Heat Transfer

Conduction

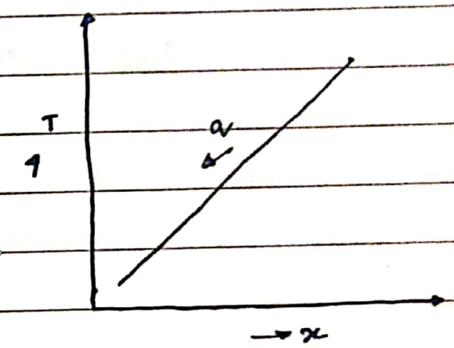
Heat loss is a vector, and is the -ve product of thermal conductivity and the gradient of T

$$\vec{q} = -k \nabla T$$

→ For further calculations, we consider k to be a constant.

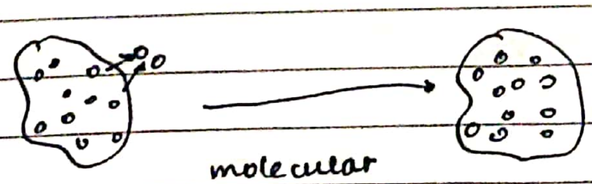
→ For steady 1D conduction

$$q = -k \frac{dT}{dx}$$



Convection

Consider a chunk of fluid molecules, convective movement means, that these fluid molecules are moved to another place. Even during this convection, there are molecules around this chunk, now this chunk collide with these molecules, thereby transferring heat. This kind of transfer of heat by molecular movement is convection



While convection occurs, conduction also happens. The degree of which dominates depends on the

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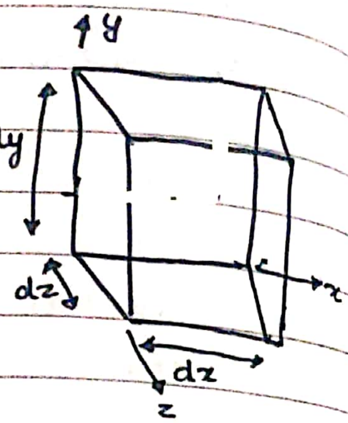
Situation:

Microscopic Energy Balance



→ This is a diff. control volume

→ Energy content / unit vol. of fluid

Enthalpy = $C_p (T - T_{ref})$

$\Rightarrow \quad h = C_p (T - T_{ref})$

$T_{ref} = 0K \quad \Rightarrow \quad h = C_p T \quad \frac{J}{kg} \cdot \rho \frac{kg}{m^3}$

$\therefore \quad h = \rho C_p T$ → Enthalpy / unit vol. of fluid

→ Flux = velocity * Density

$\frac{d}{dt} (\rho C_p T \, dx \, dy \, dz) = E_{in} - E_{out} + \text{Generation}$

$E_{in} - E_{out} \rightarrow \sum_{\text{all planes}} (E_{flux} \times A)$

$\text{Generation} \rightarrow q \, dx \, dy \, dz$

$\hookleftarrow$ ①

We consider only conduction and convection

$q = -k \nabla T \quad$ -②

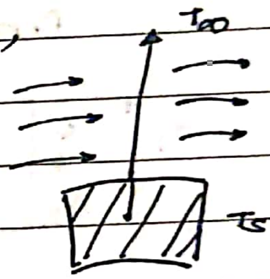
After solving the above equation, we get,

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) - \left(\frac{\partial (u_x T)}{\partial x} + \frac{\partial (u_y T)}{\partial y} + \frac{\partial (u_z T)}{\partial z}\right) + q$$

$$\alpha = \frac{k}{\rho C_p} \rightarrow \text{Thermal Diffusivity}$$

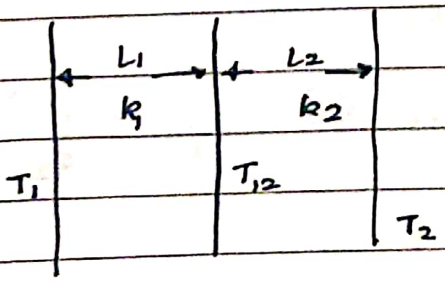
Newton's law of Cooling

Consider a diff. area, let the temp. at surface be T_s, and at a far away location be T_∞ and fluid is flowing over it. Then,



$$q_{conv.} = \underset{\substack{\downarrow \\ \text{Heat Transfer coeff}}}{h}\,(T_s - T_\infty)$$

For steady 1D conduction $\frac{dT}{dx}$ is const.



