Opgave 137
Let $X_1, X_2, \ldots$ be iid nonnegative and integer valued, and let $N \sim \mathrm{Poi}(\lambda)$, independent of X_k. If the random sum $S_N = X_1 + \ldots + X_N$ has a Poisson distribution with mean μ, what is the distribution of X_k.

Beregn $G_X(s)$ ved

$$G_{S_N} = \mathrm{e}^{\mu(s-1)} = \mathrm{e}^{\lambda(G_X(s)-1)}$$
$$\Downarrow$$
$$G_X(s) = \frac{\mu(s-1)}{\lambda} + 1$$

Siden $G_X(s)$ er et polynomium af grad 1, så har forsøget kun 2 forskellige resultater. Altså er det en Bernoullifordeling.

Opgave 139
Jobs arrive at a computer at a rate of 1.8 jobs/day. Each job requires an execution time (in milliseconds) that has a binomial distribution with $n = 10$ and $p = 0.5$ (where a zero execution time means that the job is rejected). Find **(a)** the probability that the total execution time in a day for this type of job is at least 2 ms, and **(b)** the expected total execution time in a day.

(a) Antag, at jobs/dag er Poissonfordelt. Altså haves

$$G_X(s) = (0.5 + 0.5s)^{10}$$
$$G_N(s) = \mathrm{e}^{1.8(s-1)}$$

$G_X(s)$ sættes ind i $G_N(s)$.

$$G_{S_N}(s) = G_N(G_X(s)) = \mathrm{e}^{1.8((0.5+0.5^{10})-1} \tag{1}$$

Derefter beregnes sandsynligheden

$$P(S_N \geq 2) = \sum_{k=0}^{1} \frac{G'_{S_N}(0)}{k!} = 0.831 \tag{2}$$

(b) Find forventet værdi af binomialfordelingen og gang med 1.8.

$$1.8\,E[X] = 1.8\,G'(1) = 1.8\left((0.5 + 0.5s)^{10}\right) = 9 \tag{3}$$

Opgave 151
Traffic accidents in a town occur according to a Poisson process at a rate of

two accidents per week. **(a)** What is the probability that a given day has no accidents? **(b)** Whenever there is an accident, the risk is 1 in 10 that it will cause personal injury. What is the probability that a given month has at least one such accident? **(c)** Let N be the number of accident-free weeks in a year. What is the distribution of N?

(a) Antal uheld pr. uge er Poissonsfordelt med $\lambda = 2$. Antal uheld pr. dag er Poissonfordelt med $\mu = \frac{2}{7}$. Altså fås

$$P(X(1/7) = 0) = \mathrm{e}^{-2/7}\frac{(2/7)^0}{0!} = 0.75 \tag{4}$$

(b) Brug samme formel som før med $\lambda = 2/10$ og $t = 4$.

$$P(X(4) = 0) = 1 - \mathrm{e}^{-8/10}\frac{-8/10^0}{0!} = 0.55 \tag{5}$$

Opgave 4

Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables. The *harmonic mean* is defined as

$$H_n = \left(\frac{1}{n}\sum_{k=1}^{n}\frac{1}{X_k}\right)^{-1} \tag{6}$$

Suppose that the pdf of the X_k is $f(x) = 3x^2$, $0 \leq x \leq 1$, and find the limit of H_n as $n \to \infty$.

Brug at

$$E[g(x)] = \int_0^1 g(x) f_x(x)\, dx \tag{7}$$

Altså

$$H_n \to E[g(x_k)] \text{ for } n \to \infty \tag{8}$$

Så der haves

$$H_n = \int_0^1 \frac{1}{x} 3x^2\, dx = \frac{2}{3} \text{ for } n \to \infty \tag{9}$$

Opgave 10

In any given day, a certain email account gets a number of spam emails that has a Poisson distribution with mean 200. What is the approximate probability that it receives less than 190 spam emails in a day?