Opgave 76
Consider the variance formula $\mathrm{Var}[Y] = \mathrm{Var}[E[Y|X]] + E[\mathrm{Var}[Y|X]]$. **(a)** Show that $\mathrm{Var}[E[Y|X]] = 0$ if X and Y are independent. **(b)** Show that $E[\mathrm{Var}[Y|X]] = 0$ if X and Y are totally dependent in the sense that $Y = g(X)$ for some function g (recall Corollary 3.6).

(a) Brug proposition 2.16, som siger, at

$$\mathrm{Var}[X] = 0 \text{ hvis og kun hvis } X = E[X] \tag{1}$$

Da X og Y er uafhængige

$$\mathrm{Var}[E[Y|X]] = \mathrm{Var}[E[Y]] = 0 \tag{2}$$

(b) Brug korollar 3.6, som siger

$$E[g(X)|X] = g(X) \tag{3}$$

og omskriv

$$\begin{aligned}
\mathrm{Var}[g(X)] &= E[\mathrm{Var}[Y|X]] + \mathrm{Var}[E[g(X)|X]] \\
\mathrm{Var}[g(X)] &= E[\mathrm{Var}[Y|X]] + \mathrm{Var}[g(x)] \\
&\Downarrow \\
E[\mathrm{Var}[Y|X]] &= 0
\end{aligned}$$

Opgave 85
Let U and V be independent and unif$[0,1]$ and let $X = \min(U,V)$ and $Y = \max(U,V)$. Find $\mathrm{Cov}[X,Y]$ and comment on its sign.

Opgave 88
In Definition 3.17, it is necessary that $\mathrm{Var}[X] > 0$ and $\mathrm{Var}[Y] > 0$. If this is not the case, what value is reasonable to assign to the correlation?