Joint distribution function of random variable (X, Y)

$$F(x, y) = P(X \leq x, y \leq y), \text{ for } x, y \in \mathbb{R} \tag{1}$$

Properties

- $F(x, \infty) = \lim_{y\to\infty} F(x, y)$
- $F(\infty, y) = \lim_{x\to\infty} F(x, y)$

Marginal cdf of (X, Y)

$$F_X(x) = F(x, \infty)$$
$$F_Y(y) = F(\infty, y)$$

If X and Y are independent then

$$F(x, y) = F_X(x)F_X(y) \tag{2}$$

Proposition
Let $X_1 \sim \text{Poi}(\lambda_1)$ and $X_2 \sim \text{Poi}(\lambda_2)$. Then

$$X_1 \text{ and } X_2 \text{ are independent } \Leftrightarrow \{X_1+X_2 \sim \text{Poi}(\lambda_1+\lambda_2) \text{ and } X_1(X_1+X_2) = n \sim \text{Bin}(n, \frac{\lambda_1}{\lambda_1+\lambda_2})\} \tag{3}$$

Continuouss stochastic vector
X and Y are continuous random variables with pdf f_x and f_y respectively.

Definition X and Y are jointly continuous if there ecists a function f from R^2 into R such that

$$P((X, Y) \in B) = \int\int_B f(x, y)\, dx\, dy, \text{ for all } B \subset \mathrm{R}^2 \tag{4}$$

- f is called the joint pdf og (X, Y)
- the conditional pdf of Y given $X = x$ is

$$f_y(y/x) = \begin{cases} \frac{f(x,y)}{f_x(x)} \text{ if } f_x(x) > 0 \\ 0 \text{ if else} \end{cases} \tag{5}$$

- the conditional cdf of Y given X is given by

$$F_y(y/x) = \int_{\infty}^{y} f_y(t/x)\, dt \tag{6}$$

- the conditional distribution of Y given $X = x$ is

Proposition Let (X, Y) be jointly continuous

1. If Area$(B) = 0$. $(f_x \, a \, line) \Rightarrow P((X; Y) \in B) = 0$
2. f is positive valued and $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y) = 1$
3. For all $x, y \in \mathbb{R}$. $F(x, y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(s, t) \, dt \, ds$
4. $f(x, y) = \frac{d^2}{dxdx} F(x, y) = \frac{\partial^2}{\partial x \partial y} F(x, y)$
5. $f_x(a) = \int_{-\infty}^{\infty} f(x, y) \, dy = \int_{-\infty}^{\infty} f_x(x/y) f_y(y) \, dy$
6. For $B \subset \mathbb{R}$. $P(Y \in B) = \int_{-\infty}^{\infty} P(Y \in B)/X = x) f_x(x) \, dx$
7. For $B \subset \mathbb{R}$. $P((x, y) \in B) = \int_{-\infty}^{\infty} P((x, y) \in B/X = x) f_x(x) \, dx$
8. X and Y are independent $\Leftrightarrow$ $F(x, y) = F_x(x) F_y(y)$ $\Leftrightarrow$ $f(x, y) = f_x(x) f_y(y)$ $\Leftrightarrow$ $f_y(y/x) = f_y(y)$