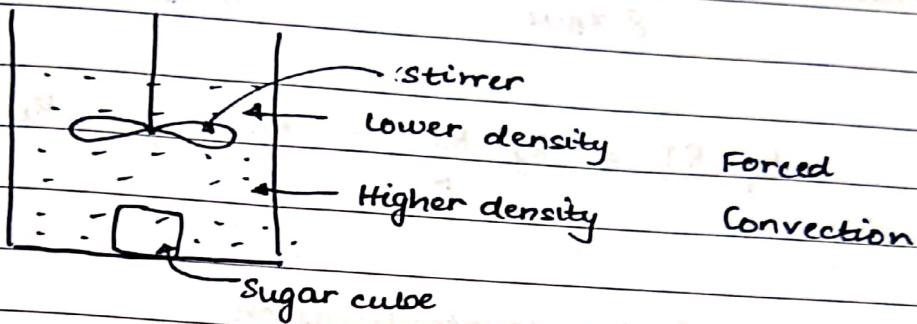
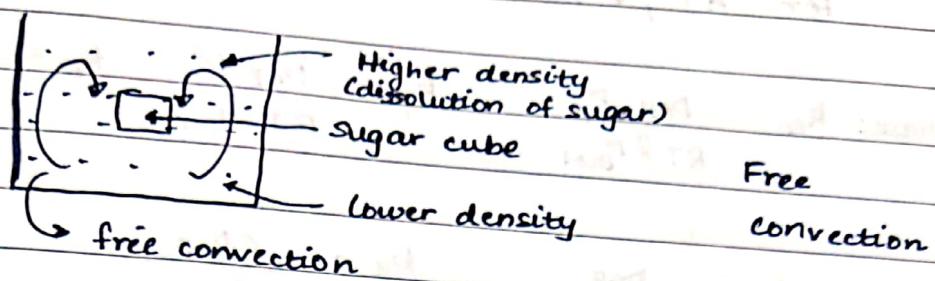


Convective Mass Transfer and HTC



Mass flux (N_A) is a driving force

$$N_A = (HTC) (D.F)$$

Mass transfer co-eff

For gases,

$$N_A = k_c (\Delta C_A)$$

$$\begin{aligned} &= k_y (\Delta y_A) \\ &= k_m (\Delta P_A) \end{aligned}$$

For liquids,

$$N_A = k_L (\Delta C_A)$$

$$= k_x (\Delta x_A)$$

Units of HTC,

$$k_c \rightarrow \text{mol / (time area)} (\Delta C_A)$$

$$k_m \rightarrow \text{mol / (time area)} (\Delta P_A)$$

$$k_y \rightarrow \text{mol / (time area)} (\Delta y_A)$$

$$k_L \rightarrow \text{mol / (time)(area)} (\Delta C_A)$$

$$k_x \rightarrow \text{mol / (time)(area)} (\Delta x_A)$$

For $A \uparrow, B \downarrow$

$$\text{Gas-Phase: } k_G = \frac{D_{AB} P}{RT \delta P_{BLN}} \quad k_y = \frac{D_{AB} P^2}{RT \delta P_{BLN}} \quad k_o = \frac{D_{AB} P}{\delta P_{BLN}}$$

$$\text{Liquid-Phase: } k_L = \frac{D_{AB}}{\delta x_{BLN}} \quad k_x = \frac{C D_{AB}}{\delta x_{BLN}}$$

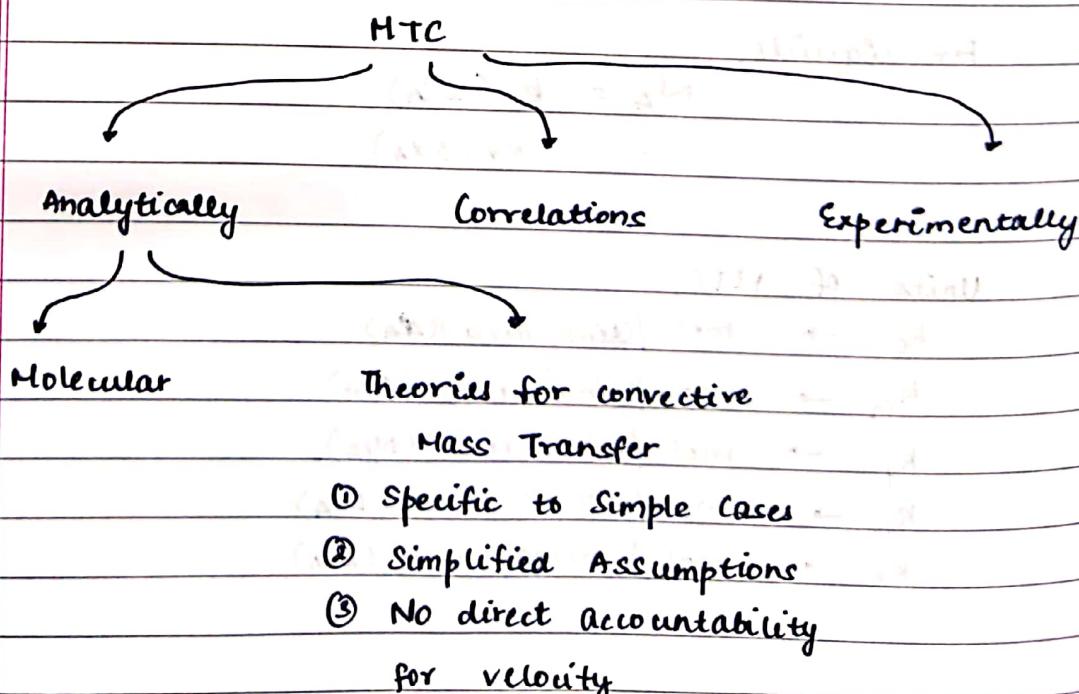
$$k_G \frac{RT}{P} = k_y \frac{RT}{P} = k_o ; \quad k_L = \frac{k_x}{C_{av}}$$

