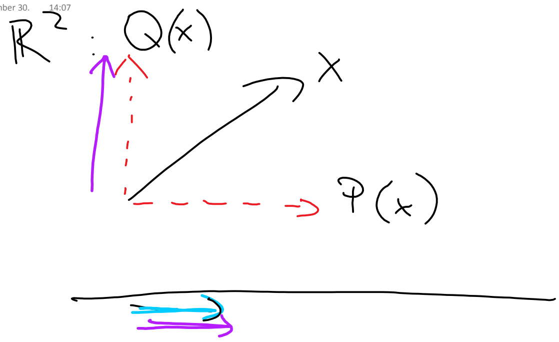
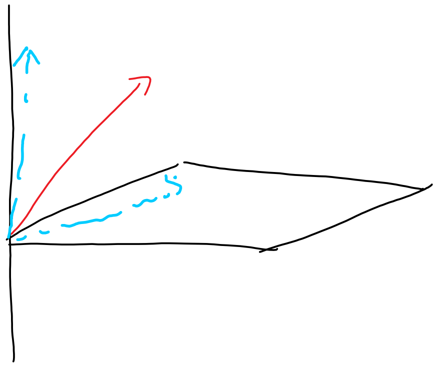


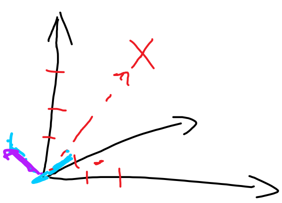


$$1.c,\quad P(x)=\frac{\langle x,u_1\rangle}{\langle u_1,u_1\rangle}\cdot u_1+\frac{\langle x,u_2\rangle}{\langle u_2,u_2\rangle}\cdot u_2+\frac{\langle x,u_3\rangle}{\langle u_3,u_3\rangle}\cdot u_3=$$

$$=\frac{\left\langle\begin{pmatrix}2\\1\\3\\1\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}{\left\langle\begin{pmatrix}1\\1\\1\\1\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}+\frac{\left\langle\begin{pmatrix}2\\1\\3\\1\end{pmatrix},\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\right\rangle}{\left\langle\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix},\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\right\rangle}\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}+\frac{\left\langle\begin{pmatrix}2\\1\\3\\1\end{pmatrix},\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}\right\rangle}{\left\langle\begin{pmatrix}-1\\0\\0\\1\end{pmatrix},\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}\right\rangle}\cdot\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}$$

$$=\frac{7}{4}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}+\frac{-1}{4}\cdot\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}+\frac{-1}{2}\cdot\begin{pmatrix}-1\\0\\0\\1\end{pmatrix}=\begin{pmatrix}2\\2\\2\\1\end{pmatrix}$$

$$Q(x)=x-P(x)=\begin{pmatrix}2\\1\\3\\1\end{pmatrix}-\begin{pmatrix}2\\2\\2\\1\end{pmatrix}=\begin{pmatrix}0\\-1\\1\\0\end{pmatrix}$$



2. $b_1=(1,1,1,1),\ b_2=(3,3,-1,-1),\ b_3=(-2,0,6,8)$

$u_1=b_1$

$$u_2=b_2-\frac{\langle b_2,u_1\rangle}{\langle u_1,u_1\rangle}\cdot u_1=\begin{pmatrix}3\\3\\-1\\-1\end{pmatrix}-\frac{\left\langle \begin{pmatrix}3\\3\\-1\\-1\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}{\left\langle \begin{pmatrix}1\\1\\1\\1\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=$$

$$=\begin{pmatrix}3\\3\\-1\\-1\end{pmatrix}-\frac{4}{4}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\begin{pmatrix}2\\2\\-2\\-2\end{pmatrix}\sim\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}$$

$$u_3=\begin{pmatrix}-2\\0\\6\\8\end{pmatrix}-\frac{\left\langle \begin{pmatrix}-2\\0\\6\\8\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}{\left\langle \begin{pmatrix}1\\1\\1\\1\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}-\frac{\left\langle \begin{pmatrix}-2\\0\\6\\8\end{pmatrix},\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}\right\rangle}{\left\langle \begin{pmatrix}1\\1\\-1\\-1\end{pmatrix},\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}\right\rangle}\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}$$

$$=\begin{pmatrix}-2\\0\\6\\8\end{pmatrix}-\frac{\overset{3}{\cancel{12}}}{\cancel{4}}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}+\frac{\overset{4}{\cancel{16}}}{\cancel{4}}\cdot\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}=$$

$$=\begin{pmatrix}-2\\0\\6\\8\end{pmatrix}-\begin{pmatrix}3\\3\\3\\3\end{pmatrix}+\begin{pmatrix}4\\4\\-4\\-4\end{pmatrix}=\begin{pmatrix}-2-3+4\\0-3+4\\6-3-4\\8-3-4\end{pmatrix}=\begin{pmatrix}-1\\1\\-1\\1\end{pmatrix}$$

