CECS 552 Programming Assignment 2

Monte Carlo Simulation, Part 2

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Approximating an Integeral

Have a program option that takes as input the number n of desired independent samples, and uses them to approximates the integral

$$\int_0^{\frac{\pi}{2}} \int_0^x e^{\cos(x+y)} dydx.$$

In addition to $\hat{\lambda}$ (the integral approximation), print the sample variance, standard error, and the 95% confidence interval.

Monte Carlo Simulation for Playing Blackjack

A simple version of the (two-player) game of Blackjack is described as follows. For this version four decks of cards are randomly shuffled together to make a **dealing deck** of $52 \times 4 = 208$ cards total. The value of a number card equals the card number, the value of a face card equals 10, while the value of an ace may either equal either 1 or 11. In what follows we let random variable X represent the result of drawing a card at random from the (entire) dealing deck. For example, $P(X = i) = 16/208$, for all $i = 1, \ldots, 9, 11\}$, while $P(X = 10) = 64/208$.

Each player is first dealt a card facing downward that is hidden from her opponent. Players are then alternately dealt cards facing upward that are visible to the opponent. Before a card is dealt to a player, she has the option of **finishing**, meaning that she is not dealt any further cards. Play ends when one of the following has occurred:

1. the sum (of the values) of a player's cards equals 21, in which case this player wins the game;

2. the sum of a player's cards exceeds 21, in which case this player loses the game; or
3. both players have finished in which case the winner is the player whose card values have the highest sum (a tie occurs if the sums are equal).

When a game ends, all dealt cards are revealed to both players, and the cards are placed in the **used pile**, and are never dealt again during the **match**, which is a sequence of games that ends once the dealing deck is exhausted (if this happens during a game, then the game ends in a tie).

Now consider two different strategies for playing the game. For the first strategy, called the **naive** strategy, player P accepts another dealt card provided one of the following is true. Let S denote the sum of P's cards.

1. P's opponent is not finished, and $P(X + S \leq 21) > 0.5$, where S denotes the sum of P's cards.
2. P's opponent is finished, and $E[X|X + S_v < 21] + S_v > S$, or $P(X + S \leq 21) \geq 2/3$, where S_v is the opponent's "visible" sum.

The second strategy, called **Monte Carlo** is for P to conduct a Monte Carlo simulation. A sample of such a simulation consists of first randomly sampling a card from the (unused part of) the dealing deck. This card represents the opponent's hidden card. Next a sequence of cards are sampled from the dealing deck, and that represent the hypothetical next cards that will be dealt to P (and the opponent if the opponent is not finished). P then assumes that its hidden card is visible to the opponent, so that there is no hidden information, and both P and the opponent make perfect decisions with respect to the hypothetical card sequence. Based on the perfect decision making, P then decides on whether or not it is better to finish or accept the next card. This sampling process is repeated 1000 times, in which case P determines which action (accepting or finishing) offers the best chance of winning. P then takes this action. Note that P has complete knowledge about which cards remain in the dealing deck. In case there are insufficient cards to complete a sample, then P takes the action of finishing.

Have a program option that simulates 500 consecutive matches between a naive player and an MC player. Report on the total number of games won by each player, along with their respective winning percentages.