Robot
- mechanically moving parts
- electrical actuations
- Autonomy → (sensing & controls, actions based on sensing)

Types of robots

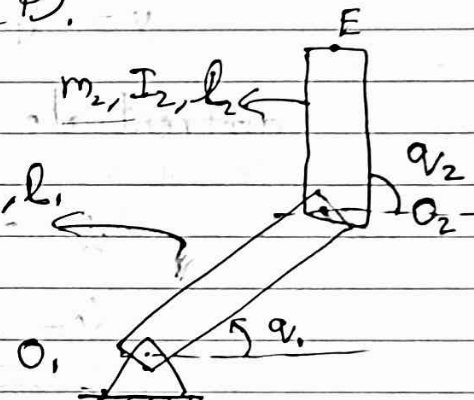
Manipulators, Mobile, Aerial, Higher level robot

Manipulators → Serial, parallel

2R manipulators (planar & elbow) -

2R → 2 Revolute joints

joints → Revolute (R), Prismatic (P).



E - end effector.
- (x, y)

$q_1, q_2 \Rightarrow$ joint angles.

$O_1 \Rightarrow$ origin

Note q_2 convention (about horizontal)

Let us assume that motors are connected to both joints O_1 & O_2 and we have the ability to control either the torque τ_1 & τ_2 applied at these joints or control the angles q_1 & q_2

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Tasks ⤵

T1 ⇒ (i) Given arbitrary trajectory of the end effector. (given x, y fn of time) make the robot follow this trajectory

T2 ⇒ Given a location of a wall, make the robot touch the wall & apply the constant predefined force on the wall.

T3 ⇒ make the robot behave like a virtual spring. [That has stiffness k & connects E to a specified point given (x_0, y_0)]

Now,

$$\left.\begin{aligned} x &= l_1 \cos q_1 + l_2 \cos q_2 \\ y &= l_1 \sin q_1 + l_2 \sin q_2 \end{aligned}\right\}$$

(conviniently.

forward kinematics

$$\left.\begin{aligned} x &= l_1 cq_1 + l_2 cq_2 \\ y &= l_1 sq_1 + l_2 sq_2 \end{aligned}\right\} \longrightarrow (1)$$

diffrentiate eq^n (1)

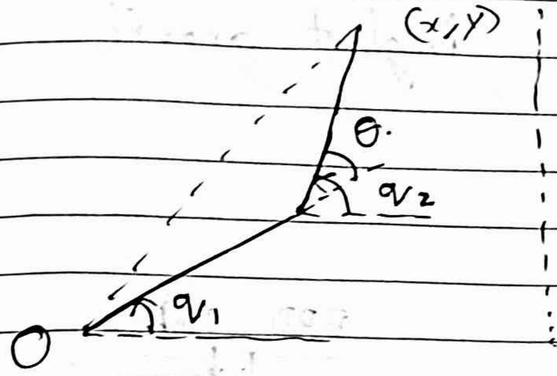
$$\left.\begin{aligned} \dot{x} &= -l_1 sq_1 \dot{q}_1 \cancel{+} l_2 sq_2 \dot{q}_2 \\ \dot{y} &= l_1 cq_1 \dot{q}_1 + l_2 cq_2 \dot{q}_2 \end{aligned}\right\} \text{end effector velocity}$$

$$\boxed{\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \cdot \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{Bmatrix}} \quad — (2)$$

· need a reverse relationships.

→ Given x & y.
→ solve for q_1 & q_2

} options.



option 1 ⇒ solve Numerically
option 2 ⇒ derive closed form equation.

Hard ; multiple solutions

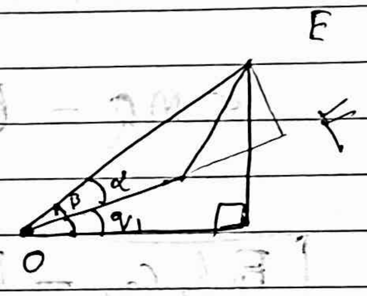
$$\theta = q_2 - q_1$$

$$\boxed{\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)}$$

$$q_1 = \beta - \gamma$$
$$= \tan^{-1}\left(\frac{y}{x}\right)$$
$$\tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right)$$
$$= \tan^{-1}\left(\frac{EF}{OF}\right)$$

$$q_2 = q_1 + \theta$$

→ ③ inverse Kinematics

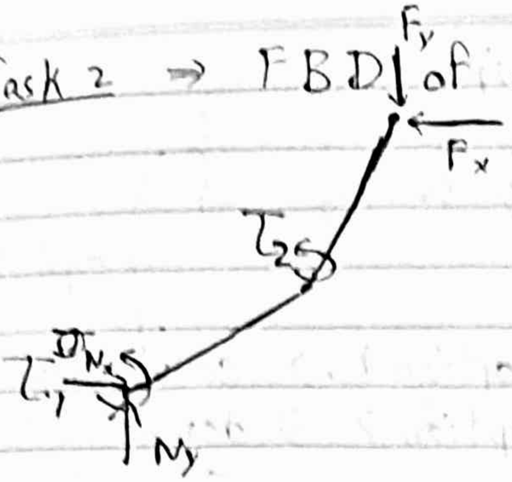


T1 ⇒ control both motors in position control mode to achieve above q_1 & q_2 at each time step.

