Tutorial-4

Q1. $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$.

$a = 3, b = 2$.

$c = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$f(n) = n^2$.

By case 3: $f(n) > n^c \Rightarrow T(n) = \theta(f(n)) = \theta(n^2)$.

Q2. $T(n) = 4T(n/2) + n^2$.

$T(n) = aT(n/b) + f(n)$.

$a = 4, b = 2$

$c = \log_2 4 = 2$.

$n^c = n^2$

$f(n) = n^2$.

By case 2: $T(n) = \theta(n^c \log n)$

$= \theta(n^2 \log n)$.

Q3. $T(n) = T(n/2) + 2^n$

$T(n) = aT(n/b) + f(n)$

$a = 1, b = 2$

$c = \log_2 1 = 0$.

$n^c = n^0 = 1. \Rightarrow f(n) = 2^n > 1$.

$T(n) = \theta(2^n)$.

Q4. $T(n) = 2^n T(n/2) + n^n$

$a = 2^n, b = 2$

$c = \log_2 2^n = n$.

$n^c = n^n = f(n)$.

$T(n) = \theta(n^n \log n)$

$T(n) = 16T(n/4) + n$.

$a = 16, b = 4$.

$c = \log_4 16 = 2$.

$n^c = n^2 > f(n)$.

$T(n) = \theta(n^2)$.

Q6. $T(n) = 2T(n/2) + n\log n$.

$a = 2, b = 2$

$c = \log_2 2 = 1$

$n^c = n < f(n)$.

$T(n) = \theta(n\log n)$.

Q7. $T(n) = 2T(n/2) + n/\log n$.

$a = 2, b = 2$

$c = \log_2 2 = 1$.

$n^c = n > f(n)$.

$T(n) = \theta(n)$

Q8. $T(n) = 2T(n/4) + n^{0.5}$

$a = 2, b = 4$.

$c = \log_4 2 = 0.5$

$n^c = n^{0.5} < f(n)$.

$T(n) = \theta(n^{0.5})$.

Q9. $T(n) = 0.5T(n/2) + 1/n$.

$T(n) = aT(n/b) + f(n)$.

$a = 0.5, b = 2$

$c = \log_2 0.5 = -1$.

$n^c = n^{-1} = 1/n = f(n)$

$T(n) = \theta(\log n/n)$

Q10. $T(n) = 16T(n/4) + n!$

$a = 16, b = 4$.

$c = \log_4 16 = 2$.

$n^c = n^2 < f(n)$.

$T(n) = \theta(n!)$.

Q11. $T(n) = 4T(n/2) + \log n$.

$a = 4, b = 2$

$c = \log_2 4 = 2$.

$n^c = n^2 > f(n)$.

$T(n) = \theta(n^2)$.

Q12. $T(n) = \text{sqrt}(n)T(n/2) + \log n$

$a = n^{1/2}, b = 2$.

$c = \log_2 n^{1/2} = 1/2 \log n$.

$n^c = n^{1/2\log n} > f(n)$.

$T(n) = \theta(n^{1/2\log n})$

Q13 $T(n) = 3T(n/2) + n$.

$a = 3, b = 2$

$c = \log_2 3 = 1.58$

$n^c = n^{1.58} > f(n)$

$T(n) = \theta(n^{1.58})$.

Q14. $T(n) = 3T(n/3) + \text{sqrt}(n)$.

$a = 3, b = 3$

$c = \log_3 3 = 1$

$n^c = n^1 > f(n)$.

$T(n) = \theta(n)$.

Q15. $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$

$c = \log_2 4 = 2.$

$n^c = n^2 > f(n).$

$T(n) = \theta(n^2).$

Q16. $T(n) = 3T(n/4) + n \log n.$

$a = 3, b = 4.$

$c = \log_4 3 = 0.79.$

$n^c = n^{0.79} < f(n).$

$T(n) = \theta(n \log n).$

Q17. $T(n) = 3T(n/3) + n/2.$

$a = 3, b = 3.$

$c = \log_3 3 = 1.$

$n^c = n > f(n).$

$T(n) = \theta(n)$

Q18. $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3$

$c = \log_3 6 = 1.63.$

$n^c = n^{1.63} < f(n)$

$T(n) = \theta(n^2 \log n)$

Q19. $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2$

$c = \log_2 4 = 2.$

$n^c = n^2 > f(n)$

$T(n) = \theta(n^2).$

Q20. $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, b = 8.$

$c = \log_8 64 = 2.$

$n^c = n^2 < f(n).$

$T(n) = \theta(n^2 \log 1/n).$

Q21. $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3.$

$c = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

$T(n) = \theta(n^2).$

Q22. $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2, c = \log_2 1 = 0.$

$n^c = n^0 = 1 < f(n).$

$T(n) = \theta(n(2 - \cos n)).$