For equimolar countercurrent diffusion,

$$\text{Gas-Phase: } k_G'' = \frac{D_{AB}}{\delta RT} \quad k_y = \frac{D_{AB} P}{\delta RT} \quad k_o = \frac{D_{AB}}{\delta}$$

$$\text{Liq-Phase: } k_L = \frac{D_{AB}}{\delta} \quad k_x = \frac{C D_{AB}}{\delta}$$

$\frac{1}{NTC}$ = Mass Transfer Resistance



Dimensionless Numbers

Sherwood Number (Sh) → Mass transfer analogue of Nusselt Number

Schmidt Number (Sc) → Mass transfer analogue of Prandtl Number

$$Sh = \frac{\text{Convective Mass flux}}{\text{Mass flux for molecular diff. through a stag. medium at thickness } L \text{ under D.F., } \Delta P_A}$$

$$= \frac{k_L \Delta P_A}{(P_{AB}^P / RT L P_{BLH}) \Delta P_A}$$

$$\therefore Sh = \frac{k_L L}{P_{AB}} \frac{P_{BLH}}{P^P} = \frac{k_L L}{P_{AB}} \left[\frac{C}{C_{BLH}} \right]$$

$$= \frac{k_L L}{P_{AB}}$$

$L \rightarrow$ characteristic length

$$Sc = \frac{\mu}{P_{AB} \rho} = \frac{\nu}{P_{AB}}$$

$$Re = \frac{\rho v L}{\mu}$$

$$\text{Stanton Number, } St_N = \frac{Sh}{(Re)(Sc)} = \frac{k_L}{v} = \frac{k_L A_C}{v A_C}$$

$$\text{Colburn factor, } j_0 = (St_N) (Sc)^{2/3} = \frac{Sh}{Re (Sc)^{1/3}}$$

Film Theory

Mass transfer from a solid surface to a flowing liquid.

Assumptions:

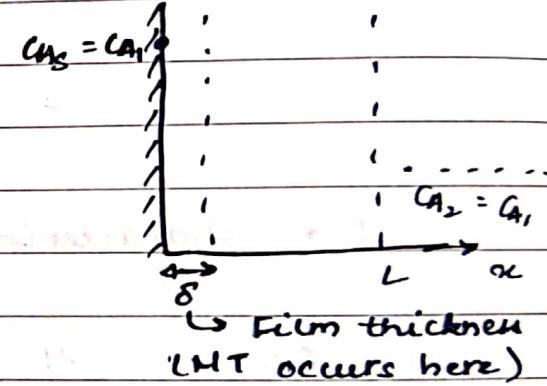
- MT occurs by purely molecular diffusion through a stagnant fluid layer at the phase boundary
- Beyond the film, the fluid is well mixed ($C = C_{AB}$)
- At steady state
- $N_A + N_B = 0$ (Flux is low, mass transfer occurs at low conc.)

From Fick's 2nd law,

$$\frac{d^2 C_A}{dx^2} = 0$$

$$C_A|_{x=0} = C_{A_1} = C_{AS}$$

$$C_A|_{x=L} = C_{A_2} = C_{AL}$$



$$\frac{dC_A}{dx} = k_1, \quad C_A = k_1 x + k_2$$

$$k_2 = C_{A,S}, \quad k_1 = \frac{C_{A,L} - C_{A,S}}{\delta}$$

$$C_A = \left(\frac{C_{A,L} - C_{A,S}}{\delta} \right) x + C_{A,S}$$

vel ↑ $N_A \uparrow, k_c \uparrow, \delta \uparrow$

$$\frac{dC_A}{dx} = \frac{(C_{A,L} - C_{A,S})}{\delta}$$

$$\therefore N_A = - \frac{D_{AB}}{\delta} (C_{A,S} - C_{A,L})$$

$$k_c = \frac{D_{AB}}{\delta}$$

Penetration Theory

HT from liquid to gas in rising bubble

Assumptions:

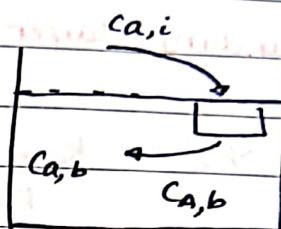
- Unsteady state HT occurs to a liquid element so long as it is in contact with the bubble
- Eq. at gas - liq. interface
- Each of liq. elements stays in contact with gas for same period of time.

