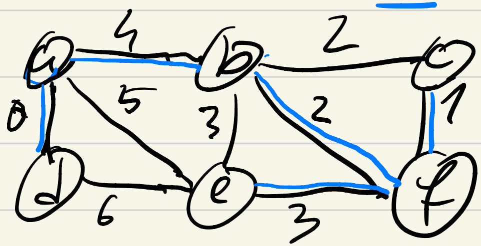
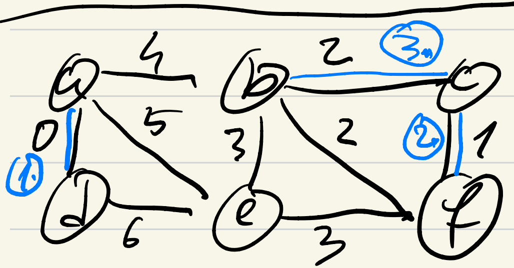
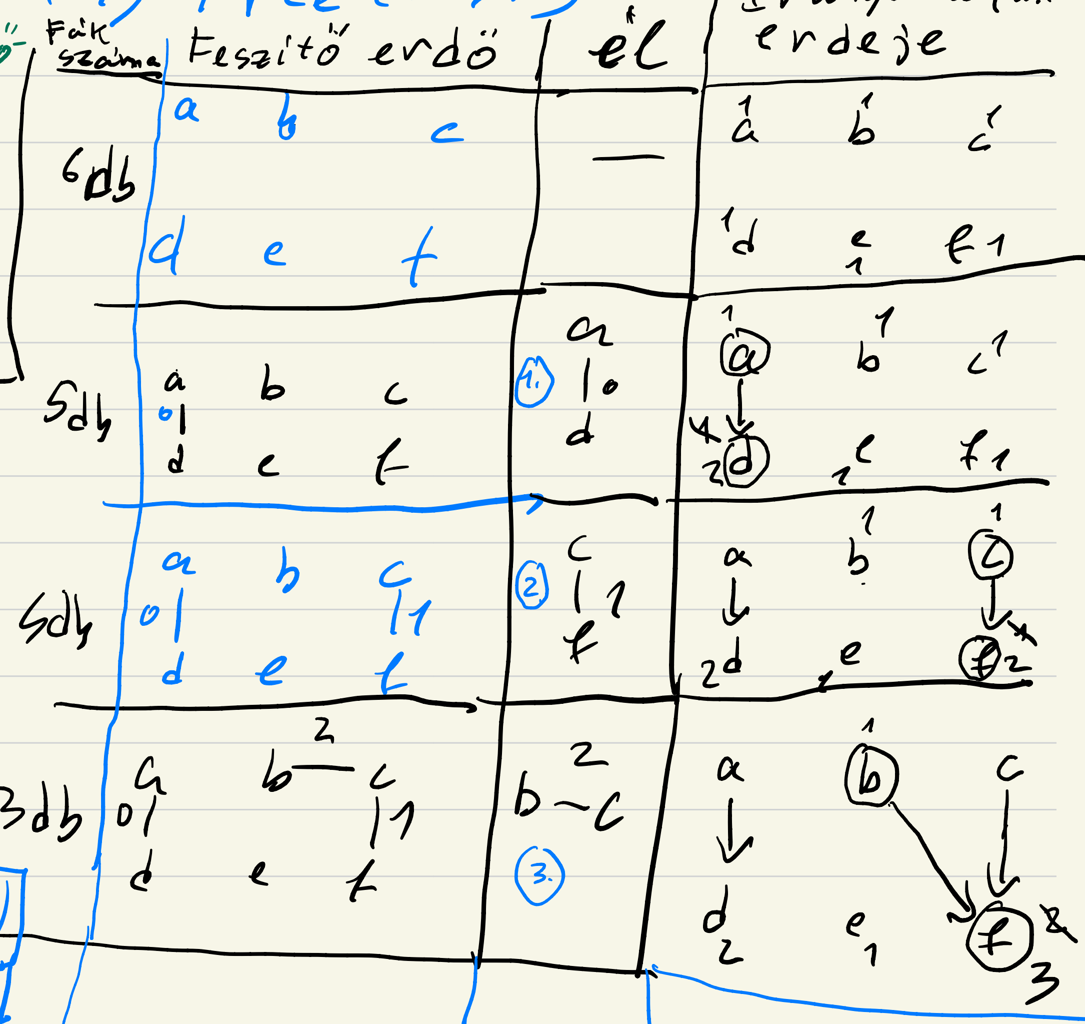
Minimum spanning Tree (MST)

