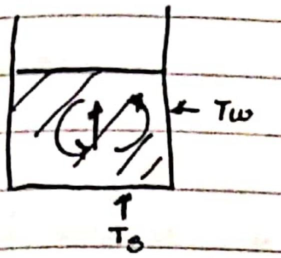
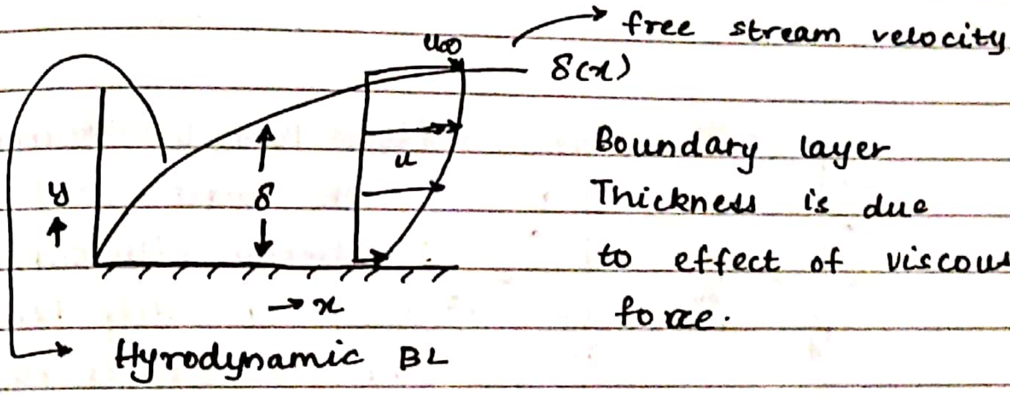
Heat Transfer

Convection $\rightarrow$ Advection + Conduction



Conduction happens at the liquid - solid interface
Now the different layers of water are at diff. temperatures

Now due to this convection happens and this convection is called natural convection or forced



Boundary layer Thickness is due to effect of viscous force.

At $y = \delta$ $\quad u = 0.99 u_\infty$

The fluid flow can be characterized in two distinct layers

1. Thin liquid layer i.e the boundary layer in which shear stress, velocity gradient are quiet dominant.

2. Region outside the BL, where the above mentioned are negligable.

With increasing distance from the leading edge, the effect of viscosity penetrates further in to the liquid and the boundary layer grows.

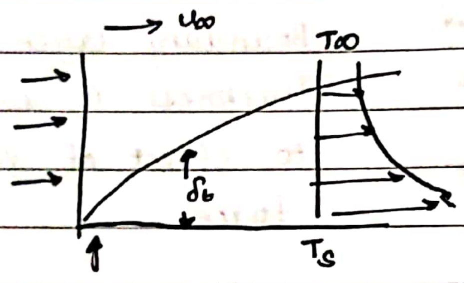
Dimensionless

$$C_f = \frac{\tau_s}{\rho U_\infty^2/2}$$

local friction co-eff

$$\tau_s = \mu \left.\frac{du}{dy}\right|_{y=0}$$ ← Newton's law of viscosity

μ — Dynamic viscosity



Fluid Particles nearer to the wall will exchange energy with surface as they are diff temp and transfer the energy to nearby layer and thus ~~forming a~~ causing a temp. gradient.

$$\left.\frac{T_s - T}{T_s - T_\infty}\right|_{y=\delta_t} = 0.99$$

Thermal boundary layer thickness (δ_t)

Hydrodynamic BL thickness (δ)

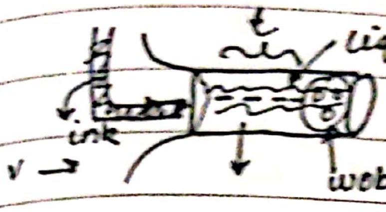
↳ Depends on velocity of fluid which dictates δ_t

δ_t is also dependent of T_s and temp. of fluid.

