On the Verification of Structured Parameterized Networks

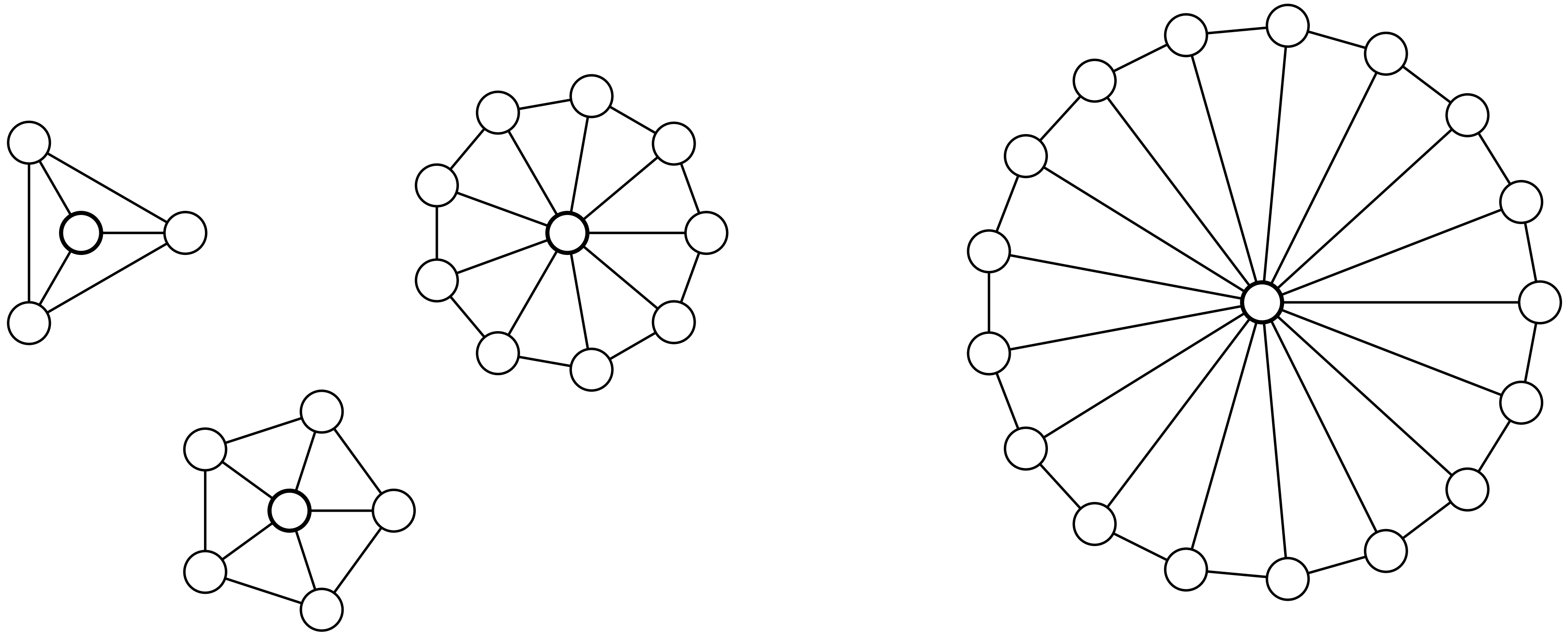
Neven Villani[1] , Marius Bozga[1], Radu Iosif[1], Arnaud Sangnier[2]

AVM'25 @ Timisoara; using material from NETYS'25, CAV'25, and work in progress

2025-09-24

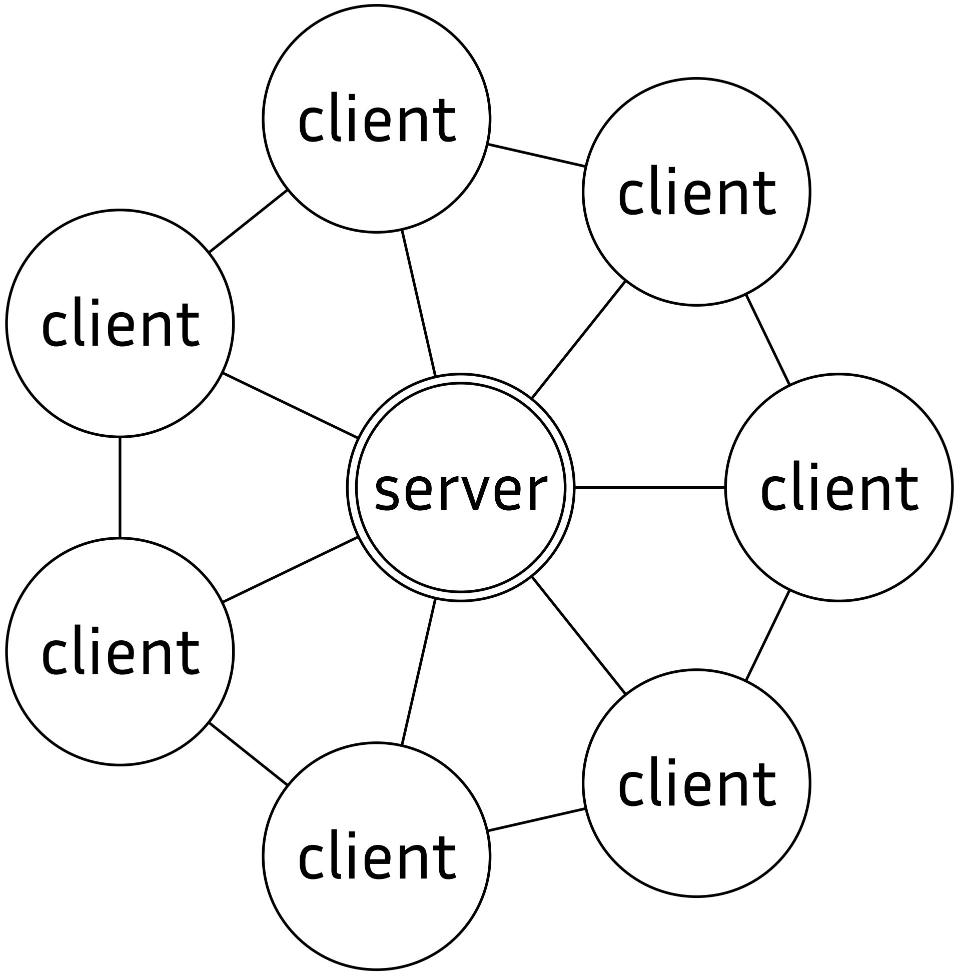
[1]VERIMAG, Univ. Grenoble Alpes, CNRS
[2]DIBRIS, Univ. di Genoa

- (automated) verification of networks
- challenge: size and architecture (communication topology)



- undecidable $\rightsquigarrow$ abstraction

Token ring with resource



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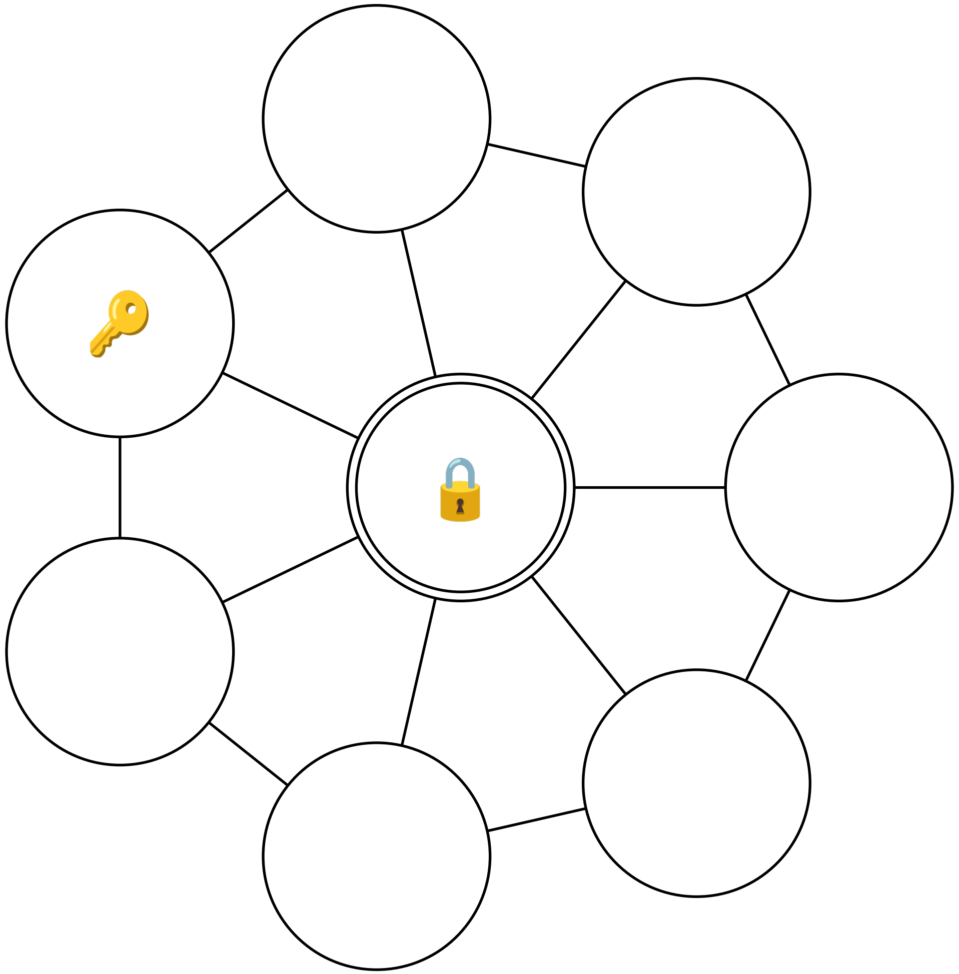

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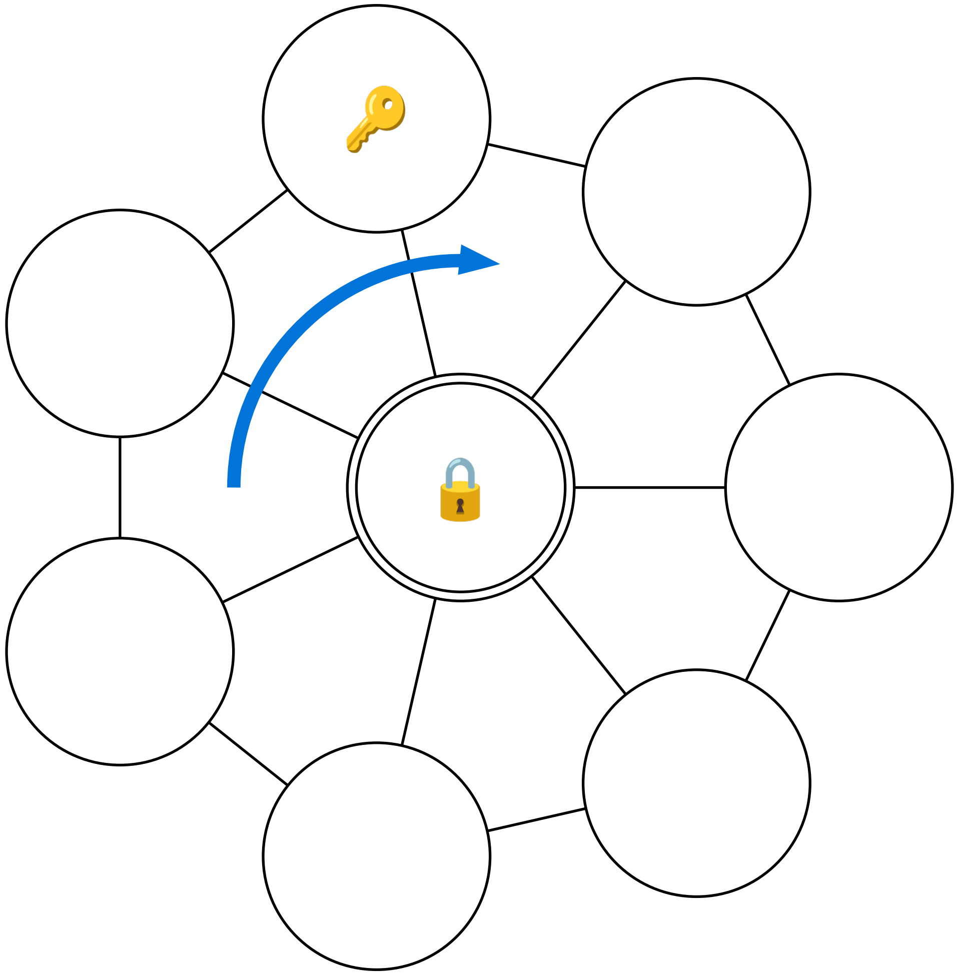

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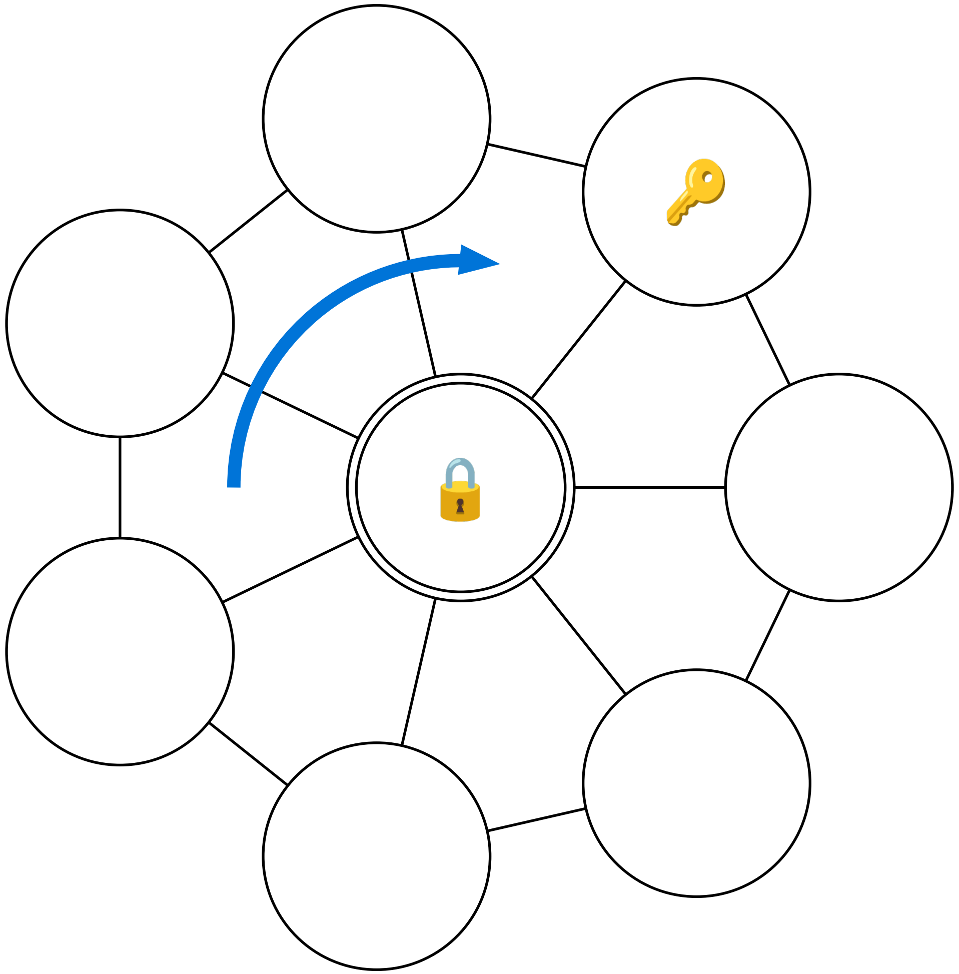

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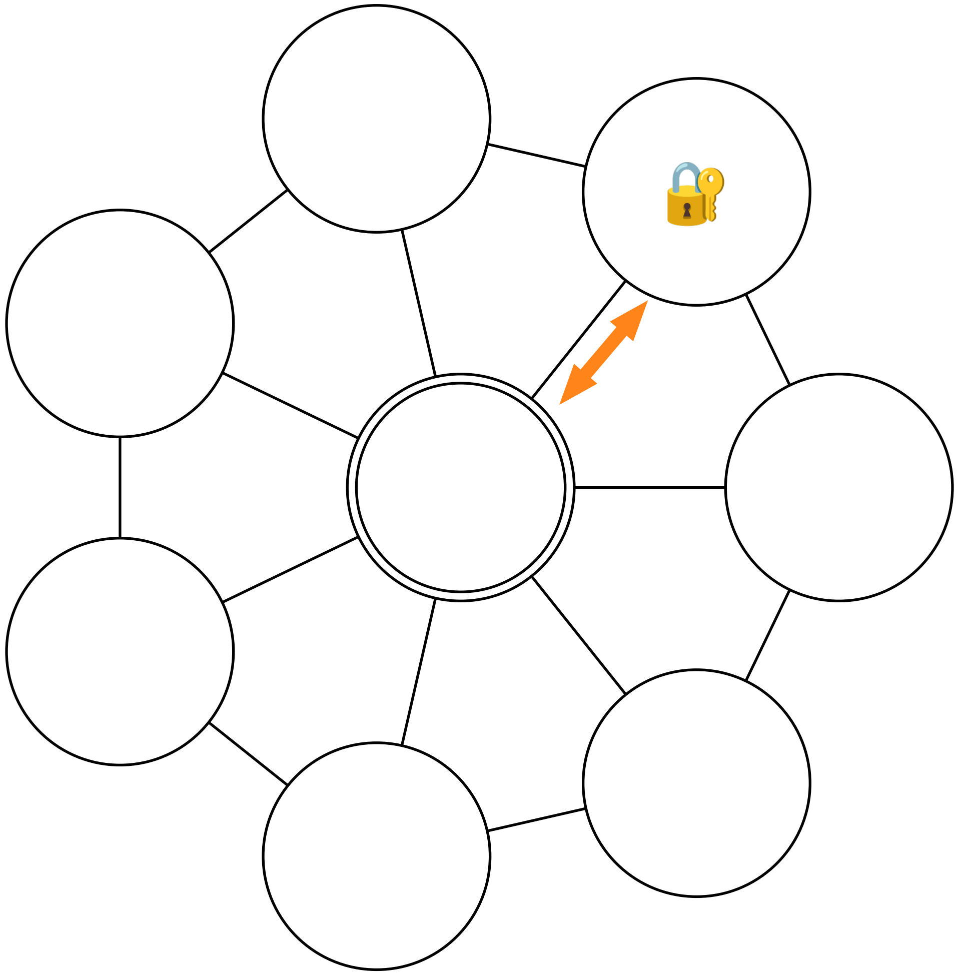

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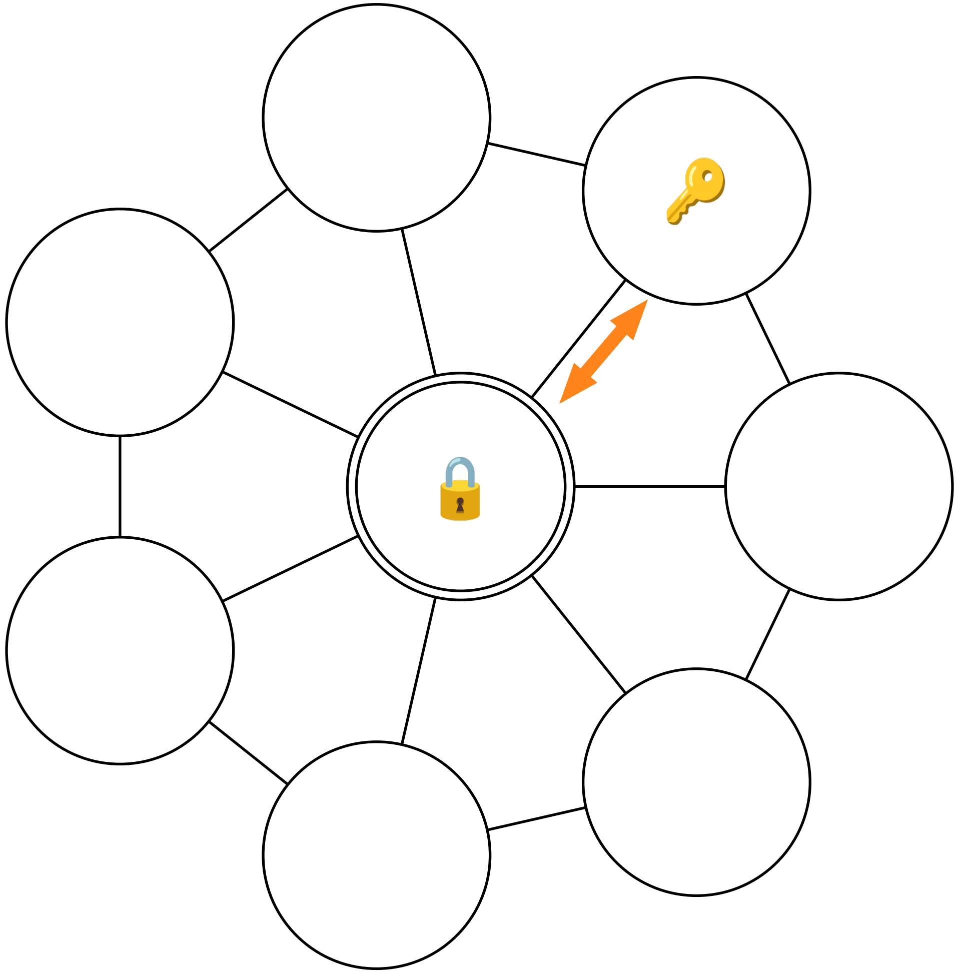

