Problem:-

a) Solution:-

$$Mz'' = fz - gM - C_z z' \rightarrow (5)$$

$$fz = Mg + K_{pz}(z_{des} - z) - K_{dz}(z') \rightarrow (6)$$

Replace (6) with (5)

$$Mz'' = Mg + K_{pz}(z_{des} - z) - K_{dz}(z') - gM - C_z z'$$

$$Mz'' = (K_{pz} \times z_{des}) - (K_{pz} \times z) - z'(K_{dz} + C_z)$$

$$Mz'' = (K_{pz} * z_{des}) - (K_{pz} * z) - (K_{dz} + C_z) z'$$

$$Mz'' + (K_{dz} + C_z) z' + (K_{pz} * z) = K_{pz} * z_{des}$$

Apply laplace transform

$z(0) = z'(0) = 0$ at Initial conditions are zero

$$Ms^2 z(s) + (K_{dz} + C_z) s\, z(s) + K_{pz} * z(s) = K_{pz} * z_{des}(s)$$

Take common $z(s)$

$$z(s)\left(Ms^2 + (K_{dz} + C_z)s + K_{pz}\right) = K_{pz} * z_{des}(s)$$

As we know that $z(s) =$ o/p & $z_{des}(s) =$ i/p

$$\boxed{\frac{z(s)}{z_{des}(s)} = \frac{K_{pz}}{Ms^2 + (K_{dz} + C_z)s + K_{pz}}} \rightarrow (7)$$

Given Values:-

$M = 1\,kg$ $\quad C_z = 3 + (A \cdot M / 5000)$ where $A \cdot M = 4452$

$K_{pz} = 5$ $\quad C_z = 3.8904$

$K_{dz} = 15$

Put the all given values in eq(7)

$$\frac{Z(S)}{Z_{des}(S)} = \frac{5}{(1)s^2 + (15+3.8904)S + 5}$$

$$\frac{Z(S)}{Z_{des}(S)} = \frac{5}{S^2 + 18.8904S + 5}$$

↳ For Zeros

Equate the numerator = 0

Zeros = (No zeros)

↳ For Poles

Equate the denominator to 0

Two Poles

★ $S^2 + 18.8904\, S + 5 = 0$

By using Quadratic formula $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$a=1$ $b=18.8904$ $c=5$

$$S_{1,2} = \frac{-18.8904 \pm \sqrt{(18.8904)^2 - 4(1)(5)}}{2(1)} = 0$$

$$S_{1,2} = \frac{-18.8904 \pm 18.3534}{2}$$

$$S_1 = \frac{-18.8904 + 18.3534}{2} \qquad S_2 = \frac{-18.8904 - 18.3534}{2}$$

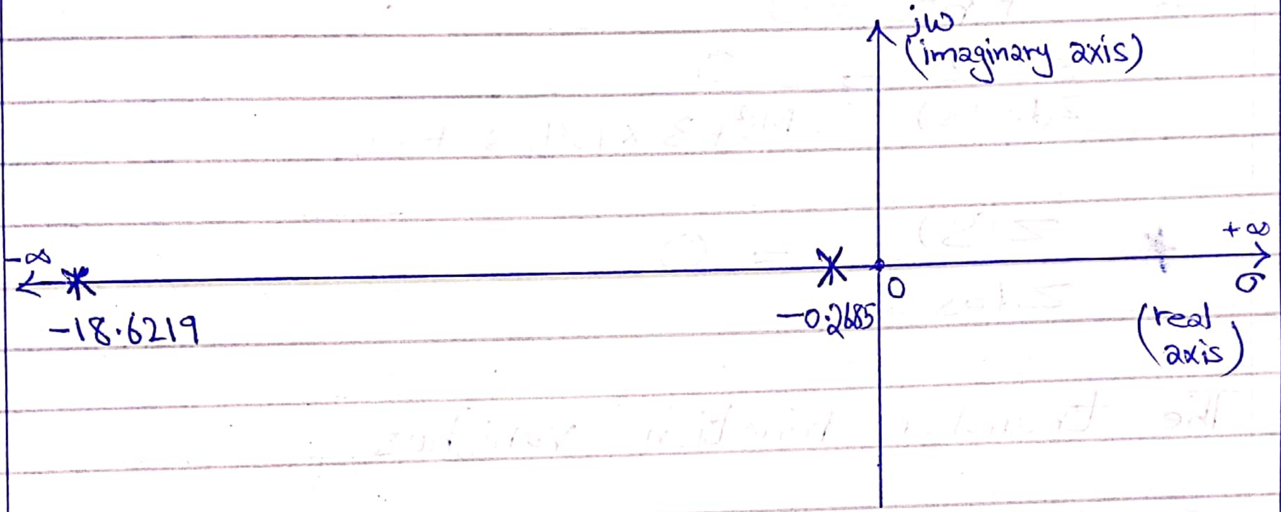
$$\boxed{S_1 = -0.2685} \qquad \boxed{S_2 = -18.6219}$$

