# Lab 05 Regression Analysis (Part 2)

Lusine Zilfimian

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## Multiple Regression Model Selection

- Let's consider the following models

```
model2 <- lm(price ~. ,data = hd)
model3 <- lm(price ~.-condition , data = hd)
model4 <- lm(price ~ sqft_living + condition, data = hd)
```

# Multiple Regression Model Selection

```r
summary(model3)
```

```
##
## Call:
## lm(formula = price ~ . - condition, data = hd)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -650.70 -160.09  -14.93  136.39 1724.98
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -247.2449    96.4261  -2.564   0.0119 *
## sqft_living         0.7269     0.5990   1.213   0.2279
## Sqft_with_garden   -0.3200     0.5942  -0.539   0.5914
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 312.7 on 98 degrees of freedom
## Multiple R-squared:  0.5717, Adjusted R-squared:  0.5629
## F-statistic:  65.4 on 2 and 98 DF,  p-value: < 2.2e-16
```

# Understanding the output of regression in R

### F test

```
(Fstat4<-(summary(model3)$r.sq/(3-1))/((1-summary(model3)$r.sq)/(101-3)))
```

```
## [1] 65.39611
```

# Regression with Categorical variables

```r
model4 <- lm(price ~ sqft_living + condition, data = hd)
summary(model4)
```

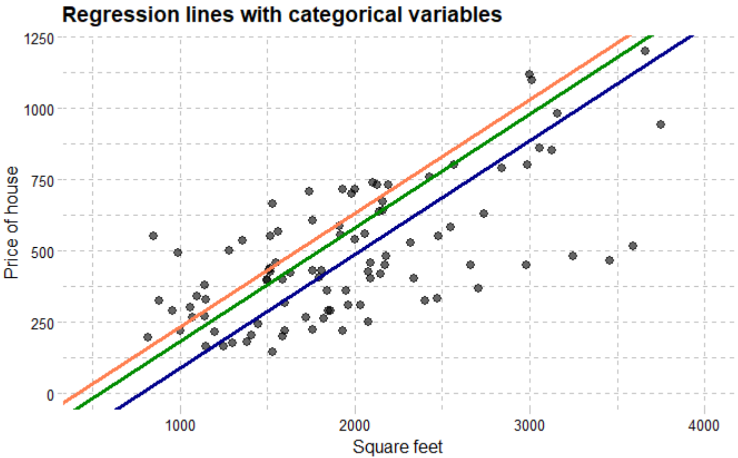
```
##
## Call:
## lm(formula = price ~ sqft_living + condition, data = hd)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -607.5 -139.8  -34.6  117.5 1717.4
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)   -164.16439  130.87350  -1.254    0.213
## sqft_living      0.39864    0.03545  11.247   <2e-16 ***
## conditionfair -144.47088  107.21565  -1.347    0.181
## conditiongood  -49.93705  112.17615  -0.445    0.657
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

## Regression with Categorical variables

- The levels of condition are fair, good, excellent. One of the categories is the base. R takes the one that comes first in alphabetical order

```r
g8 <- g1+
  # excellent
  geom_abline(intercept = coef(model4)[1],
    slope = coef(model4)[2], col = "coral", size = 1.2) +
  # fair
  geom_abline(intercept = coef(model4)[1] + coef(model4)[3],
    slope = coef(model4)[2], col = "darkblue", size = 1.2) +
  #good
  geom_abline(intercept = coef(model4)[1] + coef(model4)[4],
    slope = coef(model4)[2], col = "green4", size = 1.2) +
  ggtitle("Regression lines with categorical variables") +
  ylim(1, 1200) + xlim(500, 4000)
```

# Regression with Categorical variables




