Discrete distributions

Bernoulli distribution. Let A be an event. Then random variable I_A defined by

$$I_A = \begin{cases} 1 \text{ if } A \text{ occurs} \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

is called the *indicator* of the event A or a *Bernoulli* random variable.

$$\begin{aligned} E[I_A] &= 1p + 0(1-p) = p \\ \mathrm{Var}[I_A] &= E[I_A^2] - (E[I_A])^2 = p(1-p) \end{aligned}$$

Binomial distribution. If X has probability mass function

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \ldots, n \tag{2}$$

it is said to have a *biinomial distribution* with parameters n and p, and we write $X \sim \mathrm{bin}(n, p)$.

$$\begin{aligned} E[X] &= np \\ \mathrm{Var}[X] &= np(1-p) \end{aligned}$$

Geometric distribution. If X has probability mas function

$$p(k) = p(1-p)^{k-1}, \qquad k = 1, 2, \ldots \tag{3}$$

it is said to have *geometric distribution* with parameter p, and we write $X \sim \mathrm{geom}(p)$.

$$\begin{aligned} E[X] &= \frac{1}{p} \\ \mathrm{Var}[X] &= \frac{1-p}{p^2} \end{aligned}$$

Poisson distribution. If X has probability mass function

$$p(k) = \mathrm{e}^{-\lambda} \frac{\lambda^k}{k!}, \qquad k = 0, 1, \ldots \tag{4}$$

it is said to have *Poisson distribution* with parameter $\lambda > 0$, and we write $X \sim \mathrm{Poi}(\lambda)$.

$$\begin{aligned} E[X] &= \lambda \\ \mathrm{Var}[X] &= \lambda \end{aligned}$$

Hypergeometric distribution. If X has probability mass function

$$p(k) = \frac{\binom{r}{k}\binom{N-r}{n-k}}{\binom{N}{n}}, \qquad k = 0, 1, \ldots, n \tag{5}$$

it is said to have a *hypergeometric distribution* with parameters N, r and n, written $X \sim \text{hypergeom}(N, r, n)$.

$$\begin{aligned} E[X] &= \frac{nr}{N} \\ \text{Var}[X] &= n\frac{N-n}{N-1}\frac{r}{N}\left(1-\frac{1}{N}\right) \end{aligned}$$

Continuous distributions

Exponential distribution. If the pdf of X is

$$f(x) = \lambda \mathrm{e}^{-\lambda x}, \qquad x \geq 0 \tag{6}$$

then X is said to have an *exponential distribution* with parameter $\lambda > 0$, written $X \sim \exp(\lambda)$.

$$E[X] = \frac{1}{\lambda}$$
$$\mathrm{Var}[X] = \frac{1}{\lambda^2}$$

Normal distribution. If X has pdf

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-(x-\mu)^2/\sigma^2}, \qquad x \in \mathbb{R} \tag{7}$$

it is said to have a *normal distribution* with parameters μ and σ^2, written $X \sim N(\mu, \sigma^2)$.

$$E[X] = \mu$$
$$\mathrm{Var}[X] = \sigma^2$$

Corollary 2.24. Suppose that $X \sim N(\mu, \sigma^2)$ and let $Z = (X - \mu)/\sigma$. Then $Z \sim N(0, 1)$.