Öf. ir.lan élsúlyozott gráf min. feszítő-fáját keressük **Pl.**





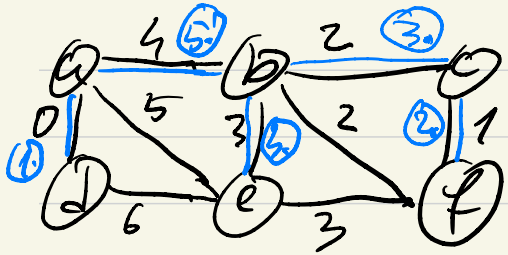


KRUSKAL alg.

$O((n+m)\log n) = O(m \log n)$

$n-1 \leq m < n^2$

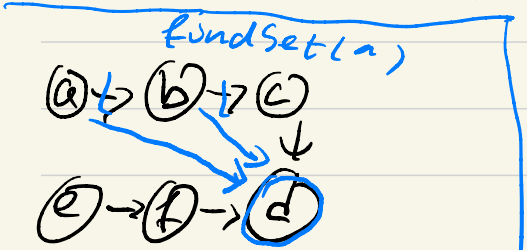
$MT(n,m)$

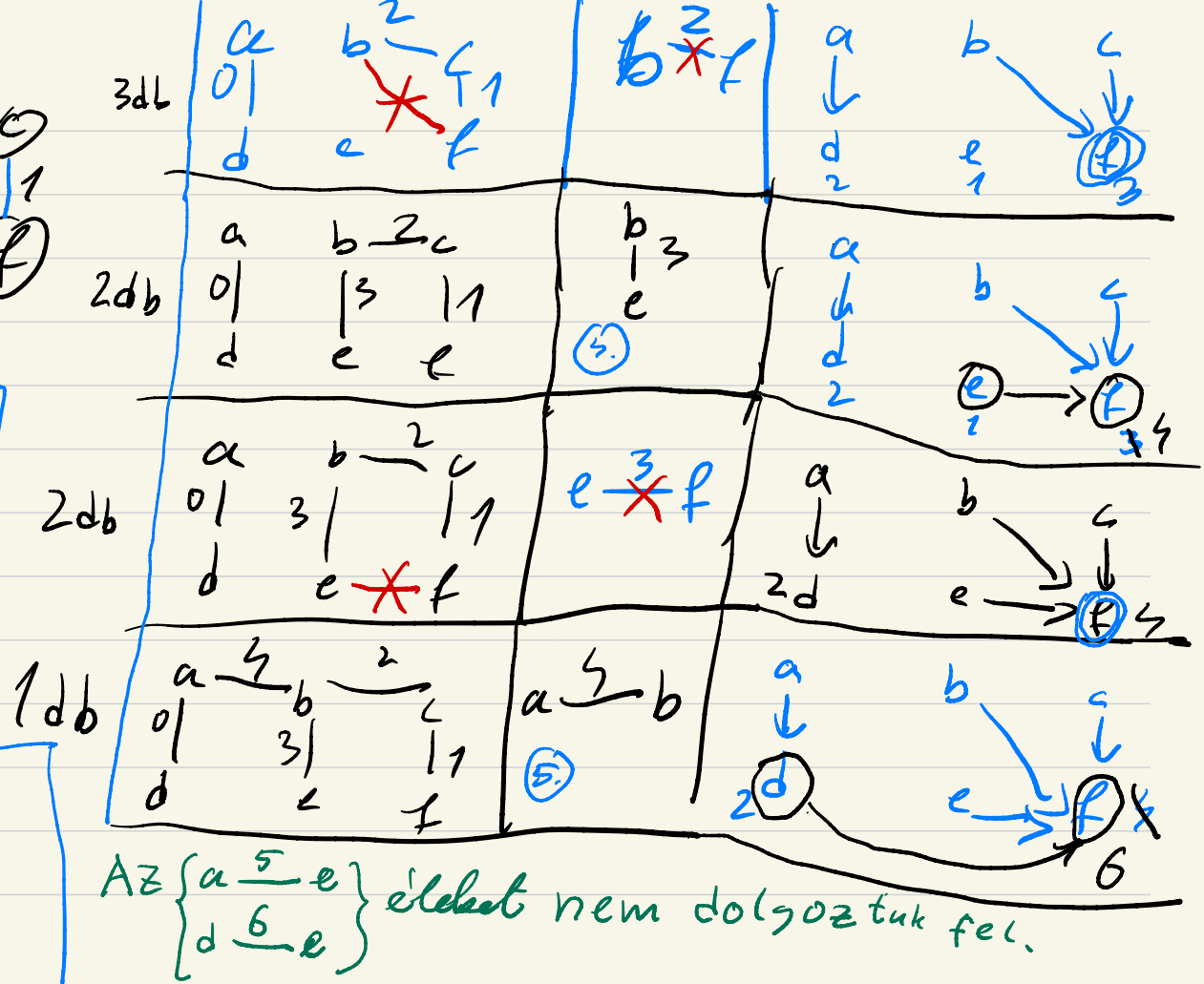


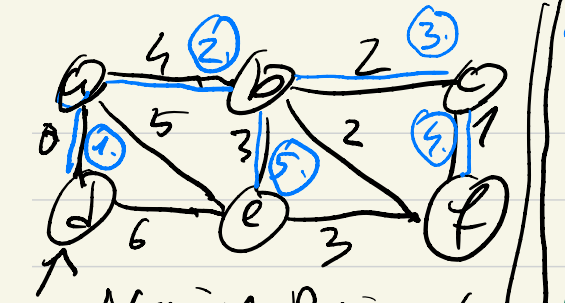
$MT(n,m)$

$\in O(m * \log n)$

(Mindkét MST algoritmusra)