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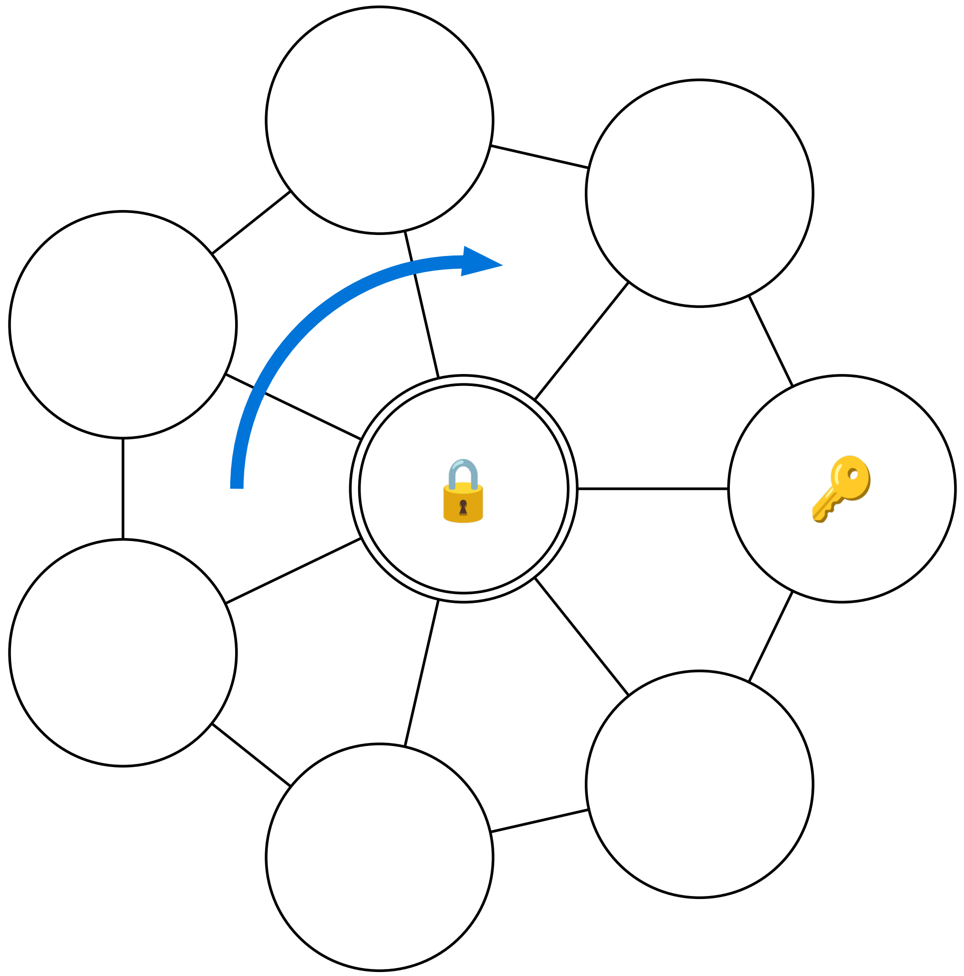

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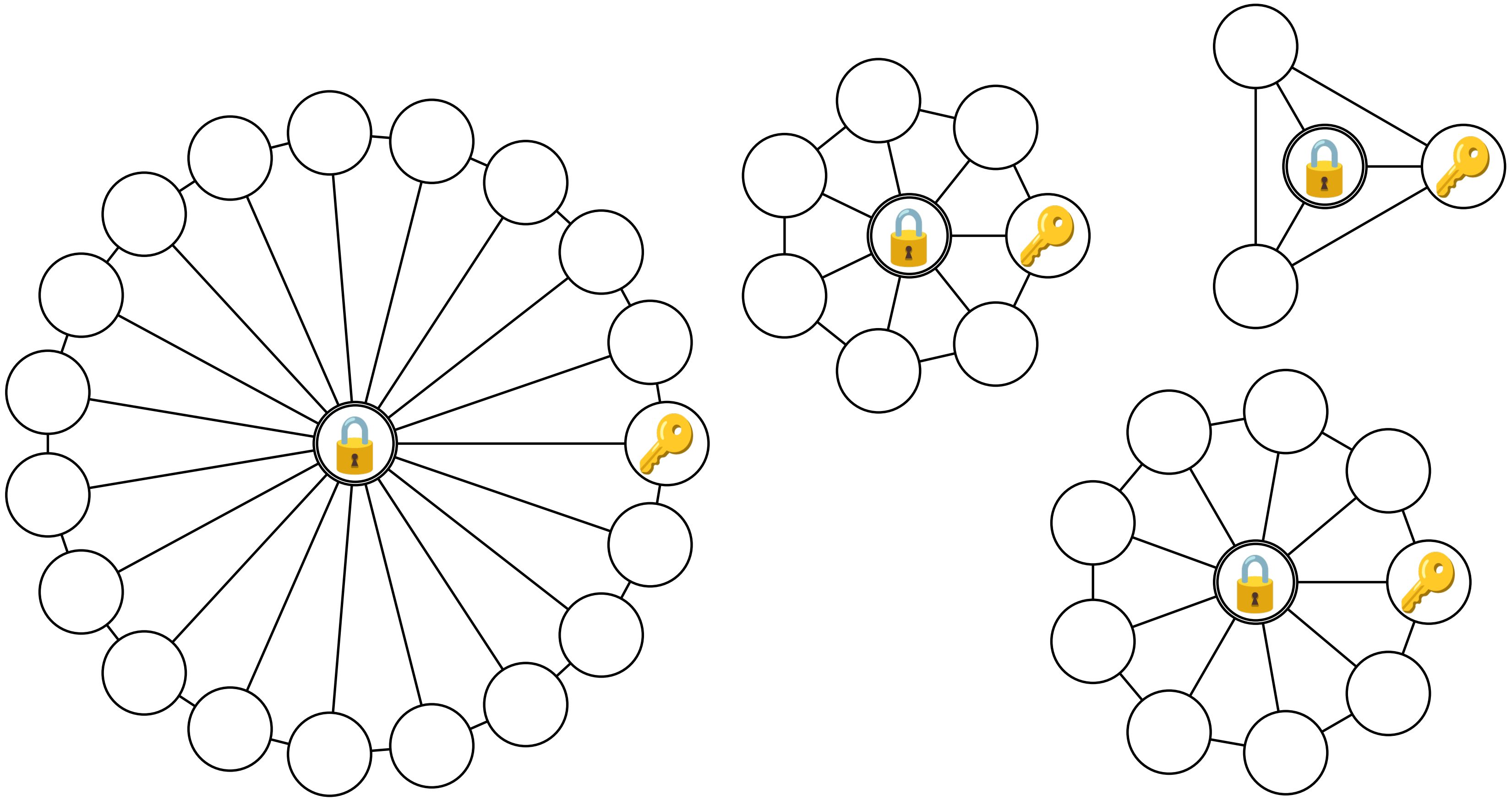

$$\forall n \geq 2$$

How would we automatically verify this ?

Techniques for non-finite-state systems...

Parameterized model checking of rendezvous systems

(B. Aminof, T. Kotek, S. Rubin, F. Spegni, H. Veith)

❌ not homogeneous (2 kinds of processes)

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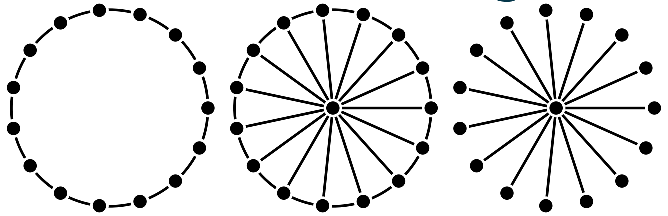
Parameterized Verification of Algorithms for Oblivious Robots on a Ring

(A. Sangnier, N. Sznajder, M. Potop-Butucaru, S. Tixeuil)

❌ not a standard architecture (clique, ring, star)



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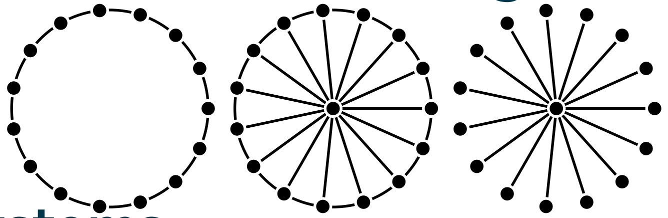


Parameterized Model Checking of Token-Passing Systems

(B. Aminof, S. Jacobs, A. Khalimov, S. Rubin)

❌ not a token-passing system (key and lock don't behave like tokens)

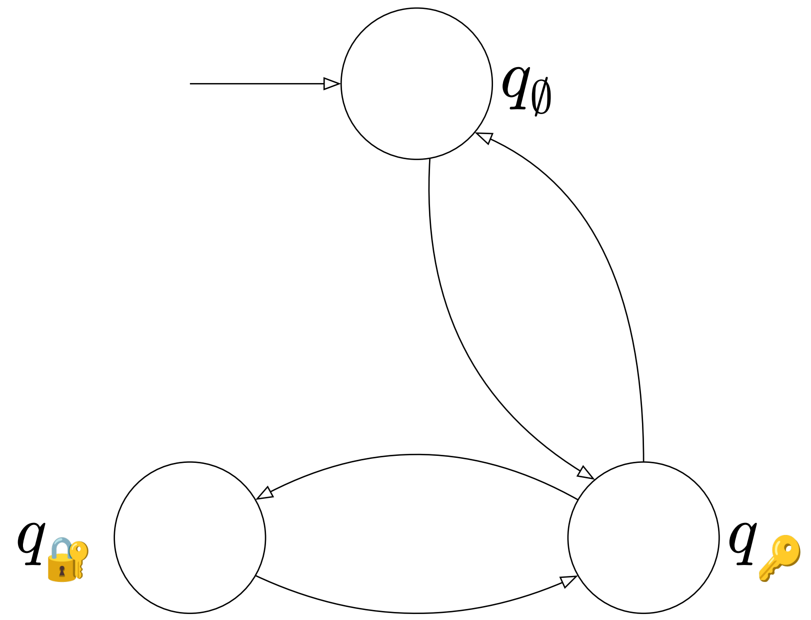
Framework requirements

We must be able to express

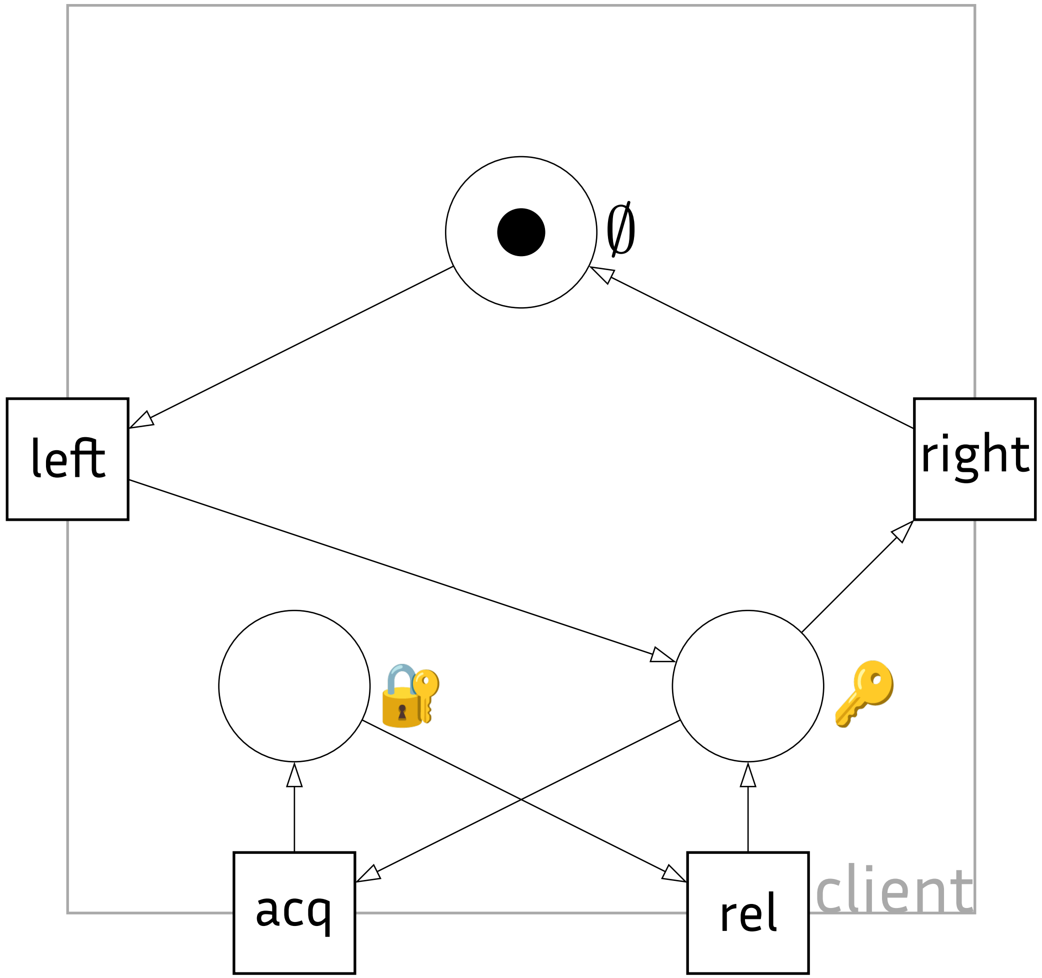
- an encoding of the **local behavior** of processes
- a description of the **interactions** and **architectures** of arbitrary size
- a **specification language** for safety properties

