Taguchi method

Example

10/04/2020

Raw material mixing ratio and strength

No.	Raw materials 1	2	3	4	5	Additives 1	2	Strength
1	34.27	7.10	20.08	24.30	9.48	1.17	3.60	49.77
2	26.78	21.71	15.23	23.84	7.00	1.74	3.70	53.73
3	17.01	26.04	19.65	23.16	9.41	1.12	3.60	54.10
4	23.77	22.25	15.40	25.67	7.00	2.21	3.70	54.29
5	22.11	21.71	19.91	23.84	7.00	1.74	3.70	56.27
6	22.14	30.49	11.15	23.88	7.00	1.74	3.60	56.45
7	22.11	21.71	19.91	23.84	7.00	1.74	3.70	59.14
8	20.81	21.05	19.25	26.56	7.00	1.63	3.69	59.89
9	12.18	31.64	19.91	23.84	7.00	1.74	3.70	60.59
10	19.66	23.15	21.35	22.75	7.00	2.37	3.71	61.51

Selected >>Close to average value

avg=56.574

Unit space

Nos. 5 and 6, the respective strengths of which figure in the vicinity of average values, as Unit Space data

	Raw materials					Additives		
No.	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**Strength**
5	22.11	21.71	19.91	23.84	7.00	1.74	3.70	56.27
6	22.14	30.49	11.15	23.88	7.00	1.74	3.60	56.45
Average	22.13	26.1	15.53	23.86	7.00	1.74	3.65	56.36

Signal data

we chose as Signal Data the remaining eight pieces of data excluding the two items of data selected for the Unit Space.

Unit space Data Average

Average	22.13	26.1	15.53	23.86	7.00	1.74	3.65	56.36

	Raw materials					Additives		
No.	1	2	3	4	5	1	2	Strength
1	34.27	7.10	20.08	24.30	9.48	1.17	3.60	49.77
2	26.78	21.71	15.23	23.84	7.00	1.74	3.70	53.73
3	17.01	26.04	19.65	23.16	9.41	1.12	3.60	54.10
4	23.77	22.25	15.40	25.67	7.00	2.21	3.70	54.29
7	22.11	21.71	19.91	23.84	7.00	1.74	3.70	59.14
8	20.81	21.05	19.25	26.56	7.00	1.63	3.69	59.89
9	12.18	31.64	19.91	23.84	7.00	1.74	3.70	60.59
10	19.66	23.15	21.35	22.75	7.00	2.37	3.71	61.51

Normalization of signal data

We normalize the individual pieces of Signal Data by subtracting the average values of each of the items in the Unit Space and for each of the strengths.

Unit space Data Average

Average	22.13	26.1	15.53	23.86	7.00	1.74	3.65	56.36

	Raw materials					Additives		
No.	1	2	3	4	5	1	2	Strength
1	12.14	−19.00	4.55	0.44	2.48	−0.57	−0.05	−6.59
2	4.65	−4.39	−0.30	−0.02	0.00	0.00	0.05	−2.63
3	−5.12	−0.06	4.12	−0.70	2.41	−0.62	−0.05	−2.26
4	1.64	−3.85	−0.13	1.81	0.00	0.47	0.05	−2.07
7	−0.02	−4.39	4.38	−0.02	0.00	0.00	0.05	2.78
8	−1.32	−5.05	3.72	2.70	0.00	−0.11	0.04	3.53
9	−9.95	5.54	4.38	−0.02	0.00	0.00	0.05	4.23
10	−2.47	−2.95	5.82	−1.11	0.00	0.63	0.06	5.15

COMPUTATION OF PROPORTIONAL COEFFICIENT b AND SN RATIO h

$$b_1 = \frac{(\text{stress})_1 \times (\text{raw material})_{1\,1} + (\text{stress})_2 \times (\text{raw material})_{2\,1} + \ldots + (\text{stress})_{10} \times (\text{raw material})_{10\,1}}{r}$$

$$h_1 = \frac{S_{b1} - V_{e1}}{r \times V_{e1}} \quad (\text{If } S_{b1} > V_{e1})$$

