Counting Abstraction
for the Verification of
Structured Parameterized Networks

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CAV'25 @ Zagreb

2025-07-25

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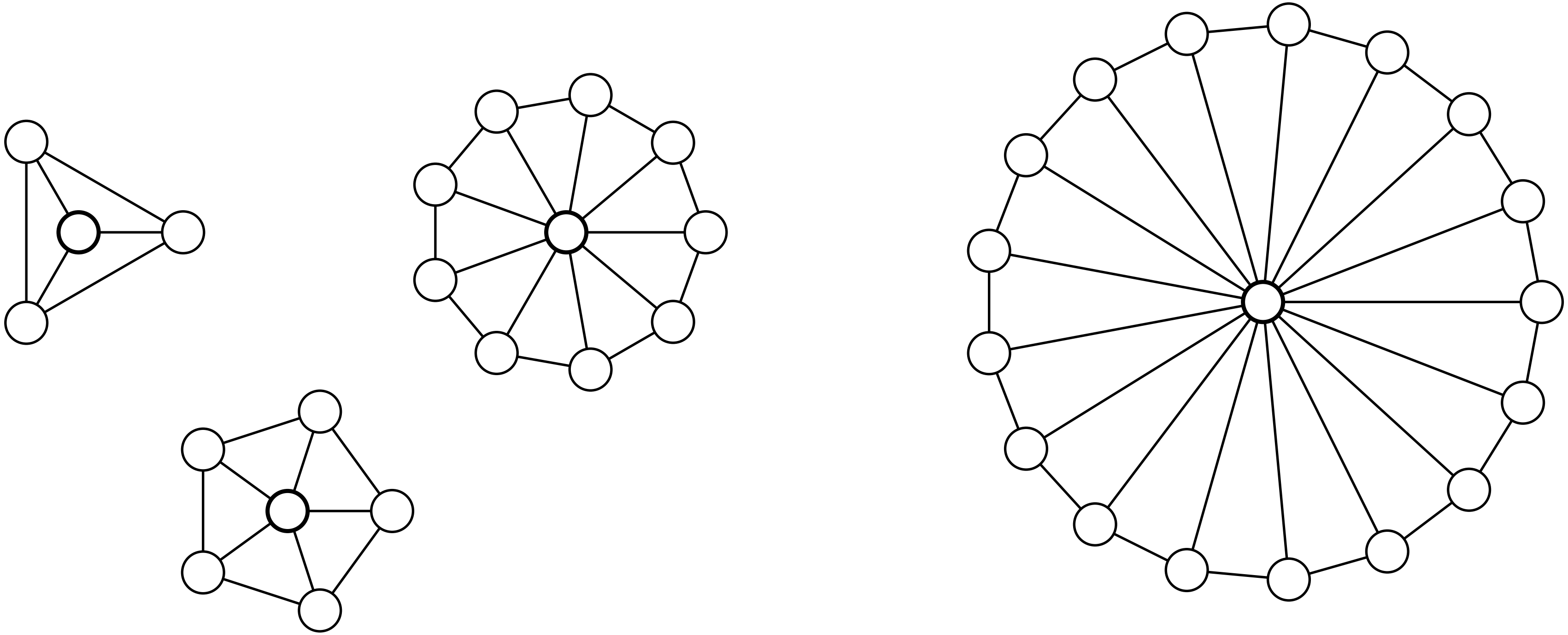
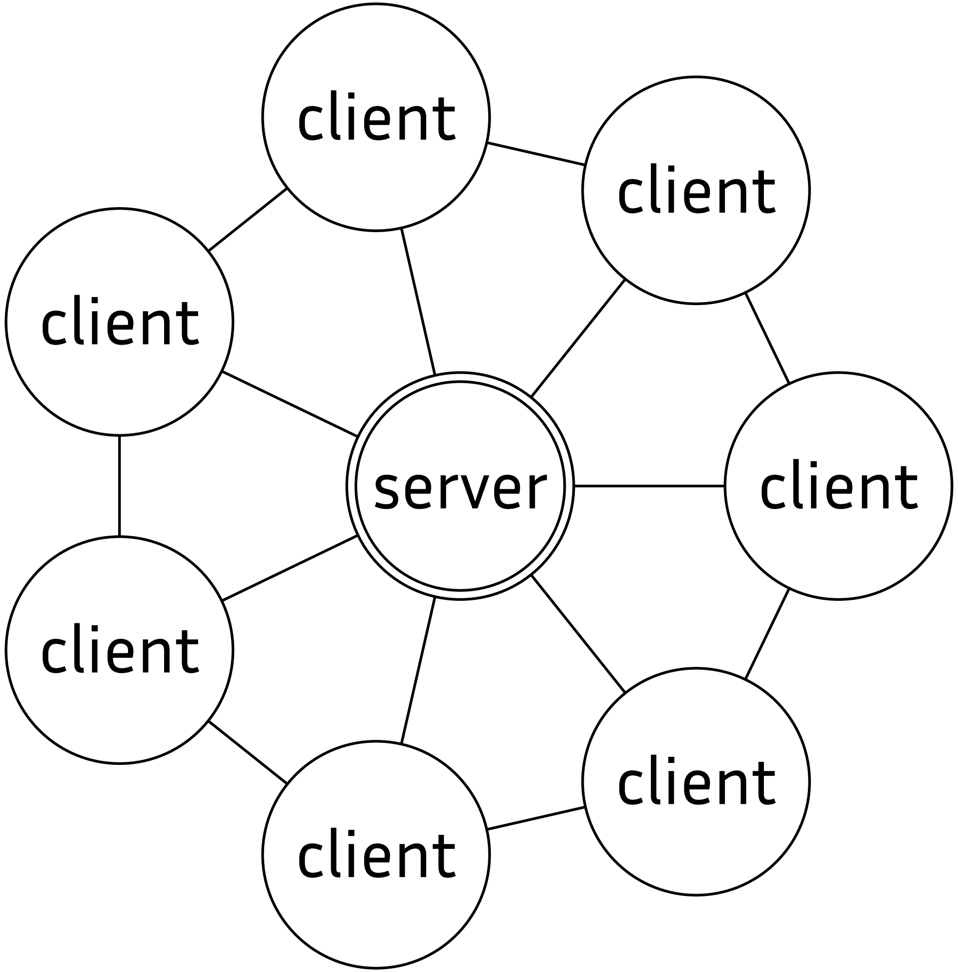
- (automated) verification of networks
- challenge: size and architecture (communication topology)



- undecidable $\Rightarrow$ abstraction

Token ring with resource



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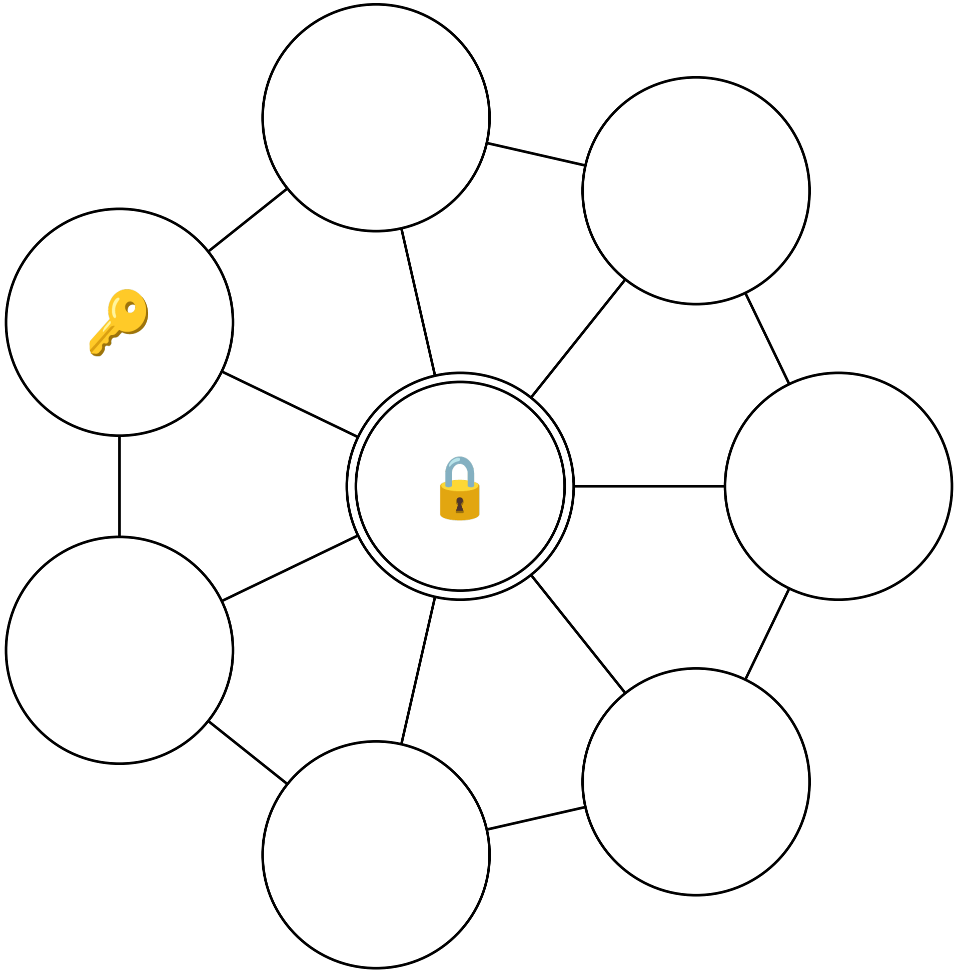

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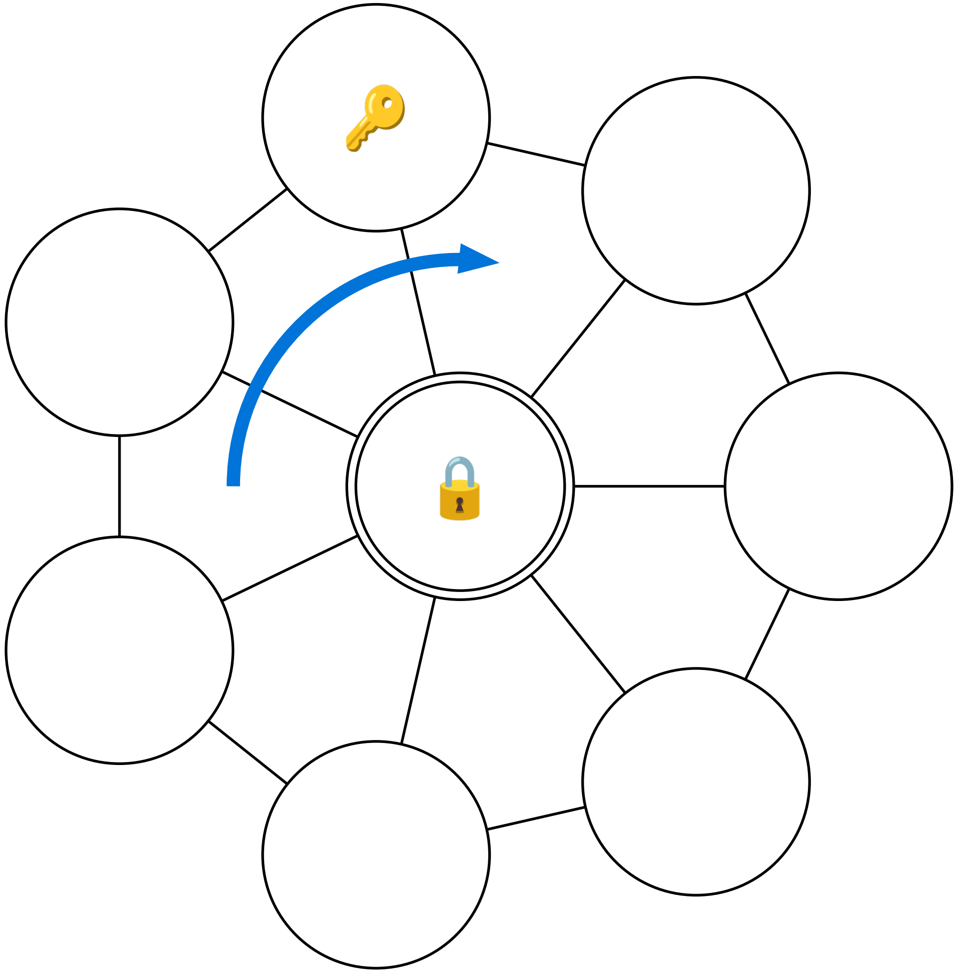

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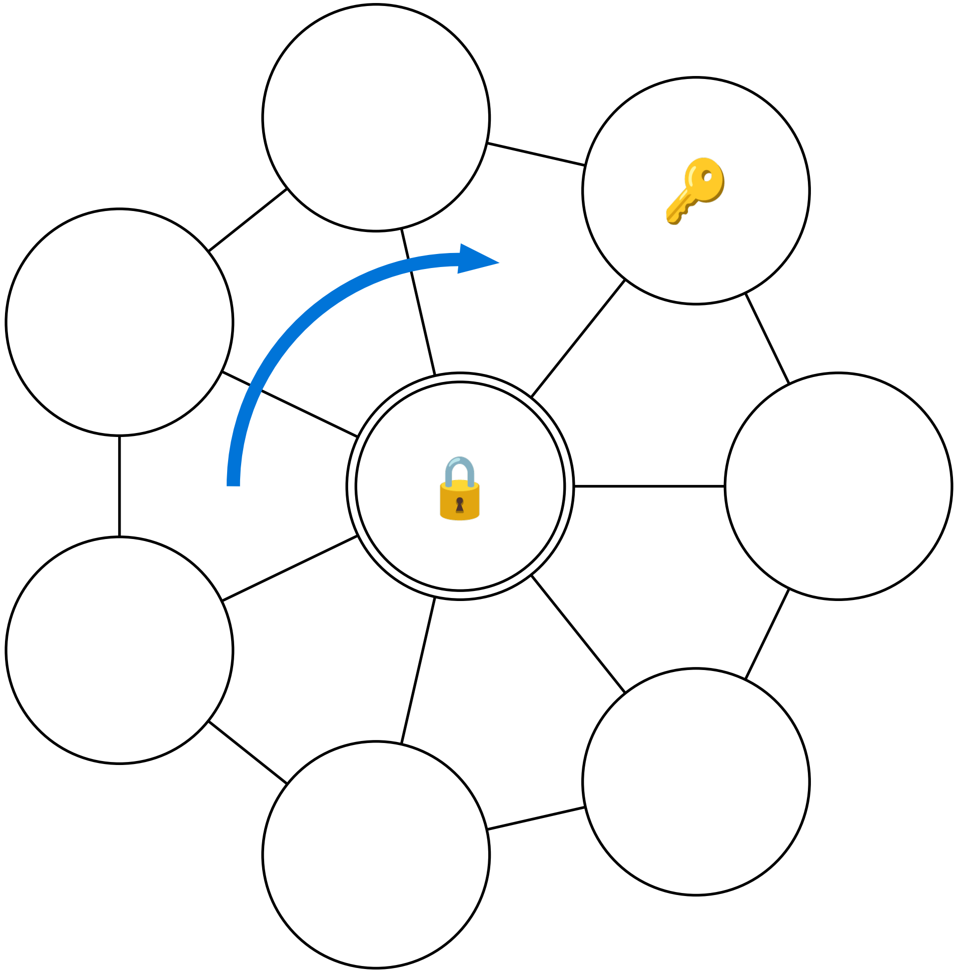

