

Experiment No.-06

Objective: Solve Constraint Satisfaction Problems.

Theory: A Constraint Satisfaction Problem (CSP) is a mathematical problem defined as a set of objects whose state must satisfy a number of constraints or limitations.

It is widely used in Artificial Intelligence, operations research, and optimization.

A CSP is represented by:

- $X = \{X_1, X_2, \dots, X_n\} \rightarrow$ a set of variables
- $D = \{D_1, D_2, \dots, D_n\} \rightarrow$ domains of possible values for each variable
- $C = \{C_1, C_2, \dots, C_k\} \rightarrow$ a set of constraints specifying allowable combinations of values

A solution to a CSP is an assignment of values to all variables that satisfies all constraints.

➤ Types of CSPs:

- Discrete CSPs: Variables take values from a finite domain (e.g., Sudoku, N-Queens).
- Continuous CSPs: Variables have continuous domains (e.g., scheduling problems).
- Dynamic CSPs: Constraints or variables may change over time.

➤ Common Techniques for Solving CSPs:

- Backtracking Search: Systematically explores possible variable assignments and backtracks when constraints are violated.
- Forward Checking: Prevents future conflicts by checking consistency during assignment.
- Arc Consistency (AC-3 Algorithm): Removes inconsistent values from domains.
- Heuristics:

o Minimum Remaining Values (MRV) — choose the variable with the fewest remaining legal values.

o Least Constraining Value (LCV) — choose the value that rules out the fewest options for neighboring variables.

➤ Applications:

- Map coloring
- Sudoku solving
- Scheduling and timetabling
- Resource allocation
- N-Queens problem

Algorithm: Steps:

Step 1: Initialization

- Define the set of variables and their possible domains.
- Define the constraints between variables.

Step 2: Selection

- Choose an unassigned variable.

Step 3: Assignment

- Assign a value from its domain.

Step 4: Constraint Checking

- Check if the assignment is consistent with the constraints.

Step 5: Backtrack

- If a constraint is violated, undo the last assignment and try a different value.

Step 6: Termination

- If all variables are assigned valid values, output the solution.
- If no valid assignment exists, report failure.

Time Complexity:

Worst case — $O(dn)$ where d = size of domain, n = number of variables.

Space Complexity:

$O(n)$ for recursion stack and variable assignments.

Program:

```
def is_safe(assignment, var, value, constraints):
    for neighbor in constraints[var]:
        if neighbor in assignment and assignment[neighbor] == value:
            return False
    return True
```

```
def backtrack(variables, domains, constraints, assignment):
    if len(assignment) == len(variables):
        return assignment # All variables assigned
```

```
var = [v for v in variables if v not in assignment][0]
```

```
for value in domains[var]:
    if is_safe(assignment, var, value, constraints):
        assignment[var] = value
        result = backtrack(variables, domains, constraints, assignment)
        if result:
            return result
    del assignment[var] # backtrack
return None
```

```
def main():
    variables = ['WA', 'NT', 'SA', 'Q', 'NSW', 'V', 'T']
```

```
domains = {
    'WA': ['Red', 'Green', 'Blue'],
    'NT': ['Red', 'Green', 'Blue'],
    'SA': ['Red', 'Green', 'Blue'],
    'Q': ['Red', 'Green', 'Blue'],
```

```
'NSW': ['Red', 'Green', 'Blue'],  
'V': ['Red', 'Green', 'Blue'],  
'T': ['Red', 'Green', 'Blue']  
}
```

```
constraints = {  
'WA': ['NT', 'SA'],  
'NT': ['WA', 'SA', 'Q'],  
'SA': ['WA', 'NT', 'Q', 'NSW', 'V'],  
'Q': ['NT', 'SA', 'NSW'],  
'NSW': ['Q', 'SA', 'V'],  
'V': ['SA', 'NSW'],  
'T': []  
}
```

```
assignment = {}  
solution = backtrack(variables, domains, constraints, assignment)
```

```
if solution:  
    print("Solution Found:")  
    for state, color in solution.items():  
        print(f"{state} → {color}")  
else:  
    print("No solution exists.")
```

```
if __name__ == "__main__":  
    main()
```

Output:

```
Solution Found:  
WA → Red  
NT → Green  
SA → Blue  
Q → Red  
NSW → Green  
V → Red  
T → Red
```