Framework

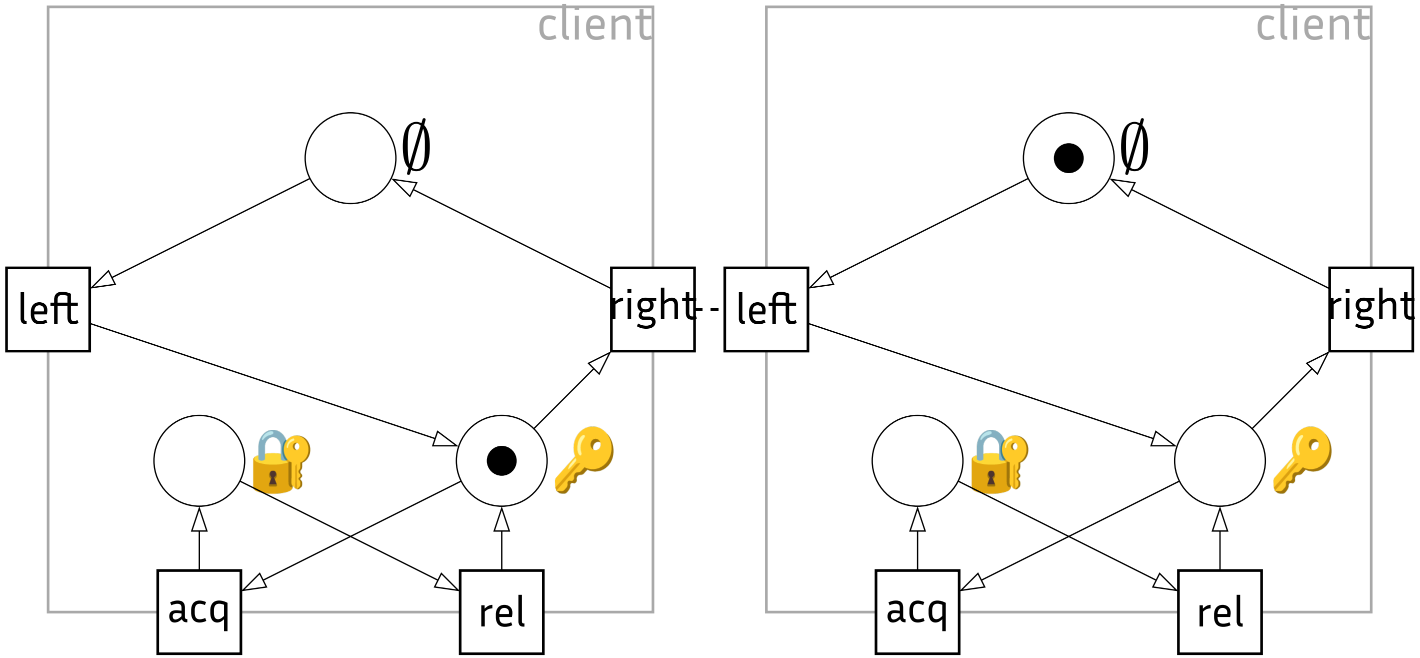
Client



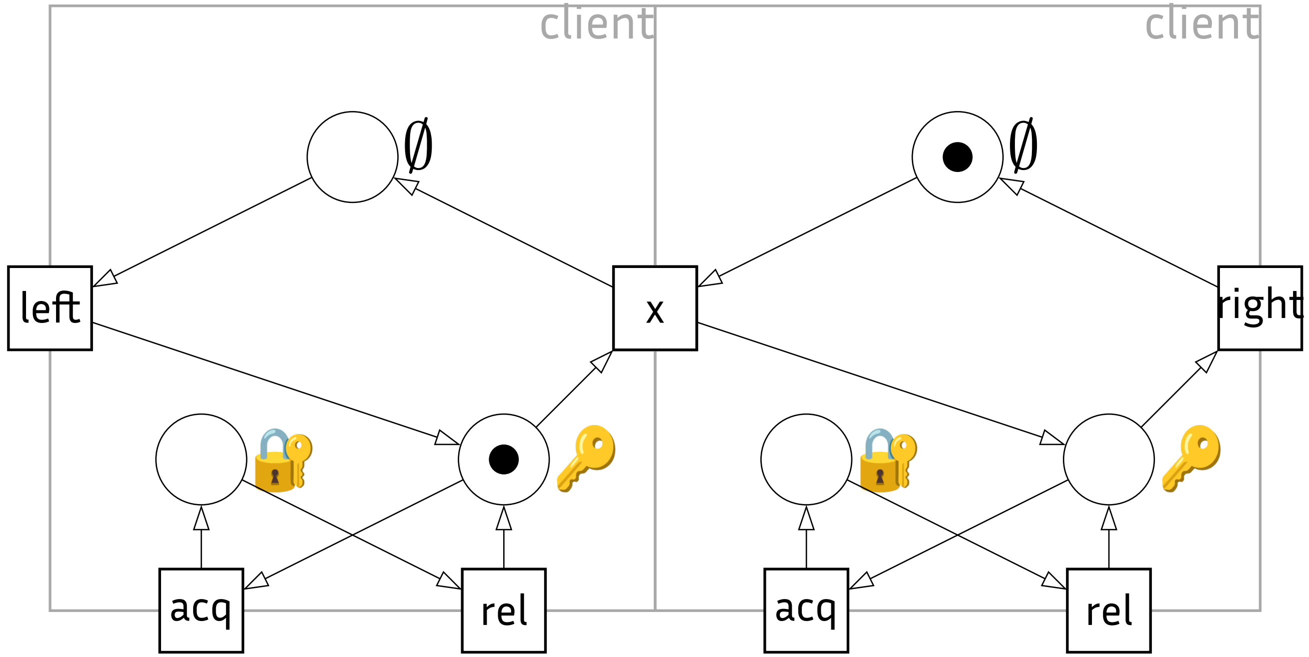
Client



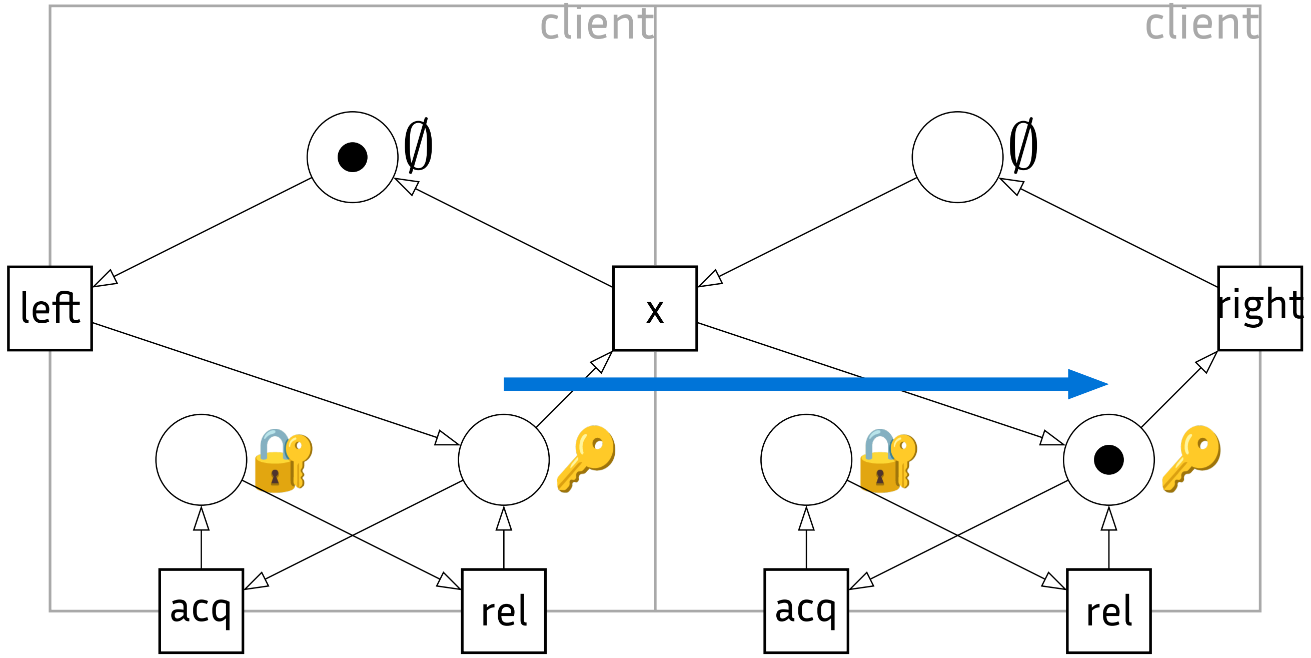
Interactions through composition



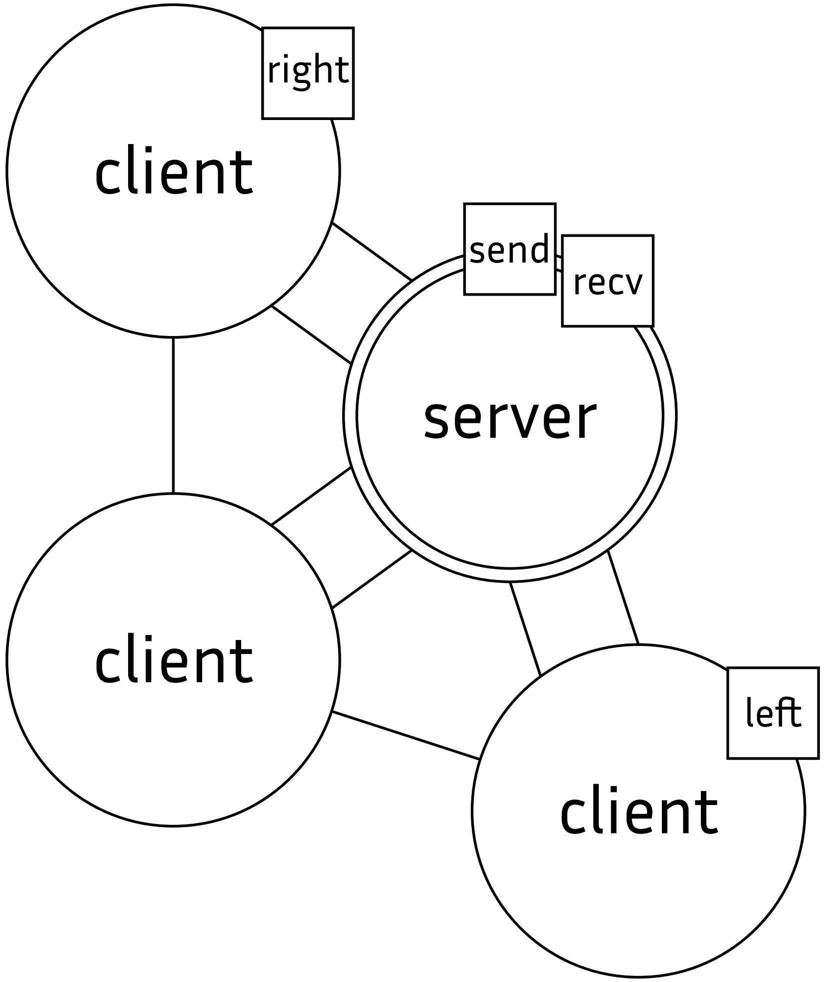
Interactions through composition



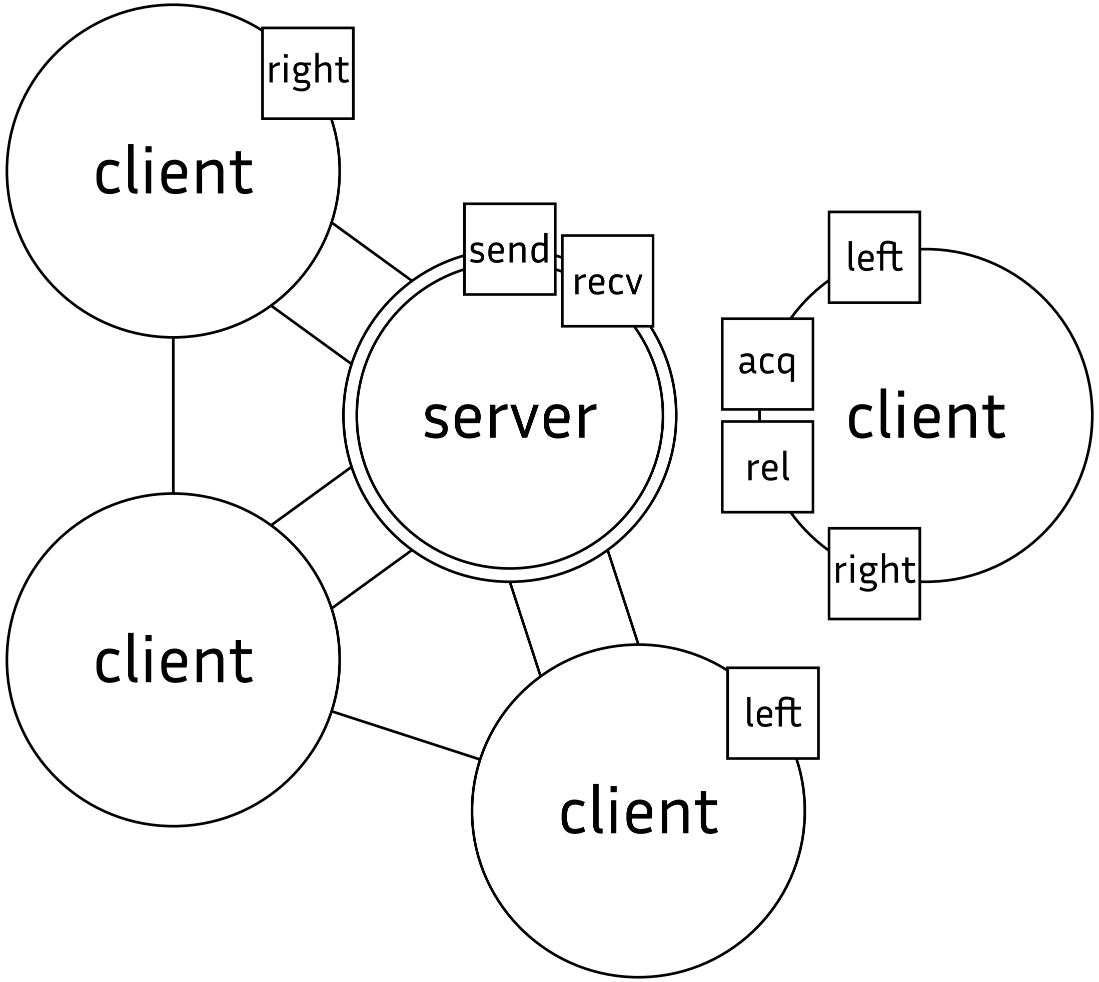
Interactions through composition



Structure: inductive step



Structure: inductive step



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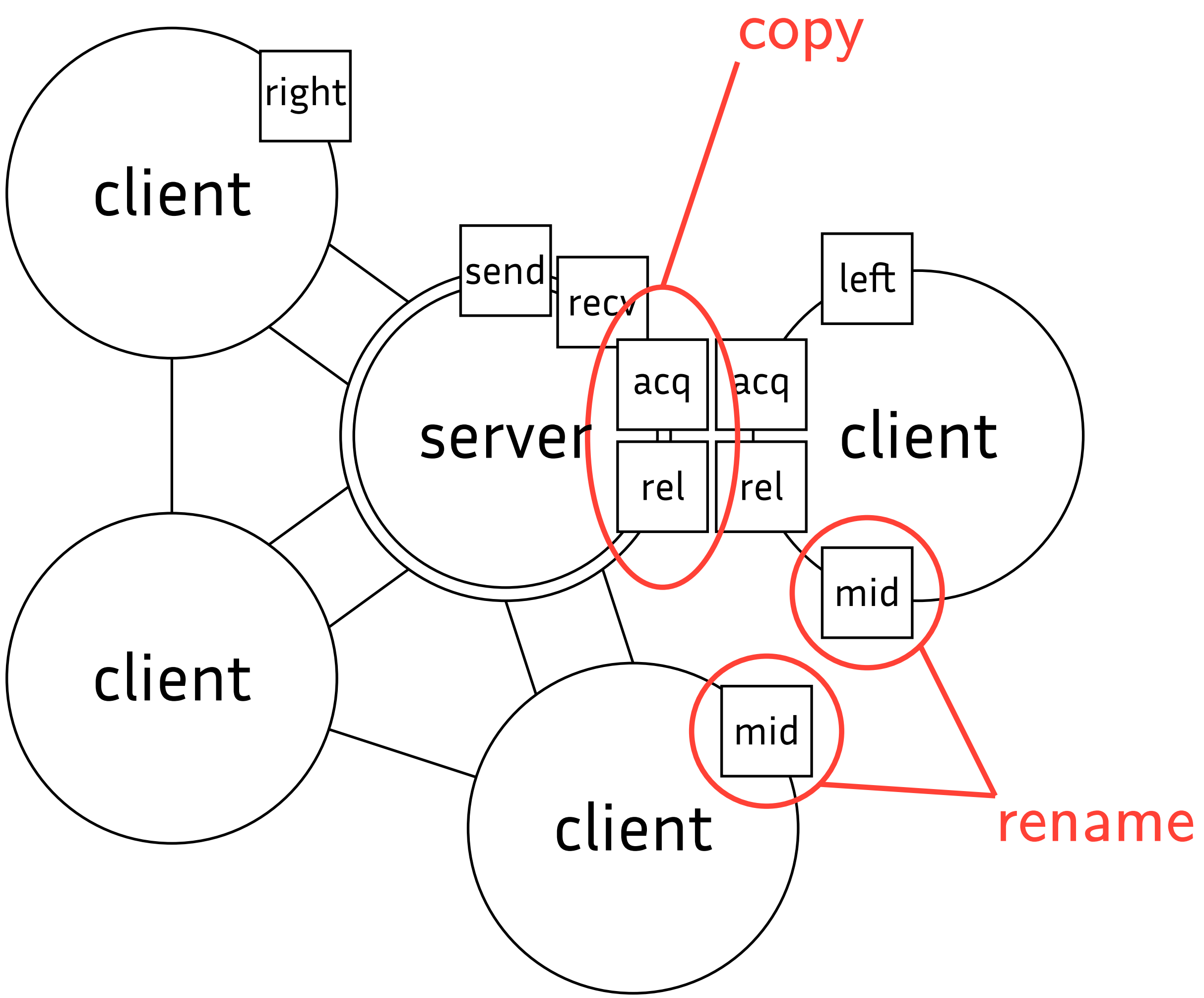

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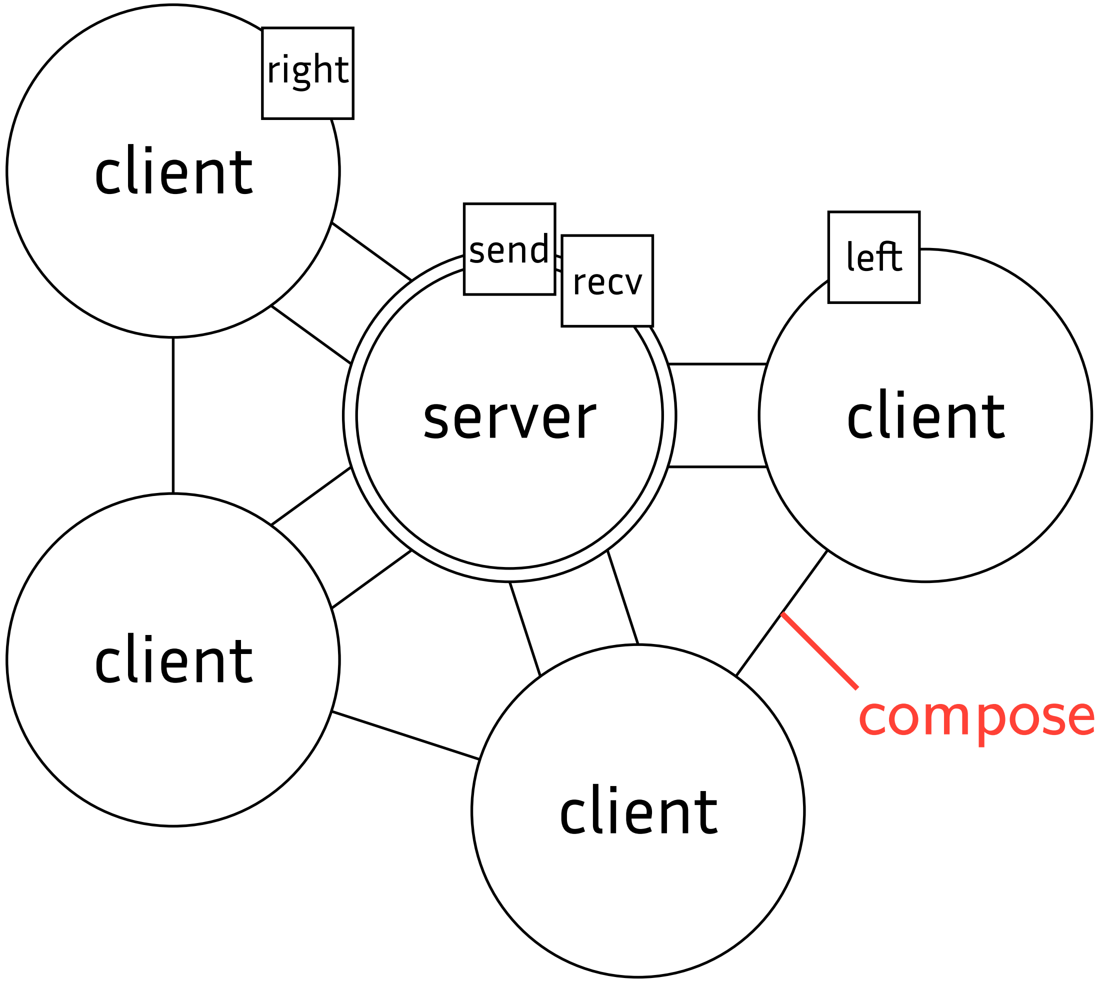
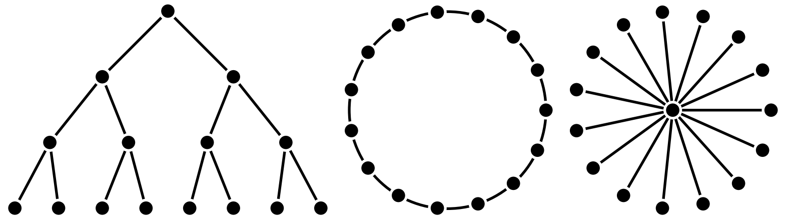


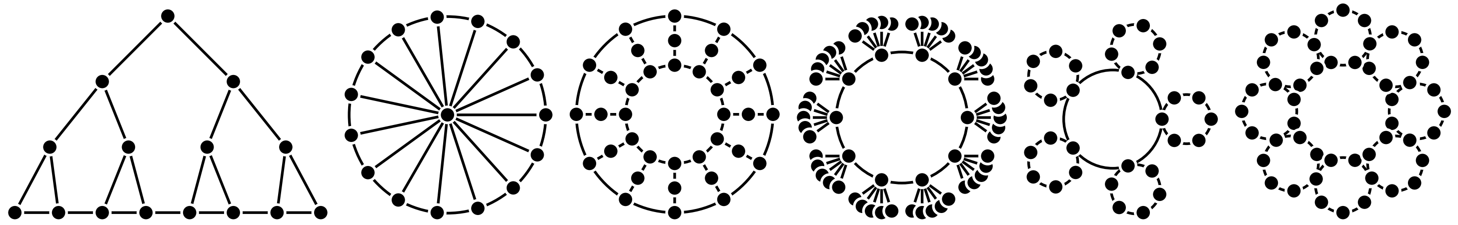
Structure represented by a grammar

$$
\begin{aligned}
\mathrm{X} \longrightarrow \mathsf{compose}\Big(& \\
&\mathsf{rename}_{\mathrm{left}\mapsto\mathrm{mid}}\big(\mathsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(X)\big), \\
&\mathsf{rename}_{\mathrm{right}\mapsto\mathrm{mid}}(\mathrm{client}) \\
\Big)&
\end{aligned}
$$

Representable architectures

Encoded as a CFG for graphs[1] $\Longrightarrow$ families of bounded TW are representable (missing: ~~square grids~~, cliques)





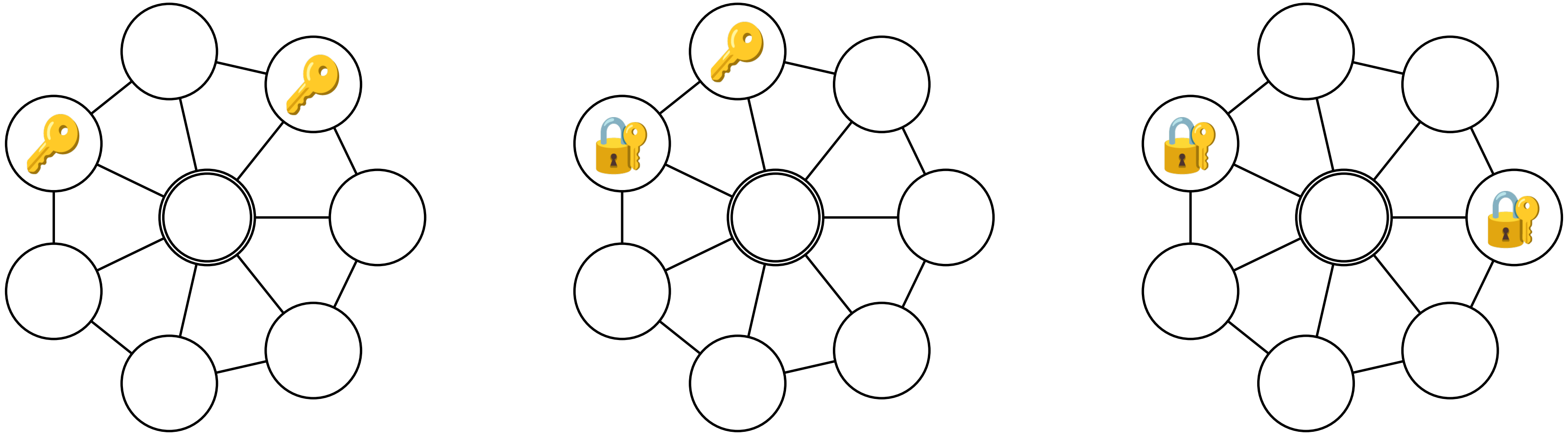
[1]Graph Structure and Monadic Second Order Logic; by B. Courcelle, J. Engelfriet

Safety specification

Safety properties

#(🔑): number of tokens on 🔑

$\sim$ number of clients who claim to own the key



$$\#(🔑) + \#(🔐) > 1$$

Proving safety $\approx$ reachability problem in an infinite family of PNs

Expressible Properties

- **mutual exclusion**
  "at most k processes can enter a critical section simultaneously"
- **uniqueness**
  "the entire system contains at most k instances of a resource"
- **uncoverability**
  "no process can reach a bad state"

Examples: leader election, semaphores, dining philosophers, …

Missing: ~~liveness~~, ~~deadlock freedom~~

An Abstraction Technique

Verification pipeline

$$\mathrm{Sys} \longrightarrow \mathsf{compose}\big(\mathrm{X}, \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{right},\ \mathrm{right}\mapsto\mathrm{left}}(\mathrm{client}')\big)$$

$$\mathrm{X} \longrightarrow \mathsf{compose}\big(\ \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{mid}}\big(\mathsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(X)\big),\ \mathsf{rename}_{\mathrm{right}\mapsto\mathrm{mid}}(\mathrm{client})\ \big)$$

$$\mathrm{X} \longrightarrow \mathsf{compose}\big(\mathsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(\mathrm{server}), \mathrm{client}\big)$$

