Ex 4.3

From (3.4)

$$\mu_{m|y}(\{m \in A \mid y\})$$

$$= \lim_{\Delta y \to 0} \frac{\mathbb{P}[\, y \le y' \le y + \Delta y \mid \{m \in A\}] \ \mathbb{P}[\{m \in A\}]}{\mathbb{P}[\, y \le y' \le y + \Delta y]}$$

$$= \lim_{\Delta y \to 0} \frac{\dfrac{\mathbb{P}[\, y \le y' \le y + \Delta y, \ \{m \in A\}]}{\mathbb{P}[\{m \in A\}]} \ \mathbb{P}[\{m \in A\}]}{\mathbb{P}[\, y \le y' \le y + \Delta y]}$$

$$= \lim_{\Delta y \to 0} \frac{\dfrac{\int_A \pi(m, y)\, dm\, \Delta y}{\int_A \pi(m)\, dm} \ \int_A \pi(m)\, dm}{\pi(y)\, \Delta y}$$

$$= \frac{\pi(y \mid m)\ \pi(m)}{\pi(y)}$$

Ex 5.2

1)

$$\int_A \pi_{y^{obs}|m}(y^{obs}|m)\, dy^{obs}$$

$$\overset{def}{=} \mu_{y^{obs}|m}(A)$$

$$= \mu_e(B) \qquad \text{where } B \triangleq \left\{ \frac{y^{obs}}{h(m)} \,\middle|\, y^{obs} \in A \right\}$$

$$= \int_B \pi_e\left(\frac{y^{obs}}{h(m)}\right) d\left(\frac{y^{obs}}{h(m)}\right)$$

$$= \int_B \underbrace{\pi_e\left(\frac{y^{obs}}{h(m)}\right) \frac{1}{h(m)}}_{\pi_{y^{obs}|m}} dy^{obs}$$

$$\Rightarrow \quad \pi_{y^{obs}|m} = \frac{\pi_e\left(\frac{y^{obs}}{h(m)}\right)}{h(m)}$$

(2)

$$\pi_{y_i^{obs}|\underline{m}} = \frac{\pi_e\left(\frac{y_i^{obs}}{h(\underline{m})}\right)}{h(\underline{m})}$$

$$= \frac{\pi_e\left(\frac{y_i^{obs}}{m_i}\right)}{m_i}$$

Thus,

$$\pi_{\underline{y}^{obs}|\underline{m}} = \prod_{i=1}^{n} \frac{1}{y_i^{obs}\sigma\sqrt{2\pi}} e^{-\frac{(\ln y_i^{obs} - w_0)^2}{2\sigma^2}}$$

Ex 5.3

$$\int_A \pi_{y^{obs}|m}(y^{obs}|m)\, dy^{obs}$$

$$\overset{def}{=} \mu_{y^{obs}|m}(A)$$

$$= \mu_e(B) \qquad \text{where } B \triangleq \{ g(y^{obs}, h(m)) \mid y^{obs} \in A \}$$

$$= \int_B \pi_e(g(y^{obs}, h(m))) \frac{\partial g}{\partial y^{obs}}\, dy^{obs}$$

$$\Rightarrow \quad \pi_{y^{obs}|m} = \frac{\partial g}{\partial y^{obs}} \pi_e(g(y^{obs}, h(m)))$$