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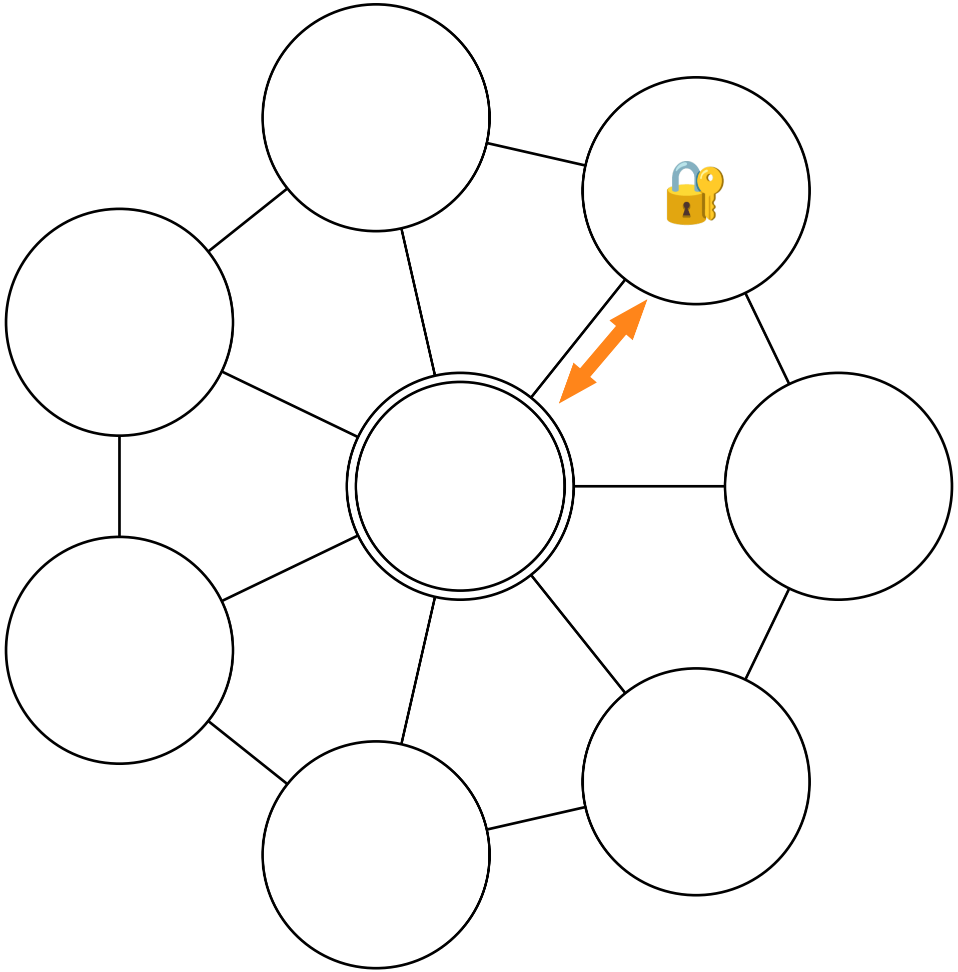

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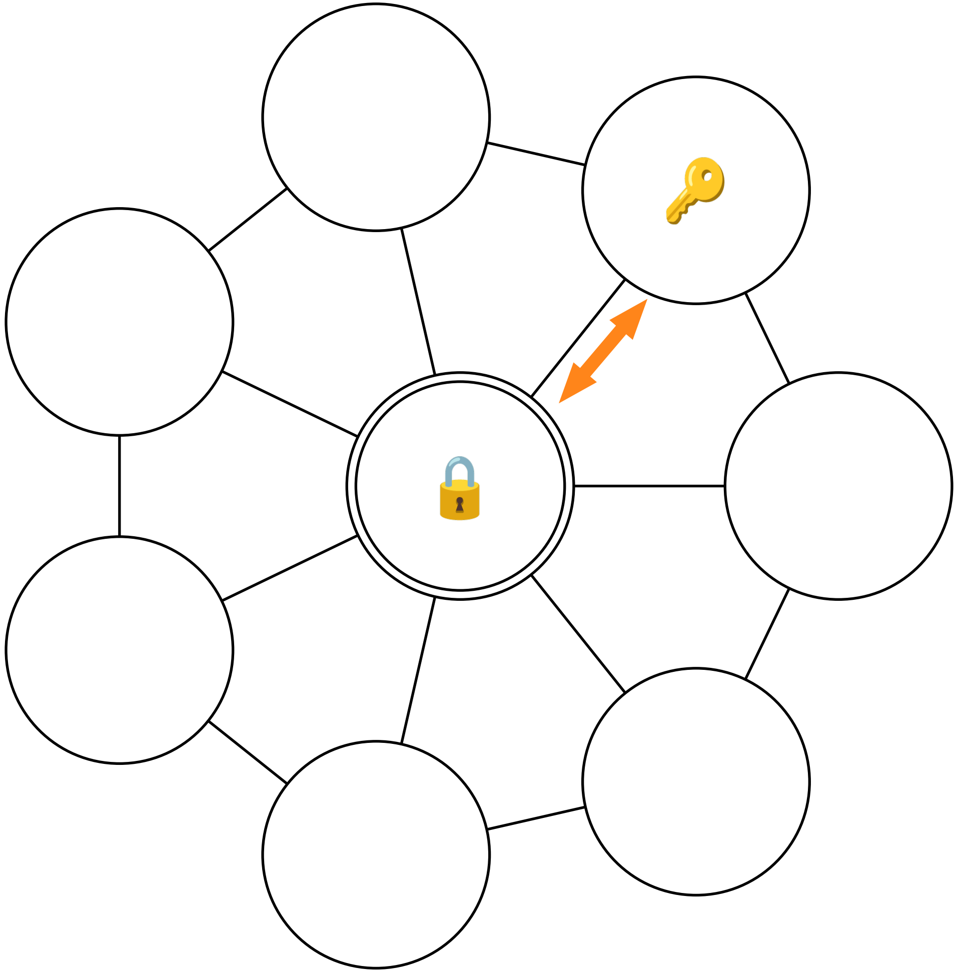

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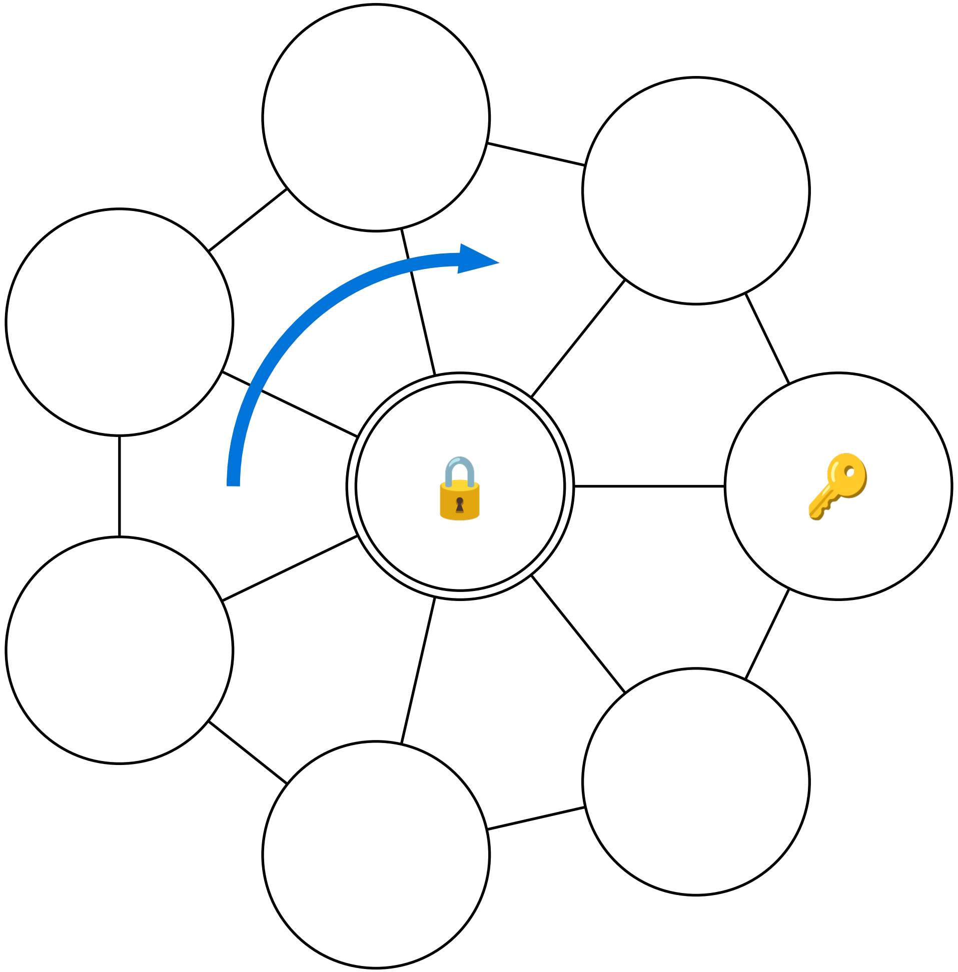

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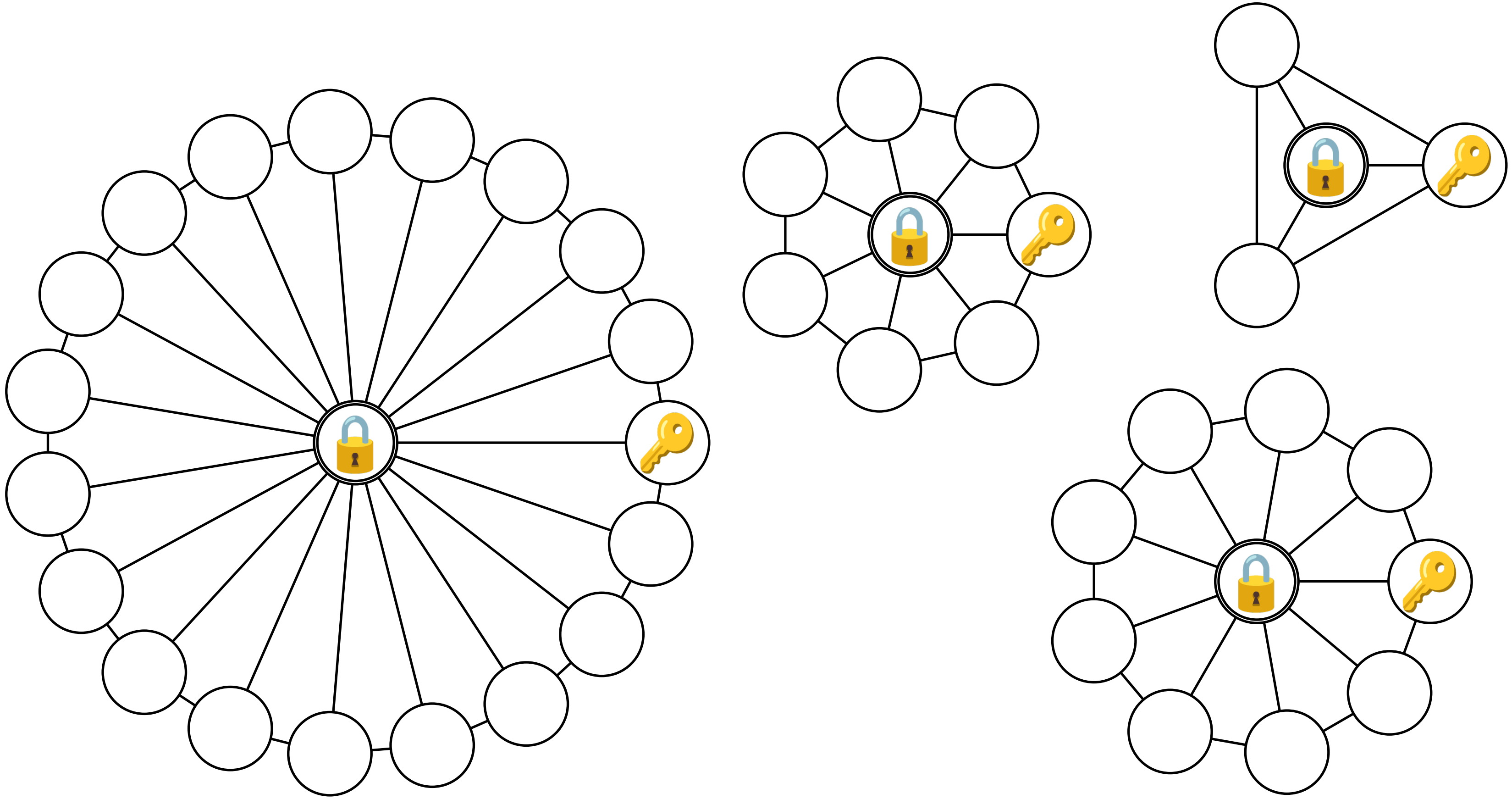

$$\forall n \geq 2$$

How would we automatically verify this ?

