Candidate Code- jtk011

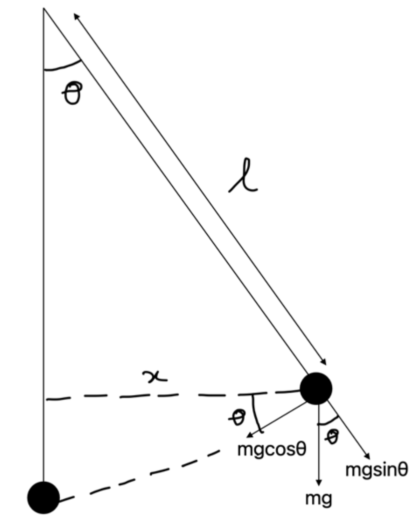
Introduction

In the beginning of IBDP 1, my physics teacher had asked my whole class to write a topic which interested them, and I wrote waves and oscillations on that sheet due to my exciting experience with this topic in MYP. He had later asked us to find interesting investigations which we could explore within the topic of interest. I couldn't find any topic which I could click with until we did the unit of circular motion and gravitation where my teacher mentioned while using small angle approximation, we could write $\sin\theta\ approx = \theta$, which made me wonder why we consider them to be the same and by how much does the value deviate. I then went on to research why it's like this and I found out that the value is very close hence $\sin\theta = \theta$, but then unit 1 helped motivated me to find out how much percentage uncertainty we will get from using $\sin\theta\ as\ \theta$. I was looking for situations where $\sin\theta$ is considered as θ and I found out that it's not just in circular motion but it's also in simple harmonic motion, which triggered a flashback of the time I was researching for a research topic within waves so this time I decided to dive deeper into a simple pendulum and I recollected how my room clock in my old house had a pendulum right at the bottom which oscillated every second. This was when I decided that I want to work and research into an investigation of the percentage deviation of the Time period of a simple pendulum with and without using small angle approximate.

Research Question

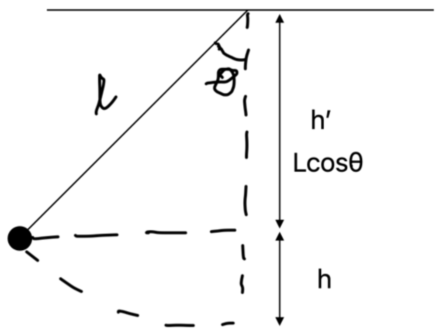
How does the utilization of the small angle approximation on the angles 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50° affect the accuracy of determining the time period compared to the actual value?

Background Research



Self-Made on Freeform

A pendulum is an object that moves back and forth around a fixed point because of gravity. A simple pendulum is a type of pendulum that consists of a bob, which represents a concentrated mass, connected to a lightweight string or rod. The string or rod hangs from a fixed point of suspension, as shown in Figure 1. A simple pendulum is a classic example of simple harmonic motion (SHM). In this situation, the pendulum experiences a restoring force that acts upon it. The intensity of this force is directly related to how much the pendulum is displaced and always points towards a specific reference point. This point is the vertical position where the bob stays still, indicating no movement. This specific point is also known as the equilibrium position, as shown in Figure 1. The equation which is used with small angle approximate is $T = 2\pi\sqrt{\frac{L}{g}}$. This equation is derived from the concept of force. With the concept of the Pythagoras theorem, we can say that $sin\theta = \frac{x}{l}$. Since we are using small angle approximate where $sin\theta = \theta$ then we can say that $\theta = \frac{x}{l}$. With reference to the diagram above, we can say that $m \times a = -mgsin\theta$ and since $a = -\omega^2 x$, we can derive $\omega^2 x = g(\frac{x}{l})$. Since $\omega = \frac{2\pi}{T}$, then $(\frac{2\pi}{T})^2 = \frac{g}{l}$. With which we can understand how $T = 2\pi\sqrt{\frac{L}{g}}$.[1] The problem with this formula is that this will not tell us the period of a simple pendulum at any given angle, and if we are going to compare the value at any given angle, we require the formula to give us the time period at different angles, the problem with a simple pendulum is that it gives use the time period of a complete period. After some more research, I found out that the formula of a large amplitude pendulum will give us the time period at different angles at different times. I came across a series, $\frac{T}{T_0} = 1 + \frac{1}{16}\alpha^2 + \frac{11}{3072}\alpha^4 + \cdots$. Where T_0 is the small angle result and α is the angle of release. This series is



Self-Made on Freeform

[1] Oscillations - Umich, www-personal.umd.umich.edu/~jameshet/IntroLabs/IntroLabDocuments/150-11%20Oscillations/Oscillations%200.5.doc. Accessed 10 June. 2023.

