Chapter - 1

[Introduction to Probability]

1.1 [Basic Terms Linked with Probability]

1. **Random Experiments :**
Experiments which do not produce the same outcome every time i.e. experiments whose outcomes cannot be predicted or determined in any advance are called random experiments.
Example : Tossing a coin (Result cannot be predicted in advance)

2. **Sample Space :**
The set of all possible outcomes of a random experiments is called sample space. It is denoted as S.
Example : $S = \{H, T\}$ (in case of tossing a coin)
$S = \{1, 2, 3, 4, 5, 6\}$ (in case of throwing a die)

3. **Event :**
A subset of sample space associated with a random experiment is called an event.
Example : $S = \{1, 2, 3, 4, 5, 6\}$ (in case of rolling a die)
Let E = be the event of getting prime number, then elements 2, 3, 5 belong to event E .

4. **Types of Events :**
(a) **Elementary event :** If a random experiment is performed, then each of its outcomes is known as an elementary event.
(b) **Sure event :** An event is called sure event if its occurrence is sure whenever experiment is performed.
Example : Event "getting a number from 1 to 6" is a sure event, in case of rolling a die.
(c) **Impossible event :** An event is called impossible event if it can never occur whenever the experiment is performed. Event "getting a number 7" is an impossible event in case of rolling a die.
(d) **Mutually Exclusive events :** 2 events A and B are said to be mutually exclusive events if events A and B cannot occur simultaneously in the same trial.
Example : Event A "getting even number" and event B "getting odd number" are two mutually exclusive events.
***Remark* : $A \cap B = \phi$ if A and B are mutually exclusive events.**
(e) **Equally likely events :** Events are said to be equally likely if there is no reason to expect any one in preference to any other.
Example : In case of throwing a die, all the six faces are equally likely to come.
(f) **Independent events :** Two events are called independent events if happening / occurrence of one event does not depend on the happening / occurrence of another, otherwise they are called dependent events.
Example : When card is replaced after drawing the first card from the pack of 52 well shuffled cards, in that case drawing of second card is in dependent of drawing first card. Events which are not independent are called dependent events.
(If the first card drawn is not replaced, the drawing of second card is dependent of drawing of first card).
(g) **Exhaustive events :** Two events A and B are said to be exhaustive events if $A_1 \cup A_2 = S$
(S = Sample space associated with random experiment).
Remark : This result can be generalised to more than two events.
Example : Consider the experiment of drawing a card from well shuffled pack of 52 cards.
Let A_1 A_2 A_3 A_4 be 4 events defined as
A_1 = Card drawn is of spade
A_2 = Card drawn is of diamond
A_3 = Card drawn is of hearts
A_4 = Card drawn is of club
Clearly $A_1 \cup A_2 \cup A_3 \cup A_4 = S$

(h) **Mutually Exclusive and Exhaustive event :** Two events A and B are said to be mutually exclusive and exhaustive events

(i) $A_1 \cup A_2 = S$ i.e. A_1 and A_2 are exhaustive events

(ii) $A_1 \cap A_2 = \phi$ i.e. A_1 and A_2 are mutually exclusive

(where S is the sample space associated with random experiment)

Remark : This result can be generalised to more than two events.

Example : Consider the experiment of rolling a die

Let A be the event of getting numbers less than 5 and B be the event of getting number more than 4.

$S = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4\}$; $B = \{5, 6\}$

Now $A \cup B = S$ and $A \cap B = \phi$

Examples

1. A coin is tossed twice. Find its sample space..

Solution:

Let 'S' be the sample space associated with random experiment.

Possible outcomes are : Head an first and Head on second
Head on first and Tail on second
Tail on first and Head on second
Tail on first and Tail on second

$\therefore$ $S = \{HH, HT, TH, TT\}$

2. A coin is tossed twice. If the first draw results in a head, a die is rolled and if the first draw results in a tail, coin is tossed again. Write the sample space for this experiments.

Solution:

Let 'S' be the sample space associated with random experiment

If the first draw results in a head, a die is rolled.