Verification pipeline

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\mathrm{Sys} &\longrightarrow \textsf{compose}\big(\mathrm{X}, \textsf{rename}_{\mathrm{left}\mapsto\mathrm{right},\ \mathrm{right}\mapsto\mathrm{left}}(\mathrm{client}')\big)\\
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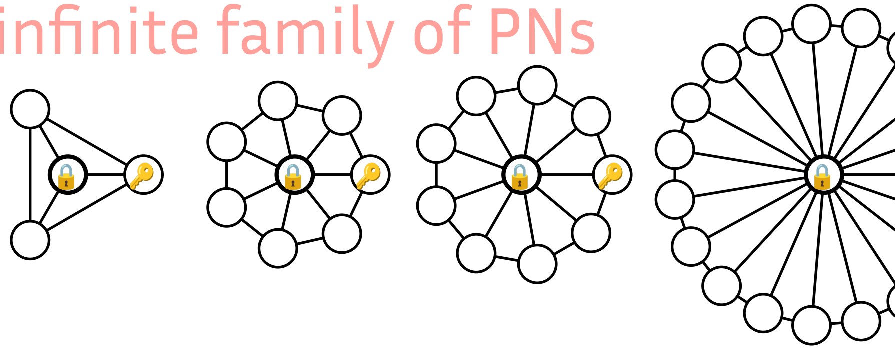
&\ \big)\\
\mathrm{X} &\longrightarrow \textsf{compose}\big(\textsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(\mathrm{server}), \mathrm{client}\big)
\end{aligned}
$$

$$\xrightarrow{\text{language}}$$

infinite family of PNs



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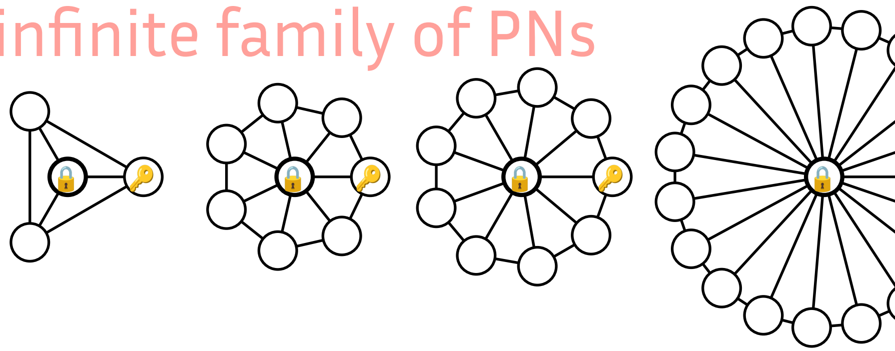

$\Big\downarrow$ safety

$$\boxed{\#(🔑) + \#(🔐) > 1}$$

Verification pipeline

$\text{Sys} \longrightarrow \mathsf{compose}(\text{X}, \mathsf{rename}_{\text{left}\mapsto\text{right},\ \text{right}\mapsto\text{left}}(\text{client}'))$

$\text{X} \longrightarrow \mathsf{compose}($

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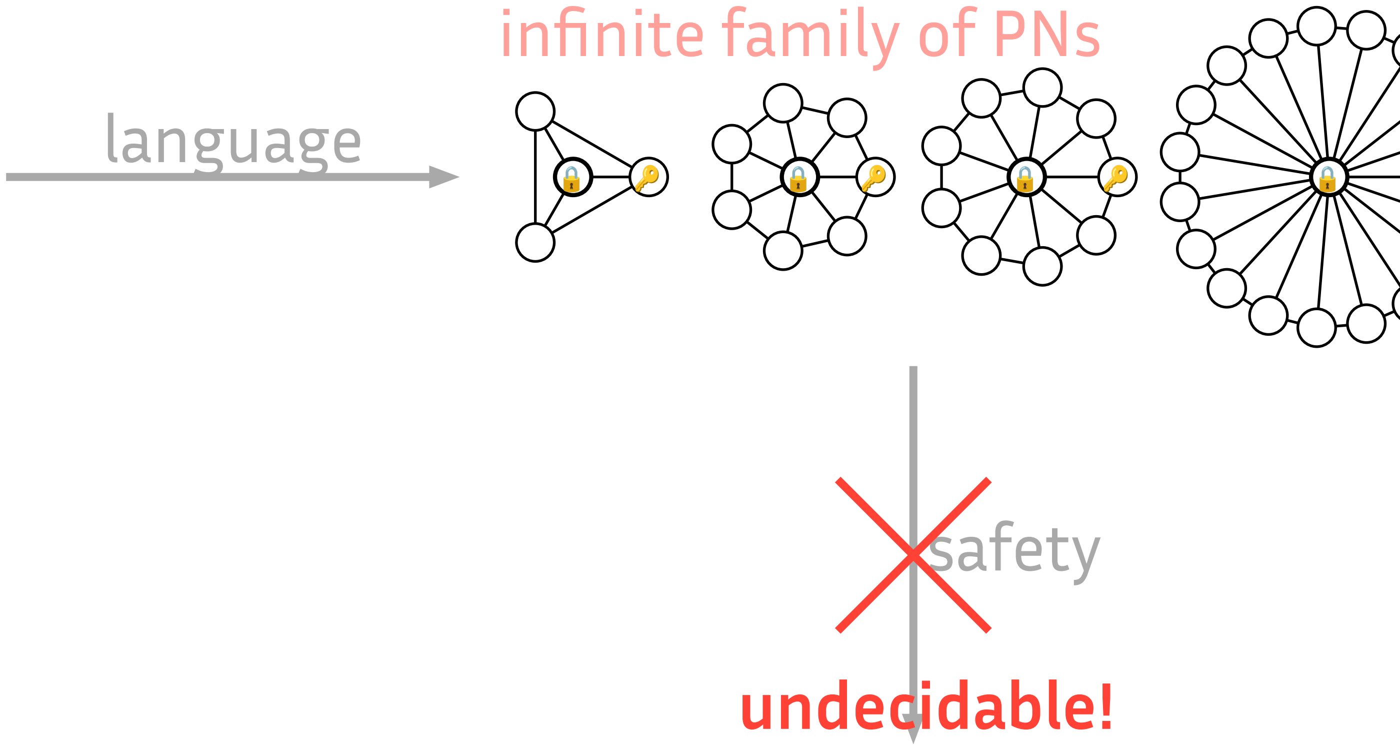

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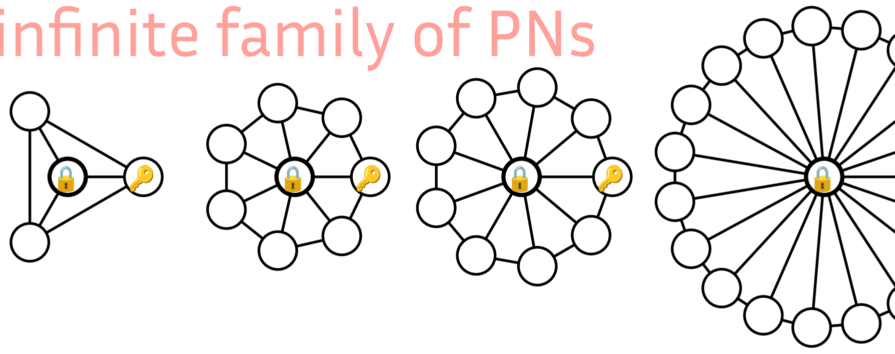
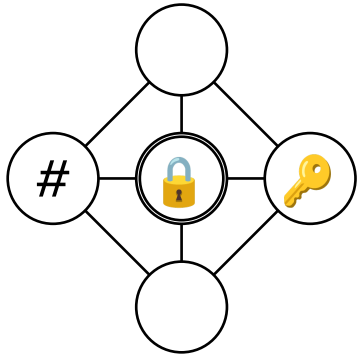
language

infinite family of PNs



abstraction



$\supseteq$

(soundness)

safety

undecidable!

$\boxed{\#(🔑) + \#(🔐) > 1}$

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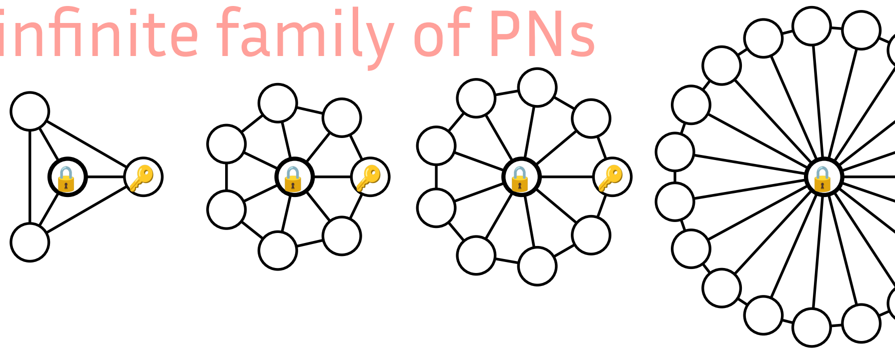
language

infinite family of PNs



abstraction

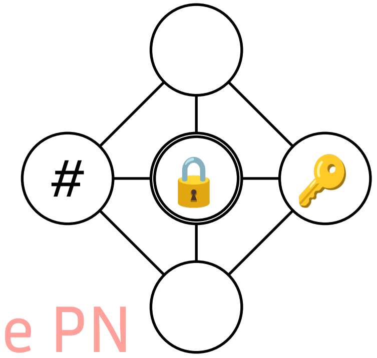
CAV Artifact Evaluation ★ ★ ★ Reusable

$\supseteq$

(soundness)

~~safety~~

undecidable!



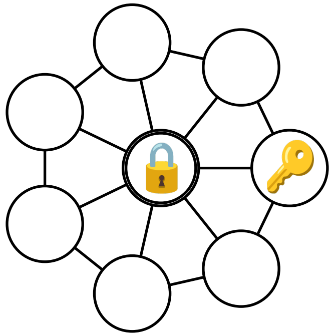
finite PN

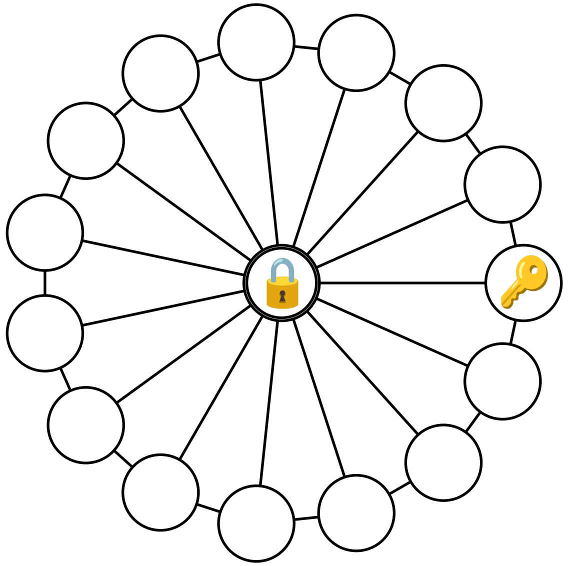
decidable

e.g. LoLA

$\boxed{\#(🔑) + \#(🔐) > 1}$

Folding abstraction





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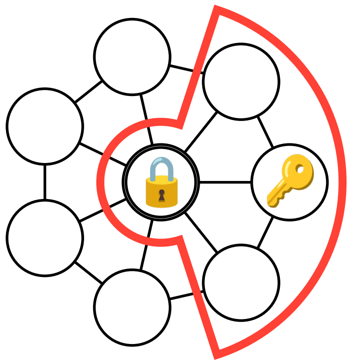

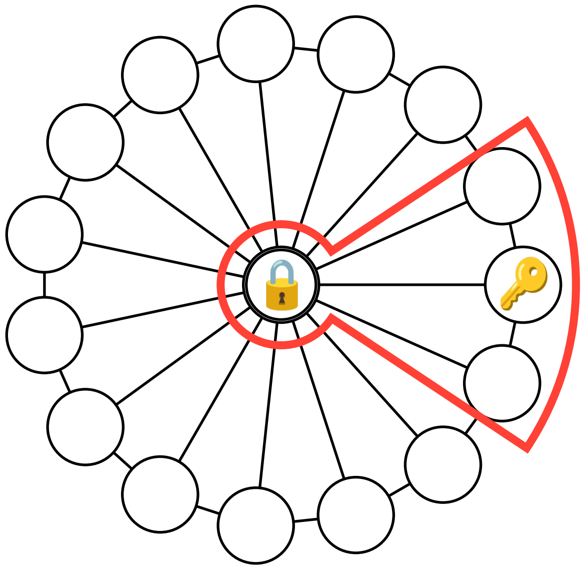

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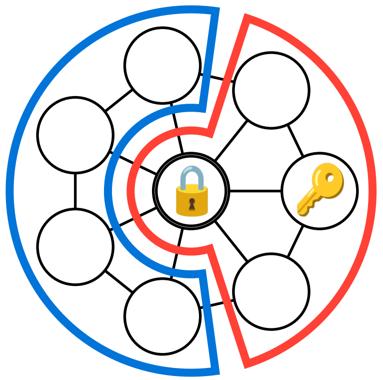

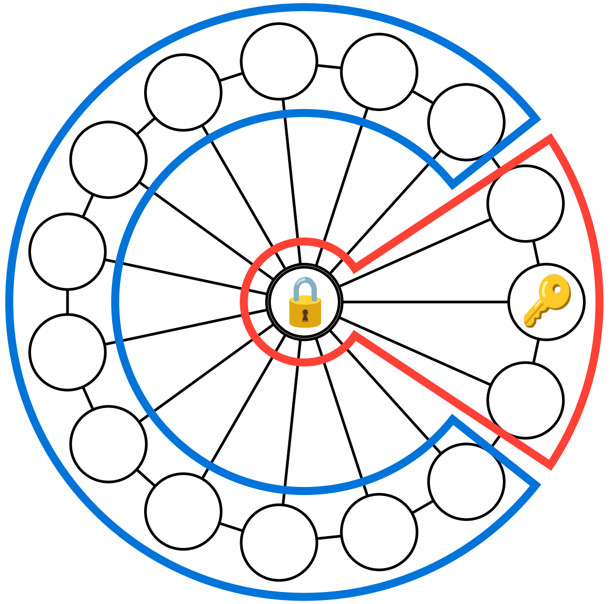

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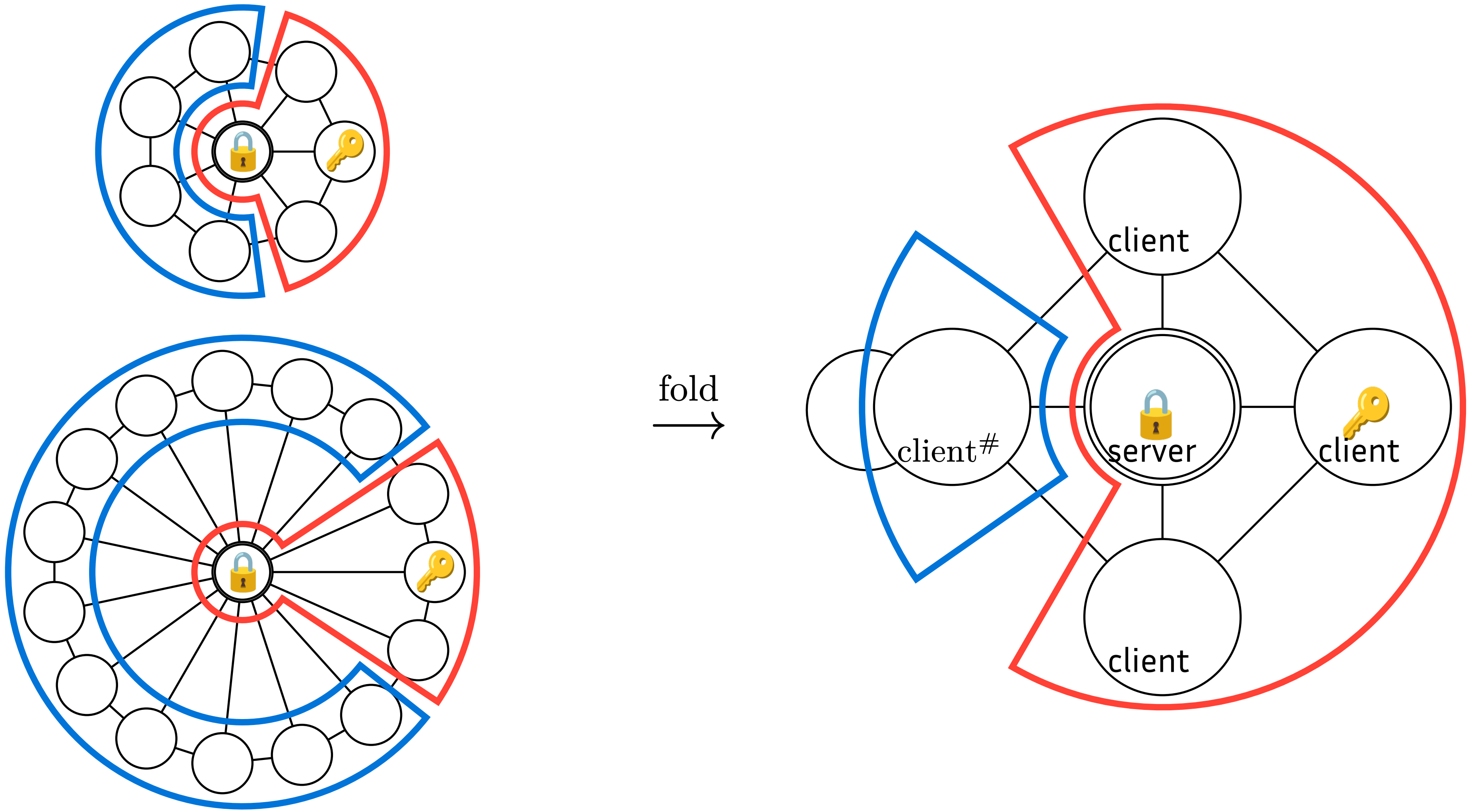
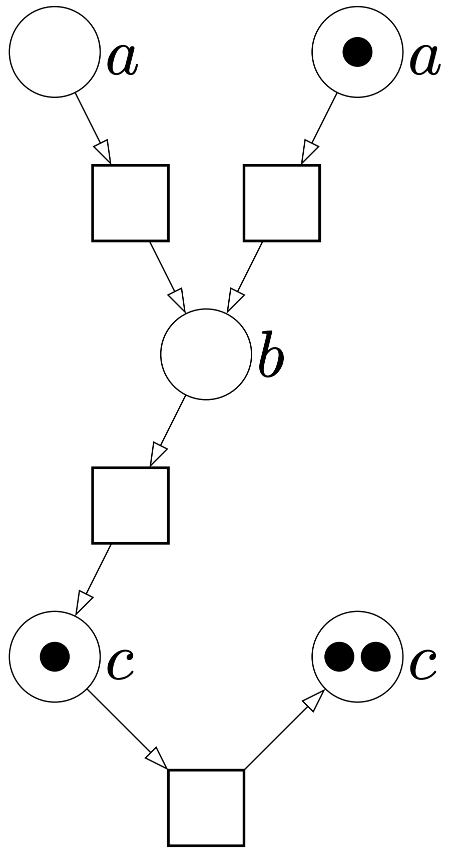


Implementing fold



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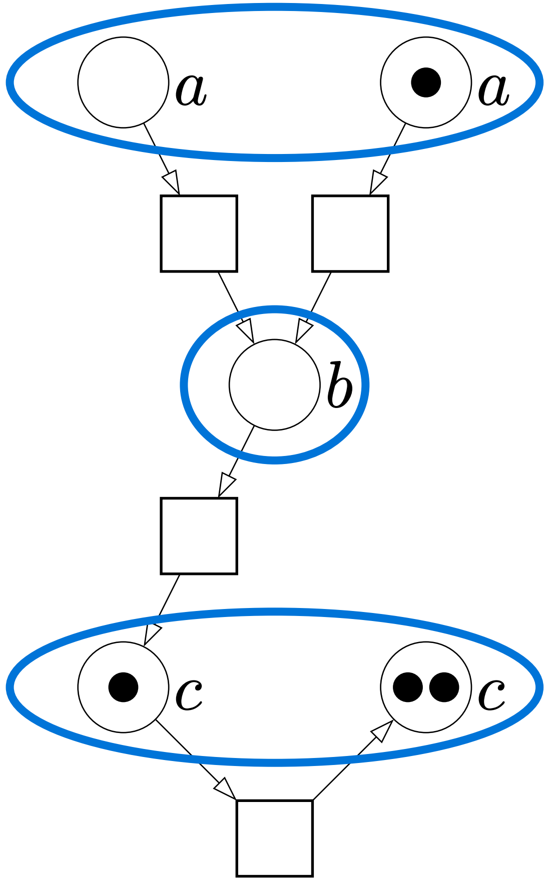

