Deformations

F_t - Shear

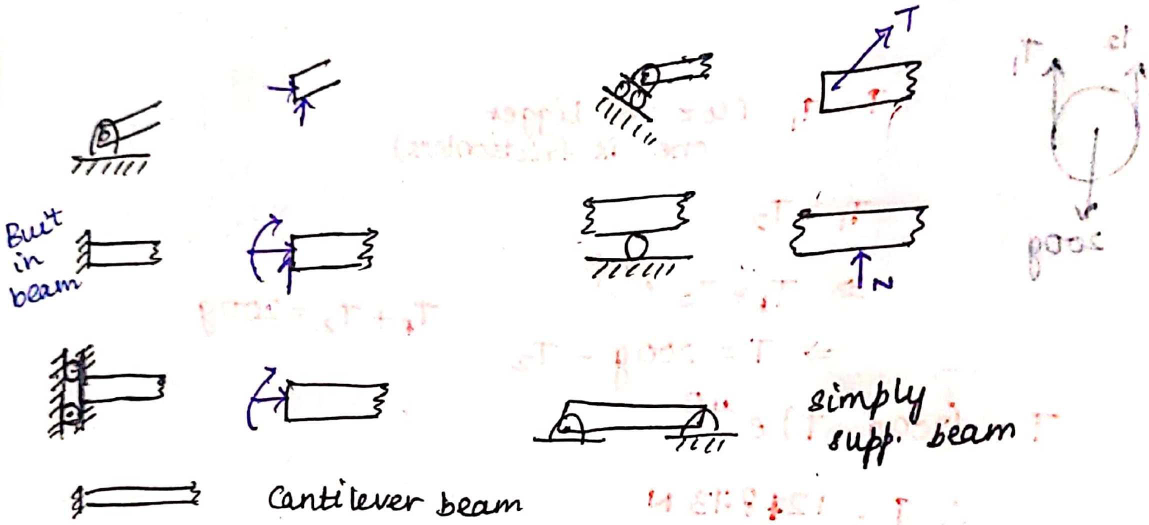
M_n / T

Normal Moment is twisting

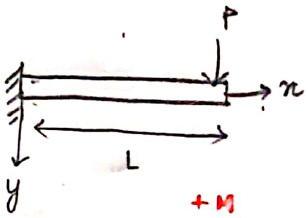
Tangential Moment is bending

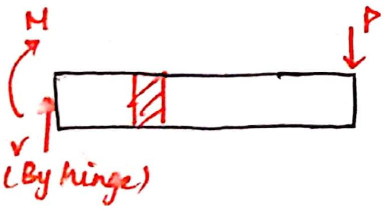
Beam carries Bending moment and shear force.

Beam



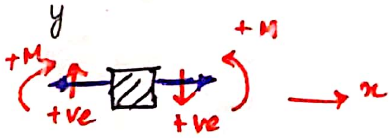
Q) Draw the shear force and bending moment diagrams for the end-loaded cantilever beam shown. Neglect the weight

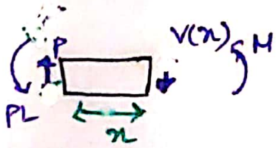


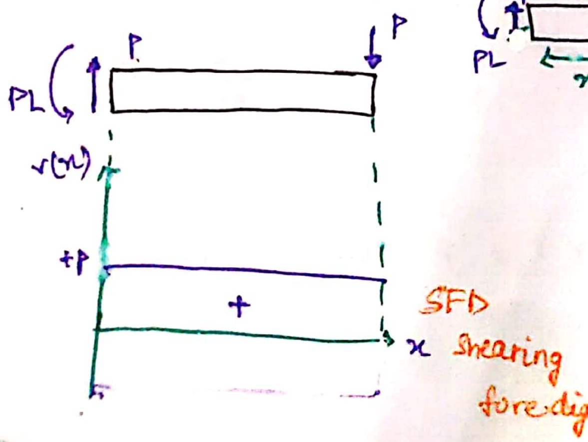


$V = P$

$M = -PL$





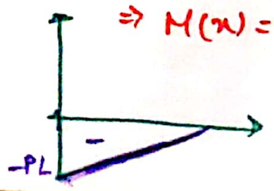


$Px - PL + M(x) = 0$

$\Rightarrow M(x) = -PL + Px$

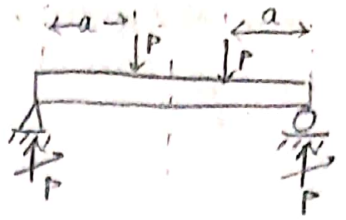
$M(x) = -P(L - x)$

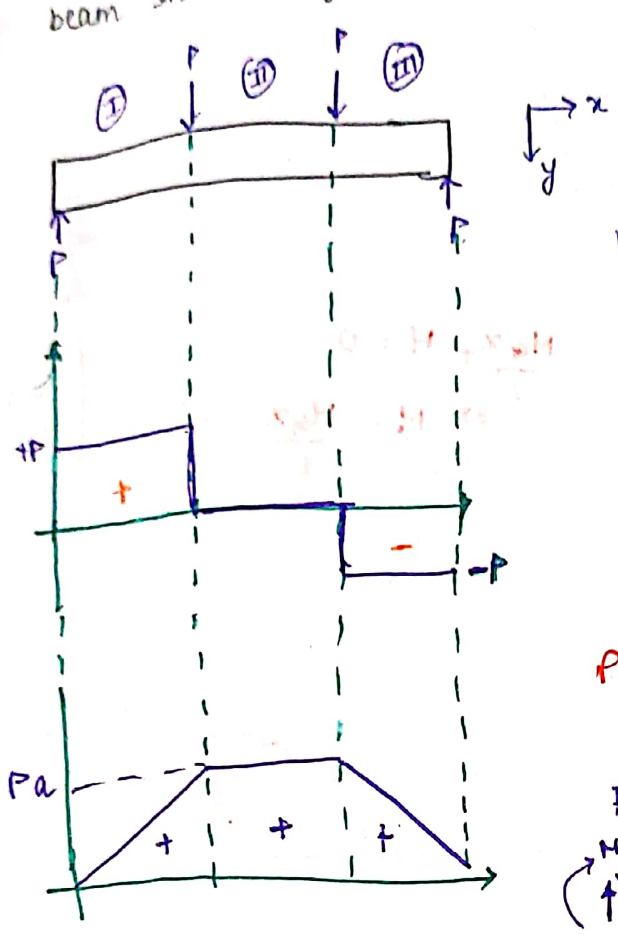
$\Rightarrow M(x) = P(x - L)$



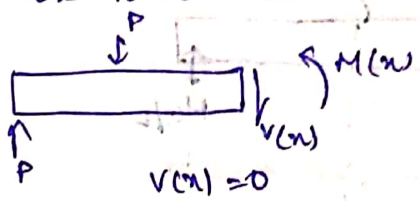
- Vardhan

8) Draw shear force and bending moment diagrams for the loaded simply supp beam shown. Neglect the weight



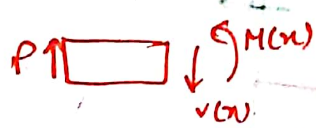


For the midde section



$V(x) = 0$

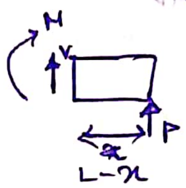
For 1st section



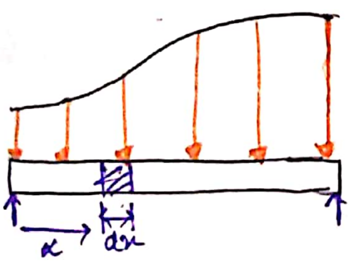
$M(x) = Px$

$M(a) = Pa$

For 2nd section

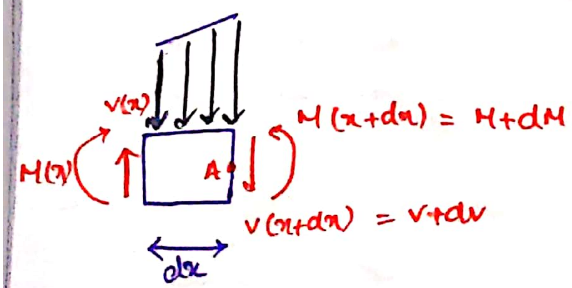


*Whenever shear force is const. , Moment has const. slope.



$W(x)$. unit - N/m

Represents distributed load



From Force Balance

$V + dV - V + w\,dx = 0$

$\Rightarrow \boxed{\dfrac{dV}{dx} = -w(x)}$ ← Relation B/w shear force and load

From moment balance — There is even $(w\,dx)\,dx$ which is negligable

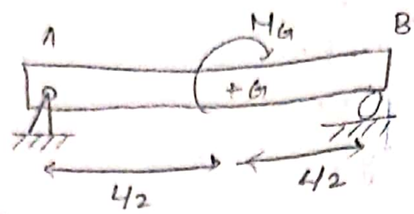
$M + dM - M - V\,dx = 0$

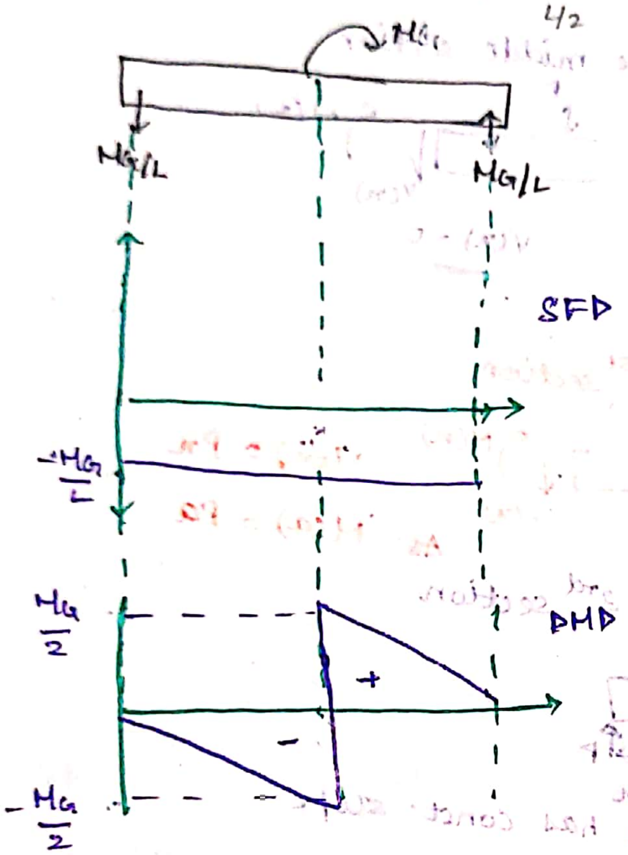
$\Rightarrow \boxed{\dfrac{dM}{dx} = V(x)}$

$\Rightarrow \boxed{\dfrac{d^2M}{dx^2} = w(x)}$

Vardhan

8) Draw shear force and bending moment diagrams for the couple moment loaded simply supp. beam as shown. Neglect weight of beam.



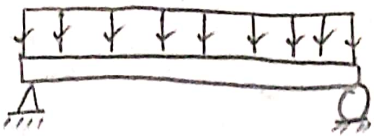


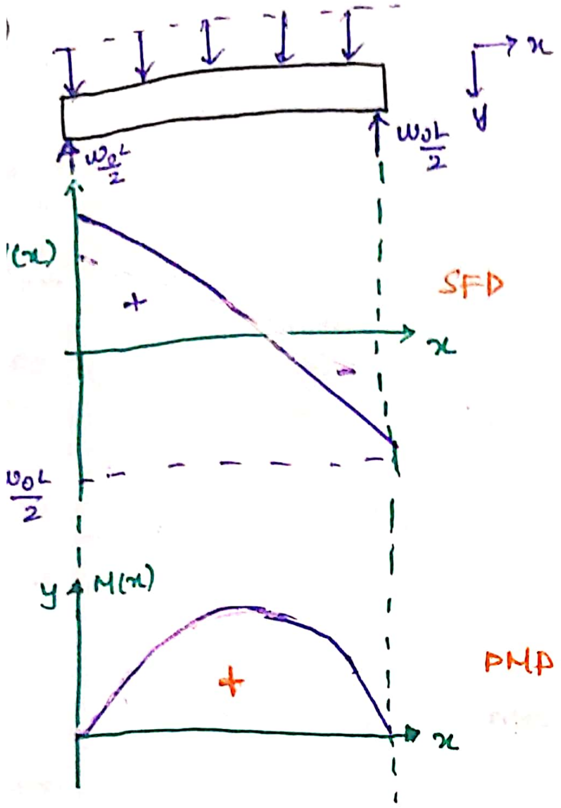
$\frac{M_G x}{L} + M = 0$

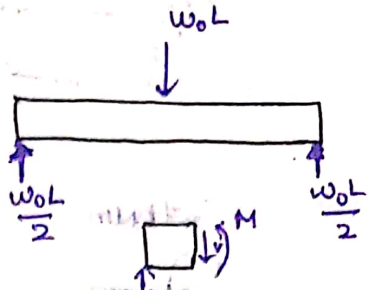
$\Rightarrow M = -\frac{M_G x}{L}$

Vardhan

Draw the shear force and bending moment diagrams for the uniformly loaded simply supp. beam as shown







$$\frac{dV}{dx} = -w(x) = -w_0$$

$$V = -w_0x + C_1$$

$$V(0) = \frac{w_0L}{2} = C$$

$$\Rightarrow V = w_0\left(\frac{L}{2} - x\right)$$

$$\frac{dM}{dx} = V(x) = w_0\left(\frac{L}{2} - x\right)$$

$$\Rightarrow M = \frac{w_0Lx}{2} - \frac{w_0x^2}{2} + D_1$$

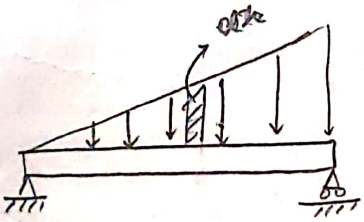
$$\Rightarrow M = \frac{w_0Lx}{2} - \frac{w_0x^2}{2}$$

(when $x = 0$, $M = 0 \Rightarrow D_1 = 0$)

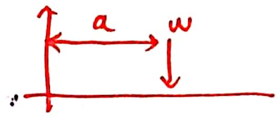
$$\Rightarrow M = \frac{w_0x}{2}(L - x)$$

Whenever shear force vanishes it results in ~~the~~ turning pt. of bending moment

b)

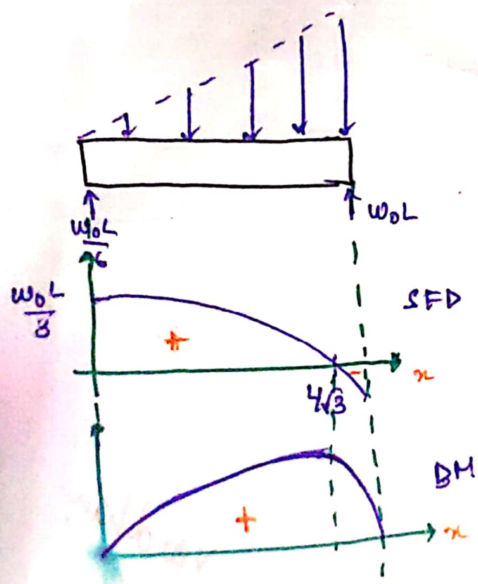


$w = w_0\frac{x}{L}$



$$W = \int_0^L w(x)dx$$

$$W = \frac{w_0L}{2}$$



$$\frac{dV}{dx} = -w(x) = -w_0\frac{x}{L}$$

$$\Rightarrow V = -\frac{w_0x^2}{2L} + C \qquad \Rightarrow V = -\frac{w_0x^2}{2L} + \frac{w_0L}{6}$$

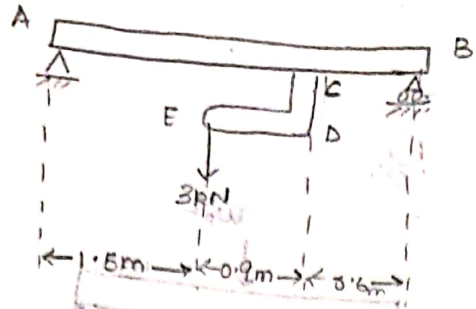
$$\Sigma M_0 = \int_0^L \frac{w_0x}{L}dx\,(x) = Wa$$

$$a = \frac{1}{W} \times \frac{1}{3}w_0L^2 = \frac{2L}{3}$$

$$M = -\frac{w_0x^3}{6L} + \frac{w_0Lx}{6}$$

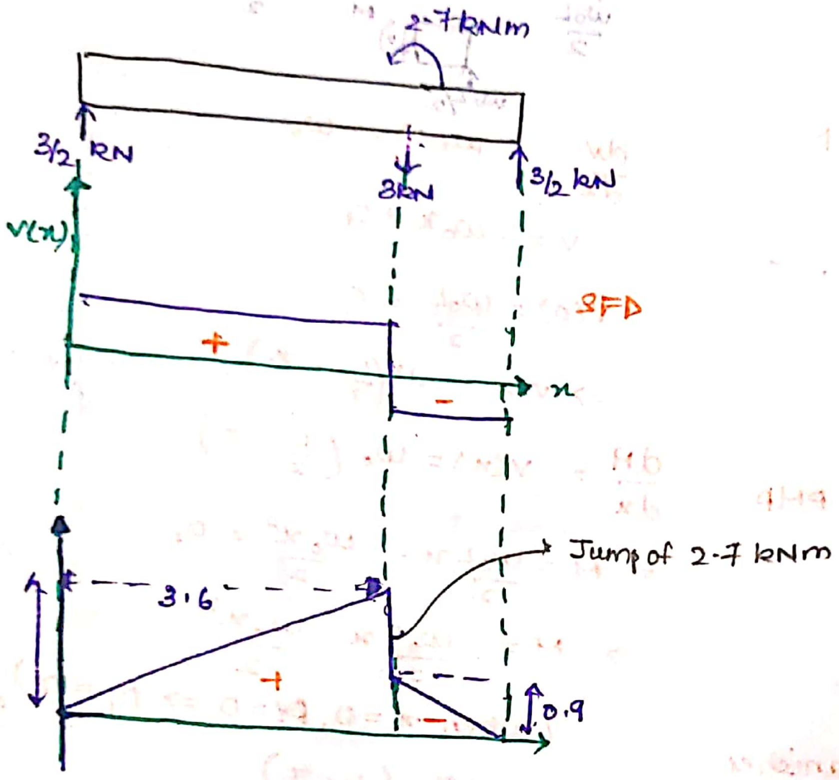
Vardhan

c)



