ARMAV Model for Energy Production and Mean Temperature

April 22. 2019

Objective
Fit vectorial ARMA (ARMAV) model to the energy production and mean temperature

Matlab function armax:

sys = armax(data,[na nb nc nk])

n_a : order of AR part
n_b : order of input
n_c : order of MA part
n_k : the number of input samples that occur before the input affects the output, delay time, = 0.
Use *help armax* to see details

Example:
Let x_1 be the temperature, x_2 be the energy production.
Then a ARMAV(n,m) we want to fit (energy production driven by the mean temp) can be written as follows:

$$\begin{bmatrix} 1 & \phi_{0,12} \\ \phi_{0,21} & 1 \end{bmatrix}\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} - \begin{bmatrix} \phi_{1,11} & \phi_{0,12} \\ \phi_{1,21} & \phi_{1,22} \end{bmatrix}\begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} - \ldots - \begin{bmatrix} \phi_{n,11} & \phi_{0,12} \\ \phi_{n,21} & \phi_{n,22} \end{bmatrix}\begin{bmatrix} x_{1,t-n} \\ x_{2,t-n} \end{bmatrix}$$
$$= \begin{bmatrix} a_{1,t} \\ a_{2,t} \end{bmatrix} - \begin{bmatrix} \theta_{1,11} & 0 \\ 0 & \theta_{1,22} \end{bmatrix}\begin{bmatrix} a_{1,t-1} \\ a_{2,t-1} \end{bmatrix} - \ldots - \begin{bmatrix} \theta_{m,11} & 0 \\ 0 & \theta_{m,22} \end{bmatrix}\begin{bmatrix} a_{1,t-m} \\ a_{2,t-m} \end{bmatrix}$$

Write this row by row, for example, the second row:

$$x_{2,t} - \phi_{1,22}x_{2,t-1} - \ldots - \phi_{n,22}x_{2,t-n}$$
$$= -\phi_{0,21}x_{1,t} + \phi_{1,21}x_{1,t-1} + \ldots + \phi_{n,11}x_{1,t-n}$$
$$a_{2,t} - \theta_{1,22}a_{2,t-1} - \ldots - \theta_{m,22}a_{2,t-m}$$

We can observe that:
order of AR: n_a=n
order of input: n_b=n
order of MA part: n_c=m
Then the following command can be used to fit the energy production:

Sys = armax(data,[n,n,m,0])

Steps:
Step1: Load the data: energy produced during month, mean temperature over month
Step2: De-trend: fit a line to de-trend (polyfit) and extract the residudals
Step3: Fit model to mean temperature with energy production as an input:

1. Create time-domain signals (iddata)
2. Fit ARMAV model (armax) using AIC
3. Check residuals

Step4: Fit model to energy production with mean temperature as an input:

4. Create time-domain signals (iddata)
5. Fit ARMAV model (armax) using AIC
6. Check residuals

Step5: Compare RSS of ARMAV model and ARMA model

Matlab Code and Results:

Contents


```
clear all
close all
clc
```

Step 1: Loading the EP_port Vectors:

```
EP_port=xlsread('Electricity_Generation.xlsx'); % energy produced during month
mean_T=xlsread('Mean_temp2.xlsx'); % mean temperature over month
mod=1:132; % use 132 data points for training
k=1:length(EP_port);
k=k';
```

Step 2: Detrending data

Fitting a linear deterministic trend: energy production trend

```
line_ep=polyfit(k,EP_port,1);

linevals_ep=line_ep(1)*k+line_ep(2);



% mean temperature trend
line_meanT=polyfit(k,mean_T,1);
linevals_meanT=line_meanT(1)*k+line_meanT(2);

% Extracting the residuals:
res_ep=EP_port-linevals_ep;
res_meanT=mean_T-linevals_meanT;
```