δ may or may not be equal to δ_t

$$q_s''\Big|_{y=0} = -k \frac{dT}{dy}\Big|_{y=0}$$

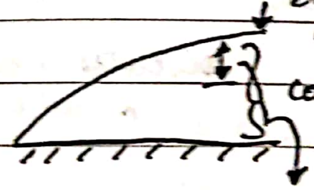
- Consider no velocity at the surface.



If we go with a slow velocity speed 't' increases when v is jet speed the wobble is created there it self.

steady, undisturbed water flow

Laminar to Turbulent Transistion



$$q_s'' = h(T_s - T_\infty)$$ ← Newton's law of cooling

$$h = -\frac{k}{(T_s - T_\infty)} \frac{\partial T}{\partial y}\Big|_{y=0}$$

$\underbrace{\text{Whole Fluid Movement}}_{\text{Advection}}$ on surface → Convection

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Heat transfer via conduction

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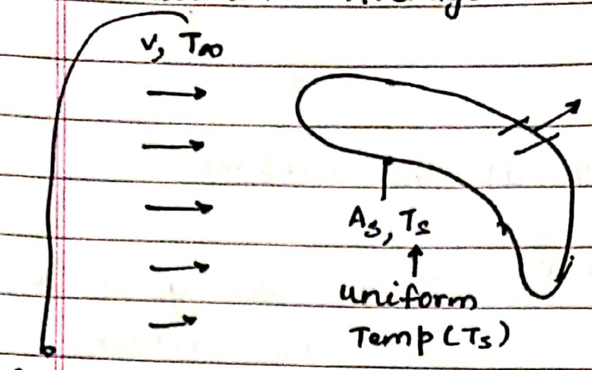
Local and Average Coefficients



If $T_\infty != T_s$, the convection will happen

In such case,
Convection heat transfer co-efficient, surface heat flux both will change along the surface.

Total heat transfer rate

$$q = \int q'' dA_s$$

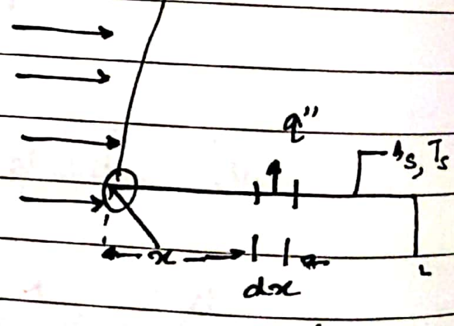
local heat flux

$$q = (T_s - T_\infty) \int h \, dA_s$$

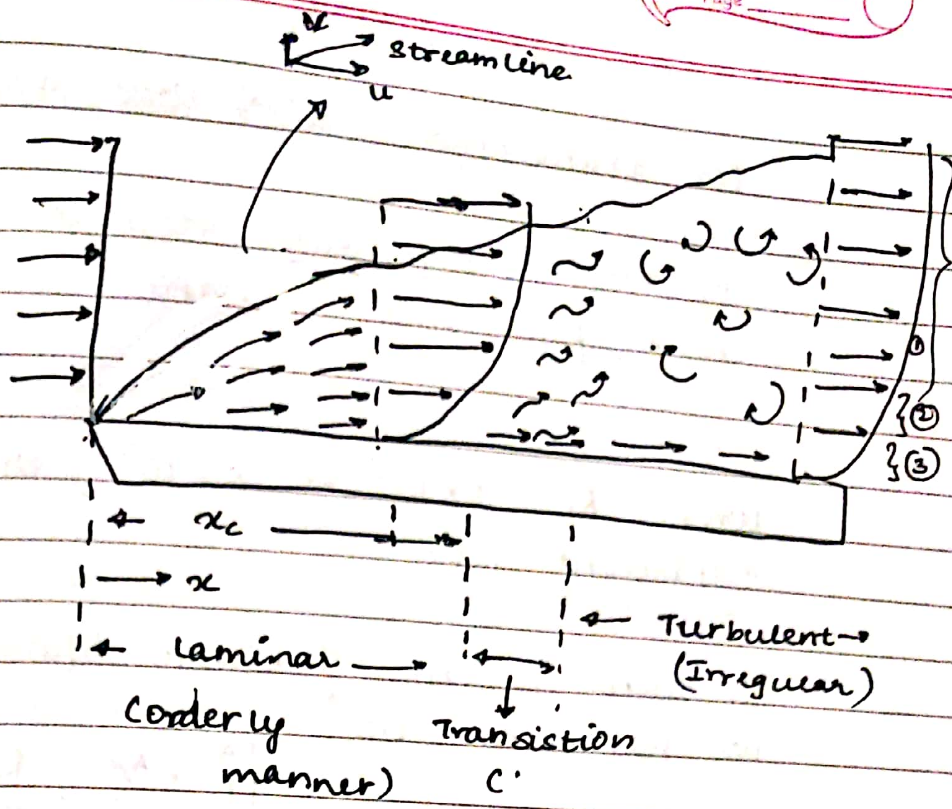
$$= \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s$$

