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Parameter Estimation Assignment

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Q1 $(x_1, x_2, \ldots x_n) \Rightarrow$ random sample of size n from Normal distribution

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$\theta_1 \rightarrow$ mean, $\theta_2 \rightarrow$ variance

To find MLE of these 2 parameters

$$L(x_1, x_2, x_3, \ldots x_n) = L(x_1) \times L(x_2) \ldots L(x_n)$$

$$\Rightarrow \frac{1}{\sqrt{2\pi\theta_2}} \times e^{\frac{-(x_1-\theta_1)^2}{2\theta_2}} \times \frac{1}{\sqrt{2\pi\theta_2}} \times e^{\frac{-(x_2-\theta_1)^2}{2\theta_2}} \ldots$$

$$\ldots \frac{1}{\sqrt{2\pi\theta_2}} \times \frac{-(x_n-\theta_1)^2}{2\theta_2}$$

$$\approx \frac{1}{\sqrt{2\pi\theta_2}} \prod_{i=1}^{n} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$$

Taking log on both side

$$\ln(L(x_1, x_2 \ldots x_n)) = \ln\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_1-\theta_1)^2}{2\theta_2}} \times \ldots \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_n-\theta_1)^2}{2\theta_2}}\right)$$

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$$\Rightarrow \ln\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_1-\theta_1)^2}{2\theta_2}}\right) + \ln\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_2-\theta_1)^2}{2\theta_2}}\right) \cdots$$

$$\cdots \ln\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_n-\theta_1)^2}{2\theta_2}}\right)$$

Now $\ln\left(\frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}\right)$ can be written as

$$\Rightarrow \ln\left(\frac{1}{\sqrt{2\pi\theta_2}}\right) + \ln\left(e^{-(x_1-\theta_1)^2/2\theta_2}\right) \rightarrow (b)$$

Now $\ln\left(\frac{1}{\sqrt{2\pi\theta_2}}\right) \Rightarrow \ln\left((2\pi\theta_2)^{-1/2}\right)$

Now simplifying (b) $\theta_2 = \sigma^2$

$$-\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x_1-\theta_1)^2}{2\theta_2}\ln(e)$$

$$\Rightarrow -\frac{1}{2}\left(\ln 2\pi + \ln \theta_2\right) - \frac{(x_1-\theta_1)^2}{2\theta_2}\ln(e)$$

$$\Rightarrow -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\theta_2) - \frac{(x_1-\theta_1)^2}{2\theta_2} \quad [\ln e = 1]$$

$$\Rightarrow -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\theta_2 - \frac{(x-\theta_1)^2}{2\theta_2}$$

This we got for b , Similarly we will get this for other n-1 terms

$$\therefore L(x_1, x_2 \ldots x_n) = \cdots - \frac{n}{2}\ln 2\pi - \frac{n}{2}\ln\theta_2 - \frac{(x_1-\theta_1)^2}{2\theta_2}$$

$$- - - - - - \frac{(x_n-\theta_1)^2}{2\theta_2}$$

Taking $\frac{\delta}{\delta\theta_1}$ on both side

By chain rule

$$\frac{\delta(L(x_1 \ldots x_n))}{\delta\theta_1} \Rightarrow 0 - 0 + \frac{x_1-\theta_1}{\theta_2} + \ldots \frac{x_n-\theta_1}{\theta_2}$$

$$\Rightarrow \frac{1}{\theta_2}(x_1-\theta_1 + \ldots \ldots x_n-\theta_1)$$

$$\frac{\delta(L(x_1 \ldots x_n))}{\delta\theta_1} = \frac{1}{\theta_2}[(x_1+x_2 \ldots x_n) - n\theta_1]$$

Now take $\frac{\delta}{\delta\theta_2}$

$$\frac{\delta L(x_1 \ldots x_n)}{\delta \theta_2} = 0 - \frac{n}{2} \times \frac{1}{\theta_2} + \frac{(x_1 - \theta_1)^2}{2\theta_2^2} + \frac{(x_2 - \theta_1)^2}{2\theta_2^2}$$

$$\ldots \frac{(x_n - \mu)^2}{2\theta_2^2}$$

$$= \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\left((x_1 - \theta_1)^2 \ldots (x_n - \theta_1)^2\right)$$

For MLE over θ_1 put $\frac{\delta L(x_1 \ldots x_n)}{\delta \theta_1} = 0$

$$0 = \frac{1}{\theta_2}\left[(x_1 \ldots x_n) - n\theta_1\right]$$

$$x_1 + \ldots x_n = n\theta_1$$

$$\Rightarrow \frac{x_1 + x_2 + \ldots x_n}{n} = \theta_1$$

$$\Rightarrow \boxed{\theta_1 = \text{mean} = \frac{x_1 + x_2 \ldots x_n}{n}}$$

For MLE over θ_2 we put $\frac{\delta L(x_1 \ldots x_n)}{\delta \theta_2} = 0$

$$0 = \frac{-n}{2\theta_2} + \frac{1}{2\theta_2^2}\left((x_1 - \theta_1)^2 \ldots (x_n - \theta_1)^2\right)$$

$$0 = -n + \frac{1}{\theta_2}\left((x_1-\theta_1)^2 - - - (x_n-\theta_1)^2\right)$$

$$n\theta_2 = (x_1-\theta_1)^2 + - - - + (x_n-\theta_1)^2$$

$$\boxed{\theta_2 = \frac{(x_1-\theta_1)^2 + - - + (x_n-\theta_1)^2}{n}}$$

(variance for measurement)

Q2 $X_1 - - - - X_n \in B(m, \theta)$

$\theta \in \Theta(0,1)$

It is Binomial distribution

m is +ve integer.

To do MLE for θ parameter

$$P(x, \theta) = {}^mC_k\, \theta^{x}(1-\theta)^{m-x}$$

~~Take log on both side~~

$$L(x_1 - - - - x_n) = \prod_{i=1}^{n} P(x_i, \theta)$$

Take log on both side

P.T.O

$$\ln L(x_1 \ldots x_n) = \sum_{i=1}^{n} \ln({}^{m}C_{x_i}) + x_i \ln\theta + (m - x_i)\log(1-\theta)$$

Now do $\dfrac{\delta(\ln L(x_1 \ldots x_n))}{\delta\theta}$

$$\frac{\delta \ln L(x_1 \ldots x_n)}{\delta\theta} \Rightarrow \sum_{i=1}^{n} 0 + \frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}$$

$$= \sum_{i=1}^{m} \frac{x_i(1-\theta) - (m + x_i)\theta}{\theta(1-\theta)}$$

$$= \sum_{i=1}^{m} \frac{x_i - m\theta}{\theta(1-\theta)}$$

To maximize Put $= 0$

$$\sum_{i=1}^{m} x_i - m\theta = 0$$

$$\sum_{i=1}^{n} x_i = m\theta n$$

$$\left(\frac{\sum x_i}{n} = \bar{x}\right)$$

$$\theta = \frac{\sum_{i=1}^{n} x_i}{mn} \Rightarrow \therefore \boxed{\theta = \frac{\bar{x}}{m}}$$

$$\therefore \bar{x} = m\theta$$

Hence MLE for $\theta = \dfrac{\bar{x}}{m}$