$\therefore$ Possible outcomes are (H1), (H2), (H3), (H4), (H5), (H6)

If the first draw results in a tail, a coin is tossed again

$\therefore$ Possible outcomes are (TH), (TT)

$\therefore$ $S = \{(H1), (H2), (H3), (H4), (H6), (TH), (TT)\}$

3. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Solution:

Let S be the sample space associated with this experiment

If the number on the die is even, coin is tossed

$\therefore$ Possible outcomes are (2 H), (2 T), (4 H), (4 T), (6 H), (6 T)

If the number on the die is odd, coin is tossed twice

$\therefore$ Possible outcomes are (1 HH), (1 HT), (1, TH), (1 TT)
(3 HH), (3 HT), (3, TH), (3 TT)
(5 HH), (5 HT), (5, TH), (5 TT)

$\therefore$ $S =$ {(2 H), (2 T), (4 H), (4 T), (6 H), (6 T), (1 HH), (1 HT), (1, TH), (1 TT), (3 HH), (3 HT), (3, TH), (3 TT), (5 HH), (5 HT), (5, TH), (5 TT)}

4. Two boys and two girls are in a room P and one boy and three girls are in a room Q. Write the sample space for the experiment in which a room is selected and then a person.

Solution:

Let S be the sample space associated with the random experiment.

Let 2 boys and 2 girls in room P be B_1, B_2, G_1, G_2

Let the 1 boy and 3 girls in room Q be B_3, G_3, G_4, G_5

Suppose the room P is selected

Possible outcomes are $(PB_1), (PB_2), (PG_1), (PG_2)$

Suppose the room Q is selected

Possible outcomes are $(QB_3), (QG_3), (QG_4), (QG_5)$

$\therefore \quad S = \{(PB_1), (PB_2), (PG_1), (PG_2), (QB_3), (QG_3), (QG_4), (QG_5)\}$

5. Three coins are tossed once. Describe the following events associated with this random experiment.

A = Getting three heads

B = Getting two heads and one tail

C = Getting a head on the first coin

Which pairs of events are mutually exclusive ?

Solution:

Clearly $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$A = \{HHH\}$

$B = \{HHT, HTH, THH\}$

$C = \{HHH, HHT, HTH, HTT\}$ Now as $A \cap B = \phi$

$\therefore$ A and B are mutually exclusive events.

6. From a group of 2 boys and 3 girls, two children are selected at random. Describe the events as follows :

A = both selected children are girls

B = selected group consists of one boy and one girl

C = at least one boy is selected

Which pairs of events are mutually exclusive ?

Solution:

Let G_1, G_2, G_3 be three girls and B_1, B_2 be two boys

Clearly $S = \{B_1B_2, G_1G_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_2G_3, G_3G_1\}$

and $A = \{G_1G_2, G_3G_1, G_2G_3\}$

$B = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

$C = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

$B \cap C = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$

Now $A \cap B = \phi$, $B \cap C \neq \phi$ and $A \cap C = \phi$

Hence, Pairs A and B, and A and C are mutually exclusive events.

7. A die is thrown twice. Each time the number appearing on it is recorded. Describe the following events :

A = both numbers are odd

B = both numbers are even

C = sum of the numbers is less than 6

which pairs of events are mutually exclusive ?

Solution:

$$S = \begin{Bmatrix} (11) & (12) & (13) & (14) & (15) & (16) \\ (21) & (22) & (23) & (24) & (25) & (26) \\ (31) & (32) & (33) & (34) & (35) & (36) \\ (41) & (42) & (43) & (44) & (45) & (46) \\ (51) & (52) & (53) & (54) & (55) & (56) \\ (61) & (62) & (63) & (64) & (65) & (66) \end{Bmatrix}$$

$A = \{(11)(13)(15), (31)(33)(35), (51)(53), (55)\}$

$B = \{(22)(24)(26), (42)(44)(46), (62)(64)(66)\}$

$C = \{(11), (12), (13), (14), (21), (22), (23), (31), (32), (41)\}$

$A \cap B = \phi$

$B \cap C = \{(22)\}$ $\therefore$ $B \cap C \neq \phi$

$A \cap C = \{(11), (13), (31)\}$ $\therefore$ $A \cap C \neq \phi$

$\therefore$ $A \cap B = \phi$

$\therefore$ A and B are mutually exclusive events.