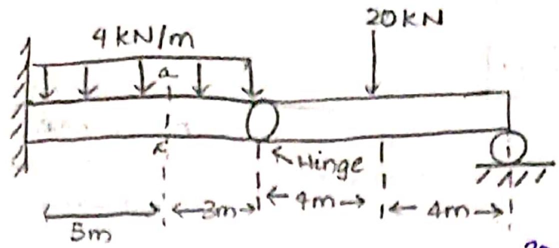
I can't transfer 3kN to CM of AB

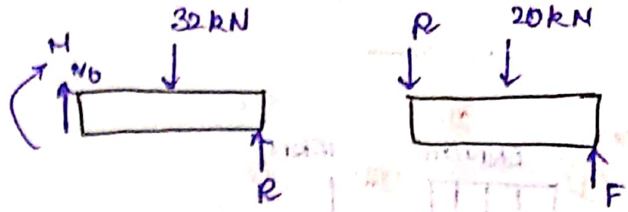
I can only transfer to C

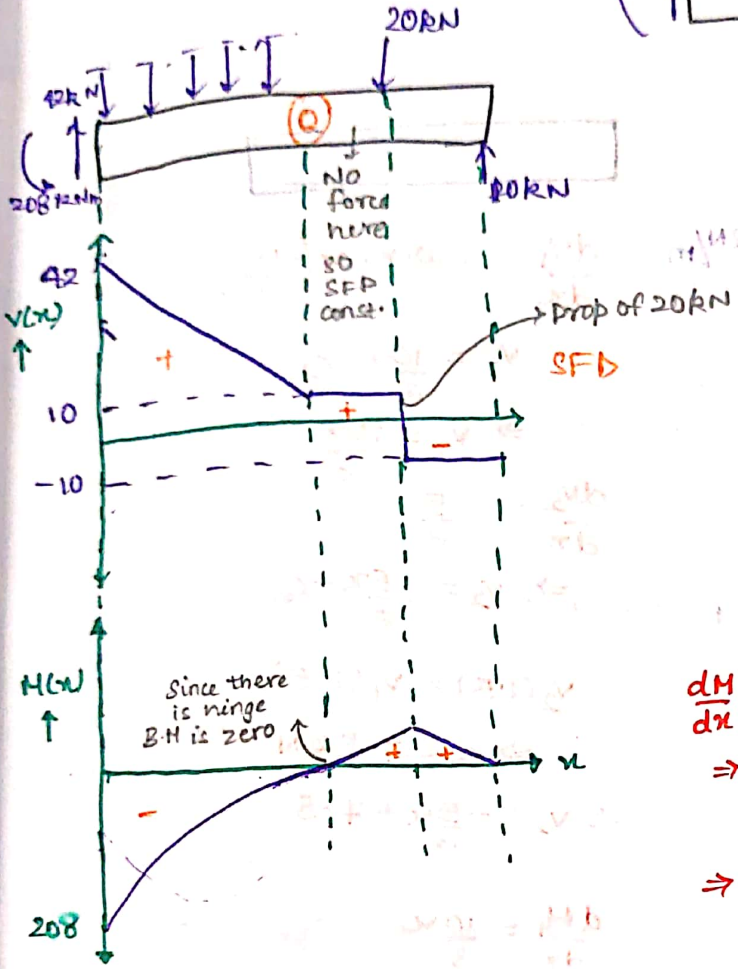


Vardhan

d)







$R = -10\,kN$

$F = 10\,kN$

$V_0 = 42\,kN$

$M_0 = -208\,kNm$

$$\frac{dV}{dx} = -w(x) = -4\ kN/m$$

$$V(x) = -4x + C_1$$

$$V(0) = 42\ kN = C_1$$

$$\Rightarrow V_1(x) = -4x + 42$$

$$\frac{dM}{dx} = -4x + 42$$

$$\Rightarrow M(x) = -2x^2 + 4x + D_1$$

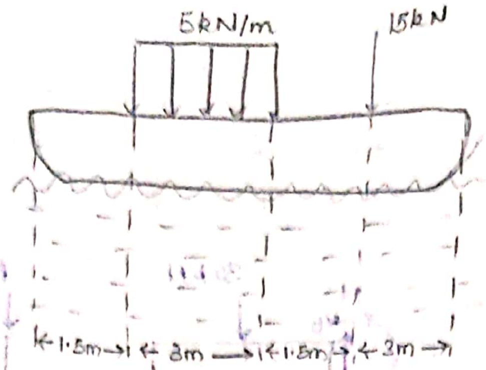
At $x = 0$, $M = -208$

$$\Rightarrow M(x) = -2x^2 + 4x - 208$$

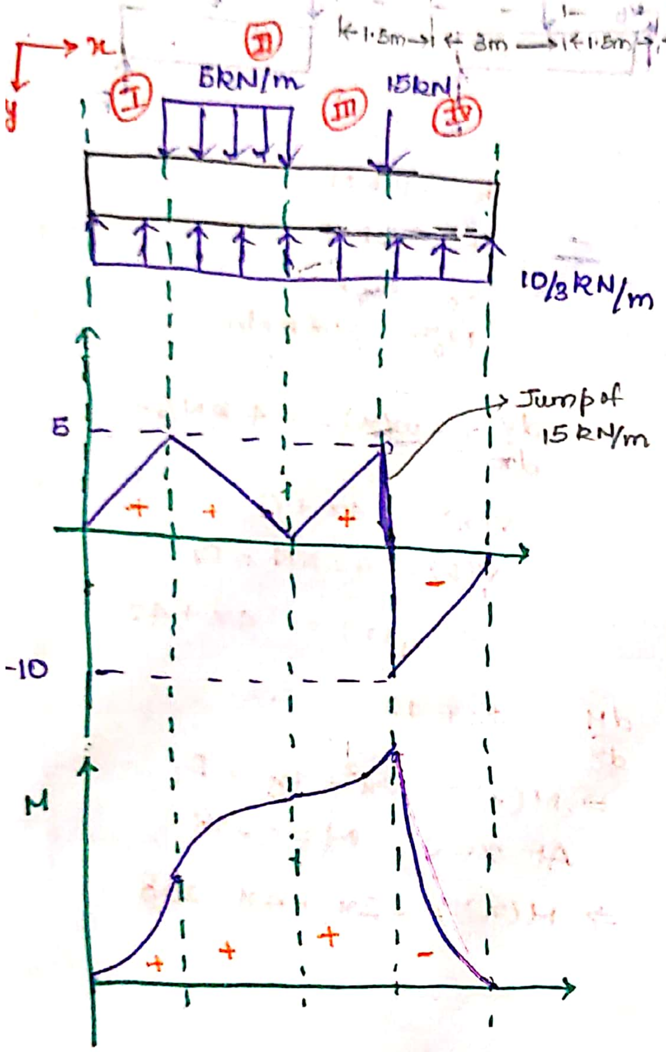
Varsha

c)

Here F_B is balancing the force



* Shear force can take a jump only when there is conc force



$$\frac{dV_1}{dx} = -w(x) = -\left(-\frac{10}{3}\right)$$

$$V_1 = \frac{10x}{3} + C_1$$

$$\Rightarrow V_1 = \frac{10x}{3}$$

$$\frac{dV_2}{dx} = -\frac{5}{3}$$

$$\Rightarrow V_2 = -\frac{5x}{3} + C_2$$

$$V_2(1.5) = V_1(1.5)$$

$$\Rightarrow C_2 = 7.5 \text{ kN}$$

$$\therefore V_2 = -\frac{5}{3}x + 7.5$$

$$\frac{dM_1}{dx} = \frac{10x}{3}$$

$$\Rightarrow M_1 = \frac{5x^2}{3}$$

$$\frac{dM_2}{dx} = -\frac{5x}{3} + 7.5$$

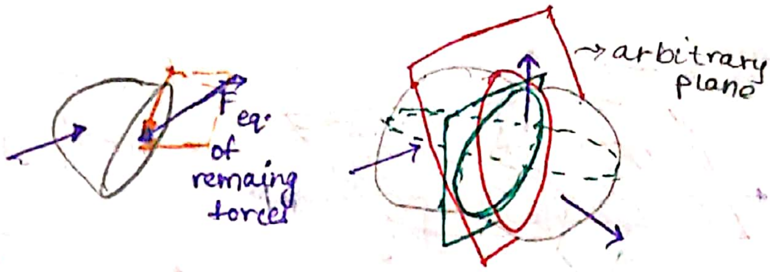
$$\Rightarrow M_2 = -\frac{5}{6}x^2 + 7.5x + D_2$$

$$M_2(1.5) = M_1(1.5)$$

$$\Rightarrow D_2 = -\frac{45}{8}$$

Vardhan

Stress

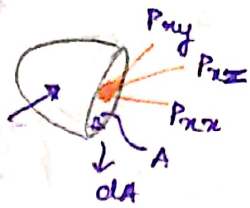


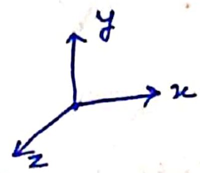
$$\bar{\sigma}_{xx} = \frac{P_{xx}}{A}$$

$$\bar{\sigma}_{xy} = \frac{P_{xy}}{A}$$

$$\bar{\sigma}_{xz} = \frac{P_{xz}}{A}$$

Avg. ~~Inst.~~ stress





Avg stress is Total Force / Area

x is always normal to the surface

$$\sigma_{xx} = \frac{dP_{xx}}{dA}$$

$$\sigma_{xy} = \frac{dP_{xy}}{dA}$$

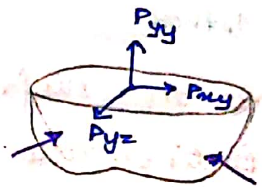
$$\sigma_{xz} = \frac{dP_{xz}}{dA}$$

Inst. Stress

P_{xx} → direction of force
↓
normal to the surface

The cut could be made in any direction

The stress in each direction will be σ_{yx}, σ_{yy}, σ_{yz}



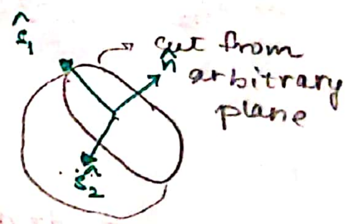
∴ So in total there are 9 components

i.e $\sigma_{xx}, \sigma_{xy}, \sigma_{xz}, \sigma_{yx}, \sigma_{yy}, \sigma_{yz}, \sigma_{zx}, \sigma_{zy}, \sigma_{zz}$

State of stress at a point

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

↑
Tensor Quantity
(Ratio of two vectors)



$\vec{t} = [\sigma]\hat{n}$ traction vector

$\sigma_{nn} = \hat{n} \cdot \vec{t}$ → Normal stress

$$\left.\begin{aligned} \sigma_{n1} &= \hat{s}_1 \cdot \vec{t} \\ \sigma_{n2} &= \hat{s}_2 \cdot \vec{t} \end{aligned}\right\} \rightarrow \text{Shear stress}$$

(Produced by component of force in the plane of area)

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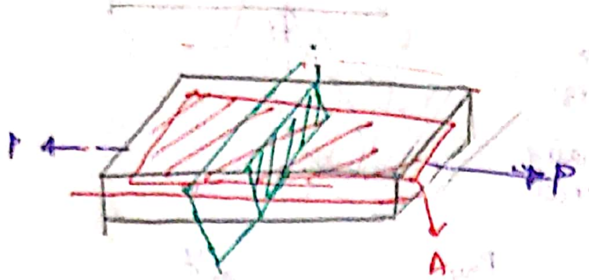
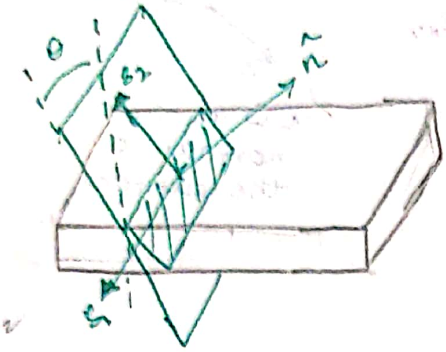


$$[\sigma] = \begin{bmatrix} P/A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

* stresses will be produced due to unbalanced forces



$$\hat{n} = \begin{Bmatrix} c\theta \\ s\theta \\ 0 \end{Bmatrix}$$

$$\vec{t} = [\sigma]\hat{n} = \begin{Bmatrix} \frac{P}{A} c\theta \\ 0 \\ 0 \end{Bmatrix}$$

$$\sigma_{nn} = \hat{n}\cdot\vec{t} = \frac{P}{A} c^2\theta$$

$$\hat{s}_1 = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \quad \hat{s}_2 = \begin{Bmatrix} -s\theta \\ c\theta \\ 0 \end{Bmatrix}$$

$$\sigma_{nn} = \hat{n}\cdot\vec{t} = \frac{P}{A} c^2\theta$$

$$\sigma_{n1} = \hat{s}_1\cdot\vec{t} = 0$$

$$\sigma_{n2} = \hat{s}_2\cdot\vec{t} = -\frac{P}{A} c\theta\, s\theta$$

Alternate Method

Normal force on surface $P_{nn} = P\cos\theta$

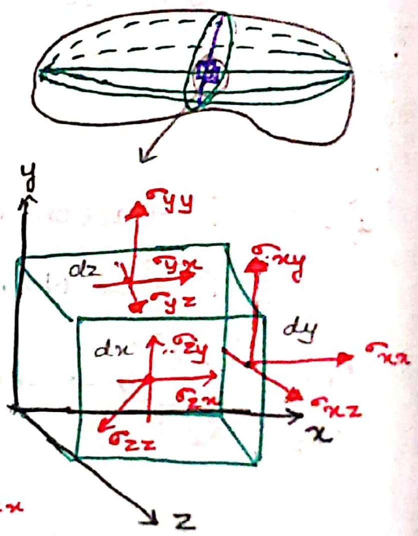
Area of inclined cute $(A') = \dfrac{A}{c\theta}$

$$\sigma_{nn} = \frac{P}{A} c^2\theta \quad , \quad \sigma_{n_1} = 0$$

σ_{n2} is the projection of force along s_2

$$P_{n2} = -P s\theta$$

$$\Rightarrow \sigma_{n2} = -\frac{P s\theta}{A} c\theta$$



$\Sigma F_x = 0$

$$\Rightarrow \sigma_{xx}\,dy\,dz + \sigma_{yx}\,dz\,dx - \sigma_{xx}\,dy\,dz + \sigma_{zx}\,dx\,dy - \sigma_{yx}\,dz\,dx - \sigma_{zx}\,dx\,dy = 0$$

$\Sigma M_x = 0$

$$-\sigma_{zy}\,dx\,dy\,dz + \sigma_{yz}\,dx\,dz\,dy = 0$$

$$\Rightarrow \sigma_{yz} = \sigma_{zy} \quad , \quad \sigma_{xy} = \sigma_{yx} , \quad \sigma_{xz} = \sigma_{zx}$$

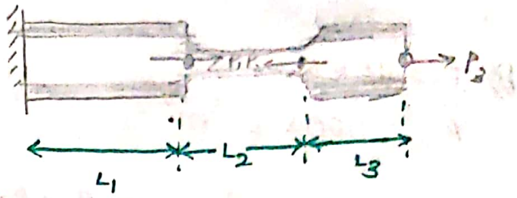
$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

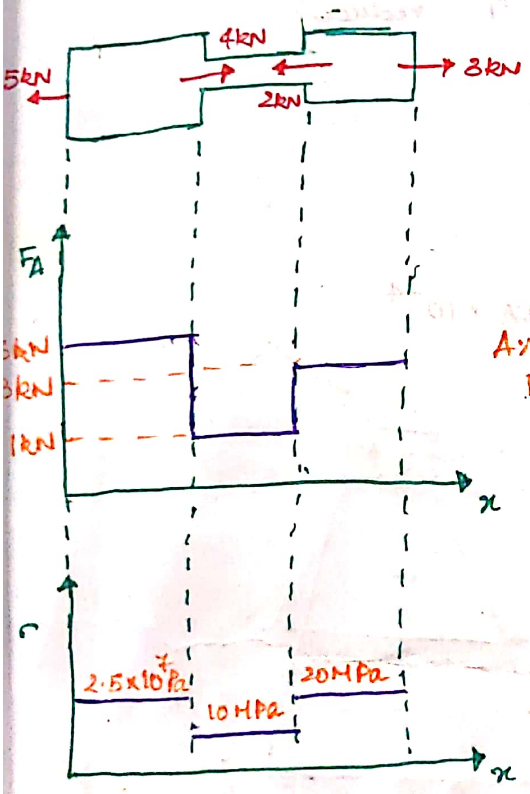
$\Rightarrow [\sigma]$ is a symmetric Tensor

$$[\sigma]^T = [\sigma]$$

$\Rightarrow$ So there are only six independent stresses

8) A bar is shown is subjected to forces $P_1 = 4kN$, $P_2 = 2kN$, $P_3 = 3kN$. Plot the axial force and axial stress along the length of the bar





Axial Force Diagram

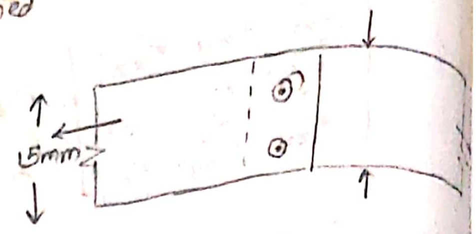
$$\sigma_I = \frac{5000\ N}{200 \times 10^{-6}}$$

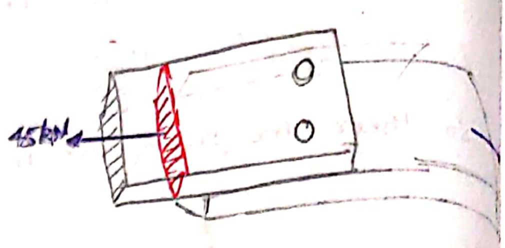
$$\sigma_{II} = \frac{1000\ N}{100 \times 10^{-6}}$$

$$\sigma_{III} = \frac{3000\ N}{150 \times 10^{-6}}$$

Vardhan

2) Two 10mm thick steel plates are fastened together by two 20mm bolts in single shear. If the joint transmits a tensile force of 45kN, determine a) avg. normal stress b) avg. normal stress in critical section c) avg. shear stress in bolts

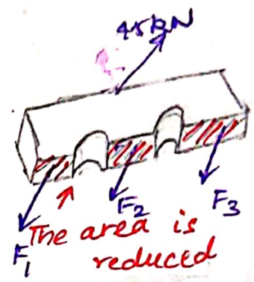




a) $F = 45kN$

$A = 1500 \times 10^{-6} m^2$

$\bar{\sigma} = \frac{45000}{1500 \times 10^{-6}} = 30 MPa$



b) $F = 45kN$

$A_c = 1100 \times 10^{-6} m^2$

$\bar{\sigma}_c = \frac{45000}{1100 \times 10^{-6}} = 40.9 MPa$

Normal Stress

c) The bolts are taking 45kN together

$F = 45kN$

$A = 2\left(\frac{\pi d^2}{4}\right) = \frac{2\pi (0.02)^2}{4}$

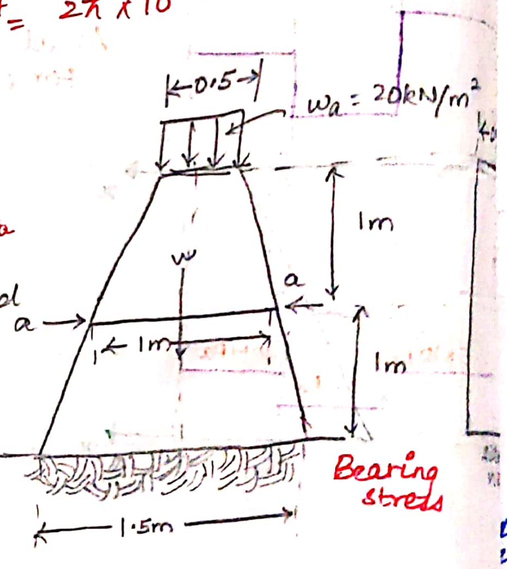
$= \frac{\pi}{2} 4 \times 10^{-4} = 2\pi \times 10^{-4}$

