

# STATISTICAL MACHINE LEARNING

- Given a data set  $S := \{ \vec{x}^i, y^i \}_{i=1}^N$

$$\vec{x}^i \in X \subset \mathbb{R}^k$$

↳ closed and bounded  
(compact)

$$y^i \in Y \subset \mathbb{R}$$

Def (ML in this class): ML is the task of learning the unknown function  $f(\vec{x}): y^i = f(\vec{x}^i)$  given an incomplete information described by the data set  $S := \{ \vec{x}^i, y^i \}_{i=1}^N$

- Define the product space  $Z := X \times Y$ .

equipped with a "Borel" probability measure

$\pi(\vec{x}, y)$ . Let  $\pi(y|\vec{x})$  be the conditional probability measure of  $y$  given  $\vec{x}$  and

$$\pi(\vec{x}) := \int_Y \pi(\vec{x}, y) dy = \int_Y \pi(\vec{x}, dy)$$

|| . . . ||

by def: Bayes' Theorem

$$\pi(\vec{x}, y) \stackrel{??}{=} \pi(y | \vec{x}) \times \pi(\vec{x})$$

least square error (risk) w.r.t.  $\pi(\vec{x}, y)$

$$\mathcal{R}(h) := \int_{\mathcal{Z}} (h(\vec{x}) - y)^2 d\pi(\vec{x}, y)$$

where  $h(\vec{x})$  is an approximation of  $y = f(\vec{x})$