1.2 [Classical Definition of Probability]

Definition:

If in '*n*' trials (which are exhaustive mutually exclusive and equally likely cases) associated with the random experiments, *m* of them are favourable to the event A, then the probability of happening of A' is defined as

$$p = P(A) = \frac{m}{n} = \frac{\text{Favourable number of cases}}{\text{Exhaustive number of cases}}$$

As '*m*' are the favourable number of cases

$\therefore$ $n - m$ are the unfavorable number of cases

$\therefore$ Probability of 'not happening of A' is defined as

$$q = P(\bar{A}) = \frac{n-m}{n} = \frac{\text{unavourable number of cases}}{\text{Exhaustive number of cases}}$$

Remark : $q = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p \Rightarrow q + p = 1$

Examples

8. Find the probability of getting '6' when a die is thrown.

Solution:

Sample space associated with random experiment

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let A = event of getting six

Now, favourable number of cases = 1 = *m* and exhaustive number of cases = 6 = *n*

$\therefore$ $P(A) = \frac{1}{6}$

9. Find the probability of getting a multiple of 2 or 3 when a die is thrown.

Solution:

Let *S* be the sample space associated with random experiment

$S = \{1, 2, 3, 4, 5, 6\}$

Let A be the event of getting multiple of 2 or 3

$\therefore$ $A = \{2, 3, 4, 6\}$

Hence, favourable number of cases = 4 = *m*

and exhaustive number of cases = 6 = *n*

$\therefore$ $P(A) = \frac{4}{6} = \frac{2}{3}$

10. In a simultaneous toss of 2 coins, find of probability of getting (i) exactly 2 heads (ii) no tail.

Solution:

Let *S* be the sample space associated with random experiment

$S = \{HH, HT, TH, TT\}$

(i) Let A be the event of getting exactly 2 heads

$\therefore$ $A = \{HH\}$

Hence, no. of favourable number of cases = 1 and exhaustive number of cases = 4

$\therefore$ $P(A) = \frac{1}{4}$

(ii) Let A be the event of getting no tails

$\therefore$ $A = \{HH\}$

Hence, favourable number of cases = 1 and exhaustive number of cases = 4

$\therefore$ $P(A) = \frac{1}{4}$

11. Three coins are tossed once. Find the probability of getting (i) exactly one tail (ii) no heads (iii) at least 2 heads (iv) exactly 2 tails

Solution:

Let S be the sample space associated with random experiment

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$$

(i) Let A be the event of getting exactly one tail $\therefore$ A = {HHT, HTH, THH}

Hence, favourable number of cases = 3

and exhaustive number of cases = 8 $\therefore \quad P(A) = \frac{3}{8}$

(ii) Let A be the event of getting no head $\therefore$ A = {TTT}

Hence, favourable number of cases = 1

and exhaustive number of cases = 8 $\therefore \quad P(A) = \frac{1}{8}$

(iii) Let A be the event of getting two heads

[getting atleast 2 heads means getting 2 heads or more than 2]

$\therefore \quad$ A = {HHH, HHT, HTH, THH}

Hence, favourable number of cases = 4

and exhaustive number of cases = 8 $\therefore \quad P(A) = \frac{4}{8} = \frac{1}{2}$

12. In a simultaneous throw of a pair of dice, find the probability of getting

(i) an even number on first (ii) a doublet

(iii) a sum less than 7 (iv) a sum greater than 8

Solution:

(i) Let S be the sample space associated with random experiment

$$S = \begin{Bmatrix} (11) & (12) & (13) & (14) & (15) & (16) \\ (21) & (22) & (23) & (24) & (25) & (26) \\ (31) & (32) & (33) & (34) & (35) & (36) \\ (41) & (42) & (43) & (44) & (45) & (46) \\ (51) & (52) & (53) & (54) & (55) & (56) \\ (61) & (62) & (63) & (64) & (65) & (66) \end{Bmatrix}$$

Let A be the event of getting even number on first

$$\therefore \quad A = \begin{Bmatrix} (21) & (22) & (23) & (24) & (25) & (26) \\ (41) & (42) & (43) & (44) & (45) & (46) \\ (61) & (62) & (63) & (64) & (65) & (66) \end{Bmatrix}$$