Techniques for non-finite-state systems...

Parameterized model checking of rendezvous systems

(B. Aminof, T. Kotek, S. Rubin, F. Spegni, H. Veith)

❌ not homogeneous (2 kinds of processes)

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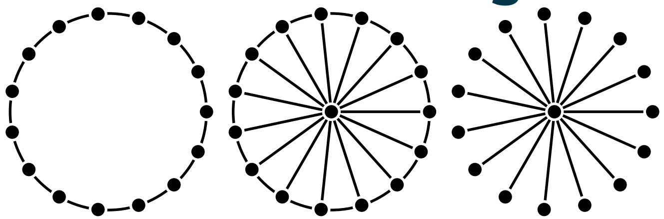
Parameterized Verification of Algorithms for Oblivious Robots on a Ring

(A. Sangnier, N. Sznajder, M. Potop-Butucaru, S. Tixeuil)

❌ not a standard architecture (clique, ring, star)



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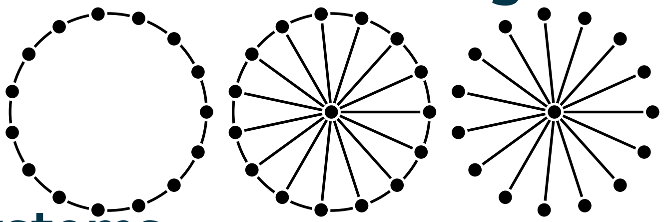


Parameterized Model Checking of Token-Passing Systems

(B. Aminof, S. Jacobs, A. Khalimov, S. Rubin)

❌ not a token-passing system (key and lock don't behave like tokens)

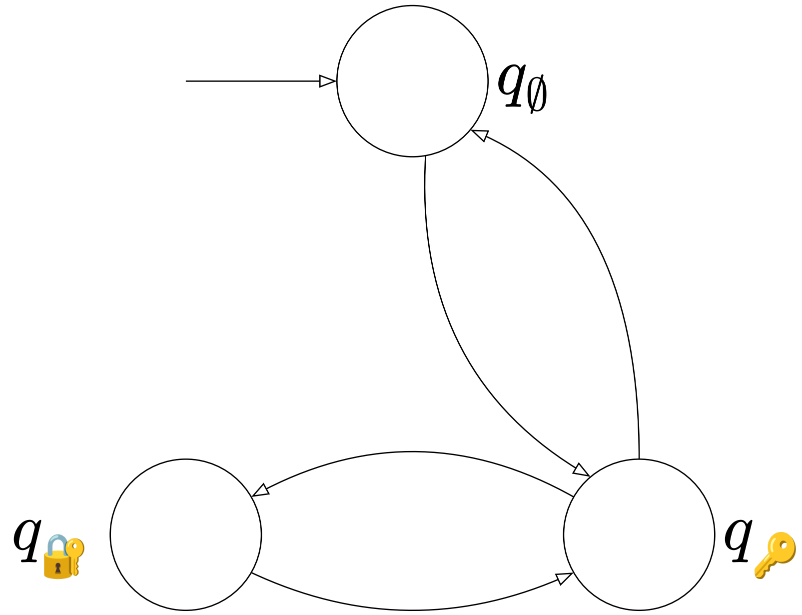
Framework requirements

We must be able to express

- an encoding of the **local behavior** of processes
- a description of the **interactions** and **architectures** of arbitrary size
- a **specification language** for safety properties

Framework

Client



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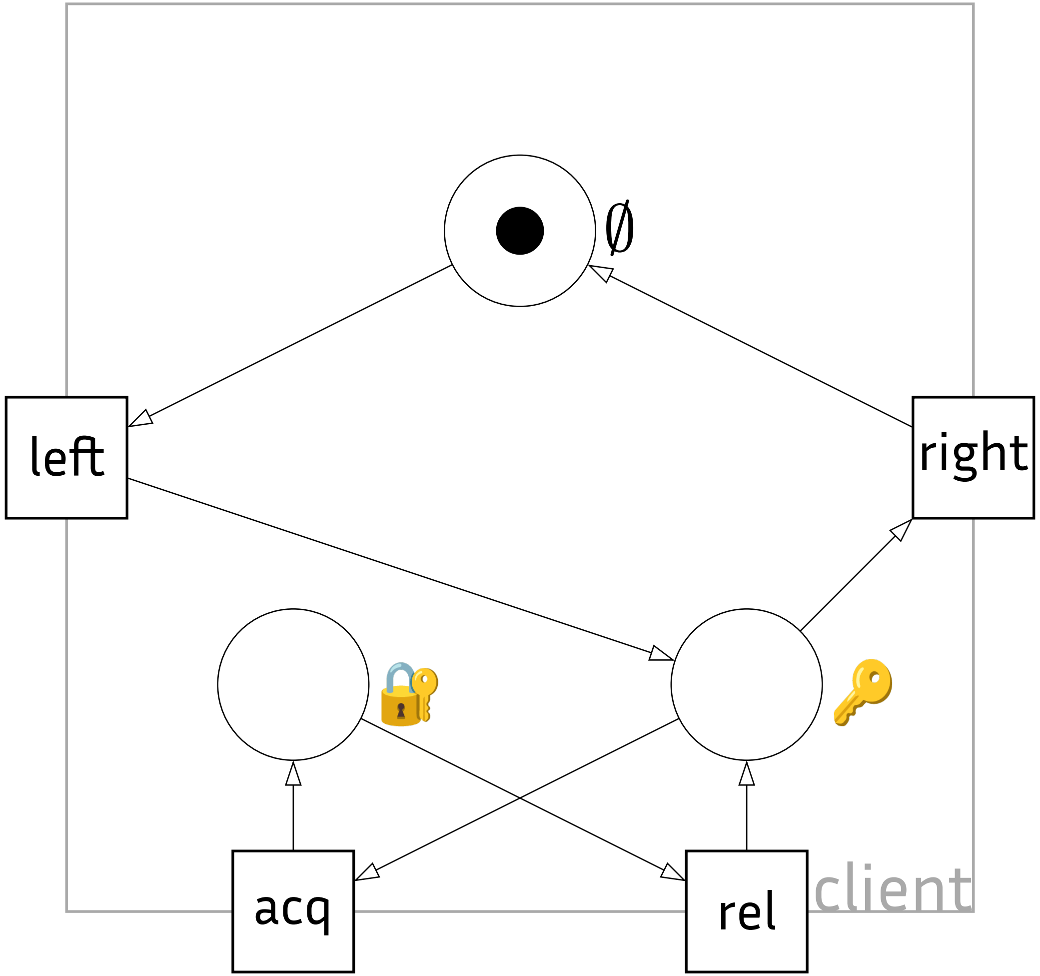
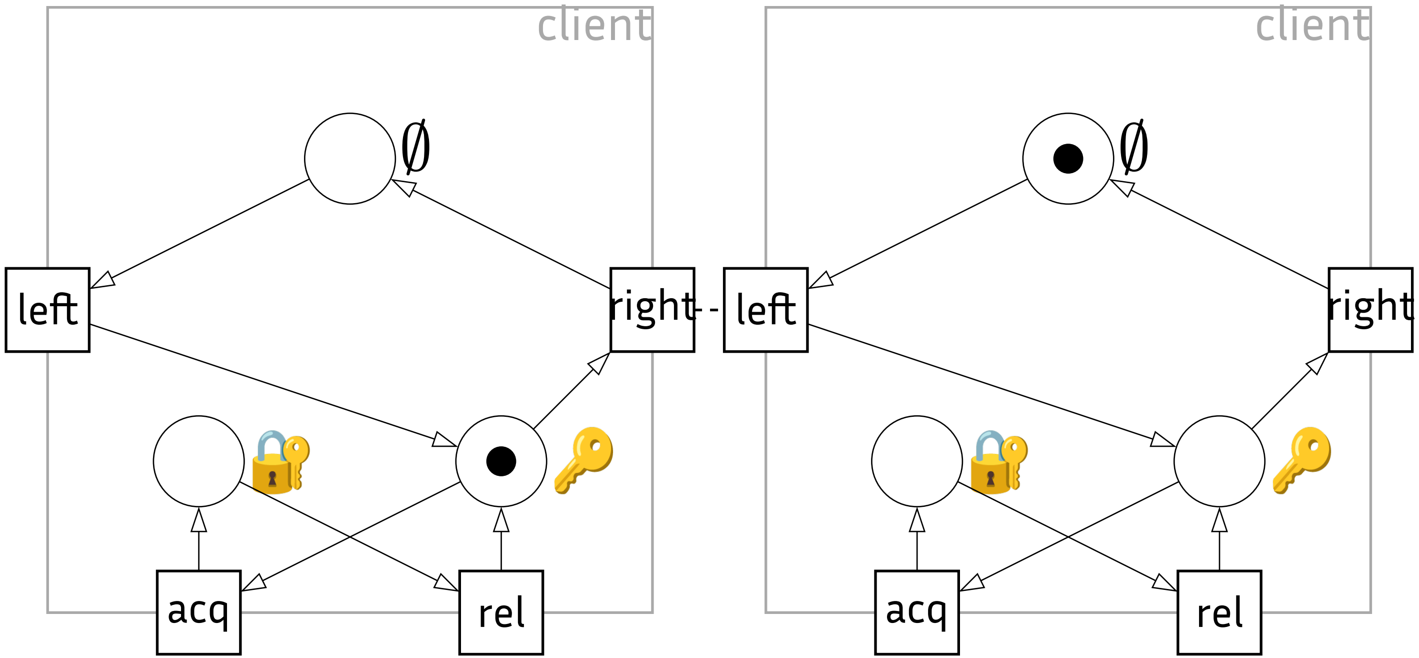


Interactions through composition



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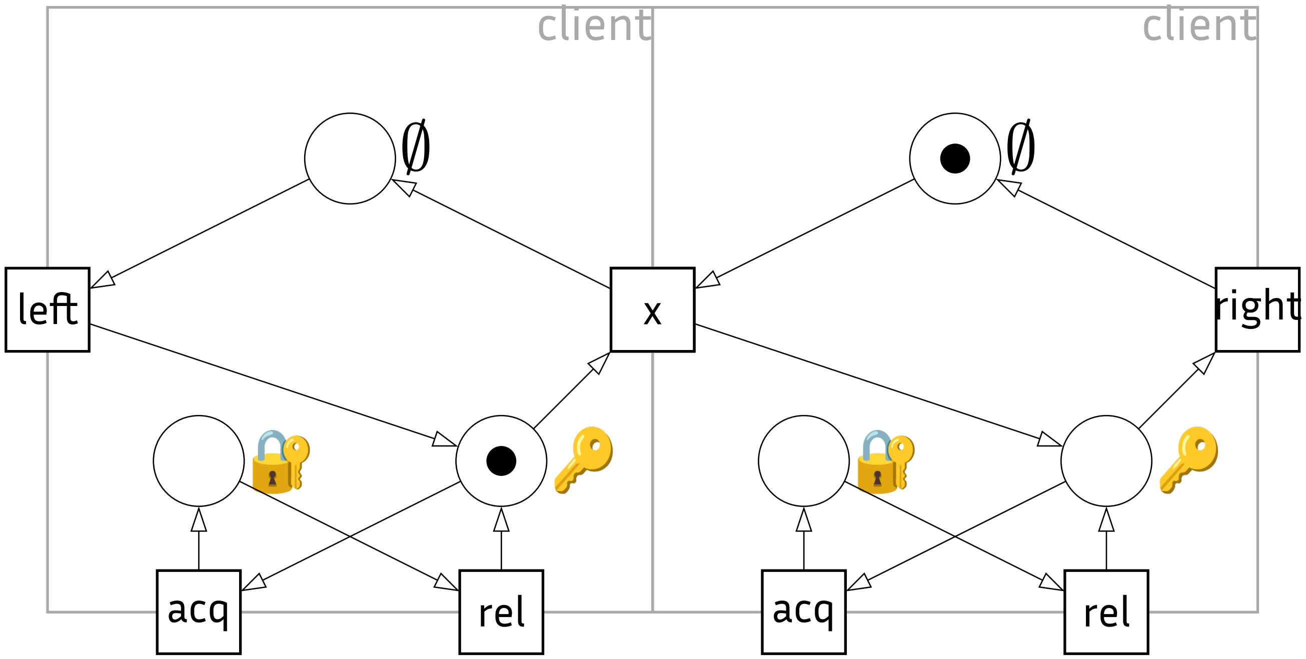

