

Tutorial Sheet-1

①. `int a=0, b=0;`
`for (i=0; i<n; i++) at = rand();`
`for (j=0; j<m; j++) bt = rand();`

Time Complexity = $O(n+m)$;
 Space Complexity = $O(1)$ Ans.

②. `int sum=0, i;`
`for (i=0; i<n; i=i+2) { sum+=i; }`
 loop will run even times.

$\therefore 0+2+4 \dots 2n$
 $= O(n/2)$
 \Rightarrow Time Complexity = $O(n)$ Ans.

③. `int sum=0, i;`
`for (i=0; i<n; i=i*2) { sum+=i; }`

Time Complexity = $O(\log n)$ Ans.

④. `int sum=0, i;`
`for (i=0; i*i<n; i++)`

$O(\log n)$ Ans.

⑤. `int j=1, i=0;`
`while (i<=n) {`
`i = i+j;`
`j++;`
`}`

Time = $O(n)$
 Space = $O(1)$ Ans.



⑥. $\begin{array}{l} \text{void recursion(int n) \{ } \\ \text{if (n == 1) return;} \\ \text{recursion(n-1);} \\ \text{print(n);} \\ \text{recursion(n-1);} \\ \} \end{array}$

$\rightarrow T(n)$
 $\rightarrow T(1)$
 $\rightarrow T(n-1)$
 $\rightarrow T(1)$
 $\rightarrow T(n-1)$

$$T(n) = T(n-1) + 1$$

$$T(n-1) = T(n-2) + 1$$

$$T(n) = T(n-2) + 2$$

$$T(1) = 1$$

$$T(n) = T(n-k) + k$$

$\rightarrow O(n)$ Ans.

⑦. The given recursive funcⁿ is of binary search.

$$\therefore \text{Time Complexity} = T\left(\frac{n}{2} + 1\right)$$

or

n is dividing by 2 every time
 $\therefore O(\log n)$ Ans.

⑧. (i) $\begin{array}{l} T(n) = T(n-1) + 1 \\ T(n-1) = T(n-1) + 1 \\ T(n) = T(n-2) + 2 \\ T(n-1) = T(n-1) + 1 \\ T(n) = T(n-k) + k \\ \rightarrow O(n) = O(n) \text{ Ans.} \end{array}$

ii) $\begin{array}{l} T(n) = T(n-1) + n \\ T(n-1) = T(n-2) + (n-1) \\ T(n) = T(n-2) + 2n - 1 \\ T(n-2) = T(n-4) + 2n - 5 \end{array}$

$$T(n) = T(n-4) + 4n - 6$$

$$T(n) = T(n-K) + Kn - (K-1)(-2)$$

iii) $T(n) = T(n/2) + 1$
 $a = 1, b = 2, k = 0, p = 0$
 $a = b^k$
 $O(n^{\log_2 1} \log^{p+1} n)$

$O(\log n)$ Ans.

iv) $T(n) = 2T(n/2) + 1$
 $a = 2, b = 1, k = 0$
 $2 > 1, (a > b^k)$
 $O(n^{\log_2 2}) = O(n)$ Ans.

v) $T(n) = 3T(n-1)$
 $T(0) = 1$
 $T(n-1) = 3T(n-2)$
 $T(n) = 3(3T(n-2))$

$T(n) = 3^k T(n-k)$
 $n-k = 0$

$\boxed{n=k}$
 $3^n \times 1 = T(n)$
 $T(n) = 3^n$
 $\boxed{O(3^n)}$ Ans.

9. $O(n)$

10. $O(N+N)$

11. $O(N \log N)$

12. X will be always a better choice for ^{large} inputs.

13. $O(\log N)$

14. $T(n) = 7T(n/2) + 3n^2 + 2$

using Master's Theorem

$a = 7, b = 2, k = 2$

$a > b^k$

$7 > 2^2$

$= \text{True}$
 $T(n) = O(n^{\log_2 7})$

$= O(n^{2.8})$ Ans.

15. $f_1(n) = n^{\sqrt{n}}$

$f_2(n) = 2^n$

$f_3(n) = (1.00001)^n$

$f_4(n) = n^{(1.0 + 2^{-(n/2)})}$

$f_4 > f_2 > f_3 > f_1$



⑩. $f(n) = \frac{2^n (2n)}{\prod (2^n n)}$

⑪. $T(n) = 2T(n/2) + n^2$
 $a=2, b=2, k=2$
 $2 < 2^2$ $b=0$

$T(n) = O(n^k \log^b n)$
 $T(n) = O(n^2)$ Ans.

⑫.

```
int ged (int n, int m) {
    if (n % m == 0) return m;
    if (n < m) swap (n, m);
    while (m > 0) { n = n % m;
                    swap (n, m); }
    return m; }
```

↳ here n is gradually decreasing

$\therefore O(\log n)$ Ans.

⑬.

```
int a=0, b=0;
for (i=0; i<N; i++) {
    for (j=0; j<N; j++) {
        a = a + j;
    }
    for (k=0; k<N; k++) {
        b = b + n;
    }
}
```

$O(n \times n + n)$
 $O(n^2 + n)$
 $= O(n^2)$ Ans.