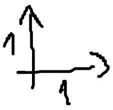
$\mathbb{R}^2$ 

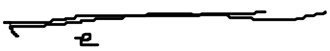
 ortogonális = merőleges

O.B.

normált = egységnyi hosszú

O.N.B.


$v = v_1 + v_2$ és $v_1 \perp v_2$




$v_1 \parallel e$

$V$ vektortér, $x \in V$

$W$ altér $e_1, e_2, \dots, e_n$ O.N.B

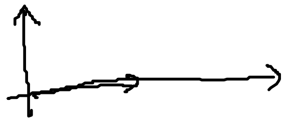
$x = x_1 + x_2$ : $x_1 \in W$ (párh.), $x_2 \perp W$

$$P(x) = x_1 = \sum_{i=1}^{n} \langle x, e_i \rangle \cdot e_i =$$

$$= \langle x, e_1 \rangle \cdot e_1 + \langle x, e_2 \rangle \cdot e_2 + \dots + \langle x, e_n \rangle \cdot e_n$$

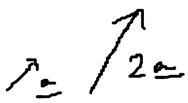
$$Q(x) = x_2 = x - x_1 = x - \sum_{i=1}^{n} \langle x, e_i \rangle \cdot e_i$$

Legyen $u_1, u_2, \dots, u_n \in W$, O.B



$\|u_1\|$: hossza

$\rightarrow \dfrac{u_1}{\|u_1\|}$ : egységnyi hosszú vektor




$$\boxed{P(x)} = \sum_{i=1}^{n} \left\langle x, \frac{u_i}{\|u_i\|} \right\rangle \cdot \frac{u_i}{\|u_i\|} = \boxed{\sum_{i=1}^{n} \frac{\langle x, u_i \rangle}{\langle u_i, u_i \rangle} \cdot u_i}$$

$$\boxed{Q(x) = x - \sum_{i=1}^{n} \frac{\langle x, u_i \rangle}{\langle u_i, u_i \rangle} \cdot u_i}$$

Kiindulás: lin. ftlen. $b_1, b_2, \dots, b_n$ $\rightarrow$ ebből felépített ort. rszt

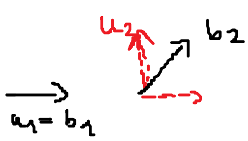
cél: • ugyanazt generálják

• ortogonálisak legyenek

• $u_1 := b_1$

$$u_2 := b_2 - \frac{\langle b_2, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1$$

$u_2$: $b_2$-nek az $u_1$-re merőleges komponense



$u_1 = b_1$

$$u_3 := b_3 - \frac{\langle b_3, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1 - \frac{\langle b_3, u_2 \rangle}{\langle u_2, u_2 \rangle} \cdot u_2$$

$\vdots$

23. 1. c) $x = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}$ $\left\langle \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle = 1+1+1+1=4$

$$P(x) = \frac{\langle x, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1 + \frac{\langle x, u_2 \rangle}{\langle u_2, u_2 \rangle} \cdot u_2 + \frac{\langle x, u_3 \rangle}{\langle u_3, u_3 \rangle} \cdot u_3 =$$

$$= \frac{\left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \right\rangle}{4} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{\left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle}{4} \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \frac{\left\langle \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle}{2} \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$= \frac{7}{4} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \frac{-1}{4} \cdot \begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} + \frac{-2}{4} \cdot \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{4} \cdot \begin{pmatrix} 8 \\ 8 \\ 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$

$$Q(x) = \begin{pmatrix} 2 \\ 1 \\ 3 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

② Kell: O.R. $b_1=(1,1,1,1)$, $b_2=(3,3,-1,-1)$, $b_3=(-2,0,6,8)$

$$u_1=b_1$$

$$u_2=b_2-\frac{\langle b_2,u_1\rangle}{\langle u_1,u_1\rangle}\cdot u_1=\begin{pmatrix}3\\3\\-1\\-1\end{pmatrix}-\frac{\left\langle\begin{pmatrix}3\\3\\-1\\-1\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}{4}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\begin{pmatrix}3\\3\\-1\\-1\end{pmatrix}-\frac{4}{4}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=$$

$$=\begin{pmatrix}2\\2\\-2\\-2\end{pmatrix}\rightsquigarrow\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}$$

$$u_3=b_3-\frac{\langle b_3,u_1\rangle}{\langle u_1,u_1\rangle}\cdot u_1-\frac{\langle b_3,u_2\rangle}{\langle u_2,u_2\rangle}\cdot u_2=\begin{pmatrix}-2\\0\\6\\8\end{pmatrix}-\frac{\left\langle\begin{pmatrix}-2\\0\\6\\8\end{pmatrix},\begin{pmatrix}1\\1\\1\\1\end{pmatrix}\right\rangle}{4}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}-\frac{\left\langle\begin{pmatrix}-2\\0\\6\\8\end{pmatrix},\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}\right\rangle}{4}\cdot\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}=$$

$$=\begin{pmatrix}-2\\0\\6\\8\end{pmatrix}-\frac{\cancel{12}^{\,3}}{\cancel{4}_{\,1}}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}-\frac{\cancel{-16}^{\,-4}}{\cancel{4}_{\,1}}\cdot\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}=\begin{pmatrix}-1\\1\\-1\\1\end{pmatrix}$$

