

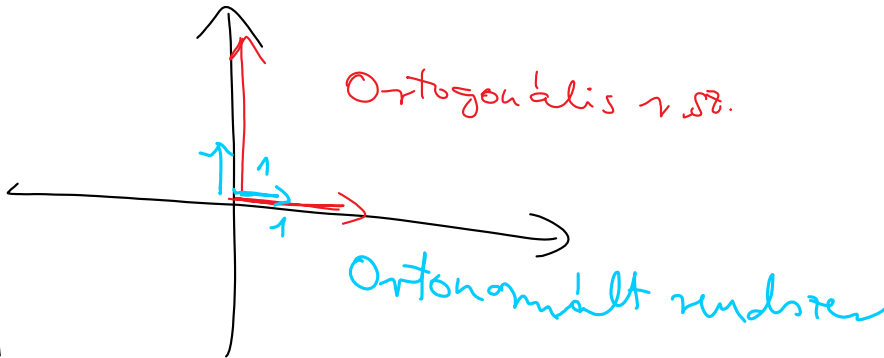
Skalárszorzás \mathbb{R}^n -ben:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$\underline{x} \cdot \underline{y} = (x_1, x_2, x_3, \dots, x_n)$$

$$= x_1 y_1 + x_2 y_2 + \dots + x_n y_n = \sum_{i=1}^n x_i y_i = \langle x, y \rangle$$

\mathbb{R}^2 -ben:



\mathbb{R}^n -ben ortonormált bázis O.N.B

$$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}, \dots, \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbb{R}^4 \quad (2) \quad x = (1, -2, -3, 5) \quad z = (2, -1, 1, 3) \\ y = (-1, 2, -1, 0) \quad x - z = (-1, -1, -4, 2)$$

$$a, \langle x, y \rangle = \left\langle \begin{pmatrix} 1 \\ -2 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \\ 0 \end{pmatrix} \right\rangle = \begin{matrix} -1 \\ -4 \\ 3 \\ 0 \end{matrix} = \underline{\underline{-2}}$$

$$b, \|x\| = \sqrt{\left\langle \begin{pmatrix} 1 \\ -2 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ -3 \\ 5 \end{pmatrix} \right\rangle} = \sqrt{1+4+9+25} = \underline{\underline{\sqrt{39}}}$$

norma

$$c, \|x - z\| = \sqrt{\left\langle \begin{pmatrix} -1 \\ -1 \\ -4 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ -4 \\ 2 \end{pmatrix} \right\rangle} = \sqrt{1+1+16+4} = \sqrt{22}$$

$$d, \underline{\langle x, z \rangle \cdot y - \langle y, z \rangle \cdot x}$$

$$\langle x, z \rangle = \left\langle \begin{pmatrix} 1 \\ -2 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix} \right\rangle = 2 + 2 - 3 + 15 = 16$$

$$\langle y, z \rangle = \left\langle \begin{pmatrix} -1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix} \right\rangle = -2 - 2 - 1 + 0 = -5$$

$$\langle y, y \rangle = \left\langle \begin{pmatrix} -1 \\ 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ -1 \\ 0 \end{pmatrix} \right\rangle = 1 + 4 + 1 + 0 = 6$$

$$16 \cdot \begin{pmatrix} -1 \\ 2 \\ -1 \\ 0 \end{pmatrix} + 5 \cdot \begin{pmatrix} 1 \\ -2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -16 + 5 \\ 32 - 10 \\ -16 - 15 \\ 0 + 25 \end{pmatrix} = \begin{pmatrix} -11 \\ 22 \\ -31 \\ 25 \end{pmatrix}$$

$$\frac{1}{6} \cdot \begin{pmatrix} -11 \\ 22 \\ -31 \\ 25 \end{pmatrix}$$

$$\frac{1}{6} \cdot \begin{pmatrix} 22 \\ -31 \\ 25 \end{pmatrix}$$

e, z irányú egységvektor

kell: $\frac{z}{\|z\|}$

$$\|z\| = \sqrt{\left\langle \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix} \right\rangle} = \sqrt{4+1+1+9} = \sqrt{15}$$

z -vel ellentétes irányú egységvektor:

$$\frac{1}{\sqrt{15}} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$$

$$-\frac{1}{\sqrt{15}} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \\ 3 \end{pmatrix}$$