Building a Numerical Model of the Influence of a Gravitational Wave on an Electromagnetic Wave

The Physics behind PhotonGravityWavePerpendicularCase.py and PhotonGravityWave4D.py

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Let us define a left-handed coordinate system along whose xy plane electromagnetic waves can travel. Consider a gravitational wave propagating along the z-axis. Let us further define the x- and y-axes to be along the polarization axes of the gravitational wave. This has the effect of stretching and contracting space along those 2 axes. So, the wave equation in the presence of a gravitational wave can be written (taking out the the time component because it will not be affected) as

$$\begin{pmatrix} \partial_x & \partial_y & \partial_z \end{pmatrix} \mathbf{M} \begin{pmatrix} \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \tag{1}$$

where $\mathbf{M}$ is the tensor represented by

$$\mathbf{M} = \begin{pmatrix} 1+f & & \\ & 1-f & \\ & & 1 \end{pmatrix} \quad \text{where } f = \epsilon \cos(kz - kt) \tag{2}$$

Note that when the amplitude ϵ is 0, Eq. 1 reduces to the standard wave equation for flat spacetime,

$$\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}. \tag{3}$$

Let us generalize this for any inclination of the gravitational wave. We need not generalize for azimuthal angle because we can always define a planar axis along the azimuthal direction of the wave. First, let us define a left-handed coordinate system attached to the gravitational wave with $x\prime, y\prime$ along its polarization axes of and $z\prime$ as its propagation direction. Second, we must also define a left-handed coordinate system $\langle X, Y, Z \rangle$ fixed to our frame of interest whose XY plane contains those electromagnetic waves. The latter is the frame our computational algorithms will use; the rows and columns of the numerical method form the discretized representation of the plane. The row indices and column indices correspond to Y-values and X-values respectively with (0,0) as the top left corner of the left-handed system. Note that our initial coordinate system $\langle x, y, z \rangle$ was equivalent to both $\langle X, Y, Z \rangle$ and $\langle x\prime, y\prime, z\prime \rangle$ as that was the case where the gravitational wave is perpendicular to the plane. In our new frame, our tensor $\mathbf{M}$ takes $f = \epsilon \cos(kz\prime - kt)$ with t unchanged.

We know that we can always find a rotation matrix $\mathbf{R}$ such that

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathbf{R} \begin{pmatrix} x\prime \\ y\prime \\ z\prime \end{pmatrix} \tag{4}$$

. We can now rewrite the left hand side of Eq. 1 as

$$LHS = \begin{pmatrix} \partial_{x'} & \partial_{y'} & \partial_{z'} \end{pmatrix} \mathbf{R}^{-1}\mathbf{R}\ \mathbf{M}\ \mathbf{R}^{-1}\mathbf{R} \begin{pmatrix} \partial_{x'} \\ \partial_{y'} \\ \partial_{z'} \end{pmatrix} u \tag{5}$$

$$= \begin{pmatrix} \partial_{x'} & \partial_{y'} & \partial_{z'} \end{pmatrix} \mathbf{R}^{-1}\mathbf{R}\ \mathbf{M}\ \mathbf{R}^{-1} \begin{pmatrix} \partial_{X} \\ \partial_{Y} \\ \partial_{Z} \end{pmatrix} u \tag{6}$$