CT216: Introduction to Communication Systems

Lab-6 LDPC Decoding

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Honour Code

I, Vansh Naimesh Joshi (Student ID 202101445), declare that the work that I am presenting is my own work. I have not copied the work (Matlab code, results, etc.) that someone else has done. Concepts, understanding and insights I will be describing are my own. Wherever I have relied on an existing work that is not my own, I have provided a proper reference citation. I make this pledge truthfully. I know that violation of this solemn pledge can carry grave consequences.

I have worked this lab along with Parth Parmar - 202101077 and Khushi Shah - 202101430.

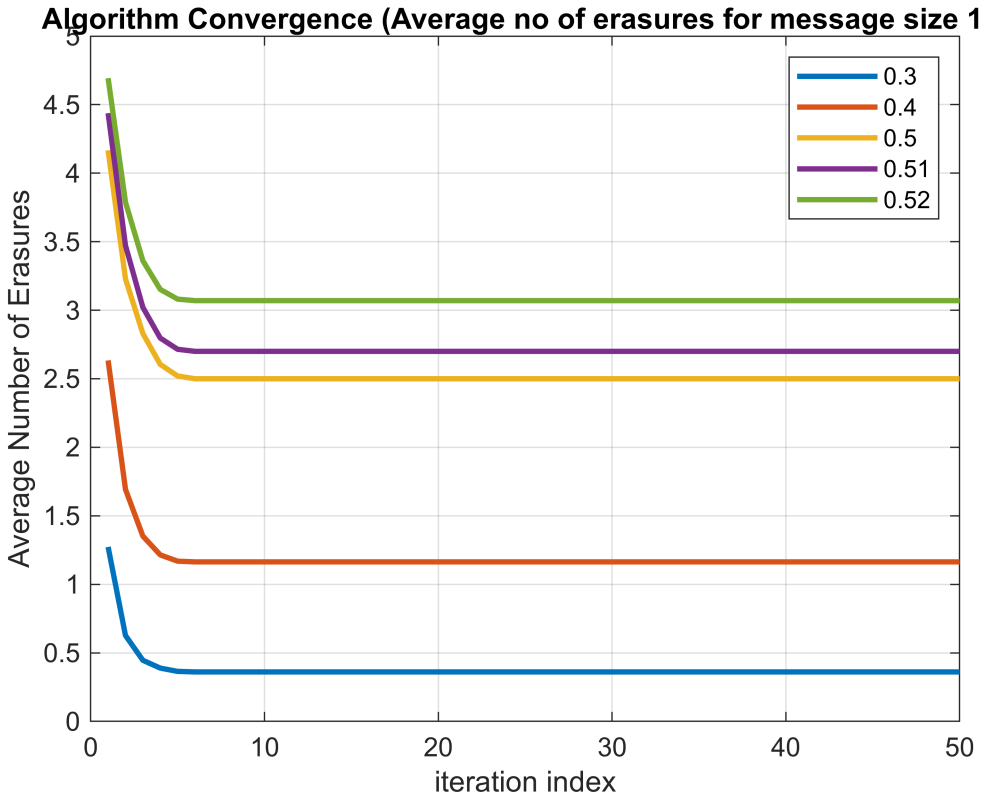
1 Hard Decision Decoding

For H matrix of rate 1/4 (n=12, k=3, u=9) LDPC code

```
clc;
close all;
clear all;

%given matrix
H = [1 0 0 0 0 1 0 1 0 1 0 0;
     1 0 0 1 1 0 0 0 0 0 1 0;
     0 1 0 0 1 0 1 0 1 0 0 0;
     0 0 1 0 0 1 0 0 0 0 1 1;
     0 0 1 0 0 0 1 1 0 0 0 1;
     0 1 0 0 1 0 0 0 1 0 1 0;
     1 0 0 1 0 0 1 0 0 1 0 0;
     0 1 0 0 0 1 0 1 0 1 0 0;
     0 0 1 1 0 0 0 0 1 0 0 1];
%columns
```

