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CECS 328: Program #1

For each of the tasks, I have included the algorithm in C# as well as a screenshot of the output. The source code for the project is available here:
https://github.com/Vardominator/CSULBProjects/tree/master/CECS328_DataStructuresAlgorithms

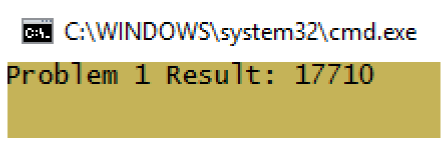
1. Write a program to calculate S(n) by calculating the values of the Fibonacci sequence recursively.

Source:

```
public static long Sn(long number)
{
    long result = 0;

    for (int i = 1; i <= number; i++)
    {
        result += Fibonacci(i);
    }

    return result;
}

public static long Fibonacci(long number)
{
    if(number <= 2)
    {
        if(number == 0)
        {
            return 0;
        }
        return 1;
    }

    return Fibonacci(number - 1) + Fibonacci(number - 2);
}
```

Sample run:

```
#region Problem 1: Calculate S(n) by calculating the values of the Fibonacci sequence
recursively
// S(n) definition: S(n) = f(0) + f(1) +...+ f(n)

int sampleN = 20;
Console.WriteLine($"Problem 1 Result: {Sn(sampleN)}\n\n");
#endregion
```



2. Write a non-recursive program to calculate S(n).

Source:

```
public static long NonRecursiveSn(long number)
{
    long previous = 1;
    long previous2 = 1;
    long current = 0;
    long sum = 2;

    for (int i = 3; i <= number; i++)
    {
        current = previous + previous2;
        sum += current;
        previous = previous2;
        previous2 = current;
    }

    return sum;
}
```

Sample run:

```
#region Problem 2: Write a non-recursive program to calculate S(n)
Console.WriteLine("Problem 2 Result: \n");
Console.WriteLine($"Fibonacci Table for n = {sampleN}:");

for (int i = 0; i <= sampleN; i++)
{
    Console.WriteLine($"f({i}) = {Fibonacci(i)}");
}
Console.WriteLine("------------");
Console.WriteLine($"Sum: {NonRecursiveSn(sampleN)}\n\n");
#endregion
```

C:\WINDOWS\system32\cmd.exe

```
Problem 2 Result:

Fibonacci Table for n = 20:
f(0) = 0
f(1) = 1
f(2) = 1
f(3) = 2
f(4) = 3
f(5) = 5
f(6) = 8
f(7) = 13
f(8) = 21
f(9) = 34
f(10) = 55
f(11) = 89
f(12) = 144
f(13) = 233
f(14) = 377
f(15) = 610
f(16) = 987
f(17) = 1597
f(18) = 2584
f(19) = 4181
f(20) = 6765
------------
Sum: 17710
```

3. Algebraically verify that g(n) is a solution of Equ 1 by substituting g(n) in Equ 1.

   (This portion is attached to the assignment packet)

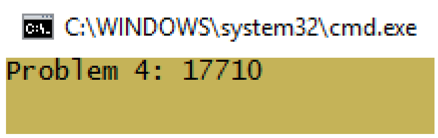
4. Write a third iterative program by summing g(n) over n.

Source:

```
public static long GrimaldiSum(long number)
{
    long result = 0;

    for (int i = 1; i <= number; i++)
    {
        result += Grimaldi(i);
    }

    return result;
}

public static long Grimaldi(long k)
{
    return (long)((1 / Math.Sqrt(5)) * (Math.Pow(((1 + Math.Sqrt(5)) / 2), k) - Math.Pow(((1 - 
Math.Sqrt(5)) / 2), k)));
}
```

Sample run:

```
#region Problem 4: Third possible way to sum Fibonnaci
Console.WriteLine($"Problem 4: {GrimaldiSum(20)}\n\n");
#endregion
```



