Opgave 1
The weather at a coastal resort is classified each day simply as "sunny" or "rainy". A sunny day is followed by another sunny day with probability 0.9 and a rainy day is followed by another rainy day with probability 0.3. **(a)** Describe this as a Markov chain. **(b)** If Friday is sunny, what is the probability that Sunday is also sunny? **(c)** If Friday is sunny, what is the probability that both Saturday and Sunday are sunny?

(a) Kæden beskrives med overgange. Lad solvejrsdag være 0 og regnvejrsdag være 1.

$$\begin{aligned}
P_{00} &= P(X_{n+1} = 0/X_n = 0) = 0.9 \\
P_{01} &= P(X_{n+1} = 1/X_n = 0) = 0.1 \\
P_{11} &= P(X_{n+1} = 1/X_n = 1) = 0.3 \\
P_{10} &= P(N_{n+1} = 0/X_n = 1) = 0.7
\end{aligned}$$

(b) Opstil overgangsmatricen:

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} \tag{1}$$

Sandsynligheden for at der er solvejrsdag søndag efter der er solvejrsdag fredag:

$$P_{00}^{(2)} = P_{ij}^2 = 0.88 \tag{2}$$

(c) Sandsynligheden for at der er solskinsvejr både lørdag og søndag givet at der er solskinsvejr fredag er 0.9^2, da der lørdag er 0.9 sandsynlighed for det og der søndag, for der var solskin lørdag, er 0.9 sandsynlighed for at det er solskinsvejr.
Altså 0.81.

Opgave 3
A machine produces electronic components that may come out defective and the process is such that defective components tend to come in clusters. A defective component is followed by another defective component with probability 0.3, whereas a nondefective component is followed by a defective component with probability 0.01. Describe this as a Markov chain and find the long-term proportion of defective components.

Markovkæden beskrives ved overgange. Lad nondefekt være 0 og defekt være 1.

$$
\begin{aligned}
P_{00} &= 0.99 \\
P_{01} &= 0.01 \\
P_{11} &= 0.30 \\
P_{10} &= 0.70
\end{aligned}
$$

P opløftes i et højt tal for at finde grænseværdierne for indgangene i P^n når $n \to \infty$. Der fås, at

$$
P^n = \begin{bmatrix} 0.986 & 0.014 \\ 0.986 & 0.014 \end{bmatrix}. \tag{3}
$$

Altså er 1.4% af produkterne defekte.

Opgave 6
Suppose that state i is transient and that $i \to j$. Can j be recurrent?

Ja - se korollar 8.2 og 8.3.

Opgave 13
Show that a limit distribution is a stationary distribution. The case of finite S is easier, so you may assume this.