derived from the concept of energy where $P.E = mgL(1 - cos\theta)$. The potential energy is maximum at the bottom of the pendulum as the hight is maximum at the bottom of the pendulum. The height of the pendulum could be found out by the equation $h = L - h'$ and since $h' = Lcos\theta$ using the Pythagoras theorem, we can say that $h = L(1 - cos\theta)$. Hence we arrive to the equation $P.E = mgL(1 - cos\theta)$. Similarly for kinetic energy the equation we got is $K.E = \frac{1}{2}m(\omega L)^2$ where ω is angular velocity. Since $V = \omega L$, kinetic energy could be represented as the above equation. The internal energy of the pendulum is $E = mgL(1 + cos\theta) + \frac{1}{2}m(\omega L)^2$. When the pendulum is released from an angle α at $t = 0$, the internal energy at that point of the pendlum can can be represented as $mgL(1 - cos\theta) = mgL(1 - cos\theta) + \frac{1}{2}m(\omega L)^2$. This equation could be rearranged as $\omega = \frac{d\theta}{dt} = \sqrt{\frac{2g}{L}[cos\theta - cos\alpha]}$ and with the trigonometric equations $cosx = 1 - 2sin^2\frac{x}{2}, \frac{d\theta}{dt} = \sqrt{\frac{4g}{L}[sin^2\left(\frac{\alpha}{2}\right) - sin^2(\frac{\theta}{2})]}$. This equation can then be integrated from θ = 0 to θ = α which is the quarter-period time interval t = 0 to t = τ/4. The result is $\int_0^{\tau/4} dt = \frac{\tau}{4} = \sqrt{\frac{L}{4g}}\int_0^{\alpha}[sin^2\left(\frac{\alpha}{2}\right) - sin^2(\frac{\theta}{2})]^{-\frac{1}{2}} d\theta$. . $\frac{\tau}{\tau_0} = 1 + (\frac{1}{2})^2k^2 + (\frac{1\times3}{2\times4})^2k^4 + (\frac{1\times3\times5}{2\times4\times6})^2k^6 + \cdots$ which helped me reached the series $\frac{T}{T_0} = 1 + \frac{1}{16}\alpha^2 + \frac{11}{3072}\alpha^4 + \cdots$.

I then compared a large amplitude period and a simple pendulum and found out how they differ from each other and my result was :

- Motion
  - A simple pendulums undergoes simple harmonic motion, while a large amplitude pendulum starts to deviate from simple harmonic motion after around 30°.
- Energy conservation
  - There will be energy losses in case of large angles will be more because of the change in velocity.
- Period
  - The period with small angle is $T = 2\pi\sqrt{\frac{L}{g}}$. However for a large amplitude pendulum it is this series, $\frac{T}{T_0} = 1 + \frac{1}{16}\alpha^2 + \frac{11}{3072}\alpha^4 + \cdots$.

After finding out a formula which I could use to find the time period of a large amplitude pendulum, I found out factors which affect a large amplitude pendulum so that I can make sure that they are controlled while experimenting. Factors effecting the Time period of a Large Amplitude Pendulum are:

- A large amplitude pendulum's length continues to be an important factor. Assuming all other variables remain constant, longer pendulums have longer time periods and shorter pendulums ones shorter time periods.
- Damping may result in a decrease in amplitude and an increase in the time period. Damping is the term for the existence of forces that prevent the pendulum from swinging.
- Mass does not directly affect the time period, but if there is more mass, there will be a larger radius and larger radius means more length. The centre of mass will change for the bigger mass. Since the centre of mass is going the change, the radius of the bob will also change. With the change in mass, the radius gets affected and thus needs to be kept constant.

Hypothesis

I hypothesis that the resulting estimated time period will be slightly shorter than the actual value. When I am using the small angle approximation, so long as the value remains between a certain angle, the value are close to correct. But if I am going to use the angle and time people as calculated with small angle approximation, it varies with the actual values.

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Design and methodology

Independent Variable	
What will be changed?	How will I change it?
Angle of release (in °).	I will be releasing the bob from different angles (10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°) to measure the trend in the time period with the help of a protractor.
Dependent Variable	
What needs to be measured?	How will it be measured?
The time period of one oscillation (in *s*).	I will be using a motion sensor by Vernier which will be set in pendulum mode and which is used for measuring the time period.

