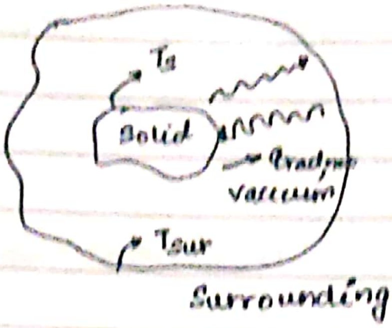
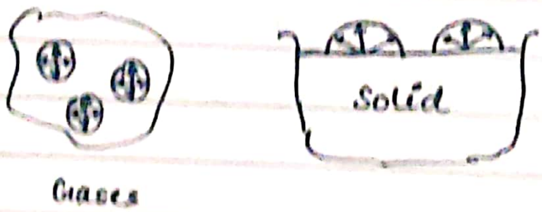
Radiation Heat Transfer

Body at high temp. radiates energy. (Sun → Earth)
This heat transfer can be done via space.



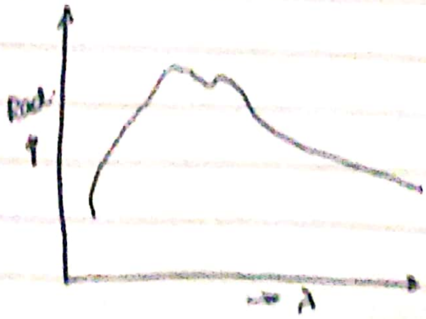

For gases, radiation comes from the volume of gase, while for solids it comes from the surface, as the radiation of the particles inside, gets absorbed by other particles in solid it self

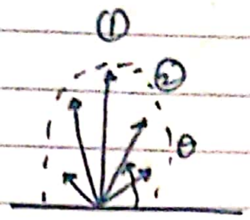



All EMR doesn't cause thermal effect, only those EMR that can cause vibration of molecules

Distribution of radiation

Spectral Distribution






← Directional Distribution

Intensity of radiation depends on $\theta$, the intensity at ① > intensity at ②, it also depends on $\lambda$

Colour depends on which direction we see.

→ $I(\theta, \phi, \lambda)$

Emissive power (E) - Rate at which radiation is emitted from a surface per unit area.

$$E = \varepsilon \sigma T_s^4$$

$\sigma$ → Stefan Boltzman const

$\varepsilon$ → Emissivity

Irradiation (G) - Rate at which radiation is incidendt upon a surface per unit area

$$G \leq E$$

Radiosity (J) - Rate at which radiation leaves a surface per unit area

$$J = E + \rho G \quad \text{(opaque surface)}$$

Net radiative Flux ($q''_{rad}$) - Net rate of radiation leaving a surface per unit area.

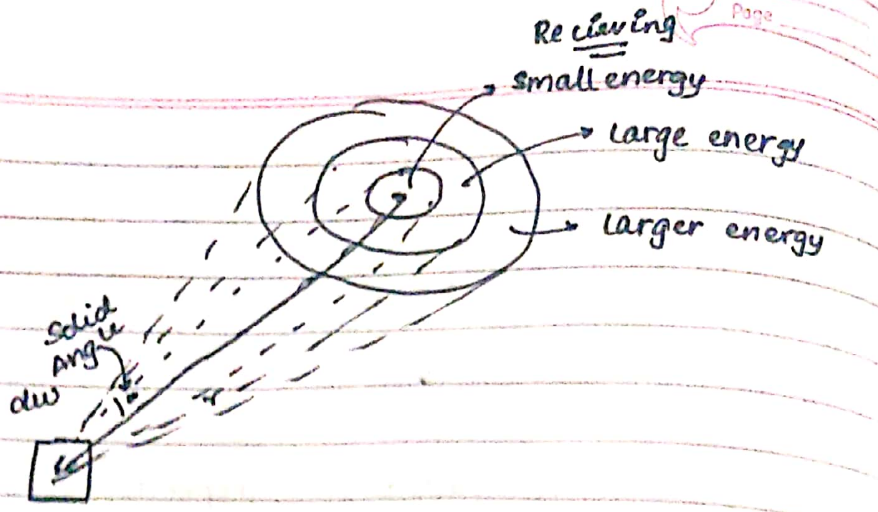
$$q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G$$