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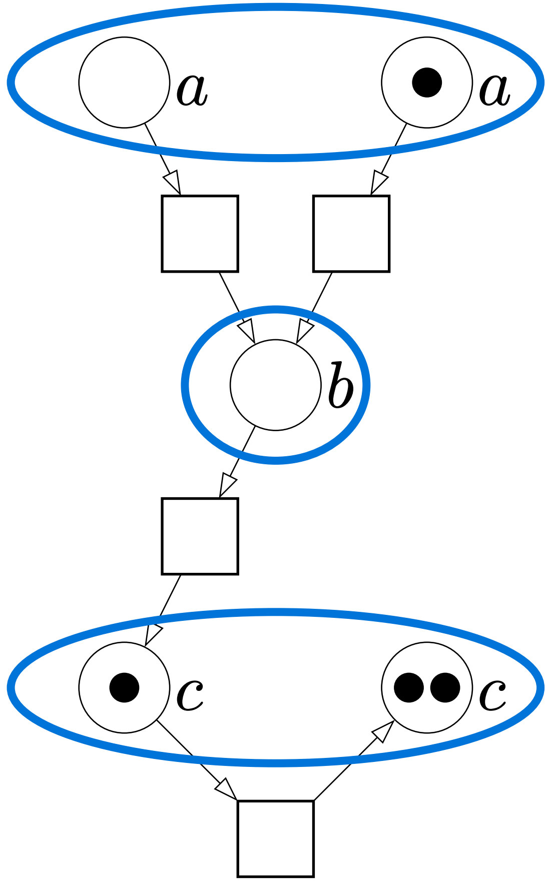
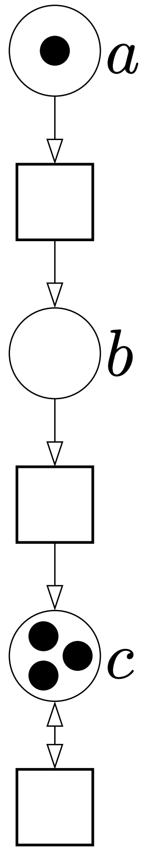


$$\xrightarrow{\text{fold}}$$

$$\dashleftarrow$$
$$\subseteq$$
(soundness)



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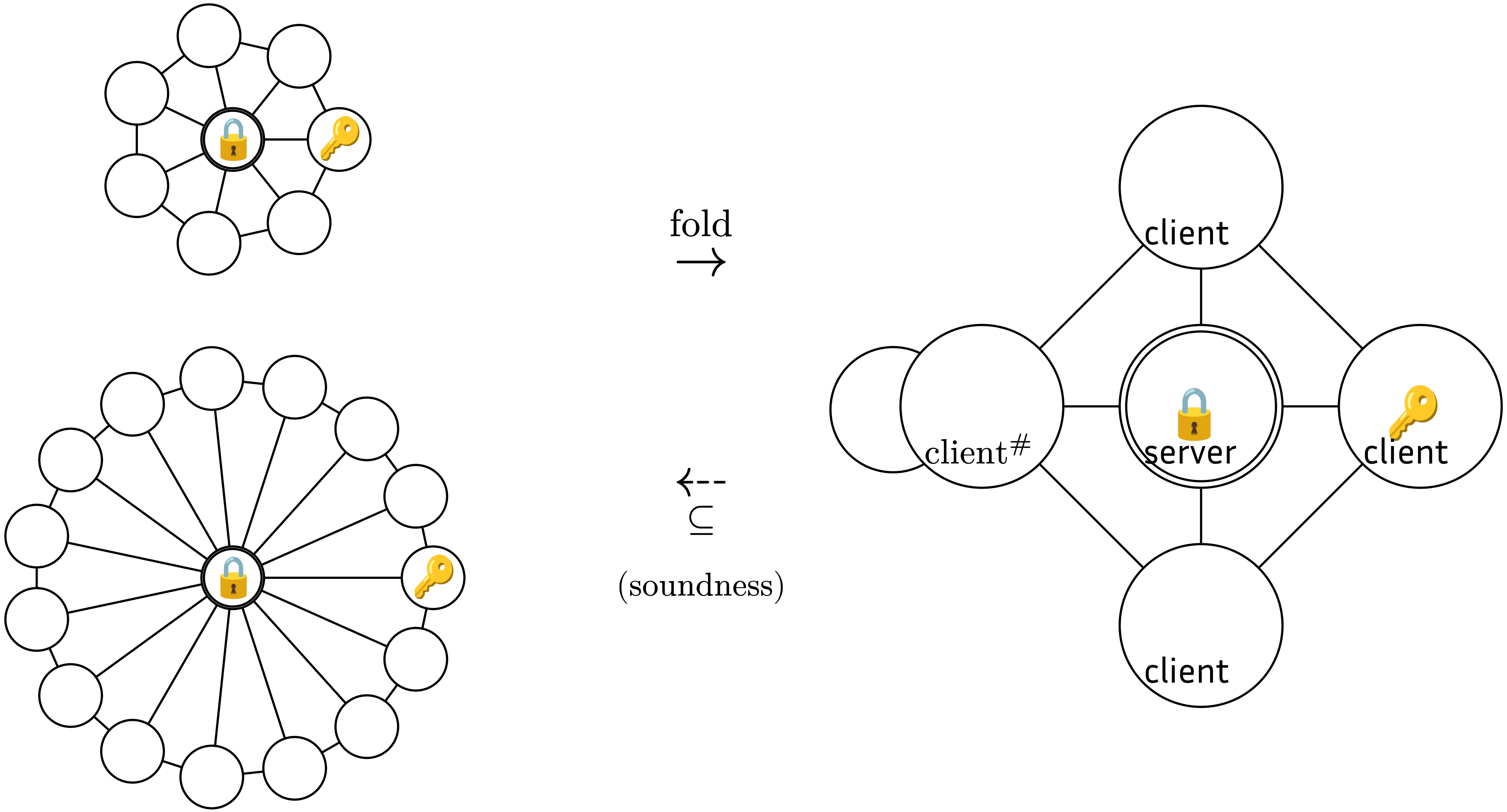
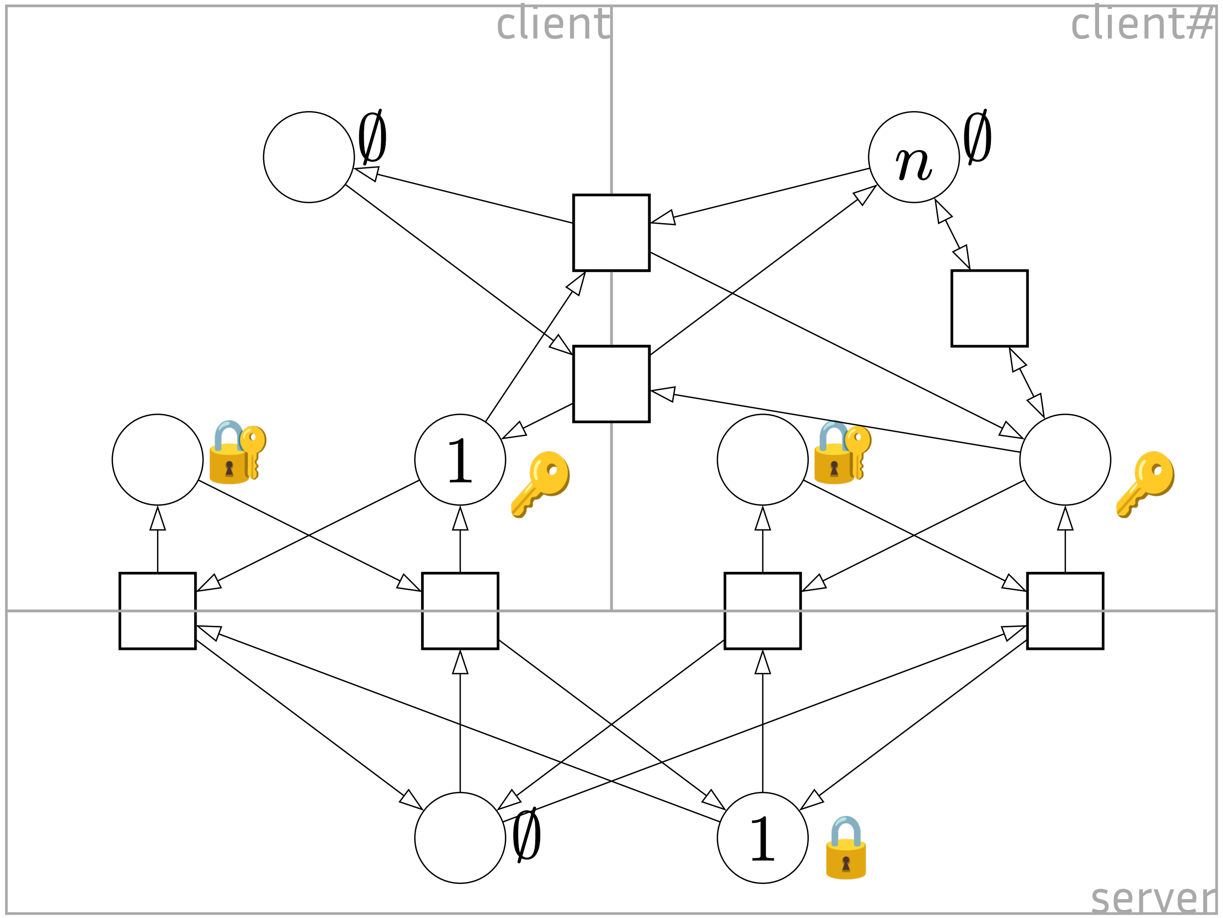
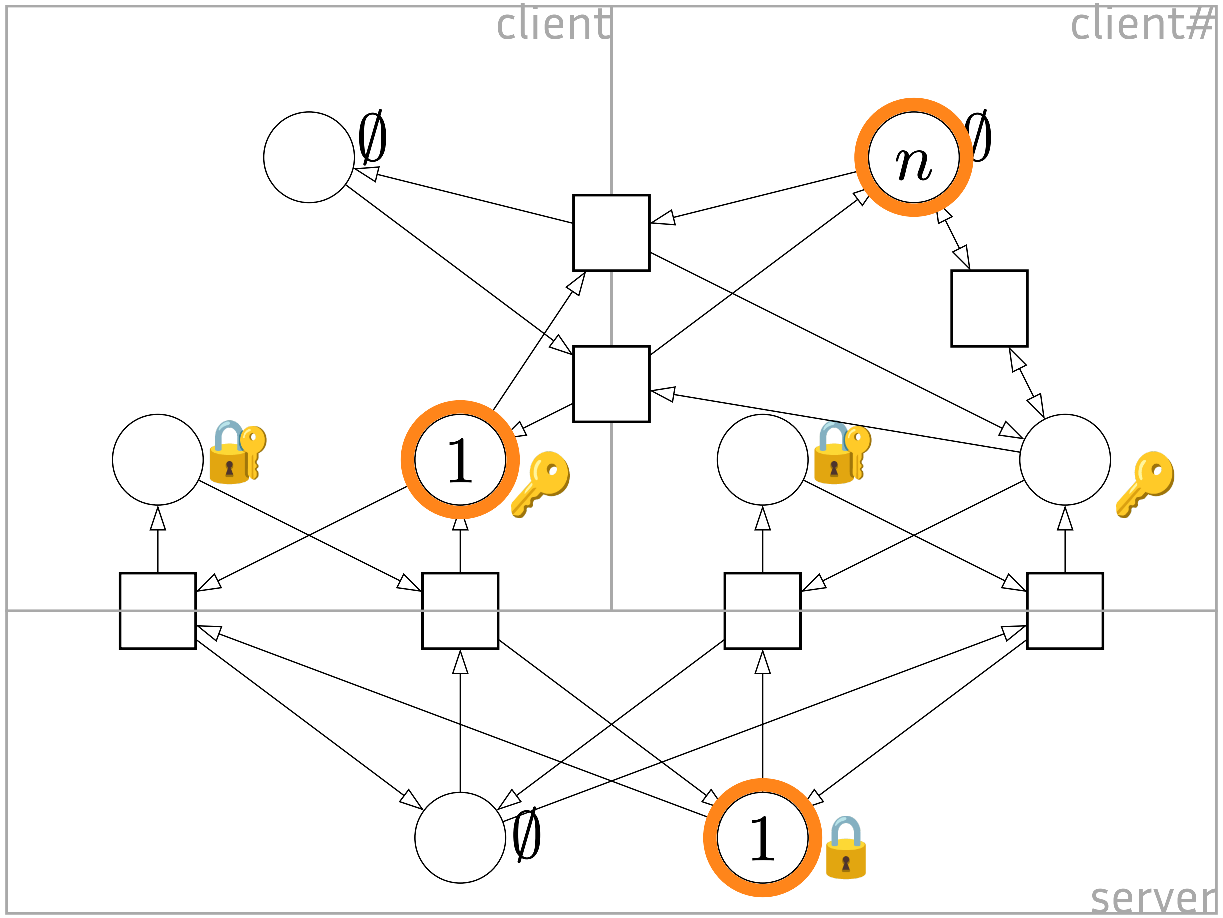


Folded system



Folded system



Initial marking

$$\mathrm{Sys} \longrightarrow \mathsf{compose}\Big(\mathrm{X}, \mathsf{rename}_{\mathrm{left}\mapsto\mathrm{right},\ \mathrm{right}\mapsto\mathrm{left}}(\mathrm{client}')\Big)$$

$$\mathrm{X} \longrightarrow \mathsf{compose}\Big($$

$$\mathsf{rename}_{\mathrm{left}\mapsto\mathrm{mid}}\Big(\mathsf{copy}_{\mathrm{send}\rightsquigarrow\mathrm{acq},\mathrm{recv}\rightsquigarrow\mathrm{rel}}(X)\Big),$$

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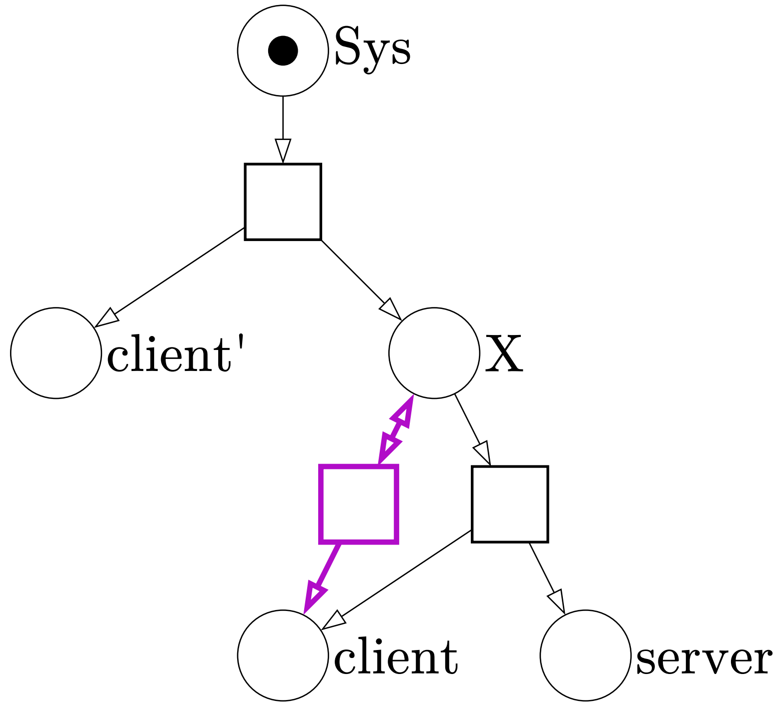
Initial marking

From the grammar

$$\begin{aligned} \mathrm{Sys} &\longrightarrow X, \mathrm{client'} \\ X &\longrightarrow X, \mathrm{client} \\ X &\longrightarrow \mathrm{server}, \mathrm{client} \end{aligned}$$



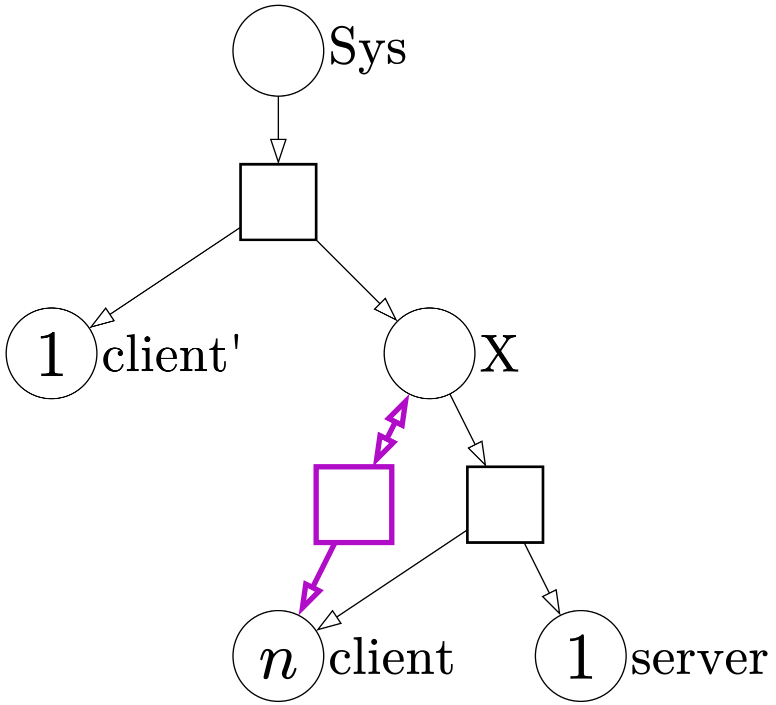
Initial marking

From the grammar

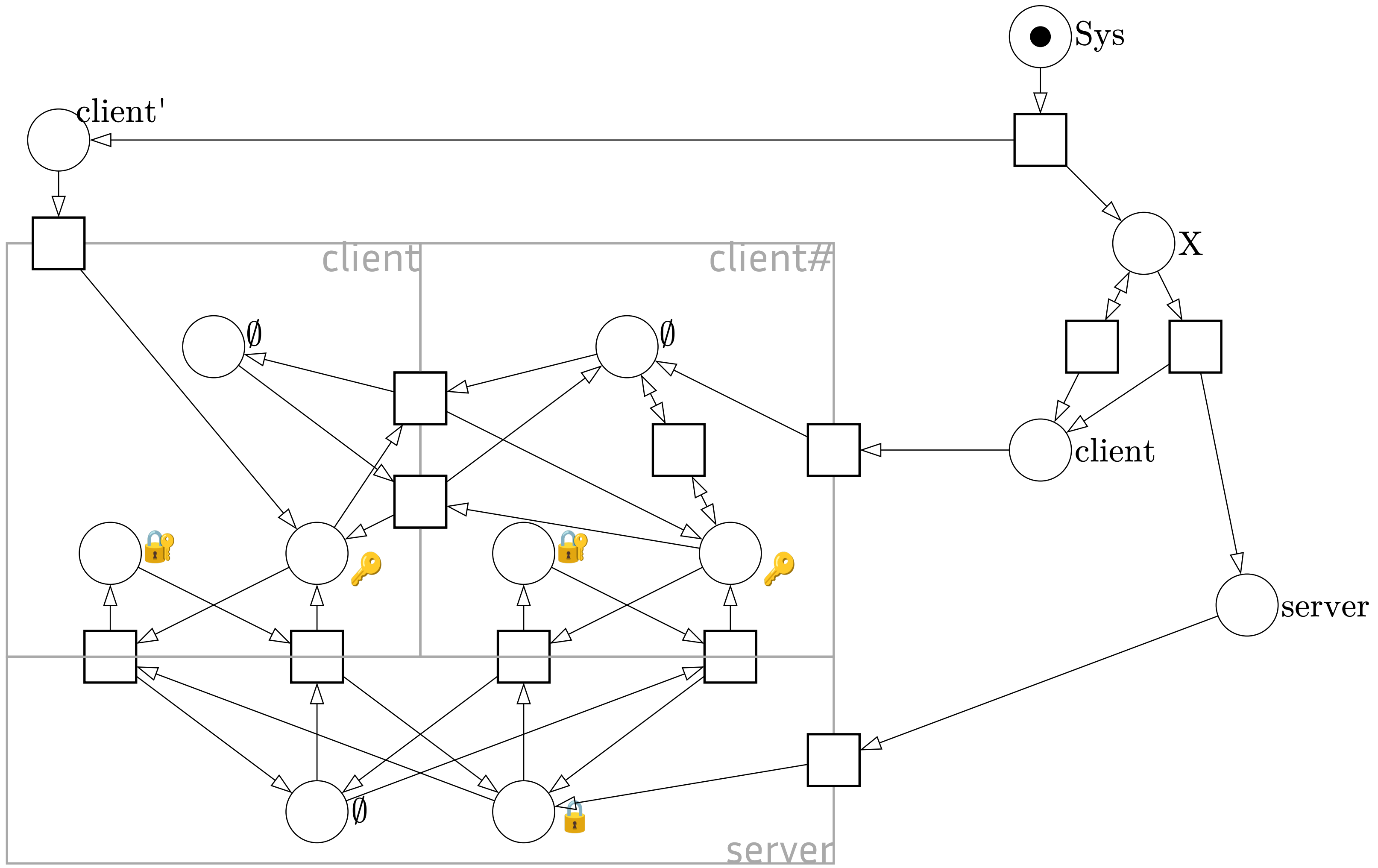
$$\text{Sys} \longrightarrow X, \text{client'}$$