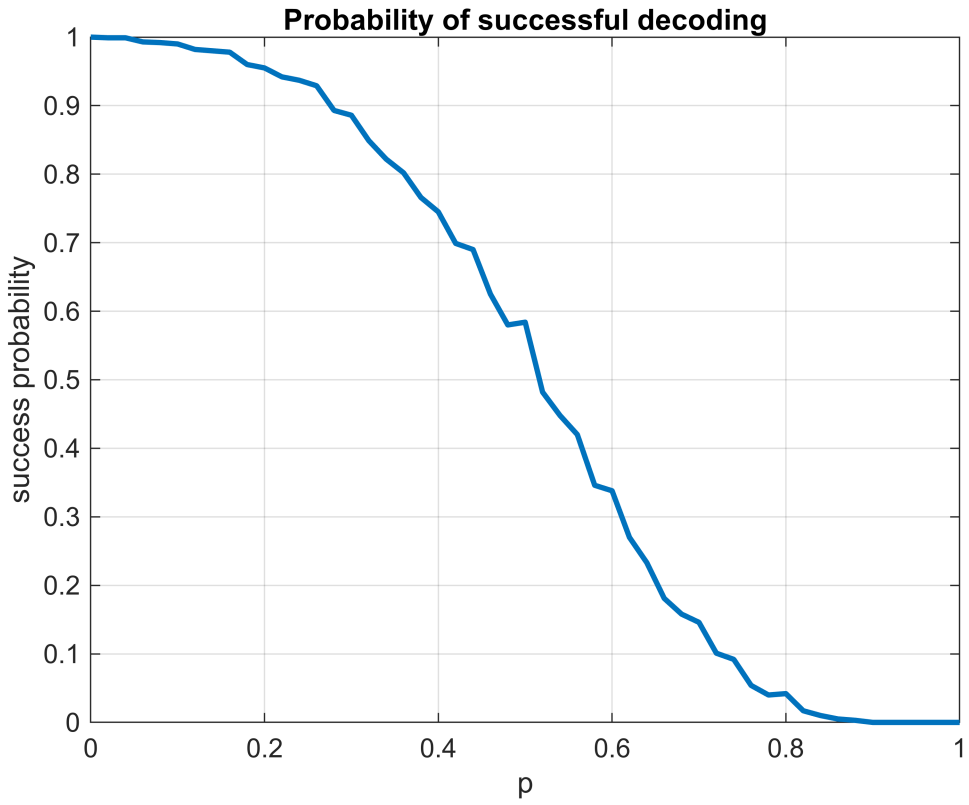
```
col = length(H);
%row
row = height(H);
%degree of checknodes
dc = getdc(H);
%degree of variable nodes
dv = getdv(H);
%connections of all the CNs
map_of_CN = getCNmap(dc,H);
%connections of all the VNs
map_of_VN = getVNmap(dv,H);
%transmitted message
transmit_msg = zeros(1,col);

%number of repititions/simulations
Nsim = 1000;

%maximum number of iterations
max_it = 50;
no_of_it = 1:1:max_it;

for p = [0.3, 0.4, 0.5, 0.51, 0.52]
    erasure = zeros(1,max_it);
    error = zeros(1,max_it);
        count = 0;
        count_erasure = 0;

        for L = 1:1:Nsim

            received_msg = get_off_bec(transmit_msg,p,col);
            currMsg = received_msg;
            sentfromCN = zeros(size(map_of_CN));
            receivedtoVN = zeros(size(map_of_VN));
            sentfromVN = zeros(size(map_of_VN));
            receivedtoCN = zeros(size(map_of_CN));
            flagy = 0;

            for it = 1:1:max_it

                prevMsg = currMsg;
                if it==1
                    sentfromCN = spc0(received_msg,map_of_CN);
                else
                    sentfromVN =  repititioncode(receivedtoVN, received_msg,map_of_VN);
                    receivedtoCN = CNreceival(sentfromVN,map_of_VN,map_of_CN);
                    sentfromCN = spc(receivedtoCN, map_of_CN);
                end

                receivedtoVN = VNreceival(sentfromCN,map_of_CN,map_of_VN);
                currMsg = repetitioncode(receivedtoVN,received_msg);
```

```
                erasureno = 0;
                myflag = 0;
                 for i=1:length(currMsg)
                     if currMsg(i) == -1
                         erasureno = erasureno+1;
                         myflag = 1;
                     end
                 end

                error(it) = error(it) + myflag;

                erasure(it) = erasure(it)+erasureno;

                if checkerasure(currMsg) == 0
                    flagy = it;
                    break;
                end

            end

        end
        erasure = erasure./Nsim;
        error = error./Nsim;


plot(no_of_it,erasure, LineWidth=2);
hold on;

end

legend('0.3','0.4','0.5','0.51','0.52');
title('Algorithm Convergence (Average no of erasures for message size 12)');
xlabel('iteration index');
grid on;
ylabel('Average Number of Erasures');
```




```
pbec = 0:0.02:1;
successProb = zeros(size(pbec));
index=1;
for p = pbec
    count_success = 0;
        for L = 1:1:Nsim

            received_msg = get_off_bec(transmit_msg,p,col);
            currMsg = received_msg;
            sentfromCN = zeros(size(map_of_CN));
            receivedtoVN = zeros(size(map_of_VN));
            sentfromVN = zeros(size(map_of_VN));
            receivedtoCN = zeros(size(map_of_CN));
            for it = 1:1:max_it
                prevMsg = currMsg;

                if it==1
                    sentfromCN = spc0(received_msg,map_of_CN);
                else
                    sentfromVN =  repititioncode(receivedtoVN, received_msg,map_of_VN);
                    receivedtoCN = CNreceival(sentfromVN,map_of_VN,map_of_CN);
                    sentfromCN = spc(receivedtoCN, map_of_CN);
                end

                receivedtoVN = VNreceival(sentfromCN,map_of_CN,map_of_VN);
```

```
                currMsg = repetitioncode(receivedtoVN,received_msg);

            end

            if sum(currMsg) == 0
                count_success = count_success + 1;
            end

        end
        successProb(1,index) = count_success/Nsim;
        index=index+1;
end
figure;
plot(pbec,successProb,LineWidth=2);
ylabel('success probability');
grid on;
title('Probability of successful decoding');
xlabel('p');
```




```
function dc = getdc(H)
    dc = 0;
    row = height(H);
    col = length(H);
    for i = 1:row
        tmp = 0;
```

```
        for j = 1:col
            if H(i,j) == 1
                tmp = tmp+1;
            end
        end
        if dc<tmp
            dc = tmp;
        end
    end
end

function dv = getdv(H)
    row = height(H);
    col = length(H);
    dv = 0;
    for i = 1:col
        tmp = 0;
        for j = 1:row
            if H(j,i) == 1
                tmp = tmp+1;
            end
        end
        if dv<tmp
            dv = tmp;
        end
    end
end

%cn1 --> vn1, vn2...      map_of_CN(2,3) = 4 => CN 3 is connected to VN 4
function map_of_CN = getCNmap(dc,H)
    row = height(H);
    col = length(H);
    map_of_CN = zeros(dc,row);
    for i = 1:row
        new_i = 1;
        for j=1:col
            if H(i,j) == 1
                map_of_CN(new_i,i) = j;
                new_i = new_i + 1;
            end
        end
    end
end

%vn1 --> cn1, cn2, ...   map_of_VN(2,4) = 3 => VN 4 is connected to CN 3
function map_of_VN = getVNmap(dv,H)
    row = height(H);
    col = length(H);
    map_of_VN = zeros(dv,col);
    for i = 1:col
```

```
        new_i = 1;
        for j=1:row
            if H(j,i) == 1
                map_of_VN(new_i,i) = j;
                new_i = new_i + 1;
            end
        end
    end
end


function received_msg = get_off_bec(transmit_msg,p,col)
    received_msg = zeros(1,col);
    becNoise = rand(1,col)<p;
    %making the random message with bec error p
    for i = 1:col
        if becNoise(i) == 1
            received_msg(i) = -1;
        else
            received_msg(i) = transmit_msg(i);
        end
    end
end


function check = checkerasure(msg)
    len = length(msg);
    check = 0;
    for i = 1:len
        if msg(i) == -1
            check = 1;
            break;
        end
    end
end

function ansu = spc0(received_msg,map_of_CN)

    ansu = zeros(size(map_of_CN));
    [row, col] = size(map_of_CN);

    for i =1:1:col

        for j = 1:1:row
            value = 0;
            for k = 1:1:row
                if k~=j

                    if received_msg(map_of_CN(k,i))==-1
                        value = -1;
```