Dictates how the BL will develop.



$$\bar{h} = \frac{1}{L} \int_0^L h \, dx$$



As BL thickness ↑, velocity gradient decreases in x-direction

③ → Viscous sublayer (Transport is dominated by diffusion, velocity profile is linear)

② → Buffer layer (Diffusion and mixing are comparable)

① → Turbulent (Mixing is dominant)

$$Re_x = \frac{\rho u_\infty x}{\mu}$$

μ ← Dynamic Viscosity

For flat plate, x is distance from leading edge

For a flow to be turbulent $Re < 2100$
< 2300
< 2400
↓
Then flow is laminar

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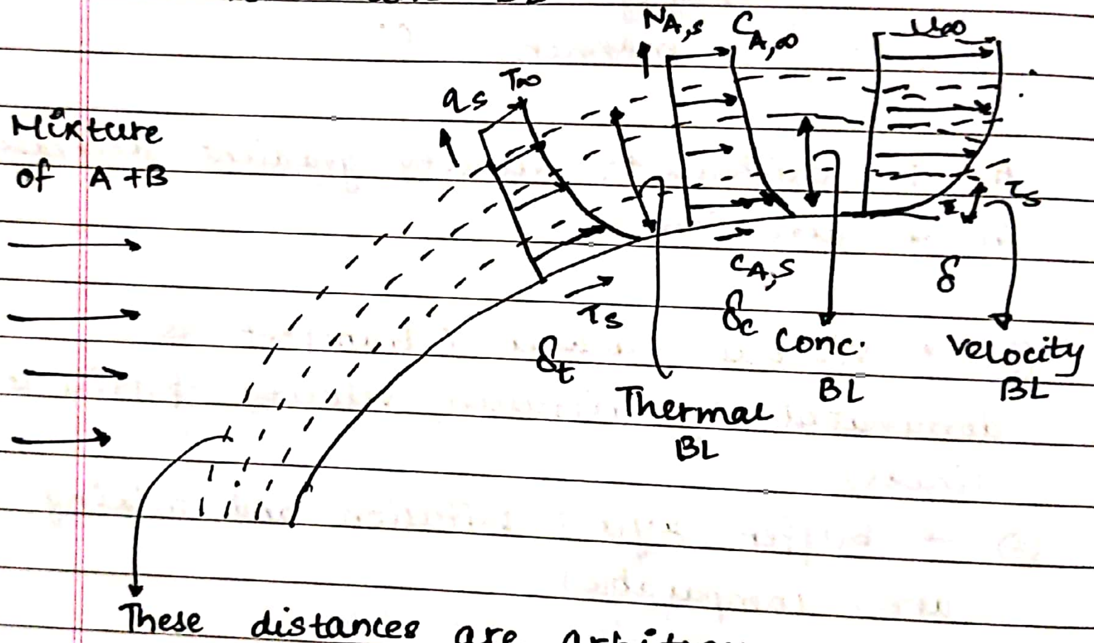
For transcistion $Re \rightarrow \{2000, 4000\}$