★ Poles are -0.2685, -18.6219

b) Solution:-

The poles that we get from part (a)

Complex Plane:-



In order to system (linear) to be stable, all of its poles must have negative real parts' as shown. i.e they mus all lie within the left half of the s-plane.

Kpz & Kdz values fluctuate make four cases
Let high value of both Kpz & Kdz = 100
low " " " " " " = 0

As we know that
In this system have no zeros & two poles in each cases.

Case 01:- $(K_{pz} = 0, K_{dz} = 0)$

Eq 7 → $\frac{Z(S)}{Z_{des}(S)} = \frac{K_{pz}}{Ms^2 + (K_{dz} + C_z)S + K_{pz}}$

Put the values

So $K_{pz} = 0$

$$\frac{Z(S)}{Z_{des}(s)} = \frac{0}{Ms^2 + 3.8904\ s + 0}$$

★ $\frac{Z(S)}{Z_{des}(S)} = 0$

The transfer function vanishes.

So, Complex plane is not drawn.

Case 02:- $(K_{pz} = 0, K_{dz} = 100)$

★The transfer function also vanishes (because of $K_{pz} = 0$)

Case 03:- $(K_{pz} = 100, K_{dz} = 0)$

Put values in eq (7)

$$\frac{Z(S)}{Z_{des}(S)} = \frac{100}{s^2 + 3.8904S + 100}$$

So, Zeros = (No zeros)

Poles:- $s^2 + 3.8904S + 100$

By applying Quadratic Formula we find the roots S_1 & S_2

$$s^2 + 3.89045 + 100 = 0$$

$a = 1 \quad b = 3.8904 \quad c = 100$

$$S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

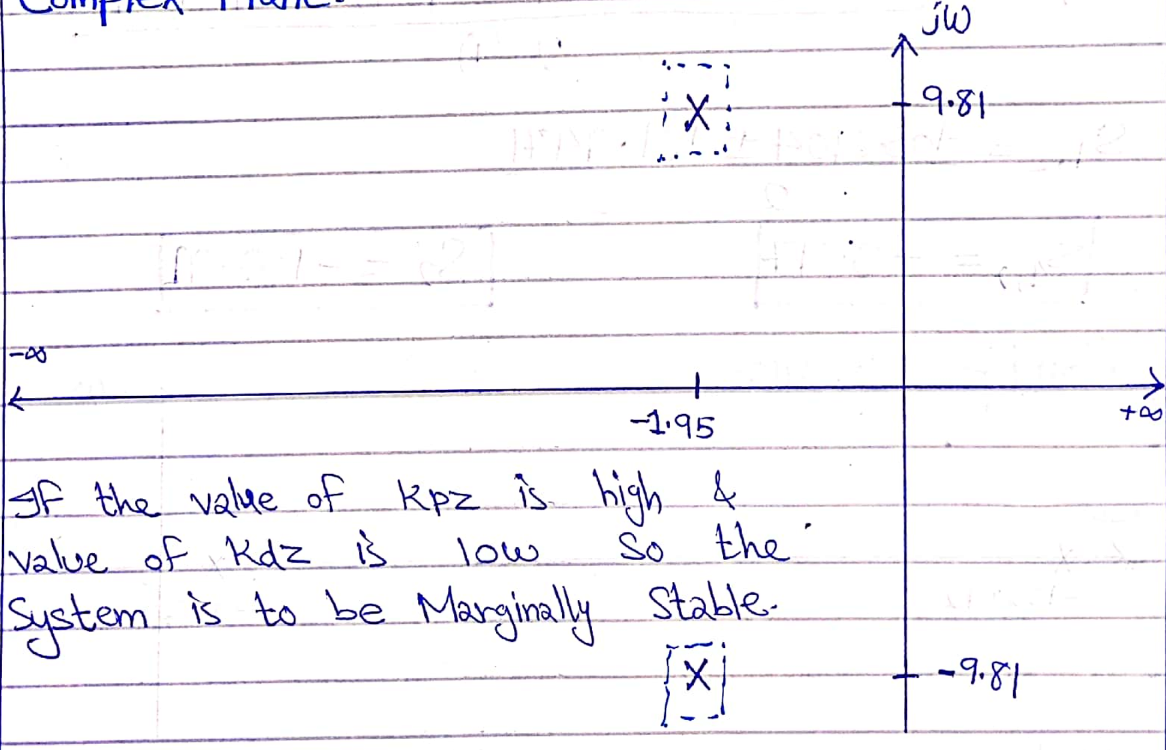
$$S_{1,2} = \frac{-3.8904 \pm \sqrt{(3.8904)^2 - 4(1)(100)}}{2(1)}$$