$q_x = ?$

← Resistances are in series

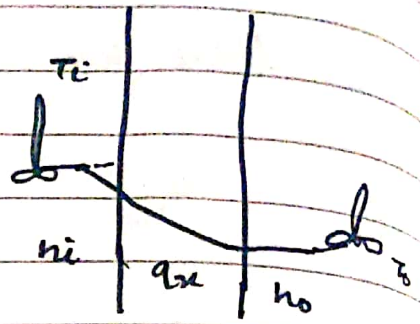
$$q_x = \frac{T_1 - T_2}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)}$$

$T_1 - T_2$ → Voltage

q_x → Current

$\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)$ → Resistance

$$q_x = h_i (T_i - T_{iw}) \quad -①$$



$$q_x = -k \frac{T_{iw} - T_{ow}}{0 - L} \quad -②$$

$$q_x = h_o (T_{ow} - T_o) \quad -③$$

$$q_L = \left(\frac{L}{k} + \frac{1}{h_i} + \frac{1}{h_o}\right)^{-1} (T_i - T_o)$$

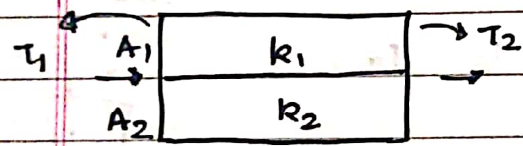
↳ overall temp coeff

↳ overall heat coeff

$$= U (T_i - T_o)$$

$$\frac{1}{U} = \frac{1}{k} + \frac{1}{h_i} + \frac{1}{h_o}$$

Parallel heat conduction



→ One-dimensional Heat conduction

$$Q_1 = q A_1 = \frac{T_1 - T_2}{\underbrace{(L/A_1 k_1)}_{R_1}}$$

$$Q_2 = \frac{T_1 - T_2}{\underbrace{(L/A_2 k_2)}_{R_2}}$$

$$Q = Q_1 + Q_2 = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_{eff} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

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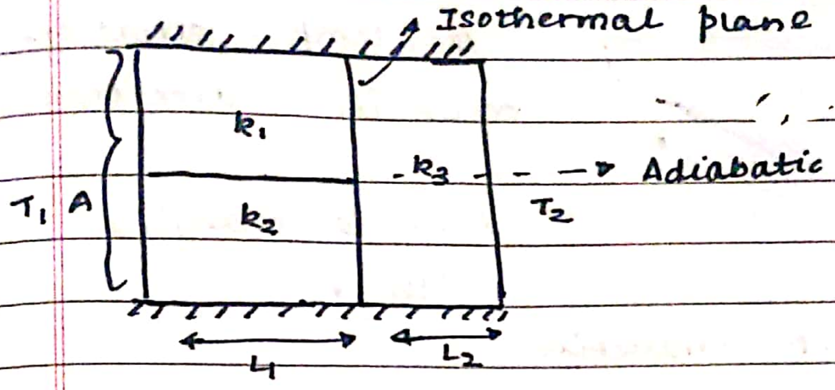
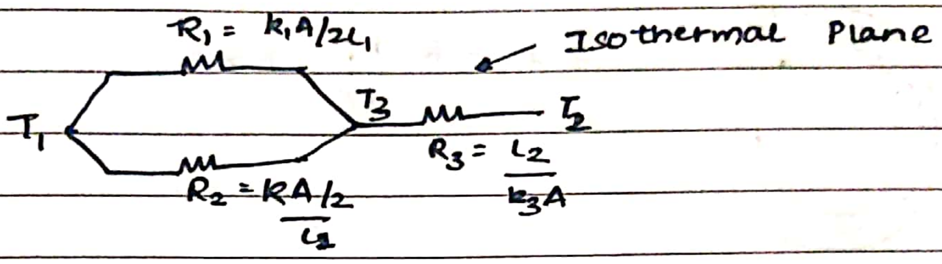
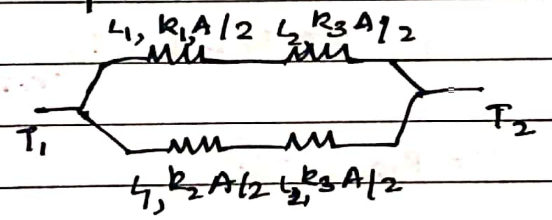


