- Given data for Semiconductor – X :

$E_g = 1.8 \text{ eV}$

$\varepsilon_r = 10$

$m_n^* = 0.25 m_o \quad m_p^* = 0.5 m_o$

$\mu_n = 800 \text{ cm}^2/\text{Vs} \quad \mu_p = 400 \text{ cm}^2/\text{Vs}$

$T = 300 \text{ K}$

A. Material characterization and carrier statistics

A1. ① Find N_c, N_v at 300 K :

- $N_c = 2 \cdot \left(\frac{2\pi m_n^* kT}{h^2} \right)^{3/2} = 2 \cdot \left(\frac{2 \times 3.14 \times 0.25\, m_o \times k \times 300}{h^2} \right)^{3/2}$

$= 3.1343 \times 10^{24} \text{ m}^{-3}$

$\boxed{N_c = 3.1343 \times 10^{18} \text{ m}^{-3}}$

- $N_v = 2 \cdot \left(\frac{2\pi m_p^* kT}{h^2} \right)^{3/2} = 2 \cdot \left(\frac{2 \times 3.14 \times 0.5\, m_o \times k \times 300}{h^2} \right)^{3/2}$

$= 8.8653 \times 10^{24} \text{ m}^{-3}$

$\boxed{N_v = 8.8653 \times 10^{18} \text{ cm}^{-3}}$

② Find intrinsic carrier conc. n_i :

$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT} \right]$

$= 3.13 \times 10^{18} \times 8.86 \times 10^{18} \times \exp\left[- \frac{1.8 \times 1.6 \times 10^{-19}}{k \times 300} \right]$

$= 17594589.34$

$\Rightarrow n_i = \sqrt{17594589.34}$

$\boxed{n_i = 4194.59 \text{ cm}^{-3}}$

B. Junction and Design

④ Choose N_A, N_D such that :
- i) $0.5 \le V_{bi} \le 1$
- ii) $W \le 1\,\mu m$
- iii) minority carrier conc. non-degenerate

Let us assume a symmetric junction, where $N_A = N_D = x$ for simplicity and symmetry :

Taking boundary conditions :

① $V_{bi} \le 1$

$$\Rightarrow \frac{2kT}{q} \ln\left(\frac{x}{n_i}\right) \le 1$$

$$\Rightarrow x \le 1.02 \times 10^{12}\ cm^{-3}$$

② $V_{bi} \ge 0.5$

$$\Rightarrow x \ge \exp\left(\frac{0.5q}{2kT}\right) \cdot n_i$$

$$\Rightarrow x \ge 6.56 \times 10^{7}\ cm^{-3}$$

$$\Rightarrow 6.563 \times 10^{7}\ cm^{-3} \le x \le 1.026 \times 10^{12}\ cm^{-3}$$

① when $x = 6.563 \times 10^{7}\ cm^{-3}$:

$$W = \sqrt{\frac{2\varepsilon V_0}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$$

$= 0.58\ cm$, which is greater than $1\,\mu m$ ✗

② when $x = 1.026 \times 10^{12}\ cm^{-3}$:

$W = 3.28 \times 10^{-3}\ cm$, which is greater than $1\,\mu m$ ✗

$\Rightarrow$ Hence given these values of N_c, N_v, n_i, E_g we cannot design such a diode

Solution : if we decrease E_g, hence increasing n_i, we may be able to achieve the req. conditions :

- @ $E_g = 1.8\ eV$, we have very small value of $n_i \Rightarrow V_{bi}$ becomes too large
- Such large V_{bi} forces depletion width W to become larger than necessary $\left[V_{bi} \propto \ln\left(\frac{x^2}{n_i^2}\right)\right]$
- Let's take $E_g \approx 1.36\ eV$, close to that of GaAS :

