Financial Mathematics

Stock Options and Put-Call Parity

Summer of Science

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Factors Affecting the Price of Stock Options

There are six major factors. These are :

1. The current price the stock is trading at, S_0,
2. The strike price, K,
3. The time until expiration, T,
4. The volatility of the price of the stock, σ,
5. The risk-free interest rate, r,
6. The dividends that are expected to be paid, q.

Factors Affecting the Price of Stock Options

Variable	Call	Put
Current stock price	+	-
Strike Price	-	+
Time to expiration	+	+
Volatility	+	+
Risk-free rate	+	-
Amount of future dividends	-	+

+ indicates that an increase in the variable causes the option price to increase

- indicates that an increase in the variable causes the option price to decrease

Assumptions and Notation

1. There are no transactions costs.
2. All trading profits are subject to the same tax rate.
3. Borrowing and lending are possible at the risk-free interest rate.

S_0 : Current stock price
K: Strike price of option
T : Time to expiration of option
S_T : Stock price on the expiration date
r : Continuously compounded risk-free rate of interest for an investment maturing in time T
c : Value of call option to buy one share
p : Value of put option to sell one share

Upper and Lower Bounds for Option Prices

Upper Bound

No matter what happens, the option can never be worth more than the stock. Hence, the stock price is an upper bound to the option price:

$$c \leq S_0$$

At maturity the option cannot be worth more than K. It follows that it cannot be worth more than the present value of K today:

$$p \leq Ke^{-rT}$$

Lower Bound for Calls on Non-Dividend-Paying Stocks

A lower bound for the price of a call option on a non-dividend-paying stock is $S_0 - Ke^{-rT}$

Portfolio A: one call option plus a zero-coupon bond that provides a payoff of K at time T
Portfolio B: one share of the stock.

If $S_T > K$, the call option is exercised at maturity and is worth S_T .
If $S_T < K$, the call option expires worthless and the portfolio is worth K.
Hence, at time T, portfolio A is worth : max(S_0,K)
Portfolio B is worth S_T at time T

$$S_0 - Ke^{-rT} \leq c$$
$$max(S_0 - Ke^{-rT}, 0) \leq c$$

Lower Bound for Puts

Portfolio C: one put option plus one share
Portfolio D: a zero-coupon bond paying off K at time T.

If $S_T < K$, then the option in portfolio C is exercised at option maturity and the portfolio becomes worth K. If $S_T > K$, then the put option expires worthless and the portfolio is worth S_T at this time. Hence, portfolio C is worth $max(S_T, K)$
Portfolio D is worth K in time T. Hence, portfolio C is always worth as much as, and can sometimes be worth more than, portfolio D in time T.
Hence, $Ke^{-rT} - S_0 \leq p$

$$max(Ke^{-rT} - S_0, 0) \leq p$$

Put - Call Parity

The components of portfolio A are worth c and Ke^{-rT} today, and the components of portfolio C are worth p and S_0 today. Hence,

$$c + Ke^{-rT} = p + S_0$$

This relationship is known as put - call parity.

Thank You