Controlled Variables		
Controlled Variable	Why it needs to be controlled	How will it be controlled
Pendulum string length, diameter of the string, nature of the string.	The length of the pendulum string directly affects the period of oscillation, which is the time it takes for the pendulum to complete one full swing back and forth. Different natures of string will react differently to air friction.	The same pendulum will be used for all my trials to ensure that a fair test has been conducted.
Internal dampening and air resistance.	Controlling internal dampening and air resistance ensures consistent conditions for accurate time period determination with the small angle approximation, minimizing variables that could affect experimental outcomes and reliability.	The same pendulum will be used for all my trials to ensure that a fair test has been conducted. Making sure that the pivot point remains untouched.
Mass and Radius of the pendulum bob.	If mass increases, Surface area increases and radius will increase which will affect the overall length of the pendulum which affect the time period.	I will use the same bob for all the trials with a fixed radius and fixed mass. The radius of the bob will be 2.4cm and the fixed mass of the bob will be 21.284g.
Initial force with which the pendulum is released.	If the initial velocity of a pendulum is increased throughout the different trials and different angles, the value given not be exact because the pendulum takes less time to reach its highest point and return to its starting position, the overall time period of the pendulum decreases.	I will place my hand Infront of the bob before release and I will release my try to release my hand with the same force for all my trials.

Apparatus

Material	Name	Quantity	Notes/ Uncertainty	How will it be used?
Pendulum Set Up	G clamp	1	N/A	All these equipment will be used for the set-up of the pendulum. The G clamp will make sure that the stand will be static and will not move. The clamp will have the cork tightened which will have will the string with the bob attached to it. The clamp will be attached to the clamp stand.
	Clamp	1		
	Split cork	1		
	String	1	50±0.1cm	
	Bob	1	Weight is 21.284±0.001g and diameter is 2.454±0.001 cm	
	Clamp Stand	1	N/A	

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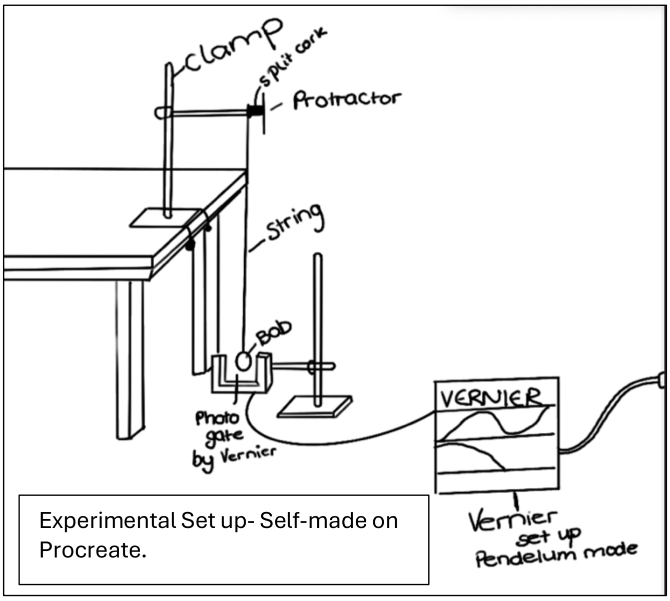
Material	Name	Quantity	Notes/ Uncertainty	How will it be used?
Measuring equipment	Measuring tape	1	200±0.05cm	This apparatus will be required to measure the length of the string.
	Digital Balance	1	±0.001g	This apparatus will be required to measure the weight of the bob.
	Vernier calliper	1	0.01cm	This apparatus will be required to measure the diameter of the bob.
	LabQuest	1	N/A	It is required for reading the data off
	Vernier Photo Gate.	1	±0.0001s	This apparatus will be required to measure the time period of a single oscillation.
	Protractor	1	±0.5 °	This apparatus will be required to measure the angle of release.
General Apparatus	Notebook and Pen	1	N/A	This apparatus will be required to note down the readings of all the trials and different angles.
	Scissors	1		Required to cut the string
	Blu Tack	1		Required for attaching the protractor to the pendulum.

Method

Set Up method

1. Place the Clamp stand near the edge of the table
2. Tighten the clamp stand to the table with the help of a G clamp. This is very important so that the pendulum stand does not quiver.
3. Measure a height of 50±0.05 cm from the base of the pendulum stand and attach the clamp to the stand.
4. Attach the split cork to the clamp.
5. Measure a 50 ± 0.05 cm string and cut it from the reel of the string.
6. Measure 43 ± 0.05 cm of the cut out piece of string and make sure that 43 ± 0.05cm is below the split cork.
7. Measure the mass of the bob with a digital balance and measure the diameter of the bob with a vernier calliper.
8. Attach the bob to the bottom of the string and tighten the clamp making sure that it is not wobbly and not easily detachable.
9. Attach the protractor to the pendulum by placing it at the bottom of the cork. We will use blu tack to attack the protractor.



Experimental Set up- Self-made on Procreate.