Ve1=Error variance

$$h_1 = 0 \quad (\text{If } S_{b1} \leq V_{e1})$$

$$r = (\text{stress})_1^2 + (\text{stress})_2^2 + \ldots + (\text{stress})_{10}^2$$

	Raw materials					Additives		
No.	1	2	3	4	5	1	2	Strength
1	12.14	−19.00	4.55	0.44	2.48	−0.57	−0.05	−6.59 r
2	4.65	−4.39	−0.30	−0.02	0.00	0.00	0.05	−2.63
3	−5.12	−0.06	4.12	−0.70	2.41	−0.62	−0.05	−2.26
4	1.64	−3.85	−0.13	1.81	0.00	0.47	0.05	−2.07
7	−0.02	−4.39	4.38	−0.02	0.00	0.00	0.05	2.78
8	−1.32	−5.05	3.72	2.70	0.00	−0.11	0.04	3.53
9	−9.95	5.54	4.38	−0.02	0.00	0.00	0.05	4.23
10	−2.47	−2.95	5.82	−1.11	0.00	0.63	0.06	5.15

(-6.59 x 12.14) +……. N10

Variation of Proportional term,

$$S_{b1} = \frac{\{(\text{stress})_1 \times (\text{raw material})_{1\,1} + (\text{stress})_2 \times (\text{raw material})_{2\,1} + \ldots + (\text{stress})_{10} \times (\text{raw material})_{10\,1}\}^2}{r}$$

$$\text{Total variation } S_{T1} = (\text{raw material})^2_{1\,1} + (\text{raw material})^2_{2\,1} + \ldots + (\text{raw material})^2_{10\ 1}$$

$$\text{Error variation, } S_{e1} = S_{T1} - S_{b1}$$

$$\text{Error variance, } V_{e1} = \frac{S_{e1}}{l - 1}$$

	Raw materials					Additives	
β, η	**1**	**2**	**3**	**4**	**5**	**1**	**2**
β	−1.155	0.990	0.286	−0.010	−0.175	0.057	0.008
η	0.059	0.011	0.000	0.000	0.018	0.016	0.030

COMPUTATION OF INTEGRATED ESTIMATED STRENGTH VALUE $\hat{M}$ FOR EACH DATA ITEM

$$\hat{M}_1 = \frac{h_1 \times \frac{(\text{raw material})_{1\,1}}{b_1} + h_2 \times \frac{(\text{raw material})_{1\,2}}{b_2} + \ldots + h_7 \times \frac{(\text{raw material})_{1\,7}}{b_7}}{h_1 + h_2 + h_3 + h_4 + h_5 + h_6 + h_7}$$

Data No.	Measured Value M	Integrated Estimate Value $\hat{M}$
1	−6.59	−10.71
2	−2.63	−0.80
3	−2.26	−2.58
4	−2.07	1.44
7	2.78	0.99
8	3.53	1.09
9	4.23	5.61
10	5.15	3.59

COMPUTATION OF INTEGRATED ESTIMATE SN RATIO (db)

Proportional equation $L = M_1\hat{M}_1 + M_2\hat{M}_2 + \ldots + M_{10}\hat{M}_{10}$

$= (-6.59)\times(-10.71) + (-2.63)\times(-0.80) + \ldots + 5.15\times 3.59$

$= 124.3422$

Total variation, $S_T = \hat{M}_1^2 + \hat{M}_2^2 + \ldots + \hat{M}_{10}^2$

$= (-10.71)^2 + (-0.80)^2 + \ldots + (3.59)^2$

$= 170.5354$

$r = (\text{stress})_1^2 + (\text{stress})_2^2 + \ldots + (\text{stress})_{10}^2$