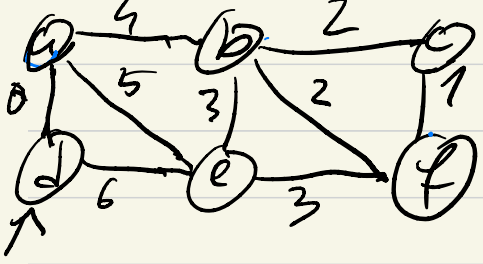






Naiv. Prim alg.

MT(n,m) ∈ Θ(n*m)



C(x) **Prim alg.**

(x, P(x))

fán kívüli csúcsok C-értékei (Q halmaz)

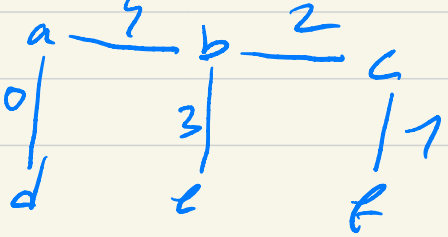
a	b	c	d	e	f	LÉPÉS	MST-be
∞	∞	∞	0	∞	∞		
0	∞	∞		6	∞	0.	d
	4	∞		5	∞	1.	a (a 0 d)
		2		3	2	2.	b (a 4 b)
				3	1	3.	c (b 2 c)
				3		4.	f (c 1 f)
						5.	e (b 3 e)
0	4	2	0	3	1	EREDMÉNY	

