

Conditional PMF

For two jointly discrete variables X and Y the conditional PMF is defined as

$$p_{Y|X}[y_j|x_i] = \frac{p_{X,Y}[x_i, y_j]}{p_X[x_i]} \quad (1)$$

Marginal PDF

The marginal PDF $p_X(x)$ of jointly distributed random variables X and Y is defined as

$$p_X(x) = \int_{-\infty}^{\infty} p_{X,Y}(x, y) dy$$

Properties

1. Joint PDF yields conditional PDFs

$$p_{Y|X}(y|x) = \frac{p_{X,Y}(x, y)}{\int_{-\infty}^{\infty} p_{X,Y}(x, y) dy}$$
$$p_{X|Y}(x|y) = \frac{p_{X,Y}(x, y)}{\int_{-\infty}^{\infty} p_{X,Y}(x, y) dx}$$

2. Conditional PDFs are related

$$p_{X|Y}(x|y) = \frac{p_{Y|X}(y|x)p_X(x)}{p_Y(y)}$$

3. Conditional PDF is expressible using Bayes' rule

$$p_{Y|X}(y|x) = \frac{p_{X|Y}(x|y)p_Y(y)}{\int_{-\infty}^{\infty} p_{X|Y}(x|y)p_Y(y) dy}$$

4. Conditional PDF and its corresponding marginal PDF yields the joint PDF

$$p_{X,Y}(x, y) = p_{Y|X}(y|x)p_X(x) = p_{X|Y}(x|y)p_Y(y)$$

5. Conditional PDF and its corresponding marginal PDF yields the other marginal PDF

$$p_Y(y) = \int_{-\infty}^{\infty} p_{Y|X}(y|x)p_X(x) dx$$