Hence, favourable number of cases = 18, exhaustive number of cases = 36

$$\therefore \quad A = \frac{18}{36} = \frac{1}{2}$$

(ii) Let A be the event of getting doublet

$\therefore \quad$ A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

Hence, favourable number of cases = 6

and exhaustive number of cases = 36

$$\therefore \quad A = \frac{6}{36} = \frac{1}{6}$$

(iii) Let A be the event of getting sum less than 7

$\therefore \quad$ A = {(11), (12), (13), (14), (15), (21), (22), (23) (24, (31), (32), (33), (41), (42), (51)}

Hence, favourable umber of cases = 15

and exhaustive number of cases = 36 $\therefore \quad A = \frac{15}{36} = \frac{5}{12}$

(iv) Let A be the event of getting sum greater than 8

$\therefore$ A = {(36), (45), (46), (54), (55), (56), (63), (64), (65), (66)}

Hence, favourable number of cases = 10

and exhaustive number of cases = 36 $\therefore$ $A = \frac{10}{36} = \frac{5}{18}$

13. A coin is tossed. If head comes, the coin is tossed again and if tail comes, a die is thrown. Find the probability of getting (i) 2 heads (ii) tail and even number

Solution:

Let S be the sample space associated with random experiment

$$S = \{(HH), (HT), (T1), (T2), (T3), (T4), (T5), (T6)\}$$

(i) Let A be the event of getting 2 heads $\therefore$ A = {HH}

Hence, favourable number of cases = 1

and exhaustive number of cases = 8 $\therefore$ $A = \frac{1}{8}$

(ii) Let A be the event of getting tail and even number

$\therefore$ A = {(T2), (T4), (T6)}

Hence, favourable number of cases = 3

and exhaustive number of cases = 8 $\therefore$ $A = \frac{3}{8}$

14. A card is drawn at random from a pack of 52 cards. Find the probability that the card drawn is (i) a diamond card (ii) not a diamond card (iii) not an ace (iv) a black king

Solution:

From 52 cards, one card can be drawn in ${}^{52}C_1$ ways

(i) Let A be the event of drawing a diamond card

There are 13 diamond cards,

out of which one card can be drawn is ${}^{13}C_1$ ways

Hence favourable number of cases = ${}^{13}C_1$

and exhaustive number of cases = ${}^{52}C_1$ $\therefore$ $P(A) = \frac{{}^{13}C_1}{{}^{52}C_1} = \frac{13}{52} = \frac{1}{4}$

(ii) $P(\overline{A}) = 1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4}$ $\left(P(\overline{A}) = 1 - P(A)\right)$,

where $\overline{A}$ is the probability of the card drawn is not a diamond card.

(iii) Let A be the card of drawing not an ace then $\overline{A}$ is the event of drawing an ace

$\therefore$ $P(A) = 1 - P(\overline{A})$

Now there are 4 Ace cards, out of which one card can be drawn in ${}^{4}C_1$ ways

Hence, $P(\overline{A}) = \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{4}{52}$ $\therefore$ $P(A) = 1 - \frac{4}{52} = \frac{48}{52} = \frac{24}{26} = \frac{12}{13}$

(iv) Let A be event of drawing black king

There are 4 king, out of which two are of black colour

$\therefore$ black king can be drawn in ${}^{2}C_1$ ways

Hence, favourable number of cases = C_1 and exhaustive number of cases = ${}^{52}C_1$

$\therefore$ $P(A) = \frac{{}^{2}C_1}{{}^{52}C_1} = \frac{2}{52} = \frac{1}{26}$

15. A bag contains 6 red, 4 white and 8 blue balls. If three balls are drawn at random, find the probability that one is red, one is white and one is blue.

Solution:

Let A be the event of drawing one red, one white and one blue ball.

From 6 red balls, one red ball can be drawn in ${}^{6}C_1$ ways

From 4 white balls, one white ball can be drawn in ${}^{4}C_1$ ways

From 8 blue balls, one blue ball can be drawn in ${}^{8}C_1$

$\therefore$ favourable number of cases = ${}^6C_1 \times {}^4C_1 \times {}^8C_1$

and exhaustive number of cases = ${}^{18}C_3$

[as we have to draw three balls from 6 + 4 + 8 = 18 balls]

$$\therefore \quad P(A) = \frac{{}^6C_1 \times {}^4C_1 \times {}^8C_1}{{}^{18}C_3} = \frac{6\times4\times8}{18!}3!\times15! = \frac{6\times4\times8\times3\times2}{18\times17\times16\times15!}\times15! = \frac{4}{17}$$

16. From 15 students in a school, there are 10 girls and 5 boys, out of these students a team of 10 students are selected for the debate competition. Find the probability that 6 are girls and 4 are boys.