P-értékek változásai

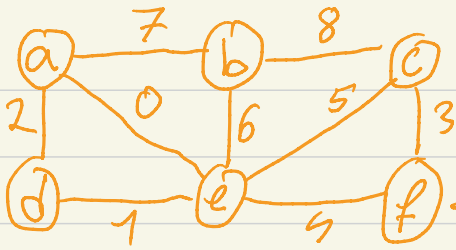
a	b	c	d	e	f
⊗	⊗	⊗	⊗	⊗	⊗
d				d	
	a			a	
		b		b	b
					c
d	a	b	⊗	b	c

MT(n,m) ∈ O(m * log n)

ha a gráfot szomszédossági listásanni Q-t min. kupaccal + indextáblával reprezentáljuk. (invertált kupaccal)



HF (nov. 22-ig)

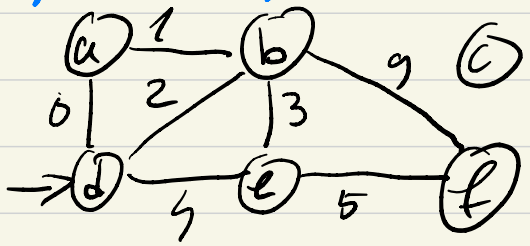


1) Kruskal alg. szemléltetése (2p)

2) Prim (root = d) ——— // ——— (2p)

LEGRÖVIDEBB UTAK EGY FORRÁSBÓL (START CSÚCSBÓL)

Dijkstra alg.

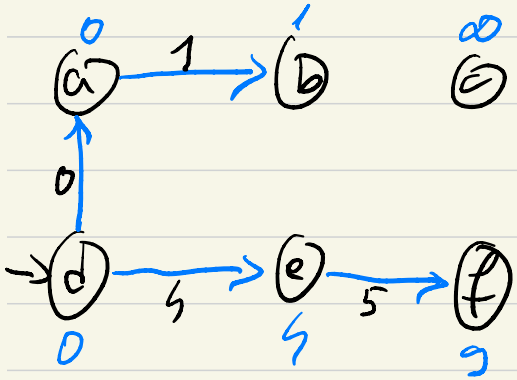


Előf.: $\forall (u,v) \in G.E: G.w(u,v) \geq 0$

nyílt csúcsok d-ek.

a	b	c	d	e	f	kiv. csúcs: d
∞	∞	∞	0	∞	∞	
0	2	∞		↯	∞	d:0
	1	∞		↯	∞	a:0
		∞		↯	10	b:1
		∞			9	e:↯
		∞				f:9
						(c:∞)
0	1	∞	0	↯	↯	EREDMÉNY