Conducting the experiment

1. Move the bob to an initial release of 5 $\pm$ 0.5° looking at the protractor.
2. Start the reading on the Vernier screen and release the bob at the exact same time.
3. Stop the reading after 60 seconds and observe the graph.
4. Note down the Time period from the LabQuest Screen. Next to the graphs, there will be a reading of the time period.

5. Repeat step’s from 2-5 with the remaining angles (15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°)
6. Repeat step 5, 5 times and take its average.

Safety Procedure and Environmental Considerations

In such an experiment there is very minimal safety requirements. Firstly making sure that the clamp stand is safely attached to the table. To avoid being hurt, taking a step backward as the pendulum swings would be ideal. The mass hanger is the only thing that might have fallen, and it was unlikely to have caused any damage. Looking at the nature of my experiment, there are not much environmental considerations to take account of. All the threads and all the blu-tac was properly disposed. Electricity was not wasted and hence no influence to the greenhouse effect.

Data Collection- Raw Data

	Time (s)(±0.0001s)				
Angle (±0.5°)	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
10	1.3797	1.3785	1.3781	1.3784	1.3778
15	1.3818	1.3823	1.3820	1.3826	1.3830
20	1.3865	1.3861	1.3855	1.3860	1.3856
25	1.3912	1.3894	1.3914	1.3934	1.3898
30	1.3976	1.3964	1.3962	1.3964	1.3964
35	1.4025	1.4044	1.3990	1.4034	1.4000
40	1.4144	1.4128	1.4057	1.4102	1.4114
45	1.4204	1.4243	1.4151	1.4233	1.4244
50	1.4277	1.4331	1.4382	1.4343	1.4344

Processing Data

For achieving the mean data I used the formula $\frac{Trial\ 1+Trail\ 2+Trial\ 3+Trial\ 4+Trial\ 5}{5}$.

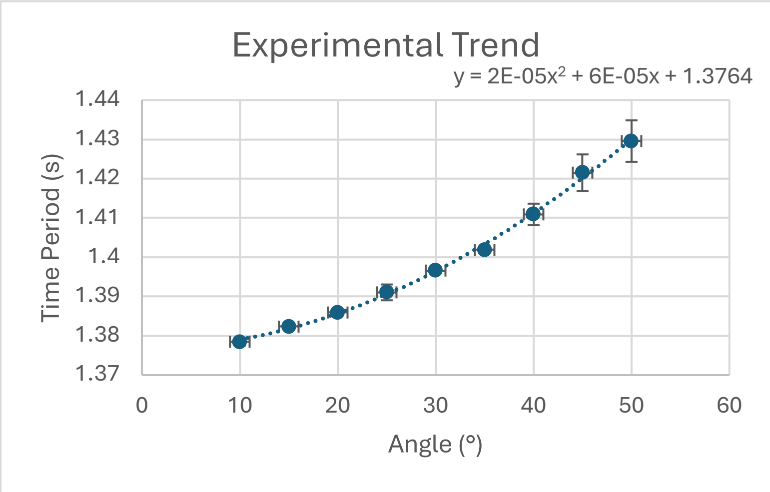
For Finding out the uncertainty I used the formula $\frac{Largest\ Value-smallest\ value}{2}$.

For example at 10±0.5°$(\frac{1.3797+1.3785+1.3785+1.3784+1.3778)}{5}) = 1.3786$

For example at 10±0.5° $\left(\frac{1.3797-1.3778}{2}\right) = 0.00095$

Processed Experimental Data

Angle(±0.5°)	Time period (s)
10	1.3786 ± 0.0010
15	1.3823 ± 0.0006
20	1.3859 ± 0.0005
25	1.3911 ± 0.0020
30	1.3966 ± 0.0007
35	1.4019 ± 0.0007
40	1.4109 ± 0.0027
45	1.4215 ± 0.0046
50	1.4296 ± 0.0053



Using the data obtained above, two graphs have been constructed to give an illustration of the relationship between the angle and the time period which was found out during the experiment and while finding out the theoretical data. The various angles of release which I used to are displayed on the x-axis as the independent variable whereas the average time period over the 5 trials are on the y-axis as the dependent variable. The values are supported with their various errors bars to show the maximum possible error. The y intercept of this trend line is 1.3616 and it is increasing with a slope of 0.0013 seconds. I used Microsoft Excel for generating the graphs based on the processed data because it facilitated the inclusion of error bars and the determination of the best-fit line. The graph illustrates the correlation between the various angles and the mean time period. This graph displays the experimental data, which was acquired during the experiment. The horizontal uncertainty in the angles is $\pm 0.5°$ as mentioned in all the data tables due to the uncertainty of a protractor being $\pm 0.5°$. The vertical uncertainty is has been plotted as per the data table, some uncertainty's are so small that they are not visible. We can analyse this data by applying standard deviation as a metric to assess the dispersion or spread of data points in relation to the mean value. Before finding the standard deviation, I will linearize the graph.