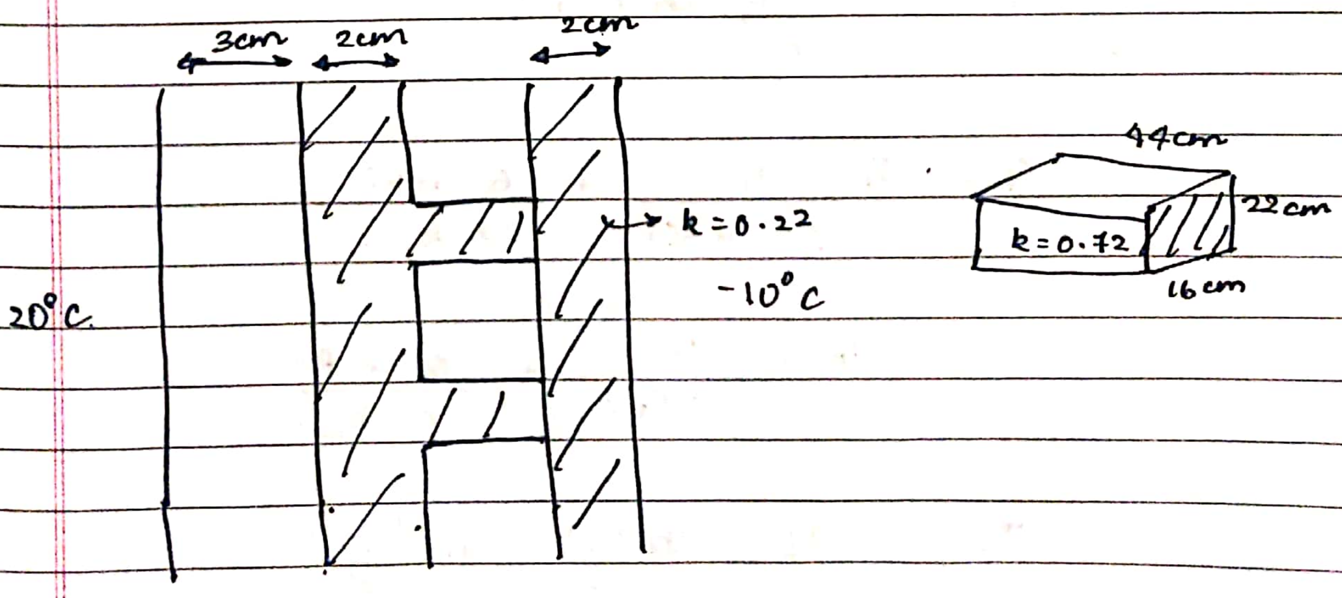


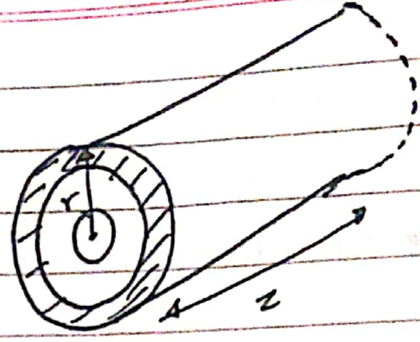
$$R_{eff} = \left(\frac{k_1 A}{2L_1} + \frac{k_2 A}{2L_2}\right)^{-1} + \frac{L_2}{k_3 A}$$

→ Planes parallel to x-axis are adiabatic



Our temperature (T_r) will lie between T_{adi} and T_{iso}

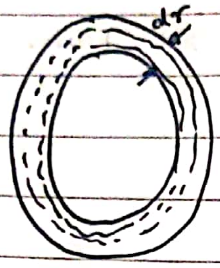




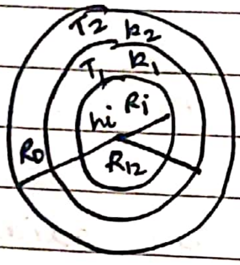
All temp. variations occur in r-direction.

→ Area is changing with r

→ steady, 1D conduction



$$q = -k \frac{dT}{dr} 2\pi r L$$



$$-k_2 \frac{dT_2}{dr} 2\pi r L = q$$

BC → $q = 2\pi R_i L h (T_i - T_1)$ At $r = R_i$

$$q = 2\pi R_o L h \left(T_2\big|_{r=R_o} - T_o\right)$$

$T_1 = T_2$ @ $r = R_{12}$ — ①

$$T_1 = -\frac{Q}{2\pi L k_1} \ln r + c_1$$

$$T_2 = -\frac{Q}{2\pi L k_2} \ln r + c_2$$

$$c_1 = T_1 - \frac{Q}{2\pi h_i R_i L} + \frac{Q \ln R_i}{2\pi k_1 L}$$

$$c_2 = T_o + \frac{Q}{2\pi R_o L} + \frac{Q \ln R_o}{2\pi k_2 L}$$

$$T_{12} = T_i - \frac{(Q/L)}{2\pi k_1} \left(\ln \frac{R_{12}}{R_i} \right)$$

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Now we use ①, to find Q

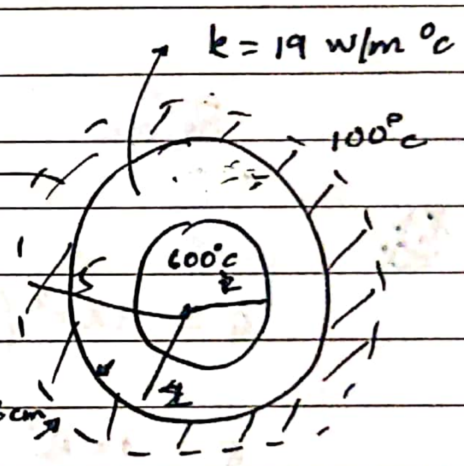
$$(T_i - T_o) = Q\left[\frac{\ln R_{12}/R_i}{2\pi L k_1} + \frac{\ln R_o/R_{12}}{2\pi L k_2} + \frac{1}{2\pi R_o L h_o} + \frac{1}{2\pi R_i L h_i}\right]$$

$$A_i = 2\pi R_i L$$

$$A_o = 2\pi R_o L$$

$$Q = \underbrace{U_o}_{\text{overall heat transfer co-eff}} A_o (T_i - T_o) = U_i A_i (T_i - T_o)$$

