

MABAC model based on linguistic (p, q)-rung orthopair fuzzy Z-number and their application in green supply chain management[☆]

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ABSTRACT

The problem and complication arise from the growing environmental inefficiencies and concerns in traditional supply chains, for instance, poor accountability, excessive waste, and lack of transparency. The green supply chain practices aim to reduce or minimize the environmental impact of supply chain activities, but these efforts often face problems, for example, difficulty in monitoring sustainability performance, data manipulation, and limited traceability across numerous stakeholders. The main problem is that without effective techniques to verify and track eco-friendly practices, enterprises struggle to utilize and enforce green initiatives reliably. The blockchain technique is being derived as a solution because of its capability to give decentralized, transparent, and immutable records of processes and transactions. By integrating the blockchain into green supply chain practices, we aim to design the model of linguistic (p, q)-rung orthopair fuzzy Z-number sets with algebraic and Sugeno-Weber operational laws for the construction of the power weighted averaging operator and power weighted geometric operator. These operators can be used in the utilization of the multi-attributive border approximation area comparison model, which is also explained step-by-step with the help of examples to simplify the supremacy and validity of the invented model by comparing their ranking values with the ranking values of the existing approaches.

1. Introduction

In this section, we present the existing techniques in sub-sections, including their implementation, extensions, and applications. For example, first, we discussed in detail the impact of blockchain technologies on green supply chain practices. Then, we revised the fuzzy technique model into the linguistic (p, q)-rung orthopair fuzzy Z-number and explained its application in decision-making, green supply chain management, and blockchain. Additionally, we reviewed the Sugeno-Weber information technique and various types of decision-making models along with their applications and utilizations in different fields.

1.1. Impact of blockchain technologies on green supply chain practices

Green supply chain practice (GSCP) is the combination of environmentally friendly techniques with the enterprise's supply chain from product design and raw data obtaining to production, profitability, transportation, usage, and last disposal. The aim is to reduce the environmental impact and improve sustainability while enhancing or maintaining profitability, efficiency, and quality. The green supply chain is the integration of the traditional supply chain and environmental sustainability, and is considered with three pillars, for instance, social, economic, and environmental. The major fields and areas of the GSCP are described in Table 1.

From the above information, we noticed that the GSCP is an efficient technique and model for the utilization of the business. The foundations

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Table 1
Main areas of the green supply chain practices.

PHASE	GREEN PRACTICE	EXPLANATIONS
Designing the Product	Eco-design for environmental	Aim to design the product with the following characteristics: 1) Consume less energy. 2) Easily recycle. 3) Biodegradable or reusable.
Procurement	Green purchasing	Aim to select the supplier according to their environmental circumstances, for instance, sourcing raw materials responsibly, like renewable and recycled inputs.
Developing	Clean and lean development	Aim to reduce energy usage, waste during production, and emissions with eco-friendly techniques.
Packaging	Supportable packaging	Aim to use recyclable, compostable, or minimal packaging materials.
Distributions	Green logistics	Aim to use the shortest route to reduce fuel, use an electric vehicle, or biofuel.
Usage and end-of-life	Product capability and recycling	Aim to guarantee that products require fewer possessions and a system for capturing back secondhand products for recycles.

and principles of the green supply chain management are Green product design and lifecycle thinking, Sustainable and ethical sourcing, Circular economy and reverse logistics, Operational energy efficiency, Transportation and logistics optimization, Waste minimization and recycling, Governance, collaboration, and transparency, and Life-cycle assessment and integrated chain management.

The main question is “why go green?” The reasons include cost savings, regulatory compliance and risk mitigation, brand value and consumer trust, operational resilience, and innovation with market leadership strategies. These techniques have been applied to many real-world problems, such as plastic recycling, corporate emission practices, circular economy/policy, retail and agriculture, and vertical integration with local sourcing. Some problems and challenges of GSCP are also described, such as greenwashing with policy limitations, data and measurement complexity, talent and capacity constraints, balancing local focus with risk, and high upfront investment. Furthermore, blockchain is a digital or decentralized ledger that records information (such as transactions) in a tamper-proof, secure, and transparent way across a network of systems and technologies. Blockchain is the core technology behind Ethereum and Bitcoin, but it is also widely used in finance, healthcare, and supply chains. The main advantages of blockchain include minimizing fraud, increasing transparency and trust, reducing costs, and improving information security with traceability. Numerous applications of GSCP and blockchain have been proposed by different scholars. For instance, Bag et al. (Bag et al., 2025) conducted an empirical study on the unveiling of metaverse potential in GSCP. Mahapatra et al. (Mahapatra et al., 2025) described smart supply chain practices with the impact of demand patterns. Manzoor et al. (Manzoor et al., 2025) evaluated an organizational theoretical overview related to blockchain technology and GSCP. Hussain et al. (Hussain et al., 2025) examined efficiency and subsidies in GSCP using duopoly game dynamics. Sarkar et al. (Sarkar et al., 2025) presented an analysis of the textile GSCP with wastewater treatment effects. Shaheen et al. (Shaheen et al., 2026) analyzed early diabetes prediction based on deep residual networks and proximity-based information. Khan et al. (Khan et al., 2025) proposed explainable early-stage diabetes mellitus prediction with a deep gated network. Khan et al. (Khan et al., 2025) discussed IoT device privacy and information integrity based on decentralized storage with blockchain. Javed et al. (Javed et al., 2024) explored vehicular energy networks using blockchain in smart healthcare systems. Khan et al. (Khan et al., 2023) evaluated blockchain-based deep-learning-driven technology with wireless sensor networks.

1.2. Fuzzy sets to (p, q) -Rung orthopair fuzzy sets

The investigation and assessment of the most preferable decision is very complex, especially in the presence of classical set theory. The decision-making technique is widely known and well-structured for evaluating decision-making problems. The multi-attribute decision-making (MADM) technique is a popular and effective tool, which is also used for interpreting the best decision. Numerous applications have been discussed by different scholars for solving problems under the consideration of crisp information. The main issue is that, during the decision-making process, a large amount of data is lost due to vagueness and complexities. The crisp set is massive, narrow, and limited in range; it contains only two human opinions, for example, zero and one. However, to address real-life problems, a stronger structure is required that can handle values between zero and one. For this reason, the model of fuzzy set (FS) theory was introduced by Zadeh (Zadeh, 1965) in 1965. FS theory is very effective and well-recognized due to its structure, where the truth degree in FSs is bounded within the unit interval, meaning there are multiple possible values for the truth function, such as 0, 1, 0.3, 0.7, etc.

Over time, many scholars observed that several problems remained unresolved due to ambiguity and uncertainty. Although FSs are reliable, their truth function represents only positive information. This raises the question of how to deal with negative information, since without including it, the structure of FSs remains incomplete. To address this, Atanassov (Atanassov, 1983, Atanassov, 1986) developed intuitionistic FSs (IFs). IFs include the condition that the sum of both functions is restricted to the unit interval. Later, in 2013, Yager (Yager, 2013) proposed Pythagorean FSs (PFSs). The structure of PFSs is similar to IFs, but its condition is stronger: the sum of the squares of both values must lie within the unit interval. This means that if we take the square of both values, the resulting value must still be within the unit interval. However, Yager (Yager, 2016) later noticed that if the truth value is 0.9 and the falsity value is 0.8, then both IFs and PFSs fail to handle such information. To solve this, in 2016, Yager developed q -rung orthopair FSs (q -ROFSs), which use the same structure as IFs and PFSs but with a different condition: the sum of the q -powers of both values is restricted to the unit interval. For $q = 1$ and $q = 2$, q -ROFSs reduce to IFs and PFSs respectively. From this assessment, it was observed that q -ROFSs are more valuable and reliable for dealing with vague data. Later, Ibrahim and Alshammari (Ibrahim & Alshammari, 2022) introduced a new structure called (p, q) -rung orthopair FSs (pq -ROFSs), which is a modified version of q -ROFSs. When $p = q$, pq -ROFSs reduce to q -ROFSs. The main motivation behind pq -ROFSs is that, in some cases, the truth value is very low while the falsity value is close to 1. To address this, q is increased for both functions in q -ROFSs, but ideally, the increase is only needed for the falsity value. However, due to the limitations of q -ROFSs, this is not possible, which leads to the development of pq -ROFSs. See Fig. 1.

Fig. 1 describes the geometrical interpretation of the system of pq -ROFSs, because in one figures, we fixed the value of “ x ” and variates it with the help of variable “ y ” and in second figure, we have fixed the value of “ y ” and variates it with the help of “ x ”, but if both values are equal, then we will variate it with respect to both.

1.3. Extensions of Z-numbers

The system of fuzzy models is a well-organized and effective tool for representing vague and uncertain data, especially in real-world problems. FSs contain only a membership function, which expresses human opinions more effectively compared to classical information. In 2011, Zadeh (Zadeh, 2011) introduced the concept of the Z-number set, which is a modified form of FSs. A Z-number consists of two components: the first is the fuzzy membership value, and the second is the supporting value of the first one. In many real-world scenarios, the truth function alone is not sufficient to reflect human opinions, as a supportive function is often needed to explain them more accurately. After the development of the Z-number, many applications and extensions have been proposed.

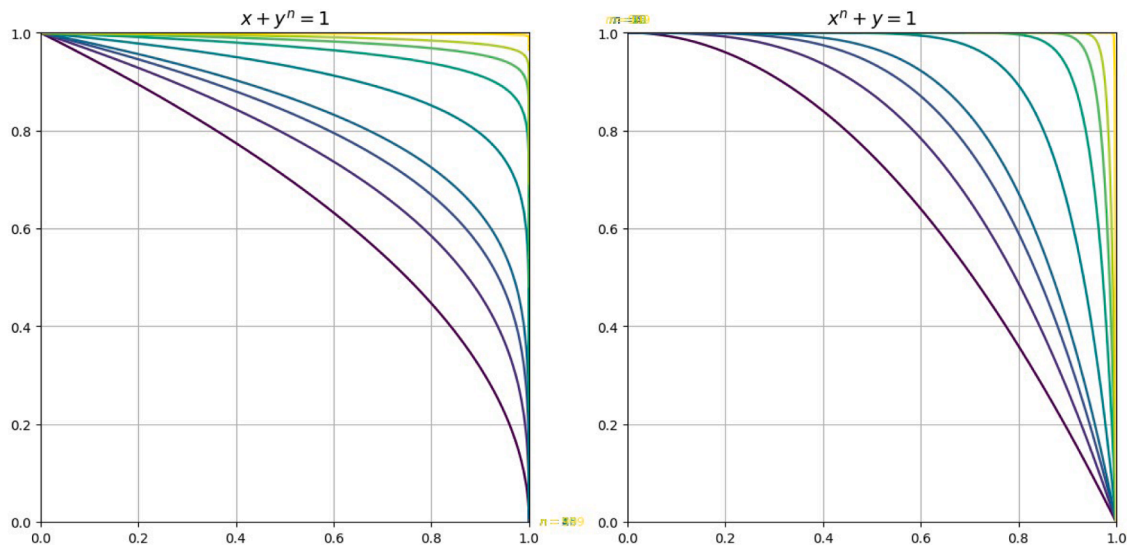


Fig. 1. Graphical form of PQ-ROFSs with different variations.

For example, Mohamad et al. (Mohamad et al., 2014) evaluated fuzzy Z-numbers (FZN) in decision-making strategies. Jaini (Jaini, 2023) applied intuitionistic FZN to supplier evaluations. Jia and Herrera-Viedma (Jia & Herrera-Viedma, 2022) studied Pythagorean FZN in decision-making contexts. Kumar and Gupta (Kumar & Gupta, 2024) discussed the use of the TOPSIS technique for q-rung orthopair FZN. All these approaches represent dominant and effective extensions and integrations of FSs with Z-number information for addressing real-world problems.

1.4. Literature review

Describing human opinions more effectively and precisely is a difficult and challenging task, since fuzzy values alone are often insufficient, especially when applied to a linguistic scale. To address this issue, Zadeh (Zadeh, 1975, Zadeh, 1975, Zadeh, 1975) introduced the concept of linguistic term sets (LTs), which incorporate linguistic variables to better represent human opinions. Tong and Bonissone (Tong & Bonissone, 1980) combined LTs with FSs and developed the technique of linguistic FSs (LFSs), which express human opinions more accurately. Building on this, Zhang (Zhang, 2014) integrated LTs with IFSs to design linguistic IFSs that include a linguistic truth function and a linguistic falsity function. Garg (Garg, 2018) further established the technique of linguistic PFSSs, also incorporating linguistic truth and falsity functions, with conditions similar to those of standard PFSSs. In 2020, Lin et al. (Lin et al., 2020) introduced linguistic q-ROFSs using the same foundation, but with modifications due to the power parameter “q.”

