Optimal Stochastic Regulation (control that keeps a constant output) Based on ARMAV Models

Minimum mean squared error control strategy

Goal: <mark>to keep the output at the target value, which may be taken as zero.</mark>

Problems: due to noise and disturbance, it is very difficult to maintain the output exactly at zero level; to keep the deviation or errors from the zero target values as small as possible.

Smallness: for random variables with zero mean, the measure of their smallness is given by their variance.

Optimal control: adjustments in the manipulable input values that yield minimum variance (minimum mean squared error) of the output.

MMSE control strategy: to adjust the input X_{1t} such that the forecast of X_{2t+L} made at time t is zero (or target value). [the earliest time at which the input can affect the output is t+L].

$$\hat{X}_{2t}(L) = 0$$

$$X_{2t+L} = \hat{X}_{2t}(L) + e_{2t}(L)$$

After implementing the control equation,

$$X_{2t+L} = e_{2t}(L)$$

This optimally controlled output, or output error or deviation from the mean, is the L-step ahead forecasting error, which is a MA(L-1) model.

Example 1 — First order model with lag 1

a) model before control

$$X_{2t} = 0.25X_{1t-1} + 0.7X_{2t-1} + a_{2t} \quad \gamma_{a22} = 0.0062$$

for papermaking process data

b) control equation

$$\hat{X}_{2t}(1) = 0.25X_{1t} + 0.7X_{2t} = 0 \quad \rightarrow \quad 0.25X_{1t} = -0.7X_{2t} \quad \text{or} \quad X_{1t} = -2.8X_{2t}$$

where,

we used zero target value since X_{2t} is the deviation of basis weight from its mean.

$$X_{1t} - X_{1t-1} = -2.8X_{2t} + 2.8X_{2t-1} \quad \nabla X_{1t} = -2.8 \nabla X_{2t}$$

This is an equivalent P-control.

c) model after control