When $E_g = 1.36$ eV :-

$$n_i^2 = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

$$= 3.13 \times 10^{18} \times 8.86 \times 10^{18} \times \exp\left[-\frac{1.36 \times 1.6 \times 10^{-19}}{k \times 300}\right]$$

$$= 4.24 \times 10^{14}$$

$$\Rightarrow n_i = \sqrt{4.24 \times 10^{14}}$$

$$\boxed{n_i = 2.05 \times 10^{7} \text{ cm}^{-3}}$$

Now, taking the condition $V_{bi} \leq 1$:-

$$\Rightarrow \frac{2kT}{q} \ln\left(\frac{x}{n_i}\right) \leq 1$$

$$\Rightarrow x \leq \exp\left(\frac{q}{2kT}\right) \times n_i$$

$$\underline{x \leq 5.036 \times 10^{15} \text{ cm}^{-3}}$$

Also $\omega = \sqrt{\frac{2\varepsilon \times 1 \times 2}{q \times x}} = 6.628 \times 10^{-5}$ cm, which is less than 1 um ✓

- Since the conditions $0.5 \leq V_{bi} \leq 1$ and $\omega \leq 1$ um are satisfied by the eqn. $x \leq 5.036 \times 10^{15}$ cm^{-3}, we will assume $\underline{N_A = N_D = 5.036 \times 10^{15} \text{ cm}^{-3}}$

- $\Rightarrow V_{bi} = \frac{2kT}{q} \ln\left(\frac{x}{n_i}\right) = \frac{2kT}{q} \ln\left(\frac{5.036 \times 10^{15}}{2.05 \times 10^{7}}\right) = \underline{0.99 V}$

$$\Rightarrow \omega = \sqrt{\frac{2\varepsilon V_0}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)} = \sqrt{\frac{2 \times 10 \times 8.84 \times 10^{-14} \times 2 \times 0.99}{1.6 \times 10^{-19} \times 5.036 \times 10^{15}}}$$

$$= 6.595 \times 10^{-5} \text{ cm}$$

$$\underline{\omega = 0.66 \text{ um}}$$

- Test for minority carrier non-degeneracy :-

Non-degeneracy requires the fermi level to be several kT away from nearest band edge :-

Conditions : ① $(E_c - E_F) > 3kT$

② $(E_F - E_v) > 3kT$

[where $3kT = 1.24 \times 10^{-20}$]

→ ~~For~~ ① $N_A = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$

$\Rightarrow E_F - E_v = -kT \ln\left(\frac{N_A}{N_v}\right) = -kT \ln\left(\frac{5.036 \times 10^{15}}{8.86 \times 10^{18}}\right)$

$(E_F - E_v) = 3.095 \times 10^{-20}$, which is greater than $3kT$ ✓

② $N_D = N_c \exp\left(-\frac{(E_c - E_F)}{kT}\right)$

$\Rightarrow (E_c - E_F) = 2.66 \times 10^{-20}$, greater than $3kT$ ✓

Conclusion : if we assume $E_g = 1.36$ eV,

- $n_i = 2.05 \times 10^7$ cm^{-3}
- ~~$V_{bi} = 0.99V$~~ • $N_A = N_D = 5.036 \times 10^{15}$ cm^{-3}
- $V_{bi} = 0.99$ V and $W = 0.66$ µm
- $(E_c - E_F)$ and $(E_F - E_v) > 3kT$, hence minority carriers non-degenerate.