O.R.: u_1, u_2, u_3

O.N.R

$$\frac{u_1}{\|u_1\|}=\begin{pmatrix}1/2\\1/2\\1/2\\1/2\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$\|u_1\|=\sqrt{\langle u_1,u_1\rangle}=\sqrt{4}=2$$

$$\frac{u_2}{\|u_2\|}=\begin{pmatrix}1/2\\1/2\\-1/2\end{pmatrix}=\frac{1}{2}\cdot\begin{pmatrix}1\\1\\-1\end{pmatrix}$$

$$\frac{}{\|u_2\|} = \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

$$\|u_2\| = 2$$

$$\frac{u_3}{\|u_3\|} = \begin{pmatrix} -1/2 \\ 1/2 \\ -1/2 \\ 1/2 \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\|u_3\| = \|(-1, 1, -1, 1)\| = 2$$

4. W – ben áll. O B ; O.N.B

$W \leq \mathbb{R}^4$

Δ $\|x\| + \|y\| \geq \|x+y\|$

$$3y_1 + 2y_2 + y_3 - 2y_4 = 0$$
$$5y_1 + 4y_2 + 3y_3 + 2y_4 = 0$$

$$\left(\begin{array}{cccc|c} 3 & 2 & \boxed{1} & -2 & 0 \\ 5 & 4 & 3 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 3 & 2 & \underline{1} & -2 & 0 \\ -4 & \boxed{-2} & 0 & 8 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -1 & 0 & \underline{1} & 6 & 0 \\ 2 & \underline{1} & 0 & -4 & 0 \end{array}\right) \begin{array}{l} \to y_3 = y_1 - 6y_4 \\ \to y_2 = -2y_1 + 4y_4 \end{array}$$

$y_1, y_4 \in \mathbb{R}$

$$y = \begin{pmatrix} y_1 \\ -2y_1 + 4y_4 \\ y_1 - 6y_4 \\ y_4 \end{pmatrix} = y_1 \cdot \underbrace{\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}}_{b_1} + y_4 \cdot \underbrace{\begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix}}_{b_2}, \quad y_1, y_4 \in \mathbb{R}$$

$$u_1 = b_1$$

$$u_2 = b_2 - \frac{\langle b_2, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1 = \begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix} + \frac{\cancel{14}^{\,7}}{\cancel{6}_{\,3}} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7/3 \\ -2/3 \\ -11/3 \\ 3/3 \end{pmatrix} =\sim \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} = u_2$$

O.B: u_1, u_2

O.B: u_1, u_2

$$u_1 = \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix}$$

O.N.B

$$e_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{6}} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \qquad e_2 = \frac{u_2}{\|u_2\|} = \frac{1}{\sqrt{183}} \cdot \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix}$$

$$\|u_1\| = \sqrt{1+4+1+0} = \sqrt{6} \qquad \|u_2\| = \sqrt{49+4+121+9} = \sqrt{183}$$

b, $x = (3, 4, -3, 5)$

$$P(x) = \frac{\left\langle \begin{pmatrix} 3 \\ 4 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\|u_1\|^2} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \frac{\left\langle \begin{pmatrix} 3 \\ 4 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} \right\rangle}{\|u_2\|^2} \cdot \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} =$$

$$= \frac{\overset{-4}{\cancel{-8}}}{\underset{3}{\cancel{6}}} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \frac{\overset{1}{\cancel{61}}}{\underset{3}{\cancel{183}}} \cdot \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 3 \\ 6 \\ -15 \\ 3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 2 \\ -5 \\ 1 \end{pmatrix}}}$$

$$Q(x) = x - P(x) = \begin{pmatrix} 3 \\ 4 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -5 \\ 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 \\ 2 \\ 2 \\ 4 \end{pmatrix}}}$$