Opgave 50

Opgave 51
The random variable X has a binomial distribution with $E[X] = 1$ and $\text{Var}[X] = 0.9$. Compute $P(X > 0)$.

Da $E[X] = 1 = np$ og $\text{Var}[X] = 0.9 = np(1-p)$ fås $p = 0.1$ og $n = 10$. Altså bruges binomialfordelingen.

$$P(X > 0) = \sum_{k=1}^{10} \frac{10!}{k!(10-k)!} 0.1^k (1-0.1)^{10-k} = 0.6513 \tag{1}$$

Opgave 52
Roll a die 10 times. What is the probability of getting **(a)** no 6s, **(b)** at least two 6s, and **(c)** at most three 6s.

(a) Her haves, at $n = 10$, $k = 0$ og $p = 1/6$.

$$\left(\frac{5}{6}\right)^{10} = 0.1615 \tag{2}$$

(b) Her haves, at $n = 10$, $k = 2, \dots, 10$ og $p = 1/6$.

$$P(X \geq 2) = \sum_{k=2}^{10} \frac{10!}{k!(10-k)!} \left(\frac{1}{6}\right) (1 - \frac{1}{6})^{10-k} = 0.51548 \tag{3}$$

(c) Her haves, at $n = 10$, $k = 0, \dots, 3$ og $p = 1/6$.

$$P(X \leq 3) = \sum_{k=0}^{3} \frac{10!}{k!(10-k)!} \left(\frac{1}{6}\right)^k (1 - \frac{1}{6})^{10-k} = 0.93 \tag{4}$$

Opgave 58
Let $X \sim \text{bin}(n,p)$ and $Y \sim \text{geom}(p)$. **(a)** Show that $P(X = 0) = P(Y > n)$. Explain intuitively. **(b)** Express the probability $P(Y \leq n)$ as a probability statement about X.

(a) Definitionerne skrives op.

$$P(X = 0) = \frac{n!}{0!(n-0)!} p^0 (1-p)^n = (1-p)^n \tag{5}$$

$$P(Y > n) = P(n \text{ consecutive failures}) = (1-p)^n \tag{6}$$

(b) $P(Y \leq n)$ er sandsynligheden for ikke at få n fiaskoer i træk. Dette kan beskrives som sandsynligheden for at få mindst én succes ud af n forsøg - $P(X > 0)$.

$$P(Y \leq n) = P(X > 0) \tag{7}$$

Opgave 61

Consider a sequence of independent trials that result in either succes or failure. Fix $r \geq 1$ and let X be the number of trials required until the rth success. Show that the pmf of X is

$$p(k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}, \qquad k = r, r+1, \ldots \tag{8}$$

This is called a *negative binomial* distribution with parameters r and p, written $X \sim \text{negbin}(r, p)$. What is the special case $r = 1$.