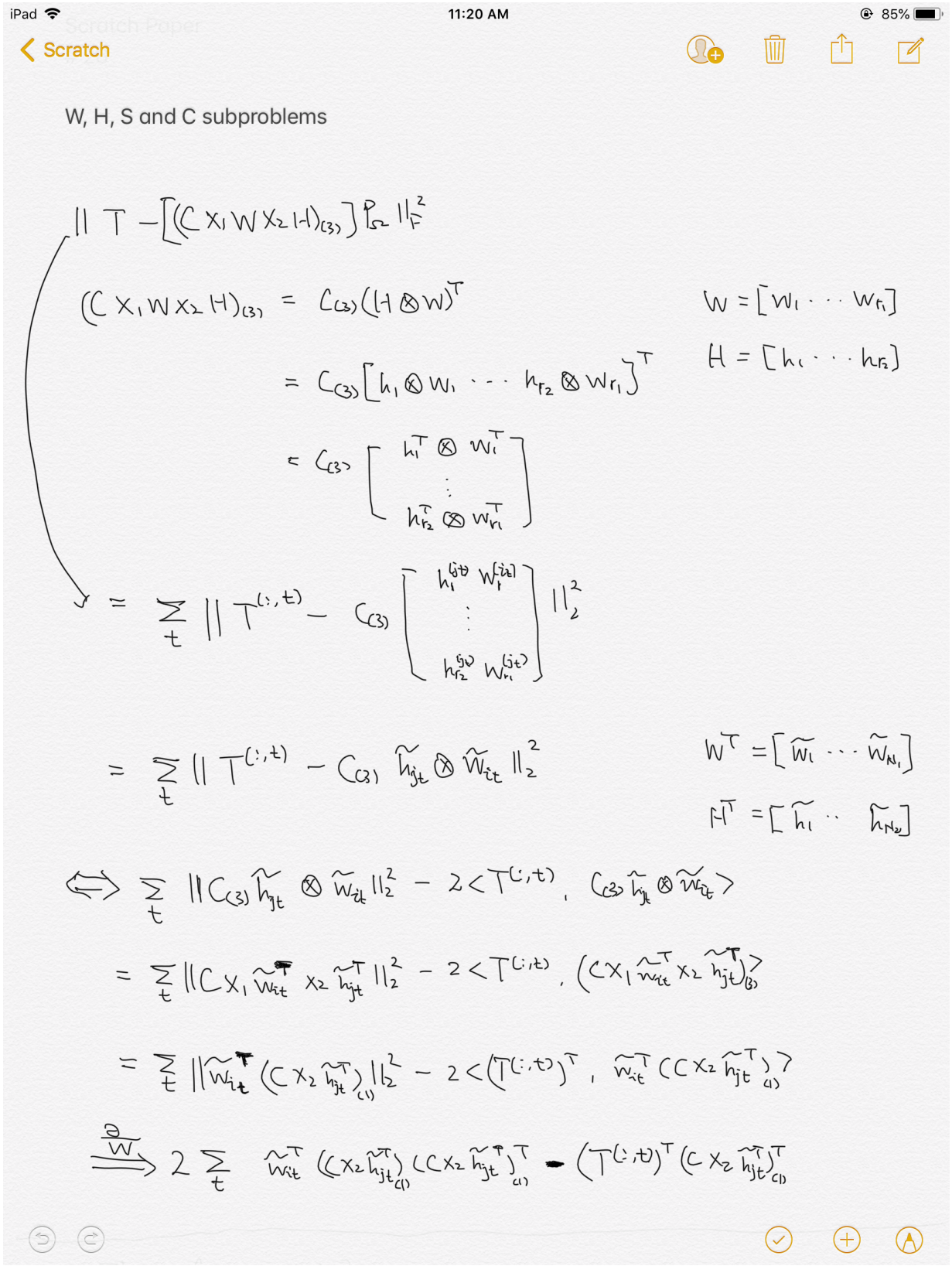
Saved Photos



$$\|T - [(C X_1 W X_2 H)_{(3)}] P_2\|_F^2$$

$$= \sum_t \| T^{(:,:,t)} - C_{(3)} \begin{bmatrix} h_1^{(j_1,t)} w_1^{(i_1,t)} \\ \vdots \\ h_{r_2}^{(j_2,t)} w_{r_1}^{(i_t,t)} \end{bmatrix} \|_2^2 \qquad W^T = [\tilde{w}_1 \cdots \tilde{w}_{N_1}]$$

$$H^T = [\tilde{h}_1 \cdots \tilde{h}_{N_2}]$$

$$= \sum_t \| T^{(:,:,t)} - C_{(3)} \tilde{h}_{j_t} \otimes \tilde{w}_{i_t} \|_2^2$$

$$\Leftrightarrow \sum_t \| C X_1 \tilde{w}_{i_t}^T X_2 \tilde{h}_{j_t}^T \|_F^2 - 2 \langle (T^{(:,:,t)})^T, C X_1 \tilde{w}_{i_t}^T X_2 \tilde{h}_{j_t}^T \rangle$$

$$= \sum_t \| \tilde{h}_{j_t}^T (C X_1 \tilde{w}_{i_t}^T)_{(2)} \|_F^2 - 2 \langle (T^{(:,:,t)})^T, \tilde{h}_{j_t}^T (C X_1 \tilde{w}_{i_t}^T)_{(2)} \rangle$$

$$\xrightarrow{\frac{\partial}{\partial H}} 2 \sum_t \tilde{h}_{j_t}^T (C X_1 \tilde{w}_{i_t}^T)_{(2)} (C X_1 \tilde{w}_{i_t}^T)_{(2)}^T - (T^{(:,:,t)})^T (C X_1 \tilde{w}_{i_t}^T)_{(2)}$$