$$X \longrightarrow X, \text{client}$$

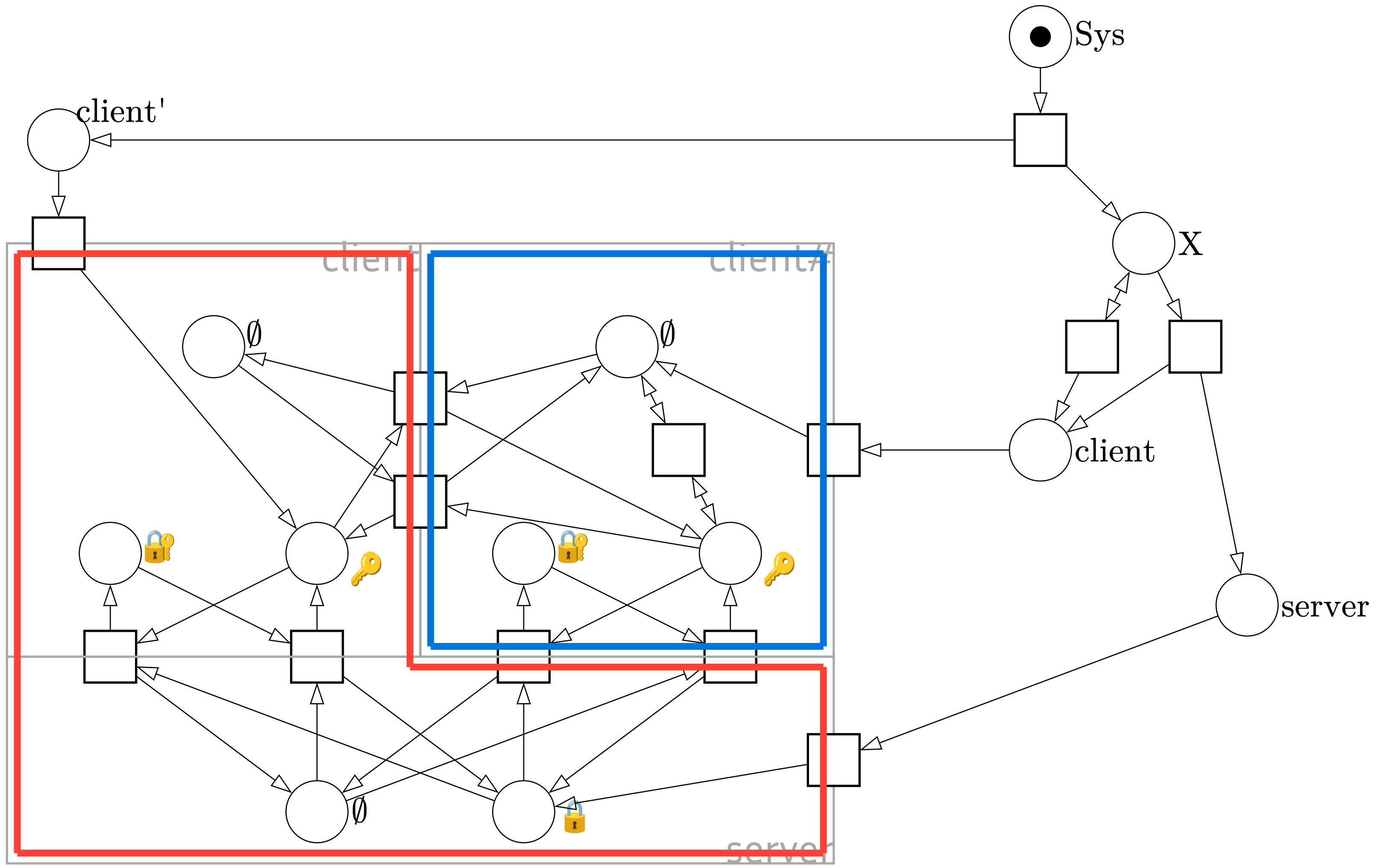
$$X \longrightarrow \text{server}, \text{client}$$



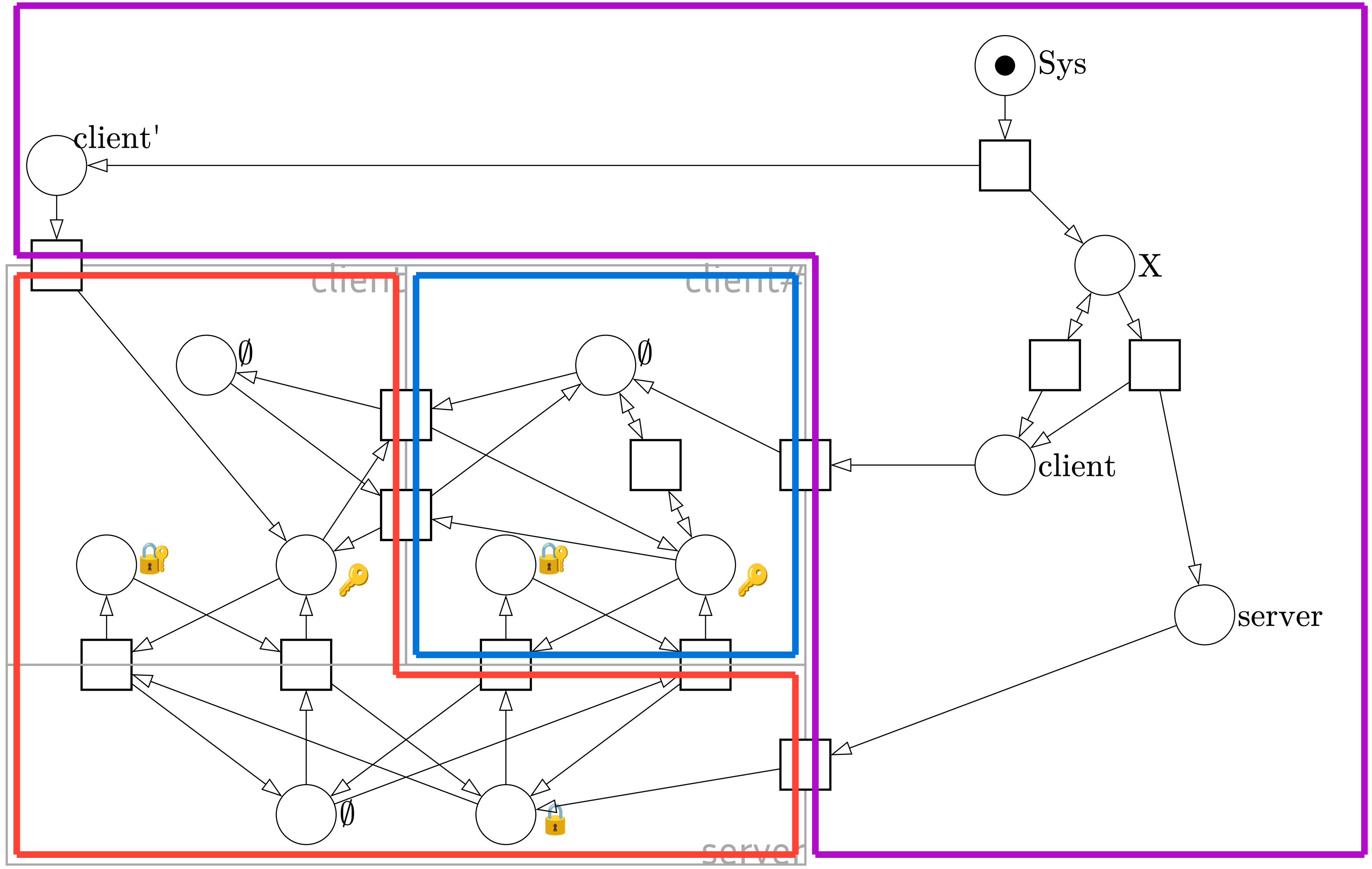
Counting abstraction



Counting abstraction

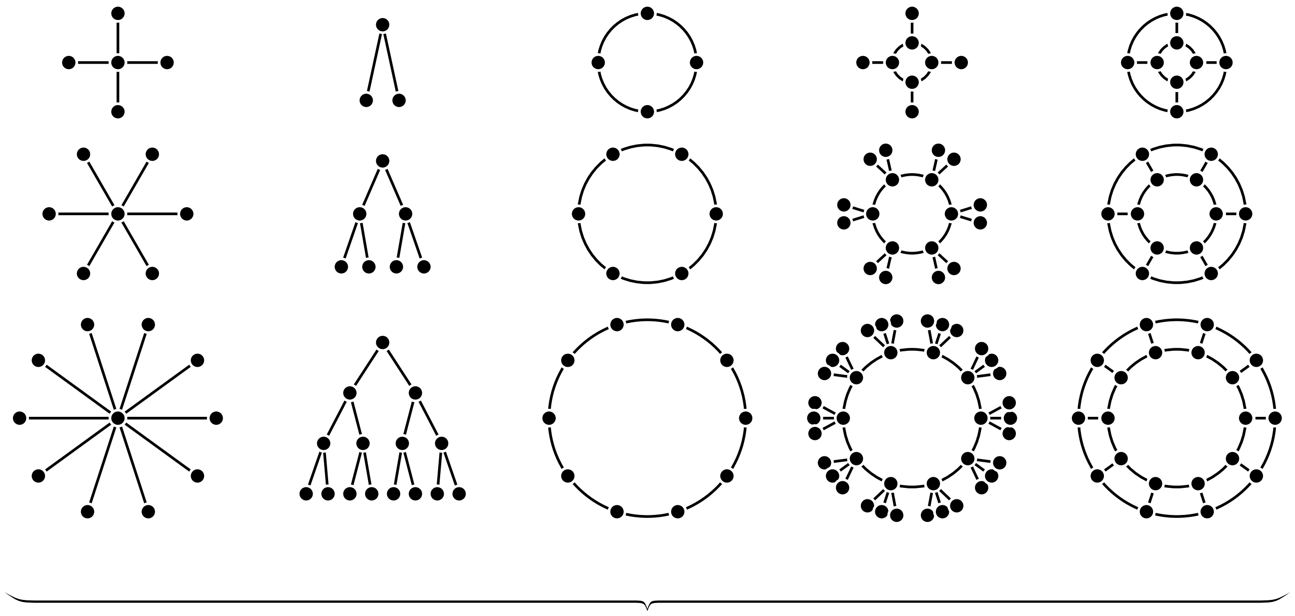


Counting abstraction



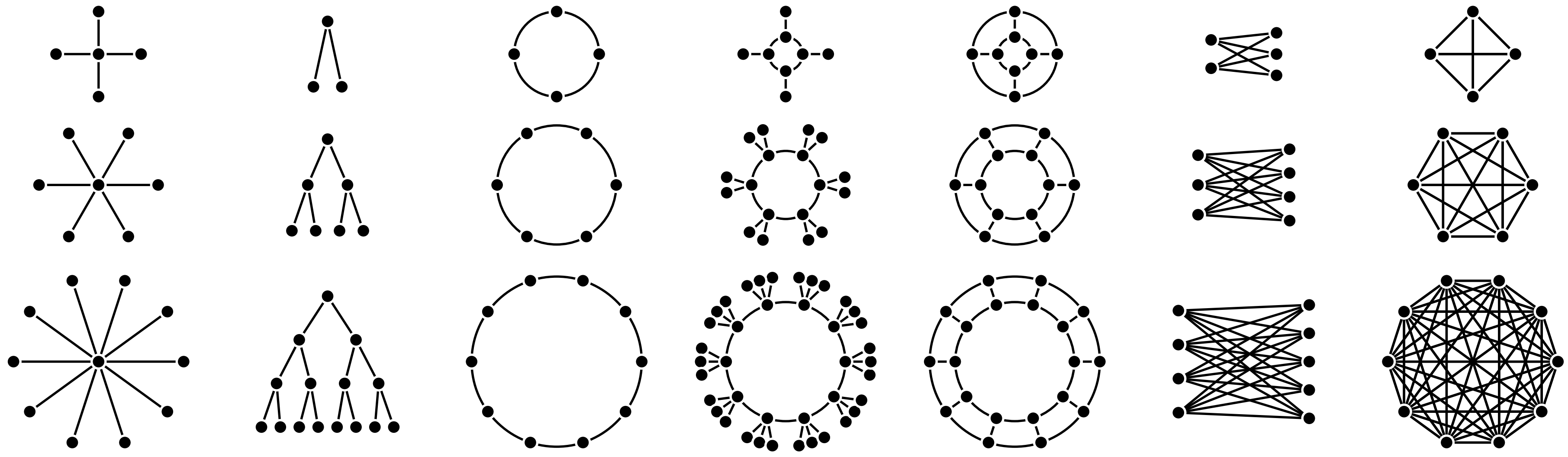
Verifying Dense Graphs

What formalism for architectures ?



Hyperedge **R**eplacement (sparse only)

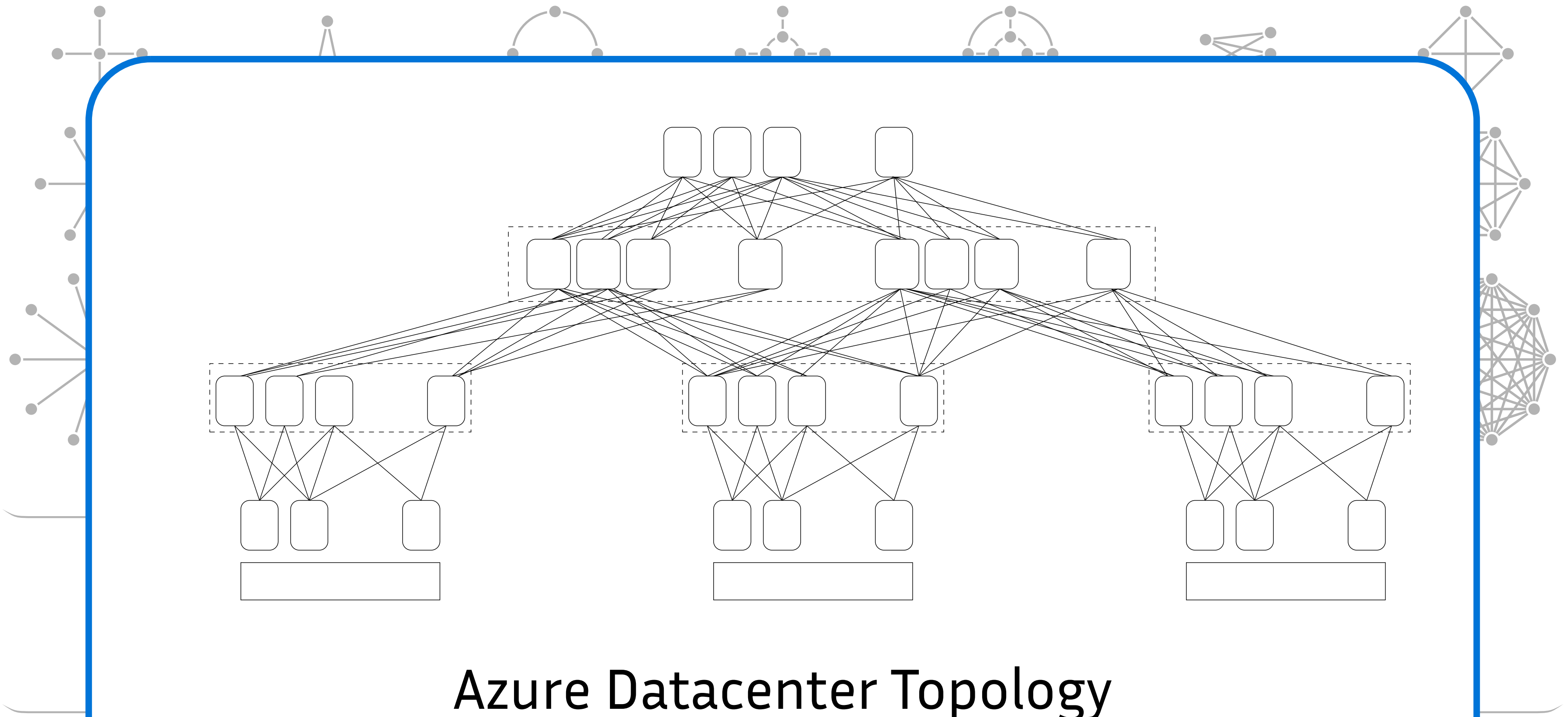
What formalism for architectures ?

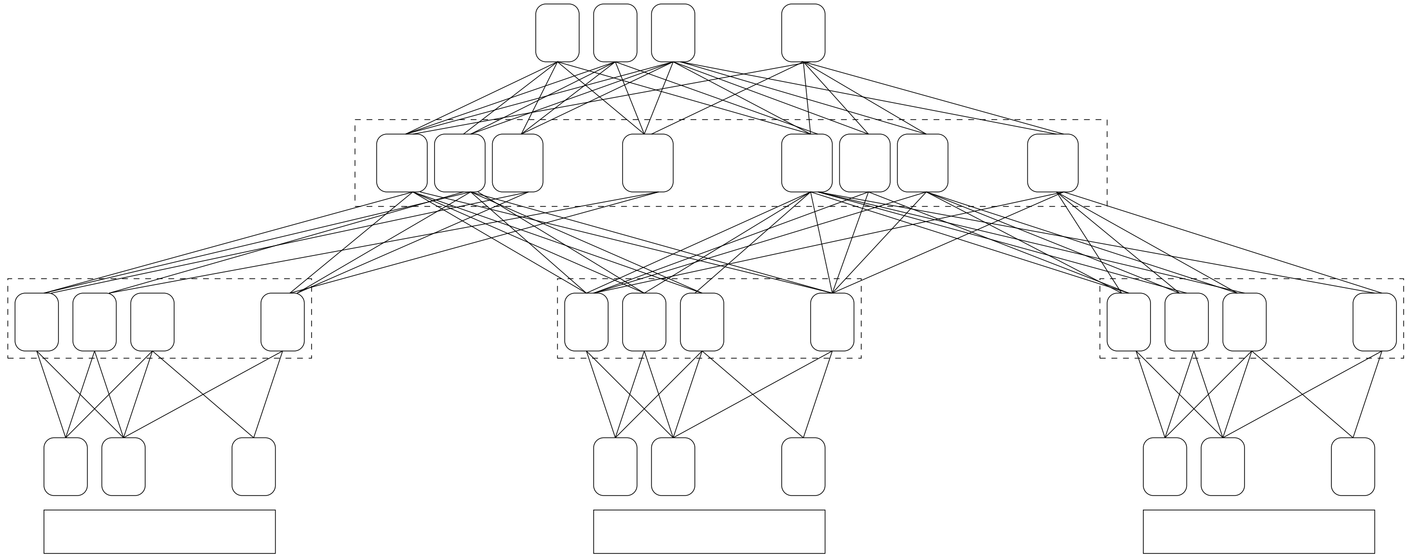


Hyperedge **R**eplacement (sparse only)

Vertex **R**eplacement (incl. some dense)

What formalism for architectures ?



