Opgave 28
I a street bet in roulette you bet on three numbers. If any of these come up, you win 11 times your wager, otherwise you lose your wager. Let X be your gain if you bet one dollar on a street bet. Find the mean and variance of X.

Der vindes på tre forskellige tal og tabes på 35 forskellige.

$$E[X] = 11\frac{3}{38} + (-1)\frac{35}{38} = -\frac{2}{38} \tag{1}$$

Varians beregnes ved nedenstående formel.

$$Var[X] = E[X^2] - E[X]^2 = 11^2\frac{3}{38} + \left((-1)\frac{35}{38}\right)^2 \tag{2}$$

Opgave 30
The game of chuck-a-luck is played with three dice, rolled independently. You bet one dollar on one of the numbers 1 through 6 and if exactly k of the dice show your number, you win k dollars k = 1, 2, 3 (and keep your wagered dollar). If no die shows your number, you lose your wagered dollar. What is your expected loss?

$$E[X] = (-1)\left(\frac{5}{6}\right)^3 + 1\frac{5^2 \cdot 3}{6^3} + 2\frac{3 \cdot 5}{6^3} + 3\left(\frac{1}{6}\right) = -0.078703 \text{ cent} \tag{3}$$

Opgave 35
A stick measuring one yard in length is broken into two pieces at random. Compute the expected length of the longest piece.

Først bestemmes en funktion $g(x)$.

$$g(x) = \max(x, 1-x) \tag{4}$$

Derefter bruges relationen for gennemsnit af $g(x)$:

$$\begin{aligned} E[g(x)] &= \int_0^1 g(x) f_X(x)\, dx & (5)\\ &= \int_0^1 g(x)\, dx & (6)\\ &= \int_0^{0.5} 1 - x\, d + \int_{0.5}^1 x\, dx & (7)\\ &= \frac{3}{4} & (8) \end{aligned}$$

Opgave 38
The random variable X has pdf $f(x) = 3x^2, 0 \geq x \geq 1$. **(a)** Compute $E[X]$ and $Var[X]$. **(b)** Let $Y = \sqrt{X}$ and compute $E[Y]$ and $Var[X]$.

(a)

$$\begin{aligned} E[X] &= \int_0^1 x f(x)\, dx && (9) \\ &= \int_0^1 3x^3\, dx && (10) \\ &= \left[\frac{3}{4}x^4\right]_0^1 && (11) \\ &= \frac{3}{4} && (12) \end{aligned}$$

(b)

$$\begin{aligned} Var[X] &= E[(X - \frac{3}{4})^2] && (13) \\ &= E[X^2 + \frac{9}{16} - \frac{3}{2}X] && (14) \\ &= E[X^2] - \frac{3}{2}E[X] + \frac{9}{16} && (15) \\ &= \frac{3}{5} - \frac{9}{8} + \frac{9}{16} && (16) \\ &= \frac{3}{80} = 0.0375 && (17) \end{aligned}$$

Opgave 39
Let $X \geq 0$ be continuous. Show that $E[X] = \int_0^\infty P(X > x)\, dx$ and $E[X^2] = 2\int_0^\infty xP(X > x)\, dx$.

Der startes med definitionen

$$
\begin{aligned}
E[X] &= \int_0^\infty x f(x)\, dx && (18)\\
&= \int_0^\infty \int_0^x f(t)\, dt\, dx && (19)\\
&= \int_0^\infty \int_t^\infty f(x)\, dx\, dt && (20)\\
&= \int_0^\infty P(x \in [t, \infty)\, dt && (21)\\
&= \int_0^\infty P(X > x)\, dx && (22)
\end{aligned}
$$