$$f(x) = \frac{x^2}{50000} + \frac{3x}{50000} + \frac{3441}{2500} \; at \; x_0 = \frac{3441}{2500}$$

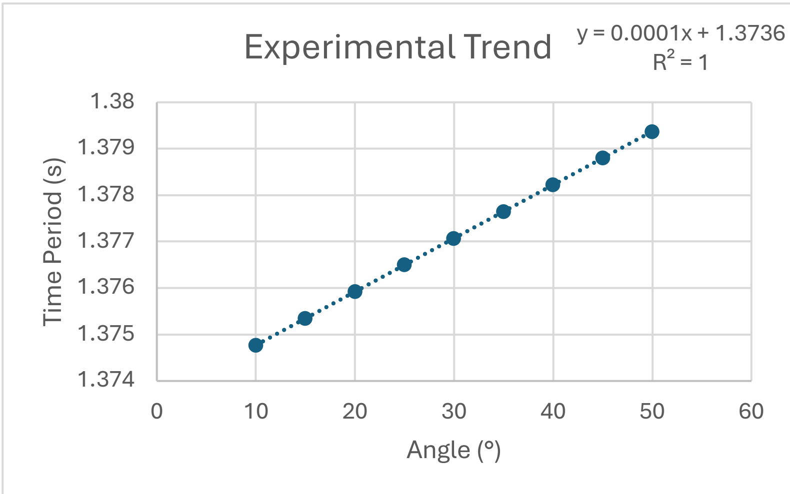
$$\frac{dy}{dx}(f(x)\,dx) = \frac{x}{25000} + \frac{3}{50000}$$

$$using\ the\ formula\ for\ \text{linear approximation}$$

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$L(x) \approx \frac{430162647981}{312500000000} + \frac{7191}{62500000}\left(x - \frac{3441}{2500}\right)$$

$$L(x) \approx \frac{7191}{62500000}x + \frac{420113159519}{312500000000} \approx 0.000115056x + 1.373621104608$$



$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}}$$

Where σ is the standard deviation,
N is the size of the angles
x_i is the value of the angle
μ is the population mean

$$\mu_{experimental} = \frac{1.3785 + 1.3823 + 1.3859 + 1.3911 + 1.3966 + 1.4019 + 1.4109 + 1.4215 + 1.4336}{9}$$

$$\mu_{experimental} = \frac{12.6023}{9} = 1.4002508889 \approx 1.4003$$

$$\sigma_{experimental} = \sqrt{\frac{(1.3785 - 1.4003)^2 + (1.3823 - 1.4003)^2 + (1.3859 - 1.4003)^2 + \cdots + (1.4336 - 1.4003)^2}{9}}$$

$$\sigma_{experimental} = 0.01755896783 \approx 0.0176$$

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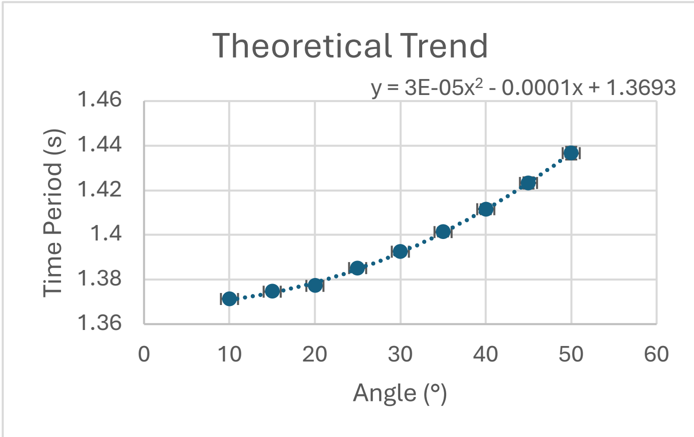
Theoretical Data

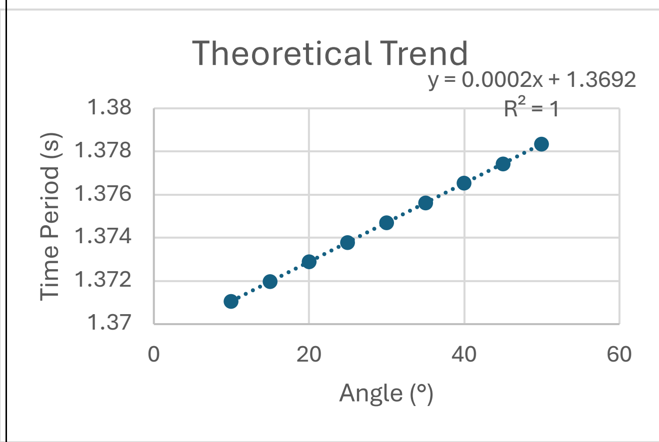
Using this series from a calculator $\frac{T}{T_0} = 1 + \frac{1}{16}\alpha^2 + \frac{11}{3072}\alpha^4 + \cdots$ to see if the theoretic value is similar to the experimental value. An online calculator[2] was used to measure the time period as this is a very long series.