$\rho$ → reflected

$\tau$ → Transmitted

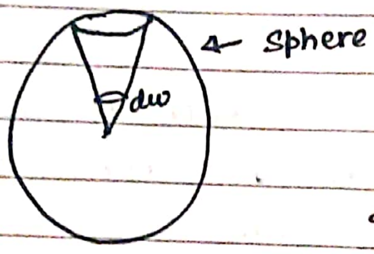
$\alpha$ → Absorbed

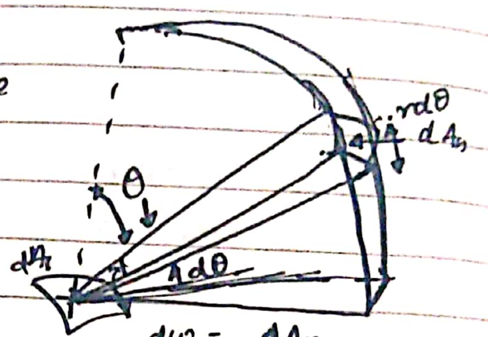
$$\boxed{\tau + \alpha + \rho = 1}$$



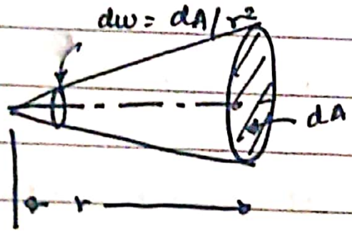

Amount of heat energy receved is prop. to the solid angle.

$$dq \propto d\omega$$



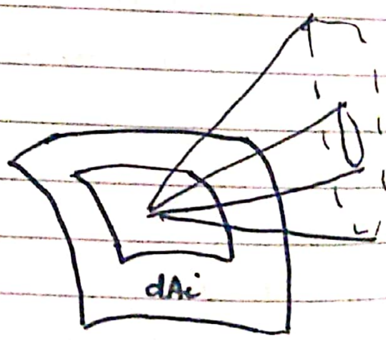

$$d\omega = \frac{dA_n}{r^2}$$




$$d\omega = \sin\theta \, d\theta \, d\phi$$

Consider a surface $dA_i$

Radiation increases with increase in

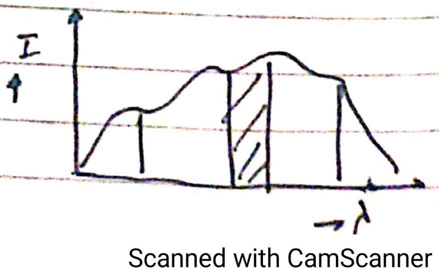



① Solid Angle

② Area of sheet

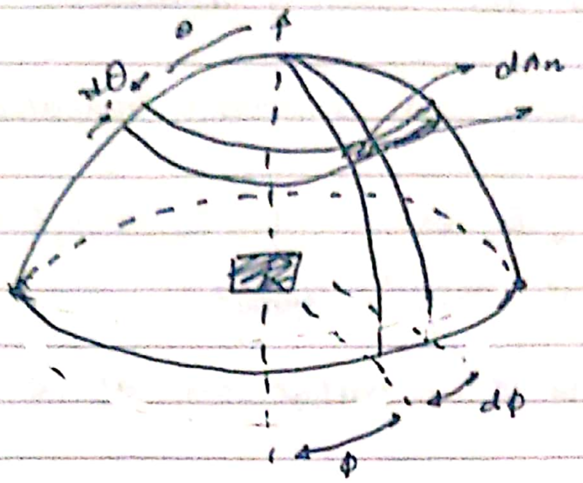
③ (Projection of Area in the direction of view)

