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Assignment - 2

(2) show columns of R_0^1 matrix are orthogonal.

we know the orthogonal means angular ~~dis~~ b/w them = 90°
& column matrix ⇒ vector

⇒

$$R_0^1 = \begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix}$$

if $C_1 \cdot C_2 = 0$, $C_2 \cdot C_3 = 0$ & $C_3 \cdot C_1 = 0$

⇒ orthogonal coumns

$$\vec{C_1} = C_\theta \hat{i} + 0\hat{j} - S_\theta \hat{k}$$
$$\vec{C_2} = 0\hat{i} + \hat{j} + 0\hat{k}$$
$$\vec{C_3} = S_\theta \hat{i} + 0\hat{j} + C_\theta \hat{k}$$

$\vec{C_1} \cdot \vec{C_2} = 0$ | $\vec{C_2} \cdot \vec{C_3} = 0$ | $\vec{C_3} \cdot \vec{C_1} = C_\theta S_\theta - C_\theta S_\theta = 0$

⇒ columns are orthogonal

(3) show $|R_0^1| = 1$

$$\det\left(\begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix}\right) = C_\theta(C_\theta - 0) + 0 + S_\theta(0 + S_\theta)$$

$$= C_\theta^2 + S_\theta^2 = \underline{1}$$

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(6) Show that $RS(a)R^T = S(R_a)$

we know $S \Rightarrow$ a skew symmetric matrix

Let $S = \begin{bmatrix} 0 & -S_1 & S_2 \\ S_1 & 0 & -S_3 \\ -S_2 & S_3 & 0 \end{bmatrix}$

& $a = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}^T$

then

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

for any $R \in SO(3)$
we know that

$$S(a)\cdot p = a \times p \quad —(1)$$
$$R(a \times b) = Ra \times Rp \quad —(2)$$

Now let a vector $R^T b$

$$S(a)R^T b = a \times R^T b \quad \text{from (1)}$$

multiply R both sides

$$R(S(a)R^T b) = R(a \times R^T b)$$

from (2)

$$= R\cdot a \times R\cdot R^T b$$

we know that $R\cdot R^T = I$

$$= Ra \times b$$

Now from (1)

$$= S[Ra]b.$$

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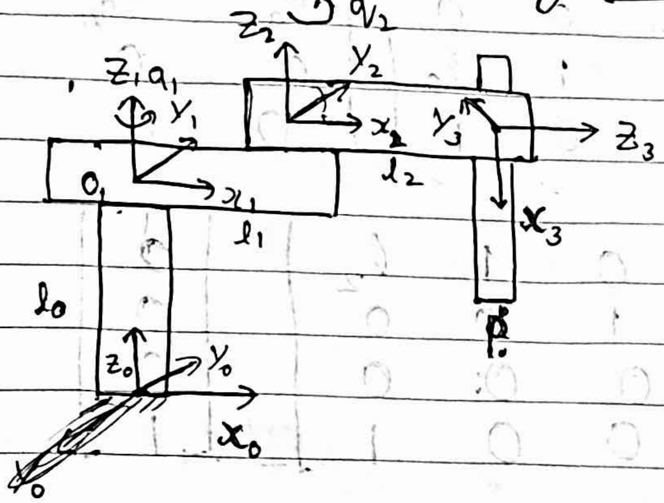
⑨ RRP ~~stanford~~ Scara configuration :-



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

for H_2^3 → $R_2^3 = R_{y, 90°}$ → $d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$

$$H_2^3 = \begin{bmatrix} 0 & 0 & 1 & l_2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for H_1^2 → $R_1^2 = R_{z_2, q_2}$ → $d_1^2 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$

$$H_1^2 = \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & l_1 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for H_0^1 → R_0^1 → $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}$

$$H_0^1 = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for $P_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} P_{ox} \\ P_{oy} \\ P_{oz} \\ 1 \end{bmatrix} = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & l_1 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & l_2 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & l_1 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

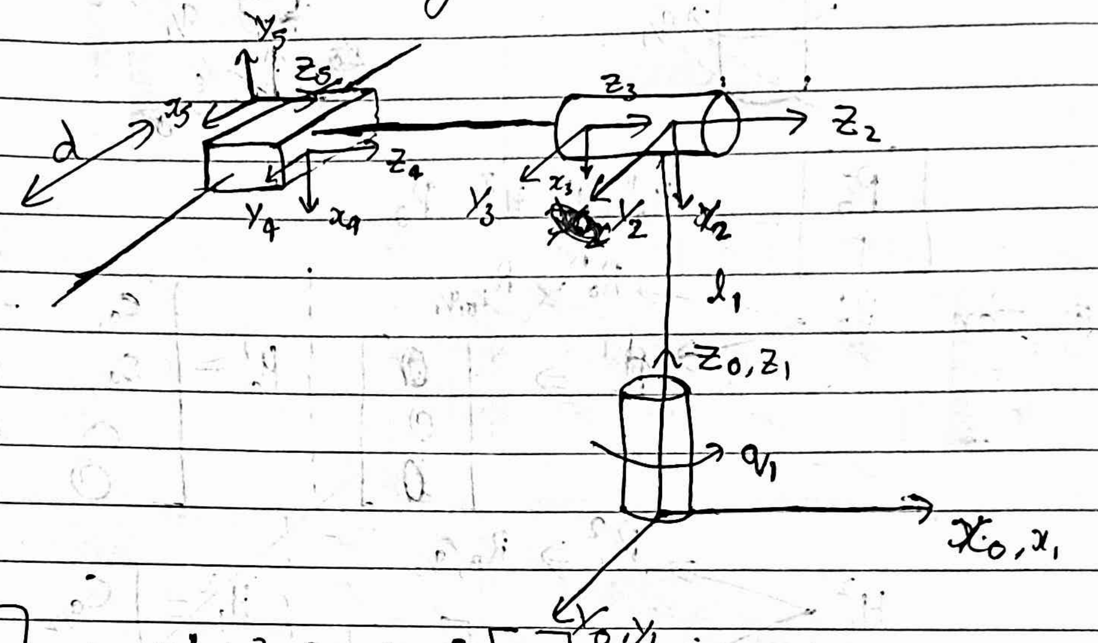
$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{q_2} l_2 + l_1 \\ S_{q_2} l_2 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} C_{q_2} l_2 + l_1 C_{q_1} - S_{q_2} S_{q_2} l_2 \\ S_{q_1} C_{q_2} l_2 + S_{q_1} l_1 + S_{q_2} C_{q_1} l_2 \\ l_0 - d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{(q_1+q_2)} l_2 + l_1 C_{q_1} \\ S_{(q_1+q_2)} l_2 + l_1 S_{q_1} \\ l_0 - d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Stanford Configuration-