The uncertainty in each angle is($\pm 0.5°$) so with the calculator 1, the time period was measured above and below the angle of measure and then they were subtracted from each other and divided by 2, for example when the angle is $10 \pm 0.5°$, the time period at 10.5 seconds and 9.5 seconds was found out. Time period at $9.5°$ is 1.3708 and at $10.5°$ is 1.3718. $\frac{1.3718-1.3708}{2} = 0.0005$, the uncertainty at $10°$ is ± 0.0005. The uncertainty was calculated in terms of range of the data. Similarly the same was done with all other angles.

Angle ($\pm 0.5°$)	Theoretical value
10	1.3713 ± 0.0015
15	1.3745 ± 0.0008
20	1.3771 ± 0.0011
25	1.3851 ± 0.0013
30	1.3925 ± 0.0016
35	1.4013 ± 0.0019
40	1.4115 ± 0.0022
45	1.4233 ± 0.0025
50	1.4366 ± 0.0028

Another graph presents the angles of release on the x-axis as the independent variable, while the average theoretical time period over five trials is on the y-axis as the dependent variable. Each data point is accompanied by error bars, representing the maximum possible error. The trend line for this graph has a y-intercept of 1.3478 and a positive slope of 0.0016 seconds. The graph demonstrates the relationship between the different angles and the mean theoretical time period, reflecting data obtained during the experiment. Regarding uncertainty, the horizontal uncertainty in the angles is ±1°, as stated in all the data tables, accounting for the inherent ±1° uncertainty associated with a protractor. Vertical uncertainty is depicted in accordance with the data table, although some uncertainties are too small to be visible. To further analyse this data, we can utilize standard deviation as a measure to gauge the extent to which data points deviate from the mean value. Before finding the standard deviation, I will linearize the graph.





$$f(x) = \frac{3x^2}{100000} + \frac{x}{10000} + \frac{13693}{10000} \; at \; x_0 = \frac{13693}{10000}$$

$$\frac{dy}{dx}(f(x)\,dx) = \frac{3x+5}{50000}$$

$$using\ the\ formula\ for\ \text{linear approximation}$$

[2] http://hyperphysics.phy-astr.gsu.edu/hbase/pendl.html#c2

$$L(x) = f(x_0) + f'(x_0)(x - x_0)$$

$$L(x) \approx \frac{13692193194747}{10000000000000} + \frac{91079}{500000000}\left(x - \frac{13693}{10000}\right)$$

$$L(x) \approx \frac{91079}{500000000}x + \frac{13692437505253}{10000000000000} \approx 0.000182158x + 1.3692437505253$$

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$$

$$\mu_{theoretical} = \frac{12.575225}{9} = 1.397247211 \approx 1.397247$$

$$\sigma_{theoretical} = \sqrt{\frac{(1.3713 - 1.3972)^2 + (1.3745 - 1.3972)^2 + (1.3792 - 1.3972)^2 + \cdots + (1.4366 - 1.3972)^2}{9}}$$