Thus for $j \in \{j_t\}$

$$\frac{\partial}{\partial H^{(j,:)}} = 2 \sum_{\{t | j = j_t\}} \left[H^{(j,:)} (C X_1 \tilde{w}_{i_t}^T)_{(2)} (C X_1 \tilde{w}_{i_t}^T)_{(2)}^T - (T^{(:,:,t)})^T (C X_1 \tilde{w}_{i_t}^T)_{(2)} \right]$$

$$L(C, C_1, C_2, C_3, V_1, V_2, V_3)$$

$$= \lambda \|C\|_1 + \mu \|C - C^{pre}\|_F^2$$

$$+ \|y - C_1 \times W^* \times_2 H^* \times_3 S\|_F^2$$

$$= \sum_t \left\| \widetilde{W}_{i_t}^T \left(C X_2 \widetilde{h}_{jt}^T \right)_{(1)} \right\|_2^2 - 2 \left\langle \left(T^{(:,t)} \right)^T, \widetilde{W}_{:t}^T \left(C X_2 \widetilde{h}_{jt}^{\backslash 1} \right)_{(1)} \right\rangle$$

$$\xrightarrow{\frac{\partial}{W}} 2 \sum_t \widetilde{W}_{:t}^T \left(X_2 \widetilde{h}_{jt}^T \right)_{(1)} \left(C X_2 \widetilde{h}_{jt}^T \right)_{(1)}^T - \left(T^{(:,t)} \right)^T \left(C X_2 \widetilde{h}_{jt}^T \right)_{(1)}^T$$

Thus for $i \in \{i_t\}$.

$$\frac{\partial}{W^{(i,:)}} = 2 \sum_{\{t \mid i = i_t\}} \left[W^{(i,:)} \left(C X_2 \widetilde{h}_{jt}^T \right)_{(1)} \left(C X_2 \widetilde{h}_{je}^T \right)_{(1)}^T - \left(T^{(:,t)} \right)^T \left(C X_2 \widetilde{h}_{jt}^T \right)_{(1)} \right].$$

Combining with other gradients.

$$A_W A_W^T W^T P_1^T P_1 + \left(B_W B_W^T + \eta I \right) W^T$$

$$= A_W Y_{(1)}^T P_1 + B_W Z_{(1)}^T + \beta W_{pre}^T$$

$$\left(P_1^T P_1 \otimes A_W A_W^T \right) \mathrm{vec}(W^T) + \left[I \otimes \left(B_W B_W^T + \eta I \right) \right] \mathrm{vec}(W^T)$$

$$= \mathrm{vec}\left(A_W Y_{(1)}^T P_1 + B_W Z_{(1)}^T + \beta W_{pre}^T \right)$$

For $i \in \{1, \cdots, N_1\} \cap \{i_t\}$.

$$\text{LHS} + \sum_{\{t \mid i = i_t\}} \left(C X_2 \widetilde{h}_{jt} \right)_{(1)} \left(C X_2 \widetilde{h}_{jt} \right)_{(1)}^T \widetilde{W}_i$$

$$\text{RHS} + \sum_{\{t \mid i = i_t\}} \left(C X_2 \widetilde{h}_{jt}^T \right)_{(1)} T^{(:,t)}$$

Thus for $j \in \{j_t\}$

$$\frac{\partial}{\partial t^{(j,:)}} = 2 \sum_{\{t | j = j_t\}} \left[H^{(j,:)} (C X_1 \tilde{W}_{(t)}^T)_{(2)} (C X_1 \tilde{W}_{(t)}^T)_{(2)}^T - (T^{(:,t)})^T (C X_1 \tilde{W}_{(t)}^T)_{(2)} \right]$$

$$L(C, C_1, C_2, C_3, V_1, V_2, V_3)$$

$$= \lambda_1 \|C\|_1 + \mu \|C - C^{pre}\|_F^2$$

$$+ \|Y - C_1 \times W^* X_2 H^* X_3 S\|_F^2$$

$$+ \beta \|C - C_1 - V_1\|_F^2$$

$$+ \|Z - C_2 X_1 W X_2 H X_3 S^*\|_F^2$$

$$+ \beta \|C - C_2 - V_2\|_F^2$$

$$+ \lambda_2 \|T - [(C_3 X_1 W X_2 (H)_{(3)}] P_n\|_F^2$$

$$+ \beta \|C - C_3 - V_3\|_F^2$$

1) update C

$$C = \underset{C}{\operatorname{argmin}} \ \lambda_1 \|C\|_1 + \mu \|C - C_{pre}\|_F^2 + \beta \|C - C_1 - V_1\|_F^2 + \beta \|C - C_2 - V_2\|_F^2 + \beta \|C - C_3 - V_3\|_F^2$$