$t_c \rightarrow$ contact time

(low enough so that

diffusing particle doesn't

reach other end)



$$\frac{dc_A}{dt} = D_{AB} \frac{\partial^2 c_A}{\partial z^2}$$

Initial cond : $t=0, z \geq 0 ; c_A = c_{Ab}$

BC-1 : $t > 0, z = 0 ; c_A = c_{Ai}$

BC-2 : $t > 0, z = \infty ; c_A = c_{Ab}$

$$\frac{c_A - c_{Ab}}{c_{Ai} - c_{Ab}} = 1 - \operatorname{erf} \eta, \quad \eta = \frac{x}{2 \sqrt{D_{AB} t}}$$

$$N_A(t) = \sqrt{\frac{D_{AB}}{\pi t}} \frac{t_c}{t_c} (c_{Ai} - c_{Ab})$$

$$N_{A,av} = \frac{1}{t_c} \int_0^{t_c} N_A(t) dt$$

$$= 2 \sqrt{\frac{D_{AB}}{\pi t_c}} (c_{Ai} - c_{Ab})$$

Inst. HTC, $k_L = \sqrt{\frac{D_{AB}}{\pi t}} \quad k_L \propto (D_{AB})^{0.5}$

Avg. HTC, $k_{L,av} = 2 \sqrt{\frac{D_{AB}}{\pi t_c}}$

Surface Renewal Theory

$$N_A = \sqrt{P_{AB} S} (C_{Ai} - C_{Ab})$$

$S \rightarrow$ Surface Renewal Rate

$v_L \uparrow N_A \uparrow k_L \uparrow S \uparrow$

→ Doesn't account for velocity profile.

Boundary layer Theory ($Sh = f(Re, Sc)$)

$$Sh_x = \frac{k_L x}{D_{AB}} = 0.332 (Re_x)^{1/2} (Sc)^{1/3}$$

$$Sh_{av} = \frac{k_L x_{av}}{D_{AB}} = 0.664 (Re_c)^{1/2} (Sc)^{1/2}$$

Film Theory $k_L \propto D$

Penetration Theory $k_L \propto D^{1/2}$

Surface Renewal Theory $k_L \propto D^{1/2}$

Boundary layer Theory $k_L \propto D^{2/3}$

$$St_M = \frac{Sh}{Re Sc} = \frac{f}{2}$$

$$j_D = St_M Sc^{2/3} = \frac{Sh}{Re Sc^{1/3}} = 0.023 Re^{-0.2}$$

→ Reynold's Analogy

Applicable for laminar flow, $Sc \approx Pr \approx 1$

$$St_M = f/2$$

Coultburn Chilton : Correlation for turbulent flow

$$0.6 < Sc < 2500$$

$$St_M Sc^{2/3} = f/2$$

$$0.6 < Pr < 100$$

$$(j_D)$$

Analogies (conditions)

- Physical properties should be const.
- No mass or energy is produced in the system
- Vel. profile is analogous

For single sphere,

$$Sh = Sh_0 + C Re^m Sc^n$$

$Sh_0 \rightarrow$ Sherwood Number at very low Re .

$$Sh = 2 + 0.3 Re^{0.5} Sc^{0.33} \quad (\text{From HT})$$

Procedure to be followed,

- ① Match the situation
- ② Match process crit. parameters (Sc, Re)
- ③ Find suitable correlations
- ④ If no correlation is available, look for analogy.
(use analogous correlation from HT)
- ⑤ If not available, then finally use Boundary layer theory

Interphase Mass Transfer

- Eq. between two phases means, no net transfer of solute from one phase to other.
At eq., chemical potential of two phases are equal.
- For a system at eq.,

$$F' = c' - p' + \frac{\text{No. of comp.}}{\text{no. of phases}}$$

↑ ↑
Degrees of Freedom no. of phases

- For a system at eq., the D.F is zero

Raoult's law

For an ideal gas- liq. or vapour liq system,

$$P_A' = x_A P_A$$

↑ ↑
eq. P. exerted by solute r.p. of solute
Mole fraction of solute A in liquid

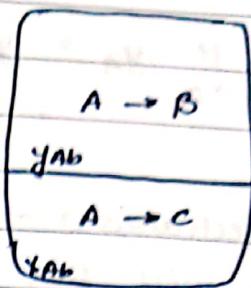
Henry's law

Eq. data for many non-ideal gases,

$$P_A' = H x_A$$

↑ const. for a given solute-solvent pair (strong fn. of temp)

Binary, Non-reactive
gas-liquid system

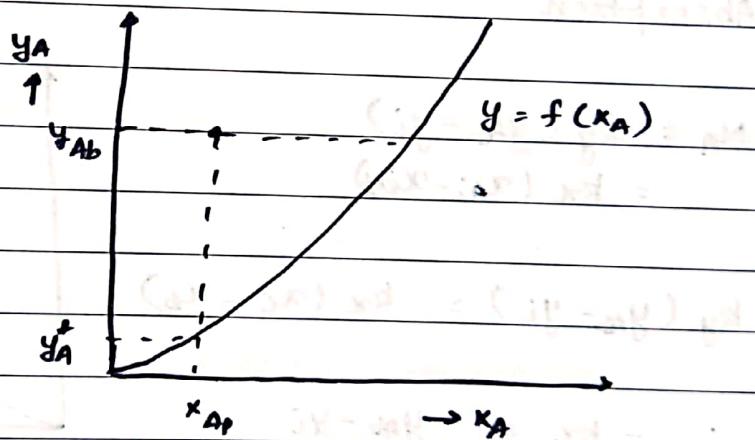


- There is transfer of A, only when $\mu_1 \neq \mu_2$
- Transfer of A occurs from higher μ to lower μ
- This transfer continues till $\frac{\partial \mu}{\partial z} = 0$ is reached

