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W, H, S and C subproblems

$\| T - [C X_1 W X_2 H]_{(3)} P_{(2)} \|_F^2$
 $(C X_1 W X_2 H)_{(3)} = C_{(3)} (H \otimes W)^T$ $W = [w_1 \dots w_{r_1}]$
 $= C_{(3)} [h_1 \otimes w_1 \dots h_{r_2} \otimes w_{r_1}]^T$ $H = [h_1 \dots h_{r_2}]$
 $= C_{(3)} \begin{bmatrix} h_1^T \otimes w_1^T \\ \vdots \\ h_{r_2}^T \otimes w_{r_1}^T \end{bmatrix}$
 $\Rightarrow = \sum_t \| T^{(:,t)} - C_{(3)} \begin{bmatrix} h_1^{(i,t)} w_1^{(i,t)} \\ \vdots \\ h_{r_2}^{(i,t)} w_{r_1}^{(i,t)} \end{bmatrix} \|_2^2$
 $= \sum_t \| T^{(:,t)} - C_{(3)} \tilde{h}_{jt} \otimes \tilde{w}_{it} \|_2^2$ $W^T = [\tilde{w}_1 \dots \tilde{w}_{r_1}]$
 $\tilde{H}^T = [\tilde{h}_1 \dots \tilde{h}_{r_2}]$
 $\Leftrightarrow \sum_t \| C_{(3)} \tilde{h}_{jt} \otimes \tilde{w}_{it} \|_2^2 - 2 \langle T^{(:,t)}, C_{(3)} \tilde{h}_{jt} \otimes \tilde{w}_{it} \rangle$
 $= \sum_t \| C X_1 \tilde{w}_{it}^T X_2 \tilde{h}_{jt}^T \|_2^2 - 2 \langle T^{(:,t)}, (C X_1 \tilde{w}_{it}^T X_2 \tilde{h}_{jt}^T)_{(3)} \rangle$
 $= \sum_t \| \tilde{w}_{it}^T (C X_2 \tilde{h}_{jt}^T)_{(1)} \|_2^2 - 2 \langle (T^{(:,t)})^T, \tilde{w}_{it}^T (C X_2 \tilde{h}_{jt}^T)_{(1)} \rangle$
 $\xrightarrow{\frac{\partial}{\partial W}} 2 \sum_t \tilde{w}_{it}^T (C X_2 \tilde{h}_{jt}^T)_{(1)} (C X_2 \tilde{h}_{jt}^T)_{(1)}^T - (T^{(:,t)})^T (C X_2 \tilde{h}_{jt}^T)_{(1)}^T$

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$$\|T - [C X_1 W X_2 H]_{(2)}] P_n\|_F^2$$

$$= \sum_t \|T^{(:,t)} - C_{(3)} \begin{bmatrix} h_{1,t}^{(i)} w_{1,t}^{(i)} \\ \vdots \\ h_{r_t,t}^{(i)} w_{r_t,t}^{(i)} \end{bmatrix}\|_2^2 \quad W^T = [\tilde{w}_1 \cdots \tilde{w}_{N_r}]$$

$$= \sum_t \|T^{(:,t)} - C_{(3)} \tilde{h}_{j_t} \otimes \tilde{w}_{i_t}\|_2^2 \quad H^T = [\tilde{h}_1 \cdots \tilde{h}_{N_r}]$$

$$\Leftrightarrow \sum_t \|C X_1 \tilde{W}_{i_t}^T \times_2 \tilde{h}_{j_t}^T\|_F^2 - 2 \langle (T^{(:,t)})^T, (C X_1 \tilde{W}_{i_t}^T \times_2 \tilde{h}_{j_t}^T) \rangle$$

$$= \sum_t \|\tilde{h}_{j_t}^T (C X_1 \tilde{W}_{i_t})_{(2)}\|_F^2 - 2 \langle (T^{(:,t)})^T, \tilde{h}_{j_t}^T (C X_1 \tilde{W}_{i_t})_{(2)} \rangle$$

$$\stackrel{2}{\Rightarrow} 2 \sum_t \tilde{h}_{j_t}^T (C X_1 \tilde{W}_{i_t})_{(2)} (C X_1 \tilde{W}_{i_t})_{(2)}^T - (T^{(:,t)})^T (C X_1 \tilde{W}_{i_t})_{(2)}$$

Thus for $j \in \{j_t\}$

$$\frac{\partial}{\partial t \tilde{h}_{j_t}^{(i)}} = 2 \sum_{\{t | j_t = j\}} \left[H^{(j,:)} (C X_1 \tilde{W}_{i_t})_{(2)} (C X_1 \tilde{W}_{i_t})_{(2)}^T - (T^{(:,t)})^T (C X_1 \tilde{W}_{i_t})_{(2)} \right]$$