$$\sigma_{theoretical} = 0.02144069828 \approx 0.0214$$

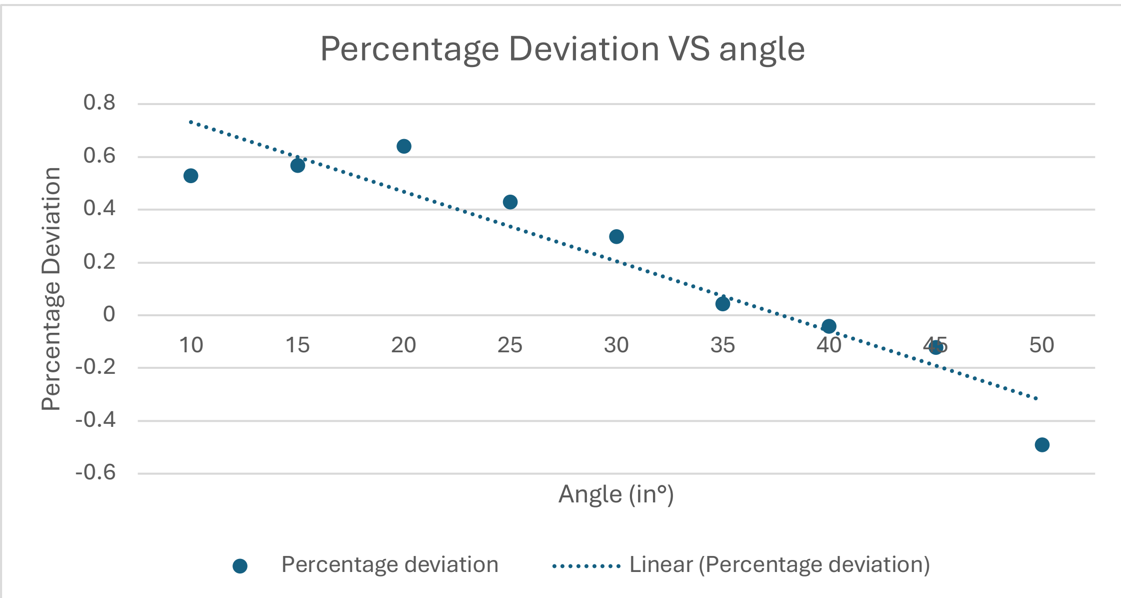
Processed Data Table

Finding out the percentage deviation

The formula I applied for finding out the Percentage deviation is $\frac{\text{Experimental Value} - \text{Theoretical Value}}{\textit{Theoretical Value}} \times 100$. This formula will help me find out by how much my value is deviating from the theoretical value and how at which angle the time $\sin\theta \neq \theta$

For example for $10 \pm 0.5°$ $\Rightarrow \frac{1.3785 - 1.3713}{1.3713} \times 100 = 0.5273$

Angle ($\pm 0.5°$)	Experimental Value (s)	Theoretical Value (s)	Experimental Value- Theoretical Value(s)	Percentage Deviation(%)
10	1.3785 ± 0.0010	1.3713 ± 0.0005	0.0072 ± 0.0015	0.5273 ± 0.0020
15	1.3823 ± 0.0006	1.3745 ± 0.0008	0.0078 ± 0.0014	0.5675 ± 0.0022
20	1.3859 ± 0.0005	1.3771 ± 0.0011	0.0088 ± 0.0016	0.6381 ± 0.0010
25	1.3911 ± 0.0020	1.3851 ± 0.0013	0.0059 ± 0.0033	0.4286 ± 0.0047
30	1.3966 ± 0.0007	1.3925 ± 0.0016	0.0041 ± 0.0023	0.2967 ± 0.0039
35	1.4019 ± 0.0007	1.4013 ± 0.0019	0.0006 ± 0.0026	0.0429 ± 0.0045
40	1.4109 ± 0.0027	1.4115 ± 0.0022	-0.0006 ± 0.0049	-0.0435 ± 0.0071
45	1.4215 ± 0.0046	1.4233 ± 0.0025	-0.0018 ± 0.0072	-0.1238 ± 0.0097
50	1.4296 ± 0.0053	1.4366 ± 0.0028	-0.0071 ± 0.0081	-0.4927 ± 0.0109



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Conclusion

The objective of this experiment was a comparison to see how the variation and trench changes if we do not use small angle approximation with the actual value, from which angle the deviation is shown also to see by how much does it deviate from the theoretical value. I hypothesised that with using the small angle approximation to calculate the time period of a pendulum, the resulting value will always be shorter than the accurate measurement. My data supports my hypothesis and it can be said that the pendulum stops following small angle approximate between 35 and 40 degrees as can be seen from graph 5, where the trend line crosses the x axis. It suggests that the theoretical value is no longer shorter than the experimental value. We can also see by the y intercepts of the experiment result (graph 2) that a very accurate experiment was conducted and seeing the difference in the standard deviation between the theoretical and experimental data to be 0.0038 shows that the experiment was showing that $\sin\theta = \theta$ until the angle becomes so large. We can also see by the percentage deviation that it is all at the first decimal and even second decimal at some places justifying why it is acceptable to use $\sin\theta\ as\ \theta$. We get to witness how as angles increase, the time period also increases by a minute value but still it increases, using small angle approximate may not always be the best way to go on about finding the time period as it tells the time period for a complete oscillation. Throughout this investigation, the research question has been addressed as we found out that the accuracy using small angle approximate is not changing much, it's only having an impact on the first or second decimal while calculating the percentage deviation.