③ Range of $\lambda$



Projection of area

$I_{\theta,\lambda}$ $\boxed{dq = I_{e,\lambda}(\lambda, \theta, \phi)\, d\omega\, A_i \cos\theta\, d\lambda}$




$$dq = I_{\lambda e}(\lambda, \theta, \phi)\, dA_1 \cos\theta\, d\omega\, d\lambda$$

$$dq_\lambda = \frac{dq}{d\lambda} = I_{\lambda e}(\lambda, \theta, \phi)\, dA_1 \cos\theta\, d\omega$$

$$dq'' = \frac{dq_\lambda}{dA_1} = I_{\lambda e}(\lambda, \theta, \phi) \cos\theta \sin\theta\, d\theta\, d\phi$$

Emmissive Power

$$E_\lambda(\lambda) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\lambda e}(\lambda, \theta, \phi) \cos\theta \sin\theta\, d\theta\, d\phi$$

$$E_T = \int_0^\infty E_\lambda(\lambda)\, d\lambda$$

If emitter is diffused, $I_\lambda(\lambda)$ only

$$E_\lambda(\lambda) = I_\lambda(\lambda) \iint \cos \sin\theta\, d\theta\, d\phi$$

$$= \frac{I_\lambda(\lambda)}{2} \int_0^{2\pi} \int_0^{\pi/2} \sin 2\theta\, d\theta\, d\phi$$

$$\therefore \quad E = \pi \int I_{\lambda,e}(\lambda)\, d\lambda \quad \leftarrow \text{Diffused emitter}$$

Only for emitted radiation

Intensity of Incident Radiation, $(I_{\lambda,i}(\lambda,\theta,\phi))$

$$G_\lambda(\lambda) = \iint_{Hemi} I_{\lambda,i}(\lambda;\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

Global

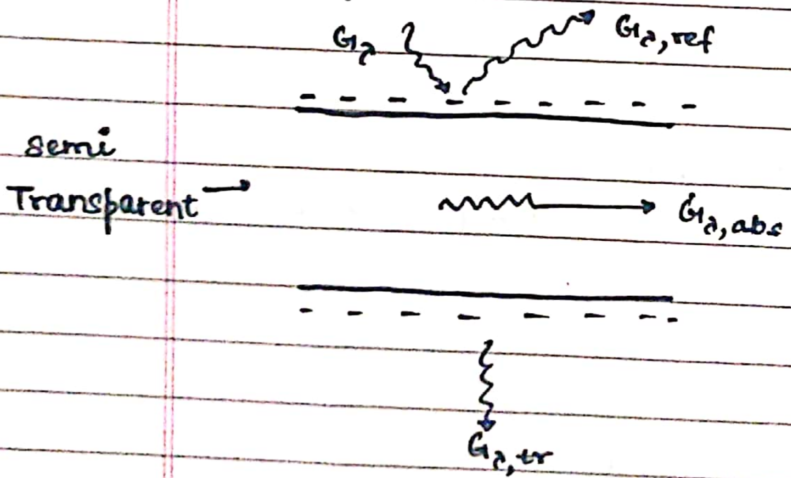
G is integrated over all $\lambda$

Radio city $\rightarrow$ Combined emitted and reflected radiation observer

$$I_{\lambda,e+r}(\lambda,\theta,\phi)$$

$$J_\lambda(\lambda) = \iint_{Hemi} I_{\lambda,e+r}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$$

Absorption




$$G_\lambda = G_{\lambda,ref} + G_{\lambda,abs} + G_{\lambda,tr}$$

$$\text{Absorpivity} = \frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)} = \alpha$$

$\downarrow$

$\alpha_{\lambda,\theta}(\lambda,\theta,\phi)$

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Spectral hemispherical Absorptivity $(\alpha_\lambda(\lambda))$ $= \dfrac{G_{\lambda,i,abs}(\lambda)}{G_{\lambda,i}(\lambda)}$

Total hemispherical absorpivity

$$\alpha = \frac{\int G_{\lambda,i,abs}(\lambda)}{\int G_{\lambda,i}(\lambda)} = \frac{G_{abs}}{G_i}$$

Similarly we have Transmittivity $(\tau)$ Reflectivity $(\rho)$

$$\tau + \rho + \alpha = 1$$

$$\tau_\lambda + \rho_\lambda + \alpha_\lambda = 1$$

( Medium shouldn't transmit energy ).


