**Stochastic (random) variables**

**Definition**
A random variable $X$ is a function that takes its values from the sample space.

$$X : S \to \mathbb{R} \tag{1}$$

For a set A we note

$$P(X \in A) = P(\{s < inS, X(s) \in A\}) \tag{2}$$

**Comulative distribution function**
Let $X$ be a random variable with cumulative distribution function $F$. Then $F$ verfies the following properties:

1. For $a, b \in \mathbb{R}$ with $a < b$
   $F(b) = P(x \leq a) + P(a < X \leq b) = F(a) + P(a < x \leq b)$
2. $F(x) = 1 - P(X > x)$, for $x \in \mathbb{R}$
3. $F$ is *cadlag*: right-continuous with limits from the left
4. $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$

The cumulative distribution funtion (cdf) $F_x$ characterizes $X$ completely.

**Probability mass function**
Let $\{x_1, x_2, ...\}$ be the values of the discrete random variable $X$.
The function $p(x_k) = P(X = x_k), \quad k = 1, 2, ...$ is called the probability mass function.

Let $X$ be a random variable with cdf $F$ and pdf $p$.
Then we have the following properties:

1. $F(x_k) = \sum$ Nåede ikke mere. Lårt.

Står i bogen.

**Continuous variable**
If $F_x$ is continuously differentaible, $X$ is called a continuous random variable.
The function $f = F_x'$ is called the qualitatively density function of $X$.

Let $X$ be a continuous random variable with cdf $F$ and pdf $f$.
Then

1. $F(x) = \int_{-\infty}^{x} f(t)dt$
2. $f(x) = F_x'(x)$
3. $P(x \in B) = \int_B f(x)dt$

$f$ characterizes $X$ completely.

**The uniform distribution**
$X$ is a unform distribution on $[a, b]$ for $a < b$ if $f(x) = \frac{1}{b-a}$ if $x \in [a, b]$.
We write $X \sim Unif([a, b])$

Let $X \sim Unif([0, 1])$ and $Y = a + (b - a)X$.
Then $Y \sim Unif([a, b])$
*Proof* For $y \in \mathbb{R}$
$F_Y(y) = P(Y \leq y) = P(a + (b - a)X \leq y) = P(X \leq \frac{y-a}{b-a})$
A $X$ in $Unif([0, 1])$; we have

1. $F_X(x) = x$ if $f \in [0, 1]$
2. $F_X(x) = 0$ if $x \leq 0$
3. $F_X(x) = 1$ if $x \geq 1$

Hence

1. $F_Y(y) = \frac{y-a}{b-a}$ if $y \in [a, b]$
2. $F_X(y) = 0$ if $y \leq a$
3. $F_Y(y) = 1$ if $y \geq 1$

Therefore $f_y(y) = F_Y'(y) = \frac{1}{b-a}$ if $y \in [a, b]$.

Let $X$ be a random variable, $a, c \in \mathbb{R}$
Then