Mass Transfer between two phases occurs when $y_{AB} \neq y_A^*$ ($y_A^* = f(x_{AB})$)

When $y_{AB} > f(x_{AB})$, mass transfer occurs from gas to liq. phase (Absorption)

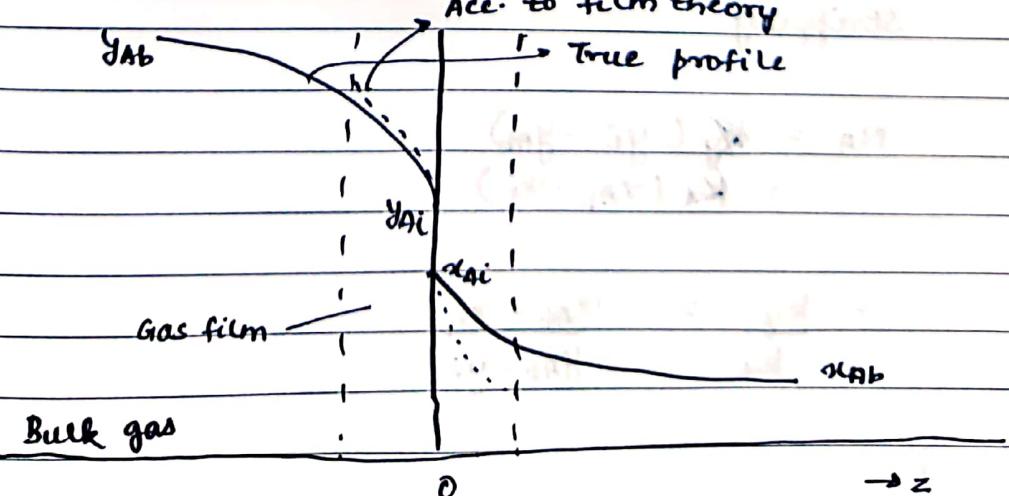
When $y_{AB} < f(x_{AB})$, mass transfer occurs from liq to gas phase.



Concentration Profiles near the interface.

Acc. to film theory

True profile



$$\lim_{z \rightarrow 0^-} y_A = y_{Ai}$$

$$\lim_{z \rightarrow 0^+} x_A = x_{Ai}$$

Mechanism -

- ① A will be transported from bulk to interface

$$y_{Ab} > y_{Ai} - \text{Gas to liq.}$$

$$x_{Ab} > x_{Ai} - \text{Liq. to gas}$$

- ② At interface eq. will be established

- ③ A will be transported from interface to bulk

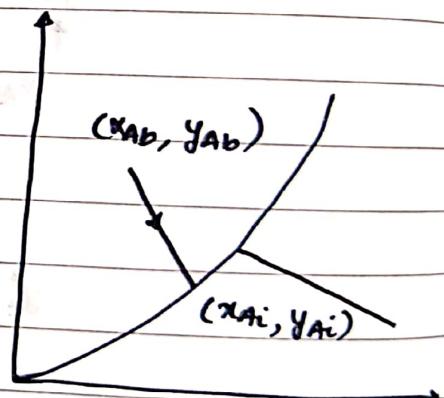
All mass transfer resistance occurs in the thin film near the interface.

Absorption

$$N_A = k_y (y_{Ab} - y_i) \\ = k_x (x_{Ai} - x_{Ab})$$

$$k_y (y_{Ab} - y_i) = k_x (x_i - x_{Ab})$$

$$\therefore -\frac{k_x}{k_y} = \frac{y_{Ab} - y_i}{x_{Ab} - x_i}$$



Stripping

$$N_A = (y_i - y_{Ab}) \\ = (x_{Ab} - x_i)$$

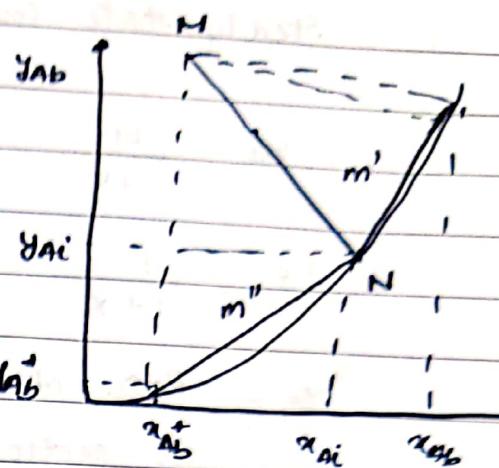
$$-\frac{k_y}{k_x} = \frac{x_{Ab} - x_i}{y_{Ab} - y_i}$$