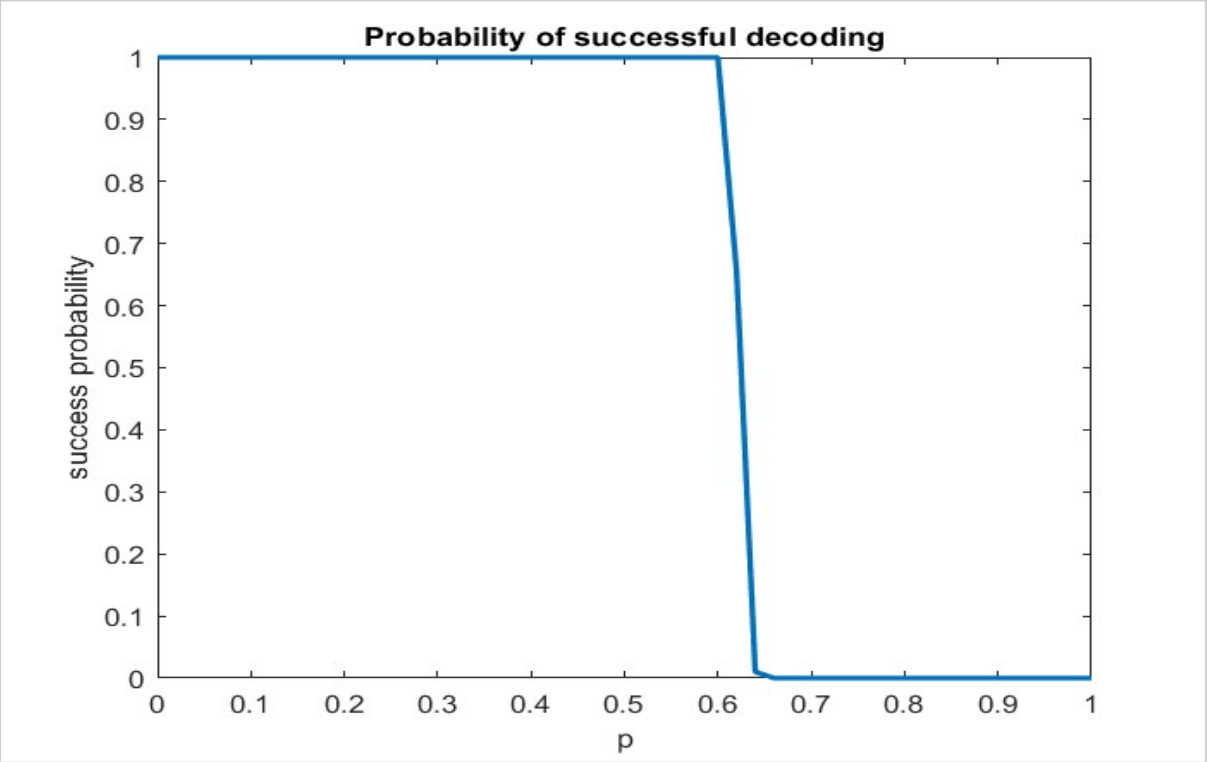
```
                        break;
                    end
                    value = mod(value + received_msg(map_of_CN(k,i)) ,2);


                end

            end
            ansu(j,i) = value;

        end
    end


end

function ansu = VNreceival(sentfromCN,map_of_CN,map_of_VN)

    ansu = size(map_of_VN);
    [row, col] = size(map_of_CN);


    iterators = ones(1,length(map_of_VN));
    for i = 1:1:col
        for j = 1:1:row

            ansu(iterators(1,map_of_CN(j,i)), map_of_CN(j,i)) = sentfromCN(j,i);
            iterators(1,map_of_CN(j,i)) = iterators(1,map_of_CN(j,i))+1;


        end
    end


end

function ansu = repetitioncode(receivedtoVN,received_msg)

    ansu = zeros(1,length(received_msg));

    [row, col] = size(receivedtoVN);

    for i = 1:1:col

        value = -1;
        if received_msg(1,i)~=-1
                ansu(1,i) = received_msg(1,i);

        else
            for j = 1:1:row
```

```
                if receivedtoVN(j,i)~=-1
                    value = receivedtoVN(j,i);
                    break;
                end


            end

           ansu(1,i) = value;

        end

    end

end


%sent from VN

function ansu = repititioncode(receivedtoVN, received_msg,map_of_VN)

    ansu = zeros(size(map_of_VN));
    [row, col] = size(ansu);

    for i = 1:col
        for j = 1:row
            ansu(j,i) = -1;
            if received_msg(1,i) ~= -1
                ansu(j,i) = received_msg(1,i);
            else
                for k = 1:row
                    if k ~= j

                        if receivedtoVN(k,i) ~= -1
                            ansu(j,i) = receivedtoVN(k,i);
                            break;
                        end

                    end
                end
            end

        end
    end

end
```

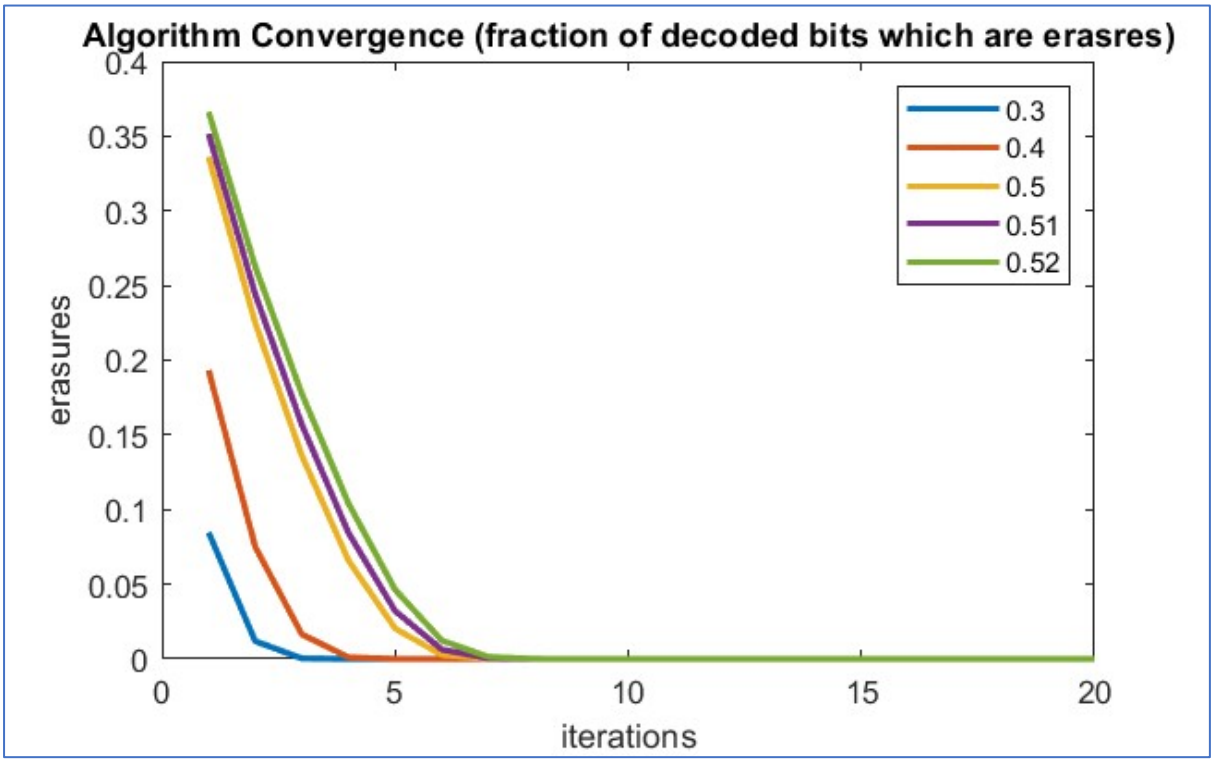
```
%received to CN calculations
function ansu = CNreceival(sentfromVN,map_of_VN,map_of_CN)

    ansu = zeros(size(map_of_CN));

    iterators = ones(1,length(map_of_CN));

    [row,col] = size(sentfromVN);

    for i=1:col
        for j = 1:row
            ansu(iterators(1,map_of_VN(j,i)),map_of_VN(j,i)) = sentfromVN(j,i);
            iterators(1,map_of_VN(j,i)) = iterators(1,map_of_VN(j,i)) + 1;
        end
    end

end


%spc1

function ansu = spc(receivedtoCN, map_of_CN)

    ansu = zeros(size(map_of_CN));

    [row,col] = size(receivedtoCN);

    for i = 1:col
        for j = 1:row
            value = 0;
            for k = 1:row
                if k ~= j
                    if receivedtoCN(k,i) == -1
                        value = -1;
                        break;
                    end
                    value = mod(value+receivedtoCN(k,i),2);
                end

            end
            ansu(j,i) = value;

        end
    end

end
```

