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PARAMETER ESTIMATION

(1) Given - Random Sample $(x_1, x_2, \dots, x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(-\left(\frac{x_i - \mu}{2\sigma^2}\right)^2\right)}$$

Taking natural log of likelihood fxn

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\left(\frac{x_i - \mu}{2\sigma^2}\right)^2 - \frac{1}{2} \ln\left(2\pi\sigma^2\right)\right)$$

To find MLE, diff. Log likelihood w.r.t θ_1, θ_2

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \left(\frac{x_i - \mu}{\sigma^2}\right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

For θ_2 $$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\left(\frac{(x_i - \theta_1)^2}{2(\theta_2)^2} + \frac{1}{2\theta_2}\right) \right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i - \theta_i}{\theta_2^2}\right)^2 - \frac{n}{\theta_2} = 0$$

$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n} \sum_{i=1}^{n} \left(x_i - \theta_1\right)^2$$

$$\theta_2 = \frac{1}{n} \sum_{i=1}^{n} \left(x_i - \theta_1\right)^2$$ Sample Variance

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(2) MLE of Binomial distribution of $B(m, \theta)$ where m is a known +ve integer.

$$L(\theta) = \prod_{i=1}^{n} \binom{m}{x_i} \theta^{x_i} (1-\theta)^{m-x_i}$$

Taking ln

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m-x_i)\ln(1-\theta) \right)$$

$$\frac{\partial}{\partial\theta} \ln(L(\theta)) = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m-x_i}{1-\theta} \right) = 0$$

Solving for θ

$$\sum_{i=1}^{n} x_i (1-\theta) = \sum_{i=1}^{n} (m-x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\theta = \frac{1}{m} \sum_{i=1}^{n} x_i$$

$\therefore$ MLE of θ is sample mean of observations.