From the eq. curve,

$$\frac{L}{K_y} = \frac{1}{k_y} + \frac{m^2}{k_x}$$

$K_y \rightarrow$ Overall MTC

$k_y \rightarrow$ Local MTC



Similarly,

$$\frac{1}{k_x} = \frac{1}{m'' k_y} + \frac{1}{k_x}$$

$\frac{1}{k_y} \rightarrow$ Individual gas-phase HTR

$\frac{1}{k_x} \rightarrow$ Individual liq-phase HTR

Fractional Resistance offered by gas phase

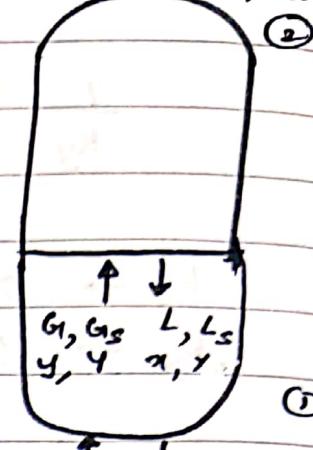
$$\frac{1/k_y}{1/k_y}$$

Fractional mass transfer resistance by liq-phase

$$= \frac{m'/k_x}{1/k_y}$$

Steady state countercurrent HT

$$G_2, G_{2s} \quad L_2, L_s \\ Y_2, Y_{2s}$$



$$G_1, G_s \quad L, L_s \\ Y, Y_s$$

$$G_1, G_s \quad L, L_s \\ Y, Y_s$$

$$G_1, G_s \quad L, L_s \\ Y, Y_s$$

G_1 - Rate of input of G_1
at section ①

G_{1s} - Rate of flow of G_1
on solute free basis

G_1 - Rate of flow of G_1
at any section

L_1 - Rate of output of L at sec ①

L_s - Rate of flow of L on solute free basis

x - Mole fraction of solute in L

X - Mole ratios of solute in L

Considering env. ①,

Total rate of input of solute = $L_s x_2 + G_{1s} Y$

Total rate of output of solute = $L_s X + G_{1s} Y$

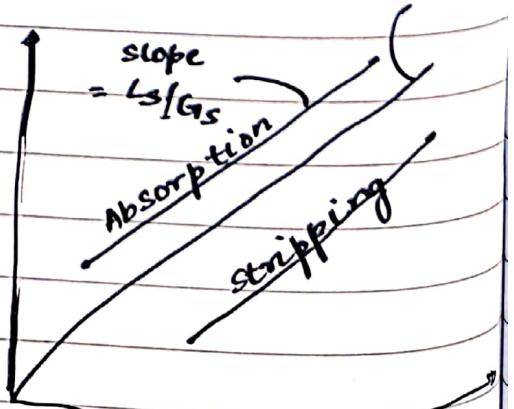
$$G_{1s} (Y - Y_2) = L_s (X - X_2) \quad \text{--- ①}$$

Considering env ②,

$$G_{1s} (Y_1 - Y_2) = L_s (X_1 - X_2)$$

① → Operating line

$$\frac{L_s}{G_{1s}} = \frac{(Y_1 - Y_2)}{(X_1 - X_2)}$$

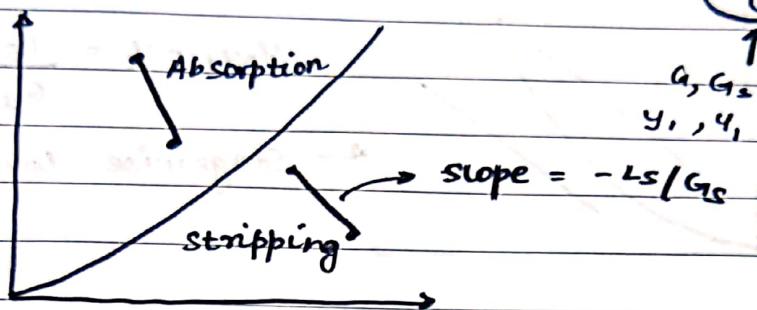
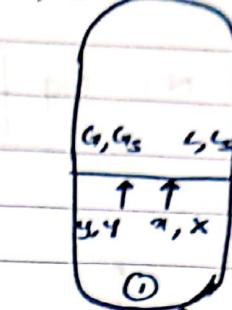
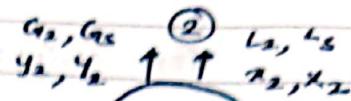


Steady state Co-current MT

$$\text{Env 1 : } G_S(Y_1 - Y) = L_S(X - X_1)$$

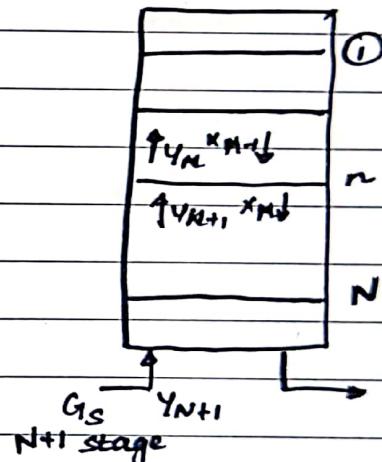
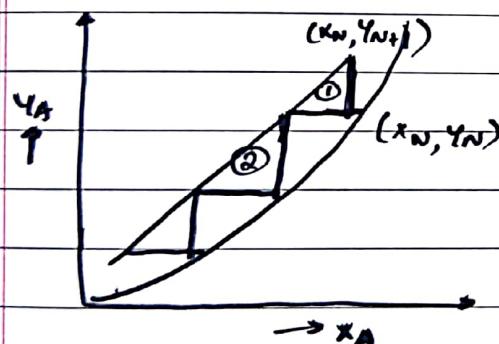
$$\text{Env 2 : } G_S(Y_1 - Y_2) = L_S(X_2 - X_1)$$

