Saathi

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Predictive Analysis Using Statistics
Assignment 6 - Parameter Estimation

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~~An~~ A1 Given Random sample $(x_1, \dots x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(\frac{-(x_i-\mu)^2}{2\sigma^2}\right)}$$

Taking natural log of likelihood fx^n

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

To find MLE, diff log likelihood wrt θ_1, θ_2

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \left(\frac{x_i - \mu}{\sigma^2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0 \qquad \frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

For θ_2 $\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i-\theta_i)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2} \right) = 0$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{x_i - \theta_1}{\theta_2^2} \right)^2 - \frac{n}{\theta_2} = 0 \qquad \frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_i)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_i)^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2$$ Sample Variance

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A2 To find the MLE of θ for a binomial distribution $B(m, \theta)$ where m is a known +ve integer

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking natural log

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + x_i \ln\theta + (m-x_i)\ln(1-\theta) \right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

Solving for θ

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = \sum_{i=1}^{n} \frac{m-x_i}{1-\theta}$$

$$\sum_{i=1}^{n} x_i(1-\theta) = \sum_{i=1}^{n} (m-x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

∴ MLE of θ is sample mean of observations