Pamucar and Cirovic (Pamučar & Čirović, 2015) derived the MABAC model for classical information. Sugeno (Sugeno, 1974) developed fuzzy integrals for practical applications, while Weber (Weber, 1983) created a general framework of fuzzy connectives with norms (Klement et al., 1997). Yager (Yager, 2001) proposed the power average (PA) operator, and Xu and Yager (Xu & Yager, 2009) introduced the power geometric (PG) operator. Hussain et al. (Hussain et al., 2024) applied Sugeno–Weber operators to IFSs, while Wang et al. (Wang et al., 2024) extended these operators to q-ROFSs. Liu and Qin (Liu & Qin, 2017) proposed power operators for linguistic IFSs, and Jiang et al. (Jiang et al., 2018) explored entropy-based power operators for IFSs with applications. Xu (Xu, 2011) also developed power operators for IFSs within decision-making frameworks. Wei and Lu (Wei & Lu, 2018) examined power operators for PFSSs, and Du (Du, 2019) designed weighted power operators for q-ROFSs. In addition, Codur and ErKayman (Çodur & ErKayman, 2025) described a blockchain-based model for supply chain

management assessment. Alam et al. (Alam et al., 2025) analyzed digital transformation by prioritizing barriers, while Dehshiri (Dehshiri, 2025) developed comparative decision-making techniques. Mehdiabadi et al. (Mehdiabadi et al., 2025) introduced the fuzzy SWARA-MABAC technique. Lo et al. (Lo et al., 2025) designed a sustainable supplier selection framework using hybrid decision-making techniques. Lukovic et al. (Luković et al., 2025) studied blockchain adoption within Industry 4.0, and Gazi et al. (Gazi et al., 2025) proposed a decision-making model for humanitarian supply chain management

1.5. Research gap and motivation

The existing systems and techniques are very efficient, especially when an expert provides information in the form of truth and falsity values within the unit interval. However, the main problem is that these values do not fully capture human opinions. For example, when providing information about the weather, linguistic variables are often required, which cannot be accurately represented by simple fuzzy values. Nevertheless, during the decision-making process, experts frequently face the following problems, such as:

Why do we develop the MABAC model?

For evaluating the best decision, numerous scholars have applied decision-making techniques. This approach is acceptable, but the main issue is that decision-making techniques often use aggregation operators and similarity measures separately. As a result, they sometimes produce two different outcomes, which creates difficulties for researchers. Therefore, we aimed to find an effective solution to this problem. For this purpose, we selected the MABAC model and developed it to incorporate both aggregation operators and similarity measures, providing a single ranking value to eliminate complications. The MABAC model is well-known for simultaneously utilizing aggregation operators and similarity measures for different purposes. It is a modified version of decision-making techniques that rely on either aggregation operators or similarity measures, designed to determine the best decision among a set of alternatives.

The techniques of q-ROFSs, LTs, and Z-numbers play an important role in real-world problems, but they do not address data containing truth and falsity values in the form of a linguistic scale. This is because LTs deal only with linguistic scales, not truth and falsity, while q-ROFSs handle truth and falsity functions but not linguistic scales. Therefore, we observe that the technique of Lpq-ROFZN is highly suitable for evaluating these types of problems and addressing such complications. The existing techniques can be considered special cases of the proposed model.

As previously discussed, the MABAC model is developed based on aggregation operators and similarity measures. Hence, we aim to design appropriate aggregation operators. For this purpose, we choose the Sugeno-Weber t-norm (SWTN) and Sugeno-Weber t-conorm (SWTCN) to define operational laws based on Lpq-ROFZN. We then develop power aggregation operators based on the proposed Sugeno-Weber operational laws for Lpq-ROFZN, as these operators facilitate the aggregation of data into singleton sets. These challenges create significant difficulties for researchers, particularly when dealing with truth and falsity values expressed as linguistic variables. To date, no techniques have been developed to address these issues due to their complexity. The mathematical framework of the SWTN is described by:

$$SWTN^{\perp}(a, b) = \begin{cases} S_D(a, b) & \text{If } \perp = -1 \\ \max\left(0, \frac{a+b-1+\perp * a * b}{1+\perp}\right) & \text{If } -1 < \perp < +\infty \\ S_{Prod}(a, b) & \text{If } \perp = +\infty \end{cases}$$

Where $\mathfrak{a}, \mathfrak{b} \in [0, 1]$. The general shape of the SWTN is described:

$$f^{\perp}(a) = \begin{cases} 1-a & \text{if } \perp = 0 \\ 1 - \log_{1+\perp}(1+a\perp) & \text{otherwise} \end{cases}$$

From the above assessments and evaluations, we observed that the existing techniques are very powerful for handling vague and uncertain data. However, it would be much better to develop an approach that can encompass all existing models. For this purpose, the technique of Lpq-ROFZN is proposed. After a thorough assessment, we found that no prior work has combined LTSs, pq-ROFSs, and Z-numbers into the Lpq-ROFZN framework. Additionally, the power Sugeno-Weber operators and the MABAC technique are also highly reliable, motivating their integration into this study.

Problems and complications arise from growing environmental inefficiencies in traditional supply chains, such as poor accountability, excessive waste, and lack of transparency. Green supply chain practices aim to reduce the environmental impact of supply chain activities, but these efforts often face challenges, including difficulty monitoring sustainability performance, data manipulation, and limited traceability across multiple stakeholders. Without effective techniques to verify and track eco-friendly practices, enterprises struggle to implement and enforce green initiatives reliably. The blockchain technique offers a solution because of its ability to provide decentralized, transparent, and immutable records of processes and transactions. By integrating blockchain into green supply chain practices, we aim to design models that:

- Introduce the new concept of Lpq-ROFZN sets, which include truth and falsity degrees in the form of linguistic variables with a supportive function. The Lpq-ROFZN system is a modified version of numerous existing models, including FSs, IFs, PFs, q-ROFSs, pq-ROFSs, LTSs, Z-numbers, and their integrations.
- Initiate algebraic and Sugeno-Weber operational laws for Lpq-ROFZN values. The primary purpose of these laws is to develop aggregation operators, and they are modified versions of numerous existing operational laws.
- Develop power aggregation operators based on Lpq-ROFZN using SWTN and SWTCN, such as
- Lpq-ROFZN power Sugeno Weber averaging (Lpq-ROFZNPSWA) operator.
- Lpq-ROFZN power Sugeno Weber weighted averaging (Lpq-ROFZNPSWWA) operator.

- Lpq-ROFZN power Sugeno Weber geometric (Lpq-ROFZNPSWG) operator.
- Lpq-ROFZN power Sugeno Weber weighted geometric (Lpq-ROFZNPSWWG) operator.

These operators can be applied within the MABAC model, which is also explained step-by-step with examples to demonstrate the effectiveness and validity of the proposed model by comparing its ranking values with those of existing approaches. The graphical abstract of the proposed theory is presented in Fig. 2.

The goal of this paper is briefly outlined section-wise. In Section 2, we review the concept of pq-ROFSs along with the revised operational laws. Additionally, the techniques of LTS and Z-number sets are discussed in detail. The PA operator, PG operator, SWTN, and SWTCN techniques are also reviewed comprehensively. In Section 3, we introduce the new concept of Lpq-ROFZN sets, a modified version of several existing models. We further establish algebraic and Sugeno-Weber operational laws for Lpq-ROFZN values and develop the Lpq-ROFZNPSWA, Lpq-ROFZNPSWWA, Lpq-ROFZNPSWG, and Lpq-ROFZNPSWWG operators with new, reliable, and valuable properties. The MABAC model is designed for these operators. In Section 4, the MABAC technique is explained step-by-step with examples to demonstrate the superiority and validity of the proposed model by comparing its ranking values with those of existing approaches in Section 5. Finally, some concluding remarks are provided in Section 6.

2. Preliminaries

This section is developed to review the prevailing techniques, such as pq-ROFSs, LTS, and Z-number, which are recognized as effective models for the development of our new techniques and models. Additionally, the PA operator, PG operator, SWTN, and SWTCN techniques are also reviewed in detail.

Definition 1. (Zadeh, 1965) For any universe of discourse \mathbb{X} . The concept of pq-ROFSs \mathbb{pqROF} is designed and mentioned by:

$$\mathbb{pqROF} = \{(\mathfrak{T}\mathfrak{F}(x), \mathfrak{F}\mathfrak{F}(x)) : x \in \mathbb{X}\} \quad (1)$$

Where $\mathfrak{T}\mathfrak{F} : \mathbb{X} \rightarrow [0, 1]$, and $\mathfrak{F}\mathfrak{F} : \mathbb{X} \rightarrow [0, 1]$, are called truth and falsity values with a unique characteristic, such that $0 \leq (\mathfrak{T}\mathfrak{F}(x))^{\Delta} + (\mathfrak{F}\mathfrak{F}(x))^{\nabla} \leq 1, \Delta, \nabla \geq 1$, where $\mathfrak{T}\mathfrak{F}(x) \in [0, 1]$ and $\mathfrak{F}\mathfrak{F}(x) \in [0, 1]$. Further, the function of refusal degree is examined by: $r(x) = (1 - ((\mathfrak{T}\mathfrak{F}(x))^{\Delta} + (\mathfrak{F}\mathfrak{F}(x))^{\nabla}))^{\frac{1}{\max(\Delta, \nabla)}}$, where the simple form of pq-ROFN is listed by: $\mathbb{pqROF}_j = (\mathfrak{T}\mathfrak{F}_j, \mathfrak{F}\mathfrak{F}_j), j = 1, 2, \dots, m$.

Definition 2. (Zadeh, 1965) For any family of pq-ROFNs $\mathbb{pqROF}_j = (\mathfrak{T}\mathfrak{F}_j, \mathfrak{F}\mathfrak{F}_j), j = 1, 2, \dots, m$. The concept of algebraic models is designed and mentioned by:

$$\mathbb{pqROF}_1 \oplus \mathbb{pqROF}_2 = \left(((\mathfrak{T}\mathfrak{F}_1)^{\Delta} + (\mathfrak{T}\mathfrak{F}_2)^{\Delta} - (\mathfrak{T}\mathfrak{F}_1)^{\Delta} (\mathfrak{T}\mathfrak{F}_2)^{\Delta})^{\frac{1}{\Delta}}, \mathfrak{F}\mathfrak{F}_1 \mathfrak{F}\mathfrak{F}_2 \right) \quad (2)$$

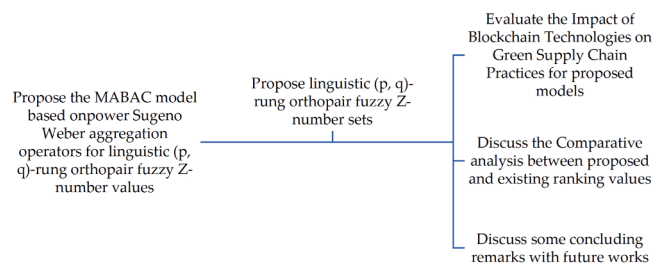


Fig. 2. Representation of proposed data.

$$\text{pqROF}_1 \otimes \text{pqROF}_2 = \left(\mathfrak{T}\mathfrak{F}_1 \mathfrak{T}\mathfrak{F}_2, ((\mathfrak{F}\mathfrak{F}_1)^\vee + (\mathfrak{F}\mathfrak{F}_2)^\vee - (\mathfrak{F}\mathfrak{F}_1)^\vee (\mathfrak{F}\mathfrak{F}_2)^\vee)^{\frac{1}{\vee}} \right) \quad (3)$$

$$\alpha \text{pqROF}_1 = \left(\left(1 - (1 - (\mathfrak{T}\mathfrak{F}_1)^\Delta)^\alpha \right)^{\frac{1}{\Delta}}, (\mathfrak{F}\mathfrak{F}_1)^\alpha \right) \quad (4)$$

$$(\text{pqROF}_1)^\alpha = \left((\mathfrak{T}\mathfrak{F}_1)^\alpha, (1 - (1 - (\mathfrak{F}\mathfrak{F}_1)^\vee)^\alpha)^{\frac{1}{\vee}} \right) \quad (5)$$

Definition 3. (Zadeh, 1965) For any family of pq-ROFNs $\text{pqROF}_j = (\mathfrak{T}\mathfrak{F}_j, \mathfrak{F}\mathfrak{F}_j), j = 1, 2, \dots, m$. The concept of Score and Accuracy values is designed and mentioned by:

$$\text{SCORE}(\text{pqROF}_j) = (\mathfrak{T}\mathfrak{F}_j)^\Delta - (\mathfrak{F}\mathfrak{F}_j)^\vee \in [-1, 1] \quad (6)$$

$$\text{ACCURACY}(\text{pqROF}_j) = (\mathfrak{T}\mathfrak{F}_j)^\Delta + (\mathfrak{F}\mathfrak{F}_j)^\vee \in [0, 1] \quad (7)$$

Definition 4. (Zadeh, 2011–Jaini, 2023) A structure of LTS is designed and mentioned by: $\| = \{ \|_i : i = 1, 2, \dots, m, \| \in [0, \sqrt{1}] \}$, with odd cardinality, but must satisfy the following condition, such as

- 1) $\|_i > \|_j \Leftrightarrow \|_j > \|_i'$.
- 2) $\text{Neg}(\|_i) = \|_i'$, where $\|_i' = \|_i \circ \mathfrak{u}$. (Negative operators)
- 3) $\max(\|_i, \|_j) = \|_i$, if $\|_i \geq \|_j'$.
- 4) $\min(\|_i, \|_j) = \|_i$, if $\|_i \leq \|_j'$.

Definition 5. (Atanassov, 1983) The structure of the Z-number set consists of two fuzzy values: the first is called the fuzzy membership value, and the second is called the supportive value of the first. These values represent the problem or limitation associated with the numbers indicated by a vague variable, as well as a measure of the reliability or certainty of the first value. The mathematical form of the Z-number set is designed by: $Z = (A, B)$.

