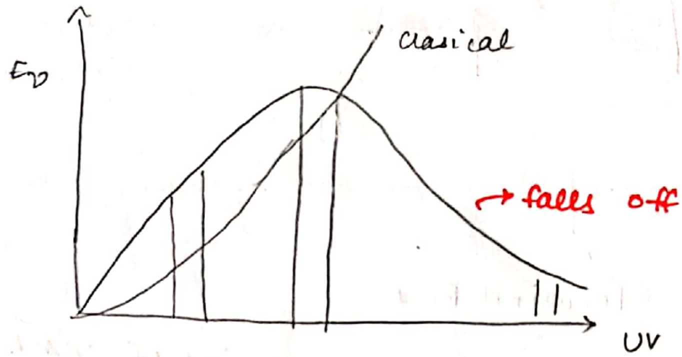
Black Body Radiation



No. of moles having freq. $[\nu, \nu+d\nu]$ $(n_\nu) = \frac{8\pi\nu^2}{c^3}$

Energy of avg. move $E_\nu = \frac{8\pi\nu^2}{c^3} kT$

$k \rightarrow$ Maxwell Boltzmann distribution

$P(\varepsilon) = e^{-E/kT}$

classical Explanation

$\tilde{\varepsilon}$ - is the smallest part of energy

$\boxed{\tilde{\varepsilon} \propto \nu}$

Plank's Explanation

$$\langle \varepsilon \rangle = \frac{\tilde{\varepsilon} x \left(\frac{1}{(1-x)^2}\right)}{\left(\frac{1}{1-x}\right)} = \tilde{\varepsilon} \frac{x}{1-x}$$

$x = e^{-\tilde{\varepsilon}/kT}$

$$\langle \varepsilon \rangle = \frac{\tilde{\varepsilon}}{e^{+\tilde{\varepsilon}/kT} - 1}$$

$\rightarrow$ Quanta

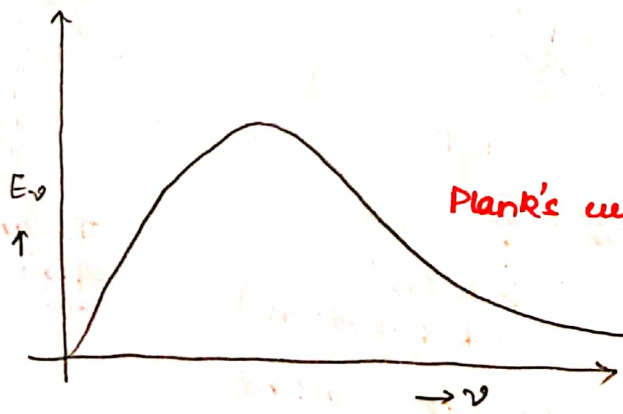
$\tilde{\varepsilon} = h\nu$

$\uparrow$

Plank's const.

$$E_\nu = n_\nu \langle \varepsilon \rangle = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

Plank did suming of energy saying Energy can take only integral multiple of $\tilde{\varepsilon}$ (smallest part of energy) whereas classical phy. ~~told~~ did integration saying energy can take all values



Plank's curve

$h \sim 6 \times 10^{-34}$ Js

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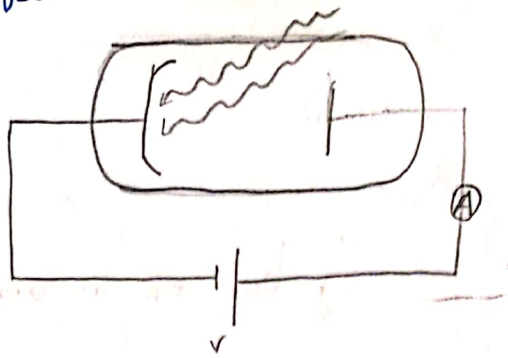
Photo electric effect



Forward Bias

→ No increase of current with intencity of light

→ $\nu_{criticial}$ below which no current for any I

Photo current

Classical

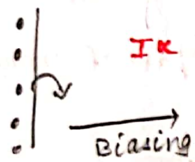


Photo current

Prop. to Intensity

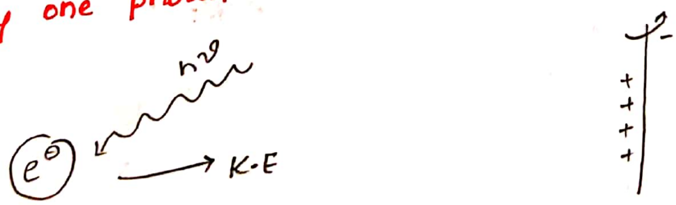
No dependancy on freq.

