrow\rho,\mu_e,\operatorname{ret},v_e}(\operatorname{RETURN}^{\operatorname{None}})$$

$$\frac{\rho,\mu,\chi,v\vdash^ee}{\rho,\mu,\chi,v\vdash^c\operatorname{CRETURN}(\operatorname{Some}e)\Rightarrow\rho,\mu_e,\operatorname{ret},v_e}(\operatorname{RETURN}^{\operatorname{None}\chi})$$

$$\frac{\chi\neq\operatorname{nil}}{\rho,\mu,\chi,v\vdash^c\operatorname{CRETURN}\operatorname{None}\Rightarrow\rho,\mu,\chi,v}(\operatorname{RETURN}^{\operatorname{None}\chi})$$

$$\frac{\rho,\mu,\chi,v\vdash^ee}{\rho,\mu,\operatorname{nil},v\vdash^c\operatorname{CRETURN}(\operatorname{Some}e)\Rightarrow\rho,\mu_e,\chi_e,v_e}$$

$$\frac{\rho,\mu,\chi,v\vdash^ee}{\rho,\mu,\operatorname{nil},v\vdash^c\operatorname{CRETURN}(\operatorname{Some}e)\Rightarrow\rho,\mu_e,\chi_e,v_e} (\operatorname{RETURN}^{\operatorname{Some}\chi})$$

2.6 Local variable declarations

First, the obvious:

$$\begin{split} & \frac{\rho, \mu, \chi, v \vdash^{c} \mathtt{CLOCAL} \ [] \Rightarrow \rho, \mu, \chi, v}{\chi \neq \mathtt{nil}} (\mathtt{LOCAL}^{0}) \\ & \frac{\chi \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CLOCAL} \ d \Rightarrow \rho, \mu, \chi, v} (\mathtt{LOCAL}^{\chi}) \\ & \frac{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \mathtt{nil}, v_{e}}{k \in alloc^{8}(\mu_{e}) \quad \rho' = \rho[w \mapsto k] \quad \mu' = \mu_{e}[k \mapsto v_{e}]} \\ & \frac{\rho', \mu', \mathtt{nil}, v_{e} \vdash^{c} \mathtt{CLOCAL} \ S \Rightarrow \rho_{s}, \mu_{s}, \chi_{s}, v_{s}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CLOCAL}((w, e) :: S) \Rightarrow \rho_{s}, \mu_{s}, \chi_{s}, v_{s}} (\mathtt{LOCAL}^{1}) \end{split}$$

2.7 Throw

If a flag is already raised, skip the CTHROW:

$$\frac{\rho, \mu, \chi, v \vdash^{e} e \Rightarrow \mu_{e}, \chi_{e}, v_{e} \qquad \chi_{e} \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CTHROW}(s, e) \Rightarrow \rho, \mu_{e}, \chi_{e}, v_{e}} (\mathtt{THROW}^{\chi})$$

Otherwise raise the new exception $s \in S$:

$$\frac{\rho,\mu,\chi,v\vdash^e e\Rightarrow \mu_e,\mathtt{nil},v_e}{\rho,\mu,\chi,v\vdash^c \mathtt{CTHROW}(s,e)\Rightarrow \rho,\mu_e,s,v_e}(\mathtt{THROW})$$

2.8 Switch

$$\frac{\rho, \mu, \chi, v \vdash^e e \Rightarrow \mu_e, \chi_e, v_e}{\frac{\rho, \mu_e, \chi_e, v_e \vdash^c \mathtt{CBLOCK}(L(v_e)) \Rightarrow \rho, \mu_l, \chi_l, v_l}{\rho, \mu, \chi, v \vdash^c \mathtt{CSWITCH}(e, L, c) \Rightarrow \rho, \mu_l, \chi_l, v_l}}(\mathtt{SWITCH})$$

Where for $L = [(j_1, l_1); \cdots; (j_n, l_n)], L(v_e)$ is defined as follows: Let $I_i = \{j_1, \cdots, j_i\}$ for $1 \le i \le n, I_{n+1} = \mathbb{Z}_{64}$. $\check{j} \triangleq \min_{1 \le i \le n+1} \{i \mid v_e \in I_i\}$, finally $L(v_e) \triangleq [l_{\tilde{j}}; \cdots; l_n; c]$.

