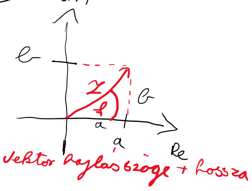
Gyak 5

2023. március 28., kedd 10:46

1 Komplex - számok:

$\hookrightarrow i^2 = -1$ $\quad \sqrt{i} = \sqrt{-1}$

$z = \underset{\substack{\uparrow \\ Re(z)}}{a} + \underset{\substack{\uparrow \\ Im(z)}}{b} \cdot i \quad \Leftarrow$ algebrai alak



szám = vektor hajlásszöge + hossza

Ha: $w = c + d \cdot i$

$z = w \Leftrightarrow a = c \wedge b = d$

Hossz: $|z| = r$

vektor hajlásszöge: $\cos\varphi = \frac{a}{r}$, $\sin\varphi = \frac{b}{r}$

$$z = r \cdot \frac{a}{r} + r \cdot \frac{b}{r} \cdot i = r \cdot \left(\frac{a}{r} + \frac{b}{r} \cdot i\right) =$$

$$= \boxed{r \cdot (\cos\varphi + i \sin\varphi)} \Leftarrow \text{trigonometrikus alak}$$

$z = |z| \cdot (\cos\varphi + i \cdot \sin\varphi)$

$w = |w| \cdot (\cos\varphi + i \cdot \sin\varphi)$

Szorzatuk: $z \cdot w = |z| \cdot |w| \cdot (\cos(\varphi + \alpha) + i \cdot \sin(\varphi + \alpha))$

Hatványozás: $z^n = |z|^n \cdot (\cos(n \cdot \varphi) + i \cdot \sin(n \cdot \varphi))$

trigonometrikus alakban

feladatok

tényleges algebrai alak: $0 \pm 4i$ $\quad 0 \pm 5i$

1. a) $\sqrt{-16} = \underline{\pm 4i}$ $\quad$ b) $\sqrt{25} = \underline{5i}$ $\quad$ d)

$(\sqrt{i} = \sqrt{-1})$ $\quad$ c) $(2i)^2 = \underline{-4}$ $\hookrightarrow -4 + 0i$ $\quad$ e) $2i + 5i = \underline{7i}$ $\to 0 + 7i$

f) $\frac{4i}{2i} = \underline{2}$ $\hookrightarrow 2 + 0i$

Konjugált

$i^2 = -1$

$(-i)^2 = (-1)^2 \cdot i^2 = i^2 = -1$

$\sqrt{i} = \sqrt{-1} = \pm$ $\Leftarrow$ csak gyököknél kell

2. $z = -2 + 7i$ $\quad z = a + b \cdot i$

$|z| = \sqrt{-2^2 + 7^2}$ $\quad |z| = a^2 + b^2$

$|z| = \sqrt{z^2} = \sqrt{a^2 + b^2}$

$= \sqrt{4 + 49} = \underline{\underline{\sqrt{53}}}$

3. $$\frac{4 + 3i}{(2 - i)^2} = \frac{4 + 3i}{4 - 4i + \underbrace{i^2}_{-1}} = \frac{4 + 3i}{3 - 4i} = \frac{4 + 3i}{3 - 4i} \cdot \frac{3 + 4i}{3 + 4i} =$$

$$= \frac{25i}{25} = \underline{i} = \underline{\underline{0+i}}$$

Konjugált szorzás: $(a+bi)\cdot(a-bi) = a^2+b^2$
(nevezetes azonosság)

(4) $$\frac{x+i-3i\bar{x}}{x-4} = i-1$$ } átírod algebraiba

$$\frac{a+bi+i-3i\cdot(a-bi)}{a+bi-4} = i-1 \quad /\cdot(a+bi-4)$$

$= ~~a+bi+i-3ai+3bi = ai+bi^2-4i-a-bi+4~~$

~~$a+4b-3ai+i = ai+bi^2-4i-a-bi+4$~~ $/-ai+b+4i+a+bi$

$i^2 = -1$, $bi^2 = -b$

~~$5i+2a+5bi-4ai+b = 4$~~

$$(2a-2b-4)+i\cdot(2b+5-4a) = 0$$

$$\left.\begin{aligned} &\text{I}\quad 2a-2b-4=0 \\ &\text{II}\quad 2b+5-4a=0 \end{aligned}\right\} +$$

$$-2a+1=0$$

$$a=\frac{1}{2} \qquad b=-\frac{3}{2}$$

$$\underline{\underline{x = \frac{1}{2} - \frac{3}{2}\cdot i}}$$ } végeredmény

Hf: (5)

argumentum = hajlásszög

$$z = \overset{a}{2} + \overset{b}{5}i$$

Hossz $= r = |z| = \sqrt{a^2+b^2} = \sqrt{4+25} = \sqrt{29}$

$$z = |z|\cdot(\cos\varphi + i\sin\varphi)$$

$$\cos\varphi = \underline{\underline{\frac{2}{\sqrt{29}}}} = \frac{a}{|z|} \qquad \sin\varphi = \frac{b}{|z|} = \underline{\underline{\frac{5}{\sqrt{29}}}}$$

trig alakja $\quad z = r\cdot(\cos\varphi + i\cdot\sin\varphi)$

trig alakja $\qquad z = r \cdot (\cos \varphi + i \cdot \sin \varphi)$

7. a) $1+i$

$r = |z| = \sqrt{1^2+1^2} = \sqrt{2}$

$\sin \varphi = \dfrac{1}{\sqrt{2}} = 45^\circ$

$\cos \varphi = \dfrac{1}{\sqrt{2}} = 45^\circ$

$z = \sqrt{2} \cdot (\cos 45^\circ + i \sin 45^\circ)$

8,

Hf: 5, 7 !