5. Calculate these values of S for n = 10, 20, 30. Calculates values of f for n = 12, 22, 32.

Sample run:

```
#region Problem 5: Calculate S for 10, 20, 30; Calculate f for 12, 22, 32
Console.WriteLine($"Problem 5 results: \n");
Console.WriteLine($"S(n) for n = 10 => {GrimaldiSum(10)}");
Console.WriteLine($"S(n) for n = 20 => {GrimaldiSum(20)}");
Console.WriteLine($"S(n) for n = 30 => {GrimaldiSum(30)}\n");
Console.WriteLine($"f(n) for n = 12 => {Fibonacci(12)}");
Console.WriteLine($"f(n) for n = 22 => {Fibonacci(22)}");
Console.WriteLine($"f(n) for n = 32 => {Fibonacci(32)}\n");
#endregion
```

```
C:\WINDOWS\system32\cmd.exe
Problem 5 results:

S(n) for n = 10 => 143
S(n) for n = 20 => 17710
S(n) for n = 30 => 2178308

f(n) for n = 12 => 144
f(n) for n = 22 => 17711
f(n) for n = 32 => 2178309
```

6. Prove the identity S(n) = f(n + 2) – 1 using induction.

   (This portion is attached to the assignment packet)

7. Example run using the alternate sum from #6.

Source:

```
public static long SnAlt2(long number)
{
    return Fibonacci(number + 2) - 1;
}
```

Sample run:

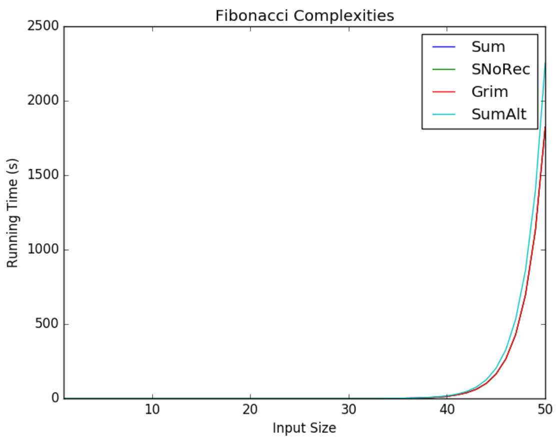
```
#region Problem 7: Fourth way to calcualte S(n)
Console.WriteLine($"Problem 7: {SnAlt2(20)}\n\n");
#endregion
```

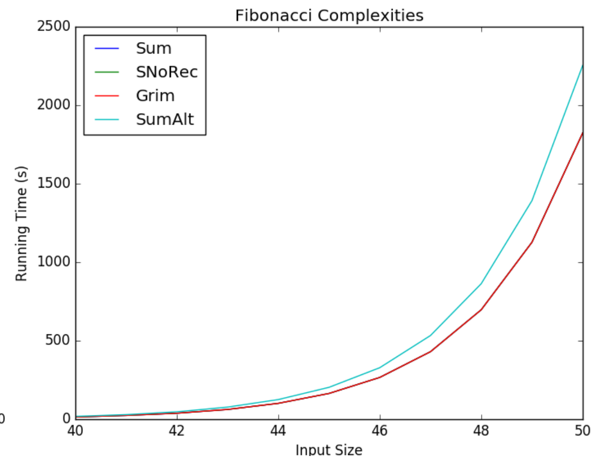
```
C:\WINDOWS\system32\cmd.exe
Problem 7: 17710
```

8. Since I am using Int64 (long in C#) to calculate the sums, the largest n that can be used is such that S(n) does not exceed 9,223,372,036,854,775,807. Another limitation would be the possibility of running out of stack space. Since the Fibonacci calls are recursive, a sufficiently large n would cause the program to crash.

9. Compare the running times of the 4 methods.

   I computed the sum with each method for values of n starting at 1 and ending at 50. I recorded the running times for each. Finally, I ran a python script to plot the growth of the running times versus the input size.





10. Summary and conclusions.

We see in the plots above that the 4th method grows the fastest by a considerable amount. However, the difference is not much. The drawbacks of the recursive methods are that they have linear growth in space complexity: $O(n)$, as opposed to the non-recursive methods that have a constant space complexity.

As far as time complexity is concerned, the recurrence relation for the Fibonacci sequence is $T(n) = T(n-1) + T(n-2) + O(1)$. Using a recursion tree, we can prove that the time complexity is $O(2^n)$. This means that the recursive Fibonacci sequence grows relatively fast. It is faster than $O(n^3)$, for example.

Finally, it is worth noting that a recursive solution to the sum is slower than the Grimaldi solution. I have shown this in the plots above.