```r
coef(model4)
```

```
##   (Intercept)   sqft_living conditionfair conditiongood
##  -164.1643864     0.3986409  -144.4708793   -49.9370479
```

## Non-linearity

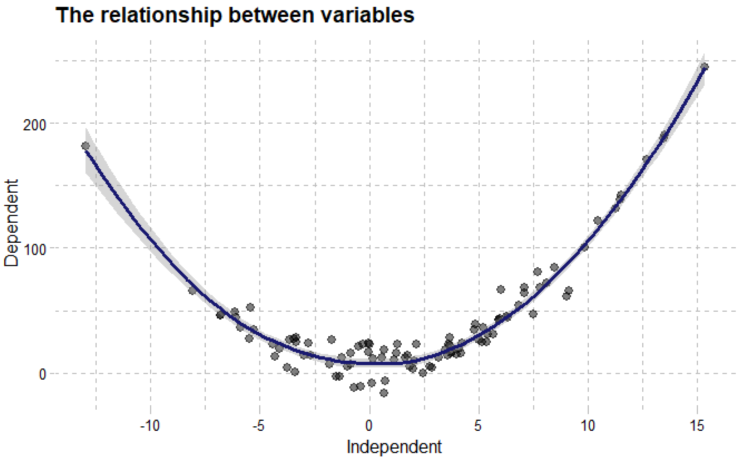
- Non-linear by x

```
set.seed(27)
x1 = rnorm(100, mean = 2, sd = 5)
y1 = x1^2 + rnorm(100, mean = 5, sd = 10)
y2 = x1^3 + x1^2 + x1 + rnorm(100, mean = 5, sd = 100)

poly_df <- data.frame(x1, y1, y2)

g9 <- ggplot(poly_df, aes(x = x1, y = y1)) +
  geom_point(size = 2.5, alpha = 0.5) +
  ggtitle("The relationship between variables") +
  xlab("Independent") +
  ylab("Dependent") +
  geom_smooth(method = "auto", col = "midnightblue", size = 1.2) +
  theme_pander()
```

# Non-linearity

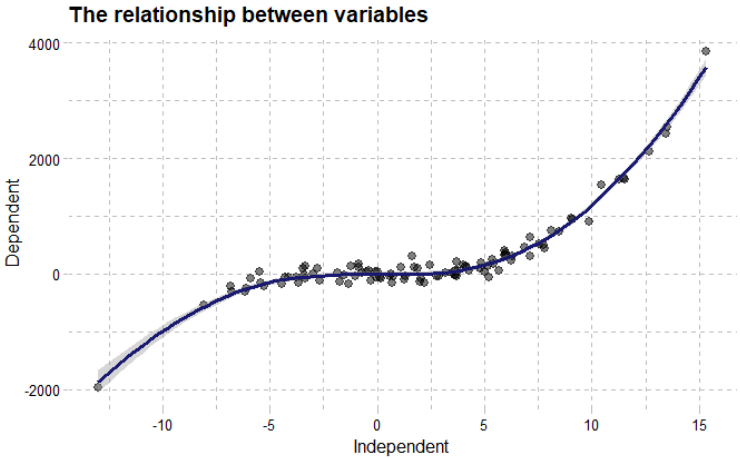


# Non-linearity in R

```
poly_1 <- lm(formula = y1 ~ I(x1^2), data = poly_df)
summary(poly_1)
```

```
##
## Call:
## lm(formula = y1 ~ I(x1^2), data = poly_df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -26.6027  -5.6713   0.4195   6.1071  24.3730
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.08906    1.21250   5.022  2.3e-06 ***
## I(x1^2)      1.00146    0.02133  46.940  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.818 on 98 degrees of freedom
## Multiple R-squared:  0.9574, Adjusted R-squared:  0.957
## F-statistic:  2203 on 1 and 98 DF,  p-value: < 2.2e-16
```

# Non-linearity in R



# Non-linearity in R

