Opgave 53
Let X be the number of 6s when a die is rolled six times, and let Y be the number of 6s when a die is rolled 12 times. Find **(a)** $E[X]$ and $E[Y]$ and **(b)** $P(X \geq E[X])$ and $P(Y \geq E[Y])$.

(a) Forventede værdi beregnes ved

$$\begin{aligned} E[X] &= \sum_{k=\infty}^{\infty} x_k P(X = x_k) = \sum_{\infty}^{\infty} x_k p(x_k) \\ &= \sum_{k=0}^{6} \binom{6}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{6-k} k \\ &= 1 \end{aligned}$$

På samme måde med $E[Y]$.

$$E[Y] = \sum_{k=0}^{12} \binom{12}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{12-k} k$$

$$= 2$$

(b) Sandsynlighederne for at få 1 eller flere 6'ere summeres.

$$P(X \geq E[X]) = \sum_{k=1}^{6} \binom{6}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{6-k} = 0.665 \tag{1}$$

Det samme gøres for Y.

$$P(X \geq E[X]) = \sum_{k=2}^{12} \binom{12}{k} \left(\frac{1}{6}\right)^k \left(\frac{5}{6}\right)^{12-k} = 0.6187 \tag{2}$$

Opgave 66
(a) Flip a coin 10 times and let X be the number of heads. Compute $P(X \leq 1)$ exactly and with the Poisson approximation. **(b)** Now instead flip four coins 10 times and let X be the number of times you get four heads. Compute $P(X \geq 1)$ exactly and with the Poisson approximation. **(c)** Compare **(a)** and **(b)**. Where does the approximation work best and why?

(a) Sandsynligheden beregnes eksakt.

$$\sum_{k=0}^{1} \binom{10}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{10-k} = 0.0107$$

Parameteret λ er gennemsnittet af krone - altså 5. Approksimationen er dermed.

$$P(X \leq 1) = \sum_{k=0}^{1} \frac{5^k}{k!} \mathrm{e}^{-5} = 0.0404 \tag{3}$$

(b) Beregn hvor tit der fås 4 kroner ved 10 kast med 4 mønter.