a	b	c	d	e	f
π					
⊘	⊘	⊘	⊘	⊘	⊘
d	d			d	
	a				
					b
					e
d	a	⊘	⊘	d	e

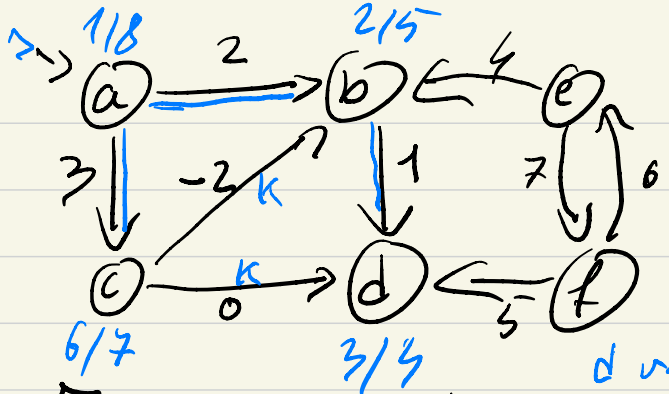


LEGRÖV. UTAK FÁJA

$MT(n,m) \in O((n+m)\log n) = O(n \log n)$

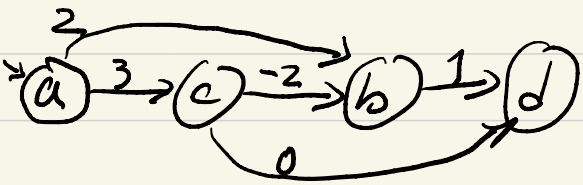
$mT(n) \in \Theta(n)$ $\quad m \in O(n)$

A nyílt csúcsok bin. kupacban + G: szomsz. listás.



DAG legrövidebb utak

Előf.: s-ből elérhető el (a).



6	a
7	c
5	b
4	d

Részleges top. sorrend

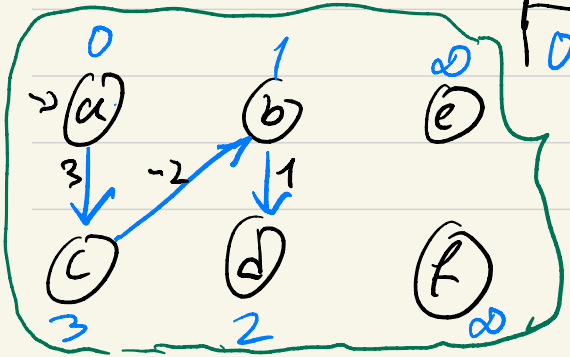
d múlt.

a	b	c	d	e	f
0	∞	∞	∞	∞	∞
	2	3			
	1		3		
			2		
0	1	3	2	∞	∞

lép.	
s : d	
a : 0	
c : 3	
b : 1	
EREDMÉNY	

Π múlt.

a	b	c	d	e	f
Θ	Θ	Θ	Θ	Θ	Θ
	a	a			
	c		c		
			b		
Θ	c	a	b	Θ	Θ



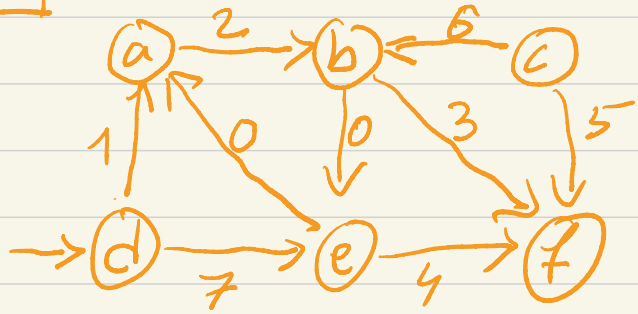
LEGRÖVIDUTAK FÁJA

$MT(n,m) \in \Theta(n+m)$

$mT(n) \in \Theta(n)$

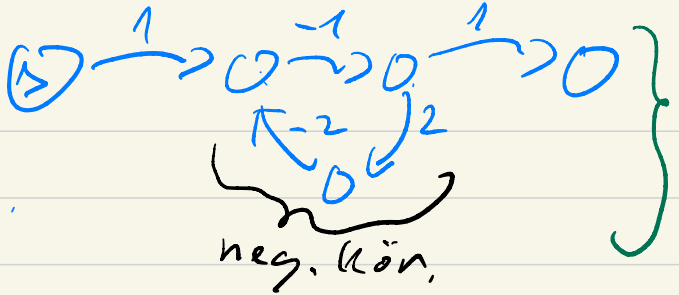
Sz HF

(2p)



Dijkstra
DAG legrövidebb utak alg.
(nov. 28. 14:00-ig)