Solution:

Let A be the event of selection 6 girls and 4 boys

From 10 girls, 6 girls can be selected in ${}^{10}C_6$ ways

From 5 boys, 4 boys can be selected in ${}^5C_4$ ways

Also, out of 15 students, 10 students can be selected in ${}^{15}C_{10}$ ways

$\therefore$ favourable number of cases = ${}^{10}C_6 \times {}^5C_4$

and exhaustive number of cases = ${}^{15}C_{10}$ $\quad \therefore \; P(A) = \dfrac{{}^{10}C_6 \times {}^5C_4}{{}^{15}C_{10}} = \dfrac{10!}{6!\times4!}\times\dfrac{5!\times10!\times5!}{4!\times15!} = \dfrac{50}{143}$

17. A class consists of 10 boys and 8 girls. Three students are selected at random, what is the probability that the selected groups has

(i) all boys (ii) all girls (iii) atmost one girl (iv) at least one girl.

Solution:

Out of 18 students, there are 10 boys and 8 girls. From 18 students, three students can be selected in ${}^{18}C_3$ ways.

(i) Let A be event that all boys are selected

$\therefore$ From 10 boys, 3 boys can be selected in ${}^{10}C_3$ ways

$\therefore$ favourable number of cases = ${}^{10}C_3$

and exhaustive number of cases = ${}^{18}C_3$ $\quad \therefore \; P(A) = \dfrac{{}^{10}C_3}{{}^{18}C_3} = \dfrac{10!\times3!\times15!}{3!\times7!\times18!} = \dfrac{5}{34}$

(ii) Let A be event that all girls are selected

$\therefore$ From 8 girls, 3 girls can be selected in ${}^8C_3$ ways

$\therefore$ favourable number of cases = ${}^8C_3$

and exhaustive number of cases = ${}^{18}C_3$ $\quad \therefore \; P(A) = \dfrac{{}^8C_3}{{}^{18}C_3} = \dfrac{8!\times3!\times15!}{3!5!18!} = \dfrac{7}{102}$

(iii) Let A be the event that atmost one girl is selected. We have the following cases:-

Case I : 0 girl and 3 boys are selected

$\therefore$ favourable number of cases = ${}^{10}C_3$

$\therefore$ Hence probability is = $\dfrac{{}^{10}C_3}{{}^{18}C_3} = \dfrac{5}{34}$

Case II : 1 girl and 2 boys are selected

From 8 girls, 1 girl can be selected in ${}^8C_1$ ways

From 10 boys, 2 boys can be selected in ${}^{10}C_2$ ways $\therefore$ favourable number of case = ${}^8C_1 \times {}^{10}C_2$

Hence probability is = $\dfrac{{}^8C_1 \times {}^{10}C_2}{{}^{18}C_3} = \dfrac{15}{34}$. Required $P(A) = \dfrac{5}{34} + \dfrac{15}{34} = \dfrac{20}{34} = \dfrac{10}{17}$

(iv) Let A be the event that at least one girl selected. We have the following cases:-

Case I : One girl and 2 boys are selected

Out of 8 girls, 1 girl can be selected in ${}^8C_1$ ways

out of 10 boys, 2 boys can be selected in ${}^{10}C_2$ ways

$\therefore$ favourable number of case $= {}^{10}C_2 \times {}^{8}C_1$. Hence probability is $= \dfrac{{}^{10}C_2 \times {}^{8}C_1}{{}^{18}C_3} = \dfrac{15}{34}$

Case II : 2 girls and 1 boy are selected

Out of 8 girls, 2 girls can be selected in ${}^{8}C_2$ ways

Out of 10 boys, 1 boy can be selected in ${}^{10}C_1$ ways

$\therefore$ favourable number of cases $= {}^{10}C_1 \times {}^{8}C_2$

Hence probability is $= \dfrac{{}^{10}C_1 \times {}^{8}C_2}{{}^{18}C_3} = \dfrac{35}{102}$

