ASSIGNMENT 4

1) To predict if the student will ace the end sem enam, based on mid semester a assignment submission

From duta, (Gram = End Sem)

- -> deed Exam = 5 False
- Acecl Exam and not submitted assignment 4 False

 Acecl Exam and not submitted assignment 1 Town
- Below Average Average Above Average

 3Fala 17 me 2 Fala 37 me 2 True 0 Fala

Formulas:

- Entropy = - [PflogiPt + PflogiPt] of for our problem)

- Authorge Entropy = n, E, + n Ez

 Sweighted ang. of all the subnooks that a parent make
 split.
- Information Gain : Eparent Aug. Echild

Roct Node Colculation:

a) Submitted Assignments

Entropy - submitted (
$$\varepsilon_s$$
) = $-\left[\frac{5}{6}\ln(\frac{5}{6}) + \frac{1}{6}\ln(\frac{1}{6})\right]$
= 0.451

Entropy-not submitted (Ens) =
$$-\left[\frac{1}{5}\ln(\frac{1}{5}) + \frac{1}{5}\ln(\frac{1}{5})\right]$$

= 0.05

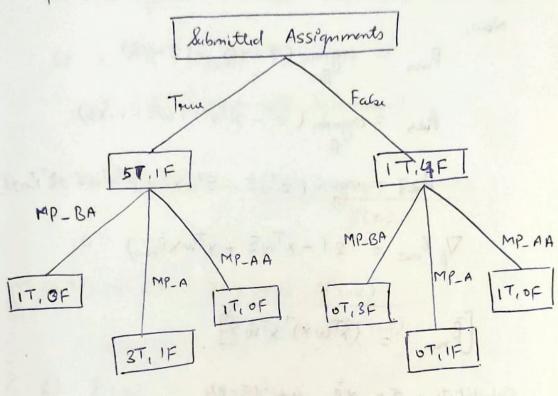
Aug. Entropy _Submission =
$$\frac{6}{11}(0.451) + \frac{5}{11}(0.5)$$

= 0.473

b) Mid - Sem Performance
Entropy - BA =
$$-\left[\frac{1}{4}\ln(\frac{1}{4}) + \frac{3}{4}\ln(\frac{3}{4})\right] = 0.562$$

Entropy - A = $-\left[\frac{3}{5}\ln(3/5) + \frac{2}{5}\ln(\frac{3}{5})\right] = 0.673$
Entropy - AA = $-\left[\frac{2}{5}\ln(\frac{3}{5}) + 0\right] = 0$

Based on the information gain the rest noch would be submitted assignments, since we have only two variables so he need for further calculation



Where, MP-BA = midsemperformance below average

MP-A = midsemperformance average

MP-AA = midsemperformance above average

-> Here only on node is not terminal mode.

i.e. [3T, IF], restall are terminal nodes.

but since we closet have any other variable, we comet

split it frather.

Problem 27 Geven

WLSE =
$$\frac{1}{4}$$
 $\frac{1}{2}$ $\frac{1}{2}$

$$\bar{\beta}_{ren} = (x^T w x)^T x^T w (x \bar{\beta}_{old} + w^T (\bar{y} - \bar{p}))$$

$$\bar{\beta}_{ren} = (x^T w x)^T ((x^T w x) \bar{\beta}_{old} + x^T (\bar{y} - \bar{p}))$$

$$\overline{\beta}_{\text{new}} = \overline{\beta}_{\text{old}} + (x^{T}wx)^{-1}x^{T}(\bar{y}-\bar{p})$$

matrices but is it not applicable in many rase. The quadratic discriminant analysis is for betongenous variouses— covarionses matrices.