Definition 6. (Garg, 2018) For any family of non-negative integers $\text{pqROF}_j, j = 1, 2, \dots, m$. The concept of the PA operator is designed and mentioned by:

$$\begin{aligned} \text{PA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) &= \frac{(1 + \mathfrak{H}(\text{pqROF}_1))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_1 \\ &\oplus \frac{(1 + \mathfrak{H}(\text{pqROF}_2))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_2 \\ &\oplus \dots \\ &\oplus \frac{(1 + \mathfrak{H}(\text{pqROF}_m))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_m \\ &= \bigoplus_{j=1}^m \frac{(1 + \mathfrak{H}(\text{pqROF}_j))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_j \end{aligned} \quad (8)$$

Further, we describe the technique $\mathfrak{H}(\text{pqROF}_j) = \sum_{i \neq j=1}^m S(\text{pqROF}_i, \text{pqROF}_j)$, where $S(\text{pqROF}_i, \text{pqROF}_j) = 1 - D(\text{pqROF}_i, \text{pqROF}_j)$, thus

- 1) $S(\text{pqROF}_i, \text{pqROF}_j) \in [0, 1]$.
- 2) $S(\text{pqROF}_i, \text{pqROF}_j) = S(\text{pqROF}_j, \text{pqROF}_i)$.
- 3) When $S(\text{pqROF}_i, \text{pqROF}_j) \geq S(\text{pqROF}_k, \text{pqROF}_1)$, then $D(\text{pqROF}_i, \text{pqROF}_j) \leq D(\text{pqROF}_k, \text{pqROF}_1)$.

Definition 7. (Lin et al., 2020) For any family of non-negative integers $\text{pqROF}_j, j = 1, 2, \dots, m$. The concept of the PG operator is designed and mentioned by:

$$\begin{aligned} \text{PG}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) &= (\text{pqROF}_1)^{\frac{(1 + \mathfrak{H}(\text{pqROF}_1))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))}} \\ &\otimes (\text{pqROF}_2)^{\frac{(1 + \mathfrak{H}(\text{pqROF}_2))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))}} \otimes \dots \\ &\otimes (\text{pqROF}_m)^{\frac{(1 + \mathfrak{H}(\text{pqROF}_m))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))}} \\ &= \bigotimes_{j=1}^m (\text{pqROF}_j)^{\frac{(1 + \mathfrak{H}(\text{pqROF}_j))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))}} \end{aligned} \quad (9)$$

Further, we describe the technique $\mathfrak{H}(\text{pqROF}_j) = \sum_{i \neq j=1}^m S(\text{pqROF}_i, \text{pqROF}_j)$, where $S(\text{pqROF}_i, \text{pqROF}_j) = 1 - D(\text{pqROF}_i, \text{pqROF}_j)$, thus

- 1) $S(\text{pqROF}_i, \text{pqROF}_j) \in [0, 1]$.
- 2) $S(\text{pqROF}_i, \text{pqROF}_j) = S(\text{pqROF}_j, \text{pqROF}_i)$.
- 3) When $S(\text{pqROF}_i, \text{pqROF}_j) \geq S(\text{pqROF}_k, \text{pqROF}_1)$, then $D(\text{pqROF}_i, \text{pqROF}_j) \leq D(\text{pqROF}_k, \text{pqROF}_1)$.

Definition 8. (Zadeh, 1975) (Tong & Bonissone, 1980), For any $\mathfrak{a} \in [0, 1]$ and $\mathfrak{b} \in [0, 1]$. The concept of SWTN and SWTCN is designed and mentioned by:

$$\text{SWTN}^\perp(\mathfrak{a}, \mathfrak{b}) = \begin{cases} S_D(\mathfrak{a}, \mathfrak{b}) & \text{If } \perp = -1 \\ \max\left(0, \frac{\mathfrak{a} + \mathfrak{b} - 1 + \perp * \mathfrak{a} * \mathfrak{b}}{1 + \perp}\right) & \text{If } -1 < \perp < +\infty \\ S_{\text{Prod}}(\mathfrak{a}, \mathfrak{b}) & \text{If } \perp = +\infty \end{cases} \quad (10)$$

and

$$\text{SWTCN}^\perp(\mathfrak{a}, \mathfrak{b}) = \begin{cases} T_D(\mathfrak{a}, \mathfrak{b}) & \text{If } \perp = -1 \\ \min\left(1, \mathfrak{a} + \mathfrak{b} - \frac{\perp * \mathfrak{a} * \mathfrak{b}}{1 + \perp}\right) & \text{If } -1 < \perp < +\infty \\ T_{\text{Prod}}(\mathfrak{a}, \mathfrak{b}) & \text{If } \perp = +\infty \end{cases} \quad (11)$$

3. Integration of methods

In this section, we described the technique of Lpq-ROFZN with operational laws. Then, we derived the power aggregation operators for Sugeno-Weber laws with Lpq-ROFZN. Finally, we developed the MABAC model based on the above information.

3.1. A novel technique of Lpq-ROFZN

This section introduces the Lpq-ROFZN system by integrating the techniques of pq-ROFSs, LTSs, and Z-number, creating a more prominent and realistic approach for handling vague and uncertain data. Furthermore, we present the operational techniques for algebraic norms and Sugeno-Weber norms.

Definition 9. For any universe of discourse \mathbb{X} . The concept of Lpq-ROFZN sets pqROF is designed and mentioned by:

$$\text{pqROF} = \left\{ \left((\|\overline{\mathfrak{T}\mathfrak{F}}(\mathfrak{x}), \|\overline{\mathfrak{F}\mathfrak{F}}(\mathfrak{x})\|), \left(\|\mathfrak{T}\mathfrak{F}(\mathfrak{x}), \|\mathfrak{F}\mathfrak{F}(\mathfrak{x})\| \right) \right) : \mathfrak{x} \in \mathbb{X} \right\} \quad (12)$$

Where $\overline{\mathfrak{T}\mathfrak{F}}(\mathfrak{x}), \overline{\mathfrak{F}\mathfrak{F}}(\mathfrak{x}) \in [0, \sqrt{1}]$, represents the truth and falsity functions. The supportive function is stated by: $\mathfrak{T}\mathfrak{F}(\mathfrak{x}), \mathfrak{F}\mathfrak{F}(\mathfrak{x}) \in [0, \sqrt{1}]$ with a unique characteristic, such that $0 \leq \left(\frac{\mathfrak{T}\mathfrak{F}(\mathfrak{x})}{\sqrt{1}} \right)^\Delta + \left(\frac{\mathfrak{F}\mathfrak{F}(\mathfrak{x})}{\sqrt{1}} \right)^\vee \leq 1, \Delta, \vee \geq 1$ and $0 \leq \left(\frac{\mathfrak{T}\mathfrak{F}(\mathfrak{x})}{\sqrt{1}} \right)^\Delta + \left(\frac{\mathfrak{F}\mathfrak{F}(\mathfrak{x})}{\sqrt{1}} \right)^\vee \leq 1, \Delta, \vee \geq 1$. Further, the function of

refusal degrees for both information is examined by: $\bar{r}(x) = \sqrt{\left(1 - \left(\left(\frac{\bar{z}\bar{r}(x)}{\sqrt{\bar{r}}}\right)^\Delta + \left(\frac{\bar{r}\bar{r}(x)}{\sqrt{\bar{r}}}\right)^\nabla\right)\right)^{\frac{1}{\max(\Delta, \nabla)}}}$ and $\underline{r}(x) = \sqrt{\left(1 - \left(\left(\frac{\underline{z}\underline{r}(x)}{\sqrt{\underline{r}}}\right)^\Delta + \left(\frac{\underline{r}\underline{r}(x)}{\sqrt{\underline{r}}}\right)^\nabla\right)\right)^{\frac{1}{\max(\Delta, \nabla)}}}$, where the simple form of Lpq-ROFZN sets is listed by: $\text{pqROF}_j = \left(\left(\|\bar{z}\bar{r}_j, \|\bar{r}\bar{r}_j\right), \left(\|\underline{z}\underline{r}_j, \|\underline{r}\underline{r}_j\right)\right), j = 1, 2, \dots, m$.

Definition 10. Let $\text{pqROF}_j = \left(\left(\|\bar{z}\bar{r}_j, \|\bar{r}\bar{r}_j\right), \left(\|\underline{z}\underline{r}_j, \|\underline{r}\underline{r}_j\right)\right), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, the concept of algebraic models is designed and mentioned by:

Definition 11. Let $\text{pqROF}_j = \left(\left(\|\bar{z}\bar{r}_j, \|\bar{r}\bar{r}_j\right), \left(\|\underline{z}\underline{r}_j, \|\underline{r}\underline{r}_j\right)\right), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, the concept of Score and Accuracy values is designed and mentioned by:

$$\text{SCORE}(\text{pqROF}_j) = \left(\frac{\left(\frac{\bar{z}\bar{r}_j}{\sqrt{\bar{r}}}\right)^\Delta + \left(\frac{\underline{z}\underline{r}_j}{\sqrt{\underline{r}}}\right)^\Delta}{2}\right) - \left(\frac{\left(\frac{\bar{r}\bar{r}_j}{\sqrt{\bar{r}}}\right)^\nabla + \left(\frac{\underline{r}\underline{r}_j}{\sqrt{\underline{r}}}\right)^\nabla}{2}\right) \in [-1, 1] \quad (17)$$

$$\text{pqROF}_1 \oplus \text{pqROF}_2 = \left(\left(\left\|\sqrt{\left(\left(\frac{\bar{z}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\Delta + \left(\frac{\bar{z}\bar{r}_2}{\sqrt{\bar{r}}}\right)^\Delta - \left(\frac{\bar{r}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\Delta - \left(\frac{\bar{r}\bar{r}_2}{\sqrt{\bar{r}}}\right)^\Delta\right)^{\frac{1}{\Delta}}}, \left\|\sqrt{\left(\left(\frac{\bar{r}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\nabla + \left(\frac{\bar{r}\bar{r}_2}{\sqrt{\bar{r}}}\right)^\nabla - \left(\frac{\underline{z}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\nabla - \left(\frac{\underline{z}\underline{r}_2}{\sqrt{\underline{r}}}\right)^\nabla\right)^{\frac{1}{\Delta}}}\right\|\right), \left(\left\|\sqrt{\left(\left(\frac{\underline{z}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\Delta + \left(\frac{\underline{z}\underline{r}_2}{\sqrt{\underline{r}}}\right)^\Delta - \left(\frac{\underline{r}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\Delta - \left(\frac{\underline{r}\underline{r}_2}{\sqrt{\underline{r}}}\right)^\Delta\right)^{\frac{1}{\Delta}}}, \left\|\sqrt{\left(\left(\frac{\underline{r}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\nabla + \left(\frac{\underline{r}\underline{r}_2}{\sqrt{\underline{r}}}\right)^\nabla - \left(\frac{\bar{z}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\nabla - \left(\frac{\bar{z}\bar{r}_2}{\sqrt{\bar{r}}}\right)^\nabla\right)^{\frac{1}{\Delta}}}\right\|\right)\right) \quad (13)$$

$$\text{pqROF}_1 \otimes \text{pqROF}_2 = \left(\left(\left\|\sqrt{\left(\frac{\bar{z}\bar{r}_1}{\sqrt{\bar{r}}}, \frac{\bar{z}\bar{r}_2}{\sqrt{\bar{r}}}\right)}, \left\|\sqrt{\left(\left(\frac{\bar{r}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\nabla + \left(\frac{\bar{r}\bar{r}_2}{\sqrt{\bar{r}}}\right)^\nabla - \left(\frac{\bar{r}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\nabla - \left(\frac{\bar{r}\bar{r}_2}{\sqrt{\bar{r}}}\right)^\nabla\right)^{\frac{1}{\nabla}}}\right\|\right), \left(\left\|\sqrt{\left(\frac{\underline{z}\underline{r}_1}{\sqrt{\underline{r}}}, \frac{\underline{z}\underline{r}_2}{\sqrt{\underline{r}}}\right)}, \left\|\sqrt{\left(\left(\frac{\underline{r}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\nabla + \left(\frac{\underline{r}\underline{r}_2}{\sqrt{\underline{r}}}\right)^\nabla - \left(\frac{\underline{r}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\nabla - \left(\frac{\underline{r}\underline{r}_2}{\sqrt{\underline{r}}}\right)^\nabla\right)^{\frac{1}{\nabla}}}\right\|\right)\right) \quad (14)$$

$$\alpha \text{pqROF}_1 = \left(\left(\left\|\sqrt{\left(1 - \left(1 - \left(\frac{\bar{z}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\Delta\right)^\alpha\right)^{\frac{1}{\Delta}}}, \left\|\sqrt{\left(\frac{\bar{r}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\Delta}\right\|\right), \left(\left\|\sqrt{\left(1 - \left(1 - \left(\frac{\underline{z}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\Delta\right)^\alpha\right)^{\frac{1}{\Delta}}}, \left\|\sqrt{\left(\frac{\underline{r}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\Delta}\right\|\right)\right) \quad (15)$$

$$(\text{pqROF}_1)^\alpha = \left(\left(\left\|\sqrt{\left(\frac{\bar{z}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\Delta}, \left\|\sqrt{\left(1 - \left(1 - \left(\frac{\bar{r}\bar{r}_1}{\sqrt{\bar{r}}}\right)^\Delta\right)^\alpha\right)^{\frac{1}{\Delta}}}\right\|\right), \left(\left\|\sqrt{\left(\frac{\underline{z}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\Delta}, \left\|\sqrt{\left(1 - \left(1 - \left(\frac{\underline{r}\underline{r}_1}{\sqrt{\underline{r}}}\right)^\Delta\right)^\alpha\right)^{\frac{1}{\Delta}}}\right\|\right)\right) \quad (16)$$

$$ACCURACY(\mathbb{pqROF}_j) = \left(\frac{\left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}} \right)^\Delta + \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}} \right)^\Delta}{2} \right) + \left(\frac{\left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}} \right)^\nabla + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}} \right)^\nabla}{2} \right) \in [0, 1] \quad (18)$$

