Vardan Mittal

classmate
Date
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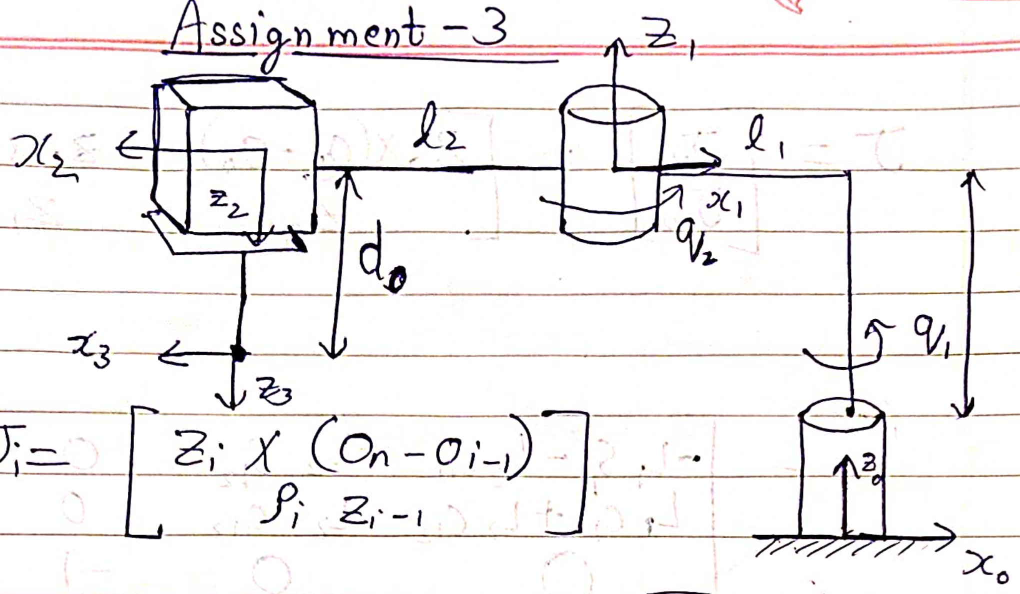
Assignment - 3

Q1



$$J_i = \begin{bmatrix} Z_i \times (O_n - O_{i-1}) \\ \rho_i \, Z_{i-1} \end{bmatrix}$$

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = \begin{bmatrix} L_1 C_1 \\ L_1 S_1 \\ h \end{bmatrix}$$

$$O_2 = \begin{bmatrix} L_1 C_1 + L_2 C_{12} \\ L_1 S_1 + L_2 S_{12} \\ h \end{bmatrix} \qquad O_3 = \begin{bmatrix} L_1 C_1 + L_2 C_{12} \\ L_1 S_1 + L_2 S_{12} \\ h - d \end{bmatrix}$$

$$Z_0 = \begin{bmatrix} c_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad Z_1$$

$$Z_1 = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_2 = Z_3 = I. \begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \\ Z_0 & Z_1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -L_1 S_1 - L_2 S_{12} & -L_2 S_{12} & 0 \\ L_1 C_1 + L_2 C_{12} & L_2 C_{12} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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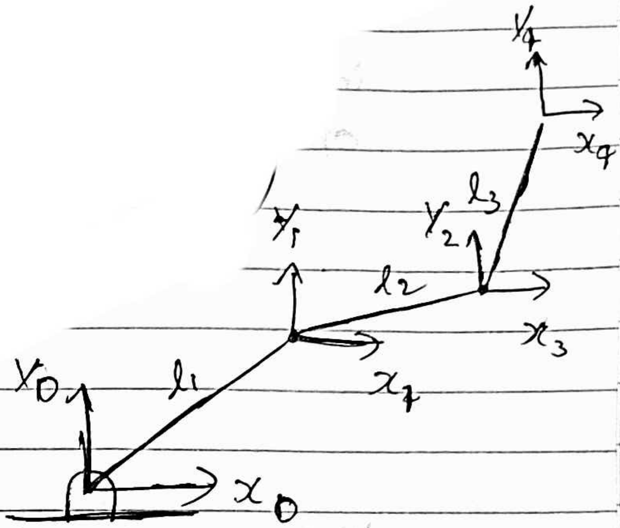

