Assignment 2

1) Given, growth rate $f(y;\alpha) = \frac{1}{\alpha^2} y e^{-y/\alpha}$
with $\alpha \in (0,\infty)$ & $y \in [0,\infty)$

- assuming a sample of n independent observations $y_1, y_2 \cdots y_n$
The likelihood is the product of the individual PDFs.

- $L(\alpha) = \prod_{i=1}^{n} \frac{1}{\alpha^2} y e^{-y/\alpha}$

- Defining log-likelihood function $l(\alpha)$

$$l(\alpha) = \log(L(\alpha)) = \sum_{i=1}^{n} \log\left[\frac{1}{\alpha^2} y e^{-y/\alpha}\right]$$

$$l(\alpha) = \sum_{i=1}^{n}\left(-2\log\alpha + \log y - \frac{y}{\alpha}\right)$$

$$l(\alpha) = -2n\log\alpha + \sum_{i=1}^{n} \log y_i - \frac{\sum_{i=1}^{n} y_i}{\alpha}$$

$\frac{d(-l(\alpha))}{\partial\alpha} = 0$, to find max likelihood we will be minimizing the $-l(\alpha)$

$$\frac{d}{d\alpha}\left[2n\log\alpha - \sum_{i=1}^{n}\log y_i + \frac{\sum_{i=1}^{n} y_i}{\alpha}\right) = 0$$

$$\frac{2n}{\alpha} - \frac{\sum_{i=1}^{n} y_i}{\alpha^2} = 0 < \begin{matrix} 1/\alpha = 0 \\ 2n - \frac{\sum_{i=1}^{n} y_i}{\alpha} = 0 \end{matrix}$$

So, maximum likelihood estimate for the parameter is,

$$\left[\alpha_{MLE} = \frac{\sum_{i=1}^{n} y_i}{2n} = \frac{\bar{y}}{2}\right]$$