$= (-6.59)^2 + (-2.63)^2 + \ldots + (5.15)^2$

$= 124.3422$

Variation of proportional term, $S_b = \frac{L^2}{r}$

$$= \frac{(124.3422)^2}{124.3422}$$

$$= 124.3422$$

Error variation $S_e = S_T - S_b$

$$= 170.5354 - 124.3422$$

$$= 46.1932$$

Error variance, $V_e = \frac{S_e}{l - 1} = \frac{46.1932}{7} = 6.5990$

$$\text{Integrated SN ratio, } h = 10 \log \left(\frac{S_b - V_e}{r \times V_e} \right)$$

$$= 10 \log \frac{124.3422 - 6.5990}{124.3422 \times 6.5990}$$

$$= 10\log(0.1435)$$

$$= -8.43$$

Evaluation of the Relative Importance of an Item

The relative importance of an item is evaluated in terms of the extent to which the Integrated Estimate SN Ratio ratio deteriorates when the item is not used.

Where the two levels of the orthogonal ray mean the following:

Level 1: Item will be used.

Level 2: Item will *not* be used.

	Item No.											
	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	Integrated Estimate
No.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**SN Ratio η(db)**
1	1	1	1	1	1	1	1	1	1	1	1	η_1
2	1	1	1	1	1	2	2	2	2	2	2	η_2
3	1	1	2	2	2	1	1	1	2	2	2	η_3
4	1	2	1	2	2	1	2	2	1	1	2	η_4
5	1	2	2	1	2	2	1	2	1	2	1	η_5
6	1	2	2	2	1	2	2	1	2	1	1	η_6
7	2	1	2	2	1	1	2	2	1	2	1	η_7
8	2	1	2	1	2	2	2	1	1	1	2	η_8
9	2	1	1	2	2	2	1	2	2	1	1	η_9
10	2	2	2	1	1	1	1	2	2	1	2	η_{10}
11	2	2	1	2	1	2	1	1	1	2	2	η_{11}
12	2	2	1	1	2	1	2	1	2	2	1	η_{12}