④



Since, the resistances are given, we neglect the ~~hea~~ h terms

$$(600 - 100) = \frac{Q}{L}\left(\frac{\ln(2/1)}{2\pi \times 19} + \frac{\ln(5/2)}{2\pi \times 0.2}\right)$$

$$\frac{Q}{L} = 680.302 \text{ W/m}$$

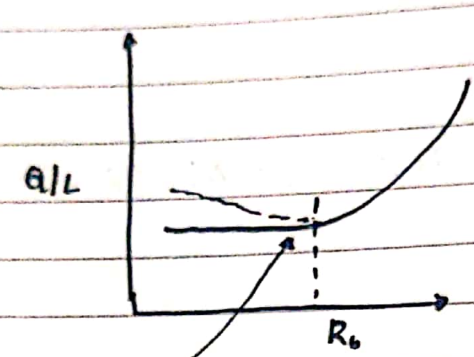
$$T_2 = -\frac{Q}{L}\left(\frac{\ln 2}{2\pi k_2}\right) + T_1$$

$$= -680.302 \frac{(\ln \overset{1\times10^{-2}}{\cancel{2}})}{2\pi \times 19} + 600$$

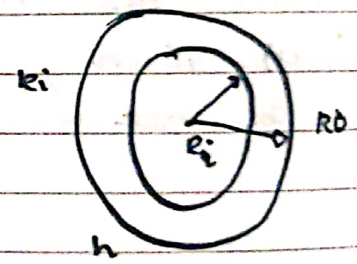
$$=$$

$$\frac{1}{U_o} = \frac{R_o \ln R_{12}/R_0}{k_1} + \frac{R_o \ln R_0/R_{12}}{k_2} + \frac{1}{h_o} + \frac{R_o}{R_i h_i}$$

$$\frac{1}{U_i} = \frac{R_i \ln R_{12}/R_i}{k_1} + \frac{R_i \ln R_o/R_{12}}{k_2} + \frac{R_i}{R_o h_o} + \frac{1}{h_i}$$



$$R_{o,opt} = \cancel{\frac{h_o}{k_2}} \; \frac{k_2}{h_o}$$



$$Q = 4\pi r^2 \left(-k_i \frac{dT}{dr}\right)$$

$$Q = 4\pi R_o^2 h (T_o - T_a)$$

$$T\big|_{r=R_o} = T_a + \frac{Q}{4\pi R_o^2 h}$$

$$\frac{dT}{dr} = -\frac{Q}{4\pi k_i} \frac{1}{r^2}$$

$$T = -\frac{Q}{4\pi k_i}\left(\frac{-1}{r}\right) + C_1$$

$$T_a + \frac{Q}{4\pi R_o^2 h} = \frac{Q}{4\pi k_i R_o} + C_1$$

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$$T = \frac{Q}{4\pi k_i r} + T_a + \frac{Q}{4\pi R_0^2 h} - \frac{Q}{4\pi k_i R_0}$$

$$T_i = \frac{Q}{4\pi k_i R_i} + T_a + \frac{Q}{4\pi R_0^2 h} - \frac{Q}{4\pi k_i R_0}$$

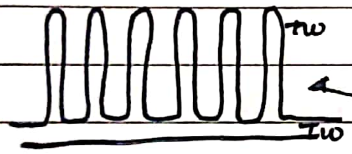
$$-Q = (T_a - T_i)\left(\frac{1}{4\pi R_0^2 h} + \frac{\left(\frac{1}{R_i} - \frac{1}{R_0}\right)}{4\pi k_1}\right)^{-1}$$

$$T_i - T_0 = Q\left[\frac{1}{h R_i 2\pi L} + \frac{1}{h_0 R_0 2\pi L} + \underbrace{\frac{\ln(R_{12}/R_i)}{k_1 2\pi L}}_{\to 0} + \frac{\ln(R_0/R_{12})}{k_2 2\pi L}\right]$$

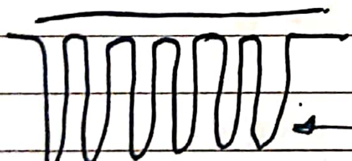
$R_{12} >> R_i$

OR $k_1 \to \infty$

Then $\to 0$

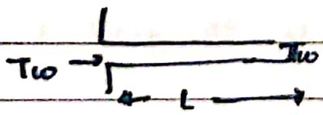


Surface that we add are at wall Temp



More surface, more heat transfer rate

Only efficient upto a certain length.



$$Q_{actual} = A_0\left(-k \frac{dT}{dx}\bigg|_{x=0}\right)$$

$$Q_{max} = A_{fin}\, h\, (T_w - T_a)$$

$$\eta = \frac{Q_a}{Q_m}$$

There will T drop along fin, and after some point, it reaches ambient temp.