Definition 12. Let $\mathbb{pqROF}_j = \left(\left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}}, \frac{\overline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}} \right), \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}}, \frac{\overline{\mathbb{z}}_{\mathbb{R}_j}}{\sqrt{\cdot}} \right) \right), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, the concept of Sugeno Weber models is designed and mentioned by:

$$\mathbb{pqROF}_1 \oplus \mathbb{pqROF}_2 = \left(\left(\left\| \sqrt{\left(\left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta + \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta - \frac{1}{1+\perp} \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta \right)^{\frac{1}{\Delta}}, \sqrt{\left(\frac{\left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla - \frac{1}{1+\perp} \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla \right)^{\frac{1}{\nabla}}} \right)^{\frac{1}{\nabla}}, \right. \right. \quad (19)$$

$$\left. \left. \left\| \sqrt{\left(\left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta + \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta - \frac{1}{1+\perp} \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta \right)^{\frac{1}{\Delta}}, \sqrt{\left(\frac{\left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla - \frac{1}{1+\perp} \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla \right)^{\frac{1}{\nabla}}} \right)^{\frac{1}{\nabla}} \right) \right)$$

$$\mathbb{pqROF}_1 \otimes \mathbb{pqROF}_2 = \left(\left(\left\| \sqrt{\left(\frac{\left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta + \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta - \frac{1}{1+\perp} \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta \right)^{\frac{1}{\Delta}}, \sqrt{\left(\frac{\left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla - \frac{1}{1+\perp} \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla \right)^{\frac{1}{\nabla}}} \right)^{\frac{1}{\nabla}}, \right. \right. \quad (20)$$

$$\left. \left. \left\| \sqrt{\left(\frac{\left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta + \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta - \frac{1}{1+\perp} \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\Delta \right)^{\frac{1}{\Delta}}, \sqrt{\left(\frac{\left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla - \frac{1}{1+\perp} \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_2}}{\sqrt{\cdot}} \right)^\nabla \right)^{\frac{1}{\nabla}}} \right)^{\frac{1}{\nabla}} \right) \right)$$

$$\alpha \mathbb{pqROF}_1 = \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta \frac{1}{1+\perp} \right)^\alpha \right)^{\frac{1}{\Delta}}, \sqrt{\left(\frac{1}{\perp} \left(1 + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla \frac{1}{1+\perp} \right)^\alpha - 1 \right)^{\frac{1}{\nabla}}} \right)^{\frac{1}{\nabla}}, \right. \right. \quad (21)$$

$$\left. \left. \left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\underline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\Delta \frac{1}{1+\perp} \right)^\alpha \right)^{\frac{1}{\Delta}}, \sqrt{\left(\frac{1}{\perp} \left(1 + \left(\frac{\overline{\mathbb{z}}_{\mathbb{R}_1}}{\sqrt{\cdot}} \right)^\nabla \frac{1}{1+\perp} \right)^\alpha - 1 \right)^{\frac{1}{\nabla}}} \right)^{\frac{1}{\nabla}} \right) \right)$$

$$(\text{pqROF}_1)^\alpha = \left(\left(\left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathfrak{z}}\overline{\delta}_1}{\sqrt{1}} \right)^\Delta + 1}{1+\perp} \right)^\alpha - 1 \right)} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \left(\frac{\overline{\delta}\overline{\delta}_1}{\sqrt{1}} \right)^\vee \frac{1}{1+\perp} \right)^\alpha} \right)^{\frac{1}{\vee}} \right), \right. \\ \left. \left(\left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \left(\frac{\left(\frac{\mathfrak{z}\delta_1}{\sqrt{1}} \right)^\Delta + 1}{1+\perp} \right)^\alpha - 1 \right)} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \left(\frac{\delta\delta_1}{\sqrt{1}} \right)^\vee \frac{1}{1+\perp} \right)^\alpha} \right)^{\frac{1}{\vee}} \right) \right) \quad (22)$$

3.2. Lpq-ROFZN power sugeno weber information

In this section, we aim to design the model of power weighted averaging operator and power weighted geometric operator based on Lpq-ROFZN, such as: Lpq-ROFZNPSWA operator, Lpq-ROFZNPSWWA operator, Lpq-ROFZNPSWG operator, and Lpq-ROFZNPSWWG operator, with numerous basic properties for each operator.

Definition 13. Let $\text{pqROF}_j = \left(\left(\left\| \frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{1}}, \left\| \frac{\mathfrak{z}\delta_j}{\sqrt{1}} \right\| \right), \left(\left\| \frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{1}}, \left\| \frac{\mathfrak{z}\delta_j}{\sqrt{1}} \right\| \right) \right), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, the concept of the Lpq-ROFZNPSWA operator is designed and mentioned by:

$$\begin{aligned} & \text{Lpq-ROFZNPSWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) \\ &= \frac{(1 + \mathfrak{H}(\text{pqROF}_1))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_1 \oplus \frac{(1 + \mathfrak{H}(\text{pqROF}_2))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_2 \\ & \quad \oplus \dots \oplus \frac{(1 + \mathfrak{H}(\text{pqROF}_m))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_m \\ &= \oplus_{j=1}^m \frac{(1 + \mathfrak{H}(\text{pqROF}_j))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_j \end{aligned} \quad (23)$$

If

$$\alpha_j = \frac{(1 + \mathfrak{H}(\text{pqROF}_j))}{\sum_{j=1}^m (1 + \mathfrak{H}(\text{pqROF}_j))}$$

thus

$$\begin{aligned} & \text{Lpq-ROFZNPSWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) \\ &= \alpha_1 \text{pqROF}_1 \oplus \alpha_2 \text{pqROF}_2 \oplus \dots \oplus \alpha_m \text{pqROF}_m = \oplus_{j=1}^m \alpha_j \text{pqROF}_j \end{aligned} \quad (24)$$

Further, we describe the technique $\mathfrak{H}(\text{pqROF}_j) = \sum_{i \neq j=1}^m S(\text{pqROF}_i, \text{pqROF}_j)$, where $S(\text{pqROF}_i, \text{pqROF}_j) = 1 - D(\text{pqROF}_i, \text{pqROF}_j)$, thus

- 1) $S(\text{pqROF}_i, \text{pqROF}_j) \in [0, 1]$.
- 2) $S(\text{pqROF}_i, \text{pqROF}_j) = S(\text{pqROF}_j, \text{pqROF}_i)$.
- 3) When $S(\text{pqROF}_i, \text{pqROF}_j) \geq S(\text{pqROF}_k, \text{pqROF}_1)$, then $D(\text{pqROF}_i, \text{pqROF}_j) \leq D(\text{pqROF}_k, \text{pqROF}_1)$.

The fundamental form of distance value is derived by:

$$\begin{aligned} D(\text{pqROF}_i, \text{pqROF}_j) &= \frac{1}{4} \left(\left| \frac{\overline{\mathfrak{z}}\overline{\delta}_i}{\sqrt{1}} - \frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{1}} \right|^\Delta + \left| \frac{\mathfrak{z}\delta_i}{\sqrt{1}} - \frac{\mathfrak{z}\delta_j}{\sqrt{1}} \right|^\Delta \right. \\ & \quad \left. + \left| \frac{\overline{\delta}\overline{\delta}_i}{\sqrt{1}} - \frac{\overline{\delta}\overline{\delta}_j}{\sqrt{1}} \right|^\vee + \left| \frac{\delta\delta_i}{\sqrt{1}} - \frac{\delta\delta_j}{\sqrt{1}} \right|^\vee \right) \end{aligned} \quad (25)$$

Theorem 1. Let $\text{pqROF}_j = \left(\left(\left\| \frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{1}}, \left\| \frac{\mathfrak{z}\delta_j}{\sqrt{1}} \right\| \right), \left(\left\| \frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{1}}, \left\| \frac{\mathfrak{z}\delta_j}{\sqrt{1}} \right\| \right) \right), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, we prove that the concept of the Lpq-ROFZNPSWA operator is also an Lpq-ROFZN value, such as

$$\begin{aligned} & \text{Lpq-ROFZNPSWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{1}} \right)^\Delta \frac{1}{1+\perp} \right)^{\alpha_j} \right)} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\overline{\delta}\overline{\delta}_j}{\sqrt{1}} \right)^\vee + 1}{1+\perp} \right)^{\alpha_j} - 1 \right)} \right)^{\frac{1}{\vee}} \right), \right. \\ & \quad \left. \left(\left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\mathfrak{z}\delta_j}{\sqrt{1}} \right)^\Delta \frac{1}{1+\perp} \right)^{\alpha_j} \right)} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\delta\delta_j}{\sqrt{1}} \right)^\vee + 1}{1+\perp} \right)^{\alpha_j} - 1 \right)} \right)^{\frac{1}{\vee}} \right) \right) \end{aligned} \quad (26)$$

The proof of the Theorem is discussed in [Appendix A](#).

Property 1. Let $\text{pqROF}_j = ((\|\overline{\mathfrak{x}}_{\overline{\delta}_j}, \|\overline{\delta}_{\overline{\delta}_j}\rangle), (\|\underline{\mathfrak{x}}_{\underline{\delta}_j}, \|\underline{\delta}_{\underline{\delta}_j}\rangle)), j = 1, 2, \dots, m$ be any group of Lpq -ROFZN sets. Then

- 1) When $\text{pqROF}_j = \text{pqROF}$, $j = 1, 2, \dots, m$, then $Lpq - \text{ROFZNPSWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \text{pqROF}$.
- 2) When $\text{pqROF}_- = \min(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m)$ and $\text{pqROF}_+ = \max(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m)$, then $\text{pqROF}_- \leq Lpq - \text{ROFZNPSWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) \leq \text{pqROF}_+$.

Definition 14. Let $\text{pqROF}_j = ((\|\overline{\mathfrak{x}}_{\overline{\delta}_j}, \|\overline{\delta}_{\overline{\delta}_j}\rangle), (\|\underline{\mathfrak{x}}_{\underline{\delta}_j}, \|\underline{\delta}_{\underline{\delta}_j}\rangle)), j = 1, 2, \dots, m$ be any group of Lpq -ROFZN sets. Then, the concept of the Lpq -ROFZNPSWWA operator is designed and mentioned by:

$$\begin{aligned} Lpq - \text{ROFZNPSWWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) &= \frac{Y_1(1 + \mathfrak{H}(\text{pqROF}_1))}{\sum_{j=1}^m Y_j(1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_1 \\ &\oplus \frac{Y_2(1 + \mathfrak{H}(\text{pqROF}_2))}{\sum_{j=1}^m Y_j(1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_2 \oplus \dots \\ &\oplus \frac{Y_m(1 + \mathfrak{H}(\text{pqROF}_m))}{\sum_{j=1}^m Y_j(1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_m \\ &= \oplus_{j=1}^m \frac{Y_j(1 + \mathfrak{H}(\text{pqROF}_j))}{\sum_{j=1}^m Y_j(1 + \mathfrak{H}(\text{pqROF}_j))} \text{pqROF}_j \end{aligned} \quad (27)$$

If

$$Lpq - \text{ROFZNPSWWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\mathfrak{x}}_{\overline{\delta}_j}}{\sqrt{\perp}} \right)^{\Delta} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^m \left(\left(\frac{\overline{\delta}_{\overline{\delta}_j}}{\sqrt{\perp}} \right)^{\vee} + 1 \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\vee}}} \right\|, \right. \right. \\ \left. \left. \left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\underline{\mathfrak{x}}_{\underline{\delta}_j}}{\sqrt{\perp}} \right)^{\Delta} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^m \left(\left(\frac{\underline{\delta}_{\underline{\delta}_j}}{\sqrt{\perp}} \right)^{\vee} + 1 \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\vee}}} \right\| \right) \right) \right) \quad (30)$$

$$\alpha_j = \frac{Y_j(1 + \mathfrak{H}(\text{pqROF}_j))}{\sum_{j=1}^m Y_j(1 + \mathfrak{H}(\text{pqROF}_j))}$$

thus

$$\begin{aligned} Lpq - \text{ROFZNPSWWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) &= \alpha_1 \text{pqROF}_1 \oplus \alpha_2 \text{pqROF}_2 \oplus \dots \oplus \alpha_m \text{pqROF}_m = \oplus_{j=1}^m \alpha_j \text{pqROF}_j \end{aligned} \quad (28)$$

Further, we describe the technique $\mathfrak{H}(\text{pqROF}_j) = \sum_{i \neq j=1}^m S(\text{pqROF}_i,$

$\text{pqROF}_j)$, where $S(\text{pqROF}_i, \text{pqROF}_j) = 1 - D(\text{pqROF}_i, \text{pqROF}_j)$, thus

- 1) $S(\text{pqROF}_i, \text{pqROF}_j) \in [0, 1]$.
- 2) $S(\text{pqROF}_i, \text{pqROF}_j) = S(\text{pqROF}_j, \text{pqROF}_i)$.
- 3) When $S(\text{pqROF}_i, \text{pqROF}_j) \geq S(\text{pqROF}_k, \text{pqROF}_i)$, then $D(\text{pqROF}_i, \text{pqROF}_j) \leq D(\text{pqROF}_k, \text{pqROF}_i)$.

The fundamental form of distance value is derived by:

$$D(\text{pqROF}_i, \text{pqROF}_j) = \frac{1}{4} \left(\left| \frac{\overline{\mathfrak{x}}_{\overline{\delta}_i}}{\sqrt{\perp}} - \frac{\overline{\mathfrak{x}}_{\overline{\delta}_j}}{\sqrt{\perp}} \right|^{\Delta} + \left| \frac{\underline{\mathfrak{x}}_{\underline{\delta}_i}}{\sqrt{\perp}} - \frac{\underline{\mathfrak{x}}_{\underline{\delta}_j}}{\sqrt{\perp}} \right|^{\Delta} + \left| \frac{\overline{\delta}_{\overline{\delta}_i}}{\sqrt{\perp}} - \frac{\overline{\delta}_{\overline{\delta}_j}}{\sqrt{\perp}} \right|^{\vee} + \left| \frac{\underline{\delta}_{\underline{\delta}_i}}{\sqrt{\perp}} - \frac{\underline{\delta}_{\underline{\delta}_j}}{\sqrt{\perp}} \right|^{\vee} \right) \quad (29)$$

Where $Y_j \in [0, 1]$ with $\sum_{j=1}^m Y_j = 1$, called the weighted vector.

