CECS 552 Programming Assignment 2

Monte Carlo Simulation, Part 2

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Approximating an Integeral

Have a program option that takes as input the number n of desired independent samples, and uses them to approximates the integral

$$\int_0^{\frac{\pi}{2}} \int_0^x e^{\cos(x+y)} dy dx.$$

In addition to $\hat{\lambda}$ (the integral approximation), print the sample variance, standard error, and the 95% confidence interval.

Monte Carlo Simulation for Playing Blackjack

A simple version of the (two-player) game of Blackjack is described as follows. For this version we use four special decks of cards in which each deck, in addition to its usual cards, has an additional four cards, each with a numerical value equal to 1. The four decks of cards are randomly shuffled together to make a **dealing deck** of $56 \times 4 = 224$ cards total. The value of a number card equals the card number, the value of a face card equals 10, while the value of an ace equals 11 (aces no longer can equal 1, since we've added the 1 cards). In what follows we let random variable X represent the result of drawing a card at random from the (entire) dealing deck. For example, $P(X = i) = 16/224 = 1/14$, for all $i = 1, \ldots, 9, 11\}$, while $P(X = 10) = 64/224 = 2/7$.

Each player is first dealt a card facing downward that is hidden from her opponent. Players are then alternately dealt cards facing upward that are visible to the opponent. Before a card is dealt to a player, she has the option of **holding**, meaning that she is not dealt any further cards. If she does not hold, then she **hits**, meaning that she is dealt an additional card. Play ends when one of the following has occurred:

1. a player hits, and the card dealt to her results in a sum (of the values of all her cards) equal to 21, in which case she immediately wins the game;
2. a player hits, and the card dealt to her results in a sum that exceeds 21, in which case she immediately loses the game; or
3. both players have decided to hold, in which case the winner is the player whose card values have the highest sum (a tie occurs if the sums are equal).

When a game ends, all dealt cards are revealed to both players, and the cards are placed in the **used pile**, and are never dealt again during the **match**, which is a sequence of games that ends once the dealing deck is exhausted (if this happens during a game, then the game ends in a tie). Note that, during a match, the players take turns at being dealt the first card.

Now consider two different strategies for playing the game. For the first strategy, called the **naive** strategy, player P hits provided one of the following is true. Let S denote the sum of P's cards.

1. P's opponent has not yet decided to hold, and $P(X + S \leq 21) > p_1$, where $p_1 \geq 0.5$ is some threshold value that is provided as input by the user. Here X represents the value of the next card that will be dealt if P decides to hit. The reasoning is that P hits provided there is sufficient probability that the next card dealt will not bring her sum over 21.
2. P's opponent has already decided to hold, and $E[X|X+S_v < 21]+S_v > S$. Here, X represents the opponent's hidden card. In words, P calculates the expected value of the hidden card, on condition that, when adding it to the visitble sum, it must give a value that is less than 21. Then, if this expected value is added to the visible sum, and it exceeds S, then P hits because she believes that her opponent has held with a better hand.
3. P's opponent has already decided to hold, and $P(X + S \leq 21) \geq p_2$, where $p_2 \geq 0.5$ is a threshold value provided as input by the user. In this case, P hits because she has a good chance of not going over 21. Of course, how "good" depends on the value of p_2.

The second strategy, called **Monte Carlo** is for P to conduct a Monte Carlo simulation. A **sampling event** for such a simulation consists of randomly sampling 20 cards, $c_0, c_1, \ldots, c_{19}$, from the (unused part of) the dealing deck. Note: if there are fewer than 20 cards remaining in the deck, then P samples the entire remaining deck. The first sampled card c_0 represents the opponent's hidden card. while the remaining cards represent the hypothetical values and order of the next 19 cards in the deck. The outcome of a sampling event is either HIT, HOLD, or NEUTRAL. To determine the sampling-event outcome, P first compares her sum S against her opponent's sum S_o (note that S_o includes c_0), and then considers the following cases.

1. Her opponent is holding and $S > S_o$. Then the outcome is HOLD.
2. Her opponent is holding and $S < S_0$. Then the outcome is HIT.

3. Her opponent is holding and $S = S_0$. Then the outcome is HIT if $S + c_1 \leq 21$, and is HOLD otherwise.
4. Her opponent is not holding and $S < S_0$. Then the outcome is HIT.
5. Her opponent is not holding, $S \geq S_0$, and $S + c_1 > 21$. Then the outcome is HOLD.
6. Her opponent is not holding, $S \geq S_0$, and $S + c_1 = 21$. Then the outcome is HIT.

In the case where her opponent is not holding, $S \geq S_o$, and $S + c_1 < 21$, P considers two subcases. The first subcase, called the **hold subcase**, is that P holds. P then checks if there is a j for which

$$S < S_o + \sum_{i=1}^{j} c_i \leq 21.$$

If such a j exists, then the outcome of this subcase is LOSE. Otherwise, the outcome is WIN. The second subcase, called the **hit subcase**, is that P hits and receives c_1. In this case the remainder of the game is simulated, in such a way that each player hits provided the next card does not bring her over 21. Otherwise she holds. Then the game simulation ends when either i) a player reaches 21, ii) both players hold, or iii) there are no more cards to be dealt (in which case the game is tied). If P wins the simulation, then the outcome of this subcase is WIN. If P loses, then the outcome is LOSE. If P ties, then the outcome is TIE. The outcome of the sampling event is now computed based on the outcomes of the two subcases, with the help of the following table.

Hold Subcase Outcome	**Hit Subcase Outcome**	**Sampling Event Outcome**
WIN	LOSE/TIE	HOLD
WIN	WIN	NEUTRAL
LOSE	LOSE	NEUTRAL
LOSE	WIN/TIE	HIT

P conducts a sequence of these sampling events and records the number of HOLD, HIT, and NEUTRAL outcomes, until one of the following occurs. Note that in what follows, N represents a user input.

1. N HIT outcomes are recorded, in which case P hits,
2. N HOLD outcomes are recorded, in which case P holds,
3. 100 *consecutive* NEUTRAL outcomes are recorded, in which case P hits (by default).

Have a program option that takes as input the values M, N, p_1, p_2, and simulates M consecutive matches between a naive player and an MC player. Report on the total number of games won by each player, along with their respective winning percentages.