O.R.:

$$u_1=\begin{pmatrix}1\\1\\1\\1\end{pmatrix},\quad u_2=\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix},\quad u_3=\begin{pmatrix}-1\\1\\-1\\1\end{pmatrix}$$

O.N.R.

$$\|u_1\|=\sqrt{\langle u_1,u_1\rangle}=\sqrt{1^2+1^2+1^2+1^2}=\sqrt{4}=2$$

$$e_1=\frac{u_1}{\|u_1\|}=\frac{1}{2}\cdot\begin{pmatrix}1\\1\\1\\1\end{pmatrix}=\begin{pmatrix}1/2\\1/2\\1/2\\1/2\end{pmatrix}$$

$$e_2=\frac{u_2}{\|u_2\|}=\frac{1}{2}\cdot\begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}$$

$$e_3=\frac{u_3}{\|u_3\|}=\frac{1}{2}\cdot\begin{pmatrix}-1\\1\\-1\\1\end{pmatrix}$$

(3.) előző feladat

$u_1, u_2, u_3$: gen. rsz.

ortogonálisak => függetlenek

} bázis : O.B.

$e_1, e_2, e_3$ : normált : O.N.B.

4. a)

$$\left.\begin{aligned} 3y_1+2y_2+y_3-2y_4&=0 \\ 5y_1+4y_2+3y_3+2y_4&=0 \end{aligned}\right\}$$

$$\left(\begin{array}{cccc|c} 3 & 2 & \boxed{1} & -2 & 0 \\ 5 & 4 & 3 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 3 & 2 & \underline{1} & -2 & 0 \\ -4 & \boxed{-2} & 0 & 8 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} -1 & 0 & \underline{1} & 6 & 0 \\ 2 & \underline{1} & 0 & -4 & 0 \end{array}\right) \begin{array}{l} \to y_3 = y_1 - 6y_4 \\ \to y_2 = -2y_1 + 4y_4 \end{array}$$

$y_2, y_3$ kötött ism.

$y_1, y_4 \in \mathbb{R}$ szabad ism.

$$y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} y_1 \\ -2y_1+4y_4 \\ y_1-6y_4 \\ y_4 \end{pmatrix} = y_1 \cdot \underbrace{\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}}_{b_1} + y_4 \cdot \underbrace{\begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix}}_{b_2} \qquad y_1, y_4 \in \mathbb{R}$$

O.B.:

$u_1 = b_1$

$$u_2 = b_2 - \frac{\langle b_2, u_1\rangle}{\langle u_1, u_1\rangle} \cdot u_1 = \begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 4 \\ -6 \\ 1 \end{pmatrix} - \frac{-14}{6} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 14/6 \\ -4/6 \\ -22/6 \\ 6/6 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix}$$

O.B.: $u_1 = (1, -2, 1, 0)$, $u_2 = (7, -2, -11, 3)$

O.N.B. $\|u_1\| = \sqrt{\langle u_1, u_1\rangle} = \sqrt{1^2 + (-2)^2 + 1^2} = \sqrt{6}$

$\|u_2\| = \sqrt{\langle u_2, u_2\rangle} = \sqrt{7^2 + 2^2 + 11^2 + 3^2} = \sqrt{49 + 4 + 121 + 9} = \sqrt{183}$

$$\|u_2\| = \sqrt{\langle u_2, u_2\rangle} = \sqrt{7^2 + 2^2 + 11^2 + 3^2} = \sqrt{49 + 4 + 121 + 9} = \sqrt{183}$$

O.N.B.:

$$e_1 = \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{6}} \cdot (1, -2, 1, 0)$$

$$e_2 = \frac{u_2}{\|u_2\|} = \frac{1}{\sqrt{183}} \cdot (7, -2, -11, 3)$$

b, $x = (3, 4, -3, 5)$

$$P(x) = \frac{\langle x, u_1\rangle}{\langle u_1, u_1\rangle} \cdot u_1 + \frac{\langle x, u_2\rangle}{\langle u_2, u_2\rangle} \cdot u_2 = \frac{\left\langle \begin{pmatrix} 3 \\ 4 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{6} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \frac{\left\langle \begin{pmatrix} 3 \\ 4 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} \right\rangle}{183} \cdot \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} =$$

$$= \frac{\cancel{-8}^{\,-4}}{\cancel{6}_{\,3}} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + \frac{\cancel{61}^{\,1}}{\cancel{183}_{\,3}} \begin{pmatrix} 7 \\ -2 \\ -11 \\ 3 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 3 \\ 6 \\ -15 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \\ 1 \end{pmatrix}$$

$$Q(x) = x - P(x) = \begin{pmatrix} 3 \\ 4 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ -5 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 4 \end{pmatrix}$$

c, $W = \mathrm{Ker}\left( \begin{bmatrix} 3 & 2 & 1 & -2 \\ 5 & 4 & 3 & 2 \end{bmatrix} \right)$