$Re > 4000$, typically flow is turbulent in a piple flow scenario

When $Re = 5\times10^5 - 5\times10^6$, then its turbulent flow

If there is conc. gradient b/w two points, we have conc. BL



$$\rho\left(u\frac{du}{dx} + v\frac{du}{dy}\right) = -\frac{dP}{dx} + \mu\left(\frac{d^2u}{dx^2} + \frac{d^2v}{dy^2}\right) + X$$

Body forces

Convective transport

Pressure grad

Diffusive transport

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Nusselt Number $= \frac{hL}{K}$

$$= \frac{q_{convection}}{q_{conduction}} = \frac{h\,\Delta T}{\frac{k\,\Delta T}{L}}$$

Nusselt Number > 1 (Since convection is x + cond.)

- Invicit Flow (Fluid with zero viscosity)
- Flow is bounder (inside a pipe) → Internal flow
- Flow on a flat plate, around a sphere → external
- Three surfaces closed, one open → open channel flow

- Compressible Flow - ρ of fluid changes
  ~~St~~ Incompressible Flow - ρ of fluid is const.

- Steady - Uniform

Prandtl Number = $\frac{\text{Molecular diffusivity of Momentum}}{\text{Molecular diffusivity of heat}}$

Kinematic viscosity (μ/ρ)

↳ ($k/\rho c_p$)

$$= \frac{\mu c_p}{k}$$ ← Dimensionless

Prandtl Number < 1 ⇒ Heat diffuses faster
⇒ ~~Thick~~ Thermal BL ~~thickness is gthi~~
⇒ Thermal BL is thicker.

Liquid metal   Pr → {0.004, 0.03}
↳ Thicker Thermal BL

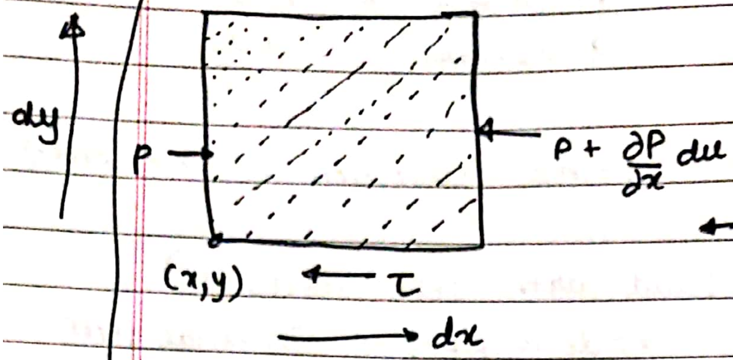
Oil   Pr → {50, 100000}
↳ Momentum diffuses faster.

$$\overset{\rho (dx\, dy)}{\delta m}\, a_x = F_S + F_B$$

$$u = u(x,y)$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$\frac{du}{dt} = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}$$



← Normal Stress

shear stress

$$F_S = \left(\frac{\partial \tau}{\partial y} dy\right)(dx) - \left(\frac{\partial P}{\partial x} dx\right)(dy)$$

$$\tau = \mu \frac{\partial u}{\partial y}$$

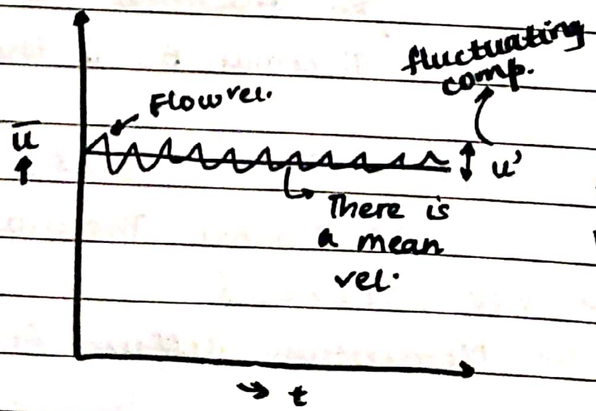
$$\rho\left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = \mu \frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

B.L Approximations

→ $u >> v$

→ $\frac{\partial v}{\partial x} \approx 0$, $\frac{\partial v}{\partial y} \approx 0$