} szemléltetése

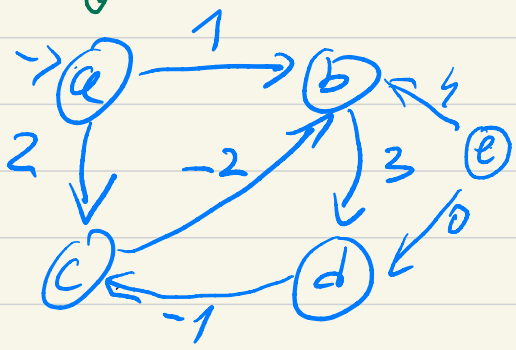


A jelöletlen csúcsokba ∄ optimális (legrövidebb) út

A LEGRÖV. UTAK EGY FORRÁSBÓL PROBLÉMA

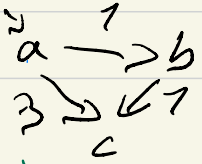
megoldható $\Leftrightarrow$ ∄ s-ből elérhető neg. kör.

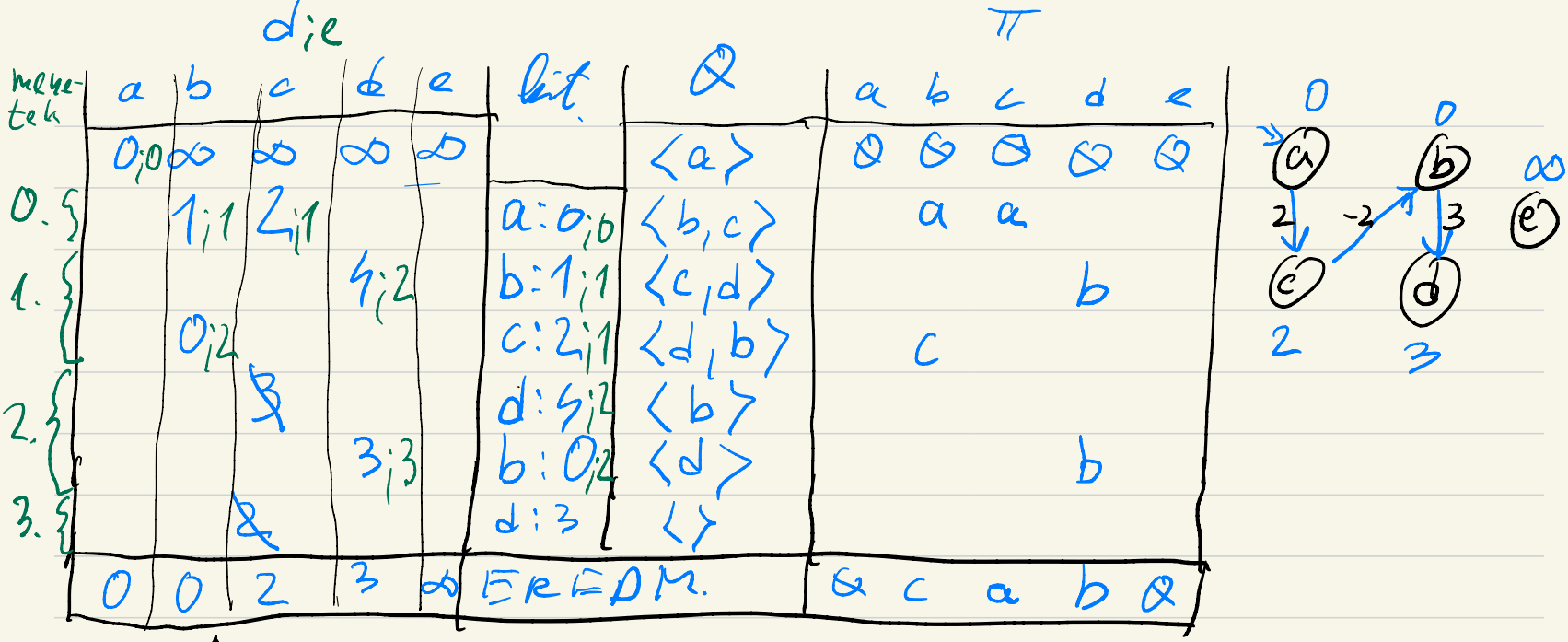
Pl.