$$\text{slope} = -L_S/G_S$$



Stage wise operation

$$(Y_{n+1} - Y_n) = \frac{L_S}{G_S} (X_n - X_0)$$



$$Y_{n+1} - \bar{A} Y_n = Y_1 - \bar{A} \alpha X_0 \quad \bar{A} = \frac{L_S}{G_S \alpha}$$

$$(Y = \alpha X)$$

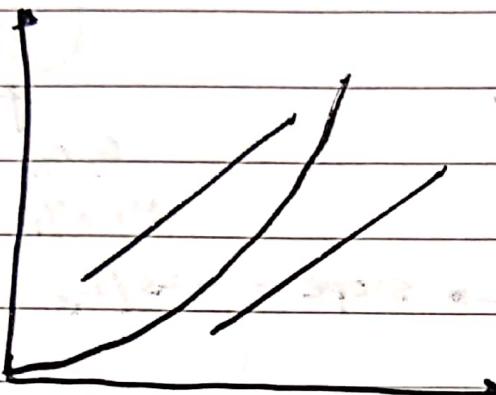
$$N = \log \left[\left(\frac{C Y_{n+1} - \alpha X_0}{Y_1 - \alpha X_0} \right) \left(1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right] \quad (\bar{A} \neq 1)$$

$$\text{For } Y_n = \frac{L_S}{G_S} X_n$$

$$N = \frac{Y_{n+1} - Y_1}{Y_1 - \bar{A} \alpha X_0} \quad (\text{Absorption})$$

For stripping

$$N = \log \left[\frac{\left(\frac{x_0 - y_{N+1}/\alpha}{x_N - y_{N+1}/\alpha} \right) (1 - \bar{A}) + \bar{A}}{\log (1/\bar{A})} \right]$$

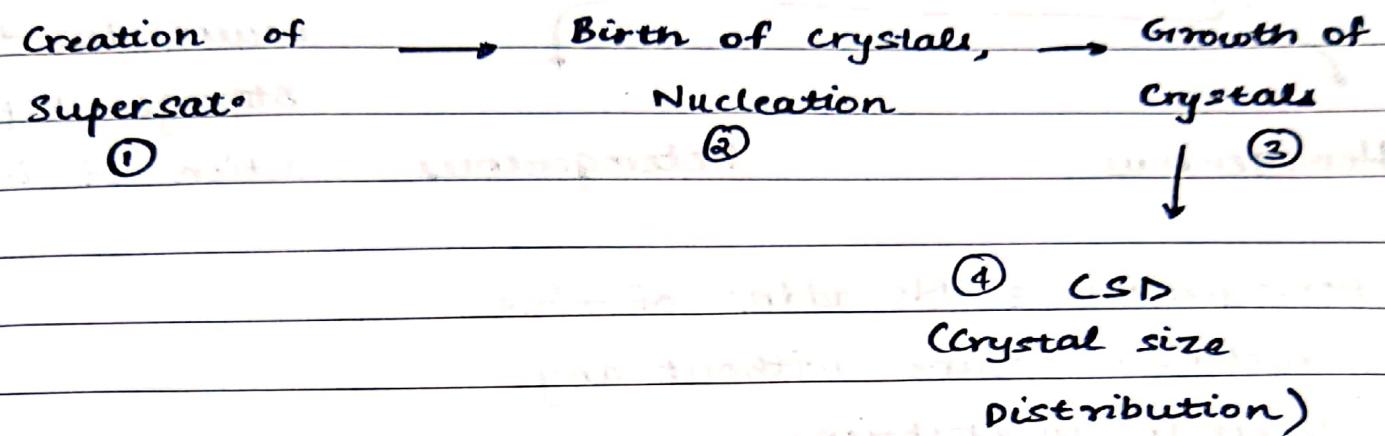


$$y_{N+1} - y_1 = \frac{L_S}{G_{12}} (x_N - x_0)$$

→ Stage wise counter current

Crystallization

separation of solute in solid form from a soln.



① creation of Supersaturation

$$\frac{c - c^*}{c^*} = S \text{ (Degree of supersaturation)}$$

$c - c^*$ → Extent of supersaturation

- Process can be done by cooling or by evaporation
- Addition of solute or solvent may shift the eq. curve

② Birth of crystals

Rate of generation of crystals of zero size = B_0

char. length of crystal = L

$$B_0 = \lim_{L \rightarrow 0} \frac{d(N/v)}{dt}, \quad A = \Phi_a L^2$$

$$v = \Phi_v L^3$$

Nucleation

Primary

Homogeneous

Heterogeneous

Secondary

occurs due to storage or prolonged action of impeller.

Homogeneous = No addn. of fine crystals. Occurs without any external interruption.