$I_e(\theta, \lambda, \phi, T)$

$\uparrow$ If temp isn't const.

Black Body

- Absorbs all incident radiation, irrespective of $\lambda, \theta, \phi$
- For a given $\lambda$, $T$, no surface can emit more radiation that
- Black body is a diffused emitter

$$I_{\lambda,b}(\lambda, T) = \frac{2hc^2\lambda^{-5}}{\exp\left(\frac{hc}{kT\lambda}\right)-1}$$

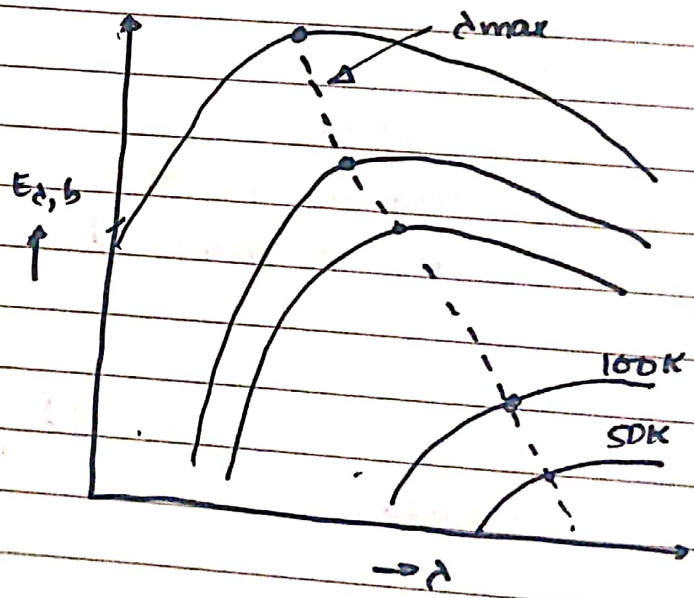
$h \rightarrow$ Plank's const.
$c \rightarrow$ speed of light
$k \rightarrow$ Boltzman const.

$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}$$

Emission / Area $= \sigma T^4$

$\hookrightarrow$ stephan's boltzman constant

$$\sigma = 5.67 \times 10^{-8} \; W/m^2K^4$$




Wein's Displacement law

$$T\lambda_{max} = C = 2898 \; \mu m K$$

## Emissivity (Real Body)

$$\epsilon_{\lambda\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda e}(\lambda, \theta, \phi, T)}{I_{\lambda b}(\lambda, T)}$$

Spectral directional Emissivity

Total directional Emissivity

$$\epsilon_{\theta,\lambda}(\theta, \phi, T) = \frac{I_e(\theta, \phi, T)}{I_b(T)}$$

Spectral hemispherical emissivity

$$\epsilon_\lambda(\lambda, T) = \frac{E_\lambda(\lambda, T)}{E_{\lambda, b}(\lambda, T)}$$

Total Hemispherical Emissivity

$$\epsilon(T) = \frac{E(T)}{E_b(T)}$$

Compare radiative and neutral convective flux from a horizontal flat plate $1 m^2$ once when the plate is at temp 350K and then for 700 K. The emissivity is 0·3, Ambient temp. 300K

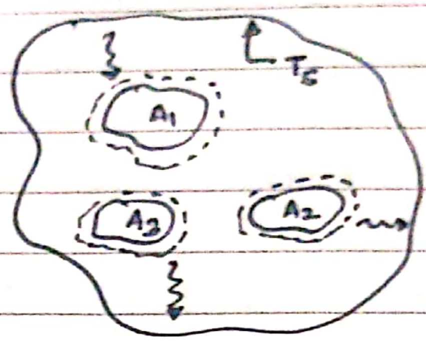
A)
$$E_{\lambda, b} = \sigma A T^4$$
$$= 5{\cdot}67 \times 10^{-8} \times 1 \times (350)^4$$
$$= 255{\cdot}26 \ W/m^2$$

$$q_s = h(T_s - T_A)$$
$$= 290 \ W/m^2$$

At 700K, $E_{\lambda, b} = 4084 \ W/m^2$ $\qquad q_s = 3389 \ W/m^2$

Kirchoff's Law

Consider large, isothermal enclosure of surface temperature $T_s$.