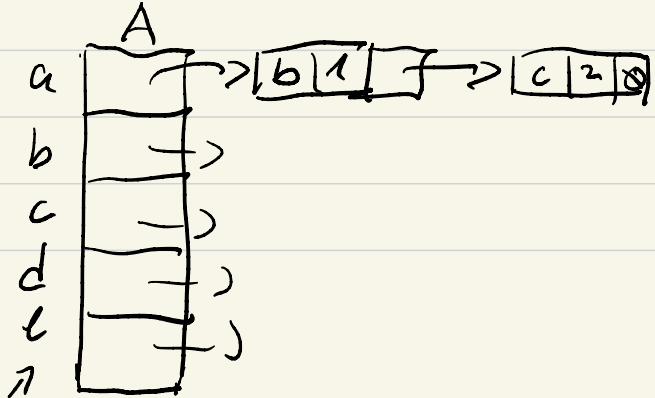


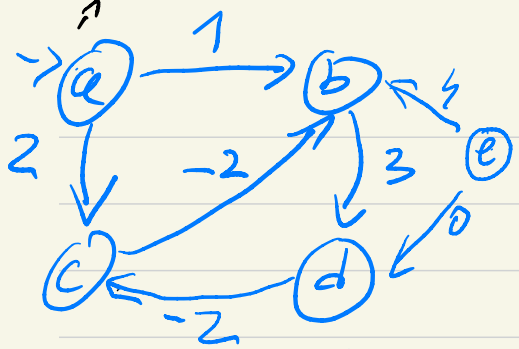
Probléma: Egyik sem alkalmazható
Dijkstra: negatív élek
DAG. legrövidebb utak: irányított kör érhető el a kezdőcsúcsból.

Mo:
Queue-based Bellman-Ford } QBF
(Breadth-first scanning)




d;e
π
Merke-tek
a b c d e
Hilf.
Q
0;0 ∞ ∞ ∞ ∞
1;1 2;1
4;2
0;2
3;3
0.
1.
2.
3.
⟨a⟩
a:0;0 ⟨b,c⟩
b:1;1 ⟨c,d⟩
c:2;1 ⟨d,b⟩
d:4;2 ⟨b⟩
b:0;2 ⟨d⟩
d:3 ⟨⟩
0 0 2 3 ∞ ERGEBN.
Ø Ø Ø Ø Ø
a a
b
c
b
0 2 3 ∞ 2 3
-3


A
a
b
c
d
e
b 1
c 2 Ø



d;e

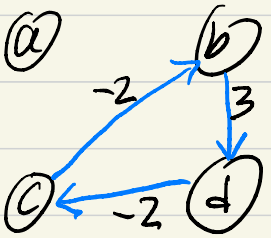
mehr-tek	a	b	c	d	e
	0;0	∞	∞	∞	∞
0.		1;1	2;1		
1.				4;2	
		0;2			
2.					
				3;3	
3.			1;4		
4.		-1;5			
		⑤			

kit.
a:0;0
b:1;1
c:2;1
d:4;2
b:1;2
d:3;3
c:1;4

Q
⟨a⟩
⟨b,c⟩
⟨c,d⟩
⟨d,b⟩
⟨b⟩
⟨d⟩
⟨c⟩
⟨⟩

π

	a	b	c	d	e
	⊘	⊘	⊘	⊘	⊘
		a	a		
				b	
		c			
				b	
			d		
		c			
	⊘	c	d	b	⊘



Edge
+v: ℕ
+w: ℝ+
+next: Edge*

SzHF 2023.12.03-ra: 3p

QBF(A/1:Edge*[n]; s:1..n; d/1:ℝ[n]; π/1:ℕ[n])

SzHF