#integration Antiderivatives are the inverses of derivatives. Let's have $f(x)$ and $F(x)$ the function of the area bounded by f from a to b. I will show that $F'(x) = f(x)$. Let $c \in (a, b)$. We know that $F(c) = k$ for some k , and $F(c + dx) = F(c) + f(c) * dx$ (intuicion de los rectangulitos cuya base tiende a 0). Despejando, $F(c + dx) - F(c) = f(c) * dx$, $\frac{dF}{dx} = f(c)\square$.

Fundamental theorem of calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Por definicion F es la antiderivada de f, y ya vimos que F es el area de la curva de su derivada. Veamos el TFC de una forma intuitiva. Por la linealidad de la integral, $\int_a^b f(x)dx = \int_0^b f(x)dx - \int_0^a f(x)dx$. Por la definicion de F, que es $F(x) = \int_0^x f(x)dx$ esto equivale a decir $F(b) - F(a)$. La logica menos circular. Pero tiene mucho sentido, si sabes calcular el area entre 0 y un numero arbitrario, el area de a a b es lo acumulado hasta b menos lo acumulado hasta a.

Uniqueness of antiderivative up to the constant

Theorem : If $F' = f$ and $G' = f$ (F and G both antiderivatives of f then $F = G + C$ (they only differ by a constant.

Proof : $(F - G)' = F' - G' = f - f = 0$ Since the derivative of $F - G$ is 0, that new function must be a constant.

Basic application

Finding F based on f and a single value of F. Simple case: we know $F(0) = k$. Then $F(t)$, for $t > 0$ is

$$F(t) = \int_0^t f(t)dt + F(0)$$

Why? Because what we want to find is F, which is the area under a curve of f, which we know. To calculate the change of the area we can use $\int_0^t f(t)dt$ as previously established. By TFC $\int_0^t f(t)dt = F(t) - F(0)$. To get $F(t)$ back we simply add back $F(0)$.

Generalizing, if we know $F(k)$ for some $k > 0$, by the earlier formula we know that

$$F(k) = \int_0^k f(t)dt + F(0)$$

Here we know everything except $F(0)$, which we can find. From there, we get a general formula for $F(t)$.

Does this make sense for k<0? There is an intuive and easy way to apply the former to different real world scenarios with time/velocity. Negative time doesn't make any sense, but is there any reasonable meaning behind using $t < 0$?

$$\int_b^a f(x)dx = F(a) - F(b)$$

$$\int_a^b f(x)dx = F(b) - F(a)$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

Por simplicidad sea $t > 0$, quiero $F(-t)$

$$F(-t) = \int_0^{-t} f(t)dt + F(0) = -\int_{-t}^0 f(t)dt + F(0)$$

Y ahora esto si tiene la forma de una integral normal. Tiene una interpretacion coherente? Sea $t = 1, -t = -1$ y $F(0) = 3$ La integral normal $\int_0^{-1} f(t)dt = 1$. Entonces en el momento "-1" F valia un cierto valor x y en $F(0)$ valia 3. En ese intervalo ascendio por 1. El resultado logico es que $F(-1) = -1 + 3 = 2$. Asi, podemos generalizar $F(k) = \int_0^k f(t)dt + F(0)$ para todos los reales, y en caso de negativos es como "contar para atras".

Metodos de integracion

[[Integration by Parts]] [[Integration by Substitution]]