Let $X \subseteq \mathbb{R}^p$ (input) $Y \subseteq \mathbb{R}^p$ (input)

Let $f_k(x) = P(x|y=k)$ $\overline{I}_k = P(y=k)$

assuming , fx(x) be a gaussian distribution

f_k(x) = (√2π)^{1/P}(Σ)^{1/2} eup (-½(x-ū_k)^T Σ⁻¹_κ(x-ū_k))

-Shortony sometion (at norm aloss (NK))

$$\frac{1}{2}\left(\frac{P(Y=k|X=X)}{P(Y=k|X=X)}\right) = 0$$
- log $\left(\frac{P(Y=k|X=X)}{P(X=k|X=X)}\right) = \log\left(\frac{P(Y=k|X=X)}{P(X=k|X=X)}\right) = 0$
- log $\left(\frac{1}{2}(X), \frac{1}{2}(X)\right) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X)\right) = 0$
- log $\left(\frac{1}{2}(X), \frac{1}{2}(X)\right) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X)\right) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X)\right)$
- $\frac{1}{2}(X-U_k)^T \sum_{k=1}^{1}(X-U_k) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X)\right) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X), \frac{1}{2}(X)\right)$
- $\frac{1}{2}\log\left(\frac{1}{2}(X)\right) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X), \frac{1}{2}(X), \frac{1}{2}(X), \frac{1}{2}(X)\right)$
- $\frac{1}{2}\log\left(\frac{1}{2}(X)\right) + \log\left(\frac{1}{2}(X), \frac{1}{2}(X), \frac{1}{2}(X)$

Ilas Prediction:
$$g = cog_{max} S_{k}(x)$$

-basically a point lies in k class if

 $log\left(\frac{P(Y=k\mid X=\bar{x})}{P(Y=k\mid Y=\bar{x})}\right) > 0 \quad \forall \ l \neq k$

47 to find probability of passing enderm given the midson performance was average and all assignment were submitted.

X = 'average' midsem performance

X1 = all assignments was submitted

Y = passing endsem.

x = 1x, x, r

assuming conditional independence of features

$$P(Y|X) = \frac{P(X|Y) P(Y)}{P(X)}$$

$$P(Y|X) = \frac{P(x_1|Y) P(x_2|Y) P(Y)}{\sum_{i=1}^{2} p(x_i|Y_i) P(x_2|Y_i) P(Y_i)}$$

$$P(Y|X_{1},X_{2}) = \frac{\binom{3}{6}(\frac{5}{6})(\frac{6}{4})}{\binom{1}{5}(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})(\frac{5}{6})}$$

$$P(Y|X_1,Y_2) = \frac{\frac{15}{6}}{\frac{2}{5} + \frac{15}{6}} = \frac{25}{29}$$

5) Given
$$p(x|y=0) \approx N(0, 1/4)$$

$$p(x|y=1) \sim N(0, 1/2)$$

$$p(y=1) = 0.5$$

$$L = \begin{bmatrix} 0 & \sqrt{2} \\ 1 & 0 \end{bmatrix}$$

Postroion for class o

$$P(Y=0|x) = \frac{P(X|Y=0) P(Y=0)}{P(X)}$$

Pastirion for class 1

$$P(Y=Q(X)) = \frac{P(X)Y=1)P(Y=1)}{P(X)}$$

- The loss for class o & 1 R(Y=0|X) = \$\frac{1}{4} \text{deft} loop(Y=0|X) + losp(Y=1|X) R(Y=1/x) = L10P(Y=0/x) + L11 P(Y=1/x)

-> We say a point belong to relaw 1 if R(Y=1/Y) < R(Y=0/x) & to class o if

POPEN R(Y=0 1X) < R(Y=11X)

So, the decision boundary is R(Y=0|Y) = R(Y=1|X)

Loop(Y=0|x) + Loip(Y=1|x) = Lop(Y=0|x) + Lip(Y=1|x)