$$= \underset{C}{\operatorname{argmin}} \ \lambda_1 \|C\|_1 + \mu \|C\|_F^2 - 2\mu \langle C, C_{pre} \rangle + \beta \|C\|_F^2 - 2\beta \langle C, C_1 + V_1 \rangle$$

$$= \arg\min_{C} \lambda_1 \|C\|_1 + \mu \|C\|_F^2 - 2\mu \langle C, C_{pre} \rangle + \beta \|C\|_F^2 - 2\beta \langle C, C_1 + V_1 \rangle + \beta \|C\|_F^2 - 2\beta \langle C, C_2 + V_2 \rangle + \beta \|C\|_F^2 - 2\beta \langle C, C_3 + V_3 \rangle$$

$$= \arg\min_{C} \lambda_1 \|C\|_1 + (3\beta + \mu) \left\| C - \frac{\mu}{3\beta+\mu} C_{pre} - \frac{\beta}{3\beta+\mu}(C_1 + V_1) - \frac{\beta}{3\beta+\mu}(C_2 + V_2) - \frac{\beta}{3\beta+\mu}(C_3 + V_3) \right\|_F^2$$

$$C = \text{soft}\left[\frac{\mu(C_1 + V_1 + C_2 + V_2 + C_3 + V_3) + \mu C_{pre}}{3\beta + \mu}, \frac{\lambda_1}{6\beta + 2\mu} \right].$$

2) update C_1

$$C_1 = \arg\min_{C_1} \|Y - C_1 X_1 W^* X_2 H^* X_3 S\|_F^2 + \beta \|C_1 + V_1 - C\|_F^2$$

3) update C_2

$$C_2 = \arg\min_{C_2} \|\Sigma - C_2 X_1 W X_2 H X_3 S^*\|_F^2 + \beta \|C_2 + V_2 - C\|_F^2$$

4) update C_3

$$C_3 = \arg\min_{C_3} \lambda_2 \|T - [(C_3 X_1 W X_2 H)_{(3)}] P_{S2}\|_F^2 + \beta \|C_3 + V_3 - C\|_F^2$$

$$\lambda_2 \sum_t \|T^{(t)} - (C_3)_{(3)} \cdot \tilde{h}_{jt} \otimes \tilde{w}_{it}\|_2^2 + \beta \|C_3 + V_3 - C\|_F^2$$

C_3

$$\lambda_2 \sum_t \| T^{(i,t)} - (C_3)_{(3)} \cdot \tilde{h}_{jt} \otimes \tilde{w}_{it} \|_2^2 + \beta \| C_3 + V_3 - C \|_F^2$$

$$\Rightarrow \lambda_2 \sum_t \left[(C_3)_{(3)} \tilde{h}_t \otimes \tilde{w}_{it} - T^{(:,t)} \right] \left[\tilde{h}_{jt} \otimes \tilde{w}_{it} \right]^T + \beta \left[(C_3)_{(3)} + (V_3)_{(3)} - C_{(3)} \right] = 0$$

$$\Rightarrow \lambda_2 (C_3)_{(3)} \sum_t (\tilde{h}_{jt} \otimes \tilde{w}_{it})(\tilde{h}_{jt} \otimes \tilde{w}_{it})^T + \beta (C_3)_{(3)}$$

$$= \lambda_2 \sum_t T^{(:,t)} (\tilde{h}_{jt} \otimes \tilde{w}_{it})^T + \beta \left[C_{(3)} - (V_3)_{(3)} \right]$$

$$\Rightarrow (C_3)_{(3)} \left[\lambda_2 \sum_t (\tilde{h}_{jt} \otimes \tilde{w}_{it})(\tilde{h}_{jt} \otimes \tilde{w}_{it})^T + \beta I \right]$$

$$= \lambda_2 \sum_t T^{(:,t)} (\tilde{h}_{jt} \otimes \tilde{w}_{it})^T + \beta \left[C_{(3)} - (V_3)_{(3)} \right]$$

$$\Rightarrow (C_3)_{(3)} = \left\{ \lambda_2 \sum_t T^{(:,t)} (\tilde{h}_{jt} \otimes \tilde{w}_{it})^T + \beta \left[C_{(3)} - (V_3)_{(3)} \right] \right\} \cdot$$

$$\left[\lambda_2 \sum_t (\tilde{h}_{jt} \otimes \tilde{w}_{it})(\tilde{h}_{jt} \otimes \tilde{w}_{it})^T + \beta I \right]^{-1}$$

5) update

$$V_1 \leftarrow V_1 + C_1 - C$$

$$V_2 \leftarrow V_2 + C_2 - C$$

$$V_3 \leftarrow V_3 + C_3 - C$$