Turbulent flow is rapid and random, this is called eddies



$u = \bar{u} + u'$

$v = \bar{v} + v'$

$P = \bar{P} + P'$

$T = \bar{T} + T'$

Steady turbulent, the mean comp. are const.

$$\tau_t = -\rho\,\overline{v'u'}$$

Reynolds stress

$$\overline{u'} = 0,\quad \overline{v'} = 0 \quad \overline{u'v'} \neq 0$$

$$= \mu_t \frac{\partial \bar{u}}{\partial y}$$

Kinematic visc.

$$q_t = \rho c_p \overline{v'T'}$$

$$= -k_t \frac{\partial T'}{\partial y}$$

turbulent thermal cond.

$$\tau_{total} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y}$$

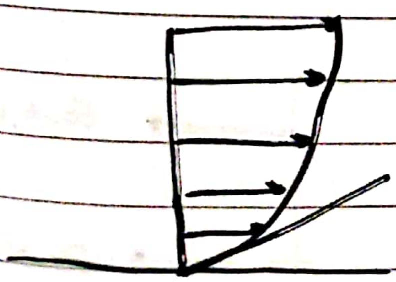
$$= \rho(\nu + \varepsilon_m)\frac{\partial \bar{u}}{\partial y} \qquad \varepsilon_m = \frac{\mu_t}{\rho}$$

Eddy diffusivity of momentum

$$q_{total} = -(k + k_t)\frac{\partial \bar{T}}{\partial y}$$

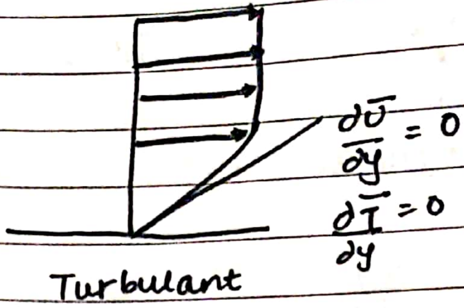
$$= -\rho c_p(\alpha + \epsilon_H)\frac{\partial \bar{T}}{\partial y}$$

$$\epsilon_H = \frac{k_t}{\rho c_p}$$ ← Eddy diffusivity of heat.



$$\left.\frac{\partial u}{\partial y}\right|_{y=0} \quad \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

laminar



→ $\frac{\partial v}{\partial x} << \frac{\partial u}{\partial y}$, $\frac{\partial T}{\partial x} << \frac{\partial T}{\partial y}$

In boundary layer

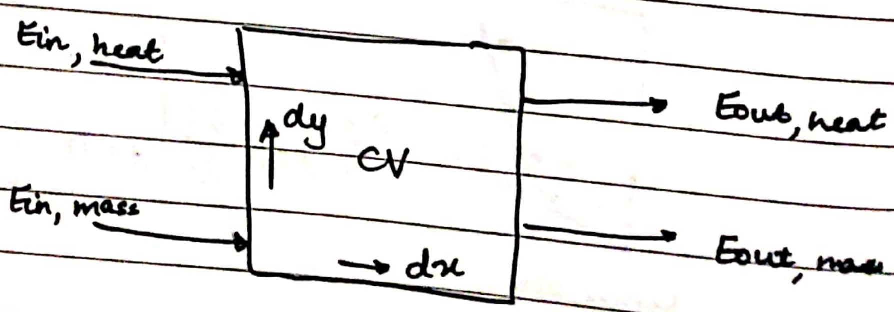
→ Neglecting gravity , $P = P(x)$, $\frac{\partial P}{\partial y} = 0$

In absence of body force, u constant, Pressure remains constant, whether it's inside or outside boundary layer, in the free stream region (?)

