Master Mind

Let's consider single-digit grading.

Complexity Estimation → we have binary digits $\{0,1\}$ and we are grading correct possitions and our word can contain n digits. So classical estimation will be:

$$\underline{2}_1\ \underline{2}_2\ \underline{2}_3\ \underline{2}_4\ \underline{2}_5\ \ldots\ \underline{2}_{n-1}\ \underline{2}_n$$

$$\Rightarrow \boxed{2^n}$$

Quantum complexity estimation

$$N_0 = 2^n$$

$$N_1 = \binom{n}{k} = \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\sqrt{2\pi k}\left(\frac{n}{e}\right)^k \sqrt{2\pi(n-k)}\left(\frac{n-k}{e}\right)^{n-k}}$$

$$k = \frac{n}{2} \Rightarrow N_n(max) = \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{2\pi\frac{n}{2}\left(\frac{n}{2e}\right)^n} = \sqrt{\frac{2}{\pi}}\frac{2^n}{\sqrt{n}} \Rightarrow \frac{N_0}{N_1} = \sqrt{\frac{\pi}{2}}\cdot\sqrt{n} \Rightarrow$$

$$\Rightarrow \boxed{\sqrt{n}}$$

Algorithm:

Assume that we have 5 digits. At first we do $|0\rangle$ on $H^{\otimes 5}$ times. We need to calculate difference btw our guesses and current state score. If diff $> 0 \Rightarrow$ xor 1's, else xor 0's and then we flipp right and wrong digits and get some state which have the same score.

We start flipping from the left least significant Bit approach.

To help ourselves lets define the following functions:

$|x\rangle$ - 0000...1111

$|r\rangle$ - reference number

k - next matches

$$f(x, r, k) = \begin{cases} 1, & \text{if } x \otimes r = n - k \\ 0, & \text{otherwise} \end{cases}$$

if $n = 5 \Rightarrow U_+ |x\rangle = (-1)^{f(x,r,k)} |x\rangle$

We can say that $x \otimes r$ contains the number of 1's, which is showing number of missmatches btw x and r.

So let's see an example.

$$H^{\otimes 5} |100000\rangle = \frac{1}{\sqrt{32}} \sum_{x=0}^{31} |x\rangle \xrightarrow{U_x|x\rangle} \frac{1}{5} \left(\sum_{f(x,r,k)=0} |x\rangle - \sum_{f(x,r,k)=1} |x\rangle \right)$$

After collapsing, we get some state and that state we can compare with our number and getting new number. We can do this again and again and we can get same number. To not have this type of situations we can keep n parallel superpositions and compare scores with xor's and when the mismatches are 0 $\Rightarrow$ that is the state.

Thank you very much :)