Step 3: Fit ARMAV model to Mean Temperature with Energy Consumption as an input:

```
%Creating iddata (objeect for time-series data)for use in modelling:
data_meanT_ep=iddata(res_meanT,res_ep,1); %iddata(y,u,Ts) y:output, u:input, Ts:time
interval between samples

%Testing models of order (n,n,n-1) - (AR,Input,MA):
sys_meanT_ep=cell(25,1);
for n=1:25
    sys_meanT_ep{n}=armax(data_meanT_ep(mod),[n,n,n-1,0]); % use help armax to see the
meaning of the parameters used
end

%Determining the AIC for each model:
maic_meanT_ep=zeros(25,1);
for n=1:25
    maic_meanT_ep(n)=aic(sys_meanT_ep{n});
end

%Localizing the least complex adequate, based on AIC:
[AIC_opt_meanT_ep,n]=min(maic_meanT_ep);
sys_opt_meanT_ep=sys_meanT_ep{n};

%Printing selected model:
fprintf('Selected Model for the Mean Temperature driven by Energy Consumption, based
on AIC, is [%d,%d]',n,n-1)
present(sys_opt_meanT_ep)

%Confirming the adequacy of the model:
figure()
resid(sys_opt_meanT_ep,data_meanT_ep(mod));
title('Confirmation of Adequacy of Chosen Models for Mean Temperature driven by Energy
Consumption','fontsize',11,'fontweight','demi')

r = resid(sys_opt_meanT_ep,data_meanT_ep(mod));
res = r.y;
RSS_v_meanT = sum(res.^2);
```