Einstein's theory of Photoelectric effect

1) Light is a beam of particles (photons)
2) $\boxed{E_\nu = h\nu}$ (The energy that photons possess)
3) Intensity of light $\boxed{I = n_\nu \times E_\nu}$ ← Energy of a single photon

$\propto E_0^2$ { no. of photons per unit area unit time

4) At one pt., only one photon can interact only with one electron



$h\nu = \frac{1}{2} m v_e^2 + W$

"Toll Tax" (Work Function)

The amt. of energy spent is so as to over come the pull back

This work fn. is dependent on cathode

$\boxed{h\nu_{cr} = W}$

when $\nu < \nu_{cr}$, the $e^{\ominus}$ won't come out even though there are many photons. It takes the energy from p/ and dissipates that energy, the same is the case with other photons

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Stopping voltage

$$\boxed{eV_{stop} = \underbrace{\tfrac{1}{2}mv_e^2}_{\text{K.E of } e^{\ominus}} = h\nu - W}$$

Particle is an objected in space time to which we can assign some energy, momentum

Particle Nature of Light

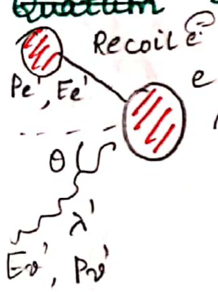
Light $\Rightarrow$ photon $\begin{cases} E_\nu = h\nu \\ p_\nu = ? \end{cases}$

Dimensional

$$[p] = \frac{[E]}{[\nu]} \Rightarrow p_\nu = \frac{E_\nu}{c}$$

$$\boxed{p_\nu = \frac{h}{\lambda}}$$

Compton ~~Quatum~~ Scattering experiment



$$\boxed{E_e' = \sqrt{P_e'^2 c^2 + (m_e c^2)^2}}$$

conserving energy & momentum

$$E_\nu + E_e = E_\nu' + E_e'$$

$$\vec{P_\nu} = \vec{P_\nu'} + \vec{P_e'}$$

photon with some momentum will hit the $e^{\ominus}$, and gives ~~the~~ some energy to the $e^{\ominus}$ and goes away with some lesser energy, so higher λ

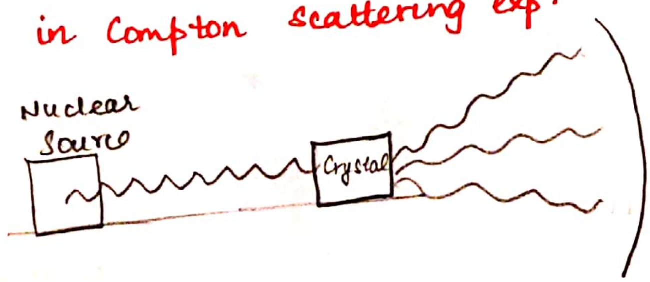
$$\boxed{\lambda' - \lambda = \lambda_{cp}[1 - \cos\theta]} \quad \text{Prediction}$$

$$\lambda_{cp} = \frac{h}{m_e c} \sim 2.5 \times 10^{-12}\, m$$

$$P_\nu = \frac{h}{\lambda} \qquad P_\nu' < P_\nu$$

$$P_\nu' = \frac{h}{\lambda'} \Rightarrow \lambda' > \lambda$$

Verified in Compton scattering exp.



The exp. is to show that "Light is a particle"
The particle is called photon.

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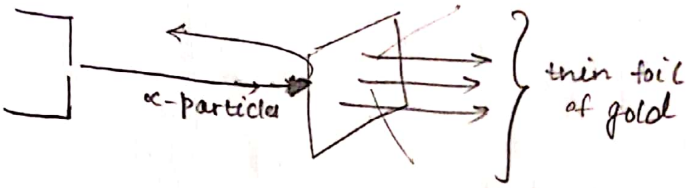
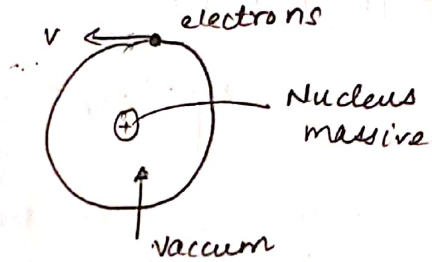
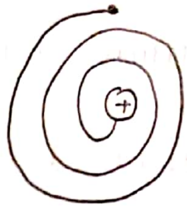




$a_e = \frac{v^2}{r}$ (Electrons have non-zero acceleration)

