Wholesales Problem

Homework 5 Solutions

$$X_t + 1.10414 X_{t-1} - 0.0594 X_{t-2} - 0.024 X_{t-3} - 0.0092 X_{t-4}$$
$$+ 0.0458 X_{t-5} - 0.0373 X_{t-6} + 0.0111 X_{t-7} + 0.0393 X_{t-8}$$
$$- 0.0446 X_{t-9} - 0.0057 X_{t-10} + 0.022 X_{t-11} + 0.9908 X_{t-12}$$
$$- 1.0796 X_{t-13} + 0.0773 X_{t-14} = a_t + 0.9876 a_{t-1} - 0.1893 a_{t-2}$$
$$- 0.2967 a_{t-3} + 0.4521 a_{t-4} - 0.2162 a_{t-5} - 0.1691 a_{t-6}$$
$$+ 0.2495 a_{t-7} + 0.0341 a_{t-8} - 0.4218 a_{t-9} + 0.3815 a_{t-10}$$
$$+ 0.0581 a_{t-11} + 0.407 a_{t-12} - 0.3067 a_{t-13}$$

AR roots of this model are

Real Roots

$\lambda_1 = -1.0009$

$\lambda_2 = 0.0770$

Complex roots

$\lambda_{3/4} = -0.8675 \pm 0.5033j$

$\lambda_{5/6} = 0.0003 \pm 1.0038j$

$\lambda_{7/8} = 0.9926 \pm 0.0522j$

$\lambda_{9/10} = -0.5071 \pm 0.8591j$

$\lambda_{11/12} = 0.5019 \pm 0.8676j$

$\lambda_{13/14} = 0.8624 \pm 0.5071j$

(b)

Though root $\lambda_1 = -1.0009 \approx -1$ happens to be a real root ON the unit circle (or, close to it), it is effectively a stochastic <u>seasonality of 2</u> (not a trend). Since there are no roots at or close to 1, we can say that there are NO STOCHASTIC trends in this model.

(c) Roots $\lambda_3, \lambda_4, \lambda_5, \ldots, \lambda_{14}$ are all near the unit circle and are roughly associated with periodicities

2.39, 2.99, 4.001, 6.013, 11.836 and 123.2

We seem to have periodicity of 12 and all its harmonics (12, 6, 4, 3, 2.4 and even 2, because $\lambda_1 \approx -1$, which corresponds to periodicity of 2).

d) Since we have periodicity of 12 and all its harmonics (AR roots are creating a symmetrical star pattern on the unit circle), we can try operator $(1-B^{12})$ which corresponds to roots $e^{j\frac{2\pi}{12}k}$, $k=0,1,\ldots,11$.

After creating $Y_t = (1-B^{12})X_t$ and fitting ARMA(2,13) to Y_t, the corresponding residual sum of squares becomes $RSS_1 = 2.1876 \cdot 10^9$ F test gives

$$F = \frac{(RSS_1 - RSS_0)/\overset{\text{restricted parameters}}{12}}{\underset{1.57791\cdot 10^9}{RSS_0} / \underset{93}{(N-r)}} = 3.0185 > F^{95\%}_{12,\infty} = 1.81$$

=> we cannot accept that all 12 roots of $1-B^{12}$ exist, though even this F test indicates that such dynamics exists (F test did not fail "miserably").

9.4 a) From Table 9.1 the periods are 7 and 30, possibly weekly and monthly in daily data.