Hence, all required conditions satisfied. ✓

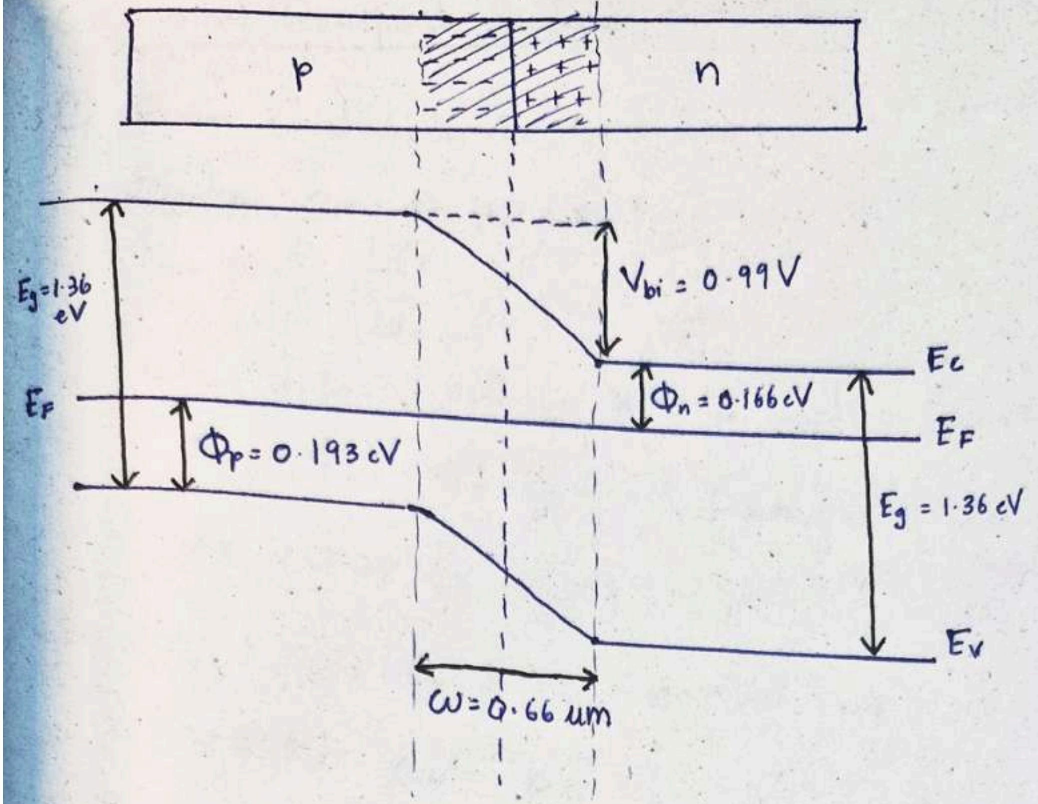
⑤ Band diagram showing E_c, E_v, E_F, V_{bi}, W :-

when testing for non degeneracy we calculated :-

$E_c - E_F = 2.66 \times 10^{-20} = 0.166$ eV $= \phi_n$

$E_F - E_v = 3.095 \times 10^{-20} = 0.193$ eV $= \phi_p$

$E_g = 1.36$ eV , $V_{bi} = 0.99V$, $W = 0.66$ µm :-



⑥ Find x_n and x_p, E_{max}, charge density distribution :-

- Since ~~$x_n = x_p$~~ W is symmetric, $\underline{x_n = x_p} = \frac{W}{2} = \boxed{0.33\ \mu m}$

- $E_{max} = \dfrac{q N_A x_p}{\varepsilon} = \dfrac{q N_D x_n}{\varepsilon} = \dfrac{1.6\times10^{-19} \times 5.036\times10^{15} \times 0.33\times10^{-4}}{10\times\varepsilon_0}$

$$\boxed{E_{max} = 3\times10^{4}\ V/cm}$$

- Space charge density distribution :-

$$\rho(x) = \begin{cases} -qN_A, & -x_p \le x \le 0 \quad (\text{p side}) \\ +qN_D, & 0 \le x \le x_n \\ 0, & \text{elsewhere} \end{cases}$$

$$\Rightarrow \boxed{\rho(x) = \begin{cases} -8.057\times10^{-4}, & -x_p \le x \le 0 \\ +8.057\times10^{-4}, & 0 \le x \le x_n \\ 0, & \text{elsewhere} \end{cases}}$$

C. Current Transport Analysis

① Derive ideal diode eqn. for X-semi :-

Starting eqn : $n_o p_o = n_i^2$

$\Rightarrow n_{p_o} = \frac{n_i^2}{N_A}$, $p_{n_o} = \frac{n_i^2}{N_D}$

When external voltage V is applied (forward bias) :-

$n_p = n_{p_o} \exp\left(\frac{qV}{kT}\right)$ [Here $V_{bi} \downarrow$, carrier injection occurs]

$p_n = p_{n_o} \exp\left(\frac{qV}{kT}\right)$

$\Rightarrow \Delta n_p = n_p - n_{p_o} = n_{p_o}\left(\exp\left(\frac{qV}{kT}\right) - 1\right)$

$\Delta p_n = p_n - p_{n_o} = p_{n_o}\left(\exp\left(\frac{qV}{kT}\right) - 1\right)$

These Δn_p, Δp_n excess carriers diffuse away from junction, recombine with majority carriers :-