↳ Acc. to this Atom collapses in 10^{-12} s

⇓ Radiate Energy



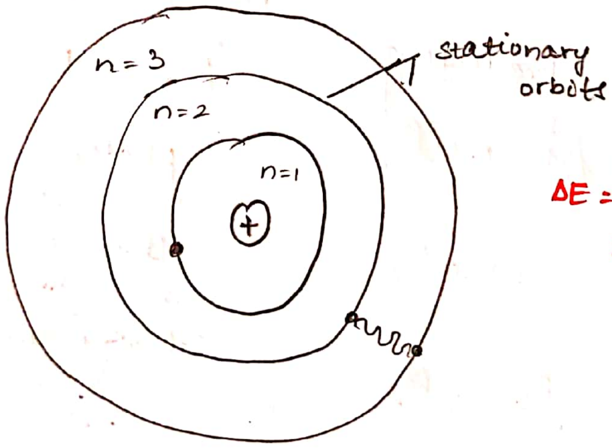
So there should be some balancing force i.e the force of attraction

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{ze^2}{r^2}$$

Bohr's Atomic Model

1) Stationary orbits in which electrons don't radiate
2) Energy of the nth stationary orbit. $E_n = \frac{E_0}{n^2}$
3) Radiation from an atom originates when an atom de-excites from $n \to m$ energy level

$$\Delta E = E_n - E_m$$



$$\Delta E = E_3 - E_2 = E_0\left[\frac{1}{3^2} - \frac{1}{2^2}\right]$$

Wave Nature of Matter: (De Broglie hypothesis).

light $\quad \lambda \to P$, $\frac{h}{\lambda} = P_\nu$ } Compton

particle $\quad P_e \to \lambda_e$

$\Rightarrow \boxed{P_e = \frac{h}{\lambda_e}}$ De Broglie's eqn.

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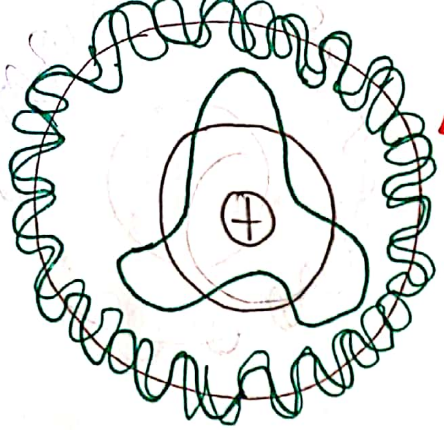


← destructive int.

$r_n \rightarrow$ radius of n^{th} orbit

$$\boxed{P_e = \frac{h}{\lambda_e} = \frac{hn}{2\pi r_n} = \frac{\hbar n}{r_n}}$$

$$\hbar = \frac{h}{2\pi} = h\text{cut}$$

$$P_e = m_e v_e = \frac{\hbar n}{r_n}$$

Force Balance

$$\frac{m_e v_e^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{(ze^2)}{r_n^2}$$

$$\Rightarrow \boxed{r_n = \left[\frac{h^2}{m_e z e^2}(4\pi\varepsilon_0)\right] n^2 \qquad r_n = r_0 n^2}$$

Energy of n^{th} stationary orbit

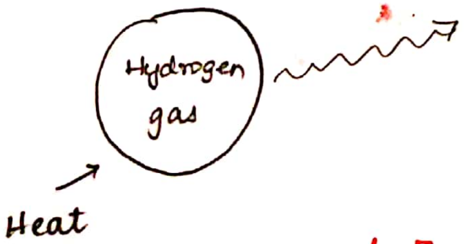
$$E_n = \frac{1}{2} m_e v_e^2 - \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n}} \rightarrow \text{Electrostatic potential energy.}$$

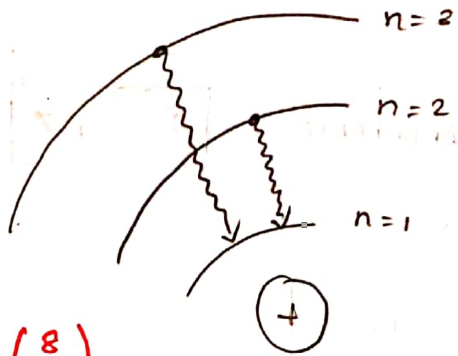
$$E_n = \frac{1}{2} m_e \left(\frac{\hbar n}{r_n m_e}\right)^2 - \frac{ze^2}{4\pi\varepsilon_0} \times \frac{1}{r_n}$$

$$E_n = \left(\frac{1}{2}\hbar^2 \times \frac{1}{m_e r_0^2} - \frac{ze^2 \times 1}{4\pi\varepsilon_0 r_0}\right)\frac{1}{n^2}$$

$$\boxed{E_n = \frac{E_0}{n^2}}$$

Atomic Spectroscopy





ΔE is always > 0

$$\Delta E_1 = \left|\frac{E_0}{3^2} - \frac{E_0}{1^2}\right| = E_0\left(\frac{8}{9}\right)$$

$$h\nu_1 = E_0\left(\frac{8}{9}\right) \qquad h\nu_2 = \Delta E_2 = \left|\frac{E_0}{2^2} - \frac{E_0}{1^2}\right| = E_0\left(\frac{3}{4}\right)$$