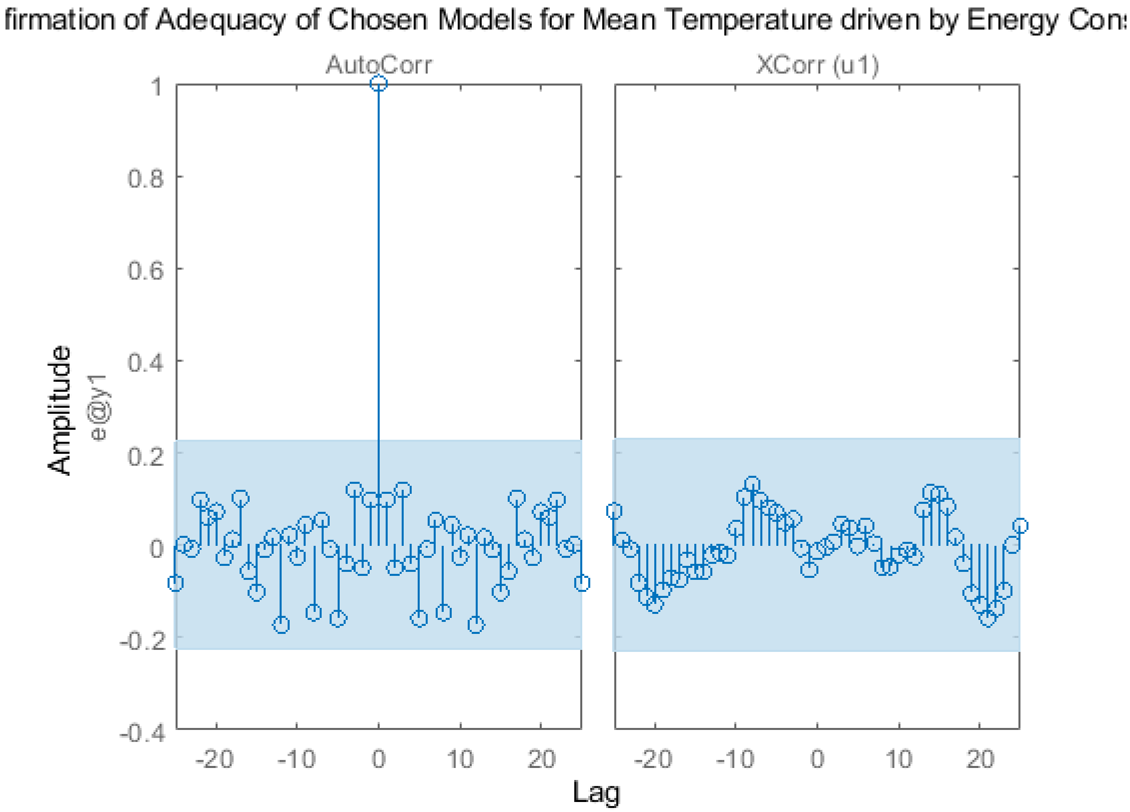
```
Selected Model for the Mean Temperature driven by Energy Consumption, based on AIC, is
[17,16]

sys_opt_meanT_ep =

Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)



  A(z) = 1 + 0.4736 (+/- 0.1628) z^-1 - 0.5664 (+/- 0.1746) z^-2 + 0.1461 (

          +/- 0.2074) z^-3 + 0.3077 (+/- 0.1615) z^-4 + 0.4713 (+/
```

```
- 0.1496) z^-5 + 0.5992 (+/- 0.1493) z^-6 - 0.1239 (+/- 0.1876) z^-7
- 0.3551 (+/- 0.1957) z^-8 - 0.07037 (+/- 0.2091) z^-9 + 0.2352 (
+/- 0.1578) z^-10 + 0.2684 (+/- 0.1388) z^-11 - 0.302 (+
/- 0.1563) z^-12 - 0.5098 (+/- 0.1326) z^-13 + 0.00954 (
+/- 0.1788) z^-14 + 0.203 (+/- 0.1217) z^-15 - 0.2358 (+
/- 0.1152) z^-16 - 0.4232 (+/- 0.08418) z^-17
```

```
B(z) = -2.975 (+/- 0.4935) - 1.947 (+/- 0.7674) z^-1 + 2.468 (+/
- 0.8534) z^-2 + 0.3338 (+/- 0.9538) z^-3 + 0.3753 (+/- 0.8627) z^-4
- 2.135 (+/- 0.7325) z^-5 - 2.331 (+/- 0.7855) z^-6 + 2.832 (
+/- 0.8767) z^-7 + 3.24 (+/- 0.8601) z^-8 + 0.08865 (+/- 1.002) z^-9
- 1.565 (+/- 0.7409) z^-10 - 2.792 (+/- 0.9327) z^-11 + 2.52 (
+/- 0.9939) z^-12 + 3.51 (+/- 0.9048) z^-13 - 2.368 (+/- 1.137) z^-14
- 0.6841 (+/- 0.752) z^-15 + 0.8195 (+/- 0.8021) z^-16
```

```
C(z) = 1 + 0.9763 (+/- 0.2121) z^-1 - 0.3135 (+/- 0.3017) z^-2 + 0.2749 (
+/- 0.2831) z^-3 + 0.7447 (+/- 0.2576) z^-4 + 0.7794 (+/
- 0.2823) z^-5 + 0.9811 (+/- 0.2891) z^-6 + 0.3731 (+/- 0.3456) z^-7
+ 0.04328 (+/- 0.3576) z^-8 + 0.172 (+/- 0.3683) z^-9 + 0.2741 (
+/- 0.3216) z^-10 + 0.7132 (+/- 0.2617) z^-11 + 0.09181 (
+/- 0.2785) z^-12 - 0.6083 (+/- 0.2423) z^-13 + 0.275 (+
/- 0.2945) z^-14 - 0.03818 (+/- 0.2426) z^-15 - 0.4543 (
```

```
                                                        +/- 0.1692) z^-16

Sample time: 1 seconds

Parameterization:
   Polynomial orders:   na=17   nb=17   nc=16   nk=0
   Number of free coefficients: 50
   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.

Status:
Termination condition: Maximum number of iterations reached..
Number of iterations: 20, Number of function evaluations: 176

Estimated using ARMAX on time domain data.
Fit to estimation data: 86.83% (prediction focus)
FPE: 3.78, MSE: 1.235
More information in model's "Report" property.
```



Step 4: Fit model to the energy production with mean temperature as an input:

