CECS 328: Program #1

For each of the tasks, I have included the algorithms in C# as well as a screenshot of the outputs. The source code for the project is available here:
https://github.com/Vardominator/CSULBProjects/blob/master/CECS328_DataStructuresAlgorithms/Project1/Project1/Project1/Program.cs

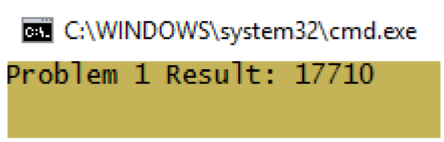
1. Write a program to calculate S(n) by calculating the values of the Fibonacci sequence recursively.

Source:

```
public static long Sn(long number)
{
    long result = 0;

    for (int i = 1; i <= number; i++)
    {
        result += Fibonacci(i);
    }

    return result;
}

public static long Fibonacci(long number)
{
    if(number <= 2)
    {
        if(number == 0)
        {
            return 0;
        }
        return 1;
    }

    return Fibonacci(number - 1) + Fibonacci(number - 2);
}
```

Sample run:

```
#region Problem 1: Calculate S(n) by calculating the values of the Fibonacci sequence recursively
// S(n) definition: S(n) = f(0) + f(1) +...+ f(n)

int sampleN = 20;
Console.WriteLine($"Problem 1 Result: {Sn(sampleN)}\n\n");
#endregion
```



2. Write a non-recursive program to calculate S(n).

Source:

```
public static long NonRecursiveSn(long number)
{
    long previous = 1;
    long previous2 = 1;
    long current = 0;
    long sum = 2;

    for (int i = 3; i <= number; i++)
    {
        current = previous + previous2;
        sum += current;
        previous = previous2;
        previous2 = current;
    }

    return sum;
}
```

Sample run:

```
#region Problem 2: Write a non-recursive program to calculate S(n)
Console.WriteLine("Problem 2 Result: \n");
Console.WriteLine($"Fibonacci Table for n = {sampleN}:");

for (int i = 0; i <= sampleN; i++)
{
    Console.WriteLine($"f({i}) = {Fibonacci(i)}");
}
Console.WriteLine("------------");
Console.WriteLine($"Sum: {NonRecursiveSn(sampleN)}\n\n");
#endregion
```

C:\WINDOWS\system32\cmd.exe

```
Problem 2 Result:

Fibonacci Table for n = 20:
f(0) = 0
f(1) = 1
f(2) = 1
f(3) = 2
f(4) = 3
f(5) = 5
f(6) = 8
f(7) = 13
f(8) = 21
f(9) = 34
f(10) = 55
f(11) = 89
f(12) = 144
f(13) = 233
f(14) = 377
f(15) = 610
f(16) = 987
f(17) = 1597
f(18) = 2584
f(19) = 4181
f(20) = 6765
------------
Sum: 17710
```

3. Algebraically verify that g(n) is a solution of Equ 1 by substituting g(n) in Equ 1.

   (This portion is attached to the assignment packet)

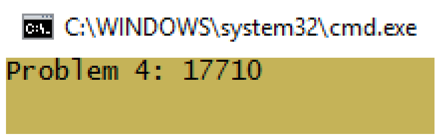
4. Write a third iterative program by summing g(n) over n.

Source:

```
public static long GrimaldiSum(long number)
{
    long result = 0;

    for (int i = 1; i <= number; i++)
    {
        result += Grimaldi(i);
    }

    return result;
}

public static long Grimaldi(long k)
{
    return (long)((1 / Math.Sqrt(5)) * (Math.Pow(((1 + Math.Sqrt(5)) / 2), k) - Math.Pow(((1 - 
Math.Sqrt(5)) / 2), k)));
}
```

Sample run:

```
#region Problem 4: Third possible way to sum Fibonnaci
Console.WriteLine($"Problem 4: {GrimaldiSum(20)}\n\n");
#endregion
```



5. Calculate these values of S for n = 10, 20, 30. Calculates values of f for n = 12, 22, 32.

Sample run:

```
#region Problem 5: Calculate S for 10, 20, 30; Calculate f for 12, 22, 32
Console.WriteLine($"Problem 5 results: \n");
Console.WriteLine($"S(n) for n = 10 => {GrimaldiSum(10)}");
Console.WriteLine($"S(n) for n = 20 => {GrimaldiSum(20)}");
Console.WriteLine($"S(n) for n = 30 => {GrimaldiSum(30)}\n");
Console.WriteLine($"f(n) for n = 12 => {Fibonacci(12)}");
Console.WriteLine($"f(n) for n = 22 => {Fibonacci(22)}");
Console.WriteLine($"f(n) for n = 32 => {Fibonacci(32)}\n");
#endregion
```

C:\WINDOWS\system32\cmd.exe

```
Problem 5 results:

S(n) for n = 10 => 143
S(n) for n = 20 => 17710
S(n) for n = 30 => 2178308

f(n) for n = 12 => 144
f(n) for n = 22 => 17711
f(n) for n = 32 => 2178309
```

