26. fejezet

1. a) $f(x) = x^2 - 4x + 3 = (x-2)^2 - 1 \quad (x \in \mathbb{R})$

$f(x) \geq -1 = f(2)$

min. hely: $x = 2$
min. érték: $f(2) = -1$

legnagyobb: nincs
felülről nem korlátos

Indirekt tfh: $K > 0$: $f(x) < K$

$(x-2)^2 - 1 < K$

$(x-2)^2 < K + 1$

$x - 2 < \sqrt{K+1}$

$x < \sqrt{K+1} + 2 \Rightarrow$ ha pl. $x = \sqrt{K+1} + 3$

$\lightning \Rightarrow$ nincs felső korlát

1. b) $f(x) = (x-2)^2 - 1 \quad \left(\frac{1}{2} \leq x \leq 3\right)$

$\frac{1}{2} \leq x \leq 3 \quad / -2$

$-\frac{3}{2} = \frac{1}{2} - 2 \leq x - 2 \leq 3 - 2 = 1 \quad / (\ )^2$

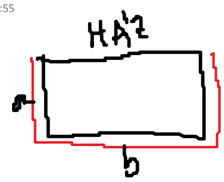
$0 \leq (x-2)^2 \leq \frac{9}{4} \quad / -1$

$-1 \leq (x-2)^2 - 1 \leq \frac{5}{4}$

legkisebb $x = 2$
$f(2) = -1$

legnagyobb $x = \frac{1}{2}$
$f\left(\frac{1}{2}\right) = \frac{5}{4}$

3.

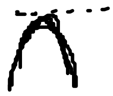


$$2a + b = 24 \Rightarrow b = 24 - 2a$$

$$T = a \cdot b = a(24 - 2a) = -2a^2 + 24a = -2(a^2 - 12a) =$$

$$= -2\left[(a-6)^2 - 36\right] = -2 \cdot (a-6)^2 + 72$$

kell: T maximális legyen



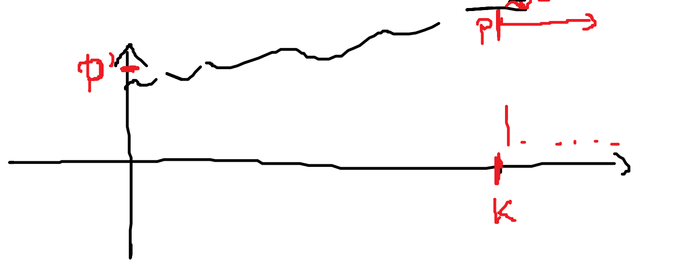
$$\Rightarrow \quad a - 6 = 0$$

$$a = 6$$

$$b = 24 - 2 \cdot 6 = 12$$

$$T = a \cdot b = 6 \cdot 12 = \underline{\underline{72\ m^2}}$$

$\lim\limits_{x\to+\infty} \ldots = +\infty$



12. a) kell.

$$\forall P>0\ \exists K>0.\ \forall x\in D_f,\ x>K:$$

$$f(x) > P$$

ha $\lim\limits_{x\to\infty} = +\infty$: kell : $f(x)$-re NRA

$$\frac{1}{2}x-2>0$$
$$x>4$$

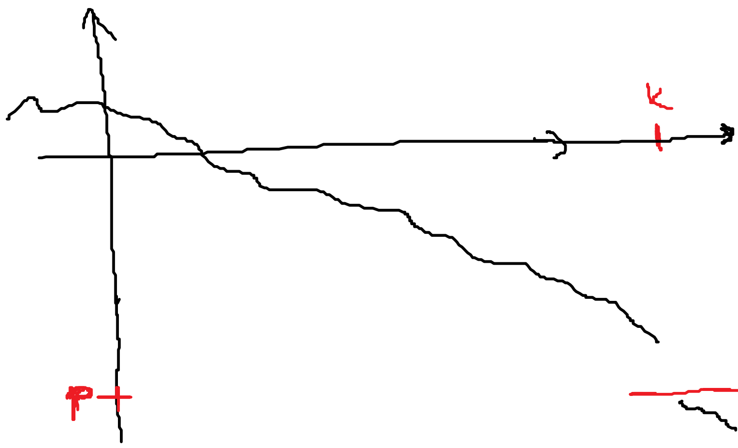
$$\frac{x^4-2x^3+x^2+7}{x^3+x+1} \geq \frac{x^4-2x^3}{3x^3} \geq \frac{\frac{1}{2}x^4+\frac{1}{2}x^4-2x^3}{3x^3} = \frac{\frac{1}{2}x^4+x^3\left(\frac{1}{2}x-2\right)}{3x^3} \geq \frac{\frac{1}{2}x^4}{3x^3} = \frac{x}{6}$$