→ For a const. x, P is same

$E_{stream} = h + K.E + P.E$

h ↓ enthalpy ; K.E ↓ $v^2/2$; P.E ↑ gz



$$(E_{in} - E_{out})_m + (E_{in} - E_{out})_{heat} + (E_{in} - E_{out})_{w} = 0$$

mass

x - direction $\quad -\rho c_p \left[u \frac{\partial T}{\partial x} + T \frac{\partial u}{\partial x} \right] dx \times dy$

y - direction $\quad -\rho c_p \left[v \frac{\partial T}{\partial y} + T \frac{\partial v}{\partial y} \right] dx \times dy$

(x) + (y) $\Rightarrow -\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] dx\, dy \quad -①$

x - direction $\quad k \frac{\partial^2 T}{\partial x^2} dx dy$

y - direction $\quad k \frac{\partial^2 T}{\partial y^2} dx dy$

$$(E_{in} - E_{out})_{heat} = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] dx dy \quad -②$$

Body forces - Pressure, viscous, shear forces

From ① & ②

$$\rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] dx dy = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] dx dy$$

$$\Rightarrow \rho c_p \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]$$

2d steady state, neg. shear force or stresses.

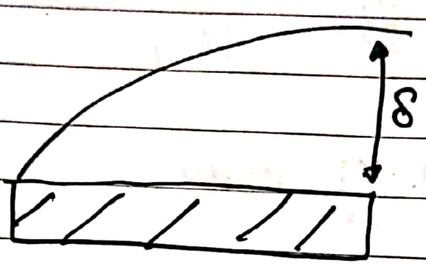
When its only conduction $\quad k \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] = 0$

When we can't neglect body forces

$$\rho c_p \left[u\frac{\partial T}{\partial x} + u\frac{\partial T}{\partial y} \right] = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \mu\phi$$

$\phi \rightarrow$ viscous dissipation fn:

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 \right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2$$



$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad u\frac{du}{dx} + v\frac{du}{dy} = \nu \frac{d^2u}{dy^2}$$

$x = 0$	$u(0,y) = u_\infty$	$T(0,y) = T_\infty$
$y = 0$	$u(x,0) = 0$	$u(x,0) = 0$
		$T(x,0) = T_s$
$y = \infty$	$u(x,\infty) = u_\infty$	$T(x,\infty) = T_\infty$

$$\delta = \frac{5.0x}{\sqrt{Re_x}} \qquad Re_x = \frac{u_\infty x}{\nu}$$

ν — Kinematic viscosity

$$C_f = \frac{\tau_\infty}{\rho v^2/2}$$

$$= 0.664\, Re_x^{-1/2}$$

$$C_w = 0.332 \frac{\rho u_\infty^2}{\sqrt{Re_x}}$$

Flow over isothermal flat plate.

$$c_f = \frac{C_w}{\rho U_\infty^2/2}$$

$$Re = \frac{LV\rho}{\mu}$$

If, Re for two flows is same, then our predictions about one flow based on the other is likely to be true

$$\theta = \frac{T(x,y) - T_s}{T_\infty - T_s}$$

T_s → Surface temp

T_∞ → Free stream temp

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2} \qquad Pr = \frac{\nu}{\alpha}$$

$$\boxed{2 \frac{\partial^2 \theta}{\partial \eta^2} + Pr \frac{\partial \theta}{\partial \eta} = 0}$$

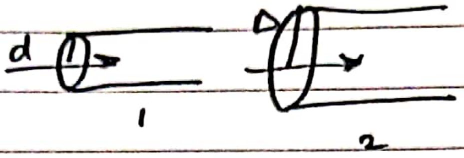
If $Pr > 0.6$, $\left.\frac{\partial \theta}{\partial \eta}\right|_{\eta = 0} = 0.332\, Pr^{1/3}$

$$Nu_x = \frac{h_x x}{k} = 0.332\, Pr^{1/3} Re_x^{1/2}, \text{ for } Pr > 0.6$$

↑ Local Nusselt Number (At any x from leading edge)

$$\delta = \delta_t \, Pr^{1/3}$$

Hydrodynamic BL ↑ (δ)   Thermal BL ↑ (δ_t)