15 b(A=11x) = b(A=01x)

$$\frac{b(x)}{b(x|\lambda=P)} = \left(\frac{b(x)}{b(x|\lambda=0)b(\lambda=0)}\right)$$

$$\sqrt{2}\left(\frac{1}{\sqrt{2\pi}\left(\frac{1}{2}\right)}e^{\mu}p\left(\frac{-\chi^{2}}{2(1/2)}\right)\right)=\left(\sqrt{2\pi}\left(\frac{1}{4}\right)e^{\mu}p\left(\frac{-\chi^{2}}{2(1/4)}\right)\right)$$

$$exp(x^2) = 1$$

given the prediction function ŷ = sign (wTx+b), wTERd, bER let t; 6 d-1,17

y: = sityh (w*x1+6)

a) In hand margin SVM we try to minimize maximize the distance of the closest point from the decision boundary.

ang maix 1/2 llw112 such that tp (WTx;+6) >11

Using lag rangian we can write

 $L(w,b,\alpha) = \min\left[\max\left(\frac{1}{2}||w||^2 - \sum_{i=1}^{\infty} \alpha_i^{i} \left(\frac{1}{2} \left(\sqrt{x_i^2 + b}\right) - 1\right)\right]$

using dual formulation

 $L(w,b,x) = \max_{\alpha > 0} \left[\min_{\alpha > 0} \left(\frac{1}{2} |w||^2 - \sum_{i=1}^{d} x_i^2 (t_i^2 (\sqrt{x_i^2 + b}) - 1) \right) \right]$

=> [w = \left] \times \ $\frac{\partial N}{\partial \Gamma} = 0$

> [\langle \ditte = 0] 3L = 0

So, we have

L(a) = max [dTI - 1 dTtTxTxtx]

So, only & to play meli in making prediction Now, to get value of 6

$$th\left[\left(\sum_{i=1}^{d} \lambda_{i} t_{i} x_{i}\right) \chi_{n} + b\right] = 1$$

$$th^{2} \left[\left(\sum_{i=1}^{d} (\lambda_{i} t_{i} x_{i})\right) \chi_{n} + b\right] \mp t_{n}$$

$$b = t_{n} + \left(\sum_{i=1}^{d} \lambda_{i} t_{i} x_{i}\right) \chi_{n}$$

averaging over all support vectors

$$\left[B_{ias} = b = \frac{1}{N_s} \sum_{n \in S} \left[t_n - \left(\sum_{i=1}^{d} \alpha_i t_i x_i\right) x_n \right] \right]$$

b) For seft margin SVM, we will introduce slack Nariables &; tiEd Qi = 0 fox datapoints that are on or inside the correct margin boundary Gi= | t: - yi | for allow points. x, E.71, Y=1 Now we have new / QE1/13: condition to yo + &i >, 1 , i=1,2 ... d ,"=0" such that 8, 70 -> Maximize the margin while penalizing the points that lie on the wrong side. ang min $\frac{1}{2} ||w||^2 + C \sum_{i=1}^{d} \epsilon_i$ such that $|(2i)_i, 0, |(i=1,2)|$ $|(2i)_i, 0, |(i=1,2)|$ $|(3i)_i, 1 \in [3,1]$ tig; + Ei >1 -> Using lagrangian [L(w, b, E, x, B) = \frac{1}{2} | | | | | | + c \frac{d}{2} (i - d) \frac{d}{i=1} (t; (w^Tx; +b) + 6i - d) \frac{d}{d} \frac{1}{7} -ZB; Ei]

min [max
$$L(w,b,x,\beta)$$
] = max $\begin{bmatrix} min & L(w,b,x,\beta) \end{bmatrix}$
 w,b $\begin{bmatrix} min & L(w,b,x,\beta) \end{bmatrix}$
 w,b $\begin{bmatrix} min & L(w,b,x,\beta) \end{bmatrix}$

bies = [b = 1 \ \times \left(\tau_n - \left(\frac{d}{2} \titixi) \chi_n \right) \]

No mem (tn - \left(\frac{d}{2} \titixi) \chi_n \right)

- (a). Because the hinge loss for SVM only consider support vectors and give zero weight to atten points but this is not true for logistic model when all points are considered here SVM's decision boundary remains unaffected by distant points.
 - b) Kenrel bricks allow to operate in the oniginal feature without computing the coordinate of decta in higher dimensional space. In SVM Kenrel is basically the dot product of the features, which is non-putationally less expension to exhaulter.