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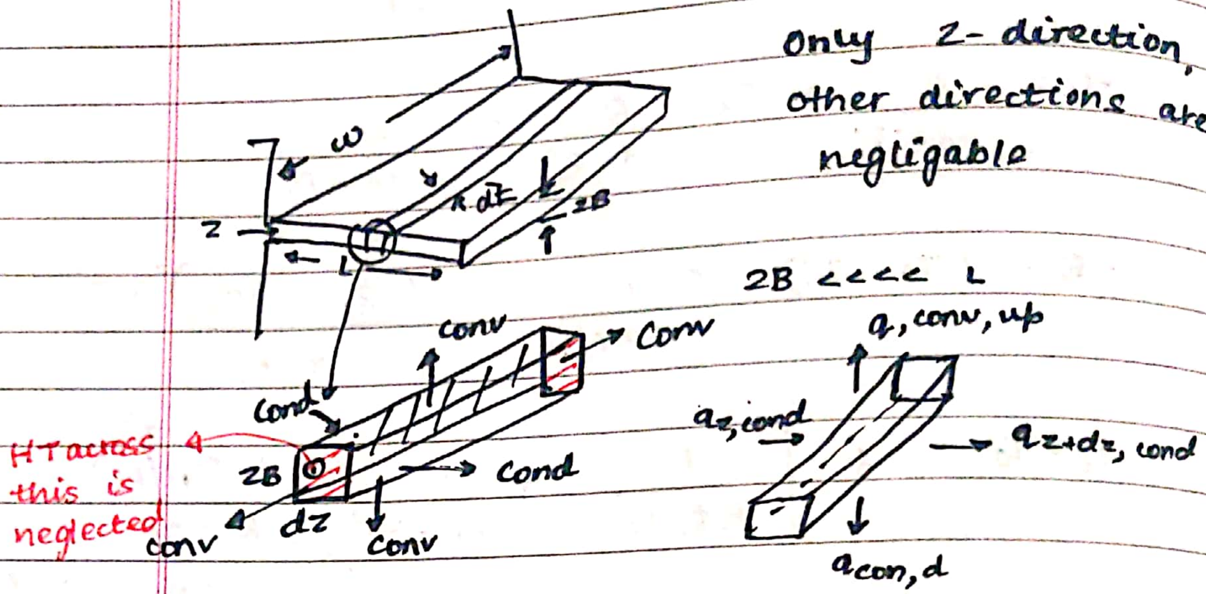
Ideal fin will have same temp. everywhere



Only Z-direction, other directions are negligable

2B <<<< L

$$q_z (2Bw) = q_{z+dz} (2Bw) + 2dz\,w\,h(T-T_a)$$

$$\frac{dq}{dz} + \frac{\cancel{2}\cancel{w}h}{\cancel{2}B\cancel{w}}(T-T_a) = 0$$

$$\frac{d^2T}{dz^2} - \frac{h}{Bk}(T-T_a) = 0$$

$$\theta = \frac{T-T_a}{T_w - T_a} \qquad 0 < \xi < 1 \qquad \xi = \frac{z}{L}$$

Non-dimensionlizing so as to give a bound

Boundary Conditions

$Z=0, \quad T = T_w$

$Z=L, \quad T = T_a$ (Inf. Long Fin)

$$-k\frac{\partial T}{\partial z}\bigg|_{z=L} = h(T-T_a)\big|_{z=L}$$

Considering Adiabatic fin,

$z = 0 \rightarrow T = T_w$

$z = L \rightarrow \frac{dT}{dz} = 0$

$$\frac{d^2\theta}{d\xi_1^2} - \frac{hL^2}{Bk}\cdot\theta = 0 \qquad \xi_1 = 0 \quad \theta = 0$$
$$\xi_1 = 1 \quad \frac{d\theta}{d\xi_1} = 0$$

$$N^2 = \frac{hL^2}{Bk} = \frac{\text{Conductive Res.}}{\text{Convective Res.}}$$

Biot No: $\theta = c_1 e^{N\xi_1} + c_2 e^{-N\xi_1}$

$$c_1 + c_2 = 1$$
$$c_1 N e^{N} - c_2 N e^{-N} = 0$$

$$c_2 = \frac{e^N}{e^N + e^{-N}} = \frac{1}{1 + e^{-2N}}$$

$$c_1 = 1 - \frac{1}{1+e^{-2N}} = \frac{e^{-2N}}{1 + e^{-2N}}$$

$$Q_{act} = (\cancel{k}w2B)\frac{dT}{dz}\Big|_{z=0} = \frac{(\cancel{k}w2B)(T_w - T_a)}{L}\frac{d\theta}{d\xi_1}\Big|_{\xi_1=0}$$

$$= \frac{w2B(T_w - T_a)}{L}\left\{\frac{Ne^{-2N}}{1+e^{-2N}} - \frac{N}{1+e^{-2N}}\right\}$$

$$Q_{ideal} = Q_{max} = 2WL\, h(T_w - T_a)$$

$$\eta = \frac{\cancel{w}2Bk\cancel{(T_w - T_a)}\{\ \ \}}{\cancel{2w}L^2 h\cancel{(T_w - T_a)}}$$

$$= \frac{\{\ \ \}}{\frac{L^2 h}{kB}} = \frac{1}{N}\left(\frac{e^{-N} - e^{N}}{e^{-N} + e^{N}}\right) = \frac{\tanh N}{N}$$