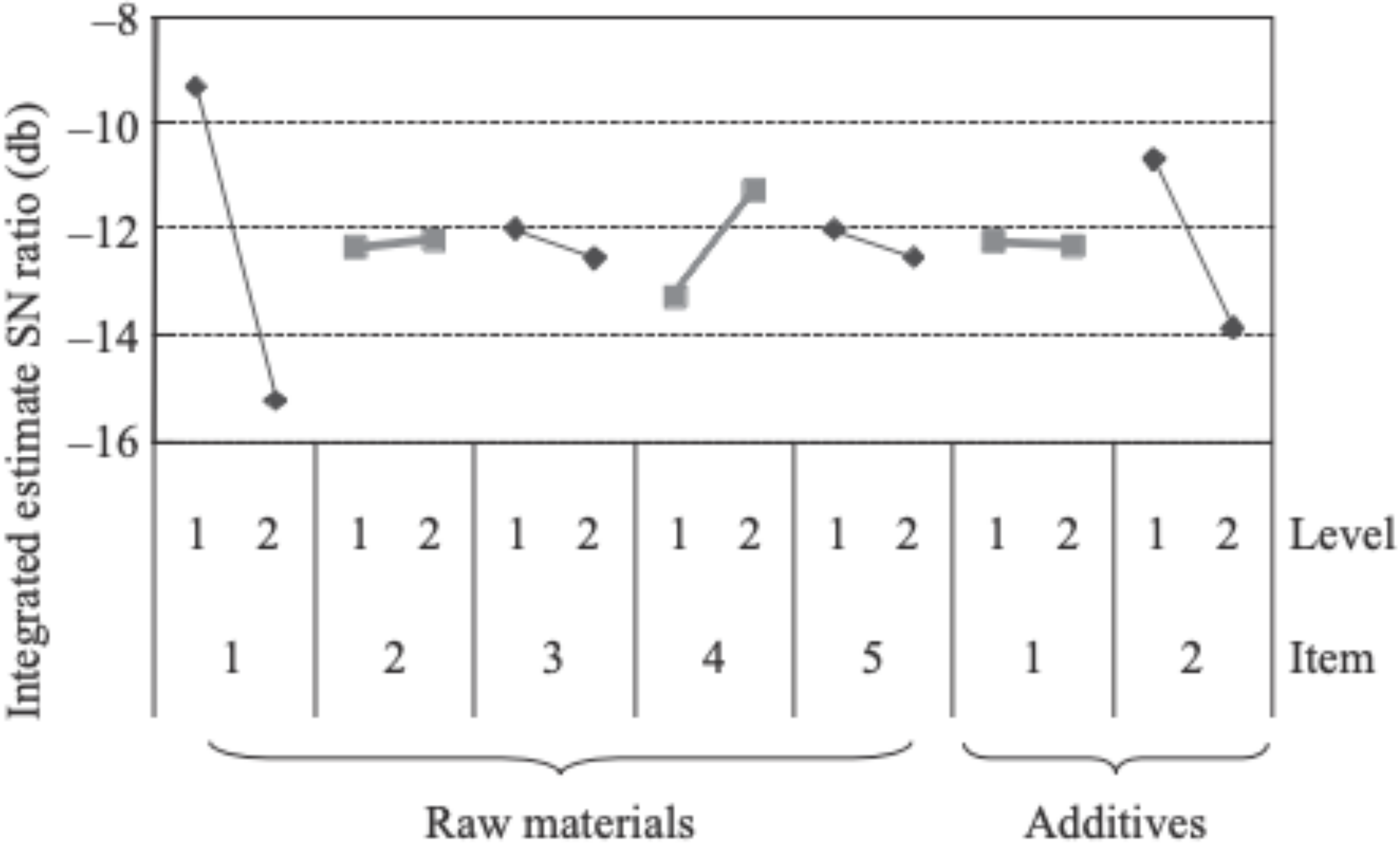
	Raw materials					Additives						Integrated estimate
No.	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**e**	**e**	**e**	**e**	**SN ratio (db)**
1	1	1	1	1	1	1	1	1	1	1	1	−8.43
2	1	1	1	1	1	2	2	2	2	2	2	−11.24
3	1	1	2	2	2	1	1	1	2	2	2	−8.37
4	1	2	1	2	2	1	2	2	1	1	2	−9.52
5	1	2	2	1	2	2	1	2	1	2	1	−8.45
6	1	2	2	2	1	2	2	1	2	1	1	−9.82
7	2	1	2	2	1	1	2	2	1	2	1	−15.10
8	2	1	2	1	2	2	2	1	1	1	2	−19.53
9	2	1	1	2	2	2	1	2	2	1	1	−11.31
10	2	2	2	1	1	1	1	2	2	1	2	−13.96
11	2	2	1	2	1	2	1	1	1	2	2	−13.46
12	2	2	1	1	2	1	2	1	2	2	1	−17.96

Creation of a Factorial Effect Chart of Integrated Estimate SN Ratios (db)

Items	Level 1	Level 2
Material 1	−9.31	−15.22
Material 2	−12.33	−12.20
Material 3	−11.99	−12.54
Material 4	−13.26	−11.27
Material 5	−12.00	−12.53
Additive 1	−12.22	−12.30
Additive 2	−10.67	−13.86

auxiliary table of (averages by level to) integrated estimate SN ratios (db)

We then create, from the auxiliary table, a factorial effect chart. From this figure, we learn that, given the significant shrinkage of the mixing ratios of raw material 1 and additive 2 from Level 1 to Level 2, the mixing ratio of raw material 1 and additive 2 plays a major part in product strength prediction.



Computation of Integrated Estimate Signal Data Value M under Optimum Conditions

Case 1: Items where the SN ratios came out positive in Table 7, namely raw material 1, 2, and 5 and additive 1 and 2.