*

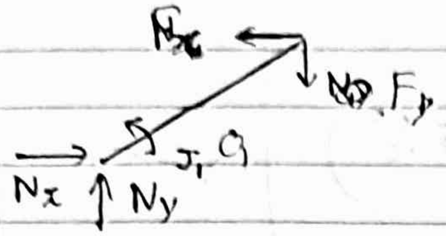
x_d, y_d
q_{1d}, q_{2d} } → desired values

Task 2 → FBD of entire robot

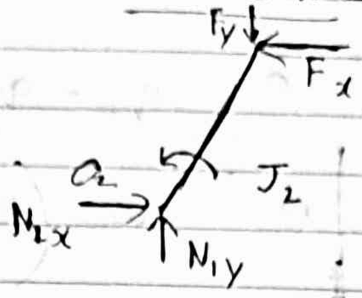


Neglect gravity

FBD Link 1



FBD Link 2.



$$\Sigma M_{O_2} = 0 \qquad \Sigma M_{O_1} = 0$$

$$\boxed{\begin{aligned} F_y l_2 c_{q_2} - F_x l_2 s_{q_2} &= \tau_2 \\ F_y l_1 c_{q_1} - F_x l_1 s_{q_1} &= \tau_1 \end{aligned}} \rightarrow (4)$$

(3) along with (4) answers τ_2

$$\begin{bmatrix} -l_2 s_{q_2} & l_2 c_{q_2} \\ -l_1 s_{q_1} & l_1 c_{q_1} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \tau_2 \\ \tau_1 \end{bmatrix}$$

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Dynamic effects -

Lagrange's eq^ns -

$$\mathcal{L} = \overset{K.E}{K} - \underset{PE}{V}$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'} \longrightarrow (5)$$

$q_i \Rightarrow$ independent DOF.

$Q_i' \Rightarrow$ generalized forces derived using principle of virtual work.

$i \Rightarrow$ No. of DOF

$$K = \underbrace{\frac{1}{2}\left(\frac{1}{3} m_1 l_1^2\right) \dot{q}_1^2}_{\text{pure rotation of } L_1} + \underbrace{\frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right) \dot{q}_2^2}_{\text{rotation of } L_2} + \underbrace{\frac{1}{2} m_2 v_{c_2}^2}_{\text{translation of } L_2}$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2} \dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2} \dot{q}_2 \, c(q_1 - q_2)$$

$$V = m_1 g \frac{l_1}{2} sq_1 + m_2 g \left(l_1 sq_1 + \frac{l_2}{2} sq_2\right)$$

$$\mathcal{L} = K - V$$

$$\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_2^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \, c(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \, s(q_2 - q_1) + m_1 g \frac{l_1}{2} c q_1$$
$$+ m_2 g l_1 \, c q_1 = \tau_1$$

$$\frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \, c(q_2 - q_1)$$
$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \, s(q_2 - q_1) + m_2 g \frac{l_2}{2} s q_2 = \tau_2$$

— (6) eq^n of motion

Task 3:

$F_x = kx$

$F_y = ky$

using (1) & (4)

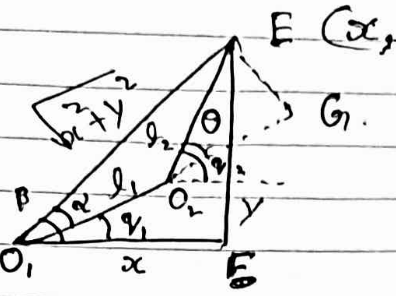
$$\tau_{2s} = k(l_1 sq_1 + l_2 sq_2) \, l_2 cq_2 - k(l_1 cq_1 + l_2 cq_2) \, l_2 sq_2$$
$$\tau_{1s} = k(l_1 sq_1 + l_2 sq_2) \, l_1 cq_1 - k(l_1 cq_1 + l_2 cq_2) \, l_1 sq_1$$

— (7)

Answer to T3

* code forward & inverse kinematic using python
→ 3 tasks.

Assignment



$y = h + k\cos(t)$

$x = a\cos t$
$y = b\sin t.$

$$\tan\alpha = \frac{EG}{O_1G} = \frac{l_2\sin\theta}{l_1 + l_2\cos\theta}.$$

$$\tan\beta = \frac{y}{x}$$

$$q_1 = \beta - \alpha$$

$$= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1 + l_2\cos\theta}\right)$$

$$q_2 = q_1 + \theta.$$

$$\theta = \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right)$$