$\therefore j = \sqrt{-1}$

$$S_{1,2} = \frac{-3.8904 \pm 19.6180j}{2}$$

$\boxed{S_1 = -1.95 + 9.81j}$, $\boxed{S_2 = -1.95 - 9.81j}$

Complex Plane:-



If the value of Kpz is high & value of Kdz is low so the system is to be Marginally Stable.

Roots are in the left half plane & on the imaginary axis, so the system is "Marginally Stable" means that system lies between Stability & Unstability

Case 04 ($K_{pz} = 100$, $K_{dz} = 100$)

Put values in eq (7)

$$\frac{Z(s)}{Z_{des}(s)} = \frac{100}{s^2 + 103.8904s + 100}$$

Zeros = (No zeros)

Poles $\Rightarrow$ $s^2 + 103.8904s + 100 = 0$

$a = 1$ $b = 103.8904$ $c = 100$

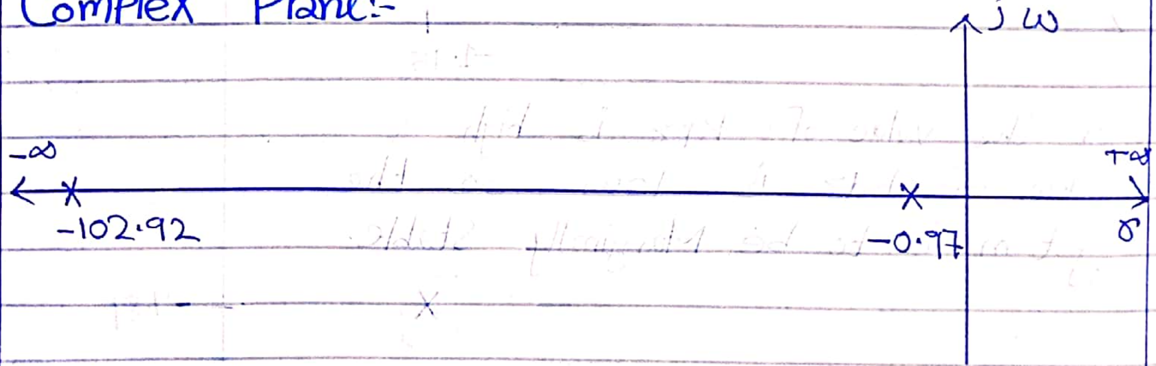
Find the roots by using Quadratic Formula

$$S_{1,2} = \frac{-103.8904 \pm \sqrt{(+103.8904)^2 - 4(1)(100)}}{2(1)}$$

$$S_{1,2} = \frac{-103.8904 \pm 101.9471}{2}$$

$\boxed{S_1 = -0.97}$ $\boxed{S_2 = -102.92}$

Complex Plane:-



Both roots are in left plane, so the system is stable.
If the value of both K_{pz} & K_{dz} are high, so the system is to be stable.