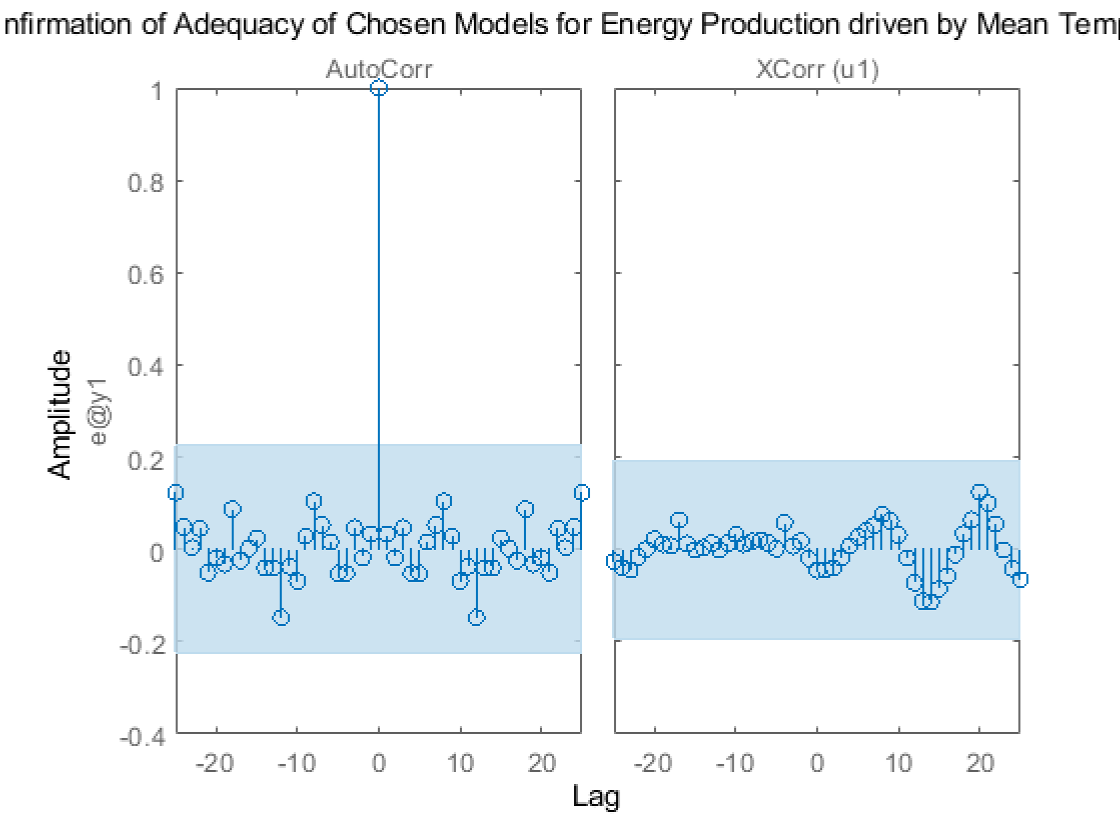
```
%Creating iddata for use in modelling:
data_ep_meanT=iddata(res_ep,res_meanT,1); %iddata(y,u,Ts) y:output, u:input, Ts:time
interval between samples

%Testing models of order (n,n,n-1) - (AR,Input,MA):
sys_ep_meanT=cell(25,1);
for n=1:25
    sys_ep_meanT{n}=armax(data_ep_meanT(mod),[n,n,n-1,0]); % use help armax to see the
meaning of the parameters used
end

%Determining the AIC for each model:
maic_ep_meanT=zeros(25,1);
for n=1:25
    maic_ep_meanT(n)=aic(sys_ep_meanT{n});
end

%Localizing the least complex adequate, based on AIC:
[AIC_opt_ep_meanT,n]=min(maic_ep_meanT);
sys_opt_ep_meanT=sys_ep_meanT{n};

%Printing selected model:
```

```
fprintf('Selected Model for the Energy Production driven by Mean Temperature, based on
AIC, is [%d,%d]',n,n-1)
present(sys_opt_ep_meanT)


%Confirming the adequacy of the model:
figure()
resid(sys_opt_ep_meanT,data_ep_meanT(mod));
title('Confirmation of Adequacy of Chosen Models for Energy Production driven by Mean
Temperature','fontsize',11,'fontweight','demi')

r = resid(sys_opt_ep_meanT,data_ep_meanT(mod));
res = r.y;
RSS_v_ep = sum(res.^2);
```

```
Selected Model for the Energy Production driven by Mean Temperature, based on AIC, is
[13,12]

sys_opt_ep_meanT =

Discrete-time ARMAX model: A(z)y(t) = B(z)u(t) + C(z)e(t)



  A(z) = 1 + 0.1959 (+/- 0.3144) z^-1 + 0.2284 (+/- 0.164) z^-2 + 0.1109 (

          +/- 0.1375) z^-3 - 0.1189 (+/- 0.11) z^-4 - 0.164 (+/- 0.1022) z^-5

          + 0.1312 (+/- 0.1042) z^-6 + 0.2687 (+/- 0.1003) z^-7 + 0.1928 (

          +/- 0.1363) z^-8 - 0.04834 (+/- 0.1224) z^-9 - 0.04304 (

          +/- 0.08704) z^-10 + 0.1508 (+/- 0.08988) z^-11 - 0.605 (

                              +/- 0.0938) z^-12 + 0.08754 (+/- 0.1704) z^-13




  B(z) = -0.1033 (+/- 0.01261) - 0.07931 (+/- 0.03117) z^-1 - 0.04947 (

          +/- 0.03525) z^-2 - 0.03953 (+/- 0.02852) z^-3 - 0.0229 (

          +/- 0.02098) z^-4 - 0.01673 (+/- 0.0146) z^-5 - 0.04053 (

          +/- 0.0134) z^-6 - 0.0553 (+/- 0.02199) z^-7 - 0.06162 (
```

```
          +/- 0.02773) z^-8 - 0.04409 (+/- 0.02856) z^-9 - 0.03122 (

          +/- 0.0223) z^-10 - 0.03883 (+/- 0.01414) z^-11 + 0.03046 (

                                                     +/- 0.01493) z^-12




  C(z) = 1 + 0.7828 (+/- 0.3527) z^-1 + 0.9291 (+/- 0.3845) z^-2 + 1.151 (

          +/- 0.3994) z^-3 + 0.9735 (+/- 0.4421) z^-4 + 0.5832 (+/

          - 0.3708) z^-5 + 0.6657 (+/- 0.2547) z^-6 + 1.045 (+/- 0.2744) z^-7

          + 1.079 (+/- 0.4083) z^-8 + 0.8305 (+/- 0.4766) z^-9 + 0.7341 (

          +/- 0.3878) z^-10 + 0.7474 (+/- 0.2988) z^-11 - 0.1795 (

                                                      +/- 0.2842) z^-12


Sample time: 1 seconds


Parameterization:

   Polynomial orders:   na=13   nb=13   nc=12   nk=0

   Number of free coefficients: 38

   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.


Status:

Termination condition: Maximum number of iterations reached..

Number of iterations: 20, Number of function evaluations: 414


Estimated using ARMAX on time domain data.
```

```
Fit to estimation data: 80.99% (prediction focus)

FPE: 0.09579, MSE: 0.0424

More information in model's "Report" property.
```



