Discrete Mathematics (2025)
Mandatory Assignment 1

1. Solve for x in the following equations.

   (a) $\frac{5}{3}\,x + \frac{2}{9} = 3$
   (b) $3\,x - \frac{3}{7} = \frac{x}{3} + 2$
   (c) $|\frac{5}{3}\,x| = \frac{3}{4}$
   (d) $\frac{7\,(1-3x)}{4} = \frac{4}{3}x + 1$

   NB: You need to show *how* you obtained the solution. It is not enough to simply give the solution for x.

2. Consider the compound proposition $(p \wedge \sim q) \to (p \vee q)$.

   (a) Construct the truth table of $(p \wedge \sim q) \to (p \vee q)$.
   (b) Is $(p \wedge \sim q) \to (p \vee q)$ a tautology, a contradiction, or neither a tautology nor a contradiction? Give a justification for your answer.
   (c) Give a compound proposition that is a contradiction and contains at least $\wedge$, $\vee$, $\sim$, p, and q.

3. Use the logical equivalences in Figure 1 on the next page to verify the following logical equivalences step by step. Supply a reason for each step, i.e. state which of the logical equivalences from Figure 1 you used.

   (a) $\sim(\sim p \vee \sim q) \equiv p \wedge q$
   (b) $p \wedge (q \vee \sim p) \equiv q \wedge p$
   (c) $(q \wedge p) \to r \equiv p \to (q \to r)$
   (d) $p \leftrightarrow q \equiv (\sim p \wedge \sim q) \vee (p \wedge q)$

   For example, to verify the equivalence $(p \wedge q) \wedge q \equiv p \wedge q$, you would write:

$$\begin{aligned} & (p \wedge q) \wedge q & \\ \equiv\ & p \wedge (q \wedge q) & (2) \\ \equiv\ & p \wedge q & (7) \end{aligned} \qquad \text{or} \qquad \begin{aligned} & (p \wedge q) \wedge q \\ \overset{(2)}{\equiv}\ & p \wedge (q \wedge q) \\ \overset{(7)}{\equiv}\ & p \wedge q \end{aligned}$$

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold:

1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. Negation laws:	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. Double negative law:	$\sim(\sim p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. Universal bound laws:	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. De Morgan's laws:	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of $\mathbf{t}$ and $\mathbf{c}$:	$\sim\mathbf{t} \equiv \mathbf{c}$	$\sim\mathbf{c} \equiv \mathbf{t}$
12. Conditional:	$\sim p \vee q \equiv p \to q$	$p \to q \equiv \sim q \to \sim p$
13. Biconditional:	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	$p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

Figure 1: Logical equivalences.