① e^- diffusion current component on p side :-

$J_n = \frac{q D_n \Delta n_p}{L_n} \rightarrow$ ⓘ $\left[\begin{array}{l} D_n, D_p : \text{diffusion coeff.} \\ L_n, L_p : \text{diffusion length} \end{array}\right]$

② hole diffusion current component on n side :-

$J_p = \frac{q D_p \Delta p_n}{L_p} \rightarrow$ ⓘⓘ

$\Rightarrow$ Total current density $J = J_n + J_p$

$= q\left(\exp\left(\frac{qV}{kT}\right) - 1\right)\left[\frac{D_n n_{p_o}}{L_n} + \frac{D_p p_{n_o}}{L_p}\right]$

But, saturation current $J_o = q\left(\frac{D_n n_{p_o}}{L_n} + \frac{D_p p_{n_o}}{L_p}\right)$

$\Rightarrow \boxed{J = J_o\left(\exp\left(\frac{qV}{kT}\right) - 1\right)}$

Multiplying by diode area A :-

$\boxed{I = I_s\left(\exp\left(\frac{qV}{kT}\right) - 1\right)}$ where $I_s = Aq\left(\frac{D_n n_i^2}{N_A L_n} + \frac{D_p n_i^2}{N_D L_p}\right)$

⑧ Calculate I_s when $A = 1 mm^2$:-

- $$I_s = Aq\left[\frac{D_n n_i^2}{N_A L_n} + \frac{D_p n_i^2}{N_D L_p}\right] \rightarrow (i)$$

- where $D_n = \mu_n \times \frac{kT}{q} = 800 \times \frac{k \times 300}{q} = \underline{20.709 \ cm^2/s}$

  $D_p = \mu_p \times \frac{kT}{q} = \underline{10.354 \ cm^2/s}$

- $L_n = \sqrt{D_n \times \tau_n} = \sqrt{20.709 \times 10^{-8}} = \underline{4.55 \times 10^{-4}} \ cm$

  $L_p = \sqrt{D_p \times \tau_p} = \sqrt{10.354 \times 10^{-8}} = \underline{3.22 \times 10^{-4}} \ cm$

[__Justification for $\tau = 10^{-8}$__ : Since we took $E_g = 1.36 \ eV$, the recombination should be mainly radiative. For direct semiconductors like GaAs, minority carrier lifetime is generally $\approx 10^{-9}s - 10^{-8}s$]

- __Substituting all values in (i)__ :-

$$I_s = 10^{-2} \times 1.6 \times 10^{-19} \times \left[\frac{20.709 \times n_i^2}{N_A \times 4.55 \times 10^{-4}} + \frac{10.354 \times n_i^2}{N_D \times 3.22 \times 10^{-4}}\right]$$

$$(3798.129) + (2683.331)$$

$$\Rightarrow \boxed{I_s = 1.047 \times 10^{-17} \ A}$$

⑨ Plot IV curve for forward biases between 0-1 V :-

__Eqn__ : $I = 1.047 \times 10^{-17}\left(\exp\left(\frac{V}{0.0258}\right) - 1\right)$

Observations from graph :-

① I increases exponentially with voltage

② At around 0.8V, sharp rise → turn on voltage

③ For X, turn on voltage $\approx 0.8V$, which is higher than Si ($\approx 0.6 V$) because of its larger E_g.

(Refer to attached plot)

D. Realistic Conditions

(11) If $T = 400K$, effect on V_{bi}, W, I_s :-

- Change in N_c, N_v and n_i :-

(1) $N_c \propto T^{3/2}$

$$\Rightarrow \frac{3.1343 \times 10^{18}}{N_c(400k)} = \left(\frac{3}{4}\right)^{3/2}$$

$$\Rightarrow \underline{N_c = 4.979 \times 10^{18}\ cm^{-3}}$$

Similarly, $N_v = \left(\frac{4}{3}\right)^{3/2} \times 8.8653 \times 10^{18}$

$$\underline{N_v = 1.364 \times 10^{19}\ cm^{-3}}$$

(2) $n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$

$$= 4.979 \times 10^{18} \times 1.364 \times 10^{19} \times \exp\left(\frac{-1.36 \times 1.6 \times 10^{-19}}{400 \times k}\right)$$

$$= 5.247 \times 10^{20}$$

$$\Rightarrow n_i = \sqrt{5.247 \times 10^{20}} \Rightarrow \underline{n_i = 2.29 \times 10^{10}\ cm^{-3}}$$