6. Prove the identity S(n) = f(n + 2) – 1 using induction.

   (This portion is attached to the assignment packet)

7. Example run using the alternate sum from #6.

Source:

```
public static long SnAlt2(long number)
{
    return Fibonacci(number + 2) - 1;
}
```

Sample run:

```
#region Problem 7: Fourth way to calcualte S(n)
Console.WriteLine($"Problem 7: {SnAlt2(20)}\n\n");
#endregion
```

C:\WINDOWS\system32\cmd.exe

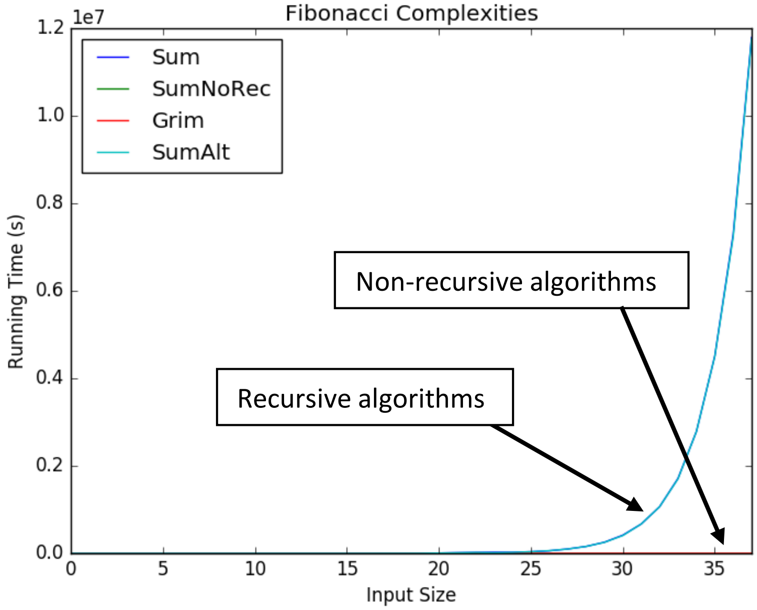
```
Problem 7: 17710
```

8. Since I am using Int64 (long in C#) to calculate the sums, the largest n that can be used is such that S(n) does not exceed 9,223,372,036,854,775,807. Another limitation would be the possibility of running out of stack space. Since the Fibonacci calls are recursive, a sufficiently large n would cause the program to crash.

9. Compare the running times of the 4 methods.

   I computed the sum with each method for values of n starting at 1 and ending at 37. I recorded the running times for each. Finally, I ran a python script to plot the growth of the running times versus the input size.

10. Summary and conclusions.



	Sum	SumNoRec	Grim	SumAlt
0	2	0	7	1
1	3	0	7	1
2	2	1	6	1
3	3	0	10	2
4	2	0	8	2
5	3	1	8	2
6	6	1	10	4
7	56	0	9	7
8	60	1	7	41
9	66	1	6	47
10	71	1	39	26
11	86	0	8	90
12	71	1	8	70
13	118	1	10	115
14	195	0	9	188
15	392	0	10	317
16	485	0	11	522
17	829	1	9	794
18	1334	0	9	1334
19	2155	0	10	2159
20	7250	1	29	8016
21	14458	3	44	13342
22	20046	2	38	20191
23	24748	1	19	14958
24	24231	2	22	22119
25	36711	1	19	37355
26	58723	1	23	61490
27	99332	1	24	98611
28	157938	1	26	157777
29	254731	1	24	251517
30	415648	1	24	407213
31	670433	1	25	670161
32	1065764	1	25	1065740
33	1713357	1	25	1704364
34	2784081	1	26	2784430
35	4510864	1	27	4504125
36	7290340	1	27	7238976
37	11789164	1	27	11757170

We see in the plots above that the 4th method grows the fastest by a considerable amount. However, the difference is not much. The drawbacks of the recursive methods are that they have linear growth in space complexity: $O(n)$, as opposed to the non-recursive methods that have a constant space complexity.

Mathematically the recurrence relation for the Fibonacci sequence is $T(n) = T(n-1) + T(n-2) + O(1)$. Using a recursion tree, we can prove that the time complexity is $O(2^n)$. This means that the recursive Fibonacci sequence grows relatively fast. In fact, a trial run using a recursive method with an input size of 50 took me about 30 minutes to run. This is incredibly inefficient and can be seen in the table to the right. I have displayed here the input size on the left most column, the results of the recursive methods on the second and fifth columns and the non-recursive methods in the middle columns (their values represent the elapsed ticks of the timer in C#). I realized that after an input size larger 35, the incredibly different growth rates became evident.

Finally, it is worth noting that a recursive solution to the sum is slower than the **Grimaldi** solution. I have shown this in the plots above. It is very clear that the recursive methods dominate in regards to computational time (in addition to computational space). Sum and SumAlt overlap because their computation times are nearly identical, but the non-recursive running times are so insignificant they are not even visible.