MATH Dataset (Ours)

Problem: Tom has a red marble, a green marble, a blue marble, and three identical yellow marbles. How many different groups of two marbles can Tom choose?

Solution: There are two cases here: either Tom chooses two yellow marbles (1 result), or he chooses two marbles of different colors ($\binom{4}{2} = 6$ results). The total number of distinct pairs of marbles Tom can choose is $1 + 6 = \boxed{7}$.

Problem: If $\sum_{n=0}^{\infty} \cos^{2n} \theta = 5$, what is $\cos 2\theta$?

Solution: This geometric series is $1 + \cos^2 \theta + \cos^4 \theta + \cdots = \frac{1}{1-\cos^2 \theta} = 5$. Hence, $\cos^2 \theta = \frac{4}{5}$. Then $\cos 2\theta = 2\cos^2 \theta - 1 = \boxed{\dfrac{3}{5}}$.

Problem: The equation $x^2 + 2x = i$ has two complex solutions. Determine the product of their real parts.

Solution: Complete the square by adding 1 to each side. Then $(x+1)^2 = 1 + i = e^{\frac{i\pi}{4}} \sqrt{2}$, so $x + 1 = \pm e^{\frac{i\pi}{8}} \sqrt[4]{2}$. The desired product is then $\left(-1 + \cos\left(\frac{\pi}{8}\right) \sqrt[4]{2}\right)\left(-1 - \cos\left(\frac{\pi}{8}\right) \sqrt[4]{2}\right) =$ $1 - \cos^2\left(\frac{\pi}{8}\right) \sqrt{2} = 1 - \frac{\left(1 + \cos\left(\frac{\pi}{4}\right)\right)}{2} \sqrt{2} = \boxed{\dfrac{1 - \sqrt{2}}{2}}$.