- Effect on V_{bi} :-

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) = \frac{2 \times k \times 400}{q} \ln\left(\frac{5.036 \times 10^{15}}{2.29 \times 10^{10}}\right)$$

$$\boxed{V_{bi} = 0.85V}$$

$\Rightarrow$ with $\uparrow$ in temp., $V_{bi} \downarrow$ &

(3) Effect on W :-

$$W = \sqrt{\frac{2\varepsilon V_{bi}}{q} \times \left(\frac{1}{N_A} + \frac{1}{N_D}\right)} = \sqrt{\frac{2 \times 10 \times 8.86 \times 10^{-14} \times 0.85 \times 2}{1.6 \times 10^{-19} \times 5.036 \times 10^{15}}}$$

$\boxed{W = 0.611\ um}$ which is approximately a $\underline{7.5\%\ decrease}$

$$\left[\%\ change = \frac{0.61 - 0.66}{0.66} \times 100\right]$$

(4) Effect on saturation current I_s :-

$$I_s = Aq\left[\frac{D_n n_i^2}{N_A L_n} + \frac{D_p n_i^2}{N_D L_p}\right]$$

- $D_n = \frac{k \times 400}{q} \times 800 = \underline{27.61 \ cm^2/s}$

$D_p = \frac{k \times 400}{q} \times 400 = \underline{13.8 \ cm^2/s}$

- $L_n = \sqrt{D_n \tau_n} = \sqrt{27.61 \times 10^{-8}} = \underline{5.254 \times 10^{-4}} \ cm$

$L_p = \sqrt{D_p \tau_p} = \sqrt{13.8 \times 10^{-8}} = \underline{3.714 \times 10^{-4}} \ cm$

- Substituting in I_s eqn :-

$$I_s = 10^{-2} \times q \times \left[\frac{27.61 \times n_i^2}{N_A \times 5.254 \times 10^{-4}} + \frac{13.8 \times n_i^2}{N_D \times 3.714 \times 10^{-4}}\right]$$

$\boxed{I_s = 1.49 \times 10^{-11} A}$ ⇒ I_s increases by a factor of $\approx 1.3 \times 10^6$ (exponential relation hence major rise)

⇒ To conclude :-

	300k	400k	conclusion
V_{bi}	0.99 V	0.85 V	↓ V_{bi} if T↑
W	0.66 µm	0.61 µm	decrease by 7.5%
I_s	$1.05 \times 10^{-17} A$	$1.4 \times 10^{-11} A$	increase by factor 10^6

(12) Change in I-V behavior and band diagram if p side doping increased by 100X ?

- Currently we have $N_A = N_D = 5.036 \times 10^{15} cm^{-3}$

acc. to the qn, we modify $N_A' = 100 N_A = \underline{5.036 \times 10^{17} cm^{-3}}$

while N_D remains $5.036 \times 10^{15} \ cm^{-3}$

This creates asymmetric p^+-n junction

- $V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A' N_D}{n_i^2}\right) = \underline{1.12 V}$

⇒ band bending increases form 0.99V to 1.12V.

- Depletion width becomes strongly one sided :

for p^+n junction , $x_p \propto 1/N_A'$ → shrinks x 100

$x_n \propto 1/N_D$ → unchanged

By charge neutrality equation :