$$O_0 = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad O_1 = \begin{Bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{Bmatrix} \qquad \cancel{O_2 = \begin{Bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{Bmatrix}}$$

$$O_3 = \begin{Bmatrix} l_1 c_1 + l_2 c_{12} + l_3 c_{123} \\ l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ 0 \end{Bmatrix} \qquad O_2 = \begin{Bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \\ 0 \end{Bmatrix}$$

$$Z_0 = Z_1 = Z_3 = Z_4 = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$J = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q5 * Singular configuration occurs on loss of ~~cont~~ one or more DOF which leads to loss of control in certain direction. Can be find by finding determinant of Jacobian matrix

$$\det(J) \begin{cases} =0 & ; \text{ looses rank (Singular)} \\ \neq 0 & ; \end{cases}$$

- if the Jacobian is highly sensitive to small change gives indication of positions close to singularity.

Q6 Denavalt - Hartenberg Representation: -

1. Locate and Label the joint axes $z_0, \ldots z_{n-1}$
2. Locate O_i where common normal to z_i & z_{i-1} intersects z_i.
   → if Parallel locate at any convinient z_i
3. Establish x_i along common normal b/w z_i & z_{i-1}
4. establish y_i using R.H. rule.
5. establish the E.E. frame assuming it as revelute joint along z_{n-1}
6. create a table with $a_i, d_i, \alpha_i, \theta_i$

a_i = distance along x_i from o_i to the intersection of x_i and z_{i-1} axes
d_i = distance along z_{i-1} from o_{i-1} to the intersection of the x_i & z_{i-1} axes
α_i = angle b/w z_{i-1} & z_i measured about X_i
θ_i = the angle b/w x_{i-1} & x_i measured about z_{i-1}.

(7) write homogeneous matrices using

$$\cancel{A = H_{z,\theta} H}$$

$$A = Rot_{z,\theta} \, T_{z,d} \, \cancel{Rot_{x,\alpha}} \, Trans_{x,a} \, Rot_{x,\alpha}$$

$$= \begin{bmatrix} C_\theta & -S_\theta & 0 & 0 \\ S_\theta & C_\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C_\alpha & -S_\alpha & 0 \\ 0 & S_\alpha & C_\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & a_1 C_\theta \\ & & & \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & a C_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & a S_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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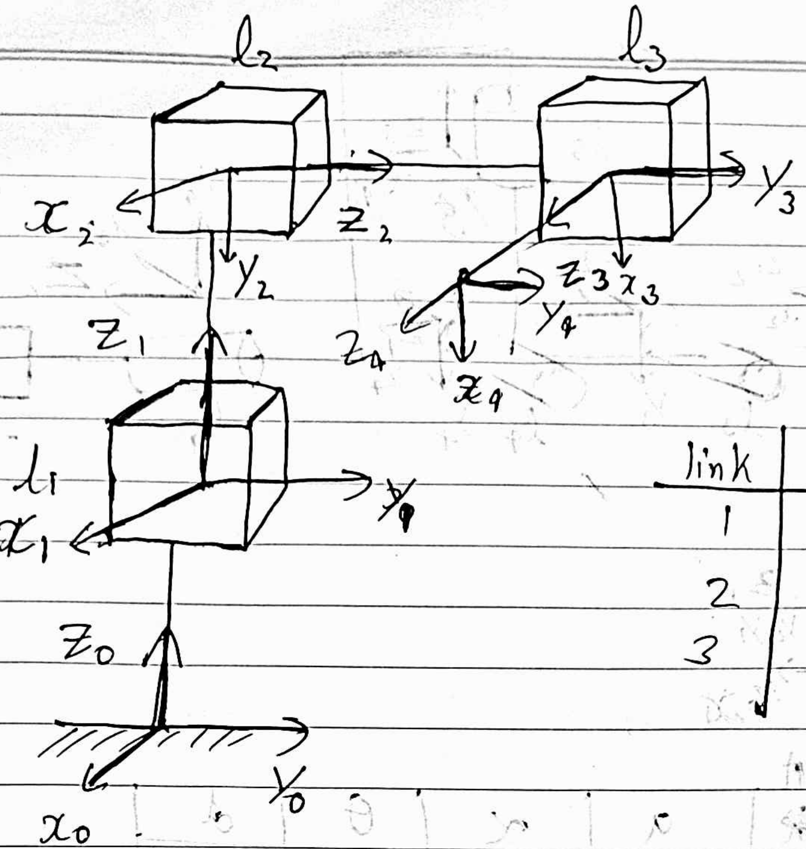