2.9 Try

Skip the block when a flag is already raised:

$$\frac{\chi \neq \mathtt{nil}}{\rho, \mu, \chi, v \vdash^{c} \mathtt{CTRY}(c, L, f) \Rightarrow \rho, \mu, \chi, v}(\mathtt{TRY}^{\chi})$$

For $L = [(e_1, x_1, c_1); \dots; (e_n, x_n, c_n)]$, let $E = \{e_i | 1 \le i \le n\} \subset S$. When no exception is raised :

$$\begin{split} & \rho, \mu, \mathtt{nil}, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, \mathtt{nil}, v_{c} \\ & \frac{\rho, \mu_{c}, \mathtt{nil}, v_{c} \vdash^{c} f \Rightarrow \rho_{f}, \mu_{f}, \chi_{f}, v_{f}}{\rho, \mu, \mathtt{nil}, v \vdash^{c} \mathtt{CTRY}(c, L, f) \Rightarrow \rho, \mu_{f}, \chi_{f}, v_{f}} (\mathtt{TRY}^{\mathtt{nil}}) \end{split}$$

When an exception is raised that is not caught by the current handler:

$$\begin{split} \rho, \mu, & \texttt{nil}, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, s_{c}, v_{c} \\ s_{c} \not\in E & _ \not\in E \\ \hline \rho, \mu, & \texttt{nil}, v_{c} \vdash^{c} f \Rightarrow \rho_{f}, \mu_{f}, \texttt{nil}, v_{f} \\ \hline \rho, \mu, & \texttt{nil}, v \vdash^{c} \Rightarrow \texttt{CTRY}(c, L, f) \Rightarrow \rho, \mu_{f}, s_{c}, v_{c} \\ \rho, \mu, & \texttt{nil}, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, s_{c}, v_{c} \\ s_{c} \not\in E & _ \not\in E \\ \hline \rho, \mu, & \texttt{nil}, v \vdash^{c} f \Rightarrow \rho_{f}, \mu_{f}, \chi_{f}, v_{f} & \chi_{f} \neq \texttt{nil} \\ \hline \rho, \mu, & \texttt{nil}, v \vdash^{c} \Rightarrow \texttt{CTRY}(c, L, f) \Rightarrow \rho, \mu_{f}, \chi_{f}, v_{f} \end{split} (\texttt{TRY}^{\chi'})$$

When the handler is able to catch the exception:

$$\rho, \mu, \operatorname{nil}, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, s_{c}, v_{c}$$

$$s_{c} = e_{i_{0}} \qquad x_{i_{0}} \neq \underline{\hspace{2cm}}$$

$$\rho[x_{i_{0}} \mapsto k], \mu_{c}[k \mapsto v_{c}], \operatorname{nil}, v_{c} \vdash^{c} c_{i_{0}} \Rightarrow \rho_{0}, \mu_{0}, \chi_{0}, v_{0}$$

$$\rho, \mu_{0}, \chi_{0}, v_{0} \vdash^{c} f \Rightarrow \rho_{f}, \mu_{f}, \chi_{f}, v_{f}$$

$$\rho, \mu, \operatorname{nil}, v \vdash^{c} \operatorname{CTRY}(c, L, f) \Rightarrow \rho, \mu_{f}, \chi_{f}, v_{f}$$

$$\rho, \mu, \operatorname{nil}, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, s_{c}, v_{c}$$

$$s_{c} = e_{i_{0}} \qquad x_{i_{0}} = \underline{\hspace{2cm}}$$

$$\rho, \mu_{c}, \operatorname{nil}, v_{c} \vdash^{c} c_{i_{0}} \Rightarrow \rho_{0}, \mu_{0}, \chi_{0}, v_{0}$$

$$\rho, \mu_{0}, \chi_{0}, v_{0} \vdash^{c} f \Rightarrow \rho_{f}, \mu_{f}, \chi_{f}, v_{f}$$

$$\rho, \mu, \operatorname{nil}, v \vdash^{c} \operatorname{CTRY}(c, L, f) \Rightarrow \rho, \mu_{f}, \chi_{f}, v_{f}$$

$$\rho, \mu, \operatorname{nil}, v \vdash^{c} c \Rightarrow \rho_{c}, \mu_{c}, s_{c}, v_{c}$$

$$s_{c} \notin E \qquad \underline{\hspace{2cm}} = e_{i_{0}}$$

$$\rho, \mu_{c}, \operatorname{nil}, v_{c} \vdash^{c} c_{i_{0}} \Rightarrow \rho_{0}, \mu_{0}, \chi_{0}, v_{0}$$