HMatrix1

- Success Probability Graph

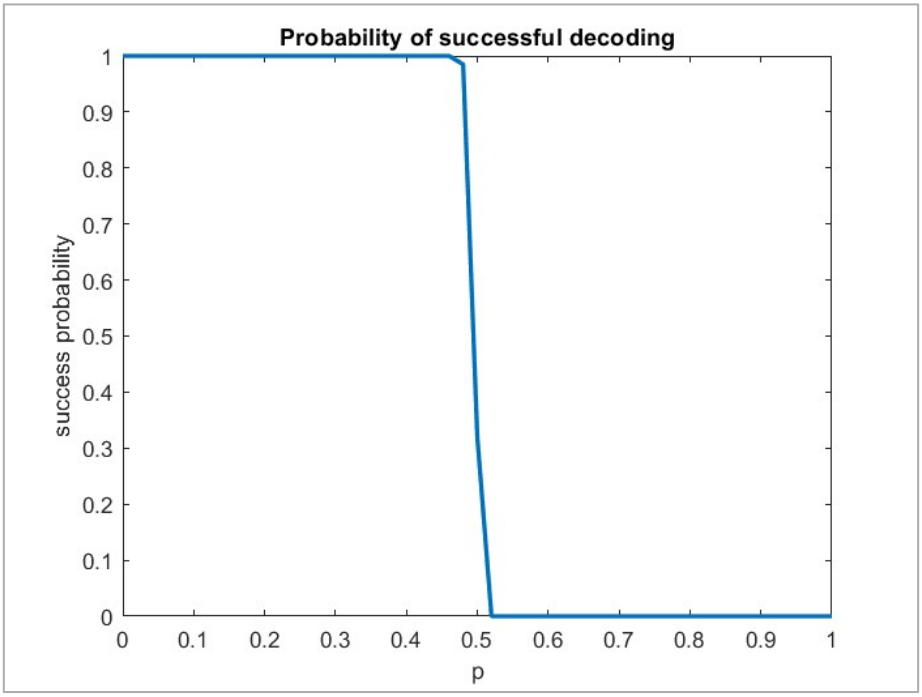


- Algorithm Convergence Graph

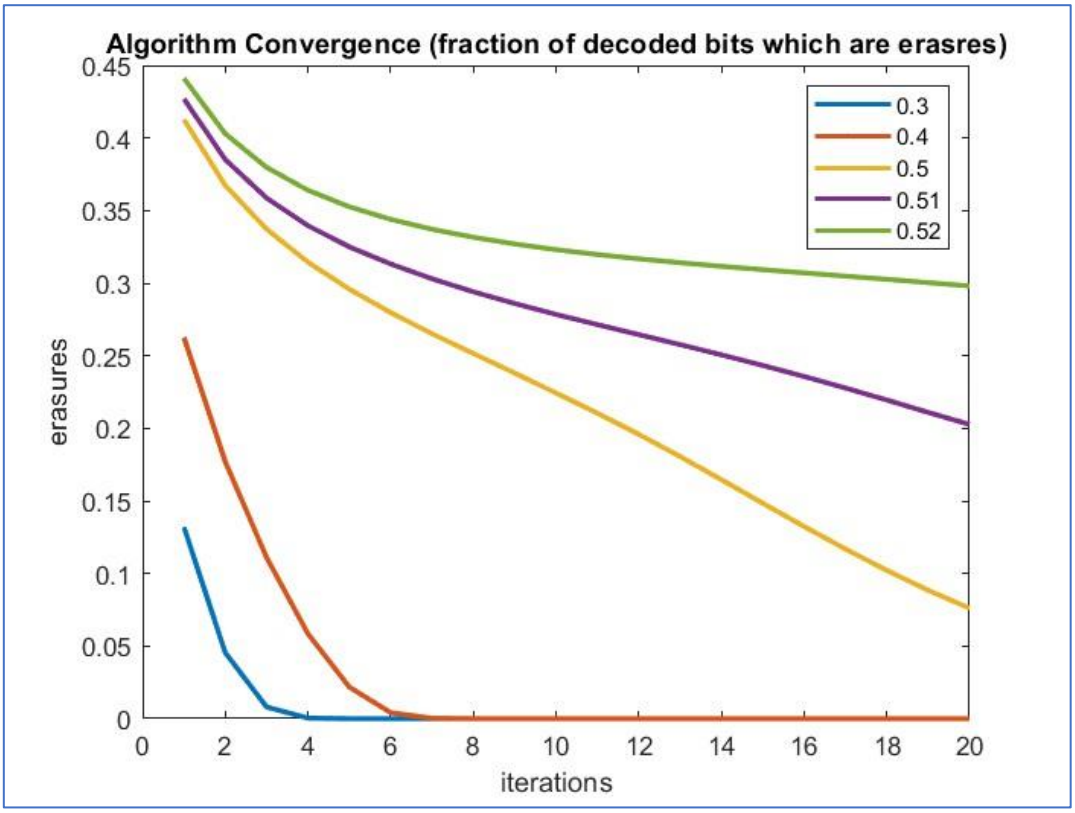


HMatrix2

- Success Probability Graph



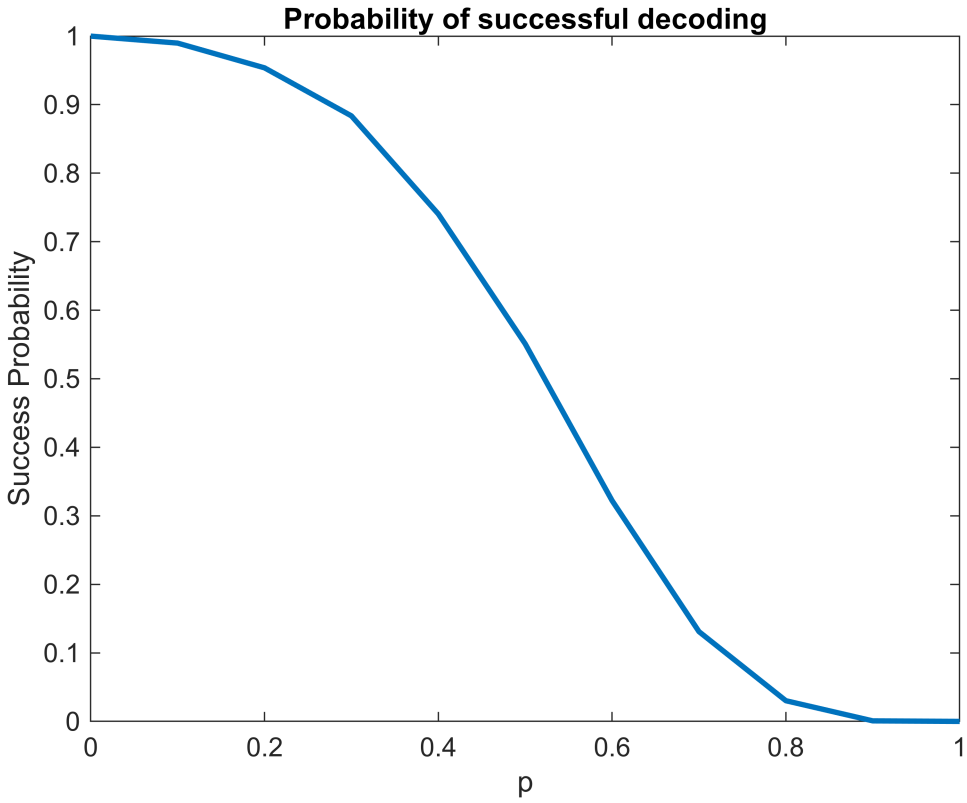
- Algorithm Convergence Graph



Soft Decision Decoding

```
%columns
col = length(H);
%rows
row = height(H);
%degree of checknodes
dc = getdc(H);
%degree of variable nodes
dv = getdv(H);
%connections of the checknodes
map_of_CN = getCNmap(dc,H);
%connections of the variable nodes
map_of_VN = getVNmap(dv,H);
%transmitted message
transmit_msg = zeros(1,col);

%number of simulaitons
Nsim = 10000;

%number of iterations
max_it = 50;
no_of_it = 1:1:max_it;

%probability of successful decoding for different values of p
pbec = 0:0.1:1;
successProb = zeros(size(pbec));
index=1;




for p = pbec
    count_success = 0;
        for L = 1:1:Nsim

            received_msg = get_off_bec(transmit_msg,p,col);
            currMsg = received_msg;
            sentfromCN = zeros(size(map_of_CN));
            receivedtoVN = zeros(size(map_of_VN));
            sentfromVN = zeros(size(map_of_VN));
            receivedtoCN = zeros(size(map_of_CN));
            for it = 1:1:max_it
```

