Plotting the decision boundary of a logistic regression model

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(2 comments)

In the notation of this previous post (/blog/logistic-regression-for-image-classification/), a logistic regression binary classification model takes an input feature vector, $\boldsymbol{x}$, and returns a probability, $\hat{y}$, that $\boldsymbol{x}$ belongs to a particular class: $\hat{y} = P(y=1|\boldsymbol{x})$. The model is trained on a set of provided example feature vectors, $\boldsymbol{x}^{(i)}$, and their classifications, $y^{(i)} = 0$ or 1, by finding the set of parameters that minimize the difference between $\hat{y}^{(i)}$ and $y^{(i)}$ in some sense.

These model parameters are the components of a vector, $\boldsymbol{w}$ and a constant, b, which relate a given input feature vector to the predicted logit (https://en.wikipedia.org/wiki/Logit) or log-odds, z, associated with $\boldsymbol{x}$ belonging to the class $y=1$ through

$$z = \boldsymbol{w}^T\boldsymbol{x} + b.$$

In this formulation,

$$z = \ln\frac{\hat{y}}{1-\hat{y}} \quad \Rightarrow \hat{y} = \sigma(z) = \frac{1}{1+\mathrm{e}^{-z}}.$$

Note that the relation between z and the components of the feature vector, x_j, is linear. In particular, for a two-dimensional problem,

$$z = w_1x_1 + w_2x_2 + b.$$

It is sometimes useful to be able to visualize the boundary line dividing the input space in which points are classified as belonging to the class of interest, $y=1$, from that space in which points do not. This could be achieved by calculating the prediction associated with $\hat{y}$ for a mesh of (x_1, x_2) points and plotting a contour plot (see e.g. this scikit-learn example (https://scikit-learn.org/stable/auto_examples/linear_model/plot_iris_logistic.html#sphx-glr-auto-examples-linear-model-plot-iris-logistic-py)).

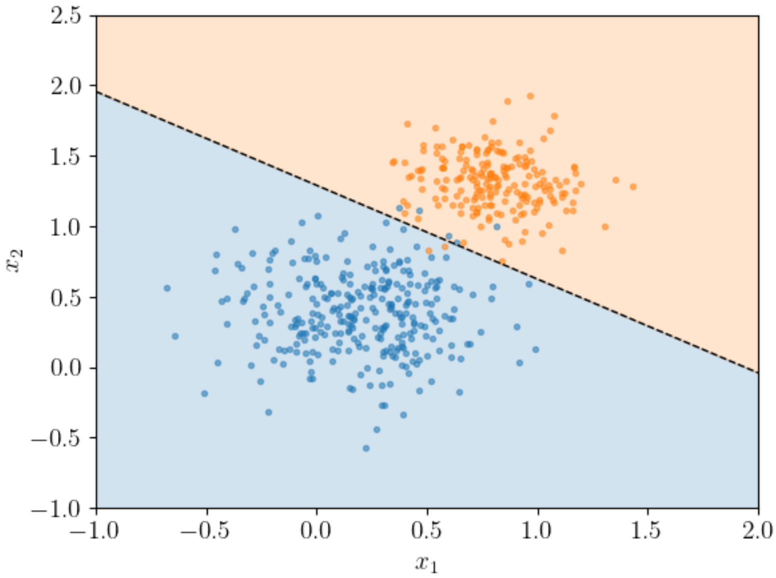
Alternatively, one can think of the decision boundary as the line $x_2 = mx_1 + c$, being defined by points for which $\hat{y} = 0.5$ and hence $z=0$. For $x_1 = 0$ we have $x_2 = c$ (the intercept) and

$$0 = 0 + w_2x_2 + b \quad \Rightarrow c = -\frac{b}{w_2}.$$

For the gradient, m, consider two distinct points on the decision boundary, (x_1^a, x_2^a) and (x_1^b, x_2^b), so that $m = (x_2^b - x_2^a)/(x_1^b - x_1^a)$. Along the boundary line,

$$\begin{aligned} &0 = w_1x_1^b + w_2x_2^b + b - (w_1x_1^a + w_2x_2^a + b)\\ \Rightarrow &-w_2(x_2^b - x_2^a) = w_1(x_1^b - x_1^a)\\ \Rightarrow &m = -\frac{w_1}{w_2}.\end{aligned}$$

To see this in action, consider the data in linpts.txt (/static/media/uploads/blog/logistic_regression/linpts.txt), which maybe classified using scikit-learn's `LogisticRegression` classifier (https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html#sklearn.linear_model.LogisticRegression). The following script retrieves the decision boundary as above to generate the following visualization.




```
import numpy as np
import matplotlib.pyplot as plt
import sklearn.linear_model
plt.rc('text', usetex=True)

pts = np.loadtxt('linpts.txt')
X = pts[:,:2]
Y = pts[:,2].astype('int')

# Fit the data to a logistic regression model.
clf = sklearn.linear_model.LogisticRegression()
clf.fit(X, Y)

# Retrieve the model parameters.
b = clf.intercept_[0]
w1, w2 = clf.coef_.T
# Calculate the intercept and gradient of the decision boundary.
c = -b/w2
m = -w1/w2

# Plot the data and the classification with the decision boundary.
xmin, xmax = -1, 2
ymin, ymax = -1, 2.5
xd = np.array([xmin, xmax])
yd = m*xd + c
plt.plot(xd, yd, 'k', lw=1, ls='--')
plt.fill_between(xd, yd, ymin, color='tab:blue', alpha=0.2)
plt.fill_between(xd, yd, ymax, color='tab:orange', alpha=0.2)

plt.scatter(*X[Y==0].T, s=8, alpha=0.5)
plt.scatter(*X[Y==1].T, s=8, alpha=0.5)
plt.xlim(xmin, xmax)
plt.ylim(ymin, ymax)
plt.ylabel(r'$x_2$')
plt.xlabel(r'$x_1$')

plt.show()
```

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