Opgave 1
Let $X_1, X_2, \ldots$ be a sequence of random variables with the same mean μ and variance σ^2, which are such that $\text{Cov}[X_j, X_k] < 0$ for all $j \neq k$. Show that $\overline{X} \xrightarrow{P} \mu$ as $n \to \infty$.

Brug korollar 3.24 til at beregne variansen, da variablene ikke er uafhængige.

$$\begin{aligned}
\text{Var}[\overline{X}] = \text{Var}\left[\sum_{k=1}^{n} a_k X_k\right] &= \sum_{k=1}^{n} a_k^2 \text{Var}[X_k] + \sum_{i \neq k} \text{Cov}[X_i, X_j] \\
&= \text{Var}\left[\sum_{k=1}^{n} \frac{1}{n} X_k\right] + \sum_{i \neq j} [X_i, X - j] \\
&= \frac{1}{n^2} \text{Var}\left[\sum_{k=1}^{n} X_k\right] + \sum_{i \neq j} [X_i, X_j] \\
&= \frac{1}{n^2} n\sigma^2 + \sum_{i \neq j} [X_i, X_j] \\
&\leq \frac{\sigma^2}{n}
\end{aligned}$$

Nu kan Chebychevs ulighed bruges - sæt $c = \varepsilon \frac{\sqrt{n}}{\sigma}$:

$$P\left(|\overline{X} - \mu| \geq c\sqrt{\text{Var}[\overline{X}]}\right) \leq \frac{1}{c^2}$$

$$P\left(|\overline{X} - \mu| \geq \varepsilon \frac{\sqrt{n}}{\sigma} \sqrt{\text{Var}[\overline{X}] \leq \varepsilon)}\right) \leq \frac{\sigma^2}{n\varepsilon^2} \to 0 \text{ for } n \to \infty$$

Opgave 3
Let $X_1, X_2, \ldots$ be iid unif$[0, 1]$ and let $g : [0, 1] \to \mathbb{R}$ be a funtion. What is the limit of $\sum_{k=1}^{n} g(X_k)/n$ as $n \to \infty$? How can this result be used?

Opgave 8
Use the central limit theorem to argue that the following random variables are approximately normal; also give the parameters: **(a)** $X \sim \Gamma(n, \lambda)$ for large n and **(b)** $X \sim \text{Poi}(\lambda)$ for large λ.