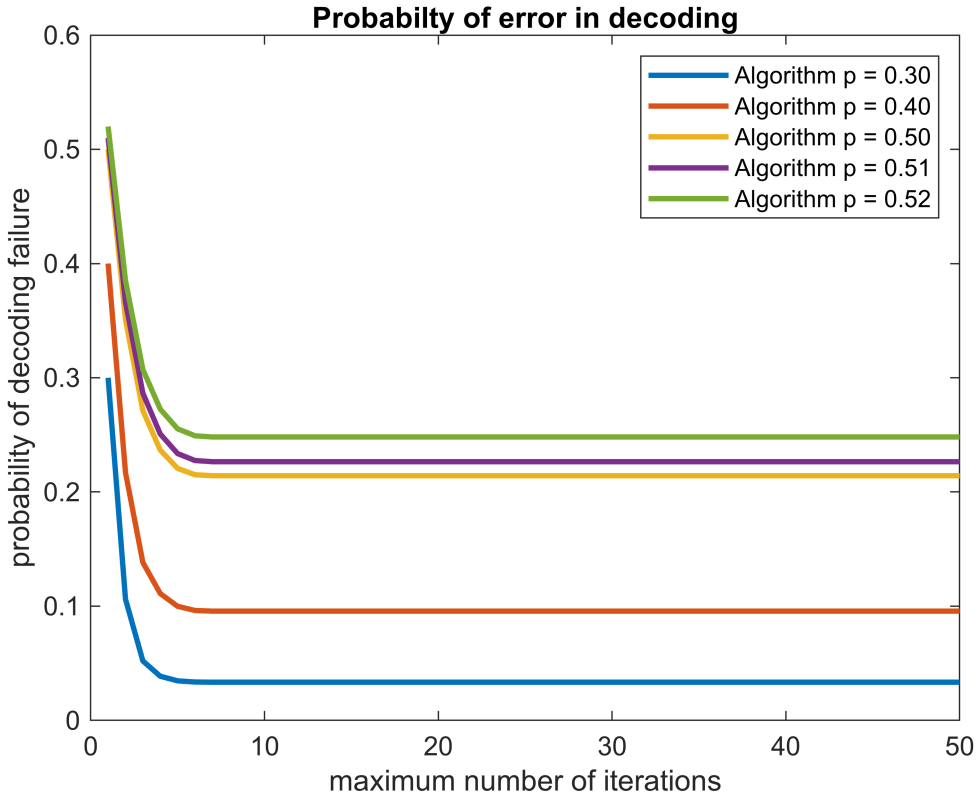
```
                prevMsg = currMsg;

                if it==1
                    sentfromCN = spc0(received_msg,map_of_CN);
                else
                    sentfromVN =  repititioncode(receivedtoVN, received_msg,map_of_VN);
                    receivedtoCN = CNreceival(sentfromVN,map_of_VN,map_of_CN);
                    sentfromCN = spc(receivedtoCN, map_of_CN);
                end

                receivedtoVN = VNreceival(sentfromCN,map_of_CN,map_of_VN);

                currMsg = repetitioncodeforcurrmsg(receivedtoVN,received_msg);

            end

            if sum(currMsg) == 0
                count_success = count_success + 1;
            end

        end
        successProb(1,index) = count_success/Nsim;
        index=index+1;
end
figure;
plot(pbec,successProb,LineWidth=2);
xlabel('p');
ylabel('Success Probability');
title('Probability of successful decoding');
```




```
%probability of error in decoding with respect to iterations
ps = [0.3, 0.4, 0.5, 0.51, 0.52];

for p = ps
    error = zeros(1,max_it);

        count = 0;
        count_error = 0;
    for L = 1:1:Nsim

        received_msg = get_off_bec(transmit_msg,p,col);
        currMsg = received_msg;
        sentfromCN = zeros(size(map_of_CN));
        receivedtoVN = zeros(size(map_of_VN));
        sentfromVN = zeros(size(map_of_VN));
        receivedtoCN = zeros(size(map_of_CN));
        error(1)=p*Nsim;
        for it = 1:1:max_it
            prevMsg = currMsg;

            if it==1

                sentfromCN = spc0(received_msg,map_of_CN);

            else
```

```
            sentfromVN =  repititioncode(receivedtoVN, received_msg,map_of_VN);
            receivedtoCN = CNreceival(sentfromVN,map_of_VN,map_of_CN);
            sentfromCN = spc(receivedtoCN, map_of_CN);

        end

        receivedtoVN = VNreceival(sentfromCN,map_of_CN,map_of_VN);

        currMsg = repetitioncodeforcurrmsg(receivedtoVN,received_msg);

        decoded = zeros(1,length(received_msg));
        erasure=0;
        for i=1:length(currMsg)
            if (currMsg(i)~=0 && received_msg(i)==1)
                erasure=erasure+1;
            end
        end
        %{
        for i= 1:length(currMsg)
            if currMsg(i) >= 1
                decoded(i) = 1;
            else
                decoded(i) = 0;
            end
        end
        erasure = 0;
        for i = 1:length(decoded)
            if decoded(i) ~= transmit_msg(i)
                erasure = 1;
                break;
            end
        end
        %}
        if it==max_it
            break;
        end
        error(it+1)  = error(it+1) + ((erasure)/length(currMsg));

    end

end
error = error./Nsim;
plot(no_of_it,error,'DisplayName',sprintf('Algorithm p = %.2f',p),LineWidth=2);
hold on;
%     p_analytical = zeros(1,length(no_of_it));
%     p_analytical(1)=1;
%     for it = 1:1:max_it-1
%         p_analytical(it+1) = p*(1-((1 -p_analytical(it))^(dc - 1))^(dv-1));
%     end
```

```
%     plot(no_of_it,p_analytical,'DisplayName',sprintf('Analytical p=%.2f',p),LineWidth=2);
%     hold on;

end
legend('show');
title('Probabilty of error in decoding');
xlabel('maximum number of iterations');
ylabel('probability of decoding failure');
```




```
function dc = getdc(H)
    dc = 0;
    row = height(H);
    col = length(H);
    for i = 1:row
        tmp = 0;
        for j = 1:col
            if H(i,j) == 1
                tmp = tmp+1;
            end
        end
        if dc<tmp
            dc = tmp;
        end
    end
end
```

```
function dv = getdv(H)
    row = height(H);
    col = length(H);
    dv = 0;
    for i = 1:col
        tmp = 0;
        for j = 1:row
            if H(j,i) == 1
                tmp = tmp+1;
            end
        end
        if dv<tmp
            dv = tmp;
        end
    end
end

%cn1 --> vn1, vn2...      map_of_CN(2,3) = 4 => CN 3 is connected to VN 4
function map_of_CN = getCNmap(dc,H)
    row = height(H);
    col = length(H);
    map_of_CN = zeros(dc,row);
    for i = 1:row
        new_i = 1;
        for j=1:col
            if H(i,j) == 1
                map_of_CN(new_i,i) = j;
                new_i = new_i + 1;
            end
        end
    end
end

%vn1 --> cn1, cn2, ...   map_of_VN(2,4) = 3 => VN 4 is connected to CN 3
function map_of_VN = getVNmap(dv,H)
    row = height(H);
    col = length(H);
    map_of_VN = zeros(dv,col);
    for i = 1:col
        new_i = 1;
        for j=1:row
            if H(j,i) == 1
                map_of_VN(new_i,i) = j;
                new_i = new_i + 1;
            end
        end
    end
end
```

```
function received_msg = get_off_bec(transmit_msg,p,col)
    received_msg = zeros(1,col);
    becNoise = rand(1,col)<p;
    %making the random message with bec error p
    for i = 1:col

        if becNoise(i) == 1
            received_msg(i) = 1;
%         elseif transmit_msg(i) == 1
%             received_msg(i) = 99;
        elseif transmit_msg(i) == 0
            received_msg(i) = 0;
        end
    end
end

%{
function check = checkerasure(msg)
    len = length(msg);
    check = 0;
    for i = 1:len
        if msg(i) == -1
            check = 1;
            break;
        end
    end
end
%}

function ansu = spc0(received_msg,map_of_CN)

    ansu = zeros(size(map_of_CN));
    [row, col] = size(map_of_CN);

    for i =1:1:col

        for j = 1:1:row
            sa = zeros(1,row-1);
            it = 1;

            for k = 1:1:row
                if k~=j
                    sa(it) = received_msg(map_of_CN(k,i))/(1+received_msg(map_of_CN(k,i)));
                    it = it + 1;
                end
            end



            if mod(row - 1,2) == 1
```