$C, C_1, C_2, C_3, V_1, V_2, V_3$

$$= \lambda \|C\|_1 + \mu \|C - C^{pre}\|_F^2 + \|y - C_1 \times W^* \times_2 H^* \times_3 S\|_F^2$$

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$$= \sum_t \|\tilde{w}_{it}^T (x_2 \tilde{h}_{jt}^T)_{(1)}\|_2^2 - 2 \langle (T^{(:,t)})^T, \tilde{w}_{it}^T (x_2 \tilde{h}_{jt}^T)_{(1)} \rangle$$

$$\xrightarrow{\frac{\partial}{\partial w}} 2 \sum_t \tilde{w}_{it}^T (x_2 \tilde{h}_{jt}^T) (x_2 \tilde{h}_{jt}^T)^T - (T^{(:,t)})^T (x_2 \tilde{h}_{jt}^T)^T$$

Thus for $i \in \{1, \dots, n\} \setminus \{i_t\}$,

$$\frac{\partial}{\partial w^{(i)}} = 2 \sum_{\{t | i \neq i_t\}} \left[w^{(i,:)} (x_2 \tilde{h}_{jt}^T) (x_2 \tilde{h}_{jt}^T)^T - (T^{(:,t)})^T (x_2 \tilde{h}_{jt}^T)^T \right].$$

Combining with other gradients.

$$A_w A_w^T w^T P_i^T P_i + (B_w B_w^T + nI) w^T$$

$$= A_w Y_{(1)}^T P_i + B_w Z_{(1)}^T + \beta W_{pre}^T$$

$$(P_i^T P_i \otimes A_w A_w^T) \text{vec}(W^T) + [I \otimes (B_w B_w^T + nI)] \text{vec}(W^T)$$

$$= \text{vec}(A_w Y_{(1)}^T P_i + B_w Z_{(1)}^T + \beta W_{pre}^T)$$

For $i \in \{1, \dots, n\} \setminus \{i_t\}$,

$$LHS + \sum_{\{t | i \neq i_t\}} (x_2 \tilde{h}_{jt}^T) (x_2 \tilde{h}_{jt}^T)^T \tilde{w}_i$$

$$RHS + \sum_{\{t | i \neq i_t\}} (x_2 \tilde{h}_{jt}^T)^T T^{(:,t)}$$

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Thus for $j \in \{j_t\}$

$$\frac{\partial}{\partial C_{(j,t)}} = 2 \sum_{\{t | j_t = j\}} \left[H^{(j,t)} (C_{X_1} \tilde{w}_t^T)_{(2)} (C_{X_1} \tilde{w}_t^T)_{(2)}^T - (T^{(j,t)})^T (C_{X_1} \tilde{w}_t^T)_{(2)} \right]$$

$C \in C_1, C_2, C_3, V_1, V_2, V_3$

$$\begin{aligned}
 &= \lambda_1 \|C\|_1 + \mu \|C - C_{\text{pre}}\|_F^2 \\
 &+ \|y - C_1 X_1 W^* X_2 H^* X_3 S^*\|_F^2 \\
 &+ \beta \|C - C_1 - V_1\|_F^2 \\
 &+ \|S - C_2 X_1 W X_2 H X_3 S^*\|_F^2 \\
 &+ \beta \|C - C_2 - V_2\|_F^2 \\
 &+ \lambda_2 \|T - [C_3 X_1 W X_2 H]_{(3)}\|_F^2 \\
 &+ \beta \|C - C_3 - V_3\|_F^2
 \end{aligned}$$

1) Update C

$$\begin{aligned}
 C = \arg \min_C & \lambda_1 \|C\|_1 + \mu \|C - C_{\text{pre}}\|_F^2 \\
 &+ \beta \|C - C_1 - V_1\|_F^2 + \beta \|C - C_2 - V_2\|_F^2 + \beta \|C - C_3 - V_3\|_F^2
 \end{aligned}$$

$$\begin{aligned}
 &= \arg \min_C \lambda_1 \|C\|_1 + \mu \|C\|_F^2 - 2\mu \langle C, C_{\text{pre}} \rangle \\
 &+ \beta \|C\|_F^2 - 2\beta \langle C, C_1 + V_1 \rangle
 \end{aligned}$$