Strengths, Limitations and Weaknesses

Strengths of the experiment	
Strength	Explanation
Number of Significant figures taken	A measurement is more accurate the more important numbers you have. Accuracy is crucial as it allow experiments that distinguish between very small changes and allows us to make meaningful comparisons. We can determine measurement uncertainty or error with the use of significant figures. The accuracy of the measuring device and possible causes of error are implicitly indicated when measurements are collected with a specified number of significant numbers.
Number of trials taken	Taking more trials helps us to lessen the impact of random errors or variations in measurements taken. Performing the experiment more than once helps to confirm the accuracy of the results. A single trials will not adequately show the is there is an issue in the trial for examples there is an outlier. Such deviations may be found and restricted with the taking more than one trial.
The corelation between the theoretical data and experimental data	The corelation between the experimental data and theoretical data is very close which shows how precise the experiment I performed is. This is a strength for my experiment as it shows that a fair experiment was conducted with fair results and since the corelation is very minute, it shows that the experiment conducted was accurate and the expected trend was followed.
Taking increments of 5 $\pm 0.5°$	Taking increments of 5 was helpful for indicating when the theoretical value become larger than the experimental value. The thing with theoretical value is that it does not consider air resistance while finding out the time period so when the value of the theoretical data overtakes the experimental data, $\sin\theta \neq \theta$. Taking increments of 5° helps us in taking that between 35° and 40 °, $\sin\theta$ stops to equal θ. Using smaller increments gave me more data points to analyse which has given a better trend to observe.
Accuracy of time period with the technology used.	Using technology to get the time period of the pendulum is beneficial as the accuracy of testing with technology is more than that of what humans and another strength is that LabQuest and the motion sensors gave a time period so precise that the corrected significant figures is 4 decimal places.

Limitations of the experiment	
Limitations	Explanation
Air resistance	If there is air resistance there will be a damping force acting on the pendulum. Damping may result in a decrease in amplitude and an increase in the time period. Air resistance may not be the same with all my trials which is another limitation to my experiment. This limitation can be resolved with experimenting in a vacuum condition.
Range of motion sensors	The range of my motion sensor was 15cm to 6m which means it can sense any action which is happening as close as 15 cm to as far as 6m. This is a limitation as if there was some movement which I did not notice happening in the background, it was recorded with my results which can be a limitation as my results would not have been accurate. Although it was made sure that the environment was clear there might have been a possibility of some movement in the background. To overcome this limitation, a less sensitive sensor could have been used or a sensor which has a shorter range.
Strength of string	When a pendulum is used continuously, the string is repeatedly stressed, which gradually reduces its flexibility. As a result of molecular wear and strain build-up, the material eventually loses its capacity to revert to its original shape, which lowers its resilience and strength.

Error Evaluation		
Error	Significance	Improvement
Protractor Alignment- Random Error	Every time the bob used to swing the whole pendulum swung due to which the protractor also swung. The protractor was stuck on to the cork with very little Blu Tac due to which the protractor alignment used to get changed and throughout the experiment it had to be fixed 3 times. This could be a limitation as there is a possibility of more error. This limitations could be resolved with attaching the protractor with a more assuring way.	To prevent parallax errors, it’s better to one eye closed and positioning the string directly above the measurement on the protractor and keeping the observation position at 1 point or a mirror could be attached to the back to assist with readings.
Position of Photo gate with respect to the pendulum- Systematic Error	If the pendulum does not cross the photo gate through the centre of the two barriers, the result could be varied as the graphs would not show an accurate representation of the displacement and time period due to distance in between the sensors and the bob.	This experiment could be reperformed in a vacuum where there is no air to interfere with the experiment and experimental set up and the pendulum could be in the right motion throughout the trial.
Releasing the pendulum and starting the vernier at different times. Random Error	Releasing the pendulum and starting the vernier at different times could cause an error while analysing the graphs at different times because LabQuest gives a second to second analysis of the graph and while analysing various the graph at of the same point at the same time could should major deviation.	An improvement on a person reaction time could be the solution to this error or a person with a better reaction type should be able to reduce this error.

Evaluation of Methodology

There could have been some reaction error in the release velocity (more/ less), in case I left my hand earlier in a trial or faster in another due to which the release velocity could be impacted. The kinetic energy will vary due to the change in release velocity and as per my background research, if there is any change in energy, then the time period will vary. Usage of computer controlled systems could be the improvement. There are computer controlled systems made with motors, servos and solenoids which can help in increasing the accuracy of the system. I can improve this by the usage of electromagnet to stop and release. This will reduce the possibility of human errors. Addition of a mirror behind a protractor could also assist in reducing random errors. Performing this experiment with another person could also reduce the chances of random error, so that the pendulum and vernier can be started at a more accurate time.

Further Scope of research

For further research I can change the various controlled variables such as I can further research on what effect external forces such as friction and changing the damping of the pendulum will have an effect on the overall time period of a pendulum. In my investigations I kept damping controlled and I changed the angle of release and compared it to the theoretical value. For further I can research on the damping constant and how it gets effected by factors such as the pendulum mass or the pendulum length or how external factors such as vibrations and temperatures affect the damping constant of a pendulum.

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