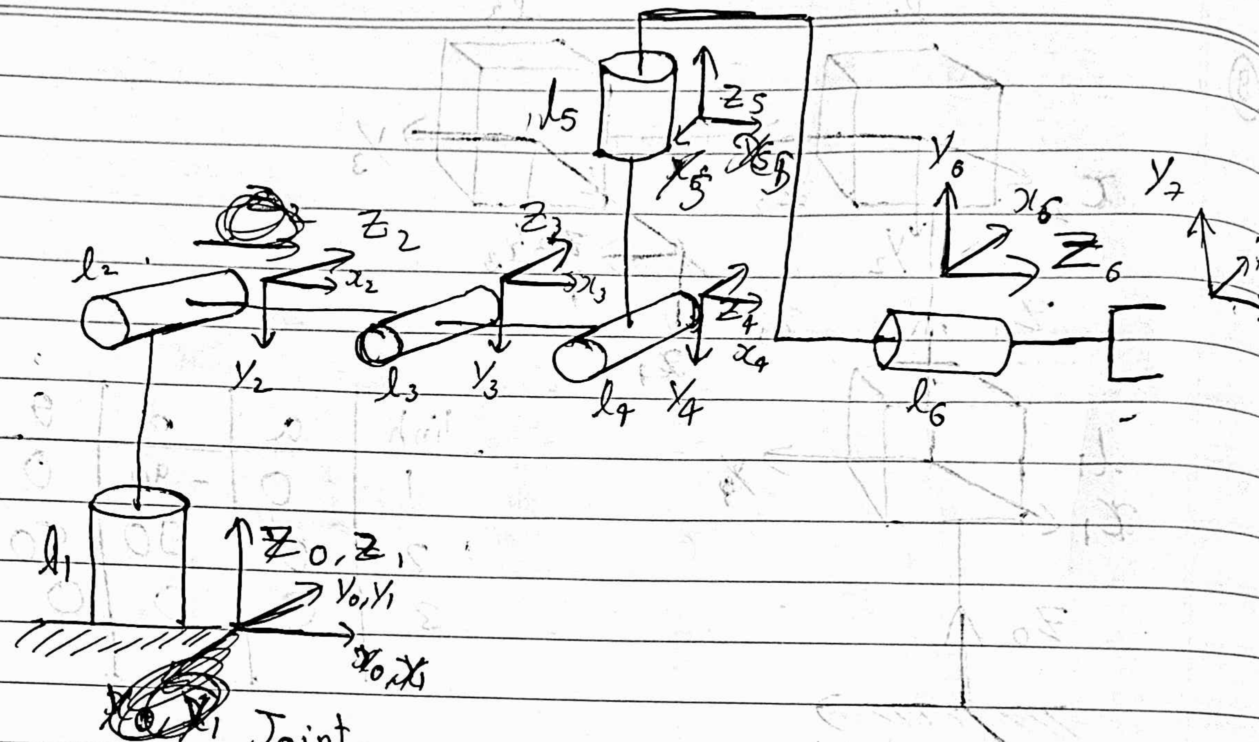
link	a	α	θ	d
1	0	-90	0	l_1
2	0	90	90	l_2
3	0	0	0	l_3

$$T_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T_0^4 = \begin{bmatrix} 0 & 0 & 1 & l_3 \\ 0 & 1 & 0 & l_2 \\ -1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