$$G = E_b(T_s)$$

Applying energy balance,

$$\alpha_1 G A_1 - E_1(T_s) A_1 = 0$$

$$\frac{E_1(T_s)}{\alpha_1} = E_b(T_s)$$

Similarly, $\frac{E_1(T_s)}{\alpha_1} = \frac{E_2(T_s)}{\alpha_2} = \cdots = E_b(T_s)$

Major implication from this law, since $\alpha \leq 1$ $E(T_s) \leq E_b(T_s)$. Hence no real surface can have emissive power exceeding that of a black surface at same temperature

$$\frac{\varepsilon_1}{\alpha_1} = \frac{\varepsilon_2}{\alpha_2} = \cdots = 1$$

Hence, for any surface enclosure,

$$\alpha = \varepsilon$$

For, diff. spectral cond., $\alpha_\lambda = \varepsilon_\lambda$

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For diff spectral cond. and directions,

$$\alpha_{\lambda,\theta} = \varepsilon_{\lambda,\theta}$$

$\varepsilon_{\lambda,\theta}$, $\alpha_{\lambda,\theta}$ are inherent surface properities

Gray Surface

$$\varepsilon_\lambda = \frac{\iint \varepsilon_{\lambda,\phi} \cos\theta \sin\theta \, d\theta \, d\phi}{\iint \cos\theta \sin\theta \, d\theta \, d\phi}$$

$$= \frac{\iint \alpha_{\lambda,\theta} I_{\lambda,i} \cos\theta \sin\theta \, d\theta \, d\phi}{\iint I_{\lambda,i} \cos\theta \sin\theta \, d\theta \, d\phi} = \alpha_\lambda$$

The above is satisfied when
1. Irradiation ($I_{\lambda,i}$) is independent of $\theta, \phi$)
2. $\varepsilon_{\lambda,\theta}, \alpha_{\lambda,\theta}$ is independent $\theta$ and $\phi$

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} \, d\lambda}{E_b(T)} = \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda) d\lambda}{G} = \alpha$$

→ $G_\lambda(\lambda) = E_{\lambda,b}(\lambda, T)$ and $G = E_b(T)$

→ $\alpha_\lambda$, $\varepsilon_\lambda$ are independent of $\lambda$ (Gray surface)

$$\therefore \quad \varepsilon = \frac{\varepsilon_{\lambda,0} \int_{\lambda_1}^{\lambda_2} E(\lambda, T)\, d\lambda}{E_b(T)} = \varepsilon_{\lambda,0}$$

$$\alpha = \frac{\alpha_{\lambda,0} \int_{\lambda_1}^{\lambda_2} G_\lambda(\lambda)\, d\lambda}{G} = \alpha_{\lambda,0}$$

Solar Radiation

At the ~~too~~ outer edge of earth's atmosphere, the flux of solar energy is decreased by a factor $(r_s/r_d)^2$

Solar const. → It is defined as flux of solar energy incident on a surface oriented normal to the sun's rays.

$$G_{s,0} = S_c \cdot \underset{\substack{\uparrow \\ \text{Correction} \\ \text{factor}}}{f} \cdot \cos\theta$$

$0.97 \leq f \leq 1.03$

$d_e = 2r_e = 1.27 \times 10^7$ , $S_c = 342 \times 4 \ W/m^2$

$G_{atm} = \sigma \underset{\substack{\uparrow \\ \text{effective sky} \\ \text{temp.}}}{T_{sky}^4}$

$G_{atm} = 324 \ W/m^2$

$T_{sky} = 275 \ K$

Kirchoff's law

$$\varepsilon_{\lambda,\theta} = \frac{I_{\lambda,\theta}(\lambda,\theta,\phi,T)}{I_\lambda(\lambda,T)}$$

$$\varepsilon_\lambda = \frac{\iint I_{\lambda,\theta}(\lambda,\theta,\phi,T)\cos\theta\, d\theta\, \sin\theta\, d\phi}{\iint I_\lambda(\lambda,T)\cos\theta\, d\theta\, \sin\theta\, d\phi}$$

$$\varepsilon_T = \frac{E(T)}{E_b(T)}$$

$$\alpha_{\lambda,\theta} = \frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}$$

