classmate

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Chapter - 9
Force Control

$$v = J_v \dot{q}$$

$$\delta x = J \delta q$$

$F \rightarrow$ force applied by the end effector
$\tau \rightarrow$ torque " at the joint

by principle of virtual work

$$\delta W = F^T \delta x - \tau^T \delta q$$
$$= (F^T J_v - \tau^T) \delta q$$

Static equilibrium

$\Rightarrow \delta W = 0$ for any arbitrary δq

$$F^T J_v - \tau^T = 0$$

$$\tau = J_v^T F$$ force end moments

$$F = (J_v^T)^{-1} \tau$$

↳ this might be problematic because

J_v^T may not be a square matrix or the singularity might arise

Manipulator ellipsoid.

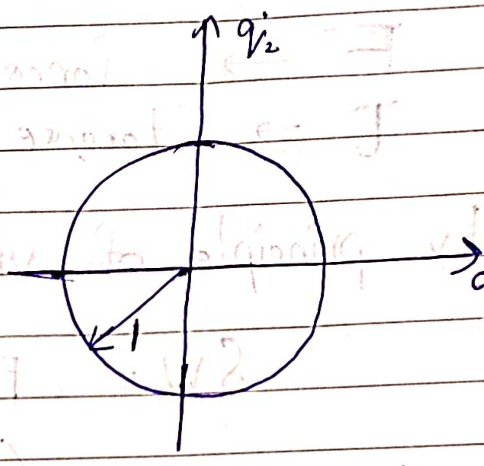
- Performance metric for velocity & force as a fn of c

$$\dot{q}^T \dot{q} = 1 \quad \text{—} ① \quad \Rightarrow \dot{q}_1^2 + \dot{q}_2^2 + \dots + \dot{q}_n^2 = 1$$

$$v = J_v \dot{q}$$

$$\therefore \dot{q} = (J_v^T J_v)^{-1} J_v^T v$$

$$= J_v^+ v \quad \text{—} ②$$



using ② & ①

$$v^T J_v \left((J_v^T J_v)^{-1}\right)^T (J_v^T J_v)^{-1} J_v^T v = 1$$

$$\Rightarrow \underbrace{v^T (J^T J_v)^{-1} v = 1}$$

$$x^T \begin{bmatrix} a^2 & 0 \\ 0 & b^2 \end{bmatrix}^{-1} x = 1$$

$$\boxed{\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1}$$

eq^n of ellipse

$x = Ay$ $\quad A_{m \times n}$ & $m \neq$

$$A^T x = A^T A y$$

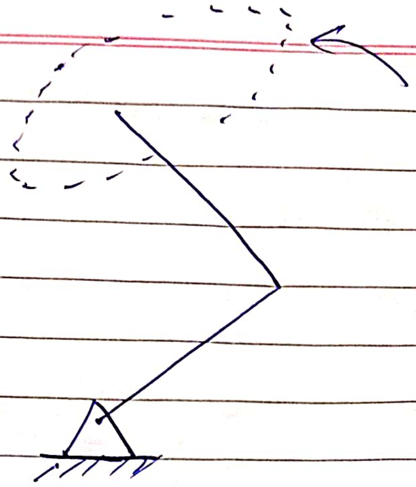
$$y = (A^T A)^{-1} A^T y.$$

$$A^+ = (A^T A)^{-1} A^T$$

→ pseudo inverse.

Eigen values ⇒ minor & major axi

Eigen vectors → major & minor axis direction.



by changing the configuration we can change this ellipsoid to a circle.

If we now start at the joint but with torques

$$\tau^T \tau = 1$$

$$\Rightarrow \boxed{F^T (J_v J_v^T) F = 1}$$

DH Representation (chapter-3)

- standardize
- Automatic coding
- Ref frames.

0 to n links (0 - base / ground)

1 to n joints i^{th} joint connects link $i-1$ & i

0 to n coordinate frames i^{th} coordinate frame rigidly connected to i^{th} link.

i^{th} joint var. is q_i

z_i's are along joint axis $\begin{cases} \text{axis of rotation for R joint} \\ \text{axis of translation for P joint} \end{cases}$

Instead of descriptive representation, need compact tabular rep.

Ideally need 6 parms to rep. any arbitrary homogeneous trans. In DH, use standardize convention so that we only need 4.