$\tau = \frac{45 \times 10^3}{2\pi \times 10^{-4}} = \frac{7.7 \times 10^7}{2\pi}$

shear stress

$= 71.619 MPa$

3) The concrete pier shown is loaded at the top with a uniformly distributed load of $20 kN/m^2$. Determine the state of stress at a level 1m above base. Concrete has weight density of $25 kN/m^3$



a) External load

$F_e = 20 \times 10^3 \times (0.25)$

$= 5 \times 10^3 N$

Self weight

$V = \frac{1.5}{2} \times 1 m^3 = 0.75 m^3 \times 0.5 = 0.375 m^3$
$\times 0.5$

$F_s = 25 \times 10^3 \times (0.375) = 9375 N$

$\sigma = \frac{F_e + F_s}{A} = \frac{14375}{0.5} = 28750 N/m^2$

$= 28.75 kPa$

Vardhan

Scanned with CamScanner

$$\sigma_{xx}(x,y,z) = \lim_{\Delta A \to 0} \frac{\Delta P_{xx}}{\Delta A} \rightarrow \text{Normal stress}$$

$$\left.\begin{aligned} \sigma_{xy}(x,y,z) &= \lim_{\Delta A \to 0} \frac{\Delta P_{xy}}{\Delta A} \\ \sigma_{xz}(x,y,z) &= \lim_{\Delta A \to 0} \frac{\Delta P_{xz}}{\Delta A} \end{aligned}\right\} \text{shear stress}$$

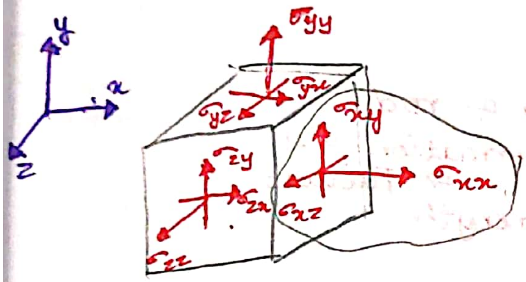
$$\bar{\sigma}_{xx} = \frac{P_{xx}}{A} \rightarrow \text{Normal stress}$$

$$\left.\begin{aligned} \bar{\sigma}_{xy} &= \frac{P_{xy}}{A} \\ \bar{\sigma}_{xz} &= \frac{P_{xz}}{A} \end{aligned}\right\} \text{shear stress}$$

Tensile normal stress : +ve
compressive normal stress : -ve

State of stress at a point

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \quad \text{Stress tensor} \qquad \hat{n} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$



$$\vec{t} = [\sigma]\hat{n} = \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yx} \\ \sigma_{zx} \end{Bmatrix} \quad \sigma_{yx} \rightarrow \sigma_{xy}, \quad \sigma_{zx} \rightarrow \sigma_{xz}$$

$$\vec{t} \cdot \hat{n} = \sigma_{xx}$$

$$\vec{t} \cdot \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} = \sigma_{xy}$$

$$\vec{t} \cdot \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \sigma_{xz}$$

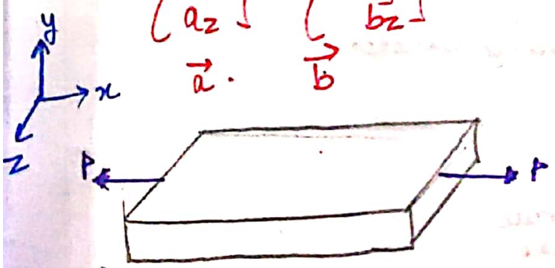
$$\vec{a} = a_x i + a_y j + a_z k$$

$$\vec{b} = b_x i + b_y j + b_z k$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

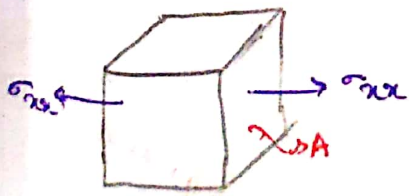
Vectors can be represented as matrices, and the dot prdt. is

$$\underset{\vec{a}}{\begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix}} \cdot \underset{\vec{b}}{\begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix}} = \underset{a^T b}{[a_x \ a_y \ a_z] \begin{Bmatrix} b_x \\ b_y \\ b_z \end{Bmatrix}}$$



$$[\sigma] = \begin{pmatrix} P/A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

"Uniaxial State of Stress"



$$\sigma_{xx} = P/A$$

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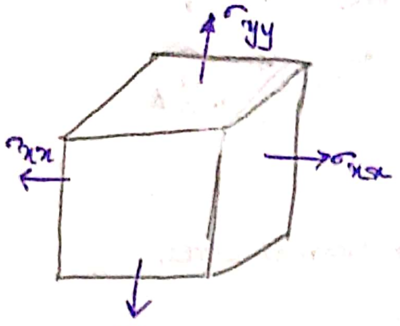
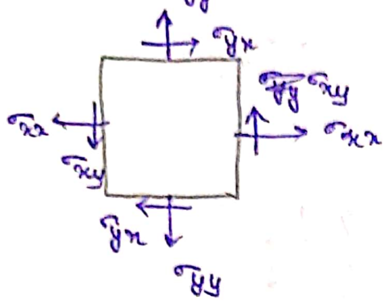
Biaxial State of Stress



$$[\sigma] = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Planar State of Stress



$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Strength based Design

Failure → Breaking of rod

Premanent Deformation also ⇒ Failure

$$\sigma_u = \frac{\text{Force at breakage}}{\text{Area}}$$ ultimate strength

$$\sigma_y = \frac{\text{Force at yielding}}{\text{Area}}$$ Yeild strength (Beyond this particular stress, it will never come back)

Steel : $\sigma_u \sim 450\,MPa$

$\sigma_y \sim 250\,MPa$

a) Fatigue (cyclic loading and de-loading)
b) Harsh environment (Too cold or Too warm)
c) Defects (in the material)

→ So as to take care of these, we use something called "Factor of safety"

$$\sigma_{allowable} = \frac{\sigma_{max}}{FS}$$

σ_{max} → max. stress the material can take

FS → Factor of safety

$\sigma = P/A \leq \sigma_{allowable}$

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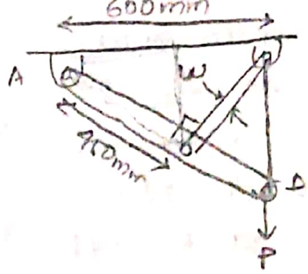
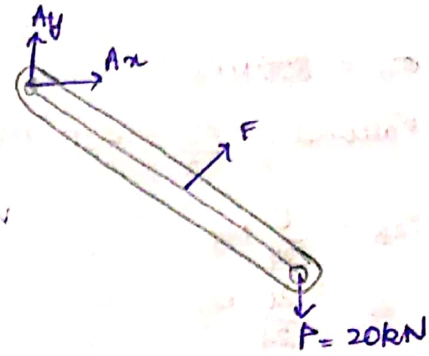
3) The link BC is 6mm thick and is made of steel with 450 MPa ultimate strength in tension. What should be its width w if the structure shown is being designed to support a 20kN load P with a FS = 3



$$\sigma_{allowable} = \frac{450}{3} = 150\ MPa$$



$\Sigma M_a = 0$

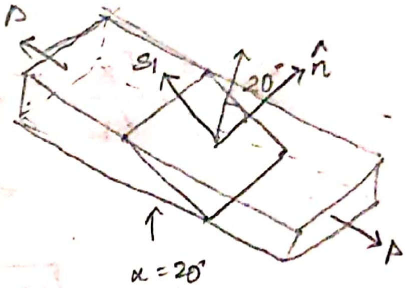
$\Rightarrow (0.48)F - (0.6)20\ kN = 0$

$\Rightarrow F = \frac{200}{8} = 25kN$

$$\sigma = \frac{F}{A} = \frac{25\times 10^3}{0.006W} \leq 150\ MPa$$

$\Rightarrow \quad W \geq 28mm$

3) Two wooden beams of cross-section 10x20mm are joined by a glued lap joint $\alpha = 20°$, as shown. Assuming that the shear strength of the glued joint is 10 MPa. Determine the max. axial force P that can be applied



$$[\sigma] = \begin{bmatrix} P/A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \hat{n} = \begin{Bmatrix} s20° \\ c20° \\ 0 \end{Bmatrix} \qquad \hat{s}_1 = \begin{Bmatrix} -c20° \\ s20° \\ 0 \end{Bmatrix}$$

$$\sigma_{n1} = \hat{s}_1 \cdot [\sigma]\hat{n} = -\frac{P}{A} s20°\ c20°$$ shear stress is in downward direction

$\Rightarrow |\sigma_{n1}| \leq 10\ MPa$

$\Rightarrow \frac{P}{A} s20°\ c20° \leq 10\ MPa$

$\Rightarrow P \leq \frac{10A}{s20°\ c20°}$

$\therefore\ P \leq 6.22\ kN$

Vardhan

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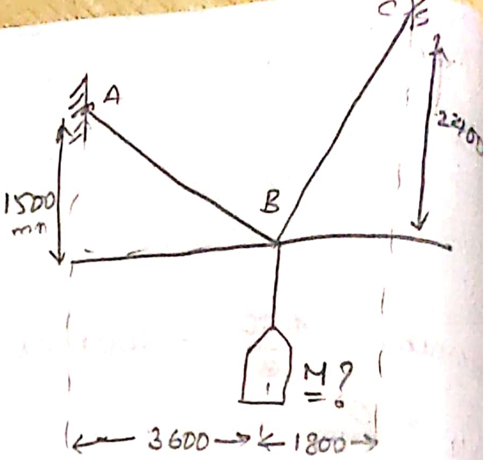
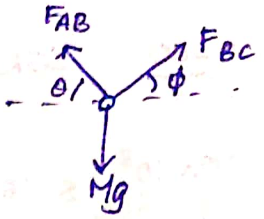
8) A high Tensile strength steel rods of area of $A_{AB} = 200\ mm^2$, $A_{BC} = 400\ mm^2$ support a mass M; If the ultimate strength is 800 MPa, the FS is 2. Determine maxm: M that can be supported.



$\sigma_u = 800\ MPa$

$\sigma_{allowed} = \frac{\sigma_u}{2} = 400\ MPa$



$F_{AB} = \frac{13}{21} Mg$

$F_{BC} = \frac{20}{21} Mg$

$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{13 Mg}{4200 \times 10^{-6}} \leq 400 \times 10^{6}\ Pa$

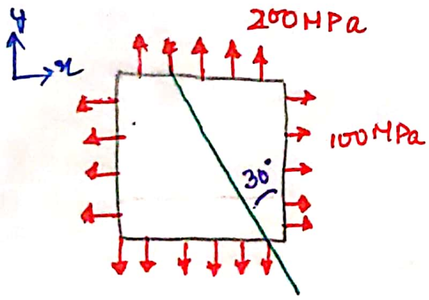
$\Rightarrow M \leq 13173 kg \quad (g = 9.8 m/s^2)$

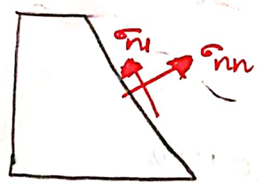
$\sigma_{BC} = \frac{F_{BC}}{A_{BC}} = \frac{20 Mg}{8400 \times 10^{-6}} \leq 400 \times 10^{6}$

$\Rightarrow M \leq 17125 kg$

$\therefore \boxed{M \leq 13173 kg}$

Biaxial state of stress





$$[\sigma] = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\hat{n} = \begin{Bmatrix} c30° \\ s30° \\ 0 \end{Bmatrix}$$

$$\hat{s}_1 = \begin{Bmatrix} -s30° \\ c30 \\ 0 \end{Bmatrix}$$

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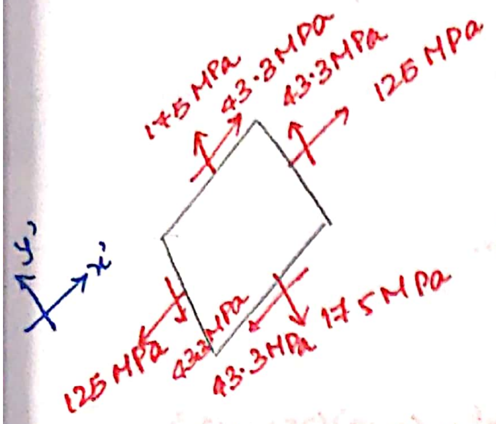
$$\sigma_{nn}^{II} = 100\, s^2 30^\circ + 200\, c^2 30^\circ$$
$$= 175 \text{ MPa}$$

$$\hat{n} = \begin{Bmatrix} -s30^\circ \\ c30^\circ \\ 0 \end{Bmatrix}$$

$$\sigma_{nl}^{II} = -43.3 \text{ MPa}$$

$$\hat{s}_1 = \begin{Bmatrix} -c30^\circ \\ -s30^\circ \\ 0 \end{Bmatrix}$$



$$[\sigma]' = \begin{pmatrix} 125 & 43.3 & 0 \\ 43.3 & 175 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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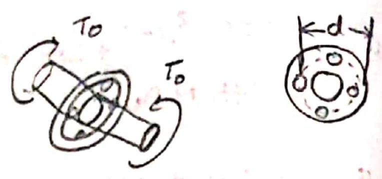
b) Two flanged shafts are connected by four 25 mm bolts as shown. Calculate the max permissible torque T_0 that can be transmitted if the allowable shear stress in the bolts is 80 MPa. The diameter of bolt circle $d = 180$ mm



A) F_s : shearing force per bolt

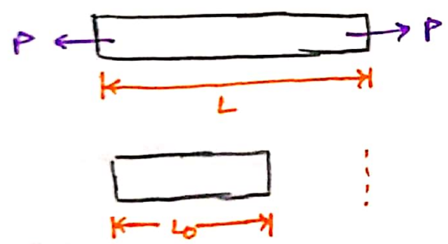
$$4 F_s \frac{d}{2} = T_0 \rightarrow F_s = \frac{T_0}{2d}$$

$$\tau = \frac{F_s}{\frac{\pi d_B^2}{4}} < 80 \times 10^6 \text{Pa} \quad \Big| \; d_B = 25 \times 10^{-3} m$$

$$\Rightarrow \frac{T_0}{\frac{2\pi d \, d_B^2}{4}} < 80 \times 10^6 \text{Pa} \Rightarrow T_0 < \frac{80 \times 10^6 \pi (0.18)(25 \times 10^{-3})^2}{2}$$

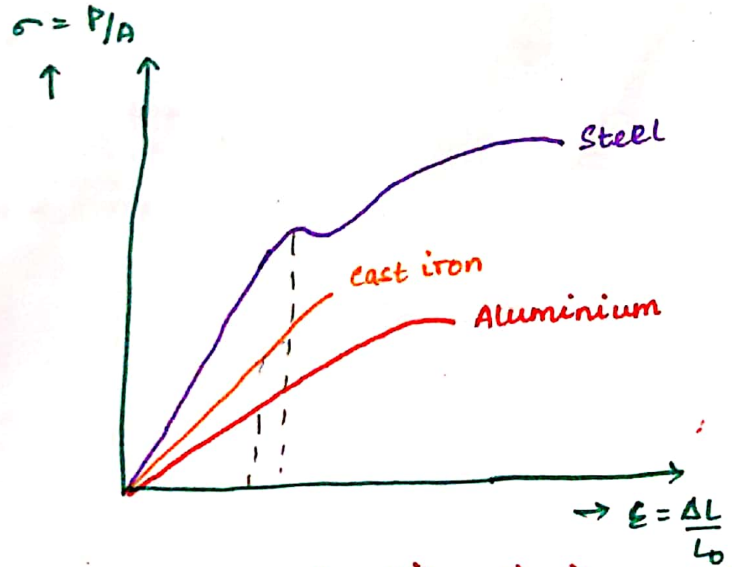
$$T_0 < 14.3 \text{ kN}$$

Strain



$(L - L_0) = \Delta L$

$$\boxed{\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}}$$



Material	E
Steel	200 GPa
Aluminium	70 GPa
C.I	90 GPa

Within proportionality limit $\sigma \propto \varepsilon$ and σ vs ε is a st. line and this prop. limit varies from material to material

Young's Modulus E is $\sigma = E\varepsilon$

Vardhan

$\Delta L = \varepsilon L_0 \Rightarrow L = L_0(1+\varepsilon)$

Note :-

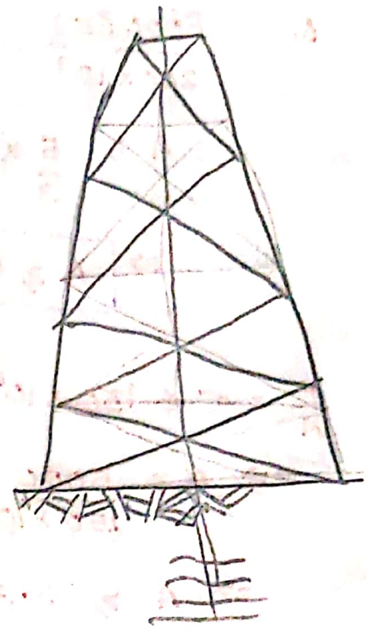
→ Strain has no dimensions

→ Micro strain is $\mu m/m$ i.e $10\ \mu m/m = 10\times10^{-6}\ m/m$

10 Micro strain $= 10\times10^{-6}$ strain

→ Strain is 0.2% ⇒ Strain $= 0.2\times10^{-2}$

8) In an oil drill rig, as shown, a very long uniform steel pipe got struck in hard clay. The engineer on the job ordered the pipe be subjected to a large tensile load. As a result, the pipe came up elastically 60 cm. At the same time the pipe elongation was measured to be 0.8 mm in a 200 mm gauge. Determine the depth at which the pipe got stuck.



$$\varepsilon = \frac{L-L_0}{L_0} = \frac{0.8}{200} = \frac{\tilde{L}-\tilde{L}_0}{\tilde{L}_0}$$

$$= \frac{60\ cm}{\tilde{L}_0}$$

$$\Rightarrow \tilde{L}_0 = \frac{60\times200}{0.8} = \frac{\cancel{6}^{3}\times\cancel{2}\times10^4}{\cancel{8}\,\cancel{4}_2}$$

$$= 150\ m$$

Stifness of a uniform bar

$$\frac{P}{A} = E\frac{\Delta L}{L_0}$$

$$K = \frac{P}{\Delta L} = \frac{EA}{L_0}$$

$\boxed{k = \frac{EA}{L_0}}$ ... for an uniform bar.