→ Problem 8

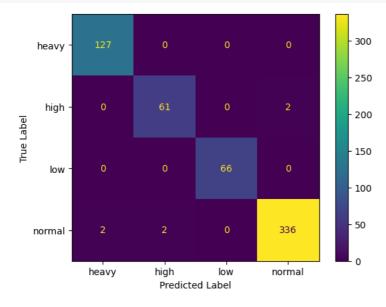
```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay, classification_report
from sklearn.ensemble import RandomForestClassifier
from sklearn.preprocessing import LabelEncoder
path = '/content/drive/MyDrive/sem 7/ID5055/Assignment 4/Traffic_data.csv'
data = pd.read csv(path)
data.head(10)
              Time Date Day of the week CarCount BikeCount BusCount TruckCount Total Traffic Situation
     0 12:00:00 AM
                      10
                                  Tuesday
                                                 31
                                                             0
                                                                                  4
                                                                                        39
     1 12:15:00 AM
                      10
                                  Tuesday
                                                 49
                                                             0
                                                                      3
                                                                                  3
                                                                                        55
                                                                                                          low
     2 12:30:00 AM
                      10
                                  Tuesday
                                                 46
                                                             0
                                                                                  6
                                                                                        55
                                                                                                          low
                                                                      2
     3 12:45:00 AM
                      10
                                  Tuesday
                                                 51
                                                             0
                                                                                  5
                                                                                        58
                                                                                                          low
         1:00:00 AM
                      10
                                  Tuesday
                                                 57
                                                             6
                                                                      15
                                                                                 16
                                                                                        94
                                                                                                       normal
     5
        1:15:00 AM
                      10
                                  Tuesday
                                                 44
                                                             0
                                                                      5
                                                                                  4
                                                                                        53
                                                                                                          low
         1:30:00 AM
     6
                      10
                                  Tuesday
                                                 37
                                                             0
                                                                      1
                                                                                  4
                                                                                        42
                                                                                                          low
     7
         1:45:00 AM
                      10
                                  Tuesday
                                                 42
                                                             4
                                                                                  5
                                                                                        55
                                                                                                          low
     8
        2:00:00 AM
                      10
                                  Tuesday
                                                 51
                                                             0
                                                                      9
                                                                                  7
                                                                                        67
                                                                                                          low
         2:15:00 AM
                                                                                  7
                      10
                                  Tuesday
                                                                                        45
                                                                                                          low
data.info()
     <class 'pandas.core.frame.DataFrame'>
     RangeIndex: 2976 entries, 0 to 2975
    Data columns (total 9 columns):
     # Column
                           Non-Null Count Dtype
                           2976 non-null
     0 Time
                                            object
                            2976 non-null
         Date
     1
                                            int64
         Day of the week 2976 non-null
     2
                                            object
                            2976 non-null
                                            int64
     3
         CarCount
         BikeCount
                            2976 non-null
                                            int64
         BusCount
                            2976 non-null
                                            int64
         TruckCount
                            2976 non-null
                                            int64
         Total
                            2976 non-null
                                            int64
         Traffic Situation 2976 non-null
                                           object
     dtypes: int64(6), object(3)
     memory usage: 209.4+ KB
df = data
# Converting string values to numeric, Monday = 0 and Sunday = 6
df['Day of the week cat'] = LabelEncoder().fit_transform(df['Day of the week'])
col = df.pop('Day of the week cat')
data.insert(2, col.name, col)
df['Time'] = pd.to_datetime(df['Time']).apply(lambda x: x.hour)
df.info()
     <class 'pandas.core.frame.DataFrame'>
     RangeIndex: 2976 entries, 0 to 2975
    Data columns (total 10 columns):
     # Column
                              Non-Null Count Dtype
     0 Time
                              2976 non-null
                                              int64
                              2976 non-null
```

Day of the week cat 2976 non-null

```
7 TruckCount 2976 non-null int64
8 Total 2976 non-null int64
9 Traffic Situation 2976 non-null object
dtypes: int64(8), object(2)
memory usage: 232.6+ KB

X = df.drop(['Traffic Situation', 'Day of the week'], axis = 1)
Y = df['Traffic Situation']
```

1. Split the dataset into train and test set (train size= 0.8, random state = 42) and train a random forest classifier using the train set. Plot a confusion matrix using the test set for prediction.