Step 5: Compare RSS of ARMAV and ARMA model

```
%--------------Fit ARMA model for temperature residuals-------------------%
%Fitting Independant series to the models of various orders to the mean temperature
residuals:
sys_meanT=cell(25,1);
% note here that we are simply fitting arma model to the temperature
for n=1:25
    sys_meanT{n}=armax(res_meanT(mod),[n n-1]);
end

%Determining the AIC for each model:
maic_meanT=zeros(25,1);
for n=1:25
    maic_meanT(n)=aic(sys_meanT{n});
end

%Localizing the least complex adequate model, based on AIC:
[AIC_opt_meanT,n]=min(maic_meanT);
```

```
sys_opt_meanT=sys_meanT{n};

%Printing selected model:
fprintf('Selected Model for the energy production, based on AIC, is [%d,%d]',n,n-1)
present(sys_opt_meanT)

%Confirming the adequacy of the model:
figure()
resid(sys_opt_meanT,res_meanT(mod));
title('Confirmation of Adequacy of Chosen Models for Mean
Temperature','fontsize',11,'fontweight','demi')

r = resid(sys_opt_meanT,res_meanT(mod));
res = r.y;
RSS_meanT = sum(res.^2);

%-----------------Fit ARMA model for energy prouction---------------------%
%Fitting Independant series to the models of various orders to energy production
residuals:
sys_ep=cell(25,1);
% note here that we are simply fitting arma model to the energy production
for n=1:25
    sys_ep{n}=armax(res_ep(mod),[n n-1]);
end

%Determining the AIC for each model:
maic_ep=zeros(25,1);
for n=1:25
    maic_ep(n)=aic(sys_ep{n});
end

%Localizing the least complex adequate model, based on AIC:
[AIC_opt_ep,n]=min(maic_ep);
sys_opt_ep=sys_ep{n};

%Printing selected model:
fprintf('Selected Model for the energy production, based on AIC, is [%d,%d]',n,n-1)
present(sys_opt_ep)

%Confirming the adequacy of the model:
figure()
resid(sys_opt_ep,res_ep(mod));
title('Confirmation of Adequacy of Chosen Models for Mean
Temperature','fontsize',11,'fontweight','demi')

r = resid(sys_opt_ep,res_ep(mod));
res = r.y;
RSS_ep = sum(res.^2);
```

```
fprintf('RSS of ARMAV model for Energy Production driven by Mean Temperature is %d
\n',RSS_v_ep)
fprintf('RSS of ARMA model for Energy Production is %d \n',RSS_ep)
fprintf('RSS of ARMAV model for Mean Temperature driven by Energy Production is %d
\n',RSS_v_meanT)
fprintf('RSS of ARMA model for Mean Temperature is %d \n',RSS_meanT)
```

```
Selected Model for the energy production, based on AIC, is [3,2]

sys_opt_meanT =

Discrete-time ARMA model: A(z)y(t) = C(z)e(t)


  A(z) = 1 - 2.085 (+/- 0.08738) z^-1 + 1.611 (+/- 0.1515) z^-2 - 0.3525 (

                                                          +/- 0.08754) z^-3



  C(z) = 1 - 1.698 (+/- 0.0183) z^-1 + 0.9635 (+/- 0.01709) z^-2



Sample time: 1 seconds



Parameterization:

   Polynomial orders:   na=3   nc=2

   Number of free coefficients: 5

   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.



Status:

Termination condition: Near (local) minimum, (norm(g) < tol)..

Number of iterations: 19, Number of function evaluations: 41
```

```
Estimated using ARMAX on time domain data.

Fit to estimation data: 72.48% (prediction focus)

FPE: 6.092, MSE: 5.395

More information in model's "Report" property.

Selected Model for the energy production, based on AIC, is [8,7]

sys_opt_ep =

Discrete-time ARMA model: A(z)y(t) = C(z)e(t)


  A(z) = 1 + 0.8032 (+/- 0.5586) z^-1 - 1.418 (+/- 0.4806) z^-2 - 1.282 (

          +/- 0.9069) z^-3 + 1.179 (+/- 0.7514) z^-4 + 1.299 (+/- 0.9115) z^-5

          - 0.1683 (+/- 0.7586) z^-6 - 0.4924 (+/- 0.3571) z^-7 - 0.2498 (

                                                           +/- 0.2882) z^-8



  C(z) = 1 + 1.722 (+/- 0.5684) z^-1 - 0.1664 (+/- 1.016) z^-2 - 1.656 (

          +/- 0.214) z^-3 + 0.06419 (+/- 1.035) z^-4 + 1.551 (+/- 0.1778) z^-5

                     + 0.8269 (+/- 0.9777) z^-6 + 0.1003 (+/- 0.4618) z^-7


Sample time: 1 seconds


Parameterization:

   Polynomial orders:   na=8   nc=7

   Number of free coefficients: 15

   Use "polydata", "getpvec", "getcov" for parameters and their uncertainties.
```