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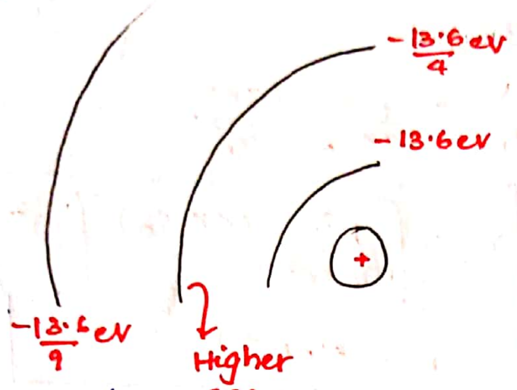
$$\boxed{\nu = \frac{E_0}{h}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

→ Emission & Absorption spectrum

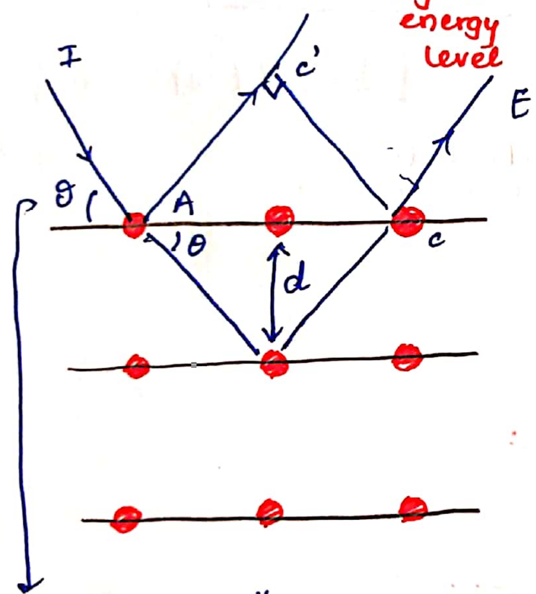
For hydrogen atom $E_0 = E_1 = -13.6\,eV$



An $e^{\ominus}$ always looks for lower energy level.

When we provide the exact ΔE, transition will take place

When 13.6 eV, the $e^{\ominus}$ will reach $n = \infty$ i.e it will be free, that energy is ionization energy



"Grazing angle"

$$AB = BC = \frac{d}{\sin\theta}$$

$$AC' = AC\cos\theta = \frac{2d}{\tan\theta}\cos\theta$$

path diff : $(AB + BC) - AC'$

$$\text{path diff} = \frac{2d}{\sin\theta} - \frac{2d}{\sin\theta}\cos^2\theta$$

$$= \frac{2d}{\sin\theta}\left[1 - \cos^2\theta\right]$$

$$= 2d\sin\theta$$

$$\delta = \frac{2\pi}{\lambda_e}\left[2d\sin\theta\right]$$

const. interference : $\boxed{\delta = 2n\pi}$

Brag's Eqn :

$$\boxed{2d\sin\theta = n\lambda_e}$$

Davisson - Germer Exp :

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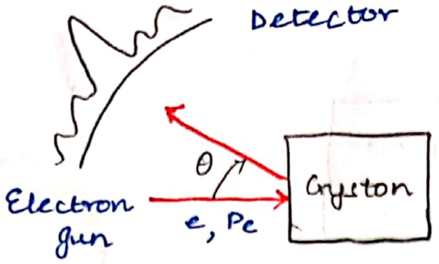
Davisson - Germer Exp



Wave - matter Duality

Black Body Rad. + Photoelectric effect + Compton Scattering ⇒ Classical waves (light) → Particle (Photon)

Bohrs Atom + De - Broglie Hyp. + D-G exp ⇒ classical particles (electron) → wave

QUANTUM MECHANICS

Postulates :-

1) All fundamental objects are particles

Light → photon
Matter → electron } particle

2) For every particle $\psi(x,t)$ wave fn:

$\psi_{e/p}(\bar{x},t) \rightarrow$ Probability Amplitude to find particle at location $\bar{x}$ at time t

$$\boxed{\psi(\bar{x},t) \times \psi^*(\bar{x},t) = P(\bar{x},t)}$$

The waves shown here represent the probability of finding the $e^{\ominus}$ there and not the path of $e^{\ominus}$