Case 2: Those items where integrated estimate SN ratios came out greater on Level 1 than on Level 2 in previous Figure, namely raw material 1 and 5 and additive 2. (Raw material 3 will not be used because its SN ratio was zero in Table 7.)

$$\hat{M}_1 = \frac{h_1 \times \dfrac{(\text{raw material})_{1\,1}}{b_1} + h_5 \times \dfrac{(\text{raw material})_{1\,5}}{b_5} + h_7 \times \dfrac{(\text{raw material})_{1\,7}}{b_7}}{h_1 + h_5 + h_7}$$

Measured value $\hat{M}$ and integrated estimate value M for strength in case 2

Data No.	Measured value M	Integrated estimate value $\hat{M}$
1	−6.59	−9.93
2	−2.63	−0.53
3	−2.26	−1.61
4	−2.07	0.97
7	2.78	1.70
8	3.53	2.18
9	4.23	6.45
10	5.15	3.14

Comparison of the integrated estimate SN ratios

Case	Used items	Integrated estimate SN ratio (db)
1	Raw materials 1,2,5 Additives 1,2	−8.43
2	Raw materials 1,5 Additives 2	−7.44

This table shows that SN ratio for case 2 is 0.99 greater than case1. Case 2 is thereby determined to represent the optimum conditions

INTEGRATED ESTIMATION OF UNKNOWN DATA

Newly acquired unknown data :—

	Raw materials					Additives		
No.	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**Strength**
<1>	23.77	22.25	15.40	25.67	7.00	2.21	3.70	Unknown
<2>	17.44	21.71	24.58	23.84	7.00	1.74	3.70	Unknown

Normalization is performed by subtracting from it the average of each item of the Unit Space.

	Raw materials					Additives		
No.	**1**	**2**	**3**	**4**	**5**	**1**	**2**	**Strength**
<1>	1.64	−3.85	−0.12	1.81	0.00	0.47	0.05	Unknown
<2>	−4.69	−4.39	9.05	−0.02	0.00	0.00	0.05	Unknown

The integrated estimate value M of unknown data, normalized, is found as follows:

$$\hat{M}_{<1>} = \frac{0.059 \times \dfrac{1.64}{(-1.155)} + 0.018 \times \dfrac{0}{(-0.175)} + 0.030 \times \dfrac{0.05}{0.008}}{0.059 + 0.018 + 0.030} = \frac{-0.084 + 0 + 0.188}{0.107} = 0.97$$

$$\hat{M}_{<2>} = \frac{0.059 \times \dfrac{(-4.69)}{(-1.155)} + 0.018 \times \dfrac{0}{(-0.175)} + 0.030 \times \dfrac{0.05}{0.008}}{0.059 + 0.018 + 0.030} = \frac{0.240 \ + 0 + 0.182}{0.107} = 3.94$$

Computation of integrated estimate value before normalization

The integrated estimate value $\hat{y}$ for their pre-normalization (actual) strength is found by adding the average value M_0 (= 56.36 MPa) of the Unit Space strength to the normalized integrated $\hat{M}$.