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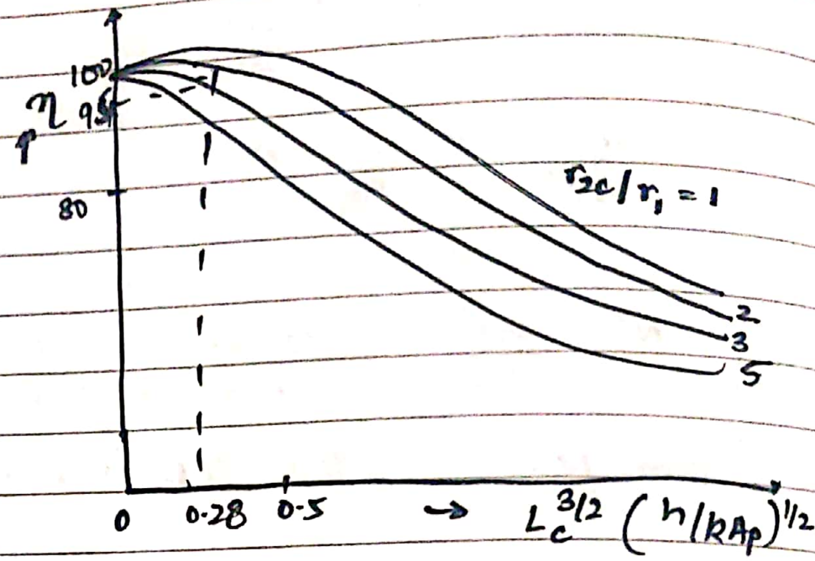
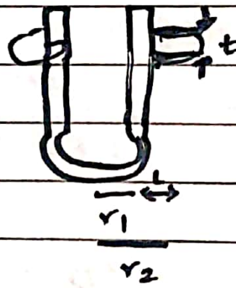
→ fin efficiency is a fn. of h, k, geometry



Efficiency of annular fins of rectangular profile



$r_{2c} = r_2 + t/2$

$L_c = L + t/2$

$A_p = L_c t$

If $L_c^{3/2}(h/kA_p)^{1/2} = 0.2$, $\frac{r_{2c}}{r_1} = 1$, Then $\eta = 95\%$

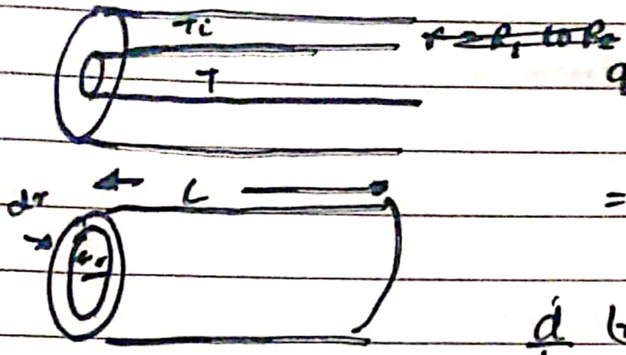
$R = \frac{\rho L}{A}$

Rate of Heat generation / vol. $= S_e = \frac{I^2 R}{AL}$

$i = \frac{I}{A}$

$= \frac{I^2 \rho}{A^2}$

$S_e = \frac{i}{A}$ ← current density (Current / Area)