(x along common Normal
origin is fixed).

DH parameters (for each joint)

θ, d, a, α

Transformation matrix : ~~$R_{z,\theta}$~~

$$\boxed{R_{z,\theta}\ Trans_{z,d}\ Trans_{x,a}\ R_{x,\alpha}}$$

Recall :

$$H = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & a C_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & a S_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Dynamics -

$$\mathcal{L} = K - P$$

$$\boxed{\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} = \tau_i}$$

equations of motion

$$\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} \underbrace{c_{ijk}(q)}_{\text{Christoffel symbols (first kind)}}\,\dot{q}_i\,\dot{q}_j + \phi_k(q) = \tau_k$$

$$\& \quad \left.\phi_k(q) = \frac{\partial P}{\partial q_k}\right\} \rightarrow \text{Conservative forces}$$

$$k = 1, 2, 3, \ldots, n$$

more common to write in matrix form

$$\boxed{D(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) = \tau}$$

$\dot{D}(q) - 2C(q, \dot{q})$ is skew symmetric

CH-4

Inverse kinematics

→ Given end effector position & orientation (or given desired position & orientation)

↓

Find joint variables / positions.

⇒ Homogeneous transform known

$$H_0^n (q_1, q_2, \ldots, q_n) = \begin{bmatrix} R_0^n & d_0^n \\ 0 & 1 \end{bmatrix}$$

given.

$R = R_0^n$; $d = d_0^n \Rightarrow$ given

find $q_1, q_2, \ldots, q_n$

Prefer closed Loop solutions.
- Computation reasons
- resolving redundancies

But tedious. ⇒ No general expressions.

two tricks:

(1) Kinematic decoupling:
manipulator with atleast 6 DOF & last 3 joints axis intersecting at a point (spherical wrist)

Let us assume exactly 6 joints & last 3 joint axis intersect at a point O → wrist centere

z_4, z_5, z_6 intersect at O

O_4 & O_5 are at O ; O_6 can in general be d_6 away from O in z_6 direction.

$\Rightarrow$ O_6 in the $O_0(x_0, y_0, z_0)$ is

$$O_6 = O + d_6 R\hat{k}$$

$$O_6 - O = d_6 R\hat{k}$$

we know $d_0^n = O_6$, R & d_6

$$\boxed{O = O_6 - d_6 R\hat{k}}$$

$$\boxed{O = P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} O_{6x} \\ O_{6y} \\ O_{6z} \end{bmatrix} \begin{bmatrix} d_x - d_6 r_{13} \\ d_y - d_6 r_{23} \\ d_z - d_6 r_{33} \end{bmatrix}}$$

Then,

1. use eq^n above to find p (or O)
2. Solve inv. position kinematics such as 4.1.1 for^n stanford to find joint variables q_1, q_2 & q_3
3. calculate R_0^3 using q_1, q_2 & q_3
4. Since $R = R_0^3 R_3^6 \Rightarrow \boxed{R_3^6 = R_0^{3T} R}$
5. Find wrist angles (Euler angles) that that give R_3^6

suppose

$$R_3^6 = \begin{bmatrix} n_1 & n_2 & n_3 \\ n_4 & n_5 & n_6 \\ n_7 & n_8 & n_9 \end{bmatrix} =$$

$$
= \begin{bmatrix} C_\phi C_\theta C_\psi - S_\phi S_\psi & -C_\phi C_\theta S_\psi - S_\phi C_\psi & C_\phi S_\theta \\ S_\phi C_\theta C_\psi - C_\phi S_\psi & -S_\phi C_\theta S_\psi + C_\phi C_\psi & S_\phi S_\theta \\ -S_\theta C_\psi & S_\theta S_\psi & C_\theta \end{bmatrix}
$$

Euler angles (θ, ϕ, ψ) convention

$R_{z\theta}, R_{y\phi}, R_{z\psi}$

Trick second:-

$$v = J_v \dot{q}$$

$$\delta x = J_v \delta q$$

$$\boxed{\delta q = J_v^{-1} \delta x}$$

$$q_{t+1} - q_t = J_v^{-1} \delta x$$

$$\boxed{q_{t+1} = J_v^{+} \delta x + q_t}$$

↳ differential form of IK

Issues -

→ dependence of joint angles at one instance

→ finding pseudo inverse of the J_v