Vardhan

8) A steel rod of length 10m is to transmit a force of 5kN without stretching more than 3mm, or exceeding an allowable stress of 150 MPa. What should be the diameter of the rod if Young's modulus $E = 200$ GPa

a) $\Delta L \leq 3mm$

$$\frac{P}{A} = E\frac{\Delta L}{L_0} \Rightarrow \Delta L = \frac{PL_0}{EA}$$

$$\Delta L = \frac{5000(10)}{200 \times 10^9} \leq 3 \times 10^{-3}$$

$$A \geq \frac{5}{3} \times \frac{10^4}{2} \times \frac{10^3}{10^{11}}$$

$$\Rightarrow \boxed{A \geq 8.33 \times 10^{-5} m^2}$$

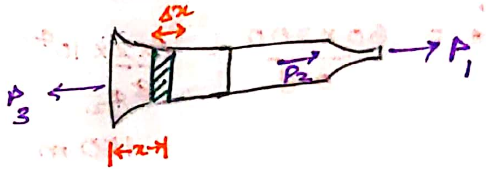
b) $\frac{P}{A} = \sigma \leq 150 \times 10^6$ Pa

$$\Rightarrow A \geq \frac{5000}{150 \times 10^6} = 33.34 \times 10^{-6} m^2$$

$$\Rightarrow A \geq 3.34 \times 10^{-5} m^2$$

By Taylor series,

$$u(x+\Delta x) = u(x) + \frac{du}{dx}\Delta x + \cdots$$



$$\Delta L = \Delta x + \left(u(x) + \frac{du}{dx}\Delta x + \cdots\right) - u(x) - \Delta x$$

$$\varepsilon = \lim_{\Delta x \to 0} \frac{\Delta L}{\Delta x}$$

$$\Rightarrow \varepsilon = \lim_{\Delta x \to 0} \frac{\Delta L}{\Delta x}$$

$$\Rightarrow \varepsilon = \lim_{\Delta x \to 0} \frac{du}{dx} + \cdots$$

$u \rightarrow$ disp. at any pt. x

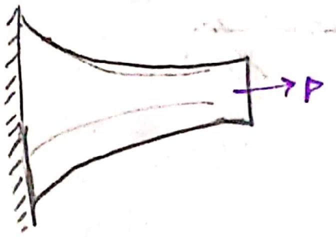
$$\varepsilon = \frac{du}{dx}$$

$$\sigma = E\varepsilon$$

$$P/A = E\frac{du}{dx} \Rightarrow \boxed{\frac{du}{dx} = \frac{P}{EA}}$$

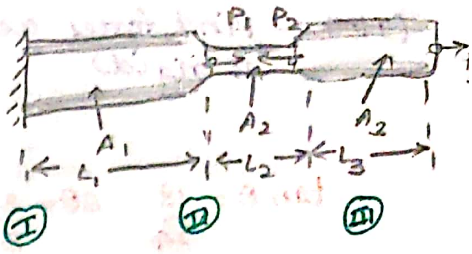
$$\int du = \int \frac{P}{EA}\,dx$$

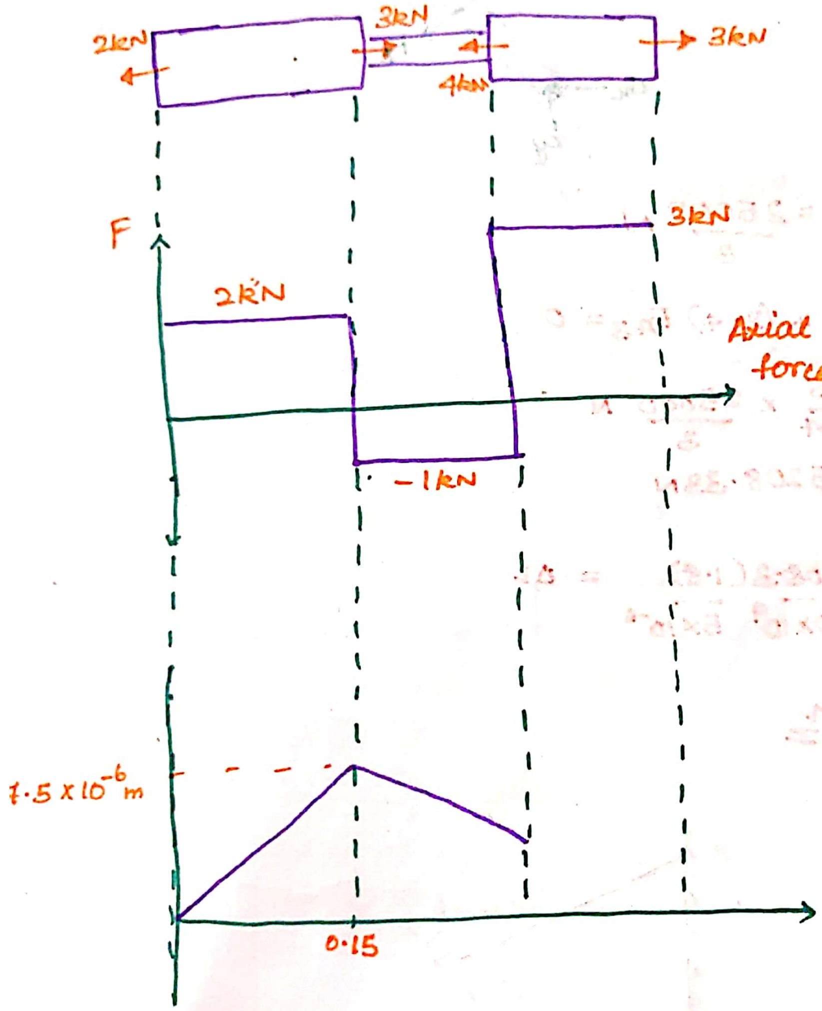
Vardhan



$$\int_0^u du = \int_0^x \frac{P}{EA(x)}\,dx$$

A bar, as shown is subjected to $P_1 = 3kN$, $P_2 = 4kN$, $P_3 = 3kN$. Plot the axial deformation along the length of bar.
Take $A_1 = 200\ mm^2$, $A_2 = 100mm^2$, $A_3 = 150mm^2$
$L_1 = 150\ mm$, $L_2 = L_3 = 100mm$ and $E = 200GPa$





$$\frac{du_1}{dx} = \frac{P_1}{EA_1}$$

$$\Rightarrow u_1(x) = \int_0^x \frac{2\times10^3}{E\times 2\times10^{-4}}\,dx$$

$$u_1(x) = \frac{10^7 x}{200\times10^9}$$

$$u_1(x) = (5\times10^{-5} m)\,x$$

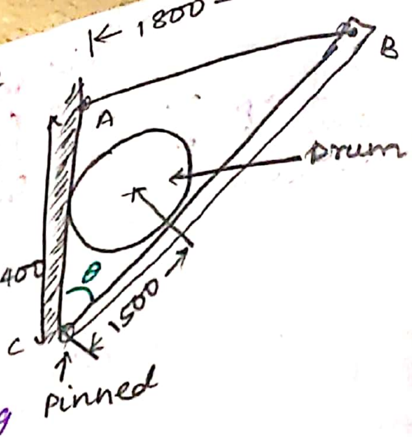
$$\frac{du_2}{dx} = \frac{-1000}{E(100\times10^{-6})}$$

$$u_2(x) - u_2(0.15) = -\int_{0.15}^{x} \frac{1000}{E(100\times10^{-6})}\,dx$$

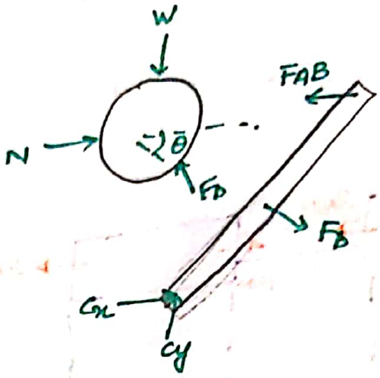
$$\Rightarrow u_2(x) = u_1(0.15) - 5\times10^{-5}(x-0.15)$$

Vardhan

(8) A 1000 mm diameter frictionless drum of mass 500 kg is supported by a rigid platform BC and an elastic rod AB as shown. Considering all joints to be pin connected, determine the elongation of the rod AB. Take $A_{AB} = 5\,mm^2$, $E = 200\,GPa$



(Note :- Find forces assuming everything is rigid)



$\tan\theta = \frac{18}{24}$ $\quad s\theta = \frac{3}{5}$

$c\theta = \frac{4}{5}$

$\Sigma F_y = 0 \Rightarrow F_D\, s\theta = W$

$F_D = \frac{5000}{s\theta} \Rightarrow F_D = \frac{25000}{3}\,N$

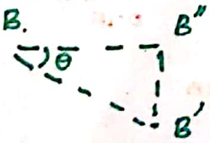
$\Sigma M_c = 0 \Rightarrow -1.5 F_D + (2.4) F_{AB} = 0$

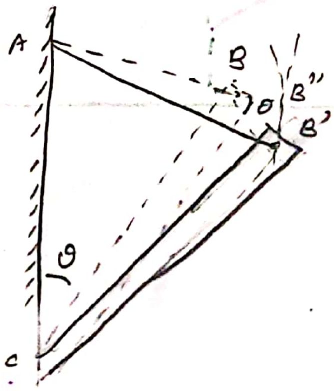
$\Rightarrow F_{AB} = \frac{15}{24} \times \frac{25000}{3}\,N$

$= 5208.33\,N$

$\Delta L = \frac{PL_0}{EA} \Rightarrow \frac{5208.3(1.8)}{200\times10^9 \;\; 5\times10^{-6}} = \Delta L$

$\Rightarrow \Delta L = 9.375\,mm$





$BB'\, c\theta = BB''$

$\Rightarrow BB' = \frac{BB''}{c\theta}$

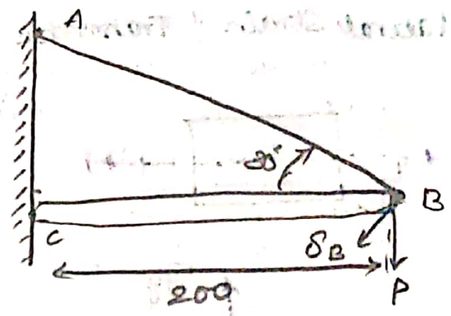
$\Rightarrow BB' = 11.72\,mm$

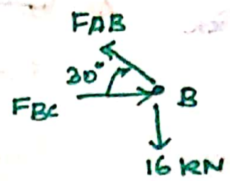
$\Delta\theta(BC) = 11.72\,mm$

$\Delta\theta = \frac{11.72}{3000} = 3.9\times10^{-3}$ radians

Vardhan

3) A jib crane consists of a steel member BC of area of cross-section 320 mm², supported by the steel cable AB of area of cross section 300 mm². Determine vertical deflection of crane at point B when a force P = 16 kN is applied. Take E = 200 GPa





$$\Sigma F_y = 0 \Rightarrow F_{AB} \, s30° = 16 \text{ kN}$$

$$\Rightarrow F_{AB} = 32 \text{ kN}$$

$$\Sigma F_x = 0 \Rightarrow F_{BC} = F_{AB} \, c30°$$

$$= 16\sqrt{3} \text{ kN}$$

$$\Delta L_{AB} = \frac{F_{AB}(AB)}{E A_{AB}} = \frac{32 \times 10^3 (4/\sqrt{3})}{200 \times 10^9} \sim 0.123 \text{ m}$$

$$\Delta L_{BC} = -\frac{F_{BC}(BC)}{E A_{BC}} = \frac{-16\sqrt{3} \times 10^3 (2)}{2 \times 10^{11} (320 \times 10^{-6})}$$

$$\vec{\delta_B} = B_x i + B_y j$$

$$\delta_B \cdot \hat{AB} = \Delta L_{AB} \qquad \vec{\delta_B} \cdot \hat{CB} = \Delta L_{BC}$$

$$\hat{AB} = c30° i - s30 j \qquad \hat{CB} = i$$

$$B_x \, c30° - B_y \, s30° = 1.23 \text{ mm}$$

Vardhan

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Lateral Strain / Transverse strains :-

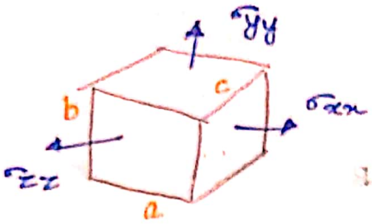
Concrete $\nu \approx 0.1$
Rubber $\nu \approx 0.5$
$\nu \approx 0.25 - 0.35$

$\varepsilon_l \propto -\varepsilon_a$

$\varepsilon_l = -\nu\varepsilon_a$

ν: Poisson's Ratio

$$\boxed{\nu = -\frac{\varepsilon_l}{\varepsilon_a}} \rightarrow \frac{\text{lateral strain}}{\text{Axial strain}}$$



Acc to Hooke's law

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\sigma_{yy} - \frac{\nu}{E}\sigma_{zz}$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}\sigma_{xx} - \frac{\nu}{E}\sigma_{zz}$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}\sigma_{xx} - \frac{\nu}{E}\sigma_{yy}$$

This doesn't depend on shear

only normal shears are corrected like this

$\Delta a = a\,\varepsilon_{xx}$

$a + \Delta a = a'$

$\Rightarrow a' = a(1+\varepsilon_{xx})$

$b' = b(1+\varepsilon_{yy})$

$c' = c(1+\varepsilon_{zz})$

$V = abc$

$$V' = a(1+\varepsilon_{xx})\,b(1+\varepsilon_{yy})\,c(1+\varepsilon_{zz})$$
$$= abc\,(1+\varepsilon_{xx})(1+\varepsilon_{yy})(1+\varepsilon_{zz})$$
$$= abc\,(1+\varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz})$$

$$\frac{\Delta V}{V} = \frac{V'-V}{V}$$

$$\Rightarrow \boxed{e = \varepsilon_{xx}+\varepsilon_{yy}+\varepsilon_{zz}} \text{ dilation}$$

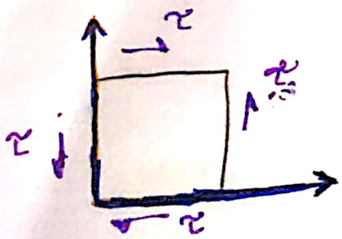
Hydro-static state of stress $\rightarrow$ $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p \Rightarrow \boxed{\varepsilon_{xx} = \frac{p}{E}(2\nu-1)}$

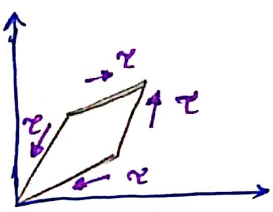
$$\boxed{e = \frac{3p(2\nu-1)}{E}}$$

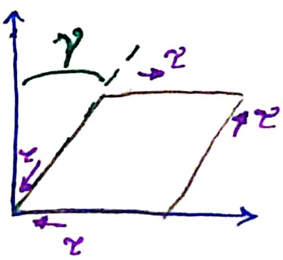
Bulk Modulus

$$K = -\frac{p}{(\Delta V/V)} = -\frac{p}{\left(\frac{3p}{E}(2\nu-1)\right)} = \frac{E}{3(1-2\nu)}$$

$$\boxed{K = \frac{E}{3(1-2\nu)}}$$







$$\gamma = \frac{\tau}{G}$$

G: shear modulus/ modulus of rigidity

$$G = \frac{E}{2(1+\nu)} \rightarrow \text{Experimental}$$

Scanned with CamScanner

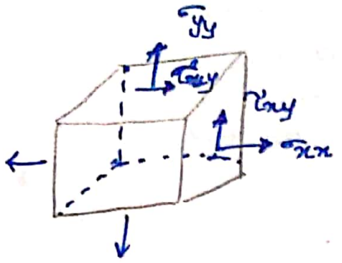
$$\gamma_{xy} = \frac{\tau_{xy}}{G} \qquad \varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy} + \sigma_{zz})$$

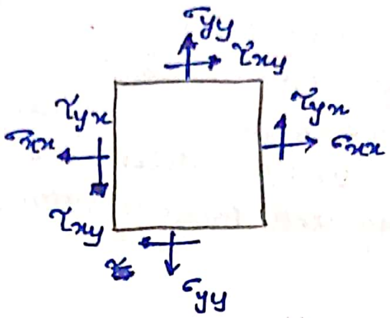
$$\gamma_{yz} = \frac{\tau_{yz}}{G} \qquad \varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz})$$

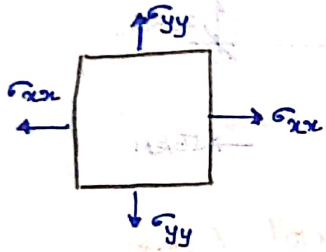
$$\gamma_{zx} = \frac{\tau_{zx}}{G} \qquad \varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$

Generalised Hooke's Law

Plane stress







Biaxial stress

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \quad \text{Plane stress}$$

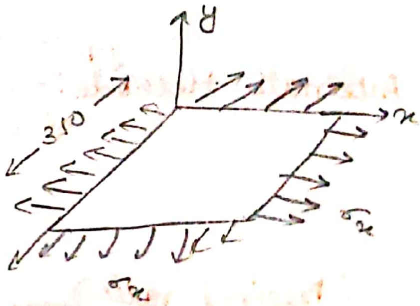
$$\sigma = \begin{pmatrix} \sigma_{xx} & 0 \\ 0 & \sigma_{yy} \end{pmatrix} \quad \text{Biaxial stress}$$

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu \frac{\sigma_{yy}}{E}$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \nu \frac{\sigma_{xx}}{E} \qquad \varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

8) A circle of D = 200mm is scribed on an unstressed aluminium plate of thickness 18mm. Forces acting in the plane of plate cause uniform stresses $\sigma_x = 85$ MPa, $\sigma_z = 150$ MPa. If $E = 70$ GPa and $\nu = 0.33$. Determine changes in AB, CD and thickness of plane and volume of plate



$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\sigma_{zz} = \frac{85 \times 10^6}{70 \times 10^9} - \frac{0.33}{70 \times 10^9}(150 \times 10^6)$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}\sigma_{xx} = \frac{150 \times 10^6}{70 \times 10^9} - \frac{0.33}{70 \times 10^9}(85 \times 10^6)$$

$$= 1.74 \times 10^{-3}$$

$\Delta AB = AB\ \varepsilon_{xx} = 10^{-4}$ m $\qquad t = 18$ mm

$\Delta CD = CD\ \varepsilon_{zz} = 3.48 \times 10^{-4}$ m $\qquad \Delta t = t\ \varepsilon_{yy}$

Vardhan

$$\varepsilon_{yy} = \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz})$$

$$= 1.108 \times 10^{-3}$$

$\Delta t = 1.98 \times 10^{-2}$ mm

$\Delta V = eV$

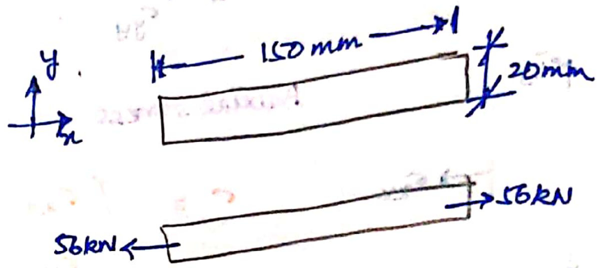
$\Delta V = (1.132 \times 10^{-3})$ (Vintial)

$\Delta V = (1.132 \times 10^{-3})(0.35 \times 0.35 \times 0.018)$

$\Delta V = (1.132 \times 10^{-3})$

$\Rightarrow \Delta V = 2.4961 \times 10^{-6}$ ← change is volume

Q) A 20mm diameter rod made of an experimental plastic is subjected to a tensile force of magnitude P = 56 kN. Knowing that an elongation of 14mm and a decrease in dia of 0.85 mm are observed in a 150mm length, determine modulus of elasticity, modulus of rigidity and Poisson's ratio for the material