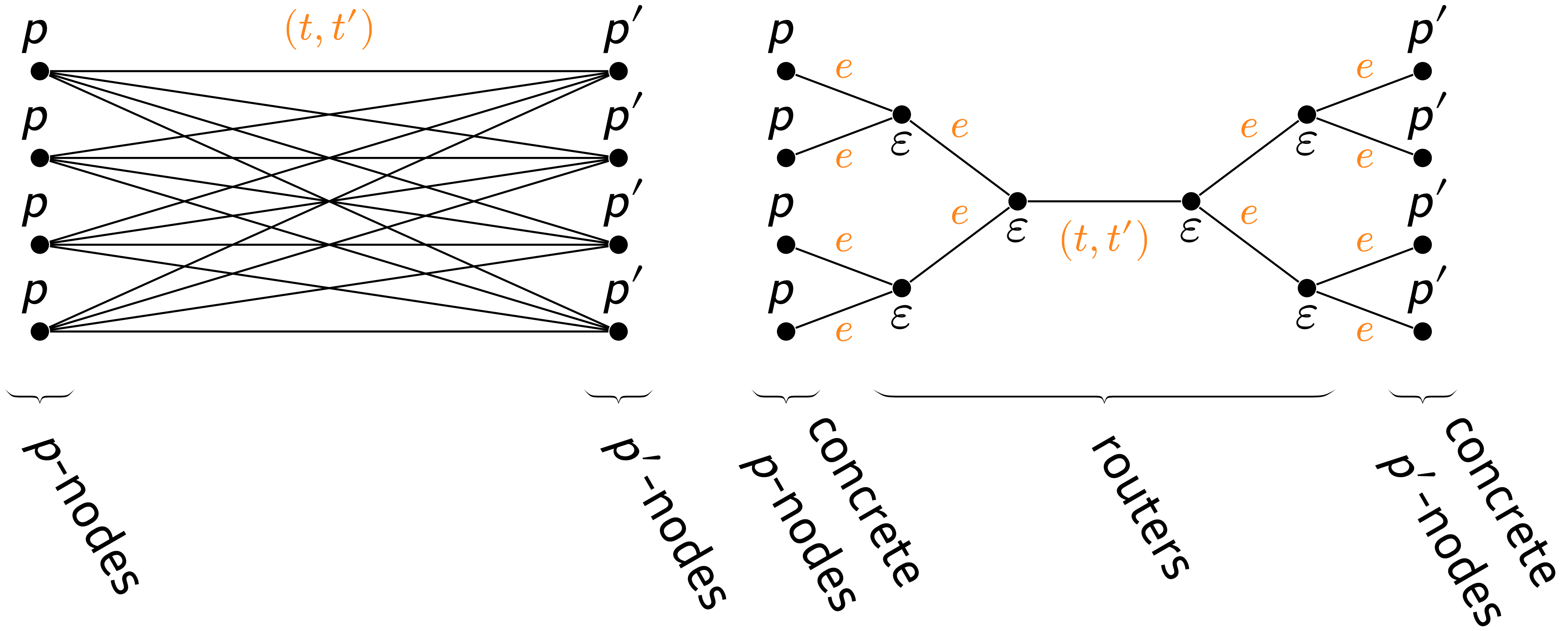


Azure Datacenter Topology
Greenberg et al., SIGCOMM 2009

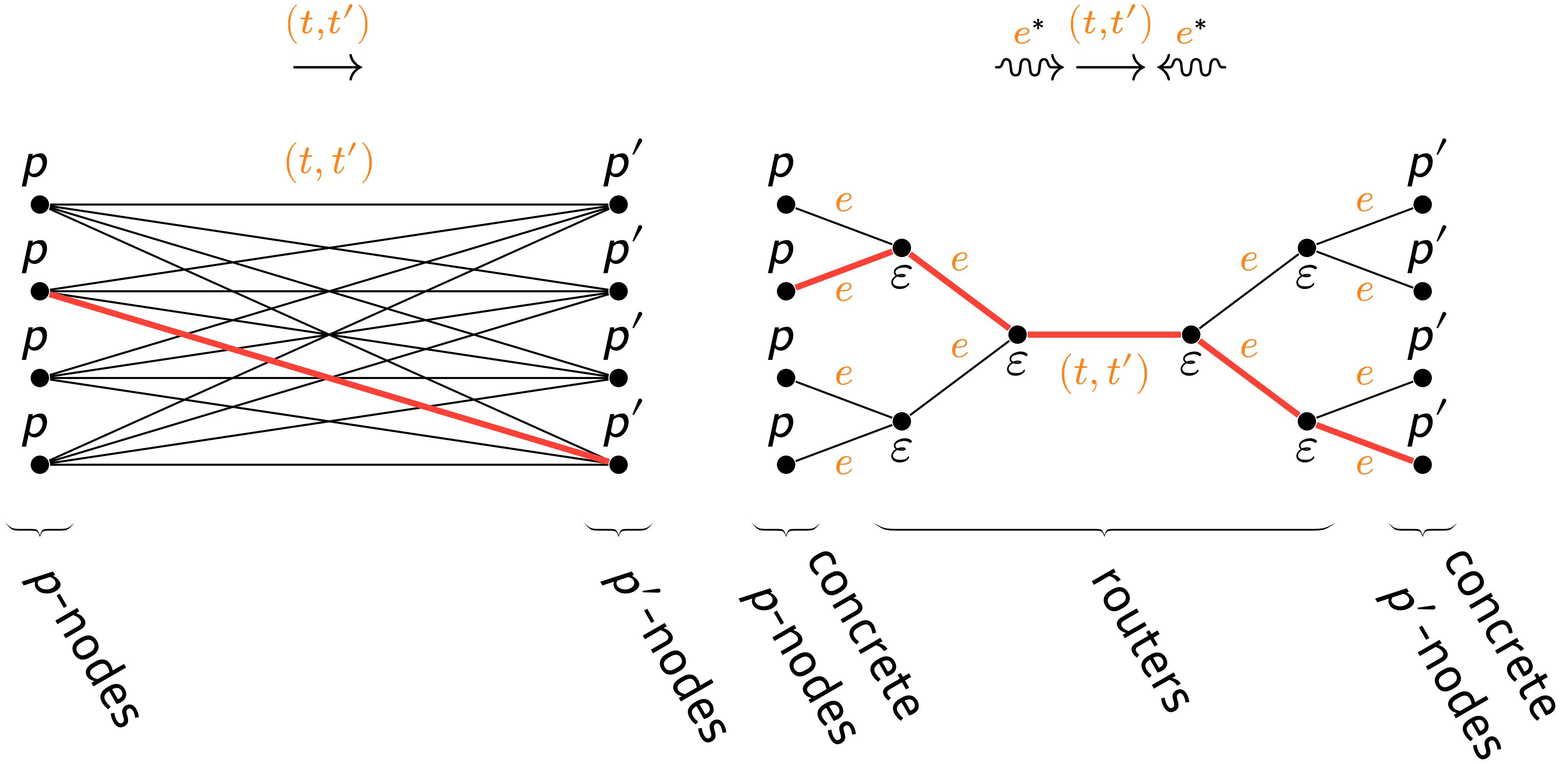
Key idea

$\xrightarrow{(t,t')}$

$\overset{e^*}{\rightsquigarrow} \xrightarrow{(t,t')} \overset{e^*}{\leftsquigarrow}$



Key idea

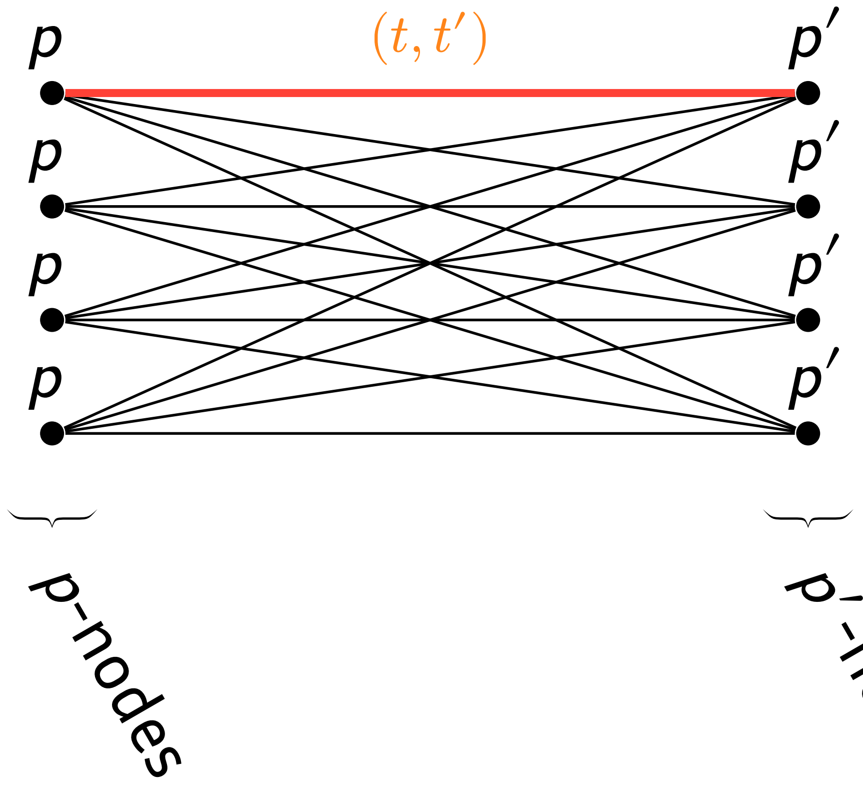


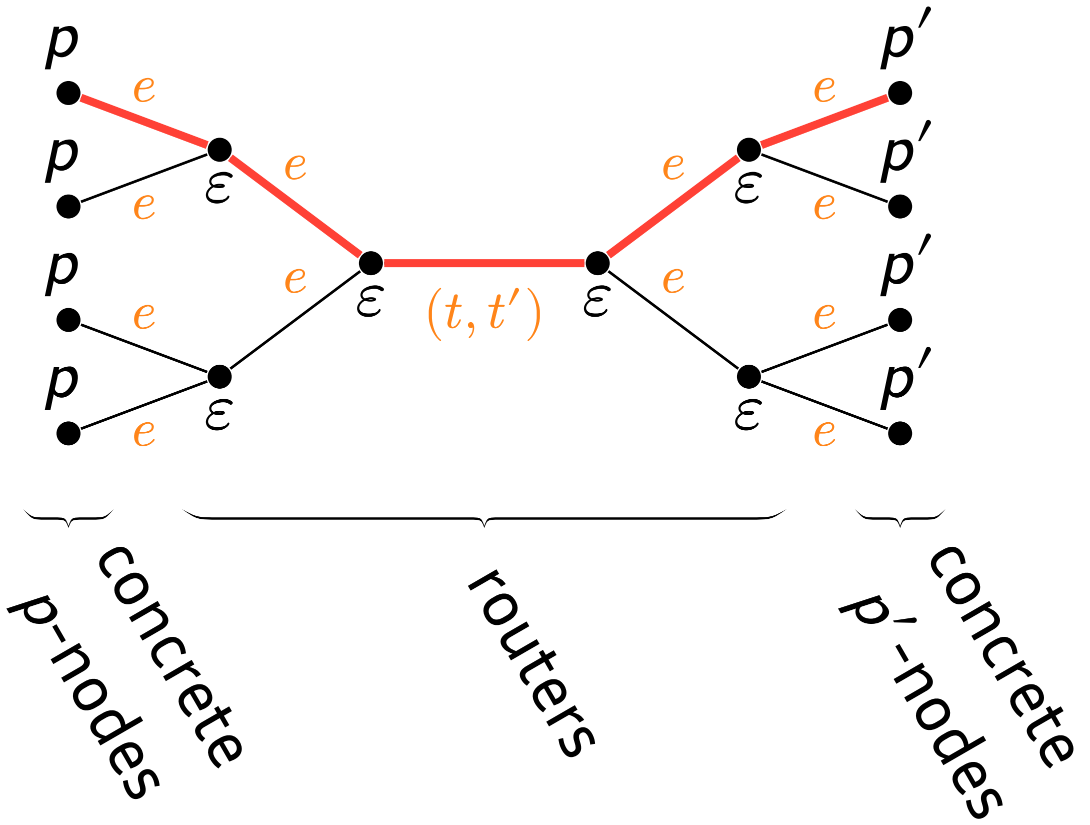
$\xrightarrow{(t,t')}$
$\overset{e^*}{\rightsquigarrow} \xrightarrow{(t,t')} \overset{e^*}{\leftsquigarrow}$
p
p'
(t,t')
e
ε
p-nodes
p'-nodes
concrete p-nodes
routers
concrete p'-nodes

Key idea

$\xrightarrow{(t,t')}$

$\overset{e^*}{\rightsquigarrow} \xrightarrow{(t,t')} \overset{e^*}{\leftsquigarrow}$





Key idea

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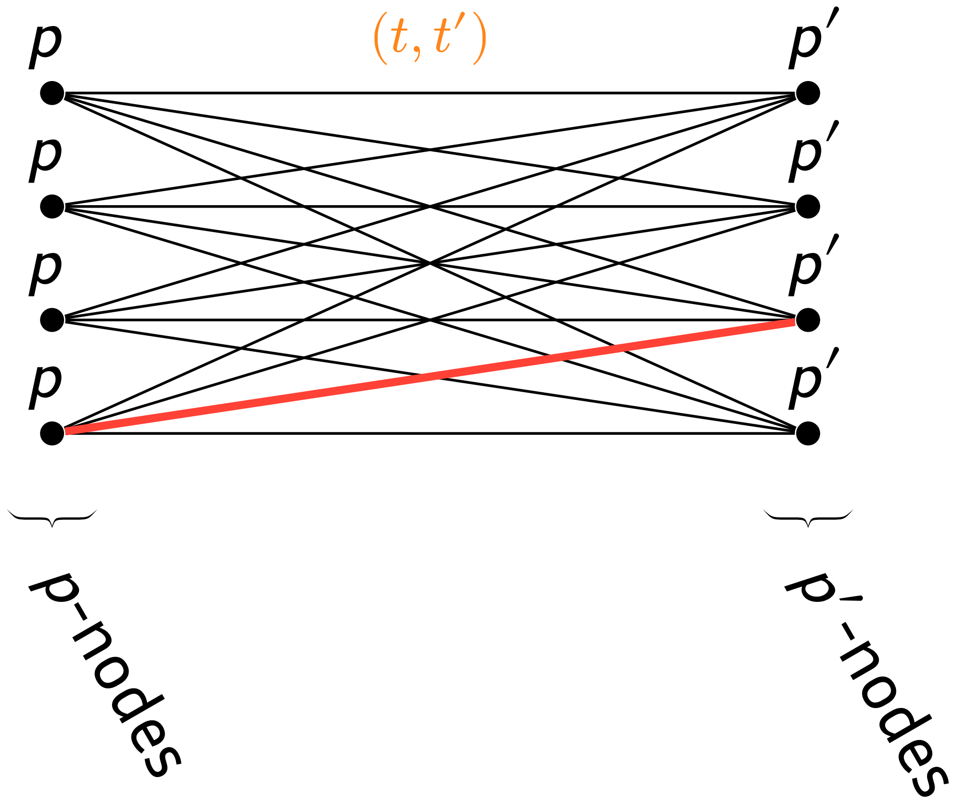
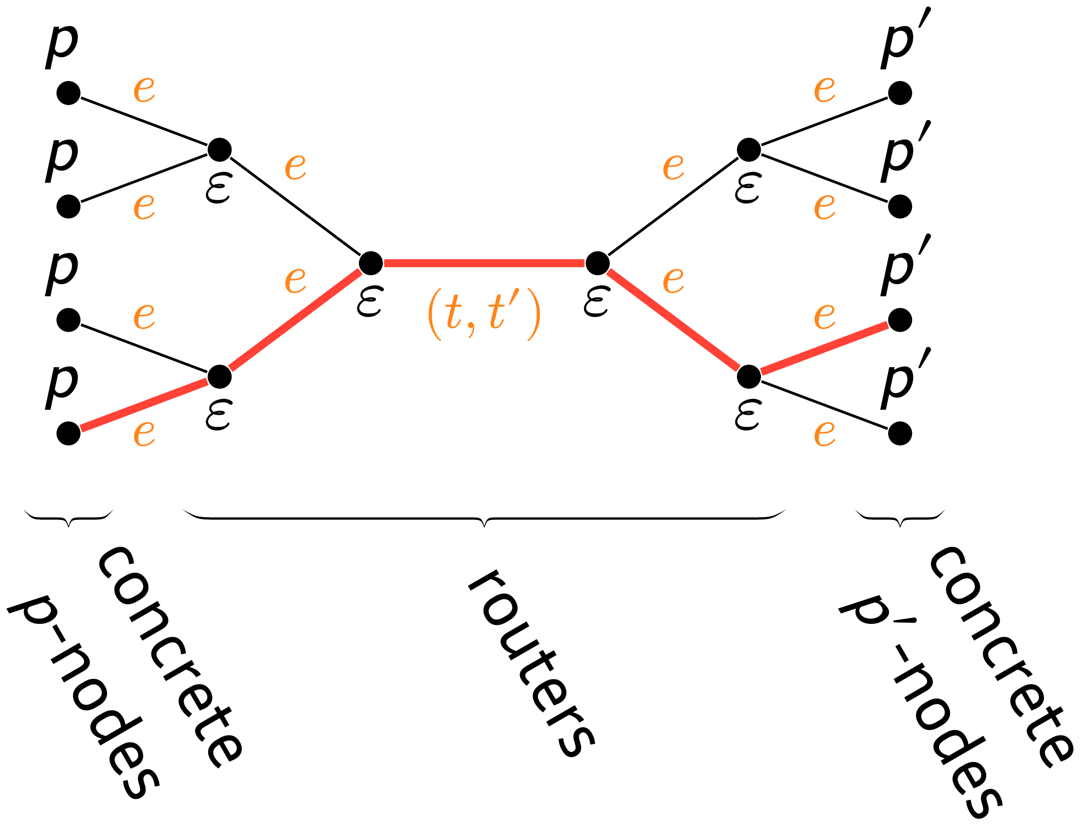
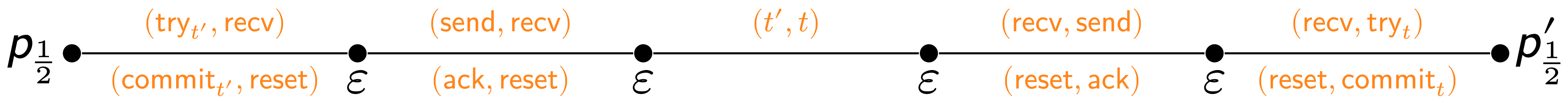
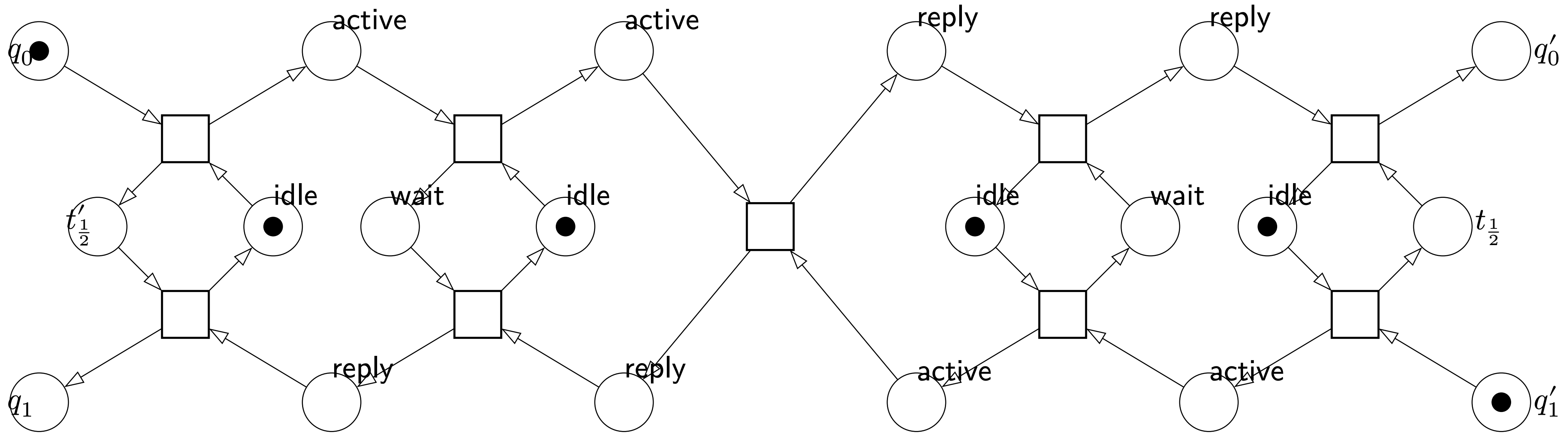




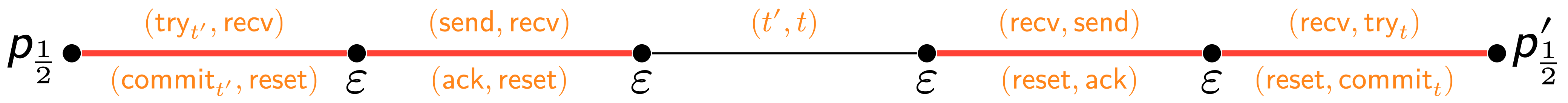
Communication through routers

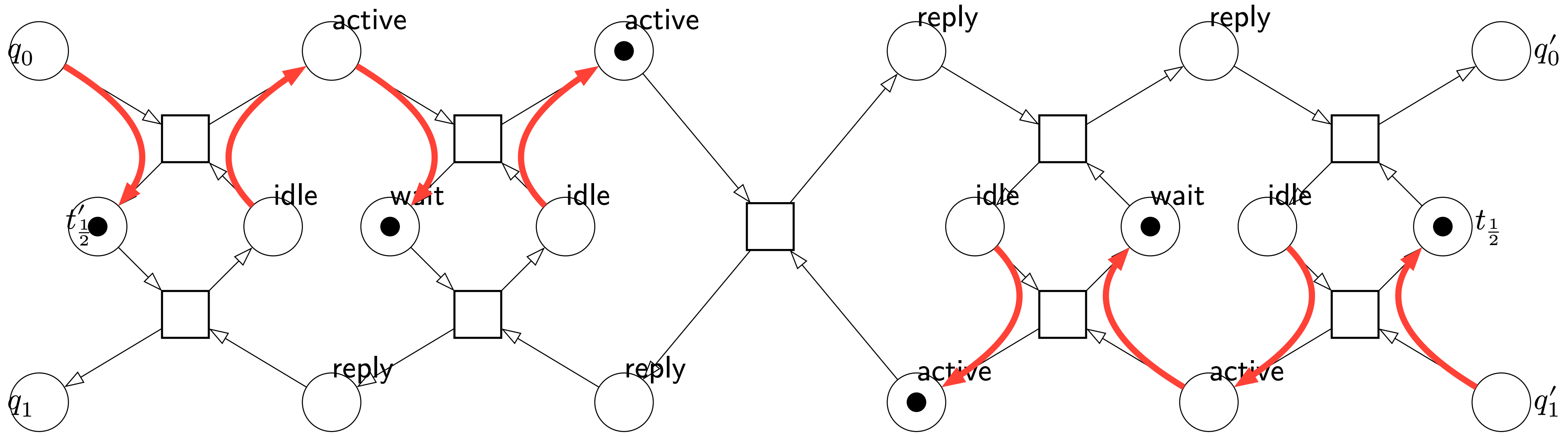


$p_{\frac{1}{2}}$
$(\text{try}_{t'}, \text{recv})$
$(\text{commit}_{t'}, \text{reset})$
ε
$(\text{send}, \text{recv})$
$(\text{ack}, \text{reset})$
ε
(t', t)
ε
$(\text{recv}, \text{send})$
$(\text{reset}, \text{ack})$
ε
$(\text{recv}, \text{try}_t)$
$(\text{reset}, \text{commit}_t)$
$p'_{\frac{1}{2}}$