```
poly_2 <- lm(formula = y2 ~ poly(x1, 3), data = poly_df)
summary(poly_2)
```

```
##
## Call:
## lm(formula = y2 ~ poly(x1, 3), data = poly_df)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -245.502  -75.148   -8.598   69.365  305.287
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    237.74      10.57   22.50   <2e-16 ***
## poly(x1, 3)1  5478.08     105.66   51.84   <2e-16 ***
## poly(x1, 3)2  2563.99     105.66   24.27   <2e-16 ***
## poly(x1, 3)3  3139.94     105.66   29.72   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 105.7 on 96 degrees of freedom
```

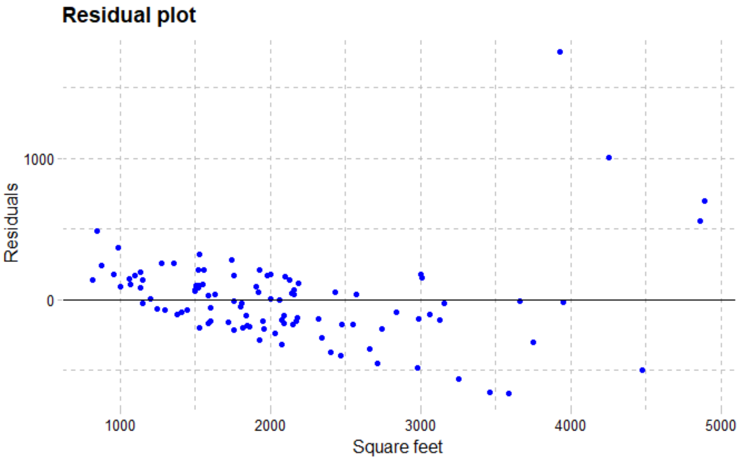
# Heteroskedasticity

- Residual plot

```
g11 <- ggplot(data = hd, aes(y = model1$residuals, x = sqft_living)) +
  geom_point(col = 'blue') +
  geom_abline(slope = 0) +
  xlab("Square feet") +
  ylab("Residuals") +
  theme_pander() +
  ggtitle("Residual plot")
```

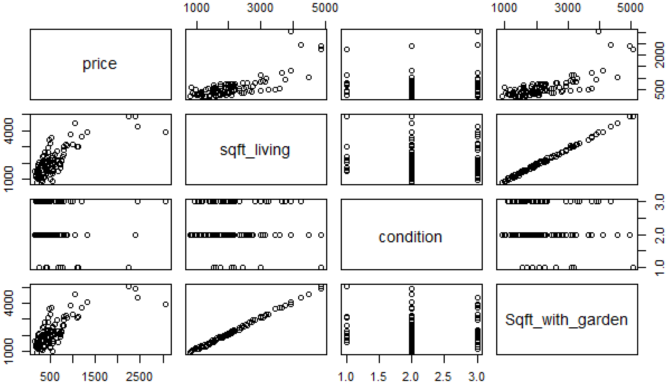
There is more variation in price for houses with a more large square of area.

## Heteroskedasticity



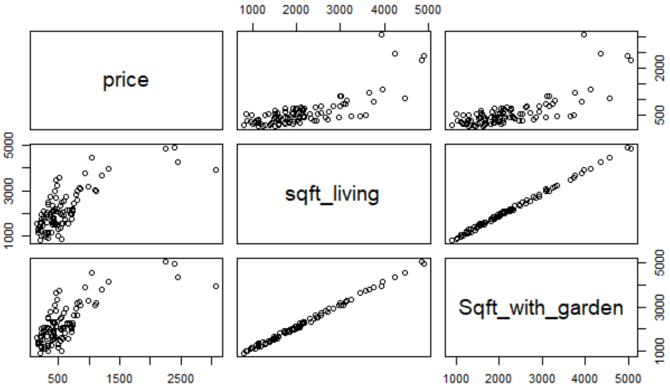
# Multicollinearity

- Suspicion of multicollinearity
- Meaningful correlation with numerics and price
- pairs(hd)



# Multicollinearity

- pairs(hd[ , c("price","sqft_living", "Sqft_with_garden")])



## Detecting the multicollinearity

```
cor(hd[ , c("price","sqft_living", "Sqft_with_garden")])
```

```
##                      price sqft_living Sqft_with_garden
## price            1.0000000   0.7552458        0.7518162
## sqft_living      0.7552458   1.0000000        0.9982481
## Sqft_with_garden 0.7518162   0.9982481        1.0000000
```