Case III : 3 girls and 0 boys are selected

Out of 8 girls, 3 girls can be selected in ${}^{8}C_3$ ways

$\therefore$ Hence probability is $= \dfrac{{}^{8}C_3}{{}^{18}C_3} = \dfrac{7}{102}$

$$\therefore \ P(A) = \frac{15}{34} + \frac{35}{102} + \frac{7}{102} = \frac{45+35+7}{102} = \frac{87}{102} = \frac{29}{34}$$

18. Find the probability that in random arrangement of the letters of the word 'FORTUNATES', two 'T' come together.

Solution:

Total number of words which can be formed from the word 'FORTUNATES'. $= \dfrac{10!}{2!}$ [2 T's are repeating]

Regarding two 'T' as one letter, number of arrangements of the word 'FORTUNATES' such that two 'T's are together = 9!

i.e. favourable number of cases = 9! exhaustive number of cases $= \dfrac{10!}{2!}$

Hence, required probability $= \dfrac{9!}{\frac{10!}{2!}} = \dfrac{9! \times 2!}{10!} = \dfrac{2}{10} = \dfrac{1}{5}$

19. A four digit number is formed by the digits 1, 2, 3, 4 without repetition. Find the probability that the number is even.

Solution:

From 4 digits, 4 digit number can be formed in ${}^{4}P_4$ ways

$\therefore$ exhaustive number of case = 4! = 24

Now number is even, when the last digit is divisible by 2.

Case I : Let the units digit be 2

$\therefore$ Remaining 3 places can be filled in ${}^{3}P_3$ ways

[from remaining 3 digits, 1, 3, 4]

Case II : Let the units digit be 4

Similarly to case I, remaining 3 places can be filled in ${}^{3}P_3$ ways

[from remaining 3 digits, 1, 2, 3]

Hence, favourable number of cases ${}^{3}P_3 + {}^{3}P_3 = 2 \times 3! = 12$

Hence, required probability $= \dfrac{12}{24} = \dfrac{1}{2}$

20. The odds in favour of an event are 7 : 9. Find the probability of occurrence and non-occurrence of this event.

Solution:

As odds in favour of an event are 7 : 9

$\therefore$ Favourable number of cases $= 7x$ and unfavourable number of cases $= 9x$

Also in exhaustive number of cases $= 16x = (7x + 9x)$

Hence, required probability of the occurrence of event $= \frac{7x}{16x} = \frac{7}{16}$

Also, probability of the non-occurrence of event $= \frac{9x}{16x} = \frac{9}{16}$

21. The odds against an event be 7 : 9, find the probability of occurrence and non-occurrence of this event.

Solution :

As odds against an event are 7 : 9

$\therefore$ Favourable number of cases = $7x$, and unfavourable number of cases = $9x$

Also in exhaustive number of cases $= (7x + 9x) = 16x$

Hence, required probability of the occurrence of event $= \frac{9x}{16x} = \frac{9}{16}$

Also, probability of the non-occurrence of event $= \frac{7x}{16x} = \frac{7}{16}$

22. Find the probability of arranging the letters of the word 'GARDEN' such that the letters A and R are always together.

Solution:

'6' distinct letter of the word GARDEN can be arranged in ${}^6P_6 = 6!$ ways

$\therefore$ exhaustive number of cases = 6! = 720

Consider A and R as single letter. Now we have to arrange 5 letters, which can be done in ${}^5P_5$ ways and 2 letters A and R can be arranged among themselves in 2! ways

$\therefore$ favourable number of cases $= {}^5P_5 \times 2! = 5! \times 2! = 120 \times 2 = 240$

Hence, required probability $= \frac{240}{720} = \frac{1}{3}$

23. Find the probability of arranging the letters of the word 'GARDEN' in such a way that letters A and R are never together.

Solution:

Probability of arranging the letters of the word GARDEN such that A and R are always together $= \frac{1}{3}$ (see previous question)

$\therefore$ Probability of arranging the letters of the word GARDEN such that A and R are never together $= 1 - \frac{1}{3} = \frac{2}{3}$

24. Find the probability of arranging the letters of the word 'NATIONAL' such that letters O and L are never together.

Solution:

In the word 'NATIONAL', we have total '8' letters, out of which, letter 'N' and letter 'A' occurs twice.