$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = -\frac{\nu}{E}\sigma_{yy}$$

$$\frac{14}{150} = \frac{1}{E} \times \frac{56 \times 10^3}{\pi \times 0.02^2/4}$$

$$\Rightarrow E = 1.9 \times 10^9 \text{ Pa}$$

$$\nu = -\frac{E}{\sigma_{xx}}\varepsilon_{yy} = -\frac{1.9 \times 10^9}{\frac{56 \times 10^3}{\pi \times 0.02^2/4}}\left(\frac{-0.85}{20}\right)$$

$$= 0.455$$

Alternate Methode :-

$$\varepsilon_{yy} = -\frac{0.85}{20}$$

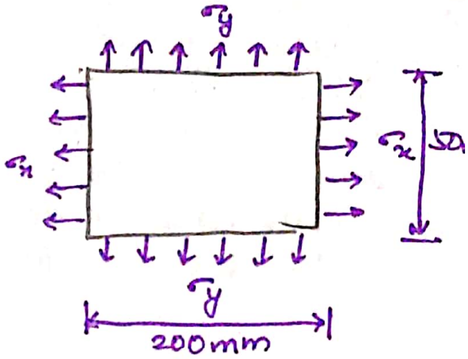
$$-\nu\,\varepsilon_{xx} = \varepsilon_{yy}$$

$$\varepsilon_{yy} = -\nu \times \frac{14}{150}$$

$$\nu = \frac{\varepsilon_{yy}}{\varepsilon_{xx}} = 0.455$$

(Possion's ratio)$_{max}$ is 0.5 and is always +ve

8) A 10mm thick steel plate, shown in fig4 is subjected to uniform edge stresses σ_x and σ_y, if the thickness reduces by 2×10^{-4} mm, estimate the change in the area of the plate. Take $E = 200$ GPa and $\nu = 0.25$



$$\varepsilon_{xx} = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_y$$

$$\Rightarrow \varepsilon_{yy} = \frac{\sigma_y}{E} - \frac{\nu}{E}\sigma_x \qquad \Delta A = A(\varepsilon_{xx} + \varepsilon_{yy})$$

$$\varepsilon_{zz} = -\frac{\nu}{E}(\sigma_x + \sigma_y) \qquad \varepsilon_{zz} = -2\times10^{-5}$$

$$\varepsilon_{xx} + \varepsilon_{yy} = \frac{1}{E}(\sigma_x + \sigma_y) - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$= \frac{1-\nu}{E}(\sigma_x + \sigma_y)$$

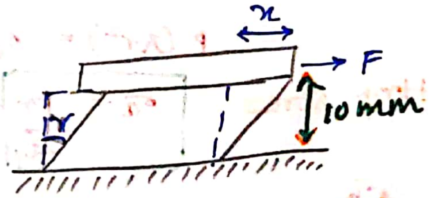
$$= \frac{1-\nu}{E}\left(-\frac{E\,\varepsilon_{zz}}{\nu}\right)$$

$$= \frac{1-0.25}{200\times10^9} \times \frac{200\times10^9 \times 2\times10^{-5}}{0.25}$$

$$= 6\times10^{-5}$$

$$\Delta A = (0.05\times0.2)$$

8) A shear mount consists of 10mm thick rubber block with a metal plate fixed on the top, as shown. If the shear modulus of rubber is 0.65 MPa, determine the req. area of block if the desired tangential stiffness $k = F/x = 100$ kN/m



$$G = 0.65\times10^6 \text{ Pa} \qquad \tau = F/A$$

$$\gamma = \frac{\tau}{G} = \frac{F}{AG}$$

$$\Rightarrow x = \gamma h = \frac{Fh}{AG}$$

$$\frac{F}{x} = \frac{AG}{h} = 100\times10^3 \text{ N/m} \Rightarrow A = 10^5 h/G$$

Vardhan

Q) Two blocks of rubber with modulus of rigidity $G = 12\,MPa$ are bounded to rigid supports and to a plate AB. Knowing that $c = 100\,mm$ and $P = 45\,kN$. Determine smallest allowable dimensions a and b of the blocks if shearing stress in the rubber isn't exceed 1.4 MPa and deflection of plate is 5mm

$$\tau = \frac{P/2}{cb} \leq 1.4 \times 10^6\ Pa$$

$$b \geq \frac{P \leftarrow \text{given}}{2c\,(1.4 \times 10^6)}$$

given

$$x = \gamma a \geq 5 \times 10^{-3}\,m$$

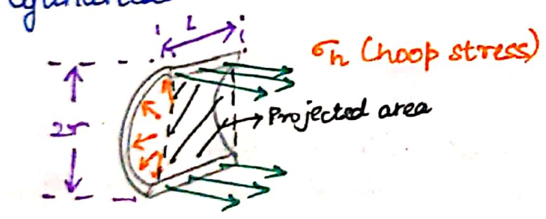
$$\gamma = \frac{\tau}{G}$$

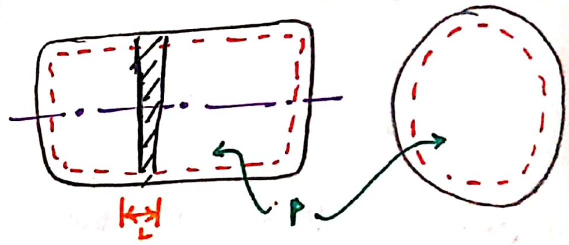
$$\frac{\tau a}{G} \geq 5 \times 10^{-3} \Rightarrow a \geq \frac{G}{\tau}(5 \times 10^{-3})$$

a is min ~~max~~ if both are equal.

Thin walled pressure Vessels

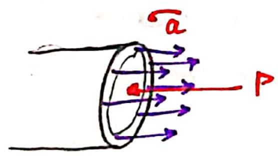
Cylindrical





$$P\,(L\,2r) = \sigma_h\,(2L\,t)$$

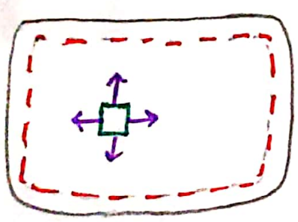
Pressure; Projected area; Thickness

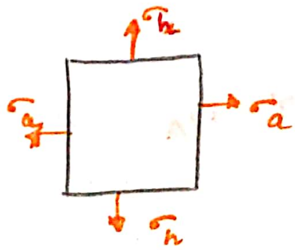


$$P(\pi r^2) = \sigma_a\,(2\pi r)\,t$$

$$\Rightarrow \boxed{\sigma_h = \frac{pr}{t}} \quad \text{Hoop stress}$$

$$\boxed{\sigma_a = \frac{Pr}{2t}} \quad \text{axial stress}$$

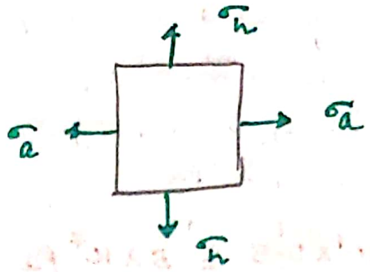




$$[\sigma] = \begin{bmatrix} \sigma_a & 0 \\ 0 & \sigma_h \end{bmatrix}$$

$$\varepsilon_a = \frac{\sigma_a}{E} - \frac{\nu}{E}\sigma_h$$

↑
axial strain
(changes the length of cyclinder)



$$\varepsilon_h = \frac{\sigma_h}{E} - \frac{\nu}{E}\sigma_a$$

↑
Hoope strain
(changes the radius i.e circumference of vessel)

$$\varepsilon_h = \frac{2\pi(r+\Delta r) - 2\pi r}{2\pi r}$$

$$= \frac{\Delta r}{r} = \frac{\Delta d}{d}$$

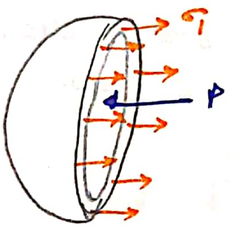
$$\frac{\Delta r}{r} = \frac{\sigma_h}{E} - \frac{\nu}{E}\sigma_a = \frac{1}{E}\frac{pr}{t} - \frac{\nu}{E}\frac{pr}{2t}$$

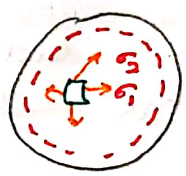
$$\Rightarrow \boxed{\frac{\Delta r}{r} = \frac{pr}{Et}\left(1 - \frac{\nu}{2}\right)}$$

Spherical

$$P(\pi r^2) = \sigma_1 (2\pi r t)$$

$$\boxed{\sigma_1 = \frac{pr}{2t} = \sigma_2}$$





↳ strain in direction 1

$$\varepsilon_1 = \frac{\sigma_1}{E} - \frac{\nu}{E}\sigma_2$$

$$\varepsilon_1 = \varepsilon_2 = \varepsilon$$

$$\boxed{\varepsilon = \frac{\sigma}{E}(1-\nu)}$$

$$\boxed{\frac{\Delta r}{r} = \frac{pr}{2Et}(1-\nu)}$$

The change in radius is more is case of spherical when compared to cylindrical

Higher the r, faster the tyre will reach the critical pressure.

Vardhan

7) A cylindrical shell 3m long and 1m diameter and 15mm thickness. Calculate the circumferential stress and longitudinal stresses included and change in dia, if subjected to an internal pressure of 1.5MPa. Take $E = 200\,GPa$ and $\nu = 0.3$

$$\sigma_h = \frac{pr}{t} = \frac{1.5\times10^6 \times 0.5}{15\times10^{-3}} = 5\times10^7\ Pa$$

$$\sigma_a = 2.5\times10^7$$

$$\varepsilon_h = \frac{\sigma_h}{E} - \frac{\nu}{E}\sigma_a = \frac{2.5\times10^7}{2\times10^9}(2-0.3)$$

$$= 1.7 \times \frac{2.5}{2} = 2.125\times10^{-4} = \frac{\Delta d}{d}$$

$$\Delta d = 2.125\times10^{-4}\ m$$

8) A cylindrical vessel is to store a gas at 2MPa. If the thickness of tank is 20mm, determine the maxm diameter of the tank if $\sigma_{allow} = 80\ MPa$. Also, determine change in diameter when the tank is filled. Take $E = 200\,GPa$ and $\nu = 0.3$

$$\sigma_h = \frac{pr}{t} \le 80\times10^6\ Pa$$

$$r \le \frac{80\times10\,(0.02)}{2\times10^6}\ m$$

$$\Rightarrow r \le 8\times10^{-1}\ m$$

$$\varepsilon_h = \frac{\sigma_h}{E} - \frac{\nu}{E}\sigma_a = \frac{\sigma_h}{E}\left(1-\frac{\nu}{2}\right)$$

$$\frac{\sigma_h}{E}\left(1-\frac{\nu}{2}\right) = \frac{\Delta d}{d}$$

Scanned with CamScanner

A steel spherical pressure vessel is being designed for a pressure of 6 MPa and an inside diameter of 600 mm. The yield stress of steel is 400 MPa. What is min. req. thickness t for a factor of safety against yeilding of 2.5?

$$\sigma_y = 400\,MPa \qquad \sigma_{allow} = \frac{\sigma_y}{FS} = \frac{400}{2.5}\,MPa$$

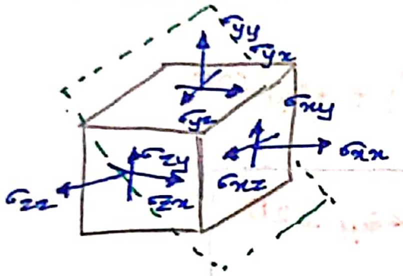
$$= 160\,MPa$$

$$\sigma = \frac{pr}{2t} \leq \sigma_{allow}$$

$$\Rightarrow t \geq \frac{pr}{2\sigma_{allow}} \qquad P = 6\times10^6\,Pa \quad r = 0.3$$

$$\Rightarrow t \geq 5.625\times10^{-3}\,m$$

Transformation of stress

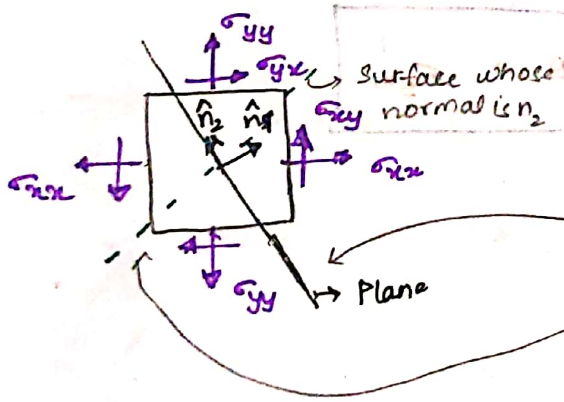


$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

$$\vec{t} = [\sigma]\hat{n}$$

$$\sigma_{nn} = \hat{n}\cdot\vec{t} \qquad \sigma_{n1} = \hat{s}_1\cdot\vec{t}$$

$$\sigma_{n2} = \hat{s}_2\cdot\vec{t}$$



$$[\sigma] = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

$$\vec{t_1} = [\sigma]\hat{n}_1$$

$$\vec{t_2} = [\sigma]\hat{n}_2$$

$$\underset{2\times2}{[\vec{t_1}\ \vec{t_2}]} = \underset{2\times2}{[\sigma]}\underset{2\times2}{(\hat{n}_1, \hat{n}_2)}$$

$$\sigma_{21} = \hat{n}_1^T[\sigma]\hat{n}_2$$

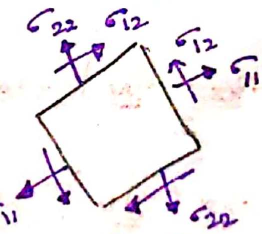
$$\sigma_{12} = \hat{n}_2^T[\sigma]\hat{n}_1$$

$$\hat{n}_1^T[\vec{t_1}\ \vec{t_2}] = [\sigma_{11}\ \ \sigma_{21}]$$

$$\hat{n}_2^T[\vec{t_1}\ \vec{t_2}] = [\sigma_{12}\ \ \sigma_{22}]$$

$$\underbrace{\begin{pmatrix} \hat{n}_1^T \\ \hat{n}_2^T \end{pmatrix}}_{[R]^T}[\sigma]\underbrace{(\hat{n}_1\ \hat{n}_2)}_{[R]} = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

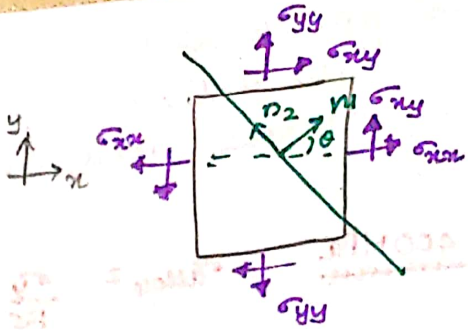
2x2 symmetric matrix



$$[\sigma]' = R^T\sigma R$$

Vardhan

$$[\sigma] = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$



$$\hat{n}_1 = \begin{pmatrix} c\theta \\ s\theta \end{pmatrix} \quad \hat{n}_2 = \begin{pmatrix} -s\theta \\ c\theta \end{pmatrix}$$

$$[\sigma]' = \begin{pmatrix} \hat{n}_1^T \\ \hat{n}_2^T \end{pmatrix} [\sigma] (\hat{n}_1, \hat{n}_2)$$

$$\begin{bmatrix} \sigma_{xx}' & \sigma_{xy}' \\ \sigma_{yx}' & \sigma_{yy}' \end{bmatrix} = \begin{bmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{bmatrix} \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{bmatrix}$$

$$\sigma_{xx}' = c\theta(\sigma_{xx} c\theta + \sigma_{xy} s\theta) + s\theta(\sigma_{yx} c\theta + \sigma_{yy} s\theta)$$

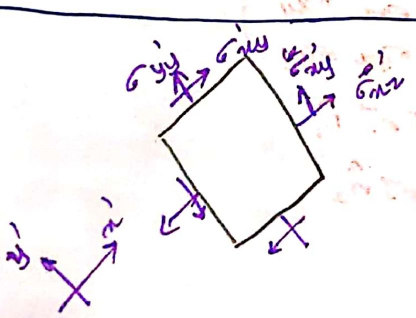
$$= \sigma_{xx} c^2\theta + \sigma_{yy} s^2\theta + 2\sigma_{xy} c\theta s\theta$$

$$= \sigma_{xx}\left(\frac{1+c2\theta}{2}\right) + \sigma_{yy}\left(\frac{1-c2\theta}{2}\right) + \sigma_{xy} s2\theta$$

$$\boxed{\sigma_{xx}' = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \frac{\sigma_{xx}-\sigma_{yy}}{2} c2\theta + \sigma_{xy} s2\theta}$$

$$\boxed{\sigma_{yy}' = \frac{\sigma_{xx}+\sigma_{yy}}{2} - \frac{\sigma_{xx}-\sigma_{yy}}{2} c2\theta - \sigma_{xy} s2\theta}$$

$$\boxed{\sigma_{xy}' = -\frac{\sigma_{xx}-\sigma_{yy}}{2} s2\theta + \sigma_{xy} c2\theta = \sigma_{yx}'}$$

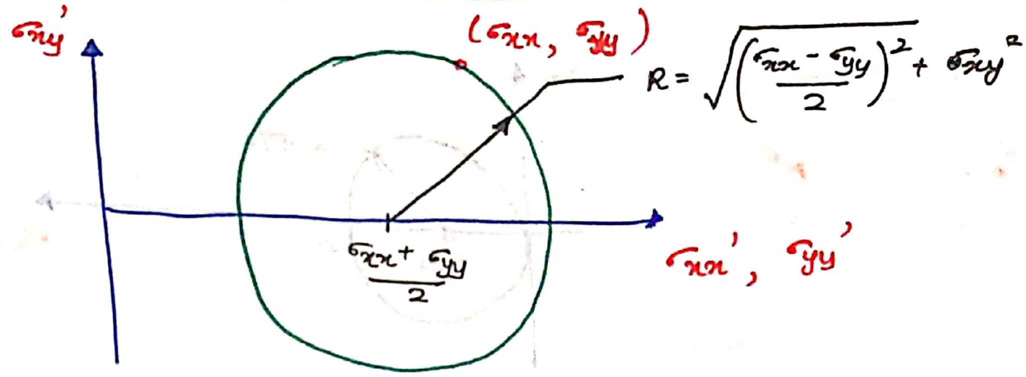


$$\sigma_{xx}' + \sigma_{yy}' = \sigma_{xx} + \sigma_{yy}$$

$$\det[\sigma'] = \det[\sigma]$$

$$\left(\sigma_{xx}' - \frac{\sigma_{xx}+\sigma_{yy}}{2}\right)^2 + (\sigma_{xy}')^2 = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2$$

$$\left(\sigma_{yy}' - \frac{\sigma_{xx}+\sigma_{yy}}{2}\right)^2 + (\sigma_{xy}')^2 = \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2$$



Center : $\left(\frac{\sigma_{xx}+\sigma_{yy}}{2}, 0\right)$

Mohr circle

Radius : $\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$

$$\frac{d\sigma_{xx}'}{d\theta} = 0 \Rightarrow -\frac{\sigma_{xx}-\sigma_{yy}}{2} \times 2\, s2\theta + 2\sigma_{xy}\, c2\theta = 0$$

$$\Rightarrow \boxed{t\, 2\theta^* = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}}}$$

θ^*, $\theta^* + \pi/2$ are soln.