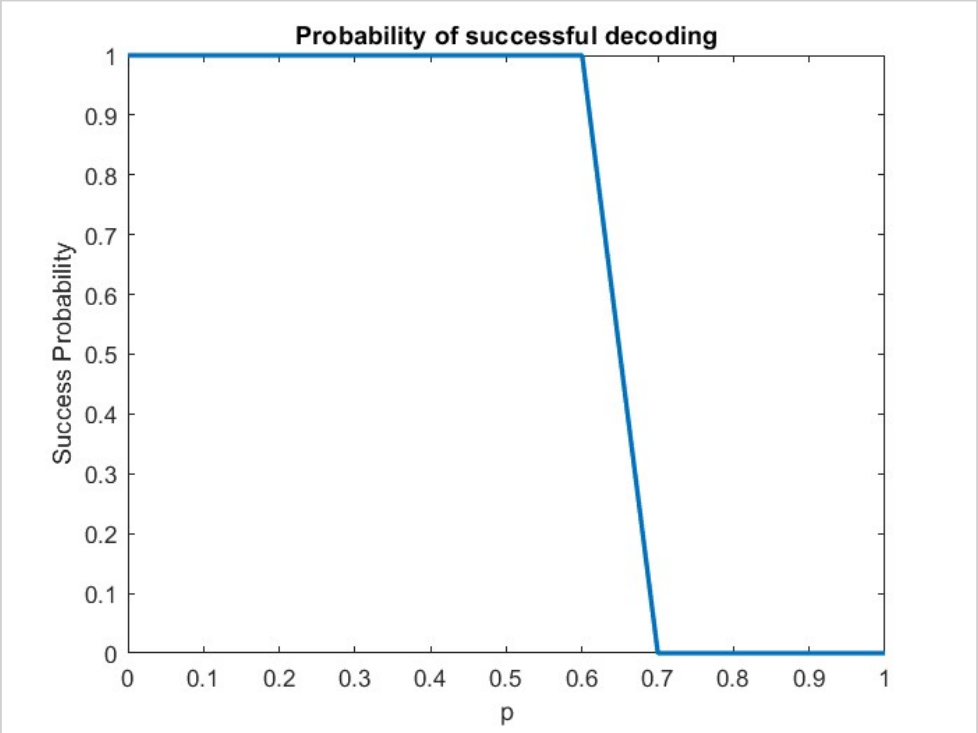
```
                p_1 = (sa(1)*(1-sa(2))*(1-sa(3))) + (sa(2)*(1-sa(1))*(1-sa(3))) +...
                    (sa(3)*(1-sa(2))*(1-sa(1))) + (sa(1)*(sa(2))*(sa(3)));
            else
                p_1 = (sa(1)*(1-sa(2))*(1-sa(3))*(1-sa(4))) + ...
                    (sa(2)*(1-sa(1))*(1-sa(3))*(1-sa(4))) +...
                    (sa(3)*(1-sa(2))*(1-sa(1))*(1-sa(4))) + ...
                    (sa(4)*(1-sa(2))*(1-sa(3))*(1-sa(1))) + ...
                    ((sa(1))*(sa(2))*(sa(3))*(1-sa(4))) + ...
                    ((sa(1))*(sa(2))*(1 - sa(3))*(sa(4))) + ...
                    ((sa(1))*(1 - sa(2))*(sa(3))*(sa(4))) + ...
                    ((1 - sa(1))*(sa(2))*(sa(3))*(sa(4)));
            end


            ansu(j,i) = p_1 / (1-p_1);

        end
    end


end

function ansu = VNreceival(sentfromCN,map_of_CN,map_of_VN)

    ansu = size(map_of_VN);
    [row, col] = size(map_of_CN);


    iterators = ones(1,length(map_of_VN));
    for i = 1:1:col
        for j = 1:1:row

            ansu(iterators(1,map_of_CN(j,i)), map_of_CN(j,i)) = sentfromCN(j,i);
            iterators(1,map_of_CN(j,i)) = iterators(1,map_of_CN(j,i))+1;


        end
    end


end

function ansu = repetitioncodeforcurrmsg(receivedtoVN,received_msg)

    ansu = zeros(1,length(received_msg));

    [row, col] = size(receivedtoVN);

    for i = 1:1:col
```

```
        value = received_msg(1,i);

        for j = 1:1:row

            value = value*receivedtoVN(j,i);

        end

        ansu(1,i) = value;

    end

end


%sent from VN

function ansu = repititioncode(receivedtoVN, received_msg,map_of_VN)

    ansu = zeros(size(map_of_VN));
    [row, col] = size(ansu);

    for i = 1:col
        for j = 1:row
            ansu(j,i) = received_msg(1,i);

            for k = 1:row
                if k ~= j