Theorem 2. Let $\text{pqROF}_j = ((\|\overline{\mathfrak{x}}_{\overline{\delta}_j}, \|\overline{\delta}_{\overline{\delta}_j}\rangle), (\|\underline{\mathfrak{x}}_{\underline{\delta}_j}, \|\underline{\delta}_{\underline{\delta}_j}\rangle)), j = 1, 2, \dots, m$ be any group of Lpq -ROFZN sets. Then, we prove that the concept of the Lpq -ROFZNPSWWA operator is also an Lpq -ROFZN value, such as

Property 2. Let $\text{pqROF}_j = ((\|\overline{\mathfrak{x}}_{\overline{\delta}_j}, \|\overline{\delta}_{\overline{\delta}_j}\rangle), (\|\underline{\mathfrak{x}}_{\underline{\delta}_j}, \|\underline{\delta}_{\underline{\delta}_j}\rangle)), j = 1, 2, \dots, m$ be any group of Lpq -ROFZN sets. Then

- 1) When $\text{pqROF}_j = \text{pqROF}$, $j = 1, 2, \dots, m$, then $Lpq - \text{ROFZNPSWWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \text{pqROF}$.
- 2) When $\text{pqROF}_- = \min(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m)$ and $\text{pqROF}_+ = \max(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m)$, then $\text{pqROF}_- \leq Lpq - \text{ROFZNPSWWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) \leq \text{pqROF}_+$.

Definition 15. Let $\text{pqROF}_j = ((\|\overline{\mathfrak{x}}_{\overline{\delta}_j}, \|\overline{\delta}_{\overline{\delta}_j}\rangle), (\|\underline{\mathfrak{x}}_{\underline{\delta}_j}, \|\underline{\delta}_{\underline{\delta}_j}\rangle)), j = 1, 2, \dots, m$ be any group of Lpq -ROFZN sets. Then, the concept of the Lpq -

ROFZNPSWG operator is designed and mentioned by:

$$\begin{aligned} Lpq - ROFZNPSWG(pqROF_1, pqROF_2, \dots, pqROF_m) \\ = (pqROF_1)^{\frac{(1+\mathbb{H}(pqROF_1))}{\sum_{j=1}^m (1+\mathbb{H}(pqROF_j))}} \otimes (pqROF_2)^{\frac{(1+\mathbb{H}(pqROF_2))}{\sum_{j=1}^m (1+\mathbb{H}(pqROF_j))}} \otimes \dots \\ \otimes (pqROF_m)^{\frac{(1+\mathbb{H}(pqROF_m))}{\sum_{j=1}^m (1+\mathbb{H}(pqROF_j))}} \\ = \otimes_{j=1}^m (pqROF_j)^{\frac{(1+\mathbb{H}(pqROF_j))}{\sum_{j=1}^m (1+\mathbb{H}(pqROF_j))}} \end{aligned} \quad (31)$$

If

$$\alpha_j = \frac{(1 + \mathbb{H}(pqROF_j))}{\sum_{j=1}^m (1 + \mathbb{H}(pqROF_j))}$$

thus

$$\begin{aligned} Lpq - ROFZNPSWG(pqROF_1, pqROF_2, \dots, pqROF_m) \\ = (pqROF_1)^{\alpha_1} \otimes (pqROF_2)^{\alpha_2} \otimes \dots \otimes (pqROF_m)^{\alpha_m} = \otimes_{j=1}^m (pqROF_j)^{\alpha_j} \end{aligned} \quad (32)$$

Further, we describe the technique $\mathbb{H}(pqROF_j) = \sum_{i \neq j=1}^m S(pqROF_i, pqROF_j)$, where $S(pqROF_i, pqROF_j) = 1 - D(pqROF_i, pqROF_j)$, thus

- 1) $S(pqROF_i, pqROF_j) \in [0, 1]$.
- 2) $S(pqROF_i, pqROF_j) = S(pqROF_j, pqROF_i)$.
- 3) When $S(pqROF_i, pqROF_j) \geq S(pqROF_k, pqROF_1)$, then $D(pqROF_i, pqROF_j) \leq D(pqROF_k, pqROF_1)$.

The fundamental form of distance value is derived by:

$$\begin{aligned} D(pqROF_i, pqROF_j) = \frac{1}{4} \left(\left| \frac{\overline{\mathfrak{z}}\overline{\delta}_i}{\sqrt{}} - \frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{}} \right|^\Delta + \left| \frac{\mathfrak{z}\delta_i}{\sqrt{}} - \frac{\mathfrak{z}\delta_j}{\sqrt{}} \right|^\Delta + \left| \frac{\overline{\delta}\overline{\delta}_i}{\sqrt{}} - \frac{\overline{\delta}\overline{\delta}_j}{\sqrt{}} \right|^\Delta \right. \\ \left. + \left| \frac{\delta\delta_i}{\sqrt{}} - \frac{\delta\delta_j}{\sqrt{}} \right|^\Delta \right) \end{aligned} \quad (33)$$

Theorem 3. Let $pqROF_j = ((\|\overline{\mathfrak{z}}\overline{\delta}_j\|, \|\overline{\delta}\overline{\delta}_j\|), (\|\mathfrak{z}\delta_j\|, \|\delta\delta_j\|))$, $j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, we prove that the concept of the Lpq-ROFZNPSWG operator is also an Lpq-ROFZN value, such as

Property 3. Let $pqROF_j = ((\|\overline{\mathfrak{z}}\overline{\delta}_j\|, \|\overline{\delta}\overline{\delta}_j\|), (\|\mathfrak{z}\delta_j\|, \|\delta\delta_j\|))$, $j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then

- 1) When $pqROF_j = pqROF$, $j = 1, 2, \dots, m$, then $Lpq - ROFZNPSWG(pqROF_1, pqROF_2, \dots, pqROF_m) = pqROF$.
- 2) When $pqROF_- = \min(pqROF_1, pqROF_2, \dots, pqROF_m)$ and $pqROF_+ = \max(pqROF_1, pqROF_2, \dots, pqROF_m)$, then $pqROF_- \leq Lpq - ROFZNPSWG(pqROF_1, pqROF_2, \dots, pqROF_m) \leq pqROF_+$.

Definition 16. Let $pqROF_j = ((\|\overline{\mathfrak{z}}\overline{\delta}_j\|, \|\overline{\delta}\overline{\delta}_j\|), (\|\mathfrak{z}\delta_j\|, \|\delta\delta_j\|))$, $j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, the concept of the Lpq-ROFZNPSWWG operator is designed and mentioned by:

$$\begin{aligned} Lpq - ROFZNPSWWG(pqROF_1, pqROF_2, \dots, pqROF_m) \\ = (pqROF_1)^{\frac{Y_1(1+\mathbb{H}(pqROF_1))}{\sum_{j=1}^m Y_j(1+\mathbb{H}(pqROF_j))}} \otimes (pqROF_2)^{\frac{Y_2(1+\mathbb{H}(pqROF_2))}{\sum_{j=1}^m Y_j(1+\mathbb{H}(pqROF_j))}} \otimes \dots \\ \otimes (pqROF_m)^{\frac{Y_m(1+\mathbb{H}(pqROF_m))}{\sum_{j=1}^m Y_j(1+\mathbb{H}(pqROF_j))}} \\ = \otimes_{j=1}^m (pqROF_j)^{\frac{Y_j(1+\mathbb{H}(pqROF_j))}{\sum_{j=1}^m Y_j(1+\mathbb{H}(pqROF_j))}} \end{aligned} \quad (35)$$

If

$$\alpha_j = \frac{Y_j(1 + \mathbb{H}(pqROF_j))}{\sum_{j=1}^m Y_j(1 + \mathbb{H}(pqROF_j))}$$

thus

$$\begin{aligned} Lpq - ROFZNPSWWG(pqROF_1, pqROF_2, \dots, pqROF_m) \\ = (pqROF_1)^{\alpha_1} \otimes (pqROF_2)^{\alpha_2} \otimes \dots \otimes (pqROF_m)^{\alpha_m} = \otimes_{j=1}^m (pqROF_j)^{\alpha_j} \end{aligned} \quad (36)$$

Further, we describe the technique $\mathbb{H}(pqROF_j) = \sum_{i \neq j=1}^m S(pqROF_i, pqROF_j)$, where $S(pqROF_i, pqROF_j) = 1 - D(pqROF_i, pqROF_j)$, thus

- 1) $S(pqROF_i, pqROF_j) \in [0, 1]$.
- 2) $S(pqROF_i, pqROF_j) = S(pqROF_j, pqROF_i)$.
- 3) When $S(pqROF_i, pqROF_j) \geq S(pqROF_k, pqROF_1)$, then $D(pqROF_i, pqROF_j) \leq D(pqROF_k, pqROF_1)$.

The fundamental form of distance value is derived by:

$$\begin{aligned} Lpq - ROFZNPSWG(pqROF_1, pqROF_2, \dots, pqROF_m) = \left(\left(\left\| \sqrt{\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\overline{\mathfrak{z}}\overline{\delta}_j}{\sqrt{}} \right)^\Delta + 1}{1+\perp} \right)^{\alpha_j}} - 1 \right)} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\delta}\overline{\delta}_j}{\sqrt{}} \right)^\Delta \right)^{\alpha_j} \right)} \right)^{\frac{1}{\Delta}} \right\|, \right. \\ \left. \left(\left\| \sqrt{\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\mathfrak{z}\delta_j}{\sqrt{}} \right)^\Delta + 1}{1+\perp} \right)^{\alpha_j}} - 1 \right)} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\delta\delta_j}{\sqrt{}} \right)^\Delta \right)^{\alpha_j} \right)} \right)^{\frac{1}{\Delta}} \right\| \right) \end{aligned} \quad (34)$$

$$D(\text{pqROF}_i, \text{pqROF}_j) = \frac{1}{4} \left(\left| \frac{\mathfrak{T}\tilde{\delta}_i}{\sqrt{\Delta}} - \frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}} \right|^\Delta + \left| \frac{\mathfrak{T}\tilde{\delta}_i}{\sqrt{\Delta}} - \frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}} \right|^\Delta + \left| \frac{\mathfrak{T}\tilde{\delta}_i}{\sqrt{\Delta}} - \frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}} \right|^\Delta + \left| \frac{\mathfrak{T}\tilde{\delta}_i}{\sqrt{\Delta}} - \frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}} \right|^\Delta \right) \quad (37)$$

Where $Y_j \in [0, 1]$ with $\sum_{j=1}^m Y_j = 1$, called the weighted vector.

Theorem 4. Let $\text{pqROF}_j = ((\|\mathfrak{T}\tilde{\delta}_j\|, \|\mathfrak{T}\tilde{\delta}_j\|), (\|\mathfrak{T}\tilde{\delta}_j\|, \|\mathfrak{T}\tilde{\delta}_j\|)), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then, we prove that the concept of the Lpq-ROFZNPSWWG operator is also an Lpq-ROFZN value, such as

their supportive function is also defined by: $\mathfrak{T}\tilde{\delta}(x), \mathfrak{T}\tilde{\delta}(x) \in [0, \sqrt{\Delta}]$ with a unique characteristic, such that $0 \leq \left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta + \left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta \leq 1, \Delta, \forall \geq 1$ and $0 \leq \left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta + \left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta \leq 1, \Delta, \forall \geq 1$. Further, the function of

refusal degrees for both information is examined by: $\bar{r}(x) =$

$\sqrt{1 - \left(\left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta + \left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta\right)^{\frac{1}{\max(\Delta, \forall)}}}$ and $r(x) = \sqrt{1 - \left(\left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta + \left(\frac{\mathfrak{T}\tilde{\delta}(x)}{\sqrt{\Delta}}\right)^\Delta\right)^{\frac{1}{\max(\Delta, \forall)}}}$, where the simple form of Lpq-ROFZN sets is listed by: $\text{pqROF}_j = ((\|\mathfrak{T}\tilde{\delta}_j\|, \|\mathfrak{T}\tilde{\delta}_j\|), (\|\mathfrak{T}\tilde{\delta}_j\|, \|\mathfrak{T}\tilde{\delta}_j\|)), j = 1, 2, \dots, m$. Therefore, the

$$\text{Lpq-ROFZNPSWWG}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \left(\left(\left\| \sqrt{\left(\frac{1}{1+\Delta}\right)^{\frac{1}{\Delta}} \prod_{j=1}^m \left(\frac{\left(\frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}}\right)^\Delta + 1}{1+\Delta}\right)^{\Delta_j}} - 1 \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{1+\Delta}\right)^{\frac{1}{\Delta}} \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}}\right)^\Delta\right)^{\Delta_j}\right)} \right)^{\frac{1}{\Delta}} \right), \right. \\ \left. \left(\left\| \sqrt{\left(\frac{1}{1+\Delta}\right)^{\frac{1}{\Delta}} \prod_{j=1}^m \left(\frac{\left(\frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}}\right)^\Delta + 1}{1+\Delta}\right)^{\Delta_j}} - 1 \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{1+\Delta}\right)^{\frac{1}{\Delta}} \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\mathfrak{T}\tilde{\delta}_j}{\sqrt{\Delta}}\right)^\Delta\right)^{\Delta_j}\right)} \right)^{\frac{1}{\Delta}} \right) \right) \quad (38)$$

Property 4. Let $\text{pqROF}_j = ((\|\mathfrak{T}\tilde{\delta}_j\|, \|\mathfrak{T}\tilde{\delta}_j\|), (\|\mathfrak{T}\tilde{\delta}_j\|, \|\mathfrak{T}\tilde{\delta}_j\|)), j = 1, 2, \dots, m$ be any group of Lpq-ROFZN sets. Then

- 1) When $\text{pqROF}_j = \text{pqROF}$, $j = 1, 2, \dots, m$, then $\text{Lpq-ROFZNPSWWG}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \text{pqROF}$.
- 2) When $\text{pqROF}_- = \min(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m)$ and $\text{pqROF}_+ = \max(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m)$, then $\text{pqROF}_- \leq \text{Lpq-ROFZNPSWWG}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) \leq \text{pqROF}_+$.