Heterogeneous = Seeds (small fine crystals) are added to facilitate formation of crystals

For primary nucleation

$$B = A' \exp \left[- \frac{16 \pi \sigma^3 v_M^2}{3 k^3 T^3 [\ln(st+1)]^2} \right]$$

crystal size
 molar ra.
 degree of supersat.
 Boltzmann const.

For heterogeneous nucleation,

$$B_0 = k_1 M_T s^n$$

Mass of suspension

k_1, n are experimentally determined

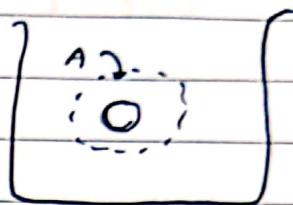
$n = 2$ if nothing is mentioned

③ Growth of crystal

$$G_1 \text{ (growth rate)} = \frac{dL}{dt}$$

a) Transfer of solute molecules from bulk to top of surface
 → mass of crystal

$$\frac{dm_c}{dt} = k_a A_c (c - c^*)$$



b) Integration of solute in bulk of crystal

$$\frac{dm_c}{dt} = k_t A_c (c_i - c^*)$$

$$k_t = \frac{1}{\frac{1}{k_a} + \frac{1}{k_r}}$$

$$m_c = v_c p_c \\ = \phi_v L^3 p_c$$

$$\frac{dm_c}{dt} = k_t A_c (c - c^*) \quad A_c = \phi_a L^2$$

$$\frac{d(\phi_v L^3 p_c)}{dt} = k_t A_c (c - c^*)$$

$$\frac{dL}{dt} = k_t \left(\frac{\phi_a c^*}{3 \phi_v p_c} \right) \left(\frac{c - c^*}{c^*} \right)$$

$$\frac{dL}{dt} = K' s$$

$$G_1 = K' s$$

In general,

$$G_1 = K' s^n$$

$$B_0 = K_1 M_T s^n$$

McCabe DL Law

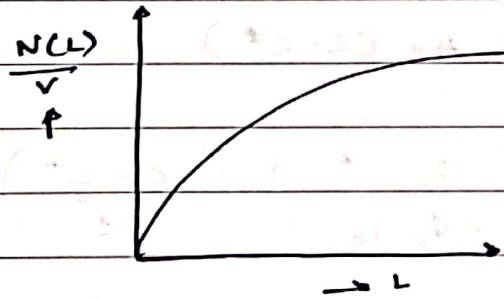
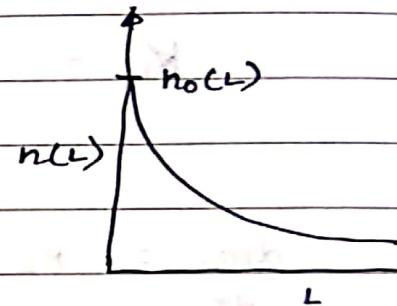
Crystal growth is independent of crystal size L , i.e. k' is independent of L

$$\frac{dG_1}{dL} = 0$$

④ Crystal Size Distribution

$n_0(L) \rightarrow$ Crystal size distribution of vanishing small size.

$$n_0(L) = \lim_{L \rightarrow 0} \left[\frac{d(NCL)/v}{dL} \right] = \frac{B_D}{G_1}$$



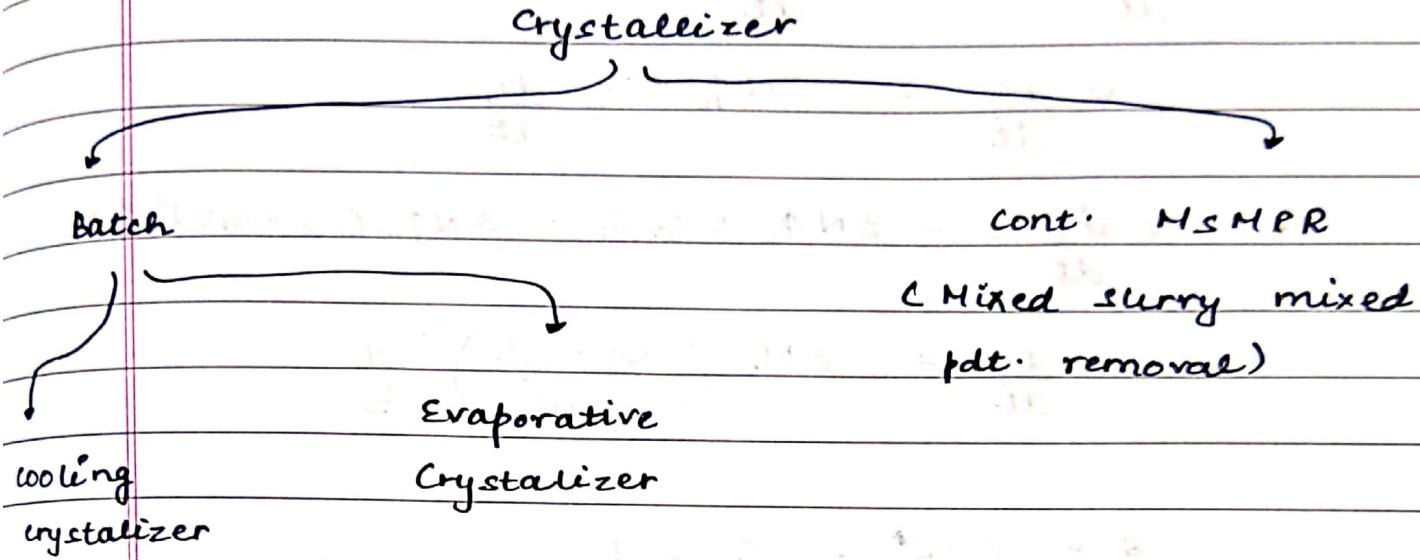
$$\frac{N_T}{v} = \int_{L_1}^{L_2} n(L) dL \quad A_c = \int_{L_1}^{L_2} n(L) \phi_a L^2 dL$$