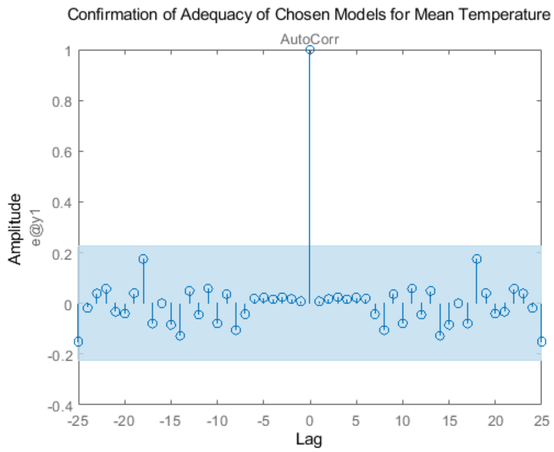
```
Status:

Termination condition: Near (local) minimum, (norm(g) < tol)..

Number of iterations: 18, Number of function evaluations: 47



Estimated using ARMAX on time domain data.

Fit to estimation data: 70.18% (prediction focus)

FPE: 0.1483, MSE: 0.1043

More information in model's "Report" property.
```

RSS of ARMAV model for Energy Production driven by Mean Temperature is 5.596554e+00

RSS of ARMA model for Energy Production is 1.376552e+01

RSS of ARMAV model for Mean Temperature driven by Energy Production is 1.629960e+02

RSS of ARMA model for Mean Temperature is 7.121965e+02



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