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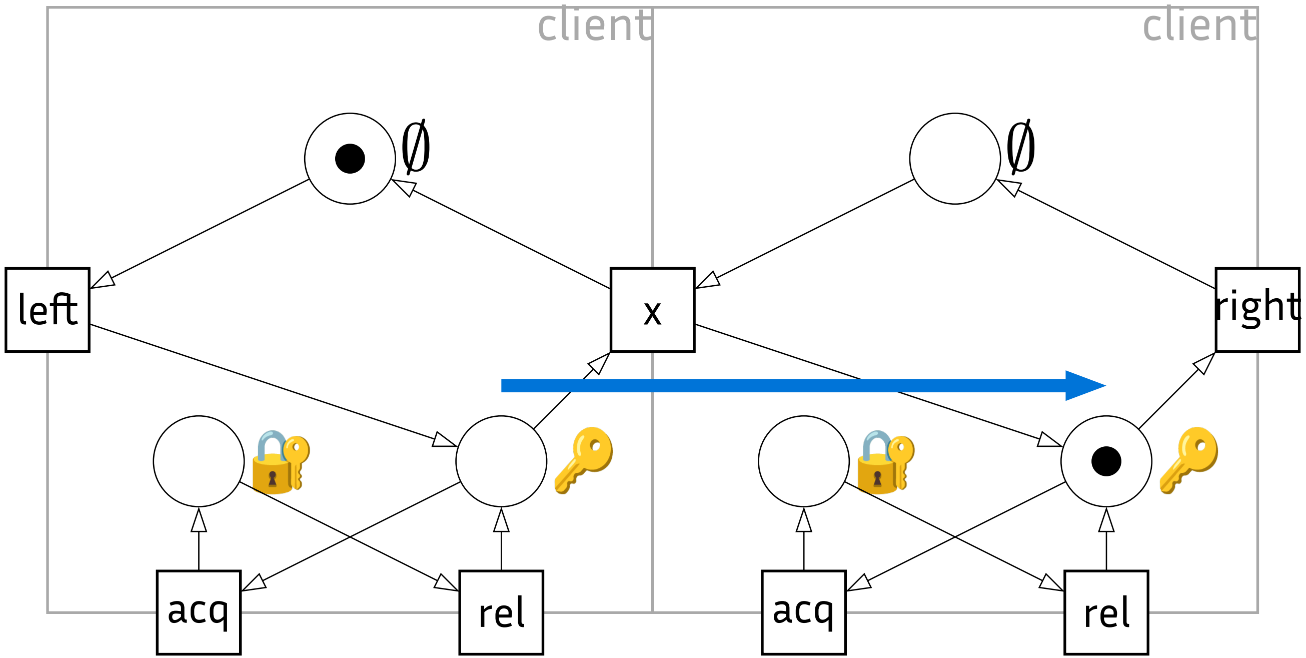
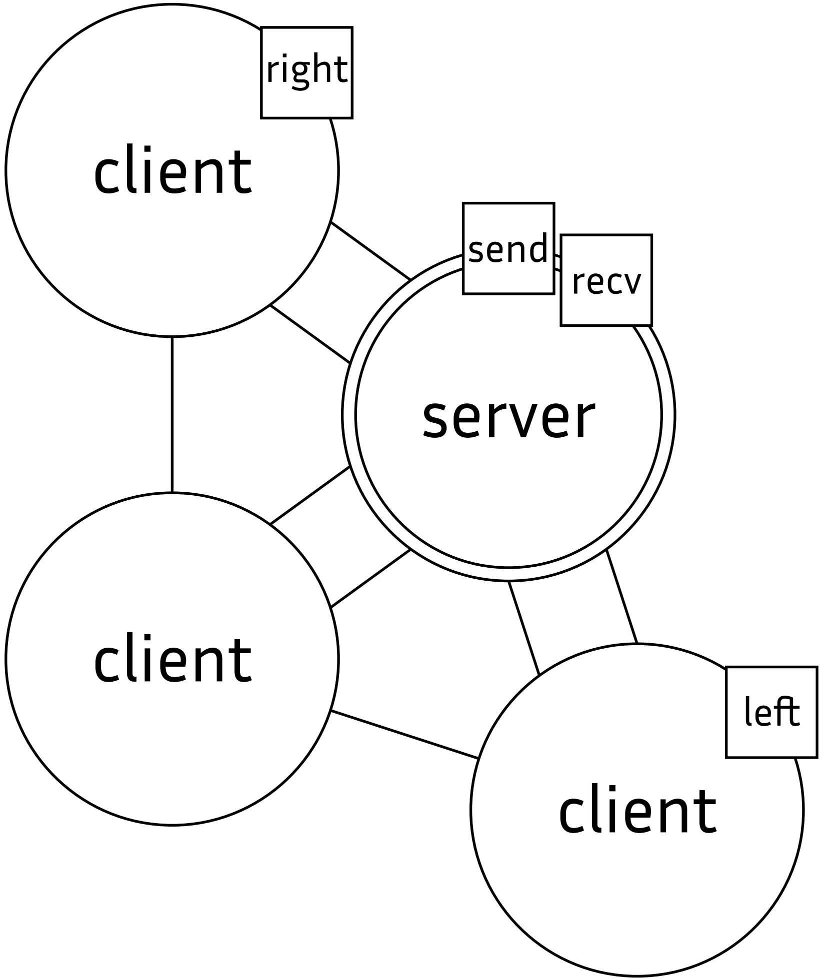


Structure: inductive step



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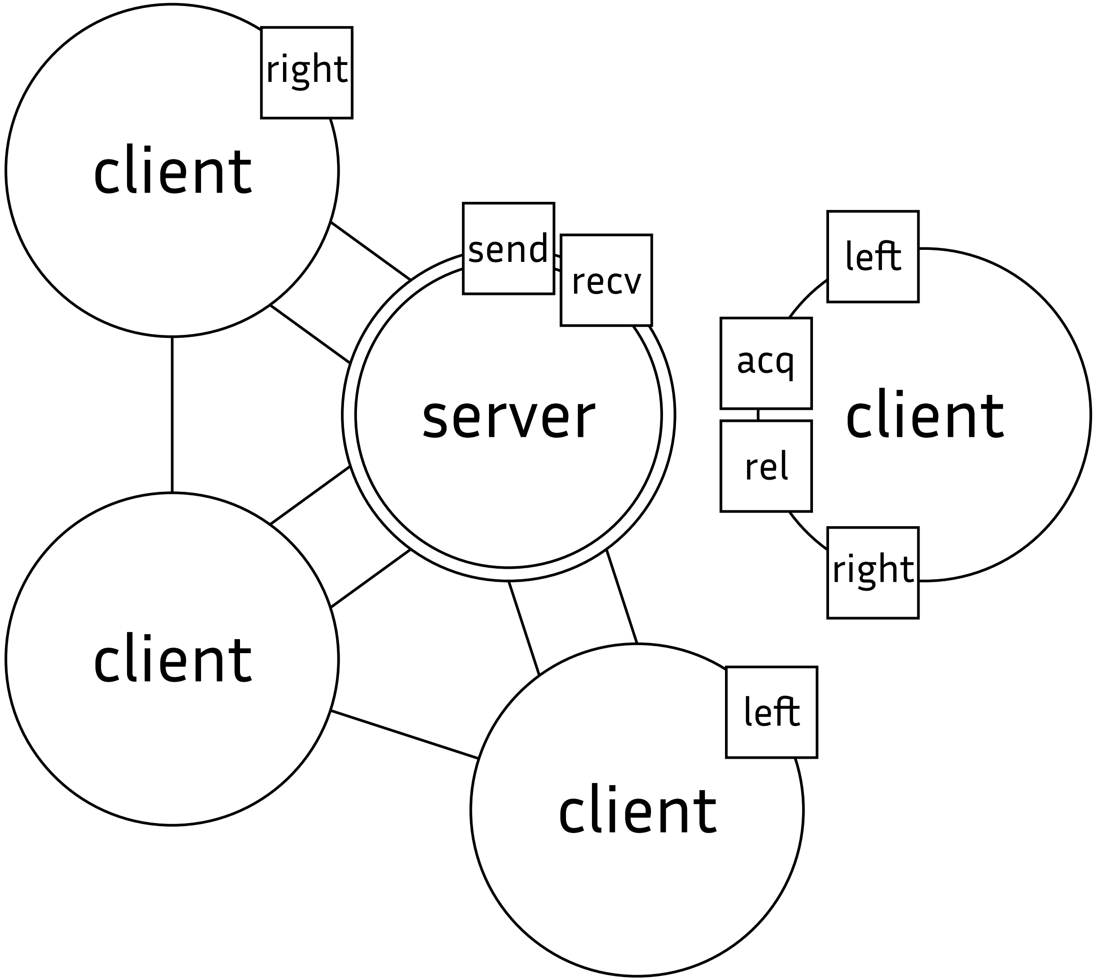

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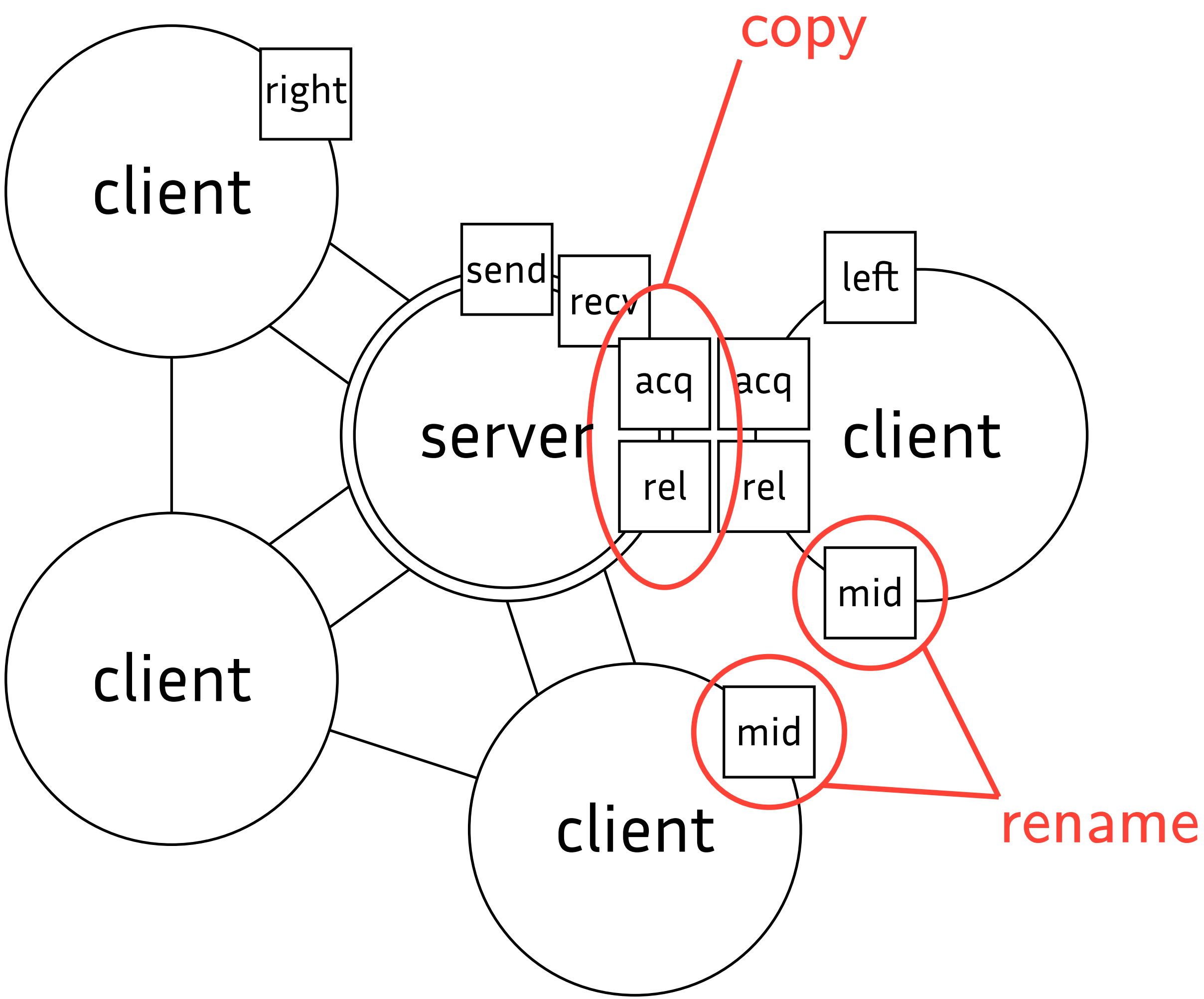

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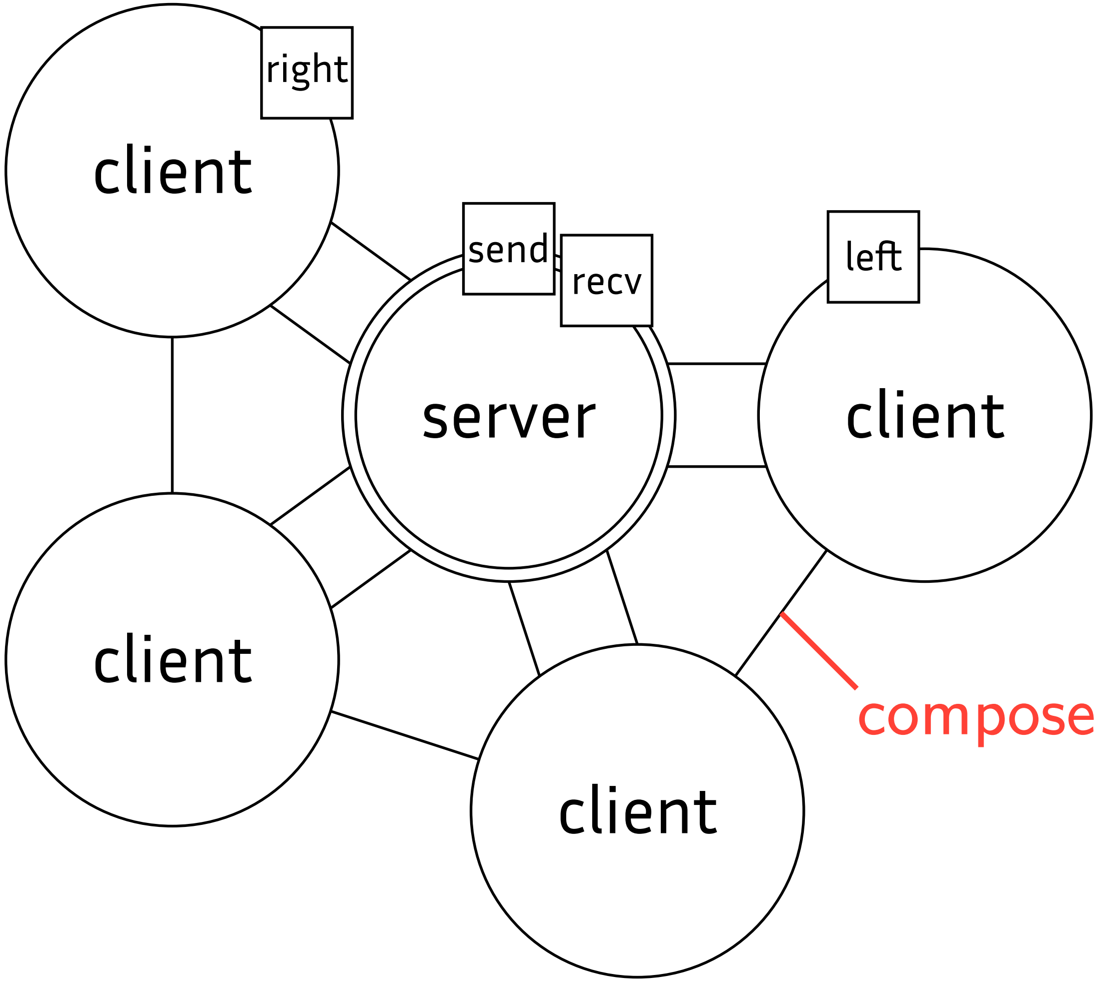
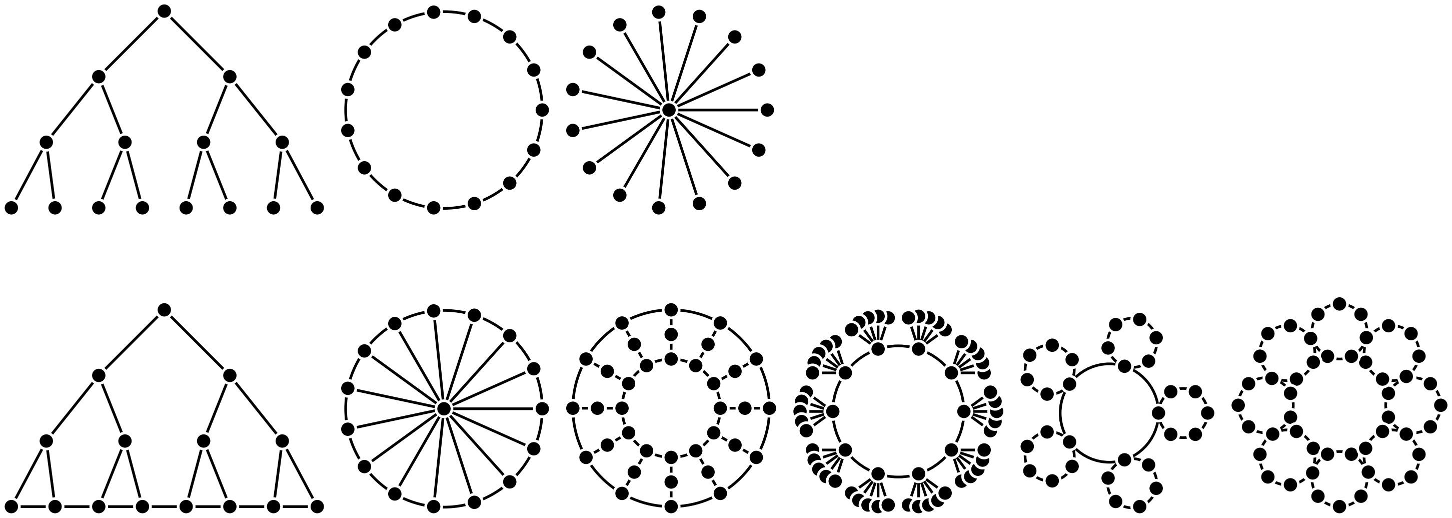