3.3. Innovation of the Lpq-ROFZN-MABAC technique

In this section, we focus on analyzing the novel MABAC system for Lpq-ROFZN and power operators based on Sugeno-Weber information. The MABAC model is a well-known tool for evaluating the best decision and is considered a subset of decision-making models. Multi-attribute decision-making (MADM) is also part of decision-making techniques, but it cannot simultaneously utilize both operators and measures to assess the best decision. The MABAC model provides efficient results by integrating operators and measures within a single framework. Although many scholars have applied the MABAC model to evaluate various problems, none have implemented it based on Sugeno-Weber information. Therefore, we have developed a MABAC technique using these operators and measures to evaluate the best decision.

Suppose we select the numerous values of alternatives $\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m$ and for each alternative, we have some attributes such as A_1, A_2, \dots, A_n . Thus, we assigned the Lpq-ROFZN values to each attribute in every alternative, such as the function of truth degree and the function of falsity degree are defined by: $\mathfrak{T}\tilde{\delta}(x), \mathfrak{T}\tilde{\delta}(x) \in [0, \sqrt{\Delta}]$ and

major and valuable procedure of the MABAC tool is explained step-by-step, such as:

Case 1. (Developed the Decision Matrix) To utilize the Lpq-ROFZN information, our primary goal is to construct the decision matrix. After developing the decision matrix, we analyzed the data and divided it into two categories. The first category contains benefit-type data, while the second includes cost-type data. For benefit-type data, normalization is not required; however, for cost-type data, normalization is necessary within the decision matrix, as shown below:

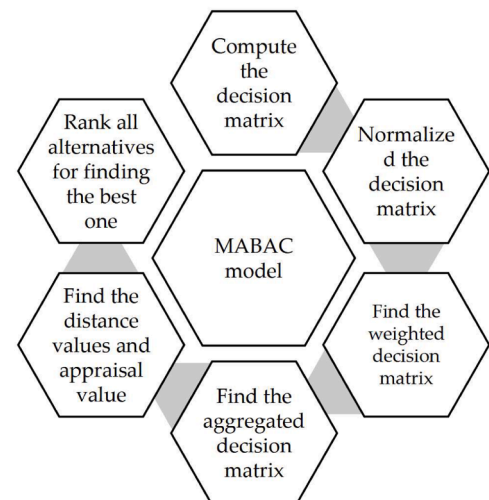


Fig. 3. Graphical form of the MABAC model.

Table 2
Lpq-ROFZN decision matrix.

ALT.	$pqROF_1^a$	$pqROF_2^a$	$pqROF_3^a$	$pqROF_4^a$	$pqROF_5^a$
$pqROF_1$	$\begin{pmatrix} (\ 5, \ 4), \\ (\ 3, \ 2) \end{pmatrix}$	$\begin{pmatrix} (\ 4, \ 3), \\ (\ 5, \ 3) \end{pmatrix}$	$\begin{pmatrix} (\ 2, \ 3), \\ (\ 4, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 5, \ 2), \\ (\ 5, \ 1) \end{pmatrix}$	$\begin{pmatrix} (\ 1, \ 2), \\ (\ 1, \ 4) \end{pmatrix}$
$pqROF_2$	$\begin{pmatrix} (\ 4, \ 5), \\ (\ 2, \ 2) \end{pmatrix}$	$\begin{pmatrix} (\ 3, \ 5), \\ (\ 3, \ 1) \end{pmatrix}$	$\begin{pmatrix} (\ 2, \ 4), \\ (\ 4, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 1, \ 3), \\ (\ 5, \ 3) \end{pmatrix}$	$\begin{pmatrix} (\ 3, \ 2), \\ (\ 6, \ 2) \end{pmatrix}$
$pqROF_3$	$\begin{pmatrix} (\ 3, \ 4), \\ (\ 4, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 4, \ 4), \\ (\ 5, \ 3) \end{pmatrix}$	$\begin{pmatrix} (\ 5, \ 3), \\ (\ 6, \ 1) \end{pmatrix}$	$\begin{pmatrix} (\ 2, \ 5), \\ (\ 5, \ 3) \end{pmatrix}$	$\begin{pmatrix} (\ 1, \ 2), \\ (\ 1, \ 2) \end{pmatrix}$
$pqROF_4$	$\begin{pmatrix} (\ 1, \ 3), \\ (\ 2, \ 3) \end{pmatrix}$	$\begin{pmatrix} (\ 2, \ 2), \\ (\ 3, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 3, \ 4), \\ (\ 4, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 4, \ 3), \\ (\ 5, \ 2) \end{pmatrix}$	$\begin{pmatrix} (\ 5, \ 2), \\ (\ 6, \ 1) \end{pmatrix}$
$pqROF_5$	$\begin{pmatrix} (\ 3, \ 2), \\ (\ 4, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 2, \ 3), \\ (\ 4, \ 5) \end{pmatrix}$	$\begin{pmatrix} (\ 4, \ 4), \\ (\ 3, \ 6) \end{pmatrix}$	$\begin{pmatrix} (\ 3, \ 5), \\ (\ 5, \ 4) \end{pmatrix}$	$\begin{pmatrix} (\ 2, \ 6), \\ (\ 2, \ 1) \end{pmatrix}$

$$pqROF = \begin{cases} \left(\left(\left(\frac{\|z_{\tilde{A}_j}}{\|\tilde{A}_j\|}, \frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right), \left(\frac{\|z_{\tilde{A}_j}\|}{\|\tilde{A}_j\|}, \frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right) \right) & \text{benefit} \\ \left(\left(\left(\frac{\|z_{\tilde{A}_j}\|}{\|\tilde{A}_j\|}, \frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right), \left(\frac{\|z_{\tilde{A}_j}}{\|\tilde{A}_j\|}, \frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right) \right) & \text{cost} \end{cases} \quad (39)$$

From the information in Eq. (39), we do not need to normalize the

data in the case of benefit types of data.

Case 2. (Develop the Weighted Decision Matrix) To design the weighted matrix by using the values from the weight vector along with the information from the decision matrix we use the following equation:

$$\alpha pqROF_1 = \begin{pmatrix} \left(\left(\left(\frac{\|z_{\tilde{A}_j}}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right), \left(\frac{\|z_{\tilde{A}_j}\|}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right) \right) & \left(\left(\left(\frac{\|z_{\tilde{A}_j}}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right), \left(\frac{\|z_{\tilde{A}_j}\|}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right) \right) \right) \\ \left(\left(\left(\frac{\|z_{\tilde{A}_j}}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right), \left(\frac{\|z_{\tilde{A}_j}\|}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right) \right) & \left(\left(\left(\frac{\|z_{\tilde{A}_j}}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right), \left(\frac{\|z_{\tilde{A}_j}\|}{\|\tilde{A}_j\|} \left(1 - \left(1 - \left(\frac{\|\tilde{A}_j\|}{\|\tilde{A}_j\|} \right)^{\frac{1}{\Delta}} \right) \right)^{\frac{1}{\Delta}} \right) \right) \right) \end{pmatrix} \quad (40)$$

or

$$(\text{pqROF}_1)^\alpha = \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathbb{Z}}\overline{\mathbb{N}}_1}{\sqrt{\perp}} \right)^\Delta + 1 \right)^\alpha - 1 \right)} \right\|_{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \left(\frac{\overline{\mathbb{N}}\overline{\mathbb{N}}_1}{\sqrt{\perp}} \right)^\vee \frac{1}{1+\perp} \right)^\alpha} \right\|_{\frac{1}{\vee}} \end{array} \right), \right. \\ \left. \left(\begin{array}{c} \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathbb{Z}}\overline{\mathbb{N}}_1}{\sqrt{\perp}} \right)^\Delta + 1 \right)^\alpha - 1 \right)} \right\|_{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \left(\frac{\overline{\mathbb{N}}\overline{\mathbb{N}}_1}{\sqrt{\perp}} \right)^\vee \frac{1}{1+\perp} \right)^\alpha} \right\|_{\frac{1}{\vee}} \end{array} \right) \right) \quad (41)$$

Case 3. (Develop the Aggregated Matrix) To analyze the aggregated matrix (Lpq-ROFZNPSWA operator and Lpq-ROFZNPSWG operator), we use the data from the decision matrix, such as

Case 4. (Develop the Distance Matrix). To design the distance matrix we use the values from the aggregated matrix and the weighted aggregated matrix, as follows:

$$Lpq - \text{ROFZNPSWA}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\mathbb{Z}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\Delta \frac{1}{1+\perp} \right)^{\alpha_j} \right)} \right\|_{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\overline{\mathbb{N}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\vee + 1 \right)^{\alpha_j} - 1 \right)} \right\|_{\frac{1}{\vee}} \end{array} \right), \right. \\ \left. \left(\begin{array}{c} \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\mathbb{Z}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\Delta \frac{1}{1+\perp} \right)^{\alpha_j} \right)} \right\|_{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\overline{\mathbb{N}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\vee + 1 \right)^{\alpha_j} - 1 \right)} \right\|_{\frac{1}{\vee}} \end{array} \right) \right) \quad (42)$$

or

$$Lpq - \text{ROFZNPSWG}(\text{pqROF}_1, \text{pqROF}_2, \dots, \text{pqROF}_m) = \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\overline{\mathbb{Z}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\Delta + 1 \right)^{\alpha_j} - 1 \right)} \right\|_{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\mathbb{N}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\vee \frac{1}{1+\perp} \right)^{\alpha_j} \right)} \right\|_{\frac{1}{\vee}} \end{array} \right), \right. \\ \left. \left(\begin{array}{c} \left\| \sqrt{\left(\frac{1}{1} \right) \left((1+\perp) \prod_{j=1}^m \left(\frac{\left(\frac{\overline{\mathbb{Z}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\Delta + 1 \right)^{\alpha_j} - 1 \right)} \right\|_{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1+\perp}{1} \right) \left(1 - \prod_{j=1}^m \left(1 - \left(\frac{\overline{\mathbb{N}}\overline{\mathbb{N}}_j}{\sqrt{\perp}} \right)^\vee \frac{1}{1+\perp} \right)^{\alpha_j} \right)} \right\|_{\frac{1}{\vee}} \end{array} \right) \right) \quad (43)$$

Table 3
Weighted decision aggregated matrix.

ALT.	$pqROF_1^a$	$pqROF_2^a$	$pqROF_3^a$	$pqROF_4^a$	$pqROF_5^a$
$pqROF_1$	$\left(\begin{array}{c} (\ 3.0314, \ 6.3166), \\ (\ 1.767, \ 5.7262) \end{array} \right)$	$\left(\begin{array}{c} (\ 2.3804, \ 6.0396), \\ (\ 3.0314, \ 6.0396) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.1720, \ 6.0396), \\ (\ 2.3804, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 3.0314, \ 5.7262), \\ (\ 3.0314, \ 5.3632) \end{array} \right)$	$\left(\begin{array}{c} (\ 0.5849, \ 5.7262), \\ (\ 0.5849, \ 6.3166) \end{array} \right)$
$pqROF_2$	$\left(\begin{array}{c} (\ 2.3804, \ 6.5655), \\ (\ 1.1720, \ 5.7262) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.767, \ 6.5655), \\ (\ 1.767, \ 5.3632) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.1720, \ 6.3166), \\ (\ 2.3804, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 0.5849, \ 6.0396), \\ (\ 3.0314, \ 6.0396) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.767, \ 5.7262), \\ (\ 3.7581, \ 5.7262) \end{array} \right)$
$pqROF_3$	$\left(\begin{array}{c} (\ 1.767, \ 6.3166), \\ (\ 2.3804, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 2.3804, \ 6.3166), \\ (\ 3.0314, \ 6.0396) \end{array} \right)$	$\left(\begin{array}{c} (\ 3.0314, \ 6.0396), \\ (\ 3.7581, \ 5.3632) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.1720, \ 6.5655), \\ (\ 3.0314, \ 6.0396) \end{array} \right)$	$\left(\begin{array}{c} (\ 0.5849, \ 5.7262), \\ (\ 0.5849, \ 5.7262) \end{array} \right)$
$pqROF_4$	$\left(\begin{array}{c} (\ 0.5849, \ 6.0396), \\ (\ 1.1720, \ 6.0396) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.1720, \ 5.7262), \\ (\ 1.767, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.767, \ 6.3166), \\ (\ 2.3804, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 2.3804, \ 6.0396), \\ (\ 4.0414, \ 5.7262) \end{array} \right)$	$\left(\begin{array}{c} (\ 3.0314, \ 5.7262), \\ (\ 3.7581, \ 5.3632) \end{array} \right)$
$pqROF_5$	$\left(\begin{array}{c} (\ 1.767, \ 5.7262), \\ (\ 2.3804, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.1720, \ 6.0396), \\ (\ 2.3804, \ 6.5655) \end{array} \right)$	$\left(\begin{array}{c} (\ 2.3804, \ 6.3166), \\ (\ 1.767, \ 6.7919) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.767, \ 6.5655), \\ (\ 3.0314, \ 6.3166) \end{array} \right)$	$\left(\begin{array}{c} (\ 1.1720, \ 6.7919), \\ (\ 1.1720, \ 5.3632) \end{array} \right)$

Table 4
Aggregated decision matrix.

