CECS 328: Program #1

You are familiar with the Fibonacci sequence from various places

Equ 1
$$f(1)=1$$
$$f(2)=1$$
$$f(n)=f(n-1)+f(n-2).$$

Let's define a sum as $S(n)=f(0)+f(1)+\cdots+f(n)$. This assignment involves experimenting with various approaches to compute *S* (*n*), as well as, demonstrating various algebraic techniques for recursive definition.

Tasks for this assignment

1. Write a program to calculate *S* (*n*) by calculating the values of the Fibonacci sequence recursively.

2. Write a non-recursive program to calculate *S* (*n*). This second program uses the recurrence definition to calculate and TABULATE the values of the Fibonacci sequence. Then, sum these values to find *S* (*n*).

3. *Discrete & Combinatorial Mathematics* by Ralph Grimaldi outlines a method to obtain the solution $g(n)=\left(\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n-\left(\frac{1}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n$.
   Algebraically verify that *g* (*n*) is a solution of Equ 1 by substituting *g* (*n*) in Equ 1.

4. From task #3, there is now a third method to calculate *S* (*n*). Write a third iterative program by summing: $S(n)=\sum_{k=0}^{n}\left(\frac{1}{\sqrt{5}}\right)\left(\frac{1+\sqrt{5}}{2}\right)^k-\left(\frac{1}{\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^k$.

5. Use your preferred program to calculate these values of *S* for *n* = 10, 20, 30. Also, compute these values of *f* for *n* = 12, 22, 32.

6. Task #7 suggests that $S(n)=f(n+2)-1$. Prove this identity (using induction).

7. Finally, there is yet a fourth way to programmatically calculate *S* (*n*).

8. Experiment with your programs to estimate the largest *n* that can be computed successfully by each program.

9. Experiment & run the recursive program for several sufficiently large values of *n*. Execute the other three programs with the same values of *n* & compare the execution times of the 4 programs.

10. Write your report and show a demo of your second program. The report should include a summary of your work, a summary & conclusion of your experiments and the results of the experiments, as well as the algebraic work and a printout of your program. What are the advantages or shortcomings of each computation?