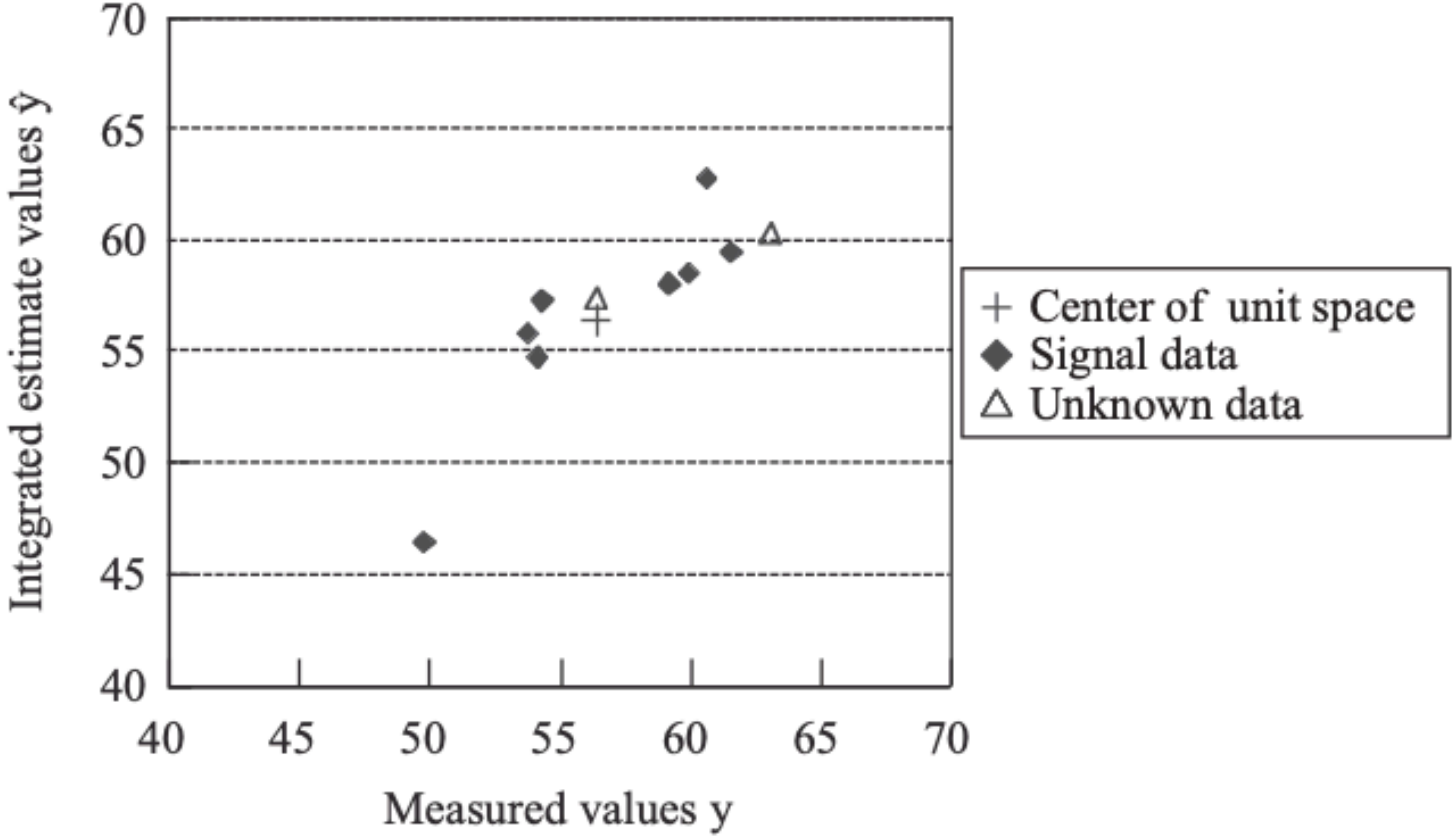
$$\hat{y} = M + M_0$$

$$= M + 56.36 \text{ (MPa)}$$

Measured value y and integrated estimate value ŷ of signal data and unknown data

Data No.		Measured value y	Integrated estimate value $\hat{y}$
Signal Data	1	49.77	46.43
	2	53.73	55.83
	3	54.10	54.75
	4	54.29	57.33
	7	59.14	58.06
	8	59.89	58.54
	9	60.59	62.81
	10	61.51	59.50
Unknown Data	<1>	56.40	57.33
	<2>	63.11	60.30

Scatter diagram of true strength values (measured values), y, and integrated estimate values, $\hat{y}$.



A look at the figure makes it clear that, given how the plotted dots are almost exactly lined up with A 45-degree straight line, both the Signal Data and unknown data are relatively closely estimated.