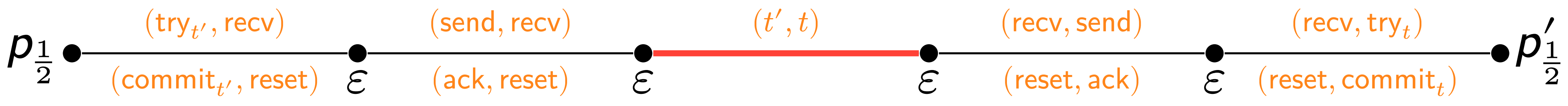


Communication through routers

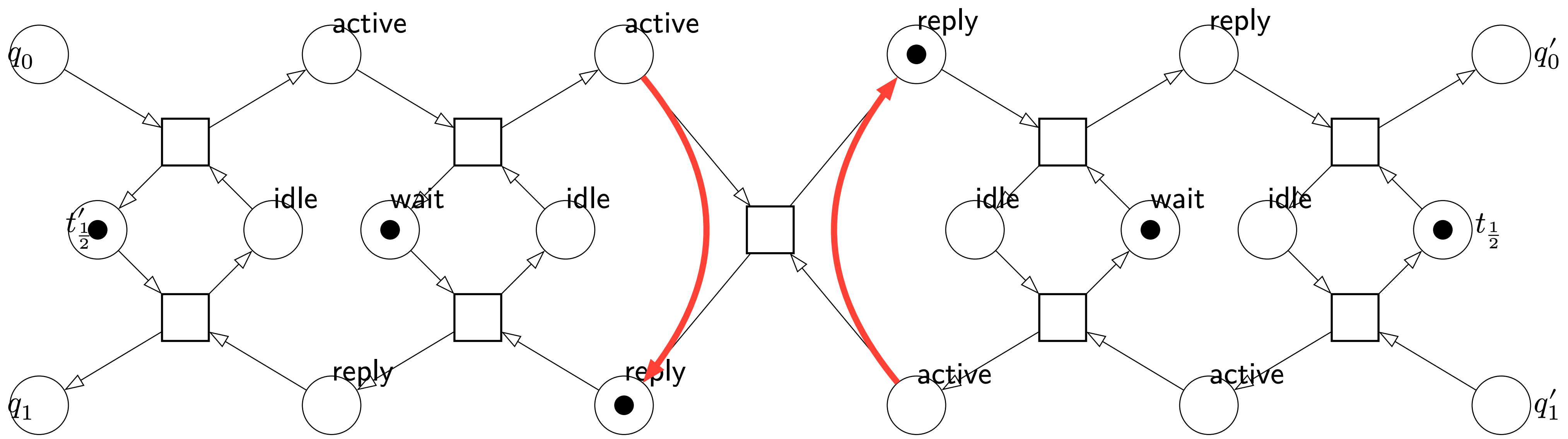

$p_{\frac{1}{2}}$
$(\mathsf{try}_{t'}, \mathsf{recv})$
$(\mathsf{commit}_{t'}, \mathsf{reset})$
ε
$(\mathsf{send}, \mathsf{recv})$
$(\mathsf{ack}, \mathsf{reset})$
ε
(t', t)
ε
$(\mathsf{recv}, \mathsf{send})$
$(\mathsf{reset}, \mathsf{ack})$
ε
$(\mathsf{recv}, \mathsf{try}_t)$
$(\mathsf{reset}, \mathsf{commit}_t)$
$p'_{\frac{1}{2}}$



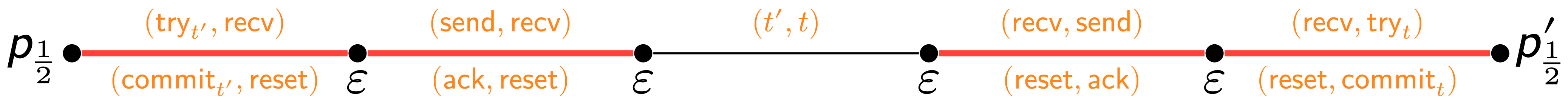
Communication through routers

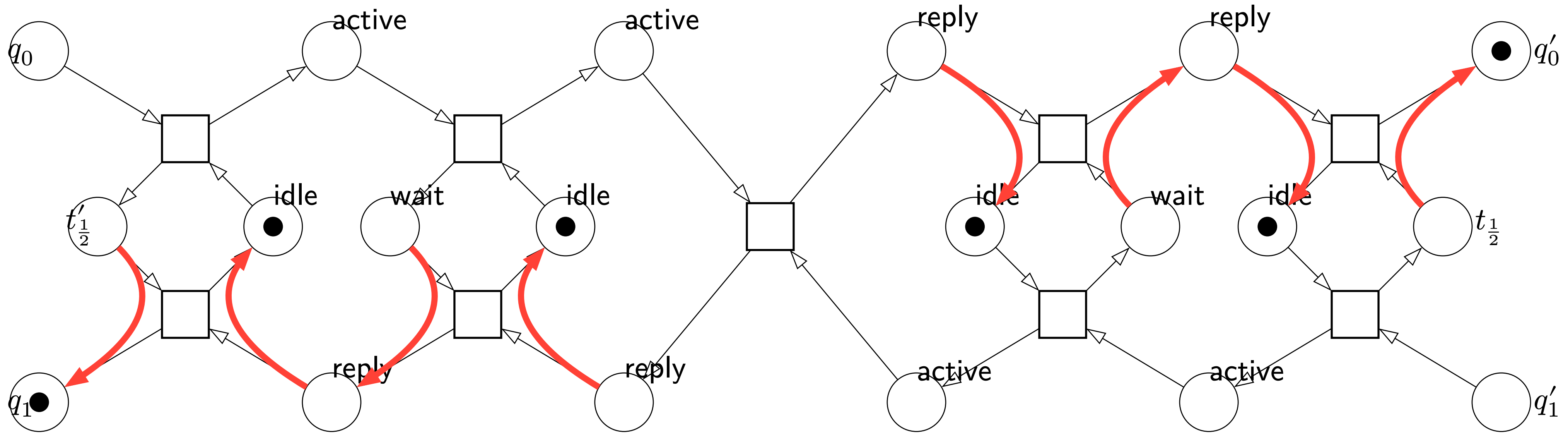


$p_{\frac{1}{2}}$
$(\text{try}_{t'}, \text{recv})$
$(\text{commit}_{t'}, \text{reset})$
ε
$(\text{send}, \text{recv})$
$(\text{ack}, \text{reset})$
ε
(t', t)
ε
$(\text{recv}, \text{send})$
$(\text{reset}, \text{ack})$
ε
$(\text{recv}, \text{try}_t)$
$(\text{reset}, \text{commit}_t)$
$p'_{\frac{1}{2}}$



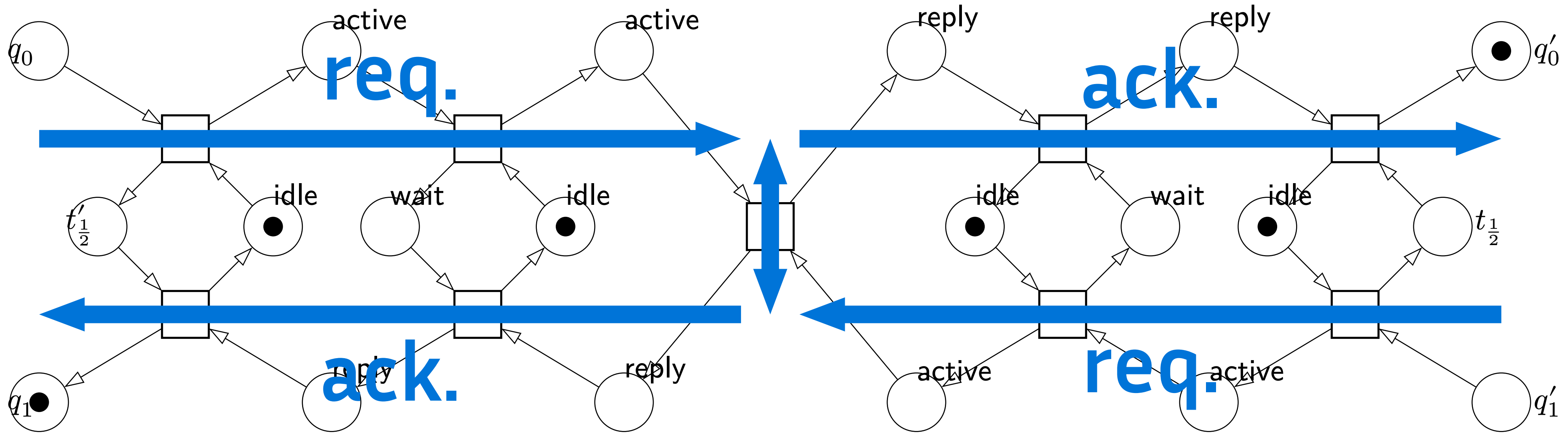
Communication through routers


$p_{\frac{1}{2}}$
$(\mathsf{try}_{t'}, \mathsf{recv})$
$(\mathsf{commit}_{t'}, \mathsf{reset})$
ε
$(\mathsf{send}, \mathsf{recv})$
$(\mathsf{ack}, \mathsf{reset})$
ε
(t', t)
ε
$(\mathsf{recv}, \mathsf{send})$
$(\mathsf{reset}, \mathsf{ack})$
ε
$(\mathsf{recv}, \mathsf{try}_t)$
$(\mathsf{reset}, \mathsf{commit}_t)$
$p'_{\frac{1}{2}}$

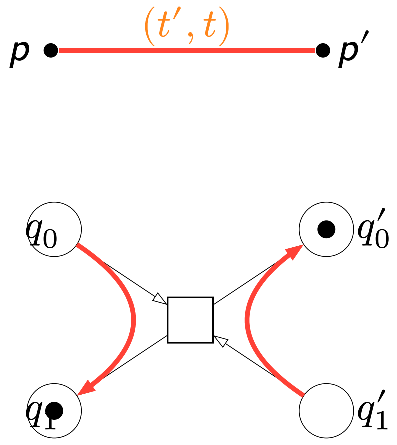


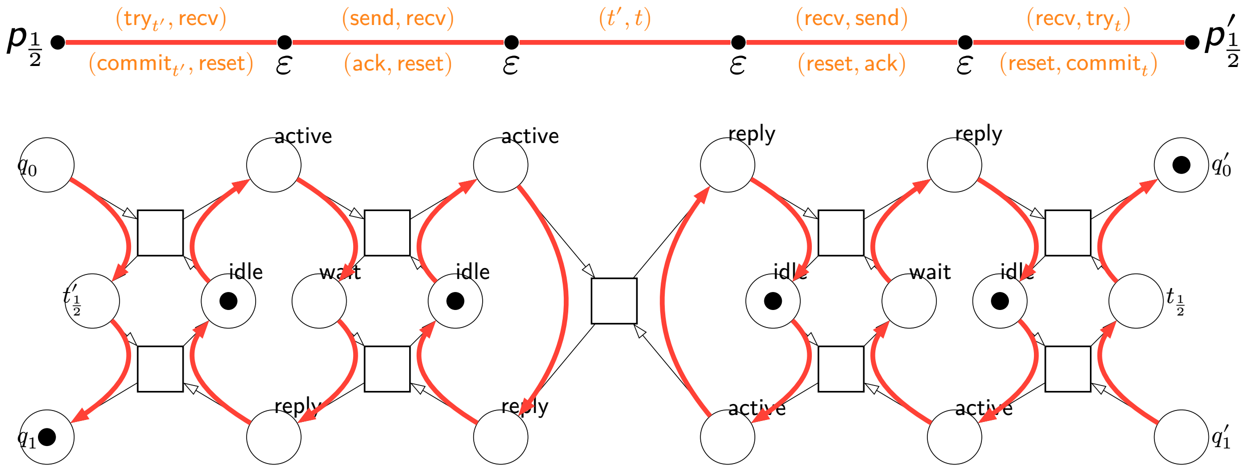
Communication through routers

$$\begin{tikzcd}
p_{\frac{1}{2}} \arrow[r, dash, "(\mathsf{try}_{t'}, \mathsf{recv})", "(\mathsf{commit}_{t'}, \mathsf{reset})"'] & \varepsilon \arrow[r, dash, "(\mathsf{send}, \mathsf{recv})", "(\mathsf{ack}, \mathsf{reset})"'] & \varepsilon \arrow[r, dash, "(t', t)"] & \varepsilon \arrow[r, dash, "(\mathsf{recv}, \mathsf{send})", "(\mathsf{reset}, \mathsf{ack})"'] & \varepsilon \arrow[r, dash, "(\mathsf{recv}, \mathsf{try}_t)", "(\mathsf{reset}, \mathsf{commit}_t)"'] & p'_{\frac{1}{2}}
\end{tikzcd}$$



Stuttering





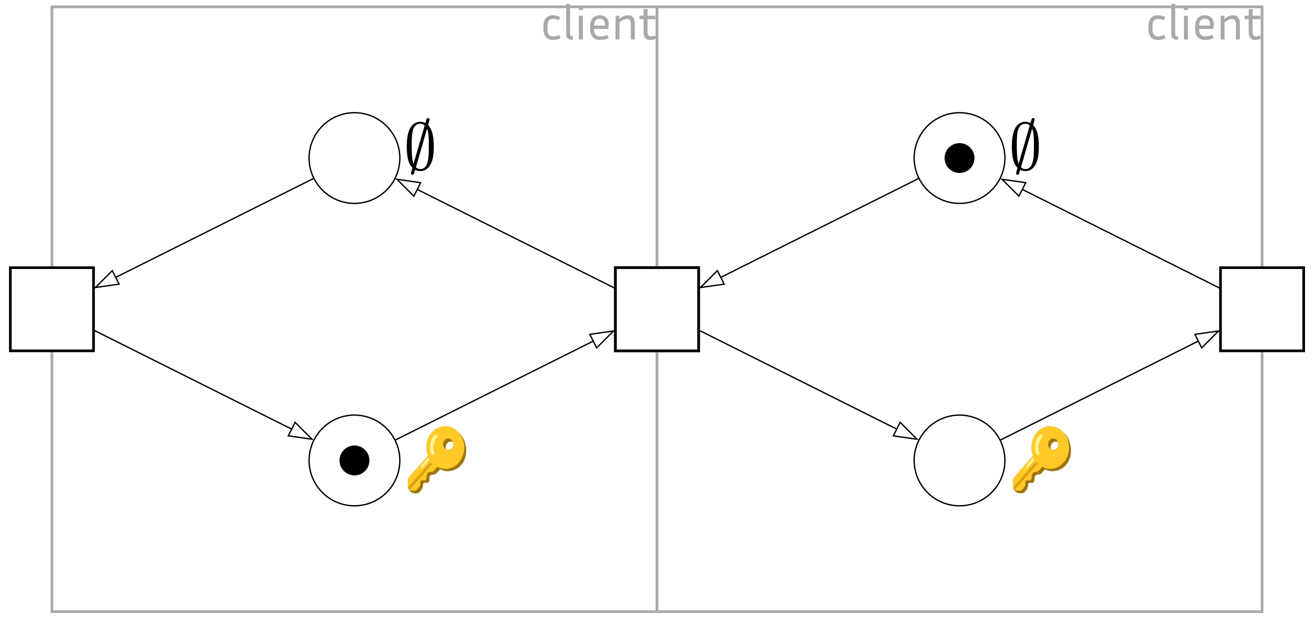
$p_{\frac{1}{2}}$
$(\text{try}_{t'}, \text{recv})$
$(\text{commit}_{t'}, \text{reset})$
ε
$(\text{send}, \text{recv})$
$(\text{ack}, \text{reset})$
ε
(t', t)
ε
$(\text{recv}, \text{send})$
$(\text{reset}, \text{ack})$
ε
$(\text{recv}, \text{try}_t)$
$(\text{reset}, \text{commit}_t)$
$p'_{\frac{1}{2}}$
q_0
active
active
reply
reply
q'_0
$t'_{\frac{1}{2}}$
idle
wait
idle
idle
wait
idle
$t_{\frac{1}{2}}$
q_1
reply
reply
active
active
q'_1

$$s_1 s_2$$

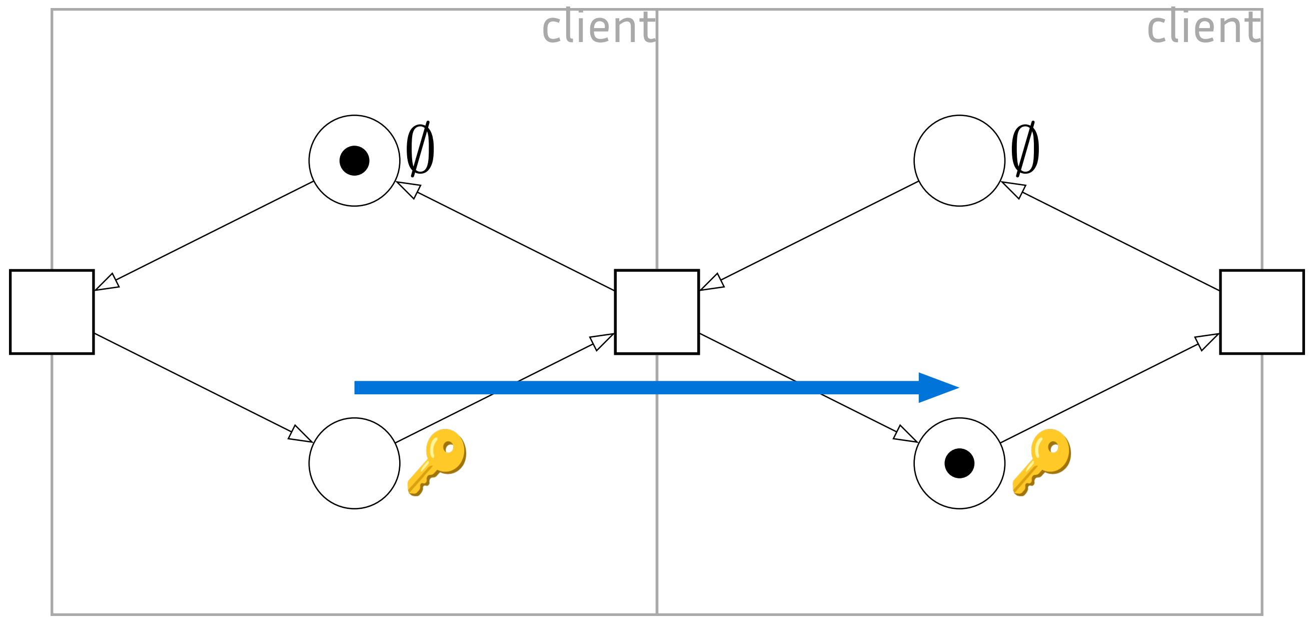
$$s_1 s_1 s_1 s_1 s_1 s_2 s_2 s_2 s_2 s_2$$

Decidable Restrictions

Token Passing



Token Passing



Decidability

	Known	***New***	***WIP***
communication	token passing		
processes	arbitrary	≤ 2 states	≤ 1 non-token state
tokens	exactly 1	arbitrary	exactly 1
architecture	MSO	HR	MSO
property	$\mathrm{CTL}^* \setminus \mathrm{X}$	cover	μ-calculus

Parameterized model checking of rendezvous systems;
by B. Aminof, T. Kotek, S. Rubin, F. Spegni, H. Veith; in Distributed Computing (2017)

Conclusion

- semi-algorithm + implementation for HR
- translation procedure for VR
- decidable classes inspired by literature

More details:

- Counting Abstraction and Decidability for the Verification of Structured Parameterized Networks; in CAV'25
- Verifying Parameterized Networks Specified by Vertex-Replacement Graph Grammars; in NETYS'25

Future work

- refinements
- extend decidable classes
- other communication models