$\frac{d^2\sigma_{xx}'}{d\theta^2} < 0$: maxm. $\qquad \frac{d^2\sigma_{xx}'}{d\theta^2} > 0$: minm.

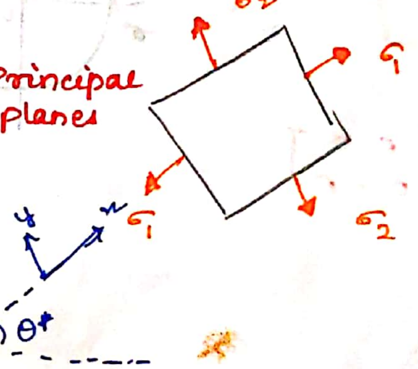
Principal

$$\sigma_1 = \frac{\sigma_{xx}+\sigma_{yy}}{2} + \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \cancel{+} - \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

τ_{max} : $\sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$

Principal planes



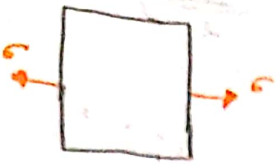
Extremum shear stress

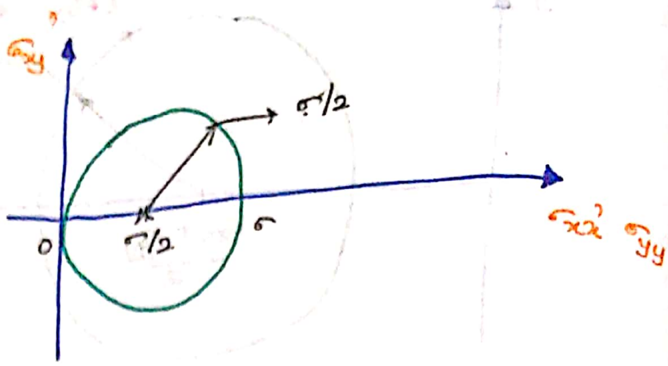
$$\frac{d\sigma_{xy}'}{d\theta} = 0$$

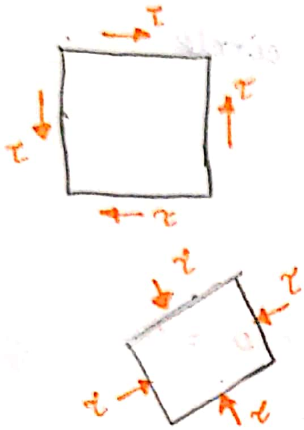
$$-\frac{\sigma_{xx}-\sigma_{yy}}{2} 2c2\theta - 2\sigma_{xy} \cancel{2} s2\theta = 0$$

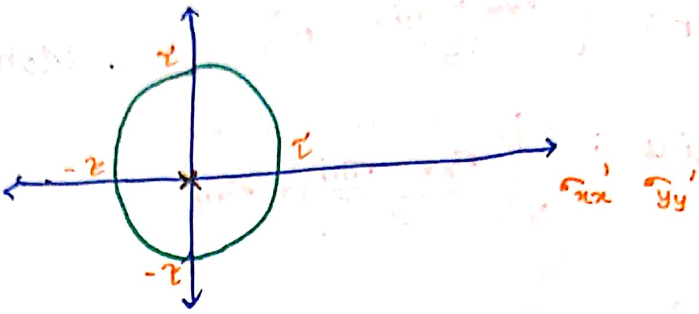
$$\Rightarrow t2\theta = -\frac{\sigma_{xx}-\sigma_{yy}}{2\sigma_{xy}}$$ → Maxm. in plane shear stress

Vardhan

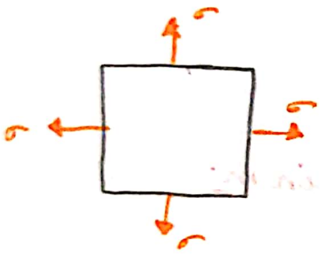


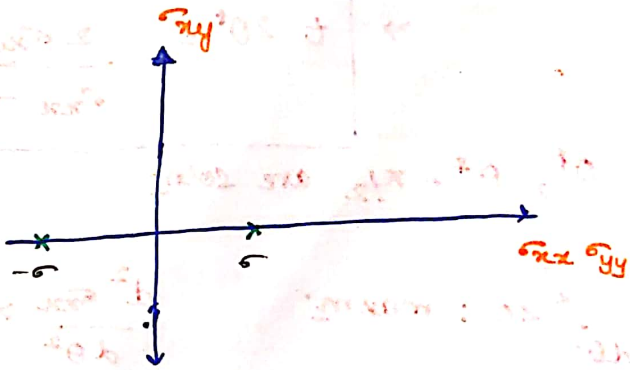




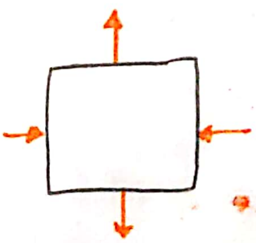


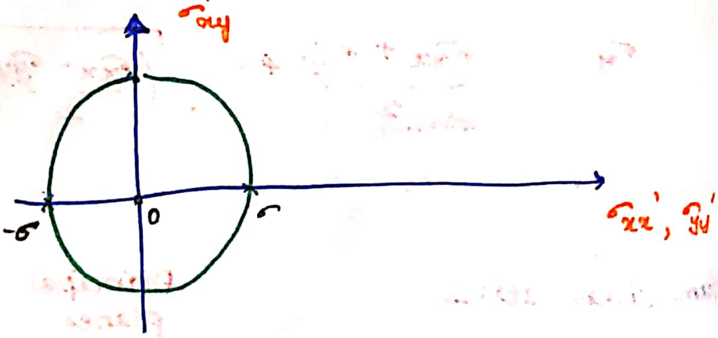
Hydrostatic state of stress





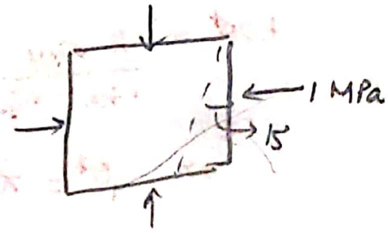
$$[\sigma] = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$$





$$[\sigma] = \begin{bmatrix} -\sigma & 0 \\ 0 & -\sigma \end{bmatrix}$$

Q) The fibers of a wooden member forms an angle of 15° with vertical as shown in figure. For the state of stress shown, determine a) in-plane shearing stress b) the normal stress



$\theta = -15°$

$\sigma_{xx}' = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2} c2\theta + \sigma_{xy} s2\theta$

$\sigma_{xx} = -4 \text{ MPa}$ $\quad \sigma_{yy} = -1.6 \text{ MPa}$

$\sigma_{xy}' = -\frac{\sigma_{xx} - \sigma_{yy}}{2} s2\theta + \sigma_{xy} c2\theta$

$[\sigma] = \begin{bmatrix} -4 & 0 \\ 0 & -1.6 \end{bmatrix}$ $\quad \hat{n} = \begin{Bmatrix} c15° \\ -s15° \end{Bmatrix}$ $\quad \vec{t} = [\sigma].\hat{n}$

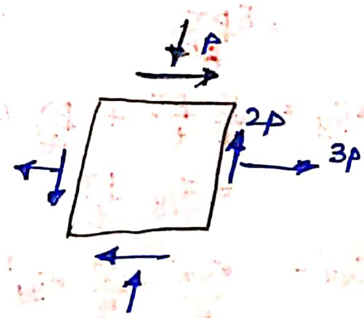
$= \begin{Bmatrix} -4c15° \\ 1.6 s15° \end{Bmatrix}$

Maxm in-plane shear stress,

$t2\theta^{+} = -\frac{\sigma_{xx} - \sigma_{yy}}{2\sigma_{xy}}$

$t(-30°) = -\frac{(-4+1.6)}{}$

Q) The state of stress at a pt. in a body is shown. If the allowable tensile stress is 150 MPa and the allowable shear stress is 60 MPa. Determine max. p.



$\sigma_{xx} = 3p$

$\sigma_{yy} = -p$ $\quad \sigma_{xy} = 2p$

$c: \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}, 0 \right) \equiv (p, 0)$

$r: \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{4p^2 + 4p^2} = 2\sqrt{2}p$

$\sigma_1 = p(1+2\sqrt{2}) \le 150 \times 10^6 \Rightarrow p \le \frac{150 \times 10^6}{1+2\sqrt{2}} \text{ Pa}$

$p \le 39.18 \times 10^6 \text{ Pa}$

Vardhan

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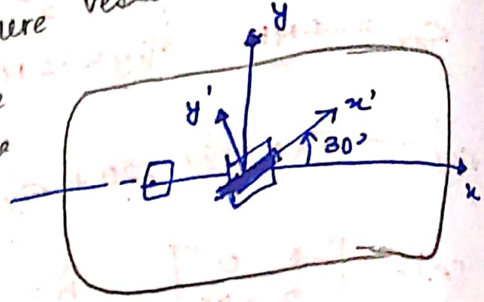
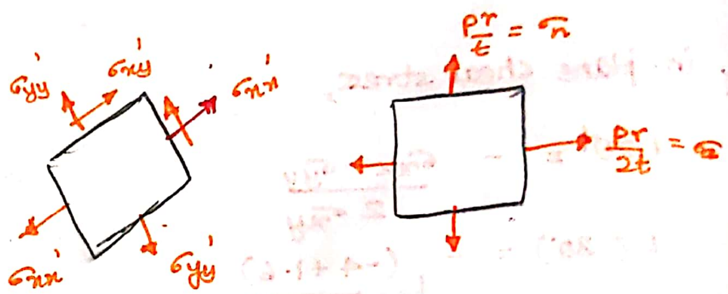
$$\tan 2\theta^* = \frac{2\sigma_{xy}}{\sigma_{xx}-\sigma_{yy}} = \frac{4}{4} = 1$$

$$\Rightarrow \quad \theta^* = \pi/8$$

$$\tau_{max} = 2\sqrt{2}\,p \le$$

8) A strain guage on a cylindrical pressure vessel measures the normal strain at an angle 30° with the axis of vessel. At a certain pressure, if the measured strain $\varepsilon_{x'x'} = 15\ \mu m/m$, determine the pressure and hoop stress. Take $r = 0.5m$, wall thickness $= 0.01\ m$. Youngs modulus as 200GPa and $\nu = 0.3$





$$\sigma_{xx} = \frac{pr}{2t} \qquad \sigma_{yy} = \frac{pr}{t} \qquad \sigma_{xy} = 0$$

$$\sigma_{xx'} = \frac{3}{4}\frac{pr}{t} \mp \frac{pr}{4t}\cos 60°$$

$$= \frac{5pr}{8t}$$

$$\sigma_{yy'} = \frac{3}{4}\frac{pr}{t} + \frac{pr}{4t}\cos 60°$$

$$= \frac{7}{8}\frac{pr}{t}$$

$$\varepsilon_{xx'} = \frac{\sigma_{xx'}}{E} - \frac{\nu}{E}\sigma_{yy'} = \frac{1}{E}\frac{pr}{t}\left[\frac{5}{8} - \nu\frac{7}{8}\right]$$

$$= \frac{pr}{8Et}(5-7\nu) = 15\times10^{-6}$$

$$\frac{p(0.5)(5-2.1)}{8\times200\times10^{9}\times10^{-2}} = 15\times10^{\cancel{-6}\,3}$$

$$= 1.65\times10^{5}\ Pa$$

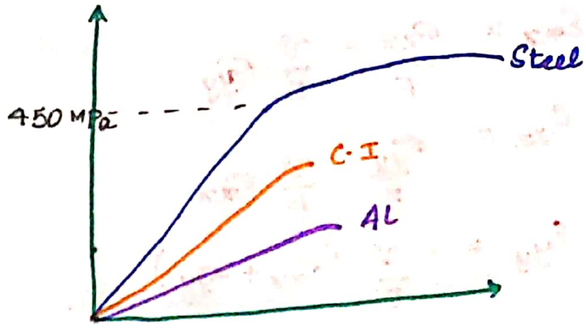
Vardh

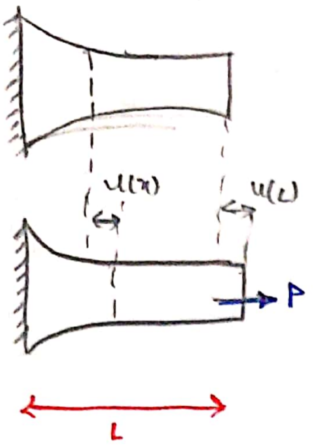
Steel $E \sim 200\,GPa$

$Al \sim 70\,GPa$

$\nu \sim 0.3$ $\nu_{rubber} = 0.5$

$$G = \frac{E}{2(1+\nu)}$$

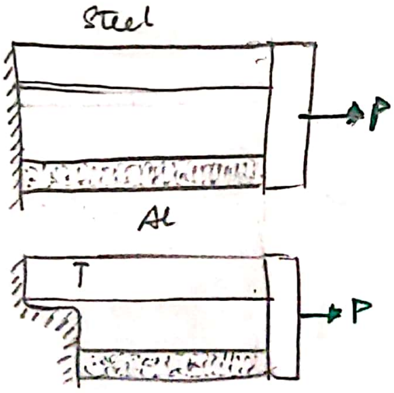




$$\frac{du}{dx} = \frac{P}{AE}$$

$$\Rightarrow \int du = \int_0^L \frac{P}{AE}\,dx$$

$$\boxed{k = \frac{P}{u} = \frac{1}{\int_0^L \frac{1}{AE}\,dx}}$$

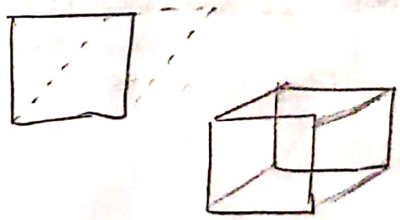


} Strain is same

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy} + \sigma_{zz})$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz})$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$



$$\gamma_{xy} = \frac{\sigma_{xy}}{G} \quad \gamma_{xz} = \frac{\sigma_{xz}}{G} \quad \gamma_{yz} = \frac{\sigma_{yz}}{G}$$

$$\frac{\Delta V}{V} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$$

Vardhan

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Plane stress:

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}\sigma_{yy}$$

$$\varepsilon_{zz} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}\sigma_{xx}$$

$$\varepsilon_{yy} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$

$$\gamma_{xy} = \frac{\sigma_{xy}}{G}$$

Plane strain: ($\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}$)

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{zz} + \sigma_{yy})$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz})$$

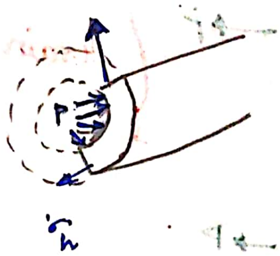
$$0 = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy})$$

$$\gamma_{xy} = \frac{\sigma_{xy}}{G}$$

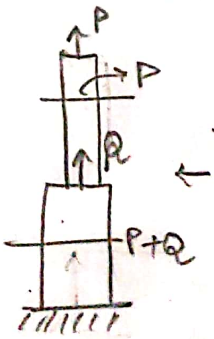
Pressure vessel (Plane stress)

$$\sigma_h = \frac{pr}{t} \qquad \sigma_a = \frac{pr}{2t}$$

$$\sigma_r = p$$



$$t << r \Rightarrow \frac{r}{t} >> 1$$

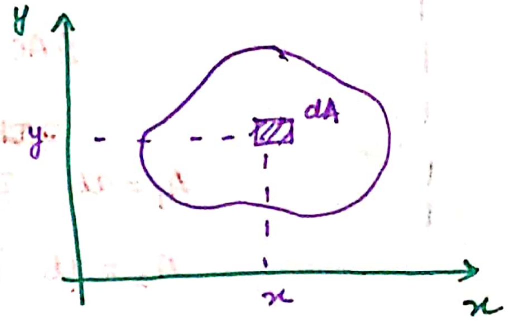


← To solve such problem use axial force diag (Like to find the force)

First Moment of Area

$$Q_x = \int_A x\, dA = \bar{x}_c A$$

$$Q_y = \int_A y\, dA = \bar{y}_c A$$

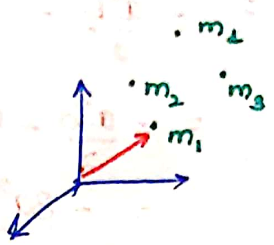


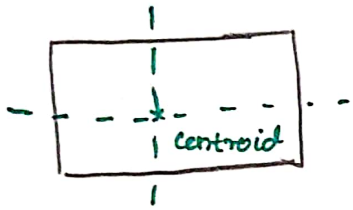
$$\bar{x}_c = \frac{1}{\int dA} \int x\, dA = \frac{1}{A} \int x\, dA$$

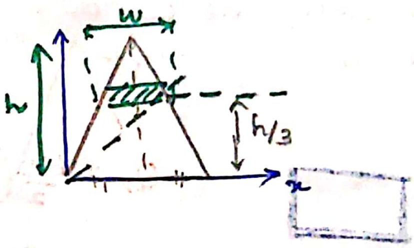
$$\bar{y}_c = \frac{1}{A} \int y\, dA$$

$$\boxed{\vec{r}_G = \frac{\sum m_i \vec{r}_i}{\sum m_i}}$$

$$\boxed{m \vec{r}_G = \sum m_i \vec{r}_i}$$







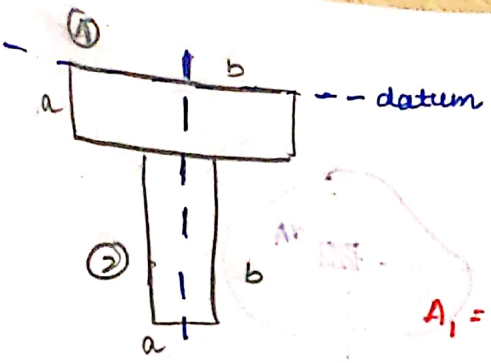
$$\left.\begin{aligned} A\bar{y}_c &= \int_A y\, dA \\ dA &= w\, dy \end{aligned}\right\} \int_0^h y\, dy$$

$$w = b\left(1 - \frac{y}{h}\right) \qquad A\bar{y}_c = \int_0^h b\left(1 - \frac{y}{h}\right) y\, dy = \frac{bh^2}{2} - \frac{bh^3}{3h}$$

$$= \frac{1}{6} bh^2$$

$$\frac{1}{2} \cancel{bh}\, \bar{y}_c = \frac{1}{\cancel{6}_3} \cancel{b} h^{\cancel{2}} \Rightarrow \bar{y}_c = \frac{h}{3}$$