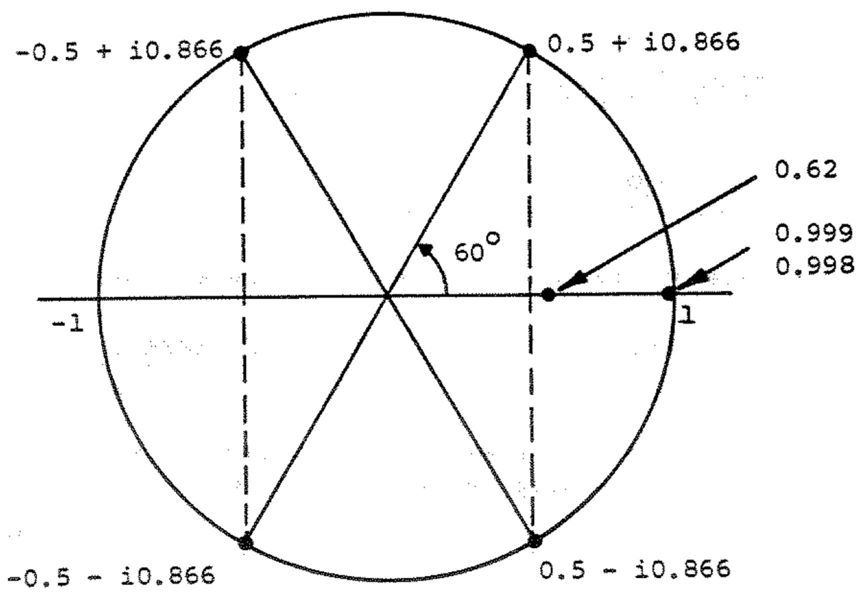
b) The operator would be

$$(1 - 1.25\ B + B^2)\ (1 - 1.956B + B^2)$$

and the parsimonious model after removing the seasonality would be MA(1).

9.5 Table 9.1 shows that $0.5 \pm i0.866$ has a period of 6 and $-0.5 \pm i0.866$ has a period of 3. These are at angles $\tan^{-1}(0.866/0.5) = 60^\circ$ and $\tan^{-1}(0.866/-0.5) = 120^\circ$ respectively, with their complex conjugates at -60° and -120°.

a)



b) Straight line trend resulting from 0.999 and 0.998 ≈ 1. For monthly data, three monthly and six monthly seasonalities.

c) No. If we had -0.998 ≈ -1 instead of 0.998 then all the 6 roots of unity required for the operator $(1-B^6)$ would be present and N would be 6. As the root -1 is absent $(1-B^6)$ cannot be used.

10.1 $\phi_1 = -0.087$, $\phi_2 = -0.140$ and $\phi_3 = 0.892$ from Table 10.2 give roots $\lambda_1 = 0.8876$ and $\lambda_{2,3} = -0.4873 \pm i0.8761$. Now Table 9.13 (or Table 9.1) shows that the stochastic part X_t of the model (10.3.7) contains a seasonality with period approximately 3 months. This 3 monthly period has no direct counterpart in the stochastic model of Section 9.2.4.

The five periods of 12 (yearly), 6 (half yearly), 4, 3 and $2\frac{2}{5}$ months in the model (10.3.7) are almost exactly reproduced in the stochastic model of Section 9.2.4 as seen from columns 7 and 9 of Table 9.13. All these periods represent the harmonies of the fundamental 12-month period.

10.4 a) Using unbiased estimates for proper comparison,

(i) First Order: $$\sigma_a^2 = \frac{\text{Residual sum of squares}}{\text{N-\# of estimated parameters}}$$

$$= \frac{1.6639}{100 - 4} = 0.0173$$

$$\text{Var } [e_t(1)] = \sigma_a^2$$

$$= 0.0173$$

$$\text{Var } [e_t(2)] = \sigma_a^2(1 + G_1^2) = \sigma_a^2(1 + \phi_1^2)$$

$$= 0.0173(1 + 0.7887^2) = 0.0281$$

(ii) Second Order: $$\sigma_a^2 = \frac{1.5774}{100-7} = 0.0170$$

$$G_1 = \phi_1 - \theta_1 = 1.6873 - 0.9788 = 0.7085$$

$$\text{Var } [e_t(1)] = \sigma_a^2 = 0.0170$$

$$\text{Var } [e_t(2)] = \sigma_a^2(1 + G_1^2) = 0.017(1 + 0.7085^2) = 0.0255$$

b) (i) $$\text{\% Improvement in Var } [e_t(1)] = \frac{0.0173 - 0.0170}{0.0173} \times 100 = 1.73\%$$

(ii) $$\text{\% Improvement in Var } [e_t(2)] = \frac{0.0281 - 0.0255}{0.0281} \times 100 = 9.25\%$$

c) $$F = \frac{1.6639 - 1.5774}{1.5774} \times \frac{100 - 7}{3}$$

$$= 1.7 < F_{0.95}(3,120) = 2.68$$

d) $$(1 - 1.69B + 0.75B^2)X_t = (1 - 0.98B)a_t$$

has by Eq. (4.3.10) $$I_1 = \phi_1 - \theta_1 = 1.69 = 0.71$$

$$I_2 = \theta_1 I_1 + \phi_2 = 0.98 \times 0.71 - 0.75 = -0.054$$

$$I_3 = \theta_1 I_2 = 0.98(-0.054) = -0.053 \text{ etc.}$$

Thus the model is close to AR(1) with $\phi_1 \approx 0.71$, which in turn is close to $\phi_1 = 0.7887$ of the AR(1) model in (10.2.2a).