

Introduction to Modeling and Simulation

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What is Simulation?

Simulation

The process of imitating the operation of a system over time.

System

- A group of entities that coexist and interact, usually towards the accomplishment of some goal.
- Systems are embedded within a larger environment, called the **system environment**.
- There is typically an exchange of information and resources between the system and its environment. Note however that the distinction between a system and its environment represents more of a logical division for the sake of model development. In most cases, a system represents an integrated part of its environment; not a separate entity.

System Model

Elements of a System Model

Entities refer to the relevant things within a system.

Rules govern how entities interact with one another. Rules help determine what will happen next in the system as the result of some system event.

Inputs represent data that is needed to populate various attributes of system entities.

Activities represent things happening in the system over a period of time.

Delays represent future activities that will begin and end in some possibly unknown time in the future. Delays are the result of not having enough resources at a given time to allow for all desired activities to take place.

Elements of a System Model

System State represents a collection of variables that is sufficient for describing the system at any given time. These variables are usually defined as a subset of the collective set of individual entity attributes.

Events are instantaneous occurrences that changes the state of the system. Two types of events include

- ① **internal:** events occurring within the system.
- ② **external:** events occurring outside the system, or in the system environment. These events usually represent the addition (subtraction) of an entity to (from) the system.

System Model Example

Elements of a Banking ATM System Model

Entities customers, ATM machines, ATM cards, queues, cash, checks, accounts. Customer attributes: arrival time to ATM, type of transaction, amount to withdraw.

Rules If ATM is in use, the customer waits in a FIFO queue.

Inputs Withdrawing customers use “Quick Cash” option 63% of the time. A withdrawing customer withdraws x dollars with probability $p(x)$. The service time for a customer at an ATM follows an exponential distribution with mean service rate of $\lambda = 0.5$.

Activities Cash is being withdrawn from the ATM. Checks are being deposited to the ATM.

Delays A customer is waiting in line for her turn to use the ATM.

System Model Example Continued

Elements of a Banking ATM System Model

System State The ATM machine is not being used. The machine currently holds \$10K in cash and 152 deposited checks. The wait queue is empty.

- Events**
- ① **internal:** A customer has finished service at the ATM.
 - ② **external:** A customer has arrived to withdraw cash from the ATM.

Why Simulation?

- Inexpensive way to learn how a system's operation and performance responds to changes
- Knowledge gained can suggest improvement for the system.
- System changes may be impossible or expensive to observe in practice.
- Can be used to reinforce and verify analytic solutions.
- Allows one to experiment with new designs or policies prior to implementation.
- Can be used for off-line employee training.
- Some systems are too complex to understand by any other means.

When to Avoid Simulation

- when a problem can be solved analytically or by using common sense
- when a problem can be solved more cheaply using direct experimentation
- when the costs of simulation exceed the savings
- resources such as time, money, or data are not available
- lack of ability to verify or validate the model
- system behavior is too complex or hard to define

Types of Systems

Static System represented by a set of variables that do not change over time

Dynamic System represented by a set of variables that change over time

Queueing System dynamic system represented by a network of servers and queues that process a population of customers

Physical System dynamic system consisting of entities that interact in space and time according to a set of rules

Types of Simulations

Monte Carlo Simulation estimating one or more statistical properties of a system via an independent sequence of state (i.e. variable) assignments

Markov-Chain Monte Carlo Simulation estimating one or more statistical properties of a system via a dependent sequence of state (i.e. variable) assignments that obey a Markov chain

Discrete-Event System Simulation driven by an event queue for which events are removed and processed in chronological order (based on when they occur in the system). A **static event** is one that is added to the queue before simulation begins, while a **dynamic event** arises during simulation, and is often triggered by some other event.

Types of Simulations Continued

Continuous System Simulation system is updated every δt time units, where δt is sufficiently small so that changes appear continuous.

Steps in a Simulation Project

Problem Formulation and Objectives state the problem and decide if a simulation project is appropriate for solving it. If so, what questions will simulation propose to answer?

Model development abstract the essential features of a system; select and modify basic assumptions that characterize the system.

Data Collection and Input Modeling statistics such as the number and frequency of various events over a period of time must be either estimated or determined using pre-existing data. Object attributes must be instantiated with realistic data.

Model Programming the model is programmed on a computer using either a general-purpose language, or a simulation language/package

Steps in a Simulation Project Continued

Verification determine if the model has been programmed correctly. Includes module and integration testing.

Validation determine if the model is an accurate representation of the system.

Experimental Design design a set of experiments whose results will help start the process of answering some of the questions set forth during the objective-setting phase.

Test Runs and Analysis implement the experiments from the previous step and analyze their results.

Documentation and reporting done throughout the project.

Implementation includes implementing changes to the system or developing an altogether new system.

Applying Simulation to Algorithms

- Monte Carlo can be used to approximate integrals, the volumes of a regions, and other quantities that are difficult if not imposible to compute analytically.
- **Simulated Annealing** is a local-search algorithm that simulates the metal annealing process, and has wide applications towards optimization problems.
- Markov-Chain Monte Carlo can be applied to machine learning by estimating the probability of events in a Bayesian network.
- The Gillespie algorithm uses simulation to generate a statistically correct solution to a stochastic differential equation.