Opgave 9
Markov chains are named for Russian mathematician A. A. Markov, who in the early twentieth century examined the sequence of vowels and consonants in the 1833 poem *Eugene Onegin* by Alexander Pushkin. He empirically verified the Markov property and found that a vowel was followed by a consonant 87% of the time and a consonant was followed by a vowel 66% of the time. **(a)** Give the transition graph and the transition matrix. **(b)** If the first letter is a vowel, what is the probability that the third is also a vowel? **(c)** What are the proportions of vowels and consonants in the text?

(a) Transitionsmatricen ser således ud

$$P = \begin{bmatrix} 0.34 & 0.66 \\ 0.87 & 0.13 \end{bmatrix} \tag{1}$$

(b) Hvis det første bogstav er en vokal, så aflæses sandsynligheden for at tredje bogstav er en vokal fra P^2:

$$P^2 = \begin{bmatrix} 0.6898 & 0.3102 \\ 0.4089 & 0.5911 \end{bmatrix} \tag{2}$$

Sandsyndligheden for at tredje bogstav er en vokal er 0.5911. **(c)** Mængden af vokaler og konsonanter findes ved

$$\frac{1}{q+p}\begin{bmatrix} q & p \\ q & p \end{bmatrix} = \begin{bmatrix} 0.57 & 0.43 \\ 0.57 & 0.43 \end{bmatrix} \tag{3}$$

Opgave 15
Consider the success run chain in Example 8.16. Suppose that the chain has been running for a while and is in state 10. **(a)** What is the expected number of steps until the chain is back at state 10? **(b)** What is the expected number of times the chain visits state 9 before it is back at 10?

(a) Brug proposition 8.5, som siger

$$E_i[\tau_i] = \frac{1}{\pi_i} \tag{4}$$

Fra eksempel 8.16 haves

$$\pi_i = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & \cdots \\ 1/2 & 0 & 1/2 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots \end{bmatrix} = \frac{1}{2^{i+1}} \tag{5}$$

Altså

$$E_{10}[\tau_{10}] = 2^{10+1} = 2048 \tag{6}$$

(b) Brug samme proposition 8.5. Der haves, at

$$E_{10}[N_9] = \frac{\pi_9}{\pi_{10}} = \frac{2^{11}}{2^{10}} = 2 \tag{7}$$

Opgave 16
Let $g : [0,1] \to R$ be a function whose integral $I = \int_0^1 g(x)\,dx$ is impossible to compute explicitly. How can you approximate I by simulation of standard uniforms $U_1, U_2, \ldots$?

Brug opgave 3 fra kursusgang 14, hvor en function approksimeres ved en sum af uniforme distributioner.

$$I = \int_0^1 g(x)\,dx \approx \lim_{n\to\infty} \sum_{k=1}^{n} g(U_k)/n \tag{8}$$