$$q_r (2\pi r L) + S_e \cdot (2\pi r \, dr \, L) = q_r (2\pi r L)\Big|_{r+dr}$$

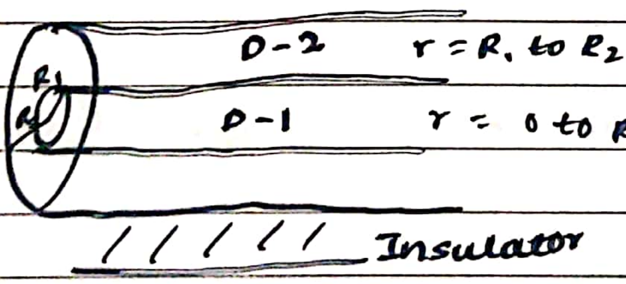
$$\frac{d}{dr}(r q_r) = r S_e$$

$$q_r = -k \frac{dT}{dr}$$

$$\Rightarrow -k \frac{d}{dr}\left(r \frac{dT}{dr} \right) = + r \cdot S_e$$

$$-k \frac{d}{dr}\left(T \frac{dT}{dr} \right) = r S_e$$

$$\therefore \frac{d}{dr}\left(r \frac{dT}{dr} \right) = -\frac{r S_e}{k_e}$$



D-1

$$T = -\frac{S_e}{k} \frac{r^2}{4} + c_1 \ln r + c_2$$

D-2

$$T = c_3 \ln r + c_4$$

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Boundary conditions

D-1

$r = 0$ T_1 is finite

$r = R_1$ $T_1 = T_2$

$r = R_1$ $-k_1 \frac{dT}{dr}\Big|_{r=R_1} = -k_2 \frac{dT_2}{dr}\Big|_{r=R_1}$

$r = R_2$ $-k \frac{dT_2}{dr}\Big|_{r=R_2} = h(T_b - T_a)$

$$c_3 = -\frac{S_e R_1^2}{2k_2}$$

$$c_4 = T_a + \frac{S_e R_1^2}{2k_2}\left(\ln R_2 + \frac{k_2}{\ln R_2}\right)$$

$$c_2 = T_a + \frac{S_e R_1^2}{2k_2}\left(\ln \frac{R_2}{R_1} + \frac{k_2}{4 R_2}\right) + \frac{S_e R_1^2}{4k_1}$$

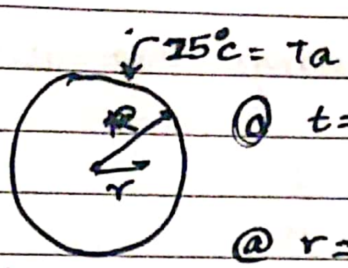
$$c_1 = 0$$

$$\therefore T_1 = T_a + \frac{S_e}{4k}(R_1^2 - r^2) + \frac{S_e R_1^2}{2k_2}\left[\ln \frac{R_2}{R_1} + \frac{k_2}{\ln R_2}\right]$$

$$T_2 = T_a + \frac{S_e R_1^2}{2k_2}\left(\ln \frac{R_2}{r} + \frac{k_i}{h R_2}\right)$$

Transient 1D - conduction

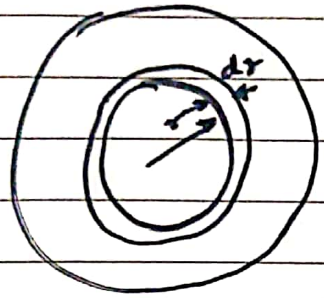
→ Temp. varies with length and time.



@ $t=0$, $T(r,t) = 5°c$ (say)

$T = 5°C \ \forall\ r$

@ $r = R$ $t > 0$, $T = T_a \ \forall\ t > 0$



$$\frac{\partial}{\partial t}(4\pi r^2 dr \rho C_p T) = E_{in} - E_{out}$$

$\hookrightarrow q_r$

$$4\pi r^2 q_r|_r - 4\pi r^2 q_r|_{r+dr}$$

$$4\pi r^2 C_p \rho \frac{\partial T}{\partial t} = -4\pi r^2 q_r|_{r+dr} + 4\pi r^2 q_r|_{r+dr}$$

$$\frac{\partial T}{\partial t} = -\frac{4\pi}{4\pi r^2 \rho C_p}\left[\frac{r^2 q_r|_{r+dr} - r^2 q_r|_r}{dr}\right]$$

$$\frac{\partial T}{\partial t} = -\frac{1}{r^2 \rho C_p}\frac{\partial}{\partial r}(r^2 q_r)$$

$$= \frac{k}{r^2 \rho C_p}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

$$= \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

$$\therefore \quad \frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

@ $t = 0$ $\quad T = T_0 \ \forall\ r$

@ $r = 0$ $\quad \frac{\partial T}{\partial r} = 0 \ \forall\ t$

@ $r = R$ $\quad -k\frac{\partial T}{\partial r} = h(T - T_a) \ \forall\ t$

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Now we try making it non-dimensional,

$$\theta = \frac{T - T_a}{T_0 - T_a} \qquad \xi_1 = \frac{r}{R}$$

↑ Fraction of temp change that already happened

$$\hat{t} = \frac{t}{t^*} \qquad t^* = \frac{R^2}{\alpha}$$

$$\therefore \hat{t} = \frac{\alpha t}{R^2}$$

After non-dimensionalising,

$$\frac{\partial \theta}{\partial \hat{t}} = \frac{1}{\xi_1^2} \frac{\partial}{\partial \xi_1}\left(\xi_1^2 \frac{\partial \theta}{\partial \xi_1} \right)$$