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$$= \underset{C}{\text{arg min}} \lambda_1 \|C\|_1 + \mu \|C\|_F^2 - 2\mu \langle C, C_{\text{pre}} \rangle$$

$$+ \beta \|C\|_F^2 - 2\beta \langle C, C_1 + V_1 \rangle$$

$$+ \beta \|C\|_F^2 - 2\beta \langle C, C_2 + V_2 \rangle$$

$$+ \beta \|C\|_F^2 - 2\beta \langle C, C_3 + V_3 \rangle$$

$$= \underset{C}{\text{arg min}} \lambda_1 \|C\|_1 +$$

$$(3\beta + \mu) \|C\|_F^2 - \frac{\mu}{3\beta + \mu} \langle C_{\text{pre}} - \frac{\beta}{3\beta + \mu} (C_1 + V_1) - \frac{\beta}{3\beta + \mu} (C_2 + V_2) - \frac{\beta}{3\beta + \mu} (C_3 + V_3) \rangle_F^2$$

$$C = \text{soft} \left[\frac{\mu(C_1 + V_1 + C_2 + V_2 + C_3 + V_3) + \mu C_{\text{pre}}}{3\beta + \mu}, \frac{\lambda_1}{6\beta + 2\mu} \right].$$

2) update C_1

$$C_1 = \underset{C_1}{\text{arg min}} \|y - C_1 \mathbf{x}_1^* \mathbf{x}_2 \mathbf{x}_2^* \mathbf{x}_3 \mathbf{x}_3^* \mathbf{x}_3\|_F^2 + \beta \|C_1 + V_1 - C\|_F^2$$

3) update C_2

$$C_2 = \underset{C_2}{\text{arg min}} \|\mathbf{z} - C_2 \mathbf{x}_1 \mathbf{w} \mathbf{x}_2 \mathbf{w}^* \mathbf{x}_3 \mathbf{x}_3^* \mathbf{x}_3\|_F^2 + \beta \|C_2 + V_2 - C\|_F^2$$

4) update C_3

$$C_3 = \underset{C_3}{\text{arg min}} \lambda_2 \|\mathbf{T} - [(\mathbf{C}_3 \mathbf{x}_1 \mathbf{w} \mathbf{x}_2 \mathbf{w}^*)_{(3,)}] \mathbf{P}_{(2)}\|_F^2 + \beta \|C_3 + V_3 - C\|_F^2$$

$$\mathbf{x}_2 \sum_t \|\mathbf{T}^t - (\mathbf{C}_3)_{(3,)} \tilde{\mathbf{h}}_{jt}^* \otimes \tilde{\mathbf{w}}_{jt}^*\|_2^2 + \beta \|C_3 + V_3 - C\|_F^2$$



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$$\lambda_2 \sum_t \| T^{(i,t)} (C_3)_{(3)} \cdot \tilde{h}_{jt} \otimes \tilde{w}_{it} \|_2^2 + \beta \| C_3 + V_3 - C \|_F^2$$

$$\Rightarrow \lambda_2 \sum_t [(C_3)_{(3)} \tilde{h}_{jt} \otimes \tilde{w}_{it} - T^{(i,t)}] [\tilde{h}_{jt} \otimes \tilde{w}_{it}]^\top + \beta [(C_3)_{(3)} + (V_3)_{(3)} - C_{(3)}] = 0$$

$$\Rightarrow \lambda_2 (C_3)_{(3)} \sum_t (\tilde{h}_{jt} \otimes \tilde{w}_{it}) (\tilde{h}_{jt} \otimes \tilde{w}_{it})^\top + \beta (C_3)_{(3)}$$

$$= \lambda_2 \sum_t T^{(i,t)} (\tilde{h}_{jt} \otimes \tilde{w}_{it})^\top + \beta [C_{(3)} - (V_3)_{(3)}]$$

$$\Rightarrow (C_3)_{(3)} \left[\lambda_2 \sum_t (\tilde{h}_{jt} \otimes \tilde{w}_{it}) (\tilde{h}_{jt} \otimes \tilde{w}_{it})^\top + \beta I \right]$$

$$= \lambda_2 \sum_t T^{(i,t)} (\tilde{h}_{jt} \otimes \tilde{w}_{it})^\top + \beta [C_{(3)} - (V_3)_{(3)}]$$

$$\Rightarrow (C_3)_{(3)} = \left\{ \lambda_2 \sum_t T^{(i,t)} (\tilde{h}_{jt} \otimes \tilde{w}_{it})^\top + \beta [C_{(3)} - (V_3)_{(3)}] \right\} \cdot$$

$$[\lambda_2 \sum_t (\tilde{h}_{jt} \otimes \tilde{w}_{it}) (\tilde{h}_{jt} \otimes \tilde{w}_{it})^\top + \beta I]^{-1}$$

5) update

$$V_1 \leftarrow V_1 + C_1 - C$$

$$V_2 \leftarrow V_2 + C_2 - C$$

$$V_3 \leftarrow V_3 + C_3 - C$$

