Expectation and variance

Expected value of a discrete random variable

$$E[X] = \sum_{-\infty}^{\infty} x_k P(X = x_k) = \sum_{-\infty}^{\infty} x_k p(x_k) \tag{1}$$

- Expectation $\Leftrightarrow$ expected value $\Leftrightarrow$ average $\Leftrightarrow$ mean
- $E[X]$ is a constant/non-random
- $E[X]$ may take a value different from X

Expectation of a continuous random variable

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx \tag{2}$$

Linearity of expected value

$$E[aX + b] = aE[X] + b \tag{3}$$

Fra sidste forelæsning

$$f_Y(y) = \left| \frac{d}{dy} g^{-1}(y) \right| f_x(g^{-1}(y)) \tag{4}$$

Expected value of function g

$$E[g(X)] = E[Y] = \int_{-\infty}^{\infty} f_y(y)\, dy = \int_{-\infty}^{\infty} y \left| \frac{d}{dy} g^{-1}(y) \right| f_x(g^{-1}(y))\, dy \tag{5}$$

$$= \int_{-\infty}^{\infty} g(x) f_x(x)\, dx \tag{6}$$

Variance of a random variable

$$Var[X] = E[(X - E[X])^2] \tag{7}$$

$\sigma = \sqrt{Var[X]}$ is called standard deviation.

Huygens formula

$$Var[X] = E[X^2] - E[X]^2 \tag{8}$$

Chebyshev's inequality

Let X be any random variable with mean μ and variance σ^2. For any constant $c > 0$, we have

$$P(|X - \mu| \le c\sigma) \le \frac{1}{c^2} \tag{9}$$

Chebyshev's inequality helps interpret variance and gives som upper bound for the deviation from the mean.

Non-linearity of variance
Let X be any random variable, and let a and b be real numbers. Then

$$Var[aX + b] = a^2 Var[a] \tag{10}$$