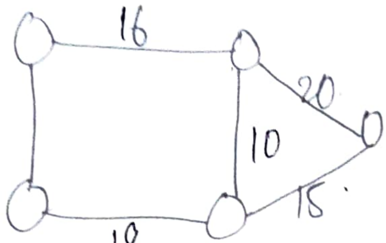
Tutorial-6

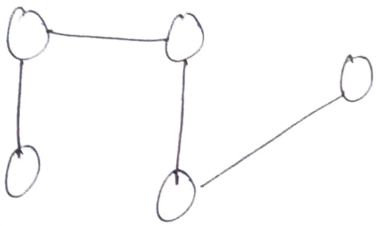
Q1. Minimum Spanning Tree

A spanning tree of an undirected graph is a subgraph that is a tree & joined by all vertices. One of those tree which has minimum total cost would be its minimum spanning tree.

ex:



Minimum cost spanning Tree.



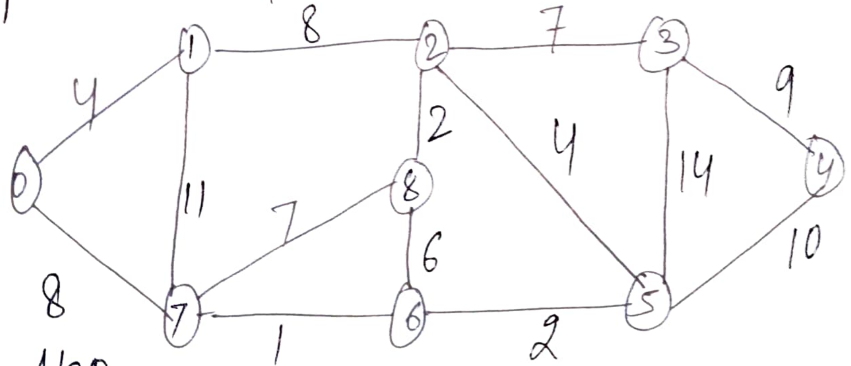
Applications of MST :-

It is direct applications in design of network including computer networks, telecommunication networks, transportation networks, etc.

Q2.

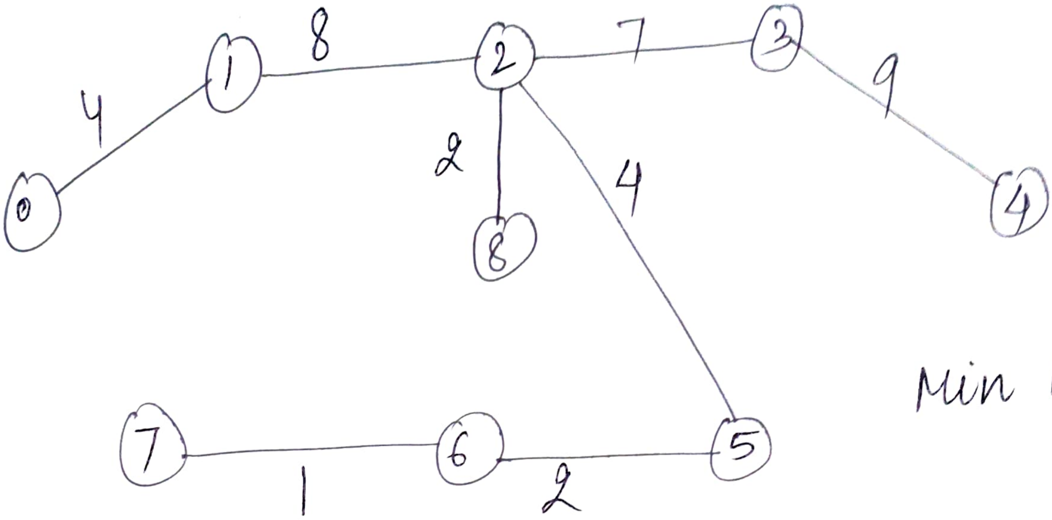
	Prim's Algo	Kruskal's Algo	Dijkstra's Algo	Bellman Ford Algo
T.C	$O(V^2)$	$O(E \log V)$	$O(V+E \log V)$	$O(VE)$
S.C.	$O(V+E)$	$O(\lvert E\rvert + \lvert V\rvert)$	$O(V^2)$	$O(V^2)$

Q3.



Prim's Algo

0	1	2	3	4	5	6	7	8
$\boxed{0}$	~~∞~~	∞	∞	∞	∞	∞	~~∞~~	∞
	$\boxed{4}$	$\boxed{8}$	$\boxed{7}$	~~10~~ $\boxed{9}$	$\boxed{4}$	~~6~~ $\boxed{2}$	8 ~~7~~ $\boxed{1}$	$\boxed{2}$



Min Weight = 37.

