Lecture 27
ARMAV Based Regulation

Let us now look at the case when we can control one time-series in order to make another time-series do what we want it to do!

<u>Goal:</u> keep the system output ("the other t-s") at a desired level, with minimal (in the mean-least-squares sense) variations!

<u>Strategy</u> Fiddle around with X_{1t} to make sure $X_{2\,t+L}$ is as close to 0 as possible

L - lag (time-delay) between input action and output reaction to that input.

P.P.1.

<u>Method</u>: We will fix X_{1t} to make sure that

$$\hat{X}_{2t}(L) = E[X_{2t+L} | t] = 0$$

After control, $X_{2t+L} = \overset{0}{\hat{X}_{2t}(L)} + \hat{e}_{2t}(L) = \hat{e}_{2t}(L)$

Steps to accomplish this:

i) Obtain the output model before control

ii) Derive a control law based on the minimal mean-squared-error forecast

iii) Obtain the model after control

iv) Evaluate control efficiency

<u>Examples</u>:

X_{1t} - gate opening X_{2t} paper weight
input   output

(i) $\rightarrow$ $X_{2t} = 0.25 \underbrace{X_{1t-1}}_{\text{control signal}} + 0.7 X_{2t-1} + a_{2t}$ Model before control

$\sigma_{a_{22}} = 0.0062$

ii) Deriving the control law

$$X_{2t+1} = 0.25X_{1t} + 0.7X_{2t} + a_{2t+1} \quad | \; E[\cdot | t]$$

$$\hat{X}_{2t}(1) = 0.25X_{1t} + 0.7X_{2t} + 0 = 0$$

we want to set it to 0 to make sure X_{2t+1} is minimized in the mean L.S sense

$$\Rightarrow 0.25X_{1t} + 0.7X_{2t} = 0 \Rightarrow X_{1t} = -\frac{0.7}{0.25}X_{2t}$$

$$X_{1t} = -2.8X_{2t}$$ CONTROL LAW

iii) Model after control

$$X_{2t+1} = \hat{X}_{2t}(1) + a_{2t+1} = a_{2t+1}$$

$$Var[X_{2t+1}] = Var[X_{2t}] = Var[a_{2t}] = \sigma_{22} = 0.0062$$

iv) Evaluating control efficiency

$$[Var\ X_{2t}]_{\text{Before control}} = ? \qquad [Var\ X_{2t}]_{\text{with control}} = 0.0062$$