                    ansu(j,i) = ansu(j,i)*receivedtoVN(k,i);

                end
            end

        end
    end

end


%received to CN calculations
function ansu = CNreceival(sentfromVN,map_of_VN,map_of_CN)

    ansu = zeros(size(map_of_CN));

    iterators = ones(1,length(map_of_CN));

    [row,col] = size(sentfromVN);

    for i=1:col
```

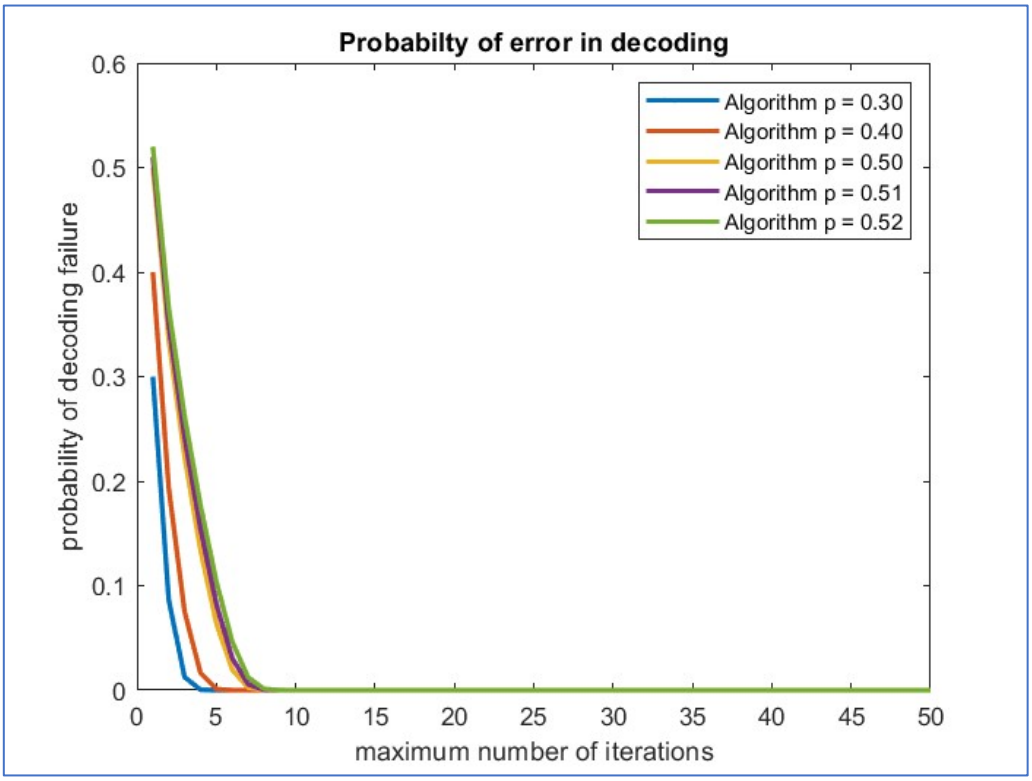
```
        for j = 1:row
            ansu(iterators(1,map_of_VN(j,i)),map_of_VN(j,i)) = sentfromVN(j,i);
            iterators(1,map_of_VN(j,i)) = iterators(1,map_of_VN(j,i)) + 1;
        end
    end

end


%spc1

function ansu = spc(receivedtoCN, map_of_CN)

    ansu = zeros(size(map_of_CN));

    [row,col] = size(receivedtoCN);

    for i = 1:col
        for j = 1:row
            sa = zeros(1,row-1);
            it = 1;
            for k = 1:row
                if k ~= j
                    sa(1,it) = receivedtoCN(k,i)/(1+receivedtoCN(k,i));
                    it = it + 1;
                end

            end
            if mod(row - 1,2) == 1
                p_1 = (sa(1)*(1-sa(2))*(1-sa(3))) + (sa(2)*(1-sa(1))*(1-sa(3))) +...
                    (sa(3)*(1-sa(2))*(1-sa(1))) + (sa(1)*(sa(2))*(sa(3)));
            else
                p_1 = (sa(1)*(1-sa(2))*(1-sa(3))*(1-sa(4))) + ...
                    (sa(2)*(1-sa(1))*(1-sa(3))*(1-sa(4))) +...
                    (sa(3)*(1-sa(2))*(1-sa(1))*(1-sa(4))) + ...
                    (sa(4)*(1-sa(2))*(1-sa(3))*(1-sa(1))) + ...
                    ((sa(1))*(sa(2))*(sa(3))*(1-sa(4))) + ...
                    ((sa(1))*(sa(2))*(1 - sa(3))*(sa(4))) + ...
                    ((sa(1))*(1 - sa(2))*(sa(3))*(sa(4))) + ...
                    ((1 - sa(1))*(sa(2))*(sa(3))*(sa(4)));
            end
            ansu(j,i) = p_1/(1-p_1);

        end
    end

end
```

