Multidimensional Discrete Fourier Transform

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1 Abstract

This paper aims to use the discrete Fourier transform on points in a multidimensional (say n-dimensional) space.

2 The Actual Stuff

Let the points be represented by vectors of the form

$$\overrightarrow{r}(t) = \sum_{k=1}^{n} \left(x_k(t)\, \hat{e_k} \right),$$

where t is a non-negative integer less than the total number of points (say N) and $x_k(t)$ is a real number for all positive integers k, less than or equal to n.
Let

$$f_{u,v}(t) = x_u(t) + i \cdot x_v(t),$$

where $i = \sqrt{-1}$.
Let

$$c_{u,v}^{(k)}(t) = \frac{1}{n-1} \sum_{t=0}^{N-1} \left(f_{u,v}(t)\, e^{-i\frac{2\pi k t}{N}} \right).$$

For each ordered pair (u, v), a set of epicycles is generated by computing $c_{u,v}^{(k)}(t)$ for all integers k in the range $\left[-\lfloor \frac{N-1}{2} \rfloor, \lfloor \frac{N}{2} \rfloor\right]$. In total, ${}^{n}C_2$ sets of epicycles are generated.

3 Future Plans

- Try introducing more complex numbers (like quaternions) to get the stuff done with only one set of epicycles.
- Try to orient each epicycle with angles like θ and ϕ to get the stuff done with only one set of epicycles.