Alt.	Lpq-ROFZNPSWA operator	Lpq-ROFZNPSWG operator
$pqROF_1$	$((\ 2.4283, \ 5.9677), (\ 2.4730, \ 5.9465))$	$((\ 2.3976, \ 5.9758), (\ 2.4470, \ 5.9680))$
$pqROF_2$	$((\ 1.7423, \ 6.2376), (\ 2.7432, \ 5.8291))$	$((\ 1.7334, \ 6.2550), (\ 2.6877, \ 5.8459))$
$pqROF_3$	$((\ 2.1419, \ 6.1906), (\ 2.9170, \ 5.8934))$	$((\ 2.1141, \ 6.2042), (\ 2.8735, \ 5.9104))$
$pqROF_4$	$((\ 2.1395, \ 5.9675), (\ 2.7408, \ 5.9466))$	$((\ 2.1116, \ 5.9757), (\ 2.6853, \ 5.9681))$
$pqROF_5$	$((\ 1.7739, \ 6.2788), (\ 2.3230, \ 6.2625))$	$((\ 1.7663, \ 6.3027), (\ 2.3044, \ 6.3009))$

Table 7
Representation of the appraisal values.

Alt.	Lpq-ROFZNPSWA operator	Lpq-ROFZNPSWG operator
$pqROF_1$	0.07693	0.07647
$pqROF_2$	0.06745	0.06699
$pqROF_3$	0.07588	0.0759
$pqROF_4$	0.07751	0.07686
$pqROF_5$	0.05588	0.0557

Table 5
Representation of the weight vectors.

Alt.	$pqROF_1^a$	$pqROF_2^a$	$pqROF_3^a$	$pqROF_4^a$	$pqROF_5^a$
$pqROF_1$	0.1997	0.20147	0.20115	0.19853	0.19917
$pqROF_2$	0.20057	0.19961	0.20005	0.20053	0.19925
$pqROF_3$	0.20105	0.20196	0.19843	0.20084	0.19772
$pqROF_4$	0.20052	0.20053	0.20007	0.20099	0.1979
$pqROF_5$	0.20031	0.20129	0.20063	0.20154	0.19623

$$pqROF_{ij} = \begin{cases} D(pqROF_i, pqROF_j) & \text{if } pqROF_i > pqROF_j \\ 0 & \text{if } pqROF_i = pqROF_j \\ -D(pqROF_i, pqROF_j) & \text{if } pqROF_i < pqROF_j \end{cases} \quad (44)$$

Where

Table 6
Representation of distance values.

Alt.	$pqROF_1^a$	$pqROF_2^a$	$pqROF_3^a$	$pqROF_4^a$	$pqROF_5^a$
$pqROF_1$	0.06708	0.06696	0.09429	0.092	0.05217
$pqROF_2$	0.02755	0.02709	0.0641	0.06181	0.02232
$pqROF_3$	0.06397	0.06165	0.05356	0.05063	0.08614
$pqROF_4$	0.07093	0.07325	0.06621	0.0685	0.05733
$pqROF_5$	0.15511	0.15338	0.05907	0.06199	0.16146

Table 8
Representation of the ranking values.

MABAC Methods	Ranking values	Best optimal
Lpq-ROFZNPSWA operator	$pqrOF_4 > pqrOF_1 > pqrOF_3 > pqrOF_2 > pqrOF_5$	$pqrOF_4$
Lpq-ROFZNPSWG operator	$pqrOF_4 > pqrOF_1 > pqrOF_3 > pqrOF_2 > pqrOF_5$	$pqrOF_4$

Table 9
Aggregated decision matrix.

Alt.	$pqrOF_1^d$	$pqrOF_2^d$
$pqrOF_1$	$((\ 4.0723, \ 2.7645), (\ 4.153, \ 2.6955))$	$((\ 3.8726, \ 2.8446), (\ 3.9879, \ 2.8948))$
$pqrOF_2$	$((\ 2.9591, \ 3.7065), (\ 4.5627, \ 2.315))$	$((\ 2.894, \ 3.9027), (\ 4.2647, \ 2.4687))$
$pqrOF_3$	$((\ 3.6165, \ 3.5517), (\ 4.8342, \ 2.5342))$	$((\ 3.4392, \ 3.6997), (\ 4.5999, \ 2.6881))$
$pqrOF_4$	$((\ 3.6013, \ 2.7614), (\ 4.5518, \ 2.6983))$	$((\ 3.422, \ 2.8421), (\ 4.2555, \ 2.8971))$
$pqrOF_5$	$((\ 3.0171, \ 3.835), (\ 3.926, \ 3.8391))$	$((\ 2.9636, \ 4.1239), (\ 3.8108, \ 4.2408))$

Table 10
Representation of the appraisal values.

Alt.	Lpq-ROFZNPSWA operator	Lpq-ROFZNPSWG operator
$pqrOF_1$	0.1435	0.1082
$pqrOF_2$	0.0839	0.0398
$pqrOF_3$	0.1446	0.0991
$pqrOF_4$	0.1462	0.1088
$pqrOF_5$	−0.0365	−0.0948

$$D(pqrOF_i, pqrOF_j) = \frac{1}{4} \left(\left| \frac{\bar{x}\bar{\delta}_i}{\sqrt{}} - \frac{\bar{x}\bar{\delta}_j}{\sqrt{}} \right|^\Delta + \left| \frac{\bar{x}\bar{\delta}_i}{\sqrt{}} - \frac{\bar{x}\bar{\delta}_j}{\sqrt{}} \right|^\Delta + \left| \frac{\bar{\delta}\bar{\delta}_i}{\sqrt{}} - \frac{\bar{\delta}\bar{\delta}_j}{\sqrt{}} \right|^\nu + \left| \frac{\bar{\delta}\bar{\delta}_i}{\sqrt{}} - \frac{\bar{\delta}\bar{\delta}_j}{\sqrt{}} \right|^\nu \right) \quad (45)$$

Case 5. (Develop the Appraisal Values). We define the appraisal values using Eq. (46).

$$S_i = \frac{1}{m} \sum_{j=1}^m D(pqrOF_i, pqrOF_j) \quad (46)$$

Case 6. (Develop the Ranking Values) Based on the values of the appraisal function, we rank all values to determine the best optimal. Furthermore, the interpretation of the proposed model is shown in

Table 11
Representation of the ranking values.

	Ranking values	Best optimal
Lpq-ROFZNPSWA operator	$pqrOF_4 > pqrOF_3 > pqrOF_1 > pqrOF_2 > pqrOF_5$	$pqrOF_4$
Lpq-ROFZNPSWG operator	$pqrOF_4 > pqrOF_3 > pqrOF_1 > pqrOF_2 > pqrOF_5$	$pqrOF_4$

Table 12
Mathematical comparison of the proposed theory.

Methods	Score values	Ranking values
Yager (Yager, 2001)	0.0,0.0,0.0,0.0,0.0	No
Xu and Yager (Xu & Yager, 2009)	0.0,0.0,0.0,0.0,0.0	No
Hussain et al. (Hussain et al., 2024)	0.0,0.0,0.0,0.0,0.0	No
Wang et al. (Wang et al., 2024)	0.0,0.0,0.0,0.0,0.0	No
Liu and Qin (Liu & Qin, 2017)	0.0,0.0,0.0,0.0,0.0	No
Jiang et al. (Jiang et al., 2018)	0.0,0.0,0.0,0.0,0.0	No
Xu (Xu, 2011)	0.0,0.0,0.0,0.0,0.0	No
Wei and Lu (Wei & Lu, 2018)	0.0,0.0,0.0,0.0,0.0	No
Du (Du, 2019)	0.0,0.0,0.0,0.0,0.0	No
MABAC-Lpq-ROFZNPSWA operator	0.07693,0.06745,0.07588,0.07751,0.05588	$pqrOF_4 > pqrOF_1 > pqrOF_3 > pqrOF_2 > pqrOF_5$
MABAC-Lpq-ROFZNPSWG operator	0.07647,0.06699,0.0759,0.07686,0.0557	$pqrOF_4 > pqrOF_1 > pqrOF_3 > pqrOF_2 > pqrOF_5$
Lpq-ROFZNPSWA operator	0.1435,0.0839,0.1446,0.1462,−0.0365	$pqrOF_4 > pqrOF_3 > pqrOF_1 > pqrOF_2 > pqrOF_5$
Lpq-ROFZNPSWG operator	0.1082,0.0398,0.0991,0.1088,−0.0948	$pqrOF_4 > pqrOF_3 > pqrOF_1 > pqrOF_2 > pqrOF_5$

Fig. 3.

4. Application: impact of blockchain technologies on GSCP

In this section, we discuss the impact of blockchain technologies on GSCP based on our proposed models for Lpq-ROFZN. Blockchain is a valuable and widely recognized technology that helps record transactions across multiple computers in a transparent, secure, and immutable way. Each transaction or piece of information is stored in a block and linked with the previous transaction, forming a continuous chain called a blockchain. Blockchain uses cryptography to ensure information security and integrity.

Furthermore, a green supply chain is a well-established approach used to integrate environmental considerations into supply chain management. It aims to reduce the environmental impact of transportation, distribution, and production processes, including practices such as minimizing energy consumption, reducing waste, and using sustainable materials. Blockchain provides decentralized and widely distributed digital knowledge that ensures the security of recorded transactions among multiple computers, guaranteeing immutability, integrity, and transparency. Unlike a traditional centralized database controlled by a single authority, blockchain stores transactions in blocks that are directly linked to form a chain.

The influence of blockchain on GSCP has been studied by various researchers to evaluate solutions under the consideration of crisp information. The focus of this application is to assess the impact of blockchain on GSCP using the proposed models and techniques. Challenges in traditional supply chains arise from growing environmental inefficiencies, such as poor accountability, excessive waste, and lack of transparency. Green supply chain practices aim to mitigate these issues, but they often face problems such as difficulty in monitoring sustainability performance, data manipulation, and limited traceability across multiple stakeholders. Without effective methods to verify and track eco-friendly practices, enterprises struggle to implement and enforce green initiatives reliably. Blockchain addresses this problem by

providing decentralized, transparent, and immutable records of processes and transactions. By integrating blockchain into green supply chain practices, we aim to identify five ways in which blockchain impacts GSCP, and our goal is to determine the most effective and the least effective among these five alternatives, such as:

1) Enhanced Transparency and Traceability

The improvement of transparency ensures that all stakeholders have access to clear, accurate, and timely data, while traceability refers to the ability to track the location, history, and application of an activity or item. Both of them are widely recognized and applied to enhance the capability to verify and monitor the status, movement, and origin of processes, products, and data throughout a system, especially in the context of digital transactions, supply chains, and manufacturing. The main benefits of this approach are improved regulatory compliance, increased consumer trust, and reduced fraud and counterfeiting.

2) Waste Reduction and Resource Optimization

Waste reduction is the process of minimizing or eliminating unnecessary materials, while resource optimization refers to using available resources in the most effective, sustainable, and efficient way to achieve a target with minimal input and waste. This approach is a well-established and reliable strategy aimed at reducing waste and maximizing resource efficiency throughout a process, system, or product lifecycle.

3) Reduction of Carbon Footprint

Reduction of carbon footprint is a valuable procedure aimed at lowering the amount of greenhouse gases, particularly carbon dioxide, released into the atmosphere due to human activities. Examples include emissions from product lifecycles, waste generation and disposal, energy consumption in buildings, and emissions from power plants, vehicles, and factories. The main benefits of this approach are compliance with environmental regulations and targets, improved corporate social responsibility, and energy conservation.

4) Smart Contracts and Incentives for Sustainable Practices

Reduction of the carbon footprint is a valuable procedure aimed at lowering the amount of greenhouse gases, particularly carbon dioxide, released into the atmosphere due to human activities. Examples include emissions from product lifecycles, waste generation and disposal, energy consumption in buildings, and emissions from power plants, vehicles, and factories. The main benefits of this approach are compliance with environmental regulations and targets, enhanced corporate social responsibility, and energy conservation.

5) Sustainable Certifications and Compliance

Sustainable certifications (official seals or labels provided by government bodies or recognized organizations) and compliance (adherence to standards, rules, and regulations) are used to ensure formal recognition of ethical, environmental, and social standards in business practices, products, and services. They validate that the organization meets specific sustainability requirements in its services and products.

For the above five alternatives, we also identified five attributes that help evaluate the best and worst options among the set of five alternatives, such as:

- 1) Decentralized Data sharing for sustainable Collaboration.
- 2) Carbon Footprint Tracking and Verification.
- 3) Traceability of Sustainable Materials.
- 4) Efficient Resource Allocation Through Data Insights.

5) Secure and Transparent Reporting for Regulatory Compliance.

Finally, we determine the most preferable and reliable decision among all the alternatives based on their criteria. For this purpose, we employ the MABAC model to accurately and effectively evaluate these problems using the invented operators, such as the Lpq-ROFZNPSWA operator and the Lpq-ROFZNPSWG operator. Accordingly, the main and valuable steps of the MABAC procedure are explained step-by-step, such as:

Case 1: (Developed the Decision Matrix). In Table 2 we defined the Lpq-ROFZN decision matrix.

