Assignment 1 (PCA)

2) Given data is not standardized and columns have zero mean. (data is centered)

- let data = $X^{m \times n}$

Now, SVD of $X = U\Sigma V^T$

Covariance of X (C) $= \left(\frac{1}{m}\right) X^T X$

$$C = \left(\frac{1}{m}\right)(U\Sigma V^T)^T (U\Sigma V^T)$$

$$C = \left(\frac{1}{m}\right)(V\Sigma U^T)(U\Sigma V^T)$$

$$\left[C = \left(\frac{1}{m}\right)(V\Sigma^2 V^T)\right] \quad \therefore (U^T U = I)$$

Since C is a symmetric matrix it's ~~eigenvalue~~ decomposition is given by

$$[C = \mathcal{V}\Lambda\mathcal{V}^T]$$

Comparing the above equations for C, we see that

- V is analogous to $\mathcal{V}$. (orthogonal matrix of eigenvectors)
- $\left(\frac{1}{m}\right)\Sigma^2$ is analogous to Λ (diagonal matrix of eigenvalues)

Thus, the eigenvalues of covariance matrix (C) are given by the squares of the singular values in Σ, scaled by (1/m).

$$\boxed{\lambda_i = \left(\frac{1}{m}\right)\sigma_i^2}$$