$$V_c = \int_{L_1}^{L_2} n(L) \phi_v L^3 dL$$

$$M_c = V_c P_c = P_c \int_{L_1}^{L_2} n(L) \phi_v L^3 dL$$

Crystallizer

- Degree of supersaturation to be maintained
- Nucleation (slurry containing fine crystals)
- Time



$$\frac{dv}{dt} = ? \quad , \quad S = \frac{c - c^*}{c^*} = \text{const.}$$

At $t=0$, $L = L_s$, $M_s = N \Phi_v L_s^3 P_c$

$t=t$, $L = L$, $M = N \Phi_v L^3 P_c$

$$\frac{d(VcM_w)}{dt} = - \frac{dM_s}{dt}$$

$$c \frac{dv}{dt} = \frac{3N\Phi_v L^2 P_c}{M_w} \frac{dL}{dt}$$

$$L = Gt + L_s \quad (\text{At } t=t)$$

$$c \frac{dv}{dt} = \frac{3N\Phi_v L^3 P_c}{M_w L_s^3} \frac{(L_s + Gt)^2 G_t}{L_s}$$

$$c \frac{dv}{dt} = \frac{3M_s}{M_w} \left(\frac{L_s + Gt}{L_s} \right)^2 \frac{G_t}{L_s} - \textcircled{1}$$

$$L_f = L_s + Gt_b \quad \text{Batch time}$$

$$\frac{dT}{dt} = ?$$

$$\text{At } t=0, L=L_s, M_s = N\phi_r L_s^3 \rho_c$$

$$t=t, L=L, M_c = N\phi_r L^3 \rho_c$$

$$\frac{d(vc)}{dt} = - \frac{dM_c}{dt}$$

$$v \frac{dc}{dt} = - 3N\phi_r L^2 \rho_c \frac{dL}{dt}$$

$$v \frac{dc}{dt} = - 3N\phi_r L^2 \rho_c G = 3N\phi_r (L_s + Gt)^2 \rho_c G$$

$$v \frac{dc}{dt} = 3M_s \left(\frac{L_s + Gt}{L_s} \right)^2 \frac{G}{L_s}$$

$$s = \frac{c - c^*}{c^*} = \frac{c}{c^*} - 1$$

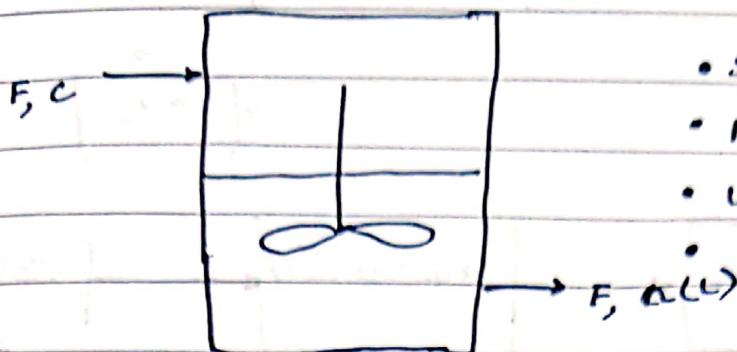
$$c = c^* (s+1) = f(T) (s+1)$$

since c^* is a fn. of temp.

$$\frac{dc}{dT} = f'(T) (s+1) \frac{dT}{dt}$$

$$v \frac{dT}{dt} = \frac{3M_s}{f'(T)(s+1)} \left(\frac{L_s + Gt}{L_s} \right)^2 \frac{G}{L_s}$$

cont. crystallizer



- Steady state
- No crystals in feed
- well mixed
- DL law applicable

$$n(L) = \frac{d(N/v)}{dL}$$

$$n_0(L) = \lim_{\Delta L \rightarrow 0} \frac{d(N/v)}{dL}$$

No. of crystals with size L in (0)

- No. of crystals with size L out ($F \cdot n(L) \Delta L$)
 - + No. of crystals grown to size L $n(L) v \Delta L$
 - No. of crystals grown to size $L + \Delta L$ $\frac{n(L + \Delta L)}{v \Delta L}$
- $$= 0$$

$$F \cdot n(L) \Delta L = n(L) v \frac{\Delta L}{\Delta t} - \frac{n(L + \Delta L)}{v \Delta L} \frac{\Delta L}{\Delta t}$$

On solving,

$$n(L) = n_0(L) \exp\left(-\frac{L}{G\tau}\right) \quad \tau = \frac{v}{F}$$

$$n_0(L) = \frac{B_0}{G} \exp\left(-\frac{L}{G\tau}\right)$$

$$M_c = \int_0^\infty n(L) \phi_v L^3 dL P_c$$