BC

$$@\ \hat{t} = 0 \qquad \theta = 1 \quad \forall\ \xi_1$$

$$@\ \xi_1 = 0 \qquad \frac{\partial \theta}{\partial \xi_1} = 0 \quad \forall\ \hat{t}$$

$$Bi = \frac{hR}{k}$$

k → ~~heat~~ thermal cond. of solid

$$@\ \cancel{r=R}\ \cancel{\xi_1}\ \xi_1 = 1 \qquad \frac{\partial \theta}{\partial \xi_1} = -Bi \times \theta$$

$$\frac{\partial \theta}{\partial \xi_1} = \frac{1}{T_0 - T_a} \frac{\partial T}{\partial r}$$

$$-k \frac{\partial \theta}{\partial \xi_1} \frac{(T_0 - T_a)}{R} = -k \frac{\partial T}{\partial r} = h(T - T_a)$$

$$-k \frac{\partial \theta}{\partial \xi_1} = \frac{h(T - T_a)}{(T_0 - T_a)}$$

$$-\frac{\partial \theta}{\partial \xi_1} = +\frac{hR}{k} \times \theta$$

$@\ r = R$

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$$Bi = \frac{hR}{k} = \frac{R/k}{1/h} = \frac{\text{Conductive Resi in solid}}{\text{Convective Resi in fluid}}$$

$Bi << 1$ (< 0.02) $\rightarrow$ Lumped capacitance model

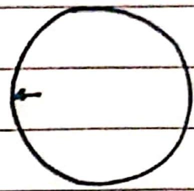
→ Temp. is uniform everywhere

→ Conductive resistance is small

$Bi >> 1$ (> 100) $\rightarrow$

→ Conductive Resistance is larger

→ $T_s \approx T_a$



@ $\xi_1 = 1$ $\quad \theta = 0$

↳ Soln: is not dependent on Biot no.

$$\theta(\xi_1, \hat{t}) = F(\xi_1) \times G(\hat{t})$$

$$\theta = A_1 e^{-\lambda_1^2 \hat{t}} \frac{\sin \lambda_1 \xi_1}{\lambda_1 \xi_1} + A_2 e^{-\lambda_2^2 \hat{t}} \frac{\sin \lambda_2 \xi_1}{\lambda_2 \xi_1}$$

$$+ A_3 e^{-\lambda_3^2 \hat{t}} \frac{\sin \lambda_3 \xi_1}{\lambda_3 \xi_1} + A_4 e^{-\lambda_4^2 \hat{t}} \frac{\sin \lambda_4 \xi_1}{\lambda_4 \xi_1}$$

$1 - \lambda_n \cot \lambda_n = Bi$ ← Plot and get 4 values

1st term approximation

$\cancel{\theta_1 = C_1 e^{-\lambda_1 t}}$ $\quad (\hat{t} > 0.2)$

$$\theta_1 = C_1 e^{-\lambda_1 t} \frac{\sin \lambda_1 \xi_1}{\lambda_1 \xi_1}$$

← Go to table for, Check $\xi_1, \hat{t}$, corresponding Bi

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For $Bi = 100$

$$\theta_1 = 1.2732\, e^{-1.57^2 \hat{t}} \frac{\sin(1.57\, \xi_1)}{1.57\, \xi_1}$$

Q) A solid sphere is cooling under $Bi = 0.2$
we want θ @ $\xi_1 = 0.5$, $\hat{t} = 5$

$$\theta_1 = 1.0592\, e^{-0.7593^2 \hat{t}} \frac{\sin(0.7593)\xi_1}{(0.7593)\xi_1}$$

$$= 1.0592\, e^{-0.7593^2 \times (5)} \frac{\sin(0.37965)}{0.37965}$$

$$= 0.0575$$

Q) An ordinary egg may be approximated as a 5cm diameter

A) $Bi = \frac{hR}{k} = \frac{1200 \times 2.5 \times 10^{-2}}{0.627}$

$$= 47.85$$

$$\sim 50$$

$Bi > 1$

Let's use approximate solution,
and $\lambda = 3$, $C_1 = 2$

$$\theta = 2 e^{-9t} \frac{\sin 3\xi_1}{3\xi_1}$$

[illegible]

$$\theta = \frac{70-95}{5-95} = \frac{-25}{-90} = 0.2778$$

$$\frac{0.277}{\cancel{2}} = e^{-9\hat{t}}$$

$$\frac{-1.9768}{9} = -\hat{t} \Rightarrow \hat{t} = 0.22$$

$$\alpha = \frac{k}{\rho C_p} = \frac{0.627}{1000 \times 4.18 \times 10^3}$$

$$= \frac{0.627}{4.18} \times 10^{-6} = 1.5 \times 10^{-\cancel{7}}$$

$$\cancel{t = \frac{5\times5\times\cancel{10^{-6}} \times \cancel{0.15\times10^{-6}} \times \cancel{0.22}}{0.15\times\cancel{10^{-6}}} = 14.5\ \text{min.}}$$

$$t = \frac{R^2\hat{t}}{\alpha} = \frac{25\times\cancel{10^{-6}}\times 0.22}{4\times0.15\times\cancel{10^{-6}}} = 14.5\ \text{min?}$$

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