After developing the decision matrix, we evaluated the data and divided it into two categories. The first category contains benefit-type data, while the second category includes cost-type data. For benefit-type data, normalization is not required, whereas cost-type data must be normalized in the decision matrix. According to the information in Eq. (39), normalization is unnecessary for benefit-type data. Since the data in the decision matrix are benefit types, no normalization is needed.

Case 2: (Develop the Weighted Decision Matrix). Using the information from Table 2 and the weight vector we can define weighted aggregated matrix, see Table 3 for $\alpha = 0.2$, $\perp = 2$, $\Delta = 3$, $\gamma = 1$.

Case 3: (Develop the Aggregated Matrix). Aggregated matrix is presented in Table 4. For defining values in Table 4 we used Lpq-ROFZNPSWA and Lpq-ROFZNPSWG operator.

The values of the weight vector, calculated using power operators, are presented in Table 5.

Case 4: (Develop the Distance Matrix). Using values from the aggregated matrix we have defined the distance matrix as shown in Table 6.

Case 5: (Develop the Appraisal Values). Based on Table 6, we can define the appraisal values, Table 7.

Case 6: (Develop the Ranking Values) Ranks are defined based on the appraisal function, where higher value means the better alternative, see Table 8.

The most preferable optimal is $pqrOF_4$ according to the technique of the MABAC model for Lpq-ROFZNPSWA operator and Lpq-ROFZNPSWG operator, called the Smart Contracts and Incentives for Sustainable Practices, but the worst one is $pqrOF_5$. Moreover, the aggregated values of the proposed operators, without using the MABAC model, are presented in Table 9. In this case, a simplified form of the decision-making technique was applied to evaluate the data in Table 2, highlighting the differences and similarities among the results. The results shown in Table 8 were obtained using the MABAC model. Now, we proceed to determine the ranking values of the data in Table 2 using a multi-attribute decision-making technique.

Thus, we use the score function of the proposed information to calculate the score values for the data in Table 9. These score values are presented in Table 10.

To rank alternatives, we utilize the values of the score function, as shown in Table 11.

The most preferable optimal is $pqrOF_4$ according to the technique of the Lpq-ROFZNPSWA operator and Lpq-ROFZNPSWG operator, called the Smart Contracts and Incentives for Sustainable Practices, the worst one is $pqrOF_5$. Finally, we assess the robustness and effectiveness of the proposed technique by comparing its ranking values with those obtained from existing approaches.

5. Comparative analysis

In this section, we conduct a comparative analysis between the proposed and existing approaches. Such an analysis is crucial for evaluating the best and worst-performing models. Many scholars have used comparative analysis to demonstrate the superiority of their methods. However, to perform this analysis effectively, it is necessary to organize the techniques, models, operators, and measures based on fuzzy sets and their extensions. Without referencing existing techniques, it is

challenging to compare the ranking values of the proposed models with those of established methods. For this comparative analysis, we selected the following existing models: Yager (Yager, 2001) for the PA operator, Xu and Yager (Xu & Yager, 2009) for the PG operator, Hussain et al. (Hussain et al., 2024) for Sugeno Weber operators for IFSs, Wang et al. (Wang et al., 2024) for Sugeno Weber operators for q-ROFSs, Liu and Qin (Liu & Qin, 2017) for power operators for linguistic IFSs, Jiang et al. (Jiang et al., 2018) for power operators for IFSs based on entropy measures, Xu (Xu, 2011) for power operators for IFSs in decision-making, Wei and Lu (Wei & Lu, 2018) for power operators for PFSs, and Du (Du, 2019) for weighted power operators for q-ROFSs. The data in Table 2 are considered for performing this comparative analysis, as summarized in Table 12.

The most preferable option is pqROF_4 according to the MABAC model using the Lpq-ROFZNPSWA and Lpq-ROFZNPSWG operators, representing Smart Contracts and Incentives for Sustainable Practices, while the least preferable is pqROF_5 . Existing techniques have several limitations and issues due to their inherent structures and characteristics.

Yager (Yager, 2001) computed the PA operator, and Xu and Yager (Xu & Yager, 2009) evaluated the PG operator. These models are based on classical information, which has no connection to Lpq-ROFZN. While the proposed model is more general and recent, the existing models are older and limited in handling our type of data. Hussain et al. (Hussain et al., 2024) introduced the Sugeno-Weber operators for IFSs, Liu and Qin (Liu & Qin, 2017) described the power operators for linguistic IFSs, Jiang et al. (Jiang et al., 2018) explored power operators for IFSs based on entropy measures, and Xu (Xu, 2011) developed power operators for IFSs in decision-making frameworks. These IFS-based models only use truth and falsity functions, which are insufficient for our data because we require truth and falsity functions expressed as linguistic variables with supportive functions, which IFS cannot provide. Wei and Lu (Wei & Lu, 2018) examined power operators for PFSs, which are a special case of the proposed model. Wang et al. (Wang et al., 2024) and Du (Du, 2019) presented Sugeno-Weber operators and weighted power operators for q-ROFSs, respectively; however, these q-ROFS-based models are also insufficient for our data due to inherent limitations.

The advantages of the proposed model are that drastic aggregation operators, periodic-norm-based aggregation operators, and Sugeno-Weber aggregation operators for FSSs, IFSs, PFSs, q-ROFSs, pq-ROFSs, Z-number, LTSs, and their integrations are all special cases of the derived theory. Moreover, analytical hierarchy process frameworks, decision-making techniques, multi-attribute decision-making techniques, and the MABAC model are also encompassed as special cases.

While existing frameworks have many limitations, the proposed model is simpler, more general, and highly reliable for handling vague and complex data, making it a novel and robust approach.

6. Conclusion

Challenges in traditional supply chains, such as poor accountability, excessive waste, and lack of transparency, create complications that green supply chain practices aim to address. However, these practices often face difficulties, including monitoring sustainability performance, data manipulation, and limited traceability across multiple stakeholders. Without effective techniques to verify and track eco-friendly practices, enterprises struggle to implement and enforce green initiatives reliably. Blockchain technology offers a solution by providing decentralized, transparent, and immutable records of processes and transactions. By integrating blockchain into green supply chain practices, we introduced the concept of Lpq-ROFZN sets, which include truth

and falsity degrees expressed as linguistic variables with supportive functions. The Lpq-ROFZN system is a modified and comprehensive extension of existing models, including FSSs, IFSs, PFSs, q-ROFSs, pq-ROFSs, LTSs, Z-numbers, and their integrations.

We further established algebraic and Sugeno-Weber operational laws for Lpq-ROFZN values to construct aggregation operators. Based on these laws, we developed power aggregation operators such as Lpq-ROFZNPSWA, Lpq-ROFZNPSWWA, Lpq-ROFZNPSWG, and Lpq-ROFZNPSWWG, which can be applied within the MABAC model. The MABAC methodology is explained step-by-step using examples to demonstrate the superiority and validity of the proposed approach by comparing its ranking values with existing techniques.

While Lpq-ROFZN is a robust and effective framework, it has limitations in cases where decision-makers provide information in forms like yes, no, abstinence, or neutral, due to its restricted features. To address this, future work will focus on developing linguistic (p, q, r)-spherical fuzzy Z-number sets and their extensions. These models will be more suitable for handling such complex scenarios and will find applications in decision-making, game theory, data mining, artificial intelligence, and software engineering. Techniques such as support vector machines, k-nearest neighbors, and clustering analysis will be employed to solve real-world problems effectively.

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This article does not contain any studies with human participants or animals. Performed by any of the authors.

Informed consent

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The data used to support the findings of this study are included in this article. However, the reader may contact the corresponding author for more details on the data.

CRediT authorship contribution statement

Zeeshan Ali: Writing – review & editing, Writing – original draft, Methodology. **Dragan Pamucar:** Writing – review & editing. **Vladimir Simic:** Writing – review & editing. **Rajesh Kumar Dhanaraj:** Writing – review & editing, Writing – original draft.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary materials

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Appendix A

For the assessment of the proposed operators, we use the technique of mathematical induction. For this, we consider “ $m = 2$ ”, then

$$\alpha_1 \text{pqROF}_1 = \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\overline{\mathfrak{z}}_{\overline{\mathfrak{N}}_1}}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_1}} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathfrak{N}}_{\overline{\mathfrak{N}}_1}}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_1} - 1 \right)} \right)^{\frac{1}{\vee}} \right\| \end{array} \right) \right)$$

and

$$\alpha_2 \text{pqROF}_2 = \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\overline{\mathfrak{z}}_{\overline{\mathfrak{N}}_2}}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_2}} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathfrak{N}}_{\overline{\mathfrak{N}}_2}}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_2} - 1 \right)} \right)^{\frac{1}{\vee}} \right\| \end{array} \right) \right)$$

thus

$$\alpha_1 \text{pqROF}_1 \oplus \alpha_2 \text{pqROF}_2$$

$$= \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\overline{\mathfrak{z}}_{\overline{\mathfrak{N}}_1}}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_1}} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathfrak{N}}_{\overline{\mathfrak{N}}_1}}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_1} - 1 \right)} \right)^{\frac{1}{\vee}} \right\| \end{array} \right) \right) \oplus \left(\left(\begin{array}{c} \left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\overline{\mathfrak{z}}_{\overline{\mathfrak{N}}_2}}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_2}} \right)^{\frac{1}{\Delta}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \left(\frac{\left(\frac{\overline{\mathfrak{N}}_{\overline{\mathfrak{N}}_2}}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_2} - 1 \right)} \right)^{\frac{1}{\vee}} \right\| \end{array} \right) \right)$$

$$= \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^2 \left(1 - \left(\frac{\tilde{z}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^2 \left(\frac{\left(\frac{\tilde{\delta}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\vee}}} \right\| \right) \right)$$

The proposed operator is held for “ $m = 2$ ”. Further, we assume that the proposed operator also holds for “ $m = m'$ ”, such as

$$Lpq - ROFZNPSWA(pqROF_1, pqROF_2, \dots, pqROF_{m'}) = \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^{m'} \left(1 - \left(\frac{\tilde{z}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^{m'} \left(\frac{\left(\frac{\tilde{\delta}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\vee}}} \right\| \right) \right)$$

Then, we derive that the proposed theory also holds for “ $m = m' + 1$ ”, such as

$$Lpq - ROFZNPSWA(pqROF_1, pqROF_2, \dots, pqROF_{m'+1}) = \alpha_1 pqROF_1 \oplus \alpha_2 pqROF_2 \oplus \dots \oplus \alpha_{m'} pqROF_{m'} \oplus \alpha_{m'+1} pqROF_{m'+1} \\ = \oplus_{j=1}^{m'} \alpha_j pqROF_j \oplus \alpha_{m'+1} pqROF_{m'+1}$$

$$= \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^{m'} \left(1 - \left(\frac{\tilde{z}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^{m'} \left(\frac{\left(\frac{\tilde{\delta}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\vee}}} \right\| \right) \right) \oplus \alpha_{m'+1} pqROF_{m'+1}$$

$$= \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^{m'} \left(1 - \left(\frac{\tilde{z}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^{m'} \left(\frac{\left(\frac{\tilde{\delta}\tilde{\delta}_j}{\sqrt{\perp}} \right)^{\vee} + 1}{1+\perp} \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\vee}}} \right\| \right) \right)$$

$$\begin{aligned}
& \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\sqrt{\frac{\delta \delta_{m'+1}}{\sqrt{\perp}}}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_{m'+1}}} \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \left(\frac{\left(\frac{\sqrt{\frac{\delta \delta_{m'+1}}{\sqrt{\perp}}}} \right)^{\Delta} + 1}{1+\perp} \right)^{\alpha_{m'+1}} - 1 \right) \right)^{\frac{1}{\Delta}}} \right\| \right) \right. \\
& \left. \left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \left(\frac{\sqrt{\frac{\delta \delta_{m'+1}}{\sqrt{\perp}}}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_{m'+1}}} \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \left(\frac{\left(\frac{\sqrt{\frac{\delta \delta_{m'+1}}{\sqrt{\perp}}}} \right)^{\Delta} + 1}{1+\perp} \right)^{\alpha_{m'+1}} - 1 \right) \right)^{\frac{1}{\Delta}}} \right\| \right) \right) \right) \\
& = \left(\left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^{m'+1} \left(1 - \left(\frac{\sqrt{\frac{\delta \delta_j}{\sqrt{\perp}}}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^{m'+1} \left(\frac{\left(\frac{\sqrt{\frac{\delta \delta_j}{\sqrt{\perp}}}} \right)^{\Delta} + 1}{1+\perp} \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\Delta}}} \right\| \right) \right. \right. \\
& \left. \left(\left\| \sqrt{\left(\frac{1+\perp}{\perp} \left(1 - \prod_{j=1}^{m'+1} \left(1 - \left(\frac{\sqrt{\frac{\delta \delta_j}{\sqrt{\perp}}}} \right)^{\Delta} \frac{\perp}{1+\perp} \right)^{\alpha_j} \right) \right)^{\frac{1}{\Delta}}}, \left\| \sqrt{\left(\frac{1}{\perp} \left((1+\perp) \prod_{j=1}^{m'+1} \left(\frac{\left(\frac{\sqrt{\frac{\delta \delta_j}{\sqrt{\perp}}}} \right)^{\Delta} + 1}{1+\perp} \right)^{\alpha_j} - 1 \right) \right)^{\frac{1}{\Delta}}} \right\| \right) \right) \right)
\end{aligned}$$

Hence, the proposed model holds for all possible values of “m”.

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