$$N_A' x_p = N_D x_n$$

Since $N_A' \gg N_D$, $\underline{x_p \approx 0 \text{ and } x_n \approx W}$

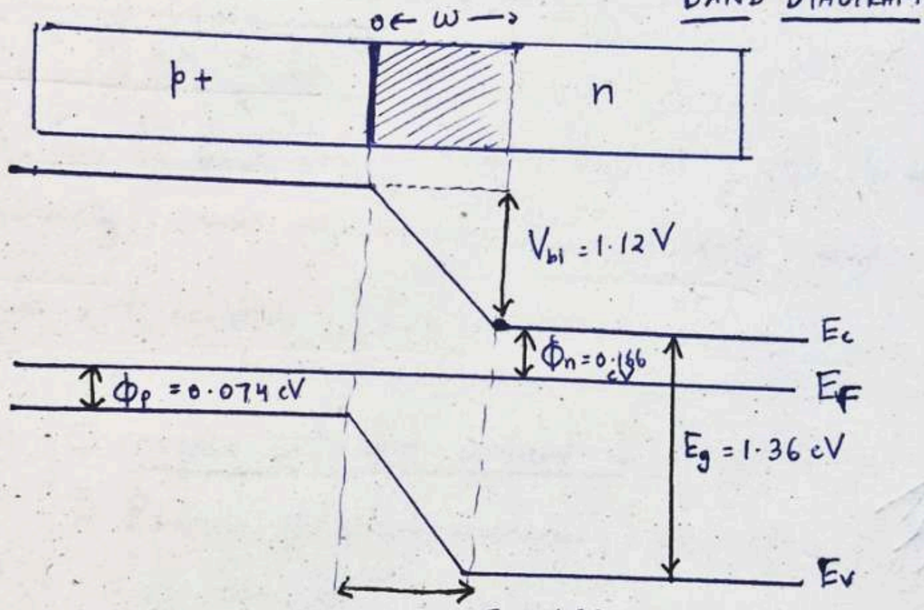
- <u>Changes in band diagram</u> :

① p side - very steep due to large doping

② n side - wider depletion region

③ Fermi lvl. on p side lies very close to E_v due to heavy doping

<u>BAND</u> <u>DIAGRAM</u>



$W \approx x_n = 0.498$ um

- Here $W = \sqrt{\dfrac{2\varepsilon V_{bi}}{q}\left(\dfrac{1}{N_A} + \dfrac{1}{N_D}\right)}$

$$= \sqrt{\frac{2\varepsilon \times 1.12}{q}\left(\frac{1}{5.036\times10^{17}} + \frac{1}{5.036\times10^{15}}\right)} = \underline{0.498 \text{ um}}$$

- $\phi_p = -kT \ln\left(\dfrac{N_A'}{N_V}\right) = -kT \ln\left(\dfrac{5.036\times10^{17}}{8.815\times10^{18}}\right) = \underline{0.074 \text{ eV}}$

$\phi_n = -kT \ln\left(\dfrac{N_D}{N_C}\right) = -kT \ln\left(\dfrac{5.036\times10^{17}}{3.13\times10^{18}}\right) = \underline{0.166 \text{ eV}}$

- Effect on I-V characteristics :-

from ⑨, we had the I-V relation :-

$$I = I_s\left(\exp\left(\frac{qV}{kT}\right) - 1\right) \quad \left[\because I_s = Aq\left(\frac{D_n n_i^2}{N_A L_n} + \frac{D_p n_i^2}{N_D L_p}\right)\right]$$

$$= 1.047 \times 10^{-17}\left(\exp\left(\frac{V}{0.0258}\right) - 1\right)$$

In p^+n junction, the total current is dominated by holes injected from p^+ side to n side, where these holes become the minority carrier on n side :-

$$\Rightarrow I_s = Aq\left(\frac{D_p n_i^2}{L_p N_D}\right)$$ → since ~~1st term~~ $N_A' \uparrow\uparrow$, $1/N_A' \downarrow\downarrow$ hence 1st term neglected

$$= 10^{-2} \times q\left(\frac{\cancel{20} \rightarrow 10.354 \times (2.05 \times 10^7)^2}{3.22 \times 10^{-4} \times 5.036 \times 10^{15}}\right)$$ [taking D_n, L_p ideal cond values]

$$\boxed{I_s = 4.29 \times 10^{-18}\,A}$$

⇒ This low n_i and heavy p side doping greatly suppresses minority carrier injection ⇒ lower leakage, ~~high~~

⇒ New I-V relation : $\boxed{I = 4.29 \times 10^{-18}\left(\exp\left(\frac{V}{0.0258}\right) - 1\right)}$

- Thus, ① reverse saturation current ↓, less leakage

② Barrier potential increases, hence turn-on voltage of the diode increases

③ One sided depletion region