

Opgave 38 Let A and B be disjoint events. Show that

$$P(A/A \cup B) = \frac{P(A)}{P(A) + P(B)} \quad (1)$$

Definition 1.4 bruges. Så fås

$$P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} \quad (2)$$

Den distributive lov bruges og regel om foreningsmængder, som ingen skæring har, bruges, og der fås

$$\frac{P(A \cap (A \cup B))}{P(A \cup B)} = \frac{P(A \cap A) \cup P(A \cap B)}{P(A \cup B)} = \frac{P(A)}{P(A) + P(B)} \quad (3)$$

Opgave 43 Show that both \emptyset and the sample space S are independent of any event. Explain intuitively.

Brug definitionen for uafhængighed og sæt S ind.

$$P(A \cap B) = P(A)P(B) \quad (4)$$

$$P(S \cap B) = P(S)P(B) \quad (5)$$

Da $P(S \cap B) = P(B)$ og $P(S) = 1$ fås

$$P(B) = P(B) \quad (6)$$

og udsagnet er altså sandt.

Det samme gøres med den tomme mængde. Da $P(\emptyset) = 0$ og $P(\emptyset \cap B) = 0$ fås

$$P(\emptyset \cap B) = P(S)P(B) \quad (7)$$

$$0 = 0 \quad (8)$$

Opgave 46 A fair coin is flipped twice. Explain the difference between the following: **(a)** the probability that both flips give heads, and **(b)** the conditional probability that the second flip gives heads given that the first flip gave heads.

Se, at

$$S = \{HH, HT, TH, TT\} \quad (9)$$

$$A = \{HH\} \quad (10)$$

$$B = \{HT, HH\} \quad (11)$$

Det ses dermed, at

$$P(A) = \frac{\#A}{\#S} = \frac{1}{4} \quad (12)$$

$$P(B) = \frac{\#B}{\#S} = \frac{1}{2} \quad (13)$$

Formlen for betinget sandsynlighed er dermed

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad (14)$$

$$= \frac{P(A)}{P(B)} \quad (15)$$

$$= \frac{1}{2} \quad (16)$$

Det ses, at de to sandsynligheder er forskellige. Rent intuitivt sker dette fordi der i **(b)** er elimineret to af mulighederne i S fra starten.

Opgave 50 You roll a dice twice and record the largest number. **(a)** Given that the first roll gives 1, what is the conditional probability that the largest number is 3? **(b)** Given that the first roll gives 3, what is the conditional probability that the largest number is 3?

Lad $S = \{(1, 1), (1, 2), \dots, (1, 6)\}$ og $A = \{(1, 3)\}$. Da have

$$P(A) = \frac{\#A}{\#S} = \frac{1}{6} \quad (17)$$

og

$$P(A/S) = \frac{P(A \cap S)}{P(S)} = \frac{P(A)}{1} = P(A) = \frac{1}{6} \quad (18)$$

Lad nu $S = \{(3, 1), (3, 2), \dots, (3, 6)\}$ og $A = \{(3, 1), (3, 2), (3, 3)\}$. Da have

$$P(A) = \frac{\#A}{\#S} = \frac{3}{6} = \frac{1}{2} \quad (19)$$

og

$$P(A/S) = \frac{P(A \cap S)}{P(S)} = \frac{P(A)}{1} = P(A) = \frac{1}{2} \quad (20)$$

Opgave 73 You roll a die and flip a fair coin a number of times determined by the number on the die. What is the probability that you get no heads?

Opgave 87