HMatrix1

- Success Probability Graph

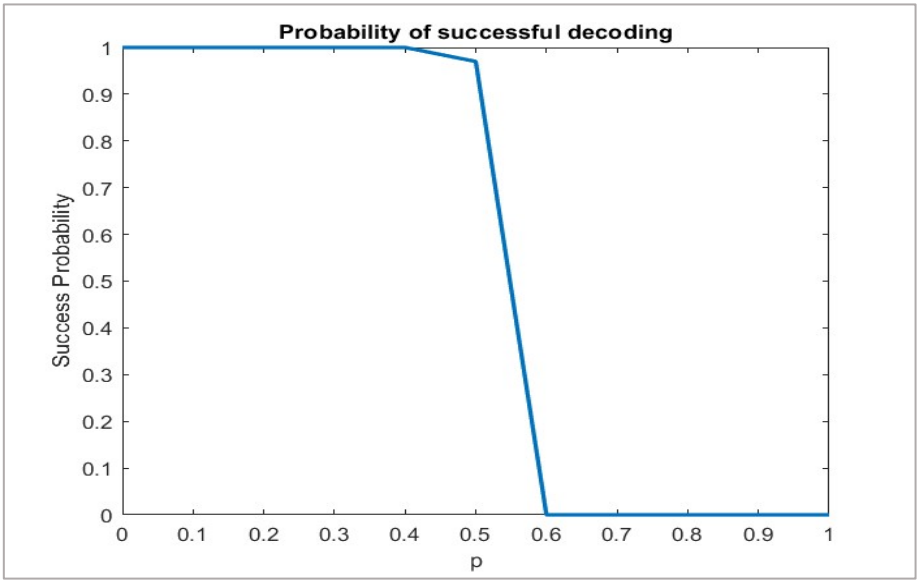


- Algorithm Convergence Graph

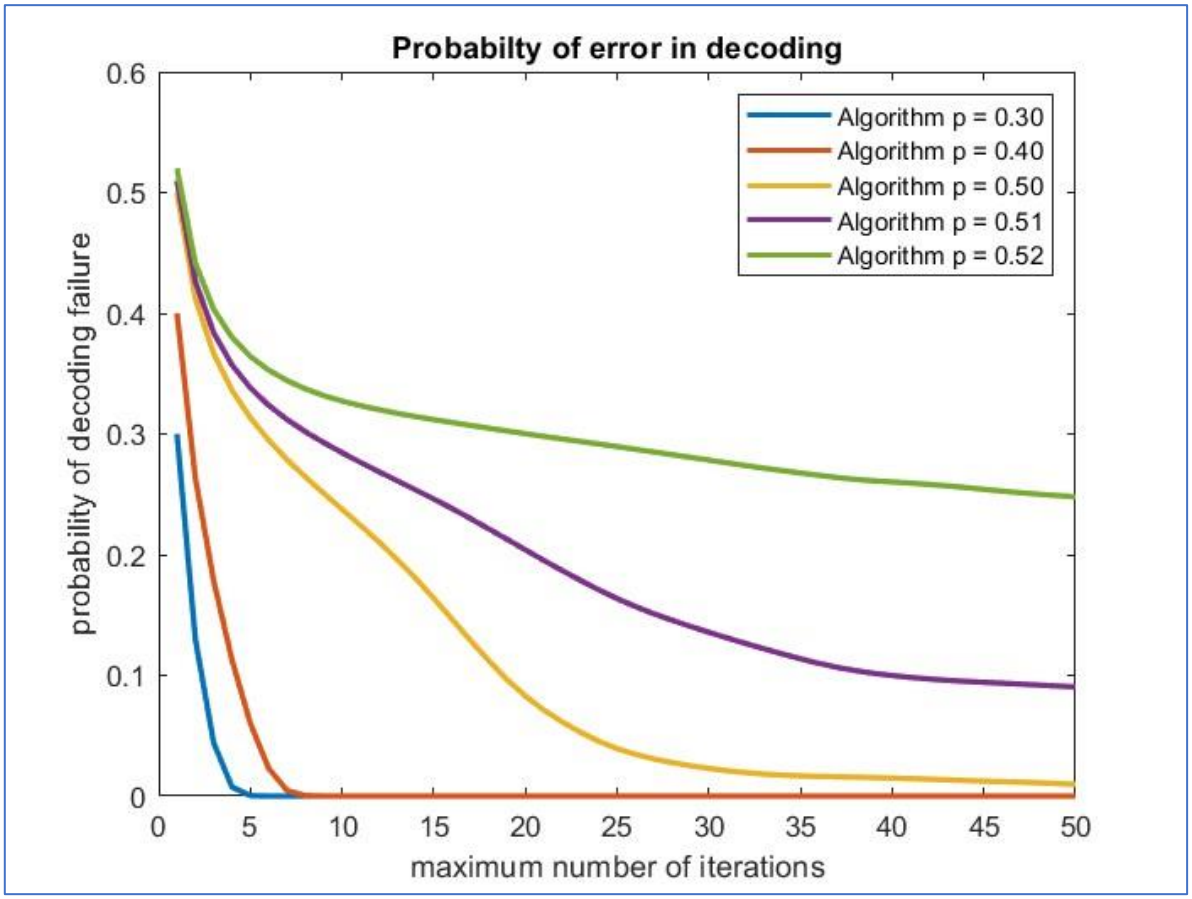


HMatrix2

- Success Probability Graph



- Algorithm Convergence Graph



Conclusion: - LDPC codes are very effective communication method. The probability of decoding a message is very high, and error probabilities are quite less in many cases.

There are two types of decoding possible, soft decision and hard decision.

Hard decisions are based on minimum hamming distance code while the soft decisions are based on minimum Euclidean distance. Thus, soft decoding schemes are more effective.

We can observe the same through the graphs of error/success probabilities as shown in the results for both the decoding schemes.

Thus, LDPC decoding is very powerful method for transmitting messages.