ME384Q.3 / ORI 390R.3: Time-Series Analysis

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Homework 6 - Solutions

Problem 11.1

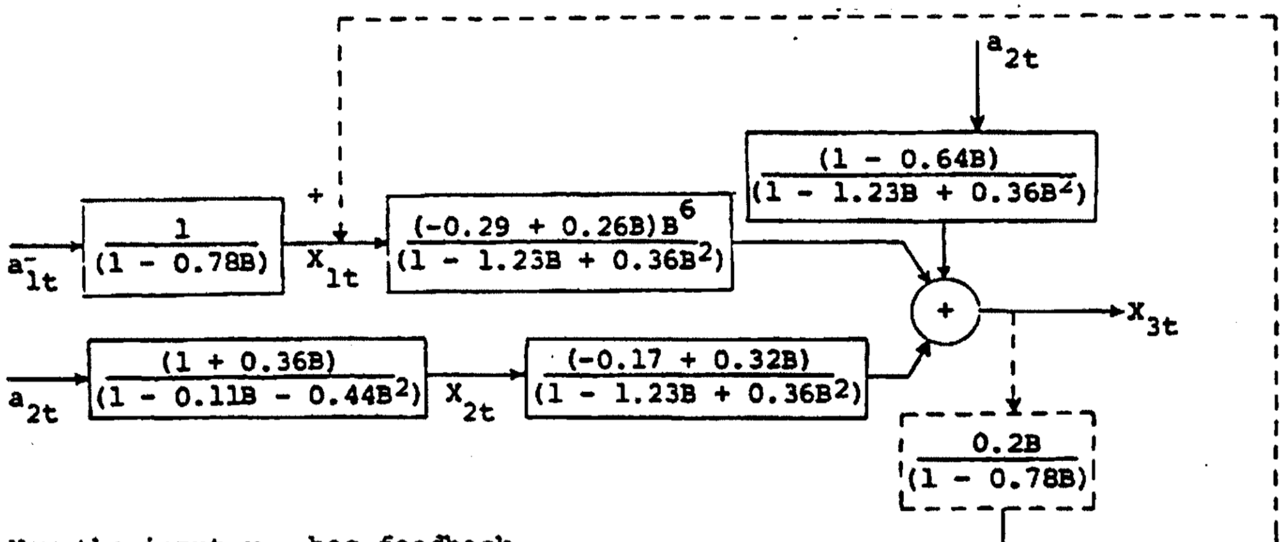
The transfer function forms of the models are

$$X_{1t} = \frac{1}{(1 - 0.78B)} a_{1t}$$

$$X_{2t} = \frac{(1 + 0.36B)}{(1 - 0.11B - 0.44B^2)} a_{2t}$$

a) $$X_{3t} = \frac{(-0.29 + 0.26B)B^6}{(1 - 1.23B + 0.36B^2)} X_{1t} + \frac{(-0.17 + 0.32B)B}{(1 - 1.23B + 0.36B^2)} X_{2t} + \frac{(1 - 0.64B)}{(1 - 1.23B + 0.36B^2)} a_{2t}$$

b)



c) Now the input X_{1t} has feedback

$$X_{1t} = \frac{0.2B}{(1 - 0.78B)} X_{3t} + \frac{1}{(1 - 0.78B)} a_{1t}$$

The change is shown by dotted line in the block diagram.

Problem 11.2

Considering a first order bivariate example in transfer function form, it is seen that for nondiagonal moving average matrix the models are

$$X_{1t} = \frac{\phi_{121}B}{(1 - \phi_{111}B)} X_{2t} + \frac{(1 - \theta_{111}B)}{(1 - \phi_{111}B)} a_{1t} + \frac{(1 - \theta_{121}B)}{(1 - \phi_{111}B)} a_{2t}$$

$$X_{2t} = \frac{\phi_{211}B}{(1 - {}_{221}B)} X_{1t} + \frac{(1 - \theta_{221}B)}{(1 - \phi_{221}B)} a_{2t} + \frac{(1 - \phi_{211}B)}{(1 - \phi_{211}B)} a_{1t}$$

Thus the noises in both the input X_{1t} and the output X_{2t} contain a_{1t} as well as a_{2t} and therefore are cross-correlated. When $\theta_{121} = \theta_{211} = 0$, then the input noise is derived only from a_{1t} and the output noise is derived only from a_{2t}. Since a_{1t} and a_{2t} are uncrosscorrelated at nonzero lags and since their zero lag correlation can be removed by including ϕ_{120} term in the transfer function, diagonal $\underset{\sim}{\theta}$ matrix essentially implies that the two noises are uncorrelated or independent. In particular, when there is no feedback, i.e. when $\phi_{121} = \phi_{120} = 0$, then the assumption of diagonal $\underset{\sim}{\theta}$ implies that the input is uncorrelated with or independent of the output noise.

Problem 11.7

Since $L = 2$ the control equation is obtained by

$$\hat{X}_{2t}(2) = 0.7X_{1t} - 0.25X_{1t-1} + 0.9\hat{X}_{2t}(1) - 0.2X_{2t} = 0$$

To find $\hat{X}_{2t}(1)$ we need optimally controlled output, which has (since there is only one input) an MA(1) model: $X_{2t} = (1 + G_1 \; B)a_{2t}$

where G_1 is the same as G_1 of the ARMA(2,1) model

$$(1 - 0.9B + 0.2B^2)X_t = (1 + 0.3B)a_t$$

i.e. $\quad G_1 = \phi_1 - \theta_1 = 0.9 - (-0.3) = 1.2$

Also $\quad G_2 = \phi_1 G_1 + \phi_2 = 0.9(1.2) - 0.2 = 0.88$

$$G_3 = \phi_1 G_2 + \phi_2 G_1 = 0.9(0.88) - 0.2(1.2) = 0.552$$

b) Thus the optimally controlled output is given by

$$X_{2t} = (1 + 1.2B)a_{2t}$$

$$Var[X_{2t}] = (1 + 1.2^2)\gamma_{a22} = 2.44\gamma_{a22}$$

a) Since the model is noninvertible, a_{2t} cannot be written in terms of X_{2t} alone. Hence, using

$$\hat{X}_{2t}(1) = 1.2a_{2t}$$

in the control equation, it takes the form

$$0.7X_{1t} - 0.25X_{1t-1} = 0.2X_{2t} - 0.9x1.2a_{2t}$$

or

$$X_{1t} = 0.357X_{1t-1} + 0.286X_{2t} - 1.543a_{2t}$$

$$a_{2t} = X_{2t} - 1.2a_{2t-1}$$

c) When $L = 4$, $X_{2t} = (1 + G_1 \; B + G_2 B^2 + G_3 B^3)a_{2t}$

$$Var\;[X_{2t}] = (1^2 + 1.2^2 + 0.88^2 + 0.552^2) = 3.519\;\gamma_{a22}$$

$$\%\ \text{Increase} = \frac{3.519 - 2.44}{2.44} \times 100 = 44\%$$