581336-0 **Theory of computation** Exercises 7

Fall 2002

1. Prove that the class NP is closed under union and intersection. It is not known to be closed under the complement. What difficulty does one face when trying to prove this property?

2. Because the class NP is not known to be closed under complementation, it is interesting to examine its dual class

$$\text{co-NP} = \{ \overline{A} \mid A \in \text{NP} \}.$$

   Prove the following properties of the class co-NP.

   (i) $\text{P} \subseteq \text{NP} \cap \text{co-NP}$;

   (ii) $\text{NP} \cup \text{co-NP} \subseteq \text{PSPACE}$;

   (iii) if $\text{NP} \neq \text{co-NP}$, then $\text{P} \neq \text{NP}$ and $\text{NP} \neq \text{PSPACE}$.

   (*Hint*: complementation.)

3. Why does the determinization construction of nondeterministic Turing machines (in the proof of Theorem 4.3) not suffice to prove that P=NP?

4. Prove the following claims (Lemmas 7.13 and 7.14):

   (i) If A is an NP-complete language and $A \in \text{P}$, then P=NP.

   (ii) If A is an NP-complete language, $B \in \text{NP}$ and $A \leq_m^p B$, then also B is NP-complete.

   (*Hint*: Apply Lemma 7.12.)