

# Example of ARMAV model analysis and decomposition:

$$\begin{bmatrix} X_{1t} \\ X_{2t} \end{bmatrix} = \begin{bmatrix} -0.0148 & 0.035 \\ 0.06584 & 0.098 \end{bmatrix} \begin{bmatrix} X_{1t-1} \\ X_{2t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}$$

$$\gamma_a = \begin{bmatrix} 0.3653 & 0 \\ 0 & 0.6016 \end{bmatrix}$$

$$\sigma_{a_{12}} = 0.6016$$

$$\sigma_{a_{11}} = 0.3653$$

Just modeling  $X_{2t}$  alone gives

$$X_{2t} = 0.0273 X_{2t-1} + a_{2t} \quad \sigma_{a_2} = 0.7604$$

$$X'_{1t} = \frac{0.035B}{1 + 0.0148B} X_{2t} + \frac{1}{1 + 0.0148B} a_{1t}$$

$$X_{2t} = \frac{0.06584B}{1 - 0.098B} X'_{1t} + \frac{1}{1 - 0.098B} a_{2t} =$$

$$= \frac{0.0023B^2}{1 - 0.0832B - 0.0015B^2} X_{2t} + \frac{0.06584B}{1 - 0.0832B - 0.0015B^2} a_{1t} + \frac{1}{1 - 0.098B} a_{2t}$$

$$\left(1 - \frac{0.0023 B^2}{1 - 0.0832 B - 0.0015 B^2}\right) X_{2t} = \frac{0.06584 B}{1 - 0.0832 B - 0.0015 B^2} q_{1t} + \frac{1}{1 - 0.0988 B} q_{2t}$$

$$X_{2t} = \frac{0.06584 B}{1 - 0.0832 B - 0.0038 B^2} q_{1t} + \frac{1 - 0.0148 B}{1 - 0.0832 B - 0.0038 B^2} q_{2t}$$

Breaking the expression above into PFE terms (partial fraction expansion) will lead to a Green's dec. (orthogonal) decomposition in terms of  $q_{1t}$  and  $q_{2t}$ .