2019 Matek zh 3

2022. december 16., péntek 12:17

1, $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

$\det(A - I\lambda) = 0 = (2-\lambda)\left((2-\lambda)^2 - (-1)\cdot(-1)\right) + \left((-1)(2-\lambda) - (-1)\cdot(-1)\right) - \left((-1)\cdot(-1) - (-1)\cdot(2-\lambda)\right)$

$4 - 4\lambda + \lambda^2 - 1$; $-2+\lambda$, -1; $1 + 2-\lambda$

$A - I\lambda = \begin{bmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix}$

$6 - 8\lambda + 2\lambda^2 - 3\lambda + 4\lambda^2 - \lambda^3 - 6 + 2\lambda$
$= -\lambda^3 + 6\lambda^2 - 9\lambda$

$\lambda(-\lambda^2 + 6\lambda - 9) = 0$

ha $\lambda_1 = 0$

ha $-\lambda^2 + 6\lambda - 9 = 0$

$\frac{-6 \pm \sqrt{36 - 4\cdot -1 \cdot -9}}{-2} = \frac{-6 \pm 0}{-2}$ — $\lambda_2 = 3$, $\lambda_3 = 3$

Multiplicitás
$a(0) = 1$
$a(3) = 2$

Sajátértékek: 0, 3

ha $\lambda = 0$

$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array}\right)$

$\left(\begin{array}{ccc|c} 0 & 3 & -3 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right)$

$\left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$x_2 - x_3 = 0$, $x_3 \in \mathbb{R}$
$x_2 = x_3$
$x_1 - x_3 = 0$
$x_1 = x_3$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$g(0) = \dim W_0 = 1$

ha $\lambda = 3$

$\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\left(\begin{array}{ccc|c} -1 & -1 & -1 & 0 \\ -1 & -1 & -1 & 0 \\ -1 & -1 & -1 & 0 \end{array}\right)$

$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

$x_1 + x_2 + x_3 = 0$
$x_1 = -x_2 - x_3$
$x_2, x_3 \in \mathbb{R}$

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$g(3) = \dim W_3 = 2$

$a(0) + a(3) = g(0) + g(3) \Rightarrow$ diagonalizálható

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ diagonális alak $\quad D = C^{-1} \cdot A \cdot C$

$C = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ diagonalizáló M.

2, $\boxed{g = u}$

$W = \mathrm{SPAN}\left(\underset{b_1}{\begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}}, \underset{b_2}{\begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}}, \underset{b_3}{\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}}\right)$

Matching táblá → általános táblázat

	q_1	q_2	q_3
b_1	1	0	0
b_2	$\frac{\langle q_1, b_2 \rangle}{\langle q_1, q_1 \rangle}$	1	0
b_3	$\frac{\langle q_1, b_3 \rangle}{\langle q_1, q_1 \rangle}$	$\frac{\langle q_2, b_3 \rangle}{\langle q_2, q_2 \rangle}$	1

$q_1 = b_1$

$q_2 = b_2 - \frac{\langle q_1, b_2 \rangle}{q_1, q_1} \cdot q_1$

$q_3 = b_3 - \frac{\langle q_1, b_3 \rangle}{\langle q_1, q_1 \rangle} \cdot q_1 - \frac{\langle q_2, b_3 \rangle}{\langle q_2, q_2 \rangle} \cdot q_2$

$q_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

$q_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} - \frac{2+1}{1+1+1} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

$\frac{3}{3} = 1$

$q_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

$q_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} - \frac{-1-1}{1+1+1} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} - \frac{-1+1+1}{1+1+1} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \Big/ \cdot 3$

$-\frac{2}{3}$; $\frac{1}{3}$

$= \begin{pmatrix} -3 \\ 3 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 2 \\ 0 \end{pmatrix} \sim \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

$q_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

ortogonális bázis

$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

ortonormált

$|q_1| = \sqrt{1^2 + 0^2 + (-1)^2 + 1^2} = \sqrt{3}$
$|q_2| = \sqrt{1^2 + 1^2 + 1^2 + 0^2} = \sqrt{3}$
$|q_3| = \sqrt{(-1)^2 + 1^2 + 1^2 + 0^2} = \sqrt{3}$

tehát bázisok:

$\frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}; \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}; \frac{1}{\sqrt{3}}\begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$

b) $x = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}$

$$\boxed{\begin{aligned} P(x) &= \sum_{i=1}^{n} \frac{\langle x, q_i \rangle}{\langle q_i, q_i \rangle} \cdot q_i \\ Q(x) &= x - \quad - \quad '' \quad - \end{aligned}}$$

$\frac{\left\langle \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \frac{\left\langle \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \frac{\left\langle \begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}; \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix}; \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \frac{2+1}{1+1+1} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \frac{2-1}{1+1+1} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \frac{-2-1}{1+1+1} \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \\ 0 \end{pmatrix} \Big/ \cdot 3 =$

$\begin{pmatrix} 3 \\ 0 \\ -3 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ -2 \\ -5 \\ 3 \end{pmatrix} \rightarrow$ párhuzamos

$\begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 7 \\ -2 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \\ 5 \\ -2 \end{pmatrix} \rightarrow$ merőleges

3) $f(x) = \frac{2x+5}{x-1} \quad (x \in (1, \infty)) \rightarrow x \neq 1$ de az nincs benne a

$f(x) - f(t) \neq 0 \rightarrow$ illyenkor invertálható (injektív)

$\frac{2x+5}{x-1} - \frac{2t+5}{t-1} = \frac{(2x+5)(t-1) - (2t+5)(x-1)}{(x-1)(t-1)} = \frac{2tx + 5t - 2x - 5 - (2}{(x-1)(t-1)}$

tehát f invertálható

$D_{f^{-1}} = R_f$

$R_f = \{ y \in \mathbb{R} \mid \exists x \in (1, \infty) : y = \frac{2x+5}{x-1} \}$

$y = \frac{2x+5}{x-1}$
$y(x-1) = 2x+5$
$y(x-1) - 2x - 5 = 0$
$xy - y - 2x - 5$
$x(y-2) - y - 5 = 0$
$x = \frac{y+5}{y-2}$

mivel $x \in (1, \infty)$

$\frac{y+5}{y-2} > 1$

$\frac{y+5}{y-2} - 1 > 0$

$\frac{y+5}{y-2} - \frac{y-2}{y-2} > 0$

$\frac{y - y + 5 + 2}{y-2} > 0$

$\frac{7}{y-2} > 0 \Rightarrow y - 2 > 0 \rightarrow y > 2$

$D_{f^{-1}} = R_f = (2; \infty)$

$f^{-1}(y) = \frac{y+5}{y-2}$

...tartományban

...rendezem)

$$\frac{\ldots 5x+5x-2t-5)}{\ldots} = \frac{2tx-2tx+5t-5x-2x+2t-5+9}{(x-1)(t-1)} = \frac{7t-7x}{(x-1)(t-1)}$$

$\Downarrow$

ez már negatívat

0

4)

$L = 2$

$\forall \varepsilon > 0 \ \exists K > 0 : \forall x \in (K, \infty) : |f(x) - L| < \varepsilon$

ezt felülről kell becsülni

nem éri el az x tengelyt

$$\left|\frac{2x^3-3x^2+x+10}{x^3+x^2+x+7} - 2\right| = \left|\frac{2x^3-3x^2+x+10}{x^3+x^2+x+7} - \frac{2x^3+2x^2+2x+14}{x^3+x^2+x+7}\right| = \left|\frac{-5x^2-x-4}{x^3+x^2+x+7}\right|$$

x^3+x^2+x+7

$$= \frac{5x^2+x+4}{x^3+x^2+x+7} \le \frac{10x^2}{x^3} = \frac{10}{x} \quad x \ge 4 \ ((x \ge 1))$$

$$\frac{10}{x} < \varepsilon \iff x > \frac{10}{\varepsilon}$$

$K := \max\left\{3; \frac{10}{\varepsilon}\right\} \to$ ezzel teljesül a definíció