```
cor(hd[ , c("price","sqft_living", "Sqft_with_garden")])[2,3]
```

```
## [1] 0.9982481
```

```
model3 <- lm(price ~ sqft_living + Sqft_with_garden, data = hd)
model3_sub <- lm(price ~ sqft_living, data = hd)
stargazer(model3, model3_sub,
  title = "Multicollinearity",
  out.header = FALSE,
  type = "latex",
  header=FALSE,
  covariate.labels = c(
    "Square feet",
    "With garder"))
```

**Table 2:** Multicollinearity

| | *Dependent variable:* | |
|---|---|---|
| | price | |
| | (1) | (2) |
| Square feet | 0.727 | $0.405^{***}$ |
| | (0.599) | (0.035) |
| With garder | $-0.320$ | |
| | (0.594) | |
| Constant | $-247.245^{**}$ | $-275.659^{***}$ |
| | (96.426) | (80.419) |
| Observations | 101 | 101 |
| $R^2$ | 0.572 | 0.570 |
| Adjusted $R^2$ | 0.563 | 0.566 |
| Residual Std. Error | 312.706 (df = 98) | 311.582 (df = 99) |
| F Statistic | $65.396^{***}$ (df = 2; 98) | $131.445^{***}$ (df = 1; 99) |
| *Note:* | | $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$ |

## Detecting the multicollinearity: VIF

```
vif(model3)
```

```
 sqft_living Sqft_with_garden
    285.6559         285.6559
```

```
mod_vif <- lm(sqft_living ~ Sqft_with_garden, data = hd)
stargazer(mod_vif, type = "latex")
```

% Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu % Date and time: Thu, Mar 12, 2020 - 10:18:46 PM

**Table 3:**

| | *Dependent variable:* |
|---|---|
| | sqft_living |
| Sqft_with_garden | $0.990^{***}$ |
| | (0.006) |
| Constant | $-80.571^{***}$ |
| | (14.005) |

# Model Selection

- Stepwise regression Based on AIC

```
set.seed(2708)
sample <- sample(nrow(hd), round(nrow(hd)*0.8))
Train <- hd[sample, ]
Test <- hd[-sample, ]
model_full <- lm(price ~ ., data = Train)
```

```r
model_step <- stepAIC(model_full, direction = "backward")
```

```
## Start:  AIC=939.91
## price ~ sqft_living + condition + Sqft_with_garden
##
##                    Df Sum of Sq     RSS    AIC
## - Sqft_with_garden  1     53748 7894711 938.47
## - sqft_living       1    184013 8024976 939.79
## <none>                          7840963 939.91
## - condition         2    398527 8239490 939.93
##
## Step:  AIC=938.47
## price ~ sqft_living + condition
##
##               Df Sum of Sq      RSS     AIC
## <none>                      7894711  938.47
## - condition    2    428167  8322877  938.75
## - sqft_living  1  11687723 19582434 1010.05
```

# Model Selection

- `938.47` - AIC if you take out the variable Sqft_with_garden
- `<none>  7840963 939.91` - The model with all the variables
- `<none> 7894711 938.47` - The model, where none of the variables is excluded has the lowest AIC, thus no more changes.

## Stepwise regression

```r
summary(model_step)
```

```
##
## Call:
## lm(formula = price ~ sqft_living + condition, data = Train)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -668.47 -153.60  -28.82  143.68 1589.04
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)      -222.07940  149.67519  -1.484    0.142
## sqft_living         0.43153    0.04042  10.677   <2e-16 ***
## conditionfair    -143.64465  123.35731  -1.164    0.248
## conditiongood       7.12732  130.40833   0.055    0.957
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 320.2 on 77 degrees of freedom
## Multiple R-squared:  0.6209, Adjusted R-squared:  0.6061
## F-statistic: 42.03 on 3 and 77 DF,  p-value: 3.406e-16
```

# RMSE

```
pred_model_step <- predict(model_step, newdata = Test)
RMSE1 <- sqrt(mean((pred_model_step - Test$price)^2))
RMSE1
```

```
## [1] 289.2494
```

```
pred_model_3 <- predict(model3, newdata = Test)
RMSE2 <- sqrt(mean((pred_model_3 - Test$price)^2))
RMSE2
```

```
## [1] 242.2185
```

```
pred_model_full <- predict(model_full, newdata = Test)
RMSE3 <- sqrt(mean((pred_model_full - Test$price)^2))
RMSE3
```

```
## [1] 293.4834
```