After solving, we see that,

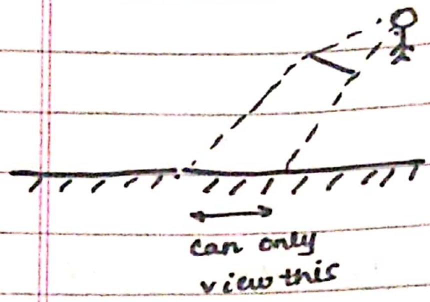
$\alpha_{\lambda,\theta} = \varepsilon_{\lambda,\theta}$ ← Surface need not ~~not~~ be diffused

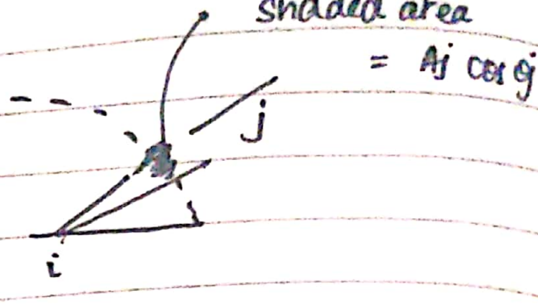
For this course, we consider, $\alpha = \varepsilon$
(under ~~such~~ certain assumptions)

If the incident radiation is diffused,
then also $\varepsilon_\lambda = \alpha_\lambda$

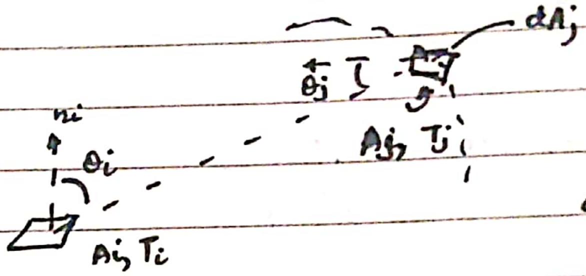
Gray surface - $\varepsilon = \alpha$ ← Diffused gray surface
↳ Surfaces independent of $\lambda$

View factor




Fraction of radiation from $i^{th}$ surface that reaches $j^{th}$ surface




Energy radiated
↓ Area, solid angle

$$dq_{i \to j} = I_{e+r, i}\, dA_i \cos\theta_i\, d\omega_i$$

Thus, this depends on

1. $d\omega$
2. $dA \cos\theta$
3. $d\lambda$

$$d\omega = \frac{\text{Shaded area}}{R^2}$$

$$\Rightarrow d\omega = \frac{A_j \cos\theta_j}{R^2}$$

$$\therefore dq_{i \to j} = I_{e+r, i} \frac{dA_i \cos\theta_i \, dA_j \cos\theta_j}{R^2}$$

$$dq_{i \to J} = \frac{J}{\pi} \frac{\cos\theta_i \, dA_i \cos\theta_j \, dA_j}{R^2}$$

$$q_{i \to j} = \int_{A_j} \int_{A_i} \frac{J \cos\theta_i \cos\theta_j \, dA_i \, dA_j}{\pi R^2}$$

$$q_{i \to j} = \frac{J}{\pi} \int_{A_j} \int_{A_i} \frac{\cos\theta_i \cos\theta_j \, dA_i \, dA_j}{R^2}$$

↳ Uniform radiosity, emitter, diffuser (Assumptions)