$D > d$

$Re_1 = Re_2$

$\Rightarrow C_{f_1} = C_{f_2}$

$$x^* = \frac{x}{L}, \quad y^* = \frac{y}{L}$$

$$u^* = \frac{u}{u_\infty} \qquad v^* = \frac{v}{v_\infty}$$

$$P^* = \frac{P}{\rho u_\infty^2} \qquad T^* = \frac{T - T_s}{T_\infty - T_s}$$

Non-dimensionalising

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^2} - \frac{\partial P^*}{\partial x^*}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$$u^*(0, y^*) = 1$$

$$C_{f,x} = \frac{\tau_s}{\rho u_\infty^2/2} \rightarrow \mu \frac{du}{dy}\Big|_{y=0} = \frac{\mu u_\infty}{L} \frac{du^*}{dy^*}\Big|_{y^*=0}$$

$$u^* = f(x^*, y^*, Re) \quad \Rightarrow \quad \frac{\mu u_\infty}{L} f(x^*, Re_L) = \tau_s$$

$$C_{f,x} = \frac{\frac{\mu u_\infty}{L} f(x^*, Re)}{\rho u_\infty^2/2} = \frac{2}{Re_L} f(x^*, Re_L)$$

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$$C_{f,x} = f_1 (x^+, Re_L)$$

$$T^* = f_2 (x^+, y^+, Re_L, Pr)$$

$$h = \frac{k}{L} \frac{\partial T^*}{\partial y^*}\Big|_{y^+=0}$$

$$\frac{hL}{k} = \frac{\partial T^*}{\partial y^*}\Big|_{y=0}$$

$$Nu = f_3 (x^+, Re_L, Pr)$$

Nusselt no. → Non-dimensional temp. grad at surface

Avg values

$$C_f = f_4 (Re_L)$$

$$Nu = c' Re^m Pr^n$$

$$Nu = f_5 (Re_L, Pr)$$

Momentum eqn. :

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

Energy :

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 T^*}{\partial y^{*2}}$$

$Pr = 1$

↓

ν / α

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \frac{\mu u_\infty}{L} f(x^*, Re_L)$$

$$C_{f,x} = \frac{\tau_s}{\rho u_\infty^2/2} = \frac{\mu u_\infty/L \; f(x^*, Re_L)}{\rho u_\infty^2/2}$$

$$= \frac{2}{Re_L} f(x^*, Re_L)$$

$$Nu_x = \frac{hL}{k} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0} = f(x^*, Re, Pr)$$

"Ringing Bells"

$$\frac{\partial u^*}{\partial y^*}\Big|_{y^*=0} = \frac{\partial T^*}{\partial y^*}\Big|_{y^*=0}$$

$$C_{f,x} \frac{Re_L}{2} = Nu_x \quad \text{(Only for } Pr = 1)$$

↳ Reynold's Analogy

$$\frac{C_{f,x}}{2} = St_x \quad (Pr = 1)$$

↑ Stanton No.

↳ Modifie Nu_x

$$St = \frac{Nu}{Re_L \, Pr} = \frac{\mu}{\rho C_p u_\infty}$$

$$C_{f,x} \frac{Re}{2} = Nu_x \, Pr^{-1/3}$$

$$\frac{C_{f,x}}{2} = St \, Pr^{2/3} = \frac{h_x}{\rho C_p u_\infty} Pr^{2/3}$$

$0.6 < Pr < 60$

$$\frac{C_{f,x}}{2} = j_H$$

← Chilton Colburn Analogy

↳ Colburn j-factor

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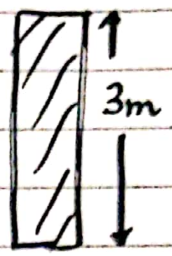
8) $T_\infty = 20°C$ $\quad$ $Pr = 0.7309$

$U_\infty = 7\ m/s$ $\quad$ $\rho = 1.204\ kg/m^3$

$F_f = 0.86$

Drag force $F_f = C_f A_s \frac{\rho U^2}{2}$



$0.86 = C_f\ (3 \times 2 \times 2)\ \frac{1.204 \times 7}{2}$

$C_f = 0.0024$

$\frac{C_f}{2} = \frac{h}{\rho C_p U} Pr^{2/3}$

$h_x = 12.7\ W/m^2$