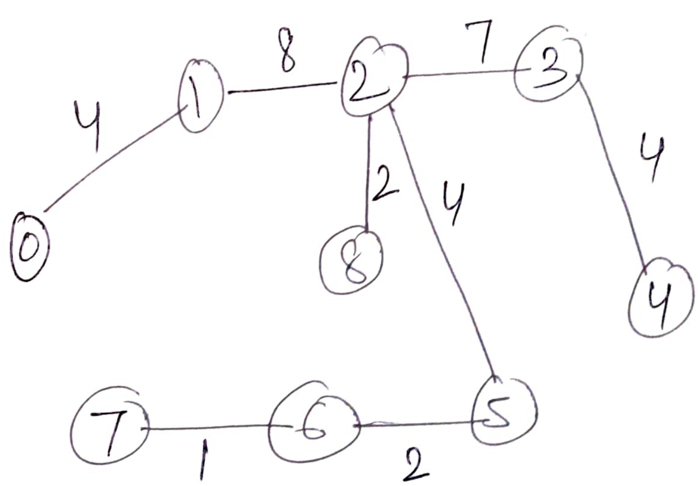
Parent:

	0	1	2	3	4	5	6	7	8
	-1	~~-1~~	~~-1~~	~~-1~~	-1	~~-1~~	-1	~~-1~~	~~-1~~
		0	1	2		2		~~0~~	2
							~~8~~	~~8~~	
					~~5~~		5	6	
					3				

Parent: -1 0 1 2 3 2 5 6 2

Kruskal's Algo.

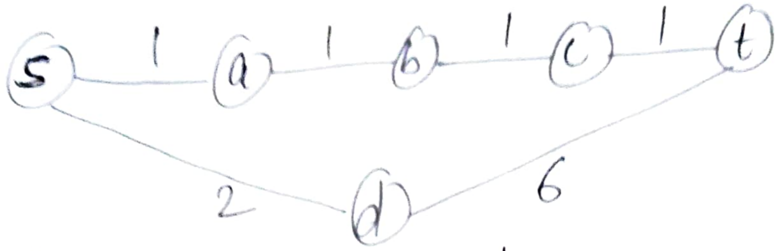
U	V	W	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	x
7	8	7	x
2	3	7	✓
1	2	8	✓
0	7	8	x
3	4	9	✓
5	4	10	x
1	7	11	x
3	5	14	x



Weight = 37

Q4 (i.) If 10 units is added to each edge, the overall weight of path may change.

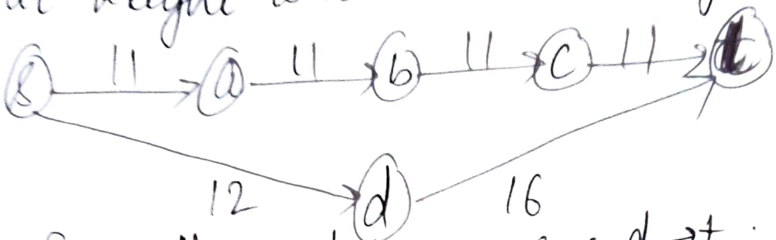
ex:



Shortest path ⇒ s → a → b → c → t

Weight ⇒ 1 + 1 + 1 + 1 = 4.

Now if 10 unit weight is added to each edge.

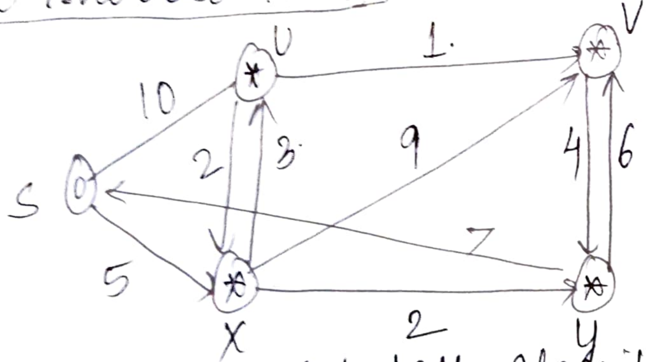


Shortest Path changed to ⇒ s → d → t.

Weight = 28

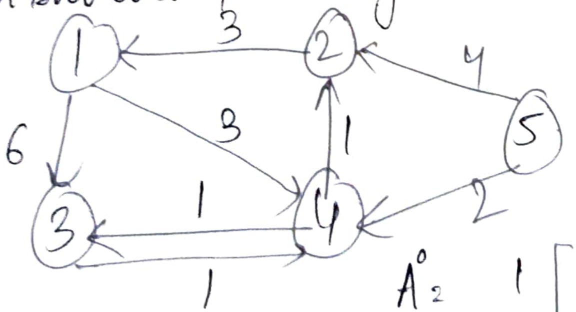
(ii.) Multiplying weight of each edge by 10 will have no impact on shortest Path.

Q5.



s	u	v	x	y
0	∞	∞	∞	∞
0	10	∞	5	∞
0	10	11	5	∞
0	10	11	5	7

Q6. All pair shortest path algorithm – Floyd Warshall.



A^0_2

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$A^0[2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3+6 = 9$

$\Rightarrow 9 < \infty$.

Similarly,

$A^0[2,4] = \infty$

$A^0[2,1] + A^0[1,4] = 3+3 = 6$

$\Rightarrow 6 < \infty$

$A^0[2,5] = \infty$

$A^0[2,1] + A^0[1,5] = 3+\infty$

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$A^1[1,3] = 6$

$A^1[1,2] + A^1[2+3] = \infty + 9$

$6 < \infty + 9$.

$$A^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^5 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & \infty \\ 2 & 3 & 0 & 7 & 6 & \infty \\ 3 & \infty & 3 & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & 7 & 3 & 3 & 2 & 0 \end{array}$$