Chapter 11 Vectorial Time-Series Models

1. Introduction

- Single series vs. multiple series
  In previous chapters, data are considered as a realization of a stationary stochastic system. In this chapter, we consider data as a realization of a vector stationary stochastic system, i.e., multiple series of data.
- Most prominent applications involving multiple series of data arose in control systems.
- Develop discrete models and strategies for optimal forecasting and control in the sense of minimum mean squared error.
- Control system formulation:
  - classical control theory, using block diagrams, transfer function, and emphasized on the systems of the single-input-single-output variety.
  - modern control theory, employing state space technique, well suited for multiple-input-multiple-output systems.
- The emphasis of this chapter will be placed on single-input-single-output as well as multiple-input-single-output cases.

2. Transfer function

Papermaking process diagram:

Output variable — the paper quality in terms of basis weight.

Input variable (s) — stock consistency, stock flow, headbox turbulence, slice opening, machine speed, etc., dominating factor, gate opening denoted by X_{1t}.

Control system: Adjust gate opening X_{1t} to maintain the output basis weight at the given target value. X_{2t} is the deviation of the basis weight at time t from the target value.

If the output depends on its past value, the simplest model will be

$$X_{2t} = 0.7\, X_{2t-1} + a_t$$

If the output also depends on the input,

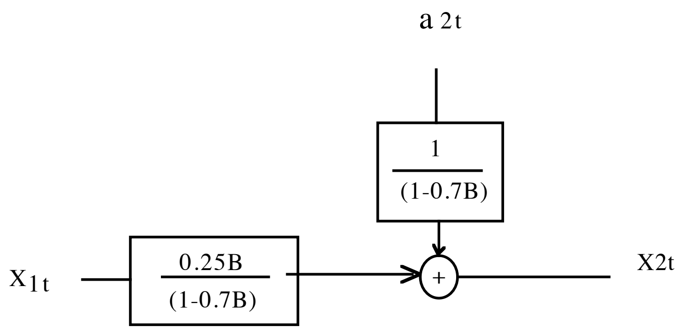
$$X_{2t} = 0.7\, X_{2t-1} + 0.25 X_{1t-1} + a_{2t}$$

The output basis weight can be decomposed into 3 parts.

$$(1-0.7B)X_{2t} = 0.25BX_{1t} + a_{2t}$$

that is,

$$X_{2t} = \frac{0.25B}{(1-0.7B)} X_{1t} + \frac{1}{(1-0.7B)} a_{2t}$$



Transfer function vs state variable approach:

Transfer function - intuitively appealing

State variable - theoretical and computational advantages.

If the input is a stationary stochastic process or time series, the input-output model can be represented by a vector model. Ex, ARV(1)

$$X_{1t} = 0.8 X_{1t-1} + a_{1t}$$

$$X_{2t} = 0.25 X_{1t-1} + 0.7 X_{2t-1} + a_{2t}$$

$$\begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix} \qquad \mathbf{X_t = \phi X_{t-1} + a_t}$$

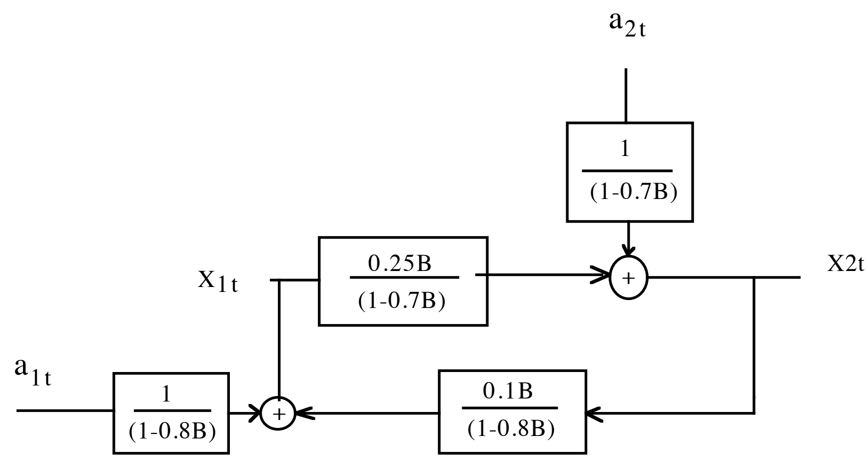
If there is feedback from output to input, e.g.,

$$X_{1t} = 0.8 X_{1t-1} + 0.1 X_{2t-1} + a_{1t}$$

$$X_{2t} = 0.25 X_{1t-1} + 0.7 X_{2t-1} + a_{2t}$$

$$X_{1t} = \frac{0.1B}{(1-0.8B)} X_{2t} + \frac{1}{(1-0.8B)} a_{1t}$$

$$X_{2t} = \frac{0.25B}{(1-0.7B)} X_{1t} + \frac{1}{(1-0.7B)} a_{2t}$$



a2t
1
(1-0.7B)
X1t
0.25B
(1-0.7B)
+
X2t
a1t
1
(1-0.8B)
+
0.1B
(1-0.8B)