Vardhan



$$\sum A_i \bar{y}_c = \sum A_i \bar{y}_{ci}$$

$A_1 = ab \quad \bar{y}_{c_1} = -a/2$

$A_2 = ab \quad \bar{y}_{c_2} = -a - \frac{b}{2}$

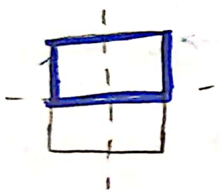
$$2ab\, \bar{y}_c = -\frac{a^2 b}{2} - ab\left(a + \frac{b}{2}\right)$$

$$= -\frac{a^2 b}{2} - a^2 b - \frac{ab^2}{2}$$

$$= -\frac{3a^2 b - ab^2}{2}$$

$$\therefore \bar{y}_c = -\frac{3a - b}{4}$$

Alternatively, we can do it by considering it as a rectangle of dimensions $a+b$, b



$$Q_x = \frac{bh}{2}\left(\frac{h}{4}\right) + \frac{bh}{2}\left(\frac{-h}{4}\right)$$

$$= 0$$

→ "First moment of area about any centroidal axis is zero"

Second moment of area

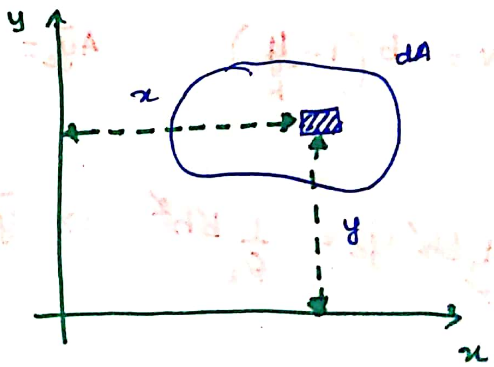
$$I_{xx} = \int_A y^2 dA$$

$$I_{yy} = \int_A x^2 dA$$

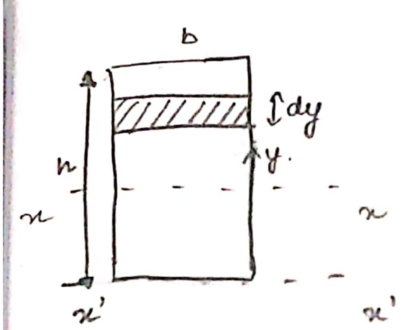
$$I_{zz} = \int_A (x^2 + y^2) dA$$

Polar moment of the area (It is also called the prep. axis theorm)

$$= \int_A r^2 dA$$



Vardha



$$I_{xx} = \int y^2\,dA$$

$$= \int y^2\,b\,dy = b\int_{-h/2}^{h/2} y^2\,dy$$

$$= \frac{bh^3}{12}$$

$$I_{yy} = \frac{hb^3}{12}$$

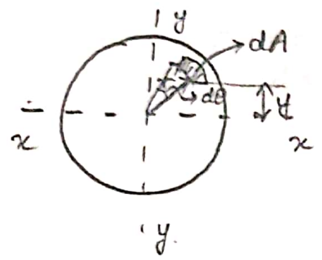
↑

Rotate I_{xx} by 90°

$$I_{zz} = \frac{hb^3}{12} + \frac{bh^3}{12} = hb\left(\frac{b^2+h^2}{12}\right)$$

Parallel Axis Theorm $\quad I_{xx}' = I_{Gxx} + Ad^2$

$$I_{x'x'} = \frac{bh^3}{12} + \frac{bh^3}{4} = \frac{bh^3}{3}$$



$dA = r\,d\theta\,dr$

$y = r\sin\theta$

$$I_{xx} = \int y^2\,dA$$

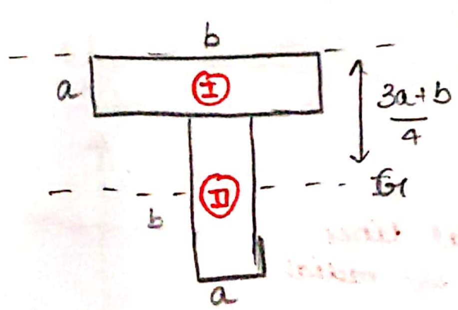
$$= \int_0^R \int_0^{2\pi} r^2 s^2\theta\; r\,d\theta\,dr$$

$$= \pi \int_0^R r^3\,dr$$

$$= \frac{\pi R^4}{4}$$

By symmetry $I_{yy} = I_{xx} = \frac{\pi R^4}{4}$

$$J = I_{zz} = \frac{1}{2}\pi R^4$$



$$I^{I}_{xx} = \frac{ba^3}{12} + ab\times\left(\frac{a}{2}\right)^2$$

$$= \frac{ba^3}{12} + \frac{ba^3}{4} = \frac{ba^3}{3}$$

$$I^{II}_{xx} = \frac{ab^3}{12} + ab\left(\frac{b}{2}+a\right)^2$$

$$= \frac{ab^3}{12} + ab\left(a^2 + \frac{b^2}{4} + ab\right)$$

$$= \frac{ab^3}{12} + a^3b + \frac{ab^3}{4} + a^2b^2$$

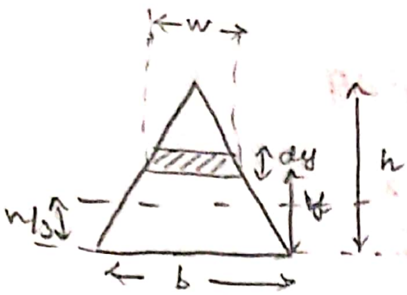
$$I_{xx} = I^{I}_{xx} + I^{II}_{xx}$$

$$= \frac{ba^3}{3} + \frac{ab^3}{3} + a^3b + a^2b^2$$

$$= \frac{4a^3b}{3} + \frac{ab^3}{3} + a^2b^2$$

$$I^{G}_{xx} = I^{G}_{xx} + 2ab\left(\frac{3a+b}{4}\right)^2$$

Vardhan



$dA = w\,dy$

$w = b\left(1-\frac{y}{h}\right)$

$dA = b\left(1-\frac{y}{h}\right)dy$

$$I_{xx} = \int y^2\,dA = \int_0^h y^2 b\left(1-\frac{y}{h}\right)dy$$

$$= b\left[\int_0^h y^2\,dy - \int_0^h \frac{y^3}{h}\,dy\right]$$

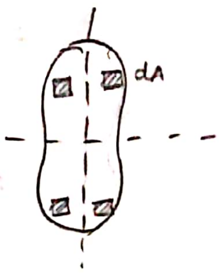
$$= \left[\frac{by^3}{3}\right]_0^h - \left[\frac{by^4}{4h}\right]_0^h = \frac{bh^3}{3} - \frac{bh^3}{4} = \frac{bh^3}{12}$$

$$I_{xx} = I_{xx}^G + \left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2$$

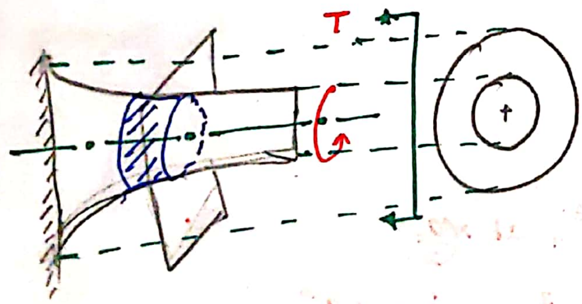
$$\Rightarrow I_{xx}^G = \frac{bh^3}{12} - \frac{bh^3}{18} = \frac{bh^3}{36}$$

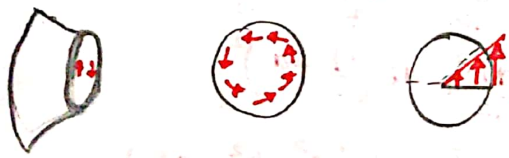
Product of Area $I_{xy} = \int xy\,dA$

For a geometric symmetric cross section, product of area is alway zero.

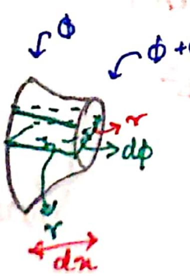


→ Torsion of Circular Bars





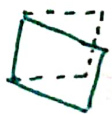
St. radial lines remain st. radial

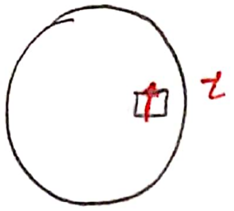


$\gamma\,dx = r\,d\phi$

$\gamma = r\frac{d\phi}{dx}$

↳ Twist per unit length





$$\tau = G\gamma = G r \frac{d\phi}{dx}$$

$$T = \int_A r \underbrace{\tau\, dA}_{\text{Force}} = \int_A r^2 G \frac{d\phi}{dx}\, dA = G\frac{d\phi}{dx}\int_A r^2 dA$$

Torque due to the whole force

$$= GJ\frac{d\phi}{dx}$$

$$T = GJ\frac{d\phi}{dx} \Rightarrow \boxed{\frac{d\phi}{dx} = \frac{T}{GJ}}$$

$$J = I_{zz} = \int r^2 dA$$

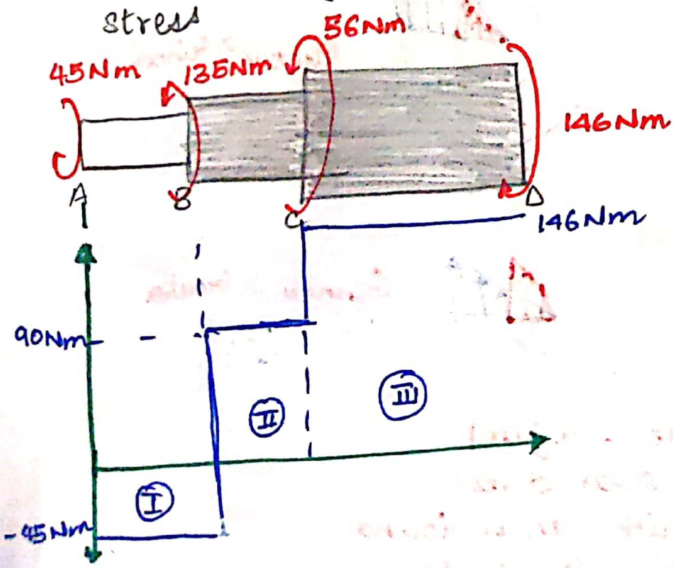
$$\boxed{\Delta\phi = \phi(x) - \phi(0) = \int^{x} \frac{T}{GJ}\, dx}$$

$$\tau = G r \frac{d\phi}{dx} = G r \frac{T}{JG} = \frac{Tr}{J}$$

$$\therefore \boxed{\tau = \frac{Tr}{J}} \qquad \boxed{\tau_{max} = \frac{Ta}{J}}$$

$$\tau_{max} = T/(J/a)$$

8) Knowing that each portion of the shaft AD consists of a solid circular rod, determine a) portion of shaft in which there is max. shearing stress b) mag. of that stress



I $\tau^{I}_{max} = \frac{Ta}{J} = \frac{-45(0.008)}{\frac{\pi}{2}(0.008)^4}$

$= \frac{-90}{\pi} \times \frac{1}{(0.008)^3}$

$= -55.9$ MPa

II $\tau^{II}_{max} = \frac{90(0.01)}{\frac{\pi}{2}(0.01)^4} = \frac{180}{\pi} \times \frac{1}{0.01^3}$

$= 57.29$ MPa

III $\tau^{III}_{max} = \frac{146(0.012)}{\frac{\pi}{2}(0.012)^4} = 53.7$ MPa

For shear stress +, - don't make any diff.

Vardhan

8) What must be the length of a 5mm diameter Al wire so that it could be twisted through one complete revolution without exceeding an allowable shear stress of 42 MPa ? G=

$$\phi = \frac{TL}{GJ} = 2\pi$$

$$\Rightarrow T = \frac{2\pi G J}{L}$$

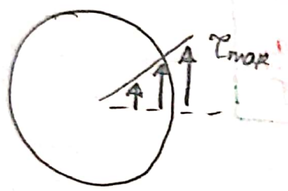
$$\tau_{max} = \frac{Ta}{J} \leq 42 \times 10^6 \text{ Pa}$$

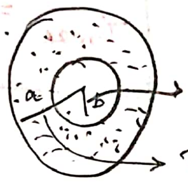
$$\frac{2\pi G J a}{JL} \leq 42 \times 10^6 \text{ Pa}$$

$$\Rightarrow \frac{2\pi G a}{L} \leq 42 \times 10^6 \text{ Pa}$$

$$\Rightarrow L \leq \frac{2\pi \times 27 \times 10^9 (0.0025)}{42 \times 10^6} = 1.01 \text{ m}$$

Note:-



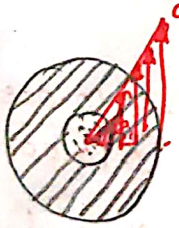


$$\tau_{max} = \frac{Tb}{J}$$

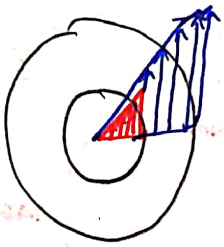
$$\tau_{max} = \frac{Ta}{J}$$

$$\tau = \frac{Tr}{J} \Rightarrow J_{hollow} = \frac{\pi}{2}(a^4 - b^4)$$

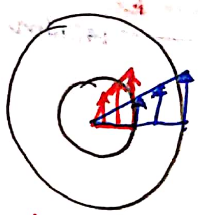
$$\tau = Gr\frac{d\phi}{dx}$$



$$\tau = Gr\frac{d\phi}{dx}$$



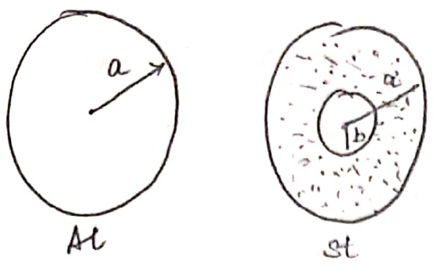
$G_{outer} > G_{inner}$



$G_{inner} > G_{outer}$

There is a jump in shear stress which can be found using $\tau = Gr\frac{d\phi}{dx}$

Q) A solid Al alloy shaft 50mm dia and 1000mm length is replaced by a tubular shaft of same outer dia such that the new shaft would neither exceed twice the max. shear stress, nor the angle of twist of Al shaft. Determine the inner dia of new shaft. For Al: $G = 28$ GPa, while for steel $G = 84$ GPa



(i) $\tau_{st}^{max} \le 2\tau_{Al}^{max}$

(ii) $\phi_{st} \le \phi_{Al}$

$$\frac{Ta}{J_{st}} \le 2\frac{Ta}{J_{Al}}$$

$$\Rightarrow J_{Al} < 2J_{st} \Rightarrow \frac{\pi}{2}a^4 \le \cancel{2}\frac{\pi}{\cancel{2}}(a^4 - b^4)$$

$$\Rightarrow b^4 \le a^4 - \frac{a^4}{2} = \frac{a^4}{2} \Rightarrow \boxed{b \le \frac{a}{2^{1/4}}}$$

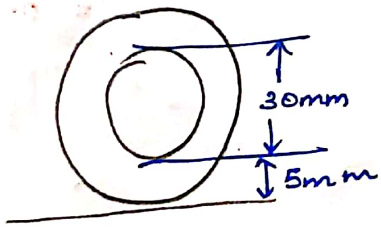
(ii) $$\frac{TL}{J_{st}G_{st}} \le \frac{TL}{J_{Al}G_{Al}} \Rightarrow J_{Al}G_{Al} \le J_{st}G_{st}$$

$$G_{Al}\frac{\pi}{2}a^4 \le G_{st}\frac{\pi}{2}(a^4 - b^4)$$

$$\Rightarrow b \le \left(\frac{2}{3}\right)^{1/4} a$$

Q) The composite shaft known is to be twisted by applying the torques. Knowing that the $G = 77$ GPa for steel and $G_{brass} = 39$ GPa. Determine $(\Delta\phi)_{max}$ of end B relative to end A. $(\tau_{steel})_{max} = 100$ MPa
$(\tau_{brass})_{max} = 55$ MPa

$$\tau = Gr\frac{d\phi}{dx}$$



$b = 15$mm
$a = 20$mm

$$\tau_{max}^{st} = G_{st}\, b\, \frac{\Delta\phi}{L} \le 100 \times 10^6 \text{ Pa}$$

$$\Delta\phi \le \frac{10^8 \times 1.8}{(77\times10^9)(0.015)} \sim 8.93$$

$$\Rightarrow \tau_{Br}^{max} = G_{Br}\, a\, \frac{\Delta\phi}{L} \le 55 \times 10^6 \text{ Pa}$$

$$\Rightarrow \Delta\phi \le \frac{55\times10^6(1.8)}{(39\times10^9)(0.02)} = 7.87^\circ$$

Vardhan

8) A power shaft distributes 4.5kW of power to two loads at 48 rpm, as shown in Fig 2. Determine angle of twist between a) A & B, b) A & C. Take shear modulus G = 27 GPa

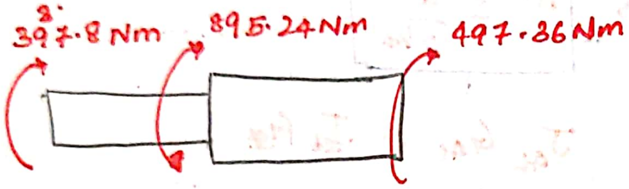
s/

$P = T\omega$ $\qquad \omega = 48 \frac{(2\pi)}{60} = \frac{8\pi}{5}$ rad/s

$T_A = -\frac{2 \times 10^3}{8\pi/5} = -397.8$ Nm

$T_B = \frac{4.5 \times 10^3}{8\pi/5} = \cancel{716.204}\ 895.24$ Nm

$T_C = -\frac{2.5 \times 10^3}{8\pi/5} = -497.36$ Nm



I) $\frac{d\phi}{dx} = \frac{T}{GJ} \Rightarrow \Delta\phi = \frac{T_{AB}(0.8)}{27 \times 10^9 \frac{\pi}{2}(0.015)^4}$

$= 0.15$ rad

II) $\phi_C - \phi_B = \frac{T_{BC}(1.0)}{27 \times 10^9 \times \frac{\pi}{2}(0.018)^4}$

$= -0.11$ rad

$\phi_C - \phi_A = \phi_C - \phi_B + \phi_B - \phi_A$

$= -0.11 + 0.15$

$= 0.04$ rad

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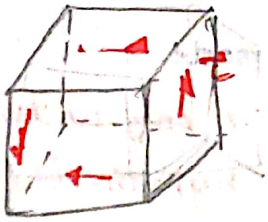
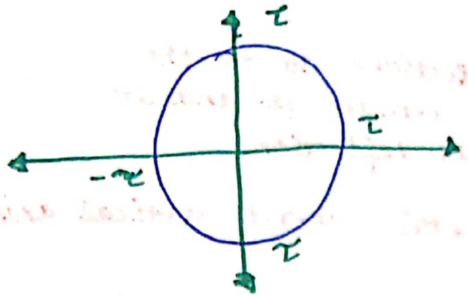
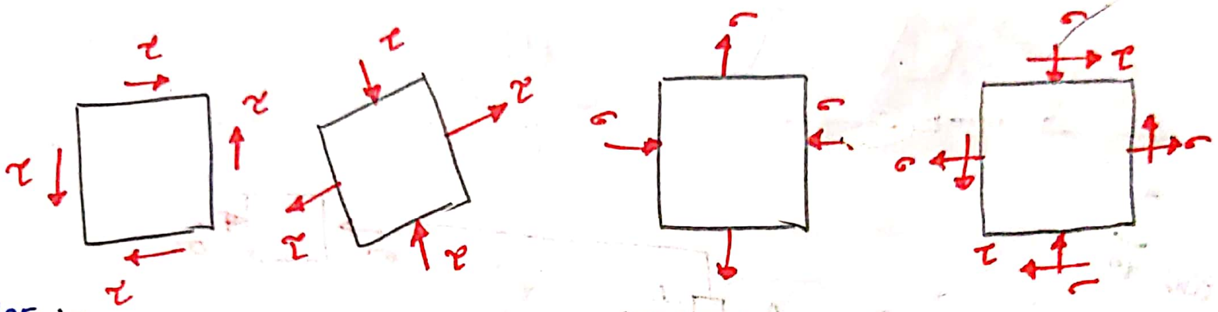


Torsion induces pure shear



$[\sigma] = \begin{bmatrix} 0 & \tau \\ \tau & 0 \end{bmatrix}$



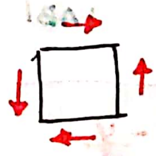
Note:-

→ Ductile materials fails under shear. (For brittle material, if under torsion the breaking surface make 45°, as it is tensile at 45°)

→ Brittle materials fails under normal tensile stress.