Structure represented by a grammar

$$
\begin{aligned}
\mathrm{X} \longrightarrow \mathsf{compose}\Big(& \\
&\mathsf{rename}_{\mathrm{left}\mapsto\mathrm{mid}}\Big(\mathsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(X)\Big), \\
&\mathsf{rename}_{\mathrm{right}\mapsto\mathrm{mid}}(\mathrm{client}) \\
\Big)&
\end{aligned}
$$

Representable architectures

Encoded as a CFG for graphs[1] $\implies$ families of bounded TW are representable (missing: ~~square grids~~, cliques[2])



[1]Graph Structure and Monadic Second Order Logic; by B. Courcelle, J. Engelfriet

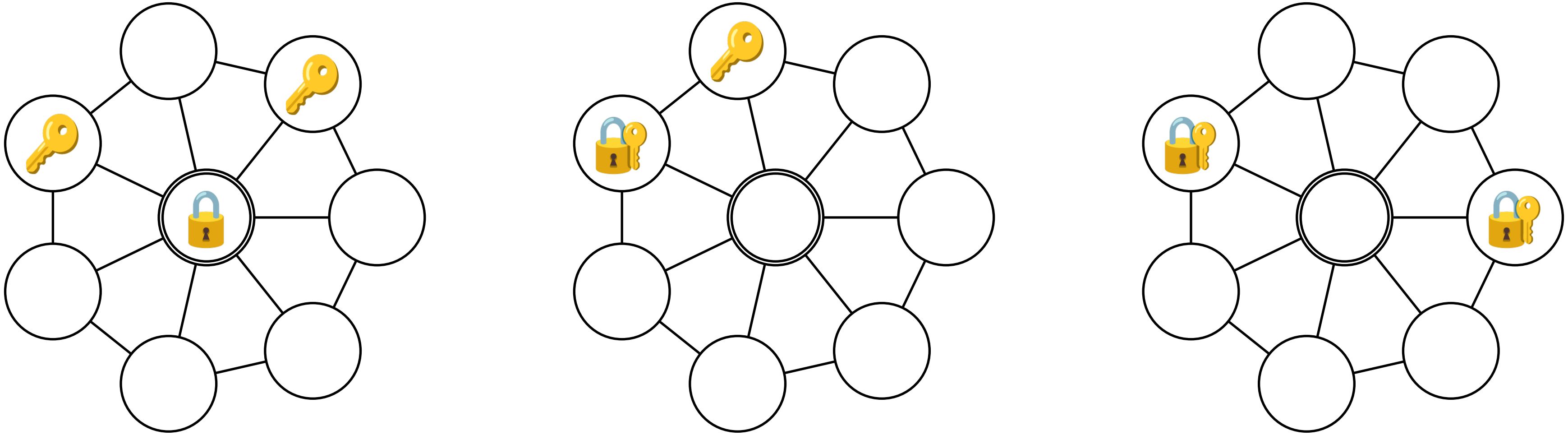
[2]Verifying Parameterized Networks Specified by Vertex Replacement Graph Grammars; by A. Sangnier, R. Iosif, N. Villani @ NETYS'25

Safety specification

Safety properties

#(🔑): number of tokens on 🔑

$\sim$ number of clients who claim to own the key



$$\#(🔑) + \#(🔐) > 1$$

Proving safety $\approx$ reachability problem in an infinite family of PNs

Expressible Properties

- **mutual exclusion**

  “at most k processes can enter a critical section simultaneously”
- **uniqueness**

  “the entire system contains at most k instances of a resource”
- **uncoverability**

  “no process can reach a bad state”

Examples: leader election, semaphores, dining philosophers, …

Missing: ~~liveness~~, ~~deadlock freedom~~

An Abstraction Technique

Verification pipeline

$$
\begin{aligned}
\mathrm{Sys} &\longrightarrow \mathsf{compose}\big(\mathrm{X}, \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{right},\ \mathrm{right}\mapsto\mathrm{left}}(\mathrm{client}')\big)\\
\mathrm{X} &\longrightarrow \mathsf{compose}\big(\\
&\qquad \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{mid}}\big(\mathsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(X)\big),\\
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&\ \big)\\
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\end{aligned}
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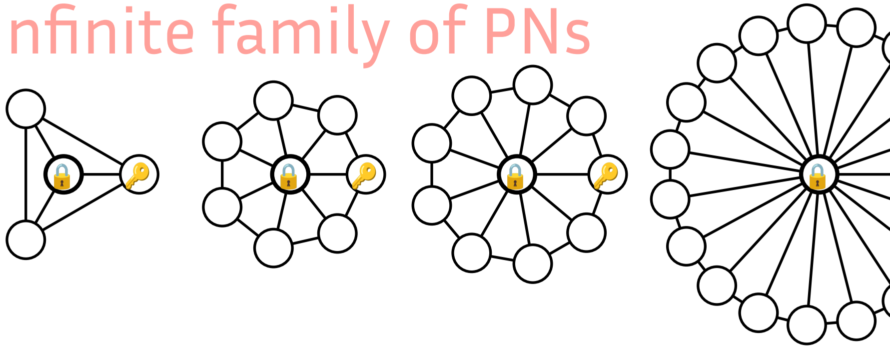
\end{aligned}
$$

$\xrightarrow{\text{language}}$

infinite family of PNs



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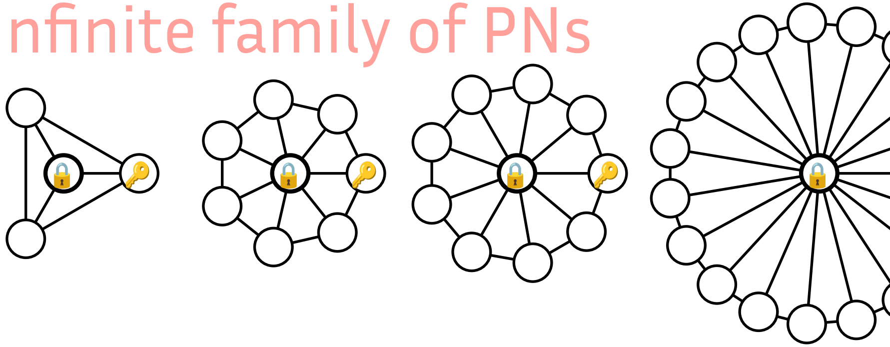

safety

$\#(🔑) + \#(🔐) > 1$

Verification pipeline

$$\mathrm{Sys} \longrightarrow \mathsf{compose}\big(\mathrm{X}, \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{right},\ \mathrm{right}\mapsto\mathrm{left}}(\mathrm{client}')\big)$$

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language

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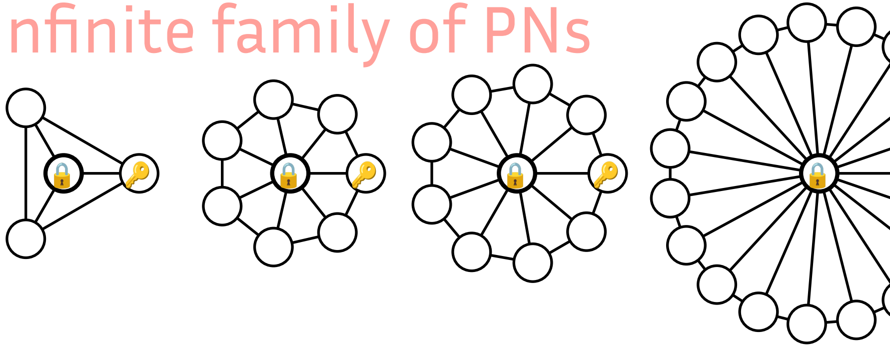

~~safety~~

undecidable!

$$\boxed{\#(🔑) + \#(🔐) > 1}$$

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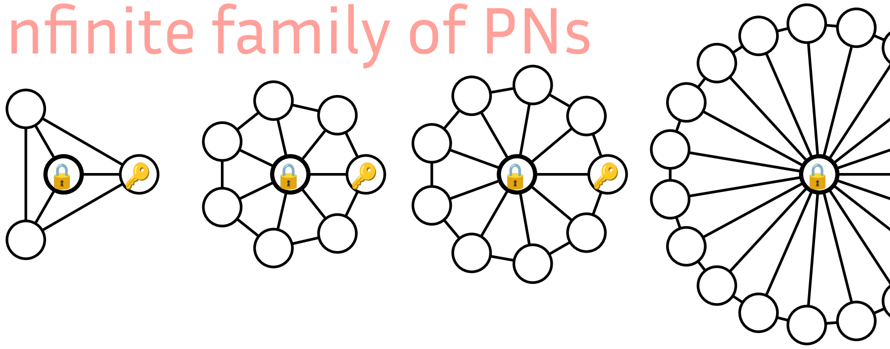
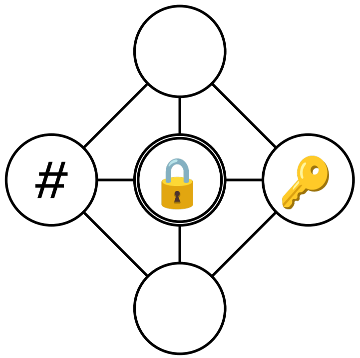


abstraction

$\supseteq$

~~safety~~

undecidable!



