Lecture12

Tuesday, October 23, 2018 9:26 AM

Basic concentration inequalities

Thm 14.1 (General Hoeffding inequality)

Assume $m_1, \dots, m_N$ are independent sub-gaussian RVs, then there exists a constant c such that the following concentration inequality (tail bound) holds:

$$\mathbb{P}\left[\left|\sum_{i=1}^{N} a_i m_i\right| > t\right] \le 2e^{-\frac{t^2}{c^2\|\vec{a}\|^2}} \quad \forall t \geq 0$$

where $\vec{a} := [a_1, \dots, a_N]^T$

Proof:

$$* \ \mathbb{P}\left[\sum a_i m_i > t\right]$$

$$\wedge \quad \text{Prop 13.2 + Chernoff}$$

$$\min_{\lambda > 0} \frac{e^{\frac{\lambda^2}{2}\sum c_i^2 a_i^2}}{e^{\lambda t}} = e^{\frac{\lambda^2}{2}\sum c_i^2 a_i^2 - \lambda t}$$

$$\| ??$$

$$e^{-\frac{t^2}{2\sum_{i=1}^{N} a_i^2 c_i^2}}$$

[illegible]

* Similarly:

$$\mathbb{P}\left[\sum a_i m_i \leq -t\right] \leq e^{-\frac{t^2}{2\sum c_i^2 a_i^2}}$$

* Using the union bound concludes the proof.