8) A uniform shaft of radius 20 mm is subjected to a torque T, as shown. A strain gauge, on the surface at 30° with the axis, measures the normal strain $\varepsilon_{x'x'} = 1.5 \times 10^{-4}$. Determine the Torque T, given youngs modulus E = 200 GPa and $\nu = 0.3$

$\tau_{max} = \frac{Ta}{J} = \sigma_{xy}$



$\sigma_{x'x'} = \tau s2\theta = \tau \frac{\sqrt{3}}{2}$

$\sigma_{y'y'} = -\tau s2\theta = -\tau \frac{\sqrt{3}}{2}$

$\varepsilon_{x'x'} = \frac{\sigma_{x'x'}}{E} - \frac{\nu}{E}\sigma_{y'y'} = \frac{\tau\sqrt{3}}{2E} + \frac{\nu\tau\sqrt{3}}{2E}$

$= \frac{\tau\sqrt{3}}{2E}(1+\nu) = 1.5 \times 10^{-4}$

$\frac{Ta\sqrt{3}}{2JE}(1+\nu) = 1.5 \times 10^{-4} \Rightarrow T = \frac{(3\times10^{-4})JE}{a\sqrt{3}(1+\nu)}$

Vardhan

Stress in Beams under flexure

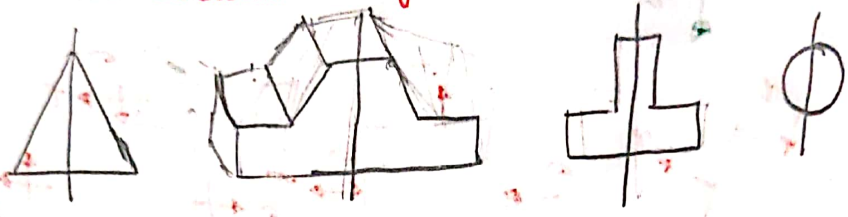
Assumptions being made :-

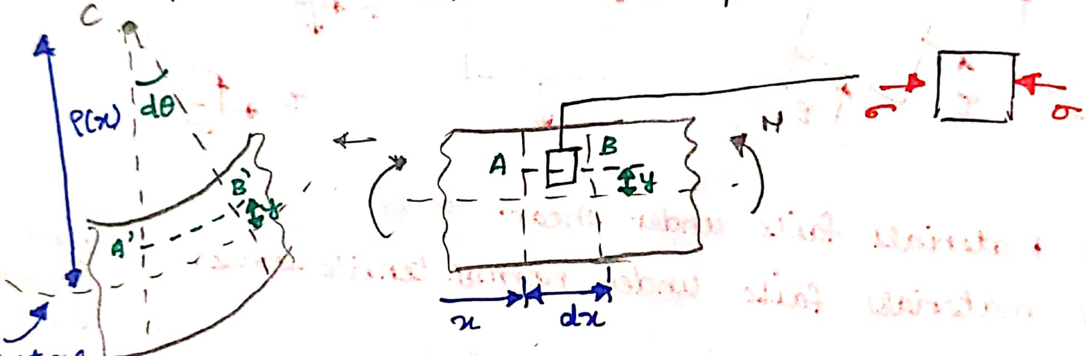
(i) No axial forces (i.e only under flexure)

(ii) Euler - Bernoulli hypothesis

St. sections ⊥r to the axis remain st. and ⊥r after deflection

(iii) Beam cross section is symmetric w.r.t vertical axis





neutral fiber (no stretching/contraction)

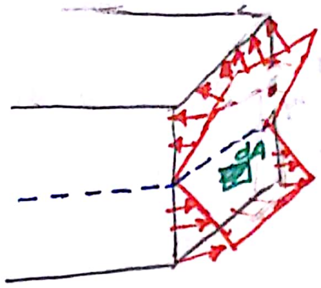
Strain in neutral fiber = 0

$$\varepsilon = \frac{|A'B'| - |AB|}{|AB|} = \frac{[\rho(x) - y]\,d\theta - \rho(x)\,d\theta}{\rho(x)\,d\theta}$$

$$\boxed{\varepsilon = -\frac{y}{\rho(x)}} \quad \text{or} \quad \varepsilon = -y\kappa(x)$$

$$\boxed{\sigma = E\varepsilon = -\kappa E y}$$

Kappa = $\frac{1}{\text{radius of curv.}}$



above is ~~tensile~~ compressive

below is tensile

provided moment is >0

$$F_a = \int_A \sigma \, dA = 0$$

$$\Rightarrow -\int_A E k y \, dA = 0$$

$$\boxed{\int_A y \, dA = 0} \quad -(1)$$

Eqn: (1) implies that first moment of area is zero

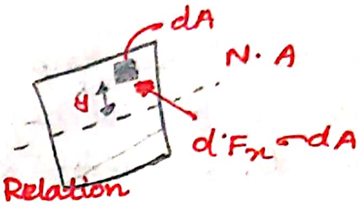
$\Rightarrow$ Neutral fiber is centroidal axis

Neutral fiber $\equiv$ Horizontal centroidal axis

N.A $\rightarrow$ Neutral axis

$$y|\sigma| dA = \cancel{y\sigma} \; dM$$

$$\boxed{\sigma = -Eky}$$ Stress Curvature Relation



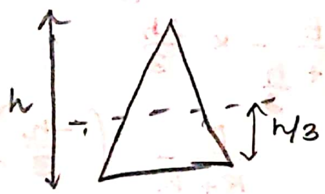
$$\Rightarrow \int dM = M = EK \int_A y^2 dA$$

$$\Rightarrow M = EK \int_A y^2 dA = EIK$$

$$\boxed{M = EIK}$$ $\rightarrow$ Moment curvature Relation

$$\Rightarrow \sigma = -\cancel{E}y \frac{M}{\cancel{E}I} \Rightarrow \boxed{\sigma = -\frac{My}{I}}$$ $\rightarrow$ flexure formula

Stresses will be maxm: at topmost and bottomost neutral fibres



$$\sigma_{max} = -\frac{M \times \frac{2h}{3}}{\frac{bh^3}{36}}$$

$$\sigma_{bottom} = -\frac{M\left(-\frac{h}{3}\right)}{\frac{bh^3}{36}}$$

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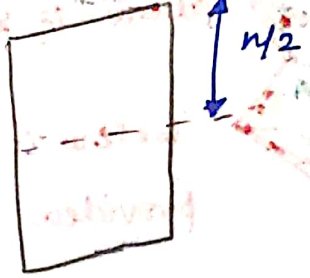
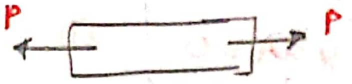
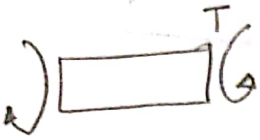
$\sigma = - M/(I/y)$



$I/(h/2)$: section modulus



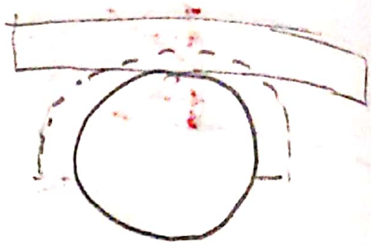
$\sigma = P/A$



$\tau = \frac{Tr}{J}$

$\Rightarrow \tau = T/(J/r)$

Q) A thin steel strip of symmetric triangular cross-section of height h is fixed on a cylinder of radius R, as shown. Assuming strip to be a beam satisfying Euler-Bernoulli hypothesis, determine the stresses at apex and base of cross-section



+ve moment produces +ve radius of curvature.

$P = -\left(R + \frac{h}{3}\right)$

$K = -\frac{1}{R + h/3}$

$\sigma = -EKy = \frac{E}{\left(R + \frac{h}{3}\right)} y$

$\sigma_{top} = \frac{E}{R + \frac{h}{3}} \times \frac{2h}{3}$

$\sigma_{bottom} = \frac{E}{R + \frac{h}{3}} \left(\frac{-h}{3}\right)$

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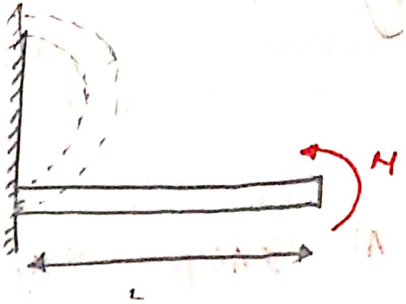
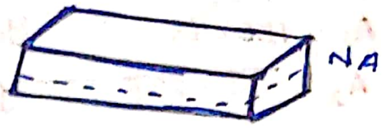
Q) Determine the moment M required at the free end of a uniform cantilever beam of length L to bend it into a semi-circle, as shown

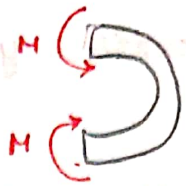


$\pi \rho = L \Rightarrow \rho = \frac{L}{\pi}$

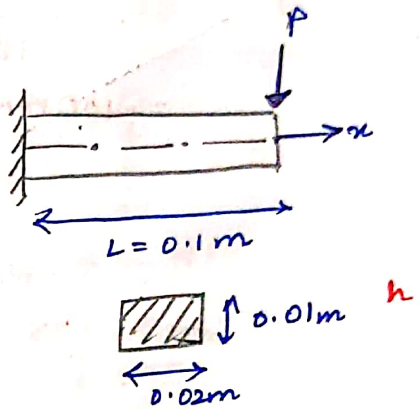


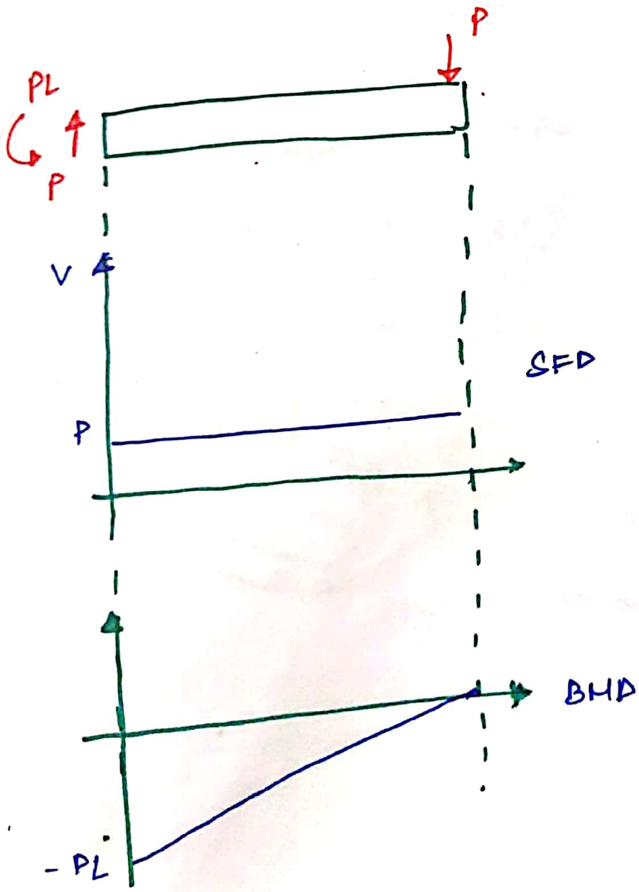
$M = EI\,K = EI/\rho$

$M = EI \times \frac{\pi}{L}$



Q) A cantilever beam type force sensor is loaded transvercely by a force P at the free end, as shown. If the strain guage registers a tensile axial strain of 50×10^{-6}, determine P. Take $E = 200$ GPa





$\frac{dv}{dx} = -q(x)$

$\frac{dM}{dx} = V$

$\sigma = -\frac{My}{I}$

$M = -PL$

$\sigma_{SG} = \sigma$

$y = h/2$

$I = \frac{bh^3}{12}$

$\sigma_{SG} = + \frac{PL\,h/2}{bh^3/12} = \frac{6PL}{bh^2}$

$\varepsilon = \frac{\sigma_{SG}}{E} = \frac{6PL}{Ebh^2} = 50 \times 10^{-6}$

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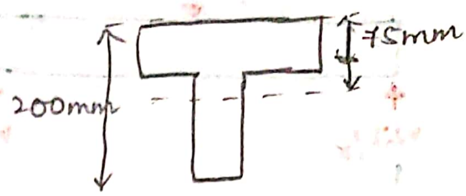
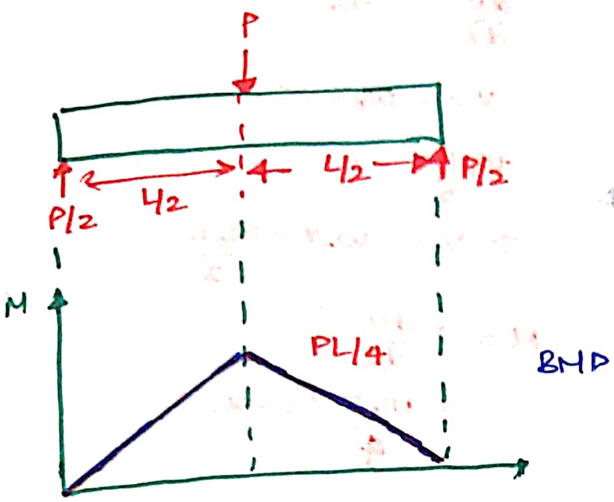
The T-beam is subjected to transverse force P in the upward or downward direction. The neutral axis of beam is at 75mm from top. If the max. fibre stress is to be restricted to 20 MPa in tension and 40 MPa in compression, determine max values of P in the two directions. Take $I = 49.6 \times 10^{-6} m^4$ about neutral axis.





$$\sigma = -\frac{My}{I}$$

$$M = \frac{PL}{4}$$

$$\sigma_{top} = -\frac{PL}{4I}(0.075)$$

$$|\sigma_{top}| \leq 40 \text{ MPa}$$

$$\Rightarrow P \leq \frac{40 \times 10^6 (4I)}{0.075(2)}$$

$$= 84.906 \text{ kN}$$

$$\sigma_{bottom} = -\frac{PL}{4I}(-0.125)$$

$$\Rightarrow |\sigma_{bottom}| \leq 20 \text{ MPa}$$

$$\Rightarrow P \leq \frac{20 \times 10^6 (4I)}{0.125(2)}$$

$$= 25.47 \text{ kN}$$

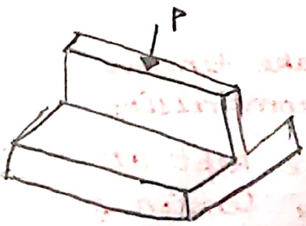
If we reverse P i.e P↑

$$|\sigma_{top}| \leq 20 \text{ MPa}$$

$$P \leq 42.4 \text{ kN}$$

$$|\sigma_{bot}| \leq 40 \text{ MPa}$$

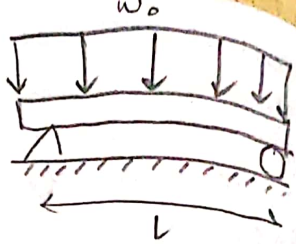
$$\Rightarrow P \leq 50.94 \text{ kN}$$

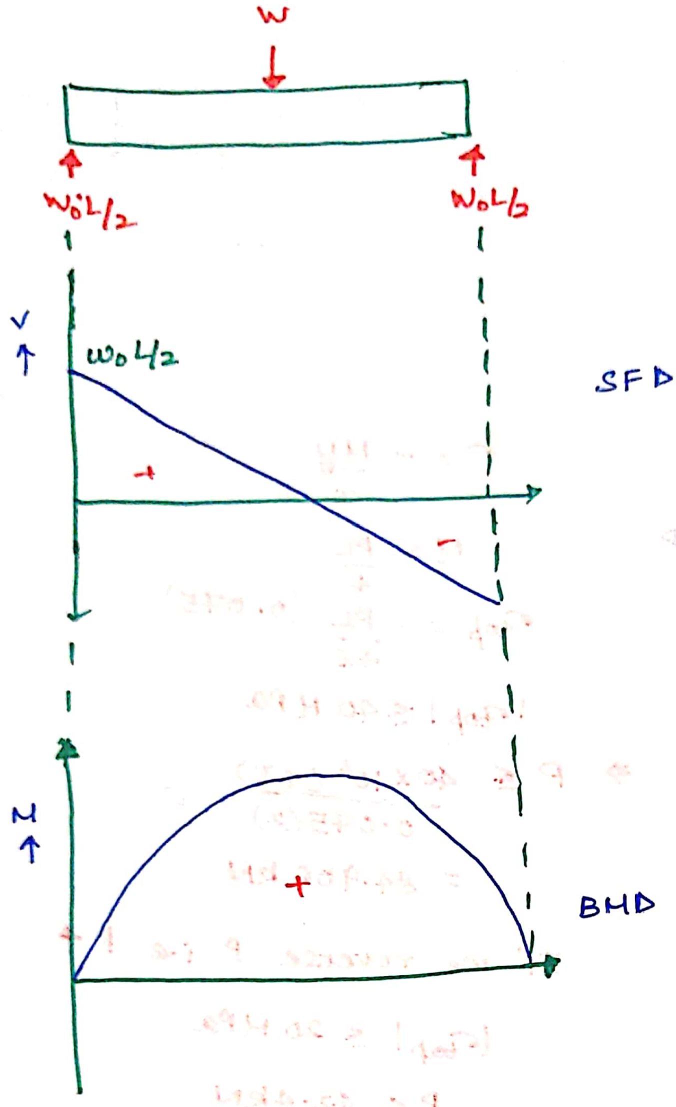


Inverted T config can take more load

Vardhan

8) Draw the shear force and bending moment diag. for the uniformly loaded simply supp. beam





$$\frac{dV}{dx} = -w_0$$

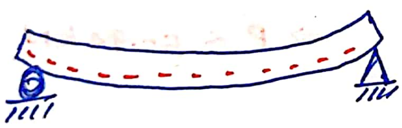
$$V = -w_0 x + c$$

$$w_0 \frac{L}{2} = c$$

$$\Rightarrow V = -w_0 x + \frac{w_0 L}{2}$$

$$M = -\frac{dV}{dx}$$

$$= \frac{w_0 x}{4}(L - x)$$



Reinforcement Beams

Concrete can't take tension
It takes only compression
So steel rods are kept at bottom for taking tension of beams in building