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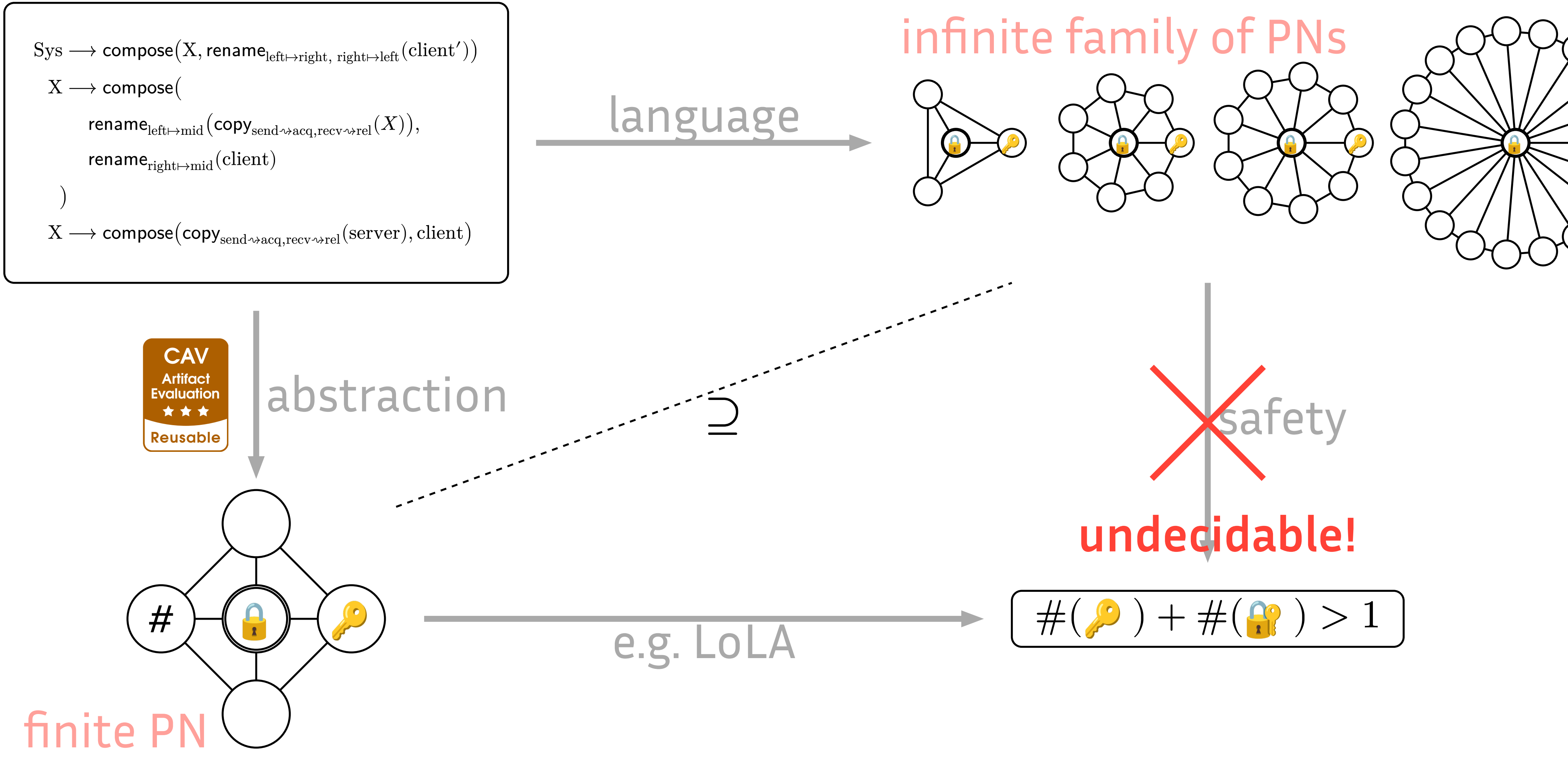
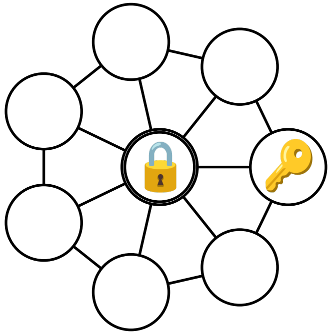
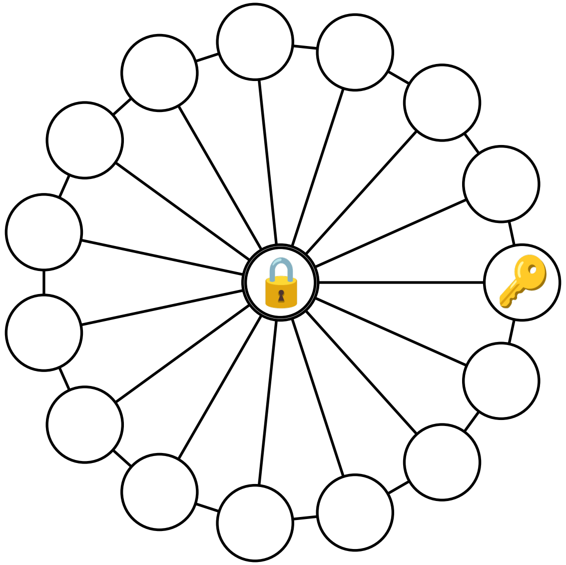


Folding abstraction





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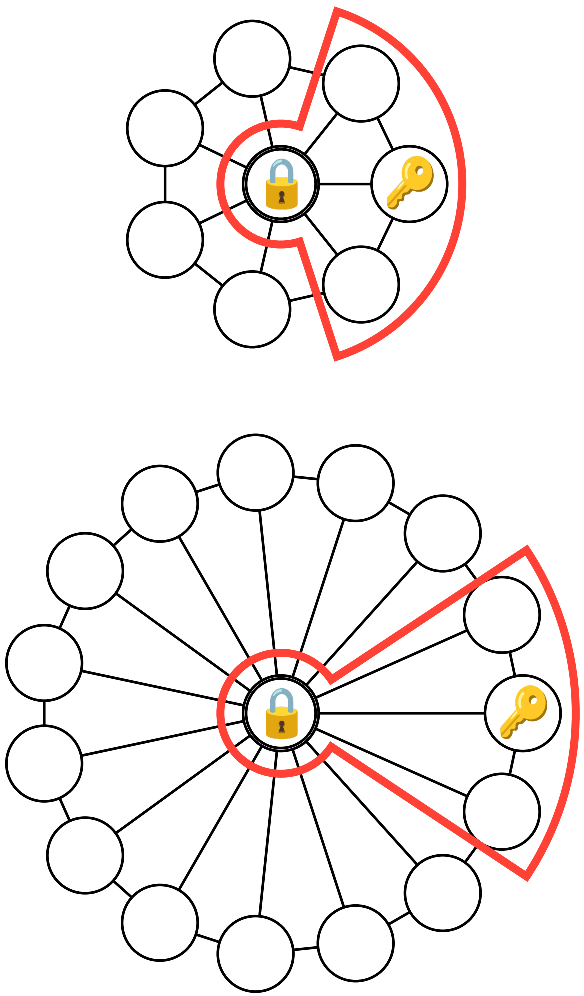

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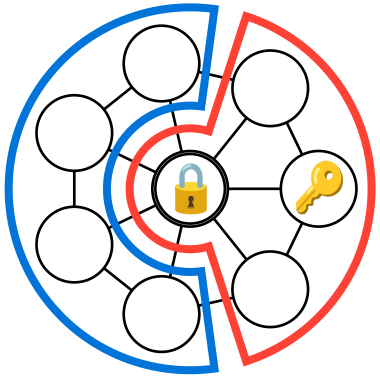

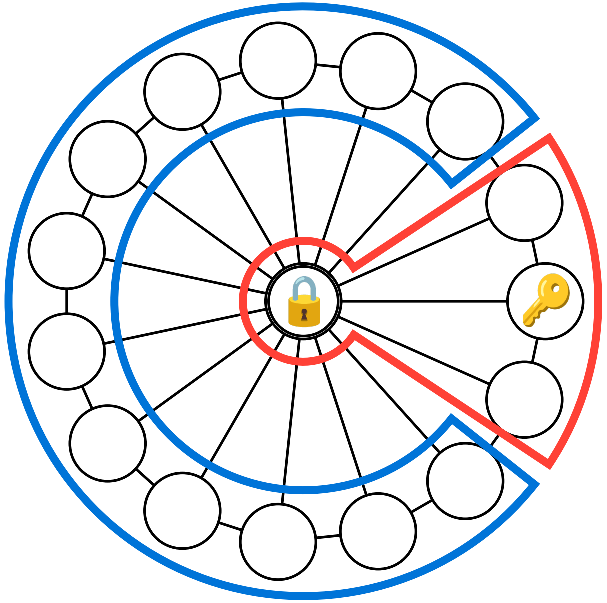

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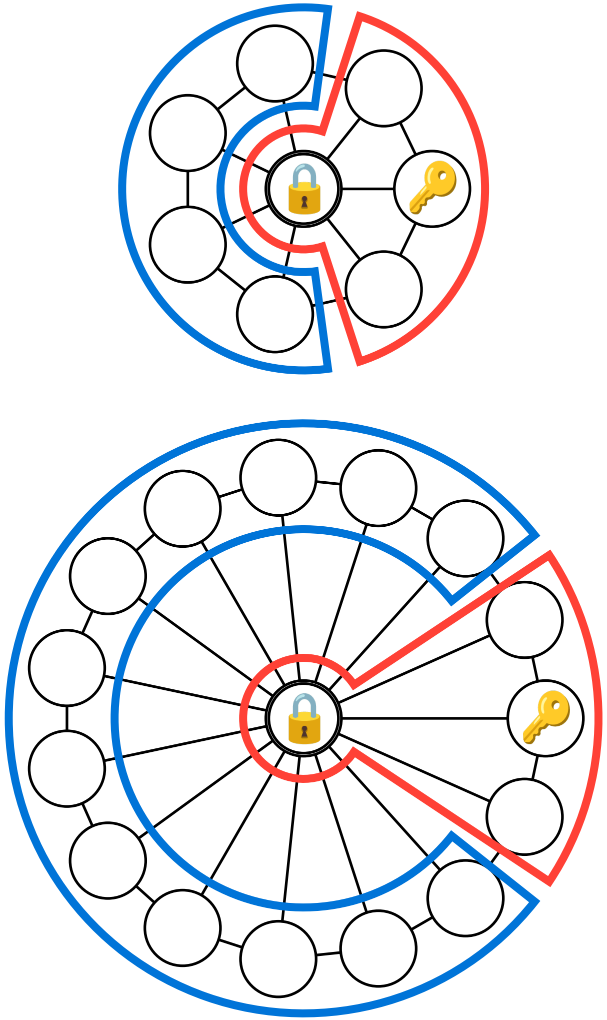
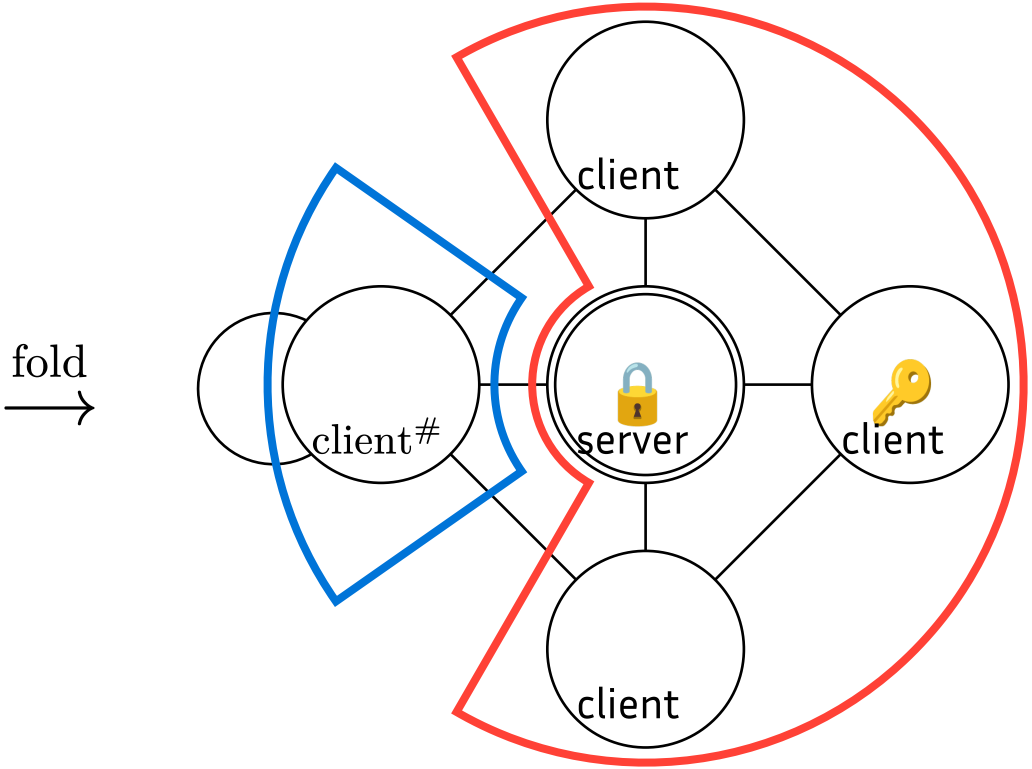
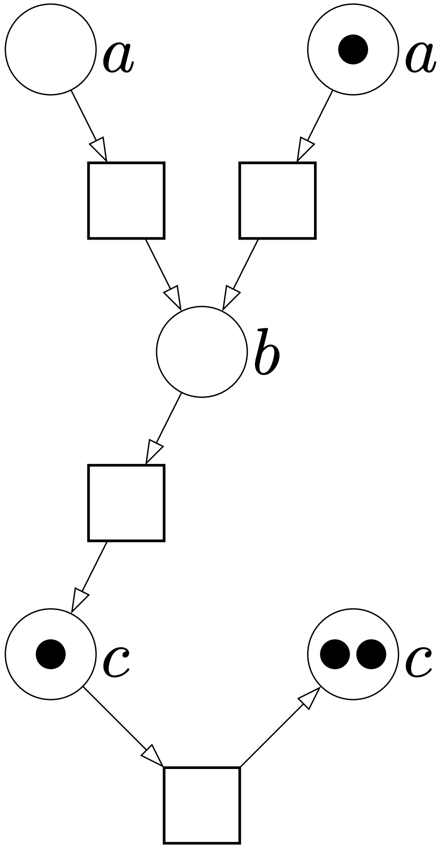




Implementing fold



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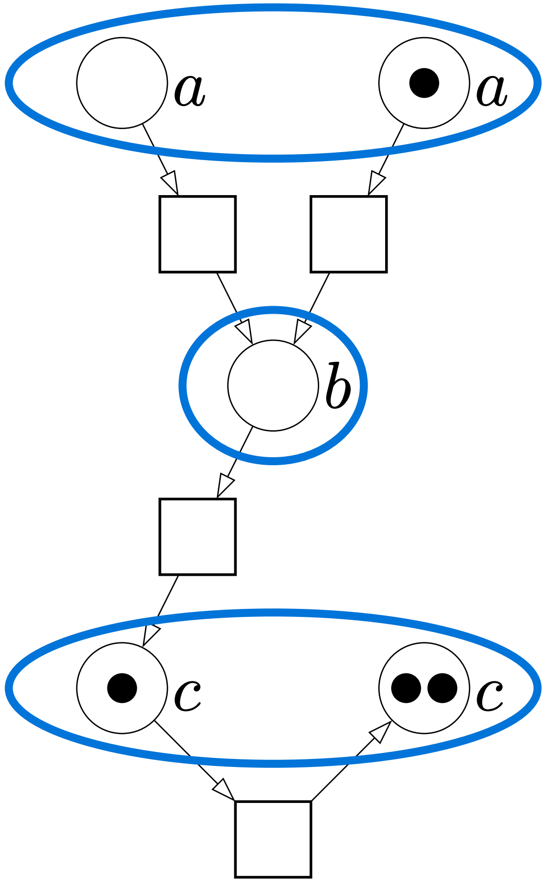