(10)



Joint	a	α	θ	d
1	0	0	θ_1	0
2	0	90	θ_2	l_1
3	0	0	θ_3	l_2
4	0	0	θ_4	l_3
5	0	-90	θ_5	l_4
6	0	0	θ_6	l_5
7	0	0	0	l_6

$$T_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_1 & S\theta_2 \\ S\theta_2 & 0 & -C\theta_2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_2^3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & 0 \\ S\theta_3 & C\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_3^4 = \begin{bmatrix} C\theta_4 & -S\theta_4 & 0 \\ S\theta_4 & C\theta_4 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T_4^5 = \begin{bmatrix} C_{\theta_5} & 0 & -S_{\theta_5} & 0 \\ S_{\theta_5} & 0 & C_{\theta_5} & 0 \\ 0 & -1 & 0 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^6 = \begin{bmatrix} C_{\theta_6} & -S_{\theta_6} & 0 & 0 \\ S_{\theta_6} & C_{\theta_6} & 0 & 0 \\ 0 & 0 & 1 & l_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^7 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_0^7 = T_0^1 \, T_1^2 \, T_2^3 \, T_3^4 \, T_4^5 \, T_5^6 \, T_6^7$$

$$T_0^7 = \begin{bmatrix} C_{\theta_1} C_{\theta_2} - S_{\theta_1} S_{\theta_2} & 0 & C_{\theta_1} S_{\theta_2} + C_{\theta_2} S_{\theta_1} & 0 \\ C_{\theta_2} S_{\theta_1} + C_{\theta_1} S_{\theta_2} & 0 & S_{\theta_1} S_{\theta_2} - C_{\theta_2} C_{\theta_1} & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{\theta_3} C_{\theta_4} - S_{\theta_3} S_{\theta_4} & -C_3 S_4 - S_3 C_4 & 0 & 0 \\ S_3 C_4 + C_3 S_4 & -S_4 S_3 + C_3 C_4 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C_5 C_6 & -S_6 C_5 & -S_5 & -S_5 l_5 \\ C_6 S_5 & -S_6 S_5 & C_5 & C_5 l_5 \\ -S_6 & -C_6 & 0 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= T_0^2 \, T_2^4 \begin{bmatrix} C_5 C_6 & -S_6 C_5 & -S_5 & -l_6 S_5 - S_5 l_5 \\ C_6 S_5 & -S_6 S_5 & C_5 & l_6 C_5 + C_5 l_5 \\ -S_6 & -C_6 & 0 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
= \begin{bmatrix}
(C_1C_2 - S_1S_2)(C_3C_4 - S_3S_4) & (C_1C_2 - S_1S_2)(-C_3S_4 - S_3C_4) & (C_1S_2 + C_2S_1) & l_3(C_1S_2 C_2S_1) \\
(C_2S_1 + C_1S_2)(C_3C_4 - S_3S_4) & (C_2S_1 + C_1S_2)(-C_3S_4 - S_3C_4) & (S_1S_2 - C_2C_1) & l_3(S_1S_2 - C_2C_1) \\
S_3C_4 + C_3S_4 & -S_4S_3 + C_3C_4 & 0 & l_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

$$
\begin{bmatrix}
C_5C_6 & -S_6C_5 & -S_5 & -l_6S_5 - S_5l_5 \\
C_6S_5 & -S_6S_5 & C_5 & l_6C_5 + C_5l_5 \\
-S_6 & C_6 & 0 & l_4 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$