$$\frac{x}{6} > P$$

$$x > 6P$$

kell. $x > \boxed{\max\{4;6P\} =: K}$

$$f(x) \geq \frac{x}{6} > P$$



12. c) Kell:

$$\forall p<0 \ \exists K>0 \ \forall x\in D_f,\ x>K:\ f(x)<p$$

kell: f(x)-re NRF

$$f(x)<p \quad /\cdot(-1)$$

$$-f(x) > -p > 0$$

$$\frac{1}{2}x^2-5>0$$
$$x^2>10$$
$$x>4$$

Kell: NRA $\left(-f(x)\right)$ -re!

NRF: $2x+3x$

$$-f(x)=\frac{x^3+x^2-2x-3}{4x^2-9} \geq \frac{x^3-\overbrace{(2x+3)}}{4x^2} = \frac{x^3-5x}{4x^2} = \frac{\frac{1}{2}x^3+\frac{1}{2}x^3-5x}{4x^2} = \frac{\frac{1}{2}x^3+x\left(\frac{1}{2}x^2-5\right)}{4x^2} \geq$$

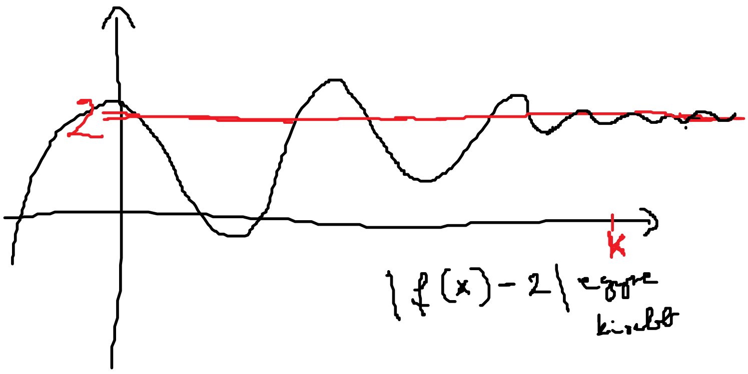
$$\geq \frac{\frac{1}{2}x^3}{4x^2} = \frac{x}{8}$$

$$-f(x) \geq \frac{x}{8} > -p$$

$$x > -8p > 0$$

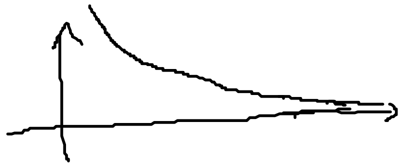
$$\boxed{K=\max\{4;\,-8p\}}$$

ha $x>K \Rightarrow f(x)<p$



$|f(x)-2|$ egyre kisebb

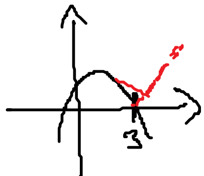
Kell: $\forall \varepsilon>0 \ \exists K: \forall x \in D_f,\ x>K: \ |f(x)-2|<\varepsilon$



Kell:

$|f(x)-2|$ -re NRF

$$|f(x)-2| = \left| \frac{2x^3-x^2+3}{x^3+2x-5} - 2 \right| = \left| \frac{2x^3-x^2+3}{x^3+2x-5} - \frac{2x^3+4x-10}{x^3+2x-5} \right| = \left| \frac{-x^2-4x+13}{x^3+2x-5} \right| =$$



ha $x>3$

$$= \frac{x^2+4x-13}{x^3+2x-5} \le \frac{5x^2}{\frac{1}{2}x^3+\frac{1}{2}x^3-5} \le \frac{5x^2}{\frac{1}{2}x^3} = \frac{10}{x} \quad (x>3)$$

$$\frac{10}{x} < \varepsilon$$

$$x > \frac{10}{\varepsilon}$$

$$K := \max\left\{3, \frac{10}{\varepsilon}\right\}$$

$$\lim_{x\to\infty} f(x) = 2$$