Discrete stochastic variables and distributions

Define

- Discrete RV
- Expected value (continuous)
- Variance

Prove

- Linearity of expected value
- Nonlinearity of variance

Definition 2.2. (Discrete RV) If the range of X is countable, then X is called a *discrete random variable*.

Definition 2.8.(Expected value) Let X be a discrete random variable with range $\{x_1, x_2, \ldots\}$ (finite or countably infinite) and probability mass function p. The *expected value* of X is defined as

$$E[X] = \sum_{k=1}^{\infty} x_k p(x_k) \tag{1}$$

Defintion 2.10. (Variance) Let X be a random variable with expected value μ. The *variance* of X is defined as

$$\mathrm{Var}[X] = E[(X-\mu)^2] \tag{2}$$

Proposition 2.11. (Linearity of E) Let X be any random variable, and let a and b be real numbers. Then

$$E[aX+b] = aE[X] + b \tag{3}$$

Bevis
Der haves for $Y = aX + b$, at

$$f_Y(y) = \frac{1}{a} f_X\left(\frac{y-b}{a}\right) \tag{4}$$

Fra definitionen af den forventede værdi fås

$$E[Y] = \sum_{k=1}^{\infty} y_k p_Y(y_k) = \frac{1}{a}\sum_{k=1}^{\infty} y_k p_X\left(\frac{y_k-b}{a}\right) \tag{5}$$

Lav variabelskifte således $y = ax + b$ hvilket giver $dy = a\,dx$ således

$$
\begin{aligned}
E[Y] &= \sum_{k=1}^{\infty}(ax_k + b)p_X(x_k) \\
&= a\sum_{k=1}^{\infty}x_k p_X(x_k) + b\sum_{k=1}^{\infty}p_X(x_k) = aE[X] + b
\end{aligned}
$$

Proposition 2.15. (Nonlinearity of Var) Let X be any random variable, and let a and b be real numbers. Then

$$
\mathrm{Var}[aX + b] = a^2\mathrm{Var}[X] \tag{6}
$$

Bevis
Brug definitionen af varians.

$$
\begin{aligned}
\mathrm{Var}[aX + b] &= E[(aX + b - E[aX + b])^2] \\
&= E[(aX + b - aE[X] - b)^2] = E[a^2(X - E[X])^2] \\
&= a^2E[(X - E[X)^2] = a^2\mathrm{Var}[X]
\end{aligned}
$$