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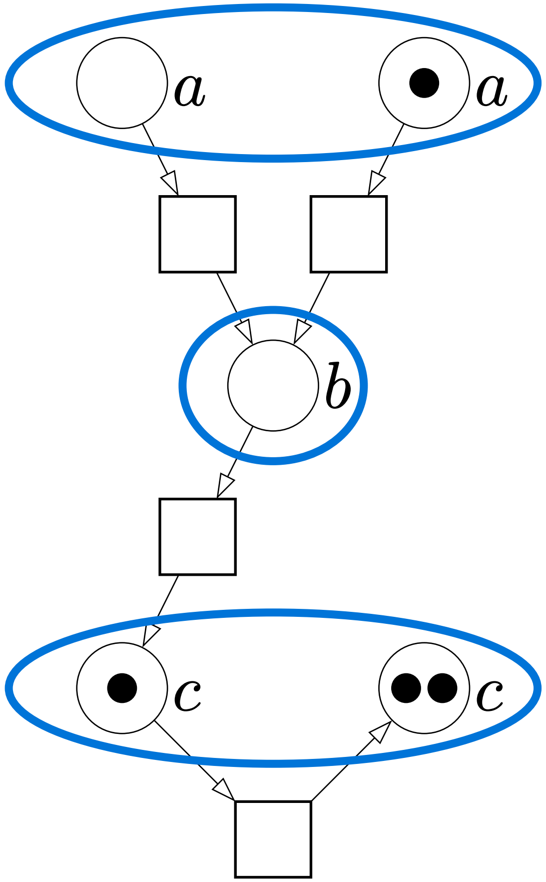
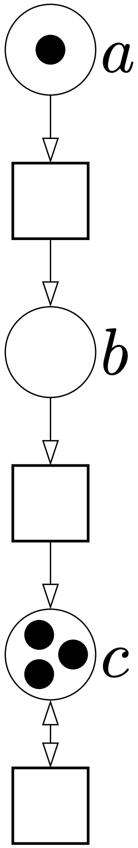


$\xrightarrow{\text{fold}}$

$\dashleftarrow$
$\subseteq$



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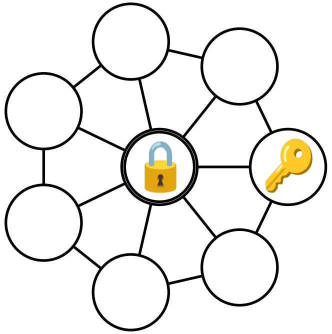

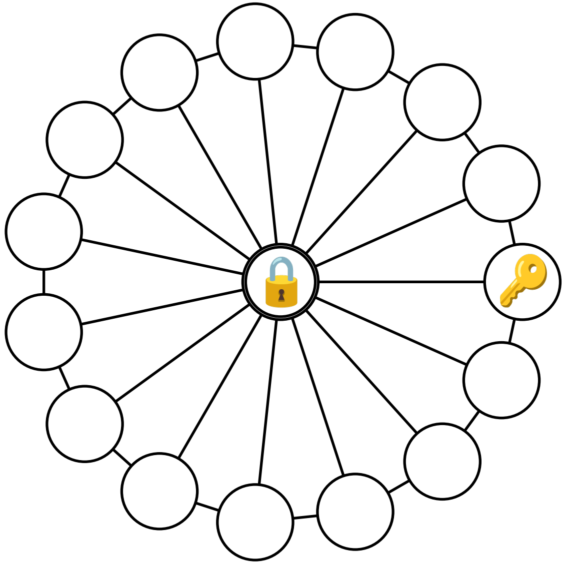

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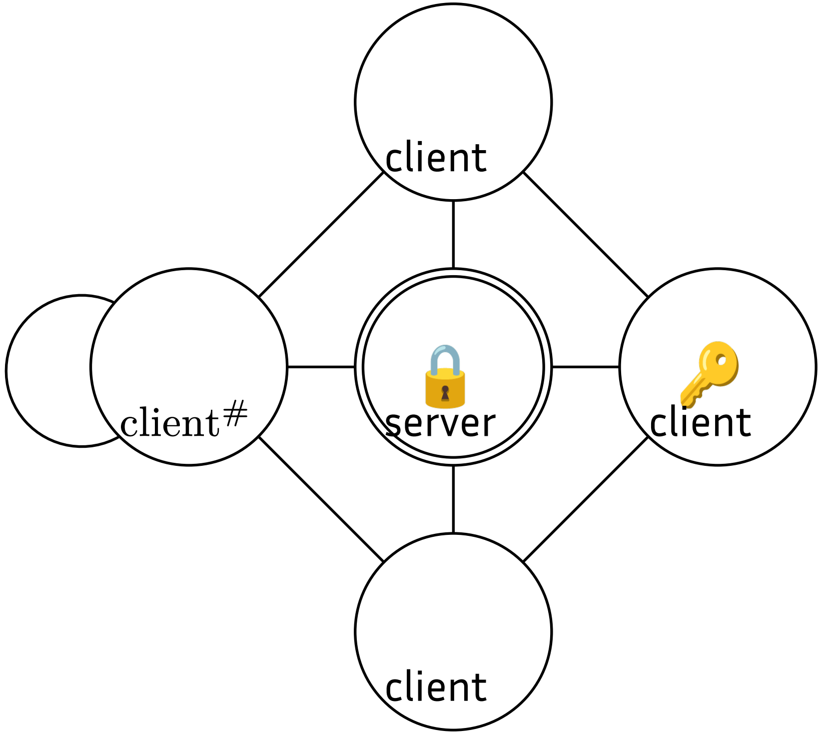
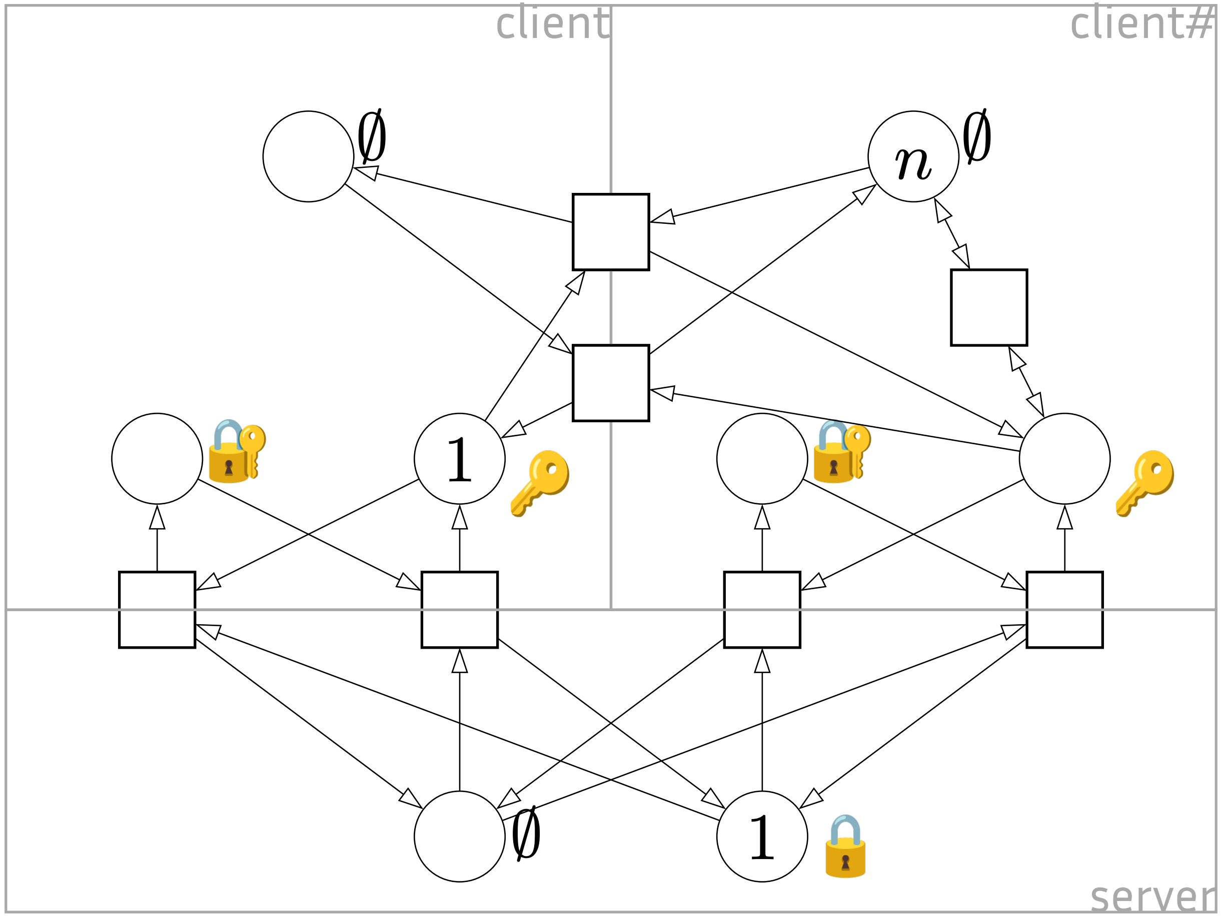


Folded system



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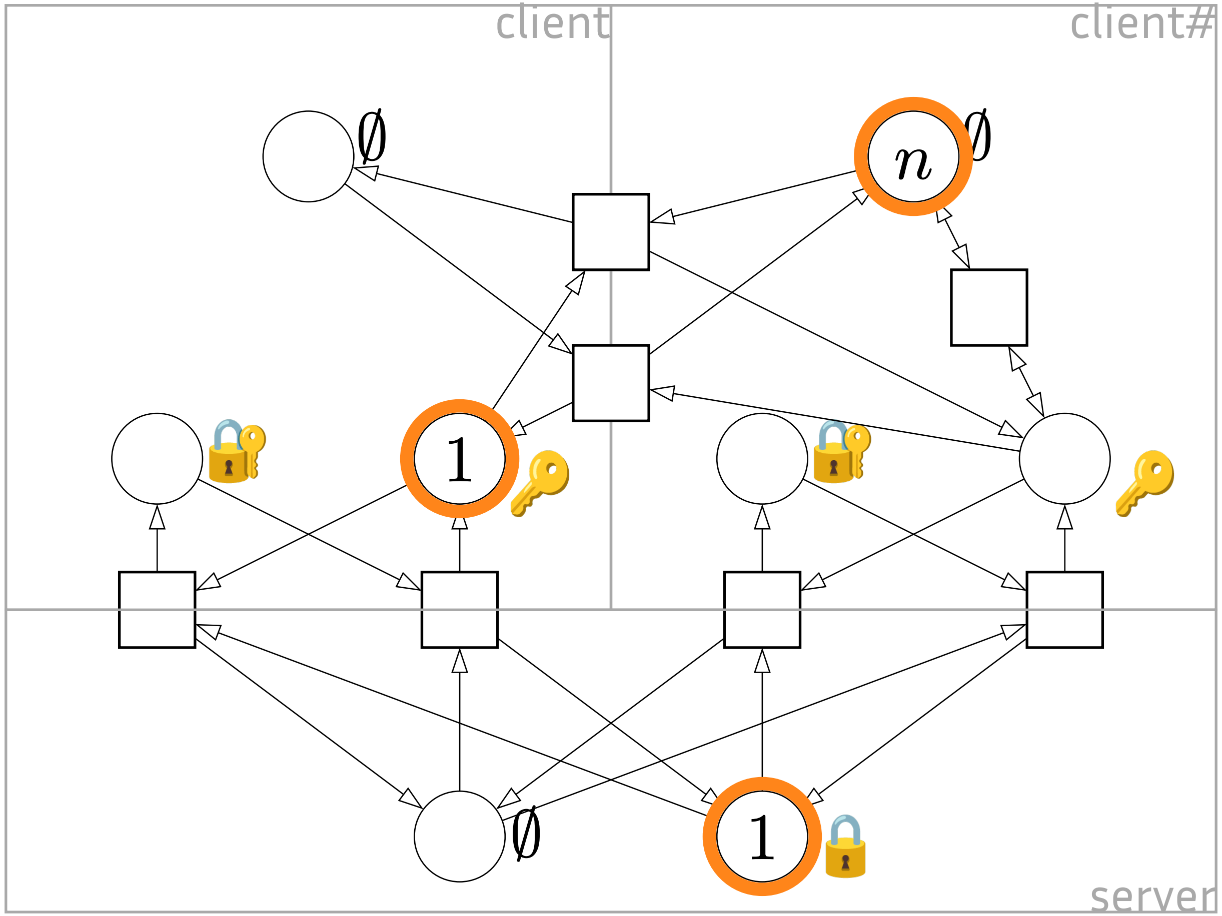

Initial marking

$$\mathrm{Sys} \longrightarrow \mathsf{compose}\big(\mathrm{X}, \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{right},\ \mathrm{right}\mapsto\mathrm{left}}(\mathrm{client}')\big)$$

$$\mathrm{X} \longrightarrow \mathsf{compose}\big($$

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From the grammar

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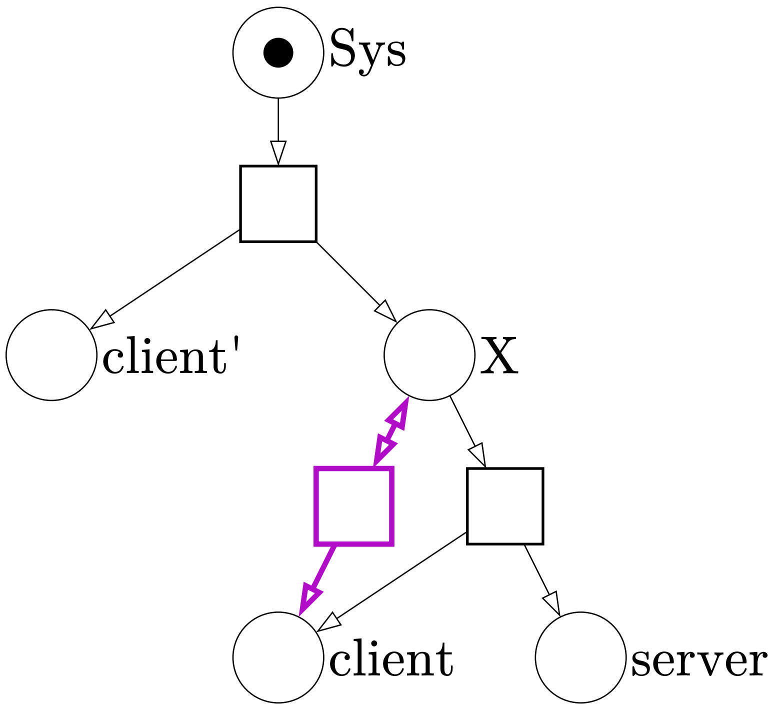
$$X \longrightarrow \mathrm{server}, \mathrm{client}$$