Wave Eqn : Schrodinger's Eqn

For one dimension

$$\frac{\hbar^2}{2m}\frac{d^2\psi(x,t)}{dx^2} + V(x,t)\,\psi(x,t) = i\hbar\,\frac{d\psi(x,t)}{dt}$$

$\sqrt{-1}$

Something related to Quantum nature

mass of particle

Potential (Dependence on system)

Complex

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2) Non-Relativistic ($v \ll c$) & Massive (m)

Wave fn: given by soln: of

Schroding Eqn:

$$\boxed{\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x,t)\right]\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t}}$$

$$\nabla^2 = \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial y^2}$$, $m \rightarrow$ mass of particle associated with $\psi(x,t)$

$$\hbar = \frac{h}{2\pi}$$

$V(x,t) \rightarrow$ potential of the surrounding of the quantum system

$\psi(x,t) \rightarrow$ every particle has a wave fn: } quantum system

particle (pt. in space-time that has p, E and a relation between them $(E = p^2/2m)$)

3) QM is Probabilistic

$\psi(x,t) \rightarrow$ probability amplitude at (x,t)

$P_q(x,t) = \psi^*(x,t)\,\psi(x,t)$

↑

probability that particle q (quantum system) is at x at time t

4) Conservation of probability

$$\boxed{\int_{l} \psi^*(x,t)\,\psi(x,t)\,dx = 1}$$

Normalization

↳ Not guranteed when Schroding eqn: is solved.

↳ to satisfy this condition

· In a room of length L there is a particle q so if we look in the whole room we will find the particle somewhere so the sum of probability of finding the particle in the room is 1

5) Superposition of probability and amplitude probability that particle q (quatum system) is $\psi_{total}(x,t) = \psi_1 + \psi_2 + \psi_3 \cdots$ at x at time t

↳ Only if ψ_1, ψ_2 are indistinguishable

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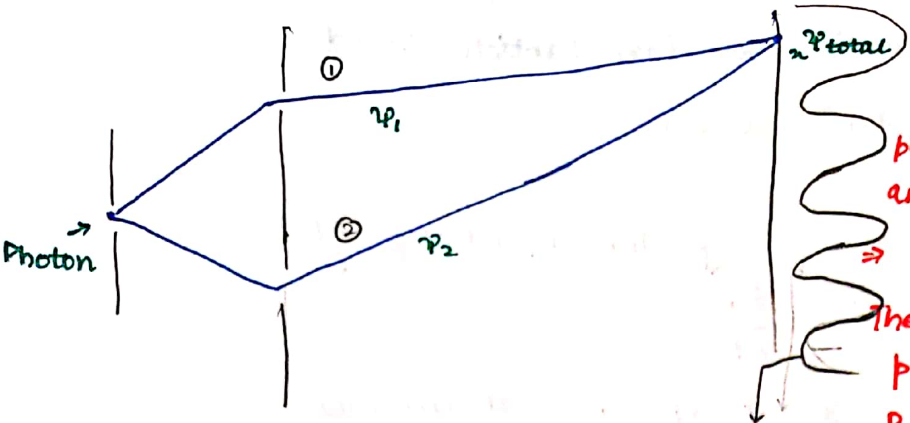

→ Some part will get more photons (bright fringe)

The probability that the photon goes by path ① is ψ_1 and by path ② is ψ_2

$\Rightarrow \psi_{total} = \psi_1 + \psi_2$

The probability of the photon reaching is

$$P_x = |\psi_1 + \psi_2|^2$$
$$= |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_2^* \psi_1$$

$$\boxed{I_x = I_0 P_x}$$

The probability of photons reaching here is min. (dark fringe)

When we place dectectors at the slits, i.e when a photon passes through a slit, it signals us, we no longer observe an interference pattern as the paths now are diff. and the principle of superposition is no longer applicable.

5) Observables

Classical observables become operators in quantum mech.

$x \rightarrow \hat{x} \equiv x$ (position operator)

$p \rightarrow \hat{p} \equiv -i\hbar \frac{\partial}{\partial x}$ (momentum operator)

$E \rightarrow \hat{H} \equiv i\hbar \frac{\partial}{\partial t}$
$= \frac{\hat{p}^2}{2m} + V(\hat{x})$ (Hamitonian operator)

$$p(\hat{o}) = \psi^*(x,t)\ \hat{o}\ \psi(x,t)$$

Avg value (Expectation value)

$$\langle \hat{o} \rangle = \int \psi^*(x,t)\ \hat{o}\ \psi(x,t)\, dx$$

(target space)

Physical Quantities in QM

$$\langle p(t_0) \rangle = \int \psi^*(x,t_0) \left(-i\hbar \frac{\partial}{\partial x}\right) \psi(x,t_0)\, dx$$

$$\langle x(t_0) \rangle = \int \psi^*(x,t_0)\ x\ \psi(x,t_0)\, dx$$

$$\langle E(t_0) \rangle = \int \psi^*(x,t_0) \left(i\hbar \frac{\partial}{\partial t} \psi(x,t)\right)\Bigg|_{t=t_0} dx$$

Can't find exact value

Can only find avg. of these values

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Solution of Schrodinger Equation for free particle in 1d

$V(x,t) = 0 \leftarrow$ Potential is zero

$\nabla^2 = \frac{d}{dx^2}$

Time dependent S-Eqn:

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \psi(x,t)}{\partial t}$$

Let $\psi(x,t) = \phi(x) f(t)$ ← Seperation of variables

$$\Rightarrow -\frac{\hbar^2}{2m} f(t) \frac{\partial^2 \phi}{\partial x^2} = i\hbar\, \phi(x) \frac{\partial f}{\partial t}$$

$$-\frac{\hbar^2}{2m} \times \frac{1}{\phi(x)} \frac{\partial^2 \phi}{\partial x^2} = i\hbar \frac{1}{f(t)} \frac{\partial f}{\partial t} = E$$

Only way $F_1(x) = F_2(t)$ is when both are equal to a const.

$$i\hbar \frac{1}{f(t)} \frac{\partial f}{\partial t} = E \quad -(A) \qquad , \qquad -\frac{\hbar^2}{2m} \times \frac{1}{\phi(x)} \frac{\partial^2 \phi}{\partial x^2} = E \quad -(B)$$

↳ Time indepedent S Eqn:

$$\int \frac{\partial f}{f} = \int -\frac{i}{\hbar} E\, dt$$

$$\Rightarrow \boxed{f(t) = f_0 e^{-i/\hbar\, Et}} \quad -(B)$$

$$\frac{1}{\phi}\frac{d^2\phi}{dx^2} = \boxed{-\frac{E(2m)}{\hbar^2}} \rightarrow k^2$$

↳ SHO

$$\frac{d^2\phi}{dx^2} = -k^2\phi \Rightarrow \boxed{\frac{\partial^2 \phi}{\partial x^2} + k^2\phi = 0}$$

$$\boxed{\phi(x) = \phi_0 e^{i[kx+\theta]}} \quad -(A)$$

The wave function of a free particle is

$$\boxed{\psi(x,t) = e^{-\frac{Et}{\hbar}}\left[A \sin kx + B \cos kx\right]}$$

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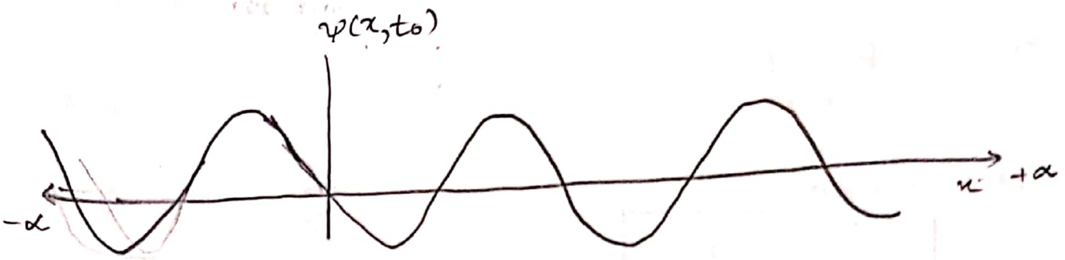


$P(x,t_0) = \psi^* \psi$

probability $= (A \sin kx + B \cos kx)^2$

$\Rightarrow$ Non-localized (No info about position)

$p = \psi^* \hat{p} \psi = \psi^* \left(i\hbar \frac{d}{dx}\right) \psi$

momentum $= \hbar k \, \psi^* \psi$

$\langle p \rangle = \int \psi^* \hat{p} \psi \, dx = \hbar k \underbrace{\int \psi^* \psi \, dx}_{\to 1 \text{ (Normalization)}}$

$\boxed{\langle p \rangle = \hbar k}$

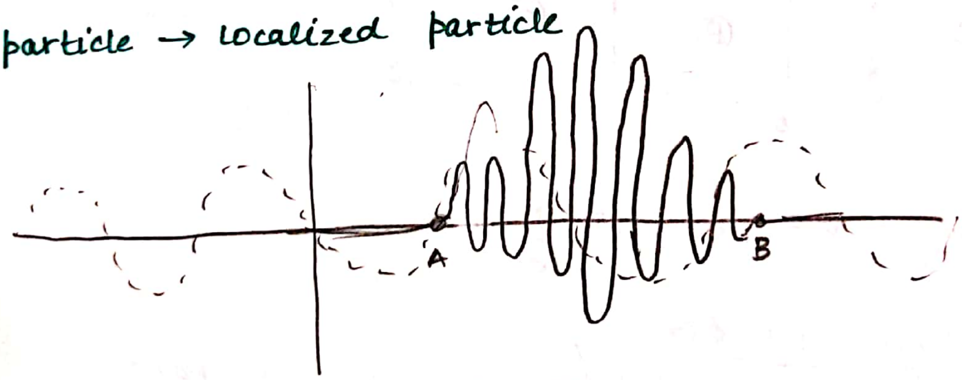
$\langle p \rangle = \cancel{\hbar} \sqrt{\frac{2mE}{\cancel{\hbar^2}}} = \sqrt{2m \times \frac{p^2}{2m}}$

$\Rightarrow \boxed{\langle p \rangle = P_{classcial}}$

$\Delta x \to \infty$

Error in measuring position

Free particle → localized particle



The particle lies only in AB

$\psi(x,t) = \sum_{n=1}^{\infty} \psi_n(t) \, e^{i(k_n x - \theta)}$

$\Delta x \to$ smaller

$\Delta k \to$ larger

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Heisenberg Uncertainity Principle

Quantum system : $p \pm \Delta p$

$x \pm \Delta x$

Simultaneous Measurement

$$\boxed{\Delta p \, \Delta x \geq \hbar/2}$$

$$\Delta x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$$

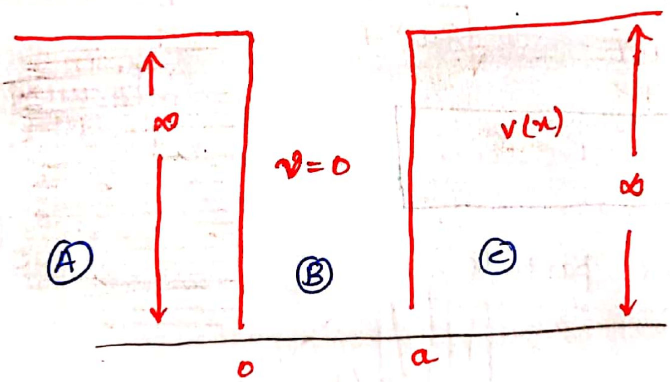
$$\langle \hat{x}^2 \rangle = \int \psi^* \, \hat{x}^2 \, \psi \, dx$$

$$\langle \hat{x} \rangle = \int \psi^* \, \hat{x} \, \psi \, dx$$

$$\Delta P = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2}$$

$$\langle \hat{p}^2 \rangle = \int \psi^* \left(-i\hbar \frac{\partial}{\partial x}\right)^2 \psi \, dx$$

Example II : Particle in a box



$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$\hookrightarrow$ $\psi(x,t) = f(t)\,\phi(x)$

$$\boxed{f(t) = f_0 \, e^{i E t/\hbar}}$$ ← f(t) will not change

Region A

$V(x)\,\phi = E\,\phi$

$\phi_A(x) = 0$

Region C

$V(x)\,\phi = E\phi$

$\phi_c(x) = 0$

Region B

$V(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{d^2 \phi}{dx^2} = E\phi$$

$\phi_B(x) = A \sin kx + B \cos kx$

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Wave fn: Matching

The wave fn. must be cont. at $x=0$, $x=a$ since S.E is a smooth diff eqn.

$$\phi_A\Big|_{x=0} = \phi_B\Big|_{x=0}$$

$$\left.\begin{aligned}\phi_B(0) &= 0\\ \phi_B(0) &= B = 0\end{aligned}\right\} \boxed{\phi_B(x) = A\sin kx}$$

Matching at $x=a$

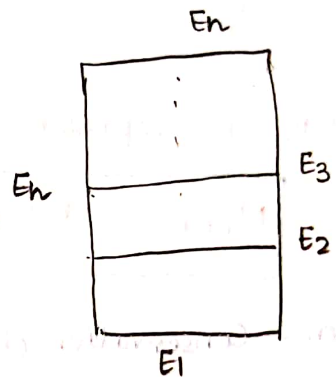
$$\phi_B\Big|_{x=a} = \phi_c\Big|_{x=a}$$

$$A\sin ka = 0 \Rightarrow \boxed{ka = n\pi}$$

Energy of particle in a box

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\frac{\hbar^2 k^2}{E} = 2m \Rightarrow E = \frac{\hbar^2 k^2}{2m}$$



$$\boxed{E_n = \frac{\hbar^2}{2m}\,\frac{n^2\pi^2}{a^2}}$$

Energy for a quantum system is quantised

$$\phi_B = A\sin kx$$

$$\boxed{\phi_B = A\sin\left[\frac{n\pi}{a}x\right]} \quad n = 1, 2, 3 \cdots$$

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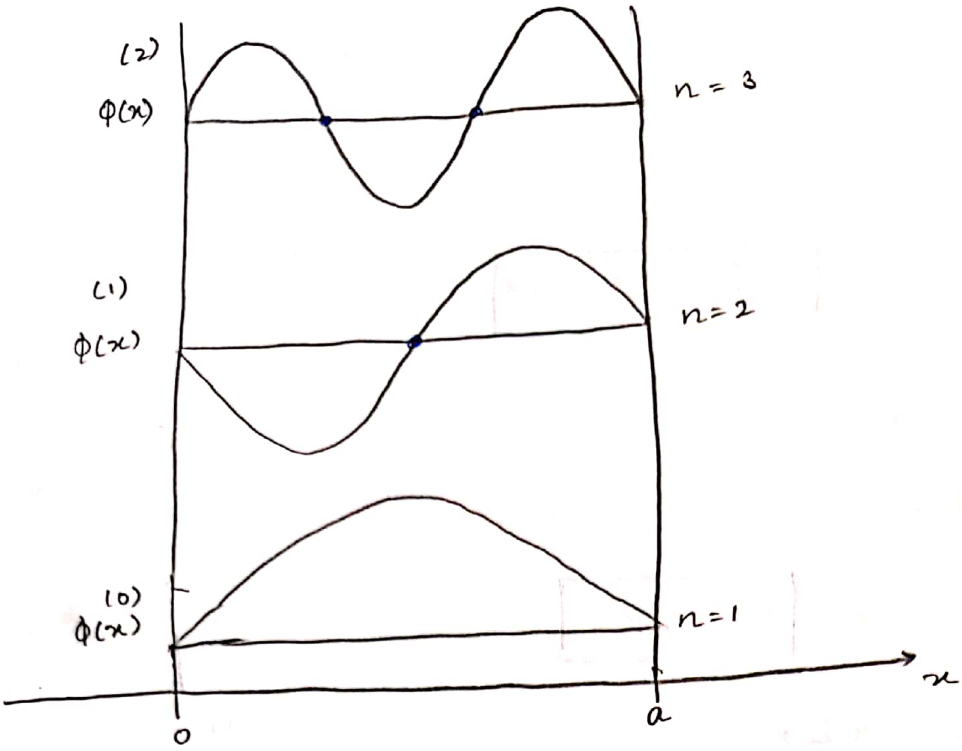

The quantity n represents the nodes − 1

$$\boxed{N_{nodes} = n-1}$$

Normalization: Conservation of probability

$$\int_0^a \phi_B^*(x,t)\,\phi_B(x,t)\,dx = 1$$

$$\Rightarrow \int_0^a A^2 \sin^2(kx) = A^2\left[\frac{a}{2} - \frac{\sin 2ka}{4k}\right]$$

Most of the time the particle will be at a/2
Equal time b/w 0 to a/2 and a/2 to a, the avg. is at a/2

$$A^2 \cdot \frac{a}{2} = 1$$

$$\Rightarrow \boxed{A = \sqrt{\frac{2}{a}}}$$

Particle in a Box Waveform

$$\boxed{\phi_B(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$$

Normalized wave fn.

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HW

Particle in a box $(n=1)$

$$\langle x \rangle = \int_0^a \phi_B(x)\, x\, \phi_B(x)\, dx = \frac{a}{2}$$

$\langle p \rangle = 0$ Half the time ~~it~~ is in one direction and the other half in other half in other direction

$$\langle p \rangle = \int_0^a \phi_B^*(x) \left(i\hbar \frac{d}{dx}\right) \phi_B(x)\, dx = 0$$

$$\langle x^2 \rangle = \int_0^a \phi_B^*(x)\, x^2\, \phi_B(x) = \frac{a^2}{3} - \frac{a^2}{2\pi^2}$$

$$\langle p \rangle = \hbar^2 \left(\frac{\pi}{a}\right)^2$$

$$\Delta x\, \Delta p = \frac{\hbar^2}{2}\left(\frac{\pi^2}{3} - 2\right)^{1/2} > \hbar/2$$

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