NOTATION, SYMBOLS AND ABBREVIATIONS

This first page provides a brief overview of the notational conventions, symbols the abbreviations used throughout the course. **It is expected that the students are familiar with all terms listed on this page.**

Sets

$\mathbb{N}$	The set of natural numbers $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	The set of integer numbers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathbb{R}$	The set of real numbers
$\mathbb{Z}^n$	The set of n-dimensional vectors with integer entries
$\mathbb{R}^n$	The set of n-dimensional vectors with real-valued entries

Abbreviations

pmf	Abbreviation for *probability mass function* (discrete random variables)
pdf	Abbreviation for *probability density function* (continuous random variables)
cdf	Abbreviation for *cumulative distribution function* (all random variables)
i.i.d.	Abbreviation for *independent and identically distributed*

Random variables (definitions)

X is a discrete RV	the range of X is countable (finite or infinite)
$P(X = x)$	the probability that X equals the outcome x
X is a continuous RV	the cdf $F_X(x)$ is a continuous function (range of X uncountable)
$P(X \in A)$	the probability that an outcome of X belongs to the set A

Functions

A function f which maps from the general set V to the general set W is written as $f : V \to W$. For example, with the cosine function we would write $\cos : \mathbb{R} \to [-1, 1]$.

List of symbols etc.

$\delta(k)$	Kronecker delta function (k being a discrete variable)
$\delta(t)$	The Dirac delta (t being a continuous variable)
$\mathbb{1}[\cdot]$	Indicator function, e.g. $\mathbb{1}[k = 0] = \delta(k)$
$\mathcal{N}(\mu, \sigma^2)$	Gaussian distribution with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$
$\mathcal{U}(a, b)$	Uniform distribution with pdf $f(\cdot) = \mathbb{1}[a \leq \cdot \leq b](b-a)^{-1}$ where $a < b$
$\mathcal{F}\{\cdot\}$	Fourier transform
$\mathbb{E}[X]$	Expected value of the random variable X
$\mathbb{V}\mathrm{ar}[Y]$	Variance of the random variable Y
$\mathbb{C}\mathrm{ov}[X, Y]$	Covariance between X and Y
$\mathbb{E}[X^n]$	The n'th moment of X
$\sim$	Means *distributed as*, e.g. $X \sim \mathcal{N}(\mu, \sigma^2)$
$\forall$	Means *for all*
$\times$	Cartesian product, e.g. $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$
$*$	Convolution
$:=$	Left hand side defined as right hand side, e.g. $\mathbb{V}\mathrm{ar}[Y] := \mathbb{E}\left[\left(Y - \mathbb{E}[Y]\right)^2\right]$

RANDOM PROCESSES AND MORE NOTATION

To get properly started, let's eliminate the doubt (if any) whether *random processes* are different from *stochastic processes*. They are not! These are simply to different words for the exact same thing.

Section 16.4 in Kay's book describes and distinguishes various types of random processes. A distinction is made whether a process is of type DTDV, DTCV, CTDV or CTCV. Also, a distinction is made between infinite processes and semi-infinite processes. Some authors also refer to discrete-time random processes as *random sequences*. **In this course we make no such distinctions whatsoever**. We simply refer to all these various quantities in one common way, namely as stochastic processes.

Another important aspect deals with the notational conventions in use. Some authors (including Kay) will often write sentences like

"Consider the random process $X[n]$",

while other authors would write

"Consider the random process $\{X[n]\}$".

This latter notation aims specifically at emphasizing that a random process is an entire collection of random variables. However, it will almost always be clear from a logical or a context point of view, whether the quantity to be considered is the entire process or the particular sample $X[n]$. Some authors (mathematicians in particular) would probably just write

"Consider the random process X",

and not even indicate whether time is discrete or continuous. Accordingly, we advise everyone to adopt familiarity with these different types of notation and also to practice in figuring out whether an entire process is under consideration or merely one of its samples.

Let's immediately get back to the sample $X[n]$ and its notation. Other but equivalent notations commonly encountered are for example $X(n)$ or X_n. Do not get confused, they are all the very same thing. Similarly, different authors use different notations when dealing with expected values, variances, covariances, etc. Hence, it makes no difference whether you encounter $\mathbb{E}[\cdot]$, $\mathbb{E}(\cdot)$, $\mathbb{E}\{\cdot\}$ or simply $\mathbb{E}$. They are all the same thing but the latter can in some rare cases turn out inappropriate, i.e. with the expression

$$\mathbb{E}(X-Y)^2,$$

it may not be completely clear whether we refer to

$$\mathbb{E}\big[(X-Y)^2\big] \qquad \text{or} \qquad \mathbb{E}\big[X-Y\big]^2.$$

Different font styles are also in use in the literature, e.g. $\mathbb{E}$, $\mathbf{E}$, $\boldsymbol{E}$, $\mathtt{E}$ or E. They could all be expected values depending on what book you are reading. Usually such notational conventions are to be found in the preliminary part of any text book.

COMMON MISCONCEPTIONS

- The Gaussian distribution and the normal distribution are the very same thing. Only the two names are different.
- When referring to probability expressions, the notations $P(\cdot)$ and $Pr(\cdot)$ are also the very same thing.
- The variance of a random variable is never a negative number.
- A random variable X is never independent or uncorrelated with itself, i.e. do not apply tricks like $\mathbb{E}\left[X^2\right] = \mathbb{E}[XX] = \mathbb{E}[X]\mathbb{E}[X]$.
- When dealing with the uniform distribution $\mathcal{U}(a, b)$, the two parameters a and b are the endpoints of the range, not the mean and the variance. That is, the usual parametrization of a Gaussian distribution $\mathcal{N}(\mu, \sigma^2)$ has absolutely nothing to do with the parametrization of a uniform distribution, and vice versa.
- Make sure not to confuse a discrete version of the uniform distribution, e.g. a fair die, with the continuous uniform distribution, e.g. $\mathcal{U}(0, 6)$ or perhaps $\mathcal{U}(1, 6)$. We recommend that you refer to the former as a discrete case which is *evenly distributed* across its range. In particular the mean and variance formulas valid for $\mathcal{U}(a, b)$ random variables do not apply for discrete random variables.
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DICTIONARY

ENGLISH	DANISH
set	mængde
sample space	udfaldsrum
event	hændelse
outcome	udfald
union ($\cup$)	foreningsmængde
intersection ($\cap$)	fællesmængde (eller snitmængde)
disjoint sets	disjunkte mængder
powerset	potensmængde
countable	tællelig (eller numerabel)
uncountable	overtællelig
probability	sandsynlighed
random variable	stokastisk variabel
distribution	fordeling (kaldes *ikke* en distribution på dansk)
cumulative distribution function (cdf)	fordelingsfunktion
probability mass function (pmf)	sandsynlighedsfunktion
probability density function (pdf)	tæthedsfunktion
expectation (or expected value)	middelværdi (eller forventningsværdi)
variance	varians
covariance	kovarians
correlation coefficient	korrelationskoefficient
marginal distribution	marginalfordeling
joint distribution	simultanfordeling
uniformly distributed	ligefordelt
Poisson distribution	Poisson fordelingen
Poisson process	Poisson proces
Poisson point process	Poisson punktproces
autocorrelation function	autokorrelationsfunktion
strict-sense stationary	stærkt stationær
wide-sense stationary	svagt stationær
power spectral density	effektspektrum (eller blot spektret)
even function (e.g. the cosine)	lige funktion (f.eks. cosinus)
odd function (e.g. the sine)	ulige funktion (f.eks. sinus)
decision rule	beslutningsregel