Day of the week

CarCount

BikeCount

BusCount

2976 non-null

2976 non-null

2976 non-null

2976 non-null

object

int64

int64

int64

3

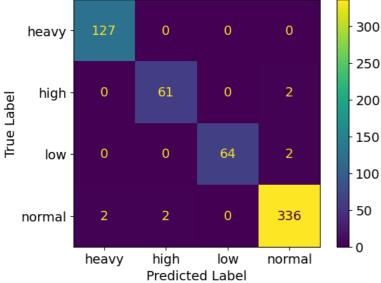
4

5

precision recall f1-score support heavy 0.98 1.00 0.99 127 high 0.97 0.97 0.97 63 1.00 1.00 low 1.00 66 normal 0.99 0.99 0.99 0.99 accuracy 596 0.99 0.99 0.99 596 macro avg 0.99 0.99 0.99 596 weighted avg

2. Use a weighted random forest classifier with weights based on the frequency of the corresponding class. Plot a confusion matrix and report your observation by comparing the results with the previous results.

```
print(f'The classes are:\n{rf.classes_}')
```



precision recall f1-score support heavy 0.98 1.00 0.99 127 high 0.97 0.97 0.97 63 low 1.00 0.97 0.98 66 normal 0.99 0.99 0.99 340 0.99 596 accuracy 0.99 0.98 0.98 macro avg 596 weighted avg 0.99 0.99 0.99 596

0.059176

0.155289

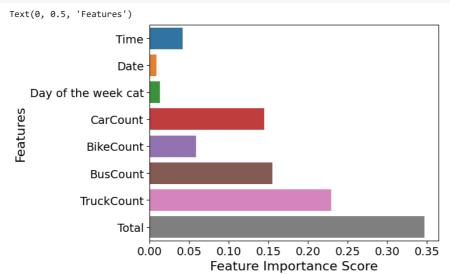
BikeCount BusCount

▼ 3. Use the trained classifier model to report the important features based on impurity metric

TruckCount 0.229021 Total 0.347152

dtype: float64

```
sns.barplot(x = feature_scores, y = feature_scores.index)
plt.xlabel('Feature Importance Score', fontsize = 16)
plt.ylabel('Features', fontsize = 16)
```

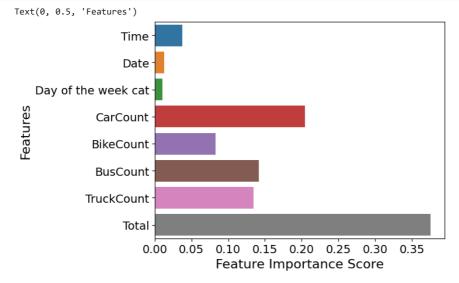


Model 2
feature_scores_w = pd.Series(rf_weighted.feature_importances_, index = X_train.columns)
print(feature_scores)

0.037775 Date 0.012922 Day of the week cat 0.010682 0.204475 CarCount BikeCount 0.082924 BusCount 0.141365 TruckCount 0.134151 Total 0.375706 dtype: float64

sns.barplot(x = feature_scores_w, y = feature_scores.index)
plt.xlabel('Feature Importance Score', fontsize = 16)

plt.ylabel('Features', fontsize = 16)



Observations

1. Since we are predicting the traffic condition, so it is reasonable why 'Total' feature has the highest feature importance.

- 2. Other important features are the count of heavy vehicles like cars, buses, and trucks, as they are main reason for traffic.
- 3. Time, Date, and Day of week got low feature score, however time is also an important feature in determining the traffic condition.