$$\underline{\hspace{2cm}} \rho, \mu_{c}, \operatorname{nil}, v_{c} \vdash^{c} c_{i_{0}} \Rightarrow \rho_{0}, \mu_{o}, \chi_{0}, v_{0}$$

$$\underline{\hspace{2cm}} \rho, \mu_{0}, \chi_{0}, v_{0} \vdash^{c} f \Rightarrow \rho_{f}, \mu_{f}, \chi_{f}, v_{f}} (\operatorname{TRY}^{s})$$

$$\underline{\hspace{2cm}} \rho, \mu, \operatorname{nil}, v \vdash^{c} \operatorname{CTRY}(c, L, f) \Rightarrow \rho, \mu_{f}, \chi_{f}, v_{f}} (\operatorname{TRY}^{s})$$

3 Declarations

3.1 Global variables

$$\begin{split} \frac{k \not\in \operatorname{dom} \rho & k \in \operatorname{alloc}^8(\mu)}{\rho, \mu, \operatorname{fun} \vdash^d \operatorname{CDECL}(x, \operatorname{None}) \Rightarrow \rho[x \mapsto k], \mu[k \mapsto 0], \operatorname{fun}}(\operatorname{DECL}^{\operatorname{None}}) \\ \frac{x \not\in \operatorname{dom} \rho & k \in \operatorname{alloc}^8(\mu)}{\rho, \mu, \operatorname{fun} \vdash^d \operatorname{CDECL}(x, \operatorname{Some}(\operatorname{CST} c)) \Rightarrow \rho[x \mapsto k], \mu[k \mapsto c], \operatorname{fun}}(\operatorname{DECL}^{\operatorname{CST}}) \\ \frac{s \text{ stored at } a \in \operatorname{Addr} & x \not\in \operatorname{dom} \rho}{\rho, \mu, \operatorname{fun} \vdash^d \operatorname{CDECL}(x, \operatorname{Some}(\operatorname{STRING} s)) \Rightarrow \rho[x \mapsto a], \mu, \operatorname{fun}}(\operatorname{DECL}^{\operatorname{STRING}}) \end{split}$$

3.2 Functions

$$\frac{f\not\in\mathrm{dom}\,\rho\quad\phi_A(b)\;\mathrm{stored}\;\mathrm{at}\;k}{\rho,\mu,\mathrm{fun}\,\vdash^d\mathrm{CFUN}(f,A,b)\Rightarrow\rho[f\mapsto k],\mu,\mathrm{fun}[k\mapsto\phi_A(b)]}(\mathrm{Fun})$$
 Let $[a_1;\cdots;a_n]=A,\;\mathrm{then}\;\phi_A(b):\mathbb{Z}^n_{64}\to\mathrm{code}\;\mathrm{is}\;\mathrm{defined}\;\mathrm{as}$
$$\phi_A(b)(v_1,\cdots,v_n)=\mathrm{CBLOCK}(\mathrm{CLOCAL}[(a_1,\mathrm{Some}\;(\mathrm{CST}\;v_1));\cdots;(a_n,\mathrm{Some}\;(\mathrm{CST}\;v_n))]::b)$$

4 Program

Finally, here's how the whole program executes:

$$\begin{split} \rho_0, \mu_0, & \operatorname{fun}_0 \vdash^d d_1 \Rightarrow \rho_1, \mu_1, \operatorname{fun}_1 \\ & \cdots \\ \rho_{n-1}, \mu_{n-1}, & \operatorname{fun}_{n-1} \vdash^d d_n \Rightarrow \rho_n, \mu_n, \operatorname{fun}_n \\ & \frac{\rho_n, \mu_n, \operatorname{nil}, 0 \vdash^e \operatorname{CALL}(\operatorname{main}, [\operatorname{argc}; \operatorname{argv}]) \Rightarrow \mu', \chi', v'}{\rho_0, \mu_0, \operatorname{fun}_0 \vdash^\pi [d_1; \cdots; d_n] \Rightarrow \mu', \chi', v'} (\operatorname{PROG}) \end{split}$$

 ρ_0 , μ_0 and fun₀ initially contain some predefined globals and constants such as NULL, stdout, EOF, true, BYTE, QSIZE, ..., as well as standard library functions (malloc, atol, rand, sleep, qsort, ...)