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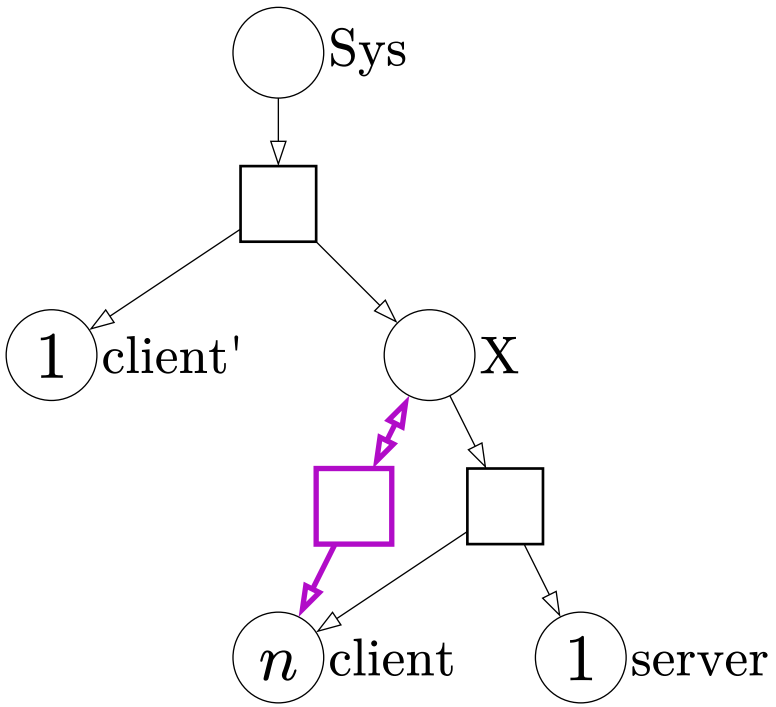
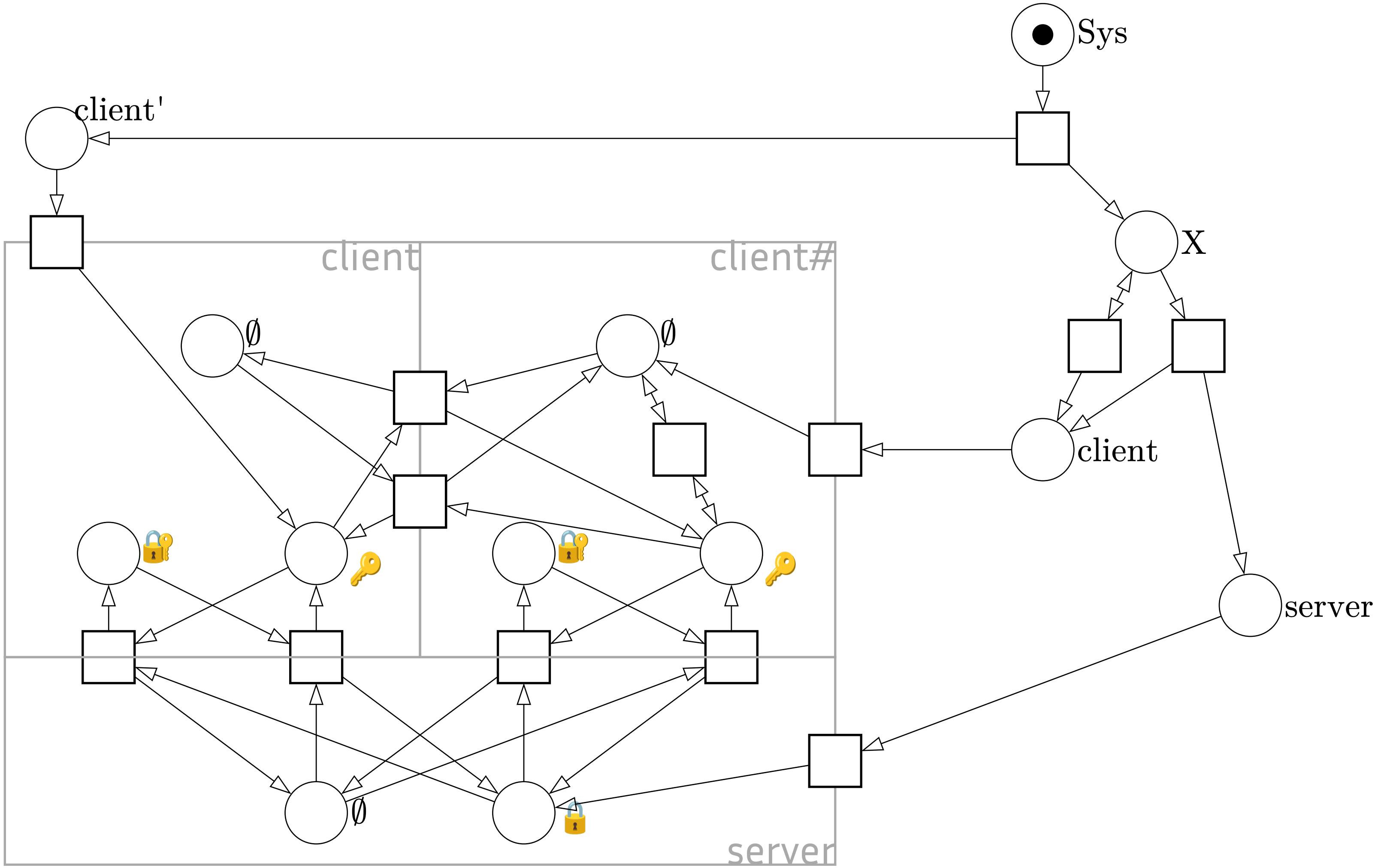


Finite abstract system



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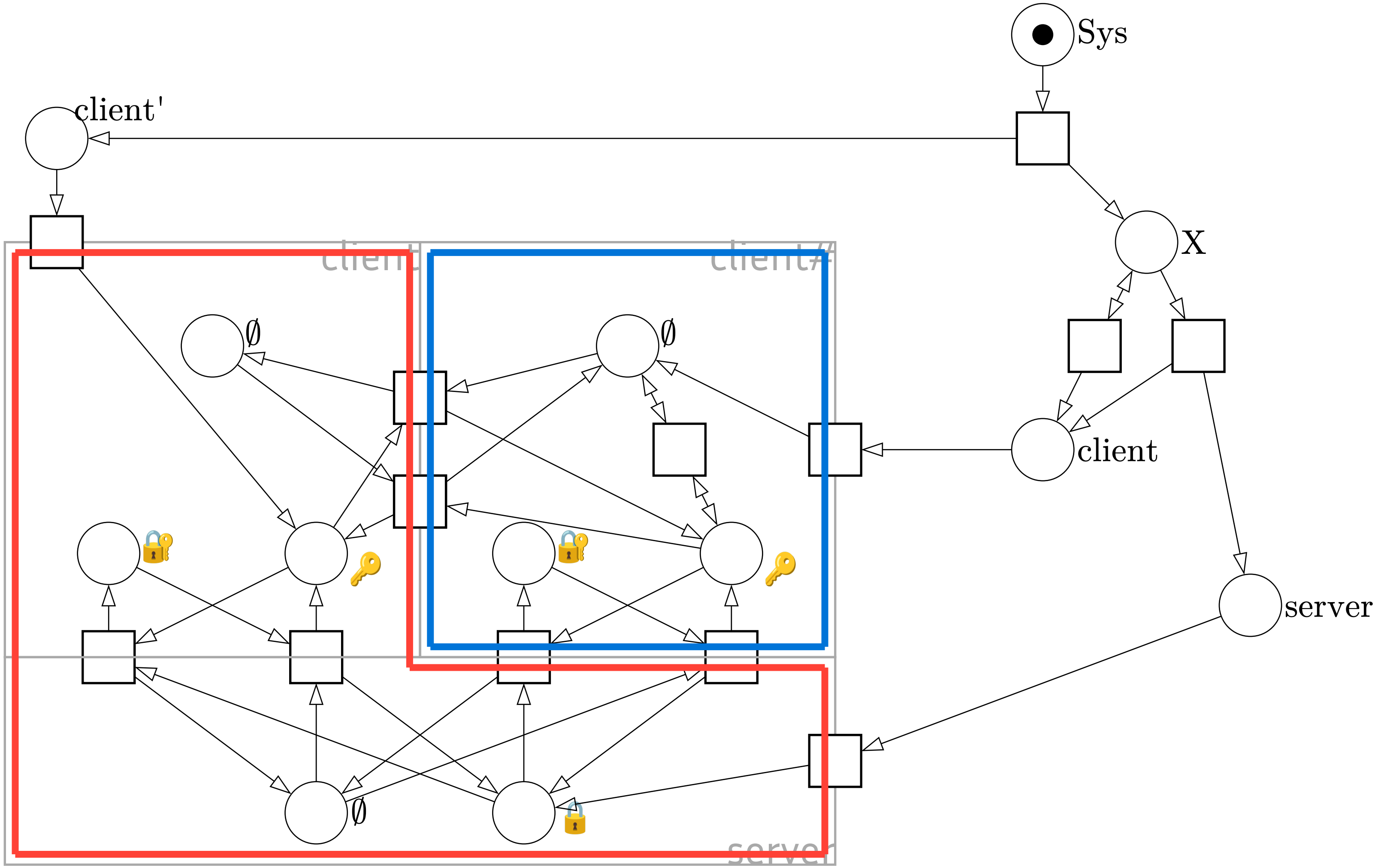
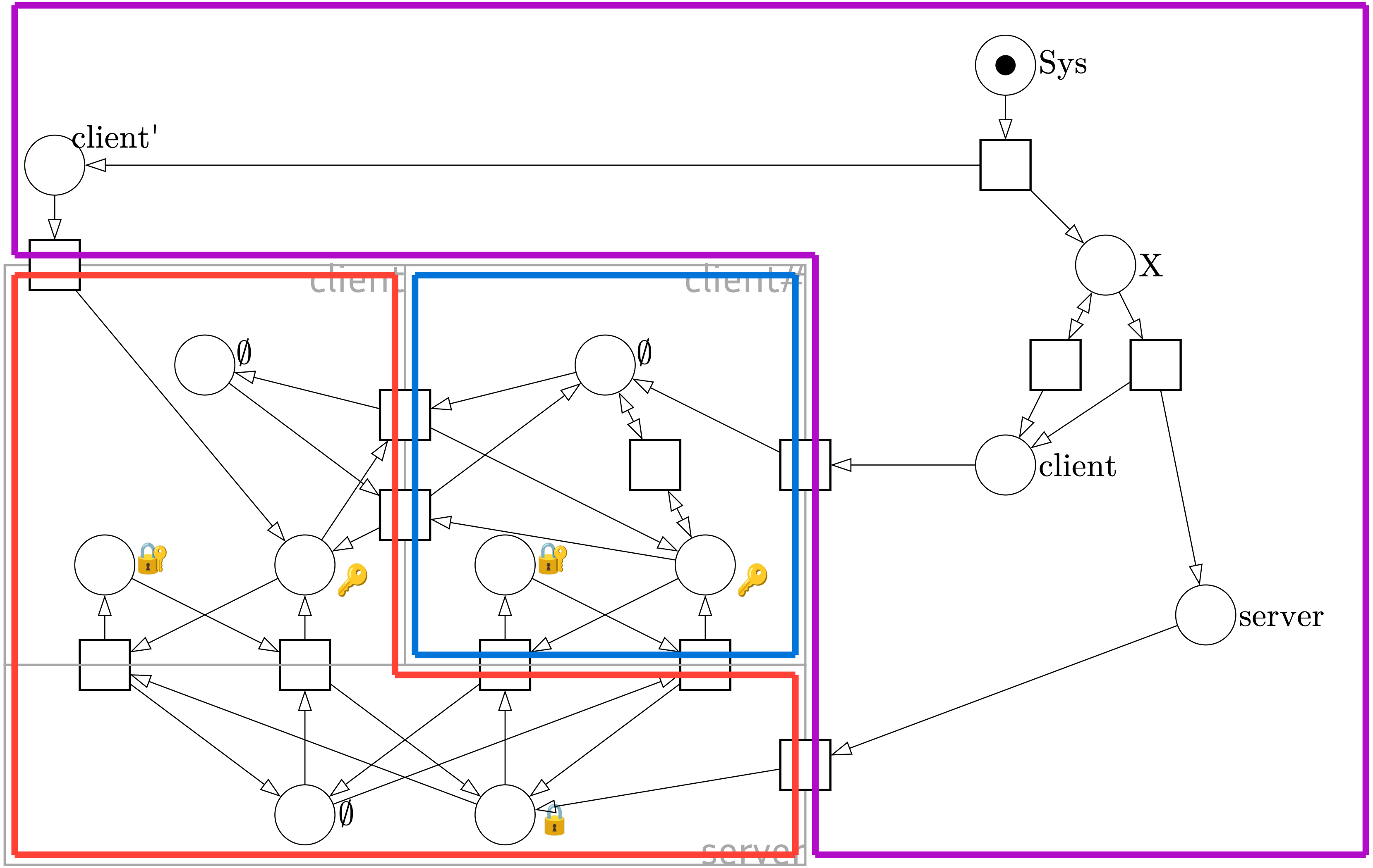


Finite abstract system



Conclusion

Refinements

Partial unfolding, boolean contracts

A decidable restriction

Pebble-Passing Systems

- multiple process types, multiple tokens
- processes only record the presence of a token

(similar to but incomparable with Token-Passing Systems)

Coverability of a control state is **2EXPTIME** and **PSPACE-hard**