$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 H_3^4 \cdot H_4^5 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$H_0^1 \to R_{z,q_1}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$H_1^2 \to R_{y,-90}$, $\begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$

$H_2^3 \to R_{z_3,q_2}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$H_3^4 \to R_{z,0}$, $\begin{bmatrix} 0 \\ 0 \\ -l_2 \end{bmatrix}$

$$H_4^5 \to R_{z_4,90}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_5 = \begin{bmatrix} -d \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} P_{0x} \\ P_{0y} \\ P_{0z} \\ 1 \end{bmatrix} = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & 0 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & 0 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -d \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & 0 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -d \\ -l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ d S_{q_2} \\ d C_{q_2} + l_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 C_{q_1} - S_{q_1} S_{q_2} d \\ l_2 S_{q_1} + S_{q_2} C_{q_1} d \\ d C_{q_2} + l_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d S_{q_2} \\ -d C_{q_2} \\ -l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ -d C_{q_2} \\ d S_{q_2} + l_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1} l_2 + S_{q_1} C_{q_2} d \\ S_{q_1} l_2 - C_{q_1} C_{q_2} d \\ S_{q_2} d + l_1 \\ 1 \end{bmatrix}$$

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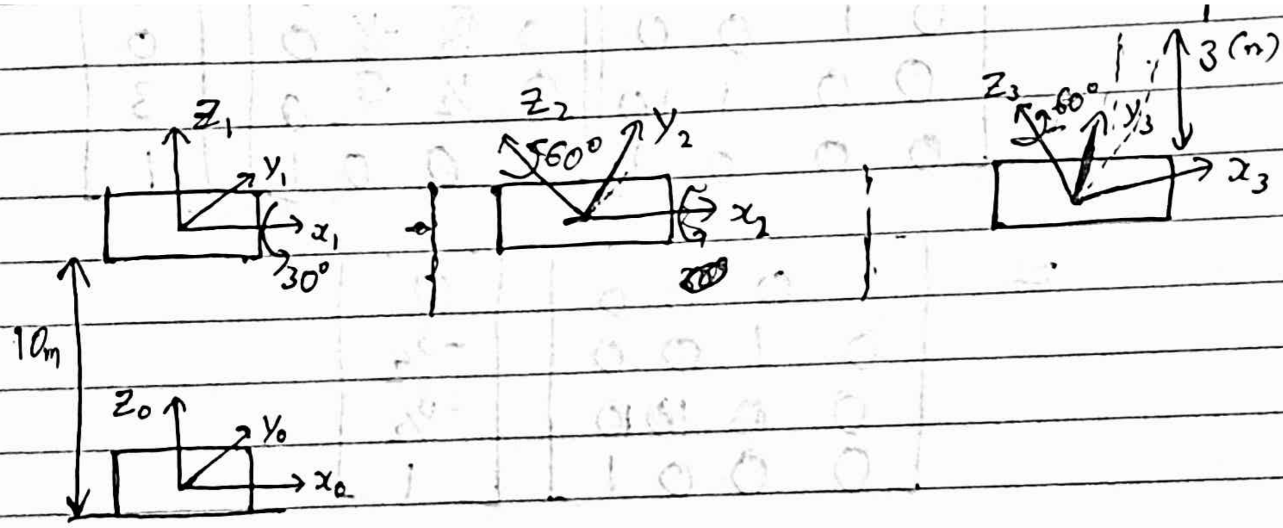

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \, P_3^3 \qquad P_3^3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$H_0^1 \begin{cases} \rightarrow R_{z,0} \\ \rightarrow d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \end{cases} \qquad H_1^2 \begin{cases} \rightarrow R_{x,30} \\ \rightarrow d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

$$H_2^3 \begin{cases} \rightarrow R_{z,60} \\ \rightarrow d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \end{cases}$$

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$$\begin{bmatrix} P_{ox} \\ P_{oy} \\ P_{oz} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

$$\boxed{P\ (0, -1.5, 12.6)}$$