Important descrete random variables

Bernoulli random variable
Consider an experiment with two outcomes: success/faliure
A random variable X is a Bernoulli random variable with parameters o if

$$P(X = 0) = 1 - p \tag{1}$$
$$P(X = 1) = p \tag{2}$$

We write $x \sim B(P)$.

Binomial distribution

- Consider an experiment with two outcomes: success/faliure
- Let p be the probability of success
- Consider n independent repetitions of the last experiment

$X = \#$ succes

Then X is a binomial random variable if

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad \text{for } k = 0, 1, 2, \ldots, n \tag{3}$$

We write $X \sim \text{bin}(n, p)$.

Expectation and variance of binomial distribution
If $X \sim \text{bin}(n, p)$, then

$$E[X] = np \text{ and } \text{Var}[X] = np(1-p)$$

Geometric distribution
Consider the independant repetition of the same experiment with two outcomes success/faliure that happen with probability p and $1 - p$ respectively.
X is the time of the first success

X in a geometric distribution with parameter p if

$$P(X = k) = (1-p)^{k-1} p, \qquad \text{for } k = 1, 2, \ldots \tag{4}$$

We write *geom*(p) or $g(p)$.

The *cdf* of a geometric distribution is

$$F_x(k) = P(X \leq k) = 1 - P(X > k) = 1 - (1-p)^k \tag{5}$$

No memory property
Let $0 < k < n$

$$P(X = n + k/x > n) = \frac{P(\{X = n + k\} \cap \{x > n\})}{P(X > n)} \tag{6}$$
$$= \frac{P(X = n + k)}{P(X > n)} \tag{7}$$
$$= \frac{(1 - p)^{n+k-1} p}{(1 - p)^n} \tag{8}$$
$$= (1 - p)^{k-1} p \tag{9}$$

Expectation and variance of geometric distribution
If $X \sim \text{geom}(p)$, then

$$E[X] = \frac{1}{p} \text{ and } \text{Var}[X] = \frac{1 - p}{p^2} \tag{10}$$

Poisson distribution
Different from above distributions, as it doesn't describe a particular experiment. It is observed during experiments where you count thing over periods of time. Ex: number of atom's disintegration by minutes.

X is a Poisson distribution with parameters $\lambda > 0$ if

$$P(X = k) = \frac{\lambda^k}{k!} \mathrm{e}^{-\lambda}, \qquad \text{for } k = 1, 2, \ldots \tag{11}$$

Nate that for all integers k, $P(X = k) \geq 0$.
We have

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = \mathrm{e}^{\lambda}, \qquad \text{thus } \sum_{k=0}^{\infty} P(X = k) = 1 \tag{12}$$

Expectation and variance of Poisson distribution
If $X \sim \text{Poi}(\lambda)$, then

$$E[X] = \text{Var}[X] = \lambda \tag{13}$$