Total arrangement of the letters of the word 'NATIONAL' $= \frac{8!}{2!2!} = 10080$

$\therefore$ exhaustive number of cases = 10080

Now consider letter O and L as one letter.

Now we have total of 7 letters and letters O and L can be arranged in 2! ways

Number of arrangements of letters of the words 'NATIONAL', is which O and L occurs together

$= \frac{7!}{2! \times 2!} \times 2! = \frac{7!}{2!}$ (O and L can be arranged in 2! ways) = 2520

$\therefore$ Required probability = 1 – (Probability that O and L are always together) $= 1 - \frac{2520}{10080} = \frac{7560}{10080} = \frac{3}{4}$

25. If n biscuits be distributed among N beggars, find chance that a particular beggar receives $p(< n)$ biscuits

Solution :

Total there are 'n' biscuits and any one biscuit can be given in N ways. Hence 'n' biscuit can be given in N^n ways. Hence total number cases = N^n.

Now to a particular beggar, 'p' biscuits can be given in $^{n}C_{p}$ ways and remaining $(n-p)$ biscuit will be distributed to $(N-1)$ beggars in $(N-1)^{n-p}$ ways

Hence favourable number cases $= {}^{n}C_{p}(N-1)^{n-p}$

$\therefore$ Required probability $= \dfrac{{}^{n}C_{p}(N-1)^{n-p}}{N^{n}}$

26. Out of $(2n+1)$ tickets consecutively numbered, three are drawn at random. Find the chance that the numbers on them are in A.P.

Solution:

From $(2n+1)$ tickets, 3 tickets can be drawn in $^{2n+1}C_3$ ways

$$^{2n+1}C_3 = \frac{(2n+1)!}{(2n+1-3)!3!} = \frac{(2n+1)!}{(2n-2)!6} = \frac{(2n+1)(2n)(2n-1)(2n-2)!}{(2n-2)!6} = \frac{(4n^2-1)(2n)}{6} = \frac{n(4n^2-1)}{3}$$

Now as the tickets drawn have numbers in the A.P. following cases arises

Let d be common difference

Case I: $d = 1$ Possibilities are of numbers :

$$\begin{matrix} 1, & 2, & 3 \\ 2, & 3, & 4 \\ \vdots & \vdots & \vdots \\ 2n-1 & 2n & 2n+1 \end{matrix}$$

i.e. total of $(2n-1)$ cases.

Case II: $d = 2$ Possibilities are of numbers

$$\begin{matrix} 1, & 3, & 5 \\ 2, & 4, & 6 \\ \vdots & \vdots & \vdots \\ 2n-3 & 2n-1 & 2n+1 \end{matrix}$$

i.e. total of $(2n-3)$ cases

Similarly if $d = n-1$. Possibilities are of numbers :

$$\begin{matrix} 1, & n, & 2n-1 \\ 2 & n+1 & 2n \\ 3 & n+2 & 2n+1 \end{matrix}$$

i.e. total of 3 cases

If $d = n$, we have only 1 case of $1, n+1, 2n+1$. Hence total number of cases:

$= (2n-1) + (2n-3) + \ldots\ldots\ldots\; 5 + 3 + 1$

$= 1 + 3 + 5 + \ldots\ldots\ldots\; (2n-3) + (2n-1)$

Above is an A.P. with d = common difference = 2 and of 'n' terms.

$\therefore$ Favourable number of cases $= \dfrac{n}{2}[2\times1+(n-1)\times 2] = \dfrac{n}{2}[2n] = n^2$

Hence required probability $= \dfrac{n^2}{\dfrac{n[4n^2-1]}{3}} = \dfrac{3n}{4n^2-1}$.

27. What is the chance that a leap year selected at random will contains 53 Tuesdays ?

Solution:

The leap year, total there are 366 days. Out of which we have , 52 weeks and 2 days

i.e. leap year has 52 Tuesdays, remaining two days can be

(i) Sunday and Monday (ii) Monday and Tuesday

(iii) Tuesday and Wednesday (iv) Wednesday and Thursday

(v) Thursday and Friday (vi) Friday and Saturday

(vii) Saturday and Sunday

i.e. exhaustive number of cases = 7 and favourable number of cases = 2 $\therefore$ required probability $= \dfrac{2}{7}$