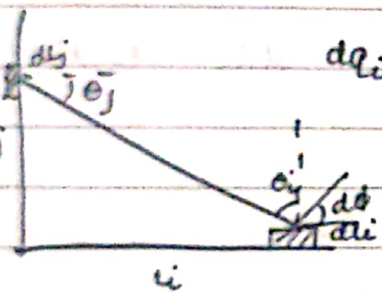
$$F_{ij} = \frac{q_{ij}}{A_i J} = \frac{1}{A_i J} \frac{J}{\pi} \iint_{A_j\, A_i} \frac{\cos\theta_i \cos\theta_j \, dA_i \, dA_j}{R^2}$$

$$\therefore \quad F_{ij} = \frac{1}{A_i \pi} \iint_{A_j\, A_i} \frac{\cos\theta_i \cos\theta_j \, dA_i \, dA_j}{R^2}$$

→ Diffused reflector / emitter
→ Uniform radiosity

$$F_{ij} = \frac{1}{A_i} \iint_{A_j\, A_i} \frac{\cos\theta_i \cos\theta_j \, dA_i \, dA_j}{\pi R^2}$$

$$A_i F_{ij} = A_j F_{ji}$$




$$dq_{i\to j} = I_{e+r,i} \, (dl \cos\theta_i) \, d\phi_j$$

$$\downarrow$$

$$\frac{dl_j \cos\theta_j}{R} \cdot d\phi_j$$

$$dq_{i\to j} = \frac{I_{e+r,i} \, dl_i \cos\theta_i \, dl_j \cos\theta_j}{R}$$

$$q_{i\to j} = I_{e+r,i} \iint_{l_2\, l_1} \frac{\cos\theta_i \cos\theta_j \, dl_i \, dl_j}{R}$$

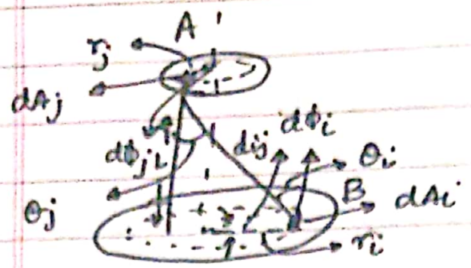
$$F_{ij} = \frac{1}{L_i} \iint_{l_i\, l_j} \frac{\cos\theta_i \cos\theta_j \, dl_i \, dl_j}{R}$$

$$\cos\theta_i \cos\theta_j = \frac{l_i l_j}{\sqrt{(l_i^2 + l_j^2)}}$$

$$F_{ij} = \frac{1}{L_i} \int_0^{l_i} \int_0^{l_j} \frac{l_i l_j \, dl_i \, dl_j}{(l_i^2 + l_j^2)^{3/2}}$$

$$= \frac{1}{L_i} (L_i - L_j) - \frac{1}{L_i} \sqrt{L_i^2 + L_j^2}$$

Obtaining view factor by inspection




$r_j\, d\phi_j\, dr_j = dA_j$

$r_i\, d\phi_i\, dr_i = dA_i$

$\cos\theta = \dfrac{L}{d_{ij}}$

$\theta_i = \theta_j$

$$F_{ij} = \frac{1}{A_i} \iint_{A_i A_j} \frac{\cos\theta_i \cos\theta_j \, dA_i\, dA_j}{\pi (d_{ij})^2}$$

$$= \frac{1}{A_i} \iint_{A_i A_j}$$

$A(r_j \cos\phi_j, r_j \sin\phi_j, L)$ $\quad dA_i = r_i\, d\phi_i\, dr_i$

$B(r_i \cos\phi_i, r_i \sin\phi_i, 0)$

$$d_{ij}^2 = (r_i \cos\phi_i - r_j \cos\phi_j)^2 + L^2 + (r_i \sin\phi_i - r_j \sin\phi_j)^2$$

$$F_{ij} = \frac{1}{\pi R_i^2} \int_{\phi_j=0}^{\phi_j=2\pi} \int_{\phi_i=0}^{2\pi} \int_{r_j=0}^{R_j} \int_{r_j=0}^{R_i} \frac{L^2}{\pi (d_{ij}^2)^2}$$

$$d_{ij}^2 = r_i^2 + r_j^2 + L^2 - 2 r_i r_j \cos(\phi_i - \phi_j)$$

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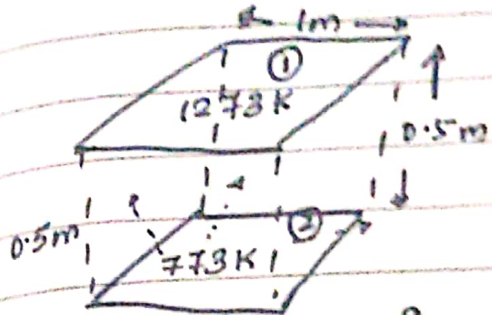
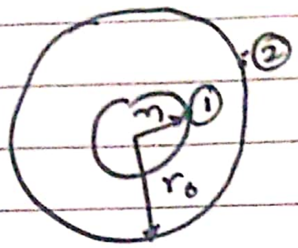



$q_{net} = q_{1\to2} - q_{2\to1}$

View factor = 0.28527

$q_{net} = 0.28 \times \sigma T_1^4 \times A_1 - 0.28 \times \sigma T_2^4 \times A_2$
$= 18.4$ kW




we have four view factors, $F_{11}$, $F_{12}$, $F_{21}$, $F_{22}$

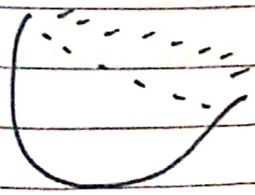
$\sum_j F_{ij} = 1$ , $A_i F_{ij} = A_j F_{ji}$

$F_{11} = 0$ ( whatever radiation leaves ①, cant reach ①, it's a convex surface)

$F_{22} \neq 0$ ( Concave surface)

$F_{11} + F_{12} = 1 \Rightarrow F_{12} = 1$

$F_{21} = \frac{r_i^2}{r_o^2}$ $F_{22} = 1 - \frac{r_i^2}{r_o^2}$



we assume this to be an of black surface, as the radiation that

Open surface

Encloser : $q_i$ = Net amount of energy leaving surface i)

$q_i = A_i (J_i - G_i)$

$J_i = E_i + \rho_i G_i = \epsilon_i E_{bi} + (1-\alpha_i) G_i$
$= \epsilon_i E_{bi} + (1-\epsilon_i) G_i$

$$\frac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i} = G_i$$

$$q_i = A_i \left( \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i} \right)$$

$$A_i G_i = \sum F_{ji} A_j J_j$$

$$= \sum_{j=1}^{N} A_i F_{ij} J_j$$

$$G_i = \sum F_{ij} J_j$$

$$q_i = A_i \left( J_i - \sum_j F_{ij} J_j \right)$$

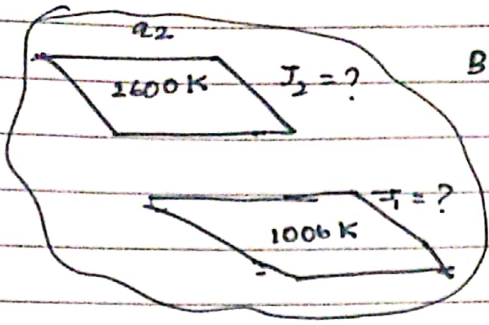
$$= A_i \left( \sum_j J_i F_{ij} - \sum_j F_{ij} J_j \right)$$

$$= A_i \sum_j F_{ij} (J_i - J_j)$$

$$\therefore \; q_i = \sum A_i F_{ij} (J_i - J_j)$$

$$A_i \frac{E_{bi} - J_i}{(1-\varepsilon_i)/\varepsilon_i} = \sum A_i F_{ij} (J_i - J_j) = q_i$$

Radiation exchange b/w opaque, diffuse, grey surfaces forming enclosure.




Black

$F_{12} = F_{21} = 0.4$

$F_{11} + F_{12} + F_{13} = 1.0$

$F_{13} = 0.6$

$F_{23} = 0.6$

$10 J_1 - 0.4 J_2 = 515766$

$-0.4 J_1 + 2.0 J_2 = 7624$

$J_2 \sim 14 \text{ kW/m}^2$

$E_{b_2} = 7.34 \text{ kW/m}^2$

$q_2 = \dfrac{10\,(7.34 - 14.0)}{(1-\varepsilon/\varepsilon)} \approx -67 \text{ kW}$