Lecture 1

Prerequisites:

* Good knowledge of basic probability & statistics (undergrad)
* Basic knowledge of differential equations & control systems

What are time series

- ordered sequence of numerical data
- one realization $\tilde{X}_t$ of a random process X_t, $t \in \mathbb{Z}$

Description of a time-series will boil down to characterizing interdependencies between X_t-s, based on the realization of that process (i.e. based on the time-series).

Basically, describe joint pdf-s

$$f_{X_{t_1}, X_{t_2}, \ldots, X_{t_n}}(x_{t_1}, x_{t_2}, \ldots, x_{t_n})$$

Very tough problem to do in general case. We'll focus on <u>stationary time series</u>, where this problem will be somewhat easier to deal with.

Wide sense stationarity

Def: X_t, $t \in \mathbb{Z}$ is wide sense stationary iff

i) $E[X_t] = m$

ii) $E[X_t \cdot X_{t+\ell}] = E[X_{t+\tau}\, X_{t+\ell+\tau}] = f(\ell)$

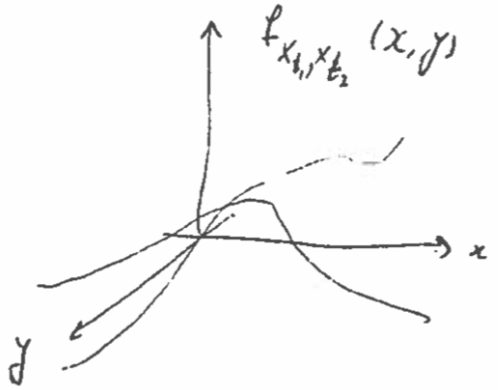
Moments of order up to 2 are origin independent

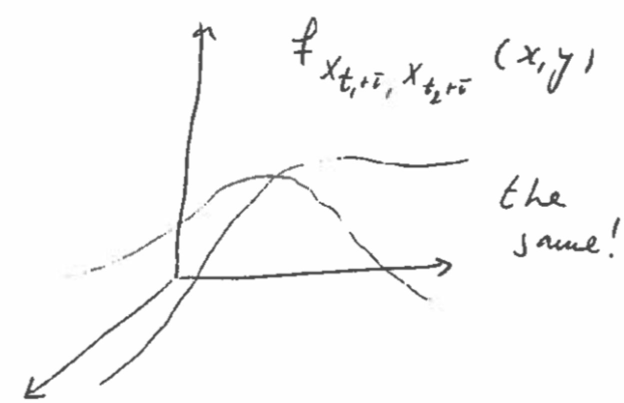
Def: X_t, $t \in \mathbb{Z}$ is strictly stationary iff for any

$X_{t_1}, X_{t_2}, \ldots, X_{t_n}$ and any $\tau \in \mathbb{Z}$

$$f_{X_{t_1}, X_{t_2}, \ldots, X_{t_n}}(x_1, x_2, \ldots, x_n) = f_{X_{t_1+\tau}, X_{t_2+\tau}, \ldots, X_{t_n+\tau}}(x_1, x_2, \ldots, x_n)$$

Joint pdf-s (all of them) are always origin independent





